

1 A PBW basis?

We will consider the Hopf algebra when $h = 0$ and $q \in \mathbb{R}$

$$\begin{aligned} [t_2^1, t_1^2] &= 0 \\ [t_1^1, t_2^2] &= (q - q^{-1})t_2^1 t_1^2 \\ [t_1^1, t_1^2]_q &= 0 \\ [t_1^1, t_2^1]_q &= 0 \end{aligned}$$

Here comes a senceful basis

$$\begin{aligned} &(t_2^1)^n (t_1^2)^m (t_3^3)^k (t_3^1)^{l_2} (t_3^2)^{l_3} & (t_2^1)^n (t_1^2)^m (t_2^2)^{l_1} (t_3^2)^{l_3} & (t_2^1)^n (t_1^2)^m (t_1^1)^{l_1} (t_3^1)^{l_3} \\ &(t_2^1)^n (t_1^2)^m (t_3^3)^k (t_3^1)^{l_2} (t_2^3)^{l_3} & (t_2^1)^n (t_1^2)^m (t_2^2)^{l_1} (t_3^3)^{l_3} & (t_2^1)^n (t_1^2)^m (t_1^1)^{l_1} (t_3^3)^{l_3} \\ &(t_2^1)^n (t_1^2)^m (t_1^1)^{l_1} (t_2^2)^{l_2} & (t_3^2)^n (t_2^3)^m (t_2^2)^{l_1} (t_3^3)^{l_2} & (t_1^3)^n (t_3^1)^m (t_3^3)^{l_1} (t_1^1)^{l_2} \end{aligned} \tag{1}$$