PART A : Consider the function g that is defined over $]0;+\infty[$ as: $g(x)=1-\dfrac{1}{x}+\ln(x)$.

Let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. G.U = 2 cm.

1. Calculate $\lim\limits_{x\to 0^+}[g(x)]$ *and* $\lim\limits_{x\to +\infty}[g(x)]$. Deduce an asymptote to the curve (C).
2. Calculate g (1), g (2) and g (e).
3. Calculate $g'(x)$, then study the variations of function g.
4. Write an equation of the tangent line (T) to (C) at a point A of abscissa 1.
5. Draw (T) and (C).

PART B: Consider the function f that is defined over $]0;+\infty[$ as: $f(x)=-1+(x-1)\ln(x)$.

The below table is the table of variations of the function f over $]0;+\infty[$:

x	0		1		$+\infty$
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$f(x)$	$+\infty$	$\searrow$	-1	$\nearrow$	$+\infty$

1. Prove that the equation f (x) = 0 has exactly two roots α and β such that: $0.2<\alpha<0.3$ and $2.2<B<2.3$.
2. Designate by (E) the region bounded by the curve (C) of the function g, the x-axis and the two straight lines x = α and x = β. Let A be the area of the region (E).
   a- Prove that for all x $\in\ ]0;+\infty[$ we have: $f'(x)=g(x)$.
   b- Prove that: $A=\int_1^{\alpha} g(x)dx+\int_1^{\beta} g(x)dx$.
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1) $\lim\limits_{x->0} g(x) = 0 - \frac{1}{0} - \infty$

$= -\infty - \infty = -\infty$

$\lim\limits_{x->+\infty} g(x) = 1 - 0 + \infty$

$=+\infty$

X = 0 is V.A at $-\infty$

2) g(1) = 1 – 1 + 0

= 0

g(2) = 1.2

➢ $g(e) = 1 - \frac{1}{e} + \ln e$

$= -\frac{1}{e} = -0.36$

3) $g'(x) = -(-\frac{1}{x^2}) + \frac{1}{x}$

$g'(x) = \frac{1}{x^2} + \frac{1}{x} > 0$ for every $x \in Df$

x	0		$+\infty$
g'(x)		+	
g(x)	$-\infty$	$\nearrow$	$+\infty$

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4) *equation of tangent*:

(T): $y - y_A = g'(x_A)(x - x_A)$

$y - 0 = 2(x - 1)$

$y = 2x - 2$

$g'(1) = 1 + 1 = 2$

$g(1) = 0$

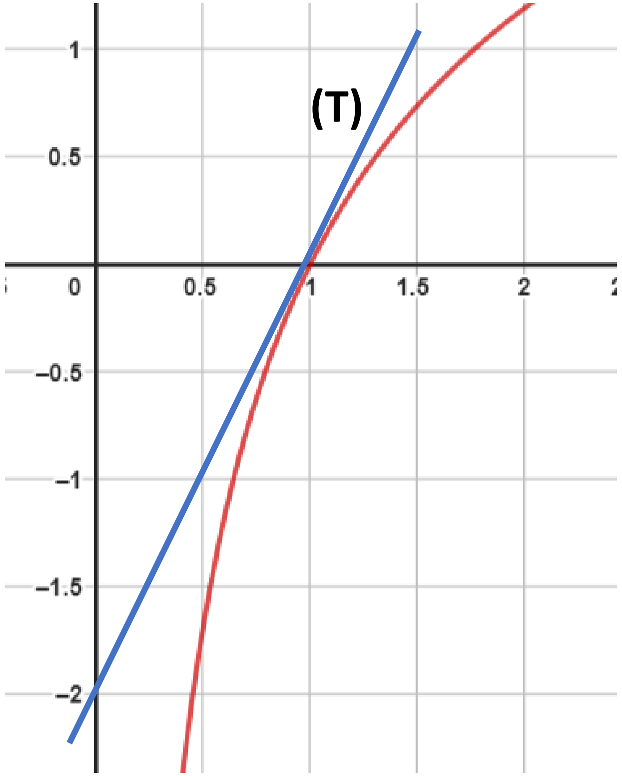
5) *Draw*

x = 0 V.A

(1,0)

(2, 1.2)

(e, 0.36)



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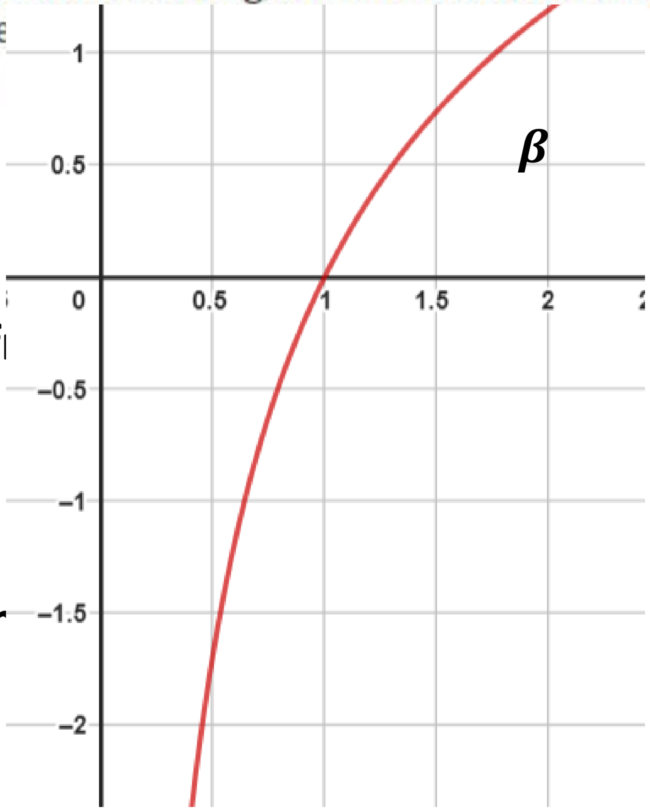

1) f(x) is continuous and strictly decreasing fr

it admit a unique root α.

- f(0.2) = 0.28 > 0
- f(0.3) = -0.15 < 0

f(x) is continuous and strictly increasing from

it admit a unique root β.

- f(2.2) = -0.05 < 0
- f(2.3) = 0.08 > 0

2) a) f '(x) = u'v + v'u   u = x-1   u' = 1

$= \ln x + \frac{1}{x}(x-1)$ v = lnx $v' = \frac{1}{x}$

$= \ln x + 1 - \frac{1}{x}$

f'(x) = g(x)

b) $A = \int_{\alpha}^{\beta} g(x)dx$

Using Chasles' rule for integrals

$A = -\int_{\alpha}^{1} g(x)dx + \int_{1}^{\beta} g(x)dx$

$= \int_{1}^{\alpha} g(x)dx + \int_{1}^{\beta} g(x)dx$

c) $A = \int_{1}^{\alpha} g(x)dx + \int_{1}^{\beta} g(x)dx$

$= [-1 + (x-1)\ln x]_1^{\alpha} + [-1 + (x-1)\ln x]_1^{\beta}$

= (0) – (-1+0) + (0) - (-1+0)

= 2 × 4 = 8 cm²