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These files have been meticulously arranged by the
'Together We Can' team,
as we wish you the best of luck on your academic journey,
filled with happiness and success

Join us in creating a better tomorrow,
hand in hand!

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وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٤ العادية

عدد المسائل :اربع

مسابقة في الرياضيات
المدة : ساعتان

الاسم :
الرقم :

ملاحظة : يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (3 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A and B such that: $z_A = 1$ and $z_B = \frac{3}{2} + i\frac{\sqrt{3}}{2}$.

Let (C) be the circle with center A and radius 1.

1) a - Write $z_B - z_A$ in the exponential form.

b – Determine a measure of the angle $(\vec{u}\ ;\vec{AB})$.

c – Show that the point B belongs to the circle (C).

2) To every point M, of non-zero affix Z, associate the point M' of affix z' such that $z' = \dfrac{\bar{z} + 2}{\bar{z}}$.

a – Prove that $\bar{z}\,(z' - 1) = 2$.

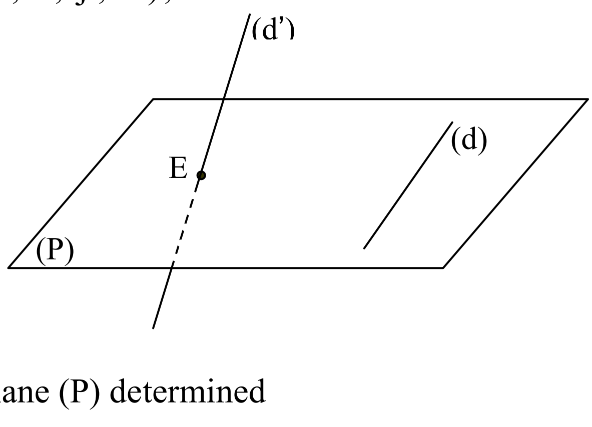
b – Deduce, when M' moves on the circle (C), that M moves on a circle (T) to be determined.

II - (4 points)

In the space referred to a direct orthonormal system $(O\ ; \vec{i}, \vec{j}, \vec{k})$, consider the lines (d) and (d') defined by :

$$(d) : \begin{cases} x = t + 1 \\ y = 2t \\ z = t - 1 \end{cases} \quad \text{and} \quad (d') : \begin{cases} x = 2m \\ y = -m + 1 \\ z = m + 1 \end{cases}$$

(t and m are two real parameters).



1) Prove that (d) and (d') are skew (not coplanar).

2) a – Show that $x - y + z = 0$ is an equation of the plane (P) determined by O and (d).

b – Determine the coordinates of E, the point of intersection of (P) and (d').

c – Prove that the straight line (OE) cuts (d).

3) a – Calculate the distance from point O to the line (d).

b – Deduce that the circle in plane (P), with center O and passing through E, is tangent to line (d).

III- (9 points)

Let f be the function defined, on $]0\,;+\infty[$ by $f(x) = x + 2\dfrac{\ln x}{x}$. (C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$; unit 2 cm .

1) a – Calculate $\lim\limits_{x\to 0} f(x)$ and give its graphical interpretation.

   b – Determine $\lim\limits_{x\to +\infty} f(x)$ and verify that the line (d) of equation $y = x$ is an asymptote of (C).

.

   c – Study according to the values of x , the relative position of (C) and (d) .

2) The table below is the table of variations of the function f ', the derivative of f.

x	0		e		$e\sqrt{e}$		$+\infty$
$f''(x)$		$-$		$-$	0	$+$	
$f'(x)$	$+\infty$	$\searrow$	1	$\searrow$	$1-e^{-3}$	$\nearrow$	1

   a – Show that f is strictly increasing on its domain of definition, and set up its table of variations.

   b – Write an equation of the line (D) that is tangent to (C) at the point G of abscissa e .

   c – Prove that the curve(C) has a point of inflection L.

   d – Show that the equation f(x) = 0 has a unique root α and verify that $0{,}75 < \alpha < 0{,}76$.

3) Draw (D) , (d) and (C) .

4) Calculate, in cm^2 , the area of the region bounded by the curve (C), the line (d) and the two lines of equations $x = 1$ et $x = e$.

IV- (4points)

Consider two urns U and V :

U contains **three** balls numbered 0 and **two** balls numbered 1 .

V contains **five** balls numbered 1 to 5 .

A - One ball is drawn randomly from each urn.
Designate by X the random variable that is equal to the product of the two numbers that are marked on the two drawn balls.

1) Prove that P(X = 0) is equal to $\dfrac{3}{5}$.

2) Determine the probability distribution of X .

B – In this part, the 10 balls that were in urns U and V are all placed in one urn W .
Two balls are drawn, simultaneously and at random, from this urn W.

1) What is the number of possible draws of these 2 balls?

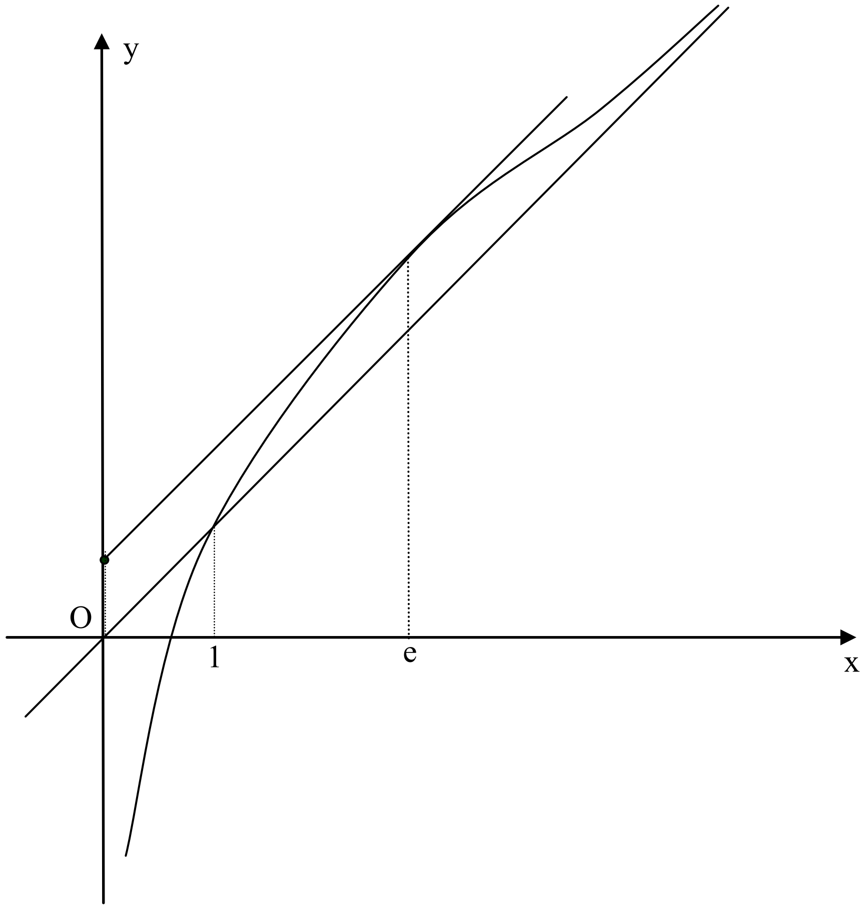
2) Let q designate the product of the too numbers that are marked on the two drawn balls. .

   a - Show that the probability P(q = 0) is equal to $\dfrac{8}{15}$.

   b – Calculate the probability P(q < 4) .

Q		Answers	M
I	1-a	$z_B - z_A = \frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{i\frac{\pi}{3}}$	
	1-b	$(\vec{u};\vec{AB}) = \arg(Z_{\vec{AB}}) = \arg(z_B - z_A) = \frac{\pi}{3}$	
	1-c	$AB = \lvert z_B - z_A \rvert = 1$ then B belongs to (C).	
	2-a	$\bar{z}(z'-1) = \bar{z}(\frac{\bar{z}+2}{\bar{z}} - 1) = \bar{z}\left(\frac{2}{\bar{z}}\right) = 2$.	
	2-b	If M' moves on (C) then AM' = 1 and $\lvert z'-1\rvert = 1$ hence $\lvert \bar{z} \rvert = 2$ then $\lvert z\rvert = 2$ and M moves on the circle of center O and radius 2.	

II	1	$\vec{V}(1\,;2\,;1)$ and $\vec{V'}(2\,;-1\,;1)$; $\vec{V}$ and $\vec{V'}$ are not collinear, then (d) and (d') are not parallel. Study of the intersection of (d) and (d') : t + 1 = 2m ; 2t = – m + 1 ; t – 1 = m + 1 Take 2t = – m + 1 ; t – 1 = m +1, we get t = 1 and m = –1, these values do not verify t + 1 = 2m. Hence (d) and (d') are skew ►Or : Let L (1 ; 0 ; –1) be a point of (d) and J (2 ; 0 ; 2) be a point of (d') ; $\vec{LJ}.(\vec{V}\wedge\vec{V'}) = \begin{vmatrix} 1 & 0 & 3 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix} = -12 \neq 0$	
	2-a	By verification : O is a point of (P) (d) lies in (P) because t + 1 –2t + t – 1 = 0 for every real number t. ►Or : M(x ; y ; z) belongs to (P) iff $\vec{OM}.(\vec{OL}\wedge\vec{V}) = 0$ which gives x – y + z = 0	
	2-b	2m + m –1 + m + 1 = 0 ; m = 0 then E (0 ; 1 ; 1).	
	2-c	(OE) is a line in plane (P), (OE) and (D) are coplanar and they are not parallel ($\vec{OE}$ and $\vec{V}$ are not collinear), therefore they intersect. ►Or : Determine a system parametric equations of (OE) and then prove that it cuts (d).	
	3-a	distance (O/ (d)) = ….. = $\sqrt{2}$.	
	3-b	OE = $\sqrt{2}$ = distance (O/ (d)) ; then (C) is tangent to (d).	

III	1-a	$\lim\limits_{x\to 0}\ln x = -\infty$ then $\lim\limits_{x\to 0} f(x) = -\infty$; y'y is an asymptote of (C).	
	1-b	$\lim\limits_{x\to+\infty}\frac{\ln x}{x} = 0$ then $\lim\limits_{x\to+\infty} f(x) = +\infty$; $\lim\limits_{x\to+\infty}[f(x) - x] = 0$ hence the line (d) of equation y = x is an asymptote of (C) at $+\infty$.	
	1-c	$f(x) - x = 2\frac{\ln x}{x}$. For x = 1, (C) cuts (d). For 0 < x < 1, f(x) – x < 0 then (C) is below (d). For x > 1, (C) is above (d).	
	2-a	$f'(x) \geq 1 - \frac{1}{e^3} > 0$ then f is strictly increasing. $\begin{array}{c\|lcr} x & 0 & & +\infty \\ \hline f(x) & \| -\infty & \nearrow & +\infty \end{array}$	
	2-b	$y = f'(e)(x - e) + f(e)$; $y = x - e + e + \frac{2}{e} = x + \frac{2}{e}$	
	2-c	f ''(x) vanishes for x = $e\sqrt{e}$ and changes sign, then (C) has a point of inflection L of abscissa $e\sqrt{e}$.	
	2-d	f is continuous and changes sign on its domain, f(x) = 0 has at least a root α, moreover f is strictly increasing, then α is unique. f (0.75) ×f (0.76) = –0.017×0.377 < 0, then 0.75 < α < 0.76.	
	3		
	4	A= $\int_1^e 2\frac{\ln x}{x}\,dx = \left[\ln^2 x\right]_1^e = 1\,u^2$, then A = 4 cm^2.	

<table>
<tr><td rowspan="6">IV</td><td>A-1</td><td>To get a product equal to 0 it's enough to draw from U a ball numbered 0, therefore the probability is equal to $\frac{3}{5}$.
►Or : Number of possible draws is equal to $5 \times 5 = 25$
$P(X = 0) = \frac{3 \times 5}{5 \times 5} = \frac{3}{5}$.</td><td></td></tr>
<tr><td>A-2</td><td>

x_i	0	1	2	3	4	5
p_i	3/5	2/25	2/25	2/25	2/25	2/25

</td><td></td></tr>
<tr><td>B-1</td><td>$C_{10}^2 = 45$.</td><td></td></tr>
<tr><td>B-2 a</td><td>To get a product equal to 0 we must obtain one of the following outcomes:
Two balls numbered 0
or
{0 ; a} with a = 1, 2, 3, 4, 5.
Number of favorable cases is $C_3^2 + C_3^1 \times C_7^1 = 24$
$P(q = 0) = \frac{24}{45} = \frac{8}{15}$</td><td></td></tr>
<tr><td>B-2 b</td><td>$P(q < 4) = P(q = 0) + P(q = 1) + P(q = 2) + P(q = 3)$
$= \frac{8}{15} + \frac{C_3^2 + C_3^1 \times C_1^1 + C_3^1 \times C_1^1}{45} = \frac{33}{45} = \frac{11}{15}$.</td><td></td></tr>
</table>

دورة سنة ٢٠٠٤ الاستثنائية	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم : الرقم :	مسابقة في الرياضيات المدة : ساعتان	عدد المسائل : اربع

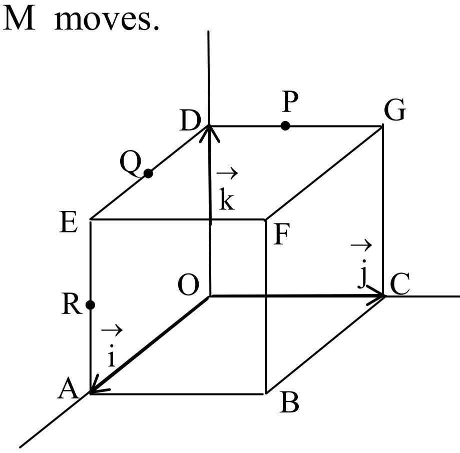
ملاحظــة: يُسمح بإستعمال آلة حاسبة غير قابلة للبرمجة أو إختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (3.5 points).

In the plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A , B and M of affixes -1 , 4 and z respectively, and let M' be the point of affix z' such that $z' = \dfrac{z-4}{z+1}$ $(z \neq -1)$.

1) In the case where $z = 1 + i$, write z' in its algebraic form, and give its exponential form.
2) Determine the values of z for which $z' = z$.
3) a- Give a geometric interpretation of $|z+1|$, and of $|z-4|$.
   b- Find, when $|z'| = 1$, the line on which the point M moves.

II- (3.5 points).



In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the cube OABCDEFG such that : A(1 ; 0 ; 0) , B(1 ; 1 ; 0) and F(1 ; 1 ; 1) . Designate by P, Q and R the midpoints of the segments [DG] , [DE] and [AE] respectively .

1) a-Show that $2x + 2y + 2z - 3 = 0$ is an equation of the plane (PQR).
   b- Prove that the plane (PQR) passes through the midpoint of [AB] .
   c- Prove that the planes (PQR) and (BEG) are parallel.

2) a- What is the nature of quadrilateral EGCA ?
   b- Let M be a variable point on the line (AC) .
   Show that $\overrightarrow{AM} \times \overrightarrow{EF} = \overrightarrow{AM} \times \overrightarrow{GF}$.

III-(4 points).

A multiple choice test is made up of **three** independent questions. The candidate is required to answer all the questions .Each question has two suggested answers out of which only one is correct.

A candidate answers randomly each of these three questions.

1) a- Show that the probability that he answers the three questions correctly is equal to $\frac{1}{8}$.

b- Consider the event E : « Among the three answers of the candidate, exactly two are correct » .
Calculate the probability of E.

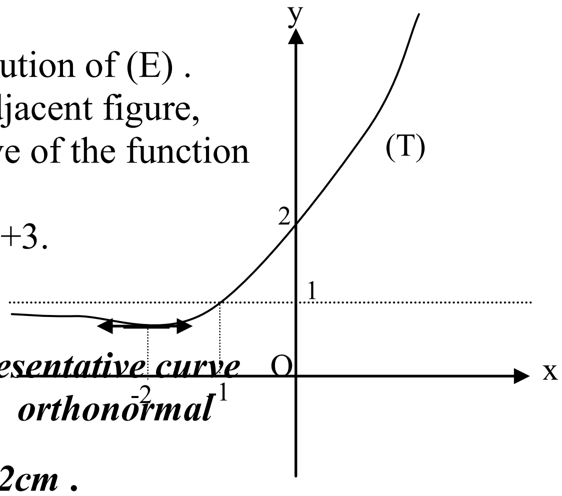
2) The test is marked as follows : **+5** points for each correct answer, and **-3** points for each wrong answer.
Designate by X the random variable that is equal to the total mark obtained by the candidate upon answering the questions of this test.
a- Determine the 4 possible values of X .
b- Determine the probability distribution of X , and calculate the mean (expected value) E (X).

IV-(9 points).

Consider the differential equation (E) : $y'' - 2y' + y = x + 1$.

1) Let $y = z + x + 3$.
a- Write a differential equation (E') satisfied by z, and solve (E').
b- Deduce the general solution of (E).

2) Let f be a particular solution of (E) .
The curve (T) , in the adjacent figure, is the representative curve of the function f ' **the derivative** of f .
Show that $f(x) = xe^x + x + 3$.



Designate by (C) ***the representative curve of the function*** f ***in an orthonormal system*** $(O; \vec{i}, \vec{j})$ ***; unit 2cm .***

3) a- Calculate f(1) and $\lim_{x \to +\infty} f(x)$.

b- Calculate $\lim_{x \to -\infty} f(x)$, and show that the line (d) of equation $y = x + 3$

is an asymptote of (C) .

c- Determine, according to the values of x, the relative positions of (C) and (d).

d- Verify that I (-2 ; $1 - \frac{2}{e^2}$) is a point of inflection of the curve (C) .

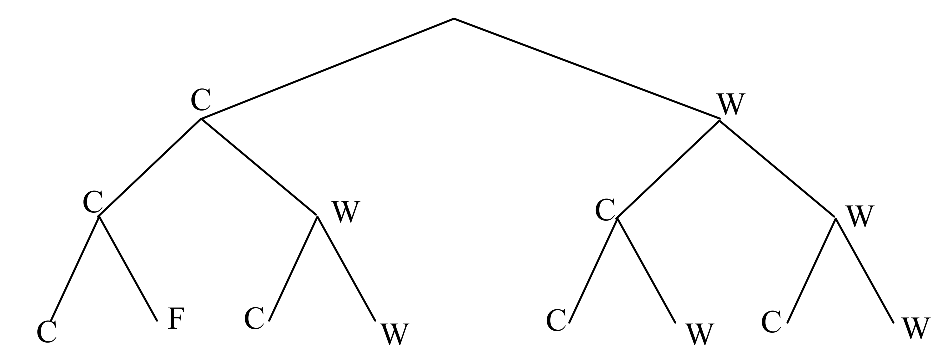
4) a- Verify that f is strictly increasing on IR , and set up its table of variations.

b- Draw (d) and (C).

c- Calculate ,in cm^2, the area of the region bounded by the curve (C), the

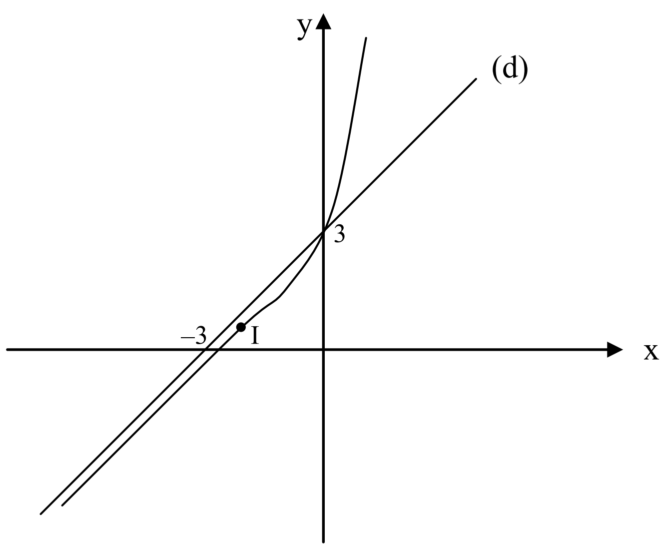
line (d) and the lines of equations $x = 0$ and $x = 1$.

Answer Key

Life Sciences		MATH	2^{nd} Session 2004
Questions		Answers	G
I	1	$z' = \frac{1+i-4}{1+i+1} = \frac{-3+i}{2+i} = -1 + i = \sqrt{2}e^{i\frac{3\pi}{4}}$	1 ½
	2	$z' = z$; $z = \frac{z-4}{z+1}$; $z^2 = -4$; $z = -2i$ or $z = 2i$.	½
	3-a-	$\lvert z+1\rvert = \lvert z_M - z_A\rvert = MA$; $\lvert z-4\rvert = \lvert z_M - z_B\rvert = MB$	½
	3-b-	$\lvert z'\rvert = \frac{MB}{MA}$; since $\lvert z'\rvert = 1$ then MB = MA. M moves on the perpendicular bisector of [AB].	1
II	1-a-	P(0 ; ½ ; 1) , Q(½ ; 0 ; 1) , R(1 ; 0 ; ½) the coordinates of P , Q and R verify the equation 2x + 2y + 2z – 3 = 0 ▶ or : M(x ; y ; z) is a point of the plane (PQR) iff $\vec{PM}.(\vec{PQ}\wedge\vec{PR}) = 0$.	½
	1-b-	I(1 ; ½ ;0) : mid point of [AB], the coordinates of I satisfy the equation of the plane (PQR).	½
	1-c-	(PQ) is parallel to (EG) ; (QR) is parallel to (DA) which is parallel to (BG) (PQR) contains two intersecting lines parallel to two intersecting lines in de (EBG) ; then (PQR) and (EBG) are parallel. ▶or: x + y + z – 2 = 0 is an equation of the plane (BEG) . The two distinct planes(PQR) and (BEG) are parallel having two collinear normal vectors.	1
	2-a-	$\vec{EA} = \vec{GC}$ (EA) is perpendicular to the plane (OABC) , then (EA)⊥(AC) EAGC is a rectangle.	½
	2-b-	$\vec{AM}\wedge\vec{EF} = \vec{AM}\wedge(\vec{EG}+\vec{GF}) = \vec{AM}\wedge\vec{EG} + \vec{AM}\wedge\vec{GF}$ $\vec{AM}$ and $\vec{EG}$ are collinear, $\vec{AM}\wedge\vec{EG} = \vec{0}$,then $\vec{AM}\wedge\vec{EF} = \vec{AM}\wedge\vec{GF}$ ▶or (AC) : x = – α + 1 ; y = α , z = 0 $\vec{AM}(-\alpha ; \alpha ; 0)$, $\vec{EF}(0 ;-1 ;0)$, $\vec{GF}(1 ;0 ;0)$; $\vec{AM}\wedge\vec{EF} = \vec{AM}\wedge\vec{GF} = \alpha\,\vec{k}$	1
III			
	1-a-	P(CCC) = ½ × ½ ×½ = 1/8	1
	1-b-	P(E) = P(CCW) + p(CWC) + p(WCC) = 1/8 +1/8 + 1/8 = 3/8	1
	2-a-	The four possible values of X are – 9 ; – 1 ; 7 ; 15 .	½
	2-b-	see table below	1 ½

$x = x_i$	–9	– 1	7	15
P_i	1/8	3/8	3/8	1/8

		$E(X) = -9/8 - 3/8 + 21/8 + 15/8 = 3$	

IV	1-a-	$y''-2y'+y = x+1$ with $y' = z'+1$ and $y'' = z''$ then $z''-2z'+z = 0$. Characteristic equation $r^2 - 2r + 1 = 0$; $r_1 = r_2 = 1$ and $z = (c_1x + c_2)e^x$.	1 ½
	1-b-	The general solution of (E) is $y = (c_1x + c_2)e^x + x + 3$.	½
	2	According to the graph: $f'(-1) = 1$ and $f'(0) = 2$ $f'(x) = c_1e^x + (c_1x + c_2)e^x + 1$ $f'(-1) = 1$ gives $\frac{c_2}{e} + 1 = 1$ so $c_2 = 0$, $f'(0) = 2$ gives $c_1 + c_2 + 1 = 2$ so $c_1 = 1$, $f(x) = xe^x + x + 3$.	1
	3-a-	$f(1) = e + 4 \approx 6.738$, $\lim\limits_{x \to +\infty} f(x) = +\infty$.	½
	3-b-	$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} xe^x + \lim\limits_{x \to -\infty} (x+3) = 0 - \infty = -\infty$. $\lim\limits_{x \to -\infty} [f(x) - (x+3)] = \lim\limits_{x \to -\infty} xe^x = 0$, then the line (d) of equation $y = x + 3$ is an asymptote of (C) .	1
	3-c-	$f(x) - (x+3) = xe^x$ For $x = 0$, (C) cuts (d) at point (0 ; 3) For $x > 0$, (C) is above (d) For $x < 0$, (C) is below (d) .	1
	3-d-	According to the graph $f''(-2) = 0$, Over $]-\infty ; -2[$: f ' is decreasing then f '' (x) < 0 Over $]-2 ; +\infty[$: f ' is increasing then f '' (x) > 0 The point I(–2 ;f(–2)= $1 - \frac{2}{e^2}$) is a point of inflection of (C).	½
	4-a-	(T) is above the axis of abscissas , then f '(x) > 0 for every x , hence f is strictly increasing over R x : $-\infty$ … $+\infty$ f '(x) : + f (x) : $-\infty \nearrow +\infty$	½
	4-b-		1 ½

	4-c-	$A = \int_0^1 xe^{x}dx = (x-1)e^{x}\Big\vert_0^1 = 1\ u^2$ then $A = 4\ cm^2$.	1

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات شهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٥ العادية

عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I - (3 points)

In the complex plane referred to a direct orthonormal system $(O\,;\,\vec{u}\,,\,\vec{v})$, consider the points E , M and M' of respective affixes i , z and z' , where $z' = i\,z + 1 + i$.

1) Find the algebraic form of z' when $z = \sqrt{2}\,e^{i\frac{\pi}{4}}$.
2) Determine the modulus and an argument of z if $z' = 1 + \sqrt{3} + 2i$.
3) Determine the value of z , for which the points M and M' are confounded .
4) a- Show that $z' - i = i\,(z - i)$.
   b- Deduce that when M moves on the circle (C) of center E and radius 3, then the point M' moves on the same circle.

II - (4 points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider :

- the plane (P) of equation $2x + y - 3z - 1 = 0$;
- the plane (Q) of equation $x + 4y + 2z + 1 = 0$;
- the line (d) defined by : $\begin{cases} x = 2t + 1 \\ y = -t - 1 \\ z = t \end{cases}$ (t is a real parameter).

1) Prove that the line (d) is included in the plane (P) .
2) Find an equation of the plane (S) that is determined by the point O and the line (d).
3) Consider the point $E\left(0\,;-\frac{1}{2};-\frac{1}{2}\right)$.
   Prove that E is the orthogonal projection of the point O on the line (d).
4) a- Show that the planes (P) and (Q) are perpendicular.
   b- Let (D) be the line of intersection of (P) and (Q) .
   Calculate the distance from E to (D).

III - (5 points)

A certain store sells only jackets, coats and shirts.
During a week, **120** customers were served in this store.
90 of those customers bought each one jacket, while the other **30** customers bought each one coat.
40% of those who bought jackets bought each also a shirt, while **20%** of those who bought coats bought each also a shirt.

A customer is chosen at random from those **120** customers and is interviewed.

1) Consider the following events :

J : « the interviewed customer has bought a jacket ».

C : « the interviewed customer has bought a coat ».

S : « the interviewed customer has bought a shirt ».

a- Verify that the probability of the event $S \cap J$ is equal to $\frac{3}{10}$.

b- Calculate the following probabilities :

$P(S \cap C)$, $P(S)$, $P(C/S)$ and $P(C/\overline{S})$.

2) The prices of the clothes in this store are as shown in the following table :

Kind	Jacket	Coat	Shirt
Price in LL	150 000	200 000	60 000

Let X designate the random variable that is equal to the amount paid by a customer.

a- Give the four possible values of X.

b- Determine the probability distribution of X .

c- Calculate the mean (expected value) E(X) .

d- Estimate the amount of sales collected by the store during that week.

IV- **(8 points)**

Consider the function f that is defined, on $I =]\,1\,;+\infty\,[$, by $f(x) = x + 1 - \dfrac{3e^x}{e^x - e}$

and let (C) be its representative curve in an orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,)$.

1) a- Prove that the line of equation $x = 1$ is an asymptote to (C) .

b- Calculate $\lim\limits_{x \to +\infty} f(x)$ and show that the line (d) of equation $y = x - 2$ is an asymptote to (C).

c- Determine the relative position of (C) and (d) .

2) Prove that $f'(x) > 0$ for all values of x in I, and set up the table of variations of f.

3) Prove that the equation f(x) = 0 has a unique root α and verify that $2.6 < \alpha < 2.7$.

4) Draw the curve (C) .

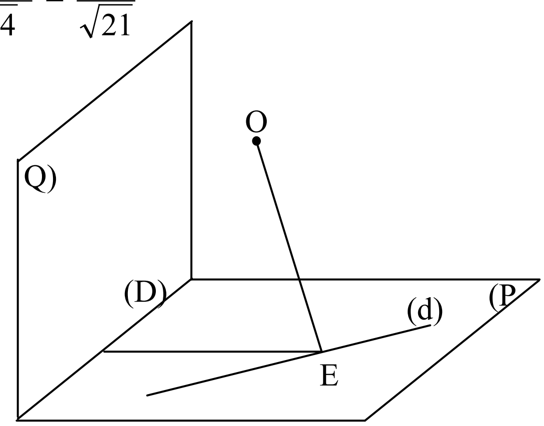
5) Designate by (D) the region that is bounded by (C) , the line (d) and the lines of equations $x = 3$ and $x = 4$.

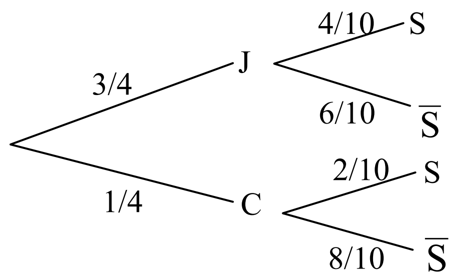
Calculate $\displaystyle\int_3^4 \frac{e^x}{e^x - e}\,dx$ and deduce the area of the region (D).

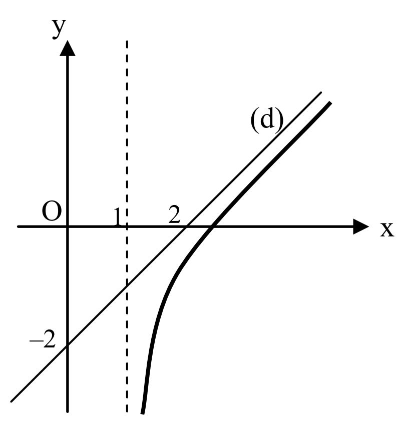
6) a- Prove that f, on the interval I, has an inverse function g.

b- Prove that the equation $f(x) = g(x)$ has no roots .

L.S.		$\mathcal{MATH}$	1^{st} session 2005
Question		Short Answers	M
I	1	$z' = i(\sqrt{2}\ e^{i\frac{\pi}{4}}) + 1 + i = i(1+i) + (1+i) = 2i$.	½
	2	$1+\sqrt{3}+2i = iz + 1 + i$; $iz = \sqrt{3} + i$; $z = 1 - i\sqrt{3}$; $\lvert z\rvert = 2$ and $\arg(z) = -\frac{\pi}{3}$	½
	3	$z' = z$ for $z = iz + 1 + i$; $z(1-i) = 1 + i$; $z = \frac{1+i}{1-i}$; $z = i$.	½
	4a	$z' - i = iz + 1 = i(z - i)$	½
	4b	$\lvert z' - i\rvert = \lvert i\rvert\,\lvert z - i\rvert = \lvert z - i\rvert$; EM' = EM. M moves on the circle (C) , EM = 3 , then EM' = 3, thus M' moves on the same circle.	1

II	1	Every point $M(2t+1; -t-1; t)$ on (d) is a point in (P) because $2(2t+1) - t - 1 - 3t - 1 = 0$; $0t=0$, hence (d) is included in (P).	½
	2	$I(1, -1, 0)$ is a point on (d), the equation of (S) is given by : $\vec{OM}.(\vec{OI}\wedge\vec{V_d}) = 0$; $\begin{vmatrix} x & y & z \\ 1 & -1 & 0 \\ 2 & -1 & 1 \end{vmatrix} = 0$; $x + y - z = 0$.	1
	3	E is a point on (d) (for $t = -½$). $\vec{OE}.\vec{V_d} = 0 + ½ - ½ = 0$. ►OR : Find the coordinates of the orthogonal projection of O on (d).	1
	4a	$\vec{n}_P(2\,;1;-3)$ and $\vec{n}_Q(1\,;4;\,2)$; $\vec{n}_P\,.\,\vec{n}_Q = 0$; (P) and (Q) are perpendicular.	½
	4b	(P) and (Q) are perpendicular, E is a point in (P) ; $d(E/(D)) = d(E/(Q)) = \frac{\lvert 0-2-1+1\rvert}{\sqrt{1+16+4}} = \frac{2}{\sqrt{21}}$ ►OR : Find a system of Parametric equations of (D) and calculate the distance from E to (D). 	1

<table>
<tr><th colspan="2">Question</th><th>Short Answers</th><th>M</th></tr>
<tr><td rowspan="7">III</td><td colspan="2">
</td><td></td></tr>
<tr><td>1a</td><td>$P(S\cap J) = P(J) \times P(S/J) = \frac{3}{4}\times\frac{4}{10} = \frac{3}{10}.$</td><td>½</td></tr>
<tr><td>1b</td><td>$P(S \cap C) = P(C) \times P(S/C) = \frac{1}{4}\times\frac{2}{10} = \frac{1}{20}.$
$P(S) = P(S \cap J) + P(S \cap C) = \frac{6}{20} + \frac{1}{20} = \frac{7}{20}.$
$P(C/S) = \frac{P(C\cap S)}{P(S)} = \frac{1/20}{7/20} = \frac{1}{7}.$
$P(C/\overline{S}) = \frac{P(C\cap \overline{S})}{P(\overline{S})} = \frac{P(C)\times P(\overline{S}/C)}{1-P(S)} = \frac{(1/4)(8/10)}{1-(7/20)} = \frac{4}{13}.$</td><td>1½</td></tr>
<tr><td>2a</td><td>A customer has bought only one out of the following four choices :
only a jacket, only a coat, a jacket and a shirt, a coat and a shirt.
$X(\Omega) = \{$ 150 000 , 200 000 , 210 000 , 260 000$\}$</td><td>½</td></tr>
<tr><td>2b</td><td><table>
<tr><td>$X = x_i$</td><td>150 000</td><td>200 000</td><td>210 000</td><td>260 000</td></tr>
<tr><td>P_i</td><td>$\frac{3}{4}\times\frac{6}{10}=\frac{18}{40}$</td><td>$\frac{1}{4}\times\frac{8}{10}=\frac{8}{40}$</td><td>$\frac{3}{4}\times\frac{4}{10}=\frac{12}{40}$</td><td>$\frac{1}{4}\times\frac{2}{10}=\frac{2}{40}$</td></tr>
</table></td><td>1 ½</td></tr>
<tr><td>2c</td><td>$E(X) = \frac{10000}{40}(15 \times 18 + 20 \times 8 + 21 \times 12 + 26 \times 2) = 183\,500$</td><td>½</td></tr>
<tr><td>2d</td><td>The sales amount during that week is equal to the product of the mean amount by the number of the customers :
183 500 × 120 = 22 020 000 LL.</td><td>½</td></tr>
</table>

Question		Short Answers	M
IV	1a	$\lim\limits_{\substack{x\to1\\x>1}} e^x = e$; $\lim\limits_{\substack{x\to1\\x>1}}(e^x - e) = 0^+$; $\lim\limits_{\substack{x\to1\\x>1}} f(x) = -\infty$ The line of equation x = 1 is an asymptote to (C) .	½
	1b	$\lim\limits_{x\to+\infty} \dfrac{3e^x}{e^x - e} = 3$, consequently $\lim\limits_{x\to+\infty} f(x) = +\infty$; $\lim\limits_{x\to+\infty}[f(x)-(x-2)] = \lim\limits_{x\to+\infty}[3 - \dfrac{3e^x}{e^x - e}] = 0$ The line (d) of equation y = x – 2 is an asymptote to(C).	1
	1c	$f(x) - (x - 2) = 3 - \dfrac{3e^x}{e^x - e} = \dfrac{-3e}{e^x - e}$ $x > 1$, $e^x > e$, then $f(x) - (x - 2) < 0$ so (C) is below (d).	½
	2	$f'(x) = 1 - 3\dfrac{e^x(e^x - e) - e^x(e^x)}{(e^x - e)^2} = 1 + 3\dfrac{e^{x+1}}{(e^x - e)^2} > 0$ $\begin{array}{c\|l} x & 1 \qquad\qquad\qquad +\infty \\ \hline f'(x) & \qquad\quad + \\ \hline f(x) & -\infty \nearrow +\infty \end{array}$	1
	3	On I, f is continuos and changes signs, thus the equation f(x) = 0 has at least one root α. But since f is strictly increasing on I ,then α is unique. f(2.6) = – 0.158 and f(2.7) = 0.0294 , thus $2.6 < \alpha < 2.7$	1
	4		1
	5	• $\displaystyle\int_3^4 \frac{e^x}{e^x - e}dx = \left[\ln(e^x - e)\right]_3^4 = \ln(e^4 - e) - \ln(e^3 - e) = \ln\frac{e^3 - 1}{e^2 - 1}$. • $\mathscr{A} = \displaystyle\int_3^4 (x - 2 - f(x))dx = \int_3^4 (-3 + 3\frac{e^x}{e^x - e})dx = \left[-3x\right]_3^4 + 3\ln\frac{e^3 - 1}{e^2 - 1}$ $= [-3 + 3\ln\dfrac{e^3 - 1}{e^2 - 1}]\ u^2 \quad \approx 0.28\ u^2$	1 ½
	6a	On I, f being continuous and strictly increasing, it has an inverse function g.	½
	6b	The equation f(x) = g(x) is equivalent to f(x) = x, so $1 - \dfrac{3e^x}{e^x - e} = 0$ gives $2\,e^x = -e$ which is impossible. ►OR : graphically , the curve (C) does not cut the first bisector y = x.	1

عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظــة : يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the complex plane referred to a direct orthonormal system $(O\,;\,\vec{u}\,,\,\vec{v})$, consider the points E and F of affixes $z_E = \frac{\sqrt{3}+1}{4} - \frac{\sqrt{3}-1}{4}i$ and $z_F = \frac{1}{2} + \frac{1}{2}i$.

1) a- Calculate $(z_E)^2$ and find the modulus and an argument of $(z_E)^2$.

b- Determine the modulus of z_E and verify that $-\frac{\pi}{12}$ is an argument of z_E.

c- Deduce the exact values of $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$.

2) Let $Z = \frac{z_E}{z_F}$.

a- Write z_E, z_F and Z in the exponential form.

b- Show that the triangle OEF is equilateral.

II- (4 points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider the points A (4 ; 0 ; 0) , B (0 ; 4 ; 0) and C (0 ; 0 ; 4).

1) Write an equation of plane (ABC).

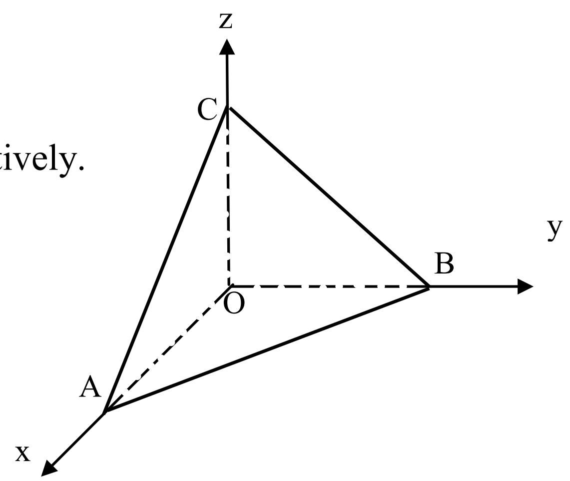
2) Calculate the area of triangle ABC.

3) Let F and G be the midpoints of [AC] and [BC] respectively.

a- Give a system of parametric equations of the straight line (FG).

b- The plane of equation z = 0 intersects the plane (OFG) along a line (d). Prove that the lines (d) and (FG) are parallel to each other.

c- Calculate the distance between the two lines (d) and (AB) .



III- (4 points)

The 80 students of the third secondary classes in a certain school are distributed into the three sections GS , LS and SE as shown in the following table :

	GS	LS	SE
Girls	8	18	10
Boys	12	14	18

The school director chooses randomly a group of 3 students, from the third secondary classes, to participate in a TV program.

1) What is the number of possible groups?
2) Designate by X the random variable that is equal to the number of boys in the chosen group. Determine the probability distribution of X .
3) Show that the probability that the chosen group contains one girl from each section is $\frac{18}{1027}$.
4) The chosen group is made up of 3 girls, What is the probability that they are from the same section ?

IV- (8 points)

Let f be the function that is defined on IR by : $f(x) = x + 2 - e^{-x}$, and (C) be its representative curve in an orthonormal system $(O ; \vec{i} , \vec{j})$.

1) a- Calculate $\lim\limits_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = x + 2$ is an asymptote of (C).
   b- Calculate $\lim\limits_{x \to -\infty} f(x)$ and give, in the decimal form, the values of $f(-1.5)$ and $f(-2)$.
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Write an equation of the line (T) that is tangent to (C) at the point A of abscissa 0.
4) Show that the equation $f(x) = 0$ has a unique root α and verify that $-0.5 < \alpha < -0.4$.
5) Draw (d), (T) and (C) .
6) Designate by g the inverse function of f, on IR.
   a- Draw, in the system $(O ; \vec{i} , \vec{j})$, the curve (G) that represents g.
   b- Designate by $A(\alpha)$ the area of the region that is bounded by the curve (C) , the axis of abscissas and the two lines of equations $x = \alpha$ and $x = 0$.
   Show that $A(\alpha) = (-\frac{\alpha^2}{2} - 3\alpha - 1)$ units of area.
   c- Deduce the area of the region that is bounded by the curve (G), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

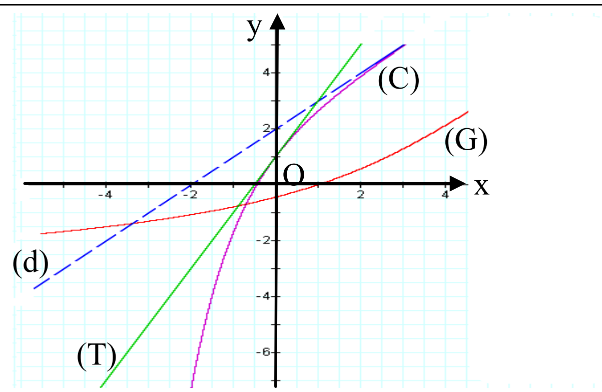
L.S-MATHS

2nd session 2005

Q1	Short Answers	M
1.a	$z_E^2 = \frac{1}{16}(3+1+2\sqrt{3}-3-1+2\sqrt{3}-4i) = \frac{1}{16}(4\sqrt{3}-4i) = \frac{\sqrt{3}}{4}-\frac{1}{4}i$ $z_E^2 = \frac{1}{2}(\frac{\sqrt{3}}{2}-\frac{1}{2}i) = \frac{1}{2}e^{-i\frac{\pi}{6}}$; $\left\|z_E{}^2\right\| = \frac{1}{2}$; $\arg(z_E{}^2) = -\frac{\pi}{6}$.	1
1.b	$\left\|z_E{}^2\right\| = \frac{1}{2}$ then $\left\|z_E\right\| = \frac{1}{\sqrt{2}}$. $\arg(z_E{}^2) = 2\arg(z_E) = -\frac{\pi}{6}+2k\pi$; $\arg(z_E) = -\frac{\pi}{12}+k\pi$, since $R_e(z_E) > 0$ and $Im(z_E)<0$, therefore $\arg(z_E) = -\frac{\pi}{12}$.	1
1.c	$z_E = \frac{1}{\sqrt{2}}[\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12})] = \frac{1}{\sqrt{2}}[\cos(\frac{\pi}{12})-i\sin(\frac{\pi}{12})] = \frac{\sqrt{3}+1}{4}-\frac{\sqrt{3}-1}{4}i$ $\cos(\frac{\pi}{12}) = \frac{\sqrt{6}+\sqrt{2}}{4}$ and $\sin(\frac{\pi}{12}) = \frac{\sqrt{6}-\sqrt{2}}{4}$.	½
2.a	$z_E = \frac{1}{\sqrt{2}}e^{-i\frac{\pi}{12}}$, $z_F = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}}$, $Z = e^{i(-\frac{\pi}{12}-\frac{\pi}{4})} = e^{-i\frac{\pi}{3}}$	½
2.b	$\|Z\| = 1 = \frac{OE}{OF}$; OE = OF $\arg(Z) = \arg(z_E) - \arg(z_F) = (\vec{u},\vec{OE}) - (\vec{u},\vec{OF})\ [2\pi] = (\vec{OF},\vec{OE})\ [2\pi] = -\frac{\pi}{3}[2\pi]$. OEF is equilateral. • OR : $EF = \|z_F - z_E\| = \frac{1}{\sqrt{2}} = OE = OF$	1

Q2	Short Answers	M
1	$\vec{AM}.(\vec{AB}\wedge\vec{AC}) = 0$; $\begin{vmatrix} x-4 & y & z \\ -4 & 4 & 0 \\ -4 & 0 & 4 \end{vmatrix} = 0$; $x+y+z-4=0$	1
2	$Area(ABC) = \frac{1}{2}\|\|\vec{AB}\wedge\vec{AC}\|\| = 2\sqrt{3}\ u^2$	½
3.a	F(2 ;0 ;2) and G(0 ; 2 ; 0) , eq. of (FG) : $x = -2t$, $y = 2t+2$, $z = 2$.	½
3.b	The plane of eq. z = 0 is the plane (AOB) ; (FG) // (AB) then (FG) // (OAB), since (d) is the line of intersection of (OFG) and (OAB) , hence (FG) //(d). • OR :the equation of (OFG) is : $x+y-z=0$. (d) : $x=m$, $y=-m$, $z=0$. $\vec{V}_d(1;-1;0)$, $\vec{FG}(-2;2;0)$ then $\vec{FG} = -2\vec{V}_d$ and the lines (d) and (FG) are distinct ; Therefore they are parallel.	1
3.c	The distance between (d) and (AB) is the distance from O to (AB) since (d) passes through O , and (d) //(AB), then $d = \frac{\|\|\vec{OA}\wedge\vec{OB}\|\|}{\|\|\vec{AB}\|\|} = 2\sqrt{2}$ u.	1

Q3	Short Answers	M
1	Number of possible cases : $C_{80}^{3} = 82\,160$.	½
2	x_i: 0, 1, 2, 3 p_i: $\frac{C_{36}^{3}}{C_{80}^{3}} = \frac{7140}{82160}$; $\frac{C_{36}^{2} \times C_{44}^{1}}{C_{80}^{3}} = \frac{27720}{82160}$; $\frac{C_{36}^{1} \times C_{44}^{2}}{C_{80}^{3}} = \frac{34056}{82160}$; $\frac{C_{44}^{3}}{C_{80}^{3}} = \frac{13244}{82160}$	1½
3	$\frac{C_{8}^{1} \times C_{18}^{1} \times C_{10}^{1}}{C_{80}^{3}} = \frac{18}{1027}$.	1
4	p(the girls are from the same section / 3 girls) = $\frac{C_{8}^{3} + C_{18}^{3} + C_{10}^{3}}{C_{36}^{3}} = \frac{248}{1785} = 0.138$	1

Q4	Short Answers	M
1.a	$\lim_{x \to +\infty} f(x) = +\infty - 0 = +\infty$; $\lim_{x \to +\infty} [f(x) - (x+2)] = \lim_{x \to +\infty} (-e^{-x}) = 0$ then the line (d) of equation $y = x + 2$ is an asymptote of (C) .	1
1.b	$\lim_{x \to -\infty} f(x) = -\infty - \infty = -\infty$; $f(-1.5) = -3.981$; $f(-2) = -7.389$.	1
2	$f'(x) = 1 + e^{-x}$ x: $-\infty$ … $+\infty$ f'(x): + f(x): $-\infty \nearrow +\infty$	1
3	(T) : $y = f'(0)x + f(0)$; $y = 2x + 1$.	½
4	f is continuous, strictly increasing on IR and varies from $-\infty$ to $+\infty$, then the equation $f(x) = 0$ has a unique solution α. $f(-0.5) \times f(-0.4) = -0.148 \times 0.1081 < 0$ then $-0.5 < \alpha < -0.4$.	1
5		1½
6.a	See the figure.	½
6.b	$A(\alpha) = \int_{\alpha}^{0} f(x)\,dx = \int_{\alpha}^{0} (x + 2 - e^{-x})\,dx = \left[\frac{x^2}{2} + 2x + e^{-x}\right]_{\alpha}^{0} = 1 - \frac{\alpha^2}{2} - 2\alpha - e^{-\alpha}$ But $f(\alpha) = 0$ i.e. $\alpha + 2 - e^{-\alpha} = 0$, therefore $e^{-\alpha} = \alpha + 2$ and $A(\alpha) = (-1 - 3\alpha - \frac{\alpha^2}{2})\,.\,u^2$	1
6.c	The region bounded by the curve (G) , the axis of abscissas and the two lines of equations x = 0 and x = 1 , is symmetric of the preceding region with respect of the line of equation y = x , therefore the required area is equal to A(α).	½

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٦ العادية

عدد المسائل: اربع

مسابقة في مادة تايضايرلا
المدة: ساعتان

الاسم:
الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I– (4 points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k}\,)$, consider the plane (P) of equation $x + y + z - 4 = 0$ and the points A (3 ;1 ; 0) , B(1; 2 ;1) , C(1 ; 1 ;2) and E(2 ; 0 ; –1).

1) Prove that the triangle ABC is right angled at B.
2) a- Verify that the points A, B and C belong to the plane (P).
   b- Write a system of parametric equations of the line (d) that is perpendicular to plane (P) at point A, and verify that the point E belongs to (d).
3) Designate by (Q) the plane that passes through A and is perpendicular to (BE). Write an equation of (Q).
4) The planes (P) and (Q) intersect along a line (D).
   a- Prove that the lines (D) and (BC) are parallel.
   b- M is a variable point on (BC), prove that the distance from M to plane (Q) remains constant.

II– (4.5 points)

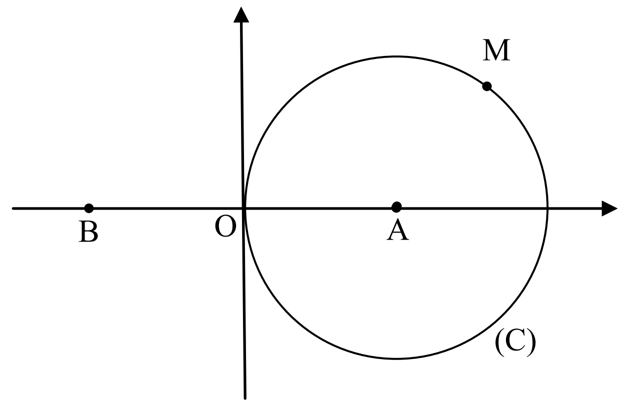
A bag **S** contains **eight** bills: **four** bills of 10 000LL, **three** of 20 000LL and **one** of 50 000LL.
Another bag **T** contains also **eight** bills : **three** bills of 10 000LL and **five** of 20 000LL.

1) **Two bills are drawn, simultaneously and randomly, from the bag S**.
   Calculate the probability of each of the following events:
   A : « the two drawn bills are of the same category »
   B : « the sum of values of the two drawn bills is 30 000LL ».
2) **One of the two bags S and T is randomly chosen, after which two bills are simultaneously and randomly drawn from this bag.**
   Consider the following events:
   E : « the chosen bag is **S** »
   F : « the sum of values of the two drawn bills is 30 000LL »
   Calculate the probabilities $P(F \cap E)$ and $P(F \cap \overline{E})$. Deduce $P(F)$.
3) **We draw, randomly, one bill from the bag S and one bill from the bag T**.
   Let X be the random variable that is equal to the sum of the values of the two drawn bills.
   a- Verify that $P(X = 60\,000) = \dfrac{3}{64}$.
   b- Determine the probability distribution of X and calculate its mean (expected value).

III– **(3.5 points)**

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B of affixes 1 and -1 respectively. Let (C) be the circle of center A and of radius 1. The exponential form of the affix z of a point M on (C), other than O, is given by $z = re^{i\theta}$.

Let M' be the point of affix z' such that $z' = \frac{1}{r}e^{i(\pi+\theta)}$.



1) Show that $z' \times \bar{z} = -1$.
2) Show that the points O, M and M′ are collinear.
3) a- Justify that $|z-1| = 1$.
   b- Prove that $|z'+1| = |z'|$, and deduce that M' moves on a line (d) to be determined.
4) Determine the points M on (C) for which $z' = -z$.

IV– **(8 points)**

A- Consider the differential equation (E): $y'' - 4y' + 4y = 4x^2 - 16x + 10$.

Let $z = y - x^2 + 2x$.

1) Write a differential equation (E') satisfied by z.
2) Solve (E') and deduce the general solution of (E).
3) Determine the particular solution of (E) whose representative curve, in an orthonormal system, has at the point A(0 ;1) a tangent parallel to the axis of abscissas.

B- Let f be the function that is defined on IR by $f(x) = e^{2x} + x^2 - 2x$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
   b- Calculate f(1) and f(–1.5) in their decimal forms.
2) The table below is the table of variations of the function f ', the derivative of f.

x	$-\infty$	0	$+\infty$
f '(x)	$-\infty$	0	$+\infty$

   a- Determine, according to the values of x , the sign of f '(x) .
   b- Set up the table of variations of f.
3) Draw the curve (C).
4) Let F be the function that is defined on $[0;+\infty[$ by $F(x) = \int_0^x f(t)dt$.
   a- Determine the sense of variations of F.
   b- What is the sign of F(x) ? Justify your answer.

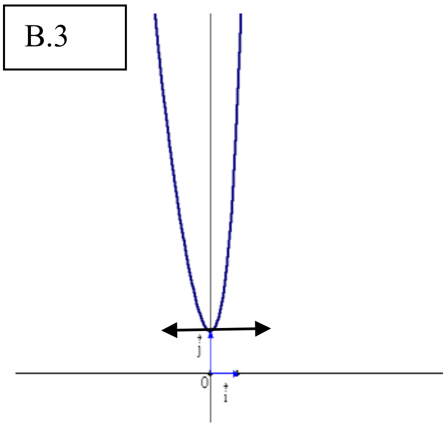
MATHEMATICS LS FIRST SESSION 2006

I	Answers	Marks
1	$\overrightarrow{BA}(2\,;-1\,;-1)$, $\overrightarrow{BC}(0\,;-1\,;1)$; $\overrightarrow{BA}\cdot\overrightarrow{BC}=0$ then ABC is right at B	½
2.a	$x_A+y_A+z_A-4=3+1+0-4=0$; $A\in(P)$. $x_B+y_B+z_B-4=1+2+1-4=0$; $B\in(P)$. $x_C+y_C+z_C-4=1+1+2-4=0$; $C\in(P)$.	½
2.b	$\overrightarrow{N_P}(1;1;1)$ is a director vector of (d) then (d) : $x=\lambda+3$; $y=\lambda+1$; $z=\lambda$. For y = 0 we have λ = –1, then x = 2 and z = – 1, hence $E\in(P)$.	1
3	$\overrightarrow{BE}(1\,;-2\,;-2)$ is normal to (Q) , so (Q) : x – 2y – 2z + r = 0 . $A\in(Q)$ then r = – 1 ; (Q) : x – 2y – 2z – 1 = 0.	½
4.a	(D)$\begin{cases}x+y+z-4=0\\x-2y-2z-1=0\end{cases}$ (D)$\begin{cases}x=3\\y=-t+1\\z=t\end{cases}$ then $\overrightarrow{V_D}(0\,;-1\,;1)=\overrightarrow{BC}$; (BC) // (D). ▶**OR :**(BC) is perpendicular to (AB) and it is orthogonal to (EA), so (BC) is perpendicular to plane (EAB) and especially to (EB) , on the other hand since (EB) is perpendicular to (Q) then (BC) is parallel to (Q) . The plane (P), which contains (BC), cuts (Q) along (D) parallel to (BC).	1
4.b	(BC) : $\begin{cases}x=1\\y=-m+2\\z=m+1\end{cases}$; M(1 ; – m + 2 ; m +1) ; $d(M;(Q))=\dfrac{\lvert 1+2m-4-2m-2-1\rvert}{\sqrt{1+4+4}}=2$. ▶**OR :** (BC) // (D) and (D)⊂(Q) ,so (BC) // (Q) . Since $M\in(BC)$; $d(M;(Q))=cst$.	½

II	Answers	Marks
1	P(A) = P[(10 000 , 10 000) or (20 000 , 20 000)] $=\dfrac{C_4^2+C_3^2}{C_8^2}=\dfrac{9}{28}$. P(B) = P(10 000 , 20 000) = $\dfrac{C_4^1\times C_3^1}{C_8^2}=\dfrac{3}{7}$.	1
2	$P(F\cap E)=P(E)\times P(F/E)=\dfrac{1}{2}\times\dfrac{3}{7}=\dfrac{3}{14}$. $P(F\cap\overline{E})=P(E)\times P(F/\overline{E})=\dfrac{1}{2}\times\dfrac{C_3^1\times C_5^1}{C_8^2}=\dfrac{1}{2}\times\dfrac{15}{28}=\dfrac{15}{56}$. $P(F)=P(F\cap E)+P(F\cap\overline{E})=\dfrac{3}{14}+\dfrac{15}{56}=\dfrac{27}{56}$.	1½
3.a	P(X = 60 000) = P(50 000 , 10 000) = 1/8 × 3/8 = 3/64.	½
3.b	(table below) $E(X)=(24+87+60+18+35)\times\dfrac{10\,000}{64}=35\,000$.	1½

$X=x_i$	20 000	30 000	40 000	60 000	70 000
p_i	$\frac{4}{8}\times\frac{3}{8}=\frac{12}{64}$	$\frac{4}{8}\times\frac{5}{8}+\frac{3}{8}\times\frac{3}{8}=\frac{29}{64}$	$\frac{3}{8}\times\frac{5}{8}=\frac{15}{64}$	$\frac{3}{64}$	$\frac{1}{8}\times\frac{5}{8}=\frac{5}{64}$

III	Answers	Marks
1	$z'\times\bar z=\frac{1}{r}e^{i(\pi+\theta)}\times re^{-i\theta}=\ e^{i\pi}\ =-1$.	½
2	$(\vec u\ ,\overrightarrow{OM})=\theta$, $(\vec u\ ,\overrightarrow{OM'})=\pi+\theta$, so $(\overrightarrow{OM},\overrightarrow{OM'})=(\pi+\theta)-\theta=\pi$, hence O , M and M' are collinear. ▶**OR :** $\frac{z'}{z}=-\frac{1}{r^2}$ **;** $z'=-\frac{1}{r^2}z$, so $\overrightarrow{OM'}=-\frac{1}{r^2}\overrightarrow{OM}$ and thus O , M , M' are collinear	½
3.a	$\lvert z-1\rvert=\lvert z_M-z_A\rvert=AM=1$	½
3.b	$\lvert z'+1\rvert=\left\lvert\frac{-1}{\bar z}+1\right\rvert=\left\lvert\frac{\bar z-1}{\bar z}\right\rvert=\frac{\lvert\overline{z-1}\rvert}{\lvert\bar z\rvert}=\frac{\lvert z-1\rvert}{\lvert\bar z\rvert}=\frac{1}{\lvert\bar z\rvert}$ and $\lvert z'\rvert=\frac{\lvert-1\rvert}{\lvert\bar z\rvert}=\frac{1}{\lvert\bar z\rvert}$; $\lvert z'+1\rvert=\lvert z'\rvert$. $\lvert z_{M'}-z_B\rvert=\lvert z_{M'}\rvert$; BM' = OM' ; M' moves on the perpendicular bisector (d) of [OB].	1
4	$z'=-z$; $-z\times\bar z=-1$; $z\times\bar z=1$; $\lvert z\rvert^2=1$; $OM^2=1$; $OM=1$, then M belongs to the circle (C') of center O , radius 1; but M belongs to (C). Then points M are the two points of intersection of (C) and (C'). ▶**OR:** $\lvert -z+1\rvert=\lvert -z\rvert$; $\lvert z-1\rvert=\lvert z\rvert$; AM = OM ; M moves on the perpendicular bisector (D) of [OA].Then points M are the two points of intersection of (D) and (C).	1

IV	Answers	Marks
A.1	$z'=y'-2x+2$ et $z''=y''-2$ $z''+2\ -4(z'+2x-2)+4(z+x^2-2x)=4x^2-16x+10$ $z''-4z'+4z=0$	½
A.2	$r^2-4r+4=0$; $r=2$ double root ; $z=(Ax+B)e^{2x}$ and $y=(Ax+B)e^{2x}+x^2-2x$	1
A.3	$y(0)=1$; $B=1$ $y'(0)=1$ with $y'(x)=Ae^{2x}+2(Ax+B)e^{2x}+2x-2$; $A+2B=2$; $A=0$; $y=e^{2x}+x^2-2x$	1
B.1.a	$\lim\limits_{x\to+\infty}f(x)=\lim\limits_{x\to+\infty}[e^{2x}+x(x-2)]=+\infty$; $\lim\limits_{x\to-\infty}f(x)=\lim\limits_{x\to-\infty}[e^{2x}+x(x-2)]=+\infty$	½
B.1.b	$f(1)=6.39$; $f(-1.5)=5.30$.	½
B.2.a	$f'(x)<0$ for $x<0$; $f'(x)>0$ for $x>0$.	1
B.2.b	$\begin{array}{c\|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & +\infty & \searrow & 1 & \nearrow & +\infty \end{array}$	B.2.b ½
B.3	B.3 	B.3 1
B.4.a	$F'(x)=f(x)>0$ for $x\geq 0$ (min (f(x)) =1) , then F is strictly increasing over $[0\ ;+\infty[$.	1
B.4.b	$f(t)>0$ and $x\geq 0$ then $\int_0^x f(t)dt\geq 0$, so $F(x)\geq 0$. ▶**OR :** F is increasing and $F(0)=0$, then $F(x)\geq 0$.	1

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة ٢٠٠٦ الأستثنائية

عدد المسائل: اربع	مسابقة في مادةتايضايرلا المدة: ساعتان	الاسم: الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I-(3.5 points)

In the complex plane, referred to a direct orthonormal system $(O\,;\,\vec{u}\,,\,\vec{v})$, consider the points A, B and M of respective affixes 2, 4 and z (where $z\neq 2$).

Let M' be the point of affix z' such that $z'=\dfrac{z-4}{z-2}$.

1) a- Give a geometric interpretation of $|z'|$, $|z-4|$ and $|z-2|$.
b- Determine the set of points M when M' moves on the circle with center O and radius 1.

2) Let $z=x+iy$ and $z'=x'+iy'$.
a- Express x' and y' in terms of x and y.
b- When z' is real, find the line on which the point M moves.

II-**(4 points)**

An urn U contains **four** balls numbered **1**, **three** balls numbered **2** and **one** ball numbered **5**.
Another urn V contains **three** balls numbered **1** and **five** balls numbered **2**.

A- Two balls are drawn, simultaneously and randomly, from the urn U.
Calculate the probability of each of the following events:
E : « the two drawn balls carry the same number »
F : « the product of the two numbers, that are marked on the two drawn balls, is 10 ».

B- We draw randomly one ball from the urn U and one ball from the urn V.
Let X be the random variable that is equal to the sum of the two numbers that are marked on the two drawn balls.
1- Give the five possible values of X.
2- Verify that the probability of having $X = 3$ is equal to $\dfrac{29}{64}$.
3- Determine the probability distribution of X.

III-**(4 points)**

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k})$, consider:

the plane (P) of equation $x+y+z-4=0$,

the points A (3 ;1 ; 0), B(1 ;2 ;1), C(1 ; 1 ;2) and E(2 ; 0 ;-1).

1) Prove that the triangle ABC is right angled at B.

2) a- Verify that (P) is the plane that is determined by A , B and C.
   b- Show that the line (AE) is perpendicular to plane (P) .

3) Designate by (Q) the plane passing through A and perpendicular to (BE).
   Write an equation of (Q).

4) The planes (P) and (Q) intersect along a line (D).
   a- Prove that the lines (D) and (BC) are parallel.
   b- Let L be any point on (BC) and H be its orthogonal projection on (Q) .
   Show that LH remains constant as L moves on the line (BC).

IV-**(8.5 points)**

A- Given the differential equation (E) : $y'-y-e^{x}+1=0$.

Let $z=y-xe^{x}-1$.

1) Find a differential equation (E') that is satisfied by z, and determine its general solution.
2) Deduce the general solution of (E), and find a particular solution y of (E) that verifies $y(0)=0$.

B- Let f be the function that is defined on IR by $f(x)=(x-1)e^{x}+1$, and designate by (C) its representative curve in an orthonormal system $(O;\,\vec{i}\,,\,\vec{j}\,)$.

1) a- Calculate $\lim\limits_{x\to+\infty} f(x)$. Give f(2) in the decimal form.
   b- Calculate $\lim\limits_{x\to-\infty} f(x)$ and deduce an asymptote (d) to (C).
   c- Verify that the curve (C) cuts its asymptote (d) at the point E(1 ; 1).

2) a- Calculate f '(x) and set up the table of variations of f.
   b- Prove that the curve (C) has a point of inflection.

3) Draw the line (d) and the curve (C).

4) a- Prove that the function f has, on $[0\,;+\infty[$, an inverse function g.
   b- Draw the curve (G) that represents g, in the system $(O;\,\vec{i}\,,\,\vec{j}\,)$.
   c- Calculate the area of the region bounded by the two curves (C) and (G).

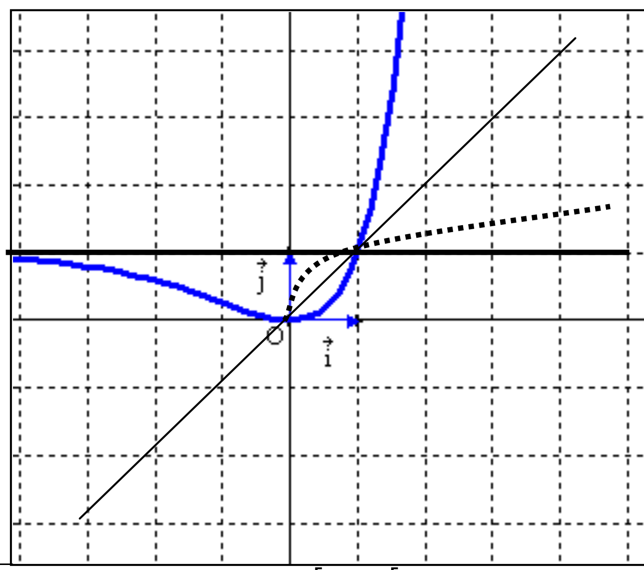
LS MATHEMATICS

SCHEME OF CORRECTION		2nd SESSION 2006
Q1		M
1.a	$\lvert z'\rvert = OM'$; $\lvert z-4\rvert = BM$ and $\lvert z-2\rvert = AM$	1
1.b	$\lvert z'\rvert = \dfrac{\lvert z-4\rvert}{\lvert z-2\rvert}$ gives $OM' = \dfrac{BM}{AM}$. $OM' = 1$ is equivalent to $BM = AM$ and thus the set of points M is the perpendicular bisector of [AB],	1
2.a	$x' + iy' = \dfrac{x+iy-4}{x+iy-2} = \dfrac{(x-4+iy)(x-2-iy)}{(x-2)^2+y^2}$;$x' = \dfrac{x^2+y^2-6x+8}{(x-2)^2+y^2}$,$y' = \dfrac{2y}{(x-2)^2+y^2}$	1
2.b	z' is real iff $y' = 0$, to get $y = 0$, so M moves on the axis of abscissas.	½

Q2		M
A	$P(E) = P(1\,;1) + P(2\,;2) = \dfrac{C_4^2}{C_8^2} + \dfrac{C_3^2}{C_8^2} = \dfrac{9}{28}$. $P(F) = P(2\,;5) = \dfrac{C_3^1 \times C_1^1}{C_8^2} = \dfrac{3}{28}$	1½
B-1	The five values of X are: 2 ; 3 ; 4 ; 6 ; 7 .	½
B-2	$P(X=3) = P(1\,;2) + P(2\,;1) = \dfrac{4}{8}\times\dfrac{5}{8}+\dfrac{3}{8}\times\dfrac{3}{8} = \dfrac{29}{64}$	½
B-3	see table below	1½

x_i	2	3	4	6	7
P_i	$\frac{4}{8}\times\frac{3}{8}=\frac{12}{64}$	$\frac{29}{64}$	$\frac{3}{8}\times\frac{5}{8}=\frac{15}{64}$	$\frac{1}{8}\times\frac{3}{8}=\frac{3}{64}$	$\frac{1}{8}\times\frac{5}{8}=\frac{5}{64}$

Q3		M
1	$\vec{AB}(-2,1,1)$, $\vec{BC}(0,-1,1)$; $\vec{AB}.\vec{BC} = 0$, thus ABC is right angled at B.	½
2.a	The coordinates of A , B and C verify the equation of (P). $\Rightarrow$ OR : $\vec{AM}.(\vec{AB}\wedge\vec{AC}) = 0$	½
2.b	$\vec{AE}(-1,-1,-1)$, $\vec{N_P}(1,1,1)$ so $\vec{AE} = -\vec{N_P}$ and (AE) is perpendicular to (P).	½
3	. $\vec{BE}(1,-2,-2) = \vec{N_Q}$ thus (Q) : $x - 2y - 2z + d = 0$, A belongs to (Q) gives $d = -1$ (Q) : $x - 2y - 2z - 1 = 0$.	½
4.a	$\vec{BC}(0,-1,1)$, $\vec{V_D} = \vec{N_P}\wedge\vec{N_Q} = 3\vec{j} - 3\vec{k}$ then $\vec{V_D} = -3\vec{BC}$ and B does not belong to (Q) , hence (D) // (BC) . $\Rightarrow$ OR : (BC) is perpendicular to plane (ABE) , then (BC) is perpendicular to (EB), and (EB) is perpendicular to (Q) , hence (BC) // (Q) [since (BC) $\not\subset$ (Q)] (P) contains (BC) then (P) cuts (Q) along (D) //(BC).	1
4.b	(D) //(BC) , then(BC) //(Q) .All the points on (BC) have the same distance from (Q) ; $\Rightarrow$ OR : Equations of (BC) : $x = 1$, $y = -m + 2$, $z = m + 1$ $d(M \to (Q)) = \dfrac{\lvert 1+2m-4-2m-2-1\rvert}{\sqrt{1+4+4}} = 2 =$ constant	1

Q4		M
A1	$y' - y - e^x + 1 = 0$; $y = z + xe^x + 1$; $y' = z' + e^x + xe^x$; $z' + e^x + xe^x - z - xe^x - 1 - e^x + 1 = 0$ $z' - z = 0$; to get $z = Ce^x$.	1
A2	$y = Ce^x + xe^x + 1$; $y(0) = C + 1 = 0$; $C = -1$, then $y = -e^x + xe^x + 1$.	1
B1.a	$\lim\limits_{x \to +\infty} f(x) = +\infty$; $f(2) = e^2 + 1 = 8.389$.	½
B1.b	$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} (xe^x - e^x + 1) = 1$, so the line (d) of equation $y = 1$ is an asymptote to (C) .	1
B1.c	$y = 1$ and $1 = (1 - 1)e^x + 1$; so (C) cuts (d) at E (1 ;1) .	½
B2.a	$f'(x) = e^x + (x - 1)e^x = xe^x$. $\begin{array}{c\|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 1 & \searrow & 0 & \nearrow & +\infty \end{array}$	1
B2.b	$f''(x) = (x + 1)e^x$; $f''(x)$ vanishes for $x = -1$ and changing signs; consequently (C) has a point of inflection .	½
B3	 	1
B4.a	f is continuous and strictly increasing on $[0;+\infty[$,so it has an inverse function g .	½
B4.b	(G) is the symmetric of (C) w.r.t. the first bisector of equation $y = x$.	½
B4.c	$\mathcal{A} = 2\int_0^1 [x - f(x)]dx = 2\int_0^1 (x - 1 + e^x - xe^x)dx = 2\left[\frac{x^2}{2} - x + e^x\right]_0^1 - 2\int_0^1 xe^x dx$ so $\int_0^1 xe^x dx = [xe^x]_0^1 - \int_0^1 e^x dx = e - (e - 1) = 1$ Thus $\mathcal{A} = 2(\frac{1}{2} - 1 + e - 1) - 2 = 2e - 5$ u^2.	1

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دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة سنة 2007 العادية

عدد المسائل: اربع

مسابقة في مادة تايضايرلا
المدة: ساعتان

الاسم:
الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I– (4 points)

In the space referred to a direct orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j}\ ,\ \vec{k})$, consider the points A (1 ; 1 ; 0), B (2; 0 ; 0), C (1; 3; –1), E (2 ; 2 ; 2) and the plane (P) of equation $x + y + 2z - 2 = 0$.

1) a- Verify that (P) is the plane determined by A, B and C.
   b- Show that the line (AE) is perpendicular to the plane (P).
   c- Calculate the area of triangle ABC and the volume of tetrahedron EABC.

2) Designate by L the midpoint of [AB] and by (Q) the plane passing through L and parallel to the two lines (AE) and (BC).
   a- Write an equation of plane (Q).
   b- Prove that the planes (P) and (Q) are perpendicular.
   c- Prove that line (d), the intersection of the planes (P) and (Q), is parallel to (BC).

II– (4 points)

The 20 employees in a factory are distributed into two departments as shown in the table below:

	Technical Department	Administrative Department
Women	3	5
Men	10	2

1) The manager of this factory wants to offer a gift to one of the employees. To do this, he chooses randomly an employee of this factory.
   Consider the following events:
   W : « the chosen employee is a woman ».
   M : « the chosen employee is a man ».
   T : « the chosen employee is from the technical department ».
   A : « the chosen employee is from the administrative department ».
   a- Calculate the following probabilities:
   $P(W / T)$, $P(W / A)$, $P(W \cap T)$ and $P(W)$.
   b- Knowing that the chosen employee is a man, what is the probability that he is from the technical department ?

2) On a different occasion, the factory manager chooses **two** employees randomly and simultaneously from the technical department and also chooses **one** employee randomly from the administrative department.
   Designate by X the random variable that is equal to the number of women chosen.
   a- Verify that $P(X = 1) = \dfrac{95}{182}$.
   b- Determine the probability distribution of X.

III– (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points E, F, G of respective affixes $z_E = 2i$, $z_F = -2i$, $z_G = -1+i$ and let M be a point of affix z.

1) a- Find the set (T) of points M such that $|z-2i| = \sqrt{2}$.

   b- Show that the point G belongs to (T).

2) a- Find the line (L) on which point M moves when $\left|\frac{z-2i}{z+2i}\right| = 1$.

   b- Determine the affix z_0 of a point W on (L) such that $|z_0 - 2i| = 3$.

3) Let A and B be the points of respective affixes z_A and z_B such that: $z_A = z_F + z_G$ and $z_B = z_F \times z_G$.

   a- Write the complex numbers z_A and z_B in the exponential form.

   b- Prove that the points O, A and B are collinear.

IV– (8 points)

Consider the function f defined over $]-\infty, 0[\cup]0,+\infty[$ by $f(x) = x-1-\frac{4}{e^x-1}$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Show that the axis of ordinates is an asymptote to (C).

   b- Calculate $\lim\limits_{x\to+\infty} f(x)$ and prove that the line (d) of equation y = x – 1 is an asymptote to the curve (C).

   c- Prove that the line (D) of equation y = x + 3 is an asymptote to (C) at $-\infty$.

2) Prove that the point S(0 ; 1) is a center of symmetry of (C).

3) a- Calculate $f'(x)$ and set up the table of variations of f.

   b- Show that the equation f(x) = 0 has two roots α and β and verify that : $1.7 < \alpha < 1.8$ and $-3.2 < \beta < -3.1$.

4) Draw (d), (D) and (C).

5) a- Prove that $f(x) = x+3-\frac{4e^x}{e^x-1}$.

   b- Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations x = 2 and x = 3.

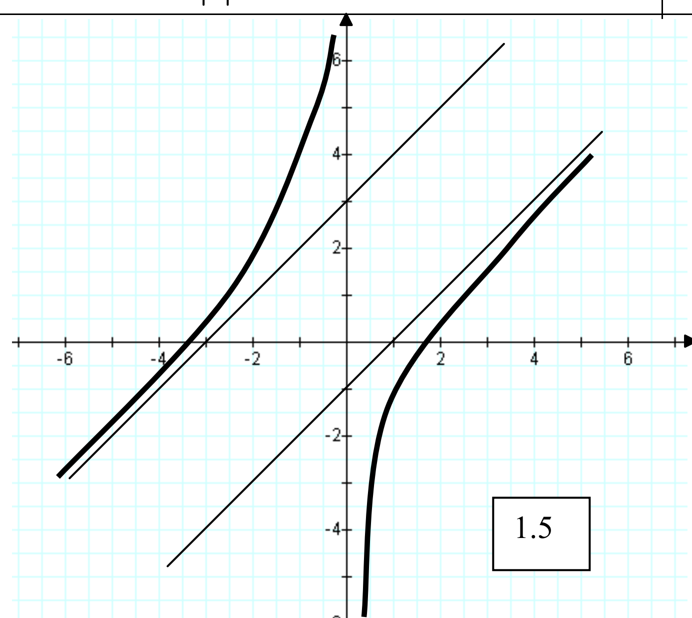
6) Let g be the inverse function of f on $]0, +\infty[$.
   Prove that the equation f(x) = g(x) has no roots.

Q1	MATH LS FIRST SESSION-2007	Marks
1-a	◇ $1+1+0-2=0$; $A\in(P)$ ◇ $2+0+0-2=0$; $B\in(P)$ ◇ $1+3-2-2=0$; $C\in(P)$ OR $\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC})=0$; $\begin{vmatrix} x-1 & y-1 & z \\ 1 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix}=0$; $x+y+2z-2=0$	1/2
1-b	$\overrightarrow{AE}(1;1;2)$ and $\overrightarrow{N_P}(1;1;2)$; (AE) is perpendicular to plane (P).	1/2
1-c	$S=\frac{1}{2}\lVert\overrightarrow{AB}\wedge\overrightarrow{AC}\rVert=\frac{\sqrt{1+1+4}}{2}=\frac{\sqrt{6}}{2}$. $V=\frac{1}{3}S\times AE=\frac{1}{3}\times\frac{\sqrt{6}}{2}\times\sqrt{6}=1$ OR : $V=\frac{1}{6}\lvert\overrightarrow{AE}.(\overrightarrow{AB}\wedge\overrightarrow{AC})\rvert=\frac{1}{6}\lvert 1+1+4\rvert=1$	1
2-a	$\overrightarrow{IM}.(\overrightarrow{AE}\wedge\overrightarrow{BC})=0$ where $I(\frac{3}{2};\frac{1}{2};0)$; So $\begin{vmatrix} x-\frac{3}{2} & y-\frac{1}{2} & z \\ 1 & 1 & 2 \\ -1 & 3 & -1 \end{vmatrix}=0$ (Q) : $7x+y-4z-11=0$.	1
2-b	$\overrightarrow{N_P}.\overrightarrow{N_Q}=-7-1+8=0$; (P) and (Q) are perpendicular.	1/2

2-c	(BC) // (Q) and (BC) is a line in (P) , so (BC) is parallel to the line of intersection of (P) and (Q).
	OR : $(d)=(P)\cap(Q)$: $\begin{cases} x+y+2z-2=0 \\ 7x+y-4z-11=0 \end{cases}$; (d) : $\begin{cases} x=t+\frac{3}{2} \\ y=-3t+\frac{1}{2} \\ z=t \end{cases}$
	$\overrightarrow{BC}(-1;3;-1)$ and $\overrightarrow{V_d}(1;-3;1)$.
	So (BC) //(d).
		1/2

<table>
<tr><td>Q2</td><td colspan="2">MATH LS FIRST SESSION-2007</td><td>Marks</td></tr>
<tr><td>1-a</td><td colspan="2">$P(W/T) = \frac{3}{13}$; $P(W/A) = \frac{5}{7}$;
$P(W\cap T) = \frac{3}{20}$; $P(W) = \frac{8}{20}$</td><td>1</td></tr>
<tr><td>1-b</td><td colspan="2">$P(T/M) = \frac{10}{12} = \frac{5}{6}$</td><td>1/2</td></tr>
<tr><td>2-a</td><td>$P(X = 1) = \frac{3\times 10}{C_{13}^2}\times\frac{2}{7} + \frac{C_{10}^2}{C_{13}^2}\times\frac{5}{7}$
$= \frac{285}{546}$
$= \frac{95}{182}$</td><td><table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>1</td><td>1</td><td>0</td><td>1</td></tr>
<tr><td>Or</td><td>0</td><td>2</td><td>1</td><td>0</td></tr>
</table></td><td>1</td></tr>
<tr><td rowspan="4">2-b</td><td>$P(X= 0) = \frac{C_{10}^2}{C_{13}^2}\times\frac{2}{7}$
$= \frac{90}{546}$
$= \frac{15}{91}$</td><td><table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>0</td><td>2</td><td>0</td><td>1</td></tr>
</table></td><td rowspan="4">1 1/2</td></tr>
<tr><td>$P(X= 2) = \frac{C_3^2}{C_{13}^2}\times\frac{2}{7} + \frac{3\times 10}{C_{13}^2}\times\frac{5}{7}$
$= \frac{156}{546}$
$= \frac{26}{91}$</td><td><table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>2</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>Or</td><td>1</td><td>1</td><td>1</td><td>0</td></tr>
</table></td></tr>
<tr><td>$P(X= 3) = \frac{C_3^2}{C_{13}^2}\times\frac{5}{7}$
$= \frac{15}{546}$
$= \frac{5}{182}$</td><td><table>
<tr><td></td><td colspan="2">T</td><td colspan="2">A</td></tr>
<tr><td></td><td>3W</td><td>10M</td><td>5W</td><td>2M</td></tr>
<tr><td></td><td>2</td><td>0</td><td>1</td><td>0</td></tr>
</table></td></tr>
<tr><td colspan="2"><table>
<tr><td>x_i</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>P_i</td><td>$\frac{15}{91}$</td><td>$\frac{95}{182}$</td><td>$\frac{26}{91}$</td><td>$\frac{5}{182}$</td></tr>
</table></td></tr>
</table>

Q3	MATH LS FIRST SESSION-2007	Marks
1-a	$\|z-2i\|=\sqrt{2}$ is equivalent to EM=$\sqrt{2}$ Thus (T) is the circle of center E and radius $\sqrt{2}$.	1
1-b	EG=$\|z_G - z_E\| = \|-1-i\| = \sqrt{2}$,thus G$\in$(T).	1/2
2-a	$\left\|\frac{z-2i}{z+2i}\right\| = 1$ is equivalent to $\|z-2i\| = \|z+2i\|$ so ME=MF and (L) is the perpendicular bisector of [EF], which is the axis of abscissas.	1/2
2-b	W$\in$(L) so $\|z_0 - 2i\| = \|z_0 + 2i\| = 3$; Let $z_0 = x + iy$. $\|x+iy-2i\| = \|x+iy+2i\|$ is equivalent to $x^2 + (y-2)^2 = x^2 + (y+2)^2$ $y = 0$ gives $x^2 + 4 = 9$; $x = \sqrt{5}$ or x=$-\sqrt{5}$, consequently $z_0 = \sqrt{5}$ or $z_0 = -\sqrt{5}$ OR : W$\in$ x'x and EW=3 so $OW^2 = EW^2 - OE^2 = 9 - 4 = 5$; OW=$\sqrt{5}$ thus $z_0 = \sqrt{5}$ or $z_0 = -\sqrt{5}$.	1/2
3-a	$z_A = -1 - i = \sqrt{2}e^{5\frac{\pi}{4}i}$, $z_B = 2 + 2i = 2\sqrt{2}e^{\frac{\pi}{4}i}$.	1
3-b	$\arg z_A = 5\frac{\pi}{4}$, $\arg z_B = \frac{\pi}{4}$; $\arg z_A = \arg z_B + \pi$ so O, A and B are collinear OR : $z_B = -2z_A$ or $\overrightarrow{OB} = -2\overrightarrow{OA}$.	1/2

Q4	MATH LS — FIRST SESSION-2007	Marks
1-a	$\lim\limits_{\substack{x\to 0\\ x>0}} f(x) = -1-\dfrac{4}{0^+} = -\infty$; $\lim\limits_{\substack{x\to 0\\ x<0}} f(x) = -1-\dfrac{4}{0^-} = +\infty$ So the axis of ordinates of equation x = 0 is an asymptote of (C) .	1/2
1-b	$\lim\limits_{x\to+\infty} f(x) = +\infty - 0 = +\infty$; $\lim\limits_{x\to+\infty} [f(x)-(x-1)] = \lim\limits_{x\to+\infty} \dfrac{4}{e^x-1} = 0$	1
1-c	$\lim\limits_{x\to-\infty}[f(x)-(x+3)] = \lim\limits_{x\to-\infty}(x-1-\dfrac{4}{e^x-1}-x-3) = \lim\limits_{x\to-\infty}(-4-\dfrac{4}{e^x-1})$ $= -4+4 = 0.$	1/2
2	The domain of f is centered at O.. $f(-x)+f(x) = -x-1-\dfrac{4}{e^{-x}-1}+x-1-\dfrac{4}{e^x-1} = -2+\dfrac{4e^x}{e^x-1}+\dfrac{4}{e^x-1}$ $= -2+4 = 2$, thus S(0 ; 1) is a center symmetry of (C).	1/2
3-a	$f'(x) = 1+\dfrac{4e^x}{(e^x-1)^2} > 0.$ $\begin{array}{c\|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & + & \| & + & \\ \hline f(x) & -\infty & \nearrow +\infty & \| & -\infty \nearrow & +\infty \end{array}$	1
3-b	f is continuous and strictly increasing from $-\infty$ to $+\infty$ on $]-\infty\,;0[$;the equation f(x) = 0 has over this interval a unique negative root β ; $f(-3.2) = -0.03 < 0$ and $f(-3.1) = 0.088 > 0$ so $-3.2 < \beta < -3.1$. Similarly : f(x) = 0 has over $]0\,;+\infty[$ a unique positive root α ; $f(1.7) = -0.154 < 0$ and $f(1.8) = +0.0078 > 0$ so $1.7 < \alpha < 1.8$. 1 point 	2.5
5-a	$x+3-\dfrac{4e^x}{e^x-1} = x-1+4-\dfrac{4e^x}{e^x-1} = x-1+\dfrac{4e^x-4-4e^x}{e^x-1} = x-1-\dfrac{4}{e^x-1}$.	1/2
5-b	$A = \displaystyle\int_2^3 (x+3-\dfrac{4e^x}{e^x-1})dx = \left[\dfrac{x^2}{2}+3x-4\ln(e^x-1)\right]_2^3 = \dfrac{11}{2}+4\ln\left(\dfrac{e^2-1}{e^3-1}\right) = 1.122\ u^2$	1
6	f(x) = g(x) equivalent to f(x) = x ; $x-1-\dfrac{4}{e^x-1} = x$; $-1 = \dfrac{4}{e^x-1}$; $e^x = -3$ is impossible since ($e^x > 0$) .So the equation f(x) = g(x) has no roots.	1/2

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	الدورة الإكمالية الإستثنائية للعام ٢٠٠٧
عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة : يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات
يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points A, M and M' of affixes i , z and z' respectively, where $z' = \frac{iz}{z-i}$ $(z \neq i)$.

1- Determine the points M such that $z' = z$.

2- If $z = \frac{\sqrt{2}}{2} e^{i\frac{3\pi}{4}}$, find an argument of z'.

3- Let $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

a) Calculate x' and y' in terms of x and y.

b) Determine the set of points M for which z' is real.

4- a) Show that $z' - i = \frac{-1}{z-i}$.

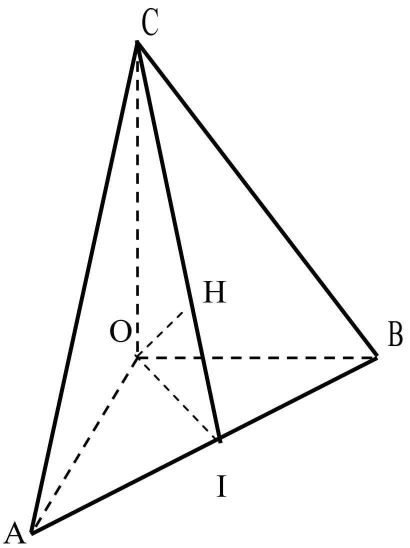
b) Show that when M moves on the circle (ω) of center A and radius 1 then M' moves on the same circle.

II- (4 points)

The adjacent figure is considered in a direct orthonormal system $\left(O; \vec{i}, \vec{j}, \vec{k}\right)$ where:

$\vec{OA} = \vec{i}$, $\vec{OB} = \vec{j}$ and $\vec{OC} = 2\vec{k}$.

Let I be the midpoint of [AB].



1- Justify that an equation of plane (ABC) is $2x + 2y + z - 2 = 0$.

2- Consider the point $H\left(\frac{4}{9}; \frac{4}{9}; \frac{2}{9}\right)$.

a) Show that C, H and I are collinear.

b) Prove that (OH) is perpendicular to the plane (ABC).

c) Prove that the two planes (OIC) and (ABC) are perpendicular.

3- a) Write a system of parametric equations of the straight line (Δ) passing through C and parallel to (OB).

b) Let F be a variable point on (Δ).

Prove that the tetrahedron FOAB has a constant volume to be calculated.

III- (4 points)

In a public library, every visitor has to either choose a book or use a computer.
70% of these visitors use the computer.

Out of those who use the computer, 45% do a research.

Out of the visitors who choose a book, 80% do a research.

A) We meet, at random, one of the visitors in this library.

Consider the following events:

C: « the visitor uses the computer».

B: « the visitor chooses a book».

R: « the visitor does a research».

1- Verify that the probability $P(C \cap R)$ is equal to 0.315 .

2- Calculate $P(B \cap R)$ then $P(R)$.

3- The visitor did a research, calculate the probability that he used the computer.

B) On a Monday morning, 30 persons visited this library. We choose, simultaneously and at random, three of these visitors. Designate by X the random variable that is equal to the number of visitors who used the computer among the three chosen visitors.

1- Determine the values of X.

2- Determine the probability distribution of X.

IV- (8 points)

Let f be the function that is defined, on $[0;+\infty[$, by $f(x) = (x+1)e^{-x}$ and designate by (C) its representative curve in an orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j}\ )$. (unit: 2cm)

1- Calculate $\lim\limits_{x \to +\infty} f(x)$ and deduce an asymptote of (C).

2- a) Calculate $f'(x)$ and set up the table of variations of f.

b) Calculate $f'(0)$ and interpret the result graphically.

3- a) Prove that the curve (C) has a point of inflection $W(1, \frac{2}{e})$.

b) Write an equation of the line (d) that is tangent to (C) at the point W.

4- Draw (d) and (C).

5- a) Calculate the real numbers a and b so that the function F defined on $[0;+\infty[$ by $F(x) = (ax+b)e^{-x}$ is an antiderivative of f.

b) Calculate, in cm^2, the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

6- Let g be the inverse function of f and designate by (G) the representative curve of g.

a) Draw (G) in the preceding system.

b) Find an equation of the tangent to the curve (G) at the point of abscissa $\frac{2}{e}$.

معيار التصحيح

مسابقة في مادة الرياضيات

I- (4 points)

Part of the Q	Answer	Mark
1	$z=\frac{iz}{z-i}$ then z(z-2i) = 0 ; z = 0 or z = 2i , so M(0;0) or M(0;2).	0.5
2	$z=\frac{\sqrt{2}}{2}e^{i\frac{3\pi}{4}}=\frac{-1}{2}+\frac{i}{2};z'=\frac{\frac{-1}{2}-\frac{i}{2}}{\frac{-1}{2}-\frac{i}{2}}=1;\ \arg z'=0(2\pi).$	0.5
3.a	$z'=\frac{-x}{x^2+(y-1)^2}+i\frac{x^2+y^2-y}{x^2+(y-1)^2};x'=\frac{-x}{x^2+(y-1)^2}\ ;\ y'=\frac{x^2+y^2-y}{x^2+(y-1)^2}.$	0.5
3.b	z' is real then $x^2+y^2-y=0$; then $x^2+(y-\frac{1}{2})^2=\frac{1}{4}$ and $z\neq i$, so the set is a circle of center (0 , 1/2) and radius 1/2 excluding point A(0;1).	1
4.a	$z'=\frac{iz}{z-i}$ then $z'-i=\frac{iz}{z-i}-i$ thus $z'-i=\frac{-1}{z-i}$.	0.5
4.b	Since AM = 1 then $\|z-i\|=1$ then $\|z'-i\|=\left\|\frac{-1}{z-i}\right\|=\frac{\|-1\|}{1}=1$, so AM' = 1 then M' moves on the same circle (ω).	1

II- (4 points)

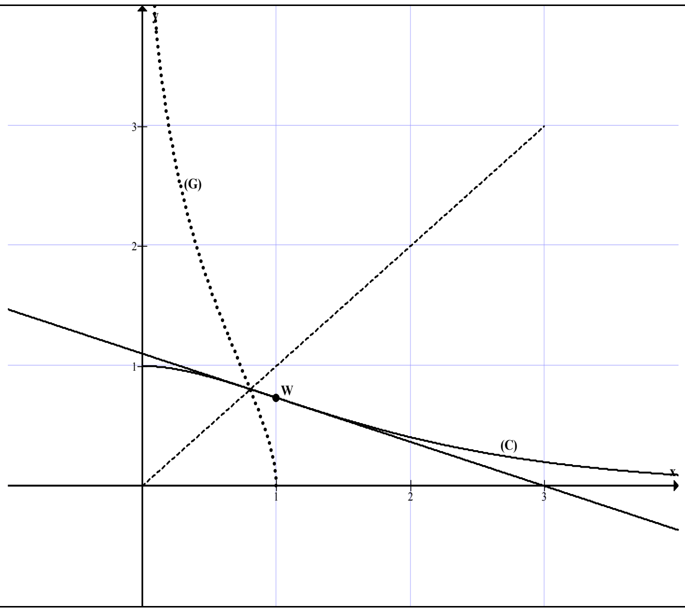
Part of the Q	Answer	Mark
1	The coordinates of A, B and C verify the given equation since: $2x_A+2y_A+z_A-2=2+0+0-2=0$. Also, $2x_B+2y_B+z_B-2=0+2+0-2=0$; and $2x_C+2y_C+z_C-2=0$.	0.5
2.a	$\vec{CH}\left(\frac{4}{9};\frac{4}{9};-\frac{16}{9}\right)$; $\vec{CI}\left(\frac{1}{2};\frac{1}{2};-2\right)$; so, $\vec{CH}=\frac{8}{9}\vec{CI}$, hence C , H and I are collinear.	0.5
2.b	$\vec{n}(2\ ;\ 2;\ 1)$ is a normal vector to plane (ABC), but $\vec{OH}\left(\frac{4}{9};\frac{4}{9};\frac{2}{9}\right)=\frac{2}{9}\vec{n}$, so $\vec{OH}$ is perpendicular to plane (ABC).	0.5

2.c	(OH) is perpendicular to plane (ABC) and (OH) $\subset$(OCI) so the plane (OCI) is perpendicular to plane (ABC). OR : $\vec{n'} = \vec{OI} \wedge \vec{OC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{vmatrix} = \vec{i} - \vec{j}$; n' is normal to plane(OIC) and $\vec{n} \cdot \vec{n'} = 0$	1
3.a	$\vec{OB}(0;1;0)$ is a direction vector of (Δ), and C is a point of (Δ). consequently, (Δ): $\begin{cases} x = 0 \\ y = t \\ z = 2 \end{cases}$	0.5
3.b	(Δ) // (OAB) , so the distance from F to (OAB) is constant hence the volume is constant. area of triangle OAB $= \dfrac{OA \times OB}{2} = 0.5u^2$ and d(F ;OAB) = OC = 2 Consequently V = $\dfrac{0.5 \times 2}{3} = \dfrac{1}{3}u^3$. ►**OR** : calculate $\vec{OF} \cdot \left(\vec{OA} \wedge \vec{OB}\right) = 2$ (independent of t), and V= $\dfrac{\left\lvert \vec{OF} \cdot \left(\vec{OA} \wedge \vec{OB}\right) \right\rvert}{6} = \dfrac{1}{3}u^3$	1

III- (4 points)

Part of the Q	Answer	Mark
A. 1	$P(C \cap R) = P(C).P(R/C) = (0.7)(0.45) = 0.315$	0.5
A. 2	$P(B \cap R) = P(B).P(R/B) = (0.3)(0.8) = 0.24$ $P(R) = P(C \cap R) + P(B \cap R) = 0.315 + 0.24 = 0.555$	1
A. 3	P(C/R)=$\dfrac{P(C \cap R)}{P(R)}$=0.567	0.5
B.1	The values of X are : 0 ; 1 ; 2 ; and 3	0.5
B.2	**If the total number is 30, then there are 21 who use the computer.** $P(X = 0) = \dfrac{C_9^3}{C_{30}^3} = \dfrac{3}{145}$; $P(X = 1) = \dfrac{C_{21}^1 C_9^2}{C_{30}^3} = \dfrac{27}{145}$ $P(X = 2) = \dfrac{C_{21}^2 C_9^1}{C_{30}^3} = \dfrac{27}{58}$; $P(X = 3) = \dfrac{C_{21}^3}{C_{30}^3} = \dfrac{19}{58}$	1.5

IV- (8 points)

Part of the Q	Answer	Mark
1	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{x+1}{e^x} = \lim\limits_{x\to+\infty} \frac{1}{e^x} = 0$, the axis of abscissas is an asymptote to (C) .	0.5
2.a	$f'(x) = -x\,e^{-x}$. x | 0 $+\infty$ f'(x) | 0 □ f(x) | 1 ↘ 0	1
2.b	f '(0) = 0. Then the tangent at A(0 ; 1) is parallel to x-axis.	1
3.a	$f''(x) = (x-1)e^{-x}$; $f''(x)$ vanishes for x = 1 and changes sign; consequently (C) has a point of inflection $W(1, \frac{2}{e})$.	1
3.b	$y - \frac{2}{e} = -\frac{1}{e}(x-1)$ or $y = -\frac{1}{e}x + \frac{3}{e}$	0.5
4		1
5.a	F'(x) = f(x) ; a – b – ax = x + 1 so a = –1 and b = – 2 .	1
5.b	$A = \int_0^1 f(x)dx = \left[(-x-2)e^{-x}\right]_0^1 = (2 - \frac{3}{e})u^2 = 0.896\ u^2 = 0.896 \times 4\ cm^2 = 3.58 cm^2$.	1
6.a	Graph	0.5
6.b	By symmetry of (d) w.r.t first bisector: $x = -\frac{1}{e}y + \frac{3}{e}$ or y = -ex + 3. ►**OR :** $g'(\frac{2}{e}) = \frac{1}{f'(1)} = -e$ **;** an equation of tangent at point $(\frac{2}{e}$; 1) to (G) is : $y - 1 = -e(x - \frac{2}{e})$; y = – ex + 3.	0.5

دورة سنة ٢٠٠٨ العادية	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل: اربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة) .

I- (4 points)

In the complex plane (P) referred to an orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B and C of affixes $a=\sqrt{3}-i$, $b=\sqrt{3}+i$ and $c=2i$ respectively.

1) Show that the three points A, B and C are on the same circle with center O.
2) Write $\frac{c-b}{a-b}$ in the algebraic and in the exponential forms.
3) Let M be a point other than O, of affix z = x + iy, in the plane (P); (x and y are real numbers).

   Let $Z=\frac{z-b}{z}$.

   a- Determine the set (E) of points M such that $|Z|=1$.

   b- Verify that A and C belong to (E).

   c- Determine the set (F) of points M such that Z is pure imaginary.

II- (4 points)

To encourage national tourism, a tourist agency proposes week-ends of two days, and offers its customers three choices:

- Full-board week-end
- Half-board week-end
- Luxury week-end.

The agency published the following advertisement:

Choice / Destination	Full-board	Half-board	Luxury
Mountain	150 000 LL	100 000 LL	200 000 LL
Beach	100 000 LL	75 000 LL	150 000 LL

This agency estimates that 65% of its customers choose mountains, and the others choose the beach; and that out of the customers to any destination 55% choose full-board and 30% choose half-board while the others choose luxury week-ends.

A customer is chosen at random and is interviewed.

Consider the following events:

A : « the interviewed customer has chosen the mountains».

B : « the interviewed customer has chosen the beach ».

C : « the interviewed customer has chosen full-board week-end ».

D : « the interviewed customer has chosen half-board week-end ».

S : « the interviewed customer has chosen the luxury week-end ».

1) a- Calculate the following probabilities: $P(A \cap C)$, $P(B \cap C)$ and $P(C)$.

b- The interviewed customer had chosen full-board, what is the probability that he chose the beach?

2) Let X be the random variable that is equal to the amount paid to the agency by a customer.

a- Show that $P(X=150\ 000) = 0.41$ and determine the probability distribution for X.

b- Calculate the mean(expected value) E(X). What does the number thus obtained represent?

c- Estimate the sum received by this agency when it serves 200 customers.

III- (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the points A(1; 2; 0), B(2; 1; 3), C(3; 3; 1), D(5; –3; –3) and E(–3; 7; 3).

1) Find an equation of the plane (P) determined by A, B and C.
2) Find a system of parametric equations of line (DE).
3) Prove that (P) is the mediator plane of [DE].
4) Prove that (BC) is orthogonal to (DE).
5) a- Calculate the area of triangle BCD.

   b- Calculate the volume of tetrahedron ABCD, and deduce the distance from A to plane BCD.

IV- (8 points)

Let f be the function defined on IR by $f(x) = (x-1)e^x + 1$ and designate by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

1) a- Calculate $\lim\limits_{x \to -\infty} f(x)$ and deduce an asymptote (d) of (C).

   b- Study, according to the values of x, the relative positions of (C) and (d).

   c- Calculate $\lim\limits_{x \to +\infty} f(x)$ and find f(2) in decimal form.

2) Calculate $f'(x)$ and set up the table of variations of f.
3) Prove that the curve (C) has a point of inflection W whose coordinates are to be determined.
4) a- Draw (d) and (C).

   b- Discuss graphically, according to the values of the real parameter m, the number of solutions of the equation $(m-1)\,e^{-x} = x - 1$.

5) Calculate the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.
6) a- Show that the function f has on $[0; +\infty[$ an inverse function g and draw (G) , the representative curve of g in the system $(O ; \vec{i}, \vec{j})$.

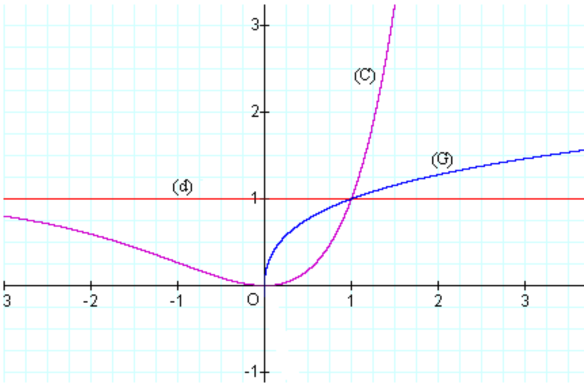
   b- Find the area of the region bounded by (G), the axis of ordinates and the line (d).

Q I	Answer	Mark
1	$\lvert a\rvert=\lvert b\rvert=\lvert c\rvert=2$ so OA = OB= OC = 2.	0.5
2	$\frac{c-b}{a-b}=\frac{-\sqrt{3}+i}{-2i}=\frac{-1-i\sqrt{3}}{2}=e^{i\left(\pi+\frac{\pi}{3}\right)}=e^{-i\frac{2\pi}{3}}$.	1
3a	$\lvert Z\rvert=1$, iff BM = OM , so M moves on the perpendicular bisector (E) of [OB].	1
3b	AB = AO and CB = CO , so A and C are two points on (E).	0.5
3c	$Z=\frac{x+iy-\sqrt{3}-i}{x+iy}=\frac{x^2+y^2-\sqrt{3}x-y}{x^2+y^2}+\frac{-x+\sqrt{3}y}{x^2+y^2}i$ Z is pure imaginary iff $\begin{cases}x^2+y^2-\sqrt{3}x-y=0\\ -x+\sqrt{3}y\neq 0\end{cases}$ M moves on a circle excluding O and B. **Or**: arg (Z) $=\frac{\pi}{2}[\pi]=(\vec{u},\overrightarrow{BM})-(\vec{u},\overrightarrow{OM})=\left(\overrightarrow{OM},\overrightarrow{BM}\right)[\pi]$. So M moves on the circle (F) with diameter [OB], excluding O and B.	1

Q II	Answer	Mark
1a	$P(A\cap C)=P(A)\times P(C/A)=0.65\times 0.55=0.3575$ $P(B\cap C)=P(B)\times P(C/B)=0.35\times 0.55=0.1925$ $P(C)=P(A\cap C)+P(B\cap C)=0.3575+0.1925=0.55$ **OR** given *P(C) = 0.55.*	1
1b	$P(B/C)=\frac{P(B\cap C)}{P(C)}=\frac{0.1925}{0.55}=0.35$.	0.5
2a	P(X=150 000)= O.65×0.55 +0.15×0.35=0.41. (see table below)	1.5
2b	$E(X)=\sum p_i x_i=0.105\times 75000+0.3875\times 100000+0.41\times 150000+0.0975\times 200000=127\ 625$ The average amount paid by a voyager is 127 625 LL.	0.5
2c	An estimation of the sum received is: 127625×200 = 25 525 000 LL.	0.5

x_i	75000	100 000	150 000	200 000
p_i	0.35×0.3=0.105	0.35×0.55+0.65×0.3=0.3875	0.41	0.65×0.15 = 0.0975

QIII	Answer	Mark
1	$\vec{N}=\overrightarrow{AB}\wedge\overrightarrow{AC}$ (4;-5;-3) is normal to (P) ; (P) : $\overrightarrow{AM}.\vec{N}=0$. Hence 4x – 5y – 3z + 6 = 0.	0.5
2	(DE): x = – 8t + 5 ; y =10t –3 ; z = 6t –3.	0.5
3	A director vector of (DE) and a normal vector of (P) have the same direction; Mid point (1; 2; 0) of [DE] belongs to (P).	1
4	$\vec{DE}$ (– 8 ; 10 ; 6) . $\vec{BC}$ (1 ; 2 ; –2) = 0 . ►**OR** since (DE) is perpendicular to plane (P).	0.5
5a	$\overrightarrow{DB}\wedge\overrightarrow{DC}$ (– 20 ; 0 ; –10), area = $\frac{1}{2}\sqrt{500}=\sqrt{125}=5\sqrt{5}$.	0.5
5b	Volume = $\frac{1}{6}\left\lvert\overrightarrow{DA}.(\overrightarrow{DB}\wedge\overrightarrow{DC})\right\rvert=\frac{50}{6}=\frac{25}{3}$. V= $\frac{\text{base}\times h}{3}$, $\frac{25}{3}=\frac{5\sqrt{5}\ h}{3}$, so h = $\sqrt{5}$.	1

Q IV	Answer	Mark
1a	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} (xe^x - e^x + 1) = 1$, so the line with equation $y = 1$ is asymptote to (C) .	0.5
1b	$f(x) - 1 = (x - 1)e^x$. □ (C) cuts (d) at point (1 ; 1) □ For $x > 1$, (C) is above (d) □ For $x < 1$, (C) is below (d).	0.5
1c	$\lim\limits_{x\to+\infty} f(x) = +\infty$ and $f(2) = 8.389$.	0.5
2	$f'(x) = e^x + (x - 1)e^x = xe^x$. $\begin{array}{c\|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 1 & \searrow & 0 & \nearrow & +\infty \end{array}$	1
3	$f''(x) = (x + 1)e^x$; $f''(x)$ vanishes for $x = -1$ and changes signs, thus (C) has a point of inflection $W(-1, 1 - \frac{2}{e})$.	0.5
4a		1.5
4b	$(m - 1)e^{-x} = x - 1$ gives $m = (x - 1)e^x + 1$. □ For $m < 0$; no solution □ For $m = 0$; one solution (double) □ For $0 < m < 1$; two solutions □ For $m \geq 1$; single solution.	1
5	$A = \int_0^1 [(x-1)e^x + 1]dx = \left[(x-2)e^x + x\right]_0^1 = (3 - e)\,u^2$.	1
6a	f is continuous and strictly increasing on $[0 ; +\infty[$, thus f has an inverse function g. (G) is symmetric of (C) wrt the line of equation $y=x$.	1
6b	The area A' of the region bounded by (G), the axis of ordinates and the line (d) is equal (by symmetry) to the area A of the region bounded by (C), the axis of abscissas and the two lines $x = 0$ and $x = 1$, consequently $A' = A = (3 - e)u^2$.	0.5

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع: علوم الحياة	دورة سنة ٢٠٠٨ الاكمالية الاستثنائية
عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I- (4 points)

In the following table, only one of the proposed answers to each question is correct.
Write the number of each question and give, with justification, the corresponding answer.

Nº	Questions	Answers a	Answers b	Answers c	Answers d
1	If $\frac{\pi}{6}$ is an argument of z, then an argument of $\frac{i}{\bar{z}^2}$ is :	$-\frac{\pi}{6}$	$\frac{\pi}{6}$	$-\frac{5\pi}{6}$	$\frac{5\pi}{6}$
2	If $z = -\sqrt{3} + e^{i\frac{\pi}{6}}$, then the exponential form of z is:	$e^{\frac{5\pi}{6}i}$	$e^{\frac{7\pi}{6}i}$	$\sqrt{3}e^{-\frac{\pi}{6}i}$	$e^{-\frac{5\pi}{6}i}$
3	If z and z' are two complex numbers such that $\lvert z \rvert = 2$ and $z' = z - \frac{1}{\bar{z}}$, then $\lvert z' \rvert =$	1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
4	If z is a complex number with $\lvert z \rvert = \sqrt{2}$, then $\lvert \bar{z} + i\bar{z} \rvert =$	$2\sqrt{2}$	2	$\sqrt{2}$	$\frac{\sqrt{2}}{2}$

II- (4 points)

In the space referred to an orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the points:
A(0 ; 1 ; – 2), B(2 ; 1; 0), C(3 ; 0; – 3) and H(2 ; 2 ; – 2).

1) Show that $x - 2y - z = 0$ is an equation of the plane (P) determined by the points H , A and B and verify that the point C does not belong to this plane.

2) a- Show that triangle HAB is isosceles of vertex H.
b- Show that (CH) is perpendicular to (P).
c- Prove that CA = CB and determine a system of parametric equations of the interior bisector (δ) of angle ACB.

3) Let T be the orthogonal projection of H on plane (ABC).
Prove that T belongs to (δ) .

III- (4 points)

In order to prevent a certain disease, we vaccinated 40% of persons of a population.
Then we noticed that 85% of the vaccinated persons were not affected by the disease and that 75% of the persons who were not vaccinated are affected by the disease.
A person is chosen randomly from this population.
Consider the following events:

D : « the chosen person is affected by the disease».
V : « the chosen person is vaccinated ».

1) a- Verify that the probability of the event $D \cap V$ is equal to $\frac{6}{100}$.

b- What is the probability that the chosen person is affected by the disease and is not vaccinated?

c- Deduce the probability P(D).

2) The chosen person is not affected by the disease.
Calculate the probability that he/she is vaccinated.

3) In this part, suppose that this population is formed of 300 persons.
We choose randomly 3 persons from this population.
What is the probability that at least one, among the 3 chosen persons, is vaccinated?

IV- (8 points)

Let f be the function defined over $]1;+\infty[$ by $f(x) = x - \frac{1}{x \ln x}$ and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim\limits_{x \to 1} f(x)$ and deduce an asymptote to (C).

2) Calculate $\lim\limits_{x \to +\infty} f(x)$. Prove that the straight line (d) of equation $y = x$ is an asymptote to (C) and study the position of (C) and (d).

3) Calculate $f'(x)$ and show that f is strictly increasing.
Set up the table of variations of f.

4) Show that the equation $f(x) = 0$ has a unique root α and verify that $1.5 < \alpha < 1.6$.

5) Draw (d) and (C).

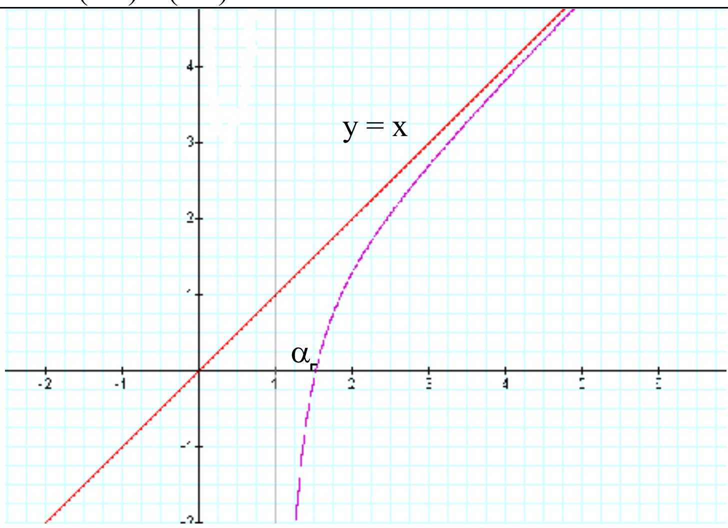
6) a- Calculate the area $A(t)$ of the region limited by the curve (C), the straight line (d) and the two straight lines of equations $x = e$ and $x = t$ where $t > e$.

b- Show that for all $t > e$, we have $A(t) < t$.

QI	Answer	Mark
1	$\arg\left(\dfrac{i}{\bar{z}^2}\right) = \arg(i) - 2\arg(\bar{z})\ [2\pi] = \dfrac{\pi}{2} + 2\left(\dfrac{\pi}{6}\right)\ [2\pi] = \dfrac{5\pi}{6}\ [2\pi]$ **d**	1
2	$z = -\sqrt{3} + \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i = -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i = e^{i\left(\frac{5\pi}{6}\right)}$ **a**	1
3	$z' = \dfrac{z\bar{z} - 1}{\bar{z}} = \dfrac{\lvert z\rvert^2 - 1}{\bar{z}} = \dfrac{3}{\bar{z}}$, so $\lvert z'\rvert = \dfrac{3}{\lvert z\rvert} = \dfrac{3}{2}$ **c**	1
4	$\lvert \bar{z} + i\bar{z}\rvert = \lvert \bar{z}(1+i)\rvert = \lvert\bar{z}\rvert \times \lvert 1+i\rvert = \sqrt{2}\times\sqrt{2} = 2$ **b**	1

QII	Answer	Mark
1	$x_A - 2y_A - z_A = 0 - 2 + 2 = 0$; $x_B - 2y_B - z_B = 2 - 2 - 0 = 0$; $x_H - 2y_H - z_H = 2 - 4 + 2 = 0$, then $x - 2y - z = 0$ is an equation of the plane (P) determined by A , B and H. $x_C - 2y_C - z_C = 3 - 0 + 3 \neq 0$,then C does not belong to (P).	1
2a	$\overrightarrow{HA}(-2;-1;0)$; $\overrightarrow{HB}(0;-1;2)$ then $HA = HB = \sqrt{5}$.	0.5
2b	$\overrightarrow{HC}(1;-2;-1) = \overrightarrow{N}_{(P)}$ then (CH) is perpendicular to (P).	0.5
2c	Triangles AHC and BHC are congruent so CA = CB and triangle ABC is isosceles of vertex C (or CA = CB = $\sqrt{11}$) hence, the bisector of angle $A\hat{C}B$ is the median relative to the side [AB]. I(1 ; 1 ; – 1) is the midpoint of [AB]; $\overrightarrow{CI}(-2;1;2)$ is a direction vector of (δ) and $C \in (\delta)$. Thus, a system of parametric equations of (δ) is : $x = -2m + 3$; $y = m$ and $z = 2m - 3$.	1
3	(CH) is perpendicular to plane (P) then (CH) is orthogonal to the straight line (AB) in (P) ; the straight line (AB) being orthogonal to (CI) and (CH) , then (AB) is perpendicular to plane (CHI) , consequently planes (ABC) and (CHI) are perpendicular , Therefore T the foot of the perpendicular through H to plane (ABC) belongs to the straight line (CI) = (δ) , intersection of the two planes. •OR: $\overrightarrow{AB} \times \overrightarrow{AC} = 2\vec{i} + 8\vec{j} - 2\vec{k}$ Then, plane (ABC) has an equation: $2x + 8y - 2z - 12 = 0$ $(HT): \begin{cases} x = 2t + 2 \\ y = 8t + 2 \\ z = -2t - 2 \end{cases}$ $(HT) \cap (ABC) = \{T\}$ then $T\left(\dfrac{5}{3}, \dfrac{2}{3}, -\dfrac{5}{3}\right)$ T belongs to (δ) for $m = \dfrac{2}{3}$.	1

QIII	Answer	Mark
1a	$P(M\cap V)=P(V)\times P(M/V)=\dfrac{40}{100}\times\dfrac{15}{100}=\dfrac{6}{100}$.	0.5
1b	$P(M\cap\overline{V})=P(\overline{V})\times P(M/\overline{V})=\dfrac{60}{100}\times\dfrac{75}{100}=\dfrac{45}{100}$.	0.5
1c	$P(M)=P(M\cap V)+P(M\cap\overline{V})=\dfrac{6}{100}+\dfrac{45}{100}=\dfrac{51}{100}$.	1
2	$P(V/\overline{M})=\dfrac{P(V\cap\overline{M})}{P(\overline{M})}=\dfrac{\frac{40}{100}\times\frac{85}{100}}{1-\frac{51}{100}}=\dfrac{34}{49}$.	1
3	Let A be the event : « at least one is vaccinated among the three persons » $P(A)=1-P(\overline{A})=1-\dfrac{C_{180}^{3}}{C_{300}^{3}}=0.785.$	1

QIV	Answer	Mark
1	$\lim\limits_{x\to1^+} f(x)=1-\infty=-\infty$: the straight line of equation $x=1$ is an asymptote to (C).	0.5
2	$\lim\limits_{x\to+\infty} f(x)=+\infty-0=+\infty$; $\lim\limits_{x\to+\infty}[f(x)-x]=0$, then the straight line (d) of equation $y=x$ is an asymptote to (C). $f(x)-x=-\dfrac{1}{x\ln x}<0$, so (C) is below (d).	1
3	$f'(x)=1+\dfrac{\ln x+1}{x^2\ln^2 x}>0$ for $x>1$, then f is strictly increasing . $\begin{array}{c\|ccc} x & 1 & & +\infty \\ \hline f'(x) & \| & + & \\ \hline f(x) & \| -\infty & \nearrow & +\infty \end{array}$	1.5
4	f is continuous and strictly increasing and f(x) increases from $-\infty$ to $+\infty$ then the equation $f(x)=0$ has a unique root α. and $f(1.5)\times f(1.6)=-0.14\times0.27<0$ then $1.5<\alpha<1.6$.	1
5		1.5

6a	$A(t) = \int_e^t [x - f(x)].dx = \int_e^t \frac{1}{x\ln x}.dx = \int_e^t \frac{(\ln x)'}{\ln x}.dx = [\ln(\ln x)]_e^t = \ln(\ln t) - \ln(\ln e) = \ln(\ln t).$	1.5
6b	$A(t) < t$ if $\ln(\ln t) < t$; $\ln t < e^t$ which is true since the representative curve of the ln function is below that of the exponential function.	1

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الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل : أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points M and M' of respective affixes z and z', where $z' = \left(1+i\sqrt{3}\right)z$.

1) In this part, suppose that z = 2i.
  a- Determine the exponential form of z'.
  b- Calculate $\left|\frac{z'}{z}\right|$ and $\arg\left(\frac{z'}{z}\right)$.
  c- Show that triangle OMM' is right angled at M.

2) Assume in this part that $z = (1+i)^3$.
  a- Write the exponential form and the algebraic form of z.
  b- Write the exponential form and the algebraic form of z'.
  c- Deduce the exact value of $\cos\frac{13\pi}{12}$.

II- (4 points)

Consider two bags **B₁** and **B₂** such that:
B_1 contains **six** cards numbered 1, 2, 3, 4, 5, 6.
B_2 contains **five** cards numbered 0, 1, 2, 4, 5.

A-

One card is drawn randomly from bag **B_1**:
- if it carries one of the numbers 1 or 2, then **three** cards are drawn randomly and simultaneously from bag **B_2**.
- But if it carries one of the numbers 3, 4, 5 or 6, then **two** cards are drawn randomly and simultaneously from bag **B_2**.

Consider the following events:

K: « the card drawn from bag **B_1** carries the one of the numbers 1 or 2 ».
L: « the card drawn from bag **B_1** carries the one of the numbers 3, 4, 5 or 6 ».
E: « The product of numbers shown on the cards drawn from bag **B_2** is zero ».

1) a- Calculate the probabilities p(K) and p(L).
   b- Show that $p(E \cap K) = \frac{1}{5}$.
   c- Calculate $p(E \cap L)$ and deduce p(E).
2) Knowing that the product of the numbers shown on the cards drawn from bag **B_2** is zero, calculate the probability that **three c**ards were drawn from **B_2**.

B-

In this part we use only the bag **B_2** and **three** cards are drawn randomly and simultaneously from this bag.
Let X be the random variable that is equal to the biggest number among those shown on the **three** drawn cards, thus the possible values of X 2, 4 and 5.

Prove that $p(X=4) = \frac{3}{10}$, and determine the probability distribution of X.

III- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the point A(1;0;1) and the two planes (P) and (Q) with equations $2x - y - 2 = 0$ and $x + 2y - z = 0$ respectively.

1) a- Verify that point A is a common to (P) and (Q).
   b- Determine a system of parametric equations of (d), the line intersection of (P) and (Q).

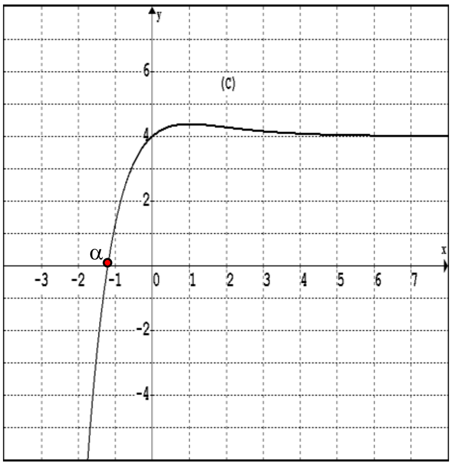
2) a- Determine a system of parametric equations of the line (D) that is perpendicular to (P) at A.
   b- Determine the coordinates of a point E on (D) such that $AE = \sqrt{5}$.

3) a- Show that the points B(0;–2;0) and C(2; 2;t) belong to (P). (t is a real number).
   b- Calculate t so that the triangle ABC is right at B and find in this case the volume of the tetrahedron EABC.

III- (8 points)

A- Consider the function f defined on IR by $f(x) = 4 + x e^{-x}$ whose representative curve (C) is shown in the adjacent figure.

(C) cuts the axis of abscissas in one point of abscissa α.



1) Use (C) to study the sign of f(x).

2) Use integration by parts to calculate $\int_0^2 x e^{-x}dx$, then calculate the area of the region bounded by the axis of ordinates, the axis of abscissas, the curve (C) and the straight line with equation x = 2 .

B- In all what follows, let $\alpha = -1.2$.

Consider the function g defined on IR, by $g(x) = 4x - 3 - (x+1)e^{-x}$ and designate by (G) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Verify that $\lim\limits_{x\to-\infty} g(x) = +\infty$ and determine g(-2.5) to the nearest 10^{-2}.

2) Calculate $\lim\limits_{x\to+\infty} g(x)$ and verify that the straight line (D) with equation y = 4x-3 is an asymptote of (G).

3) Determine the coordinates of A, the point of intersection of (G) with its asymptote (D), and study the position of (G) with respect to (D).

4) a- Verify that $g'(x) = f(x)$.
   b- Set up the table of variations of g.

5) Draw (D) and (G).

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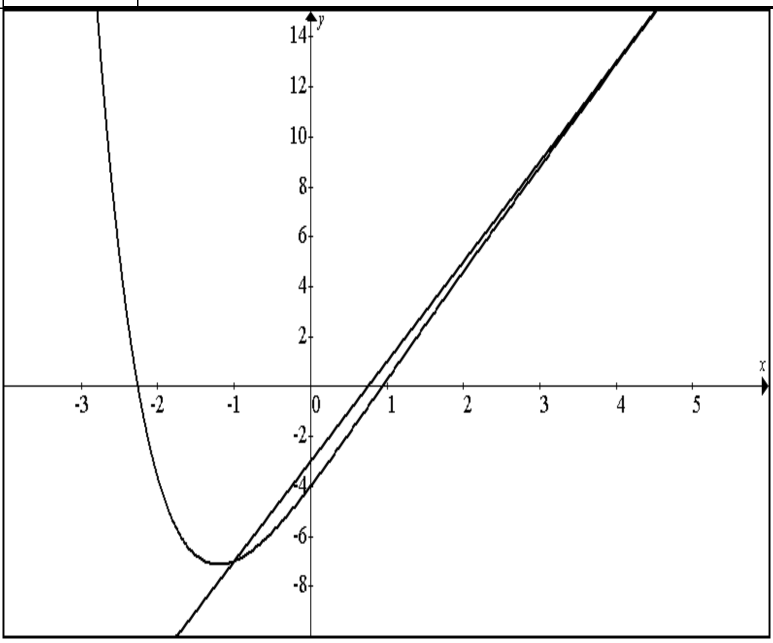
	مسابقة في مادة الرياضيات المدة ساعتان	مشروع معيار التصحيح

QI	Answer	M
1a	$1+i\sqrt{3}=2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=2e^{i\frac{\pi}{3}}$;$2i=\ 2e^{i\frac{\pi}{2}}$; $z'=4e^{i\frac{5\pi}{6}}$.	0.5
1b	$\left\|\frac{z'}{z}\right\|=2$; $\arg\left(\frac{z'}{z}\right)=\frac{\pi}{3}[2\pi]$.	0.5
1c	OM=$\|z\|=2$; $OM'=\|z'\|=4$; $MM'=\|z'-z\|=2\sqrt{3}$ so $OM'^2=OM^2+MM'^2$ **OR** : $z'=(1+i\sqrt{3})2i=-2\sqrt{3}+2i$ M and M' have the same ordinate and M belongs to y-axis so OMM' is right at M.	1
2a	$(1+i)^3=2\sqrt{2}\ e^{i\frac{3\pi}{4}}$; $(1+i)^3=-2+2i$	0.5
2b	Exponential form: $z'=(1+i\sqrt{3})(1+i)^3=2e^{i\frac{\pi}{3}}\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^3=4\sqrt{2}e^{i\left(\frac{\pi}{3}+\frac{3\pi}{4}\right)}=4\sqrt{2}e^{i\left(\frac{13\pi}{12}\right)}$. Algebraic form: $z'=\left(1+i\sqrt{3}\right)(1+i)^2(1+i)=\left(1+i\sqrt{3}\right)(2i-2)=2\left(-1-\sqrt{3}\right)+2\left(1-\sqrt{3}\right)i$	0.5
2c	Comparing the exponential and the algebraic forms of z': $4\sqrt{2}\cos\left(\frac{13\pi}{12}\right)=2\left(-1-\sqrt{3}\right)$, hence $\cos\frac{13\pi}{12}=\frac{-(\sqrt{2}+\sqrt{6})}{4}$	1

QII	Answer	M
A1a	$p(K)=\frac{2}{6}=\frac{1}{3}$; $p(L)=\frac{2}{3}$	0.5
A1b	$p(E\cap K)=p(K)\times p(E/K)=\frac{1}{3}\times\frac{C_1^1\times C_4^2}{C_5^3}=\frac{1}{5}.$	0.5
A1c	$p(E\cap L)=p(L)\times p(E/L)=\frac{2}{3}\times\frac{C_1^1\times C_4^1}{C_5^2}=\frac{4}{15}.$ $p(E)=p(E\cap K)+p(E\cap L)=\frac{1}{5}+\frac{4}{15}=\frac{7}{15}.$	1
A2	$p(K/E)=\frac{p(E\cap K)}{p(E)}=\frac{\frac{1}{5}}{\frac{7}{15}}=\frac{3}{7}.$	0.5
B	The number of possible cases is $C_5^3=10$ $p(X=4)=p(0,\ 1,\ 4\ \text{ or }\ 0,\ 2,\ 4\ \text{ or }\ 1,\ 2,\ 4)=\frac{3}{10}.$ see table below	1.5

$X=x_i$	2	4	5
p_i	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{6}{10}$

QIII	Answer	M
1a	2-0-2=0 and 1+0-1=0.	0.5
1b	(d): x = m+1; y=2m; z = 5m+1	0.5
2a	$(D):\ x=2t+1\ ;\ y=-t\ ;\ z=1$	0.5
2b	$\overrightarrow{AE}(2t,-t,0)$; $AE=\sqrt{5t^2}=\sqrt{5}$, hence $t=\pm1$. For t = 1, E(3,-1,1).	0.5
3a	0+2-2=0; 4-2-2=0 then B and C belong to P.	0.5
3b	$\overrightarrow{AB}(-1,-2,-1)$; $\overrightarrow{BC}(2,4,t)$; $\overrightarrow{AB}.\overrightarrow{BC}=0$ so $t=-10$ Area of ABC = $\frac{\sqrt{6}\times\sqrt{120}}{2}=6\sqrt{5}$ Volume of EABC = $\frac{\text{area}(ABC)\times EA}{3}=\frac{6\sqrt{5}\times\sqrt{5}}{3}=10u^3.$ ***OR*** Calculate the mixed product	1.5

QIV	Answer	M
A1	$f(x) = 0$ for $x = \alpha$; $f(x) > 0$ for $x > \alpha$; $f(x) < 0$ for $x < \alpha$	0.5
A2	Let $U = x; V' = e^{-x}$ then $U' = 1; V = -e^{-x}$ $\quad \int_0^2 xe^{-x}dx = \left[-xe^{-x}-e^{-x}\right]_0^2 = -3e^{-2}+1$ Area = $\int_0^2 4dx + \int_0^2 xe^{-x}dx = \left[4x\right]_0^2 + \left[-xe^{-x}-e^{-x}\right]_0^2 = (-3e^{-2}+9) \quad u^2$	1.5
B1	$\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} \frac{4xe^x - 3e^x - x - 1}{e^x} = +\infty \quad \lim\limits_{x\to-\infty} 4xe^x = 0.$ and $\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} \frac{-x-1}{e^x}$ g(-2.5)=5.27.	1
B2	$\lim\limits_{x\to+\infty} g(x) = \lim\limits_{x\to+\infty} (4x - 3 - xe^{-x} - e^{-x}) = +\infty \quad \lim\limits_{x\to+\infty} xe^{-x} = 0$ $\lim\limits_{x\to+\infty} (g(x)-(4x-3)) = \lim\limits_{x\to+\infty} -(x+1)e^{-x} = \lim\limits_{x\to+\infty} (-xe^{-x} - e^{-x}) = 0$ then the straight line with equation y = 4x - 3 is an asymptote of (G).	1
B3	$g(x) - (4x-3) = -(x+1)e^{-x}$ (G) intersects (D) at x = -1 thus A(-1;-7) If $x < -1$, $-(x+1)e^{-x} > 0$, then (G) is above (D) If $x > -1$, $-(x+1)e^{-x} < 0$, then (G) is below (D)	1
B4a	$g'(x) = 4 - e^{-x} + (x+1)e^{-x} = 4 + xe^{-x} = f(x)$	0.5
B4b	$\begin{array}{c\|ccccc} x & -\infty & & -1.2 & & +\infty \\ \hline g'(x) & & - & 0 & + & \\ \hline g(x) & +\infty & \searrow & -7.1 & \nearrow & +\infty \end{array}$	1
B5		1.5

الدورة الإستثنائية للعام 2009	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة: ساعتان	عدد المسائل: اربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the plane (P) of equation: $x - y + z + 2 = 0$, and the two straight lines (D) and (D') defined by the parametric equations:

$$(D)\begin{cases} x = t \\ y = -t+1 \\ z = 2t-1 \end{cases} \quad \text{and} \quad (D')\begin{cases} x = -5m-10 \\ y = 5m+11 \\ z = -2m-5 \end{cases}$$ where t and m are real parameters.

1) Show that (D) and (D') intersect at the point A(0; 1; –1) and verify that A belongs to plane (P).

2) Write an equation of the plane (Q) that contains the two straight lines (D) and (D').

3) Determine a system of parametric equations of the straight line (d), the intersection of (P) and (Q) .

4) Verify that the point B(1; 0; –3), which is on the straight line (d), is equidistant from the two straight lines (D) and (D'), and deduce that (d) is a bisector of the angle between (D) and (D').

II- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, M and M' of respective affixes 2, $-i$, z and z' where $z' = \dfrac{iz-1}{z-2}$. $(z \neq 2)$.

1) Find the coordinates of M when $z' = 1+2i$.

2) Give a geometric interpretation for $|z-2|$ and for $|iz-1|$ and determine the set of points M such that $|z-2| = |iz-1|$.

3) Let $z = x + iy$ and $z' = x' + iy'$ (x, y, x' and y' are real numbers).

a- Calculate x' and y' in terms of x and y.

b- Show that if z' is pure imaginary, then M moves on a straight line whose equation is to be determined.

c- Show that if z is real, then M' moves on a straight line whose equation is to be determined.

III- (4 points)

The following table represents the distribution of the ages of 26 men and 24 women.

Age in years	$[20;25[$	$[25;30[$	$[30;35]$
Number of men	8	8	10
Number of women	5	9	10

3 persons are randomly chosen, from these 50 people, to form a committee.
Consider the following events:

M: « the committee is formed of three men ».
F : « the committee is formed of three women ».
A: « the committee is mixed (formed of men and women) ».
B: « the age of each member of the committee is less than 30 years ».

1) Calculate each of the probabilities p(M), p(F) and p(A).

2) a- Calculate p(B) and show that $p(B \cap \bar{A}) = \frac{33}{700}$. Deduce $p(B \cap A)$.

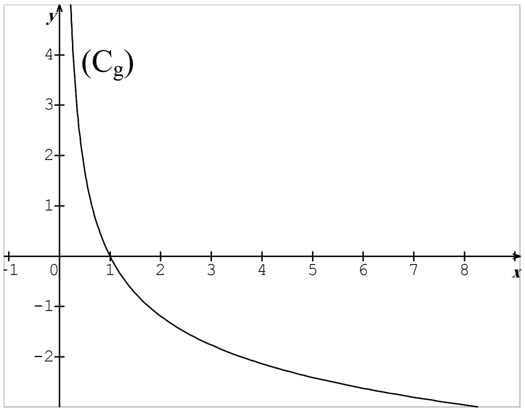
b- Calculate $p(B/A)$.

3) Designate by X the random variable that is equal to the number of women in the committee who have an age less than 25 years.
Determine the probability distribution of X.

IV- (8 points)

Consider the function f defined, on $]0;+\infty[$, by $f(x) = \frac{1+\ln x}{e^x}$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

The curve (C_g), shown in the adjacent figure, is the representative curve in an orthonormal system, of the function g defined on $]0;+\infty[$ by $g(x) = \frac{1}{x} - 1 - \ln x$.



1) Calculate the area of the region bounded by the curve (C_g), the axis of abscissas and the line of equation x = 2.

2) Show that $f'(x) = \frac{g(x)}{e^x}$ and deduce the sign of $f'(x)$ according to the values of x.

3) Calculate $\lim_{x \to 0} f(x)$ and $\lim_{x \to +\infty} f(x)$ and determine the asymptotes of the curve (C).

4) Set up the table of variations of f.

5) Solve the equation f(x) = 0.

6) Find an equation of the tangent to the curve (C) at the point of abscissa $\frac{1}{e}$.

7) Draw (C).

8) Discuss, according to the values of the real number m, the number of solutions of the equation $\ln x = me^x - 1$.

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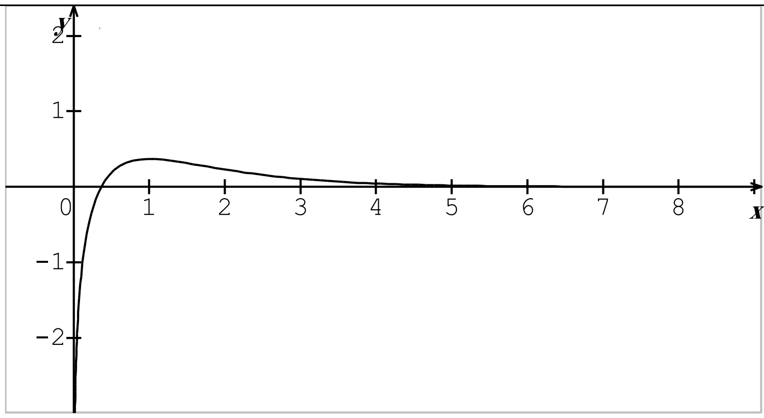
	مسابقة في مادة الرياضيات	مشروع معيار التصحيح

QI	Corrigé	Note
1	x = 0 , donc t = 0 ; y =1 et z = -1 et m= -2 ; y =1 et z = -1. A(0,1,-1) est le point d'intersection des deux droites. (P) : $x - y + z + 2 = 0$. $0 - 1 - 1 + 2 = 0$, donc A appartient au plan (P).	1
2	$\overrightarrow{V_{(D)}}(1,-1,2)$ $\overrightarrow{V_{(D')}}(-5,5,-2)$ et $A(0,1,-1)$ $M(x,y,z)$ est un point du plan (Q) si et seulement si $\det(\overrightarrow{AM},\overrightarrow{V_{(D)}},\overrightarrow{V_{(D')}}) = \begin{vmatrix} x & y-1 & z+1 \\ 1 & -1 & 2 \\ -5 & 5 & -2 \end{vmatrix} = -8x-8y+8 = 0$ une equation de (Q) : $x+y-1=0$	1
3	(P): $x - y + z + 2 = 0$ et (Q): $x + y - 1 = 0$. $(d)\begin{cases} x = \alpha \\ y = -\alpha+1 \\ z = -2\alpha-1 \end{cases}$	0.5
4	B (1 , 0 , –3) appartient à (d) et A(0 , 1 , - 1) appartient à (D) $\overrightarrow{BA}\wedge\overrightarrow{V_{(D)}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ 1 & -1 & 2 \end{vmatrix} = 4\vec{i}+4\vec{j}$. $d(B,(D)) = \dfrac{\lVert \overrightarrow{BA}\wedge\overrightarrow{V_{(D)}} \rVert}{\lVert \overrightarrow{V_{(D)}} \rVert} = \dfrac{\sqrt{32}}{\sqrt{6}} = \sqrt{\dfrac{16}{3}}$. A(0 , 1 , - 1) appartient à (D') , $\overrightarrow{BA}\wedge\overrightarrow{V_{(D')}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ -5 & 5 & -2 \end{vmatrix} = -12\vec{i}-12\vec{j}$ $d(B,(D')) = \dfrac{\lVert \overrightarrow{BA}\wedge\overrightarrow{V_{(D')}} \rVert}{\lVert \overrightarrow{V_{(D')}} \rVert} = \dfrac{\sqrt{288}}{\sqrt{54}} = \sqrt{\dfrac{16}{3}}$. donc B est équidistant de (D) et (D'). A est l'intersection de(D) et (D'), donc A est équidistant de (D) et (D') La droite (d) est contenue dans le plan (Q) et passe par A et B donc (d) est une bissectrice de l'angle de (D) et (D').	1.5

QII	Corrigé	Note
1	$1 + 2i = \dfrac{iz-1}{z-2}$; $(1 + i)z = 1 + 4i$; $z = \dfrac{5}{2} + \dfrac{3}{2}i$; $M(\dfrac{5}{2}\ ;\ \dfrac{3}{2})$.	0.5
2	$\lvert z-2\rvert = AM$ et $\lvert iz-1\rvert = \lvert i.(z+i)\rvert = \lvert i\rvert.\lvert z-(-i)\rvert = \lvert z-(-i)\rvert = BM$. $\lvert z-2\rvert = \lvert iz-1\rvert$; AM=BM ; l'ensemble des points M est la médiatrice du segment [AB].	1
3a	$x' + iy' = \dfrac{-y-1+ix}{x-2+iy} = \dfrac{2y-x+2+i(x^2+y^2-2x+y)}{(x-2)^2+y^2}$ $x' = \dfrac{2y-x+2}{(x-2)^2+y^2}$ et $y' = \dfrac{x^2+y^2-2x+y}{(x-2)^2+y^2}$	0.5
3b	z' est imaginaire pur si x'= 0 et $z' \neq 0$; $2y - x + 2 = 0$ avec $z \neq -i$ et $z \neq 2$. M se déplace sur la droite (d) d'équation : $y = \dfrac{x}{2} - 1$	1
3c	z est un réel si y = 0 : $x' = \dfrac{-1}{x-2}$ et $y' = \dfrac{x}{x-2}$; $y' = -2x'+1$ et M' se déplace sur la droite d'équation : $y = -2x + 1$.	1

QIII.	Corrigé	Note
1	$P(M) = \dfrac{C_{26}^3}{C_{50}^3} = \dfrac{2600}{19600} = \dfrac{13}{98}$. ; $P(F) = \dfrac{C_{24}^3}{C_{50}^3} = \dfrac{2024}{19600} = \dfrac{253}{2450}$. $P(A) = 1 - P(M) - P(F) = \dfrac{936}{1225} = 0{,}764$.	1
2a	$P(B) = \dfrac{C_{30}^3}{C_{50}^3} = 0{,}207$. ; $P(B \cap \overline{A}) = \dfrac{C_{16}^3 + C_{14}^3}{C_{50}^3} = \dfrac{33}{700} = 0{,}047$. $p(B \cap \overline{A}) = 0{,}047$, $p(B) = p(B \cap A) + p(B \cap \overline{A})$ Donc $p(B \cap A) = 0{,}207 - 0{,}047 = 0{,}16$.	1.5
2b	$p(B/A) = \dfrac{p(B \cap A)}{p(A)} = 0{,}21$.	0.5
3	Les valeurs possibles de X : 0,1,2,3 (voir tableau ci-dessous)	1

$X = x_i$	0	1	2	3
p_i	$\dfrac{C_5^0 \times C_{45}^3}{C_{50}^3} = \dfrac{1419}{1960}$	$\dfrac{C_5^1 \times C_{45}^2}{C_{50}^3} = \dfrac{99}{392}$	$\dfrac{C_5^2 \times C_{45}^1}{C_{50}^3} = \dfrac{9}{392}$	$\dfrac{C_5^3}{C_{50}^3} = \dfrac{1}{1960}$

<table>
<tr><th>QIV</th><th>Corrigé</th><th>Note</th></tr>
<tr><td>1</td><td>$A = -\int_1^2 g(x)dx = -\left[\ln x - x - x\ln x + x\right]_1^2 = \ln 2$ u.a.</td><td>1</td></tr>
<tr><td>2</td><td>$f'(x) = \dfrac{\frac{1}{x}e^x - e^x(1+\ln x)}{(e^x)^2} = \dfrac{g(x)}{e^x}$ et $e^x > 0$ alors le signe de $f'(x)$ est celui de g(x) donc $f'(x) > 0$ pour $0 < x < 1$, $f'(1) = 0$ et $f'(x) < 0$ pour $x > 1$</td><td>1.5</td></tr>
<tr><td>3</td><td>$\lim\limits_{x\to 0^+} f(x) = \dfrac{-\infty}{1} = -\infty$ alors y´y est une asymptote de (C).
$\lim\limits_{x\to+\infty} f(x) = \dfrac{+\infty}{+\infty}$ ind. (Hop) = $\lim\limits_{x\to+\infty} \dfrac{1}{xe^x} = \dfrac{1}{+\infty} = 0$ alors y = 0 est une asymptote de (C).</td><td>1</td></tr>
<tr><td>4</td><td><table>
<tr><td>x</td><td>0</td><td></td><td>1</td><td></td><td>$+\infty$</td></tr>
<tr><td>$f'(x)$</td><td>||</td><td>+</td><td>0</td><td>−</td><td></td></tr>
<tr><td>$f(x)$</td><td>|| $-\infty$</td><td>↗</td><td>$\frac{1}{e}$</td><td>↘</td><td>0</td></tr>
</table></td><td>1</td></tr>
<tr><td>5</td><td>f (x) = 0 ; 1+lnx = 0 ; $x = \frac{1}{e}$.</td><td>0.5</td></tr>
<tr><td>6</td><td>$x = \frac{1}{e}$ alors $f(\frac{1}{e}) = 0$ et $f'(\frac{1}{e}) = e^{1-\frac{1}{e}}$ une équation de la tangente :
$y = e^{1-\frac{1}{e}}(x - \frac{1}{e})$</td><td>1</td></tr>
<tr><td>7</td><td>
</td><td>1</td></tr>
<tr><td>8</td><td>lnx = me^x – 1 est équivalente à f(x) = m.
pour $m \le 0$ une solution .
pour $0 < m < \frac{1}{e}$ deux solutions
pour $m = \frac{1}{e}$ une solution double.
Pour $m > \frac{1}{e}$ pas de solutions.</td><td>1</td></tr>
</table>

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	الدورة العادية للعام ٢٠١٠
عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات .
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $(O ; \vec{i}, \vec{j}, \vec{k})$, consider the plane (P) with equation $3x - 4y + z = 0$ and the point $A(-1; 5 ; -3)$.

1) Determine a system of parametric equations of the straight line (d) passing through A and perpendicular to (P).

2) Let H be the orthogonal projection of A on (P). Prove that the coordinates of H are $(2 ; 1 ; -2)$.

3) Calculate the distance from O to (d).

4) a- Determine an equation of the plane (Q) containing points A and O and perpendicular to (P).

b- Determine a system of parametric equations of the line of intersection of (P) and (Q).

II. (4 points)

In the complex plane referred to a direct orthonormal system $(O ; \vec{u}, \vec{v})$, consider the points A, B and C of respective affixes:

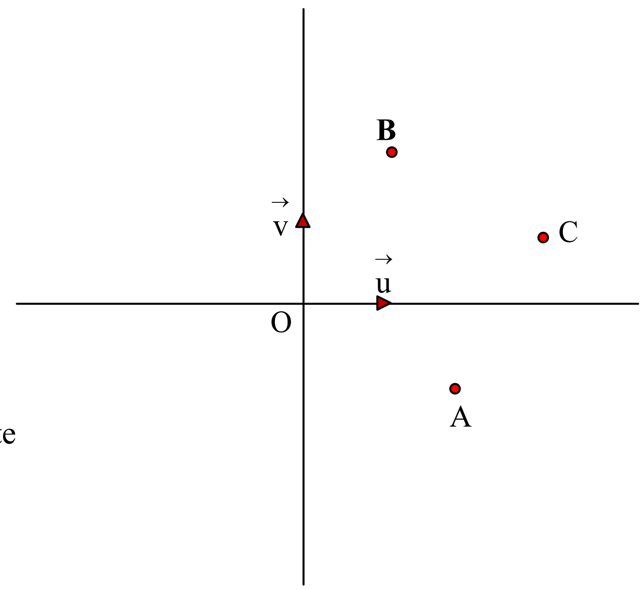
$z_A = \sqrt{3} - i$, $z_B = 1 + i\sqrt{3}$ and $z_C = z_A + z_B = \sqrt{3} + 1 + i(\sqrt{3} - 1)$.

1) Write the exponential form of z_A and of z_B.

2) a- Prove that $\frac{z_B}{z_A} = i$.

b- Show that the triangle OAB is right isosceles.

c- Verify that OACB is a square.



3) a- Use the figure to show that a measure of the angle $\left(\vec{u} ; \overrightarrow{OC}\right)$ is $\frac{\pi}{12}$.

b- Calculate the exact value of $|z_C|$, then write z_C in the exponential form.

c- Deduce the exact value of $\sin\left(\frac{\pi}{12}\right)$.

III- (4 points)

Given two urns U and V:
Urn U contains three balls each carrying the number **1** and two balls each carrying the number **3**.
Urn V contains two balls each carrying the number **1** and three balls each carrying the number **3**.

A- We draw, at random, one ball from U and one ball from V.

1) What is the probability that the two drawn balls carry the same number?
2) What is the probability that the two drawn balls carry two numbers whose sum is 4?

B- In this part, we draw simultaneously and at random two balls from U and one ball from V.

Consider the event E: « The sum of the numbers carried by the three drawn balls is 7 ».

Show that the probability of the event E is equal to $\frac{2}{5}$.

C- We place the 10 balls from the two given urns in one urn W and we draw simultaneously and at random 3 balls from W. Designate by X the random variable equal to the product of the numbers carried by the three drawn balls.

1) Find the four possible values of X.
2) Determine the probability distribution of X.

IV- (8 points)

Consider the function f defined, on]0 ; +∞[, by $f(x) = 2x - 2 + \frac{1}{e^x - 1}$ and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim_{x \to 0} f(x)$ and deduce an asymptote to (C).

   b- Determine $\lim_{x \to +\infty} f(x)$ and show that the line (d) with equation $y = 2x - 2$ is an asymptote to (C).

   c- What is the position of (C) relative to (d)?

2) a- Show that $f'(x) = \frac{(e^x - 2)(2e^x - 1)}{(e^x - 1)^2}$.

   b- Copy and complete the adjacent table of variations of f.

x	0	ln2	+∞
f '(x)		0	
f(x)			

3) Draw (d) and (C).

4) Verify that $\frac{1}{e^x - 1} = \frac{e^x}{e^x - 1} - 1$ and calculate the area of the region bounded by the curve (C), the line (d) and the two lines with equations $x = \ln 2$ and $x = \ln 3$.

5) Let g be the function defined over]0 ; +∞[by $g(x) = \ln(f(x))$

   a- Calculate $\lim_{x \to 0} g(x)$ and $\lim_{x \to +\infty} g(x)$.

   b- Set up the table of variations of g.

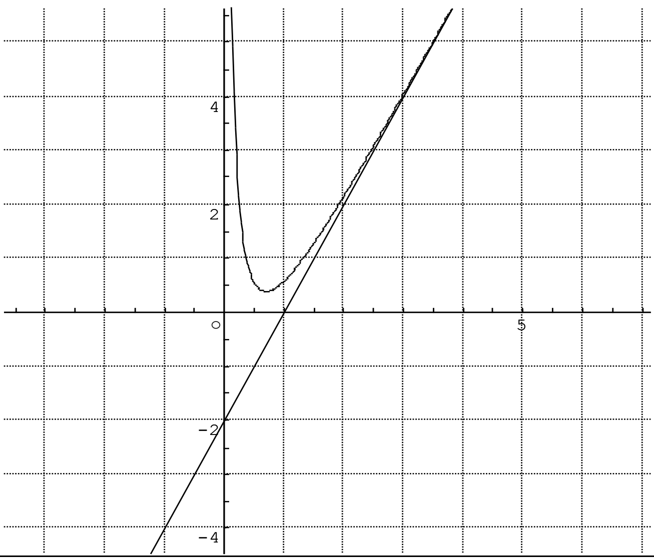
   c- Prove that the equation $g(x) = 0$ has two distinct roots.

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة مسابقة في مادة الرياضيات	الدورة العادية للعام ٢٠١٠ مشروع معيار التصحيح

QI	Answers	M
1	(d) is perpendicular to(P) then $\vec{N}_P$ is a director vector of (d). A belongs to (d) ; therefore (d): $x = 3\lambda - 1, y = -4\lambda + 5, z = \lambda - 3$	1
2	H is the orthogonal projection of A on (P), then H belongs to (d) and (P) hence $3(3\lambda-1) - 4(-4\lambda+5) + (\lambda-3) = 0$; $\lambda = 1$ and H $(2;1;-2)$. Or: $H \in (P)$ also $\overrightarrow{AH}$ and $\overrightarrow{n_P}$ are collinear.	0,5
3	O belongs to (P) then the distance from O to (d) is OH=$\sqrt{4+1+4}=3$. Or : Use distance from O to (d) = $\dfrac{\left\|\overrightarrow{V_d} \wedge \overrightarrow{OA}\right\|}{\left\|\overrightarrow{V_d}\right\|}$	1
4a	$\overrightarrow{OM}\cdot(\vec{N}_P \wedge \overrightarrow{OA}) = 0$; $\begin{vmatrix} x & y & z \\ 3 & -4 & 1 \\ -1 & 5 & -3 \end{vmatrix} = 0$; Hence (Q): $7x + 8y + 11z = 0$.	1
4b	The line of intersection of (P) and (Q) is (OH). A direction vector of (OH) is $\vec{V}(2;1;-2)$. Then a system of parametric equations of $(P)\cap(Q)$ is x=2t; y=t; z= –2t. Or (P) $\cap$(Q) : $\begin{cases} 3x - 4y + z = 0 \\ 7x + 8y + 11z = 0 \end{cases}$; let z = t, hence x= -t; y=$-\frac{1}{2}t$; z = t.	0.5

QII	Answers	M
1	$z_A = 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 2e^{-i\frac{\pi}{6}}$; $z_B = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2e^{i\frac{\pi}{3}}$	0.5
2a	$\dfrac{z_B}{z_A} = \dfrac{2e^{i\frac{\pi}{3}}}{2e^{-i\frac{\pi}{6}}} = e^{i\frac{\pi}{2}} = i$	0,5
2b	$\left\|\dfrac{z_B}{z_A}\right\| = \dfrac{OB}{OA} = 1$ then $OB = OA$ and $\arg\left(\dfrac{z_B}{z_A}\right) = \left(\overrightarrow{OA};\overrightarrow{OB}\right)[2\pi] = \dfrac{\pi}{2}[2\pi]$. Therefore the triangle OAB is right isosceles at O. OR : OA =OB =2 and $\overrightarrow{OA}\cdot\overrightarrow{OB} = 0$.	1
2c	$z_C = z_A + z_B$ then $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB}$ therefore OACB is a parallelogram. But OA = OB and $\left(\overrightarrow{OA};\overrightarrow{OB}\right) = \dfrac{\pi}{2}[2\pi]$ then OACB is a square.	0.5
3a	$\left(\vec{u};\overrightarrow{OC}\right) = \left(\vec{u};\overrightarrow{OA}\right) + \left(\overrightarrow{OA};\overrightarrow{OC}\right) = -\dfrac{\pi}{6} + \dfrac{\pi}{4} = \dfrac{\pi}{12}$ $[2\pi]$ since (OC) is the bisector of $\widehat{AOB}$.	0.5
3b	$\|z_C\| = OC = OA\sqrt{2} = 2\sqrt{2}$ and $z_C = 2\sqrt{2}e^{i\frac{\pi}{12}}$;OR: $\|z_C\| = \sqrt{\left(\sqrt{3}+1\right)^2 + \left(\sqrt{3}-1\right)^2} = 2\sqrt{2}$.	0.5
3c	$z_C = 2\sqrt{2}\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right) = \sqrt{3} + 1 + i\left(\sqrt{3} - 1\right)$ so $\sin\dfrac{\pi}{12} = \dfrac{\sqrt{3}-1}{2\sqrt{2}} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$.	0.5

QIII	Answers	M
A1	P(2 balls with same number)= P(1,1)+p(3,3) $=\frac{3}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{3}{5}=\frac{12}{25}$.	0.5
A2	$P(S=4)=p(1,3)+p(3,1)=\frac{3}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{2}{5}=\frac{13}{25}$.	1
B	P(S = 7) = p(two balls carrying 3 from U and one ball carrying 1 from V)+ P(one ball from U carrying 1 and one ball from U carrying 3 and one ball from V carrying 3) $=\frac{C_2^2}{C_5^2}\times\frac{2}{5}+\frac{C_3^1\times C_2^1}{C_5^2}\times\frac{3}{5}=\frac{2}{5}$.	1
C1	$X(\Omega)=\{1;3;9;27\}$	0,5
C2	$P(X=1)=$ p(3 balls carrying 1) $=\frac{C_5^3}{C_{10}^3}=\frac{1}{12}$ $P(X=3)=\frac{C_5^2\times C_5^1}{C_{10}^3}=\frac{5}{12}$; $P(X=9)=\frac{C_5^1\times C_5^2}{C_{10}^3}=\frac{5}{12}$; $p(X=27)=\frac{C_5^3}{C_{10}^3}=\frac{1}{12}$.	1

QIV	Answers	M
1a	$\lim_{x\to 0} f(x)=+\infty$, so *y'y* is asymptote to (C).	0,5
1b	$\lim_{x\to+\infty} f(x)=+\infty$, $\lim_{x\to+\infty}[f(x)-(2x-2)]=\lim_{x\to+\infty}\frac{1}{e^x-1}=0$. Thus the line of equation $y=2x-2$ is an asymptote to (C).	1
1c	$[f(x)-(2x-2)]=\frac{1}{e^x-1}>0$ for every x in D_f, (C) above (d).	0,5
2a	$f'(x)=2-\frac{e^x}{(e^x-1)^2}=\frac{(e^x-2)(2e^x-1)}{(e^x-1)^2}$	0,5
2b	$f'(x)\geq 0; e^x\geq 2$ or $e^x\leq\frac{1}{2}$; $x\geq\ln 2$ or $x\leq -\ln 2$(rej) $\begin{array}{c\|ccccc} x & 0 & & \ln 2 & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & +\infty & \searrow & 2\ln 2-1 & \nearrow & +\infty \end{array}$	1
3		1

4	$\dfrac{e^x}{e^x-1}-1=\dfrac{-e^x+1+e^x}{e^x-1}=\dfrac{1}{e^x-1}$. Area $=\displaystyle\int_{\ln 2}^{\ln 3}(f(x)-(2x-2))dx=\int_{\ln 2}^{\ln 3}(-1+\frac{e^x}{e^x-1})dx=\left[-x+\ln(e^x-1)\right]_{\ln 2}^{\ln 3}=(2\ln 2-\ln 3)\ u^2$.	1.5
5a	$\lim\limits_{x\to 0} g(x)=\lim\limits_{x\to+\infty}\ln(f(x))=+\infty$, $\lim\limits_{x\to+\infty} g(x)=\lim\limits_{x\to 0}\ln(f(x))=+\infty$	0,5
5b	$g'(x)$ has same sign as $f'(x)$. $\begin{array}{c\|ccccc} x & 0 & & \ln 2 & & +\infty \\ \hline g'(x) & & - & 0 & + & \\ \hline g(x) & +\infty & \searrow & & \nearrow & +\infty \\ & & & \ln(2\ln 2-1) & & \end{array}$	0,5
5c	Over]0;ln2] g is continuous and strictly decreasing from +∞ to ln(0.4) , so g(x)=0 has one root. Similarly it has also one root on [ln2; +∞[OR: $g(x) = 0$ iff $f(x) = 1$ The line of equation $y = 1$ cuts (C) in two distinct points, consequently the equation $g(x) = 0$ has two distinct roots.	1

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	الدورة الإستثنائية للعام 2010
عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the table below, only one of the proposed answers to each question is correct.
Write down the number of each question and give, with justification, the answer corresponding to it.

N	Questions	Answers			
		a	b	c	d
1	$\left(e^{i\frac{\pi}{12}}+e^{-i\frac{\pi}{12}}\right)^2=$	0	$2+\sqrt{3}$	$2-\sqrt{3}$	$\sqrt{3}$
2	The exponential form of $z=\sin\theta-i\cos\theta$ is:	$e^{i\theta}$	$e^{-i\theta}$	$e^{i(\frac{\pi}{2}-\theta)}$	$e^{i(\theta-\frac{\pi}{2})}$
3	A, B and C are the points with respective affixes $z_A=-3i$, $z_B=i$ and $z_C=3i$. The point M of affix z such that $\lvert z+3i\rvert=\lvert -i\rvert$ traces :	The perpendicular bisector of [AB].	The circle with center A and radius 1 .	The circle with center C and radius 1 .	The perpendicular bisector of [CB].
4	If $Z=\dfrac{iz}{z+1-i}$ then:	$\overline{Z}=\dfrac{i\overline{z}}{\overline{z}-1+i}$	$\overline{Z}=\dfrac{i\overline{z}}{\overline{z}+1+i}$	$\overline{Z}=\dfrac{-i\overline{z}}{\overline{z}-1+i}$	$\overline{Z}=\dfrac{-i\overline{z}}{\overline{z}+1+i}$

II- (4 points)

In a store, there are two drawers D_1 and D_2 containing neckties.
Drawer D_1 contains 15 silk neckties: 3 red, 5 green and 7 blue.
Drawer D_2 contains 10 polyester neckties: 2 red, 5 green and 3 blue.

A- We choose at random one necktie from D_1 and one from D_2.
Designate by E and F the following two events:
E: «the two chosen neckties are of the same color»
F: «the two chosen neckties are one red and one blue».

1) Prove that the probability P(E) is equal to $\frac{26}{75}$.
2) Calculate P(F).

B- In this part, one of the two drawers is randomly chosen out of which one necktie is randomly chosen.
Consider the following events:
R : «the chosen necktie is red»
D_1: «the chosen necktie is from drawer D_1».

1) Calculate $P(R/D_1)$ and $P(R\cap D_1)$.
2) Calculate P(R).

C- Suppose that these 25 neckties are placed in one drawer D and three neckties are simultaneously and randomly chosen from D. The price of a silk necktie is 50 000LL and that of a polyester necktie is 10 000LL .
Designate by X the amount equal to the sum of prices of the three chosen neckties.
Calculate $P(X\leq 100\ 000)$.

III- (4 points)

In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the point A (1; 1; 2), the plane (P) with equation $x + y - z + 2 = 0$ and the straight line (d) with parametric equations $\begin{cases} x = m + 1 \\ y = m - 2 \\ z = 2m + 1 \end{cases}$, where m is a real parameter.

1) Show that the straight line (d) lies in the plane (P) and that A is not in (P).

2) Find an equation of the plane (Q) determined by the point A and line (d).

3) Show that the point $A'\left(\frac{-1}{3}; \frac{-1}{3}; \frac{10}{3}\right)$ is the symmetric of A with respect to (P).

4) Let (Q') be the plane determined by A' and (d). Verify that an equation of (Q') is $x + 5y - 3z + 12 = 0$.

5) Let H be the orthogonal projection of A on (d) and α the acute angle between the two planes (Q) and (Q').
Show that the acute angle between the two lines (HA) and (HA') is equal to α and calculate $\cos\alpha$.

IV- (8 points)

Consider the function f defined on $]0;+\infty[$ by $f(x) = x - \frac{(\ln x)^2}{x}$.

(C) is the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$. (unit: 1 cm)

1) Determine $\lim\limits_{x \to 0} f(x)$. Deduce an asymptote to (C).

2) Determine $\lim\limits_{x \to +\infty} f(x)$, and verify that the line (D) of equation $y = x$ is an asymptote to (C).

3) The adjacent table shows the variations of the function φ defined over $]0;+\infty[$ by:
$\varphi(x) = x^2 + (\ln x)^2 - 2\ln x$.

x	0		1		$+\infty$
$\varphi(x)$	‖	↘	1	↗	

Verify that $f'(x) = \frac{\varphi(x)}{x^2}$. Deduce that f is strictly increasing.

4) a- Prove that (D) is tangent to (C) at the point A(1;1) and that (D) is above (C) for $x \neq 1$.

b- Verify that the tangent (T) to (C) at the point with abscissa e^2 is parallel to (D).

5) Prove that the equation f(x) = 0 has exactly one root α, and verify that $0.5 < \alpha < 0.6$.

6) Draw (D), (T) and (C).

7) Designate by (C') the representative curve of f^{-1}, the inverse function of f.
Draw (C') in the same system as (C).

8) a- Calculate $\int_{\alpha}^{1} f(x)dx$ in terms of α.

b- Deduce, in terms of α, the area of the region bounded by (C), (C') and the two lines of equations $x = 0$ and $y = 0$.

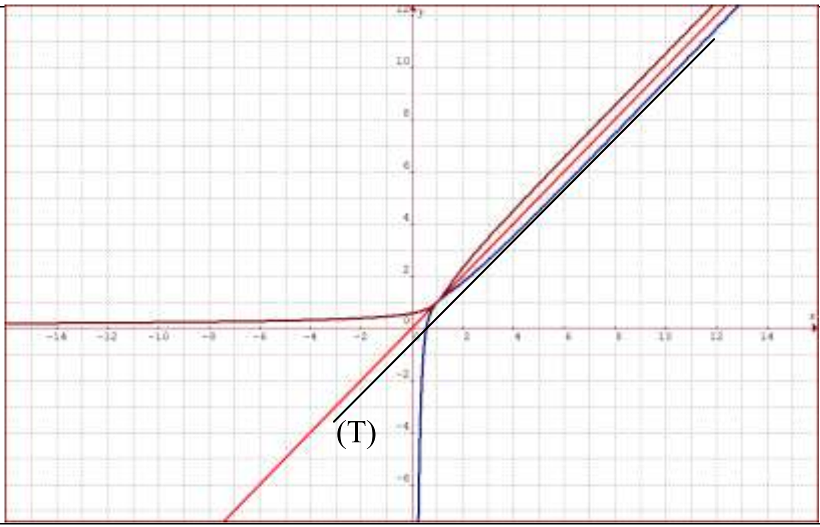
الدورة الإستثنائية للعام 2010	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات
		مشروع معيار التصحيح

QI	Answers	M
1	$(e^{\frac{i\pi}{12}}+e^{\frac{-i\pi}{12}})^2=e^{\frac{i\pi}{6}}+e^{\frac{-i\pi}{6}}+2=\frac{\sqrt{3}}{2}+\frac{1}{2}i+\frac{\sqrt{3}}{2}-\frac{1}{2}i+2=2+\sqrt{3}$ (b)	1
2	$\sin\theta-i\cos\theta=\cos(\frac{\pi}{2}-\theta)-i\sin(\frac{\pi}{2}-\theta)=\cos(\theta-\frac{\pi}{2})+i\sin(\theta-\frac{\pi}{2})=e^{i(\theta-\frac{\pi}{2})}$ (d)	1
3	$\lvert z+3i\rvert=\lvert -i\rvert$; $\lvert z_M-z_A\rvert=1$; AM = 1 ; M describes the circle of center A and radius 1 (b)	1
4	$Z=\frac{iz}{z+1-i}$; $\overline{Z}=\overline{\frac{iz}{z+1-i}}=\frac{-i.\overline{z}}{\overline{z}+1+i}$ (d)	1

QII	Answers	M
A1	p(E) = p(R,R)+ p(G,G) + p(B,B) = $\frac{3}{15}\times\frac{2}{10}+\frac{5}{15}\times\frac{5}{10}+\frac{7}{15}\times\frac{3}{10}=\frac{26}{75}$.	0.5
A2	P(F) =p(R,B) + p(B,R) = $\frac{3}{15}\times\frac{3}{10}+\frac{7}{15}\times\frac{2}{10}=\frac{23}{150}$	1
B1	$p(R/D_1)=\frac{3}{15}=\frac{1}{5}$ $P(R\cap D_1)=p(R/D_1)\times p(D_1)=\frac{1}{5}\times\frac{1}{2}=\frac{1}{10}$.	1
B2	$p(R/D_2)=\frac{2}{10}$ and $P(R\cap D_2)=p(R/D_2)\times p(D_2)=\frac{2}{10}\times\frac{1}{2}=\frac{1}{10}$. $P(R)=P(R\cap D_1)+P(R\cap D_2)=\frac{1}{10}+\frac{1}{10}=\frac{1}{5}$.	0.5
C	$P(X\le 100000)=P(X=30000)+P(X=70000)=\frac{C_{10}^3}{C_{25}^3}+\frac{C_{10}^2\times C_{15}^1}{C_{25}^3}=\frac{120+675}{2300}=\frac{159}{460}$	1

QIII	Answers	M
1	m+1+m-2-2m-1+2=0 $\Rightarrow$0=0 true, then (d) $\subset(P)$ 1+1-2+2 = 0; 2=0 impossible then A $\notin$ (P)	0.5
2	(d) // $\vec{v}(1;1;2)$; E (1;-2;1) $\in(d)$, $\overrightarrow{AE}$ (0 ;-3 ;-1) M(x,y,z)$\in(Q)$ iff $\overrightarrow{AM}.(\vec{v}\wedge\overrightarrow{AE})=0$, then an equation of the plane (Q) is then $5x+y-3z=0$	1
3	$\overrightarrow{AA'}(\frac{-4}{3},\frac{-4}{3},\frac{4}{3})$ // $\vec{N}$ which is a normal vector of (P) Let I be the midpoint of $[AA']$ so, $I(\frac{1}{3},\frac{1}{3},\frac{8}{3})$; $\frac{1}{3}+\frac{1}{3}-\frac{8}{3}+2=0$ gives therefore $I\in(P)$ Hence, A and A' are symmetric to each other with respect to (P)	1
4	We verify that A belongs to (Q') and (d) lies in (Q'). Or M (x, y, z)$\in$ (Q') iff $\overrightarrow{EM}.(\vec{v}\wedge\overrightarrow{A'E})=0$. As a result, (Q'): x+5y-3z+12=0	0.5
5	(AH)$\perp$(d) and AA'$\perp$(d) then (A'H)$\perp$(d), and (d)=(Q)$\cap$(Q'). Hence the acute angle between	

	(HA) and (HA') is equal to α . $\overrightarrow{N_1}(5,1,-3) \perp (Q)$, $\overrightarrow{N_2}(1,5,-3) \perp (Q')$ $\cos\alpha = \dfrac{\lvert \overrightarrow{N_1}.\overrightarrow{N_2} \rvert}{N_1.N_2} = \dfrac{5+5+9}{\sqrt{35}.\sqrt{35}} = \dfrac{19}{35}$ **Or:** by the calculation of the coordinates of H.	1

QIV	Answers	M
1	$\lim\limits_{x \to 0^+} \ln x = -\infty$ then $\lim\limits_{x \to 0} f(x) = -\infty$; y'y is an asymptote to (C).	0.5
2	$\lim\limits_{x \to +\infty} \dfrac{\ln^2 x}{x} = \lim\limits_{x \to +\infty} \dfrac{2\ln x}{x} = 0$ then $\lim\limits_{x \to +\infty} f(x) = +\infty$; $\lim\limits_{x \to +\infty} [f(x) - x] = 0$ hence the line (D) of equation y = x is an asymptote of (C) at $+\infty$.	0.5
3	f '(x) = $1 - \dfrac{2\left(\dfrac{\ln x}{x}\right)x - \ln^2 x}{x^2} = \dfrac{x^2 - 2\ln x + \ln^2 x}{x^2} = \dfrac{\varphi(x)}{x^2}$ with $\varphi(x) \geq 1$ so $f'(x) > 0$ and f is strictly increasing for all x in $]0;+\infty[$.	1
4a	Equation of the tangent to (C) at A: $y = f'(1)(x-1) + 1 = x$ $f(x) - x = -\dfrac{\ln^2 x}{x} \leq 0$ therefore (D) is above (C) for $x \neq 1$.	1
4b	$f'(e^2) = 1 - \dfrac{4-4}{e^2} = 1$, then the tangent (T) is parallel to (D).	0.5
5	On $]0;+\infty[$, f is continuous and changes sign, thus the equation f(x) = 0 has at least one root α but since f is strictly increasing on $]0;+\infty[$,then α is unique. f(0.5)= – 0.46< 0 and f(0.6) =0.165 > 0 , thus $0.5 < \alpha < 0.6$	1
6		1.5
7	(C') and (C) are symmetrical with respect to y = x.	0.5
8a	$\displaystyle\int_{\alpha}^{1} \left(x - \frac{\ln^2 x}{x} \right) dx = \left[\frac{x^2}{2} \right]_{\alpha}^{1} - \int_{\alpha}^{1} u^2 u' dx = \left(\frac{x^2}{2} - \frac{\ln^3 x}{3} \right)_{\alpha}^{1} = \frac{1}{2} - \frac{\alpha^2}{2} + \frac{\ln^3 \alpha}{3}$	1
8b	The area of the region bounded by (C),x'x, x= α and x = 1 is equal to the area of the region bounded by(C'), y'y, y=α and y = 1. So the required area A is the area of the square of side 1 minus twice the area calculated in the previous part , so A = $1 - 2(\frac{1}{2} - \frac{\alpha^2}{2} + \frac{\ln^3 \alpha}{3}) = \alpha^2 - \frac{2\ln^3 \alpha}{3}$ units of area	0.5

دورة 2011 العادية	**امتحانات الشهادة الثانوية العامة** **الفرع : علوم الحياة**	**وزارة التربية والتعليم العالي** **المديرية العامة للتربية** **دائرة الامتحانات**
الاسم: **الرقم:**	مسابقة في مادة الرياضيات المدة ساعتان	**عدد المسائل : أربع**

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.

- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the line (d) defined by:

$$(d):\begin{cases} x = t-1 \\ y = t+3 \\ z = t+1 \end{cases} \qquad \text{(t is a real parameter).}$$

1) Determine an equation of the plane (Q) determined by the point O and the line (d).
2) a- Calculate the coordinates of point H, the orthogonal projection of O on (d).
   b- Show that the distance from point O to line (d) is equal to $2\sqrt{2}$.
3) (P) is the plane with equation $(2m-1)x - my + (1-m)z + 6m - 2 = 0$, where m is a real parameter.
   a- Verify that H belongs to (P).
   b- Show that (P) contains the line (d).
   c- Calculate, in terms of m, the distance from point O to (P).
4) Determine m so that the line (OH) is perpendicular to plane (P).

II- (4 points)

In a school, each student of the GS and LS sections practices only one sport. The students are distributed as shown in the following table:

	Football	**Basketball**	**Tennis**
LS	1	6	3
GS	4	4	2

The name of each student is written on a separate card, where all the 20 cards used are identical.

A- The cards carrying the names of the LS students are placed in a box B_1 and those carrying the names of the GS students are placed in another box B_2.
The school principal chooses at random a box and then draws, randomly and simultaneously, two cards from the chosen box.
Consider the following events:
E : The chosen box is B_1
S : The two drawn cards carry the names of two students who practice the same sport.

1) a- Show that the probability $p(S/E)$ is equal to $\frac{2}{5}$ and deduce $p(E\cap S)$.
   b- Prove that $p(S) = \frac{31}{90}$.
2) Knowing that the two selected cards carry the names of two students who practice different sports, what is the probability that these two students are in the LS section?

B- Assume, in this part, that the 20 cards carrying the names of the students are placed together in one box B.
Three cards are drawn simultaneously and at random from this box.

1) Prove that the probability that the three drawn cards carry the names of three students, who practice the same sport, is $\frac{7}{57}$.
2) Let X be the random variable equal to the number of sports practiced by the three students whose names are written on the three drawn cards. Determine the probability distribution of X.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points M and M' with respective affixes z and z' such that $z' = (1 + i\sqrt{3})z - 2$.

1) In this part, suppose that $z = 1+i$.

   a- Show that the point M' belongs to the line with equation $y = -x$.

   b- Show that triangle OMM' is right at O.

2) Let I be the point with affix -2.

   a- Verify that $|z'+2| = 2|z|$.

   b- Prove that as M describes the circle with center O and radius 2, M' describes a fixed circle whose center and radius are to be determined.

3) Suppose that $z = x+iy$ and $z' = x'+iy'$ where x, y, x' and y' are real numbers.

   a- Express x' and y' in terms of x and y.

   b- Show that if M describes the line with equation $y = -x\sqrt{3}$, then M' describes a straight line to be determined.

IV- (8 points)

Let f be the function defined, on$]-\infty\,;+\infty[$, by $f(x) = x + 2 - \dfrac{3}{1+e^{x}}$.

(C) is the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim\limits_{x\to-\infty} f(x)$; Show that the line (d_1) with equation $y = x - 1$ is an asymptote to (C) and specify the position of (d_1) relative to (C).

   b- Calculate $\lim\limits_{x\to+\infty} f(x)$; Show that the line (d_2) with equation $y = x + 2$ is an asymptote to (C) and specify the position of (d_2) relative to (C).

2) Prove that the point $I\left(0\,;\dfrac{1}{2}\right)$ is a center of symmetry of (C).

3) Show that f is strictly increasing on $]-\infty\,;+\infty[$ and set up its table of variations.

4) Draw (d_1), (d_2) and (C).

5) a - Verify that $f(x) = x + 2 - \dfrac{3e^{-x}}{1+e^{-x}}$.

   b - Calculate the area $A(\lambda)$ of the region bounded by the curve (C), the asymptote (d_2) and the two lines with equations $x = 0$ and $x = \lambda$, where $\lambda > 0$, then calculate $\lim\limits_{\lambda\to+\infty} A(\lambda)$.

6) Designate by g the inverse function of f on$]-\infty\,;+\infty[$; (G) is the representative curve of g.

   a- Verify that E (1+ln2; ln2) is a point on (G).

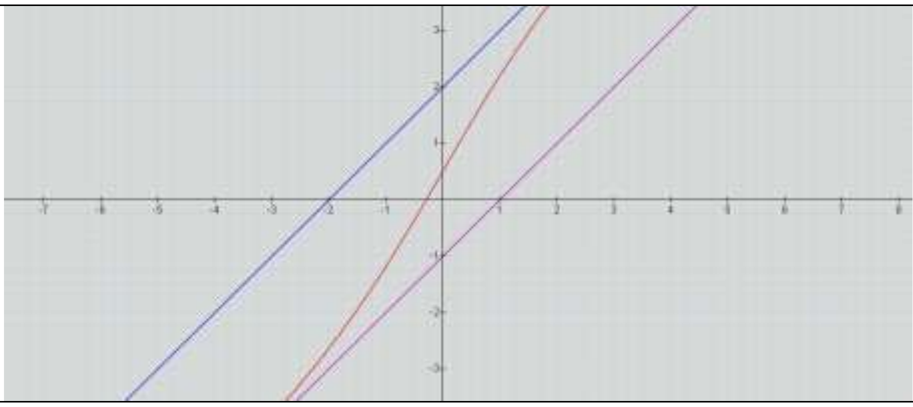
   b- Calculate the slope of the tangent to (G) at E.

Q1	Solution	G
1	For t = 0, A (–1, 3, 1) is on (d). Let M(x, y, z) be a point on (Q) ; then $\overrightarrow{OM}\cdot\left(\overrightarrow{OA}\wedge\overrightarrow{v_d}\right)=0 \Leftrightarrow$ (Q) : x + y -2z = 0	0.5
2a	H (x_H, y_H, z_H) is a point on (d) such that (OH) is perpendicular to (d) ; then $\begin{cases}H\in(d)\\ \overrightarrow{OH}\cdot\overrightarrow{v_d}=0\end{cases}$ t–1+ t + 3 + t + 1= 0, therefore t = –1 and H(– 2, 2, 0).	1
2b	$OH=\sqrt{(-2)^2+2^2}=2\sqrt{2}$.	0.5
3a	(2m – 1) (– 2) – 2m + 0 +6m – 2 = 0 The coordinates of H satisfy the equation of (P) ; hence H belongs to (P).	0.5
3b	(2m–1) (t–1) – m(t + 3) + (1– m) (t + 1) + 6m–2 = 2 m t – 2m – t + 1 – m t – 3 m + t +1– m t – m + 6 m – 2 = 0. Thus, (d) lies in (P). **OR** A belongs to (P) and H belongs to (P).	0.5
3c	$d=\frac{\lvert 6m-2\rvert}{\sqrt{(2m-1)^2+m^2+(1-m)^2}}=\frac{\lvert 6m-2\rvert}{\sqrt{6m^2-6m+2}}$.	0.5
4	(OH) is perpendicular to (P), then d= OH, so $2\sqrt{2}=\frac{\lvert 6m-2\rvert}{\sqrt{(2m-1)^2+m^2+(1-m)^2}}$, thus $12(m^2 – 2m +1) = 0$, and consequently m = 1.	0.5

Q2	Solution	G
A1 a	p(S / E) =p(both basketball or both tennis)= $\frac{C_6^2+C_3^2}{C_{10}^2}=\frac{2}{5}$ $p(E\cap S)=p(E)\times p(S/E)=\frac{1}{2}\times\frac{2}{5}=\frac{1}{5}$.	1
A1 b	$P(S) = P(S\cap E) + P(S\cap\overline{E}) = \frac{1}{5}+\frac{1}{2}\times\frac{C_4^2+C_4^2+C_2^2}{C_{10}^2}=\frac{1}{5}+\frac{13}{90}=\frac{31}{90}$.	1
A2	$p(E/\overline{S})=\frac{p(E\cap\overline{S})}{p(\overline{S})}=\frac{p(E)-P(E\cap S)}{1-p(S)}=\frac{\frac{1}{2}-\frac{1}{5}}{1-\frac{31}{90}}=\frac{\frac{3}{10}}{\frac{59}{90}}=\frac{27}{59}$.	0.5
B1	P(3 students practice the same sport)= $\frac{C_5^3+C_{10}^3+C_5^3}{C_{20}^3}=\frac{7}{57}$	0.5
B2	X(Ω) = { 1, 2 , 3} since the three students may practice the same sport , two different or three different sports. $p(X=3)=\frac{C_5^1\times C_{10}^1\times C_5^1}{C_{20}^3}=\frac{25}{114}$; $P(X = 1) = \frac{7}{57}$; $P(X=2) = 1 – p(X=1) – p(X=3) = \frac{25}{38}$.	1

Q3	Solution	G
1a	$z'=-1-\sqrt{3}+(1+\sqrt{3})i$. Then y' = - x', hence M' belongs to the line with equation y = – x.	0.5
1b	M belongs to the line with equation y = x and M' belongs to the line of equation y = -x, then triangle OMM' is right at O. **OR :** $\overrightarrow{OM}.\overrightarrow{OM'}=0$,so (OM) and (OM') are perpendicular. **OR :** $MM'^2=OM^2+OM'^2$	0.5

2a	$z'+2=\left(1+i\sqrt{3}\right)z$; thus $\lvert z'+2\rvert=\lvert 2z\rvert=2\lvert z\rvert$.	0.5
2b	M belongs to a circle with center O and radius 2, then $\lvert z\rvert=2$; thus $\lvert z'+2\rvert=4$. As a result $\lVert\overrightarrow{IM'}\rVert=4$, so M' describes the circle of center I and radius 4.	1
3a	$x'=x-\sqrt{3}\,y-2$ and $y'=y+x\sqrt{3}$.	0.5
3b	$y+x\sqrt{3}=0$, then $y'=0$, therefore z' is real, so M' describes the axis of abscissas.	1

Q4	Solution	G
1a	$\lim\limits_{x\to-\infty} f(x)=-\infty$ and $\lim\limits_{x\to-\infty}[f(x)-(x-1)]=\lim\limits_{x\to-\infty}[3-\frac{3}{1+e^x}]=\lim\limits_{x\to-\infty}\frac{3e^x}{1+e^x}=0$ then the straight line (d_1) with equation $y=x-1$ is an asymptote to (C). $f(x)-(x-1)=\frac{3e^x}{1+e^x}>0$; then (C) is above (d_1).	1
1b	$\lim\limits_{x\to+\infty} f(x)=+\infty$ and $\lim\limits_{x\to+\infty}[f(x)-(x+2)]=\lim\limits_{x\to+\infty}-\frac{3}{1+e^x}=0.$ then the straight line (d_2) with equation $y=x+2$ is an asymptote to (C). $f(x)-(x+2)=\frac{-3}{1+e^x}<0$; then (C) is below (d_2).	1
2	0 is the center of the domain of definition of f and $f(2a-x)+f(x)=f(-x)+f(x)=1$, then I is a center of symmetry of (C).	1
3	$f'(x)=1+\frac{3e^x}{(1+e^x)^2}>0$; for all x in $]-\infty\,;+\infty[$ So f is strictly increasing . $\begin{array}{c\|lr} x & -\infty & +\infty \\ \hline f'(x) & + & \\ \hline f(x) & -\infty\ \nearrow & +\infty \end{array}$	1
4		1
5a	$f(x)=x+2-\frac{3}{1+e^x}=x+2-\frac{3}{1+e^x}\times\frac{e^{-x}}{e^{-x}}=x+2-\frac{3e^{-x}}{1+e^{-x}}$.	0.5
5b	$A(\lambda)=\int_0^\lambda[(x+2)-(x+2-\frac{3e^{-x}}{1+e^{-x}})]dx=\int_0^\lambda\frac{3e^{-x}}{1+e^{-x}}dx=\left[-3\ln(1+e^{-x})\right]_0^\lambda$ $=-3\ln(1+e^{-\lambda})+3\ln2$ thus, $\lim\limits_{\lambda\to+\infty}A(\lambda)=3\ln2$.	1.5
6a	$f(\ln2)=\ln2+2-\frac{3}{1+e^{\ln2}}=\ln2+2-1=1+\ln2$ then, $g(1+\ln2)=\ln2$ and $E(1+\ln2;\ln2)$ is a point of (G).	0.5
6b	The slope of the tangent to (G) at E is: $g'(1+\ln2)=\frac{1}{f'(\ln2)}=\frac{1}{1+\frac{3\times2}{(2+1)^2}}=\frac{3}{5}$.	0.5

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الاسم: الرقم:	مسابقة في مادة الرياضيات المدة:ساعتان	عدد المسائل: اربع

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I-(4 points)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, with justification, the corresponding answer.

N°	Question	Answers a	b	c
1	The exponential form of $z=-\sin\theta+i\cos\theta$ is	$e^{i\left(\frac{\pi}{2}-\theta\right)}$	$e^{i\left(\theta-\frac{\pi}{2}\right)}$	$e^{i\left(\frac{\pi}{2}+\theta\right)}$
2	If $z=\dfrac{\cos\theta-i\sin\theta}{\cos\theta+i\sin\theta}$, then $\bar{z}=$	$e^{2i\theta}$	$e^{-2i\theta}$	1
3	If $z_A=1-2i$, $z_B=2+3i$ and $z_C=4$ then the triangle ABC is	Right and not isosceles	Isosceles and not right	Right and isosceles
4	$\lim\limits_{x\to 0}\dfrac{\int_0^x \ln(t+1)dt}{e^x-1}=$	1	0	$+\infty$
5	$\int \cos^2 x dx=$	$\frac{x}{2}-\frac{\sin 2x}{4}+c$	$\frac{\cos^3 x}{3}+c$	$\frac{x}{2}+\frac{\sin 2x}{4}+c$

II- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the points A(4 ; 0 ; 0),

B(0 ; 6 ; 0), C(0 ; 0 ; 4) and E(2 ; 3 ; 0).

1) Show that E belongs to the line (AB).

2) Let (P) be the plane passing through E and parallel to the lines (OB) and (AC).
Show that an equation of (P) is $x + z - 2 = 0$.

3) Write a system of parametric equations of the line (BC).

4) The plane (P) cuts the lines (BC), (OC) and (OA) at F, G and H respectively.
Show that the coordinates of F are (0 ;3 ;2) and specify the respective coordinates of G and H.

5) a- Prove that EFGH is a rectangle.
b- Let Γ be the circle circumscribed about the rectangle EFGH and (T) be the line in plane (P) that is tangent at E to Γ. Determine a system of parametric equations of (T).

III- (4 points)

An urn contains 8 balls:

- 4 white balls each carrying the number 0;
- 3 red balls each carrying the number 5;
- 1 white ball carrying the number 2.

We draw, simultaneously and randomly, 3 balls from the urn.

Consider the following events:

A: « the three drawn balls carry three numbers which could form the number 200».

B: « the three drawn balls carry three identical numbers ».

C: « the three drawn balls are white ».

D: « the three drawn balls are of the same color ».

1) Show that the probability p(A) is equal to $\frac{3}{28}$ and calculate p(B), p(C) and p(D).
2) Determine the probability that among the three drawn balls only one carries the number 0.
3) The three drawn balls are white; calculate the probability that the numbers carried by these balls could form the number 200.
4) let X be the random variable equal to the product of the three numbers carried by the three drawn balls.
   a- Give the 3 possible values of X.
   b- Determine the probability distribution of X.

IV- (8 points)

A- Let g be the function defined over $]0;+\infty[$ by $g(x) = x + \ln x$.

1) Calculate $\lim_{x\to 0} g(x)$ and $\lim_{x\to +\infty} g(x)$.
2) Set up the table of variations of g.
3) Prove that the equation $g(x) = 0$ has a unique solution α and verify that $0.5 < \alpha < 0.6$.
4) Determine, according to the values of x, the sign of g(x).

B- Consider the function f defined over $]0;+\infty[$ by $f(x) = x(2\ln x + x - 2)$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim_{x\to 0} f(x)$, $\lim_{x\to +\infty} f(x)$ and determine f(e).
2) Prove that $f(\alpha) = -\alpha(\alpha+2)$.
3) Verify that $f'(x) = 2g(x)$ and set up the table of variations of f.
4) Draw (C). (Take $\alpha = 0.55$)
5) Use integration by parts to calculate $\int_{0.5}^{1} x\ln x\,dx$ and deduce the area of the region bounded by the curve (C), the axis of abscissas and the two lines with equations $x = 0.5$ and $x = 1$.
6) The curve (C) cuts the axis of abscissas at a point with abscissa 1.37. Designate by F an antiderivative of f on $]0;+\infty[$; determine, according to the values of x, the variations of F.

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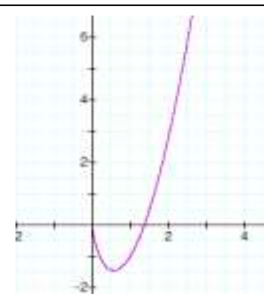
QI	Answers	M
1	$z=-\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)=\cos\left(\frac{\pi}{2}+\theta\right)+i\sin\left(\frac{\pi}{2}+\theta\right)=e^{i\left(\frac{\pi}{2}+\theta\right)}$ (c)	0.5
2	$z=\frac{e^{-i\theta}}{e^{i\theta}}=e^{-2i\theta}\Rightarrow\overline{z}=e^{2i\theta}$. (a)	0.5
3	$AB=\lvert z_B-z_A\rvert=\lvert 1+5i\rvert=\sqrt{26},\ AC=\lvert z_C-z_A\rvert=\lvert 3+2i\rvert=\sqrt{13}, BC=\lvert 2+3i\rvert=\sqrt{13}$. ABC is right and isosceles. (c)	1
4	$\lim\limits_{x\to 0}\frac{\int_0^x \ln(t+1)dt}{e^x-1}=\frac{0}{0}=\lim\limits_{x\to 0}\frac{\ln(x+1)}{e^x}=0$ (b)	1
5	$\int\cos^2 x dx=\int\frac{1+\cos 2x}{2}dx=\frac{x}{2}+\frac{1}{4}\sin 2x+c$ (c)	1

QII	Answers	M
1	(AB) : $\begin{cases}x=-2t+4\\y=3t\\z=0\end{cases}$ for $t=1: x=2, y=3$ and $z=0$, so E belongs to (AB).	0.5
2	$\overrightarrow{n_P}$ is parallel to $\overrightarrow{OB}\wedge\overrightarrow{AC}$, thus $\overrightarrow{n_P}(1;0;1)$. (P) : $x+z+r=0$; E (2 ; 3 ; 0) is a point in (P) thus $r=-2$; (P) : x +y – 2 =0.	0.5
3	$\overrightarrow{BC}(0;-6;4)$ and B is a point on (BC), (BC) : $x=0; y=-3m+6; z=2m$.	0.5
4	$x_F+z_F-2=0+2-2=0$ then F is a point in (P). for m = 1, F(0 ;3 ;2)is a point on (BC). $x_G=y_G=0$ and G is a point in (P), then $z_G=2$ and G (0 ;0 ;2). $y_H=z_H=0$ and H is a point in (P), then $x_H=2$ and H(2 ;0 ;0).	1
5a	**<u>Geometrically</u>** EF is parallel to (AC) which is parallel to (GH), so EF is parallel to (GH). Similarly (EH) is parallel to (OB) and (GF) is parallel to (OB), thus EH is parallel to (GF). Hence EFGH is a parallelogram.. Moreover $(OB)\perp(AC)\Rightarrow(EH)\perp(EF)$ then EFGH is a rectangle. **<u>By calculation</u>** $\left.\begin{array}{l}\overrightarrow{EF}(-2;0;2), \overrightarrow{HG}(-2;0;2) \text{ so } \overrightarrow{EF}=\overrightarrow{HG}.\\ \overrightarrow{FG}(0;-3;0) \text{ and } \overrightarrow{FG}.\overrightarrow{HG}=0 \text{ thus } FG\perp HG.\end{array}\right\}$ so EFGH is a rectangle.	0.5
5b	$\overrightarrow{EG}\wedge\overrightarrow{n_P}(-3;4;3)$ is a direction vector of (T), thus x= -3 λ +2 ; y = 4 λ +3 ; z = 3 λ.	1

QIII	Answers	M
1	$p(A)=\frac{C_4^2\times C_1^1}{C_8^3}=\frac{3}{28}$. $p(B)=\frac{C_3^3}{C_8^3}+\frac{C_4^3}{C_8^3}=\frac{5}{56}$, $p(C)=\frac{C_5^3}{C_8^3}=\frac{5}{28}$, $p(D)=\frac{C_5^3}{C_8^3}+\frac{C_3^3}{C_8^3}=\frac{3}{7}$.	1.5
2	$p(\text{only one ball carries } 0)=\frac{C_4^1\times C_4^2}{C_8^3}=\frac{3}{7}$	0.5
3	$p(A/C)=\frac{C_1^1\times C_4^2}{C_5^3}=\frac{3}{5}$.	0.5
4a	X(Ω)={0; 50; 125}	0.5
4b	$p(X=50)=p\left(\{2,5,5\}\right)=\frac{C_1^1\times C_3^2}{C_8^3}=\frac{3}{56}$. $p(X=125)=p\left(\{5,5,5\}\right)=\frac{1}{56}$. $p(X=0)=\frac{C_4^1\times C_4^2+C_4^2\times C_4^1+C_4^3}{56}=\frac{52}{56}=\frac{13}{14}$ Or: $1-\frac{C_4^3}{56}=\frac{52}{56}$.	1

QIV	Answers	M
A1	$\lim\limits_{x\to 0} g(x)=-\infty,\quad \lim\limits_{x\to+\infty} g(x)=+\infty.$	0.5
A2	$g'(x)=\frac{1}{x}+1$ $\begin{array}{c\|ccc} x & 0 & & +\infty \\ \hline g'(x) & & + & \\ \hline g(x) & -\infty & \nearrow & +\infty \end{array}$	1
A3	g is continuous, strictly increasing on its domain, changing signs; so it vanishes once, thus g(x)= 0 has a a unique solution α. Moreover g(0.5)=–0.193 and g(0.6)= 0.089 thus 0.5 <α< 0.6.	1
A4	g(x) > 0 for x >α , g(x) <0 for x <α and g(x) = 0 for x = α.	0.5
B1	$\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}\left(2x\ln x+x^2-2x\right)=0.\quad \lim\limits_{x\to+\infty} f(x)=+\infty$ since $\lim\limits_{x\to+\infty}\left(x^2-2x\right)=+\infty.\quad f(e)=e^2.$	1
B2	$f(\alpha)=\alpha\left(2\ln\alpha+\alpha-2\right)=\alpha\left(-2\alpha+\alpha-2\right)=-\alpha\left(\alpha+2\right)$	0.5
B3	$f'(x)=2\ln x+x-2+x\left(\frac{2}{x}+1\right)=2(\ln x+x)=2g(x).$ $\begin{array}{c\|ccccc} x & 0 & & \alpha & & +\infty \\ \hline f'(x) & & - & 0 & + & \\ \hline f(x) & 0 & \searrow & -\alpha(\alpha+2) & \nearrow & +\infty \end{array}$ 	1
B4		1
B5	$u=\ln x,\ v'=x,\ u'=\frac{1}{x},\ v=\frac{x^2}{2}.\ \int x\ln x\,dx=\frac{x^2}{2}\ln x-\int\frac{x}{2}dx=\frac{x^2}{4}\left(2\ln x-1\right).$ $\int_{0.5}^{1} x\ln x\,dx=\frac{\ln 2}{8}-\frac{3}{16}.\quad A=-\int_{0.5}^{1} f(x)dx=-\frac{\ln 2}{4}+\frac{5}{6}=0.66u^2.$	1
B6	$F'(x)\le 0$ on $]0;1.37]$, then F is increasing on this interval. $F'(x)>0$ on $]1.37;+\infty[$, then F is decreasing on this interval.	0.5

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الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل: أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points A(1; –1 ; 1), B(–2 ; 2 ; 1), $I\left(\frac{1}{2};-\frac{1}{2};1\right)$ and the line (d) defined by: $\begin{cases} x=-t+1 \\ y=-t \\ z=2t \end{cases}$ (t is a real number).

1) Write a system of parametric equations of line (AB).
2) Prove that (AB) and (d) intersect at I.
3) Show that an equation of the plane (P) determined by (AB) and (d) is $x+y+z-1=0$.
4) Consider the point $H\left(2;1;\frac{5}{2}\right)$.

   a- Prove that I is the orthogonal projection of H on (P).

   b- Verify that (AB) and (d) are perpendicular.

   c- K is a point on (d) such that IK = IA. Calculate the volume of the tetrahedron HABK.

II- (4 points)

An urn contains 4 black balls, 3 white balls and n red balls; ($n \geq 2$).

A-

In this part take **n = 2**.

We draw randomly and simultaneously 3 balls from the urn.

1) Calculate the probability of drawing three balls having the same color.
2) Designate by E the event:

   « Among the three drawn balls there are exactly two balls of the same color ».

   Prove that the probability P(E) is equal to $\frac{55}{84}$.

B-

In this part we draw randomly and simultaneously 2 balls from the **n+7** balls in the urn.

Designate by X the random variable equal to the number of red balls obtained among the three drawn.

1) Prove that $P(X=2)=\frac{n(n-1)}{(n+6)(n+7)}$.
2) Determine the probability distribution of X.
3) Calculate n so that the mathematical expectation E(X) is equal to 1.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B with affixes $z_A = 1$ and $z_B = e^{i\frac{\pi}{4}}$. Designate by E the midpoint of segment [AB].

1) Verify that $z_E = \dfrac{2+\sqrt{2}}{4} + i\dfrac{\sqrt{2}}{4}$.

2) a- Verify, for every real number θ, that $1+e^{i\theta} = e^{i\frac{\theta}{2}}\left(e^{i\frac{\theta}{2}} + e^{-i\frac{\theta}{2}}\right)$.

b- Show that $z_E = \left(\cos\dfrac{\pi}{8}\right)e^{i\frac{\pi}{8}}$.

c- Deduce from the preceding results the exact value of $\cos\dfrac{\pi}{8}$.

3) Let M be a variable point with affix z such that $\left|2z-\sqrt{2}-i\sqrt{2}\right| = 2$.
Prove that M describes a circle (C) and verify that O belongs to (C).

IV- (8 points)

Consider the function f defined on $\mathbb{R}$ by $f(x) = \dfrac{e^x}{e^x+1}$. Designate by (C) the representative curve of f in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Deduce the asymptotes of the curve (C).
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Show that $f''(x) = \dfrac{e^x(1-e^x)}{(1+e^x)^3}$. Prove that (C) has a point of inflection I to be determined.
4) Write an equation of the tangent (T) to (C) at the point I.
5) Draw (T) and (C).
6) The function f has on $\mathbb{R}$ an inverse function g.

a- Draw the representative curve (G) of g in the given system.

b- Verify that $g(x) = \ln\left(\dfrac{x}{1-x}\right)$.

c- (G) and (C) intersect at a point with abscissa α. Calculate, in terms of α, the area of the region bounded by (C), (G) and the two axes of coordinates.

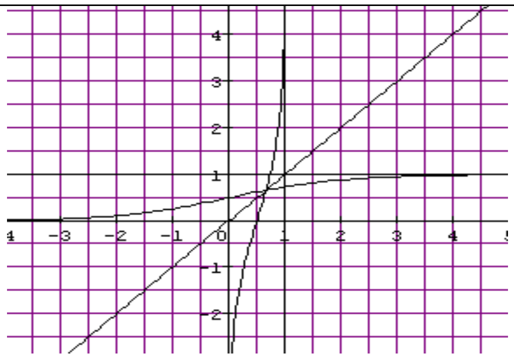
QI	Answers	M
1	$\overrightarrow{AB}(-3,3,0)$ so $\vec{V}(1,-1,0)$ is a direction vector of (AB) then : $x = m+1, y = -m-1, z = 1$.	0.5
2	For $t = \frac{1}{2}$, I belongs to (d). $\overrightarrow{AI}\left(-\frac{1}{2};\frac{1}{2};0\right)$; $\overrightarrow{BI}\left(\frac{5}{2};-\frac{1}{2};0\right)$; hence $\overrightarrow{BI} = -5\,\overrightarrow{AI}$ and B , A and I are collinear. I belongs to (AB) and (d) with $A \notin (d)$ therefore (AB) and (d) intersect at I. OR: For $m = -\frac{1}{2}$; I belongs to (AB) where (d) and (AB) are distinct.	0.5
3	The coordinates of A and B verify the given equation since: $x_A + y_A + z_A - 1 = 1 - 1 + 1 - 1 = 0$ and $x_B + y_B + z_B - 1 = 0$. Moreover $(d) \subset (P)$ since the coordinates of the point $(-t+1;-t;2t)$ verify the given equation for every t.	1
4a	$\overrightarrow{IH}\left(\frac{3}{2};\frac{3}{2};\frac{3}{2}\right)$ and $\vec{n}_P(1;1;1)$ are collinear. And I belongs to plane (P), hence I is the orthogonal projection of H on (P).	0.5
4b	$\overrightarrow{AB}.\overrightarrow{V_d} = 3-3+0 = 0$, then (d) and (AB) are perpendicular at I.	0.5
4c	Volume $= \frac{Area(ABK)\times IH}{3}$. Area of KAB $= \frac{IK\times AB}{2} = \frac{IA\times AB}{2} = \frac{\frac{\sqrt{2}}{2}\times 3\sqrt{2}}{2} = \frac{3}{2}u^2$. Therefore $V = \frac{3\times 3\sqrt{3}}{2\times 3\times 3} = \frac{3\sqrt{3}}{4}u^3$. (**Or**: Find the coordinates of point K (two possibilities) then use $V = \frac{\left\lvert \overrightarrow{AB}\cdot\left(\overrightarrow{AH}\wedge\overrightarrow{AK}\right)\right\rvert}{6}$	1

QII	Answers	M
A1	P(3 balls of same color) = P(3B) + P(3W) $= \frac{C_4^3 + C_3^3}{C_9^3} = \frac{5}{84}$	0.5
A2	P(E) = P(2 balls of same color)= $P(2R,1\overline{R}) + P(2B,1\overline{B}) + P(2W,1\overline{W}) = \frac{C_2^2\times C_7^1 + C_4^2\times C_5^1 + C_3^2\times C_6^1}{C_9^3} = \frac{55}{84}$	1
B1	p(X=2) =p(2 red) $= \frac{C_n^2}{C_{n+7}^2} = \frac{n!}{2!(n-2)!}\times\frac{2!(n+5)!}{(n+7)!} = \frac{n(n-1)}{(n+7)(n+6)}$	1
B2	p(X= 0) = p(0 red) $= \frac{C_7^2}{C_{n+7}^2} = \frac{7\times 6}{(n+7)(n+6)}$; p(X=1)= $P(1R,1\overline{R}) = \frac{C_n^1\times C_7^1}{C_{n+7}^2} = \frac{7n\times 2}{(n+7)(n+6)}$; <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>p</td><td>$\frac{42}{(n+7)(n+6)}$</td><td>$\frac{14n}{(n+7)(n+6)}$</td><td>$\frac{n^2-n}{(n+7)(n+6)}$</td></tr></table>	1
B3	E(X)= $\frac{14n + 2n^2 - 2n}{n^2 + 13n + 42} = 1$ then $n^2 - n - 42 = 0$, so $n = 7$ or $n = -6$. Therefore $n = 7$	0.5

QIII	Answers	M
1	$z_E = \frac{z_A+z_B}{2} = \frac{1}{2}\left(1+\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) = \frac{1}{2}\left(1+\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right) = \frac{2+\sqrt{2}}{4}+i\frac{\sqrt{2}}{4}$.	0.5
2a	$e^{i\frac{\theta}{2}}\left(e^{i\frac{\theta}{2}}+e^{-i\frac{\theta}{2}}\right) = e^{i\left(\frac{\theta}{2}+\frac{\theta}{2}\right)}+e^{i\left(\frac{\theta}{2}-\frac{\theta}{2}\right)} = e^{i\theta}+e^{i(0)} = 1+e^{i\theta}$.	0.5
2b	$z_E = \frac{1}{2}\left(1+e^{i\frac{\pi}{4}}\right) = \frac{1}{2}e^{i\frac{\pi}{8}}\left(e^{i\frac{\pi}{8}}+e^{-i\frac{\pi}{8}}\right) = e^{i\frac{\pi}{8}}\frac{1}{2}\left(2\cos\frac{\pi}{8}\right) = \left(\cos\frac{\pi}{8}\right)e^{i\frac{\pi}{8}}$.	1
2c	$\cos\frac{\pi}{8}e^{i\frac{\pi}{8}} = \frac{2+\sqrt{2}}{4}+\frac{\sqrt{2}}{4}i$, $\cos^2\frac{\pi}{8}+i\cos\frac{\pi}{8}\sin\frac{\pi}{8} = \frac{2+\sqrt{2}}{4}+\frac{\sqrt{2}}{4}i$ hence $\cos\frac{\pi}{8} = +\sqrt{\frac{2+\sqrt{2}}{4}}$ $\left(\cos\frac{\pi}{8}>0\right)$	1
3	$\lvert 2Z-2Z_B\rvert = 2;\ \lvert Z-Z_B\rvert = 1$ hence BM=1 , and M describes the circle with center B and radius 1 . Since BO=1 then O belongs to (C).	1

QIV	Answers	M
1	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}\frac{e^x}{e^x+1} = \frac{0}{0+1} = 0$. the line with equation y = 0 is an asymptote to (C). $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty}\frac{e^x}{e^x+1} = \frac{+\infty}{+\infty} = \lim\limits_{x\to+\infty}\frac{e^x}{e^x} = 1$. the line with equation y = 1 is an asymptote to (C).	1
2	$f'(x) = \frac{e^x(e^x+1)-e^x(e^x)}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2} > 0$	1

x	$-\infty$	$+\infty$
$f'(x)$	+	
$f(x)$	0 ↗	1

QIV	Answers	M
3	$f''(x) = \frac{e^x(e^x+1)^2-2e^x(e^x+1)e^x}{(1+e^x)^4} = \frac{e^x(1-e^x)}{(1+e^x)^3}$, so f"(x) vanishes while changing sign at x = 0. Hence the point I(0,1/2) is a point of inflection.	1.5
4	$f'(0) = \frac{1}{4};\ y-\frac{1}{2} = \frac{x}{4};\ y = \frac{x}{4}+\frac{1}{2}$	0.5
5		1
6a	(G) is symmetric of (C) with respect to the line with equation y = x.	1
6b	$e^x = \frac{y}{1-y}$ hence $x = \ln\left(\frac{y}{1-y}\right)$; so $g(x) = \ln\left(\frac{x}{1-x}\right)$	1
6c	Using the symmetry with respect to the first bisector, the area of the region is twice the area of the region bounded by (C) and the first bisector, $A = 2\int_0^{\alpha}\left(\frac{e^x}{e^x+1}-x\right)dx = 2\left[\ln\left(e^x+1\right)-\frac{1}{2}x^2\right]_0^{\alpha} = \left(2\ln\left(e^{\alpha}+1\right)-2\ln 2-\alpha^2\right)u^2$	1

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	الدورة العادية للعام ٢٠١٢
عدد المسائل : أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the space referred to a direct orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j}\ ,\ \vec{k})$, consider the following points:
A (4 ;0; 1) , B(2 ;1 ;2), C(2 ; 0 ;3) and E(3 ; –1 ;0).

1) a- Write an equation of the plane (P) determined by A , B and C.
   b- Show that A is the orthogonal projection of E on (P).
2) a- Show that triangle ABC is right.
   b- Calculate the area of the triangle ABC.
   c- Calculate the volume of the tetrahedron EABC.
3) (Q) is the plane with equation $x - 2y - 2z - 2 = 0$.
   Show that (Q) passes through A and is perpendicular to (BE).
4) a- Write a system of parametric equations of the line (BC).
   b- Let M be a variable point on (BC). Prove that the distance from M to (Q) remains constant as M moves on (BC).

II- (4 points)

A shop sells two types of earphones E_1 and E_2 and three types of batteries B_1, B_2 and B_3.
During the promotion period, some items are placed in two baskets U and V.

Basket U contains 15 earphones of type E_1 and 5 earphones of type E_2;
Basket V contains 8 batteries of type B_1, 10 batteries of type B_2 and 7 batteries of type B_3.

A- A customer selects, at random, one item from each basket.

1) Show that the probability of obtaining an earphone E_1 and a battery B_1 is equal to $\frac{6}{25}$.
2) Calculate the probability that an earphone E_1 is among the two selected items.
3) The shop announces the following prices:

Item	Earphone E_1	Earphone E_2	Battery B_1	Battery B_2	Battery B_3
Price in LL	**40 000**	**15 000**	**30 000**	**25 000**	**50 000**

X is the random variable equal to the amount paid by the customer for buying the two selected items.
   a- Prove that the probability P(X = 65 000) is equal to $\frac{37}{100}$.
   b- Determine the probability distribution of X.

B- In this question, a customer selects, at random, an earphone from basket U and selects simultaneously and at random two batteries from basket V. Calculate the probability that the customer pays an amount less than or equal to 70 000LL.

III- (4 points)

The complex plane is referred to a direct orthonormal system $(O;\ \vec{u}, \vec{v})$.

For every point M with affix z $(z \neq 0)$, we associate the point M' with affix z' such that $z' = \frac{2}{\bar{z}}$.

1) Let $z = re^{i\theta}$ $(r > 0)$, write z' in exponential form.
2) a- Show that $OM \times OM' = 2$.

   b- If $z = z'$, prove that M moves on a circle (C) whose center and radius are to be determined.
3) Let z = 1 + iy where y is a real number.

   a- Prove that $|z' - 1| = 1$.

   b- As y varies, show that M' moves on a circle (C') whose center and radius are to be determined

IV- (8 points)

Consider the function f defined over $\mathbb{R}$ by $f(x) = (x+1)^2 e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i}, \vec{j})$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-2)$.

   b- Determine $\lim\limits_{x \to +\infty} f(x)$ and deduce an asymptote to (C).

2) Show that $f'(x) = (1 - x^2)e^{-x}$ and set up the table of variations of f.

3) The line (d) with equation y = x intersects (C) at a point with abscissa α.

   Verify that $1.4 < \alpha < 1.5$.
4) Draw (d) and (C).

5) Let F be the function defined on $\mathbb{R}$ by $F(x) = (px^2 + qx + r)\, e^{-x}$.

   a- Calculate p, q and r so that F is an antiderivative of f.

   b- Calculate the area of the region bounded by (C), the axis of abscissas and the two lines with equations x = 0 and x = 1.

6) The function f has over $[1;+\infty[$ an inverse function h. Determine the domain of definition of h and draw its representative curve in the same system as (C).

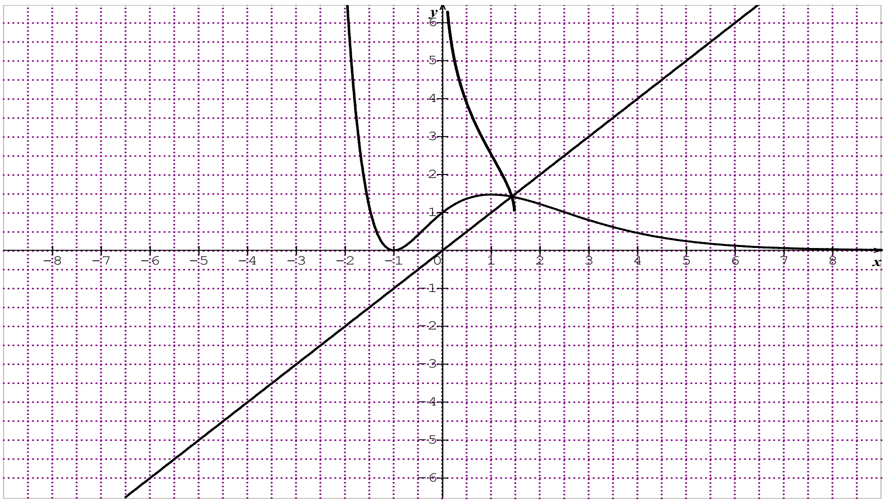
I-	Solution	Mark
1a	For every point M(x; y ; z) of (P) ; $\vec{AM}.(\vec{AB}\wedge\vec{AC})=0$. So, (P): x + y + z –5 = 0 .	0.5
1b	$\vec{AE}(-1,-1,-1)$, $\vec{N}_P(1,1,1)$ then $\vec{AE}=-\vec{N}_P$. (AE) is perpendicular to (P) and $A\in(P)$. Thus A is the orthogonal projection of E on (P).	0.5
2a	$\vec{AB}(-2,1,1)$, $\vec{BC}(0,-1,1)$; $\vec{AB}.\vec{BC}=0$ Thus, ABC is right at B.	0.5
2b	Area (ABC) = $\frac{1}{2}AB\times BC=\frac{1}{2}\sqrt{6\times2}=\sqrt{3}\,u^2$	0.5
2c	$V=\frac{A_{(ABC)}\times AE}{3}=1u^3$. OR : By calculating the triple scalar product.	0.5
3	The coordinates of A satisfy the equation of (Q): 4 – 0 –2 –2 = 0 . $\vec{BE}(1,-2,-2)$ and $\vec{N}_Q$ (1 ; -2 ; -2) then (Q) passes through A and is per. to (BE)	0.5
4a	$\vec{BC}(0,-1,1)$, (BC): $x=2;\ y=-m+1;\ z=m+2\ ;m\in\mathbb{R}$.	0.5
4b	$d(M\to(Q))=\frac{\lvert 2+2m-2-2m-4-2\rvert}{\sqrt{1+4+4}}=2$.	0.5

II-	Solution	Mark
A1	$P(E_1 , B_1)=\frac{15}{20}\times\frac{8}{25}=\frac{120}{500}=\frac{6}{25}$.	0.5
A2	$P(E_1,B)=\frac{15}{20}\times1=\frac{3}{4}$.	0.5
A3a	$P(X=65000)=P(E_1,B_2)+P(E_2,B_3)=\frac{15}{20}\times\frac{10}{25}+\frac{5}{20}\times\frac{7}{25}=\frac{37}{100}$.	0.5
A3b	$X=x_i$: 40000, 45000, 65000, 70000, 90000; p_i: $\frac{1}{10}$, $\frac{2}{25}$, $\frac{37}{100}$, $\frac{6}{25}$, $\frac{21}{100}$	1.5
B	To pay a sum less than or equal to 70000LL, we cannot choose E_1 since 2 batteries cost at least 50000LL ; thus we choose $\{E_2,B_2,B_2\}$ or $\{E_2,B_1,B_2\}$ $P(S\le70000)=\frac{5}{20}\times\frac{C_{10}^2+C_8^1\times C_{10}^1}{C_{25}^2}=\frac{5}{48}$.	1

Table for A3b:

$X=x_i$	40000	45000	65000	70000	90000
p_i	$\frac{1}{10}$	$\frac{2}{25}$	$\frac{37}{100}$	$\frac{6}{25}$	$\frac{21}{100}$

III	Solution	Mark
1	$z'=\frac{2}{re^{-i\theta}}=\frac{2}{r}e^{i\theta}$.	0.5
2a	$OM\times OM'=r\times\frac{2}{r}=2.$ OR : $\lvert z'\rvert=\left\lvert\frac{2}{\bar z}\right\rvert=\frac{2}{\lvert z\rvert}$ hence $OM'=\frac{2}{OM}$.	0.5
2b	If z = z ' then $OM^2=2$; $OM=\sqrt{2}$. M moves on a circle with center O and radius $\sqrt{2}$.	1
3a	$\lvert z'-1\rvert=\left\lvert\frac{2}{1-iy}-1\right\rvert=\left\lvert\frac{1+iy}{1-iy}\right\rvert=\frac{\sqrt{1+y^2}}{\sqrt{1+y^2}}=1.$	1
3b	Let I be the point with affix 1. IM' = 1. Thus, M' moves on the circle (C') with center I(1 ; 0) and radius 1.	1

IV	Solution	Mark
1a	$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} (x+1)^2 e^{-x} = +\infty$; $f(-2) = 7.4$.	0.5
1b	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{(x+1)^2}{e^x} = \lim\limits_{x\to+\infty} \frac{2(x+1)}{e^x} = \lim\limits_{x\to+\infty} \frac{2}{e^x} = 0.$ The x-axis is an asymptote to (C).	0.5
2	$f'(x) = 2(x+1)e^{-x} - e^{-x}(x+1)^2 = (1-x^2)e^{-x}$ $\begin{array}{c\|ccccccccc} x & -\infty & & -1 & & 1 & & +\infty \\ \hline f'(x) & & - & 0 & + & 0 & - & \\ \hline f(x) & +\infty & \searrow & 0 & \nearrow & \frac{4}{e} & \searrow & 0 \end{array}$	1.5
3	$f(1.4) = 1.42 > 1.4$; $f(1.5) = 1.39 < 1.5$ thus $1.4 < \alpha < 1.5$.	1
4		1.5
5a	$F'(x) = f(x)$ so $-p x^2 + (2p - q)x + q - r = x^2 + 2x + 1$ for all real numbers x. Hence, $p = -1$, $q = -4$, $r = -5$.	1
5b	Area= $\int_0^1 f(x)\,dx = \left[(-x^2 - 4x - 5)e^{-x}\right]_0^1 = (5 - \frac{10}{e}) = 1.321\ u^2$.	1
6	$D_h = \left]0;\frac{4}{e}\right]$, (C_h) is symmetric to (C) with respect to the straight line with equation $y = x$.	1

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عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
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I– (4 points)

In the table below, only one among the proposed answers to each question is correct.
Write down the number of each question and give, with justification, its corresponding answer.

N°	Questions	Answers a	Answers b	Answers c
1	If $z=-\frac{1}{2}-\frac{i\sqrt{3}}{2}$ and $z'=-\frac{3}{2}+\frac{i\sqrt{3}}{2}$, then an argument of $(z-z')$ is	$\frac{\pi}{3}$	$\frac{5\pi}{3}$	$\frac{2\pi}{3}$
2	z is the affix of a point M. If $\lvert z-2i\rvert=\lvert z+4i\rvert$, then M moves on	a circle	a line parallel to the axis of ordinates	a line parallel to the axis of abscissas
3	One of the values of z verifying $\lvert z+1\rvert^2+\lvert z-1\rvert^2=2\lvert z+i\rvert^2$ is	3i	2 + 3i	2
4	The exponential form of $\frac{\cos\theta-i\sin\theta}{\sqrt{3}+i}$ is	$\frac{1}{2}e^{i\left(-\theta-\frac{\pi}{6}\right)}$	$2e^{i\left(-\theta-\frac{\pi}{6}\right)}$	$\frac{1}{2}e^{i\left(\theta-\frac{\pi}{6}\right)}$

II– (4 points)

The space is referred to a direct orthonormal system $(O\,;\vec{i},\vec{j},\vec{k})$.
Consider the points A (2; –2; –1) , B(1 ; 0; –2), C(2; 1 ; –1), and the plane (P) with equation $x-2y+z+1=0$.

1) Show that $x-z-3=0$ is an equation of the plane (Q) determined by A, B and C.
2) a- Prove that (P) and (Q) are perpendicular and they intersect along the line (BC).
   b - Calculate the distance from A to (BC).
3) Let (d) be the line defined by:
   $$\begin{cases} x=t-1 \\ y=t+1 \\ z=t+2 \end{cases}$$ where t is a real parameter.
   a- Verify that (d) is contained in (P).
   b- Let M be a variable point on (d). Prove that, as M moves on (d), the area of triangle MBC remains constant.

III– (4 points)

Consider two urns **U** and **V**.
Urn **U** contains eight balls: four balls numbered 1, three balls numbered 2 and one ball numbered 4.
Urn **V** contains eight balls: three balls numbered 1 and five balls numbered 2.

1) **Two balls are selected, simultaneously and randomly, from the urn U**.

   Consider the following events:
   - A : « the two selected balls have the same number »
   - B : « the product of the numbers on the two selected balls is equal to 4 ».

   Calculate the probability P(A) of the event A, and show that P(B) is equal to $\frac{1}{4}$.

2) **One of the two urns U and V is randomly chosen, and then two balls are simultaneously and randomly selected from this urn.**

   Consider the following events:
   - E : « the chosen urn is **V** »
   - F : « the product of numbers on the two selected balls is equal to 4 ».

   a- Verify that $P(F \cap E) = \frac{5}{28}$ and calculate $P(F \cap \overline{E})$.

   b- Deduce P(F).

3) **One ball is randomly selected from U, and two balls are randomly and simultaneously selected from V.**

   Calculate the probability of the event H: « the product of the three numbers on the three selected balls is equal to 8 ».

IV– (8 points)

Let f be the function defined, over] 1 ; + ∞ [, by $f(x) = \ln\left(\frac{x+1}{x-1}\right)$.

Denote by (C) its representative curve in an orthonormal system $\left(O; \vec{i}, \vec{j}\right)$.

1) Determine $\lim\limits_{x \to 1} f(x)$ and $\lim\limits_{x \to +\infty} f(x)$. Deduce the asymptotes to (C).

2) Verify that $f'(x) = \frac{-2}{(x-1)(x+1)}$ and set up the table of variations of f.

3) Draw (C).

4) a- Prove that f has an inverse function g whose domain of definition is to be determined.

   b- Prove that $g(x) = \frac{e^x + 1}{e^x - 1}$.

   c- (G) is the representative curve of g in the same as that of (C). Draw (G).

5) Let h be the function defined over] 1 ; + ∞ [by $h(x) = x\, f(x)$.

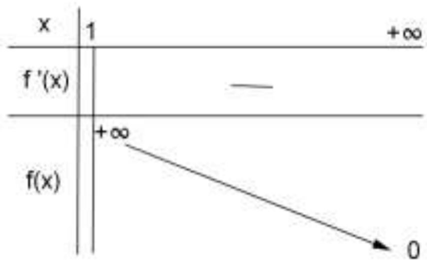
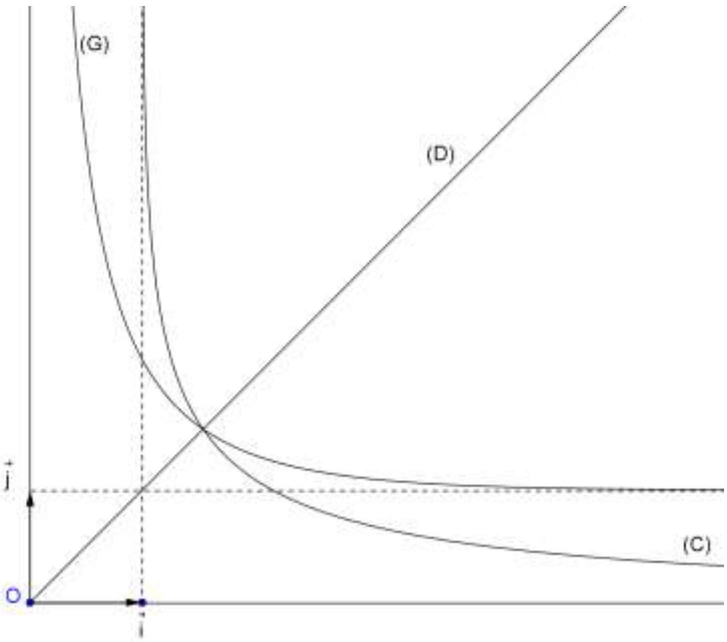
   a- Verify that $f(x) = h'(x) + \frac{2x}{x^2 - 1}$ and determine, over] 1 ; + ∞ [, an antiderivative F of f.

   b- Calculate the area of the region bounded by (C), the x-axis and the two lines with equations $x = 2$ and $x = 3$.

I	Solution		Mark
1	$z-z'=1-i\sqrt{3},\ z-z'=2e^{-i\frac{\pi}{3}}=2e^{i\frac{5\pi}{3}}$.	b	1
2	if A(2i) and B(– 4i) then $\|z-2i\|=\|z+4i\| \Leftrightarrow AM=BM$, hence M moves on the perpendicular bisector of [AB] which is parallel to the axis of abscissas.	c	1
3	If $z = 2$ then $9 + 1 = 2(2^2 + 1)$ (true),	c	1
4	$\frac{\cos\theta - i\sin\theta}{\sqrt{3}+i} = \frac{e^{-i\theta}}{2e^{i\frac{\pi}{6}}} = \frac{1}{2}e^{i\left(-\theta-\frac{\pi}{6}\right)}$	a	1

II	Solution	Mark
1	$\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC}) = 0$ so an equation of (Q) is : $x - z - 3 = 0$. **Or** : prove that the coordinates of A, B and C verify the given equation.	0.5
2a	$\vec{N}$ (1 ; –2 ; 1) is a normal vector to (P) ; $\vec{N'}$ (1 ; 0 ; –1) is a normal vector to (Q) and $\vec{N}\,.\,\vec{N'} = 0$. (P) and (Q) are perpendicular. The coordinates of B and C verify the equation of (P).	1
2b	$d_{A/(BC)} = d_{A/(P)} = \frac{\|2+4-1+1\|}{\sqrt{1+4+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}$. **Or** : $d = \frac{\left\|\overrightarrow{AB}\wedge\overrightarrow{BC}\right\|}{\left\|\overrightarrow{BC}\right\|} = \sqrt{6}$.	1
3a	$t - 1 - 2t - 2 + t + 2 + 1 = 0$ so (d) is in (P).	0.5
3b	$\overrightarrow{BC}$ (1 ; 1 ; 1) is a direction vector to(d) then (d) // (BC) and the distance from M to (BC) is constant, so the area of triangle MBC remains constant. **Or** : calculate the distance from M(t–1 ; t+1 ; t+2) to (BC) which is equal to the distance from M to (Q) and show that it is independent of t. **Or** : calculate the area of triangle MBC : $\frac{1}{2}\left\|\overrightarrow{MB}\wedge\overrightarrow{BC}\right\| = \frac{1}{2}\sqrt{54}$ = constant.	1

III	Solution	Mark
1	$P(A) = \frac{C_4^2}{C_8^2} + \frac{C_3^2}{C_8^2} = \frac{9}{28}$; $P(B) = \frac{C_3^2}{C_8^2} + \frac{C_4^1 \times C_1^1}{C_8^2} = \frac{7}{28} = \frac{1}{4}$	1
2a	$P(F\cap E) = P(E)\times P(F/E) = \frac{1}{2}\times\frac{C_5^2}{C_8^2} = \frac{5}{28}$. $P(F\cap\overline{E}) = P(\overline{E})\times P(F/\overline{E}) = \frac{1}{2}\times P(B) = \frac{1}{2}\times\frac{1}{4} = \frac{1}{8}$.	1.5
2b	$P(F) = P(F\cap E) + P(F\cap\overline{E}) = \frac{5}{28} + \frac{1}{8} = \frac{17}{56}$.	0.5
3	P(product = 8) = P(2 ;{2 ,2}) + P(4 ;{2 ,1})= $\frac{3}{8}\times\frac{C_5^2}{C_8^2} + \frac{1}{8}\times\frac{C_3^1\times C_5^2}{C_8^2} = \frac{45}{224}$.	1

IV	Solution	Mark
1	$\lim\limits_{x\to1} f(x) = \lim\limits_{x\to1} \ln\left(\dfrac{x+1}{x-1}\right) = +\infty$ and $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \ln\left(\dfrac{x+1}{x-1}\right) = 0$ The lines with equations $x = 1$ and $y = 0$ are the asymptotes to (C).	1.5
2	$f'(x) = \dfrac{u'}{u} = \dfrac{\dfrac{-2}{(x-1)^2}}{\dfrac{(x+1)}{x-1}} = \dfrac{-2}{(x-1)(x+1)} < 0$ 	1
3		1.5
4a	Over]1 ; +∞[; f is continuous and strictly decreasing, so it has an inverse function g defined over]0 ;+∞[.	0.5
4b	$f(g(x)) = x$ gives $\ln\dfrac{g(x)+1}{g(x)-1} = x$; $\dfrac{g(x)+1}{g(x)-1} = e^x$ so $g(x) = \dfrac{e^x+1}{e^x-1}$. **Or :** $g(f(x)) = \dfrac{\dfrac{x+1}{x-1}+1}{\dfrac{x+1}{x-1}-1} = \dfrac{2x}{2} = x$	1
4c	(G) is the symmetric of (C) with respect to line (D) with equation $y = x$. (See figure – question 3)	1
5a	$h'(x) = f(x) + xf'(x) = f(x) - \dfrac{2x}{(x-1)(x+1)}$ so $f(x) = h'(x) + \dfrac{2x}{x^2-1}$. $F(x) = h(x) + \ln(x^2-1) = x\ln\left(\dfrac{x+1}{x-1}\right) + \ln(x^2-1).$	1.5
5b	$A = F(3) - F(2) = 3\ln2 + \ln8 - 2\ln3 - \ln3 = 2\ln8 - 3\ln3$; $A = (2\ln8 - 3\ln3)u^2$.	0.5

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الاثنين 1 تموز 2013 عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I-(4points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\vec{j}\,,\vec{k})$, consider the points:

A (4; 2; 0), B (2; 3; 1) and C (2; 2; 2).

1) Prove that triangle ABC is right at B.
2) Show that an equation of the plane (P) determined by the three points A, B and C is $x + y + z - 6 = 0$.
3) Let (Q) be the plane passing through A and perpendicular to (AB).
   a- Determine an equation of (Q).
   b- Denote by (D) the line of intersection of (P) and (Q), show that (D) is parallel to (BC).
4) Let H(5;3;1) be a point in (Q).
   a- Show that A is the orthogonal projection of H on (P).
   b- Calculate the volume of the tetrahedron HABC.

II-(4points)

A music store sells classical and modern musical albums only.
The customers of this store are surveyed and the results are as follows:

- 20% of these customers bought each a classical album.
- Out of those who bought a classical album, 70% bought a modern album.
- 22% of the customers bought each a modern album.

A customer of the store is interviewed at random. Consider the following events:
C: «the interviewed customer bought a classical album »
M: «the interviewed customer bought a modern album ».

1) Calculate the probability $P(C \cap M)$ and verify that $P(C \cap \overline{M}) = 0.06$.
2) Prove that $P(\overline{C} \cap \overline{M}) = 0.72$.
3) Calculate the probability that the customer bought at least one album.
4) Knowing that the customer didn't buy a modern album, calculate the probability that he bought a classical album.
5) The classical album is sold for 30 000LL and the modern one is sold for 20 000LL.
   Let X be the random variable that is equal to the sum paid by a customer.
   a- Justify that the possible values of X are: 0, 20 000,30 000 and 50 000. Then, determine the probability distribution of X.
   b- During the month of June, 300 customers visited this music store. Estimate the revenue of this store during that month.

III(4 points)

In the plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A, B and C with respective affixes $z_A = i$, $z_B = 3-2i$ and $z_C = 1$.

1) Prove that the points A, B and C are collinear.
2) Consider the complex number $w = z_C - z_A$.
   Write w in exponential form and deduce that w^{20} is a real negative number.

3) Let M be a point in the plane with affix z.
   a- Give a geometric interpretation to $|z-i|$ and $|z-1|$.
   b- Suppose that $|z-i| = |z-1|$; show that the point M moves on a line to be determined.
   c- Prove that if $(z-i)\times(\bar{z}+i) = 16$, then the point M moves a circle whose center and radius to be determined.

IV-(8points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = 3 - \dfrac{4}{e^{2x}+1}$.

Let (C) be its representative curve in an orthonormal system (**unit 2 cm**).

1) Calculate $\lim\limits_{x \to -\infty} f(x)$, $\lim\limits_{x \to +\infty} f(x)$ and deduce the asymptotes to (C).

2) Prove that f is strictly increasing over $\mathbb{R}$ and set up its table of variations.

3) The curve (C) has a point of inflection W with abscissa 0. Write an equation of (T), the tangent to (C) at the point W.

4) a- Calculate the abscissa of the point of intersection of (C) with the x-axis.

   b- Draw (T) and (C).

5) a- Verify that $f(x) = -1 + \dfrac{4e^{2x}}{e^{2x}+1}$ and deduce an antiderivative F of f.
   b- Calculate, in cm^2, the area of the region bounded by the curve (C), the x- axis, the y-axis and the line with equation $x = \ln 2$.

6) The function f has over $\mathbb{R}$ an inverse function g. Denote by (G) the representative curve of g.
   a- Specify the domain of definition of g.
   b- Show that (G) has a point of inflection J whose coordinates to be determined.
   c- Draw (G) in the same system as (C).
   d- Determine g(x) in terms of x.

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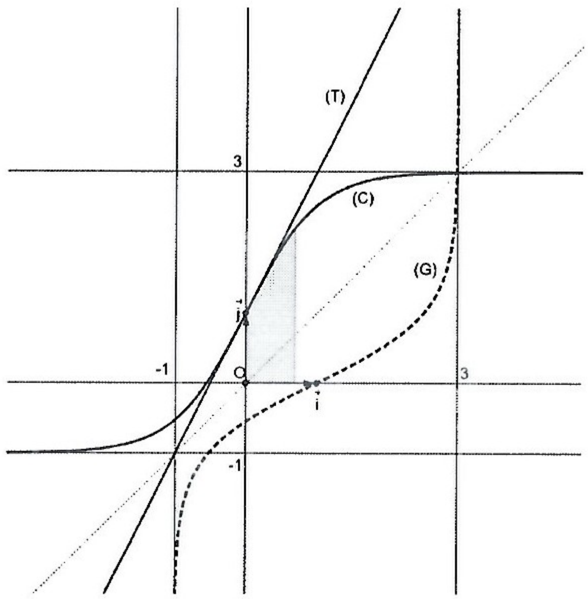
Q_1	Answers	M
1	$\overrightarrow{AB}(-2\,;1\,;1)$, $\overrightarrow{BC}(0\,;-1\,;1)$; $\overrightarrow{AB}\cdot\overrightarrow{BC}=0$, hence triangle ABC is right at B.	0.5
2	$x_A+y_A+z_A-6=0$, then A is in (P); $x_B+y_B+z_B-6=0$, then B belongs to (P) and $x_C+y_C+z_C-6=0$, then C is in (P) Therefore (P) : $x + y + z - 6 = 0$. **Or** $\overrightarrow{AM}.(\overrightarrow{AB}\wedge\overrightarrow{AC})=0$ with M(x: y; z) any point in (P).	0.5
3. a	For any point M (x, y, z) in (Q); $\overrightarrow{AM}\,.\,\overrightarrow{AB}=0$; (Q): $-2x+y+2z+6=0$.	0.5
3. b	A directing vector of (D) is $\vec{V}=\vec{n}_P\wedge\vec{n}_Q$, hence $\vec{V}(0; -3; 3)$, et $\overrightarrow{BC}(0; -1; 1)$ and $B\notin$ (Q), so $B\notin$ (D). Thus, (D) is parallel to (BC). **Or** : Since (BC) is perpendicular to (AB) and (AB) is perpendicular to (D) in A, (BC) and (D) being coplanar in (P) and perpendicular to the same line (AB), are parallel.	1
4. a	$A\in(P)$, $\overrightarrow{AH}(1;1;1)$ and $\vec{n}_P(1;1;1)$ hence (AH) is perpendicular to (P).	1
4. b	The volume of tetrahedron HABC is equal to $V=\frac{1}{3}$ HA × area of triangle ABC = $\frac{1}{6}\times BA\times BC\times\sqrt{3}=1u^3$. **Or** $V=\frac{\left\lvert\overrightarrow{AH}.(\overrightarrow{AB}\wedge\overrightarrow{AC})\right\rvert}{6}=\frac{6}{6}=1\ u^3$.	0.5

Q_2	Answers	M
1	$P(C\cap M)=P(C)\times P(M/C)=0.14$. $P(C\cap\overline{M})=P(C)\times P(\overline{M}/C)=0.06$.	1
2	$P(C\cap\overline{M})+P(\overline{C}\cap\overline{M})=P(\overline{M})=1-P(M)$ then $P(\overline{C}\cap\overline{M})=0.78-0.06=0.72$.	0.5
3	P (at least an album) = $1-P(\overline{C}\cap\overline{M})=0.28$.	0.5
4	$P(C/\overline{M})=\frac{P(C\cap\overline{M})}{P(\overline{M})}=\frac{0.06}{0.78}=\frac{1}{13}$.	0.5
5a	The four possible values are : 0 (the costumer did not buy anything), 20 000 (the costumer bought a modern album), 30 000 (the costumer bought a classical album), 50 000 (the costumer bought two albums). <table><tr><td>x_i</td><td>0</td><td>20 000</td><td>30 000</td><td>50 000</td></tr><tr><td>P_i</td><td>0.72</td><td>0.08</td><td>0.06</td><td>0.14</td></tr></table>	1
5b	$E(X)=\sum P_iX_i=0\times 0.72+20\,000\times 0.08+30\,000\times 0.06+50\,000\times 0.14=10\,400$ L L. R= $E(X)\times 300=10\,400\times 300=3\,120\,000$ LL.	0.5

Q_3	Answers	M
1	$z_A-z_B=-3+3i$ and $z_A-z_C=-1+i$. $z_A-z_B=3(z_A-z_C)$ and $z_{\overrightarrow{BA}}$ $z_{\overrightarrow{CA}}$; par suite $\overrightarrow{BA}=3\overrightarrow{CA}$ and the three points A, B and C are collinear.	0.5
2	$w=z_{\overrightarrow{AC}}=1-i=\sqrt{2}\,e^{-i\frac{\pi}{4}}$, $w^{20}=\left(\sqrt{2}\right)^{20}e^{-5i\pi}=-\left(\sqrt{2}\right)^{20}$ which is real negative.	1
3. a	$\lvert z-i\rvert=\lvert z_M-z_A\rvert=AM$; $\lvert z-1\rvert=\lvert z_M-z_C\rvert=CM$.	0.5
3. b	If z_M verifies $\lvert z-i\rvert=\lvert z-1\rvert$, so MA = MC; and the point M varies on the perpendicular bisector of segment [AC].	1
3. c	If z_M verify $(z-i)\times(\overline{z}+i)=16\Leftrightarrow(z_M-z_A)\times(\overline{z_M-z_A})=16\Leftrightarrow\lvert z_M-z_A\rvert\times\lvert\overline{z_M-z_A}\rvert=16$ $\Leftrightarrow\lvert z_M-z_A\rvert\times\lvert z_M-z_A\rvert=16$. Hence $AM^2=16$; therefore the point M belongs to the circle with center A and radius 4.	1

<table>
<tr><th>Q_4</th><th>Answers</th><th>M</th></tr>
<tr><td>1</td><td>$\lim\limits_{x\to-\infty} f(x) = 3 - 4 = -1$ and $\lim\limits_{x\to+\infty} f(x) = 3$. Hence (C) has two asymptotes with equations y = 3 and y = – 1.</td><td>1</td></tr>
<tr><td>2</td><td>$f'(x) = \dfrac{8e^{2x}}{(e^{2x}+1)^2} > 0$; f is strictly increasing over IR.

$$\begin{array}{c|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & & & 3 \\ & -1 & \nearrow & \end{array}$$

</td><td>1</td></tr>
<tr><td>3</td><td>Slope of (T) = f '(0) = 2, and (T) passes through the point W (0 ; 1), then the equation of (T) is : y = 2 x + 1.</td><td>0.5</td></tr>
<tr><td>4. a</td><td>$f(x) = 0 \Leftrightarrow 3 = \dfrac{4}{e^{2x}+1} \Leftrightarrow e^{2x} = \dfrac{1}{3} \Leftrightarrow x = -\dfrac{\ln 3}{2}$.</td><td>0.5</td></tr>
<tr><td>4.b</td><td>
</td><td>1</td></tr>
<tr><td>5-a</td><td>$f(x) = 3 - \dfrac{4}{e^{2x}+1} = \dfrac{3e^{2x}-1}{e^{2x}+1}$ et $1 + \dfrac{4e^{2x}}{e^{2x}+1} = \dfrac{3e^{2x}-1}{e^{2x}+1}$.
$F(x) = \int f(x)\,dx = \int\left(-1+\dfrac{4e^{2x}}{e^{2x}+1}\right)dx = -x + 2\int \dfrac{2e^{2x}}{e^{2x}+1}\,dx = -x + 2\ln\left(e^{2x}+1\right) + c$.</td><td>1</td></tr>
<tr><td>5-b</td><td>$A = 4A'\ cm^2$.
$A' = \displaystyle\int_0^{\ln 2} f(x)\,dx = \left[-x + 2\ln\left(e^{2x}+1\right)\right]_0^{\ln 2} = 2\ln 5 - 3\ln 2 = \ln\left(\frac{25}{8}\right)$. Thus, $A = 4\ln\left(\dfrac{25}{8}\right) cm^2$.</td><td>0.5</td></tr>
<tr><td>6. a</td><td>Dom (g) =]–1; 3[.</td><td>0.5</td></tr>
<tr><td>6. b</td><td>W (0,1) is a point of inflection of (C), then the symmetric of W with respect to the line with equation y = x is the point J (1 ; 0), which is the point of inflection of (G).</td><td>0.5</td></tr>
<tr><td>6. c</td><td>(G) is the symmetric of ©with respect to the line with equation y = x.</td><td>0.5</td></tr>
<tr><td>6. d</td><td>$y = g(x) \leftrightarrows x = f(y) \leftrightarrows x = 3 - \dfrac{4}{e^{2y}+1} \leftrightarrows \dfrac{4}{e^{2y}+1} = 3 - x \leftrightarrows e^{2y} + 1 = \dfrac{4}{3-x} \leftrightarrows$
$e^{2y} = \dfrac{4}{3-x} - 1 = \dfrac{1+x}{3-x}$. Thus, $2y = \ln\left(\dfrac{1+x}{3-x}\right)$; $y = g(x) = \dfrac{1}{2}\ln\left(\dfrac{1+x}{3-x}\right)$.</td><td>1</td></tr>
</table>

عدد المسائل اربع	مسابقة في الرياضيات المدة ساعتان	الاسم: الرقم:

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I- (4 points)

In the space referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$, consider the point A (6 ; 3 ; 2), the plane (P) with equation $x - y + 2z - 7 = 0$ and the line (d) with parametric equations:

$\begin{cases} x = t \\ y = t - 3 \\ z = -1 \end{cases}$ where t is a real parameter.

1) Show that A is a point in (P) and that (d) is parallel to (P).

2) a- Verify that the point C (1 ; –2 ; –1) is on (d).

b- Determine a system of parametric equations of line (L) passing through C and perpendicular to (P).

c- Show that the point E $(3; -4; 3)$ is the symmetric of C with respect to (P).

d- Deduce a system of parametric equations of line (Δ) symmetric of line (AC) with respect to (P).

II- (4 points)

An urn contains seven balls: four red balls and three green balls.

A player selects randomly and simultaneously three balls from this urn.

1) a- Calculate the probability that the player selects exactly two red balls.

b- Show that the probability that the player selects at least two red balls is equal to $\frac{22}{35}$.

2) After selecting three balls, the player scores:

- 9 points if he gets three red balls;
- 6 points if he gets exactly two red balls;
- 4 points if he gets exactly one red ball;
- zero if he gets three green balls.

Denote by X be the random variable that is equal to the score of the player.

a- Determine the probability distribution of X.

b- Knowing that the player scored more than 2 points, calculate the probability that his score is multiple of 3.

III- (4 points)

The complex plane is referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$.

A- Consider the points A and B with respective affixes $z_A = 2+2i$ and $z_B = \left(1+\sqrt{3}\right)\left(-1+i\right)$.

1) Determine the exponential form of the complex number $\frac{z_B}{z_A}$.

2) Prove that the triangle OAB is right at O.

B- To every point M with affix z $(z \neq 0)$, associate the point M′ with affix z′ such that $z' = 1+i-\frac{2}{z}$.

Let $z = x + iy$ with x and y are two real numbers.

1) Express, in terms of x and y, the real part and the imaginary part of the complex number z'.

2) Prove that if the real part of z' is zero, then M moves on a circle whose center and radius are to be determined.

IV- (8 points)

A- Consider the function g defined over $]0; +\infty[$ as $g(x) = x^2 - 2\ln x$.

1) Determine $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.

2) Set up the table of variations of g and deduce that $g(x) > 0$.

B- Let f be the function defined over $]0;+\infty[$ as $f(x) = \frac{x}{2} + \frac{1+\ln x}{x}$ and let (C) be its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Determine $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).

2) a- Determine $\lim\limits_{x\to +\infty} f(x)$ and show that the line (Δ) with equation $y = \frac{x}{2}$ is an asymptote to (C).

b- Study, according to the values of x, the relative positions of (C) and (Δ).

3) Show that $f'(x) = \frac{g(x)}{2x^2}$ and set up the table of variations of f.

4) Calculate the coordinates of the point B on (C) where the tangent (T) is parallel to (Δ).

5) Show that the equation $f(x) = 0$ has a unique solution α, then verify that $0.34 < \alpha < 0.35$.

6) Plot (Δ), (T) and (C).

7) Let h be the function defined over $]0;+\infty[$ as $h(x) = \frac{1+\ln x}{x}$.

a- Find an antiderivative H of h.

b- Deduce the measure of the area of the region bounded by (C), (Δ) and the lines with equations $x = 1$ and $x = e$.

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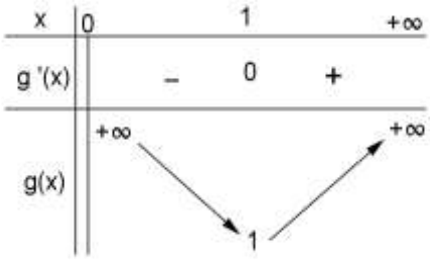
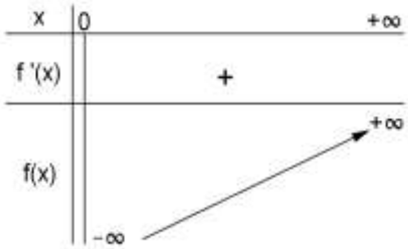
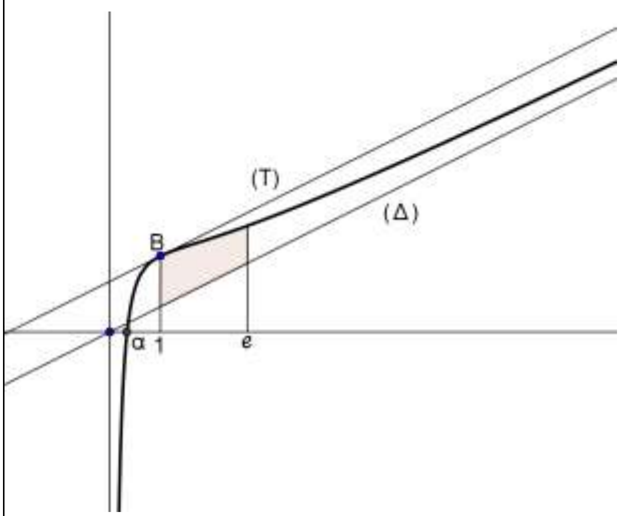
Q1	Answers	M
1	$x_A - y_A + 2z_A - 7 = 0$ then $A \in (P)$. and t –t+3 –2 –7= $-6 \neq 0$. Hence (d) is parallel to (P).	1
2a	For: $x = x_C = 1$, $t = 1$, $y = y_C = -2$, and $z = z_C = -1$; hence $C \in (d)$.	0.5
2b	$\overrightarrow{v_L}$ is parallel to $\overrightarrow{n_P}(1;-1;2)$ and (L) passes through C, hence a system of parametric equations of (L) is : x = m +1, y = – m –2, z = 2m –1 where m is a real parameter.	0.5
2c	(L) intersects (P) at point I (m +1, – m –2, 2m –1), and $I \in (P)$, hence m = 1and $I(2;-3;1)$. Moreover, I is the midpoint of [EC], hence : $x_E = 2x_I - x_C = 3$, $y_E = 2y_I - y_C = -4$ and $z_E = 2z_I - z_C = 3$. **OR :** $\overrightarrow{CE}(2;-2;4)$, $\overrightarrow{CE} = 2\overrightarrow{n_P}$, (CE) is perpendicular to(P) and the point $I(2;-3;1)$ midpoint of [CE], is in (P), hence C and E are symmetric with respect to (P).	1.5
2d	The line (Δ) passes through A and E, hence $\overrightarrow{AM} = k\overrightarrow{AE}$, x = –3k + 6, y = –7k +3, z = k + 2.	0.5

Q_2	Answers	M
1a	$P\{2R,1G\} = \dfrac{C_4^2 \times C_3^1}{C_7^3} = \dfrac{18}{35}$.	1
1b	$P\{2R,1G\} + P\{3R\} = \dfrac{18}{35} + \dfrac{C_4^3}{C_7^3} = \dfrac{22}{35}$.	1
2a	$P(X=9) = P(3R) = \dfrac{C_4^3}{C_7^3} = \dfrac{4}{35}$. $P(X=6) = P\{2R,1G\} = \dfrac{18}{35}$. $P(X=4) = P\{1R,2G\} = \dfrac{C_4^1 \times C_3^2}{C_7^3} = \dfrac{12}{35}$. $P(X=0) = P(3G) = \dfrac{C_3^3}{C_7^3} = \dfrac{1}{35}$.	1
2b	$P(\text{Score multiple of 3/ Score} > 2) = \dfrac{22}{35} \div \dfrac{34}{35} = \dfrac{11}{17}$.	1

Q_3	Answers	M
A1	$\dfrac{z_B}{z_A} = \dfrac{(1+\sqrt{3})(-1+i)}{2(1+i)} = \dfrac{1+\sqrt{3}}{2}i = \dfrac{1+\sqrt{3}}{2}e^{i\frac{\pi}{2}}$.	1
A2	$\overrightarrow{OA} \cdot \overrightarrow{OB} = -2(1+\sqrt{3}) + 2(1+\sqrt{3}) = 0$. **OR** : $\dfrac{z_B}{z_A}$ is pure imaginary hence $(\overrightarrow{OA}, \overrightarrow{OB}) = \arg\left(\dfrac{1+\sqrt{3}}{2}i\right) = \dfrac{\pi}{2}$ then $(OB) \perp (OA)$.	0.5
B1	$z' = 1 + i - \dfrac{2}{x+iy} = 1 + i - \dfrac{2x - 2iy}{x^2+y^2}$. $\mathrm{Re}(z') = 1 - \dfrac{2x}{x^2+y^2}$, $\mathrm{Im}(z') = 1 + \dfrac{2y}{x^2+y^2}$.	1
B2	$\mathrm{Re}(z') = 1 - \dfrac{2x}{x^2+y^2} = 0 \Rightarrow x^2 + y^2 - 2x = 0 \Leftrightarrow (x-1)^2 + y^2 = 0$ hence M moves on the circle with center (1 ; 0) and radius 1 deprived from O.	1.5

Q_4	Answers	M
A.1	$\lim\limits_{x\to 0} g(x)=+\infty$ $\lim\limits_{x\to+\infty} g(x)=\lim\limits_{x\to+\infty} x\left(x-2\frac{\ln x}{x}\right)=+\infty$. (since $\lim\limits_{x\to+\infty}\frac{\ln x}{x}=0$)	0.5
A.2	$g'(x)=2x-\frac{2}{x}=\frac{2(x+1)(x-1)}{x}$. g(x) has a minimum equal to 1, hence g(x) > 0 for x > 0. 	1
B.1	$\lim\limits_{x\to 0} f(x)=-\infty$ then the line with equation x = 0 is an asymptote to (C).	0.5
B.2.a	$\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\left(\frac{x}{2}+\frac{\ln x}{x}+\frac{1}{x}\right)=+\infty$ and $\lim\limits_{x\to+\infty}\left[f(x)-\frac{x}{2}\right]=\lim\limits_{x\to+\infty}\left(\frac{\ln x}{x}+\frac{1}{x}\right)=0$ .Hence the line (Δ) with equation $y=\frac{x}{2}$ is an asymptote to (C).	0.5
B.2. b	$f(x)-\frac{x}{2}=\frac{1}{x}(\ln x+1)=0$ for $x=\frac{1}{e}$ then $y=\frac{1}{2e}\Rightarrow A\left(\frac{1}{e};\frac{1}{2e}\right)$ is the point of intersection of (Δ) and (C). For $x>\frac{1}{e}$, (C) is above (Δ) and for $0<x<\frac{1}{e}$ (C) is below (Δ).	1
B.3	$f'(x)=\frac{1}{2}+\frac{\left(\frac{1}{x}\right)(x)-\ln x-1}{x^2}=\frac{1}{2}-\frac{\ln x}{x^2}=\frac{g(x)}{2x^2}$. 	1
B.4	$f'(x_B)=\frac{1}{2}\Rightarrow\frac{x^2-2\ln x}{2x^2}=\frac{1}{2}\Rightarrow x=1$ hence $B\left(1;\frac{3}{2}\right)$.	0.5
B.5	f is continuous and strictly increasing from –∞ to +∞ hence the equation f(x) = 0 has a unique solution α .Moreover f(0.34) = – 0.061 < 0 and f(0.35) = 0.032 > 0 hence $0.34<\alpha<0.35$.	1
B.6		1
B7a	$H(x)=\int h(x)\,dx=\frac{(\ln x)^2}{2}+\ln x+c$ $=\frac{(1+\ln x)^2}{2}+k$.	0.5
B7b	$A=\int_1^e\left(f(x)-\frac{x}{2}\right)dx$ $=\int_1^e\frac{1+\ln x}{x}dx$ $=H(e)-H(1)=\frac{3}{2}\ u^2$.	0.5

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة العام ٢٠١٤ العادية
الخميس ١٩ حزيران ٢٠١٤

عدد المسائل: اربع

مسابقة في مادة الرياضـــــيات
المدة: ساعتان

الاسم:
الرقم:

ملاحظة: يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
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I - (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the point A (2; 0;-2) and the line (d) with parametric equations $\begin{cases} x = t-2 \\ y = -t \\ z = 3 \end{cases}$ $(t \in \mathbb{R})$

1) a- Verify that A is not on (d).

   b- Show that $x + y + z = 0$ is an equation of (P), the plane containing (OA) and parallel to (d).

2) Let (d') be the line through O and parallel to (d).

   a- Write a system of parametric equations of (d').

   b- Prove that the point E (-1;-1; 3) is the orthogonal projection of O on (d).

   c- Calculate the distance between (d) and (d').

3) Consider in the plane (P) the circle (C) with center O and radius OA.
   Calculate the coordinates of B and L, the meeting points of (d') and (C).

II - (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points M and M' with respective affixes z and z' such that $z' = \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)z$. $(z \neq 0)$

1) Prove that OM' = OM and that $\arg(z') = \frac{\pi}{3} + \arg(z)$.

2) Let $z = x+iy$ and $z' = x' + iy'$ where x, x', y and y' are real numbers.

   a- Express x' and y' in terms of x and y.

   b- If z' is real, show that M moves on a line to be determined.

3) In what follows, suppose that $z = e^{-i\frac{\pi}{4}}$.

   a- Write z' in algebraic form.

   b- Write z' in exponential form then deduce the exact value of $\cos\left(\frac{\pi}{12}\right)$.

III - (4 points)

An urn contains two red balls numbered 0, three green balls numbered 1 and five blue balls numbered 2. Three balls are randomly and simultaneously selected from the urn. Consider the following events:

- A: « The three selected balls have the same color ».
- B: « The product of the numbers on the three selected balls is 2 ».
- C: « The product of the numbers on the three selected balls is not equal to zero ».

1) a- Calculate P(A) , the probability of the event A.

   b- Calculate P (B).

   c- Show that $P(C)=\frac{7}{15}$.

2) Knowing that the product of the numbers on the three selected balls is not equal to zero, calculate the probability that these three balls have the same color.

3) Denote by X the random variable equal to the number of red balls obtained in the selection of the three balls.

   a- What are the possible values of X?

   b- Show that $P(X=1)=\frac{7}{15}$.

   c- Determine the probability distribution of X.

IV- (8 points)

Let f be the function defined on $\mathbb{R}$ as $f(x)=x+e^{x+1}$ and (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$ and show that the line (d) with equation $y=x$ is an asymptote to (C).

   b- Specify the position of (C) with respect to (d).

   c- Determine $\lim\limits_{x\to+\infty} f(x)$. Calculate $f(-1)$, $f(0)$ and $f(1)$.

2) Show that f is strictly increasing on $\mathbb{R}$ and set up its table of variations.

3) Show that the equation f(x)=1 has a unique solution α such that $-0.6<\alpha<-0.5$.

4) Draw (d) and (C).

5) a- Show that the function f has ,on $\mathbb{R}$, an inverse function g whose domain is to be determined.

   b- Draw the curve (Γ) of g in the same system as that of (C).

   c- Calculate the area of the region bounded by (Γ), the x-axis, and the y-axis.

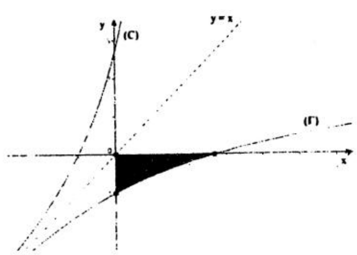
الدورة العادية 2014	تصحيح الرياضيات	علوم الحياة

Q1	Solution **4pts**
1a	$x_A = t-2$ so $t=4$; $y_A = -t$ so $t=0$ impossible then $A \notin (d)$
1b	$\vec{OA}(2;0;-2)$; $\vec{V}_d(1;-1;0)$ direction of (d) ; $(P) = \vec{OM}.(\vec{OA}\times \vec{V}_d) = 0$ $\begin{vmatrix} x & y & z \\ 2 & 0 & -2 \\ 1 & -1 & 0 \end{vmatrix} = 0$ then (P): $-2x-2y-2z=0$ $\therefore$ equation of $(P): x+y+z=0$
2a	$\vec{V}_d(1;-1;0)$ is a direction of (d') and $O \in (d')$ so $(d'): x=m$; $y=-m$; $z=0$
2b	For t=1 we have $x=-1$ $y=-1$ and $z=3$ $\therefore$ $E \in (d)$ and $\vec{OE}\bullet\vec{V}_d = -1(1)+(-1)(-1)+3(0)=0$ $\therefore$ E is the orthogonal projection of O on (d)
2c	(d) //(d') and $OE \perp (d)$; $OE \perp (d')$ so, $OE = \sqrt{1+1+3} = \sqrt{11}$ is the distance between (d) and (d')
3	$O \in (P)$; (d') passes through O and parallel to (d) and since (d)//(P) then $(d') \subset (P)$ Then the points B & L are in (P) ; O center of (C) so OA=OB=OL $B \in (d')$ so B(m;-m ;0) $OB^2 = OA^2$ then $m^2 = 4$ and $m = \pm 2$ For m=2 $B(2;-2;0)$ for $m=-2$ $L(-2;2;0)$

Q2	Solution **4pts**
1	let $a = \frac{1}{2} + i\frac{\sqrt{3}}{2} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ so; $\lvert a\rvert = 1$ & $\arg(a) = \frac{\pi}{3}$ $z' = az$ $\lvert z'\rvert = \lvert az\rvert = 1\times\lvert z\rvert$ then OM' = OM & $\arg(z') = \frac{\pi}{3} + \arg(z)$
2a	$x'+iy' = \left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)(x+iy)$ so; $x' = \frac{1}{2}(x-y\sqrt{3})$ & $y' = \frac{1}{2}(x\sqrt{3}+y)$
2b	z' is real if $y' = \frac{1}{2}(x\sqrt{3}+y) = 0$ then $y = -x\sqrt{3}$ M moves on the st-line of equation $y = -x\sqrt{3}$
3a	$z = e^{-i\frac{\pi}{4}} = \cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(1-i)$ so; $z' = \frac{1}{2}(1+i\sqrt{3})\left(\frac{\sqrt{2}}{2}(1-i)\right) = \frac{\sqrt{6}+\sqrt{2}}{4} + i\frac{\sqrt{6}-\sqrt{2}}{4}$
3b	$z' = e^{i\frac{\pi}{3}}e^{-i\frac{\pi}{4}} = e^{i\frac{\pi}{12}} = \cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)$ then $\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$ & $\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$

Q3	Solution **4 pts**
1a	P(A)= P(3 green or 3blue) = $\frac{C_3^3 + C_5^3}{C_{10}^3} = \frac{11}{120}$
1b	P(B) = P(2 balls numbered 1& a ball numbered 2) = $\frac{C_3^2 + C_5^1}{C_{10}^3} = \frac{1}{8}$

1c	P(C) = P(the balls are numbered 1 or 2) = $\frac{C_8^3}{C_{10}^3}=\frac{7}{15}$
2	$P(A/C)=\frac{P(A\cap C)}{P(C)}=\frac{P(A)}{P(C)}=\frac{11}{120}\times\frac{15}{7}=\frac{11}{56}$
3a	The possible values of X are : 0 ; 1 ; 2
3b	$P(X=1)=P(R\overline{RR})=\frac{C_2^1\times C_8^2}{C_{10}^3}=\frac{7}{15}$
3c	$P(X=0)=P(\overline{RRR})=\frac{C_8^3}{C_{10}^3}=\frac{7}{15}$; $P(X=2)=P(RR\overline{R})=\frac{C_2^2\times C_8^1}{C_{10}^3}=\frac{1}{15}$; $P(X=1)=\frac{7}{15}$

Q4	Solution	8 pts
1a	$\lim\limits_{x\to-\infty} f(x)=-\infty+0=-\infty$ $\lim\limits_{x\to-\infty}\left(f(x)-y_d\right)=\lim\limits_{x\to-\infty} e^{x+1}=e^{-\infty}=0$ so; (d) y= x (ob.As) to (C)	
1b	$f(x)-y_d=e^{x+1}>0$ then (C) is above (d)	
1c	$\lim\limits_{x\to+\infty} f(x)=+\infty+\infty=+\infty$; $f(-1)=0$; $f(0)=1+e=3.7$; $f(1)=1+e^2\approx 8.4$	
2	$f'(x)=1+e^{x+1}>0$ so; f is strictly increasing x: $-\infty$ … $+\infty$; f'(x): +; f(x): $-\infty$ ↗ $+\infty$	
3	f is continuous & strictly increasing from $-\infty$ to $+\infty$ so $f(x)=1$ admits a unique solution α Since $f(-0.6)=0.89<1$ & $f(-0.5)=1.14>1$ then $-0{,}6<\alpha<-0{,}5$ Or let $h(x)=f(x)-1$ since f is continuous & strictly increasing so h is continuous & strictly increasing & $h(-0.6)\times h(-0.5)=(-0.1)(0.14)<0$ then $-0{,}6<\alpha<-0{,}5$	
4		
5a	f is continuous & strictly increasing over IR therefore f admits an inverse function g defined on IR	
5b	See figure part 4 : (Γ) and (C) are symmetric with respect to y = x	
5c	The area limited by (Γ) the x-axis and the y-axis is equal to that limited by (C) the x-axis and the y-axis by symmetry so; $A=\int_{-1}^{0}(x+e^{x+1})dx=\left[\frac{x^2}{2}+e^{x+1}\right]_{-1}^{0}=(e-\frac{3}{2})$ u²	

وزارة التربية والتعليم العالي   امتحانات الشهادة الثانوية العامة   دورة العام 2015 العادية
المديرية العامة للتربية   فرع علوم الحياة   السبت 13 حزيران 2015
دائرة الامتحانات

عدد المسائل : اربع	مسابقة في مادة الرياضيات المدة ساعتان	الاسم: الرقم:

ارشادات عامة :- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

I-(4 points)

The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$.

Consider the plane (P) with equation $x-2y+2z-6=0$ and the two lines (d) and (d') defined as:

$$(d):\begin{cases}x=m+1\\ y=2m+1\\ z=2m+2\end{cases}\quad \text{and} \quad (d'):\begin{cases}x=2t\\ y=5t-3\\ z=4t\end{cases}\quad (m \text{ and } t \text{ are real parameters})$$

1) Find the coordinates of A, the intersection point of line (d) and plane (P).

2) Verify that A is on line (d'), and that (d') is contained in plane (P).

3) a- Write an equation of plane (Q) determined by the lines (d) and (d').

b- Show that the two planes (P) and (Q) are perpendicular.

4) Let $B(1;1;2)$ be a point on (d).

Calculate the distance from point B to line (d').

II- (4 points)

The complex plane is referred to a direct orthonormal system $(O;\ \vec{u},\ \vec{v})$.

Consider the pointsA (– i), B (– 2) and M (z) where z is a complex number different from -2.

Let M' be the point with affix z' such that $z'=\dfrac{1-iz}{z+2}$.

1) a- Find the algebraic form of the complex number $(z'+i)(z+2)$.

b- Give a geometric interpretation to $|z'+i|$ and $|z+2|$, then deduce that $AM'\times BM=\sqrt{5}$.

c- As M moves on the circle with center B and radius 1, show that M' moves on a circle whose center and radius are to be determined.

2) Suppose that $z=-2+iy$ with y a nonzero real number.

a- Find in terms of y the algebraic form of z'.

b- Determine the point M for which z' is real.

III-(4 points)

Consider two urns A and B.

- Urn Acontains two balls numbered 1 and 2.
- Urn B contains four balls numbered 1, 2, 3 and 4.

1)One of the two urns A and B is randomly chosen, after which a ball is randomly selected from thisurn.

Consider the following events:

- A: « the chosen urn is A»;
- N: « the selected ball is numbered 1».

a- Calculate the probabilities$P(N/A)$ and $P(N\cap A)$.

b- Show that $P(N)=\frac{3}{8}$ and deduce$P(A/N)$.

2) In this part, the six balls from the two urns A and B are placed in one urn W.
Two balls are selected randomly and simultaneouslyfrom the urn W.
Consider the following events:

- E: « the two selected balls carry the same number »;
- F: « the sum of numbers carried by the two selected balls is odd ».

a- Verify that $P(E)=\frac{2}{15}$.

b- Calculate P(F) and $P(F/_{E})$

IV- (8 points)

A-

Let g be the function defined on IRas $g(x)=x-1+e^{x}$.

1) Show that g is strictly increasing onIR. Set up the table of variations of g.

2) Calculate $g(0)$, then study according to the values of x the sign of $g(x)$.

B-

Let f be the function defined on IR as $f(x)=\frac{(x-2)e^{x}}{1+e^{x}}$ and (C) its representative curve in an orthonormalsystem $(O;\vec{i},\vec{j})$. Denote by (Δ) the line with equation $y=x-2$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$. Deduce an asymptote to (C).

2) Study, according to the values of x, the relative positions of (C) and (Δ).

3) Determine $\lim\limits_{x\to+\infty} f(x)$ and show that (Δ) is anasymptote to (C).

4) Showthat $f'(x)=\frac{e^{x}g(x)}{(1+e^{x})^{2}}$, then set up the table of variations of f.

5) Plot (Δ) and (C).

6) The function f has over $[0;+\infty[$ an inverse function h. Calculate $h'(0)$.

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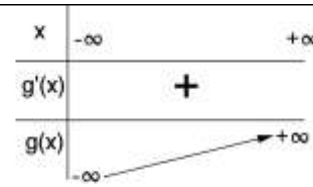
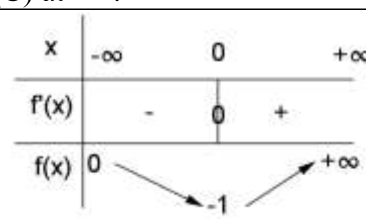
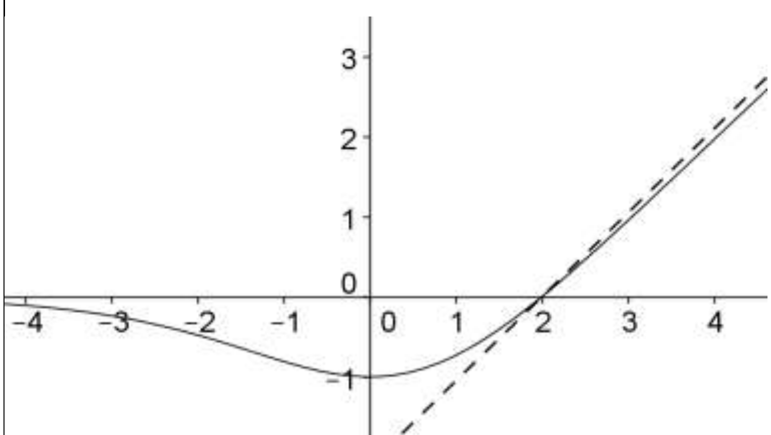
دائرة الامتحانات		

عدد المسائل : اربع	مسابقة في مادة الرياضيات	الاسم:
	المدة ساعتان	الرقم:

Q1	Answers	M
1	m + 1 – 4m- 2+4m - 6 = 0; m = 3 then A(4;7;8)	1
2	4=2t ; 7 = 5t – 3; 8 = 4t thus t = 2 unique value,hence, A belongs to (d') 2t – 10t +6 +8t -6 =0. So (d') is included in (P).	½
3a	$\overrightarrow{AM}.\left(\vec{V}\wedge\vec{V'}\right)=0$; $\begin{vmatrix} x-4 & y-7 & z-8 \\ 1 & 2 & 2 \\ 2 & 5 & 4 \end{vmatrix}=0$; $2x-z=0:(Q)$	1
3b	$\vec{N}.\vec{N'}=2+0-2=0$; $(P)\perp(Q)$	½
3c	$B\in(d)$ and $(P)\perp(Q)$ and as (d') is the intersection line of the two planes (P) and (Q) hense: $d\left(B;(d')\right)=d\left(B;(P)\right)=\dfrac{\lvert x_B-2y_B+2z_B-6\rvert}{\sqrt{1+4+4}}=\dfrac{3}{3}=1$	1

Q2	Answers	M
1a	$z'+i=\dfrac{1-iz+i\ z+2i}{z+2}=\dfrac{1+2i}{z+2}$ hence $(z'+i)(z+2)=1+2i.$	1
1b	$\lvert z'+i\rvert=AM';\lvert z+2\rvert=BM$ $AM'\times BM=\lvert z'+i\rvert\times\lvert z+2\rvert=\lvert 1+2i\rvert=\sqrt{5}.$	1
1c	$M\in C(B;1)$; $BM=1$; $AM'=\sqrt{5}$ and M' moves on the circle with center A and radius $\sqrt{5}$.	½
2a	$z=-2+iy;z'=\dfrac{2}{y}+i\dfrac{-y-1}{y}$	1
2b	z' is real ; Im(z')=0 ;-y-1=0 then $M(-2;-1)$	½

Q3		Answers	M
1	a	P (N/A)= $\dfrac{1}{2}$; $P(N\cap A)=P(A)\times P(N/A)=\dfrac{1}{2}\times\dfrac{1}{2}=\dfrac{1}{4}.$	1
	b	P(N) = $P(N\cap A)+P(N\cap\overline{A})=\dfrac{1}{4}+\dfrac{1}{8}=\dfrac{3}{8}$; P(A/N)= $\dfrac{P(A\cap N)}{P(N)}=\dfrac{1/4}{3/8}=\dfrac{2}{3}$	1
2	a	P(E) $=\dfrac{C_2^2}{C_6^2}+\dfrac{C_2^2}{C_6^2}=\dfrac{2}{15}.$	½
	b	The outcomes of F are : (1 and 2) or (1 and 4) or (2 and 3) or (3 and 4), $P(F)=\dfrac{C_2^1\times C_2^1+C_2^1\times C_1^1+C_2^1\times C_1^1+C_1^1\times C_1^1}{C_6^2}=\dfrac{3}{5}$ and $P\left(F/\overline{E}\right)=\dfrac{P(F\cap\overline{E})}{p\left(\overline{E}\right)}=\dfrac{P(F)}{1-p(E)}=\dfrac{9}{13}.$	1 ½

Q4		Answers	M
A	1	g'(x) =1+ $e^x > 0$ for all x hence g is strictly increasing over IR . 	1
	2	g is strictly increasing over IR and g (0) = 0. If x< 0 then g(x) < 0. for x > 0 then g(x) > 0.	1
B	1	$\lim\limits_{x\to-\infty} f\left(x\right)=\dfrac{\lim\limits_{x\to-\infty}(x-2)e^x}{\lim\limits_{x\to-\infty}(1+e^x)}=0.$ The x-axis is an asymptote to (C) at -∞	1
	2	f (x) –(x–2) = $\dfrac{-x+2}{1+e^x}$. Hence, (C) is above (d) for x <2, (C) is below (d) for x >2 and (C) intersects (d) at x=2.	1
	3	$\lim\limits_{x\to+\infty} f\left(x\right)=+\infty$ $\lim\limits_{x\to-\infty}[f\left(x\right)-(x-2)]=\lim\limits_{x\to+\infty}\dfrac{-x}{e^x}=0.$ So the line with equation y = x-2 is an asymptote to (C) at +∞.	1
	4	$f'(x)=\dfrac{\left[e^x+e^x(x-2)\right]\left[1+e^x\right]-e^x(x-2)e^x}{(1+e^x)^2}$ $=\dfrac{e^x(x-1+e^x)}{(1+e^x)^2}=\dfrac{e^xg(x)}{(1+e^x)^2}.$ 	1
	5		1
	6	Since f(2) = 0 henceh'(0)= $\dfrac{1}{f'(2)}=\dfrac{1+e^2}{e^2}$	1

ملاحظة : - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة)

I-(4 points)

The space is referred to a direct orthonormal system $(O; \vec{i}, \vec{j}, \vec{k})$.

Consider the plane (P) with equation $x - 2y + 2z - 6 = 0$ and

the line (d) with parametric equations $\begin{cases} x = m+1 \\ y = 2m+1 \\ z = 2m+2 \end{cases}$ $(m \in \mathbb{R})$.

Let (Q) be the plane containing (d) and perpendicular to (P) and A (1; 1; 2) a point on (d).

1) Show that $2x - z = 0$ is an equation of the plane (Q).

2) Prove that the line (Δ) with parametric equations $\begin{cases} x = 2t \\ y = 5t - 3 \\ z = 4t \end{cases}$ $(t \in \mathbb{R})$

is the line of intersection of (P) and (Q).

3) a- Determine the coordinates of B, the meeting point of (d) and (Δ).
b- Determine the coordinates of point F, the orthogonal projection of A on (Δ).
c- Calculate the cosine of the angle formed by (d) and (P).

II-(4 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$, consider the points

A, B, M and M′ with respective affixes -2 ; i ; z and z' so that $z' = \dfrac{z+2}{z-i}$ with $z \neq -2$ and $z \neq i$.

1) In this part only, assume that $z = \sqrt{2}e^{i\frac{3\pi}{4}}$.

a- Write the complex number $-1-i$ in exponential form.

b- Deduce that $(z')^{40}$ is a real number.

2) Let $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

a- Calculate x' and y' in terms of x and y.

b- Express the scalar product $\overrightarrow{AM}.\overrightarrow{BM}$ in terms of x and y.

c- Deduce that if z' is pure imaginary, then the two lines (AM) and (BM) are perpendicular.

3) a- Verify that $(z'-1)(z-i) = 2+i$.

b- Deduce that if M moves on the circle (C) with center B and radius $\sqrt{5}$, then M' moves on a circle (C') with center and radius to be determined .

III-(4 points)

An urn contains 4 red balls and 3 black balls.

A- In this part, we select randomly and one after another, three balls from this urn as follows: we select the first ball without putting it back in the urn, then we select the second ball and we put it back in the urn and finally, we select the third ball.

1) Verify that the probability to select three black balls is $\frac{1}{21}$.
2) Calculate the probability that the first ball selected is black and the two other balls are red.
3) Knowing that the first ball selected is black, calculate the probability that the two other balls are red.

B- In this part, the red balls are numbered: 1; 2; 3; 4 and the black balls are numbered: 1; 2; 3. We select randomly and **simultaneously** three balls from the urn.

1) Calculate the probability that two balls among the three balls selected have the same number.
2) Let X be the random variable that is equal to the number of selected balls which are numbered 2. Determine the probability distribution of X.

IV-(8 points)

A- Let g be the function defined on $]0;+\infty[$ as $g(x) = x^3 - 1 + 2\ln x$.

1) Determine $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.
2) Calculate $g'(x)$ then set up the table of variations of g.
3) Calculate $g(1)$ then deduce the sign of g(x) according to the value of x.

B- Consider the function f defined on $]0;+\infty[$ as $f(x) = x - \frac{\ln x}{x^2}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. Let (d) be the line with equation $y = x$.

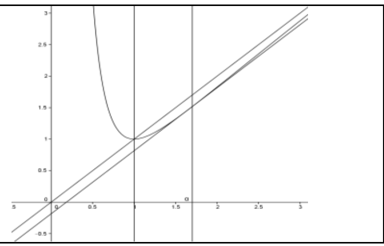
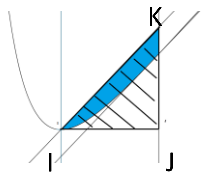
1) Determine $\lim\limits_{x\to 0} f(x)$ and deduce an asymptote to (C).
2) a- Discuss, according to the values of x, the relative positions of (C) and (d).
   b- Determine $\lim\limits_{x\to +\infty} f(x)$ and show that (d) is an asymptote to (C).
3) a- Verify that $f'(x) = \frac{g(x)}{x^3}$ and set up the table of variations of f.
   b- Determine the point E on (C) where the tangent (Δ) to (C) is parallel to (d).
   c- Plot (d), (Δ) and (C).
4) Let α be a real number greater than 1. Denote by $A(\alpha)$ the area of the region bounded by (C), (d) and the two lines with equations x = 1 and $x = \alpha$.
   a- Verify that $\int \frac{\ln x}{x^2}dx = \frac{-1-\ln x}{x} + k$, where k is a real number.
   b- Express $A(\alpha)$ in terms of α.
   c- Using the graphic, show that $A(\alpha) < \frac{(\alpha-1)^2}{2}$.

Barème SV 2015 2nd session An

Q.I	Answers	N
1	2(m + 1) – (2 m+2) = 0, $\overrightarrow{N}\cdot\overrightarrow{N_P}=2-2=0$ hence 2x – z = 0 is an equation of (Q). **OR**: $\overrightarrow{AM}\cdot\left(\overrightarrow{V_d}\wedge\overrightarrow{N_P}\right)=0\Leftrightarrow 2x-z=0.$	1
2	2t – 2t = 0 ; t –5t + 6 + 4t – 6 = 0 hence (Δ) verifies the equations of the two planes. **OR**: we solve the system $\begin{cases}2x-z=0\\x-2y+2z-6=0\end{cases}$ taking x = t.	0,5
3.a	B is the point of intersection of (d) and (P): m + 1 –4m –2 +4m + 4 – 6 = 0 $\rightarrow$ m = 3. B (4 ; 7 ; 8). **OR**: we solve the system $\begin{cases}m+1=2t\\2m+1=5t-3\\2m+2=4t\end{cases}$	1
3.b	If F is the orthogonal projection of A on (Δ) then $\overrightarrow{AF}\cdot\overrightarrow{V_\Delta}=0$.; $\overrightarrow{V_\Delta}(2;5;4)$ and $\overrightarrow{AF}(2t-1;5t-4;4t-2)$; $4t-2+25t-20+16t-8=0$ then $t=\frac{2}{3}\rightarrow F\left(\frac{4}{3};\frac{1}{3};\frac{8}{3}\right)$.	1
3.c	If α is a measure of the angle of (d) and (P) then $\cos\alpha=\frac{BF}{AB}=\frac{4\sqrt{5}}{9}$.	0,5

Q.II	Answers	N
1.a	$-1-i=\sqrt{2}\,e^{i\frac{5\pi}{4}}$	0.5
1.b	$z=\sqrt{2}e^{i\frac{3\pi}{4}}=\sqrt{2}\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=-1+i.\ z'=\frac{-1+i+2}{-1+i-i}=-1-i=\sqrt{2}e^{i\frac{5\pi}{4}}.$ $(z')^{40}=2^{20}e^{50\pi i}=2^{20}e^{0\pi i}=2^{20}$ then $(z')^{40}$ is a real number	0.5
2.a	$z'=\frac{z+2}{z-i}=\frac{x+2+iy}{x+i(y-1)}\times\frac{x-i(y-1)}{x-i(y-1)}=\frac{x^2+y^2+2x-y}{x^2+(y-1)^2}+i\frac{x-2y+2}{x^2+(y-1)^2}.$ then $x'=\frac{x^2+y^2+2x-y}{x^2+(y-1)^2}$ et $y'=\frac{x-2y+2}{x^2+(y-1)^2}$.	1
2.b	$\overrightarrow{AM}.\overrightarrow{BM}=x^2+y^2+2x-y$.	0.5
2.c	$\overrightarrow{AM}.\overrightarrow{BM}=x^2+y^2+2x-y$. If z' is pure imaginary so $\frac{x^2+y^2+2x-y}{x^2+(y-1)^2}=0$; $x^2+y^2+2x-y=0$; $\overrightarrow{AM}.\overrightarrow{BM}=0$ Therefore (AM) and (BM) are perpendicular.	0.5
3.a	$(z'-1)(z-i)=\left(\frac{z+2}{z-i}-1\right)(z-i)=\frac{2+i}{z-i}(z-i)=2+i.$	0.5
3.b	If M moves on the circle with center B and radius $\sqrt{5}$ so $\lvert z-i\rvert=\sqrt{5}$; $\lvert z'-1\rvert\lvert z-i\rvert=\lvert 2+i\rvert=\sqrt{5}$; $\lvert z'-1\rvert=\frac{\sqrt{5}}{\lvert z-i\rvert}=\frac{\sqrt{5}}{\sqrt{5}}=1$ therefore M′ moves on the circle with center H(1) and radius1.	0.5

Q.III		Answers	N
A	1	P (N, N, N) = $\frac{3}{7}\times\frac{2}{6}\times\frac{2}{6}=\frac{1}{21}$. **0.5** ⇨ **2** P (N, R, R) = $\frac{3}{7}\times\frac{4}{6}\times\frac{4}{6}=\frac{4}{21}$.	**0.5**
	3	P (the two other balls are red / the first ball selected is black) = $\frac{4}{6}\times\frac{4}{6}=\frac{4}{9}$.	**0.5**
B	1	P(two balls having the same number) = P{1,1,x}+P(2,2,x}+P{3,3,x}= $3\times\frac{C_2^2\times C_5^1}{C_7^3}=\frac{15}{35}=\frac{3}{7}$.	**1**
	2	$X=\{0;1;2\}$ $P(X=0)=\frac{C_5^3}{C_7^3}=\frac{10}{35}=\frac{2}{7}$; $P(X=1)=\frac{C_2^1\times C_5^2}{C_7^3}=\frac{20}{35}=\frac{4}{7}$; $P(X=2)=\frac{C_2^2\times C_5^1}{C_7^3}=\frac{5}{35}=\frac{1}{7}$	**1.5**

Q.IV		Answers	N
A	1	$\lim\limits_{x\to0}g(x)=-\infty$ and $\lim\limits_{x\to+\infty}g(x)=+\infty$.	0,5
	2	$g'(x)=3x^2+\frac{2}{x}>0$. $\begin{array}{c\|ccc} x & 0 & & +\infty \\ \hline g'(x) & \| & + & \\ \hline g(x) & \| -\infty & \nearrow & +\infty \end{array}$	0,5
	3	g is strictly increasing on $]0;+\infty[$ and $g(1)=0$ then : if $0<x\le1$ then $g(x)\le g(1)$ and $g(x)\le0$. if $x>1$ then $g(x)>g(1)$ and $g(x)>0$.	0,5
B	1	$\lim\limits_{x\to0}f(x)=+\infty$; the line of equation x=0 is an asymptote to (C).	0,5
	2.a	$f(x)-y=-\frac{\ln x}{x^2}$. (C) and (d) intersect at the point (1; 1). For $0<x<1$; $f(x)-y>0$; (C) is above (d) ; For $x>1$; $f(x)-y<0$, (C) is below (d).	0,5
	2.b	$\lim\limits_{x\to+\infty}\frac{\ln x}{x^2}=\lim\limits_{x\to+\infty}\frac{\frac{1}{x}}{2x}=\lim\limits_{x\to+\infty}\frac{1}{2x^2}=0\Rightarrow\lim\limits_{x\to+\infty}f(x)=+\infty$; $\lim\limits_{x\to+\infty}[f(x)-y]=\lim\limits_{x\to+\infty}-\frac{\ln x}{x^2}=0$; $y=x$ asymptote to (C).	¾
	3.a	$f'(x)=1-\frac{x-2x\ln x}{x^4}=1-\frac{1-2\ln x}{x^3}=\frac{x^3-1+2\ln x}{x^3}=\frac{g(x)}{x^3}$. $\begin{array}{c\|ccccc} x & 0 & & 1 & & +\infty \\ \hline f'(x) & \| & - & 0 & + & \\ \hline f(x) & \| +\infty & \searrow & 1 & \nearrow & +\infty \end{array}$	1
	3.b	$f'(x)=1; g(x)=x^3$; $-1+2\ln x=0$; $x=e^{\frac{1}{2}}$; $f\left(e^{\frac{1}{2}}\right)=e^{\frac{1}{2}}-\frac{1}{2e}$. $E\left(e^{\frac{1}{2}};e^{\frac{1}{2}}-\frac{1}{2e}\right)$.	¾
	3.c	 **1** ⇨ **4.a** $u=\ln x$ and $v'=\frac{1}{x^2}$; $u'=\frac{1}{x}$ and $v=-\frac{1}{x}$. $\int\frac{\ln x}{x^2}dx=\frac{-\ln x}{x}+\int\frac{1}{x^2}dx=\frac{-\ln x}{x}-\frac{1}{x}+k=\frac{-1-\ln x}{x}+k$	1
	4.b	$A(\alpha)=\int_1^\alpha[x-f(x)]dx=\int_1^\alpha\frac{\ln x}{x^2}dx=\left[-\frac{\ln x}{x}\right]_1^\alpha-\left[\frac{1}{x}\right]_1^\alpha=(1-\frac{1}{\alpha}-\frac{\ln\alpha}{\alpha})\ u^2$.	0,5
	4.c	 $A(\alpha)<\int_1^\alpha(x-1)dx$; $A(\alpha)<\frac{\alpha^2}{2}-\alpha+\frac{1}{2}$; $A(\alpha)<\frac{(\alpha-1)^2}{2}$ since the colored region is inside the right isosceles triangle IJK **OR** $A(\alpha)<$ Area of the triangle IJK; $A(\alpha)<\frac{IJ\times JK}{2}$; $A(\alpha)<\frac{(\alpha-1)^2}{2}$.	0,5

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة العام ٢٠١٦ العاديّة
الاربعاء ١٥ حزيران ٢٠١٦

عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: يُسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
يستطيع المُرشّح الاجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الوارد في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $(O\,;\,\vec{i}\,,\,\vec{j}\,,\,\vec{k}\,)$, consider the points

A $(3,1\,,\,1)$ and F $(\,2\,,2\,,-2)$. Denote by (d) the line defined as: $\begin{cases} x=-t \\ y=t+2 \\ z=t \end{cases}$ (t is a real parameter).

1) Let (P) be the plane through the point F and containing the line (d).
   Verify that: $x+z=0$ is an equation of the plane (P).
2) Let $E(1,1,-1)$ be a point on (d).
   Verify that E is the orthogonal projection of A on the plane (P).
3) Denote by L the point on the line (d) so that $x_L \neq 0$ and the triangle EFL is isosceles with principle vertex E. Calculate the coordinates of L.
4) Calculate the volume of the tetrahedron AEFL.

II- (4 points)

Consider a box V containing six cards numbered 1 ; 2 ; 3 ; 4 ; 7 ; 9, and two urns U_1 and U_2 such that:

- U_1 contains 3 red balls and 5 black balls
- U_2 contains 4 red balls and 4 black balls.

One card is randomly selected from the box V.
If this card shows an even number, then two balls are randomly and simultaneously selected from U_1. If the card shows an odd number then two balls are randomly and simultaneously selected from U_2.

Consider the following events:
E: "The card selected shows an even number"
O: "The card selected shows an odd number"
R: "The two selected balls are red"
B: "The two selected balls are black".

1) a- Calculate the probability $P\left(R/E\right)$ and deduce that $P(E\cap R)=\dfrac{1}{28}$.
   b- Calculate $P(O\cap R)$ and P(R).
2) Show that $P(B)=\dfrac{11}{42}$.
3) Knowing that the two selected balls are black, calculate the probability that these two balls come from urn U_1.

III- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points E(2i),

A(–i), M(z) and M'(z') where z and z' are two complex numbers such that: $z' = 2i - \dfrac{2}{z}$. $(z \neq 0)$.

1) a- Show that $z(z'-2i) = -2$.

   b- Calculate $\arg(z) + \arg(z'-2i)$.

2) a- Verify that: $z' = \dfrac{2i(z+i)}{z}$.

   b- Show that $OM' = \dfrac{2AM}{OM}$.

   c- As M moves on the perpendicular bisector of $[OA]$, prove that M' moves on a circle (C) whose center and radius are to be determined.

3) Suppose that $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

   a- Show that $x' = \dfrac{-2x}{x^2+y^2}$ and $y' = 2 + \dfrac{2y}{x^2+y^2}$.

   b- If x = y, show that the lines (OM) and (EM') are perpendicular.

IV- (8 points)

Consider the function f defined over $]-1;+\infty[$ as: $f(x) = e^x - \dfrac{2e^x}{x+1}$.

Denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) a- Determine $\lim\limits_{\substack{x\to -1\\ x>-1}} f(x)$. Deduce an asymptote (D) to (C).

   b- Determine $\lim\limits_{x\to+\infty} f(x)$ and calculate f(2.5).

2) Prove that $f'(x) = \dfrac{(x^2+1)e^x}{(x+1)^2}$ and set up the table of variations of the function f.

3) Let (d) be the line with equation y = x. The curve (C) intersects (d) at a unique point A with abscissa α. Verify that $1.8 < \alpha < 1.9$.

4) a- Specify the coordinates of the points of intersection of (C) with the coordinates axes.

   b- Draw (D), (d) and (C).

5) a- Prove that, over $]-1;+\infty[$, f has an inverse function f^{-1}.

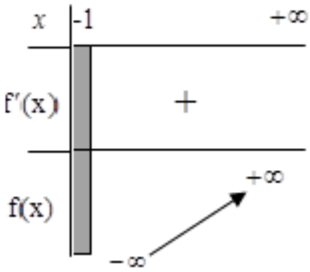
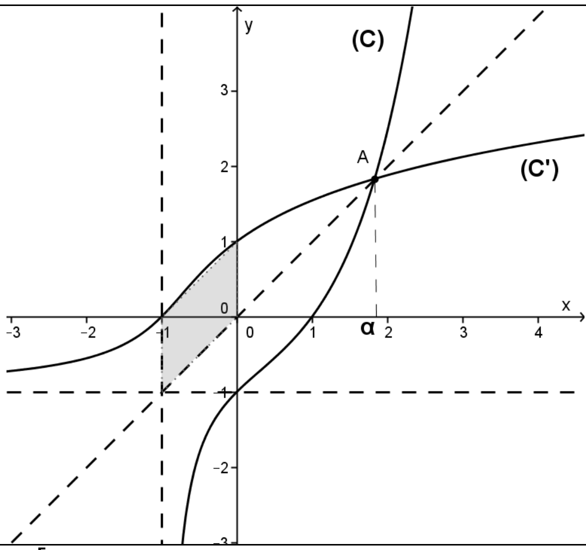
   b- Draw (C'), the representative curve of f^{-1}, in the same system as that of (C).

6) Suppose that the area of the region bounded by (C), the x-axis and the lines with equations x = 0 and x = 1 is 0.53 units of area.
   Calculate the area of the region bounded by (C'), the line (d), the y-axis and the line with equation $x = -1$.

I	Answer	M
1	Verify that F belongs to (P) and (d) lies in it.	1
2	E is on (d) then E belongs to (P) and $\overrightarrow{AE}(-1,0,-2)$ so $\overrightarrow{AE}=-2\overrightarrow{N_{(P)}}$	1
3	$EF^2=3$ and $EL^2=3(t+1)^2$ then $3(t+1)^2=3 \Rightarrow$ $t=-2$ or $t=0$ So $L(2,0,-2)$	1
4	$V=\frac{1}{6}\left\vert\overrightarrow{AL}.(\overrightarrow{AE}\wedge\overrightarrow{AF})\right\vert=\frac{\vert-8\vert}{6}=\frac{4}{3}u^3$	1

II		Answer	M
1	a	$P\left(R/E\right)=\frac{C_3^2}{C_8^2}=\frac{3}{28}$; $p(E\cap R)=p(E).\ P\left(R/E\right)=\frac{2}{6}\times\frac{C_3^2}{C_8^2}=\frac{1}{28}$	1
	b	$p(O\cap R)=p(O).\ P\left(R/O\right)=\frac{4}{6}\times\frac{C_4^2}{C_8^2}=\frac{1}{7}$ $P(R)=p(E\cap R)+p(O\cap R)=\frac{5}{28}$.	1
2		$P(B)=p(E\cap B)+p(O\cap B)=p(E).p(B/E)+p(O).p(B/O)=$ $\frac{1}{3}\times\frac{C_5^2}{C_8^2}+\frac{2}{3}\times\frac{C_4^2}{C_8^2}=\frac{11}{42}$.	1
3		$p(E/B)=\frac{p(E\cap B)}{p(B)}=\frac{5/42}{11/42}=\frac{5}{11}$	1

III		Answer	M
1	a	$z'-2i=\frac{-2}{z}$ then $z(z'-2i)=-2$	0.5
	b	$\arg(z(z'-2i))=\arg z+\arg(z'-2i)=\arg(-2)=\pi[2\pi]$	0.5
2	a	$z'=\frac{2i(z+i)}{z}$	0.5
	b	$OM'=\left\vert\frac{2i(z+i)}{z}\right\vert=\frac{\vert 2i(z+i)\vert}{\vert z\vert}=\frac{\vert 2i\vert\vert z+i\vert}{\vert z\vert}=\frac{2AM}{OM}$	0.5
	c	M belongs to perp bis of $[OA]$ then MA=MO so $OM'=2$, therefore M' Moves on circle center O and radius 2.	0.5
3	a	$x'=\frac{-2x}{x^2+y^2}$ and $y'=2+\frac{2y}{x^2+y^2}$	1
	b	x= y then $M'(\frac{-1}{x},2+\frac{1}{x})$ so $\overrightarrow{EM'}(\frac{-1}{x},\frac{1}{x})$ and $\overrightarrow{OM}(x,y)$ $\overrightarrow{EM'}.\overrightarrow{OM}=0$ so $(EM')\perp(OM)$.	0.5

<table>
<tr><th colspan="2">IV</th><th>Answer</th><th>M</th></tr>
<tr><td rowspan="2">1</td><td>a</td><td>$\lim\limits_{\substack{x\to -1\\ x>-1}} f(x) = -\infty$ then the line (D) : x = -1 is an asymptote to (C) .</td><td>0.5</td></tr>
<tr><td>b</td><td>$\lim\limits_{x\to+\infty} f(x) = (1)(+\infty) = +\infty$; f(2.5) ≈ 5.22.</td><td>1</td></tr>
<tr><td colspan="2">2</td><td>$f'(x) = e^x - \left(\dfrac{2e^x(x+1)-2e^x}{(x+1)^2}\right)$
$= \left(\dfrac{(x+1)^2-(x+1)+2}{(x+1)^2}\right)e^x$
$= \left(\dfrac{x^2+1}{(x+1)^2}\right)e^x > 0$ for all x

</td><td>1</td></tr>
<tr><td colspan="2">3</td><td>Let $\phi(x) = f(x) - x$.
$\phi(1.8) \approx -0.07 < 0$ and $\phi(1.9) \approx 0.17 > 0$</td><td>1</td></tr>
<tr><td rowspan="2">4</td><td>a</td><td>If x =0, then f(0) = -1 ,so (C) cuts the y-axis in (0,-1)
If f(x) = 0, then x = 1, so (C) cuts the x-axis in (1; 0).</td><td>0.5</td></tr>
<tr><td>b</td><td>
</td><td>1</td></tr>
<tr><td rowspan="2">5</td><td>a</td><td>On $]-1;+\infty[$, f is continuous and strictly increasing, it has an inverse function f^{-1}.</td><td>0.5</td></tr>
<tr><td>b</td><td>(C') and (C) are symmetrical with respect to y = x.
See graph.</td><td>1</td></tr>
<tr><td colspan="2">6</td><td>Due to symmetry w.r.t. y = x
The area of the region bounded by (C'), the line (d), the y-axis and the Line with equation $x = -1$ is =
0.53 + area of right isosceles triangle with side 1 =
1.03 units of area</td><td>1.5</td></tr>
</table>

دورة العام ٢٠١٦ الاستثنائيّة الإثنين ٨ آب ٢٠١٦	امتحانات الشهادة الثانوية العامة الفرع : علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسميّة
الاسم: الرقم:	مسابقة في مادة الرياضيات المدة ساعتان	عدد المسائل: أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the complex plane referred to an orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A(1), M(z) and M'(z') so that: $z' = (1-i)z + i$ with $z \neq 1$.

1) a- Verify that $z' - 1 = (1-i)(z-1)$.

   b- Verify that $AM' = AM\sqrt{2}$. Deduce that if M moves on the circle with center A and radius $\sqrt{2}$, then M' moves on a circle (C) whose center and radius should be determined.

   c- Prove that: $\left(\vec{u};\overrightarrow{AM'}\right) = -\frac{\pi}{4} + \left(\vec{u};\overrightarrow{AM}\right) + 2k\pi$ with $k \in \mathbb{Z}$.

   d- Compare $|z' - z|$ and $|z-1|$, then prove that the triangle AMM' is right isosceles.

2) Let $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers.

   a- Express x' and y' in terms of x and y.

   b- Verify that if M' moves on a line (D) with equation $y = x$, then M moves on a line (Δ) to be determined.

II- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider:

- The plane (P): $x - 2y + z = 0$; and the plane (Q) : $x + y + z + 3 = 0$
- The two points $A(1;0;-1)$ and $E(0;-1;-2)$
- In the plane (P), the circle (C) with center A and radius $R = \sqrt{3}$.

Let (Δ) be the intersection line of (P) and (Q).

1) a- Prove that (P) is perpendicular to (Q).

   b- Verify that the line (Δ) is defined by: $x = -t - 2$; $y = -1$; $z = t$ ($t \in \mathbb{R}$).

   c- Prove that E is the orthogonal projection of A on (Δ) .

   d- Deduce that (Δ) is tangent to (C) at E.

2) Denote by H the point on (Δ) with positive abscissa so that $EH = 3\sqrt{2}$. Determine the coordinates of H.

3) Let (T) be the second tangent through H to (C). Denote by F the point of tangency between (T) and (C).

   Determine a system of parametric equations of one bisector of the angle EHF.

III- (4 points)

Consider an urn U containing three dice:

- **Two** red dice where the faces of each of them are numbered from 1 to 6
- **One** black die where **two** of its faces are numbered 6 and the **four** others are numbered 1.

A player selects randomly and simultaneously two dice from the urn, then he rolls them only once.
Consider the following events:

A : «The two dice selected are red».
$\overline{A}$: «The two dice selected are one red and one black».
L : «Out of the two dice, only one shows the number 6».

1) Calculate the probability P(A).
2) a- Verify that $P(L/A) = \dfrac{5}{18}$ and calculate $P(A \cap L)$.
   b- Calculate $P(\overline{A} \cap L)$ and verify that $P(L) = \dfrac{19}{54}$.
3) Knowing that only one of the two dice shows the number 6, calculate the probability that the two dice selected are red.
4) Calculate the probability that at least one die shows the number 6.

IV- (8 points)

Let f be the function defined over $\mathbb{R}$ as $f(x) = x + xe^{-x}$, and denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Calculate $f\left(-1.5\right)$.
2) Let (d) be the line with equation $y = x$.
   a- Discuss according to the values of x, the relative position of (C) and (d).
   b- Prove that (d) is an asymptote to (C).
3) A is the point on (C) where the tangent (T) to (C) is parallel to (d). Determine the coordinates of A and write an equation of (T).
4) The following table is the table of variations of the function f ', the derivative of f .

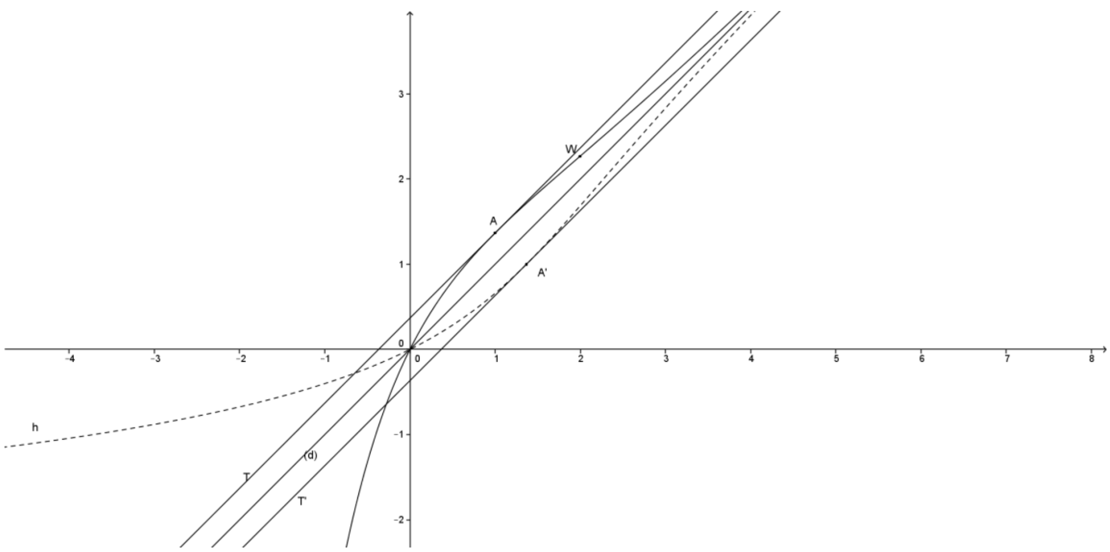
x	$-\infty$		2		$+\infty$
f ''(x)		$-$	0	$+$	
f '(x)	$+\infty$	$\searrow$	$1-e^{-2}$	$\nearrow$	1

   a- Verify that (C) admits an inflection point W whose coordinates should be determined.
   b- Verify that f is strictly increasing over $\mathbb{R}$, then set up the table of variations of the function f.
5) Draw (d), (T) and (C).
6) a- Prove that f has an inverse function h whose domain of definition should be determined.
   b- Draw the curve (C') of h in the same system as that of (C).
7) Let M be any point on (C) with abscissa $x \geq 0$, and denote by N the symmetric of M with respect to (d).
   a- Calculate MN in terms of x.
   b- Calculate the maximum value of MN.

I	Solution	M
1a	$z'-1=(1-i)z+i-1$; $z'=(1-i)(z-1)$	0.25
1b	$\lvert z'-1\rvert=\lvert(1-i)(z-1)\rvert=\lvert 1-i\rvert\times\lvert z-1\rvert$ $\lvert z_{M'}-z_A\rvert=\sqrt{2}\lvert z_M-z_A\rvert; AM'=\sqrt{2}AM$ $AM=\sqrt{2}$; $AM'=2$; $M'\in C(A;2)$	1
1c	$\arg[z'-1]=\arg[(1-i)(z-1)]=\arg(1-i)+\arg(z-1);$ $\arg(\vec{u};\overrightarrow{AM'})=-\frac{\pi}{4}+(\vec{u};\overrightarrow{AM})+2k\pi$	1
1d	$\lvert z'-z\rvert=\lvert(1-i)z+i-z\rvert=\lvert -iz+i\rvert=\lvert i\rvert\lvert z-1\rvert=\lvert z-1\rvert;$ $\arg(z'-z)=\arg[-i(z-1)]$ $\arg(\vec{u};\overrightarrow{AM'})=-\frac{\pi}{4}+(\vec{u};\overrightarrow{AM})$ then $MAM'=45^\circ$ et $\lvert z'-z\rvert=\lvert z-1\rvert; MM'=AM$ Then AMM’ is right isosceles at M.	0.75
2a	$x'=x+y$; $y'=y-x+1$	0.5
2b	$x'=y'$; then M moves on the line $(\Delta): x=\frac{1}{2}$	0.5

II	Solution	M
1a	$\overrightarrow{n_{(P)}}(1;-2;1)$ and $\overrightarrow{n_{(P)}}(1;1;1)$; $\overrightarrow{n_{(P)}}.\overrightarrow{n_{(P)}}=0$	0.5
1b	$-t-2+2+t=0$ then $(\Delta)\subset(P)$; $-t-2-1+t+3=0$ then $(\Delta)\subset(Q)$.	0.5
1c	$\overrightarrow{AE}(-1;-1;-1);\overrightarrow{AE}.\overrightarrow{V_{(\Delta)}}=0$ and for $t=-2$ $E\in(\Delta)$ so E is the projection of A on (Δ)	0.5
1d	$AE=\sqrt{3}=R;(\Delta)$ is perpendicular to (AE) at E then (Δ) is the tangent to (C) at E.	0.5
2	$H(-t-2;-1;t); EH=3\sqrt{2}; EH=\sqrt{2(t+2)^2}$; $t=1$ not accepted. $t=-5$ accepted $\Rightarrow H(3;-1;-5)$	1
3	$(AH):\begin{cases}x=2m+1\\y=-m\\z=-4m-1\end{cases}$	1

III	Solution	M
1	$p(A)=\frac{C_2^2}{C_3^2}=\frac{1}{3}$	0.5
2a	$p(L/A)=\frac{1}{6}\times\frac{5}{6}\times2=\frac{5}{18}$; $P(A\cap L)=p(L/A)\times p(A)=\frac{5}{18}\times\frac{1}{3}=\frac{5}{54}$	1
2b	$p(\overline{A}\cap L)=p(L/\overline{A})\times p(\overline{A})=\frac{2}{3}(\frac{1}{6}\times\frac{4}{6}+\frac{5}{6}\times\frac{2}{6})=\frac{14}{54}$ $p(L)=p(A\cap L)+p(\overline{A}\cap L)=\frac{5}{54}+\frac{14}{54}=\frac{19}{54}$	1.25
3	$p(A/L)=P\frac{(A\cap L)}{p(L)}=\frac{5}{19}$	0.5
4	$1-p(\text{none of the two dice shows } 6)=1-\left[\left(\frac{1}{3}\times\frac{5}{6}\times\frac{5}{6}\right)+\left(\frac{2}{3}\times\frac{5}{6}\times\frac{4}{6}\right)\right]=\frac{43}{108}$. Or $p(L)+p(\text{two dice show the number } 6)=\frac{19}{54}+\frac{2}{3}\times\left(\frac{1}{6}\times\frac{2}{6}\right)=\frac{43}{108}$	0.75

IV	Solution	M
1	$\lim\limits_{x\to-\infty} f(x) = -\infty - \infty = -\infty$; $\lim\limits_{x\to+\infty} f(x) = +\infty + 0 = +\infty$; $f(-1,5) \approx -8.22$,	0.75
2a	$f(x) - x = xe^{-x}$ $x > 0; (C)$ is above (d) $x = 0; (C)$ cuts (d) $x < 0; (C)$ is below (d)	0.5
2b	$\lim\limits_{x\to+\infty}\left[f(x) - x\right] = \lim\limits_{x\to+\infty} xe^{-x} = 0$ then the line (d): y=x is an asymptote.	0.5
3	$f'(x) = 1;\ 1 + e^{-x} - xe^{-x} = 1; x = 1; A(1; 1 + e^{-1})$ $(T): y = x + e^{-1}$	1
4a	$f''(x) = 0$ for $x = 2$ and f'' changes sign. Then $W(2; 2 + 2e^{-2})$ is a point of inflection to (C).	0.5
4b	f' admits an absolute minimum equals to $1 - e^{-2} \approx 0,8 > 0$ then f is strictly increasing. $\begin{array}{c\|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & -\infty & \nearrow & +\infty \end{array}$	0.5
5		1
6a	f is continuous and strictly increasing over $\mathbb{R}$,then it admits an inverse functiuon h $D_h =]-\infty; +\infty[$	0.5
6b	(C') is the symmetric of (C) with respect to (d).	1
7a	$M(x; x + xe^{-x}); N(x + xe^{-x}; x); MN = \sqrt{2}xe^{-x}$	1
7b	$MN = \sqrt{2}xe^{-x} = g(x)$ $g'(x) = 0; \sqrt{2}e^{-x}(1 - x) = 0;$ for $x = 1; MN = \sqrt{2}e^{-1}$ is the maximum value	0.75

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام ٢٠١٧ العاديّة السبت ١٧ حزيران ٢٠١٧
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the points

A(3; 1; 0), B(2; 0; 1) and S(3; −1; −2). Denote by (d) the line defined as: $\begin{cases} x = t \\ y = t+1 \\ z = -t \end{cases}$; $t \in IR$.

1) a- Verify that the point A is not on (d) and that the two lines (AB) and (d) are parallel.
   b- Show that y + z – 1 = 0 is an equation of the plane (P) determined by (AB) and (d).
2) a- Prove that A is the orthogonal projection of the point S on the plane (P).
   b- Denote by S′ the symmetric of S with respect to (P). Calculate the area of the triangle BSS′.
3) Consider in the plane (P) the circle (C) with center A and radius 3.
   The line (d) intersects the circle (C) in two points E and F.
   a- Find the coordinates of E and F.
   b- Write a system of parametric equations of a bisector of the angle EAF.

II- (4 points)

A bag U contains nine balls:

- three red balls numbered **0**
- two green balls numbered **1**
- four blue balls numbered **2**.

Part A

Three balls are randomly and simultaneously selected from this bag.
Consider the following events:
M: « the three selected balls have the same color »;
N: « the product of numbers on the three selected balls is equal to zero ».

1) Calculate P(M), the probability of the event M.
2) a- Verify that $P(N) = \frac{16}{21}$.
   b- Calculate $P(M \cap N)$ and verify that $P(\overline{M} \cap N) = \frac{3}{4}$.
3) Knowing that the three selected balls don't have the same color, calculate the probability that the product of numbers on these three balls is equal to zero.

Part B

In this part, one ball is randomly selected from the bag U.
This ball is not replaced back in U.

- If the selected ball is numbered **0**, then two balls are randomly and simultaneously selected from U. *(We get then 3 balls)*
- If the selected ball is not numbered **0**, then one ball is randomly selected from U. *(We get then 2 balls.)*

Calculate the probability that the sum of numbers on the selected balls is 3.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, M and M' with respective affixes i, –2i, z and z', such that $z' = \frac{-2iz}{z-i}$ with $z \neq i$.

1) a- Prove that $(z'+2i)(z-i)$ is a real number.

   b- Deduce that $AM \times BM' = 2$.

   c- If M moves on the circle with center A and radius 3, show that M' moves on a circle with center and radius to be determined.

2) In the case where $z' = 2i$, write z in exponential form.

3) Let z = x + iy and z' = x' + iy' where x, y, x' and y' are real numbers.

   a- Show that $x' = \frac{2x}{x^2+(y-1)^2}$ and $y' = \frac{-2(x^2+y^2-y)}{x^2+(y-1)^2}$.

   b- If $AM = \sqrt{2}$, prove that $x = x'$.

IV- (8 points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = (1-x)e^x + 2$.

Denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$. Deduce an asymptote (d) to (C).

   b- Determine $\lim\limits_{x\to+\infty} f(x)$, then calculate $f(1)$ and $f(2)$.

2) a- Verify that $f'(x) = -xe^x$ and set up the table of variations of the function f .

   b- Prove that the curve (C) has an inflection point I whose coordinates should be determined.

3) Draw (d) and (C).

4) Denote by (Δ) the line with equation $y = 2x$.

   a- Verify that $f(x) - 2x = (e^x + 2)(1-x)$. Study, according to the values of x, the relative positions of (C) and (Δ).

   b- Find an antiderivative F of the function f .

   c- Draw (Δ), then calculate the area of the region bounded by the curve (C), the y-axis and the line (Δ).

5) Let g be the function given as $g(x) = \ln[f(x)-2]$.

   Denote by (G) the representative curve of g in the system $(O;\vec{i},\vec{j})$.

   a- Verify that the domain of definition of g is $]-\infty\,;1[$.

   b- Is there a point on (G) where the tangent to (G) is parallel to the line (Δ)? Justify.

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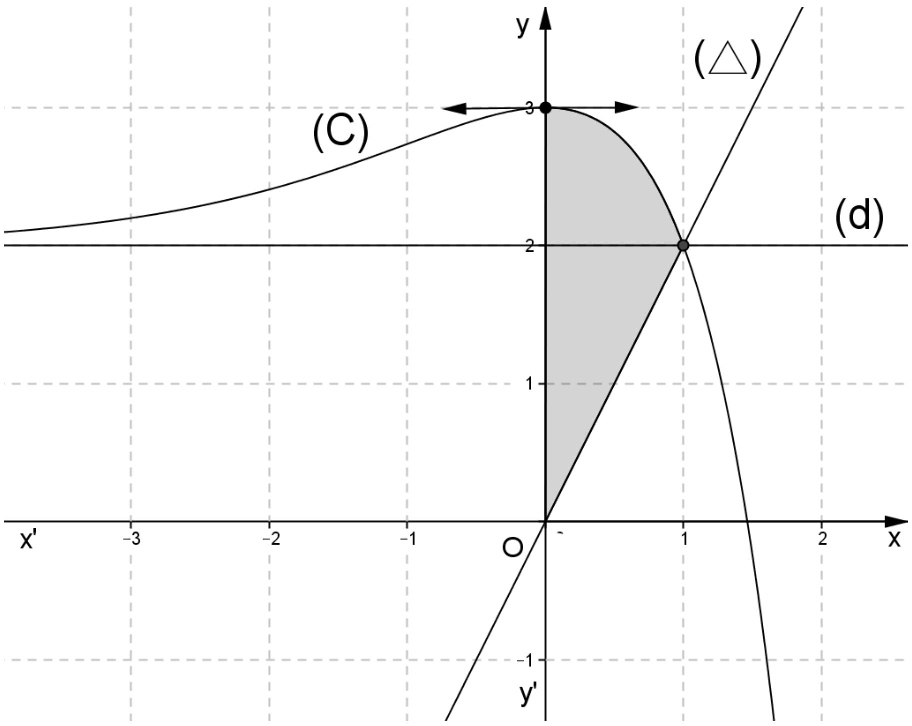
أسس تصحيح مسابقة الرياضيات

Q.I	Answers	4 pts
1.a	$3 = t$ and $1 = t + 1$ then $t = 3$ and $t = 0$ which is impossible, thus $A \notin (d)$. $\overrightarrow{AB}(-1\ ;\ -1\ ;\ 1) = -\vec{V}(1\ ;\ 1\ ;\ -1)$ thus $(AB) // (d)$.	¾
1.b	$A \in (P) : 1 + 0 - 1 = 0, 0 = 0.$ $B \in (P) : 0 + 1 - 1 = 0, 0 = 0.$ $(d) \subset (P) : t + 1 + (-t) - 1 = 0, 0 = 0.$	½
2.a	$A \in (P)$ and $\overrightarrow{AS}(0\ ;\ -2\ ;\ -2) = -2\overrightarrow{n_{(P)}}(0\ ;\ 1\ ;\ 1)$ thus $(AS) \perp (P)$ at A	¾
2.b	Area(BSS') = 2.Area(BSA) = $2.\frac{1}{2}$.AB.AS = $\sqrt{3} \times 2\sqrt{2} = 2\sqrt{6}$ square units.	½
3.a	$E \in (d)$ thus $E(t\ ;\ t + 1\ ;\ -t)$. $AE = 3$, $(t-3)^2 + t^2 + (-t)^2 = 9$ then $t = 0$ and $t = 2$. Thus, $E(0\ ;\ 1\ ;\ 0)$ and $F(2\ ;\ 3\ ;\ -2)$	1
3.b	AE = AF = radius, then AEF isosceles at A. Let I be the midpoint of [EF] then $I(1\ ;\ 2\ ;\ -1)$. $\overrightarrow{AI}(-2\ ;\ 1\ ;\ -1)$ is a directing vector of the bisector. Hence, $x = -2k + 3$, $y = k + 1$, $z = -k$ $(k \in \mathbb{R})$. **Another method :** AE = AF = 3 then $\vec{W} = \overrightarrow{AE} + \overrightarrow{AF}$ is a directing vector of a bisector. $\overrightarrow{AE}(-3\ ;\ 0\ ;\ 0)$ and $\overrightarrow{AF}(-1\ ;\ 2\ ;\ -2)$ then $\vec{W}(-4\ ;\ 2\ ;\ -2)$. Hence, $x = -4k' + 3$, $y = 2k' + 1$, $z = -2k'$ $(k' \in \mathbb{R})$.	½
Q.II	**Answers**	**4 pts**
A.1	$P(M) = \frac{C_3^3}{C_9^3} + \frac{C_4^3}{C_9^3} = \frac{5}{84}$	½
A.2.a	$P(N) = 1 - P(\bar{N}) = 1 - \frac{C_6^3}{C_9^3} = \frac{16}{21}$	½
A.2.b	$P(M \cap N) = \frac{C_3^3}{C_9^3} = \frac{1}{84}$. $P(\bar{M} \cap N) = P(N) - P(M \cap N) = \frac{16}{21} - \frac{1}{84} = \frac{3}{4}$	1
A.3	$P(N/\bar{M}) = \frac{P(\bar{M} \cap N)}{P(\bar{M})} = \frac{\frac{63}{84}}{1-\frac{5}{84}} = \frac{63}{79}$	1
B	$P(S = 3) = P(R \cap (G \text{ and } B)) + P(B \cap (G)) + P(G \cap (B))$ $= \frac{C_3^1}{C_9^1} \times \frac{C_2^1 \times C_4^1}{C_8^2} + \frac{C_4^1}{C_9^1} \times \frac{C_2^1}{C_8^1} + \frac{C_2^1}{C_9^1} \times \frac{C_4^1}{C_8^1} = \frac{20}{63}$	1
Q.III	**Answers**	**4 pts**
1.a	$(z'+2i)(z-i) = \left(\frac{-2iz}{z-i} + 2i\right)(z-i) = \frac{2}{z-i}(z-i) = 2$	¾
1.b	$AM.BM' = \lvert z-i \rvert \lvert z'+2i \rvert = 2$	½
1.c	$AM = 3, \lvert z-i \rvert = 3,$ then $BM' = \frac{2}{3}$ Thus, M' varies on the circle with center B and radius $\frac{2}{3}$	½
2	$2i = \frac{-2iz}{z-i}$ then $z = \frac{1}{2}i = \frac{1}{2}e^{i\frac{\pi}{2}}$.	¾
3.a	$x'+iy' = \frac{2i(x+iy)}{i-x-iy} = \frac{-2y+2ix}{-x+i(1-y)} = \frac{(-2y+2ix)(-x-i(1-y))}{x^2+(y-1)^2}$ $x' = \frac{2x}{x^2+(y-1)^2}$, $y' = \frac{-2(x^2+y^2-y)}{x^2+(y-1)^2}$	1
3.b	If $AM = \sqrt{2}$ then $x^2+(y-1)^2 = 2$ so, $x' = x$.	½

<table>
<tr><th>Q.IV</th><th>Answers</th><th>8 pts</th></tr>
<tr><td>1.a</td><td>$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty}(e^x - xe^x + 2) = 2$. Thus, $y = 2$ is an asymptote to (C) at $-\infty$.</td><td>½</td></tr>
<tr><td>1.b</td><td>$\lim\limits_{x\to+\infty} f(x) = +\infty$. $f(1) = 2$, $f(2) = 2 - e^2 = -5.33$.</td><td>¾</td></tr>
<tr><td>2.a</td><td>$f'(x) = -xe^x$
<table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>0</td><td></td><td>$+\infty$</td></tr>
<tr><td>f '(x)</td><td></td><td>+</td><td>0</td><td>$-$</td><td></td></tr>
<tr><td>f(x)</td><td>2</td><td>↗</td><td>3</td><td>↘</td><td>$-\infty$</td></tr>
</table></td><td>1</td></tr>
<tr><td>2.b</td><td>$f''(x) = -(x+1)e^x$ vanishes and changes sign at $x = -1$.
Thus, $I(-1\ ;\ 2e^{-1} + 2)$ is an inflection point.</td><td>¾</td></tr>
<tr><td>3</td><td>
</td><td>1</td></tr>
<tr><td>4.a</td><td>$f(x) - 2x = (1-x)e^x + 2 - 2x = (1-x)(e^x + 2)$.
<table>
<tr><td>x</td><td>$-\infty$</td><td>-1</td><td>$+\infty$</td></tr>
<tr><td>f(x) – 2x</td><td>+</td><td>0</td><td>$-$</td></tr>
<tr><td>Position</td><td>(C) is above (d)</td><td>(C) and (d) intersect at $(-1\ ;\ 2)$</td><td>(C) is below (d)</td></tr>
</table></td><td>1</td></tr>
<tr><td>4.b</td><td>$\int f(x)dx = (2-x)e^x + 2x + c$</td><td>1</td></tr>
<tr><td>4.c</td><td>L'aire = $\int_0^1 [f(x) - 2x]dx = (e-1)u^2$</td><td>¾</td></tr>
<tr><td>5.a</td><td>$f(x) - 2 > 0$ then $x \in]-\infty; 1[$</td><td>½</td></tr>
<tr><td>5.b</td><td>$g'(x) = 2$, $\frac{f'(x)}{f(x)-2} = 2$, $x = 2$ rejected since $2 \notin]-\infty; 1[$.</td><td>¾</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسميّة	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام ٢٠١٧ الاستثنائيّة الثلاثاء في ٨ آب ٢٠١٧
عدد المسائل: أربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the three points A (9; –1; 4), B(5; 1; 2) , C(3; 2; 2) and the plane (P) with equation x + 2y – 7 = 0 determined by A, B and C.

1) Let (Q) be the plane passing through A and B and perpendicular to plane (P).
Show that an equation of (Q) is 2x – y – 5z + 1 = 0.
2) Denote by (d) the line of intersection of (P) and (Q).
Write a system of parametric equations of (d).
3) (L) is the line with parametric equations: $\begin{cases} x = -t+6 \\ y = -2t+3 \\ z = 2 \end{cases}$; $(t \in \mathbb{R})$.

a- Verify that B is on (L).
b- Verify that (L) is in (Q) and that (L) is perpendicular to (d).
c- Determine the coordinates of the point E on line (L) such that the area of triangle BCE is equal to 5 square units. $(y_E > 0)$

II- (4 points)

An urn U contains ten balls:

- **five white balls** numbered 1, 2, 3, 4, 5
- **three black balls** numbered 6, 7, 8
- **two green balls** numbered 9, 10.

Part A

A player selects randomly and simultaneously two balls from the urn U.
Consider the following events:

A: “The two selected balls hold odd numbers”
B: “The two selected balls have the same color”
C: “The two selected balls hold odd numbers and have the same color”
D: “The two selected balls hold odd numbers and have different colors”.

1) Calculate the probability P(A) and verify that P(B) = $\frac{14}{45}$.
2) a- Calculate P(C).
b- Are the events A and B independent? Justify.
3) Verify that P(D) = $\frac{7}{45}$.
4) Knowing that the player has selected two balls with different colors, what is the probability that these two balls hold odd numbers?

Part B

In this part, the player selects randomly, successively and with replacement, two balls from the urn U. The player scores +1 point for each white ball selected, –1 point for each black ball selected and 0 points for each green ball selected.
Calculate the probability that the sum of scored points is equal to zero.

III- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A, M and M′ with respective affixes 2i, z and z′ such that: $z' = \dfrac{2i - z}{iz}$ with $z \neq 0$.

Let B be the midpoint of segment [OA].

1) Write z′ in algebraic form in the case where z = 1 + i.
2) a- Show that $OM' = \dfrac{AM}{OM}$.

   b- Show that, if M moves on the line (d) with equation y = 1, then M′ moves on a circle with center O and radius to be determined.
3) Verify that $z' - i = \dfrac{2}{z}$.
4) Let $z = e^{-i\frac{\pi}{4}}$.

   a- Write z′ − i in exponential form and algebraic form.

   b- Prove that the two lines (OM) and (BM′) are perpendicular.

IV- (8 points)

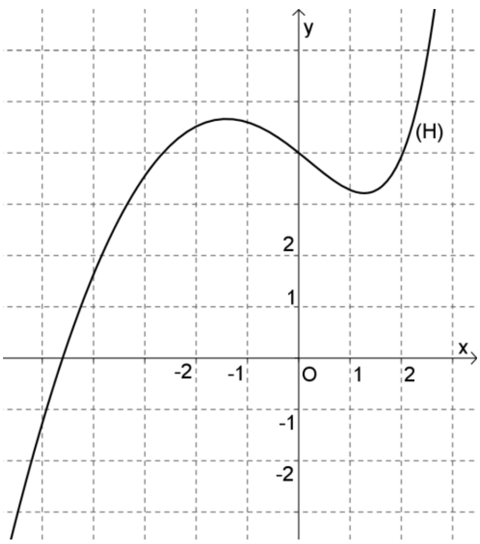
Let f be the function defined on ℝ as: $f(x) = x + 2 - 2e^{x}$. Denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

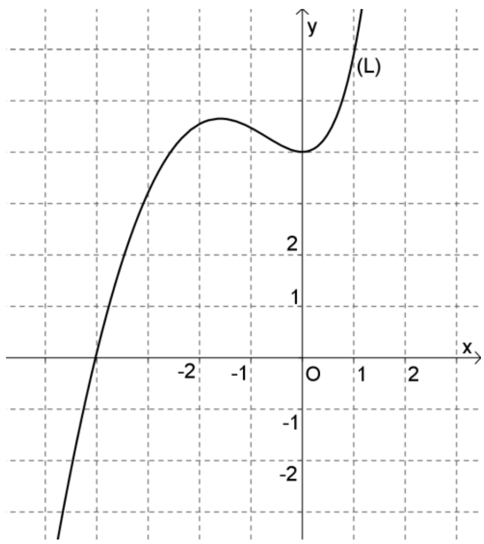
1) a- Determine $\lim\limits_{x \to -\infty} f(x)$.

   b- Show that the line (D) with equation y = x + 2 is an asymptote to (C).

   c- For all x in ℝ, show that the curve (C) is below the line (D).
2) Determine $\lim\limits_{x \to +\infty} f(x)$ and calculate f(1.5).
3) Calculate $f'(x)$ and set up the table of variations of f.
4) Show that the equation f(x) = 0 has, in ℝ, exactly two roots 0 and α.
   Verify that $-1.6 < \alpha < -1.5$.
5) Draw (D) and (C).
6) Denote by $A(\alpha)$ the area of the region bounded by (C) and the x-axis.

   Show that $A(\alpha) = \left(-\dfrac{\alpha^2}{2} - \alpha\right)$ square units.
7) Let g be a function defined on ℝ with: $g'(x) = -2f(x)$.
   One of the two curves (H) and (L) given below represents the function g.
   Choose it with justification.



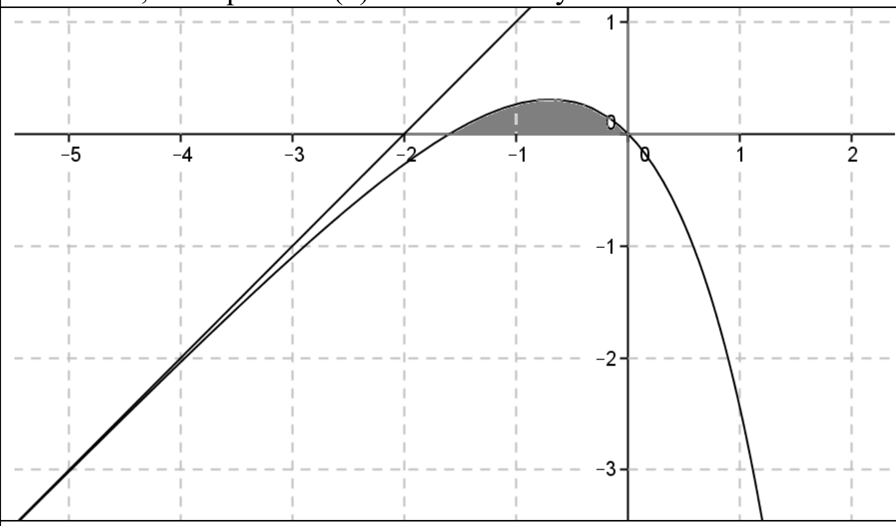


دورة العام ٢٠١٧ الاستثنائيّة الثلاثاء في ٨ آب ٢٠١٧	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسميّة
	أسس تصحيح مادة الرياضيات	عدد المسائل: أربع

I	Answers	Grade
1	$A \in (Q) : 2(x_A) - (y_A) - 5(z_A) + 1 = 0,\ 2(9) - (-1) - 5(4) + 1 = 0,\ 0 = 0$ $B \in (Q) : 2(5) - (1) - 5(4) + 1 = 0,\ 0 = 0.$ $\overrightarrow{n_Q}.\ \overrightarrow{n_P} = (2)(1) + (-1)(2) + (-5)(0) = 0.$	**0.75**
2	$A \in (Q)\cap(P)$ and $B \in (Q)\cap(P)$ then (d) is the line (AB). Hence (d) : $\begin{cases} x = -4k + 9 \\ y = 2k - 1 \\ z = -2k + 4 \end{cases}$ where $k \in \mathbb{R}$	**0.75**
3a	$B \in (L)$ for $t = 1$.	**0.5**
3b	$(L) \subset (Q) : 2(-t + 6) - (-2t + 3) - 5(2) + 1 = 0,\ 0 = 0.$ $\overrightarrow{V_L}.\overrightarrow{V_d} = (-1)(-4) + (-2)(2) + (0)(-2) = 0$	**1**
3c	$E \in (L)$ then $E(-t + 6 ; -2t + 3 ; 2)$, $\overrightarrow{BC}(-2; 1; 0)$, $\overrightarrow{EB}(t-1; 2t - 2; 0)$. Area of (EBC) = $\frac{1}{2}\left\Vert\overrightarrow{EB} \wedge \overrightarrow{BC}\right\Vert = 5$ then $\frac{1}{2}\left\Vert 5(t-1)\vec{k}\right\Vert = 5$, $\frac{1}{2}5\lvert t-1\rvert = 5$, donc $\lvert t-1\rvert = 2$, so $t = 3$ then $(3; -3; 2)$ rejected **or** $t = -1$ then $(7; 5; 2)$ accepted, hence $E(7; 5; 2)$. **Another method:** $(L)\subset(Q)$, $(Q)\cap(P) = (d)$, $(L)\perp(d)$ at B and $(P)\perp(Q)$ thus $(L)\perp(P)$ but $(BC)\subset(P)$ so $(L)\perp(BC)$ at B . Consequently, EBC is a right triangle with vertex B. Area of (EBC) = $\frac{1}{2}EB.BC = 5$. $E \in (L)$ then $E(-t + 6 ; -2t + 3 ; 2)$. $\frac{1}{2}\sqrt{(t-1)^2 + 4(t-1)^2}.\sqrt{5} = 5$ then $\lvert t-1\rvert = 2$ so $t = 3$ then $(3; -3; 2)$ rejected **or** $t = -1$ then $(7; 5; 2)$ accepted, hence $E(7; 5; 2)$.	**1**

II	Answers	Grade
A1	$P(A) = \frac{C_5^2}{C_{10}^2} = \frac{2}{9}$, $P(B) = P(ww) + P(bb) + P(gg) = \frac{C_5^2 + C_3^2 + C_2^2}{C_{10}^2} = \frac{14}{45}$	**1**
A2a	$P(C) = \frac{C_3^2}{C_{10}^2} = \frac{3}{45} = \frac{1}{15}$	**0.5**
A2b	$P(A\cap B) = P(C) = \frac{1}{15} \neq P(A).P(B) = \frac{28}{405}$ then A and B are not independent.	**0.5**
A3	$P(D) = P(A\cap \overline{B}) = P(A) - P(A\cap B) = \frac{2}{9} - \frac{1}{15} = \frac{7}{45}$	**0.5**
A4	$P(A/\overline{B}) = \frac{P(A\cap\overline{B})}{P(\overline{B})} = \frac{P(D)}{1-P(B)} = \frac{7}{31}$	**0.75**
B	$P(Sum = 0) = P(wb \text{ or } bw) + P(gg) = 2\left(\frac{5\times3}{10^2}\right) + \frac{2\times2}{10^2} = 0.34$	**0.75**

III	Answers	Grade
1	$z' = \frac{2i-(1+i)}{i(1+i)} = 1$	**0.5**
2a	$OM' = \lvert z'\rvert = \frac{\lvert z-2i\rvert}{\lvert i\rvert\lvert z\rvert} = \frac{AM}{OM}$	**0.75**
2b	$M \in (d)$ then $M(x; 1)$. $A(0; 2)$ thus $OM' = \frac{AM}{OM} = \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = 1.$ Therefore M' moves on the circle with center O and radius 1. **Another method:** (d) is the perpendicular bisector of [OA] and $M \in (d)$ then $MA = MO$ so $OM' = 1$.	**0.75**
3	$z' - i = \frac{2i-z}{iz} - i = \frac{2}{z}$	**0.5**
4a	$z' - i = 2e^{i\frac{\pi}{4}} = \sqrt{2} + i\sqrt{2}$	**0.75**
4b	$(\overrightarrow{OM}; \overrightarrow{BM'}) = (\overrightarrow{OM}; \vec{u}) + (\vec{u}; \overrightarrow{BM'})\ (2\pi) = -\arg(z) + \arg(z' - i)\ (2\pi) = \frac{\pi}{4} + \frac{\pi}{4}\ (2\pi) = \frac{\pi}{2}\ (2\pi)$ **Another method :** $B(0; 1)$, $\overrightarrow{OM}(\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2})$ and $\overrightarrow{BM'}(\sqrt{2}; \sqrt{2})$. $\overrightarrow{OM}.\overrightarrow{BM'} = 0$	**0.75**

IV	Answers	Grade
1a	$\lim\limits_{x\to-\infty} f(x) = -\infty$	**0.25**
1b	$\lim\limits_{x\to-\infty} \left(f(x)-x-2\right) = \lim\limits_{x\to-\infty}\left(-2e^{x}\right) = 0$ then (D) is an asymptote to (C)	**0.5**
1c	$f(x) - x - 2 = -2e^{x} < 0$ then (C) is below (D)	**0.5**
2	$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} e^{x}\left(\dfrac{x}{e^{x}}+\dfrac{2}{e^{x}}-2\right) = -\infty$; $f(1.5) = -5.463$	**0.75**
3	$f'(x) = 1 - 2e^{x}$ $\begin{array}{c\|ccccccccc} x & -\infty & & \alpha & & -\ln 2 & & 0 & & +\infty \\ \hline f'(x) & & + & & & 0 & & - & & \\ \hline f(x) & -\infty & \nearrow & 0 & \nearrow & 0.306 & \searrow & 0 & \searrow & -\infty \end{array}$	**1.25**
4	• On $\left]-\infty;-\ln 2\right[$: f is continuous and strictly increasing from $-\infty$ to $0.306 > 0$ then the equation $f(x) = 0$ has a unique solution α. $f(-1.6) \times f(-1.5) = (-0.003) \times (0.053) < 0$, then $-1.6 < \alpha < -1.5$. • On $\left]-\ln 2;+\infty\right[$: f is continuous and strictly decreasing from $0.306 > 0$ to $-\infty$ then the equation $f(x) = 0$ has a unique solution β. But since $f(0) = 0$, then $\beta = 0$. Therefore, the equation $f(x) = 0$ has exactly two solutions 0 and α.	**1.25**
5		**1.25**
6	$A(\alpha) = \displaystyle\int_{\alpha}^{0} f(x)dx = \left[\frac{x^2}{2}+2x-2e^{x}\right]_{\alpha}^{0} = -2-\frac{\alpha^2}{2}-2\alpha+2e^{\alpha}$. But $f(\alpha) = 0$, then $2e^{\alpha} = \alpha + 2$, therefore $A(\alpha) = \left(-\dfrac{\alpha^2}{2}-\alpha\right)$ square units.	**1.25**
7	$\begin{array}{c\|ccccccc} x & -\infty & & \alpha & & 0 & & +\infty \\ \hline g'(x) = -2f(x) & & + & 0 & - & 0 & + & \\ \hline g(x) & & \nearrow & & \searrow & & \nearrow & \end{array}$ For $x = 0$, $g'(x) = 0$ and $g'(x)$ changes sign, then the curve of g has an extremum. Consequently, (H) does not represent g. So, (L) represents g.	**1**

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the two points A(0 , 1 , 2) and B(2 , 0 , 2) and the plane (P) with equation $x + 2y - 2 = 0$.

1) Verify that the two points A and B are in plane (P).
2) Show that the plane (Q) containing the line (AB) and perpendicular to plane (P) has an equation $z-2=0$.
3) Let (L): $\begin{cases} x = t+2 \\ y = 2t \\ z = 2 \end{cases}$ $(t \in \mathbf{R})$ be the line perpendicular to plane (P) at B.

   a- Show that (L) lies in plane (Q).

   b- Let E be a point of (L) with $y_E > 0$.
   Determine the coordinates of the point E so that triangle ABE is right isosceles with vertex B.

   c- Let $I\left(\frac{3}{2},\frac{3}{2},2\right)$ be the midpoint of [AE]. Consider in plane (Q) the circle (C) with center I and passing through B. Write a system of parametric equations of the line (T) tangent to (C) at B.

II- (4 points)

The customer service department in a supermarket organizes a game to offer vouchers to its clients. For this purpose, an urn is placed at the entrance of the supermarket. The urn contains:

- three red balls each holding the number 10 000
- two white balls each holding the number 30 000
- one black ball holding the number –10 000.

A client who wants to participate in the game selects, simultaneously and randomly, three balls from the urn.

Consider the following events:

A : " the three selected balls have the same color "
B : " the three selected balls have three different colors "
C : " only two of the three selected balls have the same color "

1) a- Calculate the probabilities P(A) and P(B).

   b- Show that $P(C) = \frac{13}{20}$.

2) A client who participates in the game receives a voucher whose value, in LL, is equal to the sum of the numbers on the three selected balls.
   Let X be the random variable equal to the value of the voucher received by the client.

   a- Verify that the possible values of X are: 10 000 , 30 000 , 50 000 , 70 000.

   b- Show that $P(X = 50\ 000) = \frac{7}{20}$.

   c- Show that $P(X > 35\ 000) = \frac{1}{2}$.

   d- Knowing that a client made purchases with a voucher whose value is greater than 35 000 LL, calculate the probability that exactly one red ball is selected from the urn.

III- (4 points)

The complex plane is referred to a direct orthonormal system $(O; \vec{u}, \vec{v})$. For all points M of the plane with affix $z \neq 0$, we associate the point M' with affix z' such that $z' = \dfrac{z-5i}{z}$.

1) Write z in exponential form in the case where $z' = \dfrac{1}{2} - \dfrac{1}{2}i$.
2) Denote by E the point with affix $z_E = 1$.

   a- Verify that $z' - 1 = \dfrac{-5i}{z}$.

   b- Calculate EM' when $OM = 5$.
3) Suppose that $z = x + iy$ and $z' = x' + iy'$ with x, y, x' and y' being real numbers.

   a- Show that $x' = \dfrac{x^2+y^2-5y}{x^2+y^2}$ and $y' = \dfrac{-5x}{x^2+y^2}$.

   b- Deduce that when M' moves on the line with equation $y = x$, M moves on a circle whose center and radius are to be determined.

IV- (8 points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = 1 - 2e^{-x}$ and denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-1)$.
2) a- Determine $\lim\limits_{x\to+\infty} f(x)$ and deduce an equation of the asymptote (d) to (C).

   b- Show that (C) is below (d) for all x in $\mathbb{R}$.
3) The curve (C) intersects the x-axis at A and the y-axis at B. Find the coordinates of A and B.
4) a- Calculate $f'(x)$ and set up the table of variations of f.

   b- Draw (C) and (d).
5) a- Show that f has, on $\mathbb{R}$, an inverse function g.

   b- Determine the domain of definition of g.

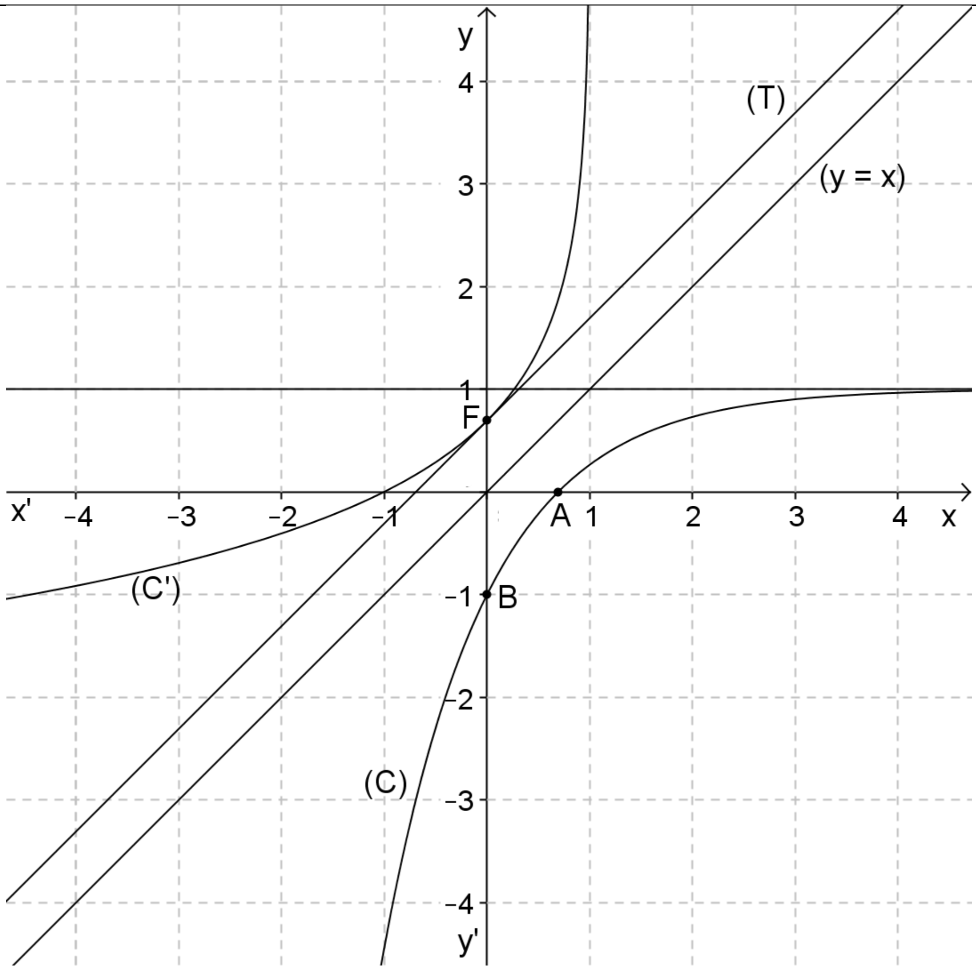
   c- Verify that $g(x) = \ln(2) - \ln(1-x)$.
6) Let (C') be the representative curve of g and let F be the point of (C') with abscissa 0.

   a- Determine an equation of the tangent (T) to (C') at F.

   b- Draw (C') and (T) in the same system as that of (C).
7) Calculate the area of the region bounded by (C'), the x-axis and the y-axis.

أسس تصحيح مسابقة الرياضيات

Q.I	Answer key	4 pts
1	$A \in (P) : (x_A) + 2(y_A) - 2 = 0$, $2(0) + 2(1) - 2 = 0$, $0 = 0$ Similarly: $B \in (P)$.	0.5
2	$A \in (Q)$ … et $B \in (Q)$ … $\overrightarrow{n_Q}.\overrightarrow{n_P} = (0)(1) + (0)(2) + (1)(0) = 0$.	0.5
3.a	$(L) \subset (Q)$ since $2 - 2 = 0$,	0.5
3.b	$E(t+2\ ;\ 2t\ ;\ 2)$, $AB = BE$ then $\sqrt{5} = \sqrt{4t^2 + t^2}$ hence $t = 1$ or $t = -1$, Therefore $E\ (3\ ;\ 2\ ;\ 2)$ accepted . $E\ (1\ ;\ -2\ ;\ 2)$ rejected.	1.5
3.c	A direction vector of (T) is : $\overrightarrow{IB} \wedge \overrightarrow{N_Q} = \frac{3}{2}\vec{i} - \frac{1}{2}\vec{j}$. hence (T): $\begin{cases} x = \frac{3}{2}m + 2 \\ y = \frac{1}{2}m \\ z = 2 \end{cases}$ **Another method:** ABE is right isosceles at B, so (BI) is perpendicular to (AE) hence (T) // (AE) and passes in B.	1
Q.II	**Answer key**	**4 pts**
1.a	$P(A) = \frac{C_3^3}{C_6^3} = \frac{1}{20}$, $P(B) = \frac{C_3^1 \times C_2^1 \times C_1^1}{C_6^3} = \frac{3}{10}$	0.5 0.5
1.b	$P(C) = 1 - P(A) - P(B) = \frac{13}{20}$ ou $P(C) = \frac{C_3^2 \times C_3^1 + C_2^2 \times C_4^1}{C_6^3} = \frac{13}{20}$	0.5
2.a	10 000 (RRN); 30 000 (RRR or RBN); 50 000 (RRB or BBN); 70 000 (RBB)	0.5
2.b	$P(X = 50\,000) = P(RRB) + P(BBN) = \frac{C_3^2 \times C_2^1 + C_2^2 \times C_1^1}{C_6^3} = \frac{7}{20}$	0.5
2.c	$P(X > 35\,000) = P(X = 50\,000) + P(X = 70\,000) = \frac{7}{20} + \frac{C_3^1 \times C_2^2}{C_6^3} = \frac{7}{20} + \frac{3}{20} = \frac{1}{2}$	1
2.d	$P(1 \text{ red} / x > 35\,000) = \frac{P(RBB)}{P(X>35\,000)} = \frac{\frac{C_3^1 \times C_2^2}{C_6^3}}{\frac{1}{2}} = \frac{3}{10}$	0.5
Q.III	**Answer key**	**4 pts**
1	$z = 5 + 5i$ then exponential form of z is $5\sqrt{2}e^{i\frac{\pi}{4}}$	0.5
2.a	$z' - 1 = \frac{z-5i}{z} - 1 = -\frac{5i}{z}$	0.5
2.b	$OM = 5$ so $\lvert z\rvert = 5$. $EM' = \lvert z' - 1\rvert = \left\lvert -\frac{5i}{z}\right\rvert = \frac{5}{\lvert z\rvert} = 1$.	1
3.a	$x' + iy' = \frac{x+iy-5i}{(x+iy)} \times \frac{x-iy}{x-iy} = \frac{x^2+y^2-5y-5ix}{x^2+y^2} = \frac{x^2+y^2-5y}{x^2+y^2} + i\frac{-5x}{x^2+y^2}$	1
3.b	$x' = y'$ then $\frac{x^2+y^2-5y}{x^2+y^2} = \frac{-5x}{x^2+y^2}$ therefore $x^2 + y^2 - 5y + 5x = 0$ hence M varies on a circle with center $I(-\frac{5}{2}\ ;\ \frac{5}{2})$ and radius $R = \frac{5\sqrt{2}}{2}$.	1

Q.IV	Answer key	8 pts
1	$\lim\limits_{x\to-\infty} f(x) = -\infty$. $f(-1) = 1 - 2e$.	0.5
2.a	$\lim\limits_{x\to+\infty} f(x) = 1$ so $y = 1$ is a horizontal asymptote to (C).	0.5
2.b	$f(x) - 1 = -2e^{-x} < 0$ therefore (C) is below (d)	0.5
3	$A(\ln 2 ; 0)$ and $B(0 ; -1)$	0.5
4.a	$f'(x) = 2e^{-x} > 0$. $\begin{array}{c\|ccc} x & -\infty & & +\infty \\ \hline f'(x) & & + & \\ \hline f(x) & & & 1 \\ & -\infty & \nearrow & \end{array}$	1
4.b		1
5.a	f is continuous and strictly increasing over $\mathbb{R}$.	0.5
5.b	$D_g =]-\infty, 1[$	0.5
5.c	$y = f(x) = 1 - 2e^{-x}$, $e^{-x} = \frac{1-y}{2}$, $-x = \ln(\frac{1-y}{2})$, $x = \ln(\frac{2}{1-y}) = \ln 2 - \ln(1-y)$ Then $g(x) = \ln 2 - \ln(1-x)$. Or $f(g(x)) = x$, $1 - 2e^{-g(x)} = x$ so $-g(x) = \ln\left(\frac{1-x}{2}\right)$ therefore $g(x) = \ln\left(\frac{2}{1-x}\right)$	1
6.a	$F(0 ; \ln 2)$, $g'(x) = \frac{1}{1-x}$ then $g'(0) = 1$ so : (T) : $y = x + \ln 2$	0.5
6.b	Figure. (C') and (C) are symmetric of each other w.r.t line $y = x$.	0.5
7	$A = -\int_0^{\ln 2} f(x)dx = -[x+2e^{-x}]_0^{\ln 2} = [\ln 2 + 2e^{\ln 0.5}] + [0+2]$ $A = 1 - \ln 2$ (sq units).	1

وزارة التربية والتعليم العالي المديريّة العامة للتربية دائرة الامتحانـات الرسمية	امتحانات الشهادة الثانوية العامة فرع علوم الحياة	دورة الـعام ٢٠١٨ الاستثنائيّة الثلاثاء ٣١ تموز ٢٠١٨

عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the point A(1 ; 1 ; 1)

and the two lines (d_1) and (d_2) defined as $(d_1):\begin{cases} x = t+1 \\ y = t+1 \ ; t \in \mathbb{R} \\ z = t+1 \end{cases}$ and $(d_2):\begin{cases} x = -k+3 \\ y = k-1 \ ; k \in \mathbb{R} \\ z = k-1 \end{cases}$.

1) a. Show that the two lines (d_1) and (d_2) are not parallel and intersect at the point A.

   b. Show that $y - z = 0$ is an equation of the plane (P) determined by (d_1) and (d_2).

2) Let $B(1\,;0\,;0)$ be a point on a bisector (δ) of an angle formed by (d_1) and (d_2) in the plane (P).

   a. Determine the coordinates of E the orthogonal projection of B on (d_1).

   b. Write a system of parametric equations of the line (Δ) perpendicular to (P) at A.

   c. Denote by F the orthogonal projection of B on (d_2) and M is a point on (Δ) with $y_M \neq 0$.

   Determine the coordinates of M so that the volume of the tetrahedron MABF is equal to $\frac{2}{9}$ units of volume.

II- (4 points)

An urn U contains **six** balls: **four** red balls and **two** blue balls.
A bag S contains **five** bills: **one** 50 000 LL bill, **two** 20 000 LL bills and **two** 10 000 LL bills.

Part A

One ball is randomly drawn from U

- If this ball is red, then **two** bills are drawn successively without replacement at random from S.
- If this ball is blue, then **three** bills are drawn simultaneously at random from S.

Consider the following events:

R: " the drawn ball is red ".
A: " the sum of the values of the bills drawn is 70 000 LL ".

1) Calculate the probabilities $P(R)$, $P\left(A/R\right)$ then verify that $P(A \cap R) = \frac{2}{15}$.

2) Calculate $P(A \cap \overline{R})$. Deduce P(A).

Part B

In this part, **two** bills are drawn successively with replacement at random from the bag S.
Designate by X, the random variable that is equal to the sum of the values of the two drawn bills.

1) Determine the six possible values of X.

2) Show that $P(X = 70\,000) = \frac{4}{25}$.

3) Calculate $P(X < 70\,000)$.

III- (4 points)

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A, B, M and M' with respective affixes $1, 2, z$ and z' so that $z'=\dfrac{z-2}{2-\overline{z}}$ with $z\neq 2$.

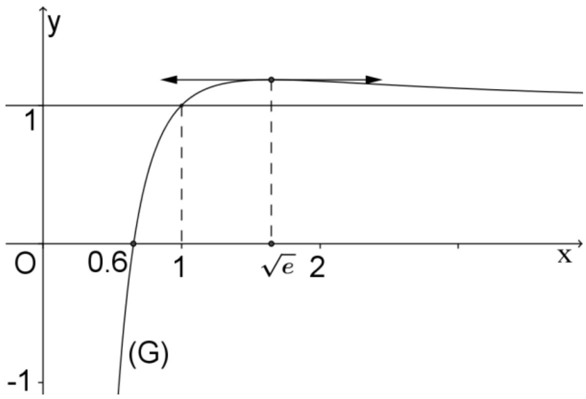
1) In this part, let $z=1-i$.
   a. Write z' in algebraic form and exponential form.
   b. Prove in this case that the quadrilateral $ABMM'$ is a parallelogram.
2) Let $z=x+iy$ where x and y are two real numbers.
   Determine the complex number z so that the points M and M' are confounded.
3) a. Show that $|z'|=1$ for all $z\neq 2$. Deduce that M' moves on a circle whose center and radius are to be determined.
   b. Show that $|z'-1|\leq 2$ for all $z\neq 2$.

IV- (8 points)

Let f be the function defined over $]0;+\infty[$ as $f(x)=x-\dfrac{1+\ln x}{x}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. (1 graphical unit = 2 cm).

Let (d) be the line with equation $y=x$.

1) a. Study, according to the values of x, the relative position of (C) and (d).
   b. Determine $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) is an asymptote to (C).
2) Determine $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$ then deduce an asymptote to (C).
3) In the adjacent figure, we have:
   - (G) is the representative curve of the function **f ', the derivative of f**.
   - (G) admits a maximum for $x=\sqrt{e}$.
   - (G) intersects the x-axis at a point of abscissa 0.6.



   a. Set up the table of variations of f.
   b. Show that the equation f(x) = 0 admits **exactly** two roots so that one of them is equal to 1.
   c. Denote by α the second root of the equation $f(x)=0$. Verify that $0.4<\alpha<0.5$.
   d. Show that (C) admits a point of inflection whose coordinates are to be determined.
   e. Determine the coordinates of the point A on (C) where the tangent (T) at A is parallel to (d).
4) Draw (d), (T) and (C).
5) a. Calculate, in cm^2, the area $A(\alpha)$ of the region bounded by (C), (d) and the two lines with equations $x=\alpha$ and $x=1$.
   b. Prove that $A(\alpha)=\left(2-2\alpha^4\right)$ cm^2.

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانات الرسمية

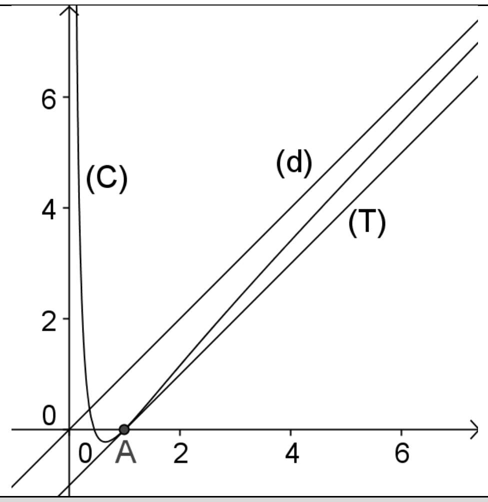
امتحانات الشهادة الثانوية العامة
فرع: علوم الحياة

دورة العام ٢٠١٨ الاستثنائية
الثلاثاء ٣١ تموز ٢٠١٨

أسس تصحيح مسابقة الرياضيات

Q.I	Answer key	4 pts
1.a	$\overrightarrow{u_1}(1;1;1)$ and $\overrightarrow{u_2}(-1;1;1)$ are not collinear. $A \in (d_1)$ for $t=0$ and $A \in (d_2)$ for $k=2$.	**1**
1.b	$(d_1) \subset (P): t+1-(t+1)=0$ for all t and $(d_2) \subset (P): k-1-(k-1)=0$ for all k. **OR** Let M(x ; y ; z) be a point in (P) and $\overrightarrow{AM}.\left(\overrightarrow{u_1} \wedge \overrightarrow{u_2}\right)=0$	**0,75**
2.a	$\overrightarrow{BE}(t\,;\ t+1\,;\ t+1)$ and $\overrightarrow{BE}.\overrightarrow{u_1}=0$ so $t=-\frac{2}{3}$ hence $E\left(\frac{1}{3}\,;\ \frac{1}{3}\,;\ \frac{1}{3}\right)$	**1**
2.b	$\overrightarrow{n_P}(0\,;\ 1\,;\ -1)$ is a direction vector of (Δ) so (Δ): $\begin{cases} x=1 \\ y=m+1 \\ z=-m+1 \end{cases}$; $m \in \mathbb{R}$	**0,5**
2.c	Area of the triangle AEB = Area of the triangle AFB since [AB) is a bisector of the angle $E\hat{A}F$ so $V_{MABE}=V_{MABF}$, $M \in (\Delta)$ so M(1 ; m+1 ; –m+1) $V_{MABE}=\frac{1}{6}\left\|\left(\overrightarrow{AM},\overrightarrow{AB},\overrightarrow{AE}\right)\right\|=\frac{2}{9}$ so $\|m\|=1$ hence m = 1 or m = –1 $M_1(1\,;\ 2\,;\ 0)$ or $M_2(1\,;\ 0\,;\ 2)$ so $M_1(1\,;\ 2\,;\ 0)$ accepted . **OR** $V_{MABE}=\frac{A_{AEB} \times AM}{3}$	**0,75**
Q.II	**Answer key**	**4 pts**
A.1	$P(R)=\frac{C_4^1}{C_6^1}=\frac{2}{3}$ $P(A/R)=P(50000 \text{ and } 20000)$ $=P\left(\left(1^{st}\ 50000 \text{ then } 2^{nd}\ 20000\right) \text{ or } \left(1^{st}\ 20000 \text{ then } 2^{nd}\ 50000\right)\right)$ $=P(1^{st}\ 50000 \text{ then } 2^{nd}\ 20000)+P(1^{st}\ 20000 \text{ then } 2^{nd}\ 50000)$ $=\frac{C_1^1}{C_5^1} \times \frac{C_2^1}{C_4^1}+\frac{C_2^1}{C_5^1} \times \frac{C_1^1}{C_4^1}=\frac{1}{5}$ $P(A \cap R)=P(R) \times P(A/R)=\frac{2}{3} \times \frac{1}{5}=\frac{2}{15}$	**1,5**
A.2	$P(A \cap \bar{R})=P(\bar{R}) \times P(A/\bar{R})=\frac{1}{3} \times \frac{C_1^1 C_2^1}{C_5^3}=\frac{1}{15}$ $P(A)=P(A \cap R)+P(A \cap \bar{R})=\frac{2}{15}+\frac{1}{15}=\frac{1}{5}$	**1**
B.1	$50000+50000=100000$; $50000+20000=70000$; $50000+10000=60000$ $20000+20000=40000$; $10000+10000=20000$; $20000+10000=30000$ $X \in \{20000\ ;\ 30000\ ;\ 40000\ ;\ 60000\ ;\ 70000\ ;\ 100000\}$	**0,5**
B.2	$P(X=70000)=P(50000 \text{ and } 20000)$ $=P((1^{st}\ 50000 \text{ then } 2^{nd}\ 20000) \text{ or } (1^{st}\ 20000 \text{ then } 2^{nd}\ 50000))$ $=P(1^{st}\ 50000 \text{ then } 2^{nd}\ 20000)+P(1^{st}\ 20000 \text{ then } 2^{nd}\ 50000)$ $=\frac{C_1^1}{C_5^1} \times \frac{C_2^1}{C_5^1}+\frac{C_2^1}{C_5^1} \times \frac{C_1^1}{C_5^1}=\frac{4}{25}$	**0,5**
B.3	$P(X<70000)=1-P(70000)-P(100000)=1-\frac{4}{25}-\frac{1}{5} \times \frac{1}{5}=\frac{4}{5}$	**0,5**

Q.III	Answer key	4 pts
1.a	$z' = -i = e^{-i\frac{\pi}{2}}$	0,75
1.b	$Z_{\overrightarrow{AB}} = Z_{\overrightarrow{MM'}} = 1$; moreover, A,B ,M and M ' are not collinear.	0,75
2	$z = \frac{z-2}{2-\bar{z}}$ so $z - z\bar{z} + 2 = 0$ hence $x - x^2 - y^2 + 2 + iy = 0$ so $x - x^2 + 2 = 0$ and $y = 0$ hence $z_1 = 2$ (rejected)or $z_2 = -1$	1
3.a	$\lvert z' \rvert = \frac{\lvert z-2\rvert}{\lvert 2-\bar{z}\rvert} = \frac{\lvert z-2\rvert}{\lvert \overline{z-2}\rvert} = \frac{\lvert z-2\rvert}{\lvert z-2\rvert} = 1$ then OM ' = 1. Hence M' moves on a circle (C) with center O and radius 1.	1
3.b	$\lvert z' - 1\rvert \leq \lvert z'\rvert + \lvert -1\rvert \leq 1+1 \leq 2$ **OR** geometric method ……	0,5

Q.IV	Answer key	8 pts
1.a	$f(x) - x = -\left(\frac{1+\ln x}{x}\right)$ $f(x) - x > 0$ so $-\left(\frac{1+\ln x}{x}\right) > 0$ so $1+\ln x < 0$, $\ln x < -1$, $x < \frac{1}{e}$ then (C) is above (d) For $x > \frac{1}{e}$ then (C) is below (d) and for $x = \frac{1}{e}$ then (C) and (d) intersect	1
1.b	$\lim\limits_{x\to+\infty} f(x) = +\infty$ and $\lim\limits_{x\to+\infty}[f(x) - y] = \lim\limits_{x\to+\infty}\left[\frac{-1}{x} - \frac{\ln x}{x}\right] = 0$ then (d) is an asymptote to (C) at $+\infty$	1
2	$\lim\limits_{\substack{x\to 0\\ x>0}} f(x) = 0 + \infty = +\infty$, so $x = 0$ is V.A	1
3.a	$\begin{array}{c\|c\|ccccc} x & 0 & & & 0.6 & & +\infty \\ \hline f'(x) & \| & & - & 0 & + & \\ \hline f(x) & \| & +\infty & \searrow & -0.2 & \nearrow & +\infty \end{array}$	0,75
3.b	• On $]0;0.6[$: f is continuous and strictly decreasing from $+\infty$ to $-0.2 < 0$ then the equation f(x) = 0 has a unique solution. • On $]0.6;+\infty[$: f is continuous and strictly increasing from $-0.2 < 0$ to $+\infty$ then the equation f(x) = 0 has a unique solution. f(1) = 0, then 1 is one of the roots. Or we have exactly two solutions 1 and another	0,5
3.c	$f(0.4) \times f(0.5) = (0{,}19) \times (-0{,}11) < 0$, then $0.4 < \alpha < 0.5$.	0,25
3.d	$f''(x)$ vanishes and changes sign for $x = \sqrt{e}$ So (C) admits an inflection point ($\sqrt{e}$; f($\sqrt{e}$)).	0,5
3.e	$f'(x_A) = 1$ so $x_A = 1$ then A(1 ;0)	0,75
4		1
5.a	$A(\alpha) = \int_{\alpha}^{1} \frac{1+\ln x}{x} dx = \left[\frac{(1+\ln x)^2}{2}\right]_{\alpha}^{1}$ $= \left[\frac{1}{2} - \frac{(1+\ln\alpha)^2}{2}\right]$ u^2 $= 2 - 2(1+\ln\alpha)^2$ cm^2	0,75
5.b	$f(\alpha) = 0$ $\alpha - \left(\frac{1+\ln\alpha}{\alpha}\right) = 0$ $1 + \ln\alpha = \alpha^2$ $A(\alpha) = 2 - 2(\alpha^2)^2 = 2 - 2\alpha^4$ cm^2	0,5

وزارة التربية والتعليم العالي
المديريّـة العامة للتربية
دائرة الامتحانات الرسمية

امتحانات الشهادة الثانوية العامة
فرع: علوم الحياة

دورة الـعام 2019 العاديّة
الثلاثاء 18 حـزيران 2019

عدد المسائل: اربع
مسابقة في مادة الرياضيات
المدة: ساعتان
الاسم:
الرقم:

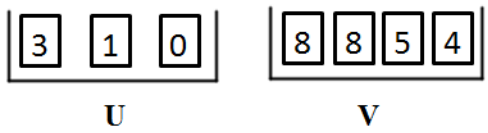
ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, given the points A(4 , 1 , 4), B(1 , 0 , 1), E(3 , −1 , 1) and the plane (P) with equation x + 2y + 3z – 4 = 0.

1) Show that the point E is the orthogonal projection of point A on plane (P).
2) a- Determine an equation of the plane (Q) determined by A, B and E.
   b- Verify that the two planes (P) and (Q) are perpendicular.
3) Let (d) be the line of intersection of (P) and (Q).
   Show that a system of parametric equations of (d) is $\begin{cases} x=-2t+1 \\ y=t \\ z=1 \end{cases}$ $(t\in\mathbb{R})$.
4) Consider, in plane (P), the circle (C) with center E and radius $\sqrt{5}$.
   Show that the line (d) intersects the circle (C) in two points whose coordinates are to be determined.

II- (4 points)



U and V are two urns such that:

- U contains three cards holding the numbers 3, 1 and 0;
- V contains four cards holding the numbers 8, 8, 5 and 4.

One card is selected randomly from urn U:

- If the selected card from U holds the number 0, then two cards are selected randomly and simultaneously from urn V;
- If the selected card from U does not hold the number 0, then three cards are selected randomly and simultaneously from urn V.

Consider the following events:

A: “The selected card from urn U holds the number 0”
S: “The sum of the numbers held on the selected cards from urn V is even”

1) a- Calculate the probabilities $P(S/A)$ and $P(S\cap A)$.
   b- Verify that $P\left(S\cap\overline{A}\right)=\frac{1}{6}$ and calculate P(S).
2) The sum of the numbers held on the selected cards from urn V is even. Calculate the probability that the selected card from urn U does not hold the number 0.
3) Let X be the random variable equal to the product of numbers held by the cards selected from the two urns U and V.
   Calculate P(X = 0) and deduce $P(X\le 160)$.

III- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points M and M' with respective affixes z and z' such that $z' = (1 + i)\,\overline{z}$.

1) In this part, let $z = e^{i\frac{\pi}{3}}$.
   a- Write z' in exponential form.
   b- Verify that $(z')^6$ is pure imaginary.
2) a- Show that $|z'| = \sqrt{2}\,|z|$.
   b- Deduce that, when M moves on the circle with center O and radius $\sqrt{2}$, M' moves on a circle whose center and radius are to be determined.
3) Let $z = x + iy$ and $z' = x' + iy'$, where x , y , x' and y' are real numbers.
   a- Express x' and y' in terms of x and y.
   b- For all $z \neq 0$, denote by N the point with affix $\overline{z}$.
   Prove that the triangle ONM' is right isosceles with principal vertex N.

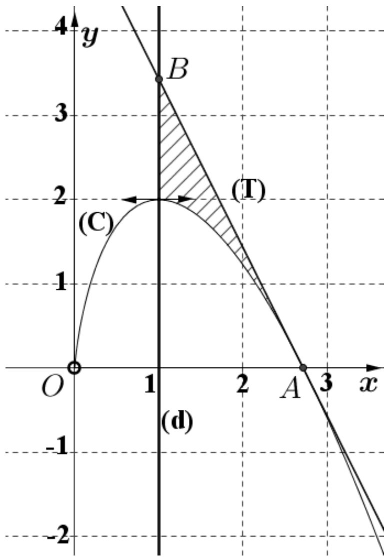
IV- (8 points)

Consider the function f defined over $]0,+\infty[$ as $f(x) = 2x(1-\ln x)$. Denote by (C) its representative curve in an orthonormal system $\left(O;\vec{i},\vec{j}\right)$.

1) Determine $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.
2) a- Let A be the point of intersection of (C) with the x-axis.
   Determine the coordinates of A.
   b- Show that $f'(x) = -2\ln x$ and set up the table of variations of f.
   c- Determine an equation of the tangent (T) to (C) at A.

In the adjacent figure:

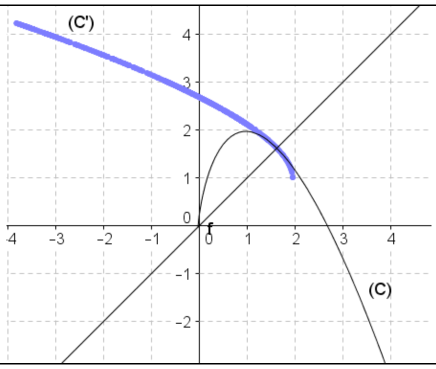
- (C) is the representative curve of f
- (T) is the tangent to (C) at A
- (d) is the line with equation $x = 1$
- $B(1\,,\,2e-2)$ is the point of intersection of (d) and (T).



3) a- Show that f has, over $]1,+\infty[$, an inverse function g whose domain of definition is to be determined.
   b- Set up the table of variation of g.
   c- Copy (C), then draw (C'), the representative curve of g in the same system.
4) a- Using integration by parts, determine $\int x\ln(x)\,dx$.
   b- Show that $\int_1^e f(x)dx = \dfrac{e^2-3}{2}$.
   c- Calculate the area of the shaded region bounded by (C), (T) and (d).

أسس تصحيح مسابقة الرياضيات

Q.I	Answer key	4 pts
1	$x_E + 2(y_E) + 3(z_E) - 4 = 0$, $3 - 2 + 3 - 4 = 0$ then $E \in (P)$. $\overrightarrow{EA}(1,2,3) = \overrightarrow{n_P}$ then E is the orthogonal projection of point A on plane (P). **OR:** $(AE): \begin{cases} x = n+4 \\ y = 2n+1 \\ z = 3n+4 \end{cases}$; E(n+4 ; 2n+1; 3n+4) ; $x_E + 2(y_E) + 3(z_E) - 4 = 0$ then n = -1 so, E(3; –1; 1)	1
2.a	Let $M(x, y, z) \in (Q)$ $\overrightarrow{AM}.(\overrightarrow{AB} \wedge \overrightarrow{AE}) = 0$ $\begin{vmatrix} x-4 & y-1 & z-4 \\ -3 & -1 & -3 \\ -1 & -2 & -3 \end{vmatrix} = 0$ Then (Q): 3x + 6y -5z + 2 = 0.	1
2.b	$\overrightarrow{n_Q}.\overrightarrow{n_P} = 3 + 12 - 15 = 0$. Then the two planes (P) and (Q) are perpendicular.	0.5
3	For every $M(-2t + 1; t ; 1) \in (d)$ $x_M + 2(y_M) + 3(z_M) - 4 = 0$ then $M \in (P)$ $3x_M + 6y_M - 5z_M + 2 = 0$ then $M \in$ **(Q)**	0.5
4	$M(-2t + 1; t ; 1)$ $\overrightarrow{EM}(-2t - 2; t + 1; 0)$ $EM = \sqrt{5}$; $(-2t - 2)^2 + (t + 1)^2 = 5$ then t = 0 or t = –2 Therefore B(1 ; 0 ; 1) (5 ; –2 ; 1).	1
Q.II	**Answer key**	**4 pts**
1.a	$P(S/A) = \frac{C_3^2}{C_4^2} = \frac{1}{2}$, $P(S \cap A) = P(S/A) \times P(A) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$	1
1.b	$P(S \cap \overline{A}) = P(S/\overline{A}) \times P(\overline{A}) = \frac{C_3^3}{C_4^3} \times \frac{2}{3} = \frac{1}{6}$ $P(S) = P(S \cap A) + P(S \cap \overline{A}) = \frac{1}{3}$	1
2	$P(\overline{A}/S) = \frac{P(S \cap \overline{A})}{P(S)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$	1
3	$P(X = 0) = \frac{1}{3} \times 1 = \frac{1}{3}$ $P(X \le 160) = P(X = 0) + P(X = 160)$ $= \frac{1}{3} + \frac{1}{3} \times \frac{C_2^1\, C_1^1\, C_1^1}{C_4^3}$ $= \frac{5}{12}$	1
Q.III	**Answer key**	**4 pts**
1.a	$z' = \sqrt{2}e^{i\frac{\pi}{4}}e^{i\frac{-\pi}{3}} = \sqrt{2}e^{i\frac{-\pi}{12}}$	0.5
1.b	$(z')^6 = (\sqrt{2}e^{i\frac{-\pi}{12}})^6 = 8e^{i\frac{-\pi}{2}} = -8i$ is pure imaginary Or: Ou $\arg(z'^6) = 6\arg(z')\ [2\pi] = 6 \times (\frac{-\pi}{12})\ [2\pi] = -\frac{\pi}{2}\ [2\pi]$, donc (z'^6) est imaginaire pur .	0.5

2.a	$\lvert z'\rvert = \lvert 1+i\rvert\lvert\bar{z}\rvert$; $\lvert z'\rvert = \sqrt{2}\lvert z\rvert$	**0.5**
2.b	$OM = \sqrt{2}$; $\lvert z'\rvert = \sqrt{2}\lvert z\rvert$; $OM' = \sqrt{2}OM = 2$ Then M' moves on a circle of center O and radius 2.	**1**
3.a	$x' + iy' = (1+i)(x-iy) = x+y+i(x-y)$ then $x' = x+y$ and $y' = x-y$.	**0.5**
3.b	$N(\bar{z})$ then $N(x; -y)$; $M'(z')$ then $M'(x+y; x-y)$ $\overrightarrow{ON}(x; -y)$; $\overrightarrow{NM'}(y; x)$ $ON = NM' = \sqrt{x^2+y^2}$ and $\overrightarrow{ON}.\overrightarrow{NM'} = xy - yx = 0$. Then ONM' is right isosceles of vertex N. **OR:** $\frac{z'}{\bar{z}} = 1+i = \sqrt{2}e^{i\frac{\pi}{4}}$, then $OM' = \sqrt{2}ON$ $(\overrightarrow{ON};\overrightarrow{OM'}) = \frac{\pi}{4}[2\pi]$ Then ONM' is right isosceles of vertex N. **OR** $\frac{z'-\bar{z}}{\bar{z}} = i$ **……** then ONM' is right isosceles of vertex N.	**1**
Q.IV	**Answer key**	**8 pts**
1	$\lim\limits_{\substack{x\to 0\\x>0}} f(x) = \lim\limits_{\substack{x\to 0\\x>0}} 2x - 2x\ln x = 0$ and $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} 2x(1-\ln x) = -\infty$	**1**
2.a	$2x(1-\ln x) = 0$; $x = 0$ rej $1 - \ln x = 0$; $\ln x = 1$ then $x = e$ hence A(e; 0)	**0.5**
2.b	$f'(x) = 2(1-\ln x) + (2x)(-1/x) = -2\ln x$ $\begin{array}{c\|ccccc} x & 0 & & 1 & & +\infty \\ \hline f'(x) & & + & 0 & - & \\ \hline f(x) & 0 & \nearrow & 2 & \searrow & -\infty \end{array}$	**1**
2.c	$f'(e) = -2$ (T): $y = -2x + 2e$	**0.5**
3.a	f is continuous and strictly decreasing over $]1;+\infty[$ then f admits an inverse function g. $D_g = \,]-\infty;2[$	**0.5**
3.b	$\begin{array}{c\|ccc} x & -\infty & & 2 \\ \hline g'(x) & & - & \\ \hline g(x) & +\infty & \searrow & 1 \end{array}$	**1**
3.c		**1.5**
4.a	$\int x\ln(x)dx = \frac{x^2}{2}\ln x - \int\frac{x}{2}dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$	**1**
4.b	$\int_1^e f(x)dx = \int_1^e 2xdx - 2\int_1^e x\ln xdx = x^2 - 2\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right) = \left[\frac{3x^2}{2} - x^2\ln x\right]_1^e = \frac{e^2-3}{2}$	**0.5**
4.c	Area $= \frac{(e-1)(2e-2)}{2} - \int_1^e f(x)dx = e^2 - 2e + 1 - \frac{e^2-3}{2} = \frac{e^2-4e+5}{2} = 0.758u^2$	**0.5**

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دائرة الامتحانـات الرسميّة

امتحانات الشهادة الثانوية العامة
فرع علوم الحياة

دورة الـعام ٢٠١٩ الاستثنائيّة
الاثنين ٢٩ تموز ٢٠١٩

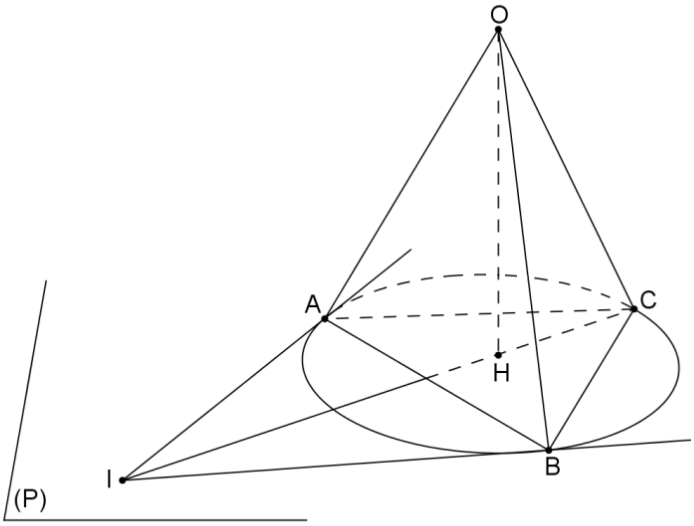
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the space referred to a direct orthonormal system $\left(O;\vec{i},\vec{j},\vec{k}\right)$, consider the points A(6 , 0 , 0), B(0 , 6 , 0) and C(0 , 0 , 6) .

Let (Ω) be the circle circumscribed about triangle ABC.



1) Show that triangle ABC is equilateral.
2) Write a cartesian equation of the plane (P) determined by the points A, B and C.
3) a- Show that point H(2 , 2 , 2) is the orthogonal projection of point O on (P).
   b- Verify that H is the center of (Ω).
   c- Show that the volume of tetrahedron OABC is triple the volume of tetrahedron OAHB.
4) Consider the line (D) with parametric equations: $\begin{cases} x = 6 \\ y = -m \\ z = m \end{cases}$; where $m \in \mathbb{R}$
   a- Show that (D) is tangent to (Ω) at A.
   b- Let (D') be the tangent to (Ω) at B, and I the point of intersection of (D) and (D'). Show that I, H and C are collinear.

II- (4 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A and B with affixes 1 and -2 respectively .

M and M' are two points with respective affixes z and z' such that $z' = \dfrac{\overline{z}+2}{\overline{z}}$ with $z \neq 0$.

1) Write z in exponential form in the case where $z' = 1+i$.
2) a- Show that $OM' = \dfrac{BM}{OM}$.
   b- If $|z'| = 1$, show that M is on a straight line to be determined.
3) a- For all $z \neq 0$, show that $\overline{z}(z'-1) = 2$.
   b- For all $z \neq 0$, verify that $\arg(z'-1) = \arg(z) + 2k\pi$ with $k \in \mathbf{Z}$.
   c- In this part, suppose that $z' = 1 + e^{i\theta}$ with $\theta \in \left]-\pi,\pi\right]$.
   Show that $\overrightarrow{OM} = 2\overrightarrow{AM'}$.

III- (4 points)

A survey conducted on a group of patients showed that these patients either have a heart disease only or a lung disease only or both diseases. It was noted that:

- 60 % of the patients are men.
- Among the men: 20 % have a heart disease only and 50% have a lung disease only.
- Among the women: 25% have a heart disease only and 40% have both diseases.

One patient is selected at random.

Consider the following events:

- M: "The selected patient is a men";
- H: "The selected patient has a heart disease only"
- L: "The selected patient has a lung disease only";
- B: "The selected patient has both diseases".

Part A

1) Calculate the probabilities $P(M \cap H)$, $P(M \cap L)$ and $P(M \cap B)$.
2) Calculate P(H), P(L) and verify that $P(B) = 0.34$.
3) Show that $P(H \cup L) = \frac{33}{50}$.
4) Knowing that the selected patient has only one disease, calculate the probability that this patient has a heart disease.

Part B

The group consists of 500 patients. The names of three patients were randomly and simultaneously selected to win an insurance policy each.

Knowing that the three selected patients have both diseases, calculate the probability that they are men.

IV- (8 points)

Part A

Consider the differential equation $(E): y' - y = -2x$.

Let $y = z + 2x + 2$.

1) Form the differential equation (E') satisfied by z.
2) Solve (E') and deduce the particular solution of (E) satisfying $y(0) = 0$.

Part B

Consider the function f defined over $]-\infty, +\infty[$ as $f(x) = 2x + 2 - 2e^x$.

Denote by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

Let (Δ) be the straight line with equation $y = 2x + 2$.

1) a- Determine $\lim\limits_{x \to -\infty} f(x)$.

   b- Show that (C) is below (Δ) for all x.

   c- Show that (Δ) is an asymptote to (C).
2) Determine $\lim\limits_{x \to +\infty} f(x)$. Calculate $f(1)$ and $f(1.5)$.
3) Calculate $f'(x)$ and set up the table of variations of f.
4) Draw (Δ) and (C).
5) a- Show that f has, over $]0, +\infty[$, an inverse function g whose domain of definition is to be determined.

   b- Denote by (G) the representative curve of g and by (T) the tangent to (C) at the point with abscissa $\ln\left(\frac{3}{2}\right)$. Show that (T) is tangent to (G) at a point L whose abscissa is $2\ln\left(\frac{3}{2}\right) - 1$.

وزارة التربية والتعليم العالي
المديرية العامة للتربية
دائرة الامتحانـات الرسمية

امتحانات الشهادة الثانوية العامة
فرع: علوم الحياة

دورة العام ٢٠٢١ العاديّة
الاثنين ٢٦ تموز ٢٠٢١

عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف الساعة	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one among the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the corresponding answer.

№	Questions	Answers		
		a	b	c
1	Let f be the function given by $f(x) = \frac{\ln(x-2)}{x}$ The domain of definition of f is	$[0 ; +\infty[$	$]2 ; +\infty[$	$]0 ; 2[\cup]2 ; +\infty[$
2	The solution of the equation $\ln(x-2) = \ln(-x+4)$ is	1	2	3
3	Let f be the function defined over $]0 ; +\infty[$ as $f(x) = \frac{\ln(x)}{x}$. An antiderivative of f is	$(\ln(x))^2$	$\frac{(\ln(x))^2}{2}$	$2(\ln(x))^2$
4	$\lim\limits_{x\to+\infty} \frac{1+\ln(x)}{\ln(x)}$ is equal to	0	1	$+\infty$

II- (6 points)

U and V are two urns such that:

- U contains 3 red balls and 5 blue balls.
- V contains 4 red balls and 3 blue balls.

Part A

One ball is randomly selected from U and one ball is randomly selected from V.

1) Show that the probability of selecting two red balls is $\frac{3}{14}$.
2) Calculate the probability of selecting two balls having the same color.
3) Calculate the probability of selecting two balls with different colors.

Part B

In this part, one ball is randomly selected from U:

- if the selected ball from U is red, then two balls are randomly and simultaneously selected from V.
- if the selected ball from U is blue, then three balls are randomly and simultaneously selected from V.

Consider the following events:

R: "The selected ball from U is red"

S: "The selected balls from V have the same color".

1) Determine the probability P(R).
2) Show that $P(S / R) = \frac{3}{7}$ and deduce $P(S \cap R)$.
3) The probability $P(S \cap \overline{R}) = \frac{5}{56}$, calculate P(S).

III- (10 points)

Consider the function f defined on $\mathbb{R}$ as $f(x) = e^x - x - 2$.

Denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Let (d) be the line with equation $y = -x - 2$.

1) Show that $\lim_{x \to +\infty} f(x) = +\infty$. Calculate $f(2)$.

2) **a-** Determine $\lim_{x \to -\infty} f(x)$.

b- Show that (d) is an asymptote to (C) at $-\infty$.

c- Show that (C) is above (d) for all $x \in \mathbb{R}$.

3) Determine $f'(x)$, then set up the table of variations of f.

4) The equation $f(x) = 0$ has two solutions $\alpha > 0$ and $\beta < 0$.

Verify that $1.1 < \alpha < 1.2$.

5) Knowing that $-1.9 < \beta < -1.8$, draw (d) and (C).

6) Let $A(\alpha)$ be the area of the region limited by the curve (C), the line (d)and the two lines with equations $x = 0$ and $x = \alpha$.

a- Verify that $e^\alpha = \alpha + 2$.

b- Prove that $A(\alpha) = (\alpha + 1)$ units of area.

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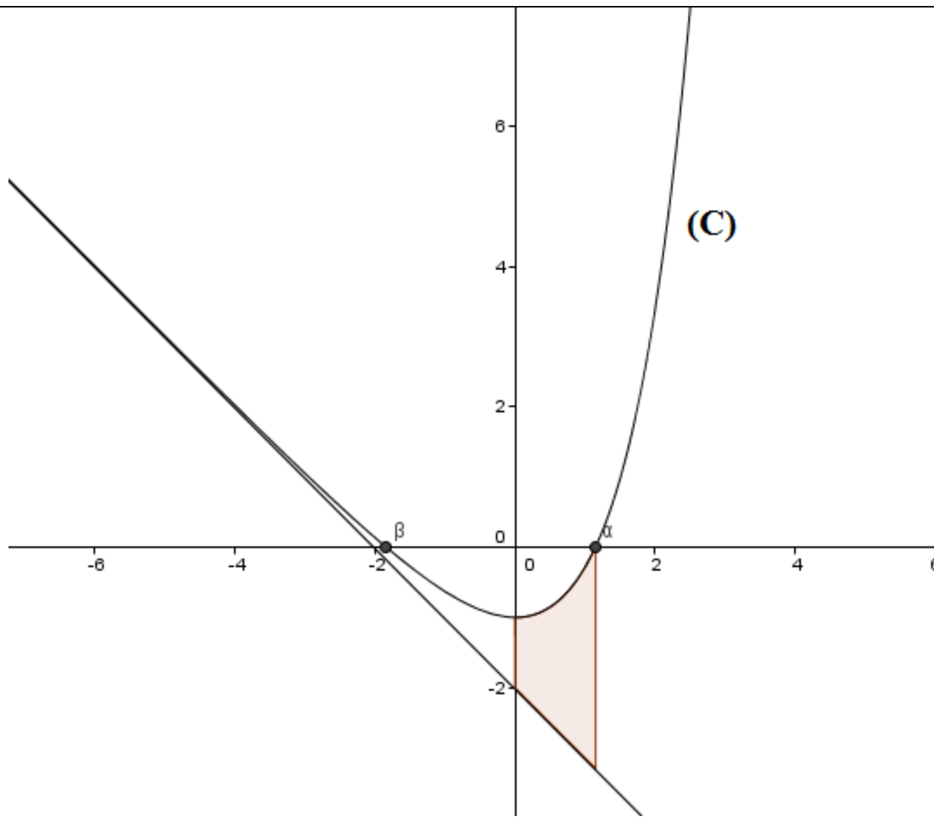
فرع علوم الحياة

دورة العام ٢٠٢١ العادية

اسس تصحيح

I	Answer key	4 pts
1	**b,** $x-2>0$ *and* $x\neq 0$; so domain = $]2,+\infty[$	1
2	**c,** $2<x<4; x-2=-x+4; x=3$ second way: by verification for x=3, ln (1) = ln (1)	1
3	**b,** let u = ln x, $u'=\frac{1}{x}$, $\int \frac{ln(x)}{x}dx=\int u'\times u dx=\frac{(ln(x))^2}{2}+c$ second way: by verification $\left(\frac{(ln(x))^2}{2}\right)'=\frac{ln(x)}{x}$	1
4	**b,** $\lim_{x\to+\infty}\frac{1+\ln(x)}{\ln(x)}=\lim_{x\to+\infty}\frac{1}{\ln(x)}+1=1$ second way: $\lim_{x\to+\infty}\frac{1+\ln(x)}{\ln(x)}=H.R\ \lim_{x\to+\infty}\frac{\frac{1}{x}}{\frac{1}{x}}=1$	1

II	Answer key	6 pts
A1	$P(2\,red\,balls)=\frac{C_3^1}{C_8^1}\times\frac{C_4^1}{C_7^1}=\frac{3}{8}\times\frac{4}{7}=\frac{3}{14}$	1
A2	$P(\text{same color})=\frac{3}{14}+\frac{C_5^1}{C_8^1}\times\frac{C_3^1}{C_7^1}=\frac{3}{14}+\frac{5}{8}\times\frac{3}{7}=\frac{27}{56}$	1
A3	$P(different\ colors)=1-\frac{27}{56}=\frac{29}{56}$	1
B1	$P(R)=\frac{C_3^1}{C_8^1}=\frac{3}{8}$	1
B2	$P(S/R)=\frac{C_4^2}{C_7^2}+\frac{C_3^2}{C_7^2}=\frac{3}{7}$ $P(R)\times P(S/R)=\frac{3}{8}\times\frac{3}{7}=\frac{9}{56}$	1
B3	$P(S)=P(S\cap R)+P(S\cap\overline{R})=\frac{9}{56}+\frac{5}{56}=\frac{1}{4}$	1

<table>
<tr><th>III</th><th>Answer key</th><th>10 pts</th></tr>
<tr><td>1</td><td>$\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} x\left(\frac{e^x}{x} - 1\right) - 2 = +\infty(+\infty - 1) - 2 = +\infty$
$f(2) = e^2 - 4 \cong 3{,}39$</td><td>1</td></tr>
<tr><td>2a</td><td>$\lim\limits_{x\to-\infty} f(x) = +\infty$</td><td>½</td></tr>
<tr><td>2b</td><td>$\lim\limits_{x\to-\infty} [f(x) - (-x-2)] = \lim\limits_{x\to-\infty} e^x = 0$
Then $y = -x - 2$ is an oblique asymptote to (C).</td><td>1</td></tr>
<tr><td>2c</td><td>$f(x) - y_d = e^x > 0$ for all x
Then (C) is above (d) for all $x \in \mathbb{R}$.</td><td>1</td></tr>
<tr><td>3</td><td>$f'(x) = e^x - 1;\ f'(x) = 0\ for\ x = 0$
<table>
<tr><td>x</td><td>$-\infty$</td><td>0</td><td>$+\infty$</td></tr>
<tr><td>f'</td><td>$-$</td><td>0</td><td>$+$</td></tr>
<tr><td>f</td><td>$+\infty \searrow$</td><td>-1</td><td>$\nearrow +\infty$</td></tr>
</table></td><td>2</td></tr>
<tr><td>4</td><td>$f(1.1) = -0.096$ (negative)
$f(1.2) = 0.12$ (positive) and f is continuous so $1.1 < \alpha < 1.2$
Or second way $f(1.1) \times f(1.2) < 0$ and f is continuous so $1.1 < \alpha < 1.2$</td><td>1</td></tr>
<tr><td>5</td><td>
</td><td>2</td></tr>
<tr><td>6a)</td><td>$f(\alpha) = 0,\ e^\alpha - \alpha - 2 = 0 \Rightarrow e^\alpha = \alpha + 2$</td><td>½</td></tr>
<tr><td>6b)</td><td>$Area = \int_0^\alpha [f(x) - y_{(d)}]\,dx = \int_0^\alpha e^x\,dx = e^x]_0^\alpha = e^\alpha - 1 = (\alpha + 1)$ units of area</td><td>1</td></tr>
</table>

عدد المسائل: ثلاث	مسابقة في مادة الرياضيات	الاسم:
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ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
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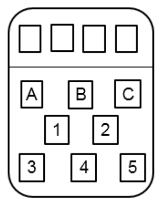
I- (4 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Proposed Answers		
		a	b	c
1	The inequality $\ln x < 1$ is verified for	$x < 0$	$0 < x < e$	$x > e$
2	The equation $\ln^2 x + \ln x - 6 = 0$ has two roots x_1 and x_2. The product $x_1 . x_2$ is equal to	-6	e^{-1}	e^{30}
3	$\lim\limits_{x\to+\infty}\left(\frac{x+\ln x}{x}+1\right) =$	$+\infty$	1	2
4	A security entrance keyboard of a building is formed of three letters A, B and C and five digits 1, 2, 3, 4 and 5. The entrance code is formed of one letter followed by a number consisting of three distinct digits. The number of all possible codes is 	15	180	375

II- (6 points)

An urn U contains red balls and black balls holding distinct natural numbers.

- 60 % of the balls are red of which 80 % hold odd numbers.
- 70 % of the black balls hold odd numbers.

Part A

One ball is selected from the urn. Consider the following events:
R: “the selected ball is red” and O: “the selected ball holds an odd number”.

1) Show that the probability $P(O \cap R)$ is equal to 0.48 and calculate $P(O \cap \overline{R})$.
2) Deduce that $P(O) = 0.76$.
3) Are the events R and O independent? Justify your answer.

Part B

Suppose in this part that the number of balls in the urn U is 50.

1) Show that the number of red balls holding odd numbers is equal to 24.
2) Copy and complete the following table :

	Red	Black	Total
Odd			38
Even			
Total	30		50

3) Three balls are selected randomly and simultaneously from the urn U.
a- Calculate the probability of selecting at least one red ball holding an odd number.
b- The even numbered balls hold the numbers 2, 4, 6, …, 24.
Knowing that the three selected balls hold even numbers, calculate the probability that each of these balls holds a number greater than 15.

III- (10 points)

The plane is referred to an orthonormal system $(O;\vec{i},\vec{j})$.

Consider the function f defined, on $\mathbb{R}$, as $f(x) = 2xe^{-x+1} + 1$ and denote by (C) its representative curve.

1) Determine $\lim\limits_{x \to -\infty} f(x)$.

2) Show that $\lim\limits_{x \to +\infty} f(x) = 1$. Deduce an asymptote (d) to (C).

3) Show that $f'(x) = 2(1 - x)e^{-x+1}$.

4) Copy and complete the following table of variations of f.

x	$-\infty$	1	$+\infty$
$f'(x)$		0	
f(x)			

5) **a-** Show that the equation f(x) = 0 has, on $\mathbb{R}$, a unique solution α.

   b- Verify that $-0.16 < \alpha < -0.15$.

6) Calculate f(–0.5) and f(0) then draw (C) and (d).

7) **a-** Show that $\int xe^{-x+1}dx = (-x-1)e^{-x+1} + K$ where K is a real number.

   b- Deduce the area limited by (C), the straight line with equation y = 3 and the two straight lines with equations x = 0 and x = 4.

عدد المسائل: ثلاث | اسس تصحيح مادة الرياضيات

I	Answers	Grade 4 pts
1	Lnx < 1 . So, x < e but x > 0 . Thus , 0 < x < e (b)	1
2	The roots of the equation : $\ln^2 x+\ln x-6=0$ are $x_1=e^{-3}$ et $x_2=e^2$. then $x_1.x_2=e^{-1}$ (b)	1
3	$\lim\limits_{x\to+\infty}\left(\frac{x+\ln x}{x}+1\right)=\lim\limits_{x\to+\infty}\left(\frac{x}{x}+\frac{\ln x}{x}+1\right)=1+0+1=2$ since $\lim\limits_{x\to+\infty}\left(\frac{\ln x}{x}\right)=0$ (c)	1
4	The number of all codes is : $3\times A_5^3=180$ (b)	1

II	Answers	Grade 6 pts
A1	$P(O\cap R)=P(O/R)\times P(R)=0.8\times 0.6=0.48$ $P(O\cap \overline{R})=P(O/\overline{R})\times P(\overline{R})=0.7\times 0.4=0.28$	1
A2	$P(O)=P(O\cap R)+P(O\cap \overline{R})=0.48+0.28=0.76$	0.5
A3	Since $P(O\cap R)=0.48\neq P(O)\times P(R)=0.76\times 0.6=0.456$ Then, the events R and O are not independent.	0.5
B1	The number of red balls holding odd numbers is $50\times 0.48=24$	1
B2	(table below)	1
B3.a	P (selecting at least one red ball holding an odd number) $=1-\frac{C_{26}^3}{C_{50}^3}=\frac{85}{98}$	1
B3.b	2 ; 4 ; 6 ; 8 ; 10 ; 12 ; 14 ; <u>16</u> ; <u>18</u> ; <u>20</u> ; <u>22</u> ; <u>24</u> P (each of the balls holds a number greater than 15/ knowing that the numbers on the balls are even) $=\frac{C_5^3}{C_{12}^3}=\frac{1}{22}$	1

B2:

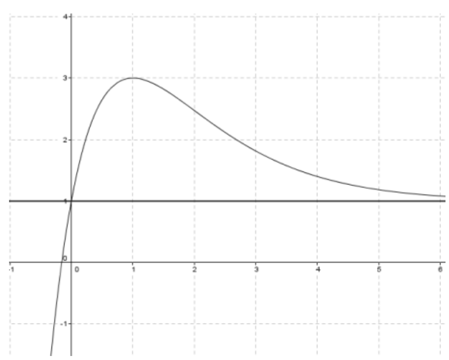
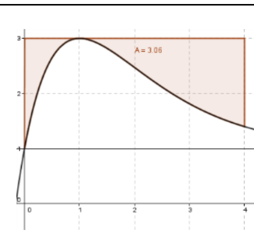
	Red	Black	Total
Odd	24	14	38
Even	6	6	12
Total	30	20	50

<table>
<tr><th>III</th><th>Answers</th><th>Grade
10 pts</th></tr>
<tr><td>1</td><td>$\lim\limits_{x\to-\infty} f(x) = (-\infty)(+\infty)+1 = -\infty$</td><td>1</td></tr>
<tr><td>2</td><td>$\lim\limits_{x\to+\infty} (2x)e^{-x+1} = +\infty \,.\, 0$ Indeterminate form ;

$\lim\limits_{x\to+\infty} \frac{2x}{e^{x-1}} = \frac{0}{0}$ I.F. then $\lim\limits_{x\to+\infty} \frac{2x}{e^{x-1}}$ Using HR $\lim\limits_{x\to+\infty} \frac{2}{e^{x-1}} = 0$.
Thus, $\lim\limits_{x\to+\infty} f(x) = 0 + 1 = 1$

(d) : $y = 1$ is an asymptote to (C).</td><td>1</td></tr>
<tr><td>3</td><td>$f'(x) = (2x)'.e^{-x+1} + (-e^{-x+1}).2x + 0 = 2(1-x)e^{-x+1}$</td><td>0.5</td></tr>
<tr><td>4</td><td><table>
<tr><td>x</td><td>$-\infty$</td><td></td><td>1</td><td></td><td>$+\infty$</td></tr>
<tr><td>$f'(x)$</td><td></td><td>$+$</td><td>0</td><td>$-$</td><td></td></tr>
<tr><td>$f(x)$</td><td>$-\infty$</td><td>$\nearrow$</td><td>3</td><td>$\searrow$</td><td>1</td></tr>
</table></td><td>1.5</td></tr>
<tr><td>5.a</td><td>

- Over $]-\infty\, ;\, 1[$: f is continuous , strictly increasing from $-\infty$ to 3 then the equation $f(x) = 0$ has one solution α
- Over $[1\, ;\, +\infty[$: f is continuousstrictly decreasing from 3 to 1 then , the equation $f(x) = 0$ has no solution.

Therefore, the equation $f(x) = 0$ has, on $\mathbb{R}$, a uniques solution α</td><td>1</td></tr>
<tr><td>5.b</td><td>$f(-0.16) \approx -0.02 < 0$
$f(-0.15) \approx +0.05 > 0$</td><td>0.5</td></tr>
<tr><td>6</td><td>$f(-0.5) = -e^{1.5} + 1 \approx -3.481$
$f(0) = 1$
</td><td>2</td></tr>
<tr><td>7.a</td><td>$(-x-1)'e^{-x+1} + (-x-1)(e^{-x+1})' = (-1 + x + 1)\, e^{-x+1} = xe^{-x+1}$</td><td>1</td></tr>
<tr><td>7.b</td><td>$A = \int_0^4 [3 - f(x)]dx = \int_0^4 [2 - 2xe^{-x+1}]dx$
$= [\, 2x + 2(x+1)e^{-x+1}]_0^4 = 8 + 10\, e^{-3} - 2\, e \approx 3.06$ (units)^2.
</td><td>1.5</td></tr>
</table>

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I- (4 points)

In the table below, only one among the proposed answers to each question is correct.

Write the number of each question and give, **with justification**, the answer that corresponds to it.

Nº	Questions	Proposed answers a	b	c
1	For all real numbers $a > 0$, $\ln\left(\frac{e}{a}\right) + \ln(ae^2) =$	0	3	2
2	The solution set of the equation $\ln(x-1) + \ln(x+1) = 0$ is	$\{-\sqrt{2}\}$	$\{-\sqrt{2}, \sqrt{2}\}$	$\{\sqrt{2}\}$
3	$\lim\limits_{x \to +\infty} [\ln(1+2x) - \ln(1+x)] =$	$\ln 2$	2	0
4	The domain of definition of the function f given by $f(x) = \ln(1 - 2e^{-2x})$ is	$\left]-\infty, \frac{\ln 2}{2}\right]$	$[0, +\infty[$	$\left]\frac{\ln 2}{2}, +\infty\right[$

II- (6 points)

Part A

Consider two urns U and V.

- U contains two red balls holding each the number 0 and two green balls holding each the number 1.
- V contains three red balls holding each the number -1 and two green balls holding each the number 1.

A game consists of choosing randomly one of the two urns U and V and then selecting 2 balls simultaneously and randomly from the chosen urn.

Consider the following events:

U: "Urn U is chosen",

V: "Urn V is chosen",

S: "The two selected balls have the same color",

Z: "The sum of the numbers on the selected balls is zero".

1) Calculate the following probabilities: $P(S/U)$ and $P(S/V)$. Deduce that $P(S) = \frac{11}{30}$.
2) The two selected balls do not have the same color. Show that the probability that they are selected from urn U is $\frac{10}{19}$.
3) Calculate $P(Z)$.
4) Show that $P(S \cup Z) = \frac{2}{3}$.

Part B

All the balls from the two urns U and V are placed in one urn W.

Three balls are selected randomly and successively without replacement from W.

1) What is the number of possible selections of the three balls?
2) Calculate the probability that the product of the numbers on the three selected balls is zero.

III- (10 points)

Part A

Consider the function g defined on $\mathbb{R}$ as $g(x) = 1 + (x - 1)e^{-x}$.

The table below is the table of variations of g.

x	$-\infty$		2		$+\infty$
$g'(x)$		$+$	0	$-$	
$g(x)$	$-\infty$	$\nearrow$	$1 + e^{-2}$	$\searrow$	1

1) Calculate $g(0)$.

2) Show that for all $x \leq 0$, $g(x) \leq 0$ and for all $x \geq 0$, $g(x) \geq 0$.

Part B

Consider the function f defined on $\mathbb{R}$ as $f(x) = x(1 - e^{-x})$ and denote by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

Let (d) be the line with equation $y = x$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-1.5)$.

2) a) Determine $\lim\limits_{x \to +\infty} f(x)$.

   b) Show that the line (d) is an asymptote to (C) at $+\infty$.

   c) Study, according to the values of x, the position of (C) with respect to (d).

3) a) Show that $f'(x) = g(x)$.

   b) Set up the table of variations of f.

4) Show that (C) has an inflection point I whose coordinates are to be determined.

5) Draw (d) and (C).

Part C

Consider the function h defined over $[0 ; +\infty[$ as $h(x) = xe^{-x}$.

1) Set up the table of variations of h.

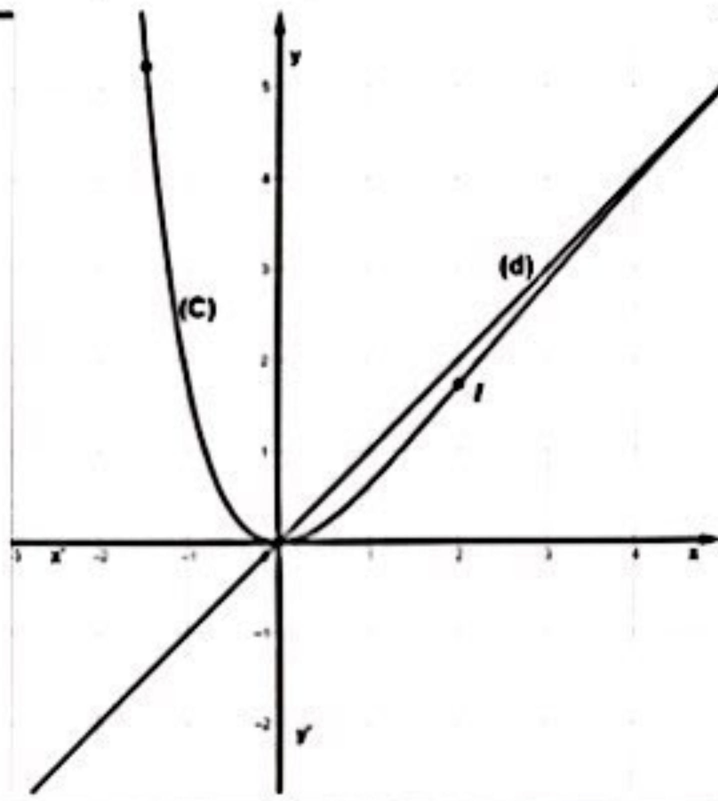
2) Let $M(x_M , f(x_M))$ and $N(f(x_M) , x_M)$ are two variables points where $x_M > 0$.
   Determine the maximum length of segment [MN] as well as the corresponding position of M.

اسس التصحيح

I	Answer key		6pts
1	$\ln e - \ln(a) + \ln e^2 + \ln(a) = 1 + 2\ln e = 3$	b	1.5
2	Conditions: $x - 1 > 0$ *then* $x > 1$ *and* $x + 1 > 0$ *then* $x > -1$ $\ln[(x-1)(x+1)] = 0$ *then* $x^2 - 1 = 1$ $x^2 = 2$ *then* $x = \sqrt{2}$ *accepted or* $x = -\sqrt{2}$ rejected	c	1.5
3	$\lim\limits_{x\to+\infty} \ln\left(\frac{1+2x}{1+x}\right) = \lim\limits_{x\to+\infty} \ln\left(\frac{2x}{x}\right) = \ln(2)$	a	1.5
4	$1 - 2e^{-2x} > 0$ *then* $-2e^{-2x} > -1$ *then* $2e^{-2x} < 1$ $e^{-2x} < \frac{1}{2}$ *then* $-2x < -\ln 2$ *then* $x > \frac{\ln 2}{2}$	c	1.5

II	Answer key	9 pts
A1	$P(S/U) = \frac{C_2^2 + C_2^2}{C_4^2} = \frac{1}{3}$; $P(S/V) = \frac{C_3^2 + C_2^2}{C_5^2} = \frac{2}{5}$ $P(S) = P(S \cap U) + P(S \cap V) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{5} = \frac{11}{30}$	2
A2	$P(U/\bar{S}) = \frac{P(U \cap \bar{S})}{P(\bar{S})} = \frac{P(U) - P(U \cap S)}{1 - P(S)} = \frac{10}{19}$	1.5
A3	$P(Z) = \frac{1}{2} \times \frac{C_2^2}{C_4^2} + \frac{1}{2} \times \frac{C_2^1 \times C_3^1}{C_5^2} = \frac{23}{60}$	1.5
A4	$P(S \cup Z) = P(S) + P(Z) - P(S \cap Z) = \frac{11}{30} + \frac{23}{60} - \frac{1}{2} \times \frac{C_2^2}{C_4^2} = \frac{2}{3}$	1.5
B1	$A_9^3 = 504$	1
B2	**First method** P(product is 0) = 1 − P(product different from 0) $= 1 - \frac{A_7^3}{A_9^3} = \frac{7}{12}$ **Second method** $P(\text{product is } 0) = \frac{A_2^1 \times A_7^2 + A_2^2 \times A_7^1}{A_9^3} \times \frac{3!}{2!} = \frac{7}{12}$	1.5

III	Answer key	15pts
A1	$g(0) = 0$	1
A2	**First method** If $x \in]-\infty, 0]$ then $g(x) \in]-\infty, 0]$ therefore $g(x) \le 0$ If $x \in [0, 2] \cup [2, +\infty[$ then $g(x) \in [0, 1 + e^{-2}] \cup]1, 1 + e^{-2}] = [0, 1 + e^{-2}]$ therefore $g(x) \ge 0$ **Second method** Over $]-\infty, 0]$, g is continuous and increasing from $-\infty$ *to* 0 then $g(x) \le 0$ Over $[0, +\infty[$, g is continuous and increasing from 0 *to* $1 + e^{-2} > 0$ then decreasing to $1 > 0$ thus $g(x) \ge 0$.	1.5

<table>
<tr><td>B1</td><td>$\lim_{x\to-\infty} f(x) = -\infty(1-e^{+\infty}) = +\infty$, f(-1.5) = 5.2</td><td>1</td></tr>
<tr><td>B2a</td><td>$\lim_{x\to+\infty} f(x) = +\infty(1-e^{-\infty}) = +\infty$</td><td>1</td></tr>
<tr><td>B2b</td><td>$\lim_{x\to+\infty} [f(x)-x] = \lim_{x\to+\infty} -x\,e^{-x} = \lim_{x\to+\infty} \frac{-x}{e^x} = \lim_{x\to+\infty} \frac{-1}{e^x} = 0$
Then (d): $y = x$ is an oblique asymptote to (C).</td><td>0.5</td></tr>
<tr><td>B2c</td><td>$f(x) - y_d = -xe^{-x}$
(C) is above (d) for all $x < 0$; (C) is below (d) for all $x > 0$; (C) intersects (d) at (0, 0)</td><td>1</td></tr>
<tr><td>B3a</td><td>$f'(x) = 1 - e^{-x} + xe^{-x} = 1 + (x-1)e^{-x} = g(x)$.
Then $f'(x)$ and g(x) have the same sign.</td><td>1</td></tr>
<tr><td>B3b</td><td><table><tr><td>x</td><td>$-\infty$ 0 $+\infty$</td></tr><tr><td>$f'(x)$</td><td>−   0   +</td></tr><tr><td>$f(x)$</td><td>$+\infty$ ↘ 0 ↗ $+\infty$</td></tr></table></td><td>1.5</td></tr>
<tr><td>B4</td><td>$f''(x) = g'(x)$
$f''(x)$ vanishes at $x = 2$ while changing its sign from positive to negative then (C) admits an inflection point $I(2\,,2-2e^{-2})$.</td><td>1.5</td></tr>
<tr><td>B5</td><td>
</td><td>2</td></tr>
<tr><td>C1</td><td>$\lim_{x\to+\infty} h(x) = 0$; $h(0) = 0$
$h'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x}$<table><tr><td>$x$</td><td>0   1 $+\infty$</td></tr><tr><td>$h'(x)$</td><td>+   0   −</td></tr><tr><td>h(x)</td><td>0 ↗ e^{-1} ↘ 0</td></tr></table></td><td>2</td></tr>
<tr><td>C2</td><td>$MN^2 = (x - xe^{-x} - x)^2 + (x - x + xe^{-x})^2 = 2x^2e^{-2x}$ then $MN = h(x)\sqrt{2}$
The length is maximum when h is maximum, from C1 that is $x = 1$
The maximum length is: $MN = \sqrt{2}\,e^{-1}$ then $M(1{,}1-e^{-1})$</td><td>1</td></tr>
</table>

وزارة التربية والتعليم العالي المديريّة العامّة للتربية دائرة الامتحانات الرسميّة	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام 2023 العاديّة الاثنين 10 تموز 2023
عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

<table>
<tr><th rowspan="2">Nº</th><th rowspan="2">Questions</th><th colspan="3">Proposed answers</th></tr>
<tr><th>a</th><th>b</th><th>c</th></tr>
<tr><td>1</td><td>The solution of the equation $2\ln(x) = \ln(25)$ is</td><td>5</td><td>$\frac{25}{2}$</td><td>-5</td></tr>
<tr><td>2</td><td>Consider the function f defined over $]e\,,+\infty[$ as
$f(x) = x - 3 - \frac{3\ln x}{1 - \ln x}$ and denote by (C) its curve in an orthonormal system $(O\,;\vec{i},\vec{j})$.
(C) admits two asymptotes with equations</td><td>$x = 1$
and
$y = x - 3$</td><td>$x = e$
and
$y = x$</td><td>$x = -e$
and
$y = x - 3$</td></tr>
<tr><td>3</td><td>The table below is the table of variations of a continuous function f over $[0\,,+\infty[$.
$$\begin{array}{c|ccccccc} x & 0 & & 1 & & e & & +\infty \\ \hline f'(x) & & - & 0 & + & 0 & - & \\ \hline f(x) & 5 & \searrow & & \nearrow & 3 & \searrow & \\ & & & -2 & & & & -1 \end{array}$$
The image of the interval $I = [1\,,+\infty[$ by f is</td><td>$[-2\,,3]$</td><td>$[-2\,,-1[$</td><td>$]-1\,,3]$</td></tr>
<tr><td>4</td><td>A code is a number formed of three digits using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
The number of possible even codes greater than or equal to 300 is</td><td>280</td><td>350</td><td>500</td></tr>
</table>

II- (6 points)

During the financial crisis in Lebanon, a study on a group of teachers showed that:

- 30% currently work abroad out of which:
  - 40% teach at private schools only.
  - 20% teach at public schools only.
  - The remaining teachers started working in another domain.
- Out of those who stayed in Lebanon:
  - 50% teach at private schools only.
  - 40% teach at public schools only.
  - The remaining teachers retired and stopped working.

A member from the group is randomly interviewed. Consider the following events:

A: “The interviewed member currently work abroad”
R: “The interviewed member teaches at private schools only”
U: “The interviewed member teaches at public schools only”
D: “The interviewed member works in another domain”
N: “The interviewed member is retired and stopped working”.

1) **a)** Calculate the probabilities $P(A \cap R)$ and $P(\overline{A} \cap R)$.
   b) Verify that $P(R) = 0.47$.
2) Calculate $P(U)$.
3) Show that the probability that the interviewed member is still teaching or is working in another domain is 0.93.
4) The interviewed member does not teach at private schools.
   Calculate the probability that the member stayed in Lebanon.
5) The group consists of 500 teachers.
   a) Show that the number of teachers that teach at private schools is 235.
   b) Three members are interviewed from this group.
   Calculate the probability of interviewing at least two members who teach at private schools.

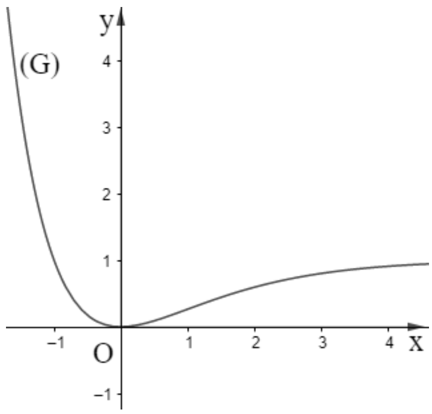
III- (10 points)

The plane is referred to an orthonormal system $(O ; \vec{i}, \vec{j})$.

Part A

The adjacent curve (G) is the representative curve of a differentiable function g over $]-\infty; +\infty[$.

(G) is tangent to the x-axis at O.



1) Using the curve (G):
   a) Verify that $g(x) \geq 0$ for all real numbers x.
   b) The function g' is the derivative of g.
   Study, according to the values of x, the sign of g'.
2) Knowing that $g(x) = (ax + b)e^{-x} + 1$ where a and b are two real numbers, show that $a = b = -1$.

Part B

Consider the function f defined, on $\mathbb{R}$, as $f(x) = (x + 2)e^{-x} + x$.

Denote by (C) the representative curve of f.

Let (D) be the line with equation $y = x$.

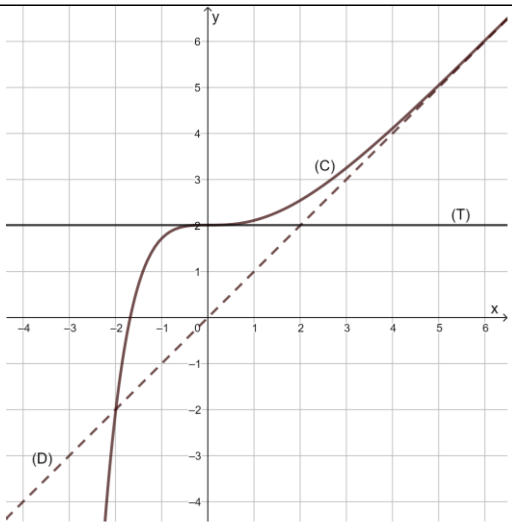
1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-2.5)$.
2) **a)** Determine $\lim\limits_{x \to +\infty} f(x)$.
   b) Show that the line (D) is an asymptote to (C).
   c) Study, according to the values of x, the position of (C) with respect to (D).
3) Show that $f'(x) = g(x)$ then set up the table of variations of f.
4) **a)** Show that the equation $f(x) = 0$ has, on $\mathbb{R}$, a unique root α.
   b) Verify that $-1.7 < \alpha < -1.6$.
5) **a)** Prove that (C) has an inflection point W whose coordinates are to be determined.
   b) Show that the line (T) with equation $y = 2$ is tangent to (C) at W.
6) Draw (T), (D) and (C).
7) Consider the function h defined over $]-2, 0[$ as $h(x) = \dfrac{\ln(x+2) - \ln(-x)}{x}$.
   Prove that $h(\alpha)$ is a natural number to be determined.

عدد المسائل: ثلاث — اسس تصحيح مادة الرياضيات

I	Answers			Answers	6 pts
1	$\ln x^2 = \ln 25$; $x > 0$ $x^2 = 25$ then $x = 5$ Answer : a	1.5	2	$\lim_{x\to e^+} f(x) = e - 3 - \frac{3}{0^-} = +\infty$, so $x = e$ is a vertical asymptote. OR: $\lim_{x\to+\infty}[f(x) - x] = -3 + 3 = 0$, so $y = x$ is an oblique asymptote at $+\infty$. Answer: b	1.5
3	f changes variation over $I = [1; +\infty[$, so $f(I) = [\min(f); \max(f)] = [-2; 3]$. Answer: a	1.5	4	Number of possible even codes greater or equal to 300 is: $7 \times 10 \times 5 = 350$. Answer: b	1.5

II	Answers	9 pts
1a	$P(A \cap R) = P(A) \times P(R/A) = 0.3 \times 0.4 = 0.12$ $P(\overline{A} \cap R) = P(\overline{A}) \times P(R/\overline{A}) = 0.7 \times 0.5 = 0.35$	0.75 0.75
1b	$P(R) = P(A \cap R) + P(\overline{A} \cap R) = 0.12 + 0.35 = 0.47$	0.75
2	$P(U) = P(A \cap U) + P(\overline{A} \cap U) = 0.3 \times 0.2 + 0.7 \times 0.4 = 0.34$	1.5
3	P(still working in teaching or other domains) $= 0.3 + 0.7 \times 0.5 + 0.7 \times 0.4 = 0.93$ **Or** $P(\overline{N}) = 1 - P(N) = 1 - 0.7 \times 0.1 = 0.93$	1.5
4	$P(\overline{A}/\overline{R}) = \frac{P(\overline{A} \cap \overline{R})}{P(\overline{R})} = \frac{P(\overline{A}) - P(\overline{A} \cap R)}{1 - P(R)} = \frac{0.7 - 0.35}{1 - 0.47} = \frac{35}{53}$	1.5
5a	Number of teachers that teach in private schools = $P(R) \times 500 = 0.47 \times 500 = 235$	0.75
5b	P(at least 2 teachers who teach at private schools) = $\frac{235C2 \times 265C1 + 235C3}{500C3} \approx 0.455$	1.5

III	Answers	15 pts
A1a	Over $]-\infty; +\infty[$, (G) is above the x-axis and intersects it at O, so $g(x) \geq 0$ for all values of x..	0.75
A1b	If $x \in]-\infty; 0[$, (G) is strictly decreasing, so $g'(x) < 0$. If $x \in]0; +\infty[$, (G) is strictly increasing, so $g'(x) > 0$. If $x = 0$, $g'(x) = 0$.	0.75
A2	$g(x) = (ax + b)e^{-x} + 1$ $g(0) = 0$, gives $b + 1 = 0$, so $b = -1$ $g'(x) = (a - ax + 1)e^{-x}$ $g'(0) = 0$, gives $a + 1 = 0$, so $a = -1$	0.75
B1	$\lim_{x\to-\infty} f(x) = -\infty \times +\infty - \infty = -\infty - \infty = -\infty$ $f(-2.5) \approx -8.59$	0.75 0.5
B2a	$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}\left(\frac{x+2}{e^x} + x\right) = \lim_{x\to+\infty}\left(\frac{1}{e^x} + x\right) = 0 + \infty = +\infty$ (Règle de l'Hôpital)	0.75
B2b	$\lim_{x\to+\infty}[f(x) - x] = \lim_{x\to+\infty}\frac{x+2}{e^x} = 0$ (proved). Thus (d): $y = x$ is an asymptote to (C).	0.75

<table>
<tr><td>B2c</td><td colspan="3">$f(x) - y_D = (x+2)e^{-x}$
If $x \in]-\infty; -2[$, (C) is below (D).
If $x \in]-2; +\infty[$, (C) is above (D).
(C) and (D) intersect at the point $(-2; -2)$.</td><td>1.5</td></tr>
<tr><td>B3</td><td colspan="3">$f'(x) = (1-x-2)e^{-x} + 1 = (-x-1)e^{-x} + 1 = g(x)$
So f' and g have the same sign and roots, so $f'(x) \geq 0$.
$$\begin{array}{c|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & + & 0 & + & \\ \hline f(x) & -\infty & & \nearrow & & +\infty \end{array}$$</td><td>1
1</td></tr>
<tr><td>B4a</td><td colspan="3">Over $]-\infty; +\infty[$: f is continuous and strictly increasing from $-\infty$ to $+\infty$, so (C) cuts the x-axis at one point only, then the equation $f(x) = 0$ has a unique root α.</td><td>0.75</td></tr>
<tr><td>B4b</td><td colspan="3">$f(-1.7) \approx -0.05 < 0$ and $f(-1.6) \approx 0.38 > 0$.
Thus, $-1.7 < \alpha < -1.6$</td><td>0.75</td></tr>
<tr><td>B5a</td><td colspan="3">$f''(x) = g'(x) = xe^{-x}$.
Thus, $f''(x)$ vanishes at $x = 0$ and changes sign from negative to positive, so (C) admits at $x = 0$ a point of inflection W of coordinates $(0; 2)$.</td><td>1.5</td></tr>
<tr><td>B5b</td><td colspan="3">$f'(0) = g(0) = 0$, so the tangent to (C) at $x = 0$ is parallel to the x-axis.
Also, $f(0) = 2$ so the equation of this tangent is $y = 2$.
Thus (T): $y = 2$ is tangent to (C) at $W(0, 2)$.</td><td>0.75</td></tr>
<tr><td>B6</td><td>
</td><td>1.75</td><td>B7b</td><td>$f(\alpha) = 0$ gives $(\alpha+2)e^{-\alpha} + \alpha = 0$, then $e^{-\alpha} = \frac{-\alpha}{\alpha+2}$, then $-\alpha = \ln(\frac{-\alpha}{\alpha+2})$,
then $\ln(\alpha+2) - \ln(-\alpha) = \alpha$.
Therefore, $h(\alpha) = \frac{\ln(\alpha+2) - \ln(-\alpha)}{\alpha} = \frac{\alpha}{\alpha} = 1$.</td><td>1.5</td></tr>
</table>

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filled with happiness and success

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hand in hand!

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