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Acoustic Metrology

Chapter 5

Modeling sound propagation in porous media

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# Microscopic and macroscopic description of sound propagation in porous media

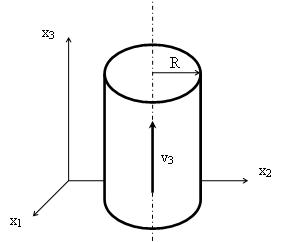
The quantities that are involved in sound propagation can be defined locally, on a microscopic scale, for instance in a porous material with cylindrical pores having a circular cross-section, as functions of the distance to the axis of the pores. On a microscopic scale, sound propagation in porous materials is generally difficult to study because of the complicated geometries of the frames. Only the mean values of the quantities involved are of practical interest. The averaging must be performed on a macroscopic scale, on a homogenization volume with dimensions sufficiently large for the average to be significant. At the same time, these dimensions must be much smaller than the acoustic wavelength (incompressibility condition). The description of sound propagation in porous material can be complicated by the fact that sound also excites and moves the frame of the material. If the frame is motionless, in a first step, the air inside the porous medium can be replaced on the macroscopic scale by an equivalent free fluid. This equivalent fluid has a complex effective density  and a complex bulk modulus . The wave number  and the characteristic impedance  of the equivalent fluid are also complex. In a second step, the porous layer can be replaced by a fluid layer of density  and of bulk modulus .

# Sound propagation in cylindrical tubes

The geometry of the pores in ordinary porous materials is not simple and a direct calculation of the viscous and thermal interaction between the air and these materials is generally impossible to perform. Even though, useful information can be obtained from the simple case of porous materials with cylindrical pores.

## Viscous effects

Let’s write the linearized Navier-Stokes equation for the motion of incompressible fluid in a single cylindrical pore subject to a constant pressure gradient:



1. Schematic representation of a cylindrical pore.



where  , , and are the spatial coordinates,  is acoustic pressure,  and  are dynamic viscosity a density of air, respectively. An unidirectional plane wave with particle velocity  has been taken into account to solve this problem. The above equation can be rewritten in cylindrical coordinate as follows



Applying boundary conditions of no slip, , at the pore’s wall, where  is the pore radius, then, the solution for  can be found:



where  is the Bessel function of first kind of order zero.

Integrating the above velocity over the cross section, the pore averaged velocity is obtained



Applying the following identity



where  is the Bessel function of first kind of order one, the following equation can be found



which, in more compact form, can be rewritten as



where



is the complex density of the equivalent fluid. This is a complex quantity and is taking into account the dissipative viscous effects introduced by the presence of the pores.

## Zwikker-Kosten and Stinson models

One of the first simplified models for describing sound propagation in porous material was proposed by Zwikker and Kosten in 1949. They understood that, compared to the free air case, the velocity in the porous medium, first changes because of the reduced volume of air and then along the pores it changes from point to point due to their irregularity. Therefore, denoting the porosity of the medium with , for a given variation of pressure , a  times smaller velocity gradient  will be found than in free air. Hence they deduced that the equation of continuity may be written as:



whereas the equation of motion for an enclosed air, is given by



In this equation three new parameters are introduced:

1) structure constant ;

2) porosity ;

3) static flow resistivity .

The presence of the term  in the equation of motion (momentum conservation equation) is accounting for viscous friction.

In the case of steady flow, the term with , cancels out and therefore the parameter  can be defined as the ratio of pressure gradient and flow velocity.

The presence of the structure constant takes into account the pore geometry such as its slope with respect to the wave propagation direction and the shape.

Later on, a method, which relates thermal and viscous effects, was proposed by Stinson in 1991.

Thermal exchanges with the frame, which during sound propagation remains isothermal, modify the bulk modulus of the air in the pore.

The linearized entropic equation, that describes the thermal conduction in the air, is given by



Where  and  are the acoustic temperature and density, ,  and , are the air density, ambient mean temperature and mean pressure. Finally, ,  and  are thermal conductivity, specific heats per unit mass at constant volume and constant pressure, respectively.

For an ideal gas, the following relationship is valid:



and the state equation reads



where  is the universal constant of the gas.

Substituting those equations into the entropic equation the following expression is obtained:



Because the variation of  in the  direction (parallel to the cylinder axis) is smaller than in the radial directions  and , the previous equation becomes



where  (specific heat ratio) and .

Finally, defining , where  is the Prandtl number, one can write



Comparing the above equation with the Navier-Stokes equation derived at the beginning of this section, it is easy to find the similarity and recognise that  is equal to



whereas  in the momentum conservation equation is



with  is the solution of the equation



The bulk modulus is given by



where the mean density  is obtained from the ideal gas law and it is equal to



In this equation  is temperature distribution averaged over a pore cross section.

Therefore, substituting this last equation into the expression of the bulk modulus, the following equation is obtained



where



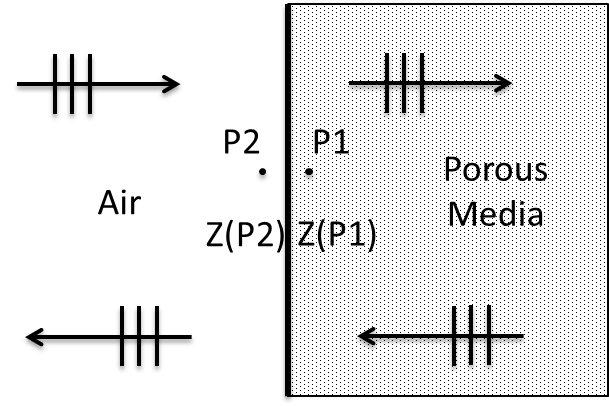
That leads to the following expression for the dynamic bulk modulus of air in a pore



Like the dynamic density, this quantity is complex because it takes into account dissipative effects due to heat exchange between the rigid frame and air in pores.

## Equivalence between fluid layer and a porous layer

Let’s consider a layer of porous material, made out of straight cylindrical pores, placed on a perfectly rigid surface and in contact with air in which a normal plane acoustic field is present. The acoustic field in the air is plane up to a small distance (smaller than the distance between two pores) from the material and identical waves propagate in each pore. Let’s choose two points at the surface of the material, *P1* in the porous material and *P2* in free air as shown in Figure 2.



1. Interface between air and a porous media.

Let’s call , and ,  mean velocity and pressure in a pore and in the free air respectively. The continuity of air flow and pressure at the surface of the porous material imply





Therefore two impedances can be defined at the surface





The wave equation in a pore is



where  and  have been defined previously and  is the complex speed of sound (this is different from phase speed defined in the previous notes: ). Therefore, characteristic impedance  and the complex wavenumber  in a pore can be calculated by the following equations





Surface impedances may be calculated by





where in both equations *d* is the thickness of the material.

Important result of this discussion is that surface impedance at normal incidence of a layer of isotropic porous medium is identical to the impedance of a layer of isotropic fluid with the same thickness if its wavenumber  is the same as the porous material, e.g. , and its characteristic impedance is  times smaller, e.g. . These conditions are satisfied if the density and bulk modulus of the fluid are given by





Determining  and  allows a full characterization of a porous media.

## Porosity, tortuosity and flow resistivity of an ideal porous media with N straight identical cylindrical pores

**Porosity**

Porosity can be estimated using its definition. In fact assuming that the porous material has axial symmetry (cylinder) or two plane of symmetry (parallelepiped) with cross section  and length then:



where  is pores surface density (number of pores per unit surface) and  is their radius

**Tortuosity**

Tortuosity for straight cylindrical pores is 1.

**Flow resistivity of a material with straight cylindrical pores**

Suppose that the porous material has  cylindrical pores with radius . According to the above equation, flow resistivity is given by



where  is mean velocity in a single pore. This equation can be derived as follows. Suppose that the sample cross-section is  and that the single pore cross-section is . Using Bernoulli equation one can write



where  is particle velocity in the *n*th-pore. Because pores are assumed to be all equal then  and  so that . Therefore .

From the Newton equation for stationary flow () in a cylindrical pore,  is equal to



and therefore flow resistivity is given by



## Example

In Figure 3 and Figure 4 are shown characteristic impedance, phase speed and attenuation of an ideal porous media made out of straight identical cylindrical pores with radius  and porosity .



1. Complex characteristic impedance ratio of an ideal medium with straight cylindrical pores. Top curve is the real part and bottom curve is the imaginary part.



1. Phase speed and attenuation coefficient of an ideal medium with straight cylindrical pores.

# Sound propagation in rigid porous materials: equivalent fluid model

Johnson, Koplik and Dashen derived, in 1987, the physically correct solutions for a Newtonian fluid saturating the pore space of a rigid isotropic porous medium subjected to an infinitesimal oscillatory pressure gradient.

They considered a homogeneous, isotropic and infinitely rigid porous solid with porosity , saturated by incompressible Newtonian fluid of density  and viscosity . They assumed that the property of the fluid is unaffected by its proximity of the walls of the solid and that the fluid has a negligible thermal expansion coefficient on any scale ensuring that pressure-density variations are decoupled from temperature variations. Then, the linear response of the fluid to a macroscopic pressure gradient  applied to the sample was evaluated.

Hypothesis on the applied pressure was related to the wavelength of the sound, it was imposed to be much larger than the characteristic size of the pores, ensuring that the fluid may be considered incompressible on the scale of the pore sizes. This hypothesis has been confirmed to be correct by the homogenization method for periodic structures: under the long-wavelength condition, the fluid inside a pore can be considered incompressible (see homogenization theory for more details).

The solution was expressed in terms of macroscopically averaged fluid velocity . The equations used were:

the Euler’s equation (momentum conservation equation)



and the so called generalized Darcy’s law



where the quantities  and  are the complex tortuosity and the complex permeability related to each other by the following relationship



which has been derived combining Euler’s equation and Darcy’s law.

Complex tortuosity is related to the complex density , mentioned in the previous paragraph, by .

In the case of low frequency limit, the complex permeability assumes its real d.c. value  conventionally measured applying a static drop pressure to the ends of the sample. This measurement is well known as flow resistivity measurement. The static flow resistivity  is related to the d.c. permeability as follows



And therefore the complex tortuosity approaches to the imaginary value



In the case of high frequency limit, the viscous skin depth defined as



becomes much smaller than any characteristic pore size. Then except for a boundary layer of thickness , the fluid motion will be given by a potential flow .

In finding this solution, they introduced a new parameter Λ called viscous characteristic length, defined as



where *V* is the volume of the pores and *S* is wall surface of the pores.

The complex tortuosity and permeability limits found were as follows



and



where  is tortuosity of the medium.

The complete function for the complex tortuosity proposed in that work was:



or



where  .

This is just one of the possible functions that can satisfy both limits and the causality property. It was chosen for its simplicity and because it provides good predictions in a wide frequency range for a wide variety of materials.

Later on, in 1993, Pride, Morgan and Gangi improved this model finding a more accurate low frequency limit for the complex tortuosity function. In fact the complex tortuosity function proposed by Johnson *et al.*, was the simplest expression for satisfying the high frequency limit and the low frequency limit of the imaginary part only. Pride *et al.* modified expression of complex tortuosity so that it can satisfy also the low frequency limit of the real part.

As result the following expression for dynamic tortuosity  was derived:



where  is defined as



And  is the solution of the static flow problem and  is the average operator over the fluid phase defined in previous equations.

## Allard, Champoux and Lafarge model

The model proposed by Johnson was based on solving the viscous problem only whereas the losses due to heat transfer between the frame and the air were not accounted for.

In 1991, Champoux and Allard and in 1997 Lafarge *et al*, approached the thermal problem and formulated the expression for the complex compressibility  and the relationship between this function and the complex tortuosity .

Complex compressibility is defined as the inverse of the bulk modulus , . However, Champoux and Allard introduced a dimensionless complex compressibility function in order to account for temperature effects.

Frequency dependence of the complex compressibility was derived from the expression of the dynamic tortuosity using similarity between these functions found by Stinson in 1991.

A new parameter called thermal characteristic length  was introduced in order to evaluate the exact high-frequency limit of the new complex function.

This parameter  was found to be twice the ratio between the pore volume *V* and the pore surface *S* and it was evaluated using the following expression



This definition was inspired by the definition of the characteristic viscous length  given by Johnson *et al* in 1987. At sufficiently high frequencies the thermal exchanges between air and frame mainly occur in a small layer close to the frame, where temperature depends on the local distance to the frame.

A normalized dynamic compressibility was so introduced



where is adiabatic bulk modulus of air and  and  are macroscopic acoustic pressure and density, respectively. The symbol  denotes the average operator over the fluid phase defined previously. When frequency increases,  tends to



where  is the thermal skin depth.

A thermal analogue of the dynamic viscous permeability was defined by the following equation



where  is macroscopic excess temperature in air (acoustic temperature). The quantity  has the same dimensions as the permeability . When the frame has a sufficient thermal capacity for the compressibility  to reach the isothermal value  at low frequencies, the excess temperature  can be considered to vanish at the pore walls (equivalent to the no-slip boundary condition for viscous flow) and a static ‘‘thermal permeability’’  exists.

Champoux, Allard and Lafarge modelled a complex thermal permeability function  in a similar manner as the permeability function . They found that expression of  at high frequency was given by



with parameters , , and porosity , wheras at low frequency was given by a real number .

Then they gave the relationship existing between  and 



Therefore complex compressibility was given by



which is the simplest function that satisfy both high and low frequency limit.

Complex bulk modulus is therefore given by



## Wilson’s model: the relaxation phenomena

Relaxation model developed by Wilson in 1993 is based on the fact that thermal and viscous diffusion process in the porous medium can be described by relaxational processes. With word relaxation it is meant the approach of a system to an equilibrium state after some perturbation has been introduced. Without losing generality, Wilson describes the relaxation phenomena in the case of thermal diffusion. He used the concept of “modal wave fields” developed in Pierce’s text. Fluctuation of pressure, density, velocity, entropy and temperature are considered to be the superposition of acoustic, vorticity and entropy “modes” (here “mode” differs from its usage in connection with eigenmode problems). So, for example the temperature decomposition is , where the acoustic mode is described by the familiar first-order equations of acoustics (the complete set of equations describing the various modes can be found in Pierce’s text).

Let’s consider a rigid frame porous medium subjected to an acoustic wave. The acoustic wave alternately heat and cool a fluid in the pores, resulting in heat conduction to and from the frame material. Supposing that the heat capacity of the frame is much larger than the saturating fluid, then the temperature in the pores attempts to obtain an equilibrium temperature equal to that of the frame. In other words, when a sudden jump in the acoustic temperature  is applied at the ends of a porous material, the temperature response in the pores , obeys the equation of heat conduction



At the pore boundaries the temperature fluctuation must be zero, so that the absolute temperature there is equal to that of the frame. This implies  at the boundaries. Given time to adjust, the average pore temperature  will also approach , cancelling out the initial jump in pore temperature. Unfortunately, the solutions to the diffusion equation are known only for very simple geometries such as straight cylinders, but there are a number of general properties that any solution must satisfy. At the initial stages of the heat diffusion, the creation of a thin boundary layer is determined by only two significant parameters that are the total perimeter length of the pore space  and the thermal diffusivity .

By dimensional analysis, the characteristic time for the diffusion process must be . From the beginning of the diffusion process, <<  , the thickness of the thermal boundary layer , along the walls of the pores, must grow according to .

Hence, the temperature response, averaged over the pore space, initially grows in proportion to the square root of time,



where  is the cross-sectional area.

When >>, an exponential approach of to  is expected. Therefore, the overall behaviour of  should be that it initially increases as , and then exponentially approaches unity. The function that has this behaviour suggested by Wilson was the error function



which is the step response of the system.

The main reasons for selecting the error function, as explained by Wilson in his paper in 1997, are that “it agrees extremely well with cases where exact solutions are known, and that it yields a particularly simple frequency-domain transfer function”.

In fact converting the step response, to an impulse response by differentiating with respect to time, the following function is obtained



where  is the Heaviside function.

The transfer function of this system follows by Fourier transformation of . The result is equal to



Hence, using the error function for the time domain step response is equivalent to using the above equation as the frequency-domain response.

Analogue derivation of the relaxation function in the case of velocity diffusion equation can be obtained by replacing the kinematic viscosity  with thermal diffusivity , the external force is a large-scale pressure gradient rather than the acoustic wave and the response of the medium is a fluid velocity rather than temperature.

Expressions of the complex density function  and the complex bulk modulus function  proposed in 1997 were





where

 and  are the relaxation functions with  and  characteristic times of the relaxational processes viscous and thermal diffusion, respectively.

Using the above set of equations, the following relationships, depending on four intrinsic parameters, e.g. , ,  and  are obtained:



and



where  is speed of sound in the air.

Relaxation times  and  can be evaluated by matching the low- and high-frequency limits of a fixed phenomenological model, i.e. Johnson *et al.* Allard-Champoux and Lafarge. The model proposed by Wilson in 1993 was intended to match the middle-frequency behaviour and not to fit the asymptotic behaviour at high and low frequencies. However, even if his model, proposed in 1997, cannot satisfy simultaneously both correct high and low frequency limits, proposed by Johnson *et al.*, it can be used for matching one limit per time.

The following set of relaxation times satisfy the low frequency limit of Johnson *et al.* Allard-Champoux and Lafarge





whereas the following set satisfy the high frequency limit of Johnson *et al.* Allard-Champoux and Lafarge





This restriction was overcome with a two relaxation time (two per each relaxation phenomenon) model proposed by Turo and Umnova in 2010.

## Meaning of low and high frequency limits for a porous media: characteristic frequency and shape factor.

Low and high frequency limits of complex density and bulk modulus are relative to a given porous media. This means that a given frequency can be considered low or high depending on the properties of the porous media under investigation. A porous media is acoustically excited at low frequency if viscous effects are dominating versus inertial ones. Viscous layer thickness is large compared with size of the pores and energy dissipation is purely viscous. On the other and when the frequency increases, viscous layer thickness, as well as the thermal one, decreases and when it becomes very small compared to the size of the pore viscous dissipation becomes negligible and inertial effect dominates the physics of the sound propagation. It is therefore clear that one can define a transition frequency (called characteristic or cut-off frequency), which is the boundary frequency between the two regimes: viscous and inertial regimes. From the thermal point of view one can similarly define an isothermal and adiabatic regime. In fact, if the temperature changes very slowly inside the pore due to an acoustic excitation at low frequency, the temperature of the air in the pore and the temperature of the wall of the pore are constantly in equilibrium and their temperature can be considered equal at all time, the process is isothermal. However, when the frequency increases, exchange of energy between air and wall of the pore becomes slower and slower until the point when the changing in temperature between air and wall of the pore happen so fast that there is no exchange of energy between the air in the pore and the wall of the pore, the process becomes adiabatic.

The characteristic (cut-off) frequencies that divide viscous-inertial and isothermal-adiabatic regimes are material dependent because strictly related to the size and shape of the pores.

From a dimensional analysis of the momentum and mass conservation equations it is possible to evaluate a viscous characteristic frequency equal to



that for straight cylindrical pores is equal to



In Figure 5, normalized complex density and complex bulk modulus have been plotted for a sample having identical straight cylindrical pores with radius . Characteristic frequency clearly divides the two regimes: viscous (where the complex density tends to be a purely imaginary quantity) and inertial (where the real component of the complex density tends to the tortuosity of the medium, ). Characteristic frequency also divides the isothermal regime (where the normalized complex bulk modulus tends to ) and the adiabatic regime (where the normalized complex bulk modulus tends to )



1. Normalized complex density and bulk modulus of an ideal sample with identical straight cylindrical pores with radius and porosity . Top plot: blue and green lines are real and imaginary parts of the normalized complex density, respectively. Dashed black line highlight the cut-off frequency of the sample.

Johnson *et al.* introduced the concept of shape factor as dimensionless number that somehow summarize physical characteristics of a porous media and was defined as



A material with identical straight cylindrical pores has , a packing of spheres has  whereas packing of grains with sharp edges has . The evaluation of this number helps to get an idea of the structure complexity of the porous media under investigation.

Using definition of , the characteristic angular frequency of a medium can be expressed as follows

.

## Cell-model and granular multi-layered media

In 2001 Umnova *et al.* proposed a model for modeling sound propagation in packing of spheres. This model allows predicting complex density and complex compressibility of a wide variety of granular materials with grains having regular shape, i.e. sand, soil and gravel. The model requires only porosity of the packing and the mean particle radius. Complete description of the model can be found in the following papers:

* O. Umnova, K. Attenborough, K. M. Li, “Cell model calculations of dynamic drag parameters in packings of spheres”, J. Acoust. Soc. Am., 107 (6), 3113-3119, (2000);
* O. Umnova, K. Attenborough, K. M. Li, “A Cell Model for the Acoustical Properties of Packings of Spheres”, Acta Acoust. united with Acustica, 87, 226-235, (2001).

However, implementation of this model can be found in the set of matlab functions provided during the class.

Cell model can be used for instance to predict the acoustic behavior of a granular multilayered media like the ground. The ground has a stratified nature. Ground can be modeled with a first layer of dirt than a layer of gravel and then a rigid rock layer. Dirt has very small granular particles (with a radius on the order of 0.1 mm) with regular shape (spherical) whereas gravel has particles with larger size (with radius that can vary between 1 to 10 mm) and with irregular shape. Even though particles of dirt and gravel are not perfectly spherical, the cell model can still offer a simple tool to predict acoustic properties of these materials. In Figure 6, a layer of ground is modeled with: a layer of 5 cm of dirt (particle radius ) placed on top of 5 cm of gravel (particle radius ) that is rigid backed by a layer of rock (rigid wall) [MULTI 1-2] and *vice versa* [MULTI 2-1].

Both layers have porosity  which is associated to a close packing of spheres (smallest porosity achievable for perfectly spherical particles).

When the dirt is on the top [MULTI 1-2], acoustic behavior of the ground is almost identical of a single layer of dirt. If the gravel is on the top [MULTI 2-1], instead, the ground offers a higher acoustic absorption.



1. Reflection and absorption coefficient estimated for a granular multilayer porous media. Single 1 is dirt (,,), Single 2 is gravel (,,), Multi 2-1 is gravel-dirt, Multi 2-1 is gravel-dirt.

However, if the dirt is dug and put back in place, increase of porosity can be achieved. Let’s assume that porosity of dirt changes from 0.36 to 0.5. In Figure 7 one can see that absorption of the ground [MULTI 1-2] is doubled respect to the previous one. However, this absorption looks also very similar to the [MULTI 2-1] observed in the previous configuration. So while a changing in absorption property of the ground can give us information about the status of the ground itself (dug or not) it can also give us information about the composition of the ground (dirt-gravel vs. gravel-dirt). Those two information should not be confused each other if they are going to be used as a detection tool for ground status or composition.



1. Reflection and absorption coefficient estimated for a granular multilayer porous media. Single 1 is dirt (,,), Single 2 is gravel (,,), Multi 2-1 is gravel-dirt, Multi 2-1 is gravel-dirt.

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