

- V) Consider two bags B_1 and B_2 such that:
 B_1 contains six cards numbered 1, 2, 3, 4, 5, 6.
 B_2 contains five cards numbered 0, 1, 2, 4, 5.

A-

One card is drawn randomly from bag B_1 :

- if it carries one of the numbers 1 or 2, then three cards are drawn randomly and simultaneously from bag B_2 .
- But if it carries one of the numbers 3, 4, 5 or 6, then two cards are drawn randomly and simultaneously from bag B_2 .

Consider the following events:

K: « the card drawn from bag B_1 carries one of the numbers 1 or 2 ».

L: « the card drawn from bag B_1 carries one of the numbers 3, 4, 5 or 6 ».

E: « the product of numbers shown on the cards drawn from bag B_2 is zero ».

- 1) a- Calculate the probabilities $p(K)$ and $p(L)$.

b- Show that $p(E \cap K) = \frac{1}{5}$.

c- Calculate $p(E \cap L)$ and deduce $p(E)$.

- 2) Knowing that the product of the numbers shown on the cards drawn from bag B_2 is zero, calculate the probability that the three cards were drawn from B_1 .

B-

In this part we use only the bag B_2 and three cards are drawn randomly and simultaneously from this bag. Let X be the random variable that is equal to the biggest number among those shown on the three drawn cards, thus the possible values of X are 2, 4 and 5.

Prove that $p(X=4) = \frac{3}{10}$ and determine the probability distribution of X .

VI)

The staff of a big factory is divided into three categories:

engineers, technicians and workers.

- 6% of the staff are engineers.
- 74% of the staff are workers.
- 80% of the workers are males.
- 10% of the engineers are females.

We interview at random a member of the staff of this factory.

Consider the following events:

E: « The person interviewed is an engineer ».

W: « The person interviewed is a worker ».

T: « The person interviewed is a technician ».

M: « The person interviewed is a male ».

- 1) a- What is the probability of interviewing a male worker?

b- What is the probability of interviewing a male engineer?

- 2) We know that 80% of the staff are males.

a- Prove that the probability of interviewing a male technician is 0.154.

b- The person interviewed is a female, what is the probability that she is a technician?

- 3) In this part, suppose that the staff of this factory counts 500 persons.

We choose randomly and simultaneously three persons from this staff.

a- Calculate the probability of choosing three persons from three different categories.

b- Calculate the probability of choosing at most one engineer.

VII) A box V contains cards such that:

- 20% of the cards are blue and the other cards are red;
- Out of the blue cards, 40% carry odd numbers;
- 32% of the total cards carry odd numbers.

1) A card is randomly selected from box V.

Consider the following events:

- B : «select a blue card »
- R : «select a red card »
- O : «select a card carrying an odd number »

a- Calculate the probabilities $p(O \cap B)$ and verify that $p(O \cap R) = 0.24$.

b- Deduce $p(O/R)$.

c- The selected card does not carry an odd number, what is the probability that it is red?

2) In this part, suppose that the number of cards in box V is 50.

Three cards are randomly and simultaneously selected from V.

Consider the following events:

- M: «among the three selected cards, exactly two carry odd numbers»
- N: «the three selected cards are blue»
- L: « among the three selected cards, exactly two carry odd numbers and one is blue».

Calculate the probability $p(M)$, $p(N/M)$ and $p(L)$.

VIII)

An urn contains 5 white balls and 2 black balls. A game consists of two consecutive drawings as follows:

- A ball is selected randomly in the first drawing. If the ball selected is white, it is put back in the urn; otherwise, it is kept outside the urn.
- Two balls are selected simultaneously and randomly in the second drawing.

Consider the following events:

- W: «The ball selected in the first drawing is white»
- E: «The 2 balls selected in the second drawing are white»
- F: «The 2 balls selected in the second drawing are black»
- G: «The 2 balls selected in the second drawing are of different colors».

1) Calculate $P(E/W)$ and $P(E/\bar{W})$. Deduce that $P(E) = \frac{26}{49}$.

2) Calculate $P(F)$ and $P(G)$.

3) Knowing that the 2 balls selected in the second drawing have the same color, calculate the probability that the ball selected in the first drawing is black.

4) In this part, we mark -3 points for each black ball selected, and +5 points for each white ball selected

Denote by S the sum of points marked for the two balls selected in the second drawing.

Calculate the probability that S is positive.

IX) Given:

- A bag B_1 containing one 20 000 LL bill and three 50 000 LL bills.
- A bag B_2 containing two 20 000 LL bills and two 100 000 LL bills.
- A six-sided fair die (numbered 1 through 6).

1) The die is rolled once.

If this die shows 5 or 6, one ball is randomly selected from bag B_1 , otherwise one ball is randomly selected from bag B_2 .

Consider the following events:

A : «obtain a 20 000 LL bill»

B : «obtain a 50 000 LL bill»

C : «obtain a 100 000 LL bill»

E : «the die shows the number 5 or 6».

a- Verify that the probability of the event A is $P(A) = \frac{5}{12}$.

b- Which bill is the most probably to be selected? Justify.

2) All bills from B_1 and B_2 are now placed in the same bag B and the same die is rolled.

If the die shows 5 or 6, then two bills are randomly and simultaneously selected from bag B, otherwise three bills are randomly and simultaneously selected from B.

a-Verify that the probability to have a total sum less than 80 000 LL is $\frac{13}{84}$.

b- The total sum obtained is less than 80 000 LL. What is the probability that the die shows the face numbered 3?

X) The following table represents the distribution of the ages of 26 men and 24 women.

Age in years	[20;25[[25;30[[30;35]	Total
Number of men	8	8	10	26
Number of women	5	9	10	24
Total	13	17	20	50

3 persons are randomly chosen, from these 50 people, to form a committee.

Consider the following events:

M: « the committee is formed of three men ».

F : « the committee is formed of three women ».

A: « the committee is mixed (formed of men and women) ».

B: « the age of each member of the committee is less than 30 years ».

1) Calculate each of the probabilities $p(M)$, $p(F)$ and $p(A)$.

2) a- Calculate $p(B)$ and show that $p(B \cap \bar{A}) = \frac{33}{700}$. Deduce $p(B \cap A)$.

b- Calculate $p(B/A)$.

3) Designate by X the ~~random~~ variable that is equal to the number of women in the committee who have an age less than 25 years.

Y) Determine the probability distribution of ~~X~~ each value of X.

a) Determine the possible values of X.

XI)

An urn contains 8 balls:

- 4 white balls each carrying the number 0;
- 3 red balls each carrying the number 5;
- 1 white ball carrying the number 2.

We draw, simultaneously and randomly, 3 balls from the urn.

Consider the following events:

A: « the three drawn balls carry three numbers which could form the number 200 ».

B: « the three drawn balls carry three identical numbers ».

C: « the three drawn balls are white ».

D: « the three drawn balls are of the same color ».

- 1) Show that the probability $p(A)$ is equal to $\frac{3}{28}$ and calculate $p(B)$, $p(C)$ and $p(D)$.
- 2) Determine the probability that among the three drawn balls only one carries the number 0.
- 3) The three drawn balls are white; calculate the probability that the numbers carried by these balls could form the number 200.
- 4) let X be the random variable equal to the product of the three numbers carried by the three drawn balls.
 - a- Give the 3 possible values of X .
 - b- Determine the probability distribution of X .
probability of each value

XII)

In a store, there are two drawers D_1 and D_2 containing neckties.

Drawer D_1 contains 15 silk neckties: 3 red, 5 green and 7 blue.

Drawer D_2 contains 10 polyester neckties: 2 red, 5 green and 3 blue.

A- We choose at random one necktie from D_1 and one from D_2 .

Designate by E and F the following two events:

E : «the two chosen neckties are of the same color»

F : «the two chosen neckties are one red and one blue».

1) Prove that the probability $P(E)$ is equal to $\frac{26}{75}$.

2) Calculate $P(F)$.

B- In this part, one of the two drawers is randomly chosen out of which one necktie is randomly chosen.

Consider the following events:

R : «the chosen necktie is red»

D_1 : «the chosen necktie is from drawer D_1 ».

1) Calculate $P(R/D_1)$ and $P(R \cap D_1)$.

2) Calculate $P(R)$.

C- Suppose that these 25 neckties are placed in one drawer D and three neckties are simultaneously and randomly chosen from D . The price of a silk necktie is 50 000LL and that of a polyester necktie is 10 000LL.

Designate by X the amount equal to the sum of prices of the three chosen neckties.

Calculate $P(X \leq 100\,000)$.

XIII

U and V are two urns such that:

- U contains 3 red balls and 5 blue balls.
- V contains 4 red balls and 3 blue balls.

Part A

One ball is randomly selected from U and one ball is randomly selected from V.

- 1) Show that the probability of selecting two red balls is $\frac{3}{14}$.
- 2) Calculate the probability of selecting two balls having the same color.
- 3) Calculate the probability of selecting two balls with different colors.

Part B

In this part, one ball is randomly selected from U:

- if the selected ball from U is red, then two balls are randomly and simultaneously selected from V.
- if the selected ball from U is blue, then three balls are randomly and simultaneously selected from V.

Consider the following events:

R: "The selected ball from U is red"

S: "The selected balls from V have the same color".

- 1) Determine the probability $P(R)$.
- 2) Show that $P(S/R) = \frac{3}{7}$ and deduce $P(S \cap R)$.
- 3) The probability $P(S \cap \bar{R}) = \frac{5}{56}$, calculate $P(S)$.

XIV

An urn U contains 10 balls: 6 blue balls and 4 red balls.

Part A

Two balls are selected randomly and simultaneously from U.

Consider the following events:

A: "The two selected balls have the same color"

B: "The two selected balls have different colors".

- 1) Verify that the number of possible outcomes is 45.
- 2) Show that the probability $P(A) = \frac{7}{15}$ and deduce $P(B)$.

Part B

In this part, a fair die numbered from 1 to 6 is rolled.

- If the die shows an even number, then two balls are selected randomly and simultaneously from U.
- If the die shows an odd number, then two balls are selected randomly and successively with replacement from U.

Consider the following events:

E: "The die shows an even number"

F: "The selected balls have the same color".

- 1) Calculate $P(F/E)$ and verify that $P(F \cap E) = \frac{7}{30}$.
- 2) Verify that $P(F \cap \bar{E}) = \frac{13}{50}$ and deduce $P(F)$.
- 3) Knowing that the two selected balls have the same color, calculate the probability that the die shows an even number.

Δ

XV An urn U contains red balls and black balls holding distinct natural numbers.

- 60 % of the balls are red of which 80 % hold odd numbers.
- 70 % of the black balls hold odd numbers.

Part A

One ball is selected from the urn. Consider the following events:

R : "the selected ball is red" and O : "the selected ball holds an odd number".

- 1) Show that the probability $P(O \cap R)$ is equal to 0.48 and calculate $P(O \cap \bar{R})$.
- 2) Deduce that $P(O) = 0.76$.
- 3) Are the events R and O independent? Justify your answer.

Part B

Suppose in this part that the number of balls in the urn U is 50.

- 1) Show that the number of red balls holding odd numbers is equal to 24.
- 2) Copy and complete the following table :

	Red	Black	Total
Odd			38
Even			
Total	30		50

- 3) Three balls are selected randomly and simultaneously from the urn U .
 - a- Calculate the probability of selecting at least one red ball holding an odd number.
 - b- The even numbered balls hold the numbers 2, 4, 6, ..., 24.
Knowing that the three selected balls hold even numbers, calculate the probability that each of these balls holds a number greater than 15.

XVI The 20 employees in a factory are distributed into two departments as shown in the table below:

	Technical Department	Administrative Department
Women	3	5
Men	10	2

- 1) The manager of this factory wants to offer a gift to one of the employees. To do this, he chooses randomly an employee of this factory.

Consider the following events:

W : « the chosen employee is a woman ».

M : « the chosen employee is a man ».

T : « the chosen employee is from the technical department ».

A : « the chosen employee is from the administrative department ».

- a- Calculate the following probabilities:

$P(W/T)$, $P(W/A)$, $P(W \cap T)$ and $P(W)$.

- b- Knowing that the chosen employee is a man, what is the probability that he is from the technical department ?

- 2) On a different occasion, the factory manager chooses **two** employees randomly and simultaneously from the technical department and also chooses **one** employee randomly from the administrative department.

Designate by X the random variable that is equal to the number of women chosen.

- a- Verify that $P(X=1) = \frac{95}{182}$.

- b- Determine the probability distribution of X .

Probabilities

I) Consider two urns U and V.

Urn U contains eight balls: four balls numbered 1, three balls numbered 2 and one ball numbered 4.
Urn V contains eight balls: three balls numbered 1 and five balls numbered 2.

1) Two balls are selected, simultaneously and randomly, from the urn U.

Consider the following events:

- A : « the two selected balls have the same number »
- B : « the product of the numbers on the two selected balls is equal to 4 ».

Calculate the probability $P(A)$ of the event A, and show that $P(B)$ is equal to $\frac{1}{4}$.

2) One of the two urns U and V is randomly chosen, and then two balls are simultaneously and randomly selected from this urn.

Consider the following events:

- E : « the chosen urn is V »
- F : « the product of numbers on the two selected balls is equal to 4 ».

a- Verify that $P(F \cap E) = \frac{5}{28}$ and calculate $P(F \cap \bar{E})$.

b- Deduce $P(F)$.

3) One ball is randomly selected from U, and two balls are randomly and simultaneously selected from V.

Calculate the probability of the event H: « the product of the three numbers on the three selected balls is equal to 8 ».

II) Consider two bags B_1 and B_2 such that:

B_1 contains six cards numbered 1, 2, 3, 4, 5, 6.

B_2 contains five cards numbered 0, 1, 2, 4, 5.

A-

One card is drawn randomly from bag B_1 :

- if it carries one of the numbers 1 or 2, then three cards are drawn randomly and simultaneously from bag B_2 .
- But if it carries one of the numbers 3, 4, 5 or 6, then two cards are drawn randomly and simultaneously from bag B_2 .

Consider the following events:

K: « the card drawn from bag B_1 carries one of the numbers 1 or 2 ».

L: « the card drawn from bag B_1 carries one of the numbers 3, 4, 5 or 6 ».

E: « the product of numbers shown on the cards drawn from bag B_2 is zero ».

1) a- Calculate the probabilities $p(K)$ and $p(L)$.

b- Show that $p(E \cap K) = \frac{1}{5}$.

c- Calculate $p(E \cap L)$ and deduce $p(E)$.

2) Knowing that the product of the numbers shown on the cards drawn from bag B_2 is zero, calculate the probability that the three cards were drawn from B_2 .

B-

In this part we use only the bag B_2 and three cards are drawn randomly and simultaneously from this bag. Let X be the random variable that is equal to the biggest number among those shown on the three drawn cards, thus the possible values of X are 2, 4 and 5.

Prove that $p(X=4) = \frac{3}{10}$ and determine the probability distribution of X.

~~XCB~~ U_1 and U_2 are two urns such that

iii)

The following table represents the distribution of the ages of 26 men and 24 women.

Age in years	$[20;25[$	$[25;30[$	$[30;35]$
Number of men	8	8	10
Number of women	5	9	10

3 persons are randomly chosen, from these 50 people, to form a committee.

Consider the following events:

M: « the committee is formed of three men ».

F: « the committee is formed of three women ».

A: « the committee is mixed (formed of men and women) ».

B: « the age of each member of the committee is less than 30 years ».

1) Calculate each of the probabilities $p(M)$, $p(F)$ and $p(A)$.

2) a- Calculate $p(B)$ and show that $p(B \cap \bar{A}) = \frac{33}{700}$. Deduce $p(B \cap A)$.

b- Calculate $p(B/A)$.

3) Designate by X the random variable that is equal to the number of women in the committee who have an age less than 25 years.

Determine the probability distribution of X .

iv)

Mr. Khalil has three sons: Sami, Farid and Zahi, all are married and have children.

The children in these three families are distributed as shown in the table below:

	Sami's Family	Farid's Family	Zahi's Family
Girls	2	1	3
Boys	2	3	1

The grandfather Khalil wants to choose randomly one child from each family to accompany him to their village.

1) What is the probability that he chooses three girls?

2) Consider the following events:

G: «The child chosen from Sami's family is a girl ».

B: «The child chosen from Sami's family is a boy ».

A: «The three chosen children are two girls and one boy ».

a- Prove that the probability $p(A/G)$ is equal to $\frac{5}{8}$.

b- Calculate $p(A/B)$ and $p(A)$.

3) Let X be the random variable that is equal to the number of girls chosen by the grandfather. Determine the probability distribution of X .

XVII U_1 and U_2 are two urns such that:

- U_1 contains one white ball and three black balls
- U_2 contains one red ball, three white balls, and two black balls.

One of the two urns is randomly selected:

- If the selected urn is U_1 , then two balls are selected randomly and successively with replacement from U_1
- If the selected urn is U_2 , then three balls are selected randomly and successively without replacement from U_2 .

Consider the following events:

T: "The selected urn is U_1 "

E: "Exactly two white balls are selected".

1) a- Calculate the probabilities $P(E/T)$ and $P(E \cap T)$.

b- Show that $P(E \cap \bar{T}) = \frac{9}{40}$.

c- Deduce $P(E)$.

2) Knowing that exactly two white balls are selected, calculate the probability that they are selected from U_2 .

3) Let X be the random variable equal to the number of white balls selected.

a- Verify that $P(X=1) = \frac{33}{80}$.

b- Determine $P(X \geq 1)$.

XVIII Consider the three urns U , V , and W such that:

- U contains three balls numbered 1, 2 and 3.
- V contains three balls numbered 1, 2 and 3.
- W contains seven balls: three red balls and four blue balls.

Part A

One ball is randomly selected from U and one ball is randomly selected from V .

Denote by X the random variable that is equal to the absolute value of the difference of the two numbers carried by the two selected balls.

1) Verify that the possible values of X are 0, 1 and 2.

2) Prove that the probability $P(X=2) = \frac{2}{9}$.

3) Determine the probability distribution of X .

Part B

One ball is randomly selected from U and one ball is randomly selected from V .

If the absolute value of the difference of the two numbers carried by the two selected balls is 2, then three balls are randomly and simultaneously selected from W ; otherwise, three balls are randomly and successively selected with replacement from W .

Consider the events:

E: "The absolute value of the difference of the two numbers carried by the two selected balls from U and V is 2"

F: "The three balls selected from W are all red"

1) Show that $P(F/E) = \frac{1}{35}$, then calculate $P(F \cap E)$.

2) Prove that $P(F) = \frac{149}{2205}$.

3) Knowing that at least one of the three balls selected from W is blue, calculate the probability that the absolute value of the difference of the two numbers carried by the two selected balls from U and V is 2.

V)

An urn contains 8 white balls and n red balls ($n \geq 2$).

A- Two balls are randomly and simultaneously drawn from the urn.

- 1) Calculate the probability of drawing two white balls.
- 2) Prove that the probability of drawing two balls having different colors is equal to $\frac{16n}{(n+7)(n+8)}$.

B- In what follows, suppose that $n = 8$.

A player, in a first draw, draws simultaneously and randomly two balls from the urn then replaces them in the urn and draws again, in a second draw, two balls randomly and simultaneously from the urn. If the two drawn balls, in the same draw, have the same color the player gets 5 points. Otherwise, he gets -3 points.

- 1) Verify that the probability of getting two balls having the same color in the first draw is $\frac{7}{15}$.
- 2) Let X be the random variable equal to the sum of points received by the player after the two draws.
 - a- Show that the probability $P(X = -6) = \frac{64}{225}$.
 - b- Specify the three possible values of X and determine its probability distribution.

VI)

Consider two dice so that one of them is a fair die and the other one is biased.

The faces of every die are numbered from 1 to 6.

If the fair die is rolled, all the faces have the same probability to appear.

If the biased die is rolled, the probability to obtain the face carrying the number 4 is equal to $\frac{1}{3}$, while all the other faces have the same probability to appear.

- 1) The biased die is rolled once.

Prove that the probability of obtaining a face not carrying the number 4 is equal to $\frac{2}{15}$.

- 2) The fair die is rolled twice.

Let X be the random variable equal to the number of times when the face carrying the number 4 appears.

Determine the probability distribution of X .

- 3) The biased die is rolled twice.

Show that the probability to obtain the face carrying the number 4 exactly one time is equal to $\frac{4}{9}$.

- 4) One of the two dice is chosen randomly, and then it is rolled twice.

(The two dice have the same probability to be chosen).

Calculate the probability to obtain exactly one face carrying the number 4.

VII

Consider the three urns U, V, and W such that:

- U contains three balls numbered 1, 2 and 3.
- V contains three balls numbered 1, 2 and 3.
- W contains seven balls: three red balls and four blue balls.

Part A

One ball is randomly selected from U and one ball is randomly selected from V.

Denote by X the random variable that is equal to the absolute value of the difference of the two numbers carried by the two selected balls.

1) Verify that the possible values of X are 0, 1 and 2.

2) Prove that the probability $P(X=2) = \frac{2}{9}$.

3) Determine the probability distribution of X.

Part B

One ball is randomly selected from U and one ball is randomly selected from V.

If the absolute value of the difference of the two numbers carried by the two selected balls is 2, then three balls are randomly and simultaneously selected from W; otherwise, three balls are randomly and successively selected with replacement from W.

Consider the events:

E: "The absolute value of the difference of the two numbers carried by the two selected balls from U and V is 2"

F: "The three balls selected from W are all red"

1) Show that $P(F/E) = \frac{1}{35}$, then calculate $P(F \cap E)$.

2) Prove that $P(F) = \frac{149}{2205}$.

3) Knowing that at least one of the three balls selected from W is blue, calculate the probability that the absolute value of the difference of the two numbers carried by the two selected balls from U and V is 2.

VIII

An urn contains four black balls and 1 white ball.

A game runs in the following manner:

A fair die is rolled;

- If the die shows an odd number, then one white ball is added to the urn.
- If the die shows an even number, then one black ball is added to the urn.

After that, three balls are randomly and simultaneously selected from the urn.

Consider the following events:

O: "The die shows an odd number"

B: "The three selected balls are black".

1) Calculate the probabilities $P(B/O)$ and $P(B \cap O)$, then verify that $P(B) = 0.35$.

2) The three selected balls are black. What is the probability that the die shows an even number?

3) Denote by X the random variable equal to the number of white balls selected during this game.

a- Show that $P(X=1) = 0.55$.

b- Determine the probability distribution of X.

4) Each of Sami and Karim played this game once. Let S be the random variable equal to the total number of white balls obtained by both Sami and Karim.

Calculate $P(S \geq 1)$.

Conditional probability

I) In a certain town, 40 % of men are smokers.

It is known that 6 % of men in this town have lung disease. Of those men having lung disease, 85 % are smokers.

A man is selected, at random, from this town.

Consider the following events:

D: "the selected man has a lung disease"

S: "the selected man is a smoker".

1) Determine the following probabilities: $p(D)$, $p(S)$ and $p(S/D)$.

2) Calculate the probability of each of the following events:

a- the selected man is a smoker and has a lung disease.

b- The selected man is a nonsmoker and has a lung disease.

c- The selected man has a lung disease knowing that he is a smoker.

d- the selected man is a smoker knowing that he does not have a lung disease.

II) A bag S contains eight bills: four bills of 10 000LL, three of 20 000LL and one of 50 000LL. Another bag T contains also eight bills: three bills of 10 000LL and five of 20 000LL.

1) Two bills are drawn, simultaneously and randomly, from the bag S.

Calculate the probability of each of the following events:

A: « the two drawn bills are of the same category »

B: « the sum of values of the two drawn bills is 30 000LL ».

2) One of the two bags S and T is randomly chosen, after which two bills are simultaneously and randomly drawn from this bag.

Consider the following events:

E: « the chosen bag is S »

F: « the sum of values of the two drawn bills is 30 000LL »

Calculate the probabilities $P(F \cap E)$ and $P(F \cap \bar{E})$. Deduce $P(F)$.

3) We draw, randomly, one bill from the bag S and one bill from the bag T.

Let X be the random variable that is equal to the sum of the values of the two drawn bills.

Verify that $P(X = 60\,000) = \frac{3}{64}$ and calculate $p(X = 30\,000)$

b- Determine the probability distribution of X and calculate its mean (expected value).