Solved Problems

N°1.

Answer by True or False with justification:

1) The solution of the inequality $\ln|1-x| \leq 0$ is the interval $[-1;0]$.
2) The function f defined over $IR - \{-1;1\}$ by $f(x) = \ln|1-x^2|$ is increasing over $[0;1[$.
3) The equation $x\ln x - x - 1 = 0$ has a unique solution for $x > 0$.
4) $\int \frac{1}{x(x+1)}dx = \ln\left|\frac{x}{x+1}\right| + C$.
5) For all real numbers x, $\ln x(x+1) = \ln x + \ln(x+1)$.
6) For all real numbers x, $\ln x^2 = 2\ln x$.

N°2.

Solve, in IR, the following equations:

1) $\ln(x+2) + \ln(x-1) = 2\ln x$
2) $\ln(2x+4) - \ln(x-1) = \ln(x)$
3) $\ln(10 - x^2) = 2\ln 3 - \ln x^2$
4) $2\ln\left|\ln\left(x^2\right)\right| = \ln 9$

N°3.

Solve, in IR, the following inequalities:

1) $\ln\left(x^2 - 3\right) - \ln 2 > \ln x$
2) $\ln\left(x^2 - 2x\right) > \ln(4x - 5)$

N°4.

Calculate the following integrals:

1) $\int_0^1 \frac{x+1}{x^2+2x+3}dx$
2) $\int_e^{e^2} \frac{dx}{x\ln x}$
3) $\int_1^e \frac{\ln x}{x}dx$
4) $\int_1^e \ln x\, dx$
5) $\int_1^e x\ln x\, dx$
6) $\int_2^3 \frac{-3x^2+4x-3}{x-1}dx$
7) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2-\cos x}dx$
8) $\int_0^{\frac{\pi}{6}} \tan(2x)dx$

Solved Problems

N°5

1) Consider the function f defined over $\mathbb{R}-\{1,2\}$ by $f(x)=\frac{2x-1}{(x-1)(x-2)}$

   a- Write $f(x)$ in the form: $f(x)=\frac{a}{x-1}+\frac{b}{x-2}$.

   b- Calculate $\int_3^4 f(x)dx$.

2) Decompose $\frac{1}{x^2-6x+5}$ into a sum of two rational fractions, then calculate: $\int_2^3 \frac{dx}{x^2-6x+5}$.

3) Calculate the following integrals:

   a- $\int_1^2 \frac{1}{x(x+1)}dx$ b- $\int_3^4 \frac{1}{x^2+x-2}dx$ c- $\int_1^2 \frac{dx}{x\left(x^2+1\right)}$

N°6

Consider the two integrals $I_1=\int_0^1 \frac{x}{1+x^2}dx$ and $I_2=\int_0^1 \frac{x^3}{1+x^2}dx$.

1) Calculate I_1.
2) Calculate I_1+I_2 and deduce the value of I_2.

N°7

Calculate the following limits:

1) $\lim\limits_{x\to+\infty}(x-\ln x)$
2) $\lim\limits_{x\to+\infty}\left(x-\frac{\ln x}{x}\right)$
3) $\lim\limits_{x\to+\infty} x\ln\left(1+\frac{1}{x}\right)$
4) $\lim\limits_{x\to+\infty}\frac{2-\ln x}{1-\ln x}$
5) $\lim\limits_{x\to+\infty}\left(\ln^2 x-2\ln x+2\right)$
6) $\lim\limits_{x\to0^+} 2x(1-\ln x)$
7) $\lim\limits_{x\to0^+}\left(\ln x+\frac{1}{x}\right)$
8) $\lim\limits_{x\to e}\frac{\ln x-1}{x-e}$

N° 8.

Solve the following systems of equations:

1) $\begin{cases} 2\ln x+\ln y=1 \\ 5\ln x+3\ln y=4 \end{cases}$ 2) $\begin{cases} (\ln x)(\ln y)=-15 \\ \ln(xy)=-2 \end{cases}$

3) $\begin{cases} x^2+y^2=10 \\ \ln x+\ln y=\ln 3 \end{cases}$ 4) $\begin{cases} \ln x+\ln y^2=4 \\ \ln^2 x-3\ln xy=-\dfrac{13}{2} \end{cases}$

N° 9.

Consider the function f defined over $]0;+\infty[$ by $f(x)=\dfrac{\ln x}{x}$, designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to 0^+} f(x)$.
2) Deduce the asymptotes to (C).
3) Study the variations of f and set up its table of variations.
4) Write an equation of the tangent (T) to (C) at the point of abscissa 1.
5) Draw (C) and (T).
6) Discuss according to the values of the real number m the number of solutions of the equation $x=e^{mx}$.
7) Deduce a comparison of the two numbers 2007^{2008} and 2008^{2007}.
8) Calculate the area of the region limited by (C), the axis $x'x$ and the two straight lines of equations $x=1$ and $x=e$.

N° 10.

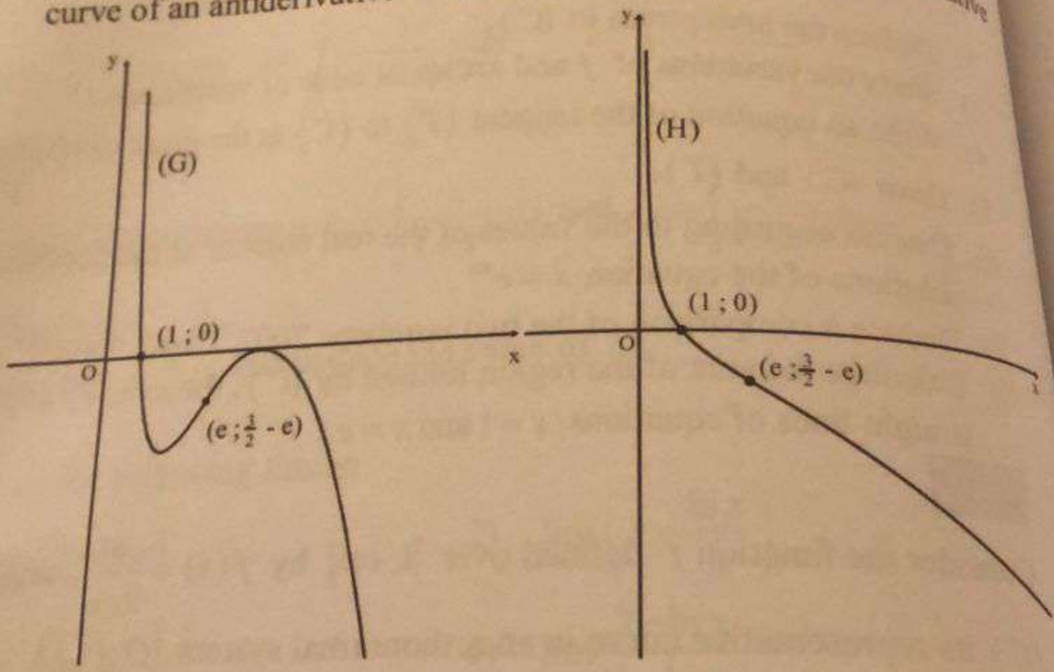
Consider the function f defined over $]1;+\infty[$ by $f(x)=\dfrac{x}{\ln x}$, designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to+\infty} f(x)$.
2) Show that (C) admits an asymptote parallel to the axis $y'y$.
3) Study the variations of f and set up its table of variations.
4) Deduce a comparison of the two numbers a^b and b^a for $e<a<b$
5) Calculate $f\left(e^2\right)$ and draw (C).

N° 11

Consider the function f defined over the interval $I =]0;+\infty[$ by $f(x) = \frac{\ln x - x}{x}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. (*unit : 2cm*).

1) a- Calculate the limits of f at the boundaries of the domain I.
   b- Determine the asymptotes of (C).
2) Calculate $f'(x)$ and set up its table of variations of f.
3) Verify that the tangent (d) to (C), at the point $A\,(1;-1)$, has an equation $y = x - 2$.
4) Draw the straight line (d) and the curve (C).
5) One of the two curves (G) and (H) drawn below is the representative curve of an antiderivative F of the function f.



a- Which of the curves above is that of the function F?

b- Without finding the expression of $F(x)$, calculate in cm^2, the area of region limited by the curve (C), the axis of abscissas and the two straight lines of equations $x = 1$ and $x = e$. Give your answer to the nearest 10^{-2}.

N°12.

Consider the function f defined over $]0;e[\cup]e;+\infty[$ by $f(x)=\dfrac{1}{x(1-\ln x)}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine the limits of at the boundaries of its domain of definition.
   b- Deduce the asymptotes to (C).
2) a- Study the variations of f and set up its table of variations.
   b- Draw (C).
3) Calculate the area of the region bounded by (C), the axis $x'x$ and the two straight lines of equations $x=\dfrac{1}{e^2}$ and $x=\dfrac{1}{e}$.

N°13.

Consider the function f defined over $]0;2[$ by $f(x)=\ln\left(\dfrac{x}{2-x}\right)$.

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine the limits of f at the boundaries of its domain of definition and deduce the asymptotes to (C).
   b- Study the variations of f and set up its table of variations.
2) a- Show that the point $A(1;0)$ is a center of symmetry of (C).
   b- Write an equation of the tangent (T) at A to (C).
3) Draw (C).
4) Show that the equation $f(x)=x$ has a unique root α and that $1.6<\alpha<1.7$.
5) a- Show that f has an inverse function f^{-1} over $]0;2[$.
   b- Determine the domain of definition of f^{-1} and draw its curve (C'), representative of f^{-1} in the same system.
   c- Determine $f^{-1}(x)$.
   d- Solve the equation $f(x)=f^{-1}(x)$.

N°14.

Part A.

Consider the function g defined over $]0;+\infty[$ by $g(x)=x+(x-2)\ln x$.

1) Show that $g'(x) = 2\dfrac{x-1}{x} + \ln x$.

2) Deduce that:
if $x > 1$ then $g'(x) > 0$ and if $0 < x < 1$ then $g'(x) < 0$.

3) Show that $g(x) \geq 1$.

Part B.

Consider the function f defined over $]0;+\infty[$ by $f(x) = 1 + x\ln x - \ln^2 x$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.
Graphical unit: 2 cm.

1) a- Verify that $f'(x) = \dfrac{g(x)}{x}$, study the limits of f at the boundaries of domain of definition and set up the table of variations of f.

b- Deduce that f has an inverse function f^{-1} defined over the interval to be determined.

2) a- Write an equation of the tangent (T) to (C) at the point of abscissa

b- Study the variations of the function h defined over $]0;+\infty[$ by $h(x) = x - 1 - \ln x$ and deduce the sign of $h(x)$.

c- Show that $f(x) - x = (\ln x - 1)h(x)$ and deduce the relative positions of (C) with respect to (T).

d- Determine the abscissa of the point B of (C) such that the tangent (T') at B to (C) is parallel to (T)

3) a- Draw (C) and (T).

b- f has an inverse function f^{-1} over $]0;+\infty[$, draw, in the same system the curve (C') representative of the function f^{-1}.

4) a- Calculate the integrals $I_1 = \int_1^e x\ln x\,dx$ and $I = \int_1^e (\ln x)^2\,dx$.

b- Designate by S the area, in cm^2, of the region limited by the curv (C) and (C'). Calculate to the nearest 10^{-3} the value of S.

N°15.

Part A.

Consider the function u defined over $]-\infty;0[\cup]0;+\infty[$ by
$u(x) = 2x^3 - 1 + 2\ln|x|$.

1) Calculate $\lim_{x \to 0} u(x)$ and $\lim_{x \to +\infty} u(x)$.
2) Study the variations of u and set up its table of variations.
3) Show that the equation $u(x) = 0$ admits over $[0.8\,;0.9]$ a unique root α.
4) Deduce the sign of $u(x)$ for $x \in \left]-\infty\,;0\right[\cup\left]0\,;+\infty\right[$.

Part B.

Consider the function f defined over $\left]-\infty\,;0\right[\cup\left]0\,;+\infty\right[$ by $f(x) = 2x - \frac{\ln|x|}{x^2}$.

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine the limits of f at the boundaries of its domain of definition. Deduce an asymptote to (C).
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Show that the straight line (d) of equation $y = 2x$ is an asymptote to (C) at $+\infty$ and at $-\infty$.
4) Study the position of (C) with respect to (d).
5) Taking $\alpha = 0.85$, calculate $f(\alpha)$ to the nearest 10^{-2} and draw (C).

N° 16.

1) Consider the function f defined over $\left]0,+\infty\right[$ by $f(x) = x + \ln x - \ln(x+1)$, and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.
   a- Show that the axis $y'y$ is an asymptote to (C).
   b- Show that the straight line (d) of equation $y = x$ is an asymptote to (C) in the neighborhood of $+\infty$.
   c- Study the variations of f and draw (C).
2) Consider the function g defined over $\left]-\infty,-1\right[\cup\left]0,+\infty\right[$ by $g(x) = x + \ln\left(\frac{x}{x+1}\right)$, and designate by (γ) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

   Show that the point $I\left(-\frac{1}{2};-\frac{1}{2}\right)$ is a center of symmetry (γ) and deduc the drawing of (γ).
3) a- Let $F(x) = (ax+b)\ln(ax+b)$, calculate $F'(x)$.

b- Deduce the area of the region limited by (C), (d) and the two lines of equations $x = 1$ and $x = e$.

N°17

Consider the function f defined over $]0;+\infty[$ by $f(x) = \frac{(\ln x)^2}{x}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine the limits of f at $+\infty$ and at 0 and deduce the asymptotes to (C).

b- Show that $f'(x)$ has the same sign as $(2-\ln x)\ln x$.

c- Study the variations of f and set up its table of variations.

d- Draw (C).

2) For $p \geq 1$, let $I_p = \int_1^{e^2} \frac{(\ln x)^p}{x^2}dx$.

a- Using integration by parts, calculate $I_1 = \int_1^{e^2} \frac{\ln x}{x^2}dx$.

b- Show that $I_{p+1} = -\frac{2^{p+1}}{e^2} + (p+1)I_p$.

c- Deduce I_2, I_3 and I_4.

3) Let (D) be the region limited by (C), $x'x$ and the two straight lines of equations $x = 1$ and $x = e^2$.

Calculate the volume of the solid generated by rotating (D) about $x'x$.

N°18

Consider the function f defined over $]0;+\infty[$ by $f(x) = x - \frac{\ln x}{x}$.

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to 0} f(x)$, $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty}(f(x)-x)$.

Deduce the asymptotes to (C).

2) The table below is the table of variations of the function f' derivative of f.

x	0		1		$e^{\frac{1}{2}}$		$+\infty$
$f''(x)$		$+$		$+$	0	$-$	
$f'(x)$	$-\infty$	$\nearrow$	0	$\nearrow$	$1+\frac{1}{2}e^{-1}$	$\searrow$	1

a- Study the variations of f and set up its table.

b- Prove that f has only one inflection point I.
Determine the coordinates of I to the nearest 10^{-2}.

c- Draw (C).

3) a- Prove that f has an inverse function f^{-1} over the interval $[1\,;+\infty[$ and determine the domain of definition of f^{-1}.

b- Draw the curve (C') representative of f^{-1} in the same system as that of (C).

c- Let $M(t; f(t))$ be a variable point of (C) with $t \geq 1$.
The straight line (d) passing through M and perpendicular to the first bisector of axes of equation $y = x$ cuts (C') at N.
Calculate MN in terms of t.

N° 19

Part A.

Consider the function f defined over $]0,+\infty[$ by $f(x)=\left(1-\dfrac{1}{x}\right)(-2+\ln x)$

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to +\infty} f(x)$, then deduce an asymptote to (C).

2) Calculate $f'(x)$.

3) Consider the function u defined over $]0,+\infty[$ by $u(x)=x-3+\ln x$.

a- Study the variations of u.

b- Show that the equation $u(x)=0$ has a unique solution α such that $2.20<\alpha<2.21$ and deduce the sign of $u(x)$.

4) Remarking that $f'(x)=\frac{u(x)}{x^2}$ study the variations of f and set up its table.

5) Express $\ln(\alpha)$ in terms of α and show that $f(\alpha)=-\frac{(\alpha-1)^2}{\alpha}$.

6) Suppose that $\alpha=2.205$, calculate $f(\alpha)$ and draw (C).

Part B.

Consider the function F the antiderivative f over $]0,+\infty[$ that vanishes at $x=1$.

1) Without calculating $F(x)$, study the variations of F over $]0,+\infty[$.

2) What can you say about the tangents to (C'), the representative curve of F at the the points of abscissas 1 and e^2?

3) Show that $f(x)=\ln(x)-\frac{\ln(x)}{x}+\frac{2}{x}-2$ and deduce the expression of $F(x)$.

4) Calculate the area of the region limited by (C), the axis $x'x$ and the straight lines of equations $x=1$ and $x=e^2$.

N°20

Consider the function f defined over $]0;+\infty[$ by $f(x)=-3-\ln x+2\ln^2 x$. Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

Part A.

1) Determine $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$ and deduce an asymptote to (C).

2) Solve the inequality $f(x)>0$.

3) Calculate $f'(x)$, study the variations of f and set up its table of variations.

4) Write an equation of the tangent (T) to (C) at the point of abscissa $e^{\frac{5}{4}}$

5) Consider the function g defined over $]0;+\infty[$ by

$$g(x)=f(x)-\left(4e^{-\frac{5}{4}}x-\frac{41}{8}\right).$$

a- Show that $g'(x)=\frac{4\ln x-1}{x}-4e^{-\frac{5}{4}}$ then calculate $g''(x)$.

b- Study the variations of g' over $]0;+\infty[$ and set up its table of variations.

c- Deduce that $g'(x) \leq 0$.

6) a- Calculate $g\left(e^{\frac{3}{4}}\right)$, then determine the sign of $g(x)$.

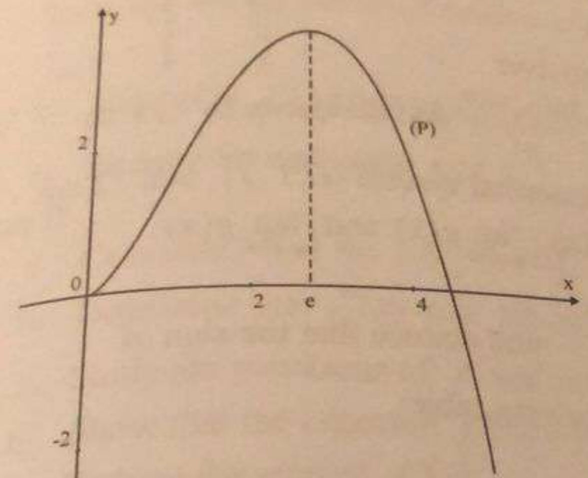
b- Deduce the position of (C) with respect to (T).

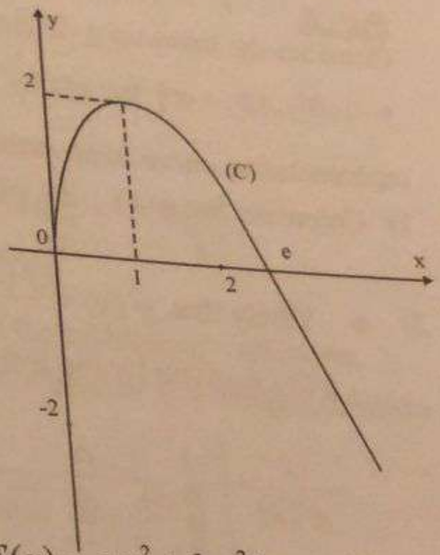
7) Draw (C) and (T).

8) Calculate the area of the domain limited by (C), the axis $x'x$ and the two straight lines of equations $x = \frac{1}{e}$ and $x = e^{\frac{3}{2}}$.

N° 21

1) The curves shown below are the representative curves of the functions f defined over $]0;+\infty[$ and its antiderivative F.
Determine the representative curve of f and that of F.





2) Suppose that the equation of F is $F(x) = ax^2 + bx^2 \ln x$ where a and b are two real numbers. Calculate a and b and deduce the expression of $f(x)$.

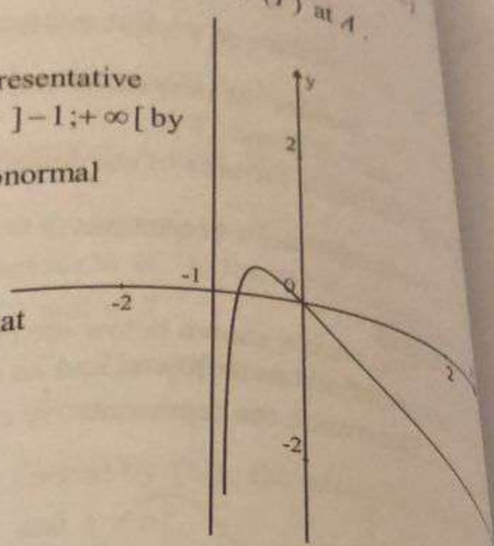
3) Discuss according to the values of the real parameter m the existence of the roots of the equation $x = e^{1-\frac{m}{2x}}$.

4) The function f has an inverse function f^{-1} for $x > 1$.

a- Calculate the coordinates of the point A, the intersection of (C) and the curve (C'), representative of f^{-1}.

b- Show that (C) and (C') have the same tangent (T) at A.

N°22



Part A.

The curve (C) to the right is the representative curve of the function f defined over $]-1;+\infty[$ by $f(x)=\frac{x}{x+1}-2\ln(x+1)$, in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Show that the equation $f(x)=0$ admits two roots 0 and α such that $\alpha\in]-0.8;-0.7[$.

2) Determine $\lim\limits_{x\to-1} f(x)$.

Part B.

Consider the function g defined over $]-1;0[\cup]0;+\infty[$ by: $g(x)=\frac{\ln(x+1)}{x^2}$ and designate by (γ) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. unit : 2 cm.

1) Calculate $\lim\limits_{\substack{x\to0\\x>0}} g(x)$, $\lim\limits_{\substack{x\to0\\x<0}} g(x)$, $\lim\limits_{x\to-1} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.

2) a- Verify that $g'(x)=\frac{f(x)}{x^3}$ and deduce that the sign of g'(x) is given by the following table:

x	-1		α		0		$+\infty$
$g'(x)$	‖	+	0	−	‖	−	

b- Set up the table of variations of g .

3) a- Show that $g(\alpha)=\frac{1}{2\alpha(\alpha+1)}$.

b- Taking $\alpha=-0.75$, find a value of $g(\alpha)$ to the nearest 10^{-1}.

c- Draw (γ).

Part C

1) Remarking that $\frac{1}{x(x+1)}=\frac{1}{x}-\frac{1}{x+1}$, calculate the integral $\int_1^{\lambda}\frac{1}{x(x+1)}dx$, with $\lambda>1$.

2) a- Calculate, in cm^2, the area A_λ of the region limited by (γ), the axis $x'x$ and the two straight lines of equations $x=1$ and $x=\lambda$.

b- Calculate $\lim\limits_{\lambda\to+\infty} A_\lambda$.

V-23 ***For the students of the G.S. section.***

Consider the function f defined over $]-1;+\infty[$ by $f(x)=x\ln(x+1)$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

Part A.

1) Consider the function h defined over $]-1;+\infty[$ by $h(x)=\frac{1}{x+1}$.

Show that the derivative of order n of h is given by $h^{(n)}(x)=\frac{(-1)^n n!}{(x+1)^{n+1}}$.

2) a- Show that $f'(x)=\ln(x+1)+1-\frac{1}{x+1}$.

b- Deduce the derivative n of f.

Part B.

1) a- Calculate $f''(x)$ for $x\in]-1;+\infty[$.

b- Determine $\lim\limits_{x\to-1} f'(x)$ and $\lim\limits_{x\to+\infty} f'(x)$.

c- Study the variations of f' and set up its table of variations.

d- Show that the equation $f'(x)=0$ admits 0 as a unique solution.

e- Deduce the sign of $f'(x)$.

2) a- Calculate $\lim\limits_{x\to-1} f(x)$, $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty}\frac{f(x)}{x}$.

b- Set up the table of variations of f.

c- Draw (C).

3) a- Show that f has an inverse function g over $[0;+\infty[$ and give the domain of definition of g.

b- Find the coordinates of the points of intersection of (C) and (C') which is the representative curve of g.

c- Draw (C'), the representative curve of g, in the given system.

4) a- Calculate a, b and c so that $\frac{x^2}{x+1} = ax + b + \frac{c}{x+1}$.

b- Using integration by parts, calculate the area of the region limited by (C) and (C').

Part C.

Let $I = [0; e-1]$, consider the sequence (u_n) defined by $u_0 = \frac{3}{2}$ and $u_{n+1} = f(u_n)$ for all $n \geq 1$.

1) Show that $f(I) = I$.
2) Prove by induction, that $u_n \in I$ for any n.
3) a- Prove that the sequence (u_n) is decreasing for any n.

   b- Deduce that (u_n) is convergent and determine its limit.

N° 24. ***For the students of the G.S. section.***

Part A.

Consider the function h defined over $]0;+\infty[$ by $h(x) = x^2 - \ln x$.

Study the variations of h and deduce that $h(x) > 0$ over $]0;+\infty[$.

Part B.

Consider the function f defined over $]0;+\infty[$ by $f(x) = \frac{1+\ln x}{x} + x - 1$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Verify that $\lim\limits_{x \to +\infty} \frac{1+\ln x}{x} = 0$ and deduce that the straight line (d) of equation $y = x - 1$ is an asymptote to (C).
2) Study the relative positions of (C) and (d).
3) Show that the axis $y'y$ is an asymptote to (C).
4) Verify that $f'(x) = \frac{h(x)}{x^2}$ and set up the table of variations of f.
5) Let F be the point of (C) of abscissa 1.

   Show that the tangent (D) at F to (C) is parallel to (d).
6) Show that the equation $f(x) = 0$ admits a unique solution $\alpha \in]0.4;0.5[$.
7) Draw (C), (d) and (D).

8) a- Show that f has an inverse function f^{-1} for $x \in]0;+\infty[$.

b- Determine the domain of definition of f^{-1} and draw (C'), the representative curve of f^{-1} in the same system.

9) Calculate, in cm^2, the area of the region limited by (C), (d) and the two straight lines of equations $x = 1$ and $x = e$.

Part C.

Consider the numerical sequence (x_n) defined by $x_n = e^{\frac{n-2}{2}}$ for all natural numbers n.

1) Show that (x_n) is a geometrical sequence whose first term and ratio are to be determined.

2) Show that (x_n) is increasing .

3) For all n , let $a_n = 4\int_{x_n}^{x_{n+1}} (f(x)-(x-1))dx$.

Show that (a_n) is an arithmetic sequence.

N°25 For the students of the G.S. section.

Part A .

Consider the two functions f and g defined over $[-1;+\infty[$ by $f(x) = -1+\sqrt{x+1}$ and $g(x) = -1-\sqrt{x+1}$.

Designate by (C) and (C') the representative curves of f and g respectively in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Study the variations of f and g and draw (C) and (C') in the same system $(O;\vec{i},\vec{j})$.

2) Let $(\gamma) = (C) \cup (C')$.

   a- Show that $(y+1)^2 = x+1$ is an equation of (γ).

   b- Determine the nature of (γ) as well as its elements.

   c- Calculate the area of the region D limited by (γ) and the straight line of equation $x = 0$.

Part B.

Define a function h by $h(x)=\ln\left(-1+\sqrt{x+1}\right)$ and designate by (Γ) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Verify that the domain of definition of h is $]0;+\infty[$.
   b- Study the variations of h and set up its table of variations.

2) Let r be the rotation of center O and angle $\frac{\pi}{2}$.

For all points M of affix z associate the point M' of affix z' image of M by r.

a- Express z' in terms of z.

b- Let A be the point of (Γ) of abscissa 3 and B is the point of (Γ) of abscissa $\frac{5}{4}$. Designate by P the orthogonal projection of B over $x'x$ and by H the orthogonal projection of B over $y'y$.
Determine the affix of B' the image of B by r and the affix of the point A' image of A by r.

c- Let $z=x+iy$ and $z'=x'+iy'$, express x and y in terms of x' and y'.

d- Show that when M describes (Γ) then the point M' describes the curve (Γ') of equation $y=e^{-2x}+2e^{-x}$.

e- Place on the preceding graph the points A, B A' and B' and draw (Γ) and (Γ') in the same system.

3) a- Calculate the integral $\int_0^{\ln 2}\left(e^{-2x}+2e^{-x}\right)dx$ and interpret this integral graphically.

b- Determine, in square units, the area α of the region limited by the segments $[OA]$, $[OH]$ and $[HB]$ and the arc limited by B and A.

c- Let $I=\int_{\frac{5}{4}}^{3}\ln\left(-1+\sqrt{1+x}\right)dx$.

Find a relation between α and I then deduce the exact value of the integral I.