قضاء البقاع الغربي	محافظة البقاع	ثانوية سحمر الرسمية (1011)
المدة: 120 دقيقة	الصف:علوم عامة	امتحان :الفصل الأول
الأستاذ: ـ علي عيسى	المادة:الرياضيات	العام الدراسي: 2022-2023

I-(10 points)

ABC is a right isosceles triangle of vertex A

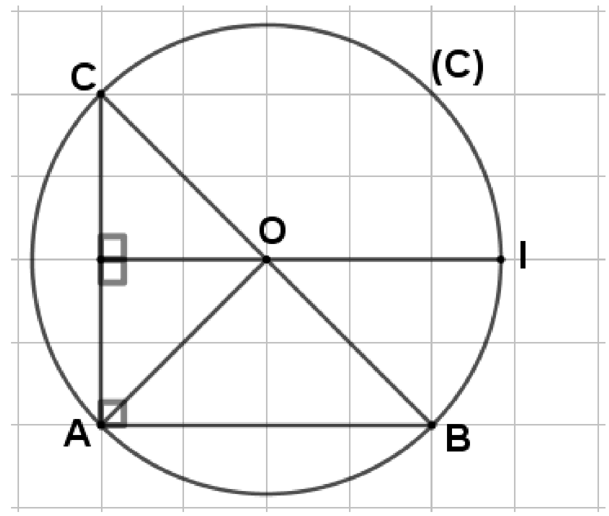
so that $\left(\overrightarrow{AB}\,;\,\overrightarrow{AC}\right) = \frac{\pi}{2} + 2k\pi$ and AB = 4

Let O be the midpoint of [BC]

Let (C) be the circle circumscribed about the triangle ABC

The line (OI) is the perpendicular bisector of [AC]

Let r be a rotation of angle $\frac{\pi}{4}$ that transforms C onto A



1) Draw a figure

2) Show that I is the center of the rotation r

3) Let D be the image of A by r

Show that I, B and D are collinear. Construct D

4)Let Q be the point on [AB] such that AQ = CO

Show that r (O) = Q

5) The two straight lines (IA) and (CB) are intersecting at E

a) Prove that r (E) = B **Hint :Show that the two triangles CEI and ABI are equal**

b) Prove that CE = AD .

c) What is the nature of the quadrilateral ADEC

6)a) Prove that the two straight lines (QD) and (AB) are perpendicular

b) Deduce that the points Q , E and D are collinear

II- (5 points)

In the below figure : ABD is right at D and BD =2cm

The points A , B and E are collinear so that AB = 3cm and AE = 8cm

(C) is a circle circumscribed about the triangle ABD

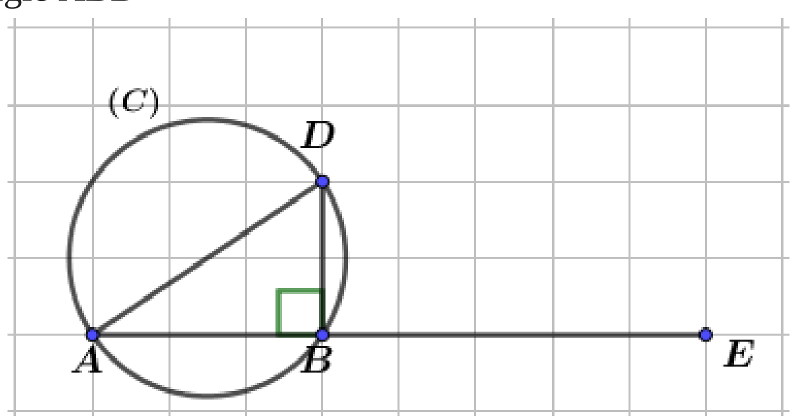
1) Determine the elements of the dilation h of center A that transforms B onto E

2) a) Construct **geometrically the image D’**, of the point D

b) Calculate ED’

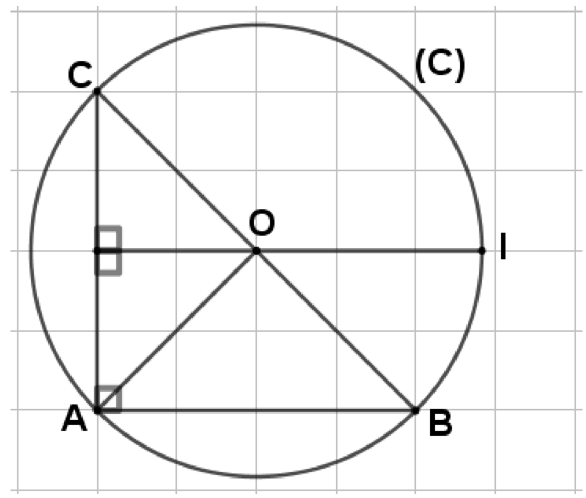
c) What is the nature of triangle AED’?

3) Construct the circle (C’), the image of the circle (C) under h and calculate its area .



Correction of exam

I-(10points)



ABC is a right isosceles triangle of vertex A
so that $\left(\overrightarrow{AB}\,;\,\overrightarrow{AC}\right) = \frac{\pi}{2} + 2k\pi$ and AB = 4
Let O be the midpoint of [BC]
Let (C) be the circle circumscribed about the triangle ABC
The line (OI) is the perpendicular bisector of [AC]
Let r be a rotation of angle $\frac{\pi}{4}$ that transforms C onto A

1) Draw a figure
2) Show that I is the center of the rotation r
3) Let D be the image of A by r
   Show that I, B and D are collinear. Construct D
4)Let Q be the point on [AB] such that AQ = CO
   Show that r (O) = Q
5) The two straight lines (IA) and (CB) are intersecting at E
   a) Prove that r (E) = B **Hint :Show that the two triangles CEI and ABI are equal**
   b) Prove that CE = AD .
   c) What is the nature of the quadrilateral ADEC ?
6)a) Prove that the two straight lines (QD) and (AB) are perpendicular
   b) Deduce that the points Q , E and D are collinear

Solution

1) Draw a figure **0.5 points**

2)

IC = IA since (OI) is the perpendicular bisectors of [AC]

Recall : measure of the inscribed angle is half of the facing arc

$\begin{cases} \widehat{ABC} = \frac{\widehat{AC}}{2} \\ \widehat{AIC} = \frac{\widehat{AC}}{2} \end{cases}$, then $\widehat{ABC} = \widehat{AIC}$. But $\widehat{ABC} = 45^0$, then $\widehat{AIC} = 45^0$

$\begin{cases} IC = IA \\ \left(\overrightarrow{IC};\overrightarrow{IA}\right) = \frac{\pi}{4} + 2k\pi \end{cases}$ **,then I is the center of r** **1 points**

3)since r (A)= D , then $\begin{cases} \boldsymbol{IA = ID} \\ \left(\overrightarrow{\boldsymbol{IA}};\overrightarrow{\boldsymbol{ID}}\right) = \boldsymbol{\frac{\pi}{4} + 2k\pi} \end{cases}$

$\left(\overrightarrow{IC};\overrightarrow{ID}\right)=\left(\overrightarrow{IC};\overrightarrow{IA}\right)+\left(\overrightarrow{\boldsymbol{IA}};\overrightarrow{\boldsymbol{ID}}\right)+2k\pi$

$\left(\overrightarrow{IC};\overrightarrow{ID}\right)=\frac{\pi}{4}+\frac{\pi}{4}+2k\pi$

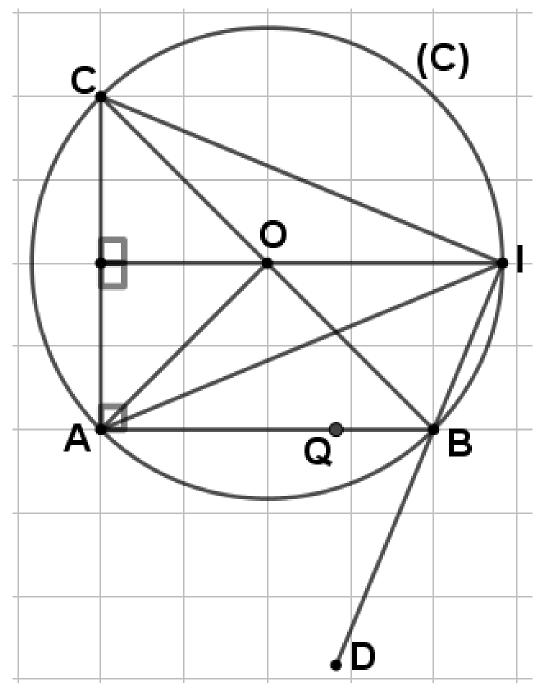
$\left(\overrightarrow{IC};\overrightarrow{ID}\right)=\frac{\pi}{2}+2k\pi$, then (ID) $\perp$ (IC)

But $\widehat{CIB}=90^0$ (Diametrically opposite)

then (IB) $\perp$ (IC)

$\begin{cases}(ID) \perp (IC)\\ (IB) \perp (IC)\end{cases}$

, then I , B and D are collinear **1 points**



Construction of D

IC = IA and IA = ID , then IC = ID

$\widehat{CID}=90^0$

D is the third vertex of the direct right isosceles triangle ICD **0.5 points**

4) Let Q be the point on [AB] such that **AQ = CO**

Show that r (O)= Q

$\left(\overrightarrow{\mathbf{CO}};\overrightarrow{\mathbf{AQ}}\right)=\left(\overrightarrow{\mathbf{CB}};\overrightarrow{\mathbf{AB}}\right)=\left(-\overrightarrow{\mathbf{BC}};-\overrightarrow{\mathbf{BA}}\right)$

$=\left(\overrightarrow{\mathbf{BC}};\overrightarrow{\mathbf{BA}}\right)=\frac{\boldsymbol{\pi}}{\mathbf{4}}+\mathbf{2}\boldsymbol{k\pi}$

We have $\begin{cases}\boldsymbol{CO}=\boldsymbol{AQ}\\ \left(\overrightarrow{\mathbf{CO}};\overrightarrow{\mathbf{AQ}}\right)=\frac{\boldsymbol{\pi}}{\mathbf{4}}+\mathbf{2}\boldsymbol{k\pi}\end{cases}$ **and r(C)= A**

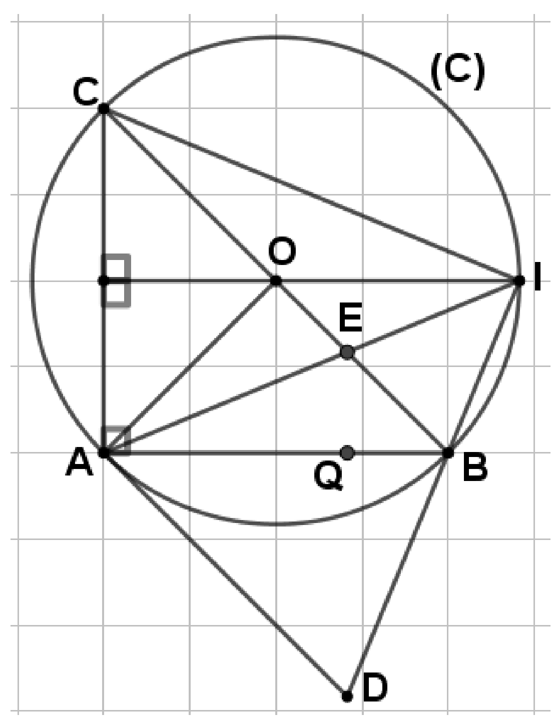
then by the **converse of characteristic property of rotation r(O) = Q** **1.5 points**

5) The two straight lines (IA) and (CB) are intersecting at E

a) Prove that r (E) = B **Hint :Show that the two triangles CEI and ABI are equal**

b) Prove that CE = AD .

c) What is the nature of the quadrilateral ADEB ?



5)a) **In two triangles CEI and ABI , we have**

***IA = IC**

* $\widehat{\boldsymbol{IAB}}=\widehat{\boldsymbol{ICE}}=\frac{\widehat{\boldsymbol{IB}}}{\mathbf{2}}$.

* $\widehat{\boldsymbol{AIB}}=\widehat{\boldsymbol{EIC}}=\mathbf{45^0}$

, then the two triangles are equal by S.A.S

Homologous elements IE = IB

$\widehat{EIB}=\widehat{AID}=45^0$

Since $\begin{cases}\boldsymbol{IE}=\boldsymbol{IB}\\ \left(\overrightarrow{\mathbf{IE}};\overrightarrow{\mathbf{IB}}\right)=\frac{\boldsymbol{\pi}}{\mathbf{4}}+\mathbf{2}\boldsymbol{k\pi}\end{cases}$ **, then r(E)= B** **1.5 points**

b) **since** $r:\begin{cases}A \to D \\ C \to A \\ E \to B\end{cases}$ **and rotation preserves distances**, then **CE = AB and AC=DA**

But AB= AC (given) , then hence **CE = AD** **1 points**

c) since $r(E) = B$, then $\begin{cases} IE = IB \\ (\overrightarrow{IE}\,;\overrightarrow{IB}) = \frac{\pi}{4} + 2k\pi \end{cases}$

since $r(A) = D$, then $\begin{cases} IA = ID \\ (\overrightarrow{IA}\,;\overrightarrow{ID}) = \frac{\pi}{4} + 2k\pi \end{cases}$

The triangles IEB and IAD are isosceles triangles of vertex I= 45^0

$\widehat{IEB} = \widehat{IAD} = \frac{180^0 - 45^0}{2} = 67^0.5$

Since the corresponding angles are equal

,then the lines (AD) and (EB) are parallels

WE have proved $\begin{cases} CE = AD \\ (CE) \parallel (AD) \end{cases}$, then **ADEC is a parallelogram** **1 points**

6)a) Prove that the two straight lines (QD) and (AB) are perpendicular

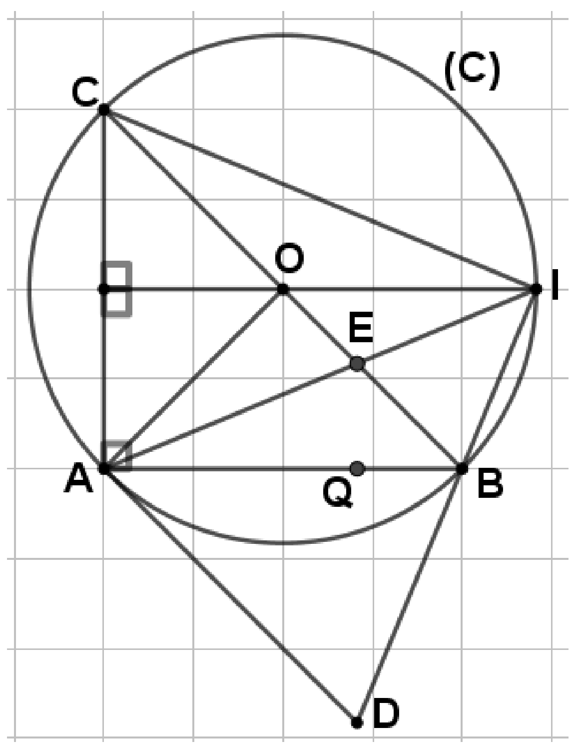
b) Deduce that the points Q , E and D are collinear

We have proved $r:\begin{cases}A \to D \\ O \to Q\end{cases}$, then $(\overrightarrow{AO};\overrightarrow{DQ}) = \frac{\pi}{4} + 2k\pi$

$(\overrightarrow{AB};\overrightarrow{DQ}) = (\overrightarrow{AB};\overrightarrow{AO}) + (\overrightarrow{AO};\overrightarrow{DQ}) + 2k\pi$

$(\overrightarrow{AB};\overrightarrow{DQ}) = \frac{\pi}{4} + \frac{\pi}{4} + 2k\pi$

$(\overrightarrow{AB};\overrightarrow{DQ}) = \frac{\pi}{2} + 2k\pi$ **, then (DQ)** $\perp$ **(AB)**



<u>Second method</u>

We have $r:\begin{cases}O \to Q \\ A \to D \\ C \to A\end{cases}$

(OA) $\perp$ (OC) , then r((OA)) $\perp$ r((OC))

, then (QD) $\perp$ (QA)

since rotation preserves perpendicularity

(QD) $\perp$ (QA) , hence **(QD)** $\perp$ (**AB)** **1.5 points**

b) Deduce that the points Q , E and D are collinear

$\begin{cases} (\boldsymbol{DE}) \parallel (\ \boldsymbol{AC}\ )\ (\boldsymbol{opposite\ sides\ of\ the\ parallelogram}) \\ (\boldsymbol{AC}) \perp (\ \boldsymbol{AB}\ )\ (\boldsymbol{given}\ ) \end{cases}$

, then $(\boldsymbol{DE}) \perp (\ \boldsymbol{AB}\ )$

$\begin{cases} (\mathrm{DE}) \perp (\ \mathrm{AB}\ ) \\ (QD) \perp (\ AB\ ) \end{cases}$, **then I, B and D are collinear** **0.5 points**

II- (5points)

In the below figure : ABD is right at B and BD =2cm

The pointsA, B and E are collinear so that AB=3cm and AE=8cm

(C) is a circle circumscribed about the triangle ABD

1) Determine the elements of the dilation h of center A that transforms B onto E

2)a) Construct **geometrically the point D'**, the image of the point D

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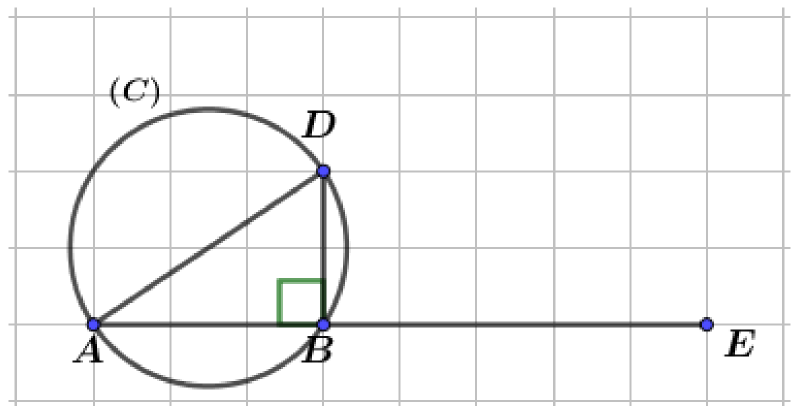
c) What is the nature of triangle AED'?

3) Construct the circle (C'), the image of the circle (C) under h and calculate its area



Solution : **First method**

AE = 8 and AB= 3

$\frac{\boldsymbol{AE}}{\boldsymbol{AB}} = \frac{\mathbf{8}}{\mathbf{3}}$, then $\boldsymbol{AE} = \frac{\mathbf{8}}{\mathbf{3}}\boldsymbol{AB}$

The vectors $\overrightarrow{AE}$ and $\overrightarrow{AB}$ have the **same direction and same sense** , then $\overrightarrow{AE} = \frac{\mathbf{8}}{\mathbf{3}}\overrightarrow{AB}$

Therefore E is the image of B under a positive dilation h of center A and ratio $\frac{\mathbf{8}}{\mathbf{3}}$

Second method

Let k be the center of h

since $\boldsymbol{h}$: $\begin{cases} \boldsymbol{A} \to \boldsymbol{A} \\ \boldsymbol{B} \to \boldsymbol{E} \end{cases}$,then $\overrightarrow{\boldsymbol{AE}} = \boldsymbol{k}\overrightarrow{\boldsymbol{AB}}$ and so $\boldsymbol{AE} = |\boldsymbol{k}|\boldsymbol{AB}$

The vectors $\overrightarrow{AE}$ and $\overrightarrow{AB}$ have the **same direction and same sense** , then $k > 0$ and $|\boldsymbol{k}| = \boldsymbol{k}$

,then $\boldsymbol{K} = \frac{\boldsymbol{AE}}{\boldsymbol{AB}} = \frac{\mathbf{8}}{\mathbf{3}}$

Therefore h is a positive dilation of center A and ratio $\frac{\mathbf{8}}{\mathbf{3}}$ **1 points**

2)a) h(D) =D' then $\overrightarrow{AD'} = \frac{8}{3}\overrightarrow{AD}$ and so **D'**$\in (\boldsymbol{AD})$

*Since h: $\begin{cases} B \to E \\ D \to D' \end{cases}$, then $\overrightarrow{ED'} = \frac{8}{3}\overrightarrow{BD}$

and hence the straight lines**(ED') and (BD) are parallel**

Conclusion : **D' is the point of intersection of the line (AD) with the parallel drawn from E to the line (BD)** **1.5 points**

b) Since $h: \begin{cases} B \to E \\ D \to D' \end{cases}$, then $\boldsymbol{ED' = \frac{8}{3}BD}$ **since dilation multiply the distance by $|k|$**

$\boldsymbol{ED' = \frac{8}{3}BD = \frac{8}{3} \times 2 = \frac{16}{3}\ cm}$ **0.5 points**

c) What is the nature of triangle AED'?

We have proved $h: \begin{cases} A \to A \\ B \to E \\ D \to D' \end{cases}$

ABD is a direct right triangle at B , the its image AED' is a direct right triangle at E

Since dilation transform a triangle onto similar one

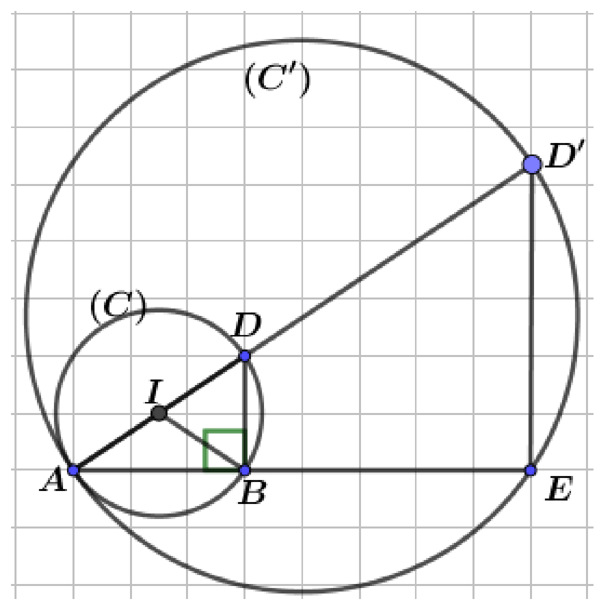
3)Let (C') be the image of the circle (C) under h

[AD] is the diameter of the circle (C) ,then

h([AD]) is the diameter of the circle h ((C))

But **h([AD]) = [AD']**

Then (C') is the circle of diameter [AD'] **0.5 points**



Area of (C') = ??

Area of (C') = $\boldsymbol{k^2}$ × area of (C)

First we should find area of the circle (C)

Let I be the midpoint of [AD]

ABD is a right triangle at B and [BI] is the median of [AD]

,then **IB = $\boldsymbol{\frac{AD}{2} = IA = ID}$**

,hence **I is the center of (C)**

Let R be the radius of (C)

, then **R** = $\boldsymbol{\frac{AD}{2} = \frac{\sqrt{AB^2+BD^2}}{2} = \frac{\sqrt{13}}{2}}$

Area of (C) = $\boldsymbol{\pi R^2 = \pi \times (\frac{\sqrt{13}}{2})^2 = \frac{13\pi}{4} cm^2}$

area of (C') = $\boldsymbol{k^2}$ × area of (C)

$\boldsymbol{= \frac{208}{9}\pi\ cm^2}$ **1.5 points**