Laboratory Exercise 1, Digital Filters 2013

The purpose of this laboratory exercise is to give some insights into the design and properties of linear-phase FIR filters. The MATLAB commands and programs that are required for the exercise are listed below. (Some of them are not actually needed, but might be useful.) For details of how to use them, see Examples 1.1 and 1.2 and Boxes 1.1, 1.2, and 2.5 (lines 21-46) in the supplementary material and the built-in help in MATLAB. The teacher will also give a short introduction at the beginning of the laboratory exercise.

Preparations to do **before the lab**: Exercises 1.1a&d, 1.2a, 1.3a&b.

Elementary matrices and matrix manipulation (elmat)
- linspace – Linearly spaced vector.

Interpolation and polynomials (polyfun)
- roots – Find polynomial roots.

Data analysis and Fourier transforms (datafun)
- max – Largest component.
- sum – Sum of elements.

Elementary math functions (elfun)
- abs – Absolute value.
- angle – Phase angle.
- log10 – Common (base 10) logarithm.

Handle graphics (graphics)
- figure – Create figure window.
- gca – Get handle to current axes.
- set – Set object properties.

Two dimensional graphs (graph2d)
- axes – Create axes in arbitrary positions.
- axis – Control axis scaling and appearance.
- grid – Grid lines.
- hold – Hold current graph.
- plot – Linear plot.
- subplot – Create axes in tiled positions.
- title – Graph title.
- xlabel – X-axis label.
- ylabel – Y-axis label.
- zoom – Zoom in and out on a 2-D plot.

M-files in MATLAB's Signal Processing Toolbox.
- `freqz` – Digital filter frequency response.
- `firpmord` – FIR order estimator.
- `firpm` – Parks-McClellan optimal equiripple FIR filter design.
- `zplane` – Z-plane zero-pole plot.

Additional M-files (available when downloading the Lab1 map)
- `Ex13cLP` – Designs filter in Exercise 1.3c (linear programming)
- `Ex13cQP` – Designs filter in Exercise 1.3c (quadratic programming)

1.1 Consider four lowpass filters with the following specifications.

$$1 - \delta_c \le |H(e^{j\omega T})| \le 1 + \delta_c, \quad \omega T \in [0, \omega_c T]$$
$$|H(e^{j\omega T})| \le \delta_s, \quad \omega T \in [\omega_s T, \pi]$$

Filter 1: $\omega_c T = 0.3\pi$, $\omega_s T = 0.4\pi$, $\delta_c = 0.01, \delta_s = 0.001$
Filter 2: $\omega_c T = 0.3\pi$, $\omega_s T = 0.4\pi$, $\delta_c = 0.001, \delta_s = 0.0001$
Filter 3: $\omega_c T = 0.39\pi$, $\omega_s T = 0.4\pi$, $\delta_c = 0.01, \delta_s = 0.001$
Filter 4: $\omega_c T = 0.69\pi$, $\omega_s T = 0.7\pi$, $\delta_c = 0.01, \delta_s = 0.001$

a) Which ones of the linear-phase filter types (I, II, III, or IV) can be used to realize the filters? Explain why.
b) Use `firpmord` to estimate the required filter orders for the four different filters. How do the passband and stopband edges, passband and stopband ripples, and transition band affect the filter order?
c) Use `firpm` to synthesize Filter 1. Does the filter satisfy the requirements? If not, explain why. What is the minimum order for a filter that satisfies the requirements?
d) Study the magnitude response of, e.g., the minimum-order filter in c). What is typical for filters optimized in the minimax sense?

1.2 Use `firpmord` and `firpm` to synthesize lowpass, highpass, bandpass, and bandstop filters, satisfying the requirements below.

a) Which ones of the linear-phase filter types (I, II, III, or IV) can be used to realize the different filters? Explain why.
b) Synthesize the filters using either Type I or II. What are the minimum orders for the filters that satisfy the requirements?
c) Study the magnitude responses and the corresponding zero locations for the different filters. How are the zeros typically located?

Filter 1) Lowpass filter with the specification

$$1 - \delta_c \le |H(e^{j\omega T})| \le 1 + \delta_c, \quad \omega T \in [0, \omega_c T]$$
$$|H(e^{j\omega T})| \le \delta_s, \quad \omega T \in [\omega_s T, \pi]$$

where $\omega_c T = 0.4\pi$, $\omega_s T = 0.5\pi$, $\delta_c = 0.01$, $\delta_s = 0.001$.

Filter 2) Highpass filter with the specification

$$1 - \delta_c \le |H(e^{j\omega T})| \le 1 + \delta_c, \quad \omega T \in [\omega_c T, \pi]$$
$$|H(e^{j\omega T})| \le \delta_s, \quad \omega T \in [0, \omega_s T]$$

where $\omega_c T = 0.6\pi$, $\omega_s T = 0.5\pi$, $\delta_c = 0.01$, $\delta_s = 0.001$.

Filter 3) Bandpass filter with the specification

$$1 - \delta_c \le |H(e^{j\omega T})| \le 1 + \delta_c, \quad \omega T \in [\omega_{c1}T, \omega_{c2}T]$$
$$|H(e^{j\omega T})| \le \delta_s, \quad \omega T \in [0, \omega_{s1}T]$$
$$|H(e^{j\omega T})| \le \delta_s, \quad \omega T \in [\omega_{s2}T, \pi]$$

where $\omega_{c1}T = 0.4\pi$, $\omega_{c2}T = 0.6\pi$, $\omega_{s1}T = 0.3\pi$, $\omega_{s2}T = 0.7\pi$, $\delta_c = 0.01$, $\delta_s = 0.001$.

Filter 4) Bandstop filter with the specification

$$1 - \delta_c \le |H(e^{j\omega T})| \le 1 + \delta_c, \quad \omega T \in [0, \omega_{c1}T]$$
$$1 - \delta_c \le |H(e^{j\omega T})| \le 1 + \delta_c, \quad \omega T \in [\omega_{c2}T, \pi]$$
$$|H(e^{j\omega T})| \le \delta_s, \quad \omega T \in [\omega_{s1}T, \omega_{s2}T]$$

where $\omega_{c1}T = 0.3\pi$, $\omega_{c2}T = 0.7\pi$, $\omega_{s1}T = 0.4\pi$, $\omega_{s2}T = 0.6\pi$, $\delta_c = 0.01$, $\delta_s = 0.001$.

1.3 Consider a lowpass filter with the following specification in the passband:

$$1 - \delta_c \le |H(e^{j\omega T})| \le 1 + \delta_c, \quad \omega T \in [0, \omega_c T]$$

where $\omega_c T = 0.3\pi$, and $\delta_c = 0.02$. Further, the filter has the stopband $\omega T \in [\omega_s T, \pi]$ where $\omega_s T = 0.4\pi$. Two different 50th-order filters are optimized according to the following:

Filter 1: Minimize the maximum of the magnitude response in the stopband subject to the constraints in the passband.

Filter 2: Minimize the energy in the stopband subject to the constraints in the passband.

a) State the optimization problem for Filter 1 as a linear programming problem (see Section 5.10 in the course book).
b) State the optimization problem for Filter 2 as a quadratic programming problem (see Section 5.11 in the course book).
c) Use the programs `Ex13cLP` and `Ex13cQP` to synthesize the two different filters. Compare the filters with respect to the magnitude responses, maximum of the magnitude response in the stopband, and energy in the stopband. What conclusions can be drawn?