Permutation

1. Factorial notation

Given a non zero natural number n ,we call factorial of n and write $n!$
the natural number defined by $n! = 1 \times 2 \times 3 \times 4 \times \dots \dots . \dots \times n$

Examples

$3! = 1 \times 2 \times 3 = 6$

$4! = 1 \times 2 \times 3 \times 4 = 24$

$1! = 1$

$0! = 1$

Notes

1. $n! = n \times (n-1)!$
2. You can calculate n! using calculator.(Press n ,Shift , x! , =)

2. Permutation

A permutation of n objects is any ordered arrangement of these n objects.
Order does matter in permutation.

There are two types of permutation.

1. 1st type :Permutation with repetition(replacement).
2. 2nd type :Permutation without repetition(replacement).

<table>
<tr><td colspan="4">Given n : number of choices and r = number of times I have to choose ($r \leq n$)</td></tr>
<tr><th colspan="2">Type</th><th>Order</th><th>Counting</th></tr>
<tr><td colspan="2">Permutation with replacement (r-list)</td><td>Is important</td><td>n^r</td></tr>
<tr><td rowspan="2">Permutation without replacement</td><td>$r < n$</td><td>Is important</td><td>$nPr = A_r^n = \frac{n!}{(n-r)!}$</td></tr>
<tr><td>$r = n$</td><td>Is important</td><td>$n!$</td></tr>
<tr><td colspan="4">Note : We can calculate nPr using calculator , Just press n ,shift ,nPr and r.</td></tr>
</table>

Examples.

1) What is the number of ways to form a code of 5 numbers.

   Answer
   Permutation with repetition (r-list).
   $n = 10$, n is the number of choices you have to choose from.
   $r = 5$, r is the number of times I have to choose from.
   Number of ways $= n^r = 10^5 = 100000.$

2) What is the number of ways to form a code of 5 numbers such that repetition is not allowed.

   Answer
   Permutation without repetition ($r < n$).
   $n = 10$, n is the number of choices you have to choose from
   $r = 5$, r is the number of times I have to choose from
   Number of ways $= nPr = A_r^n = 10p5 = \frac{10!}{5!} = 30240.$

3) What is the number of ways to form a code of 10 numbers such that repetition is not allowed.

   Answer
   Permutation without repetition ($r = n$).
   $n = 10$, n is the number of choices you have to choose from
   $r = 10$, r is the number of times I have to choose from
   Number of ways $= 10! = 3628800.$

4) Cosider the letters {A,B,C,D}.
   We want to form a code of four distinct letters using the above letters.
   Find the number of all possible codes(Outcomes).

   Answer
   Permutation without repetition ($r = n$).
   $n = 4$, n is the number of choices you have to choose from
   $r = 4$, r is the number of times I have to choose from
   Number of ways $= 4! = 24.$

5) Cosider the letters {A,A,B,C}.
   We want to form a code of four letters using the above letters.(Use the letter once)
   Find the number of all possible codes(Outcomes).

   Answer
   Permutation without repetition ($r = n$).
   $n = 4$, n is the number of choices you have to choose from(double letter A)
   $r = 4$, r is the number of times I have to choose from
   Number of ways $= \frac{4!}{2!} = 12.$

6) Cosider the letters {A,A,A,B,C}.
We want to form a code of five letters using the above letters.(Use the letter once)
Find the number of all possible codes(Outcomes).

Answer
Permutation without repetition ($r = n$).
$n = 5$, n is the number of choices you have to choose from(triple letter A)
$r = 5$, r is the number of times I have to choose from
Number of ways $= \frac{5!}{3!} = 20.$

7) Cosider the letters {A,A,A,B,B,C}.
We want to form a code of six letters using the above letters.(Use the letter once)
Find the number of all possible codes(Outcomes).

Answer
Permutation without repetition ($r = n$).
$n = 6$, n is the number of choices you have to choose from(triple letter A and double letter B)
$r = 6$, r is the number of times I have to choose from
Number of ways $= \frac{6!}{3!\ 2!} = 60.$

<u>Probability</u>

1.Random experiment.

A Random experiment is a trial given well determined conditions and form possible outcomes.

Examples : tossing a coin , drawing a card.

2.Sample Space (Ω)

A sample space is the set of all possible outcomes.

Example : If you roll a die then $\Omega = \{1,2,3,4,5,6\}$,If you toss a coin then $\Omega = \{\text{Head, tail}\}$,

3.Event.

An event is a set of outcomes.

Example ; If you roll a die then 'Obtaining an even number ' is an event given by {2,4,6}.

Note: ϕ *is the impossible set.*

Example : If you roll a die then 'Obtaining an number 7 ' is an impossible set.

4.Opertion on events.

1.The opposite event or complement of A written $\bar{A}$ is the set of outcomes that A does not occur.

Example ; If you roll a die then
The event A; ‘Obtaining an even number ‘ is given by {2,4,6}
The event $\bar{A}$ is given by {1,3,5} ,odd numbers.

2.Let A and B are two events of a certain sample space.
The event (A and B) is the set of outcomes that A and B occur simultaneously , denoted by $A\cap B$.

Example :

Experiment : rolling a die.
Let A :’the number is an even number” ={2,4,6}
B :’Then number is a multiple of 3 “={3,6}
The event A and B is $A\cap B = \{6\}$

3.Let A and B are two events of a certain sample space.
The event (A or B) is the set of outcomes that either A or B , denoted by $A\cup B$.

Example :
Experiment : rolling a die.
Let A :’the number is an even number” ={2,4,6}
B :’Then number is a multiple of 3 “={3,6}
The event A and B is $A\cup B = \{2,3,4,6\}$

4.If $A\cap B = \phi$ then A and B are mutually exclusive (Incompatible)

5.Probability.

Equiprobable spaces.
An equiprobable spaces is a space in which all the events have the same probability.
In this case if A is an event then

$$P(A) = \frac{number\ of\ elements\ in\ A}{number\ of\ all\ elements\ in\ \boldsymbol{\Omega}} = \frac{card\ A}{card\ \boldsymbol{\Omega}} = \frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes}$$

$0 \leq P(A) \leq 1$

If $\Omega = \{A_1, A_2, \dots\dots\}$

$Then\ P(A_1) + P(A_2) + \cdots = 1$

Example :

When a fair die is rolled $\Omega = \{1,2,3,4,5,6\}$

Let A :'The number obtained is greater then 2 '

B : 'The number appearing is even '

have probabilities P(A)=$\frac{4}{6} = \frac{2}{3}$ and P(B) = $\frac{3}{6} = \frac{1}{2}$

since card A = 4 A ={3,4,5,6}

card B = 3 B={2,4,6}

card Ω=6

Rules

$P(A\cup B) = P(A) + P(B) - P(A \cap B)$

$P(\bar{A}) = 1 - P(A).$

Combination

Let E be a finite set of n elements and r a natural number such that r < n.

The number of combination of p elements of E is $C_n^p = \frac{n\,!}{r!(n-r)!}$

Properties:

Type of draw	Order	Counting
Successive with repetition	Order counts	n^r
Successive without repetition	Order counts	A_n^r
Simultaneous	Order does not matter	C_n^r

Conditional probability

A and B are two events of an universe Ω ,the probability that A occurs given(knowing) that B occurs is denoted by P(A/B) we have P(A/B) = $\frac{P(A\cap B)}{P(B)}$

1- P(A ∩ B) = P(B) × P(A/ B)= P(B) × P(B/ A)
2- P($\bar{A}$/ B) = 1 − P(A/ B)
3- A and B are independent if P(A/ B)=P(A) or P(A ∩ B) = P(B) × P(B)

Total probability

Let A, B and C be three events forming a partition of a sample space **Ω**,

then for any event D of **Ω** we have: P(D) = P(D∩A) + P(D∩B) + P(D∩C)

In particular P(D) = P(D∩A) + P(D∩$\bar{A}$) .

Example

In a game we use:

- A fair die;
- An urn U that contains 4 white and 3 red balls;
- An urn V that contains 17 white and 18 red balls.

The die is rolled.

If the six appears then a ball is drawn at random from the urn U, otherwise a ball is drawn at random from the urn V.

1) Prove that the probability that the drawn ball is white and from urn U is equal to $\frac{2}{21}$
2) Calculate the probability of drawing a white ball.
3) Knowing that the drawn ball is white, calculate the probability that it is drawn from urn V.

Solution

W :" white ball"

U :" The ball is from U "

V :" The ball is from V "

1) $P(W \cap U) = P(U) \times P(W / U) = \frac{1}{6} \times \frac{4}{7} = \frac{2}{21}$

2) $P(W) = P(W \cap U) + P(W \cap V)$

$P(W \cap V) = P(V) \times P(W / V) = \frac{5}{6} \times \frac{17}{35} = \frac{17}{42}$

$P(W) = P(W \cap U) + P(W \cap V) = \frac{2}{21} + \frac{17}{42} = \frac{1}{2}$

3) $P(V/W) = \frac{P(W \cap V)}{P(W)} = \frac{\frac{17}{42}}{\frac{1}{2}} = \frac{17}{21}$

Extra sheet (Probability)

Exercise 1

In a shop there are 1000 leathers wallets, of which some are defective, these wallets are manufactured by these factories α , β , *and* γ according to the following table.

	Factory α	Factory β	Factory γ
Number of wallets	200	350	450
% of defective wallets	5%	4%	2%

1) Represent a table showing the number of defective and non-defective wallets in each factory.

2) A Wallet is chosen at random from these 1000 wallets and consider the following events

A: The chosen wallet was produced by the factory α .
B: The chosen wallet was produced by the factory β .
C: The chosen wallet was produced by the factory γ .
D: The chosen wallet is defective .

a) Show that the probability $P(D\cap A) = \frac{1}{100}$
b) Calculate $P(D\cap B)$, $P(D \cap C)$ and P(D).

Exercise 2

An urn contains 4 red balls, 3 black balls and 2 white balls.

We draw at random, three balls .

Consider the following events:

A : " The three drawn balls are red"

B : " The first drawn ball is white , the second is black and the third is red "

C : " The three drawn balls have different colors"

D : " The three drawn balls have the same color"

E: " Exactly one black ball among the three balls and it is in the first draw"

F : " At least one black balls among the three balls".

Calculate the following probabilities:

$P(A), P(B), P(C), P(D), P(E), P(E \cap C) and \ \ P(FUD)$.

Note :
Take into consideration all the cases:
Successively with and without replacement and simultaneously .

Exercise 3

The customer service department in a supermarket organizes a game to offer vouchers to its clients. For this purpose, an urn is placed at the entrance of the supermarket.

The urn contains:

- three red balls.
- two white balls .
- one black ball .

A client who wants to participate in the game selects, simultaneously and randomly, three balls from the urn.

Consider the following events:

A : " the three selected balls have the same color "

B : " the three selected balls have three different colors "

C : " only two of the three selected balls have the same color "

1) Calculate the probabilities P(A) and P(B).
2) Show that $P(C) = \frac{13}{20}$.

Exercise 4

An urn U contains six balls: four red balls and two blue balls.

A bag S contains five bills: one 50 000 LL bill, two 20 000 LL bills and two 10 000 LL bills.

One ball is randomly drawn from U

- If this ball is red, then two bills are drawn successively without replacement at random from S.
- If this ball is blue, then three bills are drawn simultaneously at random from S.

Consider the following events:

R: " the drawn ball is red ".

A: " the sum of the values of the bills drawn is 70 000 LL ".

1) Calculate the probabilities P (R), P(A/ R) then verify that P(A $\cap$ R) = $\frac{2}{15}$.
2) Calculate P(A $\cap \bar{R}$) . Deduce P(A).

Exercsie 5

Consider a box V containing six cards numbered 1 ; 2 ; 3 ; 4 ; 7 ; 9, and two urns U_1 and U_2 such that:

- U_1 contains 3 red balls and 5 black balls
- U_2 contains 4 red balls and 4 black balls.

One card is randomly selected from the box V.
If this card shows an even number, then two balls are randomly and simultaneously selected
from U_1. If the card shows an odd number then two balls are randomly and simultaneously selected
from U_2.

Consider the following events:

E: "The card selected shows an even number"
O: "The card selected shows an odd number"
R: "The two selected balls are red"
B: "The two selected balls are black".

1) a- Calculate the probability $P\left(R/E\right)$ and deduce that $P(E \cap R) = \frac{1}{28}$.

   b- Calculate $P(O \cap R)$ and P(R).

2) Show that $P(B) = \frac{11}{42}$.

3) Knowing that the two selected balls are black, calculate the probability that these two balls come from urn U_1 .

Exercsie 6

U and V are two urns such that:

- U contains three cards holding the numbers 3, 1 and 0;
- V contains four cards holding the numbers 8, 8, 5 and 4.

One card is selected randomly from urn U:

- If the selected card from U holds the number 0, then two cards are selected randomly and simultaneously from urn V;
- If the selected card from U does not hold the number 0, then three cards are selected randomly and simultaneously from urn V.

Consider the following events:

A: “The selected card from urn U holds the number 0”
S: “The sum of the numbers held on the selected cards from urn V is even”

1) a- Calculate the probabilities P (S / A) and P (S ∩ A).

   b- Verify that $P(S \cap \bar{A}) = \frac{1}{6}$ and calculate P(S).

2) The sum of the numbers held on the selected cards from urn V is even.
   Calculate the probability that the selected card from urn U does not hold the number 0.

Exercsie 7

A survey conducted on a group of patients showed that these patients either have a heart disease only or a lung disease only or both diseases.

It was noted that:

• 60 % of the patients are men. • Among the men: 20 % have a heart disease only and 50% have a lung disease only. • Among the women: 25% have a heart disease only and 40% have both diseases.

One patient is selected at random. Consider the following events:

• M: “The selected patient is a men”;
• H: “The selected patient has a heart disease only”
• L: “The selected patient has a lung disease only”;
• B: “The selected patient has both diseases”.

Part A

1) Calculate the probabilities P(M ∩ H) , P(M ∩ L) and P(M ∩ B) .
2) Calculate P(H), P(L) and verify that P(B) = 0.34 .
3) Show that P(H ∪ L) = $\frac{33}{50}$.
4) Knowing that the selected patient has only one disease, calculate the probability that this patient has a heart disease.

Part B

The group consists of 500 patients.

The names of three patients were randomly and simultaneously selected to win an insurance policy each. Knowing that the three selected patients have both diseases, calculate the probability that they are men.

Exercsie 8

A bag U contains nine balls:

• three red balls numbered 0
• two green balls numbered 1
• four blue balls numbered 2.

Part A

Three balls are randomly and simultaneously selected from this bag.

Consider the following events:

M: « the three selected balls have the same color »;

N: « the product of numbers on the three selected balls is equal to zero ».

1) Calculate P(M), the probability of the event M.
2) a- Verify that P(N) $= \frac{16}{21}$.

   b- Calculate P(M ∩ N) and verify that P($\overline{M}$ ∩ N) $= \frac{3}{4}$.
3) Knowing that the three selected balls don't have the same color, calculate the probability that the product of numbers on these three balls is equal to zero.

Part B

In this part, one ball is randomly selected from the bag U.

This ball is not replaced back in U.

• If the selected ball is numbered 0, then two balls are randomly and simultaneously selected from U. (We get then 3 balls)

• If the selected ball is not numbered 0, then one ball is randomly selected from U. (We get then 2 balls.)

Calculate the probability that the sum of numbers on the selected balls is 3.

<u>Exercsie 9</u>

An urn U contains ten balls:

- five white balls numbered 1, 2, 3, 4, 5
- three black balls numbered 6, 7, 8
- two green balls numbered 9, 10.

Part A

A player selects randomly and simultaneously two balls from the urn U.

Consider the following events:

A: “The two selected balls hold odd numbers”

B: “The two selected balls have the same color”

C: “The two selected balls hold odd numbers and have the same color”

D: “The two selected balls hold odd numbers and have different colors”.

1) Calculate the probability P(A) and verify that $P(B) = \frac{14}{45}$.

2) a- Calculate P(C).

b- Are the events A and B independent? Justify.

3) Verify that $P(D) = \frac{7}{45}$.

4) Knowing that the player has selected two balls with different colors, what is the probability that these two balls hold odd numbers?

Part B

In this part, the player selects randomly, successively and with replacement, two balls from the urn U.

The player scores +1 point for each white ball selected, –1 point for each black ball selected and 0 points for each green ball selected.

Calculate the probability that the sum of scored points is equal to zero.

<u>Exercise 10</u>

Consider an urn U containing three dice:

- **Two** red dice where the faces of each of them are numbered from 1 to 6
- **One** black die where **two** of its faces are numbered 6 and the **four** others are numbered 1.

A player selects randomly and simultaneously two dice from the urn, then he rolls them only once.

Consider the following events:

A : «The two dice selected are red».

$\overline{A}$: «The two dice selected are one red and one black».

L : «Out of the two dice, only one shows the number 6».

1) Calculate the probability P(A).

2) a- Verify that $P(L/A) = \frac{5}{18}$ and calculate $P(A \cap L)$.

b- Calculate $P(\overline{A} \cap L)$ and verify that $P(L) = \frac{19}{54}$.

3) Knowing that only one of the two dice shows the number 6, calculate the probability that the two dice selected are red.

4) Calculate the probability that at least one die shows the number 6.

Solution of extra sheet(Probability)

Exercise 1

1)

	A	B	C	
	Factory α	Factory β	Factory γ	Total
D	10	14	9	33
$\overline{D}$	190	336	441	967
Total	200	350	450	1000

Number of defective in factory $\alpha = \frac{5}{100} \times 200 = 10$

Number of non- defective in factory $\alpha = 200 - 10 = 190$

Number of defective in factory $\beta = \frac{4}{100} \times 350 = 14$

Number of non- defective in factory $\beta = 350 - 14 = 336$

Number of defective in factory $\gamma = \frac{2}{100} \times 450 = 9$

Number of non -defective in factory $\gamma = 450 - 9 = 441$

2)a) $P(D \cap A) = \frac{card(D \cap A)}{Total} = \frac{10}{1000} = \frac{1}{100}$

Note : card is cardinal(amount of).

b) $P(D \cap B) = \frac{card(D \cap B)}{Total} = \frac{14}{1000} = \frac{7}{500}$

$P(D \cap C) = \frac{card(D \cap C)}{Total} = \frac{9}{1000}$

$P(D) = \frac{card(D)}{Total} = \frac{33}{1000}$

Exercise 2

Case 1 : Successively with replacement .

$$P(A) = P(\boldsymbol{R\ R\ R}) = \left(\frac{card(R)}{Total}\right)^3 = \left(\frac{4}{9}\right)^3 = \frac{64}{729}\ .$$

$$P(B) = P(\mathbf{W\ B\ R}) = \frac{card(W)}{Total} \times \frac{card(B)}{Total} \times \frac{card(R)}{Total} = \frac{2}{9} \times \frac{3}{9} \times \frac{4}{9} = \frac{8}{243}\ .$$

$$P(C) = P(\mathrm{W\ B\ R}) \times 3\,! = \frac{card(W)}{Total} \times \frac{card(B)}{Total} \times \frac{card(R)}{Total} \times 3! = \frac{8}{243} \times 6 = \frac{16}{81}\ .$$

$$P(D) = P(same\ color) = P(\boldsymbol{RRR}) + P(\boldsymbol{BBB}) = \frac{64}{729} + \left(\frac{card(B)}{Total}\right)^3 = \frac{64}{729} + \left(\frac{3}{9}\right)^3 = \frac{91}{729}$$

$$P(E) = P(\mathbf{B}\ \overline{\boldsymbol{B}}\ \overline{\boldsymbol{B}}) = \frac{card(B)}{Total} \times \left(\frac{card(\bar{B})}{Total}\right)^2 = \frac{3}{9} \times \left(\frac{6}{9}\right)^2 = \frac{4}{27}\ .$$

$$P(F) = P(at\ least\ one\ black) = 1 - P(\overline{\boldsymbol{B}}\overline{\boldsymbol{B}}\overline{\boldsymbol{B}}) = 1 - \left(\frac{card(\bar{B})}{Total}\right)^3 = 1 - \left(\frac{6}{9}\right)^3 = \frac{19}{27}$$

$$P(E \cap C) = P(\boldsymbol{B}\ W\ R) \times 2! = \frac{card(B)}{Total} \times \frac{card(W)}{Total} \times \frac{card(R)}{Total} \times 2 = \frac{3}{9} \times \frac{2}{9} \times \frac{4}{9} \times 2 = \frac{16}{234}\ .$$

$$P(FUD) = P(F) + P(D) - P(F \cap D) = \frac{19}{27} + \frac{91}{729} - P(\boldsymbol{BBB})$$

$$= \frac{19}{27} + \frac{91}{729} - \left(\frac{card(B)}{Total}\right)^3$$

$$= \frac{19}{27} + \frac{91}{729} - \left(\frac{3}{9}\right)^3 = \frac{577}{729}$$

Case 2 : Successively without replacement .

$$P(A) = P(\boldsymbol{R\ R\ R}) = \frac{card(R)}{Total} \times \frac{card(R)}{Total} \times \frac{card(R)}{Total} = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{1}{21} .$$

$$P(B) = P(\mathbf{W\ B\ R}) = \frac{card(W)}{Total} \times \frac{card(B)}{Total} \times \frac{card(R)}{Total} = \frac{2}{9} \times \frac{3}{8} \times \frac{4}{7} = \frac{1}{21} .$$

$$P(C) = P(\mathrm{W\ B\ R}) \times 3\,! = \frac{card(W)}{Total} \times \frac{card(B)}{Total} \times \frac{card(R)}{Total} \times 3! = \frac{1}{21} \times 6 = \frac{6}{21} .$$

$$P(D) = P(same\ color) = P(\boldsymbol{RRR}) + P(\boldsymbol{BBB}) = \frac{1}{21} + \frac{card(B)}{Total} \times \frac{card(B)}{Total} \times \frac{card(B)}{Total}$$
$$= \frac{1}{21} + \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{5}{84}$$

$$P(E) = P(\mathbf{B}\ \boldsymbol{\bar{B}}\ \boldsymbol{\bar{B}}) = \frac{card(B)}{Total} \times \frac{card(\bar{B})}{Total} \times \frac{card(\bar{B})}{Total} = \frac{3}{9} \times \frac{6}{8} \times \frac{5}{7} = \frac{5}{28} .$$

$$P(F) = P(at\ least\ one\ black) = 1 - P(\boldsymbol{\bar{B}\bar{B}\bar{B}}) = 1 - \frac{card(\bar{B})}{Total} \times \frac{card(\bar{B})}{Total} \times \frac{card(\bar{B})}{Total}$$
$$= 1 - \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{16}{21}$$

$$P(E \cap C) = P(\boldsymbol{B}\ W\ R) \times 2! = \frac{card(B)}{Total} \times \frac{card(W)}{Total} \times \frac{card(R)}{Total} \times 2 = \frac{3}{9} \times \frac{2}{8} \times \frac{4}{7} \times 2 = \frac{2}{21} .$$

$$P(FUD) = P(F) + P(D) - P(F \cap D) = \frac{16}{21} + \frac{5}{84} - P(\boldsymbol{BBB})$$
$$= \frac{16}{21} + \frac{5}{84} - \frac{card(B)}{Total} \times \frac{card(B)}{Total} \times \frac{card(B)}{Total}$$

$$= \frac{16}{21} + \frac{5}{84} - \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} = \frac{17}{21}$$

Case 3 : Simultaneously

$P(A) = P(3\boldsymbol{R}) = \frac{C_4^3}{C_9^3} = \frac{1}{21}$.

$P(B) = P(C) = P(\text{W B R}) = \frac{C_2^1 C_3^1 C_4^1}{C_9^3} = \frac{2}{7}$.

$P(D) = P(same\ color) = P(3R) + P(3B) = \frac{1}{21} + \frac{C_3^3}{C_9^3} = \frac{5}{84}$

$P(E) = P(\text{B } 2\bar{B}) = \frac{C_3^1 C_6^2}{C_9^3} = \frac{15}{28}$.

$P(F) = P(at\ least\ one\ black) = 1 - P(3\bar{B}) = 1 - \frac{C_6^3}{C_9^3} = \frac{16}{21}$

$P(E \cap C) = P(B\ W\ R) = P(B) = P(C) = \frac{2}{7}$.

$P(FUD) = P(F) + P(D) - P(F \cap D) = \frac{16}{21} + \frac{5}{84} - P(3B) = \frac{16}{21} + \frac{5}{84} - \frac{C_3^3}{C_9^3} = \frac{17}{21}$

Exercise 3

1) $P(A) = P(3R) = \frac{C_3^3}{C_6^3} = \frac{1}{20}$

$P(B) = P(RWB) = \frac{C_3^1 C_2^1 C_1^1}{C_6^3} = \frac{3}{10}$

2) $P(C) = P(2R\bar{R}) + P(2W\bar{W}) = \frac{C_3^2 C_3^1}{C_6^3} + \frac{C_2^2 C_4^1}{C_6^3} = \frac{13}{20}$.

Exercise 4

1) $P\ (R) = \frac{4}{6} = \frac{2}{3}$

P(A/ R) = $P(one\ 50000\ ,0ne\ 20000) \times 2! = \frac{1}{5} \times \frac{2}{4} \times 2 = \frac{1}{5}$.

P(A ∩ R) = P(R) × P(A/ R) = $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$.

2) P(A ∩ $\bar{R}$) = P($\bar{R}$) × P(A/ $\bar{R}$) = $\frac{2}{6} \times P(one\ 50000, two\ 10000) = \frac{2}{6} \times \frac{C_1^1 C_2^2}{C_5^3} = \frac{1}{15}$

P(A) = P(A ∩ R) + P(A ∩ $\bar{R}$) = $\frac{2}{15} + \frac{1}{15} = \frac{1}{5}$

Exercsie 5

1) a- P(R/E) = P(2r from U_1) = $\frac{C_3^2}{C_8^2} = \frac{3}{28}$

P(E ∩ R) = $P(E) \times P$ (R / E) = $\frac{2}{6} \times \frac{3}{28} = \frac{1}{28}$

b- P(O∩ R)= $P(O) \times P$ (R / O) = $\frac{4}{6} \times \frac{3}{14} = \frac{1}{7}$

P(R/O) = P(2r from U_2) = $\frac{C_4^2}{C_8^2} = \frac{3}{14}$

P(R) = P(O∩ R) + P(E ∩ R) = $\frac{1}{7} + \frac{1}{28} = \frac{5}{28}$

2) P(B) = P(O∩ B) + P(E ∩ B) = $\frac{1}{7} + \frac{5}{42} = \frac{11}{42}$

P(B/E) = P(2b from U_1) = $\frac{C_5^2}{C_8^2} = \frac{5}{14}$ then P(E ∩ B) = $P(E) \times P$ (B / E) = $\frac{2}{6} \times \frac{5}{14} = \frac{5}{42}$

P(B/O) = P(2b from U_2) = $\frac{C_4^2}{C_8^2} = \frac{3}{14}$ then P(O∩ B)= $P(O) \times P$ (B / O) = $\frac{4}{6} \times \frac{3}{14} = \frac{1}{7}$

3) P(E/B) = $\frac{P(E \cap B)}{P(B)} = \frac{\frac{5}{42}}{\frac{11}{42}} = \frac{5}{11}$

Exercsie 6

1) a- P (S / A) $= P(2\ even\ cards) = \frac{C_3^2}{C_4^2} = \frac{1}{2}$

P (S ∩ A) = $P(A) \times P$ (S / A) = $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

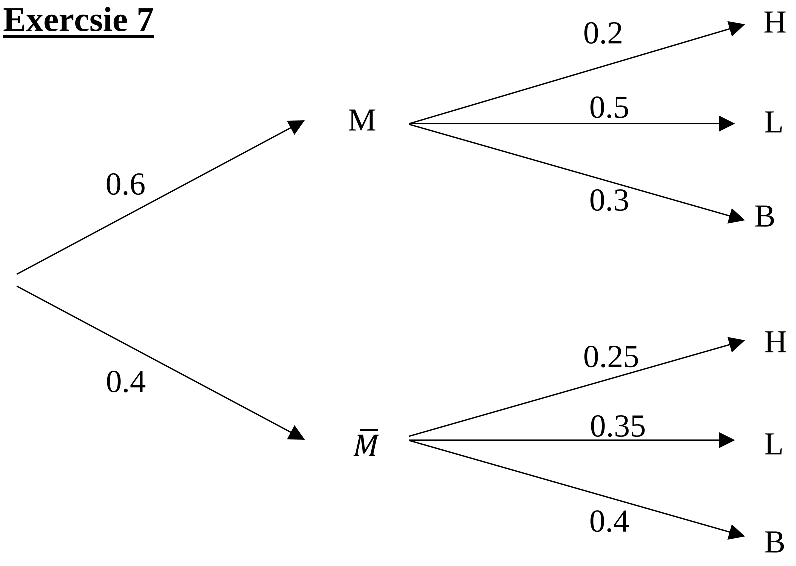
b- $P(S \cap \bar{A}) = P(\bar{A}) \times P\ (S / \bar{A}) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$

P (S / $\bar{A}$) $= P(3\ even\ cards) = \frac{C_3^3}{C_4^3} = \frac{1}{4}$

P(S) = P(S ∩A) + P(S ∩ $\bar{A}$) = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

2) P($\bar{A}$ / S) $= \frac{P(S \cap \bar{A})}{P(S)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$

<u>Exercsie 7</u>



Part A

1)$P(M \cap H) = P(M) \times P(H/M) = 0.6 \times 0.2 = 0.12$
$P(M \cap L) = P(M) \times P(L/M) = 0.6 \times 0.5 = 0.3$
$P(M \cap B) = P(M) \times P(B/M) = 0.6 \times 0.3 = 0.18$

2) $P(H) = P(H \cap M) + P(H \cap \overline{M}) = 0.12 + P(\overline{M}) \times P(H / \overline{M})$
$= 0.12 + 0.4 \times 0.25 = 0.22$
$P(L) = P(L \cap M) + P(L \cap \overline{M}) = 0.3 + P(\overline{M}) \times P(L / \overline{M}) = 0.3 + 0.4 \times 0.35 = 0.44$
$P(B) = P(B \cap M) + P(B \cap \overline{M}) = 0.18 + P(\overline{M}) \times P(B / \overline{M}) = 0.18 + 0.4 \times 0.4 = 0.34$

3) $P(H \cup L) = P(L) + P(H) - P(L \cap H) = 0.22 + 0.44 - 0 = \frac{33}{50}$

4) $P(H/\overline{B}) = \frac{P(H \cap \overline{B})}{P(\overline{B})} = \frac{P(H)}{P(\overline{B})} = \frac{0.22}{0.66} = \frac{1}{3}$

OR

$P(H/(H \cup L)) = \frac{P(H \cap (H \cup L))}{P(H \cup L)} = \frac{P(H)}{P(H \cup L)} = \frac{0.22}{0.66} = \frac{1}{3}$

Part B

$\text{Card}(B) = P(B) \times 500 = 0.34 \times 500 = 170$
$\text{Card}(M \cap B) = P(M \cap B) \times 500 = 0.18 \times 500 = 90$

$$P(3men/\,3both) = \frac{P(3\text{men} \cap 3\text{ both})}{P(3\ both)} = \frac{\frac{C_{90}^3}{C_{500}^3}}{\frac{C_{170}^3}{C_{500}^3}} = 0.146$$

Exercise 8

Part A

1) P(M) = $P(3R)+P(3B)=\frac{C_3^3}{C_9^3}+\frac{C_4^3}{C_9^3}=\frac{5}{84}$

2) a- P(N) $= P(1R\ 2\bar{R})+P(2R\ 1\bar{R})+P(3R)=\frac{C_3^1C_6^2}{C_9^3}+\frac{C_3^2C_6^1}{C_9^3}+\frac{C_3^3}{C_9^3}=\frac{16}{21}$

OR

P(N) $=1-P(3\bar{R})=1-\frac{C_6^3}{C_9^3}=\frac{16}{21}$

b- P(M $\cap$ N) $=P(3R)=\frac{C_3^3}{C_9^3}=\frac{1}{84}$

P($\bar{M}$ $\cap$ N) $=P(1R\ 2\bar{R})+P(2R\ 1\bar{R})=\frac{C_3^1C_6^2}{C_9^3}+\frac{C_3^2C_6^1}{C_9^3}=\frac{3}{4}$.

3) P($N/\overline{M}$) $=\frac{\mathrm{P}(\mathrm{N}\cap\overline{M})}{\mathrm{P}(\overline{M})}=\frac{\frac{3}{4}}{\frac{79}{84}}=\frac{63}{79}$

Part B

$P(S_3)=\ P(S_3\ \cap n_0\ )+\ P(S_3\ \cap\ \overline{n_0})=\frac{2}{21}+\frac{2}{9}=\frac{20}{63}$

$P(S_3\ \cap n_0\ )=\ P(R)\ \times P\ (GB/\ R\ \ )=\frac{3}{9}\times\frac{C_2^1C_4^1}{C_8^2}=\frac{2}{21}$

$P(S_3\ \cap\ \overline{n_0})=\ P(G)\ \times P\ (\mathrm{B}\ /\ G\ )+P(B)\ \times P\ (\mathrm{G}\ /\ B\ )=\frac{2}{9}\ \times\ \frac{4}{8}+\frac{4}{9}\ \times\frac{2}{8}=\frac{2}{9}$

Exercsie 9

Part A

1) P(A) = $\frac{C_5^2}{C_{10}^2}=\frac{2}{9}$

P(B) = $P(2W)+P(2B)+P(2G)=\frac{C_5^2}{C_{10}^2}+\frac{C_3^2}{C_{10}^2}+\frac{C_2^2}{C_{10}^2}=\frac{14}{45}$

2) a- P(C) $=P(2odd\ W)=\frac{C_3^2}{C_{10}^2}=\frac{1}{15}$

b- P(C) $=P(A\cap B)=\frac{1}{15}$

P(A) $\times$ P(B) = $\frac{2}{9}\ \times\frac{14}{45}=\frac{28}{405}$

$P(A\cap B)\neq$ P(A) $\times$ P(B) So the events A and B are dependent.

3)P(D) =P(A$\cap\bar{B}$)

P(A) = $P(A\cap B)+$ P(A$\cap\bar{B}$)

P(D) = P(A$\cap\bar{B}$) =P(A)$-P(A\cap B)=\frac{2}{9}-\frac{1}{15}=\frac{7}{45}$

4) P(A/$\bar{B}$) =$\frac{\mathrm{P}(\mathrm{A}\cap\bar{B})}{\mathrm{P}(\bar{B})}=\frac{\frac{7}{45}}{\frac{31}{45}}=\frac{7}{31}$

Part B

$P(sum\ =0)=\ P\ (WB)\times 2!\ +\ P(GG)=\frac{5}{10}\times\frac{3}{10}\times 2+\left(\frac{2}{10}\right)^2=\frac{17}{50}$

Exercise 10

1) P(A) = $\frac{C_2^2}{C_3^2} = \frac{1}{3}$

2) a- P(L/A) = P$(6,\overline{6}) \times 2 = \frac{1}{6} \times \frac{5}{6} \times 2 = \frac{5}{18}$

P(A$\cap L$) = P(A) × P(L/A) = $\frac{1}{3} \times \frac{5}{18} = \frac{5}{54}$

b- P($\overline{A} \cap$L) = P($\bar{A}$) × P(L/$\bar{A}$) = $\frac{2}{3} \times \frac{7}{18} = \frac{7}{27}$

$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

P(L/$\bar{A}$) = $P(6\ from\ red\ and\ \overline{6} from\ black) + P(\overline{6}\ from\ red\ and\ 6\ from\ black)$

$= \frac{1}{6} \times \frac{4}{6} + \frac{5}{6} \times \frac{2}{6} = \frac{7}{18}$

P(L) = P(A$\cap L$) + P($\overline{A} \cap$L) $= \frac{5}{54} + \frac{7}{27} = \frac{19}{54}$

3) P(A / L) = $\frac{P(A\cap L)}{P(L)} = \frac{\frac{5}{54}}{\frac{19}{54}} = \frac{5}{19}$

4) Let K be the event :”$None\ of\ the\ dice\ shows$ 6 “

P(at least one die shows 6) = $1 - P(K) = 1 - \frac{65}{108} = \frac{43}{108}$

$P(K) = P(K \cap A) + P(K \cap \bar{A}) = P(A) \times P$ (K/A) $+ P(\bar{A}) \times$ P(K/$\bar{A}$)

$= \frac{1}{3} \times P(\ \overline{6}\ from\ red\ , \overline{6}\ from\ red) + \frac{2}{3} \times\ P(\overline{6} fromred, \overline{6}\ from\ black)$

$= \frac{1}{3} \times \frac{5}{6} \times \frac{5}{6} + \frac{2}{3} \times \frac{5}{6} \times \frac{4}{6} = \frac{65}{108}$