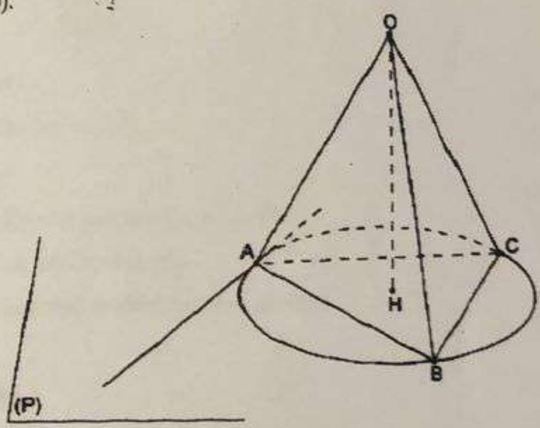
Bekaa Youth Education Center
Cycle III
Grade 12 - Life Science
Name:

Test: Math
Duration: 60 minutes
Mark: /80
Date: 9/3/2024

The space is referred to a direct orthonormal system $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$.
Consider the points $A(6, 0, 0)$, $B(0, 6, 0)$ and $C(0, 0, 6)$.
Let (Ω) be the circle circumscribed about triangle ABC.

1) Show that triangle ABC is equilateral.
2) Write a cartesian equation of the plane (P) determined by the points A, B and C.
3) a- Show that point $H(2, 2, 2)$ is the orthogonal projection of point O on (P).
   b- Verify that H is the center of (Ω).
   c- Show that the volume of tetrahedron OABC is triple the volume of tetrahedron OAHB.
4) Consider the line (D) with parametric equations:
$$\begin{cases} x = 6 \\ y = -m \text{ ; where } m \in \mathbb{R}. \\ z = m \end{cases}$$
Show that (D) is tangent to (Ω) at A.



In the space referred to a direct orthonormal system $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$, consider the plane (P) with equation: $x + y - z + 1 = 0$, the point $A(1; 0; -1)$ and the line (d) defined as : $x = t - 1$; $y = t$; $z = -t + 3$ (t is a real parameter).

1) a- Show that the line (d) is perpendicular to plane (P).
   b- Determine the coordinates of H, the point of intersection of (d) and (P).
2) Verify that the point $K(0; -1; 0)$ is the orthogonal projection of A on (P).
3) Denote by (Δ) the line passing through H, contained in the plane (P) and perpendicular to the line (KH).
   a- Verify that $\vec{V}(-2; 1; -1)$ is a direction vector of the line (Δ).
   b- Write a system of parametric equations of line (Δ).
4) Consider in the plane (P) the circle (C) with center H and radius $\sqrt{6}$. This circle intersects the line (Δ) in two points T and S. Determine the coordinates of T and S.

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In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the two straight lines (D) and (D') defined as:

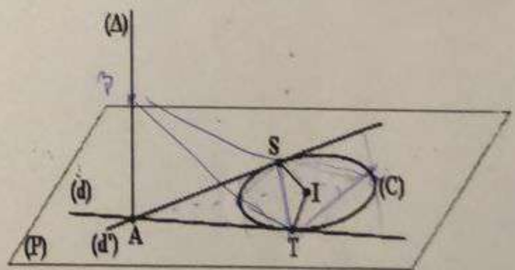
$$(D):\begin{cases}x=\lambda+1\\ y=0\\ z=\lambda+3\end{cases}\quad(\lambda\in\mathbb{R})\quad\text{and}\quad(D'):\begin{cases}x=t\\ y=3t-3\\ z=t\end{cases}\quad(t\in\mathbb{R}).$$

1) Prove that (D) and (D') are skew (non-coplanar).
2) Denote by (P) the plane containing (D') and parallel to (D).
   Show that an equation of (P) is: $x-z=0$.
3) Write an equation of the plane (Q) containing (D) and perpendicular to (P).
4) Verify that A(1 , 0 , 1) is the point of intersection of (D') and (Q).
5) a- Determine the coordinates of point B the orthogonal projection of A on (D).
   b- Let C(1 , 0 , 3) be a point on (D).
   Verify that the triangle ABC is right isosceles.
6) Determine the coordinates of the points M on (D') so that the volume of the tetrahedron MABC is equal to 2 cubic units.

The space is referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$.

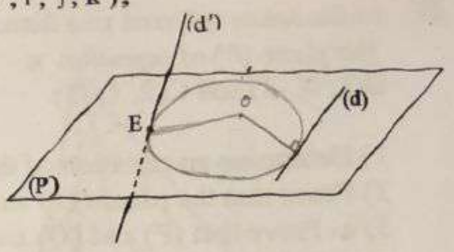
Consider the two lines (d) and (d') with parametric equations

$$(d):\begin{cases}x=m+1\\ y=2m+1\\ z=2m+1\end{cases}\text{ and }(d'):\begin{cases}x=-t\\ y=2t+3\\ z=-2t-1\end{cases}\text{ where } m,t\in\mathbb{R}.$$



1) Show that (d) and (d') intersect at the point A(1 , 1 , 1).
2) Determine a cartesian equation of the plane (P) determined by (d) and (d').
3) Let (C) be the circle, with radius $3\sqrt{5}$, tangent to (d) at T and tangent to (d') at S.
   Let (Δ) be the line perpendicular to (P) at A.
   a- Show that the point I(1 , 10 , 1) is the center of (C).
   b- Calculate the coordinates of the two points E and F on (C) that are equidistant from (d) and (d').
   c- Show that the area of the quadrilateral ATIS is $18\sqrt{5}$.
   d- Determine the coordinates of points B on (Δ) so that the volume of the solid BATIS is 30.

I) In the space referred to a direct orthonormal system $(O\ ;\vec{i}\ ,\vec{j}\ ,\vec{k}\ )$, consider the lines (d) and (d') defined by :



$$(d):\begin{cases}x=t+1\\y=2t\\z=t-1\end{cases} \quad \text{and} \quad (d'):\begin{cases}x=2m\\y=-m+1\\z=m+1\end{cases}$$

(t and m are two real parameters).

1) Prove that (d) and (d') are skew (not coplanar).

2) a- Show that $x-y+z=0$ is an equation of the plane (P) determined by O and (d).
b- Determine the coordinates of E, the point of intersection of (P) and (d').
c- Prove that the straight line (OE) cuts (d).

3) a - Calculate the distance from point O to the line (d).
b - Deduce that the circle in plane (P), with center O and passing through E, is tangent to line (d).

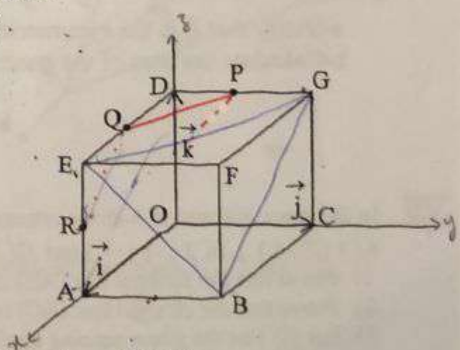
II) In the space referred to a direct orthonormal system$(O;\ \vec{i}\ ,\ \vec{j}\ ,\ \vec{k})$, consider :

- the plane (P) of equation $2x+y-3z-1=0$;
- the plane (Q) of equation $x+4y+2z+1=0$;
- the line (d) defined by : $\begin{cases}x=2t+1\\y=-t-1\\z=t\end{cases}$ (t is a real parameter).

1) Prove that the line (d) is included in the plane (P) .
2) Find an equation of the plane (S) that is determined by the point O and the line (d).
3) Consider the point $E\left(0\ ;-\frac{1}{2};-\frac{1}{2}\right)$.

Prove that E is the orthogonal projection of the point O on the line (d).
4) a- Show that the planes (P) and (Q) are perpendicular.
b- Let (D) be the line of intersection of (P) and (Q) .
Calculate the distance from E to (D).

III) In the space referred to a direct orthonormal system $(O;\vec{i}\ ,\vec{j}\ ,\vec{k})$, consider the cube OABCDEFG such that : A(1 ; 0 ; 0) , B(1 ; 1 ; 0) and F(1 ; 1 ; 1) . Designate by P, Q and R the midpoints of the segments [DG] , [DE] and [AE] respectively .



1) a-Show that $2x+2y+2z-3=0$ is an equation of the plane (PQR).
b- Prove that the plane (PQR) passes through the midpoint of [AB] .
c- Prove that the planes (PQR) and (BEG) are paralle'

2) a- What is the nature of quadrilateral EGCA ?
b- Let M be a variable point on the line (AC) .

Show that $\overrightarrow{AM}\times\overrightarrow{EF}=\overrightarrow{AM}\times\overrightarrow{GF}$.

IV In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$ consider the plane (P) of equation $x - 2y + z + 1 = 0$, and the points A (2; -2; -1), B(1; 0; -2) and C(2; 1; -1).

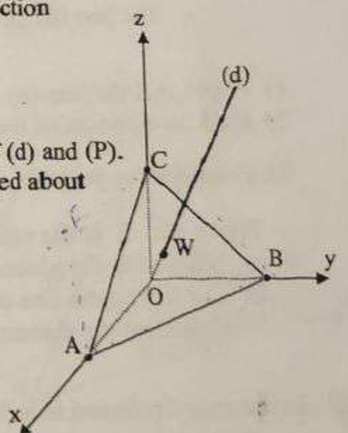
1) Determine an equation of the plane (Q) containing A, B and C.
2) Prove that the planes (P) and (Q) intersect along the straight line (BC).
3) a- Prove that (P) and (Q) are perpendicular.
   b- Calculate the distance from A to (BC).
4) Let (d) be the straight line defined by:

$$\begin{cases} x = t-1 \\ y = t+1 \\ z = t+2 \end{cases} \quad \text{where t is a real parameter.}$$

   a- Verify that (d) is included in (P).
   b- Let M be a variable point on (d). Prove that the area of triangle MBC is independent of the position of M on (d).

V In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the plane (P) of equation $x + y + z - 2 = 0$.

1) Determine the coordinates of A, B and C, the points of intersection of the plane (P) with the axes of coordinates.
2) Write a system of parametric equations of the straight line (d) passing through O and perpendicular to the plane (P).
3) a -Determine the coordinates of W, the point of intersection of (d) and (P).
   b-Prove that the point W is the center of the circle circumscribed about the triangle ABC.
4) Consider the point $E\left(\frac{4}{3}; -\frac{2}{3}; \frac{4}{3}\right)$.
   a-Verify that E is the symmetric of B with respect to W.
   b-Calculate the area of the quadrilateral ABCE.



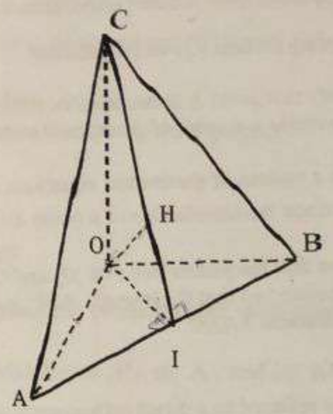
VI In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points A(1 ; 2 ; 1), B(2 ; -1 ; -1) and C(3 ; -2 ; 1), and the plane (P) of equation $x - y + 2z - 1 = 0$.

1) Prove that the straight line (AB) is contained in the plane (P).
2) Prove that the straight line (BC) is perpendicular to the plane (P).
3) Let (Q) be the plane passing through the point C and parallel to the plane (P).
   a -Write an equation of plane (Q).
   b -Calculate the distance between the planes (P) and (Q).
4) Let E be the symmetric of C with respect to the plane (P).
   Calculate the area of triangle AEC.

7) The adjacent figure is considered in a direct orthonormal system $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$ where:

$\overrightarrow{OA} = \vec{i}$, $\overrightarrow{OB} = \vec{j}$ and $\overrightarrow{OC} = 2\vec{k}$.

Let I be the midpoint of [AB].



1- Justify that an equation of plane (ABC) is $2x+2y+z-2=0$.

2- Consider the point $H\left(\frac{4}{9};\frac{4}{9};\frac{2}{9}\right)$.

a) Show that C, H and I are collinear.

b) Prove that (OH) is perpendicular to the plane (ABC).

c) Prove that the two planes (OIC) and (ABC) are perpendicular.

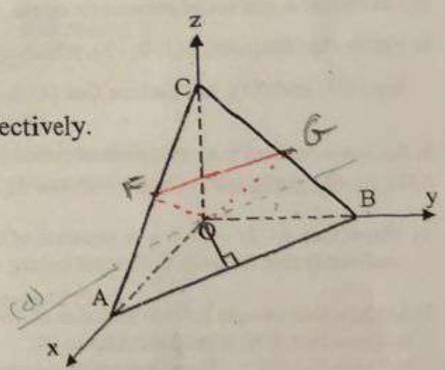
3- a) Write a system of parametric equations of the straight line (Δ) passing through C and parallel to (OB).

b) Let F be a variable point on (Δ).
Prove that the tetrahedron FOAB has a constant volume to be calculated.

8) In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points A (4 ; 0 ; 0) , B (0 ; 4 ; 0) and C (0 ; 0 ; 4).

1) Write an equation of plane (ABC).

2) Calculate the area of triangle ABC.



3) Let F and G be the midpoints of [AC] and [BC] respectively.

a- Give a system of parametric equations of the straight line (FG).

b- The plane of equation $z = 0$ intersects the plane (OFG) along a line (d). Prove that the lines (d) and (FG) are parallel to each other.

c- Calculate the distance between the two lines (d) and (AB) :

9) In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the points A(1; 2; 0), B(2; 1; 3), C(3; 3; 1), D(5; −3; −3) and E(−3; 7; 3).

1) Find an equation of the plane (P) determined by A, B and C.

2) Find a system of parametric equations of line (DE).

3) Prove that (P) is the mediator plane of [DE].

4) Prove that (BC) is orthogonal to (DE).

5) a- Calculate the area of triangle BCD.

b- Calculate the volume of tetrahedron ABCD, and deduce the distance from A to plane BCD.

10) In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the point A (1; 0; 1) and the two planes (P) and (Q) with equations $2x-y-2=0$ and $x+2y-z=0$ respectively.

1) a- Verify that point A is common to (P) and (Q).
   b- Determine a system of parametric equations of (d), the line of intersection of (P) and (Q).

2) a- Give a system of parametric equations of the line (D) that is perpendicular to (P) at A.
   b- Calculate the coordinates of a point E on (D) such that $AE=\sqrt{5}$.

3) a- Show that the points $B(0;-2;0)$ and $C(2;2;t)$ belong to (P). (t is a real number).
   b- Calculate t so that the triangle ABC is right at B and find in this case the volume of the tetrahedron EABC.

11) In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the plane (P) of equation: $x-y+z+2=0$, and the two straight lines (D) and (D') defined by the parametric equations:

$$(D)\begin{cases}x=t\\y=-t+1\\z=2t-1\end{cases}\quad\text{and}\quad(D')\begin{cases}x=-5m-10\\y=5m+11\\z=-2m-5\end{cases}$$ where t and m are real parameters.

1) Show that (D) and (D') intersect at the point $A(0;1;-1)$ and verify that A belongs to plane (P).
2) Write an equation of the plane (Q) that contains the two straight lines (D) and (D').
3) Determine a system of parametric equations of the straight line (d), the intersection of (P) and (Q).
4) Verify that the point $B(1;0;-3)$, which is on the straight line (d), is equidistant from the two straight lines (D) and (D'), and deduce that (d) is a bisector of the angle between (D) and (D').

12) In the space referred to an orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points: $A(0;1;-2)$, $B(2;1;0)$, $C(3;0;-3)$ and $H(2;2;-2)$.

1) Show that $x-2y-z=0$ is an equation of the plane (P) determined by the points H, A and B and verify that the point C does not belong to this plane.

2) a- Show that triangle HAB is isosceles of vertex H.
   b- Show that (CH) is perpendicular to (P).
   c- Prove that CA = CB and determine a system of parametric equations of the interior bisector (δ) of angle ACB.

3) Let T be the orthogonal projection of H on plane (ABC).
   Prove that T belongs to (δ).

13) In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points $A(1;1;0)$, $B(2;0;0)$, $C(1;3;-1)$, $E(2;2;2)$ and the plane (P) of equation $x+y+2z-2=0$.

1) a- Verify that (P) is the plane determined by A, B and C.
   b- Show that the line (AE) is perpendicular to the plane (P).
   c- Calculate the area of triangle ABC and the volume of tetrahedron EABC.

2) Designate by L the midpoint of [AB] and by (Q) the plane passing through L and parallel to the two lines (AE) and (BC).
   a- Write an equation of plane (Q).
   b- Prove that the planes (P) and (Q) are perpendicular.
   c- Prove that line (d), the intersection of the planes (P) and (Q), is parallel to (BC).

14) In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points $A(1;1;2)$, $B(2;-1;-1)$, $E(0;-3;-1)$ and $F(-3;2;1)$.

1) Prove that $x-y+z-2=0$ is an equation of the plane (P) passing through the points A, B and E.
2) Show that the straight lines (AE) and (BF) are skew.
3) Prove that the point $G(1;-2;5)$ is symmetric of F with respect to the plane (P).
4) Consider the line (d) defined by :
$$\begin{cases} x=1 \\ y=-t+1 \\ z=t+2 \end{cases} \quad t \text{ is a real parameter.}$$
Prove that the lines (d) and (AF) are symmetric of each other with respect to the plane (P).
5) Designate by (δ) the straight line perpendicular to (P) at A, and by M any point on this line.
Prove that M is equidistant from (d) and (AF).

15) In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the plane (P) of equation $2x-y+z=0$ and the plane (Q) of equation $x+y-z+2\sqrt{3}=0$.
Let (d) be the line of intersection of (P) and (Q).

1) Prove that the planes (P) and (Q) are perpendicular.
2) Write a system of parametric equations of the line (d).
3) Let (C) be the circle, in plane (P), of center O and radius 3.
Prove that (d) cuts (C) in two points A and B.
4) Designate by E the midpoint of the segment[AB].
a- Write a system of parametric equations of the line (OE).
b- Determine the coordinates of the point F ; the symmetric of the point O with respect to the line (d).

16) In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$,

consider the line (d) defined by : $\begin{cases} x=t+1 \\ y=-t+2 \\ z=2t \end{cases}$ (t is a real parameter)

and the plane (P) of equation $x-y-2z-5=0$.

1) Determine the coordinates of E, the point of intersection of (d) and (P).
2) a- Write an equation of the plane (Q) perpendicular to (d) at E.
b- Find a system of parametric equations of the line (D) that lies in plane (P) and perpendicular to (d) at E.
3) $I(2;1;2)$ is a point on line (d). Determine the coordinates of J, the symmetric of I with respect to (D).

17) The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$.

Consider the point $A(2\,;-3\,;5)$ and the planes (P) and (Q) of equations:

$$(P): 2x-2y-z+4=0$$

$$(Q): 2x+y+2z+1=0$$

A-1) Show that the two planes (P) and (Q) are perpendicular.

2) Show that the straight line (D) defined by: $\begin{cases} x=t \\ y=2t+3 \\ z=-2t-2 \end{cases}$ (t is a real parameter),

is the intersection of (P) and (Q).

3) Calculate the coordinates of the point H, the orthogonal projection of A on the straight line (D).

B- Designate by (R) the plane passing through the point W (1 ; 4 ; 1) and parallel to (Q).

Consider in (R) the circle (C) of center W and radius 3.

1) Find an equation of (R).

2) Show that B (3; 2; 0) is a point on (C).

3) Write a system of parametric equations of the tangent (T) at B to (C).

18) The space is referred to an orthonormal system $(O\,;\vec{i}\,,\vec{j}\,,\vec{k})$.

Consider the two straight lines $(d)\begin{cases} x=m-1 \\ y=m \\ z=m+1 \end{cases}$ and $(d')\begin{cases} x=2t \\ y=t \\ z=-3t+2 \end{cases}$

where m and t are two real parameters.

1) Prove that (d) and (d') are skew (not coplanar).

2) Let (P) be the plane containing (d) and cutting (d') at the point $E(0\,;0\,;2)$.
Prove that an equation of (P) is $x-z+2=0$.

3) Consider in plane (P) the circle (C) with center E and radius $R=1$.

a- Calculate the distance from E to (d) and prove that (C) cuts (d) at two points A and B.

b- Calculate the coordinates of the points A and B.

c- Calculate the area of triangle EAB.

19) The space is referred to a direct orthonormal system $(O\,;\vec{i}\,,\vec{j}\,,\vec{k})$.

Consider the points $A(4\,;3\,;2)$, $B(-8\,;-1\,;6)$, and the plane (P) of equation $x-y-z+4=0$.

1) a- Determine a system of parametric equations of the line (AB).

b- Determine the coordinates of I, the point of intersection of (AB) and (P).

c- Prove that A and B lie on opposite sides with respect to (P).

symmetric

2) Let (d) be the set of points in (P) which are equidistant from A and B.

a- Find an equation of (Q), the mediator plane of [AB].

b- Prove that (d) is the line defined by the system of parametric equations

$$x=m-\frac{3}{2}\ ;\ y=-m-1\ ;\ z=2m+\frac{7}{2},\ (\text{ m is a real number }).$$

3) Let J be the orthogonal projection of A on (d).

Find the coordinates of J and prove that (d) is perpendicular to the plane (ABJ).