Rotation

N°1) ABCD is a direct square with center O. $(\overrightarrow{AB}, \overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$.
Let M be the point defined by $\overrightarrow{AM} = \frac{1}{4}\overrightarrow{AB}$. consider the rotation with center O and angle $\frac{\pi}{2}$.
a) Locate the points $N = r(M)$, $P = r(N)$ and $Q = r(P)$.
b) prove that MNPQ is a square

N°2) OAB is an isosceles right triangle such that $(\overrightarrow{OA}, \overrightarrow{OB}) = -\frac{\pi}{2}$.
Denote by r the rotation $r(A, \frac{\pi}{2})$ and by r' the rotation $r'(B, \frac{\pi}{2})$. M is any point.
a) construct the points $N = r(M)$ and $P = r'(N)$.
b) prove that $(r' \circ r)(O) = O$
c) prove that O is the midpoint of [MP].

N°3) (C) is a circle with diameter [AB] and M is a point that describes (C). Construct the square AMNP such that $(\overrightarrow{AM}, \overrightarrow{AP}) = \frac{\pi}{2}$. Determine the locus of point I midpoint of [AP].

N°4) (d) is a line and A is a point not belonging to (d).
M is a variable point on (d). Construct the equilateral triangles AMN and AMN'. Determine the loci of the points N and N'

N°5) [AB] is a fixed segment and I is a variable point on line (δ). Let A' and B' be the images of A and B under the rotation $r(I, \frac{\pi}{3})$.
prove that A' and B' vary on two parallel lines when I describes (δ)

N°6) ABC is any direct triangle. points D and E are constructed in a way that BAD and EAC are direct, right isosceles triangles at A.
show that BE = DC and that (BE) and (CD) are perpendicular by using the rotation of center A.

N°7) on the sides [AB] and [BC] of a square ABCD of direct sense and of center O, locate the points E and F such that AE=BF.
Denote by H the point of intersection of line (CE) and (AF).
r is the rotation with center O and angle $\frac{\pi}{2}$.
1) Determine the images $r(A)$, $r(B)$, $r(D)$, $r(C)$ and $r(E)$.
2) prove that H is the orthocenter of triangle DEF.

N°8) Given a line (D) and A a point not belonging to (D).
M is a variable point on (D). construct the square MNPQ of center A.
What is the set of points N, P and Q when M describes (D)

N°9) ABCE is a rhombus such that $(\overrightarrow{AB}, \overrightarrow{AE}) = \frac{\pi}{3}\ (2\pi)$, O its center,
$r = r(A, \frac{\pi}{3})$, $r' = r(B, -\frac{\pi}{3})$ and $r'' = r(E, -\frac{2\pi}{3})$ three rotations
Find the nature of $r' \circ r$ and that of $r'' \circ r$ and $r'' \circ r'$

N°10) consider a triangle OAB right isosceles such that OA=OB and $(\overrightarrow{OA}, \overrightarrow{OB}) = \frac{\pi}{2}\ (\text{mod } 2\pi)$.
I, J and K are the midpoints of the segments [AB], [OB] and [OA] respectively. Let r be the rotation of center I and angle $\frac{\pi}{2}$ and by t the translation of vector $\frac{1}{2}\overrightarrow{AB}$, let $f = r \circ t$ and $g = t \circ r$
1) a) Determine $f(K)$, $f(I)$ and $f(A)$
b) precise the nature of f and determine its characteristic elements
2) a) Determine $g(J)$ and $g(B)$
b) precise the nature of g and determine its characteristic elements
3) Let $h = g \circ f^{-1}$
a) Determine $h(O)$ and find the nature of h.
b) M being any point in the plane, Let $M_1 = f(M)$ and $M_2 = g(M)$
Show that the vector $\overrightarrow{M_1M_2}$ is equal to a fixed vector