Solved Problems

N°1

In an oriented plane, consider a right triangle ABC.

1) Let I be a point of $[AC]$ such that $\overrightarrow{AI}=3\overrightarrow{CI}$.
   Determine the ratio of the dilation of center A that transforms I onto C.

2) Let h be the dilation of ratio $\frac{3}{4}$ that transforms A onto B.
   Determine the center of h.

N°2

Given a parallelogram $ABCD$; Designate by h_A the dilation of center A and ratio $\frac{1}{4}$; h_B the dilation of center B and ratio $\frac{1}{4}$ and by T the translation of vector $\overrightarrow{CB}+\frac{1}{4}\overrightarrow{BA}$.

Let $f=t\circ h_B\circ h_A$.

1) Calculate the image of A by f.
2) Show that f is a dilation whose center and ratio are to be determined.

N°3

In an oriented plane, given a segment $[CD]$.

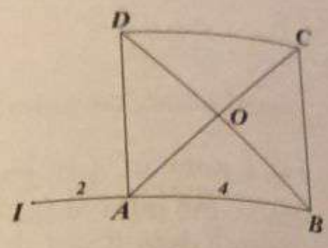
Consider the two dilations $h_1=h(C;-2)$ and $h_2=h\left(D;\frac{1}{3}\right)$.

1) Determine the nature of $h_2\circ h_1$.
2) Determine $h_2\circ h_1(C)$ and construct the center of $h_2\circ h_1$.

Solved Problems

N°4

In the figure to the right, given a square $ABCD$ of side 4, of center O and such that $\left(\overrightarrow{AB};\overrightarrow{AD}\right)=\frac{\pi}{2}\ (\text{mod } 2\pi)$.



Let r be the rotation of center O and angle $\frac{\pi}{2}$, h the dilation of center I that transforms A onto B and t the translation of vector $\overrightarrow{AD}$.

1) Determine $t\circ r(C)$ then identify $t\circ r$.
2) Determine a rotation r' such that $r'\circ t = t\circ r$.
3) Find the nature and characteristic elements of $t\circ h$ and construct its center w.

N°5

The plane is referred to a direct orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

Let f be the mapping of the plane defined by :

$$M\begin{cases}x\\y\end{cases}\xrightarrow{f} M'\begin{cases}x'=3x+4\\y'=3y-2\end{cases}.$$

1) Determine the invariant point under f then find the nature of f.
2) Let h be the dilation of center $J(-1;2)$ and ratio $k=2$.
   a- Define h analytically.
   b- Determine the image (P') of the curve (P) of equation $y=x^2$ by h
   c- Calculate the area of the domain limited by (P), the axis $x'x$ and the two straight lines of equations $x=0$ and $x=1$.
   d- Deduce the area of the domain limited by (P'), the straight line (d) of equation $y=-2$ and the two straight lines of equations $x=1$ and $x=3$.

N°6

ABC is a right triangle such that $AB=4$, $AC=3$ and

$\left(\overrightarrow{AB};\overrightarrow{AC}\right)=\frac{\pi}{2}\ (\text{mod } 2\pi)$.

D is the midpoint of $[AB]$ and E is the midpoint of $[BC]$.

Let h be the dilation of center A that transforms D onto B.

t is the translation of vector $\overrightarrow{DB}$.

1) a- Identify $t\circ h$ and locate its center I in the figure.

b- Identify $h\circ t$ and locate its center J in the figure.

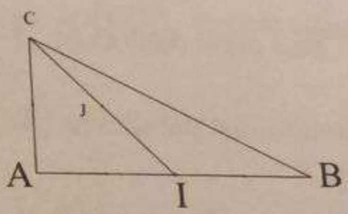
2) The plane is referred to a direct orthonormal system $\left(A;\vec{i},\vec{j}\right)$ such that $\vec{i}=\frac{1}{2}\overrightarrow{AD}$.

Define, analytically, h, t and $t\circ h$.

N°7.

ABC is a triangle right at A such that $AB=4$, $AC=2$ and $\left(\overrightarrow{AB};\overrightarrow{AC}\right)=\frac{\pi}{2}\ (\text{mod } 2\pi)$.

Let I be the midpoint of $[AB]$ and J that of $[CI]$.



1) a- Prove that $\overrightarrow{JA}+\overrightarrow{JB}+2\overrightarrow{JC}=\vec{0}$.

b- Let f be the mapping of the plane that to all points M of the plane associates the point M' defined by

$\overrightarrow{M'M}=\overrightarrow{AM}+\overrightarrow{BM}+2\overrightarrow{CM}$.

Show that f is a dilation of center J.

c- Let (γ) be the circle circumscribed about triangle ABC.

Determine the image (γ') of (γ) by f.

2) The plane is referred to a direct orthonormal system $\left(A;\vec{i},\vec{j}\right)$

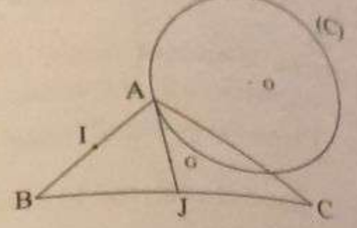
such that $\vec{i}=\frac{1}{4}\overrightarrow{AB}$.

a- Find the analytic expression of f.

b- Write an equation of (γ').

N°8
In the figure to the right, B and C are fixed and A is a point that varies on a fixed circle (C), of center O and radius R.
Let I be the midpoint of $[AB]$ and G the centroid of triangle ABC.

1) Determine the set of points I as A varies.
2) Determine the set of points G as A varies.



N°9
In an oriented plane, consider the points A and B such that $AB = 16$ and the point E such that $\overrightarrow{AE} = \frac{3}{4}\overrightarrow{AB}$.

Let C be a point distinct of A such that $\left(\overrightarrow{AB};\overrightarrow{AC}\right) = \frac{\pi}{4} \pmod{2\pi}$.

The straight line parallel to (BC) through E cuts the straight line (AC) at F.
Let I be the midpoint of $[BC]$ and J the midpoint of $[EF]$ and D the point of intersection of the straight lines (EC) and (BF).
Designate by h_A the dilation of center A that transforms B onto E and by h_D the dilation of center D that transforms E onto C.

1) Determine $h_A(C)$ and $h_D(F)$.
2) Deduce the nature and characteristic elements of $h_D \circ h_A$ Then of $h_A \circ h_D$.
3) Let E' be the image of E by h_A and E'' the image of E' by h_D. Represent E' then construct E''.
4) Determine the nature and characteristic elements of $h_D \circ h_A \circ h_A \circ h_D$.
5) Determine the nature of the quadrilateral $BECE''$.

N°10.

In a plane (P), consider a triangle ABC.

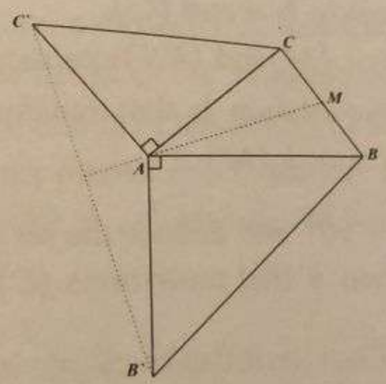
I, J and K are the respective midpoints of $[BC]$, $[AC]$ and $[AB]$ and let G be the center of gravity of triangle ABC.

For all points M of the plane, designate by P, Q and R the symmetrics of M with respect I, J and K respectively.

1) Prove that there exists a dilation h_1 that transforms A, B and C onto I, J and K respectively and determine this dilation.
2) Determine the dilation h_2 that transforms I, J and K onto P, Q and R respectively.
3) a- Precise the nature of $f = h_2 \circ h_1$.
   b- Prove that the segments $[AP]$, $[BQ]$ and $[CR]$ have the same midpoint O.
   c- Prove that the points O, G and M are collinear.

N°11.

ABC is a given triangle, M is the midpoint of $[BC]$, the triangles BAB' and CAC' are right isosceles at A.



1) h is the dilation of center B and ratio 2.
   a- Determine the images of the points A and M by h.
   b- Find a rotation r knowing that $r \circ h$ transforms A onto B' and M onto C'.
2) Deduce that the straight lines (AM) and $(B'C')$ are perpendicular and that $B'C' = 2AM$.

N° 12.

Consider a circle (γ) of center O and diameter $[AB]$.
Let C be a fixed point on $]O, A[$ and (Δ) a variable straight line through C. (Δ) cuts (γ) at M and N.
Let I be the midpoint of $[MN]$.

1) Determine the set of points I as (Δ) varies.
2) Let h be the dilation of center B and ratio 2 and H the image of I by h.
   a- Show that $BMHN$ is a parallelogram.
   b- Prove that H is the orthocenter of triangle AMN.
   c- Determine the locus of points H as (Δ) varies.

N° 13.

$[AB]$ is a fixed segment such that $AB = 12$ and let I be a point of $[AB]$ such that $AI = 4$.
Consider the circles (C) and (C') of respective diameters $[AI]$ and $[IB]$.
Let M be a point of (C) and N a point of (C') such that (MN) is a common exterior tangent to (C) and (C').

1) The two straight lines (AM) and (BN) intersect in K.
   a- Define the negative dilation h_I that transforms (C) onto (C').
   b- Determine $h_I(A)$ and $h_I(M)$.
   c- Prove that $\widehat{AKB} = 90°$ and deduce the set of points K.
2) Use the positive dilation h' that transforms (C) onto (C') and prove that $\widehat{AKB} = 90°$.
3) Prove that the two straight lines (IK) and (AB) are perpendicular.

N° 14.

$ABCD$ is a rectangle such that , $\left(\overrightarrow{AB};\overrightarrow{AD}\right) = \dfrac{\pi}{2} \pmod{2\pi}$.

On the exterior of this rectangle we construct the squares $AEFB$ and $ADGH$ and designate by I the point of intersection of the straight

lines (EG) and (FH).
Let h be the dilation of center I that transforms G onto E and h' the dilation of center I that transforms F onto H.

1) Determine the image of the straight line (CG) by h, then the image of (CG) by $h' \circ h$.
2) Determine the image of the straight line (CF) by $h \circ h'$.
3) Justify that $h \circ h' = h' \circ h$ and deduce that the straight line (AC) passes through I.

N°15.
Given a quadrilateral $ABCD$.
The straight lines (AB) and (DC) intersect in F.
The straight lines (AD) and (BC) intersect in E.
Let I, J and K be the midpoints of the segments $[BD]$, $[AC]$ and $[EF]$ respectively.
I' and J' are the points such that $AFCI'$ and $BFDJ'$ are parallelograms.

1) Draw a clear figure.
2) Designate by h_1 the dilation of center E that transforms B onto C and h_2 the dilation of center E that transforms D onto A.
   a- Determine the image of the straight line (BJ') by $h_2 \circ h_1$.
   b- Determine the image of the straight line (DJ') by $h_1 \circ h_2$.
   c- Deduce that the points E, I' and J' are collinear.
3) Prove that the points I, J and K are collinear.

N°16.
(C) and (C') are two circles of different radii, of respective centers O and O' tangent externally at a point A.
A straight line (d) passing through A cuts (C) again in M and (C') in M'.
A straight line (d') distinct of (d) passing through A intersects (C) in N and (C') in N'.

1) Designate by h the dilation of center A such that $h(O) = O'$.
   a- Show that $h(M) = M'$ and $h(N) = N'$.

b- Deduce that the straight lines (MN) and $(M'N')$ are parallel.

2) Suppose that $[MN]$ is a diameter of (C) .
   a- Show that $[M'N']$ is a diameter of (C') .
   b- Show that (MN') and $(M'N)$ intersect at a fixed point .