Report:

Teresa Algarra Ulierte
Student ID: teral436
Personnummer: 970628T129

1 Study 1: Modelling Signals

1.1 Theoretical Background:

In this study we will work with white noise, which is defined as a process with constant PSD.

$$R_0 = \frac{N_0}{2} \tag{1}$$

It is also known that the white noise is Gaussian, which means that if it is WSS, it will be SSS. As we want it to be scalled, we will use $R_x = 1$.
So first we have the noise:

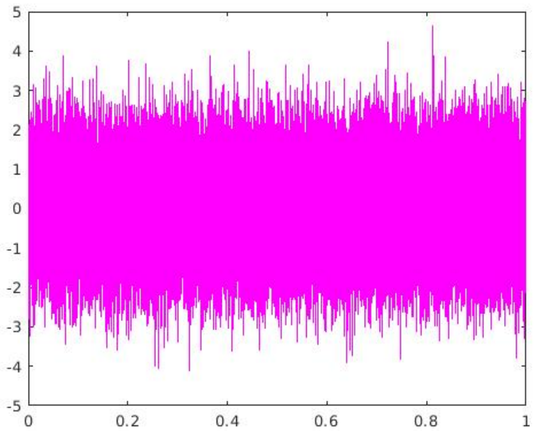


Figure 1: Noise

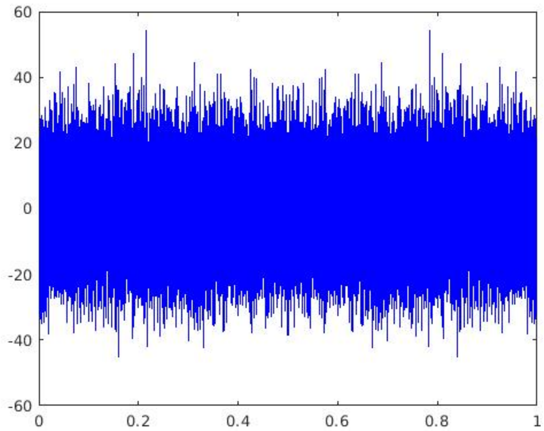


Figure 2: Noise in frequency domain

We will get this noise through some LTI filters, having:

$$R_y(f) = R_x|H(f)|^2 \tag{2}$$

1.2 Theoretical Analysis:

To get to calculate the ACF and the PSD of the result function, we have to know exactly what filters we are using.
For the ideal filter, we can use the rectangle function:

$$H[\theta] = rect(\frac{\theta}{\theta_0}) \tag{3}$$

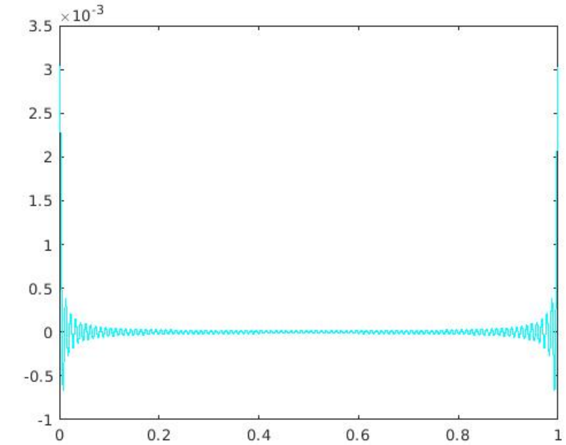


Figure 3: Filter in time domain

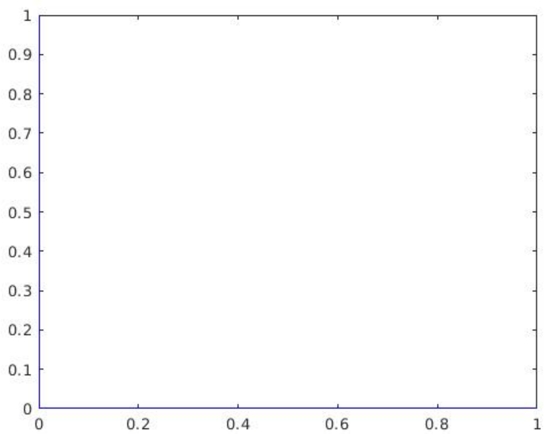


Figure 4: Filter frequency domain

The filtered signal is:

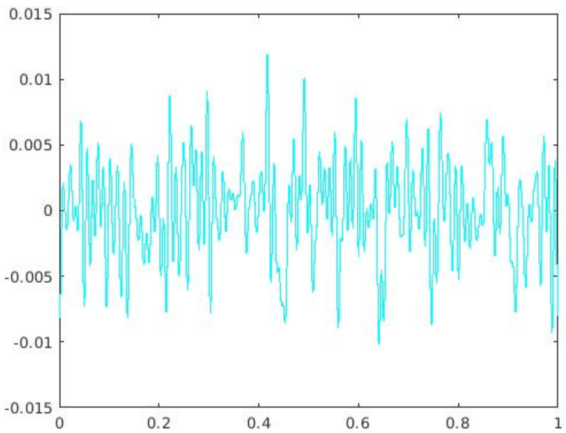


Figure 5: Filtered signal

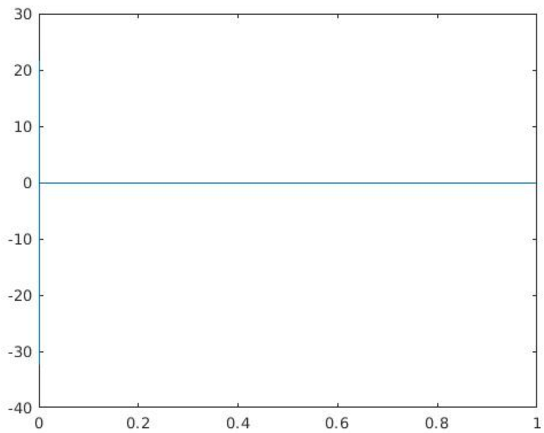


Figure 6: Filtered signal in frequency domain

For the low-degree low-pass filter, we will use a first-order Butterworth filter:

$$H(z) = \frac{b}{a_1 - a_2 e^{-j2\pi f}} \tag{4}$$

We will use $b = a_1 = 1$ and $a_2 = 0.9$.

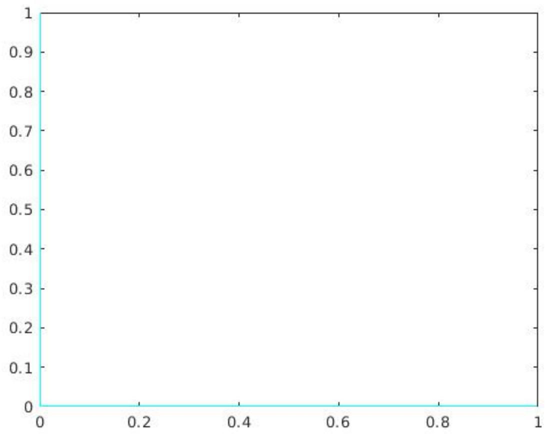


Figure 7: Filter

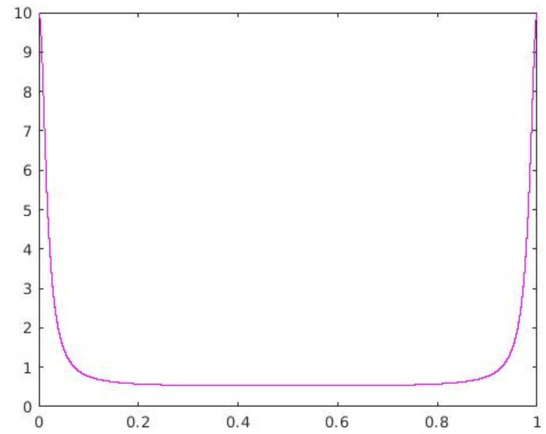


Figure 8: Filter in frequency domain

The filtered signal is:

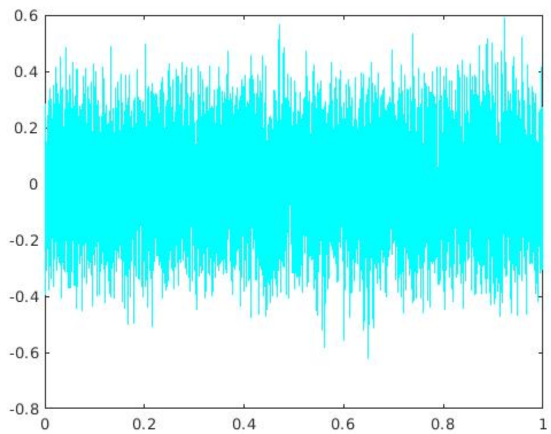


Figure 9: Filtered signal

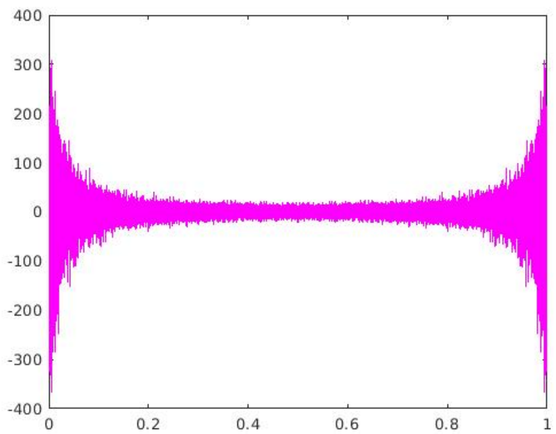


Figure 10: Filtered signal in frequency domain

So, we can start by calculating the theoretical PSD knowing the super-formula:

$$R_y[\theta] = R_x|H(\theta)|^2 \tag{5}$$

As we are working with White Gaussian Noise, its PSD it's going to be $\frac{N_0}{2}$ as said before, therefore we have:

$$R_y[\theta] = \frac{N_0}{2}|H(\theta)|^2 \tag{6}$$

For the ideal filter, the result is:

$$R_y[\theta] = \frac{N_0}{2}|rect(\frac{\theta}{\theta_0})|^2 = \frac{N_0}{2}rect(\frac{\theta}{\theta_0}) = \frac{N_0}{2}if\theta < \theta_0 \tag{7}$$

For the low-degree filter, the result is:

$$R_y[\theta] = \frac{N_0}{2}|\frac{b}{a_1 - a_2e^{-j2\pi f}}|^2 = \frac{N_0}{2|1 - 0.9e^{-j2\pi f}|^2} \tag{8}$$

Let's get on to the graphs.

1.2.1 Ideal filter:

We get the following ACF:

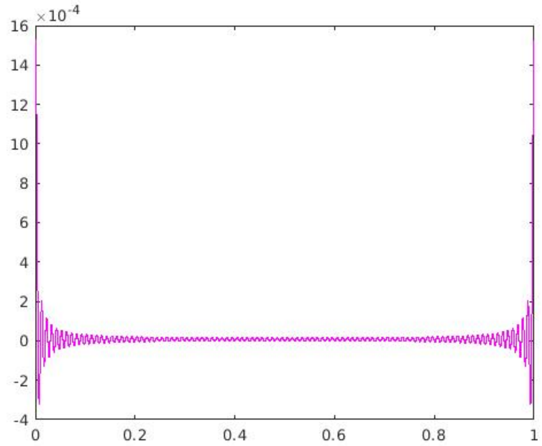


Figure 11: Theoretical ACF plotted

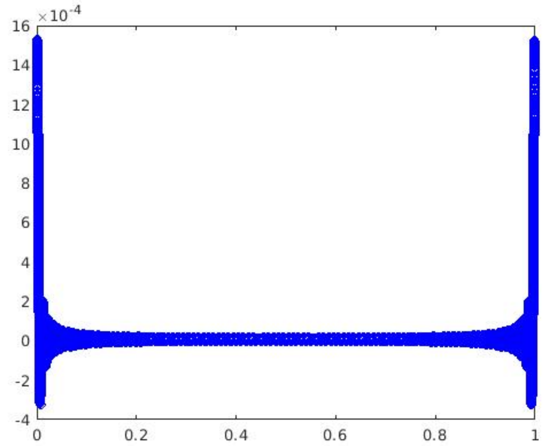


Figure 12: Theoretical ACF stemed

We get the following PSD:

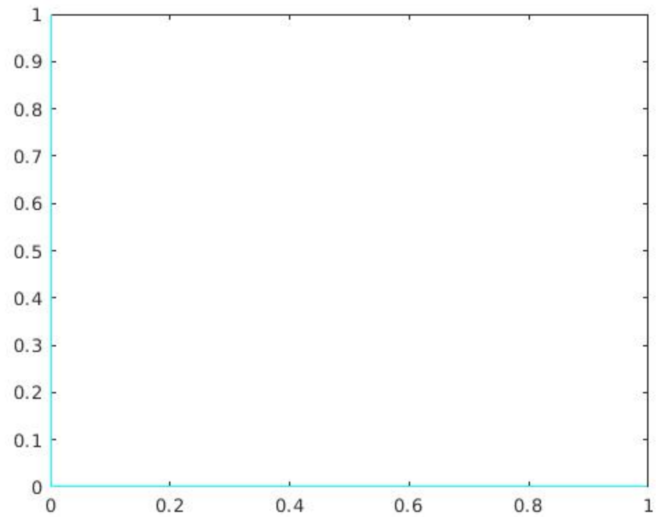


Figure 13: Theoretical PSD

1.2.2 Low-degree filter:

We get the following ACF:

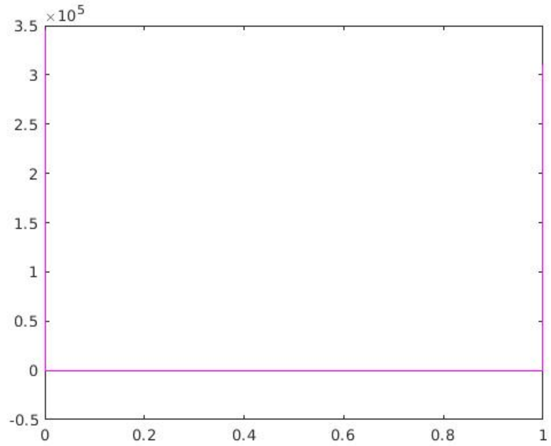


Figure 14: Theoretical ACF plotted

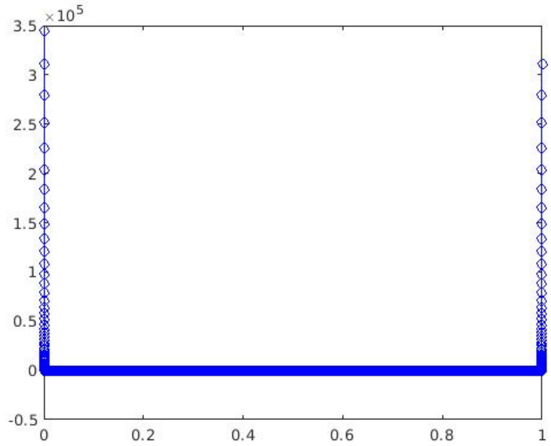


Figure 15: Theoretical ACF stemed

We get the following PSD:

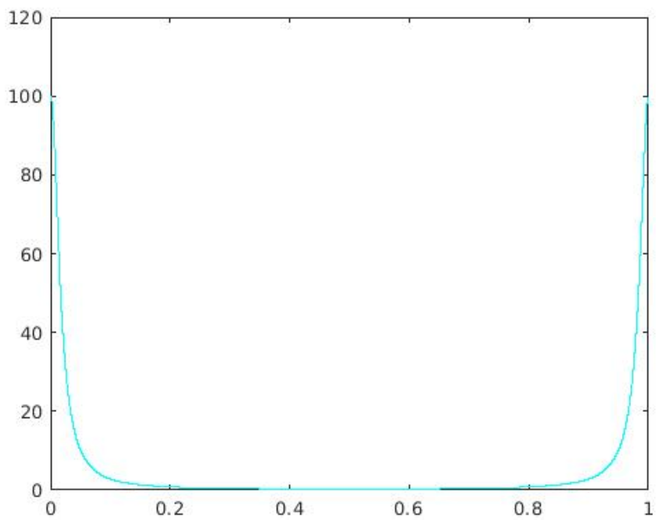


Figure 16: Theoretical PSD

1.3 Estimations:

1.3.1 Ideal filter

For the ideal filter, we will use a tenth order butterworth filter.
We get the following PSD:

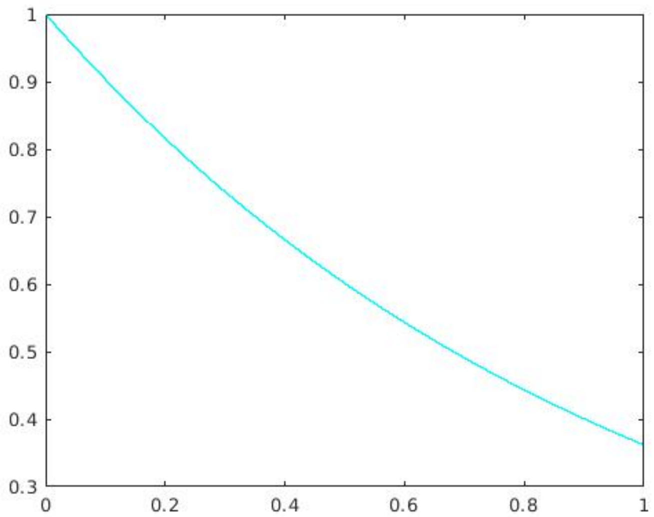


Figure 17: Estimated PSD

If we estimate by the Bartlett method, we get the following ACF:

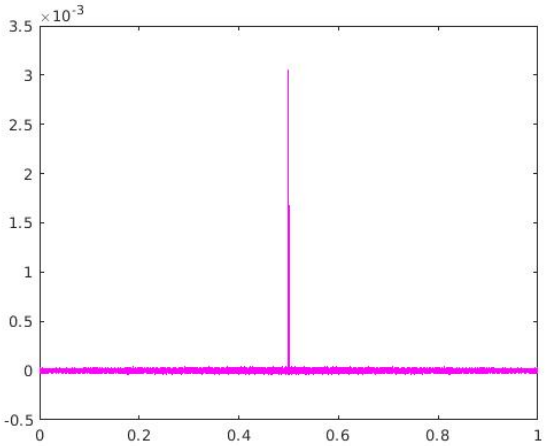


Figure 18: Estimated by Bartlett ACF plotted

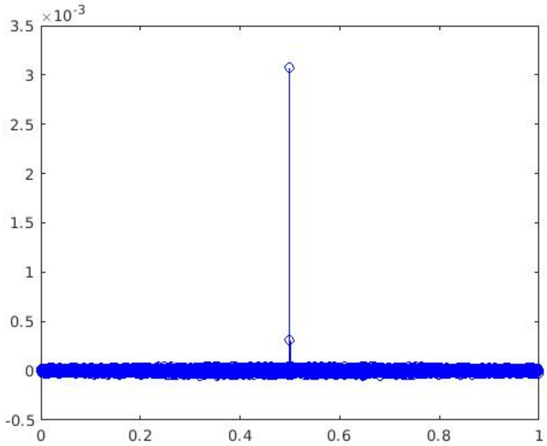


Figure 19: Estimated by Bartlett ACF stemed

If we estimate by the Blackman-Harris method, we get the following ACF:

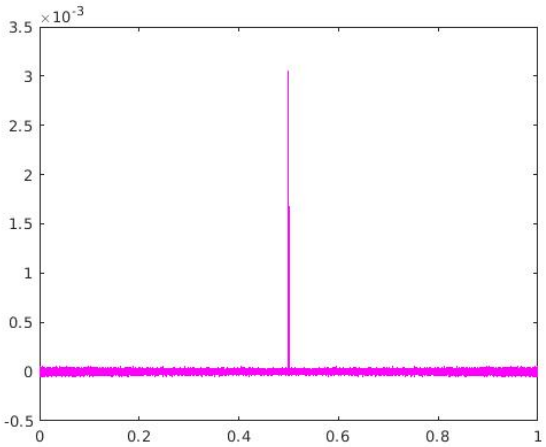


Figure 20: Estimated by Blackman-Harris ACF plotted

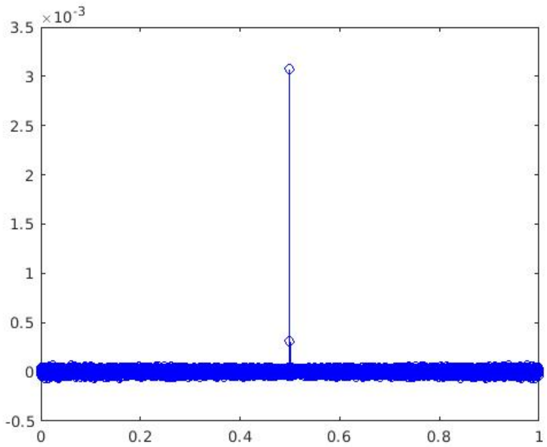


Figure 21: Estimated by Blackman-Harris ACF stemed

1.3.2 Low-degree filter

For the low-degree filter, we will use a first order butterworth filter.
We get the following PSD:

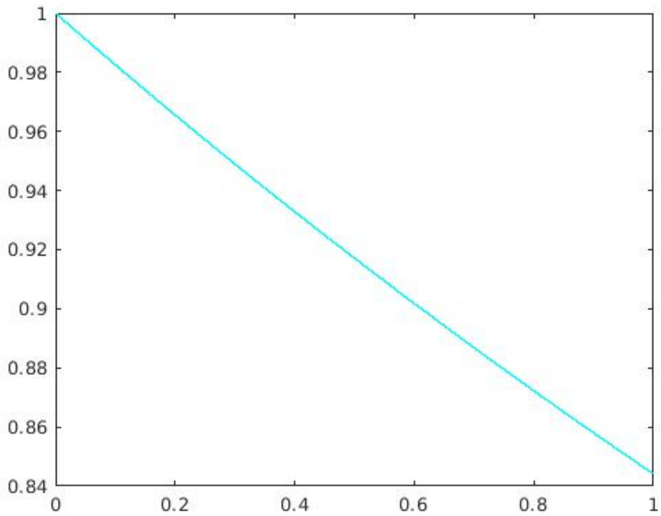


Figure 22: Estimated PSD

If we estimate by the Bartlett method, we get the following ACF:

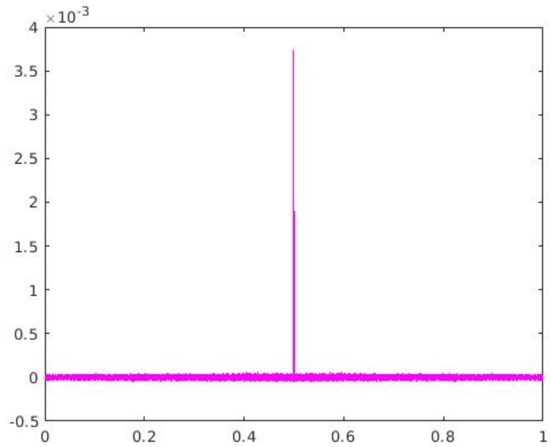


Figure 23: Estimated by Bartlett ACF plotted

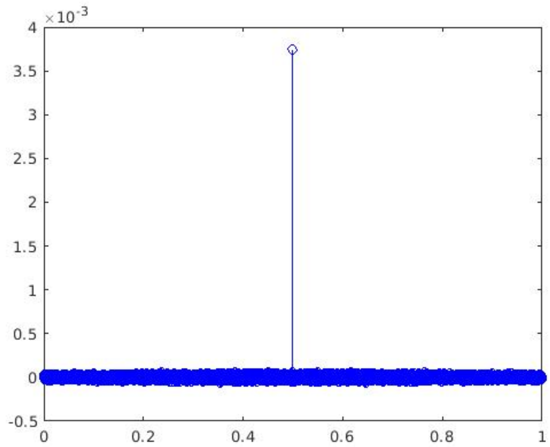


Figure 24: Estimated by Bartlett ACF stemed

If we estimate by the Blackman-Harris method, we get the following ACF:

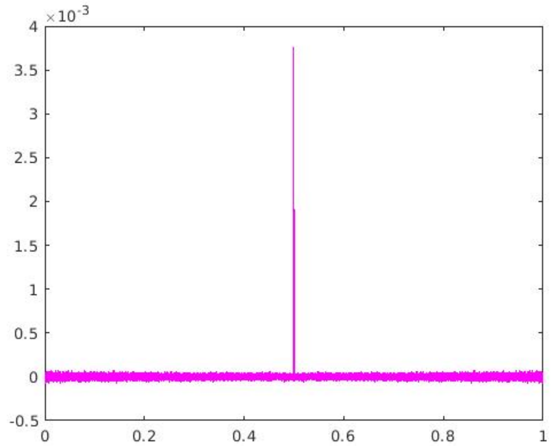


Figure 25: Estimated by Blackman-Harris ACF plotted

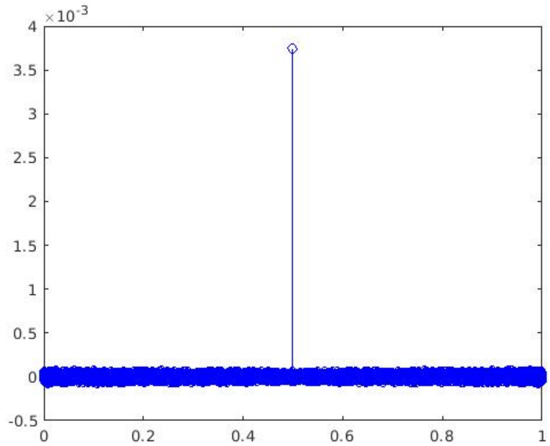


Figure 26: Estimated by Blackman-Harris ACF stemed

1.4 Final Comparation:

- Ideal filter:

  As we can see, the changes from the ideal filter to a tenth order filter are really big. Maybe it would have been more accurate to pick a 15th order filter, or something even bigger. The PSD goes from being a step to being a curve, and the ACF goes from having a peak in 0 to having it in 0.5, so the changes are bigger than desirable. There is not a big difference between the Bartlett estimation and the Blackman-Harris one.

- Low-degree filter:

  The changes from this filter to the estimated one are also really big. The PSD is not as expected, and more than a curve it's almost a down-going straight line. The ACF has the same problem as the ideal filter, the peak has been moved to 0.5 in stead of 0.

2 Study 2:

2.1 Theoretical Background:

In this second study, the aim is to improve the estimates done in the first study. We will use the same White Gaussian noise and both filters. In order to improve our estimations, we will use *windows*, in which case we have a few possibilities, of which we will explore:

- *Rectangular window*
- *Triangle window.*
- *Hamming window.*
- *Bartlett window*
- *Blackman-Harris window.*

Here we show the different windows used:

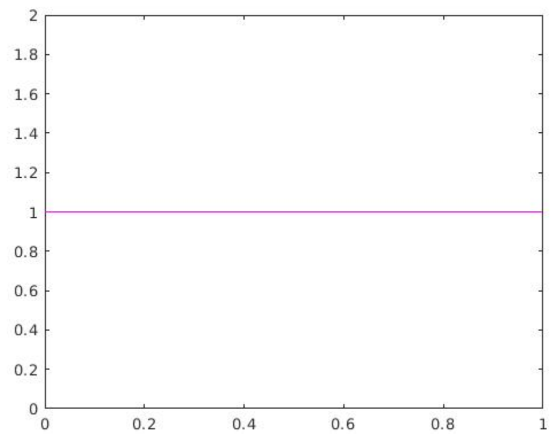


Figure 27: Rectangular window

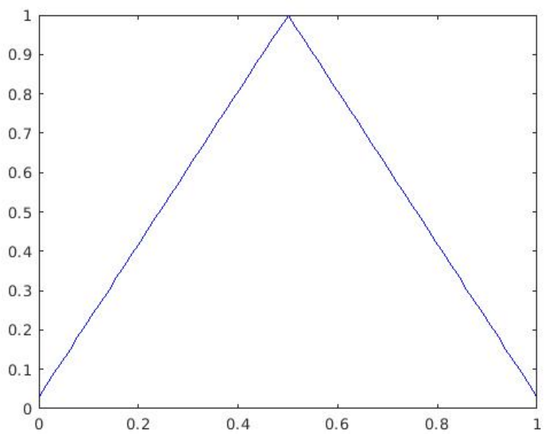


Figure 28: Triangular window

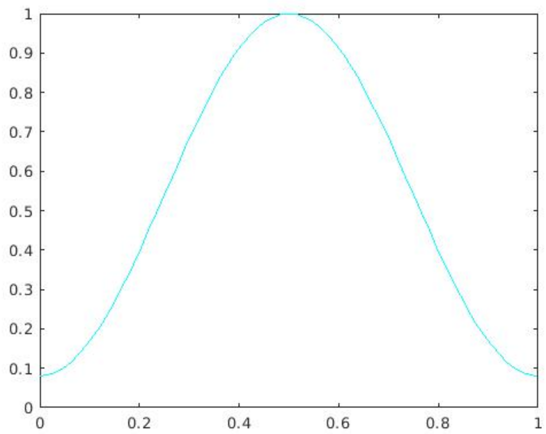


Figure 29: Hamming window

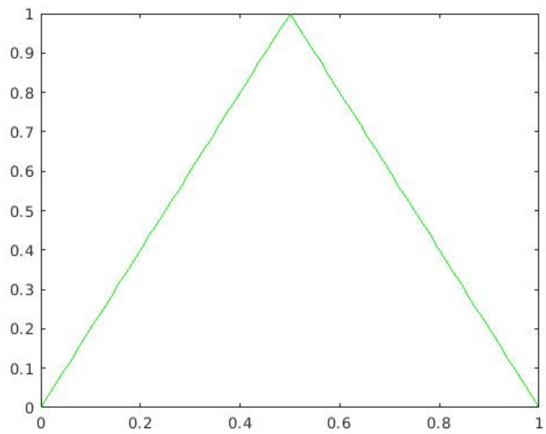


Figure 30: Bartlett window

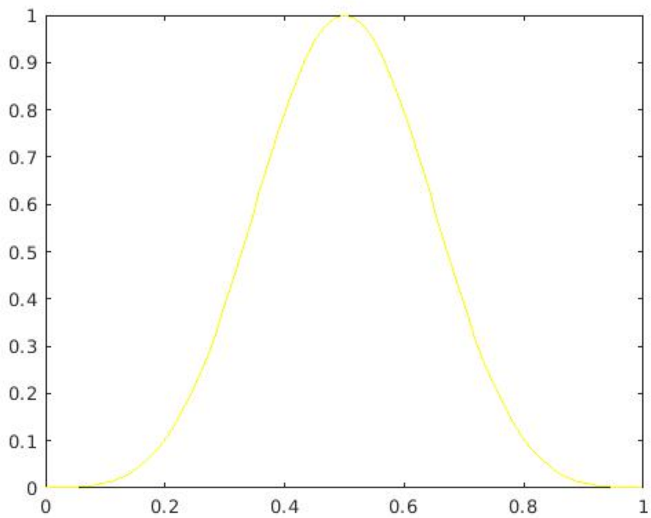


Figure 31: Blackman-Harris window

2.2 Improved Estimates: Ideal filter

For this section we will take the estimations done during the first study with the tenth degree Butterworth filter.
We have the following PSD:

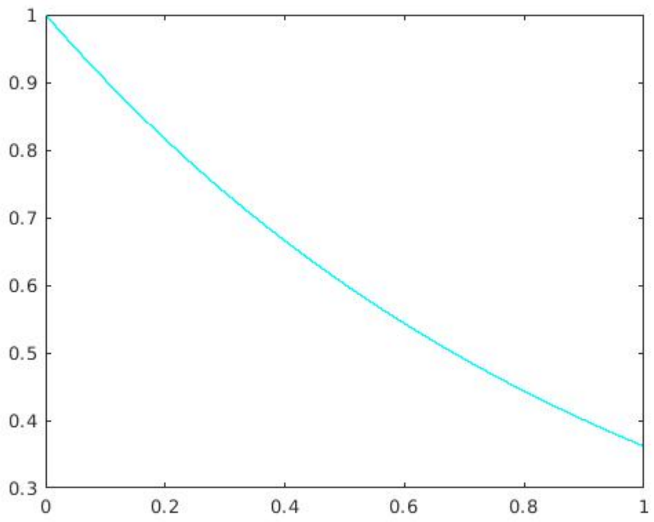


Figure 32: Estimated PSD

If we estimate by the Bartlett method, we get the following ACF:

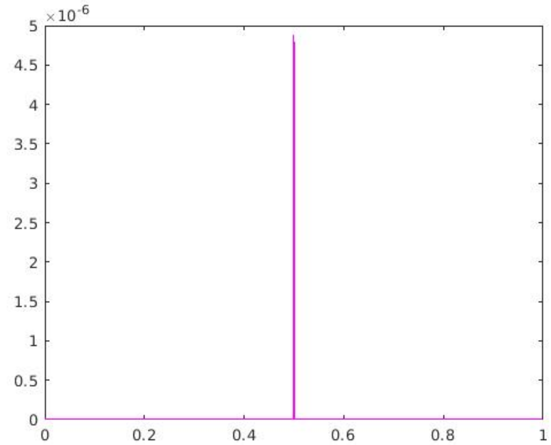


Figure 33: Estimated by Bartlett ACF plotted

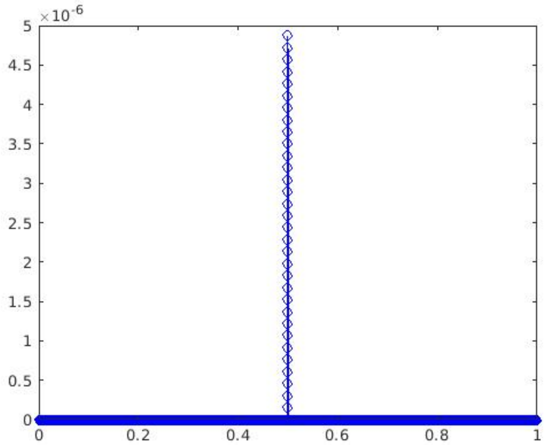


Figure 34: Estimated by Bartlett ACF stemed

If we estimate by the Blackman-Harris method, we get the following ACF:

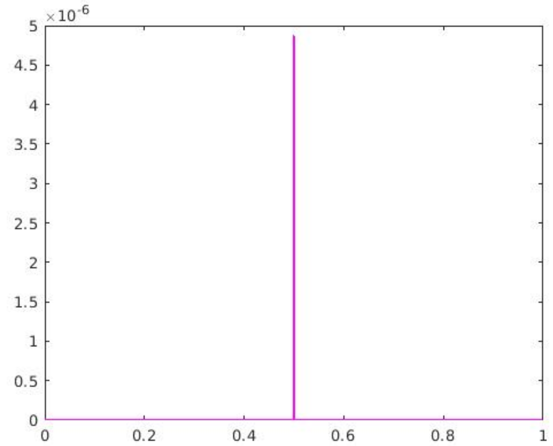


Figure 35: Estimated by Blackman-Harris ACF plotted

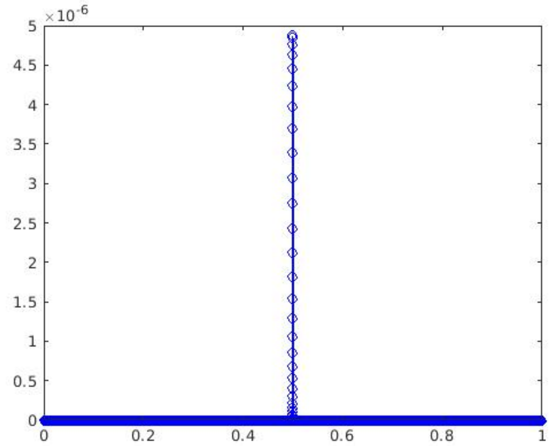


Figure 36: Estimated by Blackman-Harris ACF stemed

2.2.1 Rectangular Window:

Here we will get our filtered signal by a filter which is our rectangular window. There are the results:

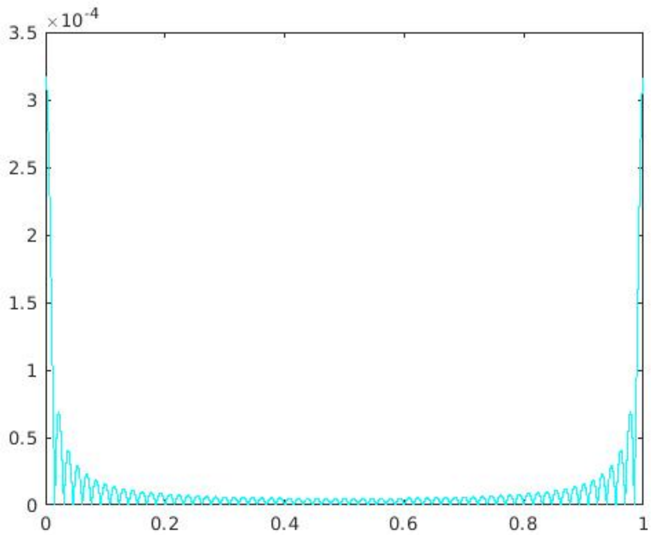


Figure 37: Improved PSD

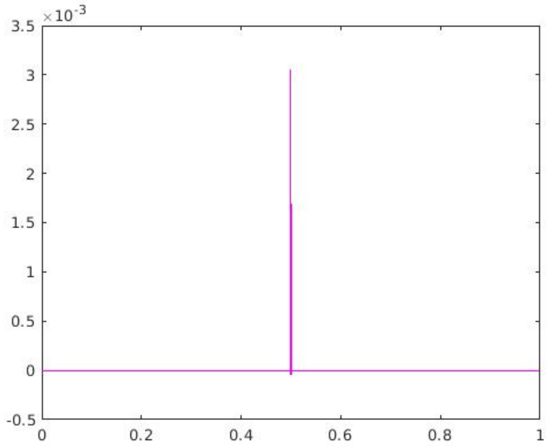


Figure 38: Improved ACF plotted (Bartlett)

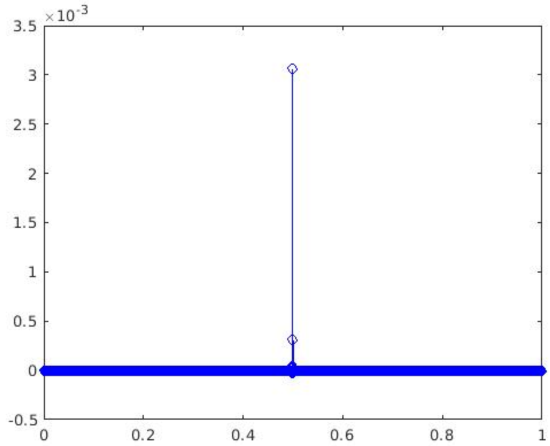


Figure 39: Improved ACF stemed (Bartlett)

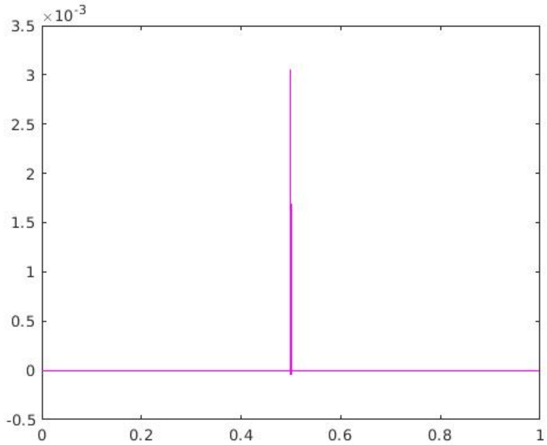


Figure 40: Improved ACF plotted (Blackman-Harris)

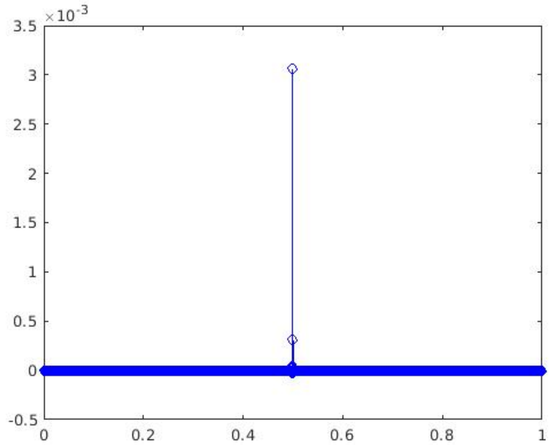


Figure 41: Improved ACF stemed (Blackman-Harris)

2.2.2 Triangular Window:

Here we will get our filtered signal by triangle window. We get:

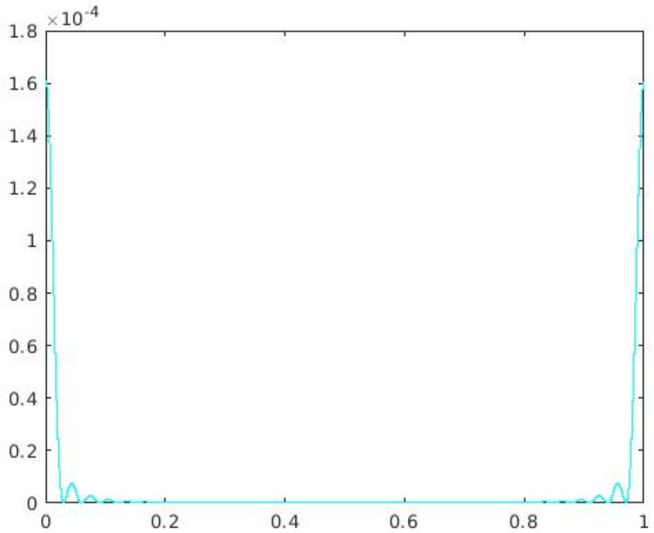


Figure 42: Improved PSD

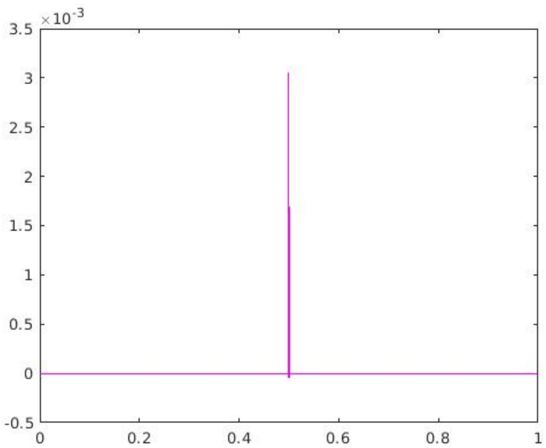


Figure 43: Improved ACF plotted (Bartlett)

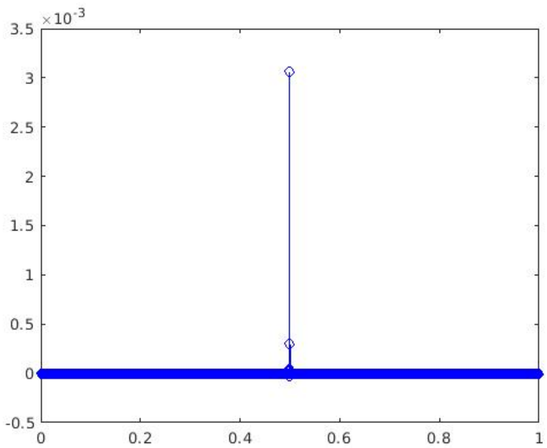


Figure 44: Improved ACF stemed (Bartlett)

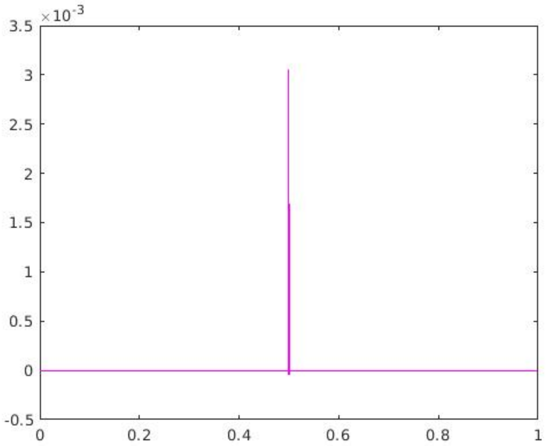


Figure 45: Improved ACF plotted (Blackman-Harris)

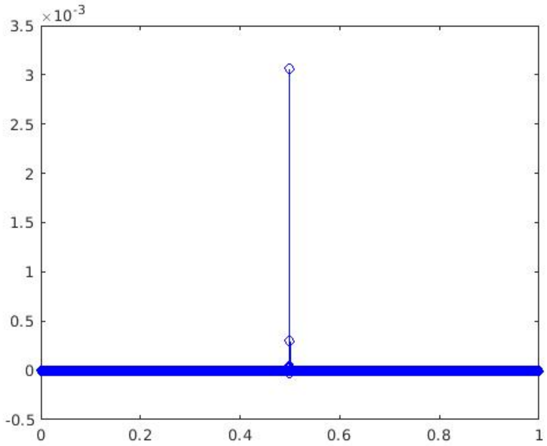


Figure 46: Improved ACF stemed (Blackman-Harris)

2.2.3 Hamming Window:

Here we will get our filtered signal by the Hamming window. Here are the results:

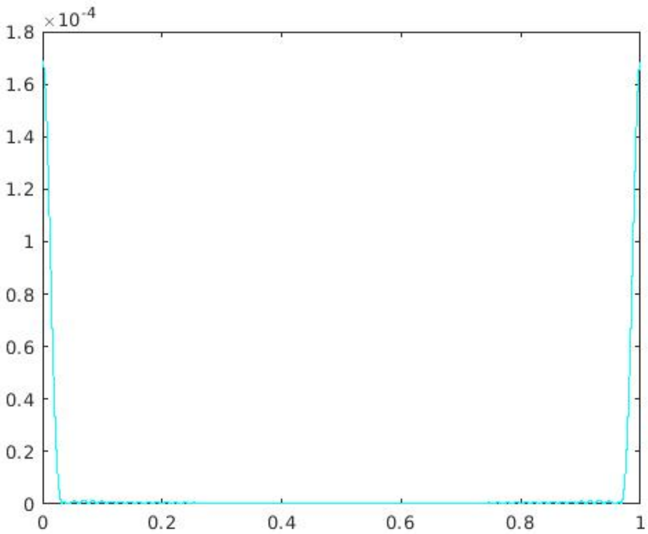


Figure 47: Improved PSD

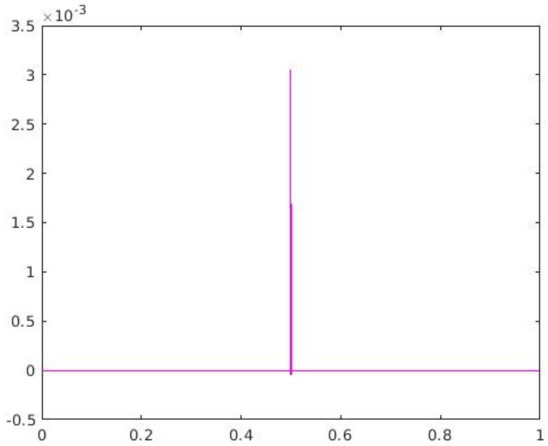


Figure 48: Improved ACF plotted (Bartlett)

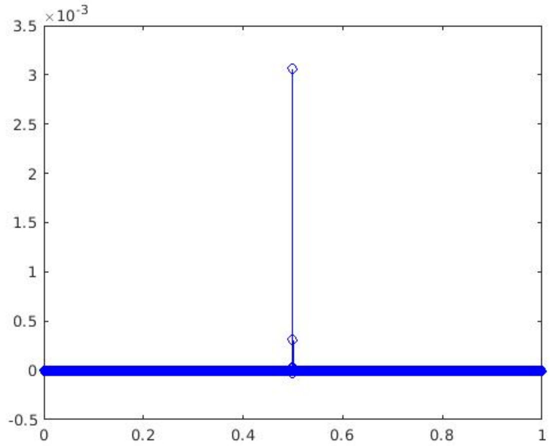


Figure 49: Improved ACF stemed (Bartlett)

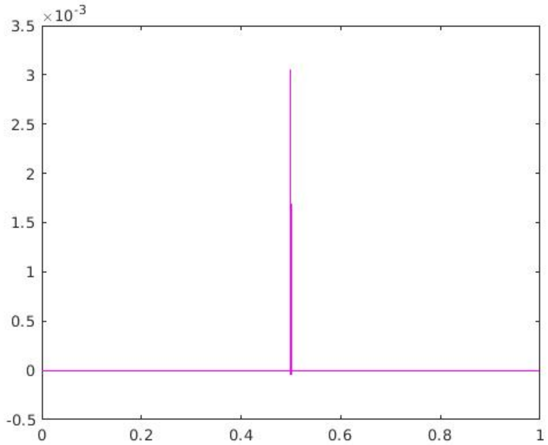


Figure 50: Improved ACF plotted (Blackman-Harris)

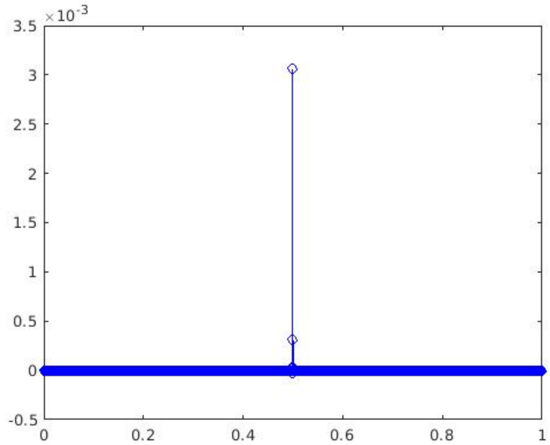


Figure 51: Improved ACF stemed (Blackman-Harris)

2.2.4 Bartlett Window:

Here we will get our filtered signal by the Bartlett rectangular window. We get:

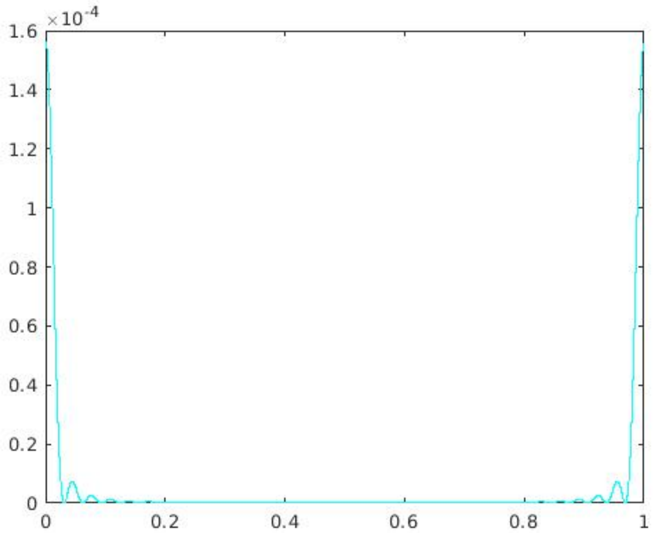


Figure 52: Improved PSD

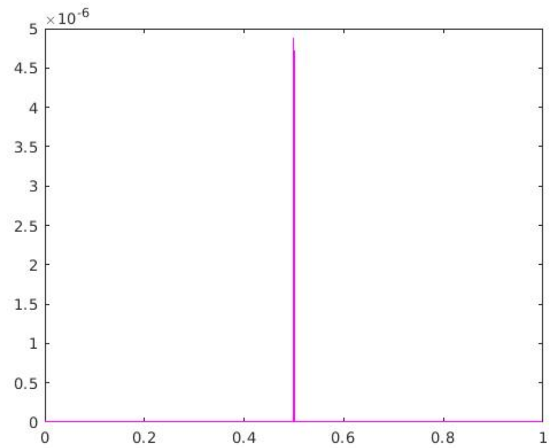


Figure 53: Improved ACF plotted (Bartlett)

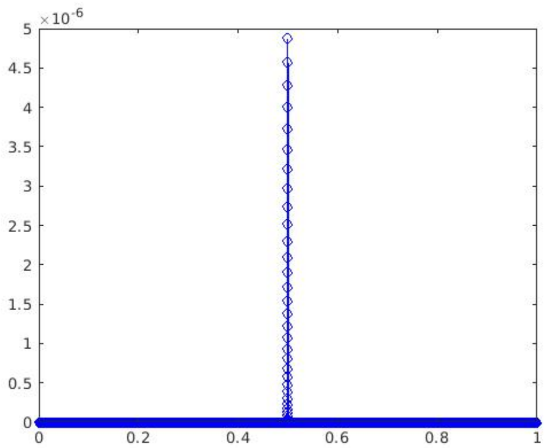


Figure 54: Improved ACF stemed (Bartlett)

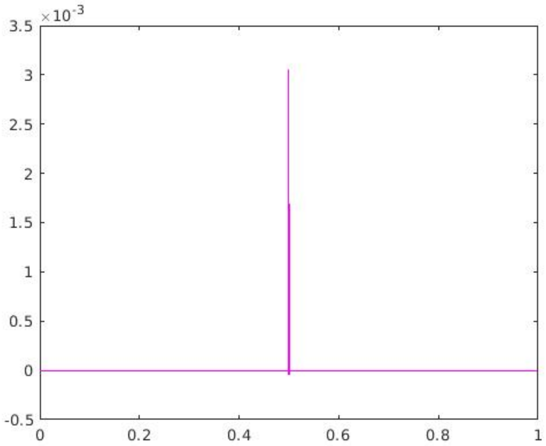


Figure 55: Improved ACF plotted (Blackman-Harris)

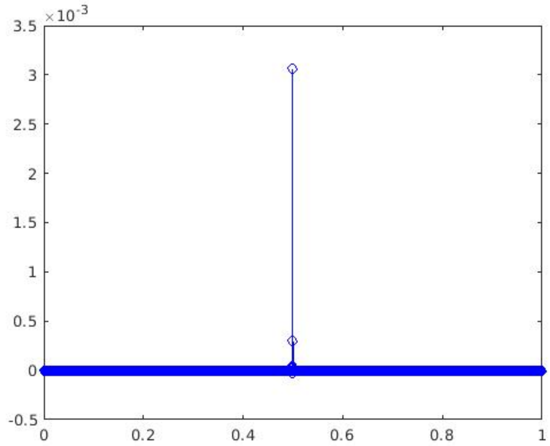


Figure 56: Improved ACF stemed (Blackman-Harris)

2.2.5 Blackman-Harris Window:

Here we will get our filtered signal by the Blackman-Harris window. Here are the results:

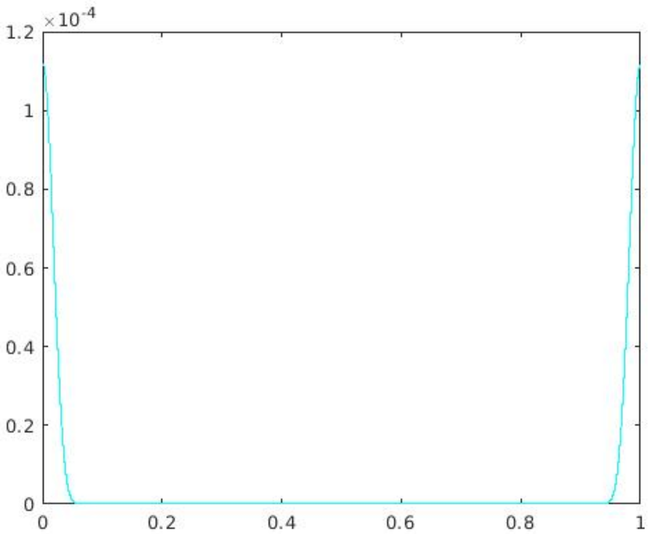


Figure 57: Improved PSD

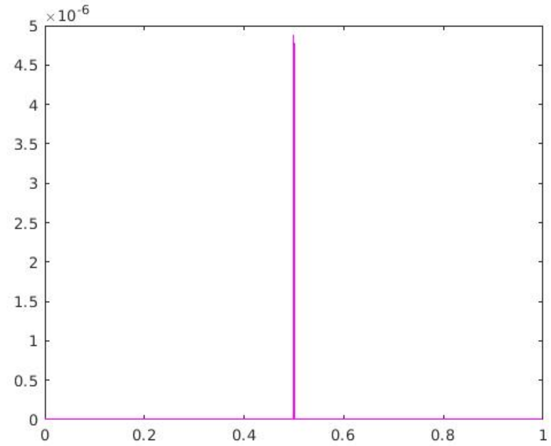


Figure 58: Improved ACF plotted (Bartlett)

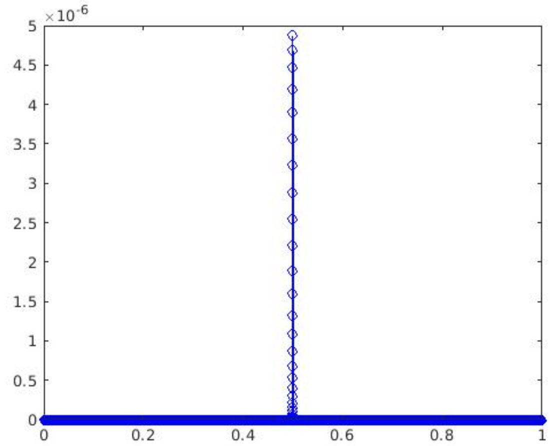


Figure 59: Improved ACF stemed (Bartlett)

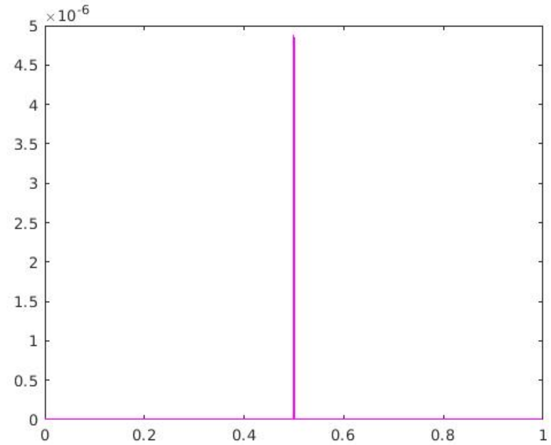


Figure 60: Improved ACF plotted (Blackman-Harris)

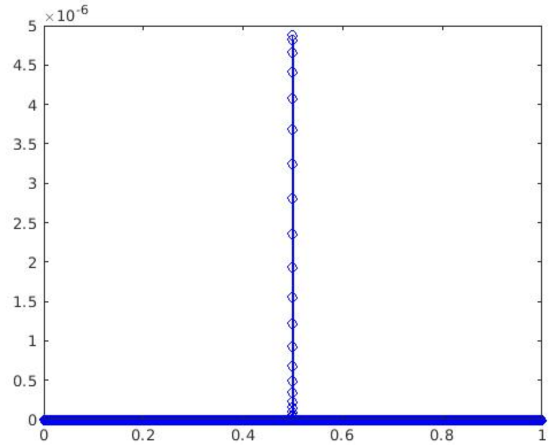


Figure 61: Improved ACF stemed (Blackman-Harris)

2.3 Improved Estimates: Low-degree filter:

Let's move on to the low-degree filter. As before, for this section we will take the estimations done during the first study with the first degree Butterworth filter.
We have the following PSD:

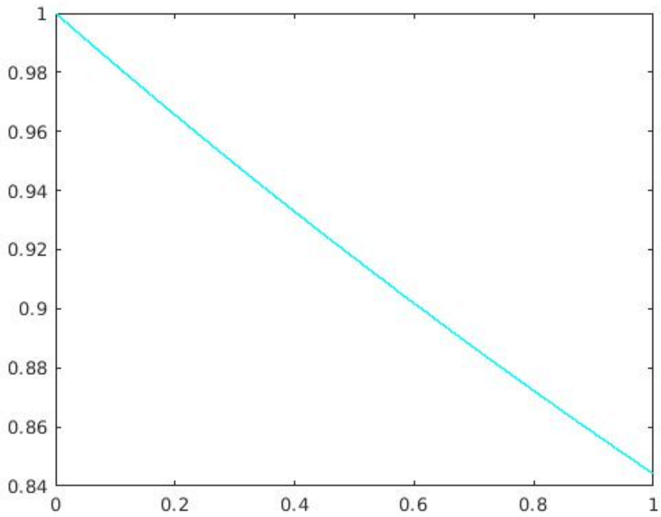


Figure 62: Estimated PSD

If we estimate by the Bartlett method, we get the following ACF:

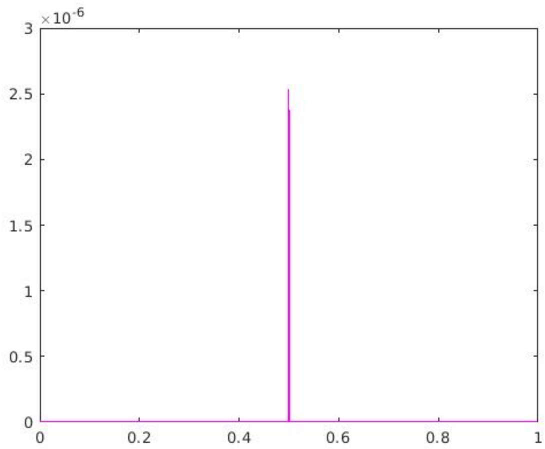


Figure 63: Estimated by Bartlett ACF plotted

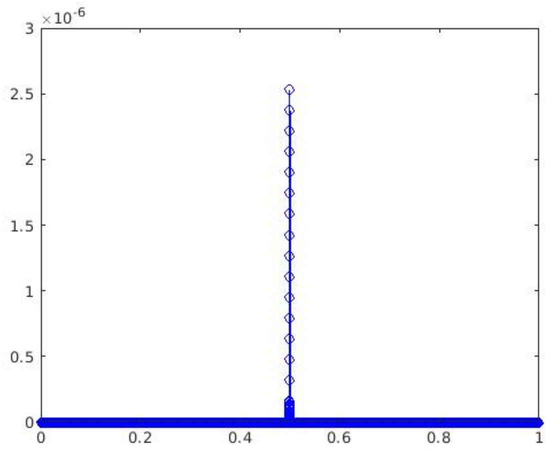
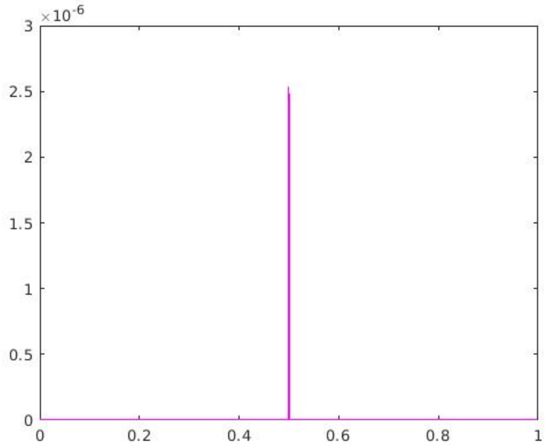


Figure 64: Estimated by Bartlett ACF stemed

If we estimate by the Blackman-Harris method, we get the following ACF:



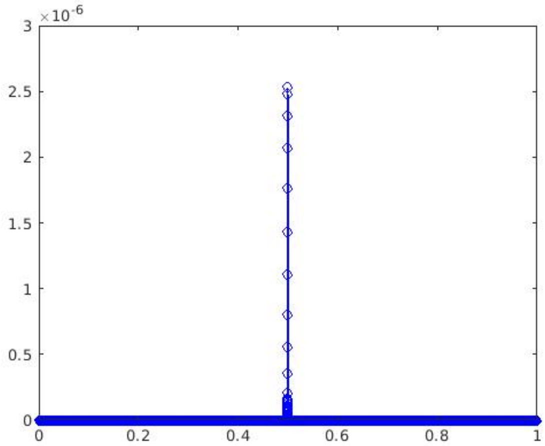


Figure 65: Estimated by Blackman-Harris ACF plotted

Figure 66: Estimated by Blackman-Harris ACF stemed

2.3.1 Rectangular Window:

Here we will get our filtered signal by a filter which is our rectangular window. There are the results:

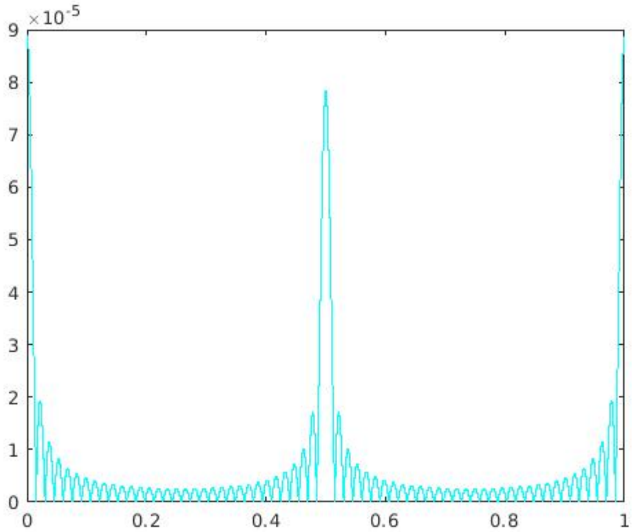


Figure 67: Improved PSD

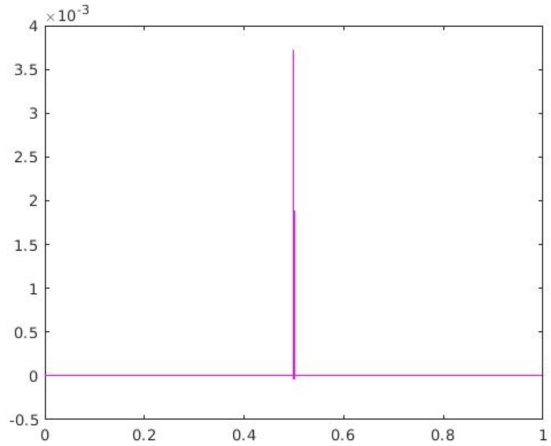


Figure 68: Improved ACF plotted (Bartlett)

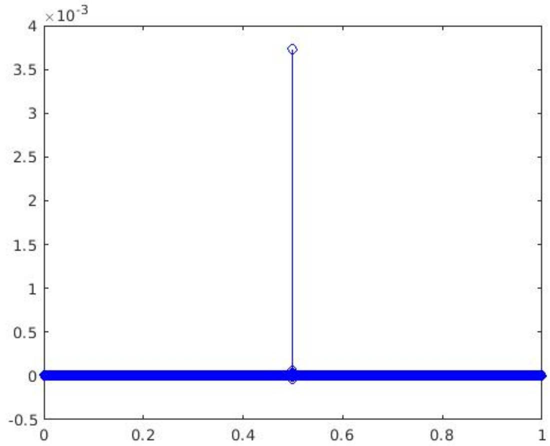


Figure 69: Improved ACF stemed (Bartlett)

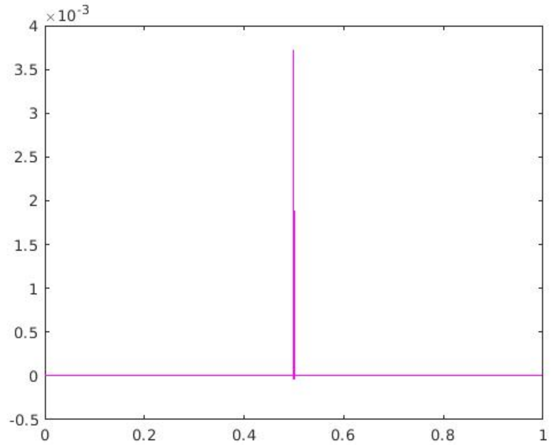


Figure 70: Improved ACF plotted (Blackman-Harris)

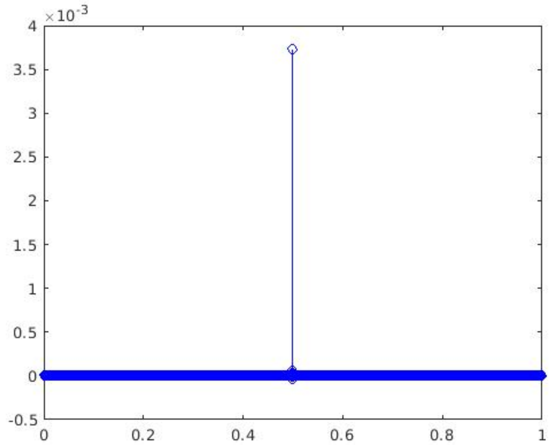


Figure 71: Improved ACF stemed (Blackman-Harris)

2.3.2 Triangular Window:

Here we will get our filtered signal by triangle window. We get:

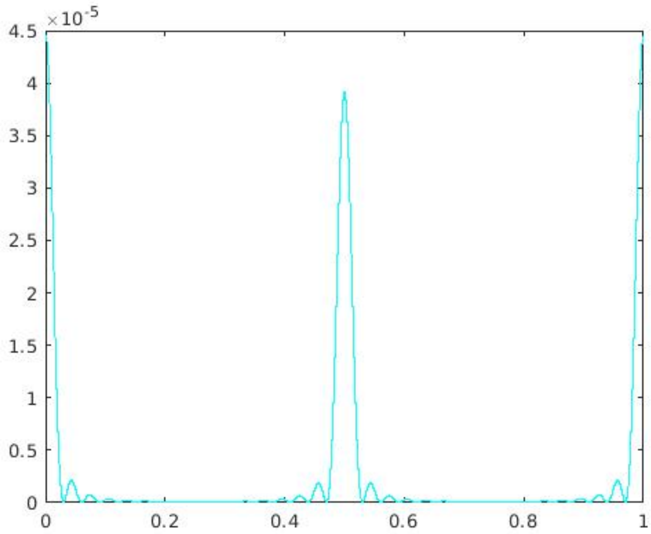


Figure 72: Improved PSD

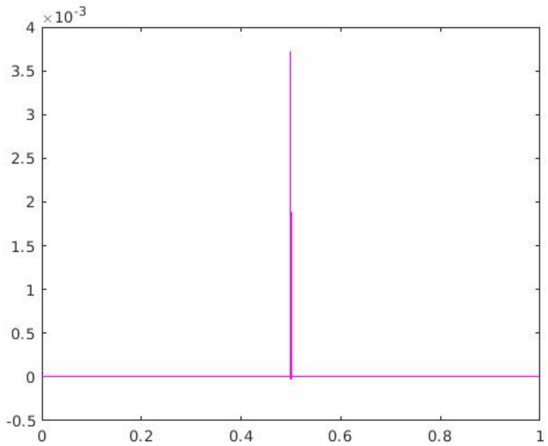


Figure 73: Improved ACF plotted (Bartlett)

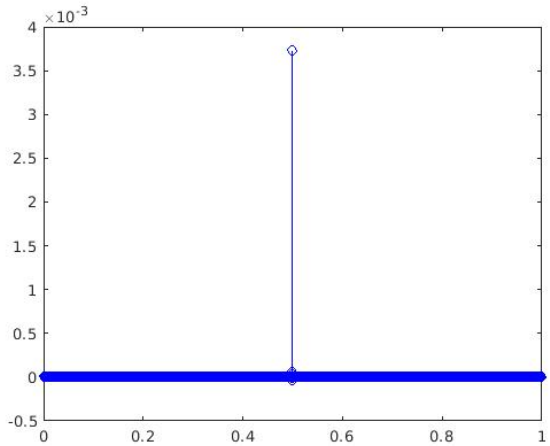


Figure 74: Improved ACF stemed (Bartlett)

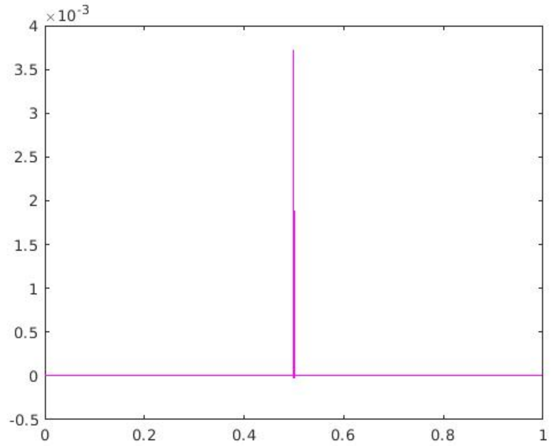


Figure 75: Improved ACF plotted (Blackman-Harris)

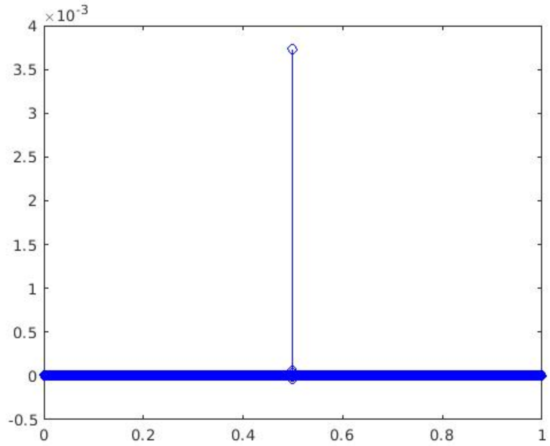


Figure 76: Improved ACF stemed (Blackman-Harris)

2.3.3 Hamming Window:

Here we will get our filtered signal by the Hamming window. Here are the results:

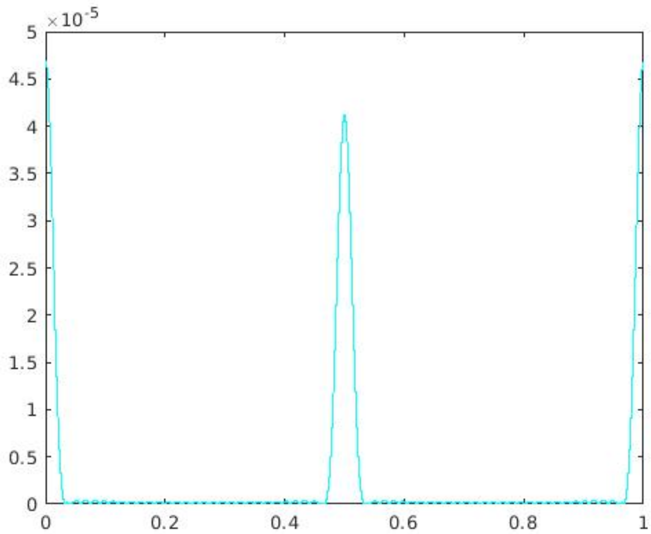


Figure 77: Improved PSD

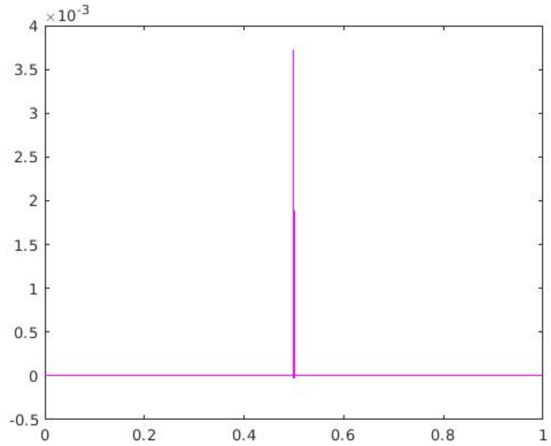


Figure 78: Improved ACF plotted (Bartlett)

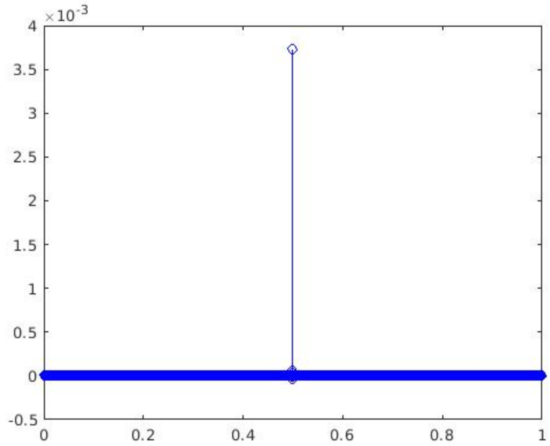


Figure 79: Improved ACF stemed (Bartlett)

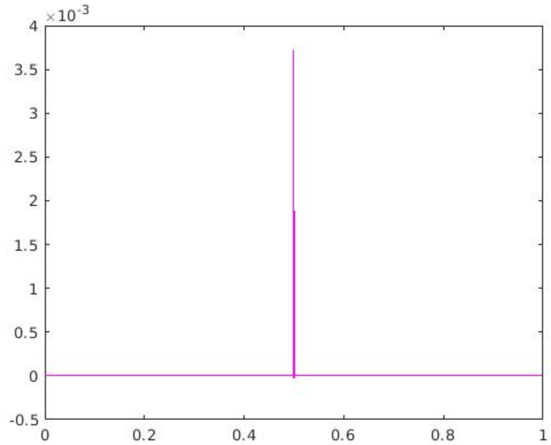


Figure 80: Improved ACF plotted (Blackman-Harris)

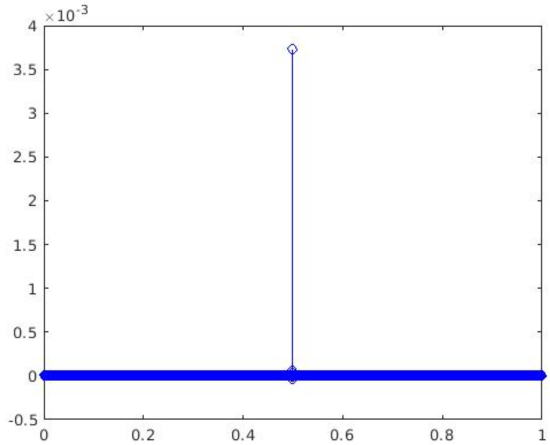


Figure 81: Improved ACF stemed (Blackman-Harris)

2.3.4 Bartlett Window:

Here we will get our filtered signal by the Bartlett rectangular window. We get:

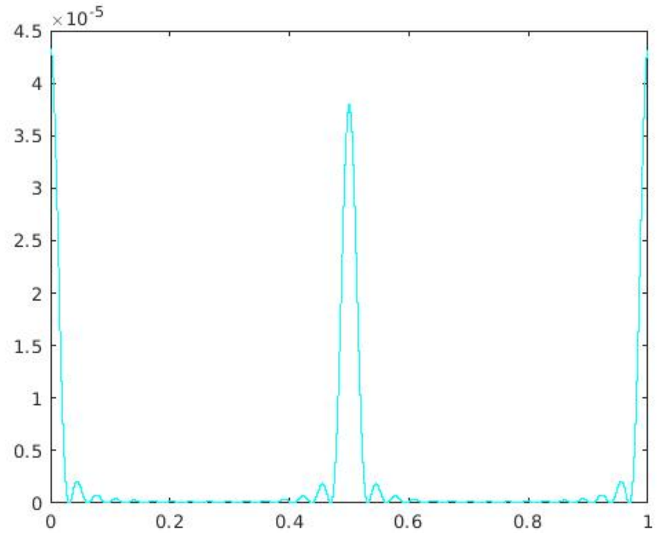


Figure 82: Improved PSD

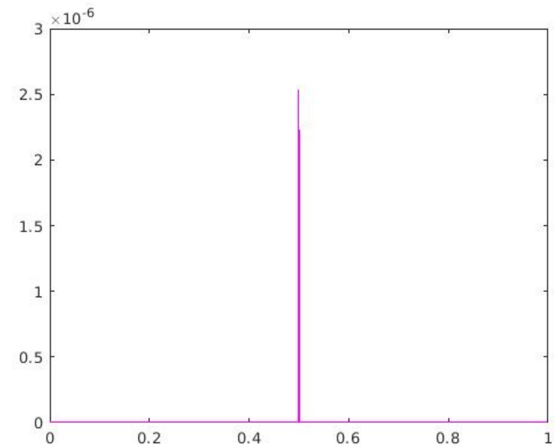


Figure 83: Improved ACF plotted (Bartlett)

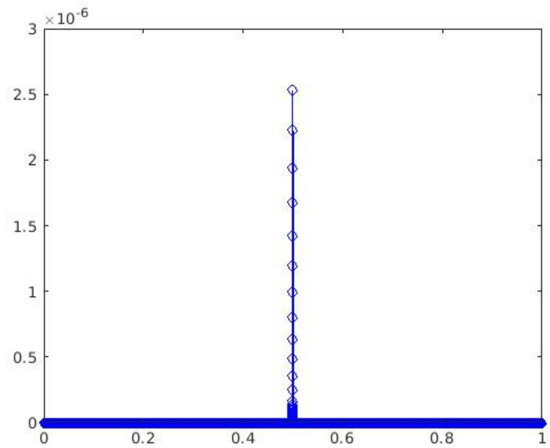


Figure 84: Improved ACF stemed (Bartlett)

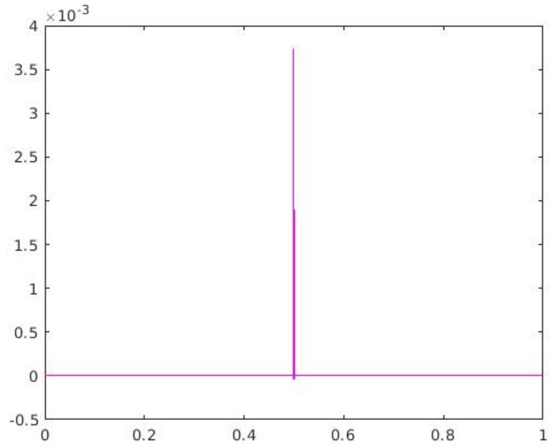


Figure 85: Improved ACF plotted (Blackman-Harris)

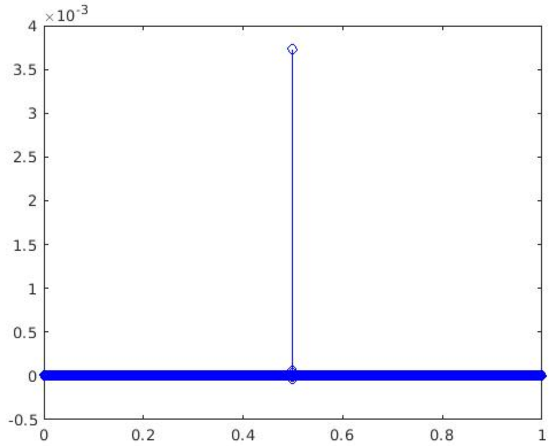


Figure 86: Improved ACF stemed (Blackman-Harris)

2.3.5 Blackman-Harris Window:

Here we will get our filtered signal by the Blackman-Harris window. Here are the results:

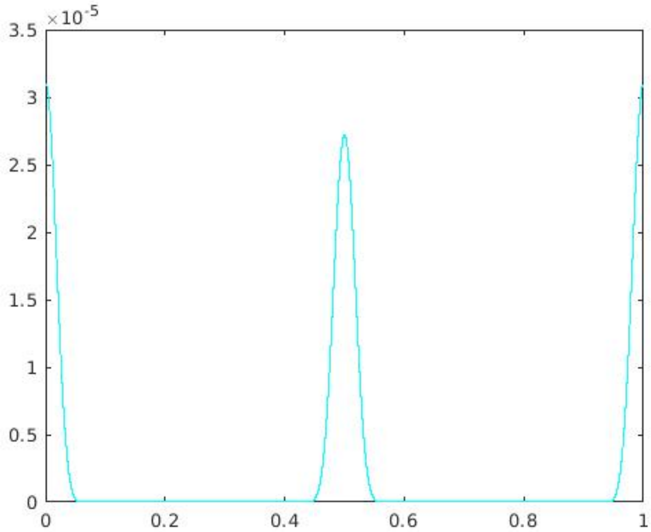


Figure 87: Improved PSD

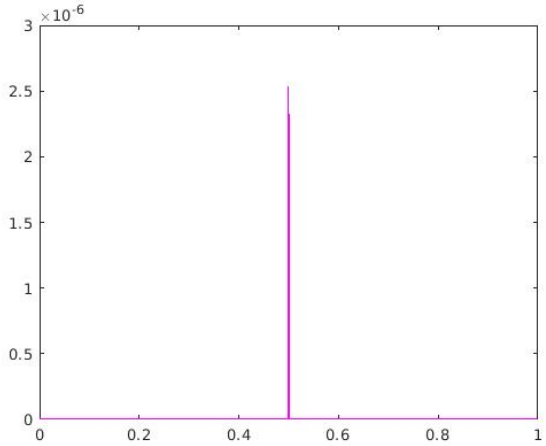


Figure 88: Improved ACF plotted (Bartlett)

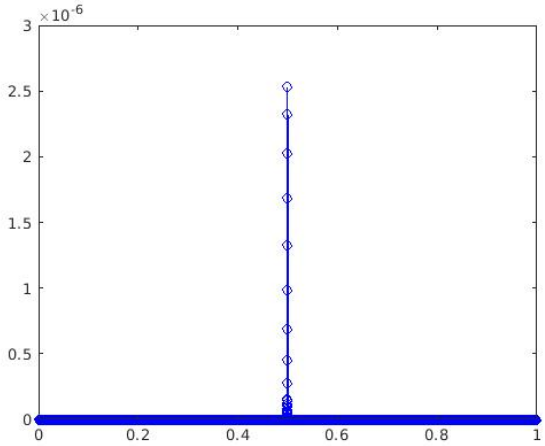


Figure 89: Improved ACF stemed (Bartlett)

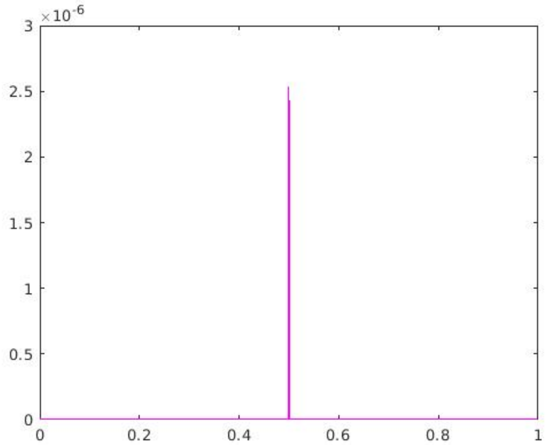


Figure 90: Improved ACF plotted (Blackman-Harris)

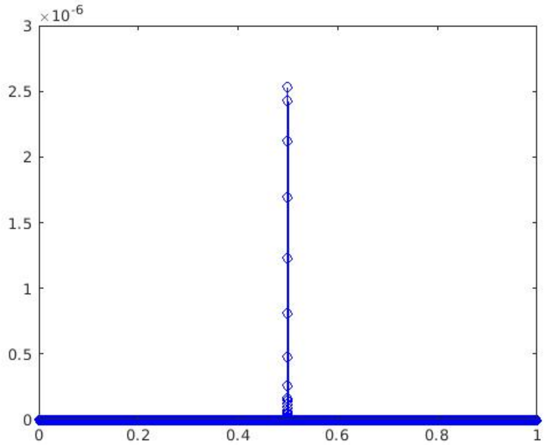


Figure 91: Improved ACF stemed (Blackman-Harris)

2.4 Final Conclusion:

As we can see, it's easier to work with smoothed graphs, for we can see the features of each plot better and it's not a messy drawing. It's like filtering the noise again.
Tha Blackman-Harris window was the most aggresive one, followed by the Hamming window. The other windows respect more the natural form of the plot.

3 Study 3:

3.1 Theoretical Background:

We have the three following systems:
A squarer:

$$Y[n] = X^2[n] \tag{9}$$

A half-wave rectifier:

$$Y[n] = \begin{cases} X[n], & n : X[n] > 0, \\ 0, & n : X[n] \leq 0, \end{cases} \tag{10}$$

An AM-SC modulator:

$$Y[n] = X[n]cos(\Omega_0 n) \tag{11}$$

As we can see, non of them is LTI, which means that the output might or might not be Gaussian, and which also means that the PSD does not follow the formula we normally use, but an specific formula for the kind of non-linearity that the system presents.
Therefore, we have three special formulas:

$$r_Y(\tau) = 2r_X^2(\tau) + r_X^2(0) \tag{12}$$

$$r_Y(\tau) = \frac{r_X(0)}{2\pi} + \frac{r_X(\tau)}{4} + \frac{r_X^2(\tau)}{4\pi r_X(o)} + ... \tag{13}$$

$$r_Y(\tau) = \frac{A^2}{2}(C^2 + r_X(\tau))cos(2\pi f_c \tau) \tag{14}$$

They belong respectively with each one of the transformations above.
Knowing the value of $R_x(\tau)$, and therefore the value of $r_x(\tau)$, we translate them to the frequency domain to have the PSD expressions:

$$R_Y[\theta] = 4\theta_c \Lambda[\frac{\theta}{2\theta_c}] + 4\theta_c^2 \delta[\theta] \tag{15}$$

$$R_Y[\theta] = \frac{1}{4\pi}\Lambda[\frac{\theta}{2\theta_c}] + \frac{1}{4}rect[\frac{\theta}{2\theta_c}] + \frac{\theta_c}{\pi}\delta[\theta] \tag{16}$$

$$R_Y[\theta] = \frac{1}{4}(rect[\frac{\theta + \Omega_0}{2\theta_c}] + rect[\frac{\theta - \Omega_0}{2\theta_c}]) \tag{17}$$

As done in the previous studies, we will use input noise. We will get it through in ideal low-pass filter, having the following result:

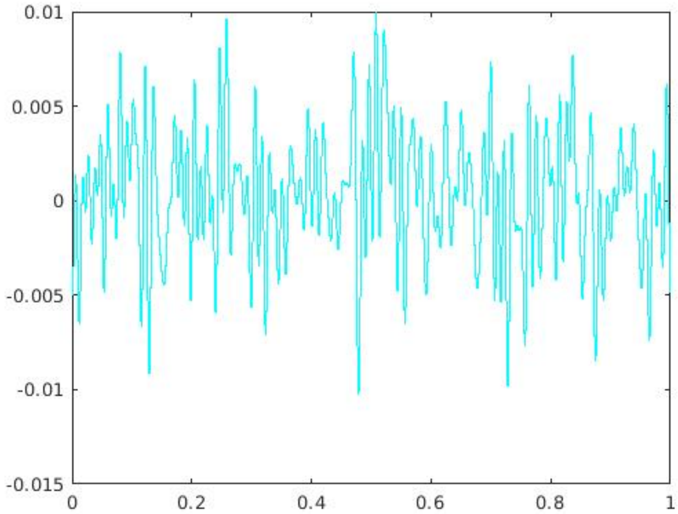


Figure 92: Filtered Signal

3.2 Theoretical Analysis:

First we can start with the filtered signal's PSD. We use the funtions that we have already used in previous studies, resulting in a PSD for the filter signal with the following plot:

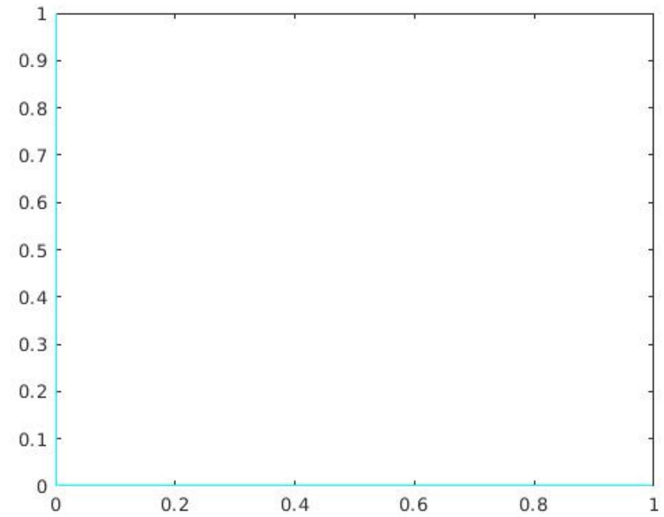


Figure 93: PSD of the filtered signal

If we get the signal through the squarer, the resulting PSD is:

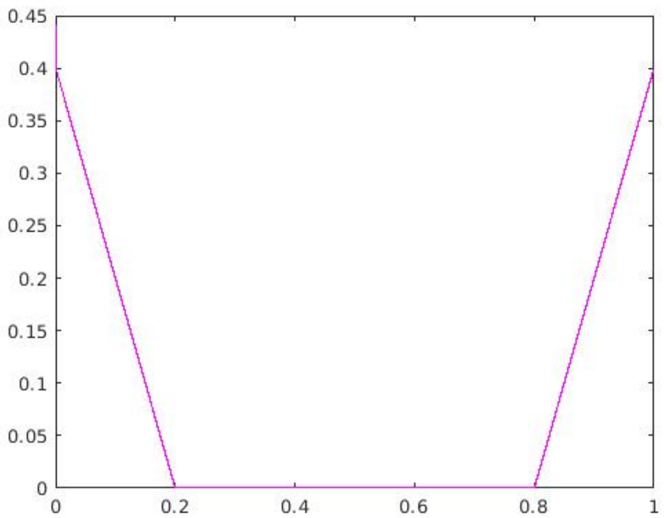


Figure 94: PSD of the signal through the squarer

The PSD that we get after applying the half-wave rectifier to the filtered signal is:

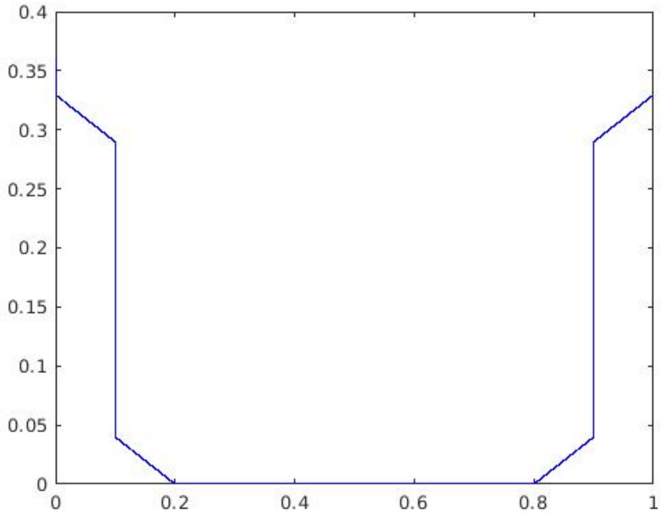


Figure 95: PSD of the signal through the half-wave rectifier

Lastly, the PSD resulting from AM-SC modulating the signal is:

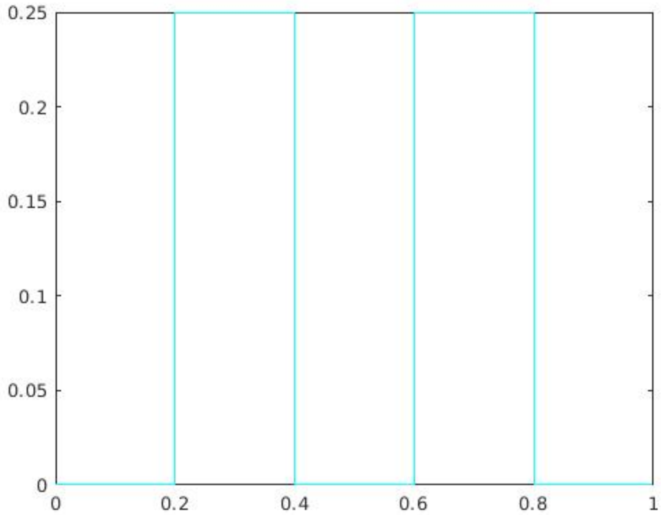


Figure 96: PSD of the signal through the AM-SC modulator

3.3 Estimations:

We can continue with the estimations the three non-linear systems.
If we get the signal through the squarer, the resulting PSD is:

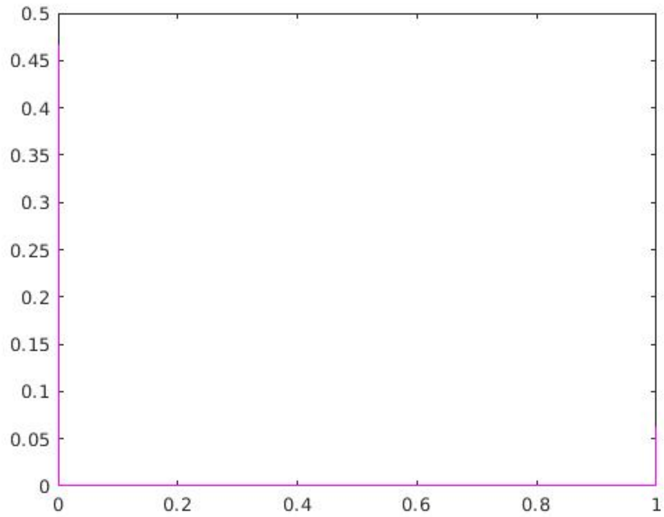


Figure 97: PSD of the signal through the squarer

The PSD that we get after applying the half-wave rectifier to the filtered signal is:

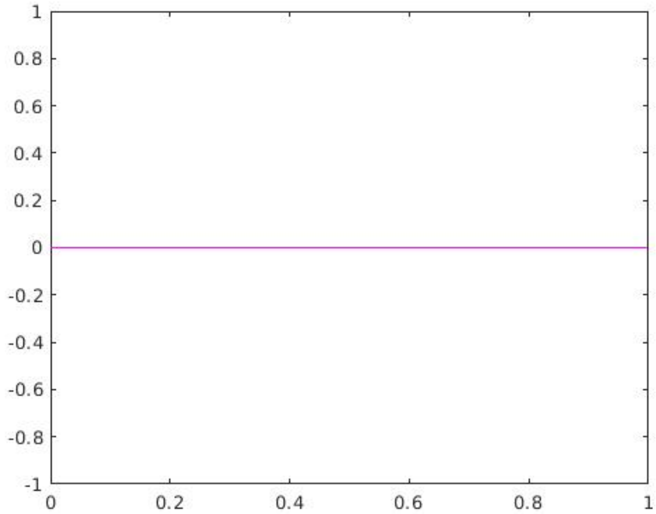


Figure 98: PSD of the signal through the Half-Wave Rectifier

Lastly, the PSD resulting from AM-SC modulating the signal is:

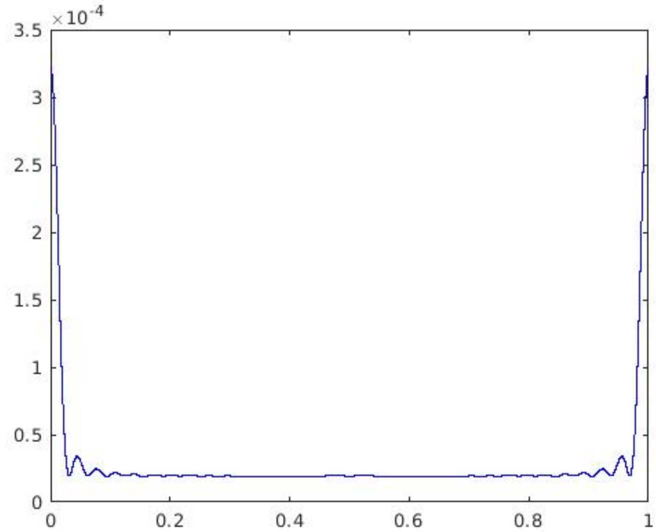


Figure 99: PSD of the signal through the AM-SC modulator

3.4 Improved Estimations:

Here we will smooth the precious plots as done in Study 2:

3.4.1 Rectangular Window:

First we have the squarer PSD:

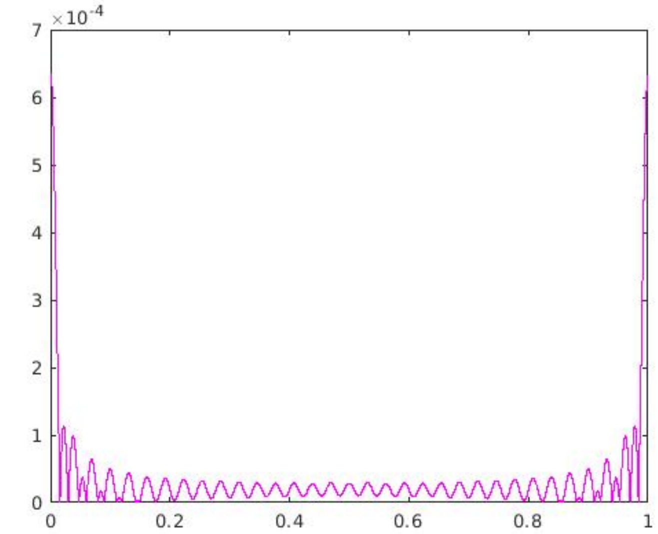


Figure 100: Improved PSD of the signal through the squarer

Then we have the half-wave rectifier PSD:

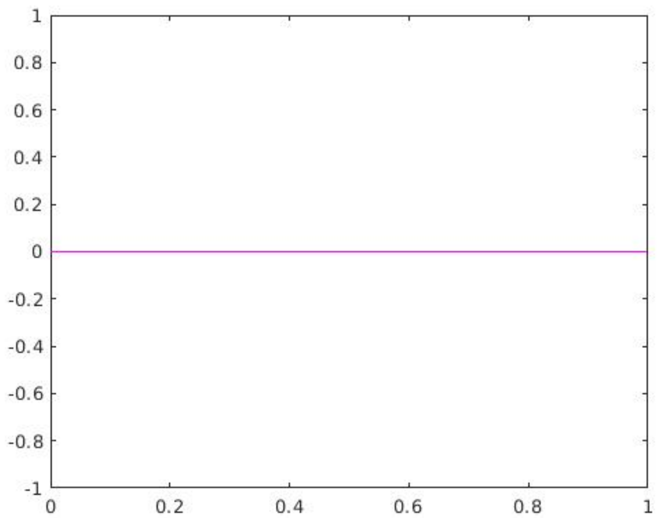


Figure 101: Improved PSD of the signal through the Half-Wave Rectifier

Lastly, we have the AM-SC modulator PSD:

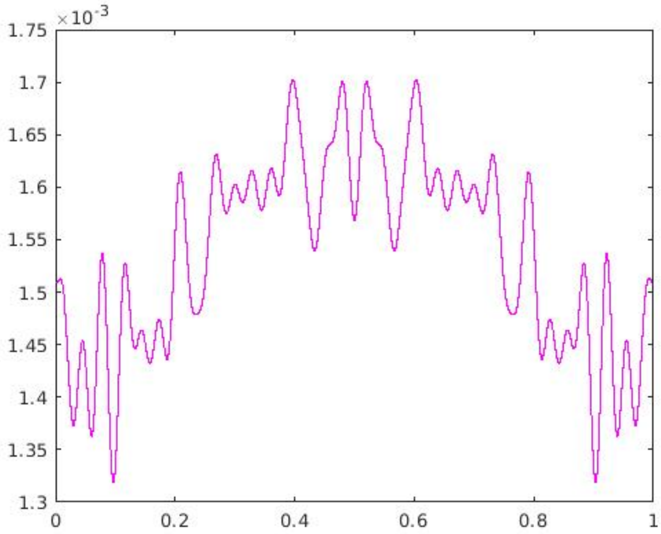


Figure 102: Improved PSD of the signal through the AM-SC modulator

3.4.2 Triangular Window:

First we have the squarer PSD:

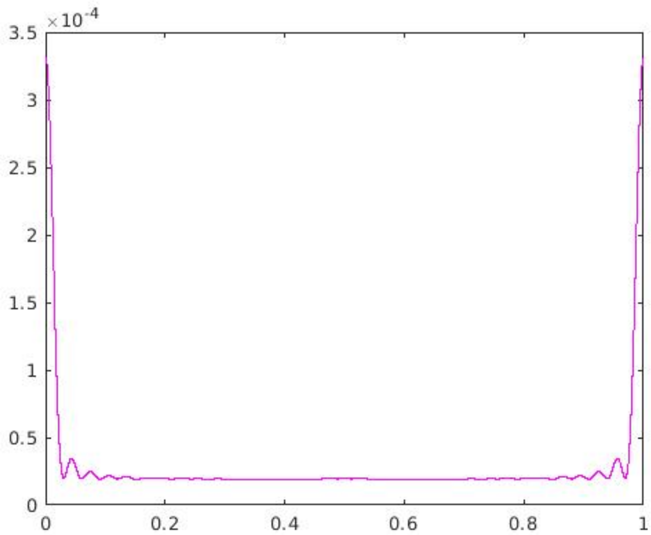


Figure 103: Improved PSD of the signal through the squarer

Then we have the half-wave rectifier PSD:

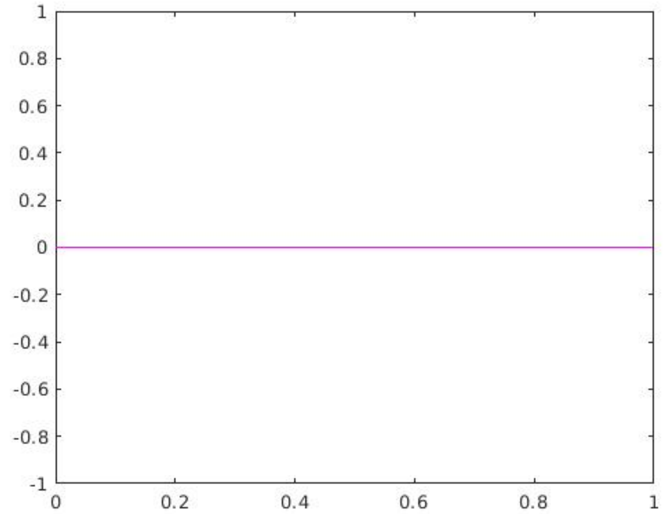


Figure 104: Improved PSD of the signal through the Half-Wave Rectifier

Lastly, we have the AM-SC modulator PSD:

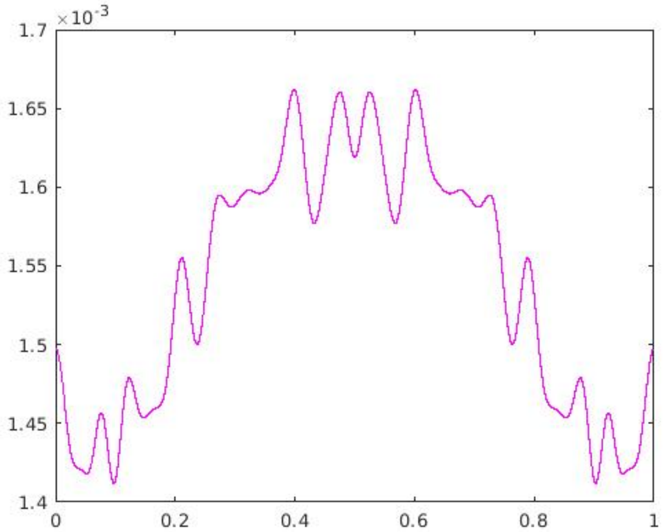


Figure 105: Improved PSD of the signal through the AM-SC modulator

3.4.3 Hamming Window:

First we have the squarer PSD:

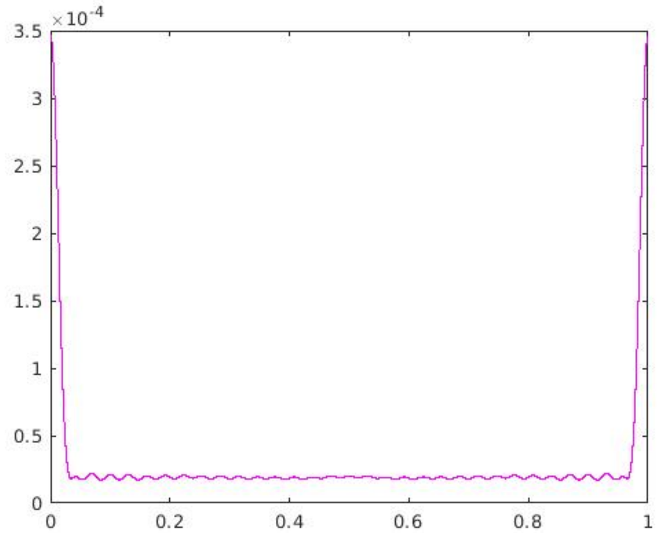


Figure 106: Improved PSD of the signal through the squarer

Then we have the half-wave rectifier PSD:

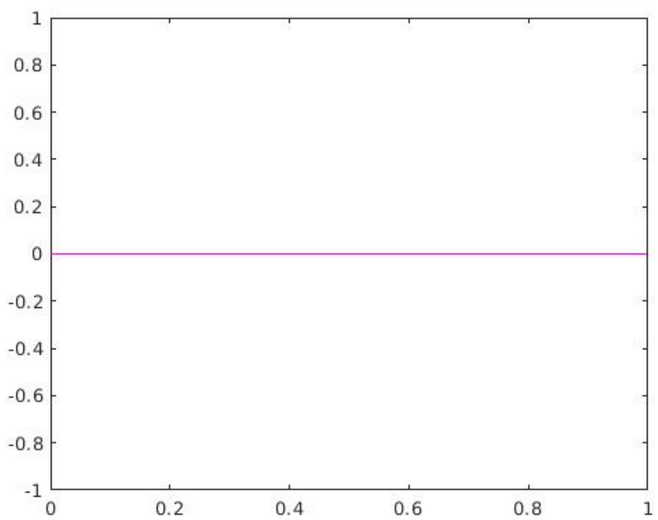


Figure 107: Improved PSD of the signal through the Half-Wave Rectifier

Lastly, we have the AM-SC modulator PSD:

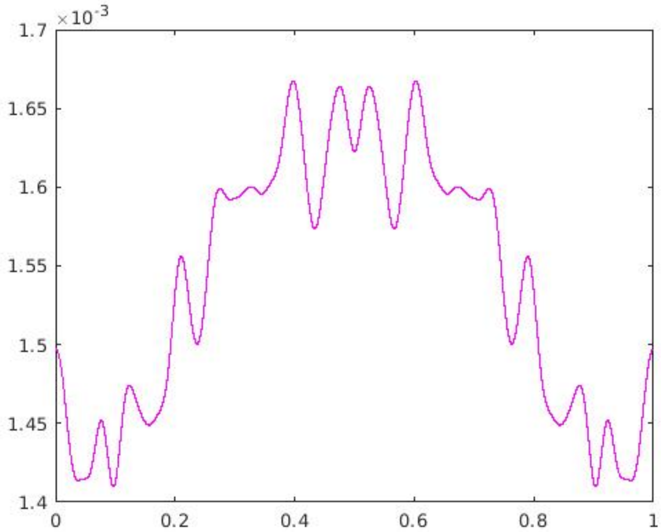


Figure 108: Improved PSD of the signal through the AM-SC modulator

3.4.4 Bartlett Window:

First we have the squarer PSD:

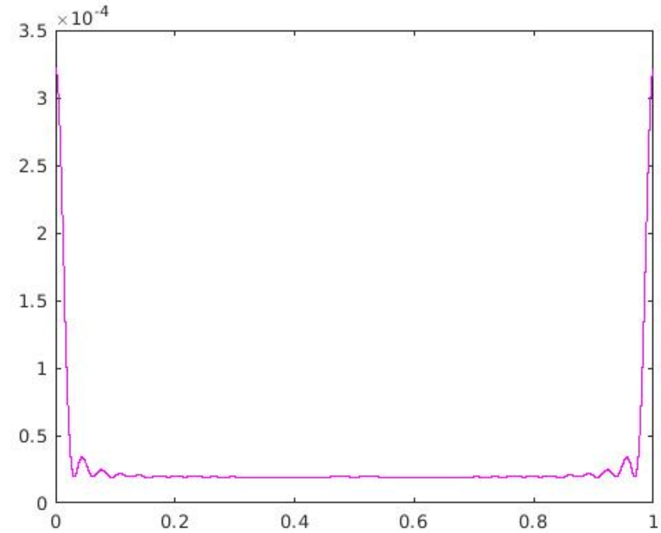


Figure 109: Improved PSD of the signal through the squarer

Then we have the half-wave rectifier PSD:

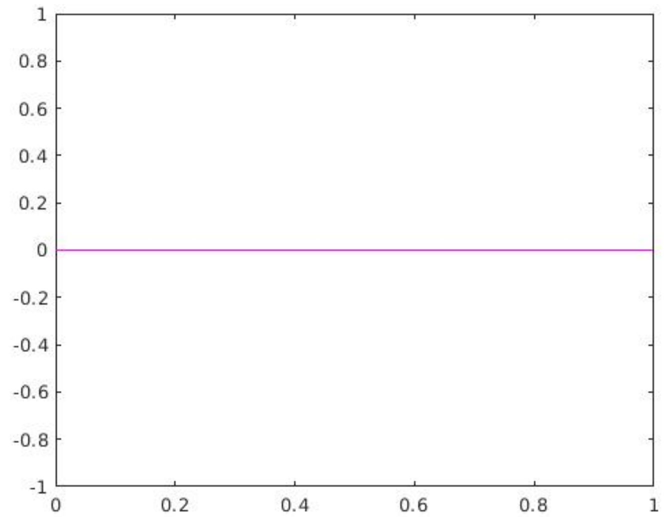


Figure 110: Improved PSD of the signal through the Half-Wave Rectifier

Lastly, we have the AM-SC modulator PSD:

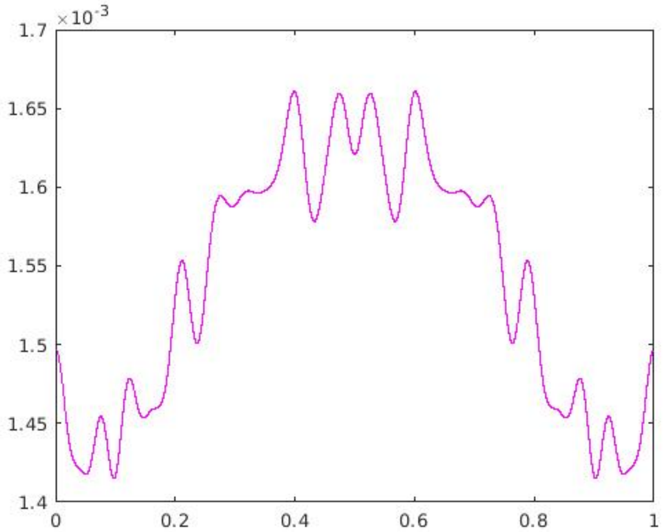


Figure 111: Improved PSD of the signal through the AM-SC modulator

3.4.5 Blackman-Harris Window:

First we have the squarer PSD:

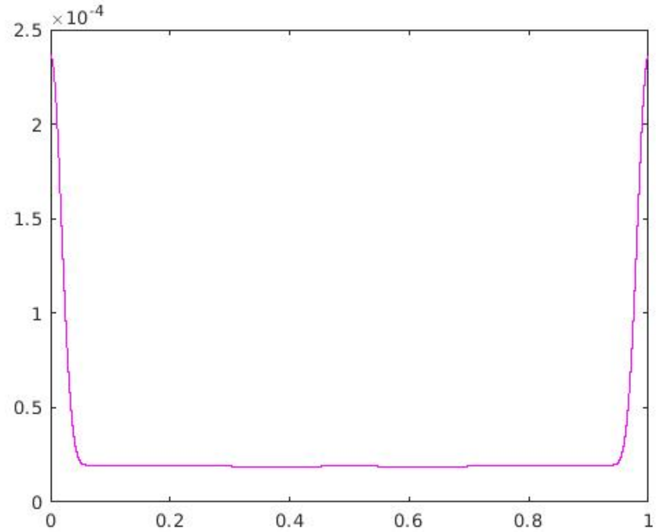


Figure 112: Improved PSD of the signal through the squarer

Then we have the half-wave rectifier PSD:

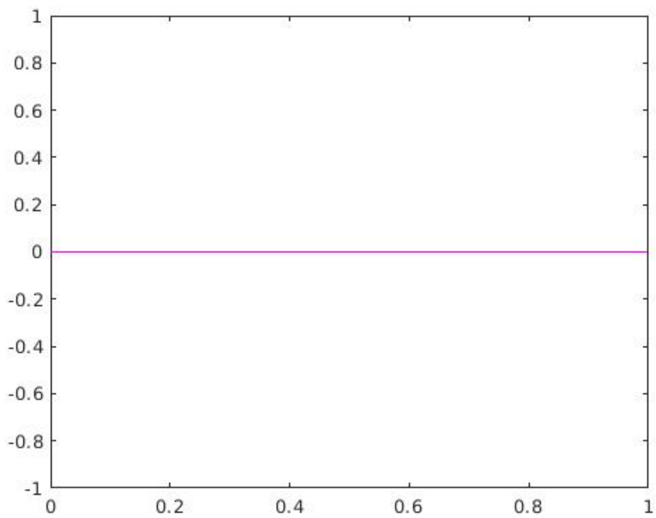


Figure 113: Improved PSD of the signal through the Half-Wave Rectifier

Lastly, we have the AM-SC modulator PSD:

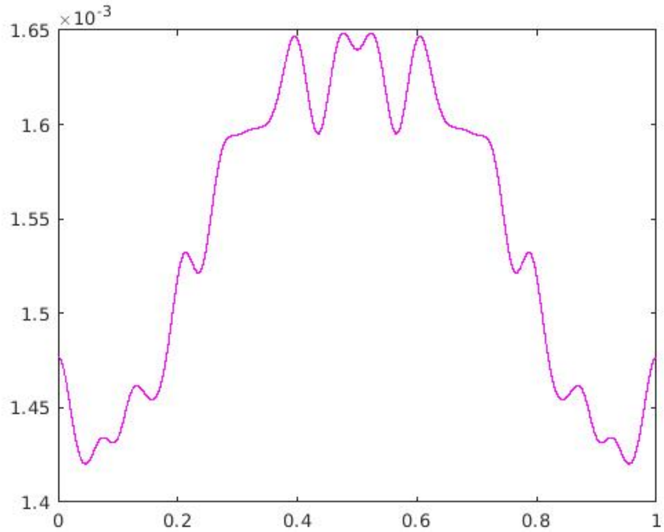


Figure 114: Improved PSD of the signal through the AM-SC modulator

3.5 Historiograms:

Here we have the historiogram of each one of our systems.
First, the historiogram of our signal Y:

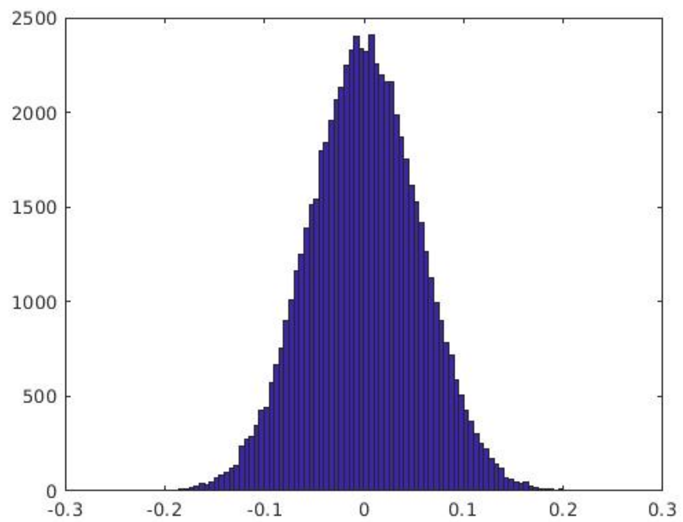


Figure 115: Historiogram of the filtered signal

If we get the signal through the squarer, the resulting Historiogram is:

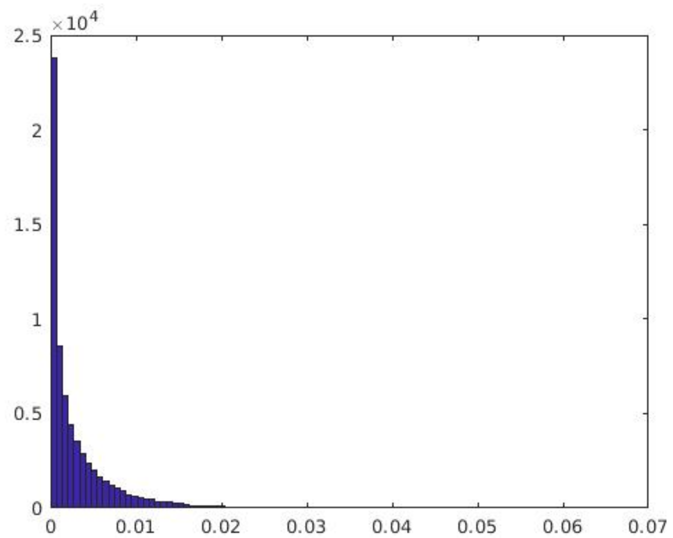


Figure 116: Historiogram of the signal through the squarer

The Historiogram that we get after applying the half-wave rectifier to the filtered signal is:

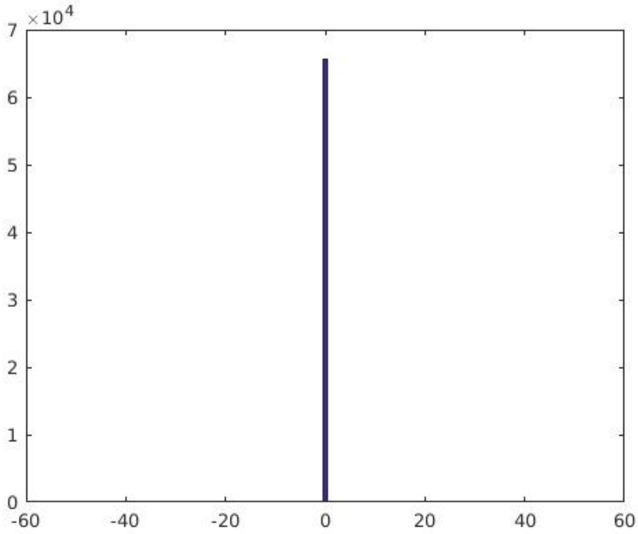


Figure 117: Historiogram of the signal through the half-wave rectifier

Lastly, the Historiogram resulting from AM-SC modulating the signal is:

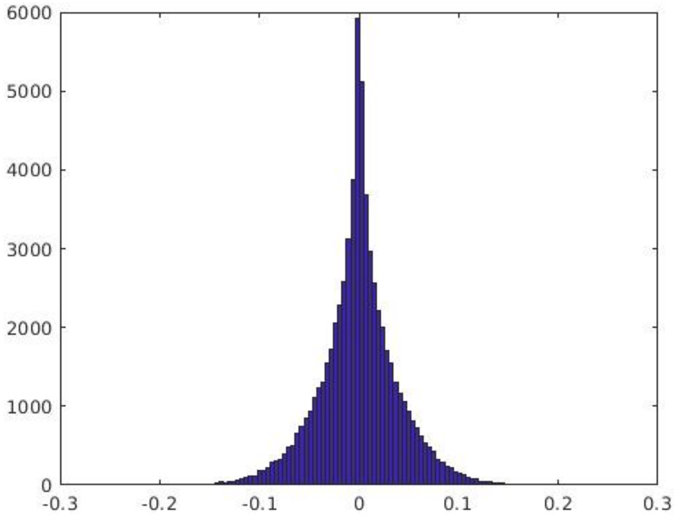


Figure 118: Historiogram of the signal through the AM-SC Modulator

3.6 Final Comparation:

My conclusion is that the half-wave rectifier is not a WSS system, because the PSD doesn't exist. The other two are WSS because the PSD exists and the plots make sense.

4 Study 4:

4.1 Theoretical Background:

We hace two special operations:

$$Y[n] = X[n](-1)^n \tag{18}$$

$$Y[n] = \begin{cases} X[n], & n : odd, \\ 0, & n : even \end{cases} \tag{19}$$

These represent two ways in which signals can be manipulated. We will get our every-study low-pass filtered White Gaussian Noise through these systems, to see if they're still WSS and to analize what happends with the PSDs of the signals that are being manipulated in this way in our every-day life situations.
The signals resulting from those systems are:

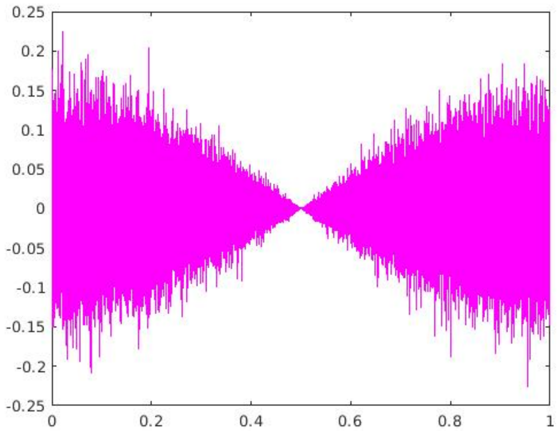


Figure 119: Signal resulting from the first system

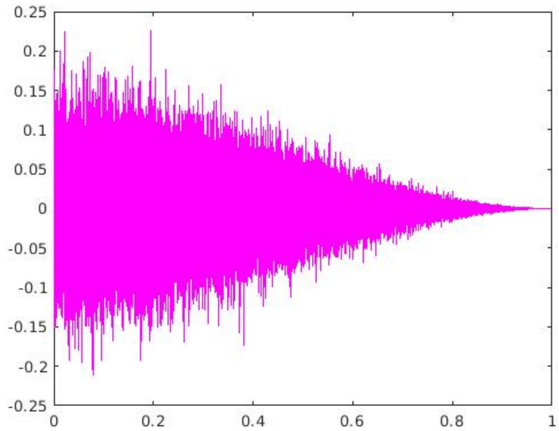


Figure 120: Signal resulting from the second system

We have to calculate the PSD from the ACF:

$$r_Y[\theta] = E\{Y[n+\theta]Y[n]\} \tag{20}$$

I will not describe all the intermediate steps calculated here, so the final PSD result after the Fourier transform is for the first alternating system:

$$R_Y[\theta] = R_X[\theta - 0.5] \tag{21}$$

And for the second system:

$$R_Y[\theta] = \frac{1}{4}(R_X[\theta - 0.5] + R_X[\theta]) \tag{22}$$

4.2 Theoretical Analysis:

First we will start showing the plots of the theoretical PSD. Here is the PSD of the signal put through the first system:

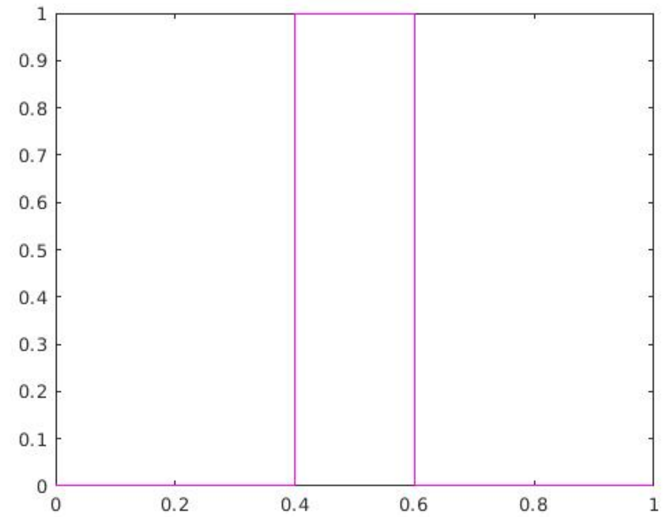


Figure 121: PSD of the signal through the first system

And here we have the PSD of the signal after the second system:

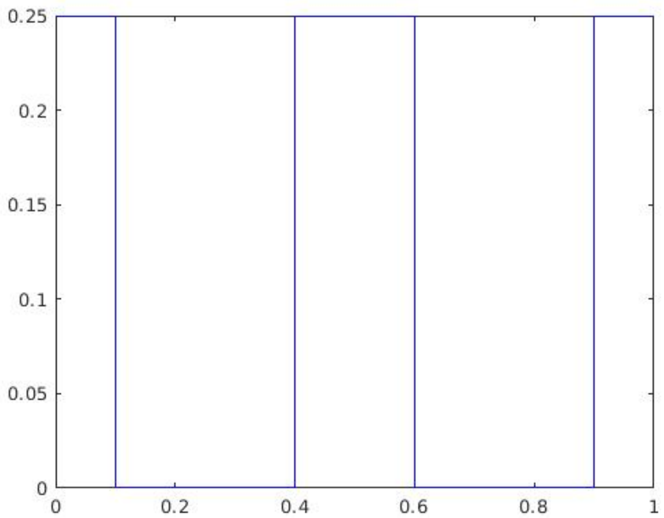


Figure 122: PSD of the signal through the second system

4.3 Estimations:

We can continue with the estimations the two systems.
If we get the signal through the first system, the resulting PSD is:

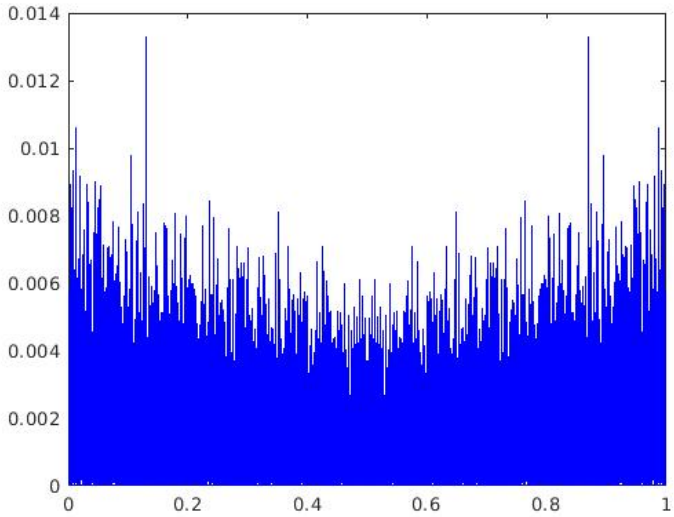


Figure 123: PSD of the signal through the first system

The PSD that we get after applying the second system to the filtered signal is:

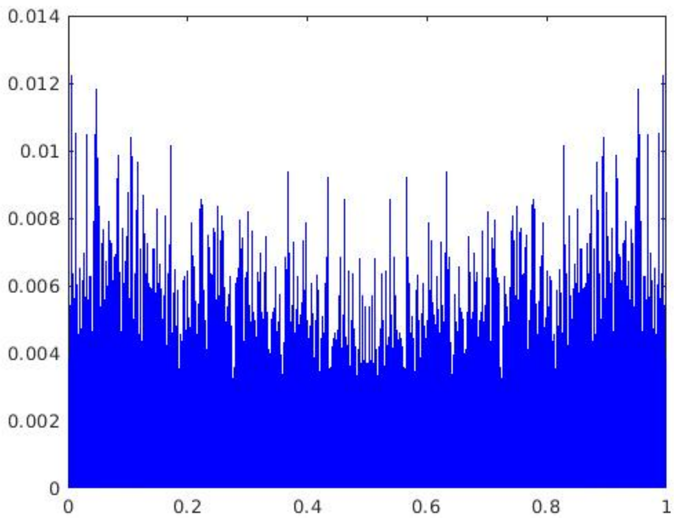


Figure 124: PSD of the signal through the second system

4.4 Improved Estimations:

Here we will smooth the precious plots as done in Study 2:

4.4.1 Rectangular Window:

First we have the first system:

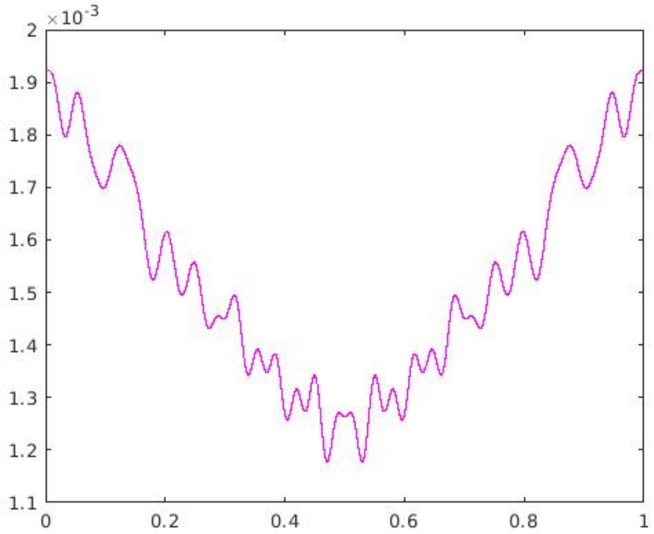


Figure 125: Improved PSD of the signal through the first system

Then we have the second system:

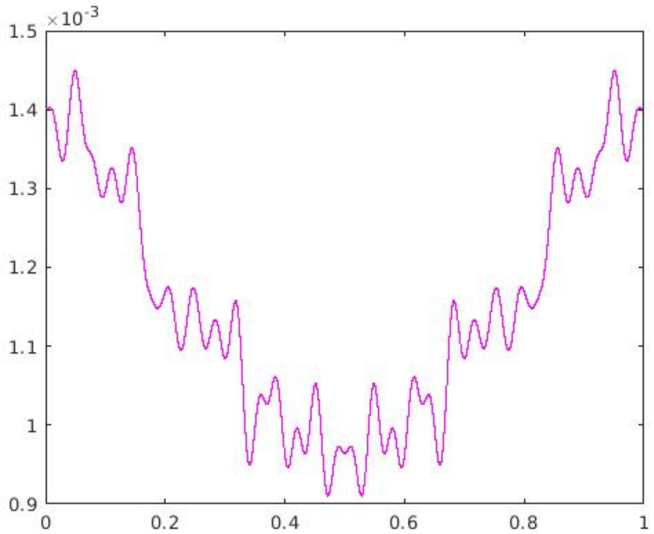


Figure 126: Improved PSD of the signal through the second system

4.4.2 Triangular Window:

First we have the first system:

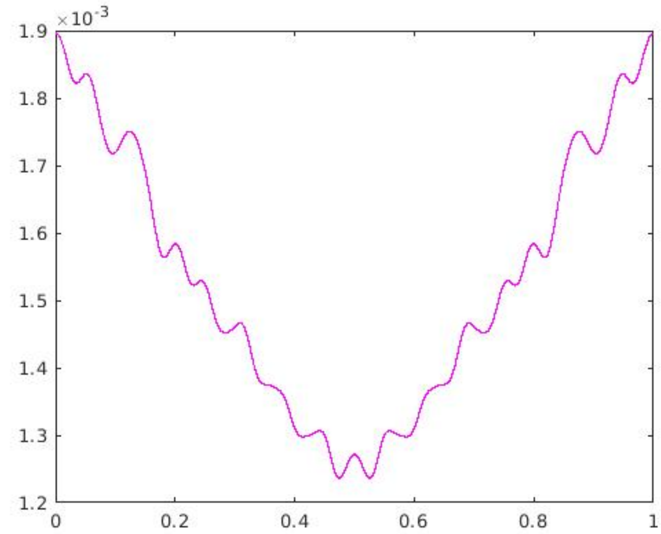


Figure 127: Improved PSD of the signal through the first system

Then we have the second system:

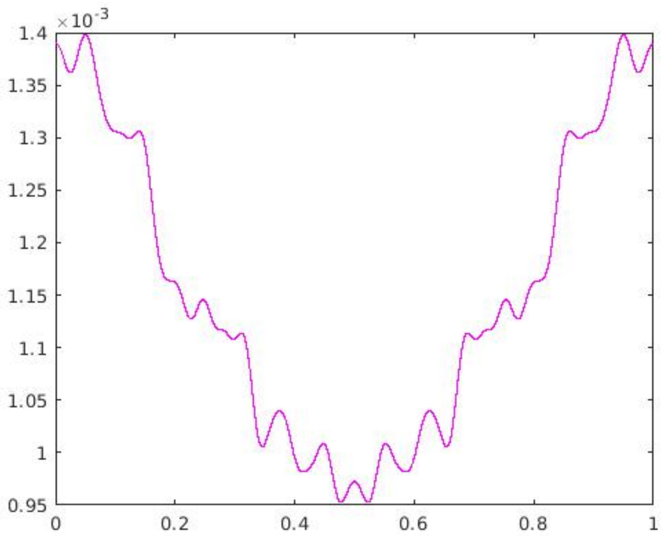


Figure 128: Improved PSD of the signal through the second system

4.4.3 Hamming Window:

First we have the first system:

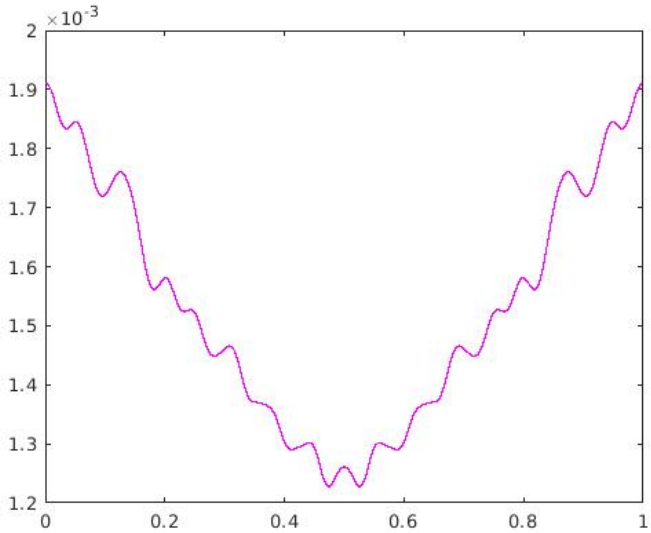


Figure 129: Improved PSD of the signal through the first system

Then we have the second system:

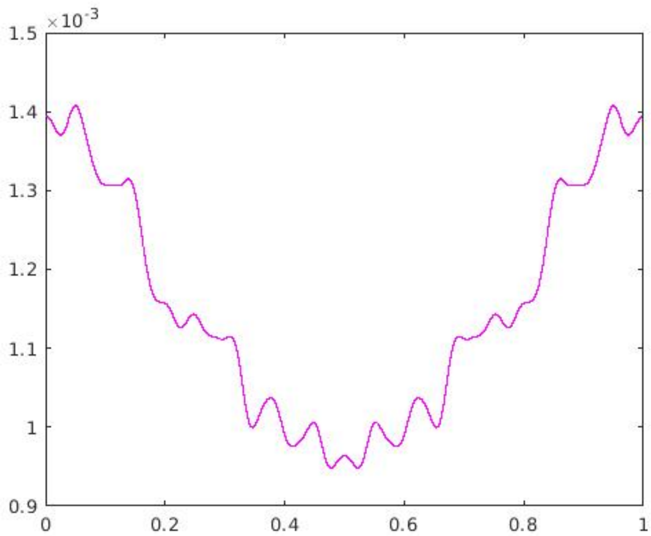


Figure 130: Improved PSD of the signal through the second system

4.4.4 Bartlett Window:

First we have the first system:

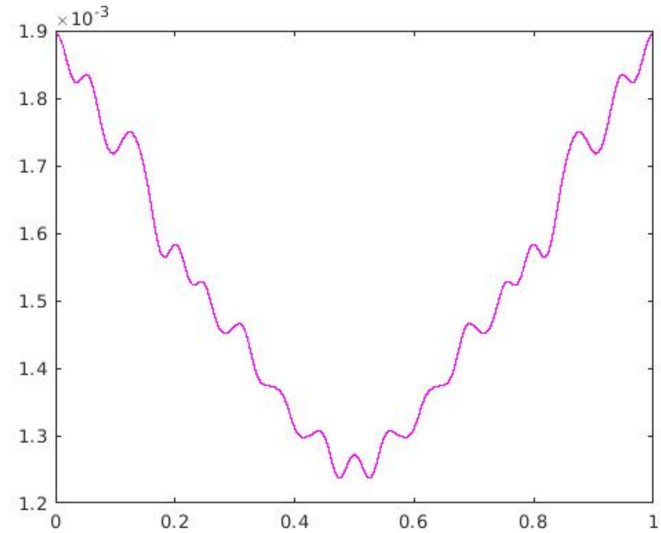


Figure 131: Improved PSD of the signal through the first system

Then we have the second system:

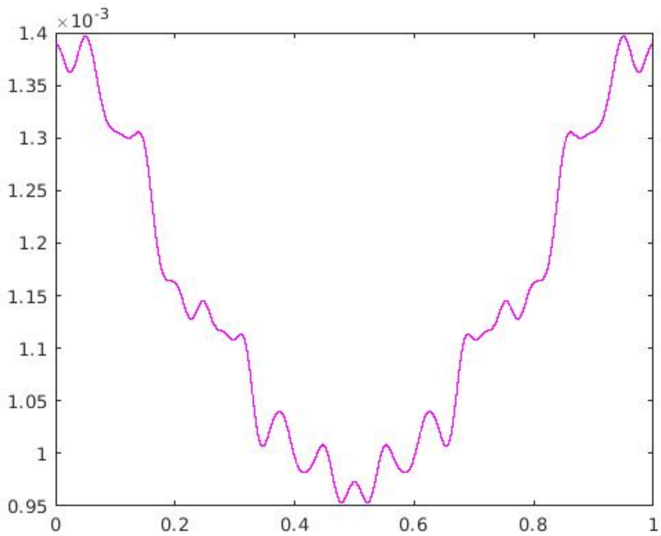


Figure 132: Improved PSD of the signal through the second system

4.4.5 Blackman-Harris Window:

First we have the first system:

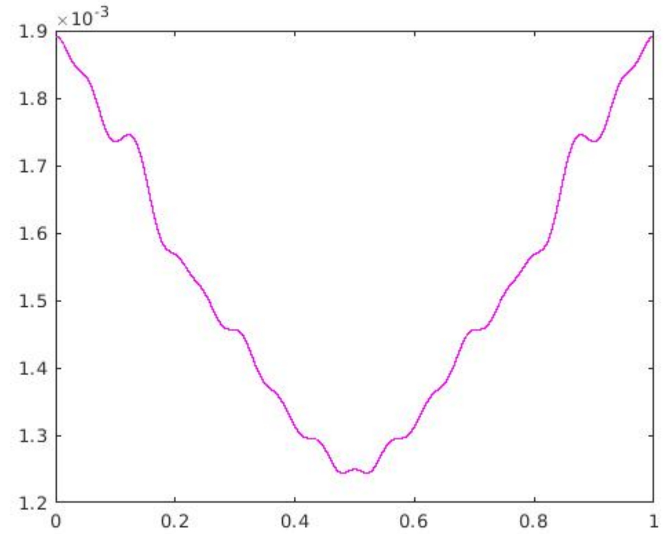


Figure 133: Improved PSD of the signal through the first system

Then we have the second system:

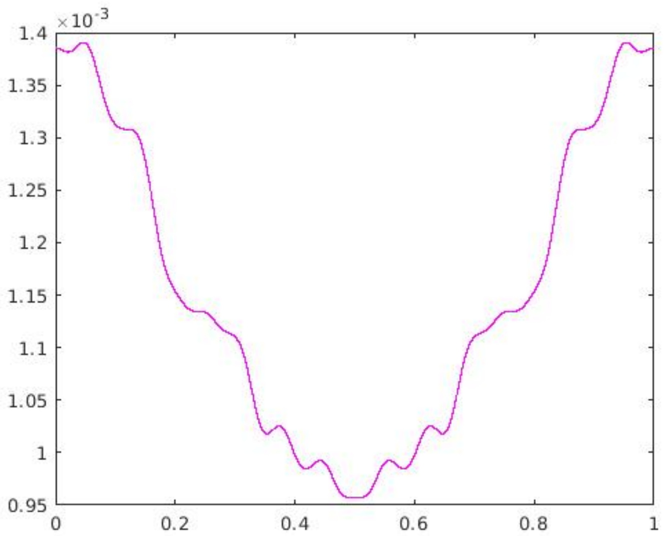


Figure 134: Improved PSD of the signal through the second system

4.5 Final Comparation:

Both systmes are relatively similar when it comes to plotting the PSD. If we don't smooth the graphics, the plots are caotic and hard to read, but after putting the through a window (I think the rectangular window is the best here) it's easier to see that the first system is a bit smoother than the second one, because there are less differences between the peaks.