

**I- (10 points)**

Let  $f$  be the function defined, on  $\mathbb{R}$ , by  $f(x) = -2e^{-2x} + e^{-x} + 1$ . Let  $(C)$  be the representative curve of  $f$  in the plane of an orthonormal system  $(O; \vec{i}, \vec{j})$  (graph unit: 2 cm).

- 1) Calculate  $\lim_{x \rightarrow +\infty} f(x)$  and  $\lim_{x \rightarrow -\infty} \frac{f(x)}{x}$  and interpret the obtained results graphically.
- 2) a) Show that  $f'(x)$  has the same sign as  $(4 - e^x)$ .  
b) Set up the table of variations of  $f$ .
- 3) Determine an equation of the tangent  $(T)$  to  $(C)$  at the point  $A(\ln 2; 1)$ .
- 4) Prove that  $(C)$  has a point of inflection whose coordinates are to be determined.
- 5) Let  $(L)$  be the line of equation  $y = 1$ . Study the relative positions of  $(C)$  and  $(L)$ .
- 6) Draw  $(L)$ ,  $(T)$ , and  $(C)$ .
- 7) The equation  $f(x) = x$  has exactly two roots 0 and  $\alpha$ . Verify that  $1.1 < \alpha < 1.2$ .
- 8) Let  $h$  be the function defined by  $h(x) = \ln(f(x))$ .  
a) Find the domain of definition of  $h$ .  
b) Set up the table of variations of  $h$  and solve the equation  $h(x) = -x$ .

