Exponential Functions

I – Part A

Let f be the function defined over R by $f(x) = a + (b - x)e^x$ and let (C) be its representative curve in an orthonormal system $(O; \vec{i}; \vec{j})$

Determine a and b if the curve (C) passes through the point A (0;3) and admits on this point a tangent parallel to the line (D) of equation y = x.

Part B

Suppose that $f(x) = 1 + (2 - x)e^x$

1) Calculate $\lim\limits_{x \to +\infty} f(x)$ and get the value of f(2.5) in the decimal form.

2) Calculate $\lim\limits_{x \to -\infty} f(x)$. Deduce an asymptote (d) to (C). Study the relative position of (C) and (d).

3) Calculate f′(x) and set up the table of variations of f.

4) Prove that the curve (C) admits an inflection point.

5) Prove that the equation f(x) = 0 admits a unique solution ∝. Verify that $2.1 < \propto < 2.2$

6) Trace (d) and (C).

7) Determine according to the values of m, the number of solutions of the equation

$(2 - x)e^x = m - 1$.

II- Part A

Let g be the function defined over R by $g(x) = e^x - x - 1$

1) Calculate $\lim\limits_{x \to -\infty} g(x)$ and $\lim\limits_{x \to +\infty} g(x)$.

2) Calculate $g'(x)$ and set up the table of variations of g.

3) Deduce the sign of $g(x)$ over R.

Part B

Let f be the function defined over R by $f(x) = x + \dfrac{x+2}{e^x}$ and let (C) be the representative curve in the orthonormal system $(O; \vec{\imath}; \vec{\jmath})$.

1) Calculate $\lim\limits_{x \to -\infty} f(x)$.

2) Calculate $\lim\limits_{x \to +\infty} f(x)$. Prove that the line (D) of equation $y = x$ is an asymptote to (C) and study the relative position of (C) and (D).

3) Verify that $f'(x) = \dfrac{g(x)}{e^x}$. Set up the table of variations of f.

4) Calculate the coordinates of point A on (C) where the tangent (T) is parallel to (D).

5) Prove that the equation $f(x) = 0$ admits a unique solution $\propto$. Prove that $-1.69 < \propto < -1.68$

6) Trace (C), (D) and (T).

III- Let f be the function defined over R by $f(x) = x + \dfrac{2e^x}{1+e^x}$. Let (C) be the representative curve in the orthonormal system $(O; \vec{\imath}; \vec{\jmath})$.

1) Calculate $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to +\infty} f(x)$.

2) Prove that the line (D_1) of equation $y = x$ is an asymptote to (C) at $-\infty$ and the line (D_2) of equation $y = x+2$ is an asymptote to (C) at $+\infty$

3) Study the relative position of (C) with (D_1) and then with (D_2).

4) Study the variations of f over R and set up the table of variations.

5) Write the equation of the tangent (T) to (C) at the point of abscissa 0.

6) Prove that the point I (0,1) is the center of symmetry of (C) of f.

7) Trace (C), (D_1), and (D_2).

ASM
ADVENTIST SCHOOL MOUSEITBEH

ASM
ADVENTIST SCHOOL MOUSEITBEH