Exercise 1 Conservation and non-conservation of E_m

Consider a system (S) formed of inextensible and massless string of length ℓ = 45 cm, having one of its ends O fixed while the other end carries a particle (P) of mass m = 100 g. (S) is shifted from its equilibrium position by $\theta_m = 90^o$; while the string is under tension and then released without initial velocity at instant $t_0 = 0$.

Take the horizontal plane containing BF as a gravitational potential energy reference for the system [(S), Earth]. Take g = 10 m/s^2.

1- Motion between A and B.

We neglect friction on the axis though O and air resistance.

1.1- Calculate the mechanical energy of the system [(S), Earth].

1.2- Determine the expression of the gravitational potential energy of the system [(S), Earth], at instant t, in terms of θ, where θ an angle making θ with the vertical at instant t.

1.3- Determine the expression of kinetic energy of (P), at instant t, in terms of θ.

1.4- Determine the value of θ_C, ($0 < \theta_C < 90^o$), for point C for which the kinetic energy of (S) is equal to the gravitational potential energy of the system [(S), Earth]. Deduce the height h_C.

1.5.1- Determine the variation of the gravitational potential energy ΔPEg of the system system [(S), Earth] between A and B.

1.5.2- Determine the work done by the weight Wmg between A and B.

1.5.3- Compare ΔPEg and Wmg.

1.6- Verify that the velocity of (P) is $V_B = 3$ m/s when it passes through its equilibrium position.

2- Motion between B and F.

Upon passing through the equilibrium position, the string is cut, and (S) moves along a plane BF, reaches point D with speed $V_D = 2$ m/s where BD = 2 m.

2.1- Prove the existence of friction force.

2.2- Determine the variation of internal energy of the system [(S), Earth, Track, Atmosphere] during the motion of (P) between B and D.

2.3- Determine the magnitude f of the force of friction $\vec{f}$, supposed constant and parallel to the displacement, exerted on (P) during its motion between B and D.

2.4- After point D, (P) continues its motion without friction, hits the spring, and compresses it by a maximum distance x_m = DE = 20 cm. Determine the value of stiffness of spring k.

2.5- Determine the value of x, ($0 < x < 20$ cm), for which the kinetic energy of (S) is equal to the elastic potential energy.