Math 20–1 — Sequences and Series

Practice Test

(Finite/Infinite Sequences · Arithmetic/Geometric · Series Sums · Applications)

Instructions. Show work in the space beside each question. Calculators permitted unless instructed otherwise. For **Numerical Response**, print your answer in the boxes from left to right (no commas or units).

Multiple Choice (1–7)

1) An example of a **finite** sequence is

   A. 3, 8, 13, 18, ...

   B. 2, 5, 8, 11

   C. $a_n = 5n - 2,\ n \in \mathbb{N}$

   D. 1, 4, 7, 10, ..., 499, $n \in \mathbb{N}$

2) Which statement about the sequence 3, 6, 12, 24, ... is correct?

   A. The sequence is arithmetic.

   B. The sequence is geometric.

   C. The sequence is both arithmetic and geometric.

   D. The sequence is neither arithmetic nor geometric.

3) The common difference of the arithmetic sequence defined by

$$t_n = \frac{9 - 4n}{5}, \qquad n \in \mathbb{N},$$

   is

   A. $\frac{4}{5}$

   B. $\frac{9}{5}$

   C. $-\frac{9}{5}$

   D. $-\frac{4}{5}$

4) As part of a training plan, Amaya does burpees and push-ups. On day 1 she does 6 burpees and 12 push-ups. Each day she increases burpees by 3 and push-ups by 5. How many of each does she do on the **20th** day?

   A. 63 burpees and 107 push-ups

   B. 60 burpees and 102 push-ups

C. 66 burpees and 112 push-ups

D. 62 burpees and 100 push-ups

5) If the 12th and 13th terms of an arithmetic sequence are 81 and 75 respectively, then the 5th term is

A. 111

B. 117

C. 129

D. 123

6) A quantity of liquid contains 60 g of impurities. Each filtration removes 15% of the impurities present. After **six** filtrations, the mass of impurities remaining is *closest* to

A. 25.0 g

B. 22.6 g

C. 29.5 g

D. 32.8 g

7) Consider the following geometric series.
Series 1: $5 + 10 + 20 + \cdots$ Series 2: $40 + 20 + 10 + \cdots$ Series 3: $4 - 4 + 4 - 4 + \cdots$
Which of these series is/are **convergent**?

A. 1 only

B. 2 only

C. 1 and 2 only

D. 2 and 3 only

Numerical Response (8–10)

Record your answer in the boxes.

8) The first three terms of a geometric sequence are 32, 16, 8, ... The value of the **10th** term, to the nearest hundredth, is ☐☐☐☐

9) On a straight street, the house numbers on one side form an arithmetic sequence. If the first two houses are numbered 7341 and 7347 and the last house is 8835, then the number of houses on this side is ☐☐☐

10) The sum of the first n terms of a sequence is $S_n = 4n + n^2$ $(n \geq 1)$. The general term t_n is

(choose in MC 10 below).

Multiple Choice (11–15)

11) The general term in Question 10 is

A. $t_n = 2n + 3$

B. $t_n = 4n - 2$

C. $t_n = n^2 - 4$

D. $t_n = 2n - 1$

12) The sums of a geometric series satisfy $S_1 = 9$ and $S_2 = 15$. The common ratio is

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

13) A shrub is 1.5 ft high when planted. During the first year it grows 1.8 ft; each subsequent year it grows $\frac{3}{5}$ of the previous year's growth. The maximum height of the shrub is

A. 6.0 ft

B. 8.5 ft

C. 7.5 ft

D. cannot be determined

14) A ball returns to 60% of its previous height after each bounce. If dropped from 12 m, the maximum height after n bounces is

A. $h = 12(0.6)^{n-1} - 1$

B. $h = 12(0.4)^n$

C. $h = 12(0.6)^{n-1}$

D. $h = 12(0.6)^n$

15) The three expressions $3x + 12,\ 5x + 6,\ 7x$ are the first three terms of an arithmetic sequence when $x = 4$. The sum of the first four terms of this sequence is

A. 96

B. 100

C. 104

D. 108

Numerical Response (16–18)

Record your answer in the boxes.

16) A pile driver forces a column into the ground. The first strike drives it 1.2 m; each successive strike drives it 85% as far as the previous strike. What is the **least** number of strikes required to drive the column at least 7.5 m? ☐☐

17) In a geometric sequence, the third term is 900 and the sixth term is 28 800. The common ratio, to the nearest tenth, is ☐☐☐

18) The sides of a triangle form an arithmetic sequence. The shortest side is 12.5 cm and the perimeter is 54 cm. The length, in cm, of the **longest** side is ☐☐☐☐

Multiple Choice (19–20)

19) If an infinite geometric series has sum 84 and common ratio $\frac{2}{7}$, then its first term is

A. 56

B. 60

C. 63

D. 72

20) A golf ball is dropped from a height of 3 m and rebounds to $\frac{3}{5}$ of the previous height each time. The height of the ball after the **7th** rebound is

A. $3\left(\frac{3}{5}\right)^7$ m

B. $3\left(\frac{5}{3}\right)^7$ m

C. $3\left(\frac{3}{5}\right)^6$ m

D. $\left(\frac{3}{5}\right)^7$ m

Written Response — 5 marks

A theme-park ride swings like a pendulum. The first swing (left to right) is 60 m long. Each subsequent swing is 92% of the length of the previous swing.

1. Determine, to the nearest hundredth of a metre, the lengths of the second and third swings. (1 mark)

2. The ride is stopped at the end of the first swing whose length is less than 20 m. How many swings take place before the ride is stopped? (1 mark)

3. Determine, to the nearest tenth of a metre, the total distance travelled before the ride is stopped. (2 marks)

4. If the ride were not stopped and continued indefinitely, determine the total distance travelled. (1 mark)

Answer Key

1) B
2) B
3) D
4) A
5) D
6) B
7) B
8) $\boxed{0.06}$
9) $\boxed{250}$
10) (used in Q11)
11) A
12) B
13) D
14) $\boxed{18}$
15) $\boxed{3.2}$
16) $\boxed{23.5}$
19) B
20) A

Notes/Justification

- 3) $t_n = \frac{9}{5} - \frac{4}{5}n \Rightarrow d = -\frac{4}{5}$.
- 4) Day n: $6 + (n-1)3$ and $12 + (n-1)5$. At $n = 20$: 63 and 107.
- 5) $d = t_{13} - t_{12} = -6$; $a + 11d = 81 \Rightarrow a = 147$; $t_5 = a + 4d = 123$.
- 6) Remaining $= 60(0.85)^6 \approx 22.6$ g.
- 7) Only Series 2 has $|r| < 1$.
- 8) $t_{10} = 32\left(\frac{1}{2}\right)^9 = 0.0625 \Rightarrow 0.06$.
- 9) $d = 6$ from $7341 \to 7347$; solve $8835 = 7341 + (n-1)6 \Rightarrow n = 250$.
- 10/11) $t_n = S_n - S_{n-1} = (4n + n^2) - (n^2 + 2n - 3) = 2n + 3$.
- 12) From $S_1 = a = 9$, $S_2 = a + ar = 15 \Rightarrow r = 2/3$.
- 13) Total extra height $= \dfrac{1.8}{1 - 3/5} = 6$; max $= 1.5 + 6 = 7.5$ ft.
- 14) $S_n = 1.2\frac{1-0.85^n}{1-0.85} = 8(1 - 0.85^n) \geq 7.5 \Rightarrow n \geq 18$.
- 15) $\dfrac{t_6}{t_3} = r^3 = \dfrac{28\,800}{900} = 32 \Rightarrow r = \sqrt[3]{32} \approx 3.2$.
- 16) Let sides $12.5,\ 12.5 + d,\ 12.5 + 2d$; sum $54 \Rightarrow d = 5.5$; longest $= 23.5$ cm.
- 19) $S_\infty = \dfrac{a}{1 - r} = 84,\ r = \frac{2}{7} \Rightarrow a = 84(1 - \frac{2}{7}) = 60$.
- 20) First rebound height is $3(\frac{3}{5})$; the 7th rebound is $3(\frac{3}{5})^7$.
- WR:
  (1) $L_2 = 60(0.92) = \mathbf{55.20}$ m, $\quad L_3 = 60(0.92)^2 = \mathbf{50.78}$ m.
  (2) Find smallest n with $60(0.92)^{n-1} < 20 \ \Rightarrow\ n = \mathbf{14}$.
  (3) $S_{14} = 60\frac{1-0.92^{14}}{1-0.92} \approx \mathbf{517.5}$ m.
  (4) $S_\infty = \dfrac{60}{1 - 0.92} = \mathbf{750}$ m.