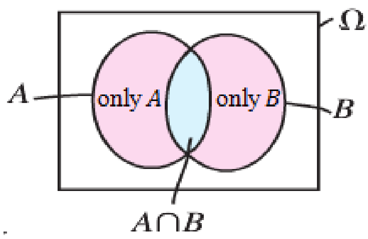
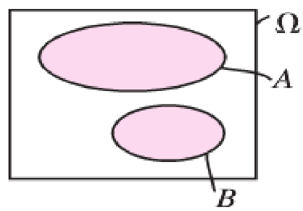
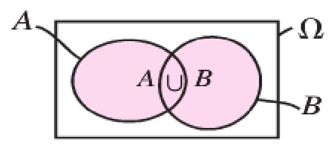
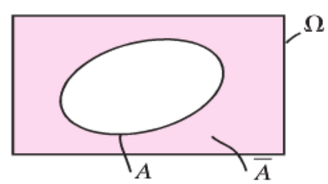
Summary: Probabilities

- Random experiment: It is an **operation of p elements from n elements randomly**.
- Sample space: It is the set of **all possible outcomes**, and denoted by $\boldsymbol{\Omega}$.
- Event: It is a part of the sample space Ω. It is usually described by a sentence.
- Certain event: It is **always occurring**.
- Impossible event: It is **never happens**.
- The event ‹‹*A and B*›› is $A \cap B$.
- The event «*A or B*» is $A \cup B$.
- The complementary event of A is $\overline{A} = \Omega - A$.
- $\boldsymbol{A \cup \overline{A} = \Omega}$; $\boldsymbol{A \cap \overline{A} = \emptyset}$; $\boldsymbol{\overline{\overline{A}} = A}$.









- The probability of A is: $\boldsymbol{P(A)} = \frac{\textbf{Card}(\boldsymbol{A})}{\textbf{Card}(\boldsymbol{\Omega})} = \frac{\text{number of possible outcomes of } A}{\text{Total number of possibilites}}$.
- If **one element** is selected, then to calculate *P(A)* we use a **fraction**.
- If **two or more elements** are selected, then to calculate *P(A)* we use a **formula** (n^p ; A_n^p or C_n^p).

- $\boldsymbol{P(\text{Certain event})} = P(\Omega) = \frac{\text{Card}(\Omega)}{\text{Card}(\Omega)} = \mathbf{1}$; $\boldsymbol{P(\text{Impossible event})} = P(\emptyset) = \frac{\text{Card}(\emptyset)}{\text{Card}(\Omega)} = \mathbf{0}$.
- For any event A of Ω, we have: $\boldsymbol{0 \leq P(A) \leq 1}$.

- $\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$.
- If A and B are two **incompatible** events, then $A \cap B = \emptyset$, therefore $\boldsymbol{P(A \cup B) = P(A) + P(B)}$.

- $\boldsymbol{P(\overline{A}) = 1 - P(A)}$.
- $P(\text{‹‹at least one ... ››}) = 1 - P(\text{‹‹not one ... ››})$.
- Note that: $\overline{A \cup B} = \overline{A} \cap \overline{B}$; $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
- $P\left(A \cap \overline{B}\right) = P(A \text{ only}) = P(A) - P(A \cap B)$; $P\left(\overline{A} \cap B\right) = P(B \text{ only}) = P(B) - P(A \cap B)$.

Conditional probability:

Consider two events A and B of a sample space Ω.

$P(A/B)$ is called the conditional probability of the event A **knowing that** the event B has occurred.

$P(A/B)$ can be calculated using the formula: $\boldsymbol{P(A/B) = \frac{P(A\cap B)}{P(B)}}$.

Remarks:

- $P(A\cap B) = P(B\cap A)$ but $P(A/B) \neq P(B/A)$.
- $P(\overline{A}/B) = 1 - P(A/B)$ but $P(A/\overline{B}) \neq 1 - P(A/B)$.

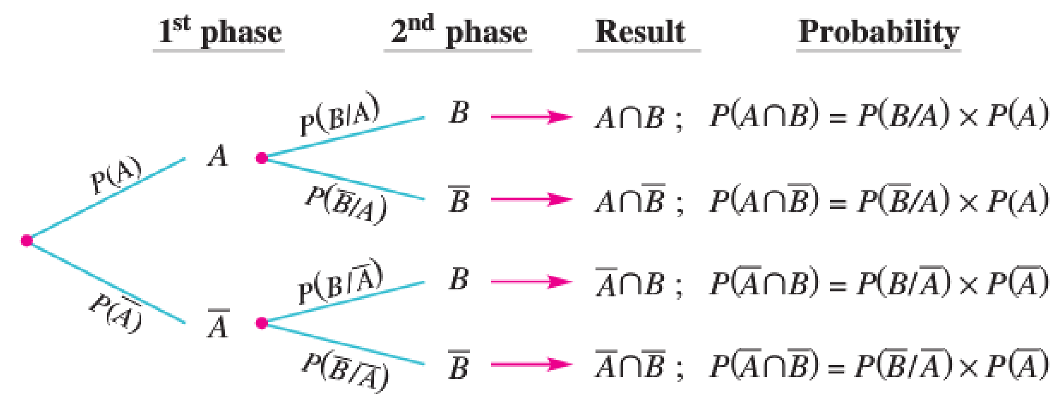
Independent events – Dependent events:

Two events A and B of a sample space Ω are said to be **independent** if the incidence of one of them **does not affect** the probability of the other, that is: $\boldsymbol{P(A/B) = P(A)}$. In this case:
$\boldsymbol{P(A\cap B) = P(A)\times P(B)}$.

Two events A and B of a sample space Ω are said to be **dependent** if the incidence of one of them **affects** the probability of the other, that is: $\boldsymbol{P(A/B) \neq P(A)}$. In this case:
$\boldsymbol{P(A\cap B) = P(A)\times P(B/A)}$ or $\boldsymbol{P(A\cap B) = P(B)\times P(A/B)}$.

Tree diagram:

The tree-diagram is a tree where the corresponding probabilities are placed on each branch as indicated below:



Rules

- Each knot corresponds to a state of the experience.
- The sum of the probabilities of the branches coming from the same knot is 1.

Total probability:

$\boldsymbol{P(B) = P(B\cap A) + P(B\cap \overline{A}) = P(A)\times P(B/A) + P(\overline{A})\times P(B/\overline{A})}$.