3) E is a vertex of the right isosceles triangle CBE with $\left(\overrightarrow{BC};\overrightarrow{BE}\right)=-\frac{\pi}{2}(2\pi)$.

Show that $\frac{z_E-z_B}{z_C-z_B}=-i$ and deduce the affix of E.

4) Calculate the affix of point F if CDF is right isosceles at D with $\left(\overrightarrow{DC};\overrightarrow{DF}\right)=\frac{\pi}{2}(2\pi)$.

5) Show that: $\frac{z_F-z_A}{z_E-z_A}=i$. Deduce the nature of triangle AEF.

N° 21

$\left(O;\vec{u},\vec{v}\right)$ is an orthonormal system of a complex plane.

A is a point of affix $a=5-i\sqrt{3}$, and B is a point such that triangle OAB is equilateral, with $\left(\overrightarrow{OA},\overrightarrow{OB}\right)=\frac{\pi}{3}(2\pi)$, let Q be the midpoint of $[OB]$.

1) a- Show that the affix of B is $b=4+2i\sqrt{3}$ and deduce the affix of Q.

b- Determine the affix z_K of point K where $ABQK$ is a parallelogram.

c- Show that: $\frac{z_K-a}{z_K}$ is pure imaginary and deduce the nature of triangle OKA, and that of quadrilateral $OQAK$.

Locate the points A, B, Q and K in the same plane.

2) Let C be a point of affix $c=\frac{2a}{3}$. Calculate $\frac{z_K-b}{z_K-c}$ and deduce the position of points B, C and K.

a- Prove that $\bar{z}(z'-1) = 2$.

b- Deduce that, as M' describes circle (C), M describes a circle (T) to be determined.

N°18

Given the complex number $z = \frac{\sqrt{3}+1}{4} - i\frac{\sqrt{3}-1}{4}$.

1) Calculate z^2 and write z^2 in trigonometric form.

2) a- Determine the modulus of z and verify that $-\frac{\pi}{12}$ is an argument of z .

b- Deduce $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$.

N°19

Consider in the complex plane referred to an orthonormal system $(O;\vec{u},\vec{v})$, the points A , B and C of respective affixes:

$z_A = 4+\frac{5}{2}i$; $z_B = 4-\frac{5}{2}i$ and $z_C = 2+\frac{3}{2}i$.

1) Locate the points A , B and C in the plane.

2) a- Write down the expression: $\frac{z_B - z_C}{z_A - z_C}$ in the exponential form.

b- Deduce the nature of triangle ABC .

3) Let (E) be the set of points M of affix z verifying the relation $|z-4| = \frac{5}{2}$.

a- Do the points A , B and C belong to (E) ? Why?

b- Determine the nature of the set (E) .

c- Find the points of (E) having real affixes.

N°20

$(O;\vec{u},\vec{v})$ is an orthonormal system of a complex plane.

1) Locate the points A, B and D of respective affixes $z_A = -2-2i$; $z_B = 2$ and $z_D = -2+2i$.

2) Calculate the affix of point C when $ABCD$ is a parallelogram and locate C .

3) Show that if M belongs to circle (C) of center O and radius $\sqrt{2}$ then M' is also on (C).

4) Let (D) be the straight line of equation $y=1$.

a- Show that if M belongs to (D) then its affix verifies the relation: $|z-2i|=|z|$ and also the affix z' of point M' verifies the relation $|z'-i|=1$.

b- Deduce that if M moves on (D) then, point M' belongs to a circle whose center and radius are to be determined.

N°16

In the complex plane referred to an orthonormal system $(O;\vec{u},\vec{v})$, consider the points E, F and G of respective affixes:

$z_E=i$, $z_F=2$ and $Z_G=\left(1+\frac{\sqrt{3}}{2}\right)+i\left(\frac{1}{2}+\sqrt{3}\right)$.

1) Express, in algebraic form, the complex number $z'=\dfrac{z_G-z_E}{z_F-z_E}$ and verify that $z'=e^{i\frac{\pi}{3}}$.

2) Prove that triangle EFG is equilateral.

3) Let M be a variable point of affix z, determine the set (T) of points M such that $|z-z_E|=\sqrt{5}$ and verify that F belongs to (T).

N°17

In the complex plane referred to an orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B such that $z_A=1$ and $z_B=\dfrac{3}{2}+i\dfrac{\sqrt{3}}{2}$.

Let (C) be the circle of center A and radius 1.

1) a- Write z_B-z_A in exponential form.

b- Determine a measure of the angle $(\vec{u};\overrightarrow{AB})$.

c- Show that the point B belongs to circle (C).

2) For all points M of affix z, $z\neq 0$, we associate the point M' of affix z' such that $z'=\dfrac{\bar{z}+2}{\bar{z}}$.

c- Calculate c^{2005}.

N° 13

In the complex plane referred to an orthonormal system $(O;\vec{u},\vec{v})$, designate by A the point of affix 1, by B the point of affix -1, by M the point of affix $z \neq 1$ and consider the point M' of affix z' such that $z' = \dfrac{z-1}{1-\bar{z}}$.

1) Prove that $|z'| = 1$ and that $(z-1)(1-\bar{z})$ is real.
2) Show that $\dfrac{z'-1}{z-1}$ is real and deduce that the points M, A and M' are collinear.
3) a- Prove that $\dfrac{z'+1}{z-1}$ is pure imaginary.
   b- Deduce that the two straight lines (BM') and (AM) are perpendicular.

N° 14

In the complex plane referred to an orthonormal system $(O;\vec{u},\vec{v})$, consider the points $A(1+i)$ and $B(2)$.
Let M be a point of affix $z \neq 2$ and let M' be the point of affix z' such that $z' = \dfrac{z-(1+i)}{z-2}$.
Show that if M moves on the perpendicular bisector of $[AB]$ then M' moves on a circle to be determined.

N° 15

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B of respective affixes i and $2-i$.
Let M be a point of affix z, ($z \neq 0$), and M' a point of affix z' such that: $z' = \dfrac{-2}{\bar{z}}$.

1) Express OM' in terms of OM.
2) a- Express $(\vec{u};\overrightarrow{OM'})$ in terms of $(\vec{u};\overrightarrow{OM})$.
   b- What can you say about $[OM)$ and $[OM')$?

Solved Problems

N° 1

In the table below, only one among the proposed answers to each question is correct. Write down the number of each question and give, with justification, the answer corresponding to it.

N°	Questions	Answers a	Answers b	Answers c
1	If $z=1-e^{-i\frac{\pi}{3}}$ then $\arg(\bar{z})=$	$\frac{\pi}{3}$	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$
2	If $z=(1+i)^n$ and n is a natural number, then z is a real positive number if:	n is even	n is odd	n is a multiple of 8
3	If an argument of z is $\frac{\pi}{6}$ then an argument of $-\frac{2}{z}$ is:	$-2\times\frac{6}{\pi}$	$-\frac{\pi}{6}$	$\frac{5\pi}{6}$
4	If $z=\frac{1+it}{1-it}$ where t is real then $\lvert z\rvert=$	1	$\sqrt{t}$	$2t$
5	The exponential form of the complex number $z=-2(\sin\theta+i\cos\theta)$ is:	$2e^{i\theta}$	$2e^{i\left(\frac{\pi}{2}+\theta\right)}$	$2e^{i\left(\frac{3\pi}{2}-\theta\right)}$
6	If z is a complex number such that $\lvert z\rvert=\sqrt{2}$, then $\lvert\bar{z}+i\bar{z}\rvert=$	$2\sqrt{2}$	2	$\sqrt{2}$
7	If $z=e^{i\frac{\pi}{2}}+e^{-i\frac{\pi}{6}}$ then:	$\arg(z)=\frac{\pi}{2}$	$\arg(z)=\frac{\pi}{2}+\frac{\pi}{6}$	$\arg(z)=\frac{\pi}{6}$
8	If $\frac{z_{\vec{t}}}{z_{\vec{s}}}$ is a real number then the two vectors $\vec{t}$ and $\vec{s}$ are:	equal	collinear	orthogonal

Solved Problems

N°2.

The complex plane is referred to an orthonormal system $(O;\vec{u},\vec{v})$.

Let M be a point of affix z. Let $Z=\frac{z-i}{z-1}$ where $z\neq 1$.

Answer by true of false and justify:

1) The set of points M, such that Z is real is a straight line deprived of a point.
2) The set of points M, such that $|Z|=1$ is a circle .
3) The set of points M, such that $\arg(Z)=0(2\pi)$ is a circle.
4) The set of points M, such that $Z+\overline{Z}=0$ is a straight line.

N°3.

Consider in the complex plane of an orthonormal system $(O;\vec{u},\vec{v})$ the variable point M fo affix z.

Determine the set of points M in each of the following cases:

1) $|z-1+2i|=2$.
2) $|z-2i|=|z+4|$
3) $z+\overline{z}=|z|$
4) $z-i=\frac{4}{\overline{z}+i}$
5) $|\overline{z}+5-i|=|z-4i|$
6) $|z+1+i|\times|\overline{z}+1-i|=4$
7) $z\overline{z}+2z-4i\overline{z}-4+2i$ is pure imaginary.
8) The points $M(z)$, $N(z^2)$ and $P(z^3)$ are collinear. $(z\neq 1 \text{ and } z\neq 0)$.

N°4.

Given the complex numbers $z_1=2\sqrt{3}+2i$ and $z_2=(1+\sqrt{3})+i(1-\sqrt{3})$ and let $z=\frac{z_1}{z_2}$.

1) Write z_1 in exponential form.
2) Find the algebraic form of z then deduce the modulus and an argument of z.
3) Deduce the modulus and an argument z_2 , then find the exact value of $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$.

N°5.

Given the complex numbers: $z_1 = 1 + i\sqrt{3}$, $z_2 = 1 + i$ and $Z = z_1 z_2$.

1) a- Write z_1 and z_2 in exponential form.

b- Deduce the exponential form of Z.

2) a- Write the algebraic form of Z.

b- Deduce the exact value of $\cos\frac{7\pi}{12}$ and $\sin\frac{7\pi}{12}$.

3) Determine the possible values of the natural number n such that z_1^n is real.

4) Determine the possible values of the natural number p such that z_2^p is pure imaginary.

N°6.

The complex plane is referred to an orthonormal system $(O;\vec{u},\vec{v})$.

For all points M of affix z we associate the point M' of affix z' such that $z' = \frac{z+2}{z-i}$.

1) a- Find the algebraic form of z' when $z = -\frac{3}{5} + \frac{1}{5}i$.

b- Show that in this case, $(z')^{40}$ is a real positive number.

2) Let $z = x + iy$ and $z' = x' + iy'$.

a- Express x' and y' in terms of x and y.

b- Deduce the set of points M when z' is pure imaginary

3) a- Show that for any z, we have $|z-i| \times |z'-1| = \sqrt{5}$.

b- Suppose that M describes the circle (C) of center A of affix i and radius $R = 1$, determine the set of points M'.

N°7.

Consider in the complex plane reffered to an orthonormal system $(O;\vec{u},\vec{v})$ the point A of affix $z_A = -2i$.

For each point M, distinct from A, we associate the point M' of affix z' such that $z' = -2\bar{z} + 2i$, θ is an argument of $z + 2i$.

1) Show that $(z+2i)(z'+2i)$ is a real non zero negative number.

2) Deduce an argument of $z'+2i$ in terms of θ.
3) What can we say about the two semi straight lines $[AM)$ and $[AM')$?

N°8

Given the complex number $z=1+\sqrt{3}+i\left(\sqrt{3}-1\right)$.

1) Write z^2 in exponential form.
2) Deduce the modulus and an argument of z.
3) Deduce the exact values of $\cos\frac{\pi}{12}$ and $\sin\frac{\pi}{12}$.

N°9

Consider in the complex plane of an orthonormal system $\left(O;\vec{u},\vec{v}\right)$ the points $A\left(-\frac{1}{2}\right)$, $M(z)$, $N(1+3z)$ and $P(1+z)$ where $z\neq 0$.

Determine the set of points M when the triangle MNP is isoceles of principal vertex M.

N°10

The complex plane is referred to an orthonormal system $\left(O;\vec{u},\vec{v}\right)$.

Consider the points A , B , M and M' of respective affixes 1, 5 , z and z' such that $z'=\frac{z-5}{z-1}$ where $z\neq 1$.

1) Show that if z' is pure imaginary then $z\bar{z}-3\left(z+\bar{z}\right)+5=0$ and show, in this case, that M varies on a circle to be determined.
2) Interpret, geometrically $|z-5|$, $|z-1|$, $|z'|$ and $\arg(z')$.
3) Deduce:
   a- The set of points M such that z' is pure imaginary.
   b- The set of points M such that z' is real.

N°11.

Consider the complex number $z=1+\cos\theta+i\sin\theta$ where $\pi<\theta<2\pi$.

1) a- Expand $e^{i\frac{\theta}{2}}\left(e^{i\frac{\theta}{2}}+e^{-i\frac{\theta}{2}}\right)$.

   b- Determine the modulus and an argument of z in terms of $\frac{\theta}{2}$.

2) Deduce an argument of the complex number $z = 1 - \frac{\sqrt{3}}{2} - \frac{1}{2}i$.

N° 12.

Consider in the complex plane of an orthonormal system $(O;\vec{u},\vec{v})$ the points $M_1(z_1)$, $M_2(z_2)$ and $M_3(z_3)$ such that $(1-i)z_1 + iz_2 - z_3 = 0$.

1) Show that $\frac{z_3 - z_1}{z_2 - z_1} = i$ and deduce the nature of the triangle $M_1M_2M_3$.
2) Given the two points A and B of affixes $z_A = -1 + 2i$ and $z_B = 2 + i$.
   Determine the affixes of the two points E and F such that the two triangles ABE and AFB are both right direct isosceles of the same principal vertex A.

N° 13.

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B of respective affixes $a = 2 + 2i$ and $b = 1 - \sqrt{3} + i\left(1 + \sqrt{3}\right)$.

1) a- Write a in trigonometric form.
   b- Calculate the modulus of b and deduce that $OA = OB$.
2) a- Let $z = b - a$, calculate $|z|$ and deduce that triangle OAB is equilateral.
   b- Using the figure calculate an argument of b, and deduce the exact values of $\cos\left(\frac{7\pi}{12}\right)$ and $\sin\left(\frac{7\pi}{12}\right)$.

N° 14.

Given the complex number $z = \cos\varphi + i\sin\varphi = e^{i\varphi}$.

1) Write $\bar{z}$ in trigonometric and exponential form.
2) Show that $\cos\varphi = \frac{1}{2}\left(e^{i\varphi} + e^{-i\varphi}\right)$ and $\sin\varphi = \frac{1}{2i}\left(e^{i\varphi} - e^{-i\varphi}\right)$.
3) Using the relations obtained in part 2) linearize $\cos^2 x \sin^4 x$ and

$\cos x \sin^4 x$.

4) θ and θ' are two real numbers, show that $a = \dfrac{e^{i\theta}+e^{i\theta'}}{1+e^{i\theta}e^{i\theta'}}$ is real.

N° 15

Let $(O;\vec{u},\vec{v})$ be a direct orthonormal system of a complex plane, and let z_1 and z_2 be any two complex numbers.

1) Show that $|z_1+z_2|^2+|z_1-z_2|^2 = 2\left(|z_1|^2+|z_2|^2\right)$.

2) Let M_1 and M_2 be two points of respective affixes $1+i$ and $1-i\sqrt{3}$.

a- Calculate the affix of point M such that OM_1MM_2 is a parallelogram.

b- Calculate: OM_1, OM_2, OM and M_1M_2.

c- Verify that: $M_1M_2^2+OM^2 = 2\left(OM_1^2+OM_2^2\right)$.

N° 16

In the complex plane $(O;\vec{u},\vec{v})$ consider the complex number $j = e^{2i\frac{\pi}{3}}$

1) Show that $j^3=1$, $1+j+j^2=0$ and $e^{i\frac{\pi}{3}}+j^2=0$.

2) ABC is an equilateral triangle such that $\left(\overrightarrow{AB};\overrightarrow{AC}\right)=\dfrac{\pi}{3}(2\pi)$, where a,b and c are the respective affixes of A,B and C .

a- Show that $\dfrac{c-a}{b-a}=e^{i\frac{\pi}{3}}$.

b- Deduce that: $a+bj+cj^2=0$.

3) For any complex number $z \neq 1$, we associate the points R,M and M' of respective affixes 1, z and $\bar{z}$.

a- Find the values of z when M and M' are distinct points.

b- Suppose that the triangle RMM' is equilateral such that $\left(\overrightarrow{RM};\overrightarrow{RM'}\right)=\dfrac{\pi}{3}(2\pi)$.

Show that M varies on a straight line (Δ) to be determined.

N°17

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A , B , C and D of respective affixes $z_A=-1+2i$, $z_B=7-2i$, $z_C=6+i$ and $z_D=2+3i$.

1) Write $\frac{z_B-z_A}{z_C-z_D}$ in exponential form and deduce that the two lines (AB) and (CD) are parallel.
2) Write $\frac{z_C-z_B}{z_A-z_D}$ in exponential form and deduce that $ABCD$ is an isosceles trapezoid.
3) The two lines (BC) and (AD) intersect in a point E .
   a- Interpret, geometrically $\left|\frac{z_B-z_E}{z_A-z_E}\right|$ and $\arg\left(\frac{z_B-z_E}{z_A-z_E}\right)$
   b- Deduce the affix of point E.
4) Let M be a variable point of affix z .
   a- Determine the set (T) of points M if $|z-5-4i|=\sqrt{10}$
   b- Precise the position of C with respect to (T).

N°18.

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points $A(-2)$, $M(z)$ and $M'(z')$ such that

$$z'=\frac{4-2\bar{z}}{\bar{z}},\ z\neq 0.$$

1) Write z' in algebraic form when $z=\sqrt{2}e^{i\frac{\pi}{4}}$.
2) a- Verify that $(z'+2)\bar{z}=4$.
   b- Deduce that $AM'\times OM=4$ and the vectors $\overrightarrow{AM'}$ and $\overrightarrow{OM}$ are parallel with same sense.
   c- Determine the set (T) of points M' when M moves on the circle (C) with center O and radius 1.

N° 19

Let $A(i)$ and $B(-i)$ be two points in a complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$. (C) is a circle of center O and radius $R=1$. M is a variable point of the plane of affix z.

1) Suppose that $z=re^{i\theta}$ where $\frac{\pi}{2}<\theta<\pi$.

   M_1 is a point of affix $z_1=(z+\bar{z})z$.

   Calculate an argument of z_1 in terms of θ and deduce that the point O belongs to $[MM_1]$.

2) For any point M of affix $z\neq -i$, associate a point M' of affix z' such that $z'=i\frac{z-i}{z+i}$.

   a- Show that: $OM'=\frac{AM}{BM}$ and $\arg(z')=\frac{\pi}{2}+(\overrightarrow{BM};\overrightarrow{AM})(2\pi)$

   b- Deduce that if M moves on the circle (C), then M' varies on the x-axis.

N° 20

Consider in the complex plane, the points A, B and C of respective affixes $z_A=-1$, $z_B=3i$ and $z_C=2-i$.

Let M be a point of affix z and M' a point of affix z' such that $z'=\frac{iz+3}{z+1}$, where $z\neq -1$.

1) Calculate $\frac{z_B-z_A}{z_C-z_A}$ and deduce the nature of the triangle ABC.

2) a- Verify that $z'=i\frac{z-3i}{z+1}$.

   b- Show that:

   $$OM'=\frac{BM}{AM} \text{ and } (\vec{u};\overrightarrow{OM'})=\frac{\pi}{2}+(\overrightarrow{AM};\overrightarrow{BM})\,(\text{mod}\,2\pi).$$

   c- Deduce:

   - The set of M when M' moves on a circle of center O and radius 1.
   - The set of points M when z' is real.

N°21

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider a point M, distinct from O of affix z and a point M' of affix z' such that $z'\bar{z}=1$, $z\neq 0$.

1) Determine the algebraic form of z' in each of the following cases:

   $z=2e^{i\frac{\pi}{6}}$; $z=\sqrt{2}e^{i\left(-\frac{\pi}{4}\right)}$.

2) a- Show that $OM \times OM'=1$.

   b- Compare $(\vec{u};\overrightarrow{OM})$ and $(\vec{u};\overrightarrow{OM'})$.

   Deduce that the points O, M and M' are collinear.

3) Show that O, $E(\bar{z})$ and $F\left(\frac{1}{z}\right)$ are collinear.

4) Prove that $\overline{z'-1}=-1+\frac{1}{z}$.

5) Suppose now that M moves on a circle of center $E(1;0)$ and radius $R=1$, M is distinct from O.

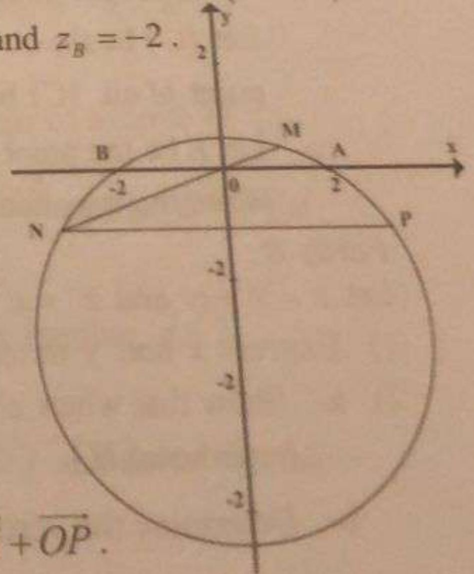
   a- Verify that $|z-1|=1$.

   b- Prove that $|z'-1|=|z'|$.

   c- Deduce the set of points M'.

N°22. ***For the students of the G.S. section***

In the complex plane referred to a direct orthonormal system $(O;\vec{u},\vec{v})$, consider the points A and B of affixes $z_A=2$ and $z_B=-2$.



Let (C) be a variable circle passing through A and B and M a variable point on (C) of affix $z=re^{i\theta}$.

(OM) recuts (C) in N, designate by P the symmetric of N with respect to $y'Oy$.

1) Determine the affixes z_N and z_P of the points N and P in terms of r and θ.

2) Let Q be the point defined by $\overrightarrow{OQ}=\overrightarrow{OM}+\overrightarrow{OP}$.

a- Show that $x_Q = \left(r+\frac{4}{r}\right)\cos\theta$ and $y_Q = \left(r-\frac{4}{r}\right)\sin\theta$.

b- Suppose that $r = 4$, show that when M varies on (C) then Q moves on the ellipse of equation $\frac{x^2}{25}+\frac{y^2}{9}=1$.

N° 23 ***For the students of the G.S. section.***

In the complex plane consider the mapping f that associates for every point M of affix z the point M' of affix z' such that $z' = z^2 - 4z$.

Part A.

1) Let A and B be the points of affixes $z_A = 1-i$ and $z_B = 3+i$.
   a- Show that the two points A and B have the same image by f.
   b- Two points have the same image by f, show that either these two points are confounded or they are symmetric to each other with respect to a point to be determined.
2) Let I be a point of affix -3.
   a- Show that $OMIM'$ is a parallelogram if and only if $z^2-3z+3=0$.
   b- Solve the equation $z^2-3z+3=0$.
3) a- Express $z'+4$ in terms of $z-2$.
   Deduce the relation between $|z'+4|$ and $|z-2|$ and also for $\arg(z'+4)$ and $\arg(z-2)$.
   b- Consider the points J and K of affixes $z_J = 2$ and $z_K = -4$. Let (C) be the circle with center J and radius 2. Show that any point M on (C) has an image M' on a circle (C').
   c- Let E be the point of affix $z_E = -4-3i$.
   Write the exponential form of $z_E + 4$.

Partie B.

Let $z = x+iy$ and $z' = x'+iy'$.

1) Express x and y in terms of x' and y'.
2) a- Show that when z' is pure imaginary then M describes a hyperbole(H).
   b- Determine the vertices, the asymptotes of (H) and draw(H).

3) Suppose that M moves on the straight line of equation $y = x - 3$.
Show that M' moves on the parabola (P) of equation
$(x'+4)^2 = 2\left(y' + \frac{1}{2}\right)$.
Determine the vertex, the focus of (P) and draw (P).

N° 24

In the complex plane consider the points A and B of affixes $z_A = 2$ and $z_B = 3$.

Part A.

1) Designate by M_1 and M_2 the points of respective affixes $z_1 = 2 + i\sqrt{2}$ and $z_2 = 2 - i\sqrt{2}$.
   a- Determine the algebraic form of the complex number $\frac{z_1 - 3}{z_1}$.
   b- Deduce that the triangle OBM_1 is right angled.
2) Prove geometrically that the points O, B, M_1 and M_2 belong to the same circle (T) to be determined.

Part B.

Let f be the mapping that associates for every point M of affix z the point M' of affix z' such that $z' = z^2 - 4z + 6$.
Let (Γ) be the circle of center A and radius $\sqrt{2}$.
M is a point on (Γ) such that $\left(\vec{u}; \overrightarrow{AM}\right) = \theta$ where $-\pi < \theta \le \pi$.

1) Verify that the affix of M is $z = 2 + \sqrt{2}e^{i\theta}$.
2) Verify that $z' = 2 + 2e^{i\theta}$ and M' that belongs to a circle (Γ').
3) Let D be the point of affix $d = 2 + \frac{\sqrt{2} + i\sqrt{6}}{2}$ and let D' be the image of D by f.
   a- Write in exponential form the complex number $d - 2$ and deduce that D belongs to (Γ).
   b- Give the measure of the angle $\left(\vec{u}; \overrightarrow{AD'}\right)$ and show that the triangle OAD' is equilateral.

Supplementary Problems

N°1

In the table below, only one among the proposed answers to each question is correct. Write down the number of each question and give, with justification, the answer corresponding to it.

N°	Questions	Answers a	Answers b	Answers c
1	An argument of $z=-4e^{-i\frac{\pi}{3}}$ is:	$\frac{\pi}{3}$	$-\frac{\pi}{3}$	$\frac{2\pi}{3}$
2	An argument of $z=-2i\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$ is :	$\frac{5\pi}{6}$	$\frac{-5\pi}{6}$	$\frac{\pi}{6}$
3	Given $z'=\frac{\bar{z}-1}{z-1}$ where $(z\neq 1)$ then $\lvert z'\rvert=$	$\lvert z\rvert$	1	$2\lvert z\rvert$
4	If $z=\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}$ then $z^{15}=$	i	1	$-i$
5	M is a point of affix $z\neq 0$ such that $\bar{z}-\frac{4}{z}=0$, Then M moves on :	The axis of abscissas	The circle of center O and radius 2	The axis of ordinates
6	If $z=e^{i\frac{\pi}{6}}-i$ then $\bar{z}=$	$e^{-i\frac{\pi}{6}}$	$e^{i\frac{\pi}{6}}$	$e^{i\frac{\pi}{3}}$

N°2

The complex plane is referred to an orthonormal system $\left(O;\vec{u},\vec{v}\right)$.

Consider the points A and B of respective affixes 1 and $2i$

Designate by:

(E) the set of points M of affix z such that $\lvert z-2i\rvert=\lvert z-1\rvert$.

(F) the set of points M, distinct from A and B, of affix z such that $\arg\left(\frac{z-2i}{z-1}\right)=\frac{\pi}{2}(2\pi)$.

Answer by true or false and justify:

1) (E) is a circle.
2) The points M of (F) move on a semi-circle deprived of two points.
3) The point C of affix $-\frac{1}{2}+\frac{1}{2}i$ belongs to (E) and to (F).
4) (F) is the set of points M such that the complex number $Z=\frac{z-2i}{z-1}$ is pure imaginary.

N°3.

Consider in the complex plane of an orthonormal system $(O;\vec{u},\vec{v})$ the variable point M of affix z.

Determine the set of points M in each of the following cases:

1) $\left|\frac{2z-4-2i}{z-i}\right|=2$ 2) $z+\bar{z}=|z|^2$

3) $|\bar{z}+i|=2$ 4) $z=\sqrt{3}e^{i\theta}$ where $0\le\theta\le\frac{\pi}{2}$.

5) The points $A(i)$, $M(z)$ and $M'(iz)$ are collinear.

N°4.

Given the complex number $z=(1+i\sqrt{3})e^{i\theta}$.

1) Write z in exponential form and in algebraic form.
2) Deduce the expression of $\cos\left(\frac{\pi}{3}+\theta\right)$ and that of $\sin\left(\frac{\pi}{3}+\theta\right)$ in terms of $\cos\theta$ and $\sin\theta$.
3) Deduce the exact values of $\cos 15^\circ$ and $\sin 105^\circ$.

N°5

Consider in the complex plane of an orthonormal system $(O;\vec{u},\vec{v})$

The points $M(z)$, $N(2z)$, $I(1)$ and $J(i)$.

Determine the set of points M when the two straight lines (IM) and (JN) are parallel.

N°6

The complex plane is referred to an orthonormal system $(O;\vec{u},\vec{v})$.

Consider the points A and E of affixes $z_A = 2$ and $z_E = \sqrt{3} + i$.

1) Write z_A and z_E in exponential form.
2) Let C be the point defined by $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OE}$.
   a- What is the nature of quadrilateral $OACE$?
   b- Calculate OC then write z_C in exponential form.
   c- Deduce the exact values of $\sin 15^\circ$ and $\cos 15^\circ$.

N°7

The complex plane is referred to an orthonormal system $(O;\vec{u},\vec{v})$.

1) Determine the set of points M of affix z such that $z\bar{z} = i\bar{z} - iz$.
2) Determine the set of points M of affix $z \neq 0$ such that the distinct points of affixes 1, z^2 and $\frac{1}{z^2}$ are collinear.

N°8

Consider in the complex plane of an orthonormal system $(O;\vec{u},\vec{v})$ the complex number $a = 2 + 2i$ and let M be a variable point of affix z such that $\arg(z+a) = \arg z + \arg a$.

Find set of points M.

N°9

The complex plane is referred to an orthonormal system $(O;\vec{u},\vec{v})$.

Let A be the point of affix $-2i$. For any point M of affix z we associate the point M' of affix z' such that $z' = -2\bar{z} + 2i$.

1) Let B be the point of affix $b = 3 - 2i$.

Determine the algebraic form of the affixes a' and b' of the points A' and B' associated to points A and B respectively and place the points A' and B' on a figure.

2) Show that if M belongs to the straight line (d) of equation $y=-2$ then M' belongs to (d).

3) Show that for any point M of affix z, $|z'+2i| = 2|z+2i|$. Interpret geometrically this equality.

4) For any point M distinct from A, designate by θ an argument of $z+2i$.
   - a- Prove that $(z+2i)(z'+2i)$ is a non zero real negative number.
   - b- Deduce an argument of $z'+2i$ in terms of θ.
   - c- What can we say about the two semi straight lines $[AM)$ and $[AM')$?

N° 10.

In the complex plane referred to an orthonormal system $(O,\vec{u},\vec{v})$, designate by I the point of affix 1 and by (C) the circle of diameter $[OI]$ and of center Ω.

Part A:

Suppose $a_0 = \frac{1}{2} + \frac{1}{2}i$ and denote by A_0 its image.

1) Show that A_0 belongs to (C).

2) Let B be the point of affix $b=-1+2i$ and B' the point of affix b' such that $b' = a_0 b$.
   - a- Calculate b'.
   - b- Prove that triangle OBB' is right at B'.

Part B:

Let a be a non-zero complex number different from 1 and let A be its image in the complex plane.

For all points M of affix z, $z \neq 0$, we associate the point M' of affix z' such that $z' = a\,z$.

1) a- Interpret, geometrically, $\arg\left(\frac{a-1}{a}\right)$.

   b- Show that $\left(\overrightarrow{M'O},\overrightarrow{M'M}\right) = \arg\left(\frac{a-1}{a}\right) + 2k\pi$ where $k \in \mathbb{Z}$.

c- Show that if A belongs to circle (C) deprived of O and I, then triangle OMM' is right at M'.

2) In this question, suppose that M is a point of the axis of abscissas different from O and denote by x its affix.
Choose x in such a way that A is a point of (C) different from I and O.
Show that the point M' belongs to the straight line (OA).
Deduce that M' is the orthogonal projection of M on this straight line.

N°11.

In the complex plane referred to a direct orthonormal system $(O,\vec{u},\vec{v})$, consider the points A, B and C of respective affixes $a=-1$, $b=2i$ and $c=-i$.

For all points M, of affix z, $z \neq -1$, we associate the point M' of affix $z'=\dfrac{-iz-2}{z+1}$.

1) Determine the affix c' of point C' associated to C, and write c' in exponential form.

2) a- Show that $|z+1|\times|z'+i|=\sqrt{5}$.

b- Deduce that if the point M belongs to circle (C) of center A and radius 2, then M' belongs to a circle whose center and radius are to be determined.

N°12.

The complex plane is referred to a direct orthonormal system $(O;\vec{u},\vec{v})$. Consider the points A, B and C of respective affixes $a=3+i$, $b=2i$ and $c=2-2i$.

1) Calculate $\dfrac{c-a}{b-a}$ and deduce the nature of triangle ABC.

2) Let M be a point of affix z and M' a point of affix z' such that $\overrightarrow{MM'}=\overrightarrow{AC}$.

a- Express z' in terms of z.

b- Calculate the affix of point D so that $ABDC$ is a rhombus.

3) a- Write c in the exponential form.

b- For what values of the natural integer n, is c^n real?

For what values of the natural integer n, is c^n pure imaginary?