N-Queen Problem

Part 1:

1.1: Variables: columns → Q_1, Q_2, Q_3, Q_4

Domains: $D_{Q_1} = D_{Q_2} = D_{Q_3} = D_{Q_4} = \{1,2,3,4\}$

Relations:

↳ adjacent (supports) → $\{(1,3),(1,4),(2,4),(3,1),(4,1),(4,2)\}$

↳ skip1 (supports) → $\{(1,2),(1,4),(2,1),(2,3),(3,2),(3,4),(4,1),(4,3)\}$

↳ skip2 (supports) → $\{(1,2),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,2),(4,3)\}$

Constraints

↳ 12 : Q_1 and Q_2 with adjacent

↳ 13 : Q_1 and Q_3 with skip1

↳ 14 : Q_1 and Q_4 with skip2

↳ 23 : Q_2 and Q_3 with adjacent

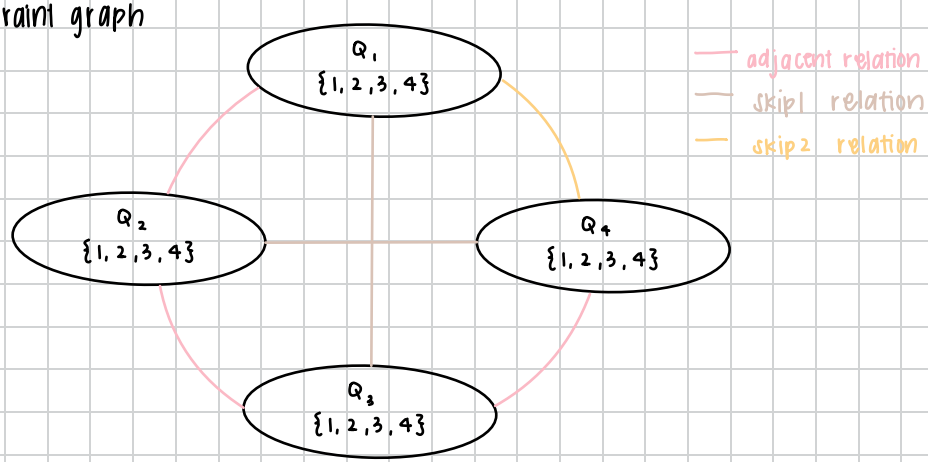
↳ 24 : Q_2 and Q_4 with skip1

↳ 34 : Q_3 and Q_4 with adjacent

1.2: C_{Q_i,Q_j} : values of Q_i and Q_j must be such that they are not in the same row or diagonal

1.3: Size of CSP: 256

1.4: Constraint graph



1.5: Manual arc-consistency was done by checking each relation to make sure each domain value has a tuple for each position since the relations are supports.
After checking, no values could be removed.

1.6: For Q_1, Q_2: all values of Q_3 and Q_4 are supported
For Q_1, Q_3: values 2, 3 are not supported for Q_2 → $D_{Q_2} = \{1, \cancel{2}, \cancel{3}, 4\}$
For Q_1, Q_4: all values of Q_2 and Q_3 are supported
For Q_2, Q_3: values 1, 4 are not supported for Q_4 → $D_{Q_4} = \{\cancel{1}, 2, 3, \cancel{4}\}$
For Q_2, Q_4: values 2, 3 are not supported for Q_3 → $D_{Q_3} = \{1, \cancel{2}, \cancel{3}, 4\}$
For Q_3, Q_4: all values of Q_1 and Q_2 are supported
For Q_1, Q_2: all values of Q_3 and Q_4 are supported
For Q_1, Q_4: all values of Q_2 and Q_3 are supported
For Q_1, Q_3: all values of Q_2 and Q_4 are supported
For Q_2, Q_3: values 1, 4 are not supported for Q_1 → $D_{Q_1} = \{\cancel{1}, 2, 3, \cancel{4}\}$

After applying (1,2)-consistency, 8 values are removed leaving each variable with new domain of size 2.

Part 2:

2.1: Number of variables : $2N - 1$

2.2: The domain of variables is $\{1, \ldots, k\} \cup \{0\}$ with k being the number of squares in in corresponding diagonal and 0 being no queens in the diagonal.

2.3: Size of CSP(4-queens): $2 \cdot 3 \cdot 4 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 2880$

2.4 (Bonus): $(N+1)!\,(N!)$