# Reduction Techniques and other Combinatorial Tools in tau-tilting Theory

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**Abstract**

# 1 Introduction

## 1.1 Setting

We closely follow the notation of [ASS06]. An algebra $A$ will always refer to a finite dimensional algebra over a field $k$, which need not be algebraically closed. All algebras encountered in this thesis will be, unless otherwise stated, on the form $k\Gamma/I$ for some finite quiver $\Gamma$ and admissible ideal $I$. The number $n$ is, unless otherwise stated, reserved for the number of indecomposable projective summands of $A$. By mod $A$ we denote the category of finite dimensional left $A$-modules (in contrast to [ASS06], who employ right modules). We denote by $D$ the duality on modules, and by Tr the transpose. The Auslander-Reiten translate $D\mathrm{Tr}$ is denoted $\tau$.

A module $M$ is called basic if no two nonzero summands are equal, that is $M = A \oplus X \oplus X$ implies $X = 0$.

# 2 Tau-tilting theory

$\tau$-tilting theory was introduced in [AIR13] as a possible generalization of tilting theory, which has had tremendous impact on the field of representation theory of finite dimensional algebras as a whole. Although first introduced in [AIR13], some of the main results of $\tau$-tilting theory stem from (cite smalø). We here recall some important definitions and results from $\tau$-tilting theory. We start by discussing $\tau$-rigid modules, which are the building blocks of $\tau$-tilting theory.

**Definition 2.1.** A module $M$ in mod $A$ is called $\tau$-rigid if Hom $(M, \tau M) = 0$.

Note that $X, Y$ being $\tau$-rigid does not imply $X \oplus Y$ being $\tau$-rigid. In fact, studying how indecomposable $\tau$-rigid modules together form larger $\tau$-rigid modules is part of $\tau$-tilting theory.

**Definition 2.2.** A pair $(M, P)$ where $M$ is basic $\tau$-rigid and $P$ basic projective such that Hom $(P, M) = 0$ is called a partial support $\tau$-tilting module.

# 3 Tau-exceptional sequences

Tau-exceptional sequences were introduced in [BM20] as a generalization of exceptional sequences to any finite dimensional algebra.

# References


[AIR13] Takahide Adachi, Osamu Iyama, and Idun Reiten. *$\tau$-tilting theory.* 2013. arXiv: `1210.1036 [math.RT]`.

[ASS06] Ibrahim Assem, Andrzej Skowronski, and Daniel Simson. *Elements of the Representation Theory of Associative Algebras: Techniques of Representation Theory.* Vol. 1. London Mathematical Society Student Texts. Cambridge University Press, 2006. DOI: `10.1017/CBO9780511614309`.

[BM20] Aslak Bakke Buan and Bethany Marsh. *$\tau$-exceptional sequences.* 2020. arXiv: `1802.01169 [math.RT]`.