

Using Proportional Reasoning

A proportion is a relationship that says two ratios are equal.
A proportion can be expressed in fraction form.

$$\frac{1}{2} = \frac{7}{14}$$

$\times 7$ $\div 5$

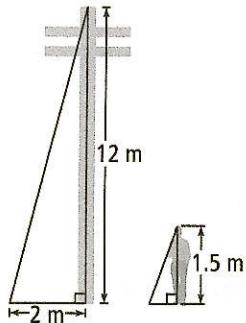
$$\frac{5 \text{ cm}}{45 \text{ cm}} = \frac{1 \text{ cm}}{9 \text{ cm}}$$

$\div 5$

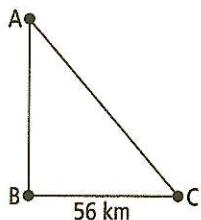
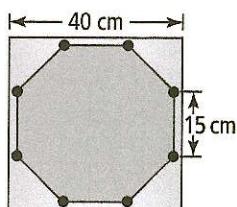


To compare items using a ratio, the units must be the same.

5. Set up a proportion for each situation.
- On a diagram of a machine part, 2 cm represents 200 cm. The actual length of the part is 100 cm. This distance is 1 cm on the diagram.
 - On a map, 1 cm represents 500 m. Linda wants to ride her bike 3500 m. This distance is 7 cm on the map.
 - A diagram of a hot tub shows the actual 3-m length of one side of the hot tub as 15 cm. The 8-m length of the square deck around the hot tub is shown as 40 cm.
6. A telephone pole that is 12 m tall casts a shadow that is 2 m long. What is the length of the shadow cast by a student who is 1.5 m tall?

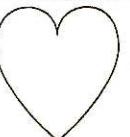
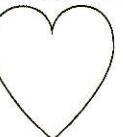
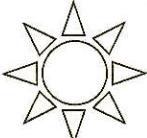
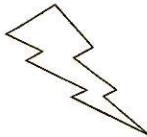


7. The distance between Town B and Town C is 56 km. The distance shown on the map is 7 cm in length. What is the actual distance between Town A and Town C if it is represented on the same map by a length of 12.5 cm?

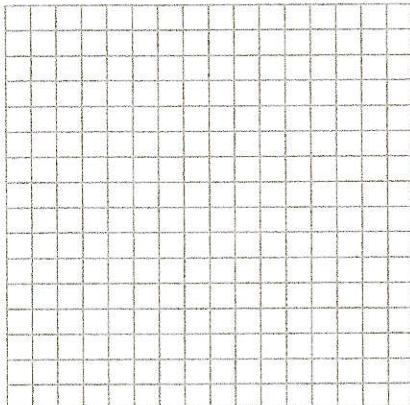


Date: _____

4. For each image in column A, state whether the image in column B has a scale factor
- greater than 1
 - less than 1
 - equal to 1

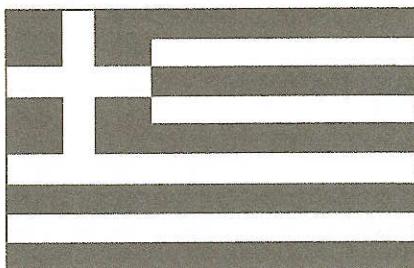
	A	B
a) _____		
b) _____		
c) _____		

5. a) Draw an enlargement of the butterfly using a scale factor of 4.



- b) Explain how you know that your drawing is correct.

6. Draw an image of the flag of Greece using a scale factor of $\frac{1}{4}$.



7. Alicia copied a headline from the school newspaper and included it on her election poster.



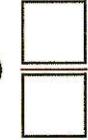
Alicia Wows Students With Campaign Promises

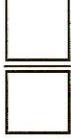
Vote for Alicia!

- a) Is the headline on the poster an enlargement or a reduction of the headline in the newspaper?

- b) What is the scale factor? How do you know?

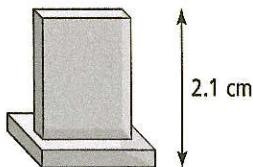
8. Determine the scale factor for each question below. Show your thinking.

a)  = $\frac{30}{225}$

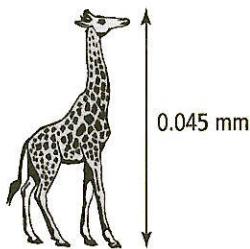
b)  = $\frac{3.8}{15.2}$

9. What scale factor is used to create each image below?

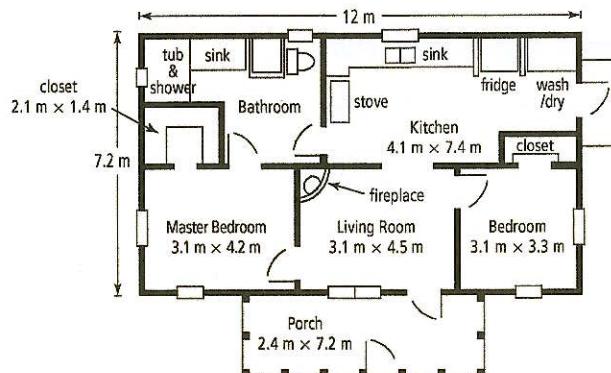
- a) The actual size of this award is 34.3 cm.



- b) The average height of a male giraffe is 6 m.



10. A blueprint is used to show all the measurements needed to build rooms in a house and the house itself.



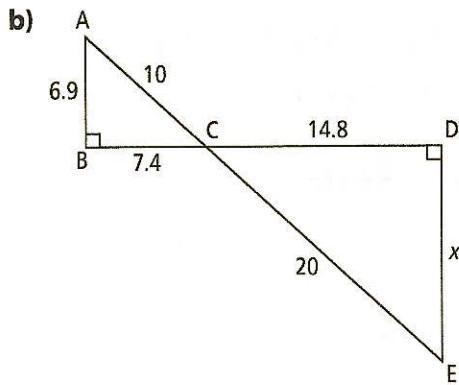
- a) What is the scale factor used to draw the blueprint? Express the denominator of the scale factor to the nearest whole number.

- b) Draw the master bedroom using a scale factor of 1:290. Express the calculations for the width and length of your drawing to the nearest tenth.

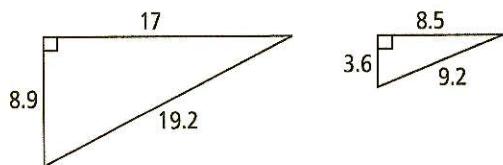
- c) What is the area of your drawing in part b)? Express your answer to the nearest hundredth.

5. Determine which pair of triangles is similar. Explain how you know.

Triangle	Angles	Sides
$\triangle PQR$	$\angle P = 90^\circ$ $\angle Q = 45^\circ$ $\angle R = 45^\circ$	$PQ = 3$ $QR = 4.2$ $PR = 3$
$\triangle STU$	$\angle S = 90^\circ$ $\angle T = 60^\circ$ $\angle U = 30^\circ$	$ST = 9.2$ $TU = 18.4$ $SV = 15.9$
$\triangle VWX$	$\angle V = 45^\circ$ $\angle W = 90^\circ$ $\angle X = 45^\circ$	$VW = 11.3$ $WX = 11.3$ $VX = 16$

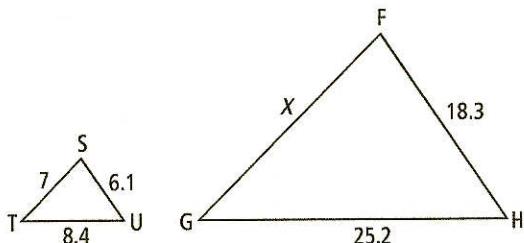


6. Are these triangles similar? Explain how you know.

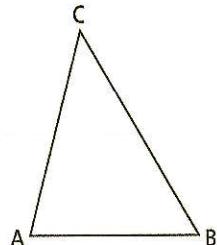


7. Determine the missing side lengths of the triangles below. Show your calculations.

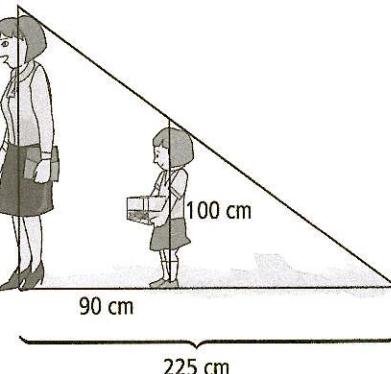
a)



8. Draw a triangle that is similar to the one shown. Label the measurements for the angles and sides on your triangle.

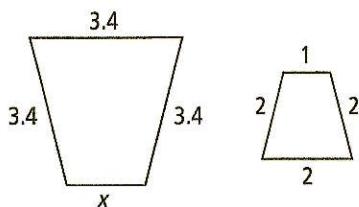


9. Kaylee is 100 cm tall and is standing so that her mother's shadow covers her shadow. She is 90 cm from her mother and her mother's shadow is 225 cm long. How tall is her mother? Express your answer to the nearest centimetre.

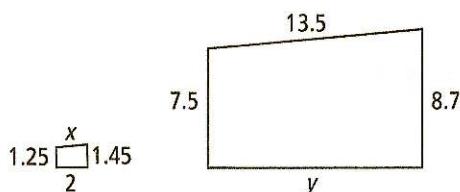


6. Use each pair of similar polygons to determine each unknown side length.

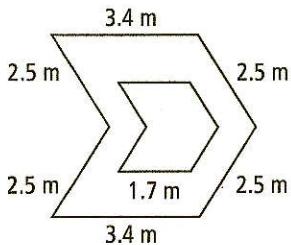
a)



b)



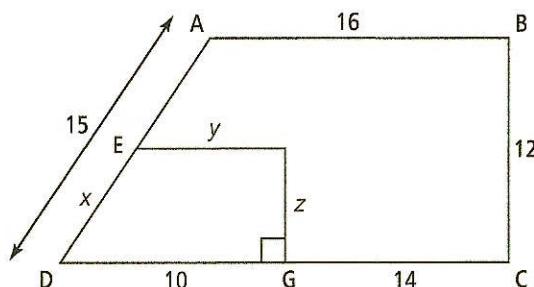
7. As part of an art project, Jamal made an outline of a shape with string. He wanted to create another shape inside the first one.



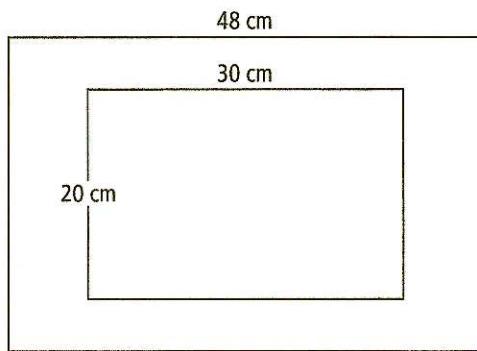
- a) Calculate the unknown side lengths of the inside shape if it is similar to the outside shape.

- b) What is the total length of string Jamal used for his art project?

8. Determine the value of the missing values to the nearest tenth. Show your thinking.



9. A pattern is cut showing the dimensions of a pair of similar trays. How much trim will you need to cover the outside edge of the larger tray? Justify your response.



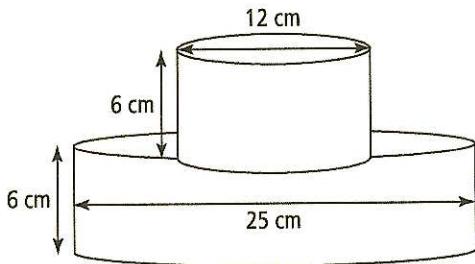
Vocabulary Link

Draw a line from the description in column A to the correct term in column B. Then, find each term in the word search.

A	B
1. compares quantities measured in the same units	a) corresponding
2. a two-dimensional, closed figure made of three or more line segments	b) enlargement
3. a decrease in the dimensions of an object by a constant factor	c) polygon
4. an adjective for figures that have the same shape but different sizes	d) proportion
5. a comparison between the actual size of an object and the size of its diagram	e) ratio
6. an angle or side with the same relative position in a geometric figure	f) reduction
7. an increase in the dimensions of an object by a constant factor	g) scale
8. the constant factor by which all dimensions of an object are enlarged or reduced in a scale drawing	h) scale diagram
9. a type of drawing that is similar to the actual figure or object	i) scale factor
10. a relationship that shows two ratios are equal	j) similar



9. Determine the surface area of the composite shape. Express your answer to the nearest hundredth.



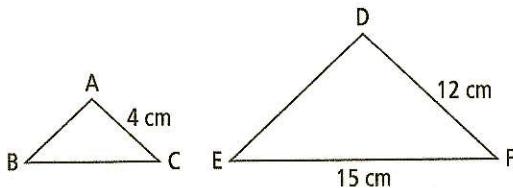
10. You bought some new bedroom furniture (f) for \$1500, including tax. The shop owner gave you the option of making no payments for 18 months from the date that you bought the furniture.

- a) If you are charged 16% interest, what would you owe at the end of the 18 months? The formula that represents this relationship is

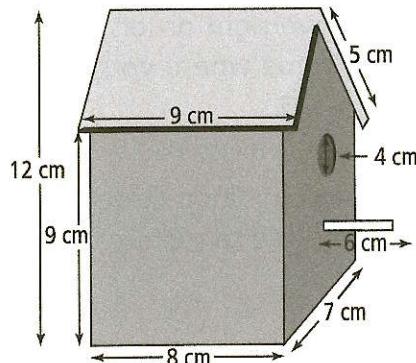
$$P = f \left(1 + \frac{0.08}{9}\right)^{18}.$$

- b) How much extra would you end up paying if you chose this option? Show your thinking.

11. In the following diagram, $\triangle ABC$ is similar to $\triangle DEF$. What is the length of BC?



12. Chloe is going to paint a bird house with the measurements shown. The cylindrical perch has a diameter of 1 cm.



How might you use symmetry to help Chloe determine the outside surface area of the bird house?

13. The label on a 1-L can of paint states that the paint will cover an area of 6 m^2 .

- a) What is the side length of the largest square area that the paint will cover? Express your answer to the nearest hundredth.

- b) If you have 4.75 L of the same paint, what is the side length of the largest square area that you could now cover?

Using Expressions

The expression $3w + 2$ consists of:

- a numerical coefficient, 3
- a variable, w
- a constant, +2

An expression can be thought of as a shorthand way of writing a word statement. For example, consider the word statement, "The length of a particular rectangle is two units more than triple its width." You could represent the rectangle's length with the expression $3w + 2$, where the variable w is its width.

5. For each expression, identify the numerical coefficient (NC), the variable (V), and the constant (C).

a) $2x - 7$ b) $-3b + 5$

c) $t - 4$ d) $3 - 6r$

6. Write an expression for each phrase. State what each variable represents.

a) Sarah is 5 years younger than her sister.

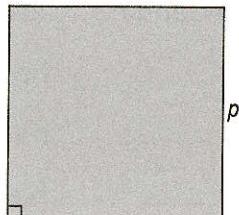
b) The width of the rectangle is 3 cm less than twice its length.

c) The perimeter of a triangle is increased by 14 cm.

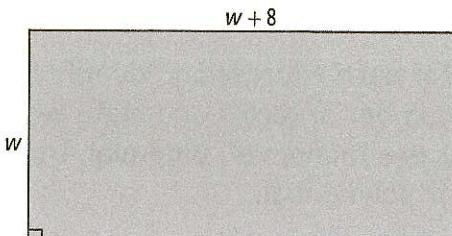
d) The school sold half of the concert tickets it expected to sell.

7. Use the information on each diagram to answer the questions below.

- a) What is the perimeter of the square?



- b) Write a word statement describing the length of the rectangle in terms of its width?



6. Refer to the following polynomials to answer the questions below.

$$4c^2 - 3c + 2$$

$$2f - 4$$

$$5p^2 - r$$

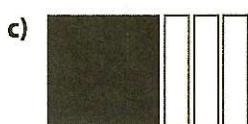
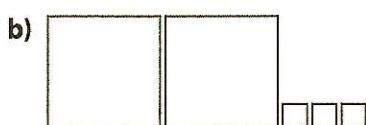
$$4ab$$

$$-12$$

$$g + h + j$$

Which of the above polynomials

- a) are trinomials?
 - b) have a degree of 2?
 - c) have a degree of 0?
 - d) are monomials?
 - e) have a coefficient of 4?
7. Write the expression represented by each set of algebra tiles. Shaded tiles are positive and white tiles are negative.



8. Sketch a model that represents the polynomial.

a) $x^2 + 3x - 2$

b) $-x^2 - 2x + 1$

9. Write an algebraic expression for each of the following:

a) the sum of 7 and x^2

b) the difference of $3x$ and 9

c) the product of x and 4

10. Use the given variables to write each statement as an algebraic expression.

- a) If n is a number, the product of the number and 5

- b) If w is the width of a rectangle and its length is 5 cm more than its width, the area of rectangle

- c) If x is the number of kilometres, the cost of renting a car, in dollars, if the charge is \$40 plus \$0.80 per kilometre

6. Circle the like terms in each group.

a) $14 \quad 3r \quad -r^2 \quad -r \quad 3s$

b) $-4y \quad 8xy \quad 2x \quad 0.3y \quad \frac{y}{2}$

c) $12c \quad cd \quad 1.2d \quad 6cd \quad cd^2$

7. Rearrange the polynomial by grouping like terms.

a) $9 - 5c - 8 + 5c^2 + c - c^2$

b) $8m - 9 + 2m^2 + 6 + 3m^2 - 6m$

c) $-5d^2 + 3d - 2 + 6d^2 - 8d + 7$

8. Rearrange each polynomial by grouping like terms. Then, simplify by adding or subtracting.

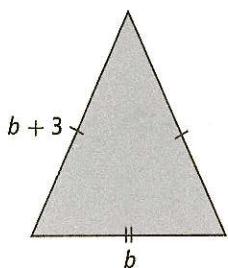
a) $-b^2 + 6 + 5b^2 - 8 + 9$

b) $7t + 14 + 6t - 5 - 3t^2 + 4t^2$

c) $5n - 3n^2 - 7 + 9n + 3 - 2n^2$

d) $3y^2 + 4 - 6y^2 - 6 + 3y - 5 + 2y$

9. Write and simplify an expression for the perimeter of the triangle by combining like terms.



10. a) Draw a figure with a perimeter that is represented by

$$(s) + (2s) + (s + 5) + (3s),$$

where each value in parentheses represents the length of one side.

Label each side length. Explain why you made each side the length that you did.

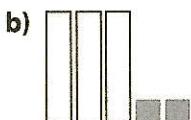
b) Simplify the expression for the perimeter by combining like terms.

11. A mechanic charges \$70 an hour plus the cost of parts to repair a vehicle. The parts cost \$215 for the repair on Tamara's car.

a) Write an expression for the total cost, C , of repairing Tamara's car for any number of hours, n .

b) Use the expression you created in part a) to calculate the cost of repairs that take $3\frac{1}{2}$ h.

6. Give the opposite of the expression.
Express your answer using both
diagrams and symbols.



7. What is the opposite of each expression?

a) $-3y^2$

b) $6g - 3$

c) $2b^2 - 4b + 7$

d) $-4d^2 - 3d - 6$

e) $-k^2 - 8k + \frac{1}{2}$

8. Change the subtraction operation to adding the opposite. Then, combine like terms.

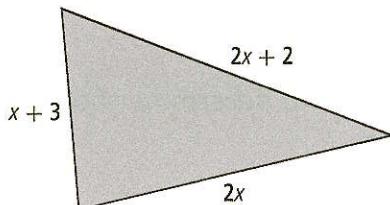
a) $(3r - 5) - (5r + 2)$

b) $(6 - 3f) - (4 - 5f)$

c) $(-4n^2 + 5) - (-n^2 - 9)$

d) $(6a^2 + 2a - 5) - (4a^2 + 5a + 7)$

9. Consider the triangle below.



- a) Write the unsimplified expression for the perimeter.

- b) Simplify the expression from part a) by combining like terms.

- c) If the perimeter of the triangle is 25 cm, calculate the value of x . Verify that your answer is correct.

10. José, Tyler, and Mike split some money they made working on the weekend. They each worked a different number of hours, so they have to split the money fairly. José receives twice the amount that Tyler receives, and Mike receives \$10 less than Tyler. Let x represent the amount that Tyler receives.

- a) Write the expression that represents the total amount that they receive.

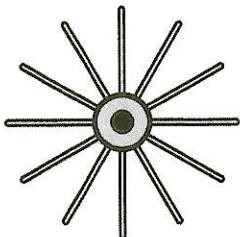
- b) Simplify the expression in part a) by combining like terms.

Vocabulary Link

Unscramble the letters of each term in column B. Use the clues in column A to help you. Each term is one to four words long.

A	B
1. an algebraic expression made up of terms connected by the operations of addition or subtraction; for example, $3x^2 - 4$ _____	LYPNAOOIML
2. terms that differ only by their numerical coefficients, such as $3x$ and $-2x$ _____	STEMILKER
3. an expression formed from the product of numbers and/or variables, such as $9x$ _____	MRTE
4. a polynomial with three terms _____	LIOITRMNA
5. a polynomial with two terms _____	OMLNIAIB
6. in algebra, terms are often arranged in this order _____	GDCSDENINE
7. a polynomial with one term _____	IALNOOMM
8. branch of mathematics that uses symbols to represent unknown numbers or operations _____	EABLGRA
9. the sum of the exponents on the variables in a single term; for example, for $3xz$, it is 2 _____	EMARDGETEORFE
10. the degree of the highest degree term in a polynomial; for example, for $8b^2 - 7b$, it is 2 _____	LYPNAOOIMLEEEFOAGRD

8. What is the order and angle of rotation symmetry for this picture? Express the angle in degrees and in fractions of a turn.



9. Canada's population is about 33 640 000. The United States has about $9\frac{1}{10}$ times Canada's population. Belgium's population is about $\frac{1}{3}$ of Canada's. Justify your answer for each of the following.

a) How many people live in the United States?

b) How many people live in Belgium?

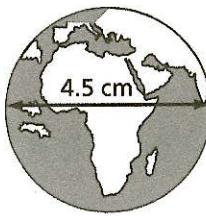
c) How many times greater is the population of the United States than the population of Belgium? Express your answer as a fraction.

10. Evaluate. For part a), keep your answer in fraction form. For part b), express your answer to the nearest thousandth.

a) $\left(\frac{2}{3}\right)^2 \times \left(\frac{-2}{-3}\right)^3 + \frac{1}{3} \div \frac{2}{5}$

b)
$$\frac{(-3)^0 (2.5)^2 - (1.1)^3 (-2)^6}{(-1.2 + 3^3)^2}$$

11. Determine the diameter of the Earth. Express your answer to the nearest tenth. The scale factor for the image of the diameter of Earth is 1 cm:2834.7 km.



12. A child is playing with five wooden blocks. Each block is a cube with an edge length of 4.5 cm. Express your answers to the nearest tenth.

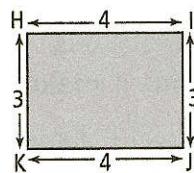
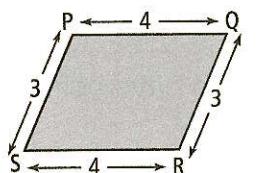
a) Calculate the total surface area of the five blocks.

b) The child builds a tower by stacking five blocks directly on top of each other. What is the surface area of the tower?



c) The child knocks the blocks down. All of the blocks become separated. What is the total visible surface area of the separated blocks?

13. Are the following polygons similar? Explain your thinking.



Patterns in a Table of Values

Linear relations can be represented using a table of values. You can sometimes tell that a relationship in a table is linear if both of the following statements are true.

- Consecutive values in one column change by the same amount.
- Consecutive values in the other column change by the same amount.

s	t
2	6
4	12
6	18
8	24

The difference between consecutive values for s is 2. The difference between consecutive values for t is 6. You can use this information to predict the next values in the table.

For s , the next value could be 10.

For t , the next value could be 30.

3. Determine if each table of values represents a linear relation. Explain how you arrived at your answer.

a)

Distance, d (m)	Speed, s (m/s)
0	2.1
15	4.2
30	6.3
45	8.4

b)

Time, t (s)	Height, h (m)
5	10
10	20
15	40
20	80

4. For each table of values in #3 that represents a linear relation, predict the next ordered pair.

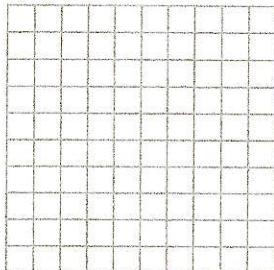
Linear Relationships

Linear relationships represented by formulas or equations can be graphed by

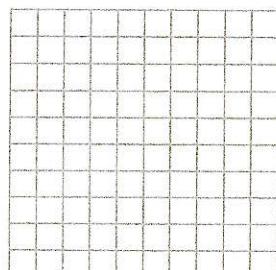
- making a table of values, and
- graphing the ordered pairs from the table of values.

5. For each equation, create a table of values and graph the linear relation.

a) $y = 3x + 2$



b) $t = -4n + 3$



Date: _____

4. a) Describe the relationship between the figure number and the number of toothpicks needed for each figure.

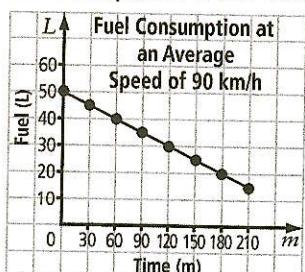


Figure 1 Figure 2 Figure 3

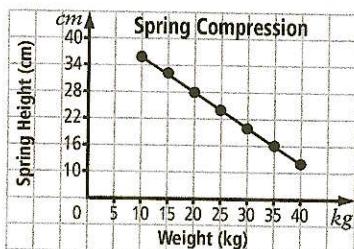
- b) Create a table of values to determine an equation for the model. Write the equation.
- c) Determine if one of the figures could have 2037 toothpicks in it. Show your thinking.
5. a) Jenny has lost part of her homework. Help her redo it by finishing the table of values for the first seven terms.
6. a) A basketball team can buy 12 warm-up jerseys for \$179.40. To put a design on any number of jerseys involves a one-time cost of \$181.80. If there are 12 people on the basketball team, develop an equation that shows the cost for one player.
- b) Assuming that the team can buy additional jerseys for the same cost per jersey as in a), and that the one-time cost for the design does not change, how much would one player pay if there are 15 people on the team?
- c) Create a table of values to show the cost of a single jersey if 1 to 15 players decide to buy a jersey.

x	y
1	-4.5
	-7
	-9.5

4. The following graph shows fuel consumption over time.

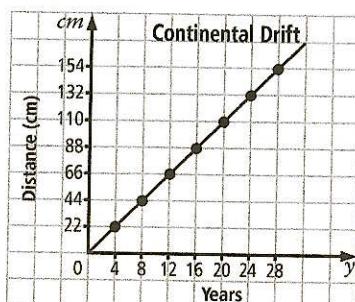


- a) Is it reasonable to extrapolate data from this graph? Explain.
- b) Approximately how much fuel has been used to travel 225 km?
5. A spring is compressed after weights are placed on it. The spring fully compressed is 12 cm long and fully extended is 40 cm long.



- a) Is it reasonable to extrapolate data from this graph? Explain.
- b) What weight fully compresses the spring?
- c) When a 25-kg weight is placed on the spring, what is the spring's length?

6. Continental drift occurs at a rate of about 1 cm to 10 cm per year. Assuming an average movement of 5.5 cm per year, use the graph to answer the following questions.

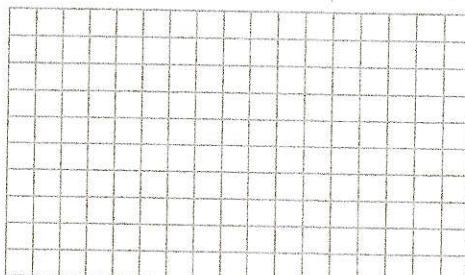


- a) Approximately how long will it take the plate to move 2 m?
- b) After 17 years, approximately how far will the plate have moved? Which method did you use to determine your answer?

7. The table of values represents the dosage of a medicine needed by body weight.

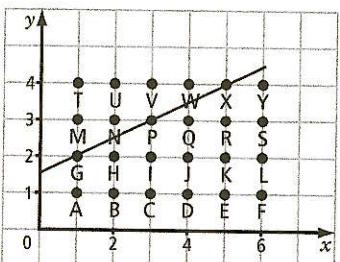
Weight, kg	18	32	46	60
Dosage, mg	60	75	90	105

- a) Plot the linear relation on a graph.

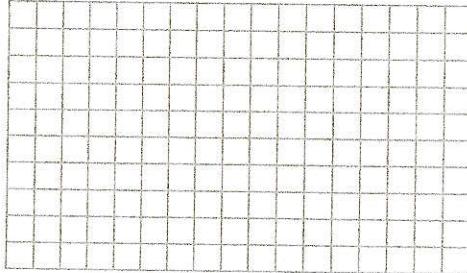


- b) From the graph, determine the approximate dosage needed for weights of 40 kg and 100 kg.
- c) From the graph, determine the approximate weights needed for dosages of 50 mg and 120 mg.

6. The line q passes through the three points G, P, and X.



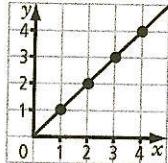
- a) What is the linear equation for the line q ?
- b) Write the linear equation of another line that passes through three letters. Identify the line.
- c) Write an equation for a line that passes through at least four letters. Identify the line.
7. An aquarium holds 1000 L. The graph shows the relationship between time, t , and the number of litres, L , of water pumped from the aquarium.
- Pumping Time to Empty an Aquarium**
- | Time in Minutes (t) | Water in Litres (L) |
|-------------------------|-------------------------|
| 60 | 901 |
| 120 | 802 |
| 180 | 703 |
| 240 | 604 |
| 300 | 505 |
| 360 | 406 |
| 420 | 307 |
| 480 | 208 |
| 540 | 109 |
| 600 | 10 |
- a) What is the linear equation?
- b) How long would it take to pump approximately 750 L of water? What method did you use?
- c) Jomari states that it would take about 15 h to empty a 1500-L aquarium. Do you agree or disagree with Jomari? Explain.
- d) Alex and Zoe live beside each other. Alex leaves home at 9:00 a.m., walking at a steady speed of 1 km per 20 min. Zoe leaves home at 9:30 a.m. and jogs after Alex at a steady speed of 1.25 km per 15 min.
- a) Create tables of values for both Alex and Zoe. Include at least five values.
- b) Graph the results of both tables. Identify each relation.



- c) At approximately what time will Zoe catch Alex?
- d) If they continued at the same pace, how far apart would they be at 10:30 a.m.?

Vocabulary Link

Draw a line from each clue in column A to the correct term in column B. Then, find each term in the word search.

A	B
1. refers to a letter that represents a value	a) coefficient
2. refers to estimating a value that is beyond a given set of values	b) commission
3. name of payment by which a salesperson receives a percentage of the value of sales	c) constant
4. refers to an equation whose graph is a straight line	d) continuous
5. a line on a graph that joins the points	e) extrapolate
6. in the equation $3n - 2$, 3 is a numerical	f) interpolate
7. Example:	g) linear equation
	h) linear relation
8. in the equation $3n - 2$, -2 is a	i) variable
9. estimating the value between two given values on a graph	



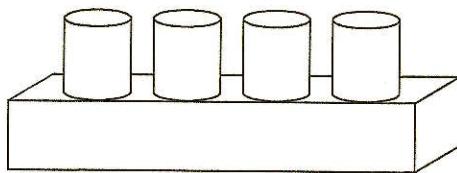
7. Determine the missing values.

a) $\frac{1}{\boxed{}} = \frac{5.9}{76.7}$ b) $\frac{1}{0.08} = \frac{2.7}{\boxed{}}$

8. Chelsea solved the math problem incorrectly. Identify the step where she made her error. Calculate the correct answer.

$$\begin{aligned} & 16 \div (-2)^2 + 6(5)^2 \\ &= 16 \div (-4) + 6 \times 25 \quad \text{Step 1} \\ &= 16 \div -4 + 150 \quad \text{Step 2} \\ &= -4 + 150 \quad \text{Step 3} \\ &= 146 \quad \text{Step 4} \end{aligned}$$

9. A large mega block is made of four cylinders that are 2 cm high and have a diameter of 2.5 cm, and one rectangular prism that measures 2.5 cm wide by 12 cm long by 0.5 cm high. Calculate the surface area, to the nearest hundredth.



10. Simplify.

a) $(a^2 - 7a - 5) - (2 - 4a^2 + 2a)$

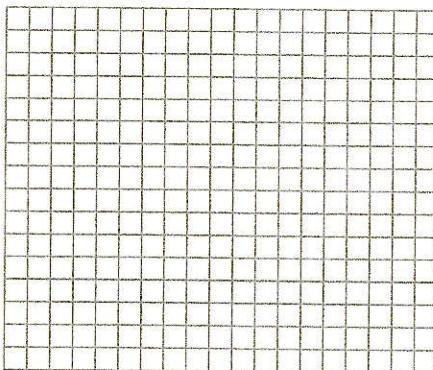
b) $4x^2y - 3xy^2 + 2x^2y - 4xy^2$

11. The surface area of a cube is 127 cm^2 . Determine the side length of the cube, to the nearest tenth.

12. A car rental company charges a flat rate of \$30 plus \$0.05 per kilometre.

- a) Create a table of values for the first 500 km.

- b) Graph the linear relation.



- c) Use the graph to approximate how much it would cost to drive the car 250 km.

- d) Using the graph, approximate how many kilometres you could drive if you had \$52.50.

- e) What equation models this situation?