Laboratory Exercise 2, Digital Filters 2013

The purpose of this laboratory exercise is to give some insights into the design and properties of IIR filters obtained by bilinearly transforming analog prototype filters. The four standard filters Butterworth, Chebyshev I, Chebyshev II, and Cauer will be considered. The procedure is to first use the functions `buttap`, `cheby1ap`, `cheby2ap`, and `ellipap`, to synthesize the analog prototype filters, and then use the function `bilinear`. This design procedure has been implemented in the functions `butter`, `cheby1`, `cheby2`, and `ellip` which thus can be used to directly design the digital filters given their requirements. However, in order to get some insights into the design procedure, and into the transformations from the analog to the digital counterparts, we will in this laboratory exercise use the former approach.

The MATLAB commands and programs that are required for the exercise are listed below. (Some of them are not actually needed, but might be useful.) For details of how to use them, we refer to Examples and Boxes 2.1-2.5 in the supplementary material, and to the built-in help in MATLAB. The teacher will also give a short introduction at the beginning of the laboratory exercise.

Elementary matrices and matrix manipulation (elmat)
linspace – Linearly spaced vector.

Interpolation and polynomials (polyfun)
roots – Find polynomial roots.

Data analysis and Fourier transforms (datafun)
max – Largest component.
sum – Sum of elements.

Elementary math functions (elfun)
abs – Absolute value.
angle – Phase angle.
log10 – Common (base 10) logarithm.
tan – Tangent.

Handle graphics (graphics)
figure – Create figure window.
gca – Get handle to current axes.
set – Set object properties.

Two dimensional graphs (graph2d)
axes – Create axes in arbitrary positions.
axis – Control axis scaling and appearance.
grid – Grid lines.
hold – Hold current graph.
plot – Linear plot.
subplot – Create axes in tiled positions.
title – Graph title.
xlabel – X-axis label.
ylabel – Y-axis label.
zoom – Zoom in and out on a 2-D plot.

M-files in MATLAB's Signal Processing Toolbox.

- `bilinear` – Bilinear transformation with optional prewarping.
- `buttap` – Butterworth analog lowpass filter prototype.
- `buttord` – Butterworth filter order selection.
- `cheb1ap` – Chebyshev type I analog lowpass filter prototype.
- `cheb1ord` – Chebyshev type I filter order selection.
- `cheb2ap` – Chebyshev type II analog lowpass filter prototype.
- `cheb2ord` – Chebyshev type II filter order selection.
- `ellipap` – Elliptic analog lowpass filter prototype.
- `ellipord` – Elliptic filter order selection.
- `freqz` – Digital filter frequency response.
- `freqs` – Laplace transform frequency response.
- `grpdelay` – Group delay of a digital filter.
- `zplane` – Z-plane zero-pole plot.
- `zp2tf` – Zero-pole to transfer function conversion.

Additional M-files (available when downloading the Lab2 map)

- `groupdelay` – Group delay of an analog filter.
- `zp2hp` – Transforms zeros and poles from analog lowpass to highpass
- `zp2bp` – Transforms zeros and poles from analog lowpass to bandpass
- `zp2bs` – Transforms zeros and poles from analog lowpass to bandstop

2.1 Consider three lowpass filters with the following specifications.

Filter 1: $\omega_c T = 0.3\pi$, $\omega_s T = 0.5\pi$, $A_{\max} = 0.1$ dB, $A_{\min} = 60$ dB

Filter 2: $\omega_c T = 0.3\pi$, $\omega_s T = 0.5\pi$, $A_{\max} = 0.01$ dB, $A_{\min} = 80$ dB

Filter 3: $\omega_c T = 0.45\pi$, $\omega_s T = 0.5\pi$, $A_{\max} = 0.1$ dB, $A_{\min} = 60$ dB

a) Determine the required filter orders for the three different cases using Butterworth, Chebyshev I, Chebyshev II, and Cauer filters. Compare the filters with respect to the filter orders. How do the passband and stopband attenuations and transition band affect the filter orders?

b) Synthesize Filter 1 using Butterworth, Chebyshev I, Chebyshev II, and Cauer filters. Study the magnitude responses and zero and pole locations for the different digital filters as well as for the analog prototype filters. What is typical for the magnitude responses? How are the zeros and poles typically located and mapped from the s-plane to the z-plane? Study the group delay responses for the digital filters. Compare the filters with respect to the delay and group delay variation in the passband of the filters.

c) Suggest a way to reduce the group delay variation in the passband. Which of the four standard filters is then likely to have the lowest order for given requirements on both the magnitude response and group delay variation in the passband?

2.2 Synthesize highpass, bandpass, and bandstop filters, satisfying the requirements below, using Cauer filters. Study the magnitude responses and zero and pole locations for the digital filters as well as for the analog prototype filters, including the lowpass prototype filters. What is typical for the magnitude responses? How are the zeros and poles typically located and mapped from the S-plane (of lowpass prototype) to the s-plane and from the s-plane to the z-plane? What is the drawback of synthesizing bandpass and bandstop filters through transformation of a lowpass prototype filter?

Filter 1: Highpass filter with the specification

$$\omega_c T = 0.75\pi,\ \omega_s T = 0.5\pi,\ A_{\max} = 0.1 \text{ dB},\ A_{\min} = 60 \text{ dB}$$

Filter 2: Bandpass filter with the specification

$$\omega_{c1} T = 0.4\pi,\ \omega_{c2} T = 0.6\pi,\ \omega_{s1} T = 0.3\pi,\ \omega_{s2} T = 0.7\pi$$

$$A_{\max} = 0.1 \text{ dB},\ A_{\min} = 60 \text{ dB}$$

Filter 3: Bandstop filter with the specification

$$\omega_{c1} T = 0.3\pi,\ \omega_{c2} T = 0.7\pi,\ \omega_{s1} T = 0.4\pi,\ \omega_{s2} T = 0.6\pi$$

$$A_{\max} = 0.1 \text{ dB},\ A_{\min} = 60 \text{ dB}$$