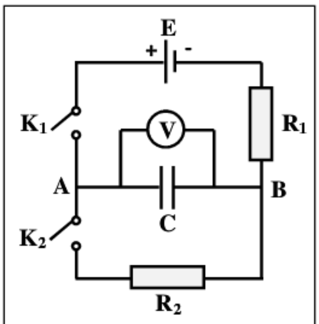
Extra sheet (Capacitor)

Exercise 1:

In order to determine the capacitance C of a capacitor, we connect up the circuit of document 1. This circuit is formed of the capacitor, a generator of e.m.f. E = 9V and of negligible internal resistance, two resistors of resistances $R_1 = 200k\Omega$ and $R_2 = 100k\Omega$ and two switches K_1 and K_2.



Doc.1

I- Charging the capacitor

The capacitor being initially uncharged, we close K_1 and keep K_2 open. The capacitor will be charged.

1- Derive the differential equation that describes the variation of the voltage $u_C = u_{AB}$ across the capacitor.

2- Knowing that the solution of this differential equation has the form $u_C = E\left(1 - e^{-\frac{t}{\tau_1}}\right)$. Determine the expression of the constant τ_1 as a function of R_1 and C.

3- At the instant $t_1 = 20s$, u_C has a value of 7.78V. Calculate the capacitance C of the capacitor.

II- Discharging the capacitor

The capacitor being charged under a voltage of 9V, we open K_1 and close K_2. The capacitor then discharges.

1- Draw a diagram of the circuit during that phase indicating the direction of the current.

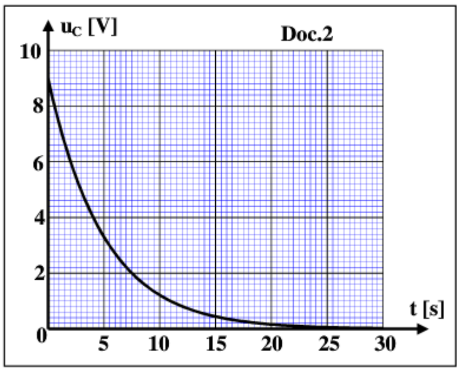
2- Derive the differential equation that describes the variation of the voltages $u_C = u_{AB}$ across the capacitor.

3- Knowing that the solution of this differential equation is of the form $u_C = Ee^{-\frac{t}{\tau_2}}$, deduce the expression of:

3.1- the current i as a function of time. Take the direction of the current as a positive direction.

3.2- the time constant τ_2 as a function of R_2 and C.

4- A convenient apparatus allows us to trace the graph of the variation of u_C as a function of time. (document 2)
Determine from the curve the value of τ_2.
Deduce the value of C.



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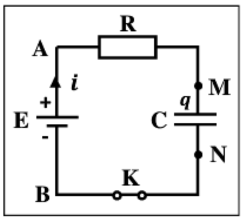
III- What conclusion can be drawn about the two values of C? Comment.

Exercise 2:

The object of this exercise is to distinguish the response of an RC series circuit when we apply across its terminals a constant voltage, from its response when it carries a constant current.

A- Case of a constant voltage

The circuit of the adjacent figure allows us to charge the capacitor of capacitance C = 10μF through a resistor of resistance R = 100kΩ, under a constant voltage E = 9V. Take the instant t = 0 the instant when the switch K is closed.



1- Denote by $u_C = u_{MN}$, the instantaneous value of the voltage across the terminals of the capacitor.

1.1- Show that the differential equation in u_C is of the form: $u_C + RC\frac{du_C}{dt} = E$.

1.2- Knowing that the solution of this equation has the form: $u_C = A\left(1 - e^{-\frac{t}{\tau}}\right)$ determine A and τ.

1.3- Trace the shape of the curve that gives the variation of u_C as a function of time.

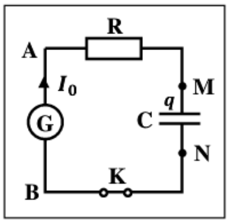
2-

2.1- Determine the expression of the voltage $u_R = u_{AM}$ as a function of time.

2.2- Trace, on the same system of axes, the shape of the curve giving the variation of u_R as a function of time.

3- What is the value of the interval if time t_A at the end of which u_C becomes practically 9V?

B- Case of a constant current



The preceding capacitor being discharged is to be recharged through the same resistor by a generator giving a constant current $I_0 = 0.1$mA.

1-

1.1- Show that the chare$q$ can be written, in SI, in the form $q = 10^{-4} \times t$.

1.2- The voltage $u_R = u_{AM}$ across the resistor remains constant. Determine its value.

1.3- Trace the shape of the graph representing u_R.

2-

2.1- Determine the expression of the voltage $u_C = u_{MN}$ as a function of time.

2.2- Trace the shape of the graph representing u_C.

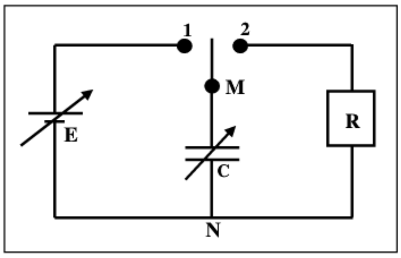
2.3- Determine the time interval t_B needed for the voltage u_C to attain the value 9V.

C- Conclusions

1- Using the preceding graphs, specify the case where the voltage across the capacitor attains, in the steady state, a limiting value.

2- A camera is equipped with a flash that is formed of the preceding RC circuit. We intend to take the largest number of photos in a given time interval. To do so we have to charge the capacitor. Which one of the two preceding charging modes is more convenient? Why?

Exercise 3:



A heart suffering from disordered muscular contractions is treated by applying electric shocks using a convenient apparatus.

In order to study the functioning of this apparatus, we use a source of DC voltage of adjustable value E, a double switch, a resistor of resistance R and a capacitor (initially neutral) of adjustable capacitance C. We connect the circuit represented in the adjacent figure.

A. Theoretical study

1- The switch is turned to position (1).

1.1- Give the name of the physical phenomenon that takes place in the capacitor.

1.2- Specify the values of the current in the circuit and the voltage u_{MN} after few seconds.

2- The switch is now turned to position (2) at an instant taken as $t_0 = 0$.

2.1- Derive, at the instant t, the differential equation giving the variation of the voltage $u_C = u_{MN}$ as a function of time.

2- The discharging starts at the instant $t_0 = 0$. At the instant t_1, the energy delivered to the patient amounts to 360J, the switch is then opened.

2.1- Calculate the energy that remains in the capacitor at the instant t_1.

2.2- Using the results of the above theoretical study; determine:

2.2.1- the value of t_1 .

2.2.2- the current at the end of the electric shock.

Exercise 4:

The object of this exercise is to determine the capacitance of a capacitor and study the effect of certain physical quantities on the duration of its charging.

The circuit of figure (1) is formed of:

-an ideal generator delivering across its terminals an adjustable DC voltage $u_{MN} = u_g = E$;

-a resistor of adjustable resistance R ;

-a capacitor of capacitance C ;

-a switch K.

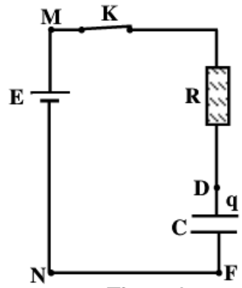


Figure 1

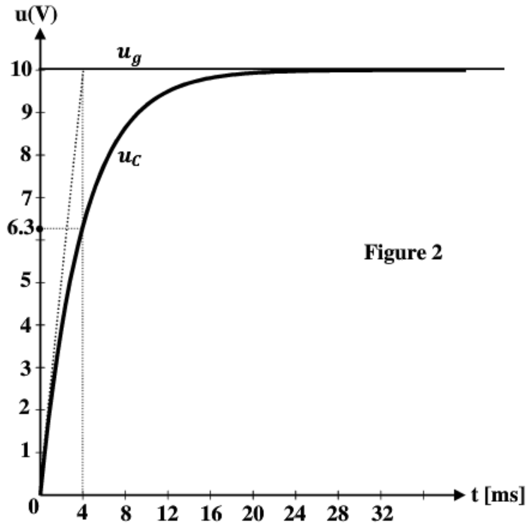


Figure 2

I- The value of E is adjusted at E = 10V and that of R at R = 2kΩ.

The capacitor being initially neutral, we close the switch at the instant $t_0 = 0$.

1-

1.1- Derive the differential equation giving the variation of the voltage $u_{DF} = u_C$ across the capacitor as a function of time.

1.2- Verify that the solution of this differential equation is $u_C = E\left(1 - e^{-\frac{t}{RC}}\right)$.

2- The voltages u_C and u_g are displayed using an oscilloscope (figure 2).

2.1- Redraw the circuit of figure (1) showing the connections of the oscilloscope.

2.2- Give the maximum value of u_C.

3- One method to determine the value of C consists of determining the duration t_1 at the end of which the voltage u_C attains 63% of its maximum value.

3.1- Show that t_1 is very close to the value of RC.

3.2- Using figure (2), determine the value of the capacitance C.

4- Another method allows us to determine C starting from the tangent at O to the curve $u_C = f(t)$ (fig.2).

4.1- Find the expression of $\frac{du_C}{dt}$, at O, in terms of E, R and C.

4.2- Show that the equation of this tangent to the curve is $u = \frac{E}{RC}t$.

4.3- Verify that this tangent intersects the asymptote to the curve at the point of abscissa t_1 = RC.

4.4- Determine then the value of the capacitance C of the capacitor.

II- The value of R is adjusted at R = 1kΩ.

1- Trace , on the same system of axes, the shape of the curve u_C in the two following cases :

Case (1): E =10V, $C = 2\times10^{-6}$ F (curve 1)

Case (2): E = 5V, $C = 2\times10^{-6}$ F (curve 2)

Scale: on the axis of abscissas:1div ↔ 4 ms; on the axis of ordinates:1 div ↔ 1 V.

2- Specify, with justification, which of the two physical quantities E or R affects the duration of charging of the capacitor.

Exercise 5:

An electric component (D), of unknown nature, which may be a resistor of resistance R or a pure coil of inductance L or a capacitor of capacitance C. To determine the nature and the characteristic of (D) we consider the following:

- An ideal generator G of constant electromotive force (e.m.f) E;
- Two resistors of resistances $R_1 = 100\Omega$ and $R_2 = 150\Omega$;
- A double switch K.

We set up the circuit of figure 1.

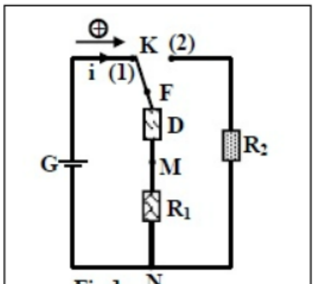


Fig.1

A- First Experiment

At an instant $t_0 = 0$, the switch K is turned to position (1). Figure 2 shows the variation of the voltage u_{FM} across the terminals of (D) as a function of time and the tangent to this curve at $t_0 = 0$.

1- The component (D) is a capacitor. Justify.
2- Indicate the value of the e.m.f E of the generator.
3- Calculate, at $t_0 = 0$, the current carried by the circuit.
4- Derive the differential equation describing the variation of the voltage$u_{FM} = u_C$.
5- The solution of the differential equation has the form: $u_C = u_{FM} = A + Be^{-\frac{t}{\tau}}$.
Determine the expressions of the constants A, B and τin terms of R_1, C and E.
6- Determine, graphically, the value of the time constant.
7- Deduce the value of C.

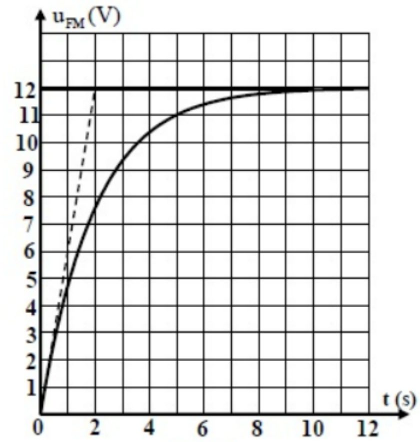


Fig. 2

B- Second Experiment

During the charging of the capacitor and at an instant t_1, we turn the switch K to the position (2) (figure 3).

1- Name the phenomenon that takes place.
2- The resistor R_2 can support a maximum power of $P_{max} = 0.24W$.
   2.1- Calculate the maximum value of the current which can pass through R_2 without damaging it (the thermal power is given by the relation: $p = Ri^2$).
   2.2- Applying the law of addition of voltages, show that the maximum voltage across the terminals of the capacitor is $u_{FM} = 10V$ so that R_2 will not be damaged.
   2.3- At the instant t_1 the current is maximum. Determine, graphically, the maximum duration $t = t_1$ of the charging process of the capacitor so that the resistor R_2 will not be damaged.

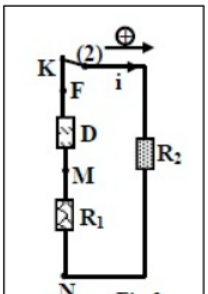


Fig.3

Exercise 6:

We intend to determine the resistance R of a resistor (R). We thus connect up the circuit represented in figure (1) that is formed of an ideal generator of e.m.f E = 5V, the resistor (R), an uncharged capacitor (C) of capacitance C = 33µF and a double switch (K).

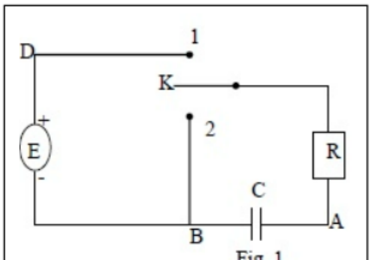


Fig. 1

A- Charging of the capacitor

1- We intend to charge the capacitor. To what position, 1 or 2, must then (K) be moved?

2- The circuit reaches a steady state after a certain time. Give then the value of the voltage u_{AB} across (C) and that of the voltage across (R).

B- Discharging of the capacitor

1- Draw a diagram of the circuit during the discharging of the capacitor and show on it the direction of the current it carries.

2- Derive the differential equation in $u_C = u_{AB}$ during the discharging.

3- The solution of this differential equation has the form: $u_C = Ee^{-\frac{t}{\tau}}$ (u_C in V, t in s) where τ is a constant.

3.1- Determine the expression of τ in terms of R and C.

3.2- Determine the value of u_C at the instant$t_1 = \tau$.

3.3- Give, in terms of τ, the minimum duration needed at the end of which the capacitor is practically totally discharged.

3.4- Derive the expression ofln u_C, the natural logarithm ofu_C, in terms of E, τ and t.

3.5- The diagram of figure 2 represents the variation of ln u_C as a function of time. Referring to the graph of figure 2, determine the value of R.

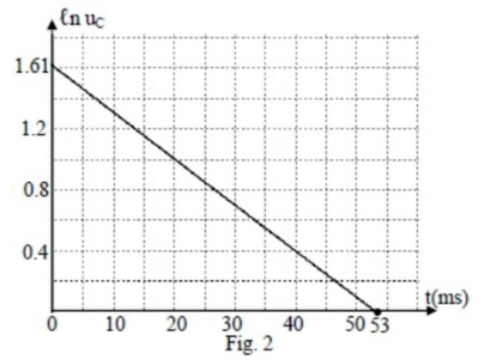


Fig. 2