Diffraction of light

1- Wavelength of a laser source in a school

A laser beam of wavelength λ falls normally on a thin rectangular slit of width *a*. A diffraction figure is produced on a screen, parallel to the slit. Let D be the distance between the slit and the screen.

1) Precise the position of the center O of the central bright fringe.

2) Let O_k be the center of the dark fringe of order k. Show that : $\overline{OO_k} = k\frac{\lambda D}{a}$.

3) Deduce that the width of the central bright fringe is : $\ell = 2\frac{\lambda D}{a}$ and that of its lateral fringe is $\ell_1 = \frac{\lambda D}{a}$.

4) For D = 2 m and *a* = 100 μm, we find : $\ell = 2.7\,\text{cm}$ and $\ell_1 = 1.4\,\text{cm}$.

a) Do the values of ℓ and ℓ_1 agree with those obtained from the theory.

Deduce the average value of λ

Correction ▼

2- Studying diffraction of light

We consider two electromagnetic monochromatic waves of frequencies: $\nu_1 = 2.10^{17}$ Hz and $\nu_2 = 5.10^{14}$ Hz falling normally on a vertical rectangular slit of width *a* = 50 μm.

Given that the speed of light in vacuum is : c = 3×10^8 m/s.

1) a) Calculate the wavelengths of the radiations.
b) Which of the above waves produces the diffraction phenomenon using the slit ? Justify.
2) The diffraction figure of the suitable wave is produced on a vertical screen placed at a distance D = 2.5 m from the slit.
a) Calculate the angular width α of the bright fringe.
b) How would the angular width become if :

- We move the screen away and parallel to itself by 50 cm from the slit ?
- The device is put in water of index of refraction $n = \frac{4}{3}$?

Correction ▼

3- Diffraction of light

A monochromatic light produced by a source S, of wavelength λ = 625 nm, falls normally, on a rectangular slit F of width *a* in an opaque screen (E_0). We observe the diffraction phenomenon on the screen (E) parallel to (E_0) and placed at a distance D = 4 m from (E_0) as shown in figure (1).

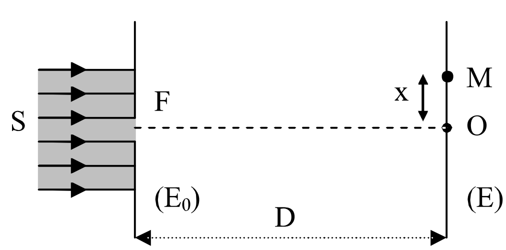


Figure (1)

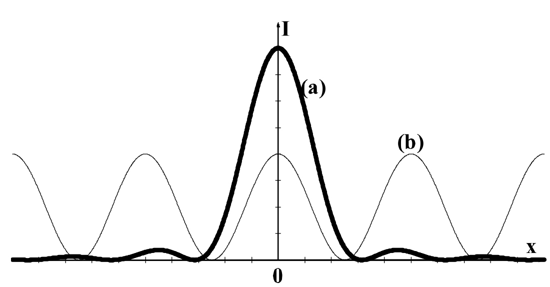


Figure (2)

1) Describe the diffraction pattern observed on (E).
2) The figure of diffraction shows :
a) An evidence of a certain aspect of light. Which one ?
b) An error in a certain principle of light. Which one ?
3) In figure (2) we represent, with scale, the variation of the luminous intensity as a function of the position x of a point M of the screen (E).
Indicate which of the two curves (a) or (b), is the corresponding curve for diffraction. Justify the answer.
4) The width of the first bright fringe (lateral bright fringe), from the central fringe, is 2.5 cm. Calculate *a*.
5) Calculate the width of the main figure observed in the cases where:
  a) $a = 1$ cm ;
  b) $a = 1$ μm

Correction ▼

4- Various applications

1) The diffraction, of a monochromatic light of a wavelength 670 nm, through a rectangular slit of width *a* produces on a screen at 82.3 cm from the slit a figure of diffraction such that the distance separating the center of the first dark fringe to the right to the center of the third dark fringe to the left is 5.2 mm. Calculate *a*.

2) In an aim to determine the diameter « d » of a hair, we place it normal to the direction of propagation of a He-Ne laser beam of wavelength 632.8 nm. A figure of the produced diffraction is found on a screen at 2.7 m where the width of the central spot is 24.2 mm. Calculate d. The hair is a similar to a slit of width « d ».

Correction ▼

5- Measuring a distance

A- Studying diffraction using a circular slit
A monochromatic light, of wavelength λ, falls normally on a circular hole (F) of center I and radius R. A diffraction pattern is produced, at a distance D (D >> R), on a screen (E) parallel to the slit.

1) Describe the figure observed on (E).
2) Precise the position O of the center of the central fringe.
3) If M is a point of the dark fringe.

The angular width of M is given by : $\theta = \hat{OIM} = p\dfrac{\lambda}{R}$ where p is a real number whose values are : …, -1.62 ; -1.12 ; -0.61 ; 0.61 ; 1.12 ; 1.62 ; ….. .

a) Let x'x be an axis passing through O and oriented in a given direction. Determine, as a function of p, λ, D and R, the linear abscissas $x = \overline{OM}$.
b) Represent the curve of the luminous intensity I_x of M as a function of x.
c) Determine the expression of the diameter d of the central fringe.

B – Measuring distance

Rayleigh Criterion : The human eye can observe distinctly the images of two luminous sources obtained by diffraction through the pupil if the same image is received by retinue's grains and if the center of one of the images is farther from the first minimum of the other.

A person can distinguish the two headlamps of a car separated by a distance Δ = 1.42 m, when it is a minimum distance D.
We suppose that the headlamps are monochromatic sources of light of an average wavelength 562 nm.
The diameter of the pupil (which plays the role of a circular slit) is 5 mm.

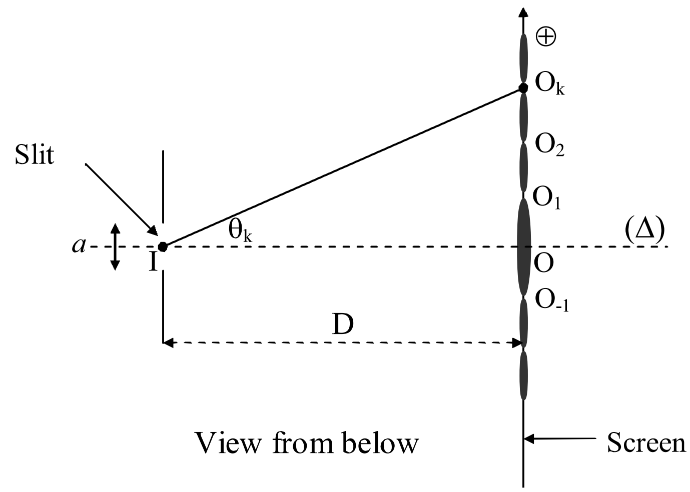
Using Raleigh's criterion**, calculate the distance D.**

Correction ▼

Corrections

1- Wavelength of a laser source in a school

1) Let (Δ) be the direction of propagation before reaching the slit. The prolongation of (Δ) cuts the screen at a point O center of the central fringe.



2) The angular width of the dark fringe of order k is given by the formula : $\sin\theta_k = k\frac{\lambda}{a}$ with k being a relative integer.

Now θ_k is small, hence $\sin\theta_k \approx \tan\theta_k \approx \theta_k$

In the triangle IOO_k, right at O, we have :

$$\tan\theta_k = \frac{\overline{OO_k}}{OI} = \frac{\overline{OO_k}}{D}$$

But : $\sin\theta_k \approx \tan\theta_k \Rightarrow k\frac{\lambda}{a} = \frac{\overline{OO_k}}{D} \Rightarrow \overline{OO_k} = k\frac{\lambda D}{a}$

3) The central fringe is limited by dark fringes of orders −1 and 1, hence its width is:

$$\ell = \overline{O_{-1}O_1} = \overline{OO_1} - \overline{OO_{-1}} = \frac{\lambda D}{a} - \left(-\frac{\lambda D}{a}\right) = 2\frac{\lambda D}{a}$$

The first bright fringe in the positive sense is limited by dark fringes of orders 1 and 2, hence its width is :

$$\ell_1 = \overline{O_1O_2} = \overline{OO_2} - \overline{OO_1} = 2\frac{\lambda D}{a} - \frac{\lambda D}{a} = \frac{\lambda D}{a}.$$

4) a) We have : $\ell = 2\frac{\lambda D}{a}$ and $\ell_1 = \frac{\lambda D}{a} \Rightarrow \ell = 2\ell_1$, hence theoretically the central fringe is double that of the lateral bright fringe.

Experimentally we have : $\ell = 2.7\,\text{cm}$ and $\ell_1 = 1.4\,\text{cm} \Rightarrow \ell \approx 2\ell_1$.

Hence the experimentation is in accordance with the theory.

b) $\ell = 2\frac{\lambda D}{a} \Rightarrow \lambda = \frac{\ell a}{2D} = \frac{2.7\times10^{-2}\times100\times10^{-6}}{2\times2} = 0.675\,\mu\text{m}$

$\ell_1 = \frac{\lambda D}{a} \Rightarrow \lambda = \frac{\ell_1 a}{D} = \frac{1.4\times10^{-2}\times100\times10^{-6}}{2} = 0.7\,\mu\text{m}$

The average value of λ, is : $\lambda_{av} = \frac{0.675+0.7}{2} = 0.6875\,\mu\text{m}$. $\boxed{\lambda_{av} = 0.6875\,\mu\text{m}}$.

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2- Studying diffraction

a) The wavelength of the first wave : $\lambda_1 = \frac{c}{\nu_1} = \frac{3\times10^8}{2\times10^{17}} = 1.5\,\text{nm}$.

The wavelength of the second wave : $\lambda_2 = \frac{c}{\nu_2} = \frac{3\times10^8}{5\times10^{14}} = 0.6\,\mu\text{m}$.

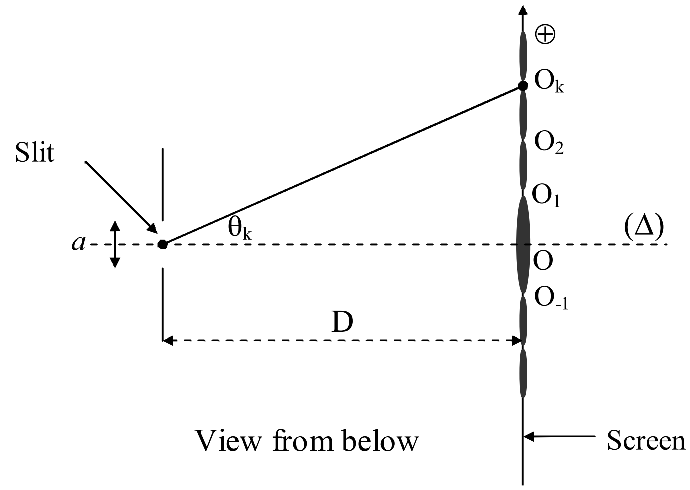
b) A wave of wavelength λ produces a diffraction phenomenon using a slit of width *a* if : *a* and λ are of the same order of magnitude.

By comparison : *a* and λ_2 have the same order of magnitude hence the frequency ν_2 is subjected to diffraction by the slit.

2) a) The angular width of the dark fringe of order k is given by the formula : $\sin\theta_k = k\frac{\lambda_2}{a}$ with k being an integer.

Now θ_k is small hence $\sin\theta_k \approx \theta_k \Rightarrow \theta_k = k\frac{\lambda_2}{a}$

The central bright fringe is limited by dark fringes of orders−1 and 1 symmetric with respect to the center of the central fringe. Hence : $\alpha = 2\theta_1 = 2\frac{\lambda_2}{a}$.



Numerically: $\alpha = 2\frac{0.6\,\mu}{50\,\mu} = 0.024\text{ rad}$. $\boxed{\alpha = 0.024\text{ rad}}$.

b) The angular width is independent of the distance between the slit and the screen hence it remains constant for all the positions of the screen with respect to the slit.

The wavelength depends on the medium. Hence the angular width changes its value as the medium changes.
The angular width in water : $\alpha' = 2\frac{\lambda'_2}{a}$ with λ'_2 is the wavelength of in water.

The frequency is independent of the medium it is a characteristic of the source:

The speed of light in air : $c = \lambda_2.\nu_2$ and that in water : $V = \lambda'_2.\nu_2$

The index of refraction of water : $n = \frac{c}{V} = \frac{\lambda_2.\nu_2}{\lambda'_2.\nu_2} = \frac{\lambda_2}{\lambda'_2} \Rightarrow \lambda'_2 = \frac{\lambda_2}{n}$.

Therefore : $\alpha' = 2\frac{\lambda_2}{na} = \frac{\alpha}{n} = \frac{0.024}{\frac{4}{3}} = 0.018\text{ rad}$. $\boxed{\alpha' = 0.018\text{ rad}}$ the angular width is smaller in water ($\alpha' < \alpha$).

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3- Diffraction of light

1) The diffraction figure is formed of diffraction fringes that are :
- Alternating bright and dark ;
- All the fringes are aligned on the screen in a direction orthogonal to the direction of the slit ;
- The width of the central fringe is double that of the lateral bright fringes ;
- The intensity of light at the central bright fringe is maximum and decreases as we move away from the central fringe.

2) a) The diffraction figure shows the evidence of the **wave aspect of light**.
b) The diffraction pattern contradicts the principle of the rectilinear propagation of light.

3) Curve (a) corresponds the diffraction phenomenon since, the maximum luminous intensity decreases as the light sensor moved away from the center O of the diffraction figure.

4) The width of the central fringe is given by : $\ell = 2\frac{\lambda D}{a}$ and that of the lateral fringe : $\ell_1 = \frac{\ell}{2} = \frac{\lambda D}{a} \Rightarrow a = \frac{\lambda D}{\ell_1} = \frac{625\times10^{-9}\times4}{2.5\times10^{-2}} = 10^{-4}\,\text{m}$. $\boxed{a = 0.1\,\text{mm}}$.

5) a) We have : *a* = 1 cm and λ = 625 nm, hence *a* and λ (a >>> λ) are not of the same order of magnitude hence we observe a rectangular spot identical to the slit of width *a* = 1 cm.

b) We have : *a* = 1 μm and λ = 0.625 μm, hence *a* and λ are of the same magnitude hence the diffraction phenomenon is observed.

The width of the central fringe is given by: $\ell = 2\frac{\lambda D}{a} = 2\frac{625.10^{-9}\times4}{10^{-6}} = 5\,\text{m}$.

Remark : For the visible wavelengths (400 nm ≤λ≤ 800 nm) and if *a* ≤ 1 mm ; there will be diffraction.

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4- Various applications

1) Let O be the center of the central bright fringe, O_k the center of the dark fringe of order k and x_k is the abscissa of the center of the dark fringe as shown in the next figure

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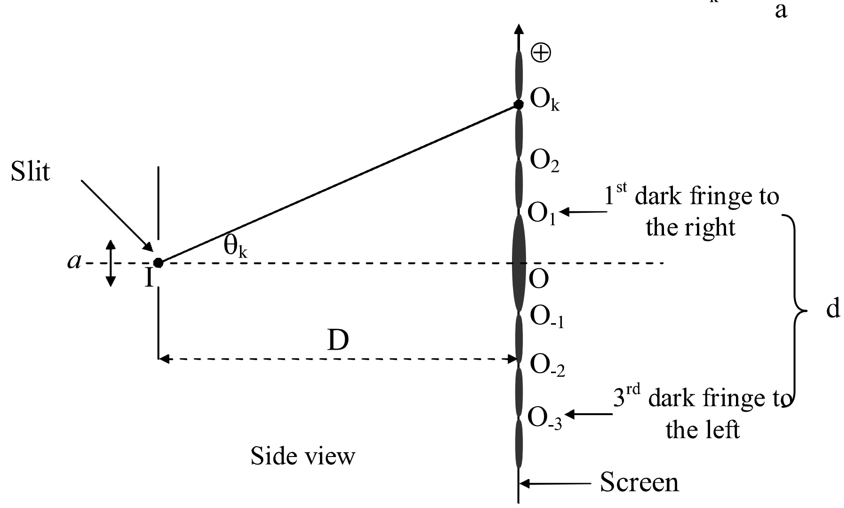
The angular width of the dark fringe of order k is given by : $\sin\theta_k = k\frac{\lambda}{a}$ with k being an integer.



Now θ_k are small, then $\sin\theta_k \approx \tan\theta_k \approx \theta_k$

In the triangle IOO_k, right at O, we have : $\tan\theta_k = \frac{\overline{OO_k}}{OI} = \frac{\overline{OO_k}}{D} = \frac{x_k}{D}$

But : $\sin\theta_k \approx \tan\theta_k \Rightarrow k\frac{\lambda}{a} = \frac{x_k}{D} \Rightarrow x_k = k\frac{\lambda D}{a}$

The distance between the first minimum to the right and the third minimum to the left is: $d = \overline{O_{-3}O_1} = x_1 - x_{-3} = \frac{\lambda D}{a} - \left(-3\frac{\lambda D}{a}\right) = 4\frac{\lambda D}{a} \Rightarrow a = 4\frac{\lambda D}{d} = 4\frac{670\times10^{-9}\times82.3\times10^{-2}}{5.2\times10^{-2}} = 42.416\times10^{-6}\,\text{m}$ $\boxed{a = 42.416\,\mu\text{m}}$.

2) The width of the central fringe is given by :
$\ell = 2\frac{\lambda D}{d} \Rightarrow d = 2\frac{\lambda D}{\ell} = 2\frac{632.8\times10^{-9}\times2.7}{24.2\times10^{-3}} = 141.2\times10^{-6}\,\text{m}$. $\boxed{d = 0.1412\,\text{mm}}$.

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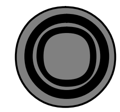
5- Measuring a distance

A – Studying diffraction using a circular slit

1) The diffraction figure is formed of a central quite bright circular spot surrounded by concentric alternate bright and dark rings.

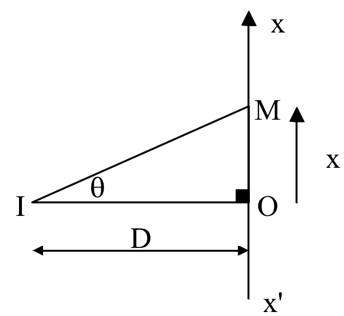
2) The center O is the intersection of the axis of the slit with the screen.



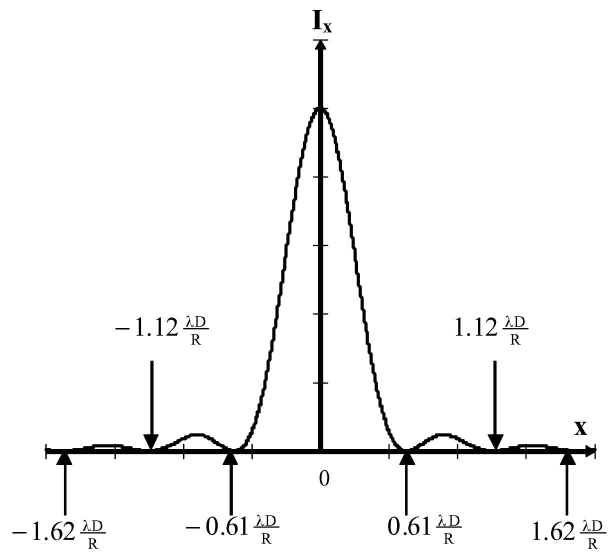
3) a) Given that θ is small, then : $\tan\theta \approx \theta$

In the triangle IOM, right at O, we have : $\tan\theta = \dfrac{\overline{OM}}{OI} = \dfrac{x}{D}$

But : $\tan\theta \approx \theta \Rightarrow p\dfrac{\lambda}{R} = \dfrac{x}{D} \Rightarrow \boxed{x = p\dfrac{\lambda D}{R}}$



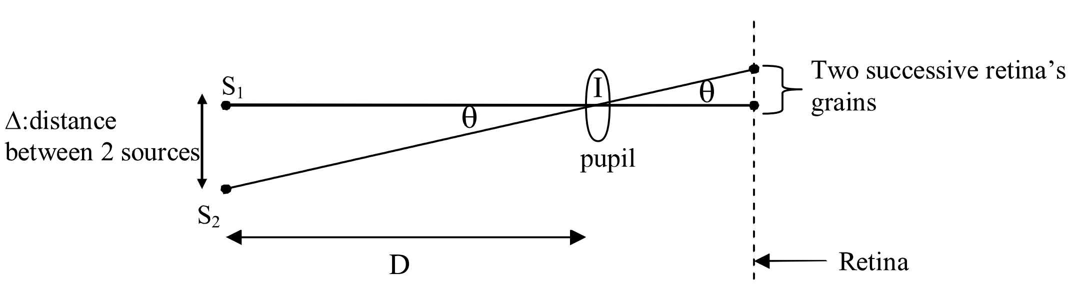
b) The curve of I_x as a function of x.



c) The diameter of the central fringe is the distance between the first minimum to the right and that on the left, this is then : $d = 2\times 0.61\dfrac{\lambda D}{R} = 1.22\dfrac{\lambda D}{R}$.

B – Measuring distance

Using Rayleigh criterion: The minimum angular between the centers of the two images represent the distance separating the center and the first minimum of an image : $\theta = 0.61\dfrac{\lambda}{R}$



But θ is small (D >> Δ), from the preceding figure : $\theta \approx \dfrac{\Delta}{D}$

$$\Rightarrow \frac{\Delta}{D} = 0.61\frac{\lambda}{R} \Rightarrow D = \frac{\Delta \times R}{0.61\lambda} = \frac{1.42 \times \frac{5.10^{-3}}{2}}{0.61 \times 562 \times 10^{-9}} = 10355.3\ \text{m} = 10.553\ \text{km}\ .\ \boxed{D = 10.553\ \text{km}}\ .$$

c) The diameter of the central fringe is the distance between the first minimum to the right and that on the left, this is then : $d = 2 \times 0{,}61\frac{\lambda D}{R} = 1{,}22\frac{\lambda D}{R}$.

B – Measuring distance

Using Rayleigh criterion: The minimum angular between the centers of the two images represent the distance separating the center and the first minimum of an image : $\theta = 0{,}61\frac{\lambda}{R}$

But θ is small (D >> Δ), from the preceding figure : $\theta \approx \frac{\Delta}{D}$

$$\Rightarrow \frac{\Delta}{D} = 0{,}61\frac{\lambda}{R} \Rightarrow D = \frac{\Delta . R}{0{,}61\lambda} = \frac{1{,}42 \times \frac{5.10^{-3}}{2}}{0{,}61 \times 562 \times 10^{-9}} = 10355{,}3\ m = 10{,}553\ km\ .\ \boxed{D = 10{,}553\ \text{km}}\ .$$

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