

3.1 – What Is a Rational Number?

Natural numbers: the counting numbers (1, 2, 3, ...)

Whole numbers: the counting numbers, plus 0 (0, 1, 2, 3, ...)

Integers: positive, negative, or zero whole numbers (... , -3, -2, -1, 0, 1, 2, 3, ...)

Rational numbers: numbers that can be written in fraction form - this includes decimal numbers that are infinite but have a recurring pattern

Ex.

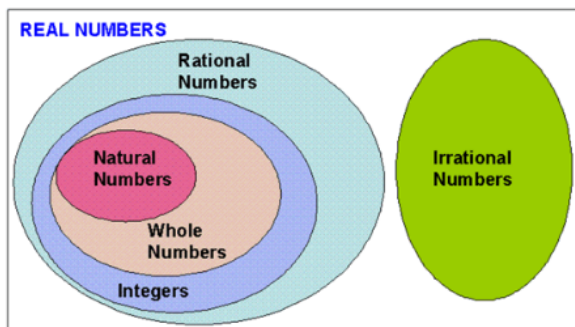
$$\frac{2}{3}, \frac{4}{5}, 5.2 \rightarrow \frac{52}{10}, 0.\bar{3} \rightarrow \frac{1}{3}$$

Irrational numbers: numbers that cannot be written as fractions – in other words, decimal numbers that are infinite and patternless

Ex.

$$\pi = 3.14159.....$$

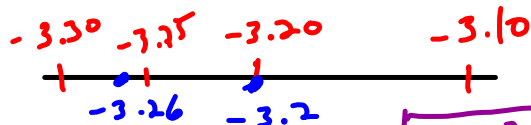
$$\sqrt{2} = 1.41421....$$



Compare with
common denominators

Ex. 1: Which rational number is greater?

(a) -3.2 vs. -3.26



$$-3.26 < -3.2$$

(b) $\frac{5}{8}$ vs. $\frac{7}{12}$

$$\frac{5}{8} \times \frac{3}{3} = \frac{15}{24}, \frac{7}{12} \times \frac{2}{2} = \frac{14}{24}$$

$$\frac{15}{24} > \frac{14}{24}$$

(c) $-1\frac{2}{3}$ vs. $-\frac{7}{5}$

$$-1\frac{2}{3} \times \frac{5}{5} = -\frac{5}{3}, -\frac{7}{5} \times \frac{3}{3} = -\frac{21}{15}$$

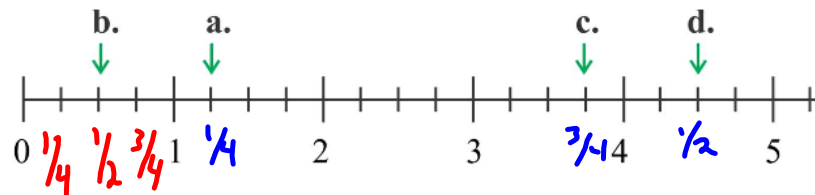
$$-\frac{25}{15} < -\frac{21}{15}$$

(d) 1.32 vs. $1.\overline{32}$

$$1.3200 \text{ vs } 1.3232$$

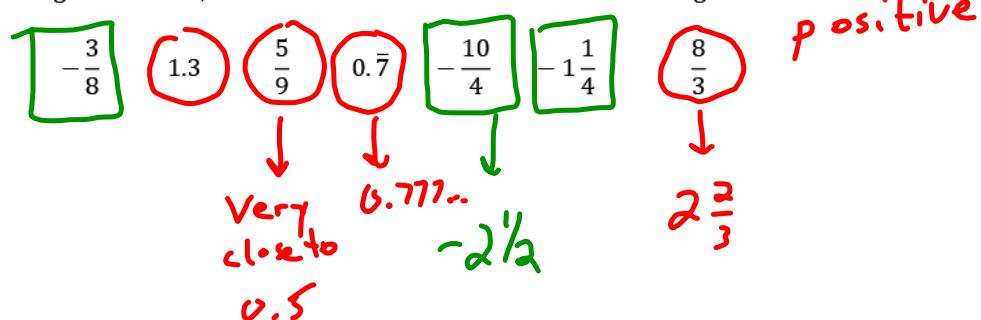
$$1.32 < 1.\overline{32}$$

Ex. 2: Write the fraction represented by each letter.

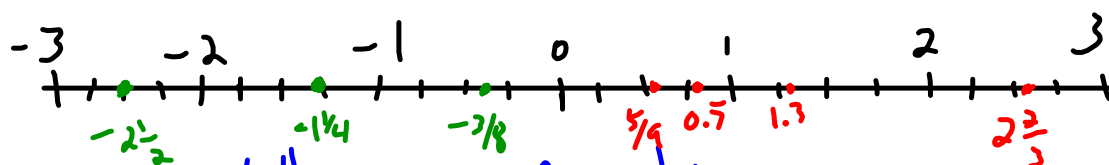


b) $\frac{1}{2}$ a) $1\frac{1}{4}$ c) $3\frac{3}{4}$ d) $4\frac{1}{2}$

Ex. 3: Without using a calculator, order these rational numbers from least to greatest.



$-2\frac{1}{2}, -1\frac{1}{4}, -\frac{3}{8}, \frac{5}{9}, 0.7, 1.3, 2\frac{2}{3}$



Assignment:

Yellow 3.1 Practice

3.2 – Adding Rational Numbers

Recall that when adding and subtracting fractions:

- 1) convert any mixed numbers to improper fractions
- 2) make the fractions compatible by creating Common denominators
- 3) add the numerators and keep the same denominator
- 4) if the answer is an improper fraction, convert it into a mixed number

Ex 1: Evaluate.

(a) $\frac{2}{3} + (-\frac{1}{5})$

Handwritten work: $\frac{2 \times 5}{3 \times 5} + (-\frac{1 \times 3}{5 \times 3}) = \frac{10}{15} + (-\frac{3}{15}) = \frac{7}{15}$

Common denominators: 3, 6, 9, 12, 15

Answer: $\boxed{\frac{7}{15}}$

(b) $-2\frac{5}{8} + (-\frac{7}{2})$

Handwritten work: $-\frac{21}{8} + (-\frac{7 \times 4}{2 \times 4}) = -\frac{21}{8} + (-\frac{28}{8}) = -\frac{49}{8} = -6\frac{1}{8}$

Answer: $\boxed{-6\frac{1}{8}}$

If you don't want to convert mixed numbers to improper fractions, you can add the whole numbers and add the fraction pieces separately, but be careful if one of the terms is negative!

Ex. 2: Evaluate using two different methods: $-3\frac{1}{4} + 2\frac{5}{6}$

~~Method 1~~ – Convert mixed to improper

Handwritten work: $-\frac{13 \times 3}{4 \times 3} + \frac{17 \times 2}{6 \times 2} = -\frac{39}{12} + \frac{34}{12}$

Common denominators: 4, 8, 12

Answer: $\boxed{-\frac{5}{12}}$

~~Method 2~~ – Add the whole numbers and add the fraction pieces

Handwritten work: $-3 + (-\frac{1}{4}) + 2 + \frac{5}{6} = (-3 + 2) + (-\frac{1 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2}) = -1 + (-\frac{3}{12} + \frac{10}{12}) = -1 + \frac{7}{12} = -\frac{12}{12} + \frac{7}{12} = -\frac{5}{12}$

Answer: $\boxed{-\frac{5}{12}}$

Use what you know about adding integers to add rational numbers in decimal form.

Ex. 3: At the beginning of November, the Frosty Snow Blower Company was \$235.46 in debt. By the end of November, the company had increased its debt by \$156.71.

- (a) Write an expression to represent this situation, then calculate how much money the company owed at the end of November.

$$\begin{array}{r} -235.46 \\ + -156.71 \\ \hline -392.17 \end{array}$$

$$-235.46 + -156.71$$

- (b) By the end of December, the company earned a net amount of \$462.58. Is the company still in debt or has it made a profit?

$$\begin{array}{r} 462.58 \\ - 392.17 \\ \hline +70.41 \end{array}$$

$$-392.17 + 462.58$$

Ex. 4: On a particular day in December, the temperature was -13.4°C in the morning and rose 7.6°C . What was the highest temperature that day?

$$\begin{array}{r} -13.4 \\ + 7.6 \\ \hline -5.8 \end{array}$$

$$-13.4 + 7.6$$

$$\begin{array}{r} -13.4 \\ \hline +7.6 \end{array}$$

Assignment: Blue 3.1 Practice

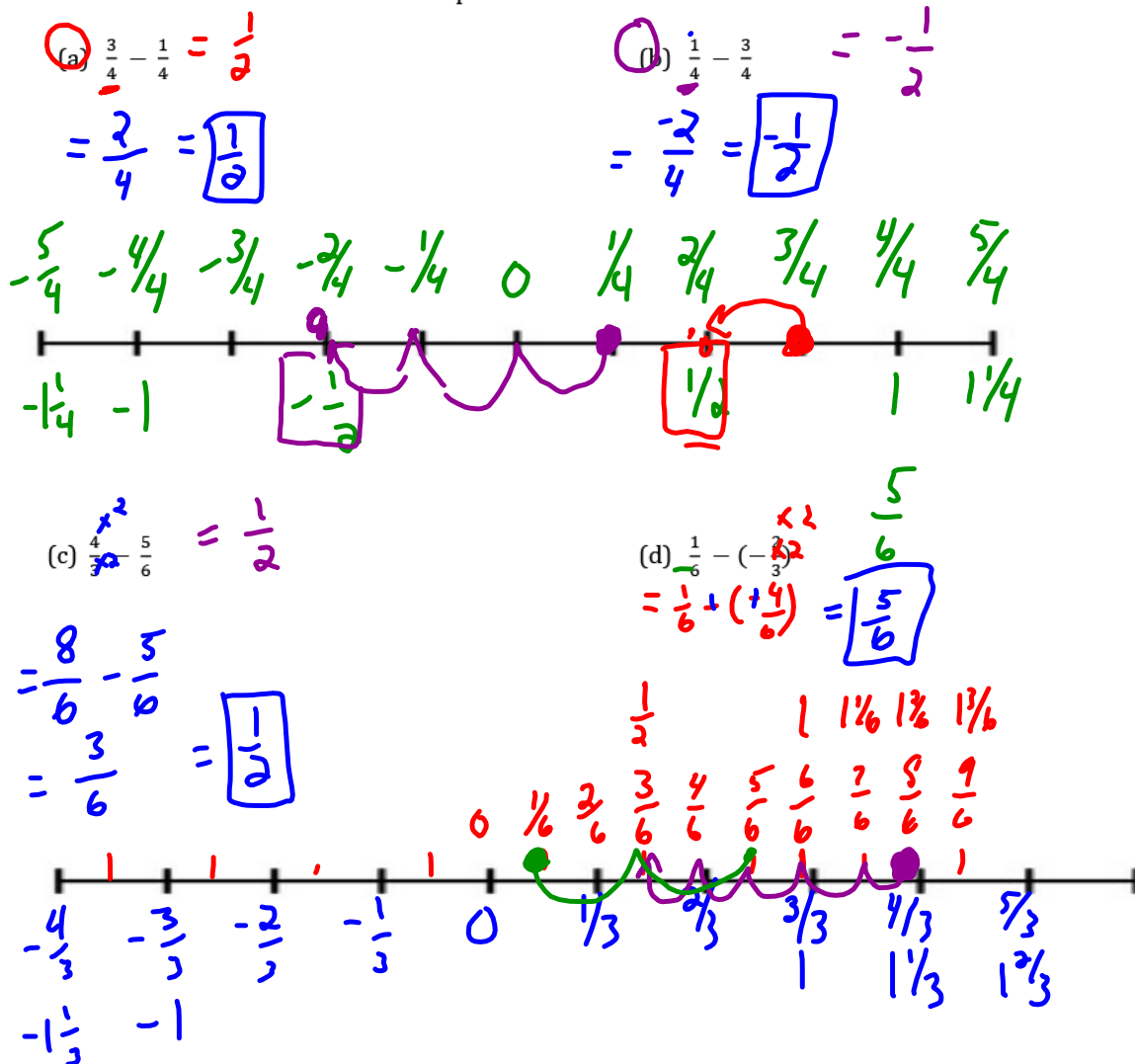
3.3 – Subtracting Rational Numbers

Recall that when adding and subtracting fractions:

- 1) convert any mixed numbers to improper fractions
- 2) make the fractions compatible by creating Common denominators
- 3) add the numerators and keep the same denominator
- 4) if the answer is an improper fraction, convert it into a mixed number

To subtract fractions, you can "add the opposite" just like we did for subtracting negative integers. You may find it helpful to visualize what is happening by using a number line.

Ex 1: Find the difference between each pair of fractions.



$$\begin{aligned}
 & \text{(e)} \quad 4\frac{3}{8} - 2\frac{1}{2} \\
 & \quad \times \quad \quad \times \\
 & -\frac{35}{8} - \frac{5}{2} \times 4 \\
 & = \frac{35}{8} - \frac{20}{8} \\
 & = \frac{15}{8} = \boxed{1\frac{7}{8}}
 \end{aligned}$$

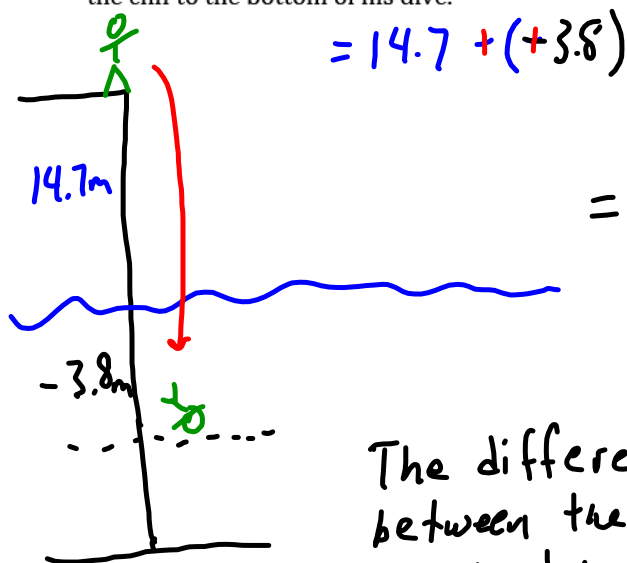
$$\begin{aligned}
 & \text{(f)} \quad \left(-\frac{5}{4}\right) - \left(-3\frac{1}{5}\right) \\
 & \quad \times 20 \quad \times 20 \\
 & = \left(-\frac{5}{4}\right) \times 20 - \left(-\frac{16}{5}\right) \times 20 \\
 & = \left(-\frac{25}{20}\right) + \left(+\frac{64}{20}\right) \\
 & = \frac{39}{20} = \boxed{1\frac{19}{20}}
 \end{aligned}$$

4, 8, 12, 16, 20
5, 10, 15, 20

Use what you know about subtracting integers to subtract rational numbers in decimal form.

Ex. 2: A diver jumps off a cliff that is 14.7 m above sea level. After hitting the water, he plunges 3.8 m below the surface of the water.

Use a drawing and rational numbers to represent the difference in heights from the top of the cliff to the bottom of his dive.



$$\begin{array}{r}
 14.7 \\
 + 3.8 \\
 \hline
 18.5
 \end{array}$$

The difference in height between the top and bottom of the dive is 18.5m.

Assignment: Blue 3.3 practice

3.4/3.5 – Multiplying/Dividing Rational Numbers

same signs $\rightarrow \oplus$ different signs $\rightarrow \ominus$

Recall that for multiplication and division:

- If there is an even number of negative signs (0, 2, 4, 6, etc.), the solution will be positive
- If there is an odd number of negative signs (1, 3, 5, etc.), the solution will be negative

When multiplying or dividing fractions, there is no need to create common denominators. You can, but you will have much more work to do simplifying your answer (lowest terms).

To multiply fractions:

- 1) Convert any mixed numbers to improper fractions.
- 2) Cross-cancel any common factors in the numerators and denominators (saves you work in step 4).
- 3) Multiply the numerators and multiply the denominators.
- 4) Be sure that your answer is fully simplified (lowest terms). If the answer is an improper fraction, convert it to a mixed number

Ex 1: Determine each product.

(a) $\left(-\frac{1}{1}\right)\left(-\frac{3}{4}\right)$

Method 1
cross-cancel 1st!
 $= \left(-\frac{1}{1}\right)\left(-\frac{3}{4}\right)$
 $= \boxed{\frac{3}{4}}$

Method 2
 $= \frac{231 \div 7}{308 \div 7} = \frac{33 \div 11}{44 \div 11} = \boxed{\frac{3}{4}}$

(b) $\left(2\frac{2}{3}\right)\left(-1\frac{5}{6}\right)$

$= \frac{18}{3} \times \left(-\frac{11}{6}\right)$
 $= \frac{-44}{9}$
 $= \boxed{-4\frac{8}{9}}$

(c) $(0.8)(-2.4)$

estimate
 $\approx 1 \times (-2) = \approx -2$

Alternative
 $= \frac{8}{10} \times \left(-2\frac{4}{10}\right)$
 $= \frac{18}{50} \times \left(-\frac{24}{5}\right)$
 $= \frac{-48}{25}$
 $= -1\frac{23}{25} \times 4 = \boxed{-1.92}$

(d) $(-1.25)(-2.84)$

Using calculator
- answer must be positive

To divide fractions:

- 1) Convert any mixed numbers to improper fractions.
- 2) Multiply the dividend by the reciprocal of the divisor ("kiss and flip").
- 3) Follow the same procedure as for multiplying fractions.

Ex 2: Determine each quotient.

(a) $\left(-\frac{5}{8}\right) \div \frac{3}{4}$

(b) $\left(-4\frac{1}{5}\right) \div \left(-3\frac{1}{3}\right)$

(c) $-1.38 \div 0.6$

(d) $-4 \div (-1.8)$

Ex. 3: On a particular day, the price of a share in CIBC decreased by \$1.640. A person owns 35 shares. By how much did those shares change in value that day?

$$\begin{array}{r}
 \underline{-1.640} \times 35 \text{ shares} \\
 \text{share} \\
 = \begin{array}{r}
 \underline{-1.640} \\
 \times \quad 35 \\
 \hline
 8200 \\
 49200 \\
 \hline
 -57.400
 \end{array}
 \end{array}$$

$\$ = -57.40$

Assignment: Textbook P. 127 #3ac, 4ac, 7, 9, 12cd

$- \times -$
 $+ \times +$

$- \times +$
 $+ \times -$

3.4/3.5 – Multiplying/Dividing Rational Numbers

Same sign $\rightarrow (+)$

different sign $\rightarrow (-)$

Recall that for multiplication and division:

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When multiplying or dividing fractions, there is no need to create common denominators. You can, but you will have much more work to do simplifying your answer (lowest terms).

To multiply fractions:

- 1) Convert any mixed numbers to improper fractions.
- 2) Cross-cancel any common factors in the numerators and denominators (saves you work in step 4).
- 3) Multiply the numerators and multiply the denominators.
- 4) Be sure that your answer is fully simplified (lowest terms). If the answer is an improper fraction, convert it to a mixed number

Ex 1: Determine each product.

(a) $\left(-\frac{1}{1}\right)\left(-\frac{3}{4}\right)$

$= \frac{3}{4}$

(b) $\left(2\frac{2}{3}\right)\left(-1\frac{5}{6}\right)$

$= \frac{48}{3} \times \left(-\frac{11}{6}\right)$

$= \frac{-44}{9}$

$= -4\frac{8}{9}$

(c) $(0.8)(-2.4)$ approximation

$\approx 1 \times (-2)$

$\approx (-2)$

$\begin{array}{r} 3 \\ -2.4 \\ \times 0.8 \\ \hline -1.92 \end{array}$

(d) $(-1.25)(-2.84)$

calculator

$= 3.55$

$\approx (-1) \times (-3)$

≈ 3

$- \times -$
 $+ \times +$

$- \times +$
 $+ \times -$

3.4/3.5 – Multiplying/Dividing Rational Numbers

• Same sign $\rightarrow (+)$

• different sign $\rightarrow (-)$

Recall that for multiplication and division:

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Ex 1: Determine each product.

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(c) $(0.8)(-2.4)$ approximation

$\approx 1 \times (-2)$

$\approx (-2)$

$\begin{array}{r} 3 \\ -2.4 \\ \times 0.8 \\ \hline -1.92 \end{array}$

(d) $(-1.25)(-2.84)$

calculator

$= 3.55$

$\approx (-1) \times (-3)$

≈ 3

3.6 – Order of Operations with Rational Numbers

Recall the order of operations:

- 1) Brackets – complete innermost brackets first if there are several
- 2) Exponents
- 3) Division/Multiplication – solve the furthest to the left first if there is more than one
- 4) Addition/Subtraction – solve the furthest to the left first if there is more than one

Ex 1: Evaluate.

$$\begin{aligned} \text{(a)} \quad & -0.8 + 1.2 \div (-0.3) \times 1.5 \\ & = -0.8 + \underline{(-4) \times 1.5} \\ & = -0.8 + \underline{(-6.0)} \\ & = \boxed{-6.8} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & -3.2 - 0.9 \div [0.7 - (-1.2)]^2 \\ & = -3.2 - 0.9 \div \underline{((1.9))^2} \\ & = -3.2 - 0.9 \div \underline{3.61} \\ & = -3.2 - \underline{0.249307479} \\ & = -3.449307479 \\ & \approx \underline{\underline{\boxed{-3.4}}} \end{aligned}$$

Ex. 2: To convert a temperature in Fahrenheit to Celsius, we use the formula: $C = \left(\frac{F - 32}{1.8}\right)$. The lowest natural temperature ever directly recorded at ground level on Earth is -128.6°F , which was in Antarctica, on July 21, 1983. What is this temperature in degrees Celsius?

$$\begin{array}{r} -128.2 \\ -32 \\ \hline -160.2 \end{array}$$

$$C = \frac{-128.2 - 32}{1.8}$$

$$C = \frac{-160.2}{1.8}$$

$$C = -89^\circ$$

Ex. 3: Evaluate $\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1}{3} + \left(-\frac{1}{4}\right)\right]$

$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{4}{12} + \left(-\frac{3}{12}\right)\right]$$

$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \frac{1}{12}$$

$$= \frac{1}{4} - \left(-\frac{2}{3}\right) \div \frac{1}{12}$$

$$= \frac{1}{4} - \left(-\frac{2}{3}\right) \times \frac{12}{1}$$

$$= \frac{1}{4} - \left(-\frac{8}{1}\right)$$

$$= \frac{1}{4} + 8$$

$$= 8\frac{1}{4}$$

Assignment: 3.6 Blue practice