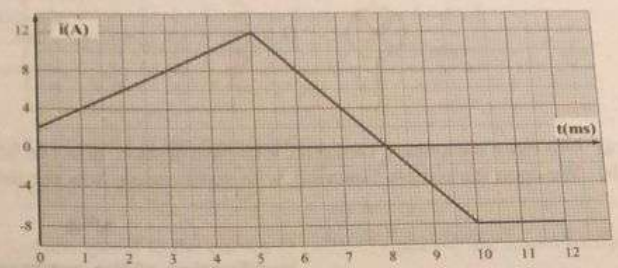
N° 1
Studying the voltage of a coil

A coil, without an iron core, of inductance $L = 200$ mH and of a resistance $r = 100\ \Omega$ is traversed by a current of variable intensity i as shown in the graph below.



1) Determine the time intervals of time where the phenomenon of self induction exists.
2) Calculate the values of the self-induced electromotive force *e* in the preceding intervals of time.
3) Represent the graph of *e* as a function of time.
4) Specify the direction of the induced current in the coil.
5) Determine the interval of time in which the coil plays the role of a generator.
6) **a)** Calculate the voltage u_{AB} at the instants $t_1 = 1$ ms, $t_2 = 3$ ms, $t_3 = 6$ ms, $t_4 = 8$ ms and $t_5 = 11$ ms.
b) Represent the graph of u_{AB} as a function of time.

N° 2
Variation of the e.m.f as a function of time

A coil of resistance $r = 5\ \Omega$ and of inductance $L = 0.45$ H is connected to a battery of constant voltage $U = 6$ V.

1) Calculate, in the steady state, the intensity of the current in the circuit.
2) Study the variation of the absolute value $|e|$ of the self-induced electromotive force *e* during the growth of the current.

N° 3
How does a lamp light up in the presence of a coil ?

A coil (L ; r) is connected across the terminals of a battery delivering a constant voltage $E = 6$ V as shown in figure (1). We close the switch at $t_0 = 0$, and register, in figure (2), the variation of the current i in the circuit as a function of time.

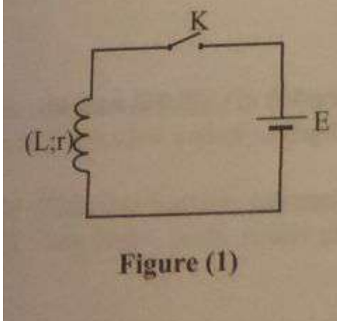


Figure (1)

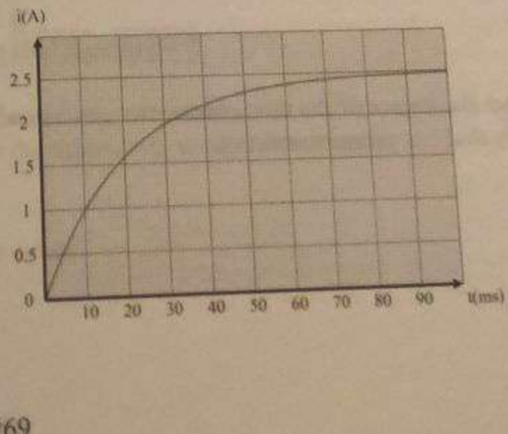


Figure (2)

1) Extract the value of I in the steady state. Deduce the resistance r of the coil.
2) Extract the time constant τ of the circuit (L ; r). Deduce the value of L.
3) Calculate the maximum magnetic energy stored in the coil.
4) The switch is open, we branch, across the terminals of the coil, a lamp L [figure (3)].

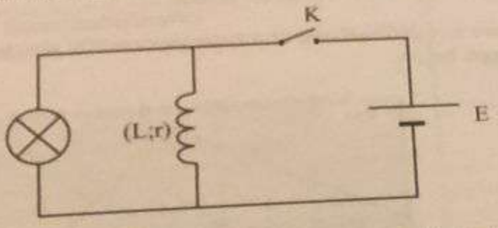
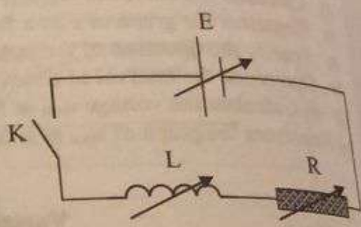


Figure (3)

a) We close the switch. Describe, with justification, how the lamp L lights up.
b) We open the switch. Describe, with justification, how the lamp L lights up.

N° 4

Verification of the expression $\tau = \frac{L}{R}$

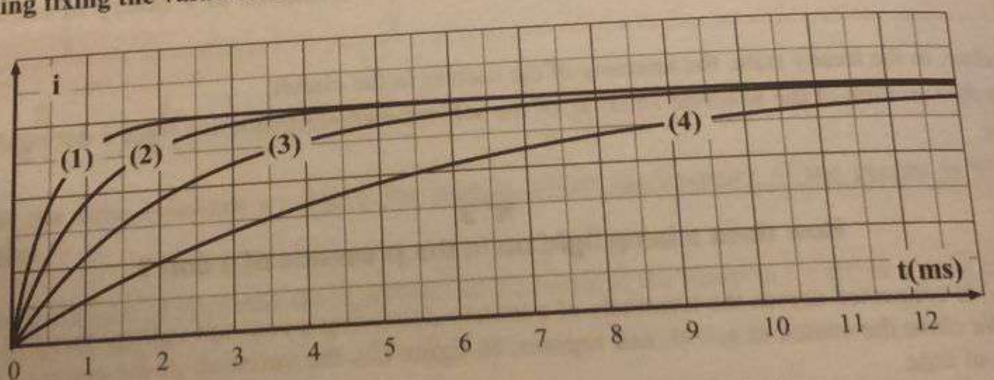


The circuit of the adjacent figure, carries a variable DC generator a coil of variable inductance L of negligible resistance and a variable resistor R.

At $t_0 = 0$, we close the switch and we record the variation of the intensity of the current as a function of time.

The time constant τ represents the time at which the intensity of the current is 63 % of its maximum value.

A – Recording fixing the value of R and E and varying L

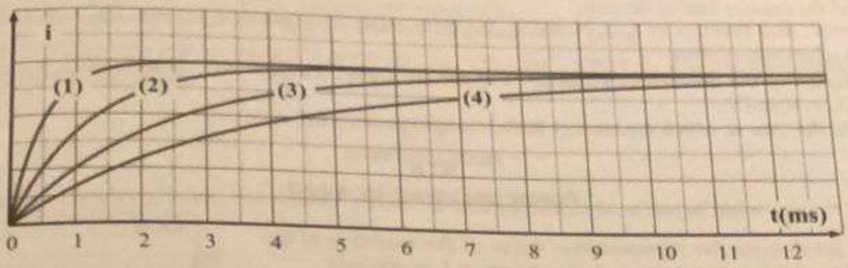


The values of the inductances for the curves (1), (2), (3) and (4) are respectively :

$\boxed{0.05\text{ H}} - \boxed{0.1\text{ H}} - \boxed{0.2\text{ H}} - \boxed{0.5\text{ H}}$.

1) Extract the values of the time constants τ_1, τ_2, τ_3 and τ_4 corresponding to the curves (1), (2), (3) and (4).
2) Verify that τ is proportional to L.

B – Recording fixing the value of L and E and varying R

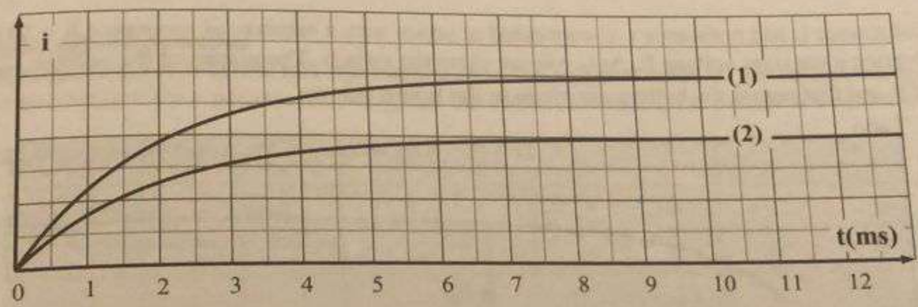


The values of the resistances of the curves (1), (2), (3) and (4) are respectively :

$\boxed{200\ \Omega}$ – $\boxed{80\ \Omega}$ – $\boxed{40\ \Omega}$ – $\boxed{25\ \Omega}$.

The values of the time constants τ_1, τ_2, τ_3 and τ_4 correspond to the curves (1), (2), (3) and (4) are respectively: 0.5 ms ; 1.25 ms ; 2.5 ms and 4 ms. Show that τ and R are inversely proportional.

C – Recording fixing the values of L and R and varying E



Show that the time constant τ is independent of E.

D – Conclusion

Studying the preceding parts **A** – and **B** – give the following : $\tau = \alpha \frac{L}{R}$, where α is a constant proportionality.

1) Show that $\frac{L}{R}$ corresponds to time. Deduce that α is unitless (dimensionless).
2) Knowing that for L = 0.2 H and R = 25 Ω, we find τ = 8 ms. Calculate α.

N⁰ 5
Establishing the current

A coil (L = 0.6 H ; r = 2 Ω) is placed in series with a resistor, of resistance R = 10 Ω, across a battery e.m.f. E = 12 V and of negligible internal resistance. At the time $t_0 = 0$, we close the circuit.

1) Calculate the time constant τ of the circuit.
2) Determine, in the steady phase, the current I_0 in the circuit.

3) Calculate the value of the current at instant $t = \tau$. Deduce, at the same instant, the voltage across the terminals of the resistor and of the coil.

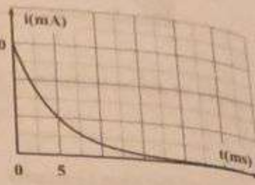
4) Determine, as a function of L, r, R and E, the differential equation that governs the evolution of the current i as a function of time.

5) The solution of the differential equation is under the form : $i = ae^{-bt} + c$. Determine a, b and c as a function of L, r, R and E.

6) Calculate the time at which the current in the circuit becomes 0.8 A ?

N° 6
Power delivered by a coil

The adjacent graph gives the representative curve of the variation of the intensity of the current traversing a coil of inductance L = 400 mH.

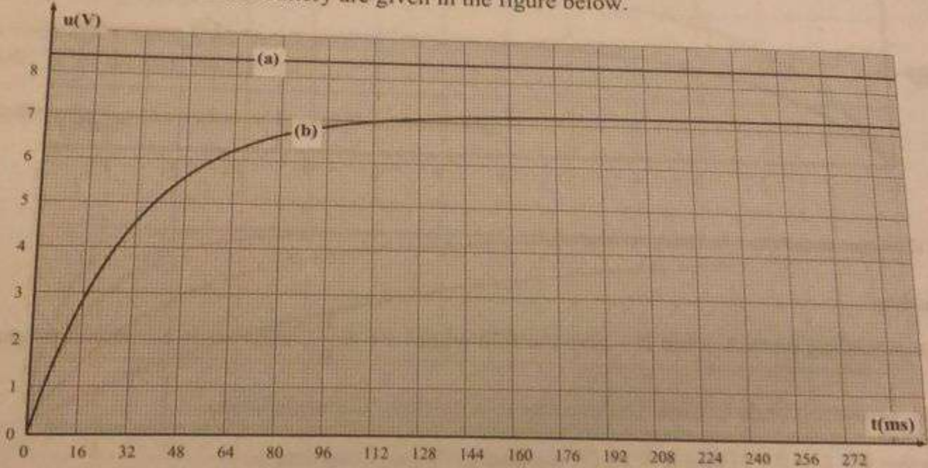


1) Calculate the magnetic energies stored in the coil at the instants $t_0 = 0$ and $t_1 = 5$ ms.

2) Deduce the average power delivered of the coil between t_0 and t_1. How does this power appear to the exterior ?

N° 7
Calculating the inductance and the resistance of a coil

A coil, of inductance L and resistance r, is connected in series with a resistor, of resistance R = 50 Ω, across a battery delivering a constant voltage E. At $t_0 = 0$, we close the switch. The graphs of the voltage u_R across the terminals of R and that across the battery are given in the figure below.



To answer the following questions we can use the graphs.

1) Draw a figure of the electric circuit.

2) The voltage across R is the image of the current in the circuit. Justify.

3) a) Associate, with justification, to each of the curves (a) and (b) the corresponding voltage.

b) Extract the value of E.

4) a) In the steady state :

— extract the value U_R of u_R.

— calculate the intensity I_0 of the current

— find the voltage across the terminals of the coil.

b) Deduce the value of r.
5) Extract the time constant of the circuit. Deduce the value of L.
6) Represent the voltage across the terminals of the coil as a function of time.
7) Knowing that the voltage across the coil is given by : $u_b = A + Be^{\alpha t}$ where A, B and α are constants ($\alpha < 0$). Calculate : A, B and α.

N° 8
Electrocution

During a session of practical work, the teacher carries on an experiment by a coil of inductance $L = 3.4$ H and of resistance $r = 1.1\ \Omega$.

This coil is branched in series with a lamp of average resistance $R_L = 5\ \Omega$ and a switch.

The set up (coil, lamp) is branched across a battery of constant voltage equal to 9 V.

A – The teacher closes the switch
1) The circuit is a seat to a phenomenon. Name this phenomenon.
2) Give an indication that this phenomenon exists.
3) Calculate the time constant of the circuit.
4) Determine, after few seconds, the intensity I_0 of the current in the circuit and the magnetic energy stored in the coil.

B – The teacher gets electrocuted
The threshold of perception is $I_S = 2$ mA corresponding to the minimal value of the current which provokes sensation to electrocution for a person. Knowing that the current traversing the person at a time t is given by :

$i = I_0 e^{-\frac{t}{\tau}}$. The average resistance of human body : $R = 240\ \Omega$.

1) The teacher is electrocuted. Justify.
2) Calculate the duration of electrocution.

N° 9
Inductance of a coil and its effects

We consider a coil which carries the following indications : **number of joint turns : N = 1000 turns ; area : S = 20 cm^2 ; length : ℓ = 40 cm.**

A – Calculating the inductance of the coil by two methods (The coil is without the soft iron core).

First method : Show that the inductance of the coil is given by the formula : $L = \mu_0 \frac{N^2 S}{\ell}$ ($\mu_0 = 4\pi.10^{-7}$ SI).

Verify that : L = 6.25 mH (given : $4\pi = 12.5$).

Second method : The coil is traversed by a current of intensity i variable as a function of time as shown in figure (a) (in the next page). The voltage u_{AB} of the coil, oriented positively between A and B, is represented as a function of time in figure (b) (in the next page). Neglect the internal resistance of the coil.

1) Write u_{AB} as a function of L and $\frac{di}{dt}$

2) Deduce the value of L.

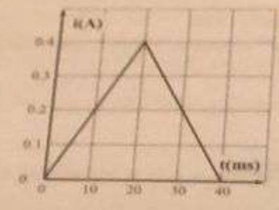


Figure (a)

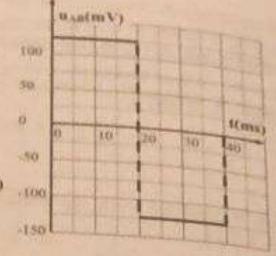


Figure (b)

B – Influence of the inductance on the brightness of a lamp
When we introduce to the coil a soft iron core the inductance becomes L' and it is few times larger than L.

Using the coil we carry on the experiments in figures (c) and (d).

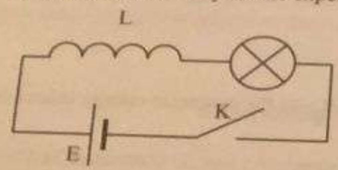


Figure (c)

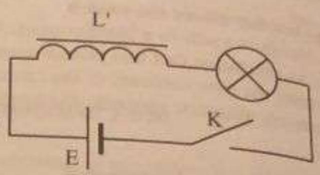


Figure (d)

The lamps in the two experiments are identical. Compare the brightness of each of the lamps when the switch is closed.

N° 10
Characteristics of a coil

We get a coil with the indications : ($r = 20\ \Omega$ and $L = 20$ mH)

The aim of this exercise is to verify the characteristics, inductance L and resistance r, of a coil and using two methods.

A – First method
The coil is placed in series with a resistor, of resistance $r' = 80\ \Omega$, across a battery delivering a constant voltage $U_0 = 12$ V as shown in figure (1).
At $t = 0$, we close the switch k, an appropriate device records, as in figure (2), the variation of the intensity of the current i in the circuit as a function of time.

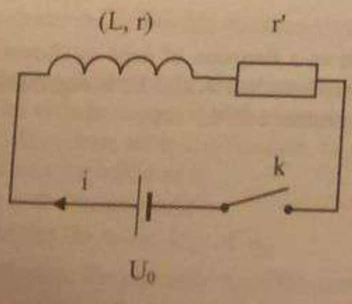


Figure (1)

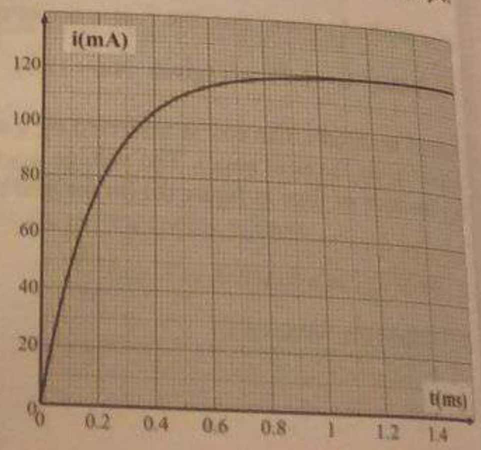


Figure (2)

1) Using the graph, determine the time at which the permanent phase is established in the circuit. Deduce the value of r.
2) Define the time constant τ of the circuit and give its expression as a function of L, r and r'.
3) Extract τ and deduce the value of L.

B – Second method

This method is formed of two experiments, one of which is to calculate r and the other is to calculate L eliminating the role of r.

First experiment: We connect in the circuit in figure (3), D the resistor, k the switch and a battery delivering the voltage.

We close the switch, the ammeter A and the voltmeter V indicate, after a certain time, the respective values 40 mA and 800 mV.

1) Why is there a certain time before the above indications are achieved?
2) Calculate the value of r.

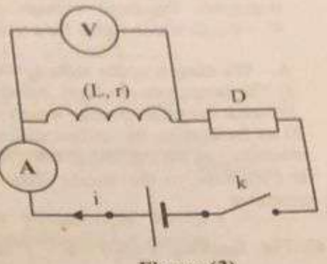


Figure (3)

Second experiment: We connect the circuit in figure (4), where G is a generator and R is a resistor of resistance $R = 20\ k\Omega$.

An oscilloscope displays, on channel ,1 the voltage across the coil, and on channel 2, the voltage across R as shown in figure 5.

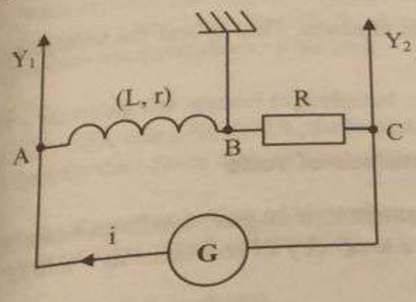


Figure (4)

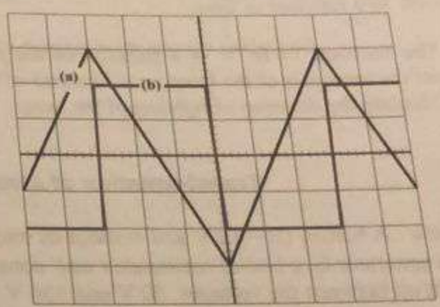


Figure (5)

The sensitivities on the oscilloscope : On both channels : $S_V = 200$ mV/div ;Time base : $V_b = 1$ µs/div.

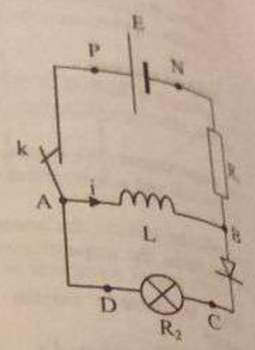
1) Name the voltages displayed on the oscilloscope.
2) a) Why can we neglect the value of r in front of R.

b) Verify that : $u_{AB} = -\frac{r}{R} u_{CB} - \frac{L}{R}\frac{du_{CB}}{dt} \approx -\frac{L}{R}\frac{du_{CB}}{dt}$

3) Show that it is impossible to correspond u_{CB} to curve (b).
4) Deduce the value of L.

N° 11
Lighting of a lamp

The circuit of the adjacent figure contains : a battery of e.m.f. $E = 1.5$ V and of negligible internal resistance, a coil of inductance $L = 1.2$ H and of negligible internal resistance, a resistor of resistance $R_1 = 0.2\ \Omega$, a diode, of negligible threshold voltage, and a lamp similar to a resistor, of resistance $R_2 = 0.5\ \Omega$ which lights for a voltage greater than or equal to 1.5 V.



A – We close k at the time $t_0 = 0$.
1) The lamp does not light. Justify.
2) Name the phenomenon which appears in the circuit.
3) a) Determine the differential equation which governs the variation of the intensity i of the current in the circuit as a function of time and of E, R and L.
b) Calculate, in the steady state, the intensity I of current delivered by the generator.
c) The function $i = I\left(1 - e^{-\frac{t}{\tau_1}}\right)$ is a solution of the preceding differential equation. Find τ_1 and calculate the magnetic energy stored in the coil.

B – After a certain time of closing k we open i tat a time considered as new origin of time.
1) The lamp has the advantage to light up. Justify.
2) a) Determine the differential equation which governs the variation of the intensity i of the current in the circuit as a function of time.
b) The function $i = Ie^{-\frac{t}{\tau_2}}$ is a solution of the preceding differential equation. Find τ_2 and the voltage u_{CD} across the terminals of the lamp, as a function of time.
c) Calculate the duration of lighting of the lamp.

N° 12
Transformation of a voltage 12 V to hundreds of volts

We bring : A battery ($E = 12$ V and without an internal resistance), a switch K, a magnet, a milli-ammeter, a current generator G, a coil of resistance r and constant inductance L, a lamp (L) of resistance $R_L = 220\ \Omega$ can light up between the voltages 50 V and 150 V.

Part A : Calculating r and L
Using the coil we carry on two separate experiments schematized in figures 1.a and 1.b. This coil is traversed by the respective magnetic flux Φ_1 and Φ_2 which are represented in figures 2.a and 2.b.

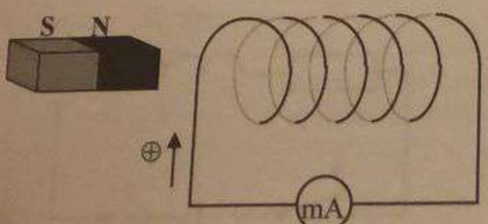


Figure 1.a: the displacement of the magnet creates the magnetic flux Φ_1 in the coil.

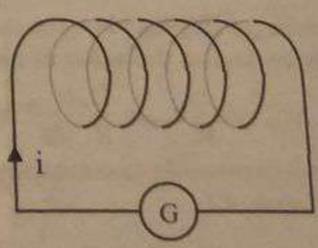


Figure 1.b: The current, of increasing intensity with respect to time, delivered by the generator creates a magnetic flux Φ_2 in the coil..

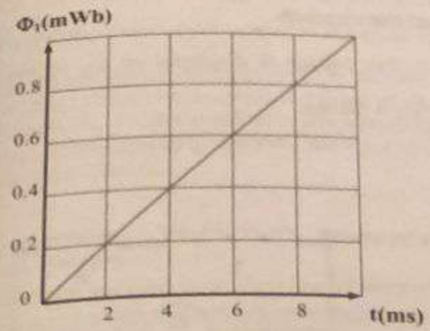


Figure 2.a: Variation of Φ_1 as a function of time.

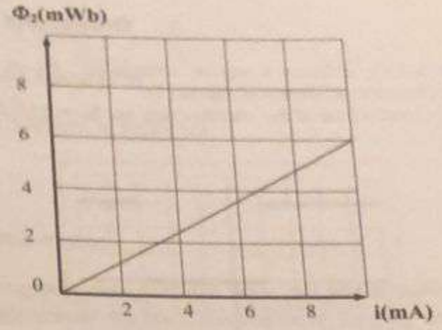


Figure 2.b: Variation of Φ_2 as a function of the intensity of current.

I – Specify each of the phenomena in the above experiments.

II – Studying the first experiment

1) Calculate, using figure 2.a, the induced e.m.f in the coil.
2) Knowing that the ammeter indicates a current of $i = -5mA$.
a) What explains the negative sign of i ?
b) Applying Lenz's law, Specify the direction of motion of the magnet with respect to the coil.
c) Deduce the resistance r of the coil.

III – Studying the second experiment

1) Calculate, using figure 2.b, the inductance L of the coil.
2) Specify the role of the coil (generator or receiver).

Part B : Transformation of the voltage

We connect the circuit in figure (3).

1) The switch K is closed.
a) The lamp doesn't light up. Justify?
b) Calculate, in the steady phase, the intensity of the current and the magnetic energy W_0 dissipated in the coil.
2) We open K at an instant taken as an origin of time ($t_0 = 0$).
a) What is the intensity of the current I_0 which circulates in the circuit at the moment of opening the switch K.
b) The lamp lights up. Justify.
c) Determine the differential equation which describes the variation of the intensity of the current as a function of time.

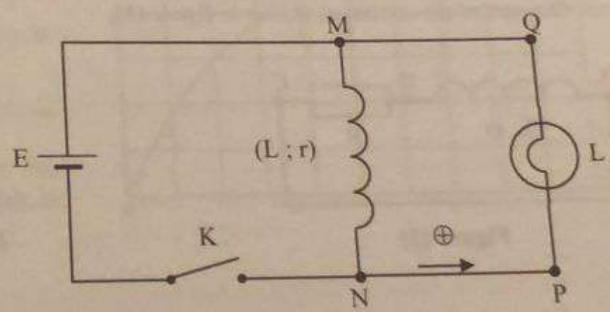


Figure (3)

d) Verify that the solution of the preceding differential equation is under the form : $i = Ae^{-B.t}$ where A and B are constant to be determined.
e) Calculate the instant when the lamp turns off.

N° 13
Growth and decay of the current

An L.F.G. delivers a square voltage (0 ; E) of period T, this voltage is displayed on the screen of the oscilloscope as shown in figure (1).
The sensitivities of the oscilloscope are $S_V = 2$ V/div and $S_h = 5$ ms/div.

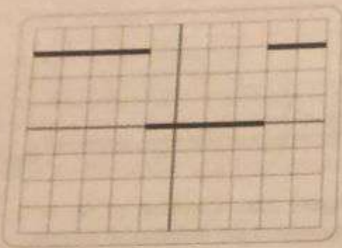


Figure (1)

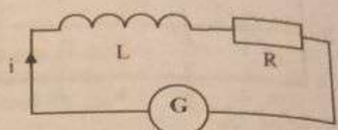


Figure (2)

A resistor, of resistance R = 100 Ω, is branched in series across a purely inductive coil of inductance L = 100mH. The coil and the resistor are connected across an L.F.G as shown in figure (2).

1) Represent, on a figure, the connections of the oscilloscope for the voltage of the L.F.G to be displayed on the channel Y_1 and of the resistor to be displayed on Y_2.
2) The oscillogram of the voltage across the terminals of the resistor is the image of the electric current in the circuit. Justify.
3) Extract T, E and calculate time constant τ of the circuit.
4) Represent, with justification, on the screen of the oscilloscope the curve of the voltage across the terminals of the resistor.
5) Resolve part **4)** for R = 1000 Ω and L = 10 H.

N° 14
Graphical methods to evaluate the time constant of an (L;R) circuit

A part of an electric circuit carries, in series, a coil of inductance L, and of resistance r = 5 Ω, and a resistor of resistance R = 10 Ω. This part is traversed by a current of constant intensity I_0 . At an instant $t_0 = 0$ we short circuit this part of the circuit, as shown in figure (1).

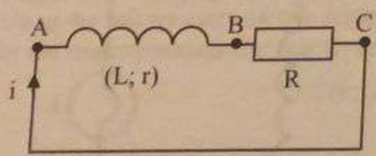


Figure (1)

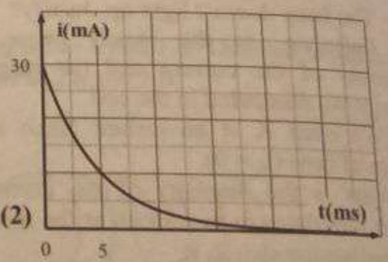


Figure (2)

1) Determine, as a function of L, r, R, the differential equation which governs the variation of the current i in the circuit with respect to time.

2) The solution of the differential equation is under the form : $i = \alpha e^{-\frac{t}{\tau}}$. Determine the expressions of α and τ as a function of L, r, R and I_0.
3) Show that τ is expressed in s in SI.
4) Calculate i, at instant t = τ, as a function of I_0.

5) Let (C) be the curve of variation of i as a function of time.

a) Show that the slope of the tangent to (C) at the instant t = 0, is : $\left.\frac{di}{dt}\right)_{t=0} = -\frac{I_0}{\tau}$

b) Write the equation of the tangent (T) to the curve (C) at the instant t = 0.

c) Show that (T) cuts the time axis at a point of abscissa t = τ.

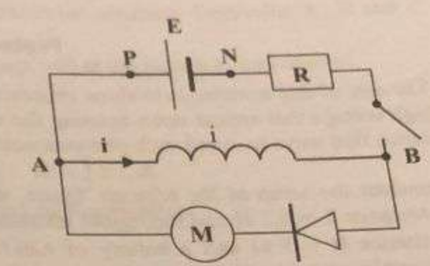
6) The graph of (C) corresponding to the preceding portion is shown in figure (2).

a) Extract, in two graphical methods, the value of τ.

b) Deduce the value of L.

N° 15
Study of an electric motor

The aim of this exercise is to show an evidence of the magnetic energy stored by a coil which makes an electric motor function, whose characteristics are to be determined (its back-electromotive force E' and its internal resistance r).



We use a battery, of electromotive force E = 12 V and of negligible internal resistance, a resistor, of resistance R = 2 Ω, a coil, of inductance L and of negligible resistance, an electric motor (M), a pulley on which we enroll a string holding an object of weight W = 2.5 N, a diode of negligible threshold voltage, a switch, connection wires and an oscilloscope with memory.

By these dipoles we realize the adjacent circuit.

A – Closing the switch

We close the switch at the time t = 0.

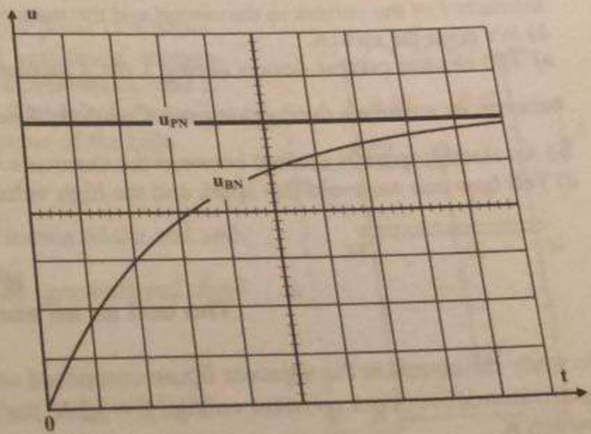
1) The motor does not function. Justify.

2) a) Represent, on a figure, the branching of the oscilloscope in order to visualize the voltages u_{PN} and u_{BN}.

b) The voltage u_{BN} is the image of i. Justify.

c) Verify the following differential equation: $\tau\frac{du_{BN}}{dt} + u_{BN} = E$ where τ is a physical quantity to be identified.

d) Deduce the value of u_{BN} and the intensity I_0 of current in steady state.



3) In the adjacent document, we represent the oscillograms visualized by the oscilloscope that is regulated to a vertical sensitivity S_V horizontal S_h = 20 ms.

a) Calculate S_V.

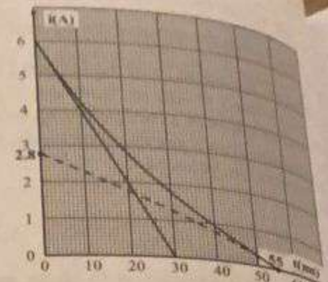
b) By the aid of the document find τ and verify that : L = 0.12 H.

c) Calculate the magnetic energy stored in the coil in the steady state.

B – Opening the switch

We open the switch, the current circulates from B to A in the motor and making it function for a duration.

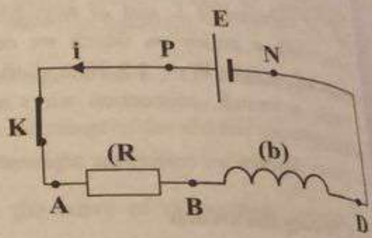
1) What is the origin of the energy allowing the motor to function ?
2) During the rotation of the motor, the object rises by a height $h = 31$ cm.
a) Calculate mechanical energy delivered by the motor.
b) The efficiency ρ of the motor is the ratio of the supplied mechanical energy to the received electric energy. Calculate ρ.
3) The motor has a back-electromotive force E' and an internal resistance r (the voltage across the terminals of the motor is : $u_{BA} = E' + ri$).
a) Verify the following differential equation : $\frac{di}{dt} + \frac{r}{L}i = -\frac{E'}{L}$.
b) In the adjacent figure we represent the graph of i as a function of time and of its tangents at the times $t_0 = 0$ and $t_1 = 55$ ms.
Calculate E' and r.



N° 16
Protection against high voltage

The aim of this exercise is to show evidence of the phenomenon of high voltage that appear upon opening the switch in an (RL) series circuit find ways to avoid such phenomenon..

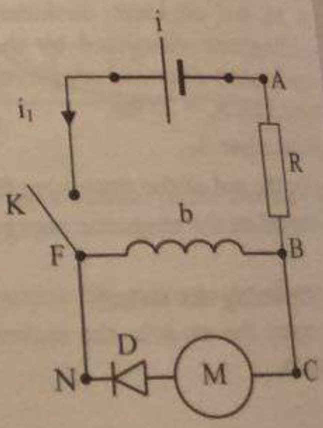
Consider the setup of the adjacent figure, where : (b) is a coil, of inductance $L = 0.2$ H and negligible resistance, (R) is a resistor of resistance $R = 5\ \Omega$ and a battery of e.m.f. $E = 12$ V and with negligible internal resistance.



1) The switch is closed. Calculate, in the permanent phase, the intensity I of the current in the circuit and the magnetic energy stored in the coil.
2) We open the switch.
a) The electric current decays during 1 ms. Calculate the average power liberated by the coil and the voltage between its terminals during this time. Conclude. Use for small durations : $\frac{di}{dt} \approx \frac{\Delta i}{\Delta t}$.
b) An electric spark is created between the terminals of the switch. Justify.
c) Tell how can we avoid this spark and the high voltage during the opening of the switch.

N° 17
The coil as an energy storing device

We study the circuit in the adjacent figure composed of :
A generator delivering a constant voltage $E = 12$ V ;
a switch K ;
a resistor of resistance $R = 10\ \Omega$;
a coil of negligible resistance and of inductance $L = 2$ H ;
a small electric motor (M) of a back electromotive force E' and of resistance $r = 5\ \Omega$;
an ideal diode D ($u_D \cong 0$V).



1) We close K at $t_0 = 0$. The generator delivers a constant current i1.
a) The motor can't function, why ?

b) Name the phenomenon which appears in the circuit. Is this phenomenon permanent ?

c) Determine the differential equation which describes the variation of the current i_1 in the circuit with respect to time and as a function of E, R and L.

d) The function $i_1 = I_0(1 - e^{-\frac{t}{\tau}})$ is a solution of the preceding differential equation. Give the expression of I_0 and τ as a function of E, L and R.

e) Calculate the intensity of the current in the steady phase and the maximum magnetic energy W_a dissipated in the coil.

2) The steady phase is established in the circuit. We open K, at a new instant taken as an origin of time.

a) The motor functions. Why ?

b) Knowing that the voltage across the motor is: $u_{CN} = E' + ri_2$, write the differential equation which describes the variation of the current i_2 as a function of time.

c) The function : $i_2 = Ae^{-\frac{t}{\tau'}} + B$ is a solution of the preceding differential equation. Determine A , B and τ' as a function of E, E', L, r and R.

d) Calculate, as a function of r, R and E, the voltage of the coil $u_{BF}(t = 0)$ at the instant K was opened.

e) The motor starts functioning for any value of E'. Why ?

f) We suppose that $E' = 0.5$ V, $r = 5\Omega$.

i. Calculate the duration of functioning of the motor.

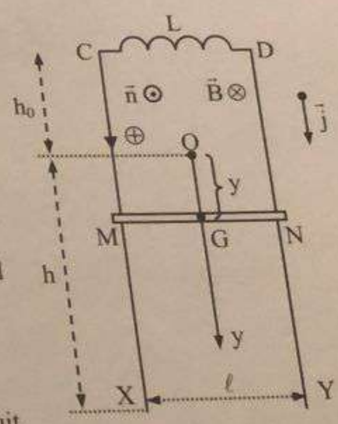
ii. Knowing that the mechanical energy dissipated by the motor : $W' = \int E'i_2 dt$.

Calculate the mechanical energy dissipated by the motor during its functioning.

iii. Deduce the output of the motor .

N° 18
Electromagnetic Induction and self-induction

A rectilinear rod, of length ℓ, of mass m, launched from point O without initial speed at $t_0 = 0$, performs a translational and rectilinear motion downward along two vertical rails (CX) and (DY). The extremities C and D form a coil of inductance L and negligible resistance. The whole setup is placed in a uniform magnetic field perpendicular to the plane of the rails. We neglect friction and the resistance in the circuit.



We designate by $y = \overline{OG}$ the ordinate of the center of inertia of the rod and by $\vec{V} = V.\vec{j} = \frac{dy}{dt}.\vec{j}$ its speed at time t and by g the gravitational field strength.

1) a) Verify that the rod is the seat of an i.e.m.f. such that : $u_{MN} = -B\ell V$.

b) Specify the direction of current of intensity i which flows in the circuit during the descent of the rod.

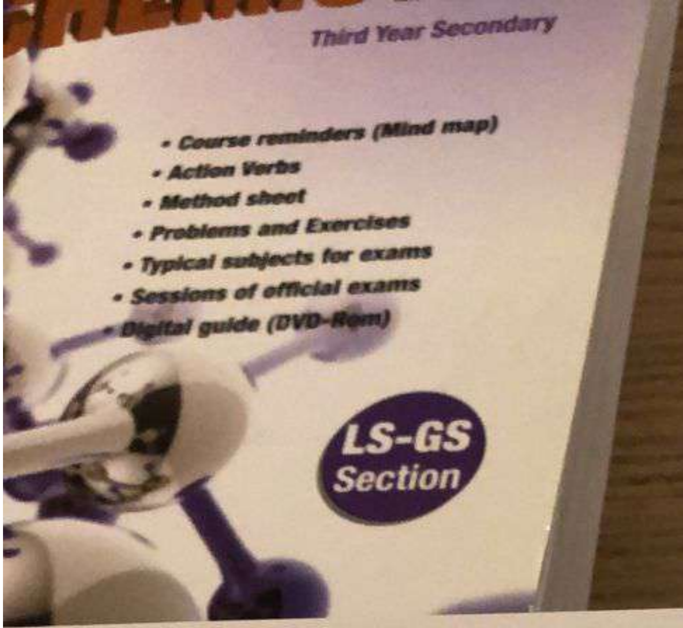
2) a) The rod is subjected to two forces. Name these forces and give their literal expressions.

b) Applying Newton's 2nd law on the rod, show that : $y'' = \frac{dV}{dt} = g - \frac{B\ell}{m} i$.

3) a) Applying the law of addition of voltage, find V as a function of L, B, ℓ and $\frac{di}{dt} = i'$.

b) Deduce y as a function of i, B, L and ℓ .

4) a) Establish from above the following differential equation : $y'' + \frac{B^2\ell^2}{mL} y = g$.



b) The solution of the preceding differential equation is : $y = \frac{mgL}{B^2\ell^2}(1-\cos\omega t)$.

i – Determine the expressions of A and ω as a function of B, ℓ , m and L.
ii – Specify the nature of motion of G.

b) The total energy $\mathcal{E}_T$ of the system [rod ; coil ; Earth] is the sum of the mechanical energy of the system [rod ; Earth] and of the magnetic energy stored in the coil. The reference level of the gravitational potential energy is the horizontal plane at a height h below O.

i – Verify that at the moment of launching of the rod : $\mathcal{E}_T = mgh$.
ii – To what forms of energy is the initial potential energy of the rod transformed to?
iii – Verify that $\mathcal{E}_T$ is conserved.