Lycee Pascal Anout	**Student's Name:** ---------------------------------------	**Subject:** Mathematics	**Date:** / / 2023
	Class: Gr12 (LS)	**Grade:** ______/ ______	**Duration: 2 hours**

Exercise I: (7pts)

A) Consider the function g defined over $]-\infty\ ;\ +\infty[$ by g(x) = $1 + (x-1)e^{-x}$.

1) Determine $\lim_{x\to-\infty} g(x)$ and $\lim_{x\to+\infty} g(x)$.
2) Calculate $g'(x)$ and set up the table of variations of g.
3) Calculate g(0). Deduce the sign of g.

B) Given the function f defined over $]-\infty\ ;\ +\infty[$ by $f(x) = x(1-e^{-x})$.
Designate by (C) the representative curve of f in an orthonormal system $(O\ ;\ \vec{\imath}\ ;\vec{\jmath})$.

1) Determine $\lim_{x\to-\infty} f(x)$ and calculate $f(-1.5)$.
2) Determine $\lim_{x\to+\infty} f(x)$ and show that the straight line (d): $y = x$ is an asymptote to (C) .
3) Study the relative position of (C) and (d).
4) Show that $f'(x) = g(x)$ and set up the table of variations of f.
5) Show that (C) admits an inflection point I whose coordinates are to be determined.
6) Draw (C) and (d).

C) Let h be a function defined by $h(x) = \ln(f(x))$.

1) Determine the domain of definition of h.
2) Draw the table of variations of h.

Exercise II: (7 pts)

Let f be a function defined over $\mathbb{R}$ by $f(x) = x + 2 - e^{-x}$.
Designate by (C) the representative curve of f in an orthonormal system $(O\ ;\ \vec{\imath}\ ;\vec{\jmath})$.

1) a) Calculate $\lim_{x\to+\infty} f(x)$ and show that the straight line (d) of equation $y = x + 2$ is an asymptote to (C)..
   b) Calculate $\lim_{x\to-\infty} f(x)$ and calculate $f(-1.5)$ and $f(-2)$.
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Write the equation of (T) the tangent to (C) at the point A of abscissa 0.
4) Show that the equation $f(x) = 0$ admits a unique solution α and verify that : $-0.5 < \alpha < -0.4$.
5) Draw (d) , (T) and (C).
6) Designate by $A(\alpha)$ the area of region limited by the curve (C) , axis of abscissas and the two lines of equations $x = \alpha$ and $x = 0$.
   Show that $A(\alpha) = \left(-\frac{\alpha^2}{2} - 3\alpha - 1\right)$ units of area

Exercise III: (3pts)

A)

Let $f(x) = (x+2)e^{-x}$ and $F(x) = (ax+b)e^{-x}$ where a and b are two real numbers. Calculate a and b if F is the primitive of f.

B) Let $g(x) = x + a + b\,lnx$ where a and b are two real numbers ($x > 0$). Calculate a and b if the representative curve (C) of g passes through the point A(1 ; 2) and admits at a point B of abscissa 2 a horizontal tangent.

Exercise IV:

Choose , with justification , the correct answer. (3 pts)

Noº	Questions	Answers		
		a	b	c
1	The domain of definition of $f(x) = \frac{ln(2-e^{-x})}{x-3}$ is	$]-ln2\,;\,+\infty[$	$]0\,;ln2[\ \cup\ ]ln2\,;\,+\infty[$	$\mathbb{R}$
2	The solution of the inequality $ln(x-1) < ln(2x-6)$	$]1\,;\,e^2[$	$\mathbb{R}$	$]5\,;\,+\infty[$
3	$\lim\limits_{x\to 2} \frac{\int_2^x (1+2t)dt}{x-2} = \cdots$	1	5	0
4	$\int_{-7}^{7} (x^3 - x)dx = \cdots$	0	2000001	200000
5	If $a > 0$ then $ln\left(\frac{e}{a}\right) + \ln\,(ae^2)$=…	0	3	2
6	$\lim\limits_{x\to +\infty} [\ln(1+2x) - \ln\,(1+x)] = \cdots$	2	$ln2$	0