- *This exam consists of three problems inscribed on two pages.*
- *The use of a non-programmable calculator is allowed.*

I- (4 points)

In the table below, only one of the proposed answers is correct.
Choose the correct answer and justify your choice.

N°	Question	Proposed answers		
		A	B	C
1)	The number of solutions of the equation $\ln(x+3)=2\ln x-\ln 3$ is:	0	1	2
2)	Let f be the function defined over $]0\,;\,+\infty[$ by $f(x)=x(\ln x)^2$ and denote by (C) its representative curve in an orthonormal system. The curve (C) admits:	at point of abscissa e a tangent of slope -3	an inflection point of abscissa e	an inflection point of abscissa $\frac{1}{e}$
3)	The domain of definition of the function f defined by $f(x)=\ln\left(e^{2x}-3e^{x}-4\right)$ is:	$]\ln 4\,;\,+\infty[$	$]-\infty\,;\,\ln 4[$	$]-\infty\,;\,\ln 4[\,\cup\,]\ln 4\,;\,+\infty[$
4)	We draw simultaneously three balls from an urn containing 5 red balls, 2 yellow and 3 white. The total number of outcomes is:	720	1000	120

II- (6 points)

A survey was done on group of citizens in a certain city in Lebanon reveals the following results:

- 75% of the citizens exchanged their salaries from Lebanese Lira to dollars.
- Out of those who didn't exchange their salaries to dollars, 35% paid more money during buying items.
- 20 % of the citizens paid more money during buying items.

A citizen who responded to this survey was randomly chosen.

Consider the following events:

C: « The chosen citizen exchanged his salary from Lebanese Lira to dollars »;

M: « The chosen citizen paid more money during buying items ».

1) Calculate $\mathrm{P}\left(\overline{C}\cap M\right)$ and verify that $\mathrm{P}(C\cap M)=\dfrac{9}{80}$.

2) Calculate $P(M/C)$.

3) A citizen paid more money during buying items, what is the probability that he didn't exchange his salary from Lebanese Lira to dollars?

4) The group consists of 800 citizens. 3 citizens were chosen randomly and simultaneously. Calculate the probability that at least one of them paid more money during buying items (give the answer rounded to the nearest hundredths).

III- (10 points)
Part A

Consider the function g defined over $\mathbb{R}$ by $g(x)=(1+x)e^x-4$.

The table below represents the table of variations of g.

x	$-\infty$		-2		$+\infty$
$g'(x)$		$-$	0	$+$	
$g(x)$	-4	$\searrow$	$-e^{-2}-4$	$\nearrow$	$+\infty$

1) The equation $g(x)=0$ admits a unique solution α. Prove that $0.7<\alpha<0.8$.

2) Use the above table to deduce the sign of $g(x)$ over $\mathbb{R}$.

Part B

Consider the function f defined over $]-\infty;+\infty[$ by $f(x)=(x-1)e^x-2x^2$ and let (C) be its representative curve on the orthonormal system $(O\,;\,\vec{i}\,;\,\vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

2) Calculate $f(2.5)$ and $f(-1)$ and give the results to the nearest 10^{-1}.

3) Prove that, for every $x\in\mathbb{R}$, $f'(x)=x\left(e^x-4\right)$ then setup the table of variations of f.

4) Show that the equation $f(x)=0$ admits a unique solution β and prove that $2<\beta<2.1$.

5) **a)** Prove that for every $x\in\mathbb{R}$, $f''(x)=g(x)$.

b) Deduce that the curve (C) admits an inflexion point I of abscissa α.

c) Show that the director coefficient of the tangent (T) to the curve (C) at point I can be written as: $\dfrac{-4\alpha^2}{\alpha+1}$.

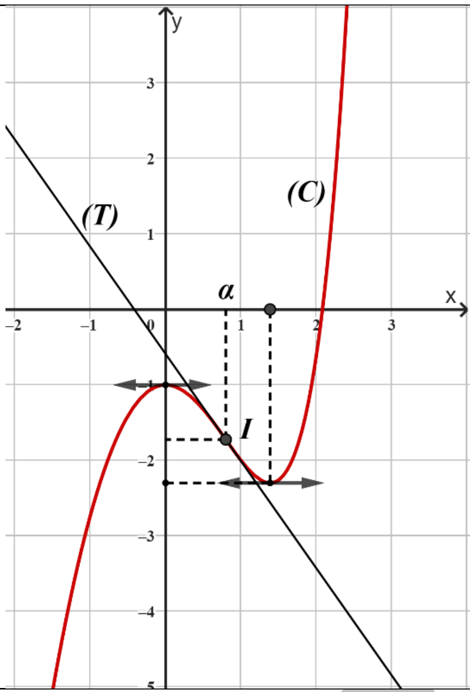
6) Draw (T) and (C) (take $\alpha\approx 0.75$).

QI	Answers	4 pts
1)	The equation exists if $\begin{cases} x+3>0 \\ x>0 \end{cases}$ then if $\begin{cases} x>-3 \\ x>0 \end{cases}$ then for $x \in]0\ ;\ +\infty[$; $\ln(x+3) = 2\ln x - \ln 3$; $\ln(x+3) = \ln\left(\dfrac{x^2}{3}\right)$; since the function $x \mapsto \ln x$ is continuous and strictly increasing over its domain then $x+3 = \dfrac{x^2}{3}$; $x^2 - 3x - 9 = 0$; then $x = \dfrac{3+3\sqrt{5}}{2} \in]0\ ;\ +\infty[$ (accepted) or $x = \dfrac{3-3\sqrt{5}}{2} \notin]0\ ;\ +\infty[$ (rejected). The correct answer is **B**.	1
2)	f is differentiable over $]0\ ;\ +\infty[$, $f'(x) = (\ln x)^2 + 2\ln x \dfrac{1}{x} x = (\ln x)^2 + 2\ln x$; f' is differentiable over $]0\ ;\ +\infty[$, $f''(x) = \dfrac{2\ln x + 2}{x}$ have same sign of $2\ln x + 2$ since $x \in]0\ ;\ +\infty[$; $f''(x) = 0$ if $2\ln x + 2 = 0$ then $x = e^{-1} = \dfrac{1}{e}$; Signe of f'': $\begin{array}{c\|ccccc} x & 0 & & \frac{1}{e} & & +\infty \\ \hline f''(x) & \| & - & 0 & + & \end{array}$ $f''(x) = 0$ if $x = \dfrac{1}{e}$ and changes sign then the curve (C) of f admits an inflection point of abscissa $\dfrac{1}{e}$. The correct answer is **C**.	1
3)	f is defined if $e^{2x} - 3e^x - 4 > 0$; Let $t = e^x > 0$ $t^2 - 3t - 4 > 0$ for $t < -1$ or $t > 4$ and since $t > 0$ then $t > 4$ then $x > \ln 4$. The domain of definition of the function f is $]\ln 4\ ;\ +\infty[$. The correct answer is **B**.	1
4)	The total number of outcomes is $C_{10}^{3} = 120$. The correct answer is **C**.	1

QII	Answers	6 pts
1)	$P(\overline{C} \cap M) = P(M / \overline{C}) \times P(\overline{C}) = \dfrac{35}{100} \times \dfrac{25}{100} = \dfrac{7}{80}$; $P(C \cap M) = P(M) - P(\overline{C} \cap M) = \dfrac{20}{100} - \dfrac{7}{80} = \dfrac{9}{80}$.	2
2)	$P(M / C) = \dfrac{P(M \cap C)}{P(C)} = \dfrac{9/80}{75/100} = \dfrac{3}{20}$.	1
3)	$P(\overline{C} / M) = \dfrac{P(M \cap \overline{C})}{P(M)} = \dfrac{7/80}{20/100} = \dfrac{7}{1600}$.	1

4)	The number of citizens who paid more money during buying items is $800\times P(M)=160$. Let A be the event "at least one of the three citizens paid more money during buying items". $P(A)=1-P(\overline{A})=1-\dfrac{C_{640}^{3}}{C_{800}^{3}}\approx 0.49$.	**2**

QIII	Answers	10 pts
A.1.	$g(0.7)\approx -0.56<0$ and $g(0.8)\approx 0.005>0$, then $0.7<\alpha<0.8$.	½
A.2.	Using the table of variations of g: $g(x)<0$ if $x\in\left]-\infty\ ;\ \alpha\right[$; $g(x)=0$ if $x=\alpha$; $g(x)>0$ if $x\in\left]\alpha\ ;\ +\infty\right[$.	1
B.1.	$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\left(xe^{x}-e^{x}-2x^{2}\right)=0-0-\infty=-\infty$; $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to+\infty} e^{x}\left(x-1-\dfrac{2x^{2}}{e^{x}}\right)=+\infty(+\infty-1-0)=+\infty$ since $\lim\limits_{x\to+\infty}\dfrac{2x^{2}}{e^{x}}\overset{HR}{=}\lim\limits_{x\to+\infty}\dfrac{4x}{e^{x}}=0$.	1
B.2.	$f(2.5)\approx 5.7$ and $f(-1)\approx -2.7$ (the results are to the nearest 10^{-1} by default).	½
B.3.	$f'(x)=(1)e^{x}+(x-1)e^{x}-4x=xe^{x}-4x=x\left(e^{x}-4\right)$; $\begin{array}{c\|ccccccc} x & -\infty & & 0 & & \ln 4 & & +\infty \\ \hline f'(x) & & + & 0 & - & & + & \\ \hline f(x) & -\infty & \nearrow & -1 & \searrow & 3e^{4}-32 & \nearrow & +\infty \end{array}$	1½
B.4.	Over $\left]-\infty\ ;\ \ln 4\right[$, $f(x)<0$; Over $\left]\ln 4\ ;\ +\infty\right[$, f is continuous, strictly increasing and change sign then the equation $f(x)=0$ admits a unique solution $\beta\in\left]\ln 4\ ;\ +\infty\right[$. **Conclusion**: The equation $f(x)=0$ admits a unique solution β over $\left]-\infty\ ;\ +\infty\right[$; In addition: $f(2)\approx -0.6<0$ and $f(2.1)\approx 0.1>0$ then $2<\beta<2.1$.	1
B.5.a.	$f''(x)=(1)\left(e^{x}-4\right)+x\left(e^{x}\right)=(x+1)e^{x}-4=g(x)$.	¾
B.5.b.	$f''(x)$ have the same sign of $g(x)$ over $\mathbb{R}$, then using part **A.2** $f''(x)=0$ if $x=\alpha$ and change sign then the curve (C) of f admits an inflection point I of abscissa α.	¾
B.5.c.	The director coefficient of the tangent (T) is $f'(x_I)=f'(\alpha)=\alpha\left(e^{\alpha}-4\right)$; But α is the solution of the equation $g(x)=0$ then $g(\alpha)=0$ then $(\alpha+1)e^{\alpha}-4=0$ then $e^{\alpha}=\dfrac{4}{\alpha+1}$; Then $f'(\alpha)=\alpha\left(e^{\alpha}-4\right)=\alpha\left(\dfrac{4}{\alpha+1}-4\right)=\alpha\left(\dfrac{4-4\alpha-4}{\alpha+1}\right)=\dfrac{-4\alpha^{2}}{\alpha+1}$.	1

B.6.	y x (T) (C) α I 3 2 1 −1 −2 −3 −4 −5 −2 −1 0 1 2 3	2