Università degli Studi di Trento – Dipartimento di Fisica

# Computational Physics 1

Recitation class, March 10, 2021

# Classical scattering

## Indice



## 1 Deflection angle

A classical particle of mass $m$ and kinetic energy $E$ is deflected by a Lennard-Jones potential. Compute and plot the deflection angle as a function of the impact parameter $b$ for energies $E^* = E/\varepsilon \in (0.1, 0.5, 1, 2, 3, 5)$. Check your answers by comparing the results obtained using the explicit formula for $\theta(E, b)$ with those obtained by integration of the Newton equations of motion in 2D.

### 1.1 No deflection

Compute the value of $b$ for which $\theta(E, b) = 0$ as a function of the same energies considered previously, and plot the corresponding trajectories.

### 1.2 Orbiting

For some specific values of $E$ the function $\theta(E, b)$ diverges for a certain value of the impact parameter $b$. This corresponds to the fact that impinging particles perform some orbits before being scattered. Assuming $E^* = 0.1$, find the value of $b$ for which $|\theta(E, b)| = 4\pi$; that is, the particle makes *two* orbits before being scattered.

## 2 Cross section of the Lennard-Jones potential

Compute and plot the "reduced" differential cross section $\overline{\sigma}(E^*)$ in the case of the Lennard-Jones potential at energies $E^* \in (0.1, 0.5, 1, 2, 3, 5)$. Choose the binning-slot size $\delta\theta$ in order to have a resolution of 1°.

## 2.1 Rainbow scattering

For sufficiently high energies, the function $\theta(E,b)$ has a *minimum* in the variable $b$, hence the differential cross section – which is proportional to $\left|\frac{db}{d\theta}\right|$ – diverges. Compute the value of $b$ for which $\theta(E,b)$ has a minimum in the case of $E^* \in (2,3,5)$, and verify that the differential cross section has an asymptote at the corresponding deflection angle.

## 2.2 Small-angle scattering

Analyze the behavior of the reduced scattering cross section $\overline{\sigma}(E,\theta)$ as $\theta \to 0$ in the case of the Lennard-Jones potential. Verify the validity of the asymptotic regime

$$\overline{\sigma}(E,\theta) \underset{\theta\to 0}{\propto} \frac{\theta^{-4/3}}{E^{1/3}}. \tag{1}$$

Notice that $\overline{\sigma}(E,\theta)$ has a non-integrable divergence for $\theta \to 0$ hence the *total* classical scattering cross section for the Lennard-Jones potential is infinite.