Grade 12 GS

Probability ex 9

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➡ **Exercise 9:**

An urn contains **three** white balls and **two** black balls.

A player draws **randomly** and **successively** three balls from this urn, respecting the following rule:

In each draw: if the drawn ball is black, he **replaces** it back in the urn ;

if it is white, he **doesn't replace** it back in the urn.

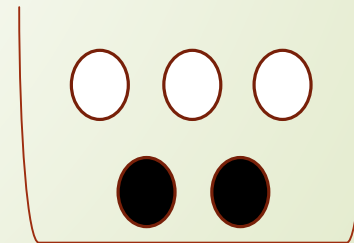1) a) Calculate the probability of drawing, in the following order: one black, one black then one white ball.

Solution:

$$P(bbw) = \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{125}$$

b) Show that the probability of obtaining one white ball only, among the three drawn balls, is equal to $\frac{183}{500}$.

Solution:

$$P(wbb \ or \ bwb \ or \ bbw) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{4} \ + \ \frac{2}{5} \times \frac{3}{5} \times \frac{2}{4} \ + \ \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5}$$

$$= \frac{183}{500}$$

2) The player now draws **randomly** and **successively** n balls from the urn (n>3) respecting the same rule.

a) Calculate, in terms of n, the probability of the event: "the player draws n black balls".

Solution:

- $P(bbb\ldots b) = \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} \ldots \times \dfrac{2}{5} = \left(\dfrac{2}{5}\right)^n$

b) Calculate, in terms of n, the probability P_n of the event: "the player obtains at least one white ball".

Solution:

- $P_n = P(at\ least\ one\ white) = 1 - P(none\ white) = 1 - P(n\ black) = 1 - \left(\dfrac{2}{5}\right)^n$

c) What is the minimum number of balls to be drawn by the player so that $P_n \geq 0.99$?

Solution:

- $1 - \left(\dfrac{2}{5}\right)^n \geq 0.99 \longrightarrow -\left(\dfrac{2}{5}\right)^n \geq -0.01 \longrightarrow \left(\dfrac{2}{5}\right)^n \leq 0.01$

$\xrightarrow{ln} n\ln\left(\dfrac{2}{5}\right) \leq \ln(0.01) \longrightarrow n \geq \dfrac{\ln(0.01)}{\ln\left(\dfrac{2}{5}\right)} \longrightarrow n \geq 5.02$

$\longrightarrow n = 6$