Logarithmic
Functions
(Official Exams)

Organized by

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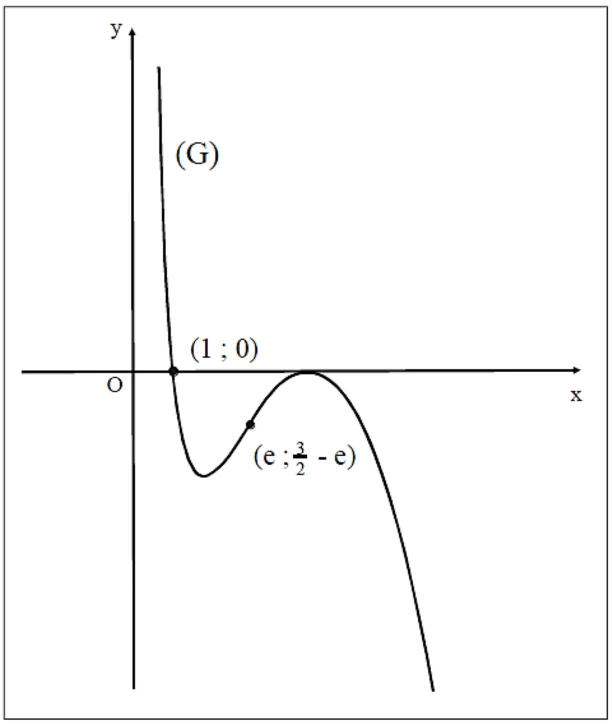
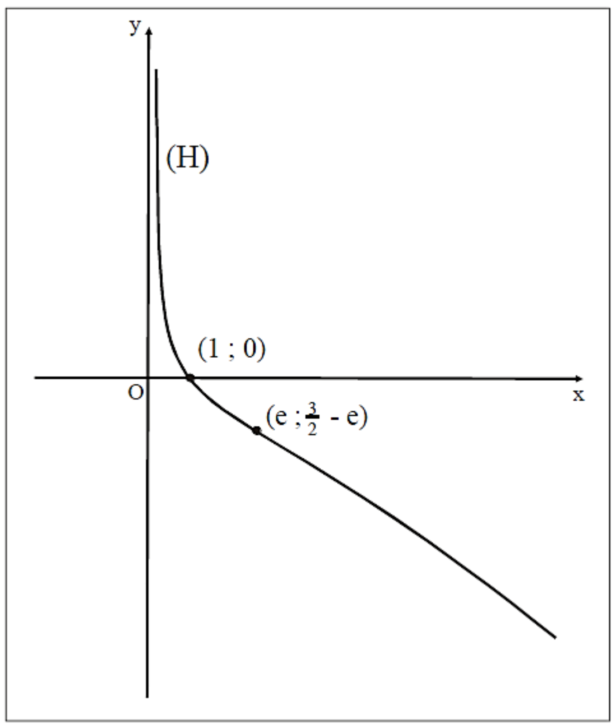
2002-1st

Let f be the function defined, on the interval $I =]0 ; +\infty[$, by : $f(x) = \frac{\ln x}{x} - 1$.

Designate by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.
(unit : 2cm).

1) a- Calculate the limits of f at the boundaries of I .
   b- Determine the asymptotes of (C) .
2) Calculate $f'(x)$ and set up the table of variations of f .
3) Verify that $y = x - 2$ is an equation of the straight line (d), tangent to (C) at the point A(1 ; – 1) .
4) Plot the line (d) and the curve (C) .
5) One of the two curves (G) and (H), shown in the figure below, represents an **antiderivative** (primitive) F of the given function f .





a- Among these two curves, which one represents the function F ?
Justify your answer.

b- Without finding the expression of F(x) , calculate in cm^2 the area of the domain bounded by the curve (C) of f , the axis of abscissas and the two lines of equations $x = 1$ and $x = e$. Give the answer to the nearest 10^{-2} .

$2003 - 2^{nd}$

Let f be the function defined on $]0; +\infty[$, by $f(x) = \frac{1}{2}x + \frac{1+\ln x}{x}$

and (C) be its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$.

1) Prove that the line of equation $x = 0$ is an asymptote of (C).

2) a- Calculate $\lim\limits_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = \frac{1}{2}x$ is an asymptote of (C).

b- Determine the coordinates of E, the point of intersection of the line (d) with the curve (C).

3) Verify that $f'(x) = \frac{x^2 - 2\ln x}{2x^2}$

4) The adjacent table is the table of variations of the function h defined by :

$$h = x^2 - 2\ln x$$

x	0		1		$+\infty$
$h'(x)$	‖	$-$	0	$+$	
$h(x)$	‖ $+\infty$	$\searrow$	1	$\nearrow$	$+\infty$

a- Verify that f is strictly increasing on $]0; +\infty[$

b- Consider, on the curve (C), a point W of abscissa 1

Write an equation of the line (D) tangent to (C) at the point.

5) Draw the curve (C) and the lines (d) and (D) and plot the points E and W.

6) Calculate the area of the region bounded by the curve (C), the asymptote (d) and the straight lines of equations $x = 1$ and $x = e$.

2004 – 1^{st}

Let f be the function defined, on $]0;+\infty[$ by $f(x) = x + 2\dfrac{\ln x}{x}$. (C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$; unit 2 cm .

1) a – Calculate $\lim\limits_{x\to 0} f(x)$ and give its graphical interpretation.

   b – Determine $\lim\limits_{x\to +\infty} f(x)$ and verify that the line (d) of equation $y = x$ is an asymptote of (C).

   c – Study according to the values of x , the relative position of (C) and (d) .

2) The table below is the table of variations of the function f ', the derivative of f.

x	0		e		$e\sqrt{e}$		$+\infty$
$f''(x)$		–		–	0	+	
$f'(x)$	$+\infty$	↘	1	↘	$1-e^{-3}$	↗	1

   a – Show that f is strictly increasing on its domain of definition, and set up its table of variations.

   b – Write an equation of the line (D) that is tangent to (C) at the point G of abscissa e .

   c – Prove that the curve(C) has a point of inflection L.

   d – Show that the equation $f(x) = 0$ has a unique root α and verify that $0{,}75 < \alpha < 0{,}76$.

3) Draw (D) , (d) and (C) .

4) Calculate, in cm^2 , the area of the region bounded by the curve (C), the line (d) and the two lines of equations $x = 1$ et $x = e$.

$2010 - 2^{nd}$

Consider the function f defined on $]0;+\infty[$ by $f(x) = x - \dfrac{(\ln x)^2}{x}$.

(C) is the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$. (unit: 1 cm)

1) Determine $\lim\limits_{x \to 0} f(x)$. Deduce an asymptote to (C).

2) Determine $\lim\limits_{x \to +\infty} f(x)$, and verify that the line (D) of equation $y = x$ is an asymptote to (C).

3) The adjacent table shows the variations of the function φ defined over $]0;+\infty[$ by:

   $\varphi(x) = x^2 + (\ln x)^2 - 2\ln x$.

x	0		1		$+\infty$
$\varphi(x)$	$\Vert$	$\searrow$	1	$\nearrow$	

   Verify that $f'(x) = \dfrac{\varphi(x)}{x^2}$. Deduce that f is strictly increasing.

4) a- Prove that (D) is tangent to (C) at the point A(1;1) and that (D) is above (C) for $x \neq 1$.

   b- Verify that the tangent (T) to (C) at the point with abscissa e^2 is parallel to (D).

5) Prove that the equation f(x) = 0 has exactly one root α, and verify that $0.5 < \alpha < 0.6$.

6) Draw (D), (T) and (C).

7) Designate by (C') the representative curve of f^{-1}, the inverse function of f.
   Draw (C') in the same system as (C).

8) a- Calculate $\displaystyle\int_{\alpha}^{1} f(x)dx$ in terms of α.

   b- Deduce, in terms of α, the area of the region bounded by (C), (C') and the two lines of equations $x = 0$ and $y = 0$.

$2011-2^{nd}$

A- Let g be the function defined over $]0;+\infty[$ by $g(x)=x+\ln x$.

1) Calculate $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.
2) Set up the table of variations of g.
3) Prove that the equation $g(x)=0$ has a unique solution α and verify that $0.5<\alpha<0.6$.
4) Determine, according to the values of x, the sign of g(x).

B- Consider the function f defined over $]0;+\infty[$ by $f(x)=x(2\ln x+x-2)$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to 0} f(x)$, $\lim\limits_{x\to+\infty} f(x)$ and determine f(e).
2) Prove that $f(\alpha)=-\alpha(\alpha+2)$.
3) Verify that $f'(x)=2g(x)$ and set up the table of variations of f.
4) Draw (C). (Take $\alpha=0.55$)
5) Use integration by parts to calculate $\int_{0.5}^{1} x\ln x\,dx$ and deduce the area of the region bounded by the curve (C), the axis of abscissas and the two lines with equations $x=0.5$ and $x=1$.
6) The curve (C) cuts the axis of abscissas at a point with abscissa 1.37. Designate by F an antiderivative of f on $]0;+\infty[$; determine, according to the values of x, the variations of F.

$2012 - 2^{nd}$

Let f be the function defined, over $]1;+\infty[$, by $f(x)=\ln\left(\frac{x+1}{x-1}\right)$.

Denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Determine $\lim\limits_{x\to 1} f(x)$ and $\lim\limits_{x\to +\infty} f(x)$. Deduce the asymptotes to (C).

2) Verify that $f'(x)=\frac{-2}{(x-1)(x+1)}$ and set up the table of variations of f.

3) Draw (C).

4) a- Prove that f has an inverse function g whose domain of definition is to be determined.

b- Prove that $g(x)=\frac{e^x+1}{e^x-1}$.

c- (G) is the representative curve of g in the same as that of (C). Draw (G).

5) Let h be the function defined over $]1;+\infty[$ by $h(x)=x\,f(x)$.

a- Verify that $f(x)=h'(x)+\frac{2x}{x^2-1}$ and determine, over $]1;+\infty[$, an antiderivative F of f.

b- Calculate the area of the region bounded by (C), the x-axis and the two lines with equations $x=2$ and $x=3$.

$\mathbf{2013 - 2^{nd}}$

A- Consider the function g defined over $]0;+\infty[$ as $g(x) = x^2 - 2\ln x$.

1) Determine $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.
2) Set up the table of variations of g and deduce that $g(x) > 0$.

B- Let f be the function defined over $]0;+\infty[$ as $f(x) = \dfrac{x}{2} + \dfrac{1+\ln x}{x}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).
2) a- Determine $\lim\limits_{x\to +\infty} f(x)$ and show that the line (Δ) with equation $y = \dfrac{x}{2}$ is an asymptote to (C).
   b- Study, according to the values of x, the relative positions of (C) and (Δ).
3) Show that $f'(x) = \dfrac{g(x)}{2x^2}$ and set up the table of variations of f.
4) Calculate the coordinates of the point B on (C) where the tangent (T) is parallel to (Δ).
5) Show that the equation f(x) = 0 has a unique solution α, then verify that $0.34 < \alpha < 0.35$.
6) Plot (Δ), (T) and (C).
7) Let h be the function defined over $]0;+\infty[$ as $h(x) = \dfrac{1+\ln x}{x}$.
   a- Find an antiderivative H of h.
   b- Deduce the measure of the area of the region bounded by (C), (Δ) and the lines with equations $x = 1$ and $x = e$.

2015 – 2^{nd}

A- Let g be the function defined on $]0;+\infty[$ as $g(x)=x^3-1+2\ln x$.

1) Determine $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.
2) Calculate $g'(x)$ then set up the table of variations of g.
3) Calculate $g(1)$ then deduce the sign of $g(x)$ according to the value of x.

B- Consider the function f defined on $]0;+\infty[$ as $f(x)=x-\dfrac{\ln x}{x^2}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. Let (d) be the line with equation $y=x$.

1) Determine $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).
2) a- Discuss, according to the values of x, the relative positions of (C) and (d).
   b- Determine $\lim\limits_{x\to +\infty} f(x)$ and show that (d) is an asymptote to (C).
3) a- Verify that $f'(x)=\dfrac{g(x)}{x^3}$ and set up the table of variations of f.
   b- Determine the point E on (C) where the tangent (Δ) to (C) is parallel to (d).
   c- Plot (d), (Δ) and (C).

4) Let α be a real number greater than 1. Denote by $A(\alpha)$ the area of the region bounded by (C), (d) and the two lines with equations $x=1$ and $x=\alpha$.

   a- Verify that $\int \dfrac{\ln x}{x^2}dx=\dfrac{-1-\ln x}{x}+k$, where k is a real number.

   b- Express $A(\alpha)$ in terms of α.

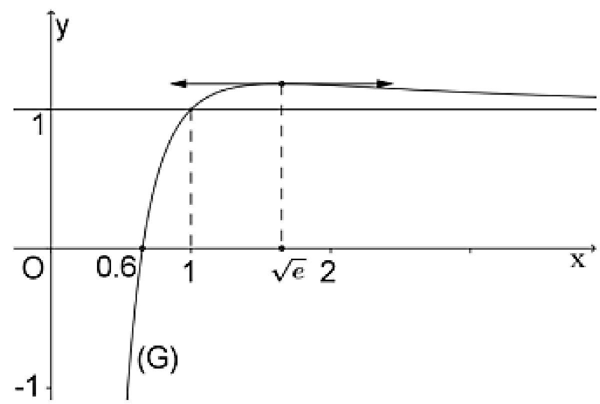
   c- Using the graphic, show that $A(\alpha)<\dfrac{(\alpha-1)^2}{2}$.

$2018-2^{nd}$

Let f be the function defined over $]0;+\infty[$ as $f(x)=x-\dfrac{1+\ln x}{x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. (1 graphical unit = 2 cm).

Let (d) be the line with equation $y=x$.

1) a. Study, according to the values of x, the relative position of (C) and (d).
   b. Determine $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) is an asymptote to (C).
2) Determine $\lim\limits_{\substack{x\to0\\x>0}} f(x)$ then deduce an asymptote to (C).
3) In the adjacent figure, we have:
   - (G) is the representative curve of the function **f ', the derivative of f.**
   - (G) admits a maximum for $x=\sqrt{e}$.
   - (G) intersects the x-axis at a point of abscissa 0.6.



   a. Set up the table of variations of f.
   b. Show that the equation f(x) = 0 admits **exactly** two roots so that one of them is equal to 1.
   c. Denote by α the second root of the equation $f(x)=0$. Verify that $0.4<\alpha<0.5$.
   d. Show that (C) admits a point of inflection whose coordinates are to be determined.
   e. Determine the coordinates of the point A on (C) where the tangent (T) at A is parallel to (d).
4) Draw (d), (T) and (C).
5) a. Calculate, in cm^2, the area $A(\alpha)$ of the region bounded by (C), (d) and the two lines with equations $x=\alpha$ and $x=1$.
   b. Prove that $A(\alpha)=\left(2-2\alpha^4\right)\,cm^2$.

$2019 - 1^{st}$

Consider the function f defined over $]0,+\infty[$ as $f(x)=2x(1-\ln x)$. Denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

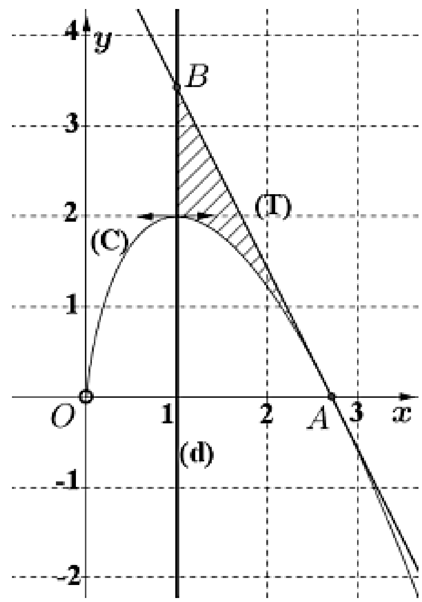
2) a- Let A be the point of intersection of (C) with the x-axis.
   Determine the coordinates of A.

   b- Show that $f'(x)=-2\ln x$ and set up the table of variations of f.

   c- Determine an equation of the tangent (T) to (C) at A.

In the adjacent figure:

- (C) is the representative curve of f
- (T) is the tangent to (C) at A
- (d) is the line with equation $x=1$
- B(1 , 2e – 2) is the point of intersection of (d) and (T).



3) a- Show that f has, over $]1,+\infty[$, an inverse function g whose domain of definition is to be determined.

   b- Set up the table of variation of g.

   c- Copy (C), then draw (C'), the representative curve of g in the same system.

4) a- Using integration by parts, determine $\int x\ln(x)\,dx$.

   b- Show that $\int_1^e f(x)dx=\dfrac{e^2-3}{2}$.

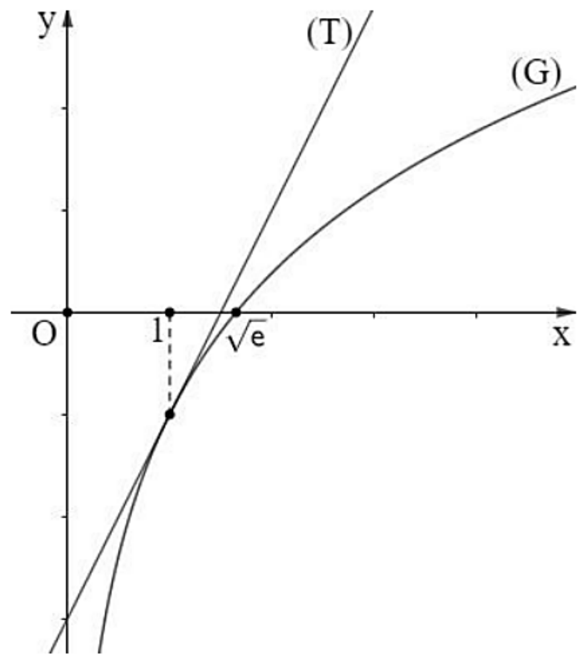
   c- Calculate the area of the shaded region bounded by (C), (T) and (d).

$2022 - 2^{nd}$

Part A

In the adjacent figure:

- (G) is the representative curve over $]0, +\infty[$ of a function g in an orthonormal system $(O\ ;\ \vec{i}, \vec{j})$.
- The line (T) with equation $y = 2x - 3$ is tangent to (G) at the point with abscissa 1.
- The curve (G) intersects the x-axis at the point with abscissa $\sqrt{e}$.



1) Determine $g(\sqrt{e})$ and $g'(1)$.

2) **a)** Study, according to the values of x, the relative positions of (G) and the x-axis.

b) Deduce the sign of g(x) for all x in $]0, +\infty[$.

Part B

Consider the function f defined over $]0, +\infty[$ as $f(x) = (\ln x)^2 - \ln x$.

Denote by (C) the representative curve of f in an orthonormal system $(O\ ;\ \vec{i}, \vec{j})$.

1) Show that the y-axis is an asymptote to (C).

2) **a)** Determine $\lim\limits_{x\to\infty} f(x)$.

b) Calculate f(6) to the nearest 10^{-1}.

3) Knowing that $f'(x) = \dfrac{1}{x} g(x)$.

Set up the table of variations of f.

4) Determine the points of intersection of (C) and the x-axis.

5) Draw (C).

Part C

Consider the function h defined over $]0, +\infty[$ as $h(x) = e^{-f(x)}$.

1) Determine $\lim\limits_{x\to 0^+} h(x)$ and $\lim\limits_{x\to\infty} h(x)$.

2) Set up the table of variations of h.