Laboratory Exercise 4, Digital Filters 2014

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1 Purpose

The purpose of this lab is to increase the understanding of coefficient sensitivity and roundoff noise in digital filters. We also study the properties of the direct-form I filter as well as the ladder wave digital filter (ladder WDF). Note that you need some additional MATLAB files from the course homepage.

2 Preparation

Recapitulate the relevant sections from Chapter 4 of the course book. In particular, read Section 4.8 in the course book, which will be used in one of the tasks in this lab.

3 Coefficient Sensitivity

In this section, we will study how the passband and stopband responses of a direct-form I filter and a ladder WDF are affected by coefficient quantization.

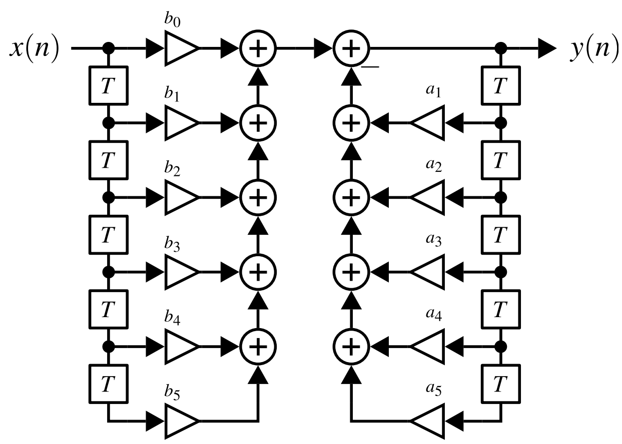
1. The transfer function $H(z)$ of a fifth-order IIR filter is given by

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5}}$$

   where $b_0 = 0.019774987772262$, $b_1 = 0.057313048668255$, $b_2 = 0.092548801756269$, $b_3 = 0.092548801756269$, $b_4 = 0.057313048668256$, $b_5 = 0.019774987772263$, $a_1 = -1.895781448174162$, $a_2 = 2.202301183298649$, $a_3 = -1.416777230207082$, $a_4 = 0.540523628526560$, and $a_5 = -0.090992457050392$.

   - Use the `freqz` function in MATLAB to verify that the above filter meets the specification given below.

Passband edge	:	0.3π rad
Stopband edge	:	0.584π rad
Passband attenuation, $A_{\max}$	:	0.011 dB
Stopband attenuation, $A_{\min}$	:	61 dB



The vectors `b` and `a`, containing the numerator and denominator coefficients, respectively, are available in the MAT file `fifthOrderFilter.mat`. In order to load these variables into your MATLAB workspace, use

```
load('fifthOrderFilter.mat');
```

2. The MATLAB function `directFormFilter` provides a direct-form I implementation of the above filter.

   - The frequency response of the direct-form I filter should be plotted in order to verify that the filter satisfies the given specification. One alternative method to get the approximate frequency response is by using a long enough impulse response of the direct-form I structure and then applying it to the `freqz` function to obtain the frequency response. The impulse response can be obtained by using the function `directFormFilter.m` which can be used as

   ```
   h = directFormFilter(impulse)
   ```

   where `impulse` is a sufficiently long impulse vector. The frequency response can then be plotted using the following MATLAB code.

   ```
   wT = linspace(0, pi, 1000);
   H = freqz(h, 1, wT);
   plot(wT/pi, 20*log10(abs(H)), 'r')
   ```

   Simulate the above direct-form I structure in MATLAB and verify that the filter meets the specification. Compare with the results in task 1.

   What should be the minimal length of the impulse response such that the computed frequency response matches well with the actual frequency response obtained in task 1?

- In order to verify the filter, apply an input $x(n)$ given by

$$x(n) = 0.5\sin(0.2171\pi n) + 0.5\sin(0.631711\pi n)$$

to the above direct-form I filter and obtain the output $y(n)$. The signal $x(n)$ is quantized to 12 fractional bits and can be generated using the following MATLAB code.

```
NFFT = 8192; % length of FFT
n = 0:NFFT-1;
x = 0.5*(sin(0.2171*pi*n) + sin(.631711*pi*n));
x = quant(x, 2^-12); % quantize to 12 fractional bits
```

Compute and plot the spectrum (using an 8192-point DFT) of the signals $x(n)$ and $y(n)$. Before computing the DFT of the signal, multiply the signal with a Blackman-Harris window. For this, use the MATLAB function `blackmanharris()`. The following code snippet can be used to compute the FFT of a signal `x` in MATLAB.

```
% x -- signal whose FFT should be computed
xw = x.*blackmanharris(NFFT).';
X = fft(xw);
wT = 0:2*pi/NFFT:pi-2*pi/NFFT; % Convert FFT index k to wT
plot(wT/pi, 20*log10(2*abs(X(1:length(wT))/NFFT)));
xlabel('\omegaT [\times\pi rad]','FontName','times');
ylabel('|X(k)| [dB]','FontName','times');
```

How large are the magnitudes of the two tones at the input and output of the filter?

Tone	Magnitude at the input (dB)	Magnitude at the output (dB)
0.2171π		
0.631711π		

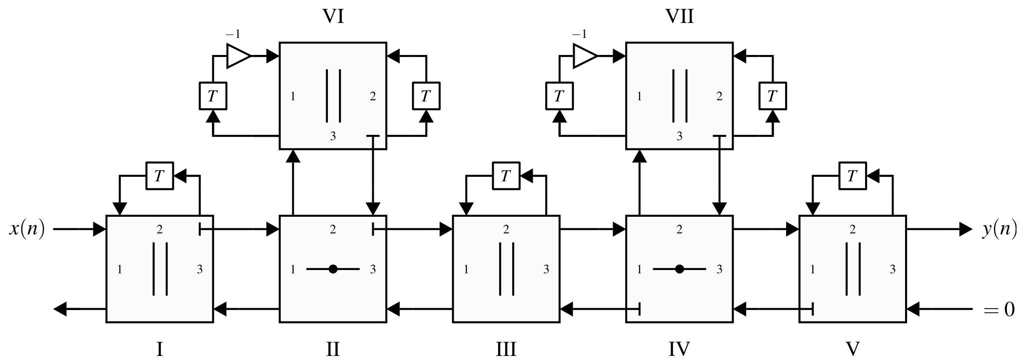
- Plot the frequency response of the direct-form I structure with the filter coefficients quantized to six fractional bits. In order to get the impulse response of the filter using the quantized coefficients, the function `directFormFilter.m` can be invoked as

```
h = directFormFilter(impulse, 0, Wf)
```

where `Wf` represents the number of fractional bits for the filter coefficients.

Compare the frequency response of the filter with and without coefficient quantization. How is the passband and stopband attenuation affected?

Through simulations, determine the number of fractional bits required to have $A_{\max} = 0.011$ dB and $A_{\min} = 60$ dB.



The total number of bits required to have $A_{\text{max}} = 0.011$ dB and $A_{\text{min}} = 60$ dB = ... bits. (Hint: Remember to take into account the integer bits required for the coefficients.)

- Compute the L_2-norm at the output of the filter.
  L_2-norm at the output = ...
  Is the filter scaled or unscaled or over scaled?

  How should the scaling constant be inserted in the direct-form I filter?

  The impulse response of the scaled direct-form I filter can be obtained by using:

  ```
  h = directFormFilter(impulse, 1)
  ```

  Compute the L_2-norm at the output of the scaled filter.
  L_2-norm at the output of the scaled filter = ...

  Plot the frequency response of the scaled direct-form I filter. Does the scaled filter satisfy the specification?

3. In this task, we will use the fifth-order ladder WDF shown above. The adaptor coefficients are as given in the table below which realizes the same $H(z)$ as before.

Adaptor	α_1	α_2
I	0.410366383870482	
II	0.167585550872200	
III	0.211034363678024	0.271498091115082
IV	0.228344874180013	
V	0.434623166811736	
VI	0.176471886871252	
VII	0.348963206587482	

- Using a sufficiently long impulse response, obtained using the function `fifthOrderLadderWDF.m`, plot the frequency response of the ladder WDF and verify that it meets the same specification as given in Task 1. The function `fifthOrderLadderWDF.m` can be invoked as

  ```
  [h, cnodes] = fifthOrderLadderWDF(impulse)
  ```

  where `impulse` and `h` represent the impulse vector and the output of the filter, respectively. Here, `cnodes` represents the node values at the input of the multipliers in the three-port adaptors.

- The ladder WDF is generally scaled as outlined in Section 11.13 of the course book. In this lab, to simplify the implementation, the ladder WDF is over scaled in the L_2-norm scaling sense. Verify this by computing the L_2-norm at the input of the multipliers in the three-port adaptors and at the output of the ladder WDF. The node values at the input of each multiplier in the adaptors can be obtained by indexing the rows of `cnodes` as shown in the table below.

Adaptor	Mult.1	L_2-norm	Mult. 2	L_2-norm
I	`cnodes(1,:)`			
II	`cnodes(2,:)`			
III	`cnodes(3,:)`		`cnodes(4,:)`	
IV	`cnodes(5,:)`			
V	`cnodes(6,:)`			
VI	`cnodes(7,:)`			
VII	`cnodes(8,:)`			

  L_2-norm at the output of the ladder WDF = ...

- Plot the frequency response of the ladder WDF filter with coefficients quantized to six fractional bits. In order to get the impulse response of the filter using the quantized coefficients, use

  ```
  [h, cnodes] = fifthOrderLadderWDF(impulse, Wf)
  ```

  where `Wf` represents the number of fractional bits for the filter coefficients. Compare the frequency response of the filter with and without coefficient quantization. How is the passband and stopband attenuation affected?

  Through simulations, determine the number of fractional bits required to have $A_{\max} = 0.011$ dB and $A_{\min} = 60$ dB.
  The total number of bits required to have $A_{\max} = 0.011$ dB and $A_{\min} = 60$ dB = ... bits. (Hint: Initial fractional bits that are zero can be removed.)

  Compare the total number of bits required for the coefficients in the ladder WDF and in the direct-form I filter. Which of these two filters is more sensitive to coefficient quantization?

4 Roundoff Noise

In practice, the internal data wordlength used in the filter is also finite. In this section, we will study the effect of roundoff noise due to the rounding at the output of each multiplier to a finite number of fractional bits, `Wr`.

1. In this task, we will assume that the coefficients are quantized to eight fractional bits. That is, `Wf= 8`.

   - Measure the roundoff noise in the fifth-order direct-form I filter as well as in the fifth-order ladder WDF when the internal data wordlength is 12 bits (`Wr=12`). Utilize the method suggested in Section 4.8 of the course book. For the input signal $x(n)$, use a vector containing uniformly distributed random numbers within the range $[-1,1]$. In MATLAB, this can be done using

     ```
     x = 2*(rand(1,1000)-0.5);
     ```

     For simulating the ideal over scaled filters without rounding at the output of the multipliers, use

     ```
     yI = directFormFilter(x, 0, Wf)
     ```

     ```
     yI = fifthOrderLadderWDF(x, Wf)
     ```

     In order to simulate the over scaled filters with rounding at the output of the multipliers, use

     ```
     yQ = directFormFilter(x, 0, Wf, Wr)
     ```

     ```
     yQ = fifthOrderLadderWDF(x, Wf, Wr)
     ```

     Compute the signal-to-noise ratio (SNR) for the two structures.
     SNR for the over scaled direct-form I filter = ... dB.
     SNR for the over scaled ladder WDF = ... dB.

   - Measure the roundoff noise for the scaled direct-form I filter and compute the SNR.

     SNR for the scaled direct-form I filter = ... dB.

     Compared to the over scaled direct-form I filter, why is the SNR of the scaled filter higher?

     For these two cases, what is the SNR difference in terms of number of bits?