Exponential functions

I- Determine the domain of definition of each of the following functions

$f(x)=\dfrac{e^x}{1+x^2}$	$f(x)=\dfrac{e^{2x}+e^x-2}{e^x-1}$	$f(x)=e^{2x}-2e^x-4$	$f(x)=\dfrac{e^x+3}{2e^x-5}$	$f(x)=\ln\left(e^x-1\right)$

II- Solve each of the following equations and inequations

1) $e^x = 3$.
2) $e^x = -5$
3) $e^{4x+2} = e^2$
4) $e^{x^2+2x} = \dfrac{1}{e^{-3}}$
5) $e^{2x} - 3e^x + 2 = 0$
6) $e^x + 3e^{-x} = 4$
7) $\ln(e^x + 1) = \ln 3 - 2\ln 2$
8) $2e^x + 2 = e^{-x}$
9) $e^{2x} - 3e^x + 2 < 0$

III- Determine the derivative of each of the following functions

1) f(x) = e^{2x^2-3x+4}.
2) f(x) = $2xe^x$.
3) f(x) = $\dfrac{x}{e^x}$.
4) f(x) = $(x^2+2)e^{-3x}$.
5) f(x) = $\dfrac{e^x+e^{-x}}{2}$.
6) f(x) = $2x + 1 - xe^{-0.5x+1}$.
7) $f(x) = e^x \ln x$

IV- Calculate each of the following limits

1) $\lim\limits_{x\to+\infty} \dfrac{e^x}{1+x^2}$.
2) $\lim\limits_{x\to+\infty} \dfrac{e^x+3}{2e^x-5}$.
3) $\lim\limits_{x\to0} \dfrac{e^{2x}+e^x-2}{e^x-1}$.
4) $\lim\limits_{x\to+\infty} (x^3-e^x)$.
5) $\lim\limits_{x\to+\infty} (1-2x)e^{-x}$.
6) $\lim\limits_{x\to+\infty} \dfrac{e^x-1}{e^x}$.
7) $\lim\limits_{x\to+\infty} e^{2x}-2e^x-4$.
8) $\lim\limits_{x\to+\infty} (x^2-2x+2)e^{-x}$.
9) $\lim\limits_{x\to+\infty} 2x+1-xe^{-0.5x+1}$.
10) $\lim\limits_{x\to+\infty} (x-2-x^2e^{-x})$.

V- Let f be a function defined over [0;+∞ [by $f(x) = 1 - (x - 2)e^{-x}$. Let (G) be its representative curve in an orthonormal system (unit = 1 cm).

1) Show that $\lim_{x \to +\infty} f(x) = 1$. What does the line (d) of equation y = 1 represent for the curve (G)?
2) Verify that $f'(x) = (x - 3)e^{-x}$.
3) Set up the table of variations of f.
4) Verify that the line (T) of equation $y = -3x + 3$ is tangent to (G) at the point of abscissa zero.
5) Draw (d), (T) and (G).

VI- Let f be the function defined on $\mathbb{R}$ as: $f(x) = x + 2 - 2e^x$. Denote by (C) its representative curve in an orthonormal system $\left(O ; \vec{i} ; \vec{j}\right)$.

1)
   a- Determine $\lim_{x \to -\infty} f(x)$.
   b- Show that the line (D) with equation y = x + 2 is an asymptote to (C).
   c- For all x in $\mathbb{R}$, show that the curve (C) is below the line (D).
2) Determine $\lim_{x \to +\infty} f(x)$ and calculate f(1.5).
3) Calculate $f'(x)$ and set up the table of variations of f.
4) Show that the equation f(x) = 0 has,, in $\mathbb{R}$, exactly two roots 0 and α.
   Verify that $-1.6 < \alpha < -1.5$.
5) Draw (D) and (C).

VII- Part A

Let g be the function defined on $\mathbb{R}$ as $g(x) = x - 1 + e^x$.

1) Show that g is strictly increasing on $\mathbb{R}$. Set up the table of variations of g.
2) Calculate g(0), then study according to the values of x the sign of g(x).

Part B

Let f be the function defined on $\mathbb{R}$ as $f(x) = \dfrac{(x-2)e^x}{1+e^x}$ and (C) its representative curve in an orthonormal system $\left(O ; \vec{i} ; \vec{j}\right)$. Denote by (Δ) the line with equation $y = x - 2$.

1) Determine $\lim_{x \to -\infty} f(x)$. Deduce an asymptote to (C).
2) Study, according to the values of x, the relative positions of (C) and (Δ).
3) Determine $\lim_{x \to +\infty} f(x)$ and show that (Δ) is an asymptote to (C).
4) Show that $f'(x) = \dfrac{e^x g(x)}{\left(1+e^x\right)^2}$, then set up the table of variations of f.
5) Plot (Δ) and (C).

VIII- Consider the function f defined on $\mathbb{R}$ as $f(x) = (1-x)e^x + 2$.

Denote by (C) its representative curve in an orthonormal system $\left(O ; \vec{i} ; \vec{j}\right)$.

1)
   a- Determine $\lim_{x \to -\infty} f(x)$. Deduce an asymptote (d) to (C).
   b- Determine $\lim_{x \to +\infty} f(x)$, then calculate f(1) and f(2).

2) Verify that $f'(x) = -xe^{x}$ and set up the table of variations of the function f .
3) Prove that the curve (C) has an inflection point I whose coordinates should be determined.

4) Draw (d) and (C).

5) Let g be the function given as $g(x) = \ln\left[f(x)-2\right]$.

   Denote by (G) the representative curve of g in the system $\left(O\ ;\vec{i}\ ;\vec{j}\right)$.

   a- Verify that the domain of definition of g is $\left]-\infty\ ;1\right[$.
   b- Is there a point on (G) where the tangent to (G) is parallel to the line (Δ)? Justify.

IX- Let f be the function defined, on $\left]-\infty\ ;+\infty\right[$, by $f(x) = x+2-\dfrac{3}{1+e^{x}}$. (C) is the representative curve of f in an orthonormal system $\left(O\ ;\vec{i}\ ;\vec{j}\right)$.

1)
   a- Calculate $\lim\limits_{x\to-\infty} f(x)$; Show that the line ($d_1$) with equation y = x – 1 is an asymptote to (C) and specify the position of (d_1) relative to (C).
   b- Calculate $\lim\limits_{x\to+\infty} f(x)$; Show that the line (d_2) with equation y = x + 2 is an asymptote to (C) and specify the position of (d_2) relative to (C).
2) Prove that the point $I\left(0\ ;\dfrac{1}{2}\right)$ is a center of symmetry of (C).
3) Show that f is strictly increasing on $\left]-\infty\ ;+\infty\right[$ and set up its table of variations.
4) Draw (d_1), (d_2) and (C).

X- Let f be the function defined over $\mathbb{R}$ as: $f(x) = x+xe^{-x}$, and denote by (C) its representative curve in an orthonormal system $\left(O\ ;\vec{i}\ ;\vec{j}\right)$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Calculate f (− 1.5).
2) Let (d) be the line with equation y = x.
   a- Discuss according to the values of x, the relative position of (C) and (d).
   b- Prove that (d) is an asymptote to (C).
3) A is the point on (C) where the tangent (T) to (C) is parallel to (d). Determine the coordinates of A and write an equation of (T).
4) The following table is the table of variations of the function f ′, the derivative of f.

x	$-\infty$		2		$+\infty$
$f''(x)$		−	0	+	
$f'(x)$	$+\infty$	↘	$1-e^{-2}$	↗	1

   a- Verify that (C) admits an inflection point W whose coordinates should be determined.
   b- Verify that f is strictly increasing over $\mathbb{R}$, then set up the table of variations of the function f.
5) Draw (d), (T) and (C).

XI- Let f and g be two functions defined on $\mathbb{R}$ as $f(x)=x+(x-1)e^{-x}$ and $g(x)=1+(2-x)e^{-x}$.

Denote by (C) the representative curve of the function f in an orthonormal system $(O;\vec{i};\vec{j})$.

1) a- Set up the table of variations of g. (The limits of g at $-\infty$ and at $+\infty$ are not required).
   b- Deduce that $g(x) > 0$ for all x.
2) a- Determine $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.
   b- Determine $\lim\limits_{x\to-\infty} \frac{f(x)}{x}$. Interpret this result graphically.
3) Let (L) be the line with equation: $y = x$.
   a- Study, according to the values of x, the relative positions of (L) and (C).
   b- Show that the line (L) is an asymptote to (C) at $+\infty$.
4) Verify that $f'(x) = g(x)$ and set up the table of variations of f.
5) Determine the coordinates of the point A on (C) where the tangent to (C) at A is parallel to (L).
6) Prove that the equation $f(x) = 0$ has a unique root α and verify that $0.4 < \alpha < 0.5$.
7) Draw (L) and (C).

XII- Consider the function f defined on $\mathbb{R}$ as $f(x) = 2-(x+2)e^{-x}$. Denote by (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1) a- Determine $\lim\limits_{x\to-\infty} f(x)$.
   b- Determine $\lim\limits_{x\to+\infty} f(x)$. Deduce an asymptote (d) to (C).
2) a- Calculate $f'(x)$, and set up the table of variations of f.
   b- Show that the equation $f(x) = 0$ has two roots α and 0.
   Verify that $-1.6 < \alpha < -1.5$.
3) a- Show that (C) has an inflection point whose coordinates are to be determined.
   b- Write an equation of (Δ), the tangent to (C) at its inflection point.
4) Let (d′) be the line with equation $y = -x$.

a- Verify that $f(x) + x = (x+2)(1-e^{-x})$.

b- Study, according to the values of x, the relative positions of (d′) and (C).

5) Draw (d), (Δ), (d′) and (C).
6) Let g be the function defined as $g(x) = \ln(-x-f(x))$.
   Determine the domain of definition of g.