2 Exercises and problems

Given for all exercises, unless indicated otherwise :
Planck's constant : $h = 6.63\times10^{-34}$ J.s ; Speed of light in vacuum : $c = 3\times10^{8}$ m/s ; 1 eV = 1.6×10^{-19} J ;
of the electron $m_e = 9.1\times10^{-31}$ kg.
The spectrum, of wavelengths, of visible waves, in vacuum, is between 400 nm and 800 nm.

N° 1
Diverse questions

The Hydrogen atom can exist in different energy levels, the energies of these levels are given by
$E_n = -\frac{13.6}{n^2}$ with E_n in eV and n is a positive integer.

1) Calculate the energy of the atom when it is in the ground state and when it is in the first excited state.
2) The atom can exist in the energy state (-3.2×10^{-19} J) ? Why ?
3) The atom is in the ground state.
a) Calculate the energy emitted by the atom during its de-excitation to the fundamental state.
b) Calculate the wavelength of the emitted radiation.
4) The atom in its ground state can absorb a photon of frequency 2.7×10^{15} Hz ? Why ?

N° 2
Spectrum analysis of the Hydrogen atom

The spectrum analysis of the Hydrogen atom, shows that this atom can only emit a finite number of the visible monochromatic radiations whose spectrum of emission is represented in **figure (1)**. The **figure (2)** represents the diagram of the energy levels of the Hydrogen atom.

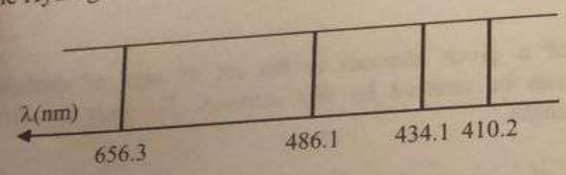


Figure (1)

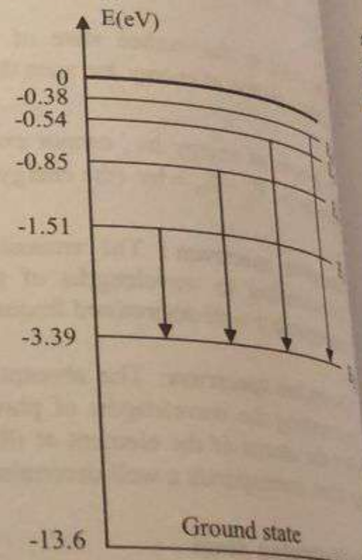


Figure (2)

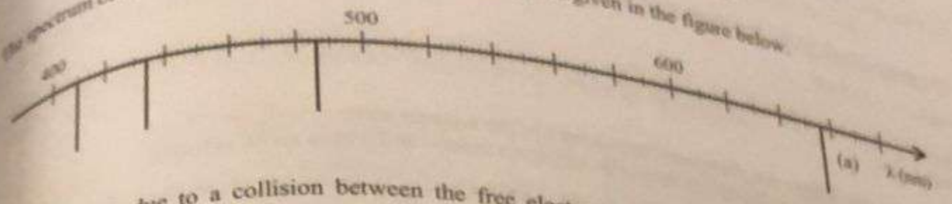
A – Referring to figure (1)
1) Name the instrument by which we observe the spectrum in figure (1).
2) Calculate in eV, the energies of the photons of the emitted visible radiations.
3) Do these energies have continuous or discrete values ?
4) The energies emitted by the atom are quantified. Justify.

B – Referring to figure (2)
1) What do the energy levels E_1, E_2 and $E = 0$ represent.
2) a) Do the arrows in figure (2) correspond to an emission or an absorption spectrum ? Why ?
b) Calculate the energy variations corresponding to each of the flashes.
c) Tell whether the energy diagram of the Hydrogen atom verifies the emission spectrum in **figure (1)** or not.
3) Calculate the minimum wavelength of the electromagnetic wave emitted by the Hydrogen atom. Indicate the domain of the electromagnetic spectrum which this wave belongs to.

N° 3

Excitation of a hydrogen atom by an electron

The spectrum of the rays emitted by a lamp of hydrogen gas is given in the figure below.



This emission is due to a collision between the free electrons liberated by the cathode and the confined hydrogen gas in the lamp.

Given: $h = 6.6 \times 10^{-34}$ J.s.

1) Is the spectrum of the obtained rays continuous or discrete ?
2) Using the preceding figure, calculate, in eV, the energy of a photon of ray (a).
3) The energy levels of hydrogen atom are given by : $E_n = -\frac{13.6}{n^2}$ where $n \in N^*$ and E_n in eV.
a) Do the emitted rays in the figure belong to the **Lyman** series ? **Balmer** series ? **Paschen** series ? **Brackett** series ? **Pfund** series ?
b) Determine the level of the transition of the hydrogen atom corresponding to ray (a).
c) The hydrogen atom, initially in its ground state, is excited by an electron (e^-) of kinetic energy 11.2 eV.
i. Is the kinetic energy of a free electron quantized ?
ii. Determine, with justification, the new state of the hydrogen atom.
iii. Calculate the speed of the electron (e^-) after its interaction with the hydrogen atom.

N° 4

Exciting a Hydrogen atom by a photon or an electron

The energy levels of the Hydrogen atom are given by the relation : $E_n = -\frac{13.6}{n^2}$ with E_n in eV, n is an integer larger or equal to 1.

1) The preceding expression shows that the energy levels of the Hydrogen atom are quantized. Justify.
2) Represent the energy diagram of the first four energy levels of the Hydrogen atom.
3) The atom is in the ground state, it receives a radiation of wavelength $\lambda = 102.82$ nm.
a) Specify the new state of the Hydrogen atom.
b) Find the possible wavelengths during the de-excitation of the atom.
c) Determine the emission spectrum produced by a Hydrogen gas under the preceding radiation.
4) A photon of energy 10.8 eV hits a Hydrogen atom initially in its ground state. In this photon absorbed?
5) An electron of kinetic energy 10.8 eV excites a Hydrogen atom, initially in its ground state to the first excited state.
a) This observation shows that the energy of the electron is not quantized. Justify.
b) Calculate the speed of the electron after its interaction with the atom.

N° 5
Rydberg's constant for a Hydrogen atom

The energy levels of the hydrogen atom are given by the expression : $E_n = \frac{E_1}{n^2}$, where n is a non zero integer and $E_1 = -13.6\ eV$.

The **Lyman** series is the set of emitted rays corresponding to the transitions of the from excited states to the ground state.

1) Calculate the largest and the lowest wavelength in the **Lyman** series.
2) Indicate the domain of the electromagnetic spectrum where the **Lyman** series is found.
3) **a)** Show that the wavelength of a ray in the **Lyman** series is given by expression : $\frac{1}{\lambda} = R_H\left(1 - \frac{1}{n^2}\right)$ where R_H is **Rydberg's constant** for the Hydrogen atom whose expression is to be determined as a function of h, c and E_1.
b) Calculate R_H and precise its unit.

N° 6
Studying a Nebula

Using a telescope, an observer observes colored clouds called the Nebula. These Nebulas are lit up by stars. They absorb certain radiations and emit a colorful light which can be considered as an emission spectrum.

The emission spectrum is formed of the following wavelengths, expressed in nm :

668 – 656 – 642 – 627 – 588 – 556 – 518 – 505 – 493 – 470 – 447 – 439.

Given :

Color	Violet	blue	green	yellow	Orange	red
Limit λ(nm)	[400 ; 450]	[450 ; 525]	[525 ; 555]	[555 ; 600]	[600 ; 650]	[650 ; 700]

The wavelengths of the rays of the Balmer series of the hydrogen atom:
$H_\alpha = 656$ nm ; $H_\beta = 486$ nm ; $H_\gamma = 434$ nm; $H_\delta = 410$ nm.

The wavelengths of the rays of the helium atom:
405 nm – 447 nm – 471 nm – 493 nm – 502 nm – 505 nm – 588 nm – 668 nm.

The wavelengths of the rays of the argon atom:
668 nm – 642 nm – 603 nm – 470 nm – 451 nm – 433 nm – 420 nm.

1) Describe the emission spectrum observed and determine the number of rays obtained in this spectrum.
2) Name few chemical elements forming the Nebula.

N° 7
Franck-Hertz Experiment

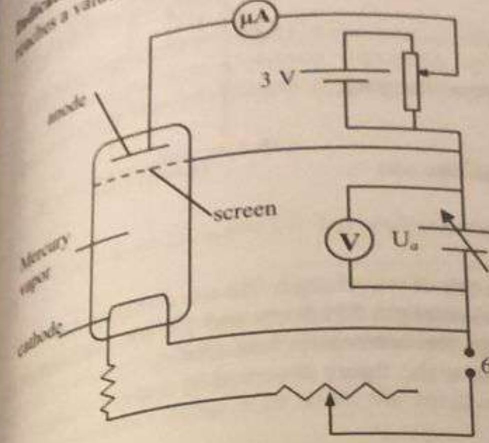
In 1914, a group of two German physicians, **James Franck** and **Gustav Hertz**, discovered a new significance of the atomic structure. It shows experimentally how the atoms absorb the energy by colliding with fast electrons.

Franck and **Hertz** used the experimental set up in figure (a). The electrons are emitted by the cathode, with a practically zero speed, and accelerate, under small pressure of the mercury vapor, by a positive voltage, called

…accelerating voltage U_a, applied between the screen and the cathode. The many emitted electrons pass through the screen and attain the anode producing an electric current detected by a micro ammeter.

The experiment consists of measuring the intensity of the current as a function of the accelerating voltage U_a gives the graph of figure (b).

Indication : An electron, leaves from rest, accelerates by a voltage U_a, its kinetic energy increases and reaches a value $E_K = eU_a$ on the anode (e is the elementary charge of the electron).



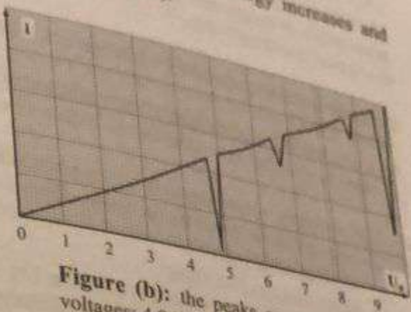


Figure (b): the peaks correspond to the voltages: 4.9 V – 6.7 V – 8.8 V – 9.8 V.

Figure (a)

1) Determine, in eV, the kinetic energies of the electrons corresponding to the peaks in figure (b).

2) a) The existence of a peak in the graph shows that a certain number of electron cannot reach the anode. Why ?

b) For what value of U_a an electron can't practically reach the anode ?

c) The final kinetic energy of an electron which can't reach the anode is zero. Where does the variation in the kinetic energy of the electron appear ?

3) a) What do the values of the kinetic energies corresponding to the peaks represent ?

b) These values show a certain characteristic of energy exchange between the atom and the exterior. Name this characteristic.

c) When the electron is accelerating by a voltage $U_a = 6$ V, it reaches the anode with a kinetic energy equal to 1.1 eV. This note shows a certain characteristic of the kinetic energy of the electron. Name this characteristic.

4) Complete the points in the figure below with the corresponding kinetic energy.

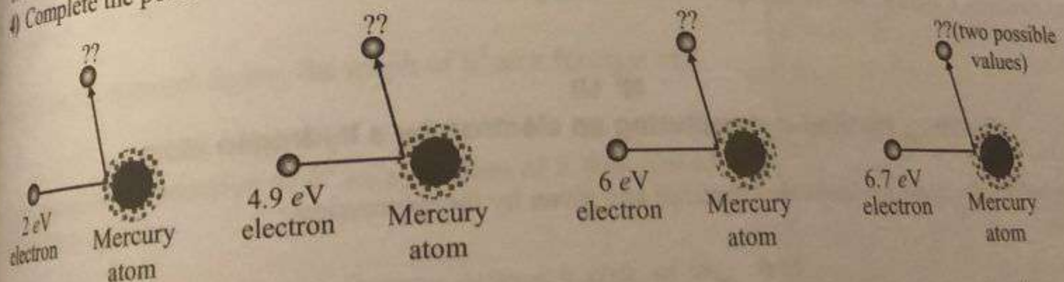


Figure: (c-1) **Figure: (c-2)** **Figure: (c-3)** **Figure: (c-4)**

5) Knowing that the minimum kinetic energy of an electron capable of ionizing mercury atom, initially in the fundamental state, is 10.4 eV.

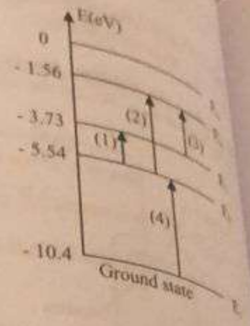
a) What is the value E_1 of the energy of the ground state of mercury ?

b) Represent the diagram of energy levels of the mercury atom.

N° 8

Studying the Mercury atom

The energy levels of the mercury atom are represented in the adjacent figure.



1) The transitions (1), (2), (3) and (4) had undergone by the mercury atom form a spectrum. Is the spectrum formed an emission or an absorption spectrum? Why ?

2) Is the preceding spectrum continuous or discrete ? Justify.

3) Calculate the wavelength of the radiation which verifies the transition (4). Specify whether this radiation is visible or not.

4) The mercury atom interacts with a photon of wavelength 686 nm.

a) This photon is received by the atom when it is initially:

- In its ground state ?
- In its first excited state?

b) To what transition, between (1) or (2) or (3), does the photon of wavelength 686 nm correspond to?

c) The two other transitions correspond to the photons of wavelength 311,8 nm and 572 nm. Correspond each of the preceding wavelengths and <u>*without any calculation*</u>, the convenient transition.

d) A mercury lamp lights up in a dark room. Describe and name the figure observed by a spectroscope.

N° 9

Extracting an electron from an atom

The energy levels of the hydrogen atom are given by the expression : $E_n = \frac{E_1}{n^2}$, where n is a none zero integer and $E_1 = -13.6$ eV.

1) Give a definition of the value $|E_1| = 13.6$ eV.

2) Name the state which corresponds to the energy of the atom when $n = \infty$.

3) The atom is initially in the ground state, receives a photon of wavelength $\lambda = 85$ nm.

a) Show that the atom is ionized.

b) Calculate the kinetic energy of the liberated electron.

4) The hydrogen atom being ionized, receives an electron of kinetic energy $\mathcal{E}_K = 0.44$ eV, the atom becomes in the energy state E_2 by emitting a photon. Applying the conservation of energy on the system (atom - projected electron), Calculate the wavelength of the emitted photon.

N° 10

Exciting, ionizing and capturing an electron by a hydrogen atom

The energy levels of a hydrogen atom are quantized and given by the expression :

$$E_n = -\frac{13.6}{n^2} \quad (E_n \text{ is expressed in eV and } n \in \mathbb{N}^*).$$

1) Calculate :

a) the energy levels of hydrogen corresponding to the fundamental state and to the third excited state.

b) the wavelength of the radiation emitted by the atom when it is de-excited from the third excited state to the fundamental state. Indicate the domain of the electromagnetic spectrum where this radiation is found.

c) the maximum wavelength that must not be exceeded by electromagnetic radiation which is capable to ionize this atom when it is initially in the : **i)** fundamental state ; **ii)** third excited state.

3) The atom is in its fundamental state.
... the new state of the atom when it interacts with photons of energies :

$$E_a = 11.4\ eV\ ;\ E_b = 10.2\ eV\ ;\ E_c = 14\ eV.$$

4) An electron, of kinetic energy of 12.4 eV, is directed to the hydrogen atom in the fundamental state. After ... the atom is in a new state and the incident electron is deviated with a kinetic energy of 0.31 eV. ... applying the conservation of the energy of the system (Atom-electron), the new state of the atom. We suppose that during the collision the atom is always at rest.

5) The hydrogen atom is ionized, it receives an electron, of kinetic energy 10 eV to become in the first excited ... after the emission a photon. Calculate, applying the conservation the energy of the system (Atom-electron-photon), the wavelength of the emitted photon. We suppose that the atom is always at rest.

N° 11
Bohr's model

In 1913, Niels Bohr proposed three postulates regarding the motion of an electron around the nucleus :

First postulate : *The electron rotates around the nucleus in well determined orbits (here circular) and each orbit is characterized by a well determined energy.*

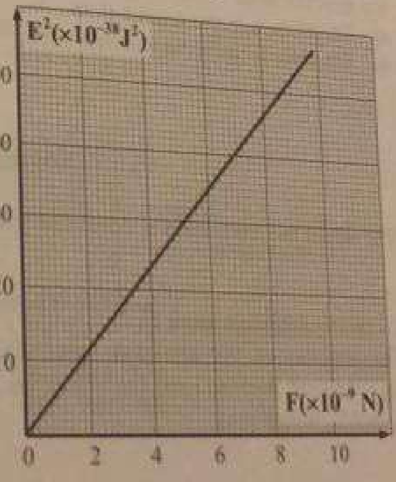
Second postulate : *Being in its orbit, the electron (or the atom) cannot emit radiations.*

Third postulate : *When passing from an orbit of high energy to an orbit of a lower one, the electron emits a photon whose energy is equal to the difference in the energies of the orbits. The atom cannot absorb an energy unless it is equal to the energy difference between the two orbits.*

A – In the preceding Bohr's postulates, the quantization of the emitted and absorbed energies by the atom are mentioned. Explain ?

B – Using Bohr's model, we suppose that the electron in the hydrogen atom describes a circular orbit of radius r and of energy E ($E < 0$), under the action of an electrostatic force $\vec{F}$ of magnitude: $F = k\frac{e^2}{r^2}$ with: $k = 9\times10^9$ SI and $e = 1.6\times10^{-19}$ C.

Given, in the adjacent figure, the graph of E^2 as a function of F.



1) Determine the equation of E^2 as a function of F then that of E as a function of r.

2) The point of the origin of reference defines a state of the hydrogen atom. Name this state.

3) Bohr proved that the smallest orbit an electron can describe is the radius: $r_0 = 5.3\times10^{-11}$ m.
a) Name of the state corresponding to r_0.
b) Determine the value E_0 of E corresponding to r_0.

4) We give, below, three expressions of r (expressed in SI) one of which is correct :

(a): $r = 5.3\times10^{-11}n^2$ where $n \in R^*$;

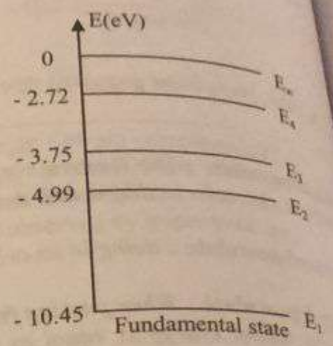
(b): $r = 5.3\times10^{-11}n^2$ where $n \in N^*$;

(c): $r = \frac{5.3\times10^{-11}}{n^2}$ where $n \in N^*$.

Referring to Bohr's postulates, why are the expressions (a) and (c) wrong.

5) Determine hence the expression of the energy levels of the hydrogen atom.

N° 12
Transition of the mercury atom



Studying the energy exchange between the mercury atom and the exterior permits us to construct the adjacent diagram concerning some energy levels of this atom.

1) The diagram shows that the energy levels of the mercury atom are quantized. Justify.

2) The atom is in an energy level E_n and moves to another level E_p (n and p are non zero integers).

a) Compare E_n and E_p when the atom is in the emission spectrum. Same question if the atom is in the absorption spectrum

b) Indicate the nature of the energy emitted by the atom during is de-excitation between the levels E_n and E_p ?

c) Studying the spectrum of the atom gives an evidence of a certain aspect of light. Name this aspect.

3) An electron, of kinetic energy 7.4 eV, hits a mercury atom in its fundamental state.

a) Determine the new state of the atom if the incident electron scatters with minimum kinetic energy.

b) Calculate the minimum kinetic energy with which the incident electron scatters.

c) The mercury atom, excited, de-excites by emitting a radiation of wavelength 1002.52 nm. Indicate the domain of the electromagnetic spectrum where this radiation is found. Determine the transition which corresponds to this radiation.

4) A photon, of energy 7.4 eV, is projected to a mercury atom. Describe, with justification, the new state of the mercury atom in the following cases:

a) The atom is in the ground state.

b) The atom is in the first excited state.

N° 13
Energy levels of the Lithium atom

The aim of this exercise is to determine the energy diagram of an isolated lithium atom another ionized atom.

I – Isolated lithium atom Li

The analysis of the emission spectrum of the lithium atom « Li » shows a group of rays of wavelength $\lambda_{42} = 610$ nm, $\lambda_{21} = 671$ nm, $\lambda_{32} = 812$ nm (λ_{pn} represents the transition wavelength between the energy level E_p and E_n with $p > n$).

1) The emission spectrum gives an evidence of the principle of the energy exchange between the atom and the exterior. Name this principle.
2) Interpret the formation of the emission rays.
3) Calculate, in electron-volt, the energies of the photons forming the emission rays of given above.
4) The minimum kinetic energy of an electron capable of ionizing the lithium atom, from its ground state, is 5.4 eV.
a) Specify the energy level of the ground state of the lithium atom.
b) Deduce the energies of the first three excited states of the lithium atom.
c) Why can't we say that the expression of the energy levels of the lithium atom is :

$$E_n = -\frac{k}{n^2}$$; k is a real constant and n is a positive integer ?

II - The lithium atom, ionized twice, Li^{2+}

The emission spectrum of the ionized lithium atom Li^{2+} shows the existence of a ray of wavelength $\lambda_{41} = 450.91$ nm.

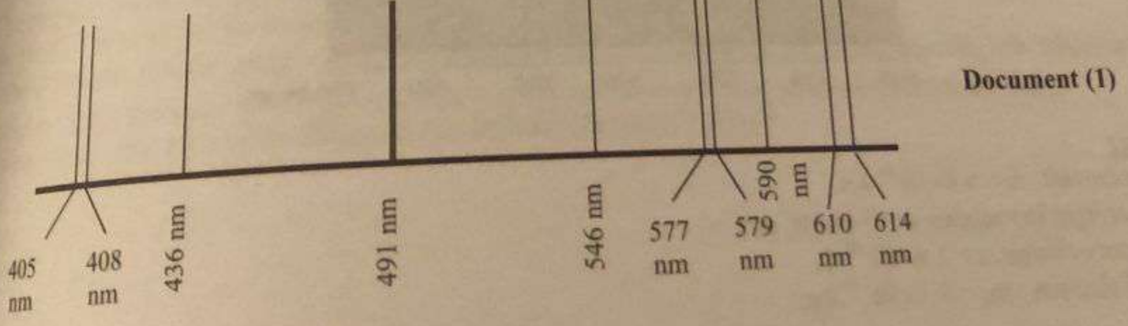
Knowing that values of the energy levels of the ion Li^{2+} are under the form : $E_n = \frac{A}{n^2}$ where A is constant and n is a none zero integer. E_n is expressed in electron-volt.

1) Give the unit and the sign of A.
2) Verify that the wavelength emitted by the atom, during its transition between E_p and E_n, is given by $\frac{1}{\lambda_{pn}} = \frac{A}{hc}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$ (where $p > n$).
3) Deduce the value of A and that of Rydberg's constant $R_{Li^{2+}}$ for Li^{2+}.
4) Calculate the minimum energy necessary for ionizing the ion Li^{2+}.
5) Compare the ionization energies of Li and Li^{2+}. Explain.

N° 14
Fluorescent Tube

Fluorescent tubes , used for domestic lighting, contains mercury (Hg), in which an electric discharge is launched and in which electrons emitted by the cathode are accelerated and enter in collisions with the atoms of mercury gas. These electrons excite the mercury atoms which emit invisible radiations of wavelengths 185nm and 254 nm and others which are visible. The invisible radiations are transformed into visible radiations by a powder containing Europium (Eu) and Terbium (Tb) deposited on the inner wall of the tube.

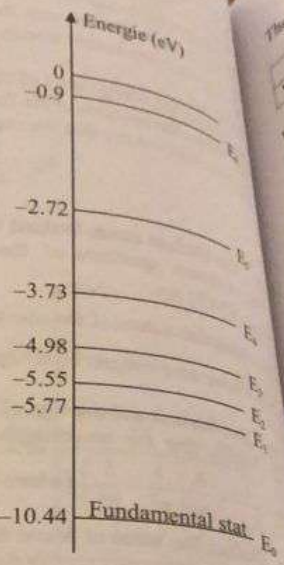
In document (1), we represent the emission spectrum of a fluorescent tube.



Document (1)

In document (2), we represent some energy levels of mercury atom.

1) a) Give two characteristics of the emission spectrum of document (1).
b) Specify the doublets which are found in this spectrum.
c) The energy levels of the mercury atom are of discrete values . Justify.
d) The radiations 491 nm, 610 nm and 614 nm did not come from mercury vapor. Where did they come from?
2) a) Indicate the domains, of the electromagnetic spectrum, which radiations 185 nm and 254 nm, emitted by the mercury vapor lamp, belong to.
b) Verify that the radiation 254 nm corresponds to the de-excitation $E_2 \to E_0$.
c) Specify if the radiations 185 nm and 254 nm have more or less energy than that of the visible radiations.
d) Give an advantage and an inconvenience fluorescent tube .
3) a) The radiation of wavelength $\lambda' = 405\text{nm}$ corresponds to the transition $E'_5 \to E_1$. Determine the energy level E'_5.
b) Specify the transition corresponding to the radiation $\lambda'' = 408\text{nm}$.
c) The de-excitations from two very near levels E_a and E_b to the level E_c produces rays forming a doublet if $\left|\dfrac{\Delta E_{a\to b}}{\Delta E_{a\to c} + \Delta E_{b\to c}}\right| < 0.5\%$. Verify that the rays of wavelengths 405 nm and 408 nm form a doublet.
4) a) Calculate the minimal energy necessary to ionize the atom, initially in the fundamental state.
b) Deduce the threshold wavelength λ_0 of the corresponding photon.
5) An electron, coming from the cathode, of kinetic energy $E_K = 7.94\,\text{eV}$, excites an atom (x) of mercury from the fundamental state to the energy level E_4.

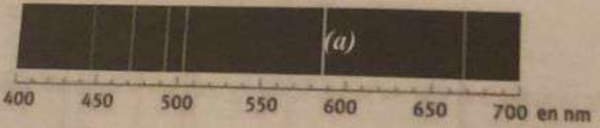


Document (2)

a) Calculate the kinetic energy of the electron after its collision with the mercury atom.
b) One of the photons, emitted by an already excited mercury atom, has a wavelength 254 nm. Justify.
c) The photon of wavelength 254 nm, excites an atom (y) of mercury which is initially at rest, upon de-excitation, can emit two photons where one is ultraviolet and the other is infrared. Specify the wavelength of each photon.

N⁰ 15
Spectrum of a gas bulb

The aim of this exercise is to study the nature of a gas that is found in a bulb and the emitted radiations. In the laboratory of a High School, the emission spectrum of a gas bulb "G" is observed through an instrument X . We get the following spectrum:



Given :
Plank constant : $h = 6.6\times10^{-34}$ J.s
Speed of light in vacuum: $c = 3.0\times10^{8}$ m/s
Elementary charge: $e = 1.6\times10^{-19}$ C.
Mass of electron : $m_e = 9.1\times10^{-31}$ kg.

The limits of the wavelengths (in nm) of the colors of a white light spectrum are as the following:

Colour	violet	blue	green	yellow	orange	red
Wavelength in (nm)	[400 - 424]	[424 - 491]	[491 - 575]	[575 - 585]	[585 - 647]	[647 - 700]

Wavelengths (in nm) of some lines emitted by different chemical elements in a gaseous state is given in the following table :

	Wavelength (nm)
Hydrogen	397 ; 410 ; 434 ; 486 ; 656
Helium	447 ; 471 ; 492 ; 501 ; 587 ; 668
Mercury	432 ; 547 ; 575 ; 580 ; 670 ; 690
Neon	439 ; 443 ; 585 ; 597 ; 618 ; 640

A - Identification of the gas « G »

1) a) Is the emission spectrum continuous or discontinuous?

b) Name the instrument X.

2) a) Indicate the colors of emitted radiations by the bulb.

b) Is the light emitted by the bulb white light? Monochromatic? Polychromatic? Justify .

c) Identify the gas « G » of the bulb.

B - Study of the emitted radiations.

The ionization energy of the atom of gas "G" is : 24.58 eV ;

The energies of some excited states of the atom of gas « G » are given in the following table :

Number of the excited state	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Energy level (eV)	-4.68	-3.58	-1.88	-1.58	-1.48	-0.78

1) *"The ionization energy of an atom is the minimum energy capable of ionizing it from its ground state ».*
Find the energy level E_1 of the ground state of the atom of gas « G ».

2) a) Determine the wavelengths λ_1 and λ_2 of the emitted radiations by the atom of the gas « G » when it is de-excited from the 6^{th} to the 4^{th} excited state and from the 2^{nd} excited state to the ground state, respectively.

b) Are all the emitted radiations by the bulb detected by the X instrument? Justify.

3) The emission spectrum of the bulb is due to the collisions between the accelerated electrons from the electrodes of the bulb when they are subjected to an electric voltage.

a) Determine the transition which corresponds to the emission of the line (a) indicated in the emission spectrum of the above given gas.

b) An electron, of Kinetic energy 20 eV, is capable of exciting the atom of gas « G » from the ground state ? Justify.

4) The professor approaches to the bulb a gold plate of work function $W_0 = 5.1$ eV.

a) Calculate the threshold wavelength λ_0 of the gold plate.

b) Specify the condition satisfied with the wavelength of a photon that is capable of performing the extraction of the electron from the gold metal.

c) The photons emitted by the bulb which satisfies the above condition results from the transition of the atom of the gas "G" from an excited state E_n to the ground state of E_1 ($E_n > E_1$).

i - Show that the kinetic energy of an electron ejected from the gold metal is: $E_K = E_n + b$ where b is a constant to be determined. Deduce the minimum speed of the ejection of an electron from the metal.

ii - What Can we say the kinetic energy of the emitted electrons ? Why ?