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الجامِعتة اللبث نانية كليثة الهندسة

Entrance exam 2017-2018 Physics

July 2017 Duration 2 h

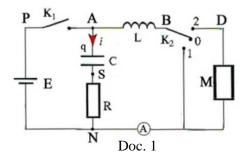
Exercise I: Capacitor, Humidity Sensor (15 pts)

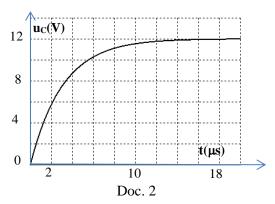
The adjacent electric set (Doc. 1) is carried out where E = 12 V, L is a constant and $R = 20 \Omega$.

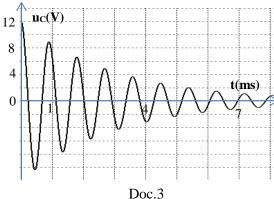
1. Switch (K_2) is in position 0; Switch K_1 is closed. The graph in Doc.2 represents the variation of the voltage $u_C = u_{AS}$ across the capacitor as a function of time.

Determine graphically (Doc. 2) the time constant of the (R, C) series circuit and show that the capacitance C of the capacitor is C = 150 nF.

- **2.** Switch K_1 is now open; Switch K_2 is put in position 1. The graph in Doc.3 shows the variation of u_C as a function of time.
- **2.1.** Derive the differential equation in u_C.
- **2.2.** Determine graphically the pseudo-period T.
- **2.3.** Deduce, with justification, the value of the inductance L of the coil.
- **3.** The capacitor is charged again and the switch K_2 is placed in position 2 at an instant chosen as a new origin of time. The voltage across the terminals of the electronic device (M) is $u_g = u_{DN}$.
- **3.1.** For a certain adjustment of (M), u_C becomes of the form $u_C = E \cos(2\pi ft)$ with f the frequency of these oscillations. Specifying the role of the device, deduce the expression of u_g .
- **3.2.** Determine the average power delivered by this device between the instants $t_0 = 0$ and $t_1 = 4T$.
- 3.3. Determine the expression of the current i carried by the circuit. Deduce that the amplitude I_m of i is given by $I_m = 2\pi f$ CE.
- **4.** The used capacitor is a humidity sensor whose capacitance C varies with the humidity rate H according to the relation: C = (aH + b) with C in nF and H in %. To determine the values of the constants a and b, the following two measurements are made

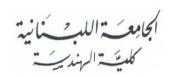






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under the same conditions as those of the first measurement, the percentage of humidity being read on a hygrometer.

4.1. Determine the values of a and b.

4.2. Determine the percentage of humidity H during the first measurement.

Measurement	Н%	I _m (mA)	f(kHz)_	С
n°2	30.4	13.7	1.16	\mathbf{C}_2
nº3	54.8	14.9	1.07	C_3

Exercise II: Use of radioactive nuclei in medicine (15 pts)

Speed of light in vacuum $c = 3.0 \times 10^8 \text{ m} \cdot \text{s}^{-1}$; Planck's constant: $h = 6.62 \times 10^{-34} \text{ J} \cdot \text{s}$; $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$; $1 \text{ u} = 931.5 \text{ MeV/}c^2 = 1.66 \times 10^{-27} \text{ kg}$; radioactive constant of thallium $201 : \lambda_{TI} = 2.6 \times 10^{-6} \text{ s}^{-1}$.

1. A thallium nucleus 201 is obtained by β^+ radioactive decay of a lead nucleus 201, itself obtained by bombarding a thallium

Particle or nucleus	Hg 201	proton	neutron	electron
Mass in u	200.970 032	1.00728	1.00866	0.00055

203 target with protons, according to the reaction:

$$^{203}_{81}\text{Tl} + ^{1}_{1}\text{H} \longrightarrow ^{201}_{82}\text{Pb} + y_z^{a}\text{X}. (1)$$

1.1. Identify the particle $\binom{a}{z}X$) and calculate y.

1.2. Write the equation (2) of the decay of the lead nucleus 201 into a thallium nucleus 201. It will be assumed that the daughter nucleus is obtained in the ground state.

2. The thallium nucleus 201, having a binding energy per nucleon $E_{\ell}/A = 7.684$ MeV/nucleon, absorbs an electron and transforms into a mercury nucleus ($^{201}_{80}$ Hg) according to the equation:

$$^{201}_{81}\text{Tl} + ^{0}_{-1}\text{e} \longrightarrow ^{201}_{80}\text{Hg} + ^{0}_{0}\text{v} + \gamma (3).$$

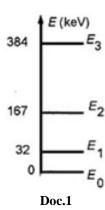
This nuclear reaction is accompanied by the emission of several photons, among which the photons γ_1 and γ_2 that are of respective energies $W_1 = 135$ keV and $W_2 = 167$ keV. Document 1 represents the energy diagram of the first levels of the mercury nucleus 201.

2.1. Represent on this diagram the transition corresponding to each of the two photons γ_1 and γ_2 .

2.2. Calculate the values of the wavelengths λ_1 and λ_2 , in vacuum, of the two photons γ_1 and γ_2 .

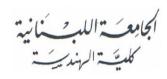
2.3. Deduce the wavelength of the photon corresponding to the transition from level E_1 to level E_0 .

2.4. Calculate, in MeV, the energy released by the nuclear reaction (3).



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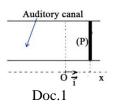




- **3.** Thallium 201 is used in nuclear medicine to perform a diagnosis following heart pain. Examination of a 70 kg patient requires intravenous injection of a thallium chloride solution with an initial activity at $t_0 = 0$, $A_0 = 78$ MBq.
- **3.1.** Calculate the number N_0 of thallium nuclei received by this patient at the instant of injection.
- **3.2.** Since thallium has some toxicity, a limit energy absorbed per unit of body mass has been established. It is 15 mg·kg⁻¹ (per unit of body mass). Check by a calculation that the energy absorbed per unit of body mass injected to the patient is safe.
- **3.3.** It is believed that the results of the examination can be used as long as the activity of thallium 201 is greater than 3 MBq. Determine, in days, the time after which a new injection is necessary if we want to continue the examination to ensure the diagnosis.

Exercise III: Functioning of the eardrum of a human ear (30 pts)

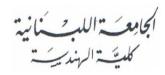
The eardrum of a human ear, under certain conditions, can be modeled by a flat membrane (P), of mass $m=1.5\times 10^{-5}$ kg, which can oscillate, parallel to itself, on either side of its stable equilibrium position at O. The membrane (P), displaced from O in the positive direction by $x_0=10^{-9}$ cm, is released from rest at the instant $t_0=0$. At an instant t, (P), of abscissa x and of velocity $\vec{v}=v\vec{i}$ where $v=\frac{dx}{dt}=\dot{x}$, undergoes from the support of the eardrum a force \vec{F}_1 of expression $\vec{F}_1=-kx\vec{i}$, where k=3500 N·m⁻¹.



- **A-** Any dissipative force is neglected.
- 1. Derive the differential equation in x that describes the motion of (P).
- **2.** Determine the expression of the proper angular frequency ω_0 of the supposed ideal oscillations and calculate its value.
- 3. Determine the time equation of motion of (P) and deduce the expression of v as a function of time.
- $\textbf{B-} \text{ In fact, (P) undergoes, in addition to the force } \vec{F}_1, \text{ a dissipative force } \vec{F}_2 = \text{ h} \vec{v}, \text{ where } h = 0.10 \text{ N} \cdot \text{s} \cdot \text{m}^{-1}.$
- **1.** Determine the differential equation in x that describes the motion of (P).
- **2.** The motion of (P) is then pseudo-periodic of pseudo angular frequency ω'_0 . Calculate its value knowing that: $(\omega'_0)^2 = (\omega_0)^2 \delta^2$ where $\delta = \frac{h}{2 \text{ m}}$.
- **3.** The solution of this differential equation is of the form: $x = x_0 e^{-\delta t} [\cos(\omega'_0 t) + \frac{\delta}{\omega'_0} \sin(\omega'_0 t)]$. Show that the expression of v is: $v = -\frac{x_0 \omega_0^2}{\omega'_0} e^{-\delta t} \sin(\omega'_0 t)$.

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4.1. Deduce the instant at which the eardrum becomes practically motionless.

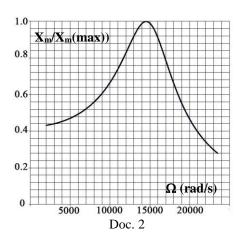
4.2. Draw the shape of the curve v = v(t) between the instants $t_0 = 0$ and t = 2 ms.

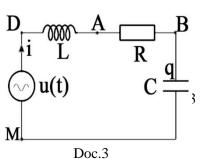
C- A sound produces physically a change in pressure. Thus, the membrane (P) is now subjected to \vec{F}_1 , \vec{F}_2 , and to a pressure force \vec{F}_3 of expression $\vec{F}_3 = F_m \sin(\Omega t + \alpha) \vec{i}$, of adjustable frequency f, where $\Omega = 2\pi f$. In steady state, (P) performs forced oscillations of time equation: $x = X_m \sin(\Omega t)$.

1. Show that $\ddot{x} + \frac{h}{m} \dot{x} + \frac{k}{m} x = \frac{F_m}{m} \sin(\Omega t + \alpha)$

2. Show, by giving
$$\Omega t$$
 two particular values, that
$$X_m = \frac{F_m}{\sqrt{h^2 \Omega^2 + [k - m\Omega^2]^2}}.$$

2.2. Document 2 shows the relative variation of X_m as a function of the angular frequency Ω , $X_m(max)$ being the maximum value of X_m. Deduce the interval of the angular frequencies for which $X_m \ge \frac{X_m(max)}{\sqrt{2}}$.





D- Electric analogy

The eardrum membrane, preceded by the auditory canal, is modeled by the circuit of document 3, where L = 20 mH, $R = 100 \Omega$ and $u = U_m \sin(\omega t + \varphi)$ is an excitation voltage of adjustable angular frequency ω. The circuit then carries the current $i = I_m \sin(\omega t)$.

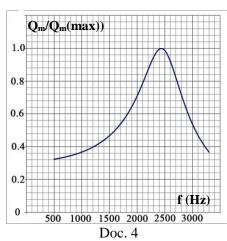
1. There is thus an electromechanical analogy. Express the same proper angular frequency ω_0 found in part (A-2) of this oscillator in terms of L and C and calculate the value of C.

2.1. Show, by applying the law of addition of voltages and giving ot two particular values, that the amplitude Q_m of the charge q is:

$$Q_m = \frac{U_m}{\sqrt{R^2\omega^2 + (L\omega^2 - \frac{1}{C})^2}}. \label{eq:Qm}$$

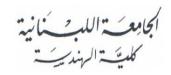
2.2. Deduce the expression of the maximum value $Q_m(max)$ of Q_m .

3. Determine, referring to document 4, the frequency range for which $Q_m \ge \frac{Q_m(max)}{\sqrt{2}}$.



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Entrance exam 2017-2018

Physics Solution

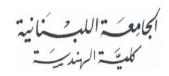
July 2017 Duration 2 h

Exercise I: Capacitor, humidity sensor

Q		Note
1.	The time constant : $u_C(\tau) = 0.63 \times 12 = 7.56 \text{ V} \Rightarrow \tau = 3.0 \mu \text{s} = 3.0 \times 10^{-6} \text{ s}.$	1
	The capacitance C of the capacitor. $\tau = R \cdot C \Rightarrow C = \tau/R = 3.0 \times 10^{-6}/20 = 1.50 \times 10^{-7} \text{ F. ou} 150 \text{ nF.}$	1
2.1	$Law \ of \ addition \ of \ voltges: \ u_{AS} + u_{SN} = u_{AB}. \ u_{AS} = u_{C}. \ i = \frac{dq}{dt} = C\frac{du_{C}}{dt} \ et \ u_{SN} = RC\frac{du_{C}}{dt}.$	2
	$u_{AB} = -L\frac{di}{dt} = -LC\frac{d^2u_C}{dt^2}. \ Thus, \ u_C + RC\frac{du_C}{dt} = -LC\frac{d^2u_C}{dt^2}. \ Therefore, \ LC\ddot{u}_C + RC\frac{du_C}{dt} + u_C = 0 \Rightarrow 0$	
	$\ddot{u}_C + \frac{R}{L}\dot{u}_C + \frac{1}{LC}u_C = 0. \label{eq:uc}$	
2.2	4.5 pseudo-periods cover 4 ms, thus $T = 4/4.5 = 0.89$ ms	1
2.3	The damping is weak, then $T_0 \approx T = 0.89 \text{ ms} = 2\pi\sqrt{LC} \Rightarrow L = 0.133 \text{ H}$	1 1/2
3.1	As oscillations become undamped, then the role of the device is to sustain oscillations, to compensate the	1 1/2
	losses due to Joule's effect: $u_g = u_{DN} = u_{SN} = +Ri$.	
3.2	The average power : $P_m = \frac{\Delta W_{C(max)}}{\Delta t} = \frac{\frac{1}{2}C(u_0^2 - u_4^2)}{4T} = \frac{\frac{1}{2}150 \times 10^{-9}(144 - 3.8^2)}{4 \times 0.89 \times 10^{-3}} = 2.73 \times 10^{-3} \text{ W}.$	2 1/2
3.3	$i = C \frac{du_C}{dt} = - CE2\pi f \sin(2\pi f t)$, thus the amplitude of i is $I_m = CE2\pi f$.	1
4.1	The capacitance C is given by: $C = \frac{I_m}{2\pi fE}$	2 1/2
	For the second measurement: $C_2 = \frac{13.7 \times 10^{-3}}{2\pi \times 1.16 \times 10^3 \times 12} = 1.57 \times 10^{-7} \text{ F ou } 157 \text{ nF.}$	
	For the third measurement: $C_3 = \frac{14.9 \times 10^{-3}}{2\pi \times 1.07 \times 10^3 \times 12} = 1.85 \times 10^{-7} \text{ F ou } 185 \text{ nF.}$	
	As C = aH + b	
	Then, $157 = 30.4$ a + b et $185 = 54.8$ a + b $\Rightarrow 185 - 157 = (54., 8 - 30.4)$ a \Rightarrow a = 1.15 nF·% ⁻¹ .	
4.2	$b = (185 - 54.8) \times 1.15 = 122 \text{ nF}$ $H = \frac{\text{C-b}}{2} = \frac{150 - 122}{1.15} = 24.3\%.$	1
	$\Pi = \frac{1}{a} = \frac{1.15}{1.15} = 24.3\%.$	
		15

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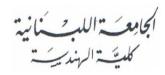
Exercise II: Use of radioactive nuclei in medicine

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Q		Note
1.1	Law of conservation of the charge number: $81 + 1 = 82 + y \cdot z \Rightarrow yz = 0$. The only possibility is that $z = 0$.	1½
	Then $_{z}^{a}X$ is a neutron with $a=1$.	
	Law of conservation of the mass number: $203 + 1 = 201 + y \cdot 1 \Rightarrow y = 3$.	
1.2	$^{201}_{82}\text{Pb} \longrightarrow ^{201}_{81}\text{Tl} + ^{0}_{+1}\text{e} + ^{0}_{0}\text{v} (2)$	1/2
2.1	$W_1 = 135 \text{ keV}$ corresponds to the transition $E_2 \rightarrow E_1$ since E_2 - $E_1 = 167 - 32 = 135 \text{ keV}$ $W_2 = 167 \text{ keV}$ corresponds to the transition $E_2 \rightarrow E_0$ since E_2 - $E_0 = 167 \text{ keV}$.	1 1/2 1/2
	-0.	
2.2	$W_1 = 135\ 000 \times 1.6 \times 10^{-19} = 2.16 \times 10^{-14} \ J \ ; \ \lambda_1 = hc/W_1 = 6.62 \times 10^{-34} \times 3.0 \ 10^8 / 2.16 \cdot 10^{-14} = 9.19 \ 10^{-12} \ m.$	2
	$\mathbf{W}_2 = 167\ 000 \times 1.6 \times 10^{-19} = 2.67 \times 10^{-14} \mathrm{J} \; ; \; \boldsymbol{\lambda_2} = \mathbf{hc/W_2} = 6.62 \times 10^{-34} \times 3.0 \; 10^8 / 2.67 \cdot 10^{-14} = 7.44 \; \mathbf{10^{-12}} \; \mathbf{m}.$	
2.3	$W_2 - W_1 = W_3 \Rightarrow hc/\lambda_2 - hc/\lambda_1 = hc/\lambda_3 \Rightarrow 1/\lambda_2 - 1/\lambda_1 = 1/\lambda_3 \Rightarrow 1/7.44 - 1/9.19 = 0.0256 \Rightarrow \lambda_3 = 39.1 \times 10^{-12} \text{ m.}$	1 ½
2.4	$m(Tl) = 81 m_p + 120 m_n - 201 \times E_{\ell/A} \times 931.5 = 81 \times 1.00728 + 120 \times 1.00866 - (201 \times 7.684/931.5) = 200.970819$ u.	2 1/2
	$E_{lib} = [m(Tl) + m(e^{-}) - m(Hg)] \times 931,5$	
	$E_{lib} = (200.970819 + 0.00055 - 200.970032) \times 931.5 = 0.00134 \times 931.5 = 1.245 \; MeV.$	
3.1	$A_0 = \lambda_{Ti} \times N_0 \Longrightarrow N_0 = A_0/\lambda_{Ti} = 78 \times 10^6/2.6 \times 10^{-6} = 3.0 \times 10^{13} \ noyaux$	1
3.2	The injected mass: $m_0 = N_0 \times m(Tl) = 3.0 \times 10^{13} \times 201 \times 1.66 \times 10^{-27} = 1.0 \times 10^{-11} \text{ kg ou } m_0 = 1.0 \times 10^{-5} \text{ mg}.$	2
	Dose is: $1.0 \times 10^{-5} / 70 = 1.43 \times 10^{-7} \text{ mg/kg} < \grave{a} 15 \text{ mg/kg}.$	
3.3	Law of radioactive decay is: $A = A_0 e^{-\lambda t}$; $3 = 78 \exp(-2.60 \times 10^{-6} t)$; $\ln(26) = -2.60 \times 10^{-6} t \Rightarrow$	2
	The time, at the end of which a new injection is necessary: $t = 1.25 \times 10^6 \text{ s} = 14.5 \text{ days}$.	
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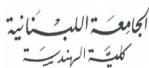




Exercise III: Functioning of the eardrum of a human ear

Q		Note
A.1	Mechanical energy of the system : $ME = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 = constant \ \forall \ t$.	2
	$\frac{dME}{dt} = mv\dot{v} + kx\dot{x} = 0 \text{ avec } v = \dot{x} \neq 0 \Rightarrow \text{The differential equation in } x : \ddot{x} + \frac{k}{m}x = 0$	
2	The solution of the differential equation is of the form : $\ddot{x} + \omega_0^2 x = 0$. By identification : $\omega_0^2 = \frac{k}{m}$ and the	2
	expression of the proper angular frequency ω_0 is : $\omega_0 = \sqrt{\frac{k}{m}}$ therefore its value is $\omega_0 = \sqrt{\frac{3500}{1.5 \times 10^{-5}}} = 1.53 \times 10^4$	
	rad/s.	
3.	The time equation of motion of (P) is of the form : $x = A \cos(\omega_0 t + \phi)$ with $\dot{x} = -A\omega_0 \sin(\omega_0 t + \phi)$.	2 1 ½
	At $t_0 = 0$, $v = \dot{x} = -A\omega_0 \sin(\phi) = 0 \Rightarrow \phi = 0$ or π rad.	1 72
	At $t_0 = 0$, $x = x_0 = A \cos(\phi) > 0$ and $A > 0$. For $\phi = 0$, $A = x_0 > 0$ to be considered and foe $\phi = \pi$, $A = -x_0$	
	< 0 to be rejected. Thus, $x = x_0 \cos(1.53 \times 10^4 \text{ t}) \text{ x in m}$ and t in s.	
	The expression of the velocity v is $v = \dot{x} = -1.53 \times 10^4 \text{ x}_0 \sin(1.53 \times 10^4 \text{ t}) \text{ v in m/s and t in s.}$	
B.1	$\frac{dME}{dt} = mv\dot{v} + kx\dot{x} = P(\vec{F}_2) = -hv^2; \text{ with } v = \dot{x} \neq 0 \Longrightarrow : m\ddot{x} + k x = -h \dot{x}.$	1 ½
	\Rightarrow The differential equation in x that describes the motion of (P) is: $\ddot{x} + \frac{h}{m} \dot{x} + \frac{k}{m} x = 0$	
2	We have: $(\omega'_0)^2 = (\omega_0)^2 - \delta^2$ with $\delta = \frac{h}{2m}$. Thus, $\delta = \frac{0.1}{2 \times 1.5 \times 10^{-5}} = 3.33 \times 10^3$ rad/s.	2
	Therefore: $(\omega'_0)^2 = (1.53 \times 10^4)^2 - (3.33 \times 10^3)^2 \Rightarrow \omega'_0 = 1.49 \times 10^4 \text{ rad/s}.$	
3	Therefore: $(\omega'_0)^2 = (1.53 \times 10^4)^2 - (3.33 \times 10^3)^2 \Rightarrow \omega'_0 = 1.49 \times 10^4 \text{ rad/s}.$ $x = x_0 e^{-\delta t} \left[\cos(\omega'_0 t) + \frac{\delta}{\omega'_0} \sin(\omega'_0 t) \right]. v = \frac{dx}{dt} = x_0 e^{-\delta t} \left[-\delta \cos(\omega'_0 t) - \frac{\delta^2}{\omega'_0} \sin(\omega'_0 t) - \omega'_0 \sin(\omega'_0 t) \right].$	1 ½
	$+\delta\cos(\omega_0')$.	
	$V = -\frac{\log \omega}{\omega_0} e^{-\delta t} \sin(\omega_0' t)$	
4.1	$V = -\frac{x_0 \omega_0^2}{\omega t_0} e^{-\delta t} \sin(\omega t)$ $V_{\text{max}} = \frac{x_0 \omega_0^2}{\omega t_0} e^{-\delta t} = \frac{10^{-9} \times 2.33 \times 10^8}{1,49 \times 10^4} e^{-3300t} = 1,56 \times 10^{-7} e^{-3330t} \text{ (V in m/s and t in s)}$	2
	The time constant: $\tau = 1/3330 = 3.0 \times 10^{-4}$ s. It stops after $5\tau = 1.5 \times 10^{-3}$ s or 1.5 ms.	
4.2	$T'_0 = 0.42 \text{ ms}, V_{\text{max}} = 1.56 \times 10^{-5} \text{ m/s et } 5\tau = 1.5 \text{ ms}.$	2
	/ \ \ \ \ \ t(ms	
	0.5 4.0 1.5	
C.1	$\frac{dME}{dt} = mv\dot{v} + kx\dot{x} = P(\vec{F}_2) + P(\vec{F}_3) = -h\dot{x}^2 + F_m \sin(\Omega t + \alpha)\dot{x} \text{ with } v = \dot{x} \neq 0 \Rightarrow$	2
	$m\ddot{x} + k x + h \dot{x} = F_m \sin(\Omega t + \alpha). \text{ Thus, } \ddot{x} + \frac{h}{m} \dot{x} + \frac{k}{m} x = \frac{F_m}{m} \sin(\Omega t + \alpha)$	
2.1	With $\dot{\mathbf{x}} = \mathbf{X}_{\mathrm{m}} \Omega \cos(\Omega t)$ and $\ddot{\mathbf{x}} = -\mathbf{X}_{\mathrm{m}} \Omega^2 \sin(\Omega t)$.	2 1/2
	$-X_{\rm m} \Omega^2 \sin(\Omega t) - \frac{k}{m} X_{\rm m} \Omega \cos(\Omega t + \varphi) + \frac{k}{m} X_{\rm m} \sin(\Omega t) = \frac{F_{\rm m}}{m} \sin(\Omega t + \alpha).$	
	For $t = 0 \Rightarrow \frac{h}{m} \Omega X_m = \frac{F_m}{m} \sin(\alpha) \Rightarrow \frac{F_m}{m} \sin(\alpha)$. For $\Omega t = \pi/2 \Rightarrow -X_m \Omega^2 + \frac{k}{m} X_m = \frac{F_m}{m} \cos(\alpha)$	
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
	$ \frac{F_m^2}{m^2} = X_m^2 \{ \frac{h^2 \Omega^2}{m^2} + [\frac{k}{m} - \Omega^2]^2 \} \Longrightarrow X_m = \frac{F_m}{\sqrt{h^2 \Omega^2} + [k - m\Omega^2]^2} $	
2.2	The angular frequency interval for which $X_m \le \frac{X_m(max)}{\sqrt{2}}$ is for 10500 rad/s $\le \Omega \le 17800$ rad/s.	1 ½
	$\frac{1}{\sqrt{2}}$	





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		*
D.1	$\omega_0 = \sqrt{\frac{1}{LC}} = 1.53 \times 10^4 \text{ rad/s.} \Rightarrow LC = 2.34 \times 10^8 \Rightarrow C = \frac{1}{20 \times 10^{-3} \times 2.34 \times 10^8} = 2.14 \times 10^{-7} \text{ F.}$	1 1/2
2.1	$i = I_m sin(\omega t)$.	3
	Then: $u_{DA} = L \frac{di}{dt} = L \omega I_m \cos(\omega t)$	
	$u_{AB} = Ri = R I_m sin(\omega t).$	
	$i = C \frac{du_{BM}}{dt} \Rightarrow u_{BM} = \frac{1}{C} \int i \ dt \ constant$. Constant = 0 since u_{BM} is alternating sinusoidal.	
	Thus, $u_{BM} = -\frac{1}{C\omega} I_{m} cos(\omega t)$	
	Law of addition of voltages : $U_m sin(\omega t + \phi) = L\omega I_m cos(\omega t) + R I_m sin(\omega t) - \frac{I_m}{C\omega} cos(\omega t)$.	
	$ \text{ For } \omega t = 0 \Rightarrow U_m sin(\phi) = L\omega I_m - \frac{I_m}{C\omega} . \qquad \qquad \text{For } \omega t = \frac{\pi}{2} \Rightarrow U_m cos(\phi) = R \; I_m . $	
	Therefore: $U_m^2 = I_m^2 [R^2 + (L\omega - \frac{1}{C\omega})^2]$. Thus, $I_m = \frac{U_m}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}}$.	
	$As \ u_{BM} = -\frac{1}{C\omega} I_m cos(\omega t), \ then \ Q_m = \frac{I_m}{\omega} = \frac{U_m}{\omega \sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}} = \frac{U_m}{\sqrt{R^2 \omega^2 + (L\omega^2 - \frac{1}{C})^2}}.$ $Q_m = \frac{U_m}{\sqrt{R^2 \omega^2 + (L\omega^2 - \frac{1}{C})^2}} \ ; \ Q_m \ maximum \ for \ \frac{dQ_m}{d\omega} = 0 \Rightarrow 2R^2\omega + 2(L\omega^2 - \frac{1}{C})(2L\omega) = 0 \ ;$	
2.2	$Q_m = \frac{U_m}{\sqrt{R^2\omega^2 + (L\omega^2 - \frac{1}{C})^2}} \; ; \; Q_m \; \text{maximum for} \; \frac{dQ_m}{d\omega} = 0 \\ \Rightarrow 2R^2\omega + 2(L\omega^2 - \frac{1}{C})(2L\omega) = 0 \; ; \label{eq:Qm}$	2 1/2
	$R^{2} + 2(L\omega^{2} - \frac{1}{C})(L) = 0 \implies R^{2} + 2L^{2}\omega^{2} - \frac{2L}{C} = 0 \implies \omega_{r}^{2} = \omega_{0}^{2} - \frac{R^{2}}{2L^{2}} \text{ with } \omega_{0}^{2} = \frac{1}{LC}.$	
	$Q_{m}(max) = \frac{U_{m}}{\sqrt{R^{2}\omega_{0}^{2} - \frac{R^{4}}{L^{4}} + (L\omega_{0}^{2} - \frac{R^{2}}{2L} - \frac{1}{C})^{2}}} = \frac{U_{m}}{\sqrt{R^{2}\omega_{0}^{2} - \frac{R^{4}}{4L^{2}}}}.$	
3	The interval of frequencies for which $Q_m \le \frac{Q_m(max)}{\sqrt{2}}$ is 2000 Hz $\le f \le 2800$ Hz.	1 1/2
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