الأنروا - برنامج التربية و التعليم / لبنان
مركز التطوير التربوي
وحدة التقييم

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عدد المسائل: أربع — مسابقة في مادة الرياضيات

العلامة: 20 — المدة: ساعتان — الإسم: --------------------------

ملاحظــة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

- يستطيع التلميذ الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one among the proposed answers to each question is correct.

Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Proposed Answers		
		a	b	c
1	If $f(x)=\frac{1}{\ln x}$, then $f'(e^2)=$	$\frac{-1}{4e^2}$	$-4e^2$	$\frac{1}{2}$
2	The domain of definition of the function h defined by: $h(x)=\frac{x^2-4}{\ln(x-1)}$ is	$]0;\ +\infty[$	$]1;2[\cup]2\ ;\ +\infty[$	$]1;+\infty[$
3	$\lim\limits_{x\to-\infty}\frac{\ln(e^x+1)}{e^x}$ is equal to	1	$+\infty$	0
4	For any real number x; $\ln(e^{-x}+1)+x$ is equal to	0	$\ln(e^x+1)$	2x

II- (4 points)

The questions 1), 2), 3) and 4) are independent.

1) (C) is the representative curve of the function f defined on $\mathbb{R}$ as $f(x)=\frac{2e^x}{e^x+1}-x$.

Prove that the point $W(0\ ;1)$ is the center of symmetry of (C).

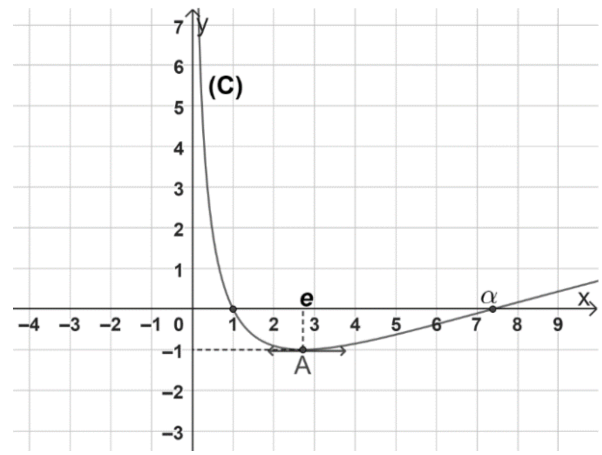
2) Solve the following inequality: $(\ln x)^2-2\ln x<0$.

3) Consider the function g defined on $\mathbb{R}$ as $g(x)=-2x+3+e^{-2x+1}$.
Determine the image of the interval $[0;\ +\infty[$ by g.

4) Does the system of equations $\begin{cases} e^{x+1}\ .\ e^{y-2}=2 \\ \ln x+\ln y=\ln(x-1)+\ln(y-1) \end{cases}$ have a solution? Justify.

III- (3 points)

- In the adjacent orthonormal system $(O; \vec{i}\,; \vec{j})$, (C) represents the curve of f which is defined over $]0; +\infty[$ as: $f(x) = a(\ln x)^2 + b\ln x$. (a and b are two integers)
- (C) admits at point A a tangent parallel to the axis of abscissas.



1) Find graphically: $\lim\limits_{x\to 0^+} f(x)$ and $f'(e)$.

2) **a-** Calculate $f'(x)$ in terms of a and b.

b- Show that $a = 1$ and $b = -2$.

3) Consider the function g defined over $]0; +\infty[$ as $g(x) = e^{f(x)}$ and (G) be its representative curve in an orthonormal system.

Find the equation of the tangent (T) to (G) at point B of abscissa 1.

IV- (9 points)

Part A

Let g be the function defined on $\mathbb{R}$ as $g(x) = xe^x - 1$.

1) Verify that $\lim\limits_{x\to-\infty} g(x) = -1$ and determine $\lim\limits_{x\to+\infty} g(x)$.

2) Copy and complete the adjacent table of variations of g.

x	$-\infty$		-1		$+\infty$
$g'(x)$		$-$	0	$+$	
$g(x)$					

3) **a-** Show that the equation $g(x) = 0$ has, on $\mathbb{R}$, a unique solution α, then verify that $0.56 < \alpha < 0.58$.

b- Determine, according to the values of x, the sign of $g(x)$.

Part B

Consider the function f defined on $\mathbb{R}$ as $f(x) = (x - 1)(e^x - 1)$.

Designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}\,; \vec{j})$.

1) **a-** Determine $\lim\limits_{x\to-\infty} f(x)$ and show that the straight line (d) with equation $y = -x + 1$ is an asymptote to (C).

b- Study, according to the values of x, the relative position of (C) and (d).

2) Determine $\lim\limits_{x\to+\infty} f(x)$, then calculate $f(2)$.

3) Show that $f(\alpha) = 2 - \alpha - \dfrac{1}{\alpha}$.

4) Verify that $f'(x) = g(x)$ and set up the table of variations of the function f.

5) Prove that the curve (C) has a point of inflection M whose coordinates to be determined.

6) Calculate the coordinates of the points of intersection between (C) and the abscissa axis.

7) Suppose that $\alpha = 0.57$. Draw (d) and (C).