Summary

* Math:-

① Integrals :-

⊛ Rules:-

① $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

② $\int u' u^n \, dx$ where u' : is the derivative of u.

$= \frac{u^{n+1}}{n+1} + c$

③ $\int \frac{1}{x} \, dx = \ln|x| + c$

④ $\int \frac{u'}{u} \, dx = \ln|u| + c.$

⑤ $\int u' e^u \, dx = e^u + c$

⑥ $\int e^{ax+b} \, dx = \frac{1}{a} e^{ax+b} + c$

⑦ $\int u.v \, dx$ where $u \neq v$ and both are functions.

we use integration by part.

⊛ Areas:-

① $\int_a^b f(x) \, dx = F(b) - F(a)$ where $F(x)$ is the anti-derivative of $f(x)$.

② Case $\int_{-a}^{a} f(x)\,dx$

1st Case:- if $f(x)$ is odd, then, $\int_{-a}^{a} f(x)\,dx = 0$.

2nd Case:- if $f(x)$ is even, then, $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx = 2F(a)$

* Notes about Integrals:-

① The unit of Area is (u^2).

② $\int_{a}^{b} f(x)\,dx \longrightarrow$ should: $b > a$.

② Equations:-

① Asymptotes:- $y = a$ & $x = b$

$y = a$: Horizontal Asymptote

$x = b$: Vertical Asymptote.

② Linear:- $y = ax + b$

③ Quadratic:- $y = ax^2 + bx + c$

④ Cubic:- $y = ax^3 + bx^2 + cx + d$.

⑤ Rational:- $y = \frac{ax+b}{cx+d}$ or $y = \frac{f(x)}{g(x)}$

⑥ Irrational:- $y = \sqrt{ax+b}$ or $y = \sqrt{g(x)}$

⑦ Circle:- $(x-a)^2 + (y-b)^2 = R^2$ (R: radius & (a,b) center).

or $x^2+y^2-2ax-2by+c=0 \Leftrightarrow R = \sqrt{a^2+b^2-c}$ & (a,b) center.

*Notes:-

① Function is even when, $f(-x) = f(x)$ or $f(-x) - f(x) = 0$

② Function is odd when, $f(-x) = -f(x)$ or $f(-x) + f(x) = 0$.

③ $x = b$ an axis of symmetry, $f(2b-x) = f(x)$.

④ (α, β) center of symmetry, $f(2\alpha - x) + f(x) = 2\beta$.

⑤ To determine if a point is on, outside or inside the circle.

$P(A)$ = equation of circle ; if, $P(A) > 0 \rightarrow$ point is outside circle.

$P(A) < 0 \rightarrow$ point is inside circle.

$P(A) = 0 \rightarrow$ point is on circle.

③ Trigonometry:-

$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin x}{\cos x} = \frac{1}{\cot x}$

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$.

* Some Rules:-

1) $\sin(-x) = -\sin x$

2) $\cos(-x) = \cos x$

3) $\sin(\frac{\pi}{2} - x) = \cos x$

4) $\cos(\frac{\pi}{2} - x) = \sin x$

5) $\cos(\frac{\pi}{2} + x) = -\sin x$

6) $\sin(\frac{\pi}{2} + x) = \cos x$

7) $\sin(\pi - x) = \sin x$

8) $\sin(\pi + x) = -\sin x$

9) $\cos(\pi - x) = -\cos x$

10) $\cos(\pi + x) = -\cos x$.

* Derivatives :-

1) $(\sin U)' = U' \cos U$

2) $(\cos U)' = -U' \sin U$

3) $(\tan U)' = U'(1 + \tan^2 U)$

4) $(\cot U)' = -U'(1 + \cot^2 U)$.

* Double Angle:-

1) $\cos 2x = \cos^2 x - \sin^2 x$

2) $\cos 2x = 2\cos^2 x - 1$

3) $\cos 2x = 1 - 2\sin^2 x$

4) $\sin 2x = 2 \sin x \cos x$.

* Notes:-

1) $\cos^2 x + \sin^2 x = 1$

2) $\cot x \cdot \tan x = 1$

④ Logarithmic Functions:-

* Form:- $\log_b a = p$ or $b^p = a$ where:
b: base a: answer p: power.

* Rules:- (Case of same bases.)

1) $\log_b m = \log_b n \iff m = n$.

2) $\log_b m + \log_b n \iff \log_b (m \times n)$.

3) $\log_b m - \log_b n \iff \log_b (m/n)$.

4) $n \log_b m \iff \log_b m^n$

5) To remove $\log_b \iff \log_b m = x$ we put the power base (b) $\Rightarrow \cancel{b} \log_{\cancel{b}} m = x \Rightarrow \boxed{m = b^x}$.

6) $-\log_b m \iff \log_b \left(\frac{1}{m}\right)$.

*Note: $\log_e x = \ln x$

* $\log_b b = 1$ * $\ln e^x = e^{\ln x} = x$

⑤ Solving Equations:-

① Absolute:-

a) $|U| = a \Rightarrow U = a$ or $U = -a$. where U: function.

b) $|U| = |V| \Rightarrow U = -V$ or $U = V$ where V & U are functions.

c) $|u| < a$ or $|u| \leq a \rightarrow -a < u < a$ or $-a \leq u \leq a$.

d) $|u| > a$ or $|u| \geq a \rightarrow u > a$ or and $u < -a$ or
$u \geq a$ and $u \leq -a$

② Square Root:-

a) $\sqrt{x^2} = |x| = \pm x$ $\begin{cases} \text{if } x > 0 \rightarrow +x \\ \text{if } x < 0 \rightarrow -x. \end{cases}$

b) Domain of it, $\sqrt{u} \Rightarrow u \geq 0$ by sign table.

c) Domain of it, $\frac{V}{\sqrt{u}} \Rightarrow u > 0$ by sign table.

⑥ Angles to Radians:-

① From Angles to Radians:-

from $\alpha \rightarrow$ to $\beta\pi$ (rd) $\rightarrow$ radian.

we say:- if $1\pi \longrightarrow 180°$
$?? (\beta\pi) \xleftarrow{\times} \alpha$

$\frac{\alpha\pi}{180} = \beta\pi \Rightarrow \beta = \frac{\alpha}{180}$

② From Radian to Angle:-

Case 1:- if $\frac{n\pi}{m}$ $(n < m) \rightarrow \alpha = \frac{n \times 180°}{m}$ (Just put instead of $\pi \Rightarrow 180°$).

Case 2:- if $\frac{n\pi}{m}$ $(n > m) \rightarrow$ Return to principle to be as $\frac{n\pi}{m} = \frac{a\pi}{m} \pm \frac{b\pi}{m}$ where $\frac{a}{m}$ is even. then $\frac{a\pi}{m} = 0$

$\Rightarrow$. then it $\pm \frac{b\pi}{m}$, we use the case 1 to determine angle.

(7) Complex forms:-

* Form:- $Z = a + bi$ where a: real part & b: imaginary part.

* $i^2 = -1 \Leftrightarrow \sqrt{-1} = i$

* Conjugate of $Z = a+bi$ is $\bar{Z} = a - bi$.

* Modulus:- $|Z| = \sqrt{a^2+b^2}$ (length of Z)

* Argument:- arg(Z) = in calculator (① ~~Mode~~ Mode → [2]
② Shift → [2] → [1] then, put the complex form of i is Shift → ENG)

* To determine the Modulus and Argument (length & angle) of Z by Calculator we do:-

① We put the complex form on calculator. (Shift → [2])

② Shift → [2] → [3] → [=]

③ Then, before the sign (∠) is Modulus and after (∠) is angle in radian (you should put the angle in radian).

* To put the angle in radian press (shift → mode → [4]).

(8) Rules in Coordinate System:-

① Midpoint:- $x_{midpt} = \frac{x_A + x_B}{2}$ & $y_{midpt} = \frac{y_A + y_B}{2}$

② To prove collinear:- (A,B,C) : $\frac{x_A - x_B}{x_C - x_A} = \frac{y_A - y_B}{y_C - y_A}$

OR $\vec{AB} = k\vec{AC}$ $\begin{bmatrix} x_B - x_A = k(x_C - x_A) \\ y_B - y_A = k(y_C - y_A) \end{bmatrix}$.

③ length of a segment:- (AB)

$$\sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}.$$

④ Prove 2 lines are $\perp$:-

Slope of 1st line × Slope of 2nd line $= -1$. | slope $= \frac{\Delta y}{\Delta x}$

⑤ Prove 2 lines are $\parallel$:-

Slope of 1st line = Slope of 2nd line.

⑥ Vectors are $\perp$:-

$\vec{a} \cdot \vec{b} = 0$ or $a \times b \times \cos(\vec{a}, \vec{b}) = 0$

then, $(\vec{a}, \vec{b}) = \frac{\pi}{2}$ or $90°$. $\Rightarrow \vec{a} \perp \vec{b}$.

⑨ Statistical Rules-

① **Mean ($\bar{x}$)** : is the sum of all numbers divide by their total.

$$\bar{x} = \frac{\Sigma \text{ of all numbers}}{n}$$

② **Mode**: is the number which is most repeated.

③ **Median**: is the middle number after arranging the numbers either ascending or descending

- Median
  - total number (n) is even → $\text{Med} = \frac{n+1}{2}$
  - total number (n) is odd → $\text{Med} = \frac{n}{2} = x$ or we take also $x+1$ then $\frac{2x+1}{2} = \text{Med}$.

④ **Range**: Is the difference between the higher number to the lower number.

⑩ Derivatives :-

① $(x^n)' = nx^{n-1}$	⑤ $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	⑨ $(\ln u)' = \frac{u'}{u}$
② $(u^n)' = nu^{n-1} \cdot u'$	⑥ $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$	⑩ $(e^x)' = e^x$
③ $\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$	⑦ $(\sqrt{x^n})' = \frac{nx^{n-1}}{2\sqrt{x^n}}$	⑪ $(e^u)' = u'e^u$
④ $\left(\frac{1}{x^n}\right)' = \frac{-n}{x^{n+1}}$	⑧ $(\ln x)' = \frac{1}{x}$	⑫ $(u \cdot v)' = u'v + v'u$

⑬ $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$.

⑭ Areas & Perimeters:-

*Areas:-

① Square: $A = s^2$

② Rectangle: $A = l \times w$

③ Triangle: 1st case: if given base & height then, $A = \frac{b \times h}{2}$.

2nd case: if not given base & height & given all sides (a, b, c)

then, first find S where $S = \frac{a+b+c}{2}$

then, $A_\Delta = \sqrt{S(S-a)(S-b)(S-c)}$ (Heron's formula)

④ Circle:- $A = \pi r^2$

⑤ Semi-circle: $A = \pi r^2/2$

⑥ Trapezoid: $A = \frac{(b+B) \times h}{2}$ B: big base b: small base.

⑦ Rhombus: $A = \frac{d_1 \times d_2}{2}$

⑧ Parallelogram: $A = b \times h$

⑨ Sphere:- $A = 4\pi r^2$

* Perimeters:-

① Square: $P = 4s$

② Rectangle: $P = 2l + 2w$

③ Triangles:- 1st case: Isos. Δ: $P = 2s + s'$

2nd case: Equil. Δ: $P = 3s$.

3rd case: Scalene Δ: $P = s + s' + s''$.

④ Circle: $P = 2\pi r$

⑤ Semi-circle: $P = \pi r$

⑥ Trapezoid: $P = B + b + s + s'$

⑦ Rhombus: $P = 4s$

⑧ Parallelogram: $P = 2l + 2w$

⑨ Sphere: $P = 2\pi r$ (same as a circle).

⑫ General Notes about Derivatives:-

① if $y = x$ ⇒ then to derive it, the variable should be x

dy/dx is derived.

② if $y = x$ and the variable of derivation is different than x

then $dy/dt = y$.

③ $y = x \rightarrow$ y is the function of $x \Rightarrow y = f(x)$.

so, the variable of $f(x)$ is x.

④ Main Rule :-

To derive a function, $y = f(\text{variable})$.

(y may be other than y.)

$\Rightarrow \dfrac{dy}{d\text{variable}}$ is derived.

⑬ Probability :-

① Permutations :- $nPr = \dfrac{n!}{(n-r)!}$

② Combinations :- $nCr = \dfrac{n!}{(n-r)!} \times \dfrac{1}{r!} = \dfrac{nPr}{r!}$

③ If the question is said "simultaneously" $\Rightarrow$ in probability use Combinations.

④ If the question is said "successively" $\rightarrow$ without Rep we use "nPr" [permutation].

⑤ If the question with Rep. $\rightarrow$ we use n^r [permutations].