Università degli Studi di Trento – Dipartimento di Fisica

Computational Physics 1

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Three bodies and drag

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1 Peculiar solutions of the three-body problem

Consider three bodies of equal mass $m = 1$ iteracting gravitationally and use a system of units where $G = 1$. Given the initial condition

$$\begin{aligned}
\mathbf{x}_1 &= (-1, 0) \\
\mathbf{x}_2 &= (1, 0) \\
\mathbf{x}_3 &= (0, 0) \\
\alpha_1 &= 0.347111 \\
\alpha_2 &= 0.532728 \\
\mathbf{v}_1 &= (\alpha_1, \alpha_2) \\
\mathbf{v}_2 &= (\alpha_1, \alpha_2) \\
\mathbf{v}_3 &= (-2\alpha_1, -2\alpha_2)
\end{aligned}$$

plot the trajectory of the three bodies in the xy plane. Is the motion periodic? If so, what is the period?

Repeat the same calculations changing the initial condition to

$$\begin{aligned}
\alpha_1 &= 0.306893 \\
\alpha_2 &= 0.125507,
\end{aligned}$$

and changing it to

$$\begin{aligned}
\alpha_1 &= 0.464445 \\
\alpha_2 &= 0.396060.
\end{aligned}$$

References

1. The three-body problem on Scholarpedia
2. Annals of Mathematics **15** (2000), page 881
3. Physical Review Letters **110** (2013), page 114301

2 Satellite decommissioning *

Let us consider a satellite in a circular orbit at a distance r_0 from the Earth surface. Apart from the gravitational force, the satellite is subject to a drag in the direction opposite to the instantaneous velocity and having modulus

$$F_{\text{drag}} = C_d \rho(h) v^2,$$

where $C_d/m = 8 \times 10^{-4}$ m^2/kg and $\rho(h)$ is the air density at an heigth h from the surface, given by

$$\begin{aligned} \rho(h) &= 1.225 \exp\left[-\frac{h}{k_1} - \left(\frac{h}{k_2}\right)^{3/2}\right] \text{ kg/m}^3 \\ k_1 &= 1.2 \times 10^4 \text{ m} \\ k_2 &= 2.2 \times 10^4 \text{ m.} \end{aligned}$$

If the satellite is initially at a height $r_0 = 120$ km, compute the time after which it crashes on the surface.

Another satellite, who is also in a circular orbit with $r_0 = 120$ km is decomissioned. At a certain time $t = 0$ some rockets are fired for $\Delta T = 5$ s, and exert a force ma_0 in the direction *opposite* to the instantaneous velocity with $a_0 = 5$ m/s^2. Calculate the satellite trajectory and the time needed to crash on the surface.

*Freely inspired from the notes of prof. A.W. Sandvik's from the University of Boston.