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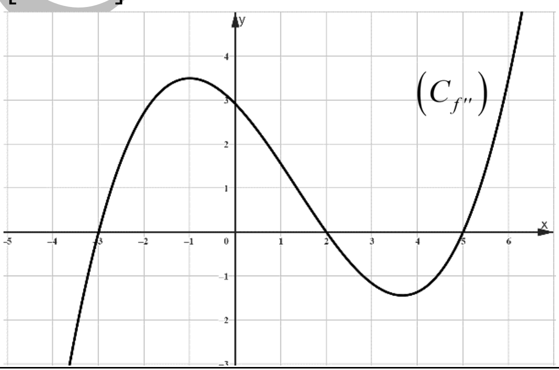
| **ExaMath Groups** | **Mathematics Exam**<br>**Class: LS** | **Prepared by: Georges H. Maamari**<br>**Edited by: Hassan Ahmad** |
| **Number of questions: 3** | **Sample 01 – year 2023**<br>**Duration: 1½ hours** | **Name:**<br>**N° :** |



- *This exam consists of three issues on two pages.*
- *The use of a non-programmable calculator is allowed.*

**I- (5 points)**
In the table below, only one of the proposed answers is correct.
Choose the correct answer and justify your choice.

| **N°** | **Question** | **Proposed answers** A | B | C |
|---|---|---|---|---|
| **1)** | The solution set of the equation: $\ln(x+2)+\ln(x-2)=\ln 3+\ln 4$ is: | $S=\{\sqrt{11};-\sqrt{11}\}$ | $S=\{4;-4\}$ | $S=\{4\}$ |
| **2)** | $\lim\limits_{\substack{x\to 0\\ x>0}} x\ln\left(\dfrac{1}{x}\right)=$ | 2 | 0 | $+\infty$ |
| **3)** | Let $g$ be the function defined over $I=\left[1\,;\,e^2\right]$ by $g(x)=(\ln x)^2-2\ln x$. The image of the interval $I$ by $g$ is $g(I)=$ | $[-1\,;\,0]$ | $[0\,;\,1]$ | $[-1\,;\,1]$ |
| **4)** | A company manufactures microchips. Each item may have two defects: $A$ and $B$. We know that $2,8\%$ of items have the defect $A$, $2,2\%$ have the defect $B$ and $95,4\%$ have no defect. The probability that an item has both defects is: | 0.005 | 0.004 | 0.046 |
| **5)** | Below is the curve $(C_{f''})$ representing the second derivative function $f''$ of a function $f$ defined over the interval $[-3.5\,;6]$.  | $f$ is convex over $[-3\,;\,3]$ | The representative curve of $f$ has three inflection points | The derivative function $f'$ of $f$ is decreasing over $[0\,;\,2]$ |

**II- (6 points)**
In this exercise, the results of the requested probabilities will, if necessary, be rounded to the thousandth.
Feline leukosis is a disease affecting cats; It is caused by a virus.
In a large veterinary center, it is estimated that 40% of cats carry the disease.
Cats present in this veterinary centre are tested for the disease.
This test has the following characteristics:

- When the cat is a carrier of the disease, its test is positive in 90% of cases.
- When the cat is not a carrier of the disease, its test is negative in 85% of cases.

A cat is randomly selected from the veterinary center and we consider the following events:
$M$ : "The cat is a carrier of the disease";
$T$ : "The cat's test is positive";
$\overline{M}$ and $\overline{T}$ denote the contrary events of the events $M$ and $T$ respectively.

**1)** **a)** Translate the situation into a weighted tree.
**b)** Calculate the probability that the cat is a carrier of the disease and that its test is positive.
**c)** Show that the probability of the cat's test being positive is 0.45.
**d)** A cat is chosen from among those who test positive. Calculate the probability that it is a carrier of the disease.

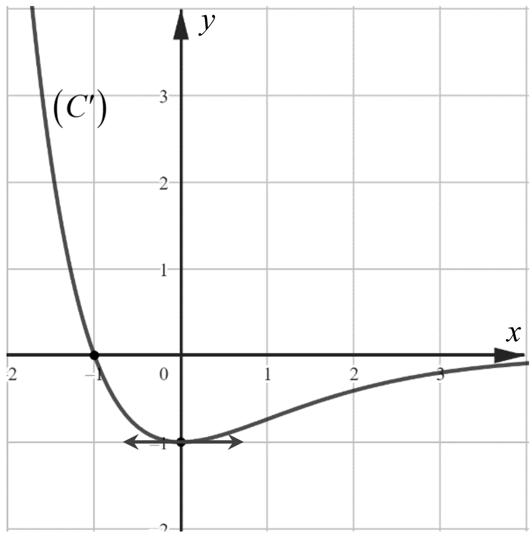
**2)** A sample of 20 cats is chosen at random from the veterinary center, including 15 females (event $F$).
It is accepted that this choice can be assimilated to a draw with replacement.
We recall that $M$ : "The cat is a carrier of the disease" and that $p(M)=0,4$.
**a)** Complete the table below:

| | $F$ | $\overline{F}$ | Total |
|---|---|---|---|
| $M$ | 6 | | 8 |
| $\overline{M}$ | | | |
| **Total** | 15 | | 20 |

**b)** Calculate the probability that the chosen cat is a female who carries the disease.

**3)** Among the 15 females, 3 cats are chosen simultaneously and randomly.
**a)** What is the probability that at least one of these three is a carrier of the disease?
**b)** What is the probability that exactly two of the three cats chosen have the disease?

**III- (9 points)**
**Part A**
In the plane referred to an orthonormal system, given the curve $(C')$ representing the derivative function $f'$ of a function $f$ differentiable over $\mathbb{R}$.




**1)** Using the curve $(C')$, determine with justification:
**a)** The sense of variations of the function $f$ over $\mathbb{R}$.
**b)** The convexity of the function $f$ over $\mathbb{R}$.

**2)** Suppose that the function $f$ is defined over $\mathbb{R}$ by $f(x)=(x+a)e^{-x}$ where $a$ is a real number.
**a)** Express $f'(x)$, the derivative function of $f$ over $\mathbb{R}$ as a function of $a$.
**b)** Determine graphically $f'(0)$ and then deduce the value of $a$.

**Part B**

In this part we take $a=2$ then $f(x)=(x+2)e^{-x}$.

Denote by $(C)$ the representative curve of $f$ in an orthonormal system $(O\,;\vec{i}\,;\vec{j})$.

**1)** **a)** Calculate $\lim_{x\to+\infty} f(x)$. Deduce an asymptote to $(C)$.

**b)** Calculate $\lim_{x\to-\infty} f(x)$.

**2)** **a)** Show that, for every real number $x$, $f'(x)=(-x-1)e^{-x}$.

**b)** Set up the table of variations of the function $f$.

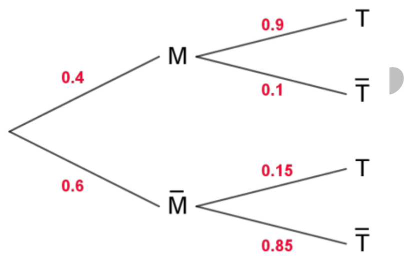
**c)** Show that the equation $f(x)=2$ admits a unique solution $\alpha$ on the interval $[-1.6; -1.5]$.

**3)** **a)** Determine, for every real number $x$, the expression of $f''(x)$ and study the convexity of the function $f$. What does its point $A$ of abscissa 0 represent for the curve $(C)$?

**b)** Write the equation of the tangent $(T)$ to $(C)$ at the point $A$.

**4)** Draw $(C)$ and $(T)$ in the same system.

| QI | Answers | 5 pts |
|---|---|---|
| 1) | Condition of existence: $\begin{cases} x+2>0 \\ x-2>0 \end{cases}$, $\begin{cases} x>-2 \\ x>2 \end{cases}$, $x \in ]2\ ;\ +\infty[$.<br>The equation is equivalent to $\ln\left(x^2-4\right)=\ln 12$ ; $x^2=16$ therefore ( $x=4$ $\in ]2\ ;\ +\infty[$ accepted) or ( $x=-2 \notin ]2\ ;\ +\infty[$ rejected).<br>The correct answer is **c**. | **1** |
| 2) | $\lim\limits_{\substack{x\to 0\\ x>0}} x\ln\left(\dfrac{1}{x}\right)=\lim\limits_{\substack{x\to 0\\ x>0}} x\left[\ln 1-\ln x\right]=-\lim\limits_{\substack{x\to 0\\ x>0}}\left(x\ln x\right)=0$.<br>The correct answer is **b**. | **1** |
| 3) | Table of variations of $g$ over $I=\left[1\ ;\ e^2\right]$:<br>$\begin{array}{c\|ccccc} x & 1 & & e & & e^2 \\ \hline g'(x) & & - & 0 & + & \\ \hline g(x) & 0 & \searrow & -1 & \nearrow & 0 \end{array}$<br>So $g(I)=g([1\ ;\ e])\cup g\left(\left[e\ ;\ e^2\right]\right)=[-1\ ;\ 0]\cup[-1\ ;\ 0]=[-1\ ;\ 0]$.<br>The correct answer is **a**. | **1** |
| 4) | $P(A\cap B)=P(A)+P(B)-P(A\cup B)=\dfrac{2.8}{100}+\dfrac{2.2}{100}-\left(\dfrac{100}{100}-\dfrac{95.4}{100}\right)=\dfrac{0.4}{100}=0.004$.<br>The correct answer is **b**. | **1** |
| 5) | In $[-3{,}5\ ;6]$, $f''(x)$ is equal to 0 three times and change sign each time, so the representative curve of $f$ admits three inflection points.<br>The correct answer is **b**. | **1** |

<table>
<tr><th>QII</th><th>Answers</th><th>5 pts</th></tr>
<tr><td>1) a)</td><td>
</td><td>1</td></tr>
<tr><td>1) b)</td><td>$P(M\cap T)=P(M)\times P(T/M)=0.4\times 0.9=0.36$.</td><td>½</td></tr>
<tr><td>1) c)</td><td>$P(T)=P(M\cap T)+P(\bar{M}\cap T)=0.36+0.6\times 0.15=0.45$.</td><td>½</td></tr>
<tr><td>(1) (d)</td><td>$P(M/T)=\dfrac{P(M\cap T)}{P(T)}=0.8$ .</td><td>½</td></tr>
<tr><td>2) a)</td><td>
<table>
<tr><th></th><th>$F$</th><th>$\bar{F}$</th><th>Total</th></tr>
<tr><td>$M$</td><td>6</td><td>2</td><td>8</td></tr>
<tr><td>$\bar{M}$</td><td>9</td><td>3</td><td>12</td></tr>
<tr><td>Total</td><td>15</td><td>5</td><td>20</td></tr>
</table>
</td><td>1</td></tr>
<tr><td>2) b)</td><td>$P(F\cap M)=\dfrac{6}{20}=0.3$.</td><td>½</td></tr>
</table>

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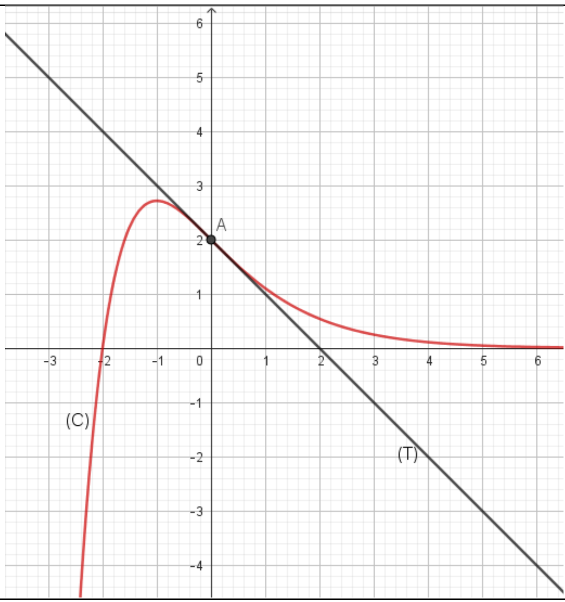
|---|---|---|
| **3) a)** | $P($ at least one of the three cats is a carrier of the disease$)=1-P($ no cat is a carrier of the disease$)=1-\frac{C_9^3}{C_{15}^3}=0.815$. | ½ |
| **3) b)** | $p=\frac{C_6^2\times C_9^1}{C_{15}^3}=0.297$. | ½ |

| QIII | Answers | 10 pts |
|---|---|---|
| **A.1.a** | • The function $f'$ is positive on $]-\infty;1]$, then the function $f$ is increasing on this interval;<br>• The function $f'$ is negative on $[-1;+\infty[$, so the function $f$ is decreasing on this interval. | ¾ |
| **A.1.b** | • The function $f'$ is decreasing on $]-\infty\ ;\ 0[$, so $f''(x)<0$ on this interval, so the function $f$ is concave on $]-\infty\ ;\ 0[$;<br>• The function $f'$ is increasing on $]0\ ;\ +\infty[$, so $f''(x)>0$ on this interval, so the function $f$ is convex on $]0\ ;\ +\infty[$. | ¾ |
| **A.2.a.** | $f'(x)=e^{-x}-(x+a)e^{-x}=(1-x-a)e^{-x}$. | ½ |
| **A.2.b.** | $f'(0)=-1\ ;1-a=-1\ ;a=2$. | ½ |
| **B.1.a.** | $\lim\limits_{x\to+\infty}f(x)=\lim\limits_{x\to+\infty}\frac{x+2}{e^x}=0$, so the line $(x'x):y=0$ is a horizontal asymptote to $(C)$ at $+\infty$. | ¾ |
| **B.1.b.** | $\lim\limits_{x\to-\infty}f(x)=\lim\limits_{x\to-\infty}(x+2)e^{-x}=-\infty\times(+\infty)=-\infty$. | ¼ |
| **B.2.a.** | According to Part **A** $f'(x)=(1-x-a)e^{-x}=(1-x-2)e^{-x}=(-x-1)e^{-x}$. | ½ |
| **B.2.b.** | <table><tr><td>$x$</td><td>$-\infty$</td><td></td><td>$-1$</td><td></td><td>$+\infty$</td></tr><tr><td>$f'(x)$</td><td></td><td>$+$</td><td>$0$</td><td>$-$</td><td></td></tr><tr><td>$f(x)$</td><td>$-\infty$</td><td>↗</td><td>e</td><td>↘</td><td>0</td></tr></table> | 1 |
| **B.2.c.** | On the interval $[-1.6;-1.5]$, the function $f$ is continuous and strictly increasing. $f(-1.6)<2$ and $f(-1.5)>2$ therefore, according to the corollary of the intermediate value theorem, the equation $f(x)=2$ admits a unique solution $\alpha$ on the interval $[-1.6;-1.5]$. | ¾ |
| **B.3.a.** | $f''(x)=(-1)\times e^{-x}+(-x-1)\times(-1)\ e^{-x}=(-1+x+1)\ e^{-x}=x\,e^{-x}$;<br>$e^{-x}>0$ for all $x$, so $f''(x)$ has the sign of $x$.<br>• Over $]-\infty\ ;\ 0[$ $f''(x)<0$ so the function $f$ is concave.<br>• Over $]0;+\infty[$, $f''(x)>0$ so the function $f$ is convex.<br>• In $x=0$, the second derivative is equal to 0 and changes sign so the point $A$ of abscissa 0 of $(C)$ is the inflection point of this curve. | 1½ |
| **B.3.b.** | $y_A=f(0)=2$<br>$(T)$: $y=f'(x_A)(x-x_A)+y_A=-1(x-0)+2$ so $(T)$: $y=-x+2$. | ¾ |

| | | |
|---|---|---|
| **B.4.** |  | **2** |