Linear momentum

1- Graphical study and Newton's second law

A trolley of mass m = 100 g is pulled, on an inclined plane, by a light and inextensible string wound on the periphery of a pulley and carrying at its other extremity a small mass as shown in figure (a). We neglect frictional force and we give $g = 10\ m/s^2$, $\alpha = 30^0$.

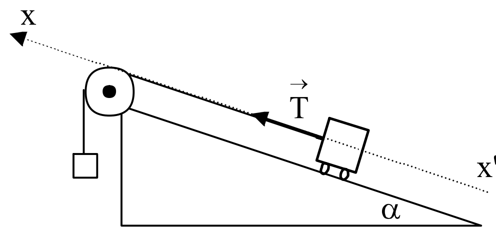


Figure (a)

1) Represent, on a figure, the forces acting on the trolley.
2) Applying Newton's second law, calculate the tension T in the string as a function of m, g, α and $\frac{dP}{dt}$ where P is the linear momentum of the chariot at an instant t.
3) Given the variation of the linear momentum of the trolley as a function of time in the graph of figure (b).
a) What are graphically :

- The velocity of the trolley at t = 0?
- The instant of releasing the trolley?

b) Deduce, using the graph, the value of T.

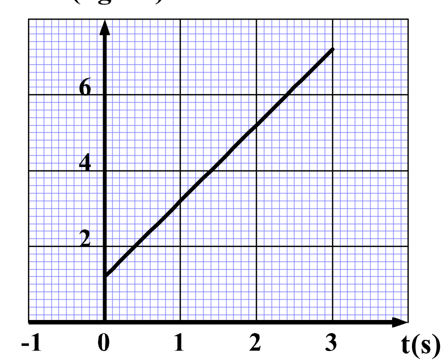


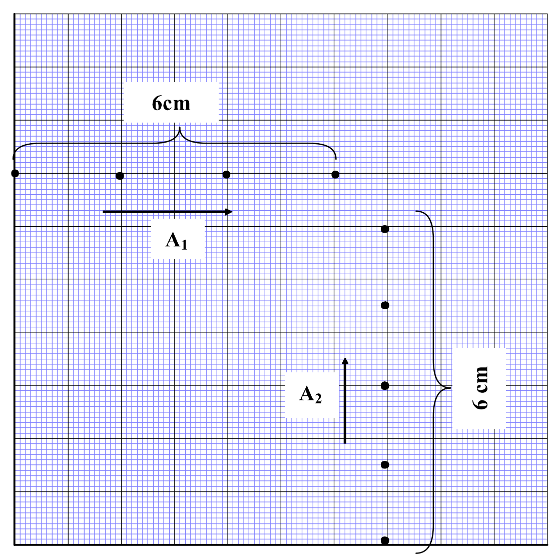
Figure (b)

Correction ▼

2- Experimental study of a collision

Two pucks A_1 and A_2 of respective masses m_1 = 250 g and m_2 = 200 g are launched, towards each other, on a horizontal air table. The pulse generator indicates a time constant τ = 20 ms.

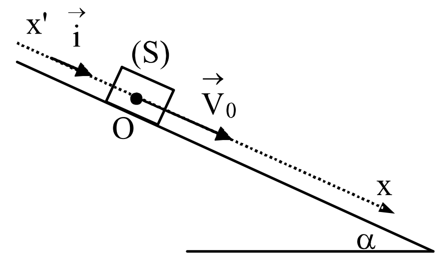
The recordings of the successive positions of the centers of mass of A_1 and A_2 before the collision are represented in the adjacent figure.



1) What is the nature of the motion of each puck before collision? Justify.
2) Calculate the velocities of the pucks before the collision.
3) Represent, to a scale 1 cm for 0.05 kgm/s, the linear momentum vector $\vec{P}$ of the system (A_1 ; A_2) before the collision. Deduce the magnitude of $\vec{P}$.
4) Why is the linear momentum vector of the system (A_1 ; A_2) conserved during the collision ?
5) After the collision, the two pucks merge together and form one body. Calculate the velocity V of this system.
6) Represent, to the same scale, the variation $\Delta\vec{P}_1$ of the puck A_1. Deduce the direction of the force by which A_2 acts on A_1 at the instant of collision (we can use the approximation $\frac{d\vec{P}_1}{dt} = \frac{\Delta\vec{P}_1}{\Delta t}$)

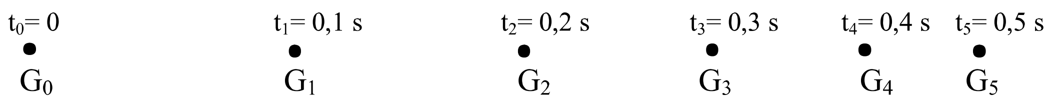
Correction ▼

3- Measuring the magnitude of the frictional force



A puck (S) of mass M = 600 g is placed, at a point O, on an air table inclined by an angle $\alpha = 30^0$ with respect to the horizontal. When (S) is released from rest it remains at rest. This observation proves the existence of a frictional force between (S) and the table. Given g = 10 m/s^2.
At an instant $t_0 = 0$ taken as an origin of time, we launch (S) with a velocity $\vec{V}_0 = V_0 . \vec{i}$ where $\vec{i}$ is the unit vector of the axis x'Ox parallel to the line of greatest slope of the table.

An appropriate device records the successive positions G_i of the center of mass of (S) during each interval of the time constant τ = 0.1 s, as shown in the figure below.



Scale 1/2

1) Complete the empty boxes in the table below:

t(s)	0	0,1	0,2	0,3	0,4	0,5
V(m/s)						
P(kgm/s)						

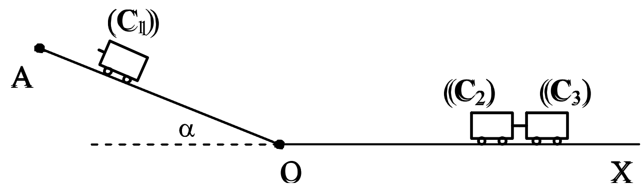
V and P are the velocity and the linear momentum of (S) respectively at a time t.

2) Draw the graph of P as a function of time.
Scale: ordinate: 1 cm ↔ 0.1 kgm/s and abscissa: 1 cm ↔ 0.05 s.
3) The slope β of the previous curve is constant. Why?
4) Calculate the value of β. Interpret this value.
5) Deduce the value of the acceleration of the motion of (S).
6) Calculate, using the graph, the value V_0 and the instant at which (S) will stop.
7) Calculate the value of the force of friction.

Correction ▼

4- Separation of masses after an elastic collision

Consider three trolleys (C_1), (C_2) and (C_3), of respective masses $m_1 = m_2 = 150$ g and $m_3 = 100$ g. (C_2) and (C_3) are attached by a light and inextensible string which conserves the compression of a spring placed between them, The system is at rest on a horizontal track OX.



(C_1) is at rest at the top A of an inclined track AO (AO = 90 cm) making an angle $\alpha = 30^0$ with the horizontal.
We release, at $t_0 = 0$, (C_1) without speed, it reaches the rail OX and undergoes a perfectly elastic collision between the system $[(C_1) ; (C_2)]$. Given $g = 10\ m/s^2$. We neglect the forces which resist the motion of the trolleys.

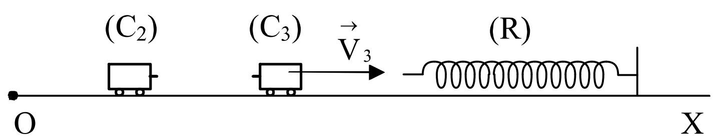
1) Applying the principle of conservation of mechanical energy, calculate the velocity V_1 of (C_1) when it reaches the point O.

2) Applying Newton's second law « $\sum \vec{F} = \frac{d\vec{P}}{dt}$ », determine, as a function of time, the linear momentum of (C_1) when it is between O and A.

3) At what instant does (C_1) reach O ?

4) Calculate the velocities V_1' and V' of (C_1) and of the system $[(C_2) ; (C_3)]$ respectively after the collision.

5) The system $[(C_2) ; (C_3)]$ moves with a velocity V' on the track OX. At a given instant, the string is cut and the trolleys (C_2) and (C_3) separate with respective velocities V_2 and V_3.



(C_3) moves towards a horizontal spring (R) of stiffness K = 90 N/m and compresses it to a distance x_0 = 10 cm.

a) Applying the principle of conservation of mechanical energy, calculate the value of V_3.

b) Calculate the value of V_2.

c) Is the kinetic energy of the system $[(C_1) ; (C_2)]$ conserved ? Interpret.

Correction ▼

5- Studying a game

Children are playing a game, made from an elastic spring, placed on a horizontal table and a ball (of mass m = 20 g) placed in front of the spring. Each child compresses the spring by the ball, once released, the spring elongates and takes its unstretched length and the ball leaves it at O with a velocity $\vec{V}_0$.

The child registers a goal if the ball falls into the box at a distance D_0 = 1.7 m from the table (see the figure).

We designate by :

h = 44 cm : the height of the table with respect to the ground,

k : stiffness of the spring,

d : the compression of the spring.

D : the abscissa of the impact point of the ball with the ground.

The zero level of gravitational potential energy is the level of the table. We neglect friction. Given g = 9.8 m/s^2.

I – Theoretical study

1) Calculate, as a function of m, k, and d, the velocity V_0 of the ball.

2) We suppose that at the instant $t_0 = 0$, the ball passes through O. At a later instant t > 0 the ball is in the air and undergoing free fall.

a) Applying Newton's second law, calculate, at a time t, the components P_x and P_y of the linear momentum $\vec{P}$ of the ball as a function of m, V_0, g and t.

b) Deduce the components V_x and V_y of the velocity vector $\vec{V}$ as a function of V_0, g and t.

c) Deduce the coordinates x and y de of the ball as a function of V_0, g and t.

d) Find a relation between h, D, d, m, k and g.

II – Calculating the value of k and the convenient value of d=d$_0$

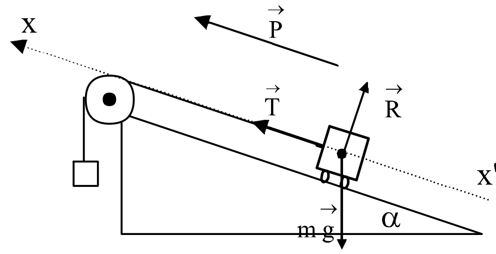
1) The first child compresses the spring using the ball by d = 1.1 cm and then he releases it, the ball doesn't enter the box and it falls next to the table at a distance of D = 27 cm. Calculate k.

2)If the second child wants to achieve a goal, calculate the corresponding value d_0 of d?

Correction ▼

Corrections

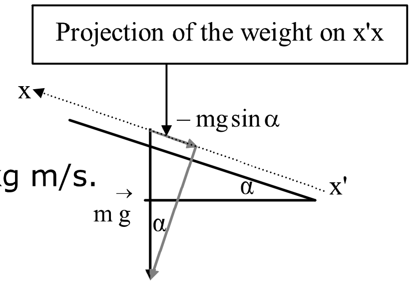
1- Graphical exploitation and Newton's second law



1) The trolley is under the action of its weight $m\vec{g}$, the reaction $\vec{R}$ normal to the support and the tension $\vec{T}$ of the string.

2) Applying Newton's second law on the chariot :

$$\sum \vec{F}_{ex} = \frac{d\vec{P}}{dt} \Leftrightarrow m\vec{g} + \vec{R} + \vec{T} = \frac{d\vec{P}}{dt}$$



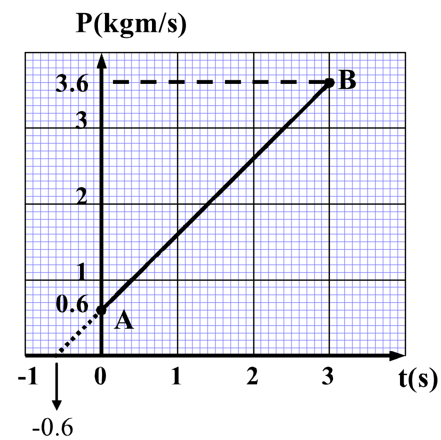
Projecting this relation on x'x:

$-mg\sin\alpha + 0 + T = \frac{dP}{dt} \Rightarrow \boxed{T = mg\sin\alpha + \frac{dP}{dt}}$.

3) At t = 0, the linear momentum of the trolley is, from the graph, P_0 = 0.6 kg m/s.

Where $P_0 = mV_0 \Rightarrow V_0 = \frac{P_0}{m} = \frac{0.6}{0.1} = 6\ \text{m/s}$. Hence : $\boxed{V_0 = 6\ \text{m/s}}$.

At the start, the velocity is zero, hence P = 0. The instant of start represents the abscissa of the point of intersection of the prolongation of the line representing P with the time axis, it is then : t = – 0,6 s.



4) The graph of P as a function of time is a straight line, hence :

$\frac{dP}{dt} =$ the slope of this line, hence :

$$\frac{dP}{dt} = \frac{\Delta P}{\Delta t} = \frac{P_B - P_A}{t_B - t_A} = \frac{3.6 - 0.6}{3 - 0} = 1\ \text{N}$$

then : $T = mg\sin\alpha + \frac{dP}{dt} = 0.1 \times 10 \times \sin 30^0 + 1 = 1.5\ \text{N}$. $\boxed{T = 1.5\ \text{N}}$

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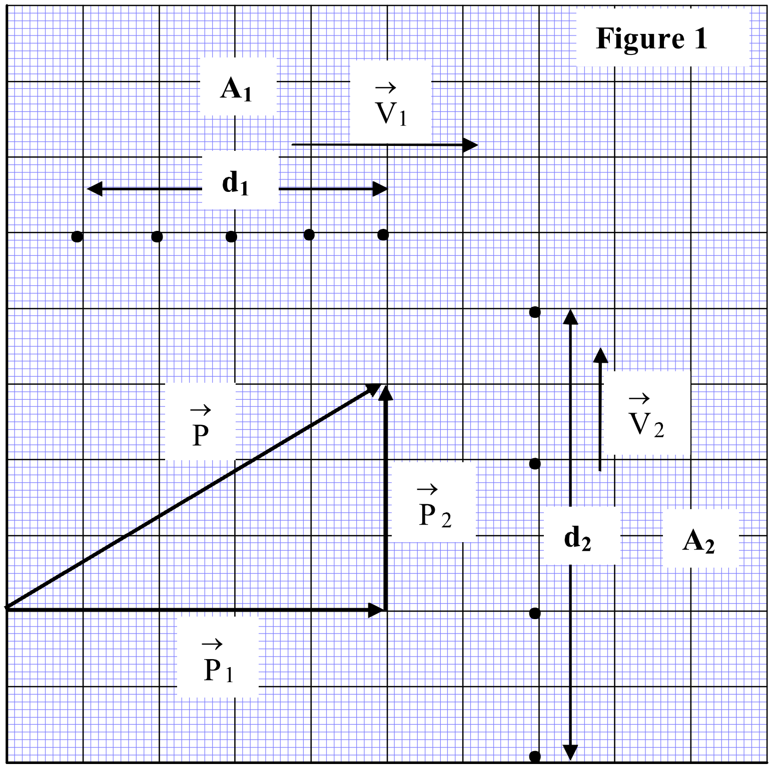
2- Experimental study of a collision

1) The dot prints, of the successive positions of the centers of inertia, of each puck are rectilinear and equidistant. Hence the motion of each puck is uniformly rectilinear.

2) The motion is uniform, hence the velocity of (see figure 1):

A_1 : $V_1 = \frac{d_1}{4\tau} = \frac{6 \times 10^{-2}\text{m}}{3 \times 20 \times 10^{-3}\text{s}} = 1\ \text{m/s}$.

A_2 : $V_2 = \frac{d_2}{4\tau} = \frac{6 \times 10^{-2}\text{m}}{4 \times 20 \times 10^{-3}\text{s}} = 0.75\ \text{m/s}$.



3)

- The linear momentum vector of A_1 , before the collision : $\vec{P}_1 = m_1\vec{V}_1$

This vector is parallel and in the same direction as $\vec{V}_1$ and of magnitude : $P_1 = m_1V_1 = 0.25\times 1 = 0.25$ kgm/s

The representative length of the vector $\vec{P}_1$ using the scale is : 5 cm (see figure 1).

- The linear momentum vector of A_2 , before the collision : $\vec{P}_2 = m_2\vec{V}_2$

This vector is parallel and in the same direction as $\vec{V}_2$ and of magnitude : $P_2 = m_2V_2 = 0.2\times 0.75 = 0.15$ kgm/s

The representative length of the vector $\vec{P}_2$ using the scale is : 3 cm (see figure 1).

- The linear momentum vector of the system (A_1 ; A_2) is : $\vec{P} = \vec{P}_1 + \vec{P}_2$

This vector is the resultant of the vectors $\vec{P}_1$ and $\vec{P}_2$. We construct $\vec{P}$ by the vectors (see figure 1).

Drawing the vector $\vec{P}$ corresponds to a length : 5.8 cm, which is equivalent to a magnitude: $\boxed{P = 0{,}29 \text{ kg.m/s}}$.

4) The external forces acting on the system (A_1; A_2) are the weights of the pucks and the normal reactions of the support (see figure 2), hence the sum is: $\sum \vec{F}_{ext} = \underbrace{m_1\vec{g} + \vec{R}_1}_{=\vec{0}} + \underbrace{m_2\vec{g} + \vec{R}_2}_{=\vec{0}} = \vec{0}$

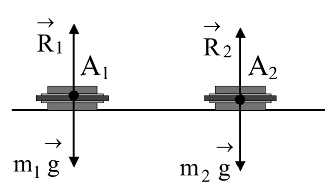


Figure 2

Hence the system (A_1 ; A_2) is isolated and hence its linear momentum is conserved.

5) After the collision the two pucks form one body moving with a velocity-vector $\vec{V}$, hence its linear momentum is : $\vec{P}' = (m_1 + m_2)\vec{V}$.

The linear momentum of the system (A_1 ; A_2), during the collision, in conserved : $\vec{P} = \vec{P}'$

Hence : $\vec{V} = \dfrac{\vec{P}}{m_1 + m_2}$ and in magnitude : $V = \dfrac{P}{m_1 + m_2} = \dfrac{0.195}{0.25 + 0.2} = 0.644\ \text{m/s}$. $\boxed{V = 0.644\ \text{m/s}}$.

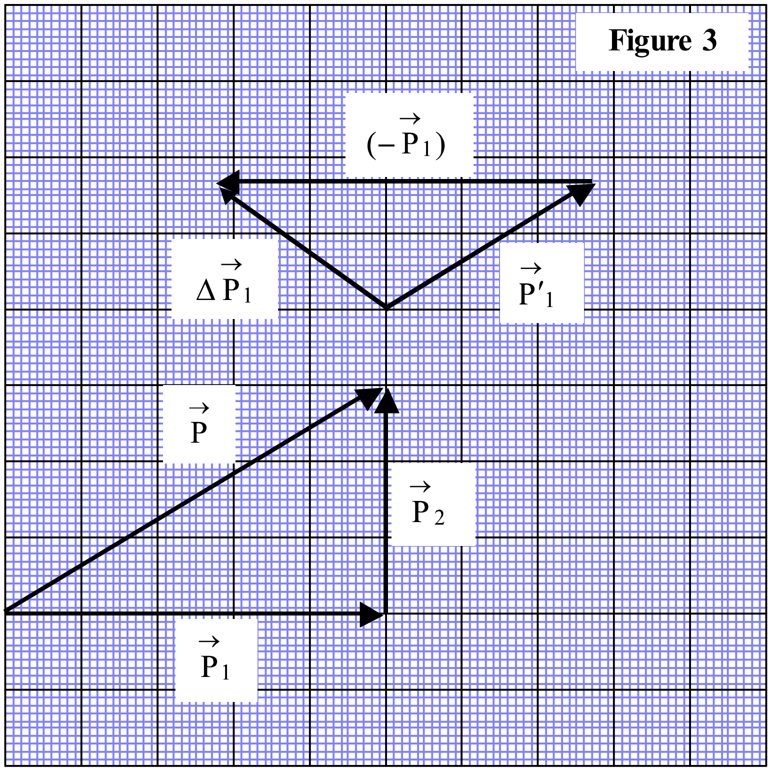
6) After the collision, A_1 moves with a speed $\vec{V}$, hence its linear momentum vector is : $\vec{P'}_1 = m_1 \vec{V}$

This vector is parallel and in the same direction as $\vec{V}$ and of magnitude : $P'_1 = m_1 V = 0.25 \times 0.644 = 0.161\ \text{kgm/s}$.

The representative length of the vector $\vec{P'}_1$ using the scale : 3,2 cm (see figure 3).

(Note that the vectors $\vec{V}$ and $\vec{P}$ are parallel and in the same direction)

Where : $\Delta \vec{P}_1 = \vec{P'}_1 - \vec{P}_1 = \vec{P'}_1 + (-\vec{P}_1)$. We construct $\Delta \vec{P}_1$ by the method of successive vectors (see figure 3).



The forces acting on A_1 at the instant of the collision are the weight, the normal reaction of the support and the force $\vec{F}$ exerted by A_2 on A_1.
Applying, at the instant of collision , Newton's second law on A_1 :

$\sum \vec{F}_{ext/A_1} = \dfrac{d\vec{P}_1}{dt} \Rightarrow \underbrace{m_1 \vec{g} + \vec{R}_1}_{=\vec{0}} + \vec{F} = \dfrac{\Delta \vec{P}_1}{\Delta t} \Rightarrow \vec{F} = \dfrac{\Delta \vec{P}_1}{\Delta t}$ with $\Delta t > 0$, we can deduce that $\vec{F}$ and $\Delta \vec{P}_1$ are parallel and in the same direction.

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3- Measuring the magnitude of the force of friction

The velocity of the puck when it passes by the position G_i is : $V_i = \dfrac{G_{i-1}G_{i+1}}{2\tau}$

In the position G_1 or at t_1 = 0.1 s : $V_1 = \dfrac{G_0G_2}{2\tau} = \dfrac{6.5\times10^{-2}\times \overbrace{2}^{\text{scale}}}{2\times0.1} = 0.65 \text{ m/s}$;

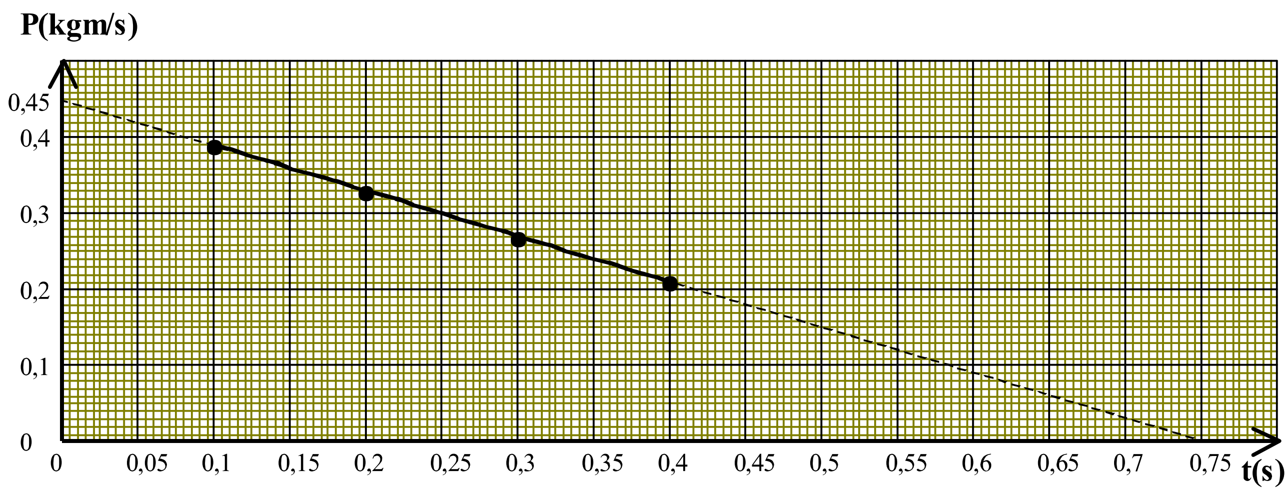
In the position G_2 or at t_2 = 0,2 s: $V_2 = \dfrac{G_1G_3}{2\tau} = \dfrac{5.5\times10^{-2}\times \overbrace{2}^{\text{scale}}}{2\times0.1} = 0.55 \text{ m/s}$; and so on.

The magnitude of the linear momentum in the position G_i : $P_i = MV_i$

In the position G_1 or at t_1 = 0,1 s : $P_1 = MV_1 = 0.6\times0.65 = 0.39 \text{ kgm/s}$;
In the position G_2 or at t_2 = 0,2 s : $P_2 = MV_2 = 0.6\times0.55 = 0.33 \text{ kgm/s}$; and so on

t(s)	0	0,1	0,2	0,3	0,4	0,5
V(m/s)		**0,65**	**0,55**	**0,45**	**0,4**	
P(kgm/s)		**0,39**	**0,33**	**0,27**	**0,24**	

1)



2) The graph of P as a function of time is a straight line hence its slope is always constant.

3) The slope of the line : $\beta = \dfrac{\Delta P}{\Delta t} = \dfrac{P_4 - P_1}{t_4 - t_1} = \dfrac{0.24-0.39}{0.4-0.1} = -0.5 \text{ N}$

Interpretation of β:

We have : $\sum \vec{F}_{ex} = \dfrac{d\vec{P}}{dt} \Rightarrow \left\|\sum \vec{F}_{ex}\right\| = \left\|\dfrac{d\vec{P}}{dt}\right\| = \left|\dfrac{dP}{dt}\right| = \left|\dfrac{\Delta P}{\Delta t}\right| = |\beta| = 0.5 \text{ N}$

4) Hence $|\beta|$ represents the magnitude of the resultant force acting on (S).

Note that for a rectilinear motion, $\vec{P}$ is parallel to the motion hence : $\left\|\dfrac{d\vec{P}}{dt}\right\| = \left|\dfrac{dP}{dt}\right|$ and hence the graph of P as a function of time is a straight line, hence: $\dfrac{dP}{dt} = \dfrac{\Delta P}{\Delta t}$.

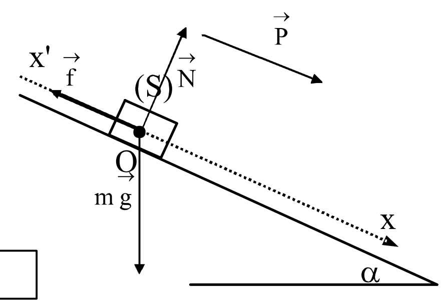
5) We have : $\sum \vec{F}_{ex} = \dfrac{d\vec{P}}{dt} = M\vec{a}$ algebraically, since along the direction of motion:

$\dfrac{dP}{dt} = Ma \Rightarrow \dfrac{\Delta P}{\Delta t} = Ma \Rightarrow \beta = Ma \Rightarrow a = \dfrac{\beta}{M} = \dfrac{-0.5}{0.6} = -0.833 \text{ m/s}$. $\boxed{a = -0.833 \text{ m/s}}$.

6) At $t_0 = 0$, we can find from the graph the linear momentum of (S), which is : $P_0 = 0.45$ kg.m/s (see the graph).

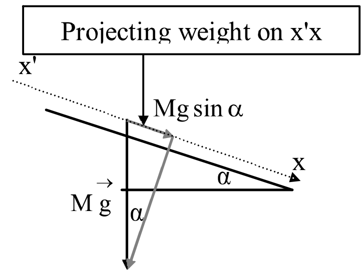
then $P_0 = MV_0 \Rightarrow V_0 = \frac{P_0}{M} = \frac{0.45}{0.6} = 0.75\text{ m/s}$. $\boxed{V_0 = 0.75\text{ m/s}}$.

At stopping, P = 0 and that corresponds to $\boxed{t = 0.75\text{ s}}$ (see the graph).



7) The forces acting on (S), are : the weight $M\vec{g}$, the normal reaction $\vec{N}$ of the support and the force of friction $\vec{f}$.

Applying Newton's second law $\sum \vec{F}_{ex} = \frac{d\vec{P}}{dt} \Rightarrow M\vec{g} + \vec{N} + \vec{f} = \frac{d\vec{P}}{dt}$



Projecting this relation on x'x :

$$Mg\sin\alpha + 0 - f = \frac{dP}{dt} = \beta \Rightarrow f = Mg\sin\alpha - \beta$$

Numerically: $f = 0.6 \times 10 \times \sin 30^0 - (-0.5) = 3.5\text{ N}$. $\boxed{f = 3.5\text{ N}}$.

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4- Separation of masses after an elastic collision

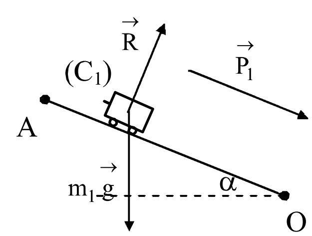
1) The trolley (C_1) moves without friction on the part AO; hence the mechanical energy of the system [Earth ; (C_1)] is conserved ; hence $E_{mA} = E_{mO}$.

If we take the horizontal plane passing through OX as a zero reference of gravitational potential energy then :

$$E_{mA} = E_{KA} + E_{PGA} = 0 + m_1.g.\underbrace{h_A}_{OA\sin\alpha} = mg.OA\sin\alpha = 0.15 \times 10 \times 0.9 \times \sin 30^0 = 0.675\text{ J}.$$

$$E_{mO} = E_{KO} + E_{PGO} = \tfrac{1}{2}m_1V_1^2 + 0 = 0.075V_1^2.$$

Hence $0.675 = 0.075V_1^2$ where $\boxed{V_1 = 3\text{ m/s}}$.



2) The forces acting on (C_1), during its downward motion, are : the weight $m_1\vec{g}$, the normal reaction to the support $\vec{R}$.

Applying Newton's second law : $\sum \vec{F} = \frac{d\vec{P}_1}{dt}$

$\Leftrightarrow m_1\vec{g} + \vec{R} = \frac{d\vec{P}_1}{dt}$ projecting this relation on an axis along $\vec{AO}$:

$$m_1 g\sin\alpha + 0 = \frac{dP_1}{dt} \Rightarrow \frac{dP_1}{dt} = m_1 g\sin\alpha$$

Integrating: $P_1 = m_1 g\sin\alpha.t + P_{01}$ with P_{01} is the linear momentum of (C_1) at $t_0 = 0$.

But at t = 0, the speed of (C_1) is zero hence $P_{01} = 0$ and so : $\boxed{P_1 = m_1 g\sin\alpha.t = 0.75t}$

3) When (C_1) reaches O its speed is : V_1 = 3 m/s hence its linear momentum is:
$P_1 = m_1V_1 = 0.15\times3 = 0.45$ kg.m/s.

From the preceding expression of the linear momentum : 0.75t = 0.45, hence t = 0.6 s.

Hence (C_1) reaches O at the instant $\boxed{t = 0.6\text{ s}}$ ..

4) The collision between (C_1) and [(C_2) ; (C_3)] is elastic, we then have conservation of linear momentum and of the kinetic energy of the system [(C_1) ; (C_2) ; (C_3)].

The trolley (C_1) moves along OX with a velocity $\vec{V}_1$ and undergoes a collision with this speed with the system [(C_1); (C_2)].

Just before the collision : Let m = m_2 + m_3 = 250 g is the mass of the system [(C_2) ; (C_3)]

The linear momentum of the system is : $\vec{P}_{before} = m_1\vec{V}_1$ (only (C_1) has a speed)

The kinetic energy of the system is : $E_{Kbefore} = \frac{1}{2}m_1V_1^2$

Just after the collision :

The linear momentum of the system is : $\vec{P}_{after} = m_1\vec{V}_1' + m\vec{V}'$

The kinetic energy of the system is : $E_{Kafter} = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}mV'^2$

Conservation of the linear momentum:

$\vec{P}_{before} = \vec{P}_{after} \Rightarrow m_1\vec{V}_1 = m_1\vec{V}_1' + m\vec{V}'$,

since the collision is collinear, we can replace the vectors by **<u>their algebraic</u>** values : $\boxed{m_1V_1 = m_1V_1' + mV'}$ (1) or $m_1(V_1 - V_1') = mV'$.

Taking these algebraic values along $\vec{OX}$, then : V_1 = + 3 m/s.

Conservation of kinetic energy:

$E_{Kbefore} = E_{Kafter} \Rightarrow \frac{1}{2}m_1V_1^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}mV'^2$ or $m_1V_1^2 = m_1V_1'^2 + mV'^2$ and hence $m_1(V_1 - V_1')(V_1 + V_1') = mV'^2$

where the system : $\begin{cases} m_1(V_1 - V_1')(V_1 + V_1') = mV'^2 \\ m_1(V_1 - V_1') = mV' \end{cases}$;

Dividing each side of the two equation by each other we get : $\boxed{V_1 + V_1' = V'}$ (2).

(1) and (2) with V_1 = +3 m/s can be written : $\begin{cases} 0.15V_1' + 0.25V' = 0.45 \\ V_1' - V' = -3 \end{cases}$;

We obtain that : $\boxed{V_1' = -0.75\text{ m/s}}$ and $\boxed{V' = 2.25\text{ m/s}}$.

Finally and just after the collision, the velocity of (C_1) is 0.75 m/s and directed along the opposite direction of $\vec{OX}$ (since $V_1' < 0$) ; and that of [(C_1) ; (C_2)] is 2.25 m/s and directed along $\vec{OX}$ (since $V' > 0$).

5) a) (C_3) moves with the velocity $\vec{V}_3$ and reaches the spring. The mechanical energy of the system [(C_3) ; (R)] after the separation is conserved.

Applying the conservation of mechanical energy between two instants :

t_1 : during the compression where the velocity of (C_3) is $\vec{V}_3$ and the elastic potential energy of the spring is zero : $E_{m1} = \frac{1}{2}m_3V_3^2$

t_2 : the end of the compression where the velocity of (C_3) is zero : $E_{m2} = \frac{1}{2}Kx_0^2$

Where : $E_{m1} = E_{m2} \Rightarrow \frac{1}{2}m_3V_3^2 = \frac{1}{2}Kx_0^2 \Rightarrow V_3 = \sqrt{\frac{K}{m_3}}x_0 = \sqrt{\frac{90}{0.1}} \times 0.1 = 3\ m/s$. $\boxed{V_3 = 3\ m/s}$.

b) During the separation, the linear momentum of the system [(C_2) ; (C_3)] is conserved :

Just before the separation : $\vec{P}_{before} = m\vec{V}'$

Just after the separation : $\vec{P}_{after} = m_2\vec{V}_2 + m_3\vec{V}_3$

$\vec{P}_{before} = \vec{P}_{after} \Rightarrow m\vec{V}' = m_2\vec{V}_2 + m_3\vec{V}_3$ the separation is collinear, we can replace the velocity- vectors by their algebraic values along $\vec{OX}$: $V' = +2.25\ m/s$ and $V_3 = +3\ m/s$

$mV' = m_2V_2 + m_3V_3 \Rightarrow 0.25 \times 2.25 = 0.15V_2 + 0.1 \times 3 \Rightarrow V_2 = 1.75\ m/s$. $\boxed{V_2 = 1.75\ m/s}$.

c) Just before the separation :

$E_{Kbefore} = \frac{1}{2}mV'^2 = \frac{1}{2}0.25 \times 2.25^2 = 0.633\ J.$

Just after the separation :

$E_{Kafter} = \frac{1}{2}m_2V_2^2 + \frac{1}{2}m_3V_3^2 = \frac{1}{2}0.15 \times 1.75^2 + \frac{1}{2}0.1 \times 3^2 = 0.68\ J.$

$E_{Kafter} \neq E_{Kbefore}$; hence the kinetic energy of the system [(C_2) ;(C_3)] is not conserved.

Before the separation, the energy stored in the system [(C_2) ; (C_3)] : $E_1 = E_{Kbefore} + E_{Pe}$ (compressed spring)
After the separation, the energy stored in the system [(C_2) ; (C_3)] : $E_2 = E_{Kafter}$ (spring takes its free length)
Using the principle of conservation of energy (without friction): $E_1 = E_2$

$\Rightarrow E_{pe} = E_{Kafter} - E_{Kbefore} = 0.047\ J$.

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5- Studying a game

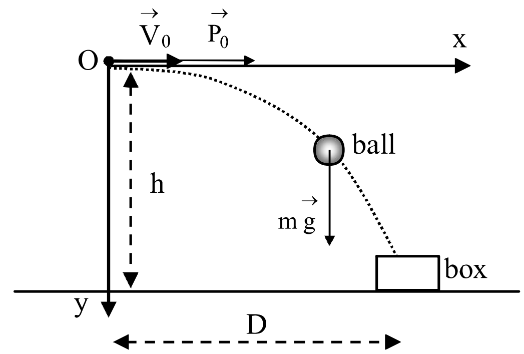
I – Theoretical study

1) The mechanical energy of the system [Earth ; ball ; spring] is conserved.
Applying the conservation of mechanical energy between two instants :

t_1 : the instant the ball was released where its velocity is zero and the elastic potential energy of the spring is $\frac{1}{2}kd^2$: $E_{m1} = \frac{1}{2}kd^2$
t_2 : instant at which the ball leaves O with a velocity V_0 and where the spring gains its free length back : $E_{m2} = \frac{1}{2}mV_0^2$

$E_{m1} = E_{m2} \Rightarrow \frac{1}{2}kd^2 = \frac{1}{2}mV_0^2 \Rightarrow \boxed{V_0 = \sqrt{\frac{k}{m}}d}$

2) a) The only force acting on the ball using its free fall is its weight $m\vec{g}\begin{cases} mg_x = 0 \\ mg_y = mg \end{cases}$



Applying Newton's second law : $\sum \vec{F} = \dfrac{d\vec{P}}{dt} \Leftrightarrow m\vec{g} = \dfrac{d\vec{P}}{dt}$

Projecting this relation on Ox :

$mg_x = \dfrac{dP_x}{dt} \Rightarrow \dfrac{dP_x}{dt} = 0 \Rightarrow P_x = \text{constant} = P_{0x}$

Projecting this relation on Oy : $mg_y = \dfrac{dP_y}{dt} \Rightarrow \dfrac{dP_y}{dt} = mg \Rightarrow P_y = mg.t + P_{0y}$.

On the other hand, at t = 0, the ball is at O with a velocity $\vec{V}_0$ directed along Ox, the same as its linear momentum vector : $\vec{P}_0 = m\vec{V}_0\begin{cases} P_{0x} = mV_0 \\ P_{0y} = 0 \end{cases}$ .Hence : $\boxed{\vec{P}\begin{cases} P_x = mV_0 \\ P_y = mgt \end{cases}}$.

b) We have : $\begin{cases} P_x = mV_x \Rightarrow V_x = V_0 \\ P_y = mV_y \Rightarrow V_y = gt \end{cases}$

d) We have : $\begin{cases} V_x = \dfrac{dx}{dt} \Rightarrow \dfrac{dx}{dt} = V_0 \Rightarrow x = \int v_0 dt \Rightarrow x = V_0 t + \underbrace{x_0}_{=0} \Rightarrow x = V_0 t \\ V_y = \dfrac{dy}{dt} \Rightarrow \dfrac{dy}{dt} = gt \Rightarrow y = \int gt dt \Rightarrow y = \frac{1}{2}gt^2 + \underbrace{y_0}_{=0} \Rightarrow y = \frac{1}{2}gt^2 \end{cases} \Rightarrow \boxed{\begin{cases} x = V_0 t \\ y = \frac{1}{2}gt^2 \end{cases}}$ (x_0=0, y_0=0) represent the coordinates of the ball at t=0

d) We have: $\begin{cases} x = V_0 t \Rightarrow t = \dfrac{x}{V_0} \\ y = \frac{1}{2}gt^2 \end{cases}$; replacing t by its expression in y, then: $y = \frac{1}{2}g\left(\dfrac{x}{V_0}\right)^2 \Rightarrow y = \frac{1}{2}\dfrac{gx^2}{V_0^2}$

When the ball reaches **the ground** its coordinates are (x = D and y = h) and we find $V_0 = \sqrt{\dfrac{k}{m}}d$, hence :

$h = \frac{1}{2}\dfrac{gD^2}{\left(\sqrt{\dfrac{k}{m}}d\right)^2} = \dfrac{mg}{2k}\left(\dfrac{D}{d}\right)^2$. Finally : $\boxed{h = \dfrac{mg}{2k}\left(\dfrac{D}{d}\right)^2}$.

II – Calculating the value of k and the convenient value of d=d_0

1) For the first child, we have D = 27 cm for a compression of d = 1.1 cm.

We have : h = 44 cm, m = 20 g, hence : $k = \dfrac{mg}{2h}\left(\dfrac{D}{d}\right)^2 = \dfrac{0.02\times10}{2\times0.44}\left(\dfrac{27}{1.1}\right)^2 \cong 137 \text{ N/m}$.

2) The second child should compress the spring by a distance d = d_0 such that : D = D_0 = 1.7 m.

$d_0 = \sqrt{\dfrac{mg}{2k.h}}D = \sqrt{\dfrac{0.02\times10}{2\times137\times0.44}}\times1.7 = 0.0692 \text{ m} \cong 7 \text{ cm}$. Hence : $\boxed{d_0 \cong 7 \text{ cm}}$.

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