$$u = \sqrt{1+\sigma^2 \sin^2 2\theta}\, \sin\theta \cos\phi_1$$
$$v = \sqrt{1\pm\sigma^2 \sin^2 2\theta}\, \sin\theta \sin\phi_1$$
$$s = \sqrt{1\pm\sigma^2 \sin^2 2\theta}\, \cos\theta \cos\phi_2$$
$$t = \sqrt{1\pm\sigma^2 \sin^2 2\theta}\, \cos\theta \sin\phi_2$$

$$du^2 = \left(-\sqrt{1\pm\sigma^2\sin^2 2\theta}\, \sin\theta \sin\phi_1 d\phi_1 + \left(\frac{-\sigma^2 \sin 2\theta \cos 2\theta}{\sqrt{1\pm\sigma^2 \sin^2 2\theta}} \sin\theta + \sqrt{1\pm\sigma^2\sin^2 2\theta}\,\cos\theta \right) \cos\phi_1 d\theta \right)^2$$

$$dv^2 = \left(\sqrt{1\pm\sigma^2 \sin^2 2\theta}\, \sin\theta \cos\phi_1 d\phi_1 + \left(\quad - \quad '' \quad '' \quad - \quad \right) \sin\phi_1 d\theta \right)^2$$

$$ds^2 = \left(-\sqrt{1\pm\sigma^2\sin^2 2\theta}\, \cos\theta \sin\phi_2 d\phi_1 + \left(\frac{-\sigma^2 \sin 2\theta \cos 2\theta}{\sqrt{1\pm\sigma^2\sin^2 2\theta}} \cancel{\cos\theta} - \sqrt{1\pm\sigma^2 \sin^2 2\theta}\, \sin\theta \right) \cos\phi_2 d\theta \right)^2$$

$$dt^2 =$$

Now notice

$$du^2 + dv^2 + ds^2 + dt^2 = (1\pm\sigma^2\sin^2 2\theta)\sin^2\theta\, d\phi_1^2 + (1\pm\sigma^2 \sin^2 2\theta)\cos^2\theta\, d\phi_2^2$$
$$+ (1\pm\sigma^2\sin^2 2\theta)^2 d\theta^2$$
$$= d\theta^2 + \sin^2\theta\, d\phi_1^2 + \cos^2\theta\, d\phi_2^2 + \sigma^2 \sin^2 2\theta \left(d\theta^2 - \cancel{\sin^2}\theta\, d\phi_1^2 - \cos^2\theta\, d\phi_2 \right)$$
$$- 2\sigma^2 \sin^2 2\theta\, d\theta^2$$

Thus everything but last term is the sigma action

this means the action can be written as

$$\gamma^{\alpha\beta} \left(\partial_\alpha u \partial_\beta u + \partial_\alpha v \partial_\beta v + \partial_\alpha s \partial_\beta s + \partial_\alpha t \partial_\beta t + 2\sigma^2 \sin^2 2\theta\, \partial_\alpha\theta \partial_\beta\theta \right)$$

with

$$u^2 + v^2 + s^2 + t^2 = 1 \pm \sigma^2 \sin^2 2\theta$$

A deformed sphere ~~cost~~

$$\sin^2 2\theta\, \partial_\alpha\theta \partial_\beta\theta \sim \partial_\alpha(u^2+v^2)\partial_\beta(u^2+v^2) + \mathcal{O}(\sigma^2)$$

We check if we can write first part as

$$\mathrm{Tr}[g^{-1}\partial g\, g^{-1}\partial g]$$