



Chapter 1: Energy

Grade 12 GS

I- Prerequisites:

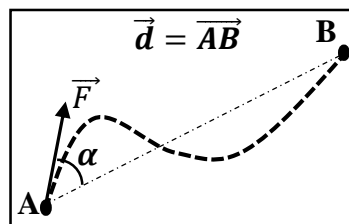
- Scalar or dot product $\vec{v}_1 \cdot \vec{v}_2 = v_1 \times v_2 \times \cos(\alpha)$
- Derivative : $(u^m)' = m u^{m-1} u'$
e.g.: $(x^2)' = 2xx'$ for rectilinear motion $x' = v$ so: $(x^2)' = 2xv$
 $(v^2)' = 2vv'$ for rectilinear motion $v' = a$ so: $(v^2)' = 2va$
- Rectilinear motion:

1. URM: $a = 0$, $v = \text{constant}$: $x = vt + x_0$

2. URAM: $a \cdot v > 0$, $|v| \nearrow$: $x = \frac{1}{2}at^2 + v_0t + x_0$;
3. URDM: $a \cdot v < 0$, $|v| \searrow$: $v = at + v_0$; $v^2 - v_0^2 = 2a(x - x_0)$

II- Work done by a force:

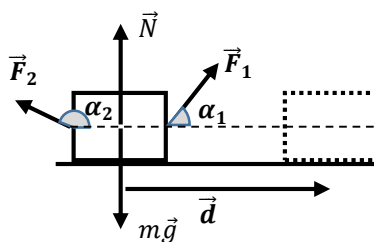
- A force works if it participates in the motion of a system.
- The work done by a **constant force** \vec{F} acting on a solid (S), whether (S) moves along a **rectilinear** displacement $\vec{d} = \overrightarrow{AB}$ or along a **curvilinear** trajectory from A to B (adjacent figure), is: $W_{\vec{F}} = \vec{F} \cdot \vec{d} = F \times d \times \cos \alpha$
- In SI units **F** is in Newton (N), **d** is in meter (m) and **W** is in Joules (J).
- Particular cases:



- A **constant force** is a force that keeps the **same direction and magnitude**
- The work done by a constant force is **independent** of the followed path.

Note 1

Angle α	Work of \vec{F}	Participation in motion
Acute	$W_{\vec{F}} > 0$	The work is motive, and the force tends to accelerate the motion (e.g.: work of \vec{F}_1)
Obtuse	$W_{\vec{F}} < 0$	The work is resistive, and the force tends to decelerate the motion (e.g.: work of \vec{F}_2)
90°	$W_{\vec{F}} = 0$	No participation in motion (e.g.: work of $m\vec{g}$ and \vec{N})

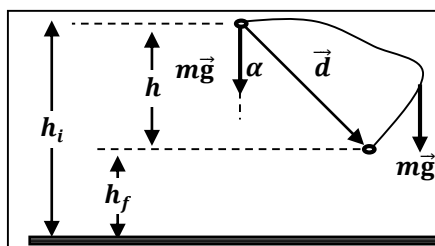


1. Work done by weight

- Along any path:**

When an object moves from an initial height h_i to a final height h_f , the work of the weight as a constant force is independent of the followed path

$$W_{m\vec{g}} = m\vec{g}d \cos \alpha = mgh = mg(h_i - h_f)$$



- If the body moves up: $h_i < h_f \Rightarrow W_{\vec{m}\vec{g}} = -mgh < 0$ then the weight resists the motion.
- If the body moves down: $h_i > h_f \Rightarrow W_{\vec{m}\vec{g}} = mgh > 0$ then the weight helps the motion.

• **Along an inclined plane:**

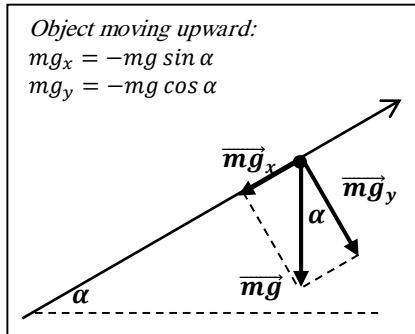
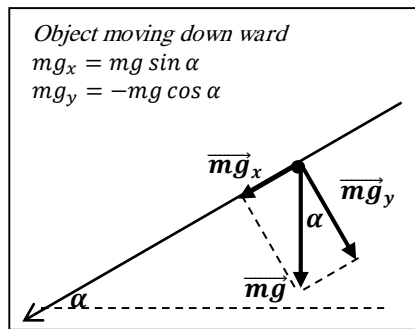
$$\begin{aligned} W_{\vec{m}\vec{g}} &= W_{\vec{m}\vec{g}_x} + W_{\vec{m}\vec{g}_y} \\ &= \vec{m}\vec{g}_x \cdot \vec{d} + \vec{m}\vec{g}_y \cdot \vec{d} \\ &= mg_x \times d \times \cos(\vec{m}\vec{g}_x; \vec{d}) + 0 \end{aligned}$$

- Downwards motion:

$$\begin{aligned} \alpha(\vec{m}\vec{g}_x; \vec{d}) &= 0^\circ \Rightarrow \\ W_{\vec{m}\vec{g}} &= +mgd \sin \alpha \end{aligned}$$

- Upwards motion:

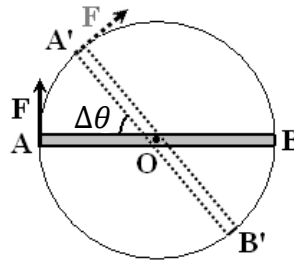
$$\begin{aligned} \alpha(\vec{m}\vec{g}_x; \vec{d}) &= 180^\circ \Rightarrow \\ W_{\vec{m}\vec{g}} &= -mgd \sin \alpha \end{aligned}$$



2. **Work done by a force having a constant moment M**

$$W_{\vec{F}} = \vec{M} \times \Delta\theta$$

($\Delta\theta = \theta - \theta_0$ is the angle described by the object in rotation.



III- **Power due to the force**

- **Definition:** The power is the time rate at which work is done by a force
- The average power: $P_{av} = \frac{\text{work done}}{\text{taken time}} = \frac{W}{\Delta t}$ or $P_{av} = \frac{W}{\Delta t}$
- The instantaneous power: $\vec{P} = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta W}{\Delta t} \right) = \frac{dW}{dt} = \vec{F} \cdot \vec{V}_t$
- **In rotational motion** $\vec{P} = \vec{M} \times \vec{\theta}'$ (where $\vec{\theta}' = \frac{d\theta}{dt}$ is the angular velocity in **rad/s**)
- In S.I units, **W** is in joules (**J**), **t** is in (s) and **P** is in Watt (**W**).
- $1 kWh = 1000 \times W \times 3600s = 36 \times 10^5 J$

if the power P is constant then
 $P = P_{av}$

Note 2

IV-**Energy:**

Energy is the ability to produce work. The SI unit of energy is the Joule (J).

1. **Kinetic Energy:** is the energy stored in a body or a system due to its velocity.

- Case of a particle or a rigid body in translation: $E_k = \frac{1}{2} m v^2$
- Case of a system of particles:

$$E_{k(\text{system})} = \sum E_{k(\text{particles})} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots$$

- For a rigid body in rotation: $E_{k(\text{system})} = \frac{1}{2} I \theta'^2$

Work-energy theorem
 $\Delta E_k = E_{kf} - E_{ki} = \sum W_{\vec{F}_{ext}}$

Note 3

2. Potential energy: is the energy stored in the system, due to the interaction between its particles.

Potential energy could be elastic, gravitational, torsion...

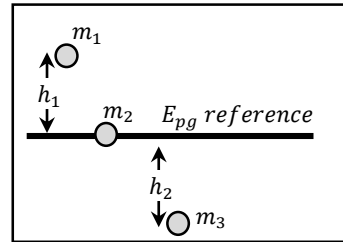
- **Gravitational potential energy (E_{pg}):** $E_{pg} = \pm m g h$

e.g. (on the adjacent figure)

$$E_{pg1} = +m_1 g h_1$$

$$E_{pg2} = +m_2 g h_2 = 0$$

$$E_{pg3} = -m_3 g h_3$$



Variation of E_{pg}

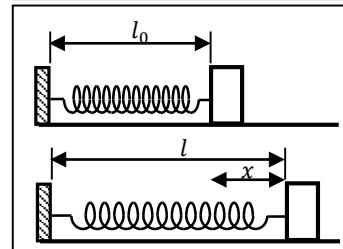
$$\Delta E_{pg} = m g (h_f - h_i) \\ = -W_{m\vec{g}}$$

Note 4

- **Elastic potential energy**

$$E_{pe} = \frac{1}{2} k (\Delta l)^2 = \frac{1}{2} k x^2$$

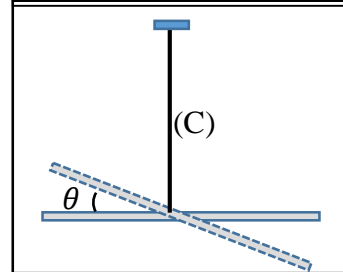
k: spring constant (or stiffness) expressed in N/m (SI)



- **Torsion Potential energy**

$$E_{pt} = \frac{1}{2} C (\theta)^2$$

C: is the torsion constant of the wire is in Nm/rad (SI)



Analogy	
Spring	Torsion wire
k: spring constant	C: torsion constant
$T = -kx$	$M = -C\theta$
$E_{pe} = \frac{1}{2} k x^2$	$E_{pt} = \frac{1}{2} C \theta^2$

3. Mechanical Energy: is the sum of the kinetic energy and potential energy of a system $E_m = E_k + E_{pg} + E_{pe}$

- **Conservation of mechanical energy:** If no non - conservative forces acting on the system: $E_m = \text{constant}$ and $\Delta E_m = 0$ so $E_{mi} = E_{mf}$
- **Non conservation of mechanical energy:** If non – conservative forces exist $\Delta E_m = \sum W_{\vec{F}(\text{non-conservative forces})}$
 - Non conservative forces: friction, traction, push...
 - Conservative forces: weight, normal, tension of spring.

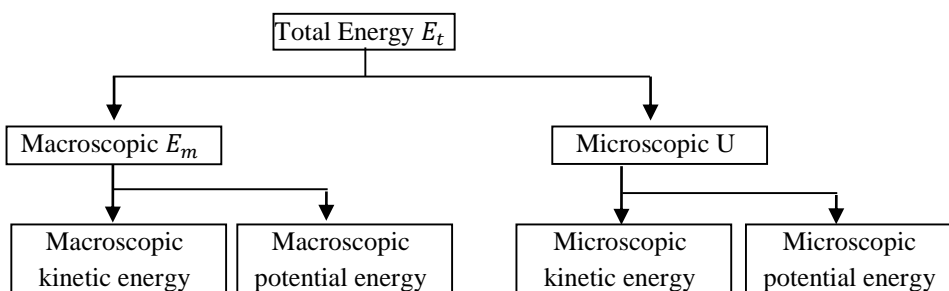
$$W_{\vec{F}(\text{non-conservative forces})} \neq 0 \\ \Rightarrow E_m \neq \text{const.}$$

$$|\Delta E_m| = |E_{mf} - E_{mi}|$$

This is the amount of energy gained by the system in the form of E_m or lost in thermal form.

Note 5

4. Total Energy $E_t = E_m + U$



Energy-isolated system:
 $E_t = E_m + U = \text{constant}$
 $\Rightarrow \Delta E_m = -\Delta U$ (e.g.: the system [(S), Earth, atmosphere].)

In addition, if $\Delta U = 0$
 then $\Delta E_m = 0$
 therefore $E_m = E_K + E_P = \text{constant}$
 As a result, $\Delta E_K = -\Delta E_P$

Note 6