Bayes' Theorem

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1 Introduction

Statistics is a field of mathematics that has a high demand. Many industries and other types of research are seeking people who have a strong understanding and is capable of incorporating statistics into their specified fields. For the various presentations in this seminar, I have decided to talk about famous theorems used in statistics in order to develop a stronger understanding of the material. In the final presentation, I chose to study the Bayes' Theorem, which is significant in studying probability.

2 Abstract

Bayes' Theorem is a formula that describes the probability of a certain event given some initial conditions that happened. First discovered by Thomas Bayes, the theorem was not official until after several mathematicians researched and described the notion of this concept. The theorem is derived

from the basic ideas of conditional probability. In real-world applications, the theorem is consistently used to update predicted hypotheses as one finds more evidence or given outcomes. In this paper, I will discuss the definition, history, and applications of Bayes' Theorem.

3 Definition

We want to find the probability of an event B occurring given that event A occurred. The relationship between the two events are:

$$\Pr(B|A) = \frac{\Pr(A|B)\Pr(B)}{\Pr(A)} \tag{1}$$

This equation is derived by conditional probability. If we want to find event B given event A, we use the formula

$$\Pr(A|B) = \frac{\Pr(B \cap A)}{\Pr(B)} \tag{2}$$

Similarly, if given event B and want to find event A, we have the formula

$$\Pr(A|B) = \frac{\Pr(B \cap A)}{\Pr(A)} \tag{3}$$

Because (2) and (3) share the same numerator, the equations can be rearranged and substituted into another to get the result.

4 History

The discovery of Bayes' theorem started in 1740, Thomas Bayes was interested in finding a solution to the "probability of cause." He noticed that it was possible to calculate P(effect—cause), but questionable to calculate P(cause—effect). On this note, he ran an experiment using a square-surface table and balls. He randomly threw the first ball and took note of the location, and for every ball thrown after, he observed whether it landed on the right or left side of the first ball. This experiment did not continue 1763 until Richard Price discovered Bayes' works and continued the research. He publishes the idea in this paper "An Essay Toward Solving a Problem in the Doctrine of Chances." It was not until a mathematician named Pierre Simon LaPlace encounters the same problem while studying astronomy, where he formally expresses the theorem.

5 Applications

Bayes' Theorem is often used to assess the probability of a person on drugs. Given their drug test result, people want to know if they are actually on drugs. Let's provide a scenario:

-Suppose a drug kit is 0.98 accurate in detecting a person on drugs, and 0.9 accurate in detecting a person not on drugs. Let 0.15 of the population be on drugs. Using the formula we substitute with the variables:

$$\Pr(D|+) = \frac{\Pr(+|D)\Pr(D)}{\Pr(+|D)\Pr(D) + \Pr(+|n)\Pr(n)} \tag{4}$$

where D=person on drugs, n=person not on drugs, and + indicates a positive test. Substituting the numbers we have

$$\frac{(0.98)(0.15)}{(0.98)(0.15)+(0.1)(0.85)} = 0.63 \tag{5}$$

It is interesting to note that if we knew the person is on drugs, then the probability is 0.98, but by reversing the order and having a probability of 0.63, the status of the person becomes questionable.