Logarithmic and Exponential Functions (Official Exams)

2001-2023

Organized by

2001-1st

Consider the functions f and g , defined on $]0 ; +\infty[$ by :

$f(x) = 2x\ln x$ and $g(x) = e^{\frac{1}{2x}}$

Designate by (C) the representative curve of f and by (G) that of g , in an orthonormal system $(O; \vec{i}, \vec{j})$. (*unit: 2 cm*).

1) a - Calculate the limits of f at 0 and at $+\infty$ and specify f(e).
   b - Set up the table of variations of f and draw (C).
2) Calculate the area of the domain bounded by the curve (C), the axis of abscissas and the lines of equations $x = 1$ and $x = \sqrt{e}$
3) a - Calculate the limits of g at 0 and at $+\infty$.
   Specify the asymptotes of (G).
   b - Set up the table of variations of g and draw (G) (*in the same system as* (C)).
4) a - Prove that the function g admits, on $]0 ; +\infty[$, an inverse function g^{-1}
   b - Specify the domain of definition of g^{-1} and determine $g^{-1}(x)$ in terms of x.
5) The line of equation $y = 1$ cuts (C) at a point A of abscissa a , and the line of equation $y = x$ cuts (G) at a point B of abscissa b.
   Prove that $a = b$ and verify that $1.4 < a < 1.5$

2002-1st

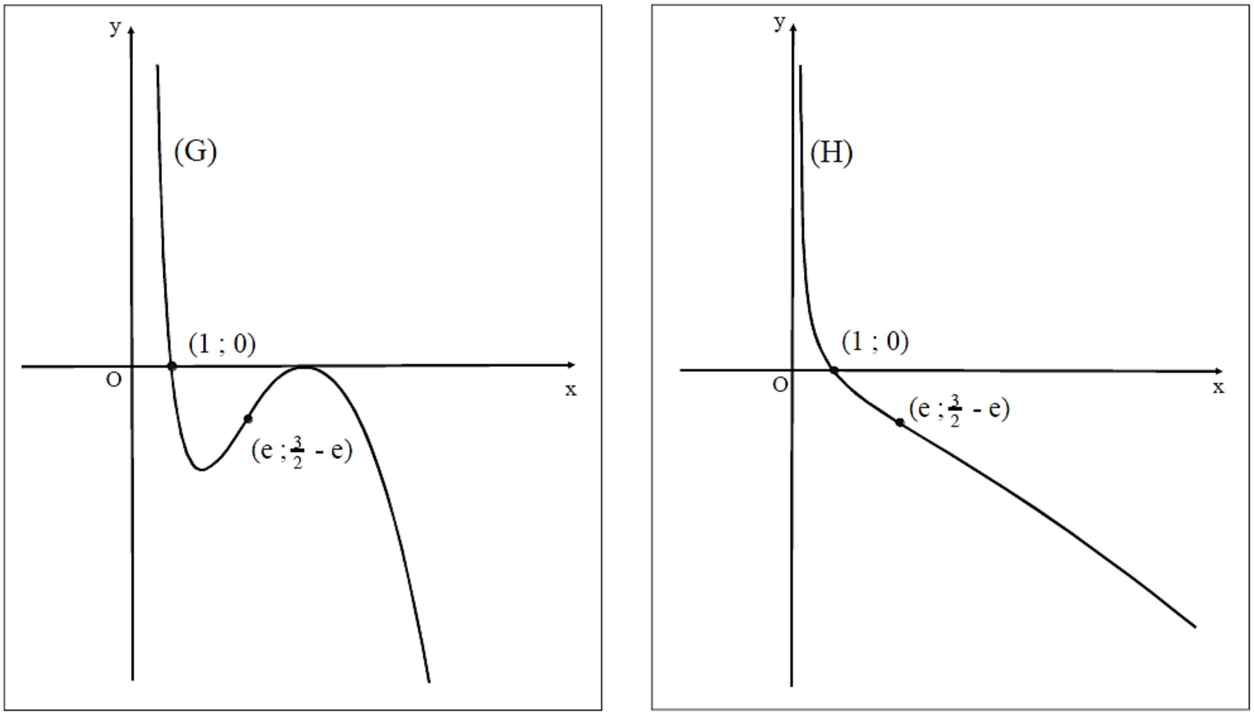
Let f be the function defined, on the interval $I =]0 ; +\infty[$, by : $f(x) = \dfrac{\ln x}{x} - 1$.

Designate by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.
(unit : 2cm).

1) a- Calculate the limits of f at the boundaries of I .
   b- Determine the asymptotes of (C) .
2) Calculate $f'(x)$ and set up the table of variations of f .
3) Verify that $y = x - 2$ is an equation of the straight line (d), tangent to (C) at the point $A(1 ; -1)$.

4) Plot the line (d) and the curve (C) .

5) One of the two curves (G) and (H), shown in the figure below, represents an **antiderivative** (primitive) F of the given function f .

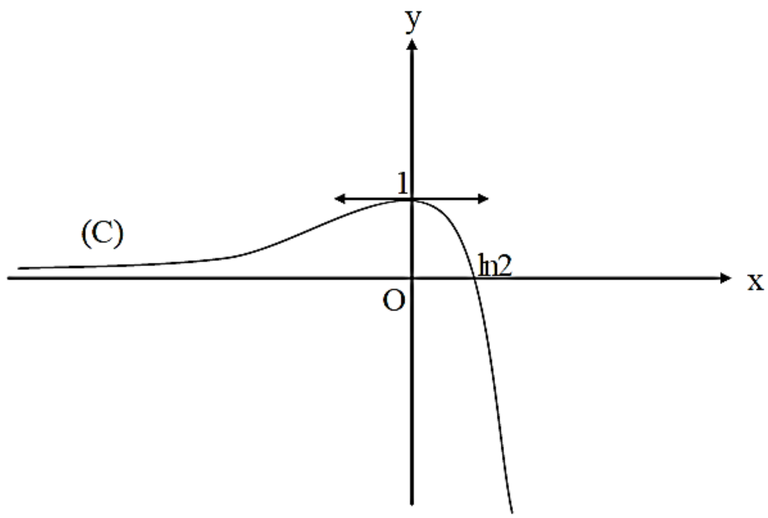


a- Among these two curves, which one represents the function F ?
Justify your answer.

b- Without finding the expression of F(x) , calculate in cm^2 the area of the domain bounded by the curve (C) of f , the axis of abscissas and the two lines of equations $x = 1$ and $x = e$. Give the answer to the nearest 10^{-2} .

$2003 - 1^{st}$

The curve (C) shown in the figure below is the representative curve, in an orthonormal system $(O; \vec{i}, \vec{j})$, of a function f defined over $\mathbb{R}$.



1) a -Prove that f admits, over $[0; +\infty[$, an inverse function g.
   b-Specify the domain of definition of g, and draw its representative curve (G).

2) The function f, represented by the curve (C) in the figure above, is a particular solution of the differential equation $y'' - 3y' + 2y = 0$.
   Determine f(x).

   In all what follows, let $f(x) = 2e^{x} - e^{2x}$.

3) Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = \ln 2$.

4) Verify that $g(x) = \ln(1 + \sqrt{1-x})$.

5) Calculate $g'(x)$, and deduce the slope of the tangent to the curve (C) at the point A of abscissa ln2.

$2003-2^{nd}$

Let f be the function defined on $]0;+\infty[$, by $f(x)=\frac{1}{2}x+\frac{1+\ln x}{x}$

and (C) be its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1) Prove that the line of equation $x = 0$ is an asymptote of (C).

2) a- Calculate $\lim\limits_{x\to+\infty} f(x)$ and prove that the line (d) of equation $y=\frac{1}{2}x$ is an asymptote of (C).

b- Determine the coordinates of E, the point of intersection of the line (d) with the curve (C).

3) Verify that $f'(x)=\frac{x^2-2\ln x}{2x^2}$

4) The adjacent table is the table of variations of the function h defined by :

$$h=x^2-2\ln x$$

x	0		1		$+\infty$
$h'(x)$	‖	$-$	0	$+$	
$h(x)$	‖ $+\infty$	$\searrow$	1	$\nearrow$	$+\infty$

a- Verify that f is strictly increasing on $]0;+\infty[$

b- Consider, on the curve (C), a point W of abscissa 1

Write an equation of the line (D) tangent to (C) at the point.

5) Draw the curve (C) and the lines (d) and (D) and plot the points E and W.

6) Calculate the area of the region bounded by the curve (C), the asymptote (d) and the straight lines of equations $x = 1$ and $x = e$.

2004 – 1st

Let f be the function defined, on $]0;+\infty[$ by $f(x) = x + 2\dfrac{\ln x}{x}$. (C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$; unit 2 cm .

1) a – Calculate $\lim\limits_{x\to 0} f(x)$ and give its graphical interpretation.

   b – Determine $\lim\limits_{x\to +\infty} f(x)$ and verify that the line (d) of equation $y = x$ is an asymptote of (C).

   c – Study according to the values of x , the relative position of (C) and (d) .

2) The table below is the table of variations of the function f ', the derivative of f.

x	0		e		$e\sqrt{e}$		$+\infty$
$f''(x)$	‖	$-$	⋮	$-$	0	$+$	
$f'(x)$	‖ $+\infty$	↘	1	↘	$1-e^{-3}$	↗	1

   a – Show that f is strictly increasing on its domain of definition, and set up its table of variations.
   b – Write an equation of the line (D) that is tangent to (C) at the point G of abscissa e .
   c – Prove that the curve(C) has a point of inflection L.
   d – Show that the equation $f(x) = 0$ has a unique root α and verify that $0{,}75 < \alpha < 0{,}76$.

3) Draw (D) , (d) and (C) .

4) Calculate, in cm^2 , the area of the region bounded by the curve (C), the line (d) and the two lines of equations $x = 1$ et $x = e$.

2005 – 1st

Consider the function f that is defined, on $I =]1 ; +\infty[$, by $f(x) = x + 1 - \dfrac{3e^x}{e^x - e}$

and let (C) be its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

1) a- Prove that the line of equation $x = 1$ is an asymptote to (C).
   b- Calculate $\lim\limits_{x \to +\infty} f(x)$ and show that the line (d) of equation $y = x - 2$ is an asymptote to (C).
   c- Determine the relative position of (C) and (d).
2) Prove that $f'(x) > 0$ for all values of x in I, and set up the table of variations of f.
3) Prove that the equation $f(x) = 0$ has a unique root α and verify that $2.6 < \alpha < 2.7$.
4) Draw the curve (C).
5) Designate by (D) the region that is bounded by (C), the line (d) and the lines of equations $x = 3$ and $x = 4$.

   Calculate $\displaystyle\int_3^4 \frac{e^x}{e^x - e}\,dx$ and deduce the area of the region (D).
6) a- Prove that f, on the interval I, has an inverse function g.
   b- Prove that the equation $f(x) = g(x)$ has no roots.

2005 – 2nd

Let f be the function that is defined on IR by : $f(x) = x + 2 - e^{-x}$, and (C) be its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = x + 2$ is an asymptote of (C).
   b- Calculate $\lim\limits_{x \to -\infty} f(x)$ and give, in the decimal form, the values of $f(-1.5)$ and $f(-2)$.
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Write an equation of the line (T) that is tangent to (C) at the point A of abscissa 0.
4) Show that the equation $f(x) = 0$ has a unique root α and verify that $-0.5 < \alpha < -0.4$.
5) Draw (d), (T) and (C).
6) Designate by g the inverse function of f, on IR.
   a- Draw, in the system $(O ; \vec{i}, \vec{j})$, the curve (G) that represents g.
   b- Designate by $A(\alpha)$ the area of the region that is bounded by the curve (C), the axis of abscissas and the two lines of equations $x = \alpha$ and $x = 0$.

      Show that $A(\alpha) = (-\dfrac{\alpha^2}{2} - 3\alpha - 1)$ units of area.
   c- Deduce the area of the region that is bounded by the curve (G), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

2006 – 1^{st}

A- Consider the differential equation (E): $y'' - 4y' + 4y = 4x^2 - 16x + 10$.

Let $z = y - x^2 + 2x$.

1) Write a differential equation (E') satisfied by z.
2) Solve (E') and deduce the general solution of (E).
3) Determine the particular solution of (E) whose representative curve, in an orthonormal system, has at the point A(0 ;1) a tangent parallel to the axis of abscissas.

B- Let f be the function that is defined on IR by $f(x) = e^{2x} + x^2 - 2x$.

Designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$.

b- Calculate f(1) and f(–1.5) in their decimal forms.

2) The table below is the table of variations of the function f ', the derivative of f.

x	$-\infty$		0		$+\infty$
$f'(x)$					$+\infty$
		↗	0	↗	
	$-\infty$				

a- Determine, according to the values of x, the sign of f '(x).

b- Set up the table of variations of f.

3) Draw the curve (C).

4) Let F be the function that is defined on $[0 ; +\infty[$ by $F(x) = \int_0^x f(t)dt$.

a- Determine the sense of variations of F.

b- What is the sign of F(x) ? Justify your answer.

2006 – 2^{nd}

A- Given the differential equation (E) : $y' - y - e^{x} + 1 = 0$.

Let $z = y - xe^{x} - 1$.

1) Find a differential equation (E') that is satisfied by z, and determine its general solution.
2) Deduce the general solution of (E), and find a particular solution y of (E) that verifies $y(0) = 0$.

B- Let f be the function that is defined on IR by $f(x) = (x - 1)e^{x} + 1$, and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Calculate $\lim_{x\to+\infty} f(x)$. Give f(2) in the decimal form.

b- Calculate $\lim_{x\to-\infty} f(x)$ and deduce an asymptote (d) to (C).

c- Verify that the curve (C) cuts its asymptote (d) at the point E(1 ; 1).

2) a- Calculate f '(x) and set up the table of variations of f.

b- Prove that the curve (C) has a point of inflection.

3) Draw the line (d) and the curve (C).

4) a- Prove that the function f has, on $[0 ; +\infty[$, an inverse function g.

b- Draw the curve (G) that represents g, in the system $(O; \vec{i}, \vec{j})$.

c- Calculate the area of the region bounded by the two curves (C) and (G).

$2007-1^{st}$

Consider the function f defined over $]-\infty, 0[\cup]0, +\infty[$ by $f(x) = x - 1 - \dfrac{4}{e^x - 1}$.

Designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Show that the axis of ordinates is an asymptote to (C).
   b- Calculate $\lim\limits_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = x - 1$ is an asymptote to the curve (C).
   c- Prove that the line (D) of equation $y = x + 3$ is an asymptote to (C) at $-\infty$.

2) Prove that the point S(0 ; 1) is a center of symmetry of (C).

3) a- Calculate $f'(x)$ and set up the table of variations of f.
   b- Show that the equation $f(x) = 0$ has two roots α and β and verify that : $1.7 < \alpha < 1.8$ and $-3.2 < \beta < -3.1$.

4) Draw (d), (D) and (C).

5) a- Prove that $f(x) = x + 3 - \dfrac{4e^x}{e^x - 1}$.
   b- Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations $x = 2$ and $x = 3$.

6) Let g be the inverse function of f on $]0, +\infty[$.
   Prove that the equation $f(x) = g(x)$ has no roots.

2007 – 2^{nd}

Let f be the function that is defined, on $[0;+\infty[$, by $f(x) = (x+1)e^{-x}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. (unit: 2cm)

1- Calculate $\lim_{x\to+\infty} f(x)$ and deduce an asymptote of (C).

2- a) Calculate $f'(x)$ and set up the table of variations of f.

b) Calculate $f'(0)$ and interpret the result graphically.

3- a) Prove that the curve (C) has a point of inflection $W(1, \frac{2}{e})$.

b) Write an equation of the line (d) that is tangent to (C) at the point W.

4- Draw (d) and (C).

5- a) Calculate the real numbers a and b so that the function F defined on $[0;+\infty[$ by $F(x) = (ax+b)e^{-x}$ is an antiderivative of f.

b) Calculate, in cm^2, the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

6- Let g be the inverse function of f and designate by (G) the representative curve of g.

a) Draw (G) in the preceding system.

b) Find an equation of the tangent to the curve (G) at the point of abscissa $\frac{2}{e}$.

2008 – 1st

Let f be the function defined on IR by $f(x) = (x-1)e^{x} + 1$ and designate by (C) its representative curve in an orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,)$.

1) a- Calculate $\lim\limits_{x\to-\infty} f(x)$ and deduce an asymptote (d) of (C).

   b- Study, according to the values of x, the relative positions of (C) and (d).

   c- Calculate $\lim\limits_{x\to+\infty} f(x)$ and find f(2) in decimal form.

2) Calculate f '(x) and set up the table of variations of f.

3) Prove that the curve (C) has a point of inflection W whose coordinates are to be determined.

4) a- Draw (d) and (C).

   b- Discuss graphically, according to the values of the real parameter m, the number of solutions of the equation $(m-1)\,e^{-x} = x - 1$.

5) Calculate the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

6) a- Show that the function f has on $[0;\,+\infty[$ an inverse function g and draw (G), the representative curve of g in the system $(O\,;\,\vec{i}\,,\,\vec{j}\,)$.

   b- Find the area of the region bounded by (G), the axis of ordinates and the line (d).

<u>2008 – 2nd</u>

Let f be the function defined over $]1;+\infty[$ by $f(x) = x - \dfrac{1}{x\ln x}$ and designate by (C) its representative curve in an orthonormal system $(O;\,\vec{i}\,,\,\vec{j}\,)$.

1) Calculate $\lim\limits_{x\to1} f(x)$ and deduce an asymptote to (C).

2) Calculate $\lim\limits_{x\to+\infty} f(x)$. Prove that the straight line (d) of equation $y = x$ is an asymptote to (C) and study the position of (C) and (d).

3) Calculate f '(x) and show that f is strictly increasing.
   Set up the table of variations of f.

4) Show that the equation $f(x) = 0$ has a unique root α and verify that $1.5 < \alpha < 1.6$.

5) Draw (d) and (C).

6) a- Calculate the area A(t) of the region limited by the curve (C), the straight line (d) and the two straight lines of equations $x = e$ and $x = t$ where $t > e$.

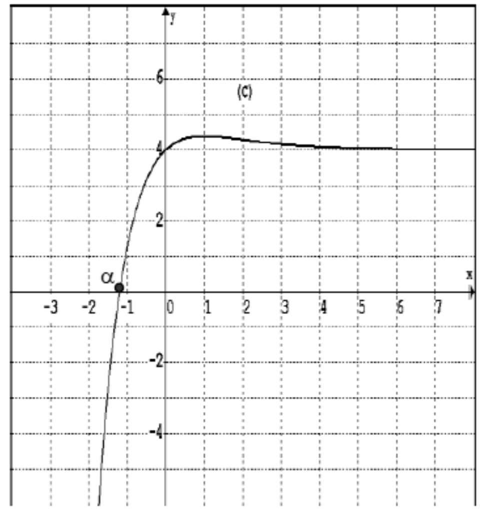
   b- Show that for all $t > e$, we have $A(t) < t$.

$2009 - 1^{st}$

III- (8 points)

A-Consider the function f defined on IR by $f(x) = 4 + x e^{-x}$ whose representative curve (C) is shown in the adjacent figure.

(C) cuts the axis of abscissas in one point of abscissa α.



1) Use (C) to study the sign of $f(x)$.

2) Use integration by parts to calculate $\int_0^2 x e^{-x} dx$, then calculate the area of the region bounded by the axis of ordinates, the axis of abscissas, the curve (C) and the straight line with equation $x = 2$.

B- In all what follows, let $\alpha = -1.2$.

Consider the function g defined on IR, by $g(x) = 4x - 3 - (x+1)e^{-x}$ and designate by (G) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Verify that $\lim\limits_{x \to -\infty} g(x) = +\infty$ and determine g(-2.5) to the nearest 10^{-2}.

2) Calculate $\lim\limits_{x \to +\infty} g(x)$ and verify that the straight line (D) with equation $y = 4x-3$ is an asymptote of (G).

3) Determine the coordinates of A, the point of intersection of (G) with its asymptote (D), and study the position of (G) with respect to (D).

4) a- Verify that $g'(x) = f(x)$.

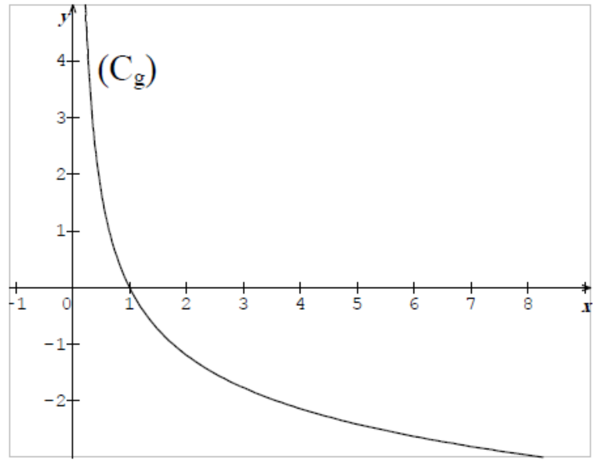
b- Set up the table of variations of g.

5) Draw (D) and (G).

$2009-2^{nd}$

Consider the function f defined, on $]0;+\infty[$, by $f(x)=\dfrac{1+\ln x}{e^x}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

The curve (C_g), shown in the adjacent figure, is the representative curve in an orthonormal system, of the function g defined on $]0;+\infty[$ by $g(x)=\dfrac{1}{x}-1-\ln x$.



1) Calculate the area of the region bounded by the curve (C_g), the axis of abscissas and the line of equation x = 2.

2) Show that $f'(x)=\dfrac{g(x)}{e^x}$ and deduce the sign of $f'(x)$ according to the values of x.

3) Calculate $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to +\infty} f(x)$ and determine the asymptotes of the curve (C).

6) Find an equation of the tangent to the curve (C) at the point of abscissa $\dfrac{1}{e}$.

7) Draw (C).

8) Discuss, according to the values of the real number m, the number of solutions of the equation $\ln x = me^x - 1$.

2010 – 1st

Consider the function f defined, on $]0\,;+\infty[$, by $f(x)=2x-2+\dfrac{1}{e^x-1}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).

b- Determine $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) with equation $y=2x-2$ is an asymptote to (C).

c- What is the position of (C) relative to (d)?

2) a- Show that $f'(x)=\dfrac{(e^x-2)(2e^x-1)}{(e^x-1)^2}$.

b- Copy and complete the adjacent table of variations of f.

x	0	ln2	$+\infty$
f '(x)		0	
f(x)			

3) Draw (d) and (C).

4) Verify that $\dfrac{1}{e^x-1}=\dfrac{e^x}{e^x-1}-1$ and calculate the area of the region bounded by the curve (C), the line (d) and the two lines with equations $x=\ln 2$ and $x=\ln 3$.

5) Let g be the function defined over $]0\,;+\infty[$ by $g(x)=\ln(f(x))$

a- Calculate $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.

b- Set up the table of variations of g.

c- Prove that the equation $g(x)=0$ has two distinct roots.

$\mathbf{2010 - 2^{nd}}$

Consider the function f defined on $]0;+\infty[$ by $f(x)=x-\dfrac{(\ln x)^2}{x}$.

(C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$. (unit: 1 cm)

1) Determine $\lim\limits_{x\to 0} f(x)$. Deduce an asymptote to (C).

2) Determine $\lim\limits_{x\to +\infty} f(x)$, and verify that the line (D) of equation $y=x$ is an asymptote to (C).

3) The adjacent table shows the variations of the function φ defined over $]0;+\infty[$ by:

$\varphi(x)=x^2+(\ln x)^2-2\ln x$.

x	0		1		$+\infty$
$\varphi(x)$	$\|\|$	↘	1	↗	

Verify that $f'(x)=\dfrac{\varphi(x)}{x^2}$. Deduce that f is strictly increasing.

4) a- Prove that (D) is tangent to (C) at the point A(1;1) and that (D) is above (C) for $x\neq 1$.

b- Verify that the tangent (T) to (C) at the point with abscissa e^2 is parallel to (D).

5) Prove that the equation f(x) = 0 has exactly one root α, and verify that $0.5<\alpha<0.6$.

6) Draw (D), (T) and (C).

7) Designate by (C') the representative curve of f^{-1}, the inverse function of f.
Draw (C') in the same system as (C).

8) a- Calculate $\displaystyle\int_{\alpha}^{1} f(x)dx$ in terms of α.

b- Deduce, in terms of α, the area of the region bounded by (C), (C') and the two lines of equations $x=0$ and $y=0$.

2011 – 1^{st} الاستكمالية

Consider the function f defined on $\mathbb{R}$ by $f(x)=\dfrac{e^x}{e^x+1}$. Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Deduce the asymptotes of the curve (C).
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Show that $f''(x)=\dfrac{e^x(1-e^x)}{(1+e^x)^3}$. Prove that (C) has a point of inflection I to be determined.
4) Write an equation of the tangent (T) to (C) at the point I.
5) Draw (T) and (C).
6) The function f has on $\mathbb{R}$ an inverse function g.

   a- Draw the representative curve (G) of g in the given system.

   b- Verify that $g(x)=\ln\left(\dfrac{x}{1-x}\right)$.

   c- (G) and (C) intersect at a point with abscissa α. Calculate, in terms of α, the area of the region bounded by (C), (G) and the two axes of coordinates.

2011 – 1^{st}

Let f be the function defined, on$]-\infty\,;+\infty[$, by $f(x) = x + 2 - \dfrac{3}{1+e^x}$.

(C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim\limits_{x\to-\infty} f(x)$; Show that the line ($d_1$) with equation $y = x - 1$ is an asymptote to (C) and specify the position of (d_1) relative to (C).

b- Calculate $\lim\limits_{x\to+\infty} f(x)$; Show that the line ($d_2$) with equation $y = x + 2$ is an asymptote to (C) and specify the position of (d_2) relative to (C).

2) Prove that the point I $\left(0\,;\dfrac{1}{2}\right)$ is a center of symmetry of (C).

3) Show that f is strictly increasing on $]-\infty\,;+\infty[$ and set up its table of variations.

4) Draw (d_1), (d_2) and (C).

5) a - Verify that $f(x) = x + 2 - \dfrac{3e^{-x}}{1+e^{-x}}$.

b - Calculate the area $A(\lambda)$ of the region bounded by the curve (C), the asymptote (d_2) and the two lines with equations $x = 0$ and $x = \lambda$, where $\lambda > 0$, then calculate $\lim\limits_{\lambda\to+\infty} A(\lambda)$.

6) Designate by g the inverse function of f on$]-\infty\,;+\infty[$; (G) is the representative curve of g.

a- Verify that E (1+ln2; ln2) is a point on (G).

b- Calculate the slope of the tangent to (G) at E.

$2011 - 2^{nd}$

A- Let g be the function defined over $]0;+\infty[$ by $g(x)=x+\ln x$.

1) Calculate $\lim_{x\to 0} g(x)$ and $\lim_{x\to +\infty} g(x)$.
2) Set up the table of variations of g.
3) Prove that the equation $g(x)=0$ has a unique solution α and verify that $0.5<\alpha<0.6$.
4) Determine, according to the values of x, the sign of g(x).

B- Consider the function f defined over $]0;+\infty[$ by $f(x)=x(2\ln x+x-2)$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim_{x\to 0} f(x)$, $\lim_{x\to +\infty} f(x)$ and determine f(e).
2) Prove that $f(\alpha)=-\alpha(\alpha+2)$.
3) Verify that $f'(x)=2g(x)$ and set up the table of variations of f.
4) Draw (C). (Take $\alpha=0.55$)
5) Use integration by parts to calculate $\int_{0.5}^{1} x\ln x\,dx$ and deduce the area of the region bounded by the curve (C), the axis of abscissas and the two lines with equations $x=0.5$ and $x=1$.
6) The curve (C) cuts the axis of abscissas at a point with abscissa 1.37. Designate by F an antiderivative of f on $]0;+\infty[$; determine, according to the values of x, the variations of F.

2012 – 1^{st}

Consider the function f defined over □ by $f(x) = (x+1)^2 e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-2)$.

   b- Determine $\lim\limits_{x\to+\infty} f(x)$ and deduce an asymptote to (C).

2) Show that $f'(x) = (1-x^2)e^{-x}$ and set up the table of variations of f.

3) The line (d) with equation $y = x$ intersects (C) at a point with abscissa α.

   Verify that $1.4 < \alpha < 1.5$.

4) Draw (d) and (C).

5) Let F be the function defined on□ by $F(x) = (px^2 + qx + r)\,e^{-x}$.

   a- Calculate p, q and r so that F is an antiderivative of f .

   b- Calculate the area of the region bounded by (C), the axis of abscissas and the two lines with equations $x = 0$ and $x = 1$.

6) The function f has over $[1;+\infty[$ an inverse function h. Determine the domain of definition of h and draw its representative curve in the same system as (C).

2012 – 2^{nd}

Let f be the function defined, over $]1 ; +\infty[$, by $f(x) = \ln\left(\frac{x+1}{x-1}\right)$.

Denote by (C) its representative curve in an orthonormal system $\left(O; \vec{i}, \vec{j}\right)$.

1) Determine $\lim_{x \to 1} f(x)$ and $\lim_{x \to +\infty} f(x)$. Deduce the asymptotes to (C).

2) Verify that $f'(x) = \frac{-2}{(x-1)(x+1)}$ and set up the table of variations of f.

3) Draw (C).

4) a- Prove that f has an inverse function g whose domain of definition is to be determined.

b- Prove that $g(x) = \frac{e^x + 1}{e^x - 1}$.

c- (G) is the representative curve of g in the same as that of (C). Draw (G).

5) Let h be the function defined over $]1 ; +\infty[$ by $h(x) = x\,f(x)$.

a- Verify that $f(x) = h'(x) + \frac{2x}{x^2 - 1}$ and determine, over $]1 ; +\infty[$, an antiderivative F of f.

b- Calculate the area of the region bounded by (C), the x-axis and the two lines with equations $x = 2$ and $x = 3$.

2013 – 1^{st}

Consider the function f defined on $\mathbb{R}$ as $f(x) = 3 - \dfrac{4}{e^{2x}+1}$.

Let (C) be its representative curve in an orthonormal system (**unit 2 cm**).

1) Calculate $\lim\limits_{x \to -\infty} f(x)$, $\lim\limits_{x \to +\infty} f(x)$ and deduce the asymptotes to (C).

2) Prove that f is strictly increasing over $\mathbb{R}$ and set up its table of variations.

3) The curve (C) has a point of inflection W with abscissa 0. Write an equation of (T), the tangent to (C) at the point W.

4) a- Calculate the abscissa of the point of intersection of (C) with the x-axis.

b- Draw (T) and (C).

5) a- Verify that $f(x) = -1 + \dfrac{4e^{2x}}{e^{2x}+1}$ and deduce an antiderivative F of f.

b- Calculate, in cm^2, the area of the region bounded by the curve (C), the x- axis, the y-axis and the line with equation $x = \ln 2$.

6) The function f has over $\mathbb{R}$ an inverse function g. Denote by (G) the representative curve of g.
a- Specify the domain of definition of g.
b- Show that (G) has a point of inflection J whose coordinates to be determined.
c- Draw (G) in the same system as (C).
d- Determine g(x) in terms of x.

$2013-2^{nd}$

A- Consider the function g defined over $]0;+\infty[$ as $g(x)=x^2-2\ln x$.

1) Determine $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.

2) Set up the table of variations of g and deduce that $g(x)>0$.

B- Let f be the function defined over $]0;+\infty[$ as $f(x)=\frac{x}{2}+\frac{1+\ln x}{x}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).

2) a- Determine $\lim\limits_{x\to+\infty} f(x)$ and show that the line (Δ) with equation $y=\frac{x}{2}$ is an asymptote to (C).

b- Study, according to the values of x, the relative positions of (C) and (Δ).

3) Show that $f'(x)=\frac{g(x)}{2x^2}$ and set up the table of variations of f.

4) Calculate the coordinates of the point B on (C) where the tangent (T) is parallel to (Δ).

5) Show that the equation $f(x)=0$ has a unique solution α, then verify that $0.34<\alpha<0.35$.

6) Plot (Δ), (T) and (C).

7) Let h be the function defined over $]0;+\infty[$ as $h(x)=\frac{1+\ln x}{x}$.

a- Find an antiderivative H of h.

b- Deduce the measure of the area of the region bounded by (C), (Δ) and the lines with equations $x=1$ and $x=e$.

$2014 - 1^{st}$

Let f be the function defined on $\mathbb{R}$ as $f(x) = x + e^{x+1}$ and (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim_{x \to -\infty} f(x)$ and show that the line (d) with equation $y = x$ is an asymptote to (C).

b- Specify the position of (C) with respect to (d).

c- Determine $\lim_{x \to +\infty} f(x)$. Calculate $f(-1)$, $f(0)$ and $f(1)$.

2) Show that f is strictly increasing on $\mathbb{R}$ and set up its table of variations.

3) Show that the equation $f(x)=1$ has a unique solution α such that $-0.6 < \alpha < -0.5$.

4) Draw (d) and (C).

5) a- Show that the function f has ,on $\mathbb{R}$, an inverse function g whose domain is to be determined.

b- Draw the curve (Γ) of g in the same system as that of (C).

c- Calculate the area of the region bounded by (Γ), the x-axis, and the y-axis.

$2015 - 1^{st}$

A-

Let g be the function defined on IRas $g(x) = x - 1 + e^x$.

1) Show that g is strictly increasing onIR. Set up the table of variations of g.

2) Calculate $g(0)$, then study according to the values of x the sign of $g(x)$.

B-

Let f be the function defined on IR as $f(x) = \frac{(x-2)e^x}{1+e^x}$ and (C) its representative curve in an orthonormalsystem $(O ; \vec{i}, \vec{j})$. Denote by (Δ) the line with equation $y = x - 2$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$. Deduce an asymptote to (C).

2) Study, according to the values of x, the relative positions of (C) and (Δ).

3) Determine $\lim\limits_{x \to +\infty} f(x)$ and show that (Δ) is anasymptote to (C).

4) Showthat $f'(x) = \frac{e^x g(x)}{(1+e^x)^2}$, then set up the table of variations of f.

5) Plot (Δ) and (C).

6) The function f has over $[0;+\infty[$ an inverse function h. Calculate $h'(0)$.

$2015 - 2^{nd}$

A- Let g be the function defined on $]0;+\infty[$ as $g(x) = x^3 - 1 + 2\ln x$.

1) Determine $\lim_{x\to 0} g(x)$ and $\lim_{x\to +\infty} g(x)$.
2) Calculate $g'(x)$ then set up the table of variations of g.
3) Calculate $g(1)$ then deduce the sign of $g(x)$ according to the value of x.

B- Consider the function f defined on $]0;+\infty[$ as $f(x) = x - \frac{\ln x}{x^2}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. Let (d) be the line with equation $y = x$.

1) Determine $\lim_{x\to 0} f(x)$ and deduce an asymptote to (C).
2) a- Discuss, according to the values of x, the relative positions of (C) and (d).
   b- Determine $\lim_{x\to +\infty} f(x)$ and show that (d) is an asymptote to (C).
3) a- Verify that $f'(x) = \frac{g(x)}{x^3}$ and set up the table of variations of f.
   b- Determine the point E on (C) where the tangent (Δ) to (C) is parallel to (d).
   c- Plot (d), (Δ) and (C).
4) Let α be a real number greater than 1. Denote by $A(\alpha)$ the area of the region bounded by (C), (d) and the two lines with equations $x = 1$ and $x = \alpha$.
   a- Verify that $\int \frac{\ln x}{x^2} dx = \frac{-1-\ln x}{x} + k$, where k is a real number.
   b- Express $A(\alpha)$ in terms of α.
   c- Using the graphic, show that $A(\alpha) < \frac{(\alpha-1)^2}{2}$.

2016 – 1st

Consider the function f defined over $]-1;+\infty[$ as: $f(x)=e^x-\dfrac{2e^x}{x+1}$.

Denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) a- Determine $\lim\limits_{\substack{x\to -1\\ x>-1}} f(x)$. Deduce an asymptote (D) to (C).

   b- Determine $\lim\limits_{x\to +\infty} f(x)$ and calculate f(2.5).

2) Prove that $f'(x)=\dfrac{(x^2+1)e^x}{(x+1)^2}$ and set up the table of variations of the function f.

3) Let (d) be the line with equation $y=x$. The curve (C) intersects (d) at a unique point A with abscissa α. Verify that $1.8<\alpha<1.9$.

4) a- Specify the coordinates of the points of intersection of (C) with the coordinates axes.
   b- Draw (D), (d) and (C).

5) a- Prove that, over $]-1;+\infty[$, f has an inverse function f^{-1}.

   b- Draw (C'), the representative curve of f^{-1}, in the same system as that of (C).

6) Suppose that the area of the region bounded by (C), the x-axis and the lines with equations $x = 0$ and $x = 1$ is 0.53 units of area.
   Calculate the area of the region bounded by (C'), the line (d), the y-axis and the line with equation $x=-1$.

$2016 - 2^{nd}$

Let f be the function defined over $\mathbb{R}$ as $f(x) = x + xe^{-x}$, and denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Calculate $f(-1.5)$.
2) Let (d) be the line with equation $y = x$.
   a- Discuss according to the values of x, the relative position of (C) and (d).
   b- Prove that (d) is an asymptote to (C).
3) A is the point on (C) where the tangent (T) to (C) is parallel to (d). Determine the coordinates of A and write an equation of (T).
4) The following table is the table of variations of the function f', the derivative of f.

x	$-\infty$		2		$+\infty$
$f''(x)$		$-$	0	$+$	
$f'(x)$	$+\infty$	$\searrow$	$1-e^{-2}$	$\nearrow$	1

   a- Verify that (C) admits an inflection point W whose coordinates should be determined.
   b- Verify that f is strictly increasing over $\mathbb{R}$, then set up the table of variations of the function f.
5) Draw (d), (T) and (C).
6) a- Prove that f has an inverse function h whose domain of definition should be determined.
   b- Draw the curve (C') of h in the same system as that of (C).
7) Let M be any point on (C) with abscissa $x \geq 0$, and denote by N the symmetric of M with respect to (d).
   a- Calculate MN in terms of x.
   b- Calculate the maximum value of MN.

2017 – 1st

Consider the function f defined on $\mathbb{R}$ as $f(x) = (1-x)e^{x} + 2$.

Denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$. Deduce an asymptote (d) to (C).

   b- Determine $\lim\limits_{x \to +\infty} f(x)$, then calculate $f(1)$ and $f(2)$.

2) a- Verify that $f'(x) = -xe^{x}$ and set up the table of variations of the function f.

   b- Prove that the curve (C) has an inflection point I whose coordinates should be determined.

3) Draw (d) and (C).

4) Denote by (Δ) the line with equation $y = 2x$.

   a- Verify that $f(x) - 2x = (e^{x} + 2)(1-x)$. Study, according to the values of x, the relative positions of (C) and (Δ).

   b- Find an antiderivative F of the function f.

   c- Draw (Δ), then calculate the area of the region bounded by the curve (C), the y-axis and the line (Δ).

5) Let g be the function given as $g(x) = \ln[f(x) - 2]$.

   Denote by (G) the representative curve of g in the system $(O;\vec{i},\vec{j})$.

   a- Verify that the domain of definition of g is $]-\infty\,;1[$.

   b- Is there a point on (G) where the tangent to (G) is parallel to the line (Δ)? Justify.

$2017 - 2^{nd}$

Let f be the function defined on $\mathbb{R}$ as: $f(x) = x + 2 - 2e^x$. Denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$.

   b- Show that the line (D) with equation $y = x + 2$ is an asymptote to (C).

   c- For all x in $\mathbb{R}$, show that the curve (C) is below the line (D).

2) Determine $\lim\limits_{x \to +\infty} f(x)$ and calculate f(1.5).

3) Calculate $f'(x)$ and set up the table of variations of f.

4) Show that the equation $f(x) = 0$ has, in $\mathbb{R}$, exactly two roots 0 and α.

   Verify that $-1.6 < \alpha < -1.5$.

5) Draw (D) and (C).

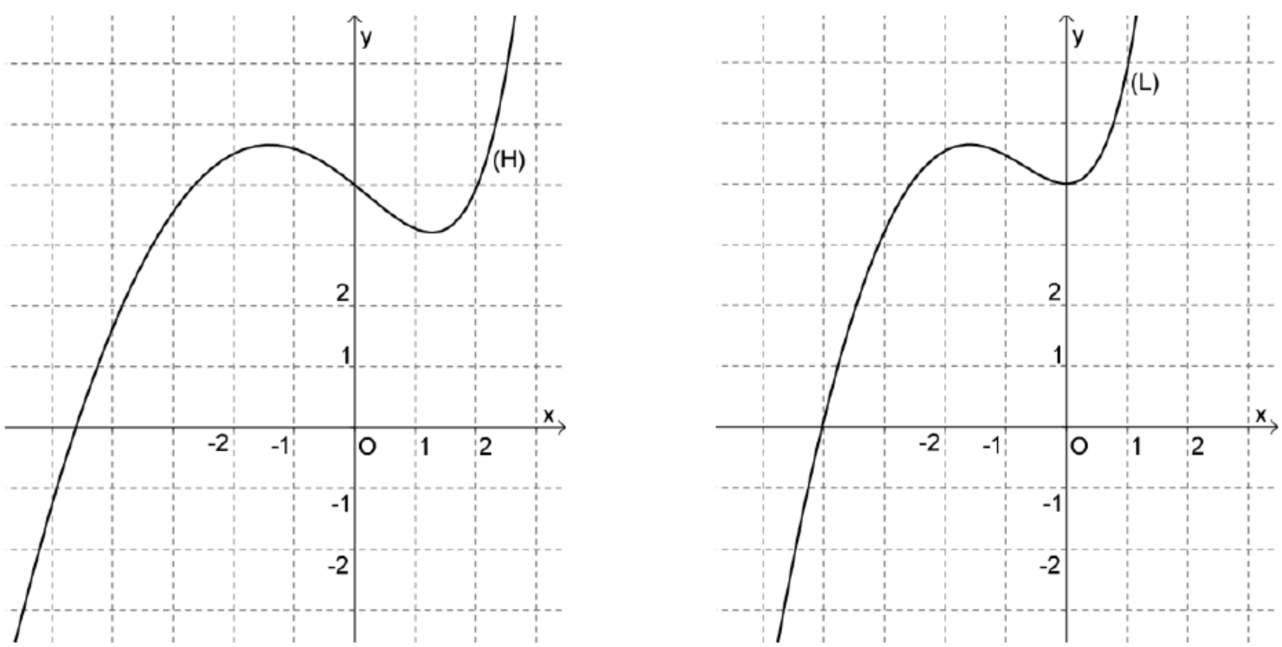
6) Denote by $A(\alpha)$ the area of the region bounded by (C) and the x-axis.

   Show that $A(\alpha) = \left(-\dfrac{\alpha^2}{2} - \alpha\right)$ square units.

7) Let g be a function defined on $\mathbb{R}$ with: $g'(x) = -2f(x)$.

   One of the two curves (H) and (L) given below represents the function g.

   Choose it with justification.



$2018 - 1^{st}$

Consider the function f defined on $\mathbb{R}$ as $f(x) = 1 - 2e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-1)$.
2) a- Determine $\lim\limits_{x \to +\infty} f(x)$ and deduce an equation of the asymptote (d) to (C).

   b- Show that (C) is below (d) for all x in $\mathbb{R}$.
3) The curve (C) intersects the x-axis at A and the y-axis at B. Find the coordinates of A and B.
4) a- Calculate $f'(x)$ and set up the table of variations of f.

   b- Draw (C) and (d).
5) a- Show that f has, on $\mathbb{R}$, an inverse function g.
   b- Determine the domain of definition of g.
   c- Verify that $g(x) = \ln(2) - \ln(1-x)$.
6) Let (C′) be the representative curve of g and let F be the point of (C′) with abscissa 0.

   a- Determine an equation of the tangent (T) to (C′) at F.

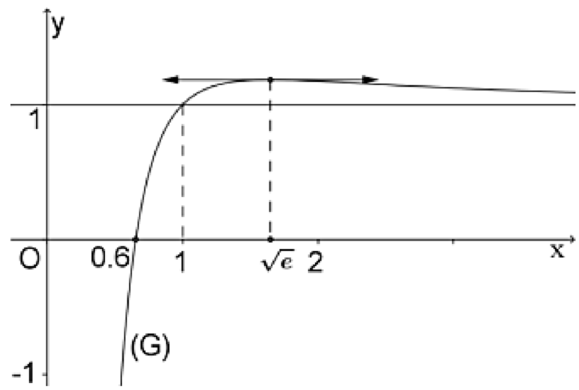
   b- Draw (C′) and (T) in the same system as that of (C).
7) Calculate the area of the region bounded by (C′), the x-axis and the y-axis.

$2018 - 2^{nd}$

Let f be the function defined over $]0;+\infty[$ as $f(x) = x - \frac{1+\ln x}{x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. (1 graphical unit = 2 cm).

Let (d) be the line with equation $y = x$.

1) a. Study, according to the values of x, the relative position of (C) and (d).
   b. Determine $\lim_{x\to+\infty} f(x)$ and show that the line (d) is an asymptote to (C).
2) Determine $\lim_{\substack{x\to0 \\ x>0}} f(x)$ then deduce an asymptote to (C).
3) In the adjacent figure, we have:
   - (G) is the representative curve of the function **f ', the derivative of f.**
   - (G) admits a maximum for $x = \sqrt{e}$.
   - (G) intersects the x-axis at a point of abscissa 0.6.



   a. Set up the table of variations of f.
   b. Show that the equation f(x) = 0 admits **exactly** two roots so that one of them is equal to 1.
   c. Denote by α the second root of the equation $f(x) = 0$. Verify that $0.4 < \alpha < 0.5$.
   d. Show that (C) admits a point of inflection whose coordinates are to be determined.
   e. Determine the coordinates of the point A on (C) where the tangent (T) at A is parallel to (d).
4) Draw (d), (T) and (C).
5) a. Calculate, in cm^2, the area $A(\alpha)$ of the region bounded by (C), (d) and the two lines with equations $x = \alpha$ and $x = 1$.
   b. Prove that $A(\alpha) = (2 - 2\alpha^4)$ cm^2.

$2019 - 1^{st}$

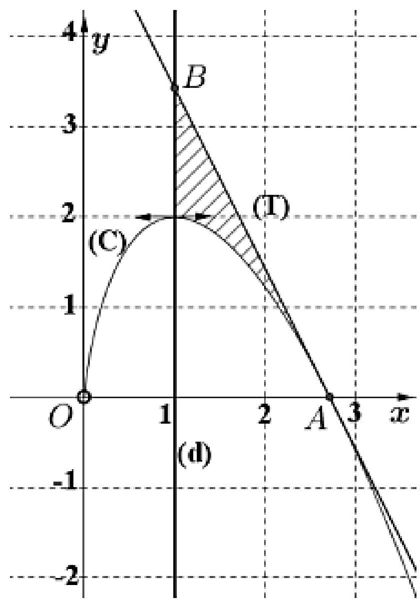
Consider the function f defined over $]0,+\infty[$ as $f(x)=2x(1-\ln x)$. Denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

2) a- Let A be the point of intersection of (C) with the x-axis.
   Determine the coordinates of A.
   b- Show that $f'(x)=-2\ln x$ and set up the table of variations of f.
   c- Determine an equation of the tangent (T) to (C) at A.

In the adjacent figure:

- (C) is the representative curve of f
- (T) is the tangent to (C) at A
- (d) is the line with equation $x=1$
- $B(1\,,\,2e-2)$ is the point of intersection of (d) and (T).



3) a- Show that f has, over $]1,+\infty[$, an inverse function g whose domain of definition is to be determined.
   b- Set up the table of variation of g.
   c- Copy (C), then draw (C'), the representative curve of g in the same system.

4) a- Using integration by parts, determine $\int x\ln(x)\,dx$.
   b- Show that $\int_1^e f(x)dx=\dfrac{e^2-3}{2}$.
   c- Calculate the area of the shaded region bounded by (C), (T) and (d).

$2019-2^{nd}$

Part A

Consider the differential equation (E): $y' - y = -2x$.

Let $y = z + 2x + 2$.

1) Form the differential equation (E′) satisfied by z.
2) Solve (E′) and deduce the particular solution of (E) satisfying $y(0) = 0$.

Part B

Consider the function f defined, over $]-\infty ; +\infty[$, as $f(x) = 2x + 2 - 2e^{x}$.

Denote by (C) the representative curve of f in an orthonormal system $(O ; \vec{i} ; \vec{j})$.

Let (Δ) be the straight line with equation $y = 2x + 2$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$.
   b- Show that (C) is below (Δ) for all x.
   c- Show that (Δ) is an asymptote to (C).
2) Determine $\lim\limits_{x \to +\infty} f(x)$. Calculate f(1) and f(1.5).
3) Calculate $f'(x)$ and set up the table of variations of f.
4) Draw (Δ) and (C).
5) a- Show that f has, over $]0 ; +\infty[$, an inverse function g whose domain of definition is to be determined
   b- Denote by (G) the representative curve of g and by (T) the tangent to (C) at the point L with abscissa $\ln\left(\frac{3}{2}\right)$. Show that (T) is tangent to (G) at a point L′ whose abscissa is $2\ln\left(\frac{3}{2}\right) - 1$.

الخاصة **2020**

Part A

Consider the function g defined on $\mathbb{R}$ as $g(x) = 1 + (x - 1)e^x$.

1) Calculate g' (x). Copy and complete the table of variations of g.

x	$-\infty$	0	$+\infty$
g'(x)		0	
g(x)	1		$+\infty$

2) Deduce that $g(x) \geq 0$ for all x.

Part B

Consider the function f defined on $\mathbb{R}$ as $f(x) = x + (x - 2)e^x$.

Denote by (C) the representative curve of f in an orthonormal system $(O;\vec{i};\vec{j})$.

Let (Δ) be the line with equation $y = x$.

1) Determine $\lim\limits_{x \to +\infty} f(x)$ and calculate f(2.5).
2) a- Determine $\lim\limits_{x \to -\infty} f(x)$.

   b- Show that the line (Δ) is an asymptote to (C).

   c- Study, according to the values of x, the relative positions of (C) and (Δ).
3) Show that $f'(x) = g(x)$ and set up the table of variations of f.
4) a- Show that the equation $f(x) = 0$ has a unique solution α.

   b- Verify that $\alpha \in]1.68 ; 1.69[$.
5) Draw (Δ) and (C).
6) Let S be the area of the region bounded by (C), the x-axis and the y-axis. Show that $S \leq 2\alpha$.

$2021-1^{st}$

Consider the function f defined on $\mathbb{R}$ as $f(x)=e^{x}-x-2$.

Denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

Let (d) be the line with equation $y=-x-2$.

1) Show that $\lim\limits_{x\to+\infty} f(x)=+\infty$. Calculate f(2).
2) a- Determine $\lim\limits_{x\to-\infty} f(x)$.

   b- Show that (d) is an asymptote to (C) at $-\infty$.

   c- Show that (C) is above (d) for all $x\in\mathbb{R}$.
3) Determine $f'(x)$, then set up the table of variations of f.
4) The equation $f(x)=0$ has two solutions $\alpha>0$ and $\beta<0$.

   Verify that $1.1<\alpha<1.2$.
5) Knowing that $-1.9<\beta<-1.8$, draw (d) and (C).
6) Let $A(\alpha)$ be the area of the region limited by the curve (C), the line (d)and the two lines with equations $x=0$ and $x=\alpha$.

   a- Verify that $e^{\alpha}=\alpha+2$.

   b- Prove that $A(\alpha)=(\alpha+1)$ units of area.

$2021 - 2^{nd}$

The plane is referred to an orthonormal system $(O; \vec{i}, \vec{j})$.

Consider the function f defined, on $\mathbb{R}$, as $f(x) = 2xe^{-x+1} + 1$ and denote by (C) its representative curve.

1) Determine $\lim\limits_{x \to -\infty} f(x)$.

2) Show that $\lim\limits_{x \to +\infty} f(x) = 1$. Deduce an asymptote (d) to (C).

3) Show that $f'(x) = 2(1 - x)e^{-x+1}$.

4) Copy and complete the following table of variations of f.

x	$-\infty$	1	$+\infty$
$f'(x)$		0	
f(x)			

5) **a-** Show that the equation f(x) = 0 has, on $\mathbb{R}$, a unique solution α.

   b- Verify that $-0.16 < \alpha < -0.15$.

6) Calculate f(–0.5) and f(0) then draw (C) and (d).

7) **a-** Show that $\int xe^{-x+1}dx = (-x-1)e^{-x+1} + K$ where K is a real number.

   b- Deduce the area limited by (C), the straight line with equation y = 3 and the two straight lines with equations x = 0 and x = 4.

2022 – 1st

The plane is referred to an orthonormal system $(O\,;\vec{i}\,;\vec{j})$.

Part A

Consider the function g defined on ℝ as $g(x)=2-xe^{-x+1}$.

1) Verify that $g'(x)=(x-1)e^{-x+1}$.

2) Copy and complete the following table of variations of g.

x	$-\infty$	1	$+\infty$
$g'(x)$			
$g(x)$	$+\infty$		2

3) Deduce the sign of g(x) on ℝ.

Part B

Consider the function f defined on ℝ as $f(x)=2x+(x+1)e^{-x+1}$ and denote by (C) its representative curve.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-1.5)$.

2) **a)** Determine $\lim\limits_{x\to+\infty} f(x)$.

b) Show that the line (d) of equation $y=2x$ is an asymptote to (C).

c) Study, according to the values of x, the relative positions of (C) and (d).

3) **a)** Knowing that $f'(x)=g(x)$, show that f is strictly increasing for all x ∈ ℝ.

b) Set up the table variations of f.

4) **a)** Show that the equation $f(x)=0$ has a unique solution α on ℝ.

b) Verify that $-0.8<\alpha<-0.7$.

5) Draw (d) and (C).

Part C

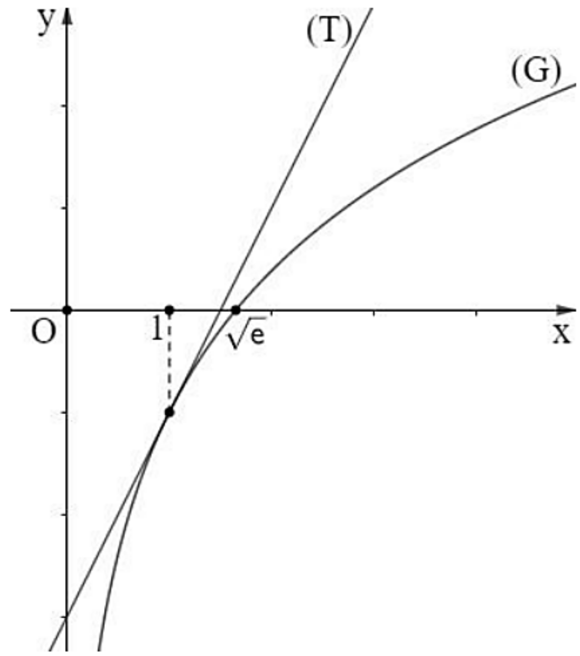
Consider the function h defined on $]-1\,,+\infty[$ as $h(x)=\ln[f(x)-2x]$ and denote by (H) its representative curve.

Find the abscissa of point A on (H) where the tangent to (H) is parallel to the x-axis.

$2022 - 2^{nd}$

Part A

In the adjacent figure:



- (G) is the representative curve over $]0, +\infty[$ of a function g in an orthonormal system $(O\ ;\ \vec{i}, \vec{j})$.
- The line (T) with equation $y = 2x - 3$ is tangent to (G) at the point with abscissa 1.
- The curve (G) intersects the x-axis at the point with abscissa $\sqrt{e}$.

1) Determine $g(\sqrt{e})$ and $g'(1)$.

2) **a)** Study, according to the values of x, the relative positions of (G) and the x-axis.

b) Deduce the sign of g(x) for all x in $]0, +\infty[$.

Part B

Consider the function f defined over $]0, +\infty[$ as $f(x) = (\ln x)^2 - \ln x$.

Denote by (C) the representative curve of f in an orthonormal system $(O\ ;\ \vec{i}, \vec{j})$.

1) Show that the y-axis is an asymptote to (C).

2) **a)** Determine $\lim\limits_{x \to \infty} f(x)$.

b) Calculate f(6) to the nearest 10^{-1}.

3) Knowing that $f'(x) = \dfrac{1}{x} g(x)$.

Set up the table of variations of f.

4) Determine the points of intersection of (C) and the x-axis.

5) Draw (C).

Part C

Consider the function h defined over $]0, +\infty[$ as $h(x) = e^{-f(x)}$.

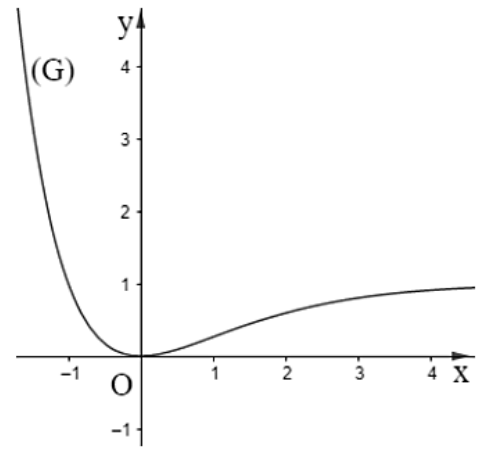
1) Determine $\lim\limits_{x \to 0^+} h(x)$ and $\lim\limits_{x \to \infty} h(x)$.

2) Set up the table of variations of h.

$2023 - 1^{st}$

The plane is referred to an orthonormal system $(O ; \vec{i}, \vec{j})$.

Part A



The adjacent curve (G) is the representative curve of a differentiable function g over $]-\infty; +\infty[$.

(G) is tangent to the x-axis at O.

1) Using the curve (G):

a) Verify that $g(x) \geq 0$ for all real numbers x.

b) The function g' is the derivative of g.
Study, according to the values of x, the sign of g'.

2) Knowing that $g(x) = (ax + b)e^{-x} + 1$ where a and b are two real numbers, show that $a = b = -1$.

Part B

Consider the function f defined, on $\mathbb{R}$, as $f(x) = (x + 2)e^{-x} + x$.

Denote by (C) the representative curve of f.

Let (D) be the line with equation $y = x$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-2.5)$.

2) **a)** Determine $\lim\limits_{x \to +\infty} f(x)$.

b) Show that the line (D) is an asymptote to (C).

c) Study, according to the values of x, the position of (C) with respect to (D).

3) Show that $f'(x) = g(x)$ then set up the table of variations of f.

4) **a)** Show that the equation $f(x) = 0$ has, on $\mathbb{R}$, a unique root α.

b) Verify that $-1.7 < \alpha < -1.6$.

5) **a)** Prove that (C) has an inflection point W whose coordinates are to be determined.

b) Show that the line (T) with equation $y = 2$ is tangent to (C) at W.

6) Draw (T), (D) and (C).

7) Consider the function h defined over $]-2 , 0[$ as $h(x) = \dfrac{\ln(x+2) - \ln(-x)}{x}$.

Prove that $h(\alpha)$ is a natural number to be determined.