ثانوية سحمر الرسمية (1011)	محافظة البقاع	قضاء البقاع الغربي
امتحان :الفصل الأول	الصف:علوم عامة	المدة:90 دقيقة
العام الدراسي: 2023-2024	المادة:الرياضيات	الأستاذ: - علي عيسى

The Exam is made up of two pages **The grades are distributed over 20**

I- (6 points)

In the table below, each question have exactly one correct answer among the proposed answers. Indicate the number of each question and choose, by justification, the correct answer.

Nº	Questions	Answers		
		a	b	c
1)	Let $f(x) = \ln(\frac{2e^{-x+5}-1}{3+e^{x-100}})$, then $\boldsymbol{D_f} =$	$]-\infty\,; 5+ln2\,[$	$]-\infty\,; ln2\,[$	IR
2)	The set solution of the inequality $(2x-1)\ln(2-x) \geq 0$ is S =	$[\frac{1}{2}\,; 2]$	$[\frac{1}{2}\,; 1]$	$]-\infty\,; \frac{1}{2}]$
3)	$\lim\limits_{x\to-\infty} \frac{\ln(3x^2e^{2x})}{x} =$	2	$+\infty$	0
4)	The equation $\ln(4-x)-x-2=0$ admits	no root	one root	two roots

II-(4 points)

The derivative f' of the function f is defined over $]\,0\,; +\infty\,[$ by $f'(x) = \frac{2alnx+2x^2-bx}{x}$,where a , b are integers

The below table is the table of variations of f'

x	0		1		$+\infty$
$f''(x)$	‖	$+$	4	$+$	
$f'(x)$	‖ $-\infty$	↗	1	↗	$+\infty$

1) Calculate a and b and show that $f'(x) = \frac{2lnx+2x^2-x}{x}$

2) a)Show that the equation $f'(x) = 0$ admits a unique root α and verify $0.78 < \alpha < 0.8$

   b) Study the sign of $f'(x)$, according to the values of x **with justification**

3) Show that $f''(\alpha) = \frac{2-\alpha+4\alpha^2}{\alpha^2}$

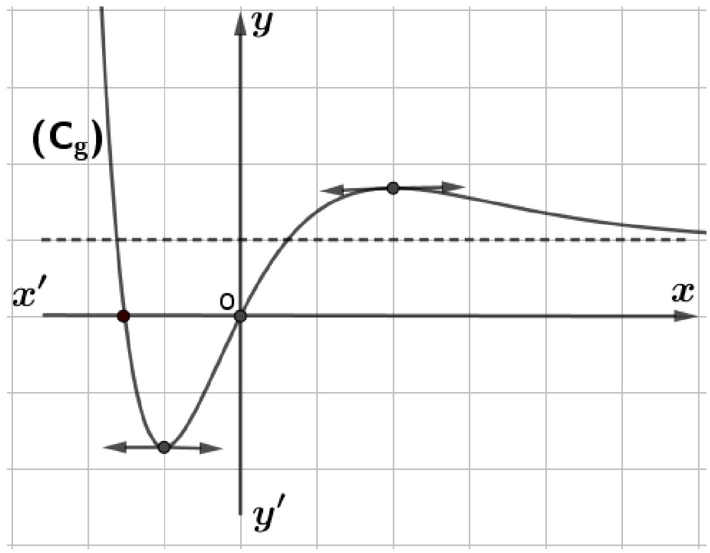
III-(10 points)

Part A

In the adjacent figure (C_g) is the graph of the function g defined over IR by $g(x) = 1 + (x^2 + x - 1)e^{-x}$

The graph(C_g) intersects the axis of abscissas $x'ox$ at two points of abscissas 0 and α

The line $y = 1$ is an asymptote to (C_g) at $+\infty$



1) Given that
   $g(-1) = 1 - e$ and $g(2) = 1 + 5e^{-2}$
   Set up the table of variations of g
2) Show that α verifies two conditions
   $-1.52 < \alpha < -1.5$ and $\alpha^2 + \alpha = 1 - e^{\alpha}$
3) Study the sign of $g(x)$, according to the values of x
4) Let $h(x) = \sqrt{g(x)} + \ln(-x)$. Find domain of definition of h

Part B

Consider a function f defined on IR by $f(x) = -x + (x^2 + 3x + 2)e^{-x}$

Let (C) be the representative curve of f in an orthonormal system (O , $\vec{\imath}$, $\vec{\jmath}$)

1) Calculate $\lim\limits_{x \to -\infty} . f(x)$ and $\lim\limits_{x \to -\infty} . \frac{f(x)}{x}$
2) a)Calculate $\lim\limits_{x \to +\infty} . f(x)$
   b) Show that the line (D):$y = -x$ is an oblique asymptote to (C) at $+\infty$
   c) Study the relative positions of (C) and (D)
3) a) Show that $f'(x) = -g(x)$
   b) Set up the table of variations of f
4) Show that (C) admits two points of inflection of coordinates to be determined
5) Plot (C) . Choose $\alpha = -1.51$
6) Let $F(x) = -\frac{x^2}{2} + (ax^2 + bx + c)e^{-x}$
   a) Determine a, b and c so that F ia an antiderivative of f
   b) Calculate the area of the region bounded between (C) , $x'ox$, $y'oy$ and a line $x = 1$.

WITH MY BEST WISHES

<table>
<tr><th>I</th><th>Correction</th><th>6</th></tr>
<tr><td>1)</td><td>

f is defined for $\frac{2e^{-x+5}-1}{3+e^{x-100}} > 0$
$\mathbf{2e^{-x+5} - 1 > 0}$ **since $3+e^{x-100} > 0$ for all $x \in IR$**
$2e^{-x+5} > 1$
$e^{-x+5} > \frac{1}{2}$
$lne^{-x+5} > ln\frac{1}{2}$
$-x+5 > -ln2$
$-x > -5-ln2$
$\boldsymbol{x < 5 + ln2}$
$\boldsymbol{x \in]-\infty\,;5+ln2\,[}$
$\boldsymbol{D_f =]-\infty\,;5+ln2\,[}$ **(a)**

</td><td>1.5</td></tr>
<tr><td>2)</td><td>

$(2x-1)\ln(2-x) \geq 0$
$D_{inequaility} =]-\infty\,;2[$
$2x-1 \geq 0\,,then\ \boldsymbol{x \geq \frac{1}{2}}$
$\ln(2-x) \geq 0, then\ \boldsymbol{x \leq 1}$
$\mathbf{S = [\frac{1}{2}\ ;1]}$

<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>1/2</td><td></td><td>1</td><td></td><td>2</td></tr>
<tr><td>$2x-1$</td><td></td><td>$-$</td><td>0</td><td>$+$</td><td>|</td><td>$+$</td><td>||</td></tr>
<tr><td>$ln(2-x)$</td><td></td><td>$+$</td><td>|</td><td>$+$</td><td>0</td><td>$-$</td><td>||</td></tr>
<tr><td>$(2x-1)ln(2-x)$</td><td></td><td>$-$</td><td>0</td><td>$+$</td><td>0</td><td>$-$</td><td>||</td></tr>
</table>

(b)

</td><td>1.5</td></tr>
<tr><td>3)</td><td>

$\lim\limits_{x\to-\infty}\frac{\ln(3x^2e^{2x})}{x} = \lim\limits_{x\to-\infty}\frac{\ln 3+2ln|x|+2x}{x} = \lim\limits_{x\to-\infty}\left(\frac{\ln 3}{x}+\frac{2ln|x|}{x}+2\right) = 2$ **(a)**

</td><td>1.5</td></tr>
<tr><td>4)</td><td>

Let $f(x) = \ln(4-x) - x - 2$
$D_f =]-\infty\,;4[$
$\lim\limits_{x\to-\infty}.f(x) = +\infty$
$\lim\limits_{x\to4^-}.f(x) = -\infty$
$f'(x) = \frac{x-5}{4-x}$
$x \in]-\infty\,;4[$
,then $x < 4$, then $x - \mathbf{5} < 4 - \mathbf{5}$, then $x - \mathbf{5} < -1$, then $\boldsymbol{x-5<0}$
$f'(x) < 0$
f is continuous and strictly decreasing on $]-\infty\,;4[$ from $+\infty > \mathbf{0}$ onto $-\infty < \mathbf{0}$
, then f **changes its sign** from **positive onto negative**
, hence the equation $f(x) = 0$ admits a **unique root α so that α** $\in]-\infty\,;\mathbf{4}[$ **(b)**

<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>4</td></tr>
<tr><td>$f'(x)$</td><td></td><td>$-$</td><td>||</td></tr>
<tr><td>$f(x)$</td><td>$+\infty$</td><td>$\searrow$</td><td>$-\infty$ ||</td></tr>
</table>

</td><td>1.5</td></tr>
</table>

II	Correction	4

1)

x	0		1		$+\infty$
$f''(x)$	‖	$+$	4	$+$	
$f'(x)$	‖ $-\infty$	$\nearrow$	1	$\nearrow$	$+\infty$

From table of variations of f' , we have $\begin{cases} \boldsymbol{f'(1) = 1} \\ \boldsymbol{f''(1) = 4} \end{cases}$

Plan: To calculate a and b **, we should apply the above conditions**

$f'(x) = \frac{2alnx+2x^2-bx}{x}$

$\boldsymbol{f'(1) = 1}$, then $\frac{2aln1+2(1)^2-b(1)}{1} = 1$. then $2 - b = 1$, then $b = 1$

$, then\ \boldsymbol{f'(x) = \frac{2alnx+2x^2-x}{x}}$

$f''(x) = \frac{\left(\frac{2a}{x}+4x-1\right)(x)-(2alnx+2x^2-x)}{x^2}$

$f''(x) = \frac{2a+4x^2-x-2alnx-2x^2+x}{x^2}$

$\boldsymbol{f''(x) = \frac{2a+2x^2-2alnx}{x^2}}$

$\boldsymbol{f''(1) = 4}$, then $\frac{2a+2(1)^2-2aln(1)}{1^2} = 4$. then $2a + 2 = 4$, then $\boldsymbol{a = 1}$ **1.5**

$\boldsymbol{f'(x) = \frac{2lnx+2x^2-x}{x}}$ **and** $\boldsymbol{f''(x) = \frac{2+2x^2-2lnx}{x^2}}$

2)a)

* f' is continuous and strictly increasing on $]0\,;+\infty[$ from $-\infty$ $\boldsymbol{< 0}$ onto $+\infty$ $\boldsymbol{> 0}$

, then f' **changes its sign from negative onto positive**

, hence the equation $f'(x) = 0$ admits a **unique root $\boldsymbol{\alpha}$ so that $\boldsymbol{\alpha}$** $\in]0\,;+\infty[$

verify 0.78 $< \boldsymbol{\alpha} <$ 0.8 ?? **0.5**

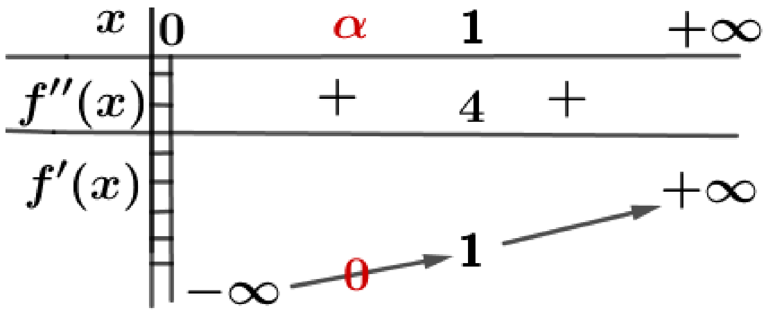
$\boldsymbol{f'(x) = \frac{2lnx+2x^2-x}{x}}$

$f'(0.78) \approx -0.077$

$f'(0.8) \approx 0.042$

$f'(0.78) \times f'(0.8) < 0$

,then $\boldsymbol{0.78 < \alpha < 0.8}$

2)b)	 **<u>Sign of $f'(x)$, with justification</u>** $*f'(x) = 0$ for $x = \alpha$ * For $0 < x < \alpha$, then $f'(x) < f'(\alpha)$ **since f' is continuous and strictly increasing on $]0\,;\alpha]$** **, then** $f'(x) < 0$ For $x > \alpha$, then $f'(x) > f'(\alpha)$ **since f' is continuous and strictly increasing on $[\alpha\,; +\infty;[$** **, then** $f'(x) > 0$	1
3)	$f''(\alpha) = \frac{2-\alpha+4\alpha^2}{\alpha^2}$?? We have proved $\boldsymbol{f'(x) = \frac{2lnx+2x^2-x}{x}}$ and $\boldsymbol{f''(x) = \frac{2+2x^2-2lnx}{x^2}}$ $f''(\alpha) = \frac{2+2\alpha^2-2ln\alpha}{\alpha^2}$ **α is a root of the equation $f'(x) = 0$** , then $f'(\alpha) = 0$, then $\frac{2ln\alpha+2\alpha^2-\alpha}{\alpha} = 0$, then $2ln\alpha + 2\alpha^2 - \alpha = 0$, then $\boldsymbol{2ln\alpha = -2\alpha^2 + \alpha}$ Substitute $\boldsymbol{2ln\alpha = -2\alpha^2 + \alpha}$ in $\boldsymbol{f''(\alpha) = \frac{2+2\alpha^2-2ln\alpha}{\alpha^2}}$ We get $f''(\alpha) = \frac{2+2\alpha^2-(\boldsymbol{-2\alpha^2+\alpha})}{\alpha^2} = \frac{2-\alpha+4\alpha^2}{\alpha^2}$	1.5

III	**Correction**	**10**
	Part A	
1)	Table of variations of g $$\begin{array}{c|ccccccc} x & -\infty & & -1 & & 2 & & +\infty \\ \hline g'(x) & & - & 0 & + & 0 & - & \\ \hline g(x) & +\infty & & & & 1+5e^{-2} & & \\ & & \searrow & & \nearrow & & \searrow & \\ & & & 1-e & & & & 1 \end{array}$$	**1**
2)	$g(-1.52) = 0.041$ $g(-1.5) = -0.1204$ $g(-1.52) \times g(-1.5) < 0$,then $\boldsymbol{-1.52 < \alpha < -1.5}$ α is a root of the equation $g(x) = 0$ $g(\alpha) = 0$, then $1 + (\alpha^2 + \alpha - 1)e^{-\alpha} = 0$, then $\boldsymbol{\alpha^2 + \alpha = 1 - e^{\alpha}}$	**0.25** **0.25**
3)	Table of sign of g(x) $$\begin{array}{c|ccccccc} x & -\infty & & \alpha & & 0 & & +\infty \\ \hline g(x) & & + & 0 & - & 0 & + & \end{array}$$	**0.5**
4)	h is defined for $\begin{cases} \boldsymbol{g(x) \geq 0} \\ \boldsymbol{-x > 0} \end{cases}$, then $\begin{cases} x \in]-\infty\,;\,\alpha] \cup [0\,;\,+\infty[\\ x \in]-\infty\,;0[\end{cases}$, then $x \in]-\infty\,;\,\alpha]$, hence $\boldsymbol{D_h =]-\infty\,;\,\alpha]}$	**0.5**
	Part B	
1)	$f(x) = -x + (x^2 + 3x + 2)e^{-x}$ $f(x) = -x + x^2(1 + 3/x + 2/x^2)e^{-x}$ is the **new form** of f $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} [-x + x^2(1 + 3/x + 2/x^2)e^{-x}]$ $= +\infty + \infty(1 + 0 + 0)e^{+\infty}$ $= +\infty$ and $\lim\limits_{x\to-\infty} \frac{f(x)}{x} = \lim\limits_{x\to-\infty} [-1 + \left(x + 3 + \frac{2}{x}\right) e^{-x}]$ $= -1 + (\infty + 3 + 0)e^{+\infty} = +\infty$	**0.5** **0.5**

<table>
<tr><td>2)a)</td><td>

$f(x) = -x + (x^2 + 3x + 2)e^{-x}$

$f(x) = -x + \frac{x^2+3x+2}{e^x}$

$\lim\limits_{x\to+\infty} . f(x) = \lim\limits_{x\to+\infty} . (-x) + \lim\limits_{x\to+\infty} . \frac{x^2+3x+2}{e^x} = -\infty + 0 = -\infty$

$\lim\limits_{x\to+\infty} . \frac{x^2+3x+2}{e^x} = 0$ (**applying l'hospital rule two times**)

</td><td>0.5</td></tr>
<tr><td>2)b)</td><td>

$f(x) - y_D = (x^2 + 3x + 2)e^{-x}$

$\lim\limits_{x\to+\infty} . [f(x) - y_D] = \lim\limits_{x\to+\infty} . \frac{x^2+3x+2}{e^x} = 0$ (**proved before**)

, then the line (D):$y = -x$ is an oblique asymptote to (C) at $+\infty$

</td><td>0.5</td></tr>
<tr><td>2)c)</td><td>

$f(x) - y_D = (x^2 + 3x + 2)e^{-x}$

$f(x) - y_D = 0$, then $x^2 + 3x + 2 = 0$, then $x = -1$ or $x = -2$

Sign of $f(x) - y_D$ is that of the trinomial $x^2 + 3x + 2$

<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>-2</td><td></td><td>-1</td><td></td><td>$+\infty$</td></tr>
<tr><td>$f(x) - y_D$</td><td></td><td>$+$</td><td>0</td><td>$-$</td><td>0</td><td>$+$</td><td></td></tr>
<tr><td>Relative position</td><td></td><td>(C) is above (D)</td><td>$\cap$</td><td>(C) is below (D)</td><td>$\cap$</td><td>(C) is above (D)</td><td></td></tr>
</table>

(C) $\cap$ $(D) = \{E(-2\,;2\,), F(-1\,;1\,)\}$

</td><td>0.5</td></tr>
<tr><td>3)a)</td><td>

$f(x) = -x + (x^2 + 3x + 2)e^{-x}$

$f'(x) = -1 + (2x + 3)e^{-x} - e^{-x}(x^2 + 3x + 2\,)$

$f'(x) = -1 - e^{-x}(x^2 + x - 1)$

$f'(x) = -[1 + e^{-x}(x^2 + x - 1)]$

$\boldsymbol{f'(x) = -g(x)}$

</td><td>0.5</td></tr>
<tr><td>3)b)</td><td>

Sense of variations of f

$f'(x) = 0$, then $g(x) = 0$, then $x = 0$ or $x = \alpha$ (**PartA -3**)

$\boldsymbol{f(0) = 2}$

$f'(x) > 0$, **then $-g(x) > 0$, then $g(x) < 0$**

, then $x \in]\alpha\,; 0[$ (PartA -3)

</td><td></td></tr>
</table>

Table of variations of f

x	$-\infty$		α		0		$+\infty$
$f'(x)$		$-$	0	$+$	0	$-$	
$f(x)$	$+\infty$	$\searrow$	$f(\alpha)$	$\nearrow$	2	$\searrow$	$-\infty$

1

4)

First method

We have proved $f'(x) = -g(x)$

, then $f''(x) = -g'(x)$

$*f''(x) = 0$, then $-g'(x) = 0$

, then $g'(x) = 0$, then $x = -1$ or $x = 2$ (**from graph of g**)

$f''(x) > 0$, then $-g'(x) > 0$

, then $g'(x) < 0$, then $x < -1$ or $x > 2$ (**from graph of g**)

x	$-\infty$		-1		2		$+\infty$
$f''(x)$		$+$	0	$-$	0	$+$	
Concavity		$\cup$	$f(-1)$	$\cap$	$f(2)$	$\cup$	

$f(-1) = 1$ *and* $f(2) = -2 + 12e^{-2}$

Since f'' **vanishes and changes signs twice at** $x = -1$ and $x = 2$

Then f admits two points of inflection $F(-1\,;1)$ *and* $G(2\,;-2+12e^{-2})$

0.5

Second method

$f'(x) = -[1 + e^{-x}(x^2 + x - 1)]$

$f''(x) = (x^2 - x - 2)e^{-x}$

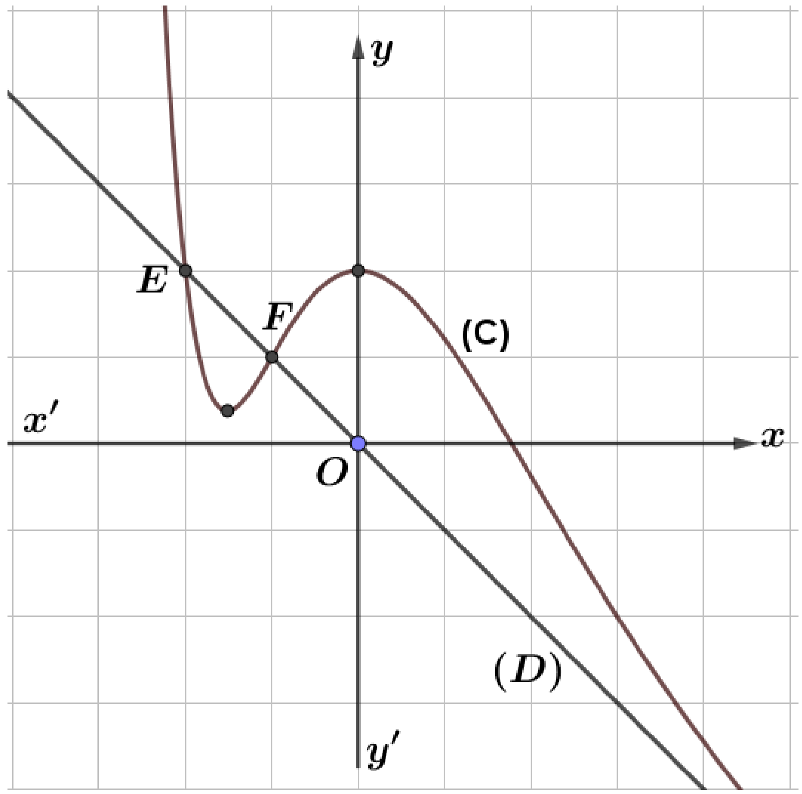
$f''(x) = 0$, , then $x = -1$ or $x = 2$

x	$-\infty$		-1		2		$+\infty$
$f''(x)$		$+$	0	$-$	0	$+$	
Concavity		$\cup$	$f(-1)$	$\cap$	$f(2)$	$\cup$	

$f(-1) = 1$ *and* $f(2) = -2 + 12e^{-2}$

Since f'' **vanishes and changes signs twice at** $x = -1$ and $x = 2$

Then f admits two points of inflection $F(-1\,;1)$ *and* $G(2\,;-2+12e^{-2})$

5)	$f(\alpha) = 0.378$ **Using a calculator**  	**1.5**

6)a)	$F(x) = -\frac{x^2}{2} + (ax^2 + bx + c)e^{-x}$ $f(x) = -x + (x^2 + 3x + 2)e^{-x}$ $\boldsymbol{F'(x) = f(x)}$ $-x + (2ax + b)\, e^{-x} - \; e^{-x}(ax^2 + bx + c\,) \;\; = -x + (x^2 + 3x + 2)e^{-x}$ $-ax^2 + (2a - b)x + b - c = \;\; x^2 + 3x + 2$, then $(\,a\,, b\,, c\,) = (-1\,;\, -5\,; -7)$, then $\boldsymbol{F(x) = -\frac{x^2}{2} + (-x^2 - 5x - 7)e^{-x}}$	**1**
6)b)	$\mathcal{A} = \int_0^1 f(x)dx = \; F(x)\|_0^1 = F(1) - F(0) = (\frac{23-26e^{-1}}{2})$ $\boldsymbol{unit^2}$	**0.5**