Grade 12 – GS **Academic year 2022 – 2023**

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MATHEMATICS TEST

Exercise – 1 – (2.5 points)

The table consists of five multiple choices questions. Only one of the choices is correct.

Choose the correct answer with justification:

	Questions	Possible answers (a)	(b)	(c)
1	Consider the complex number z defined by: $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ An argument of z is:	$\arg(z) = \frac{\pi}{3}$	$\arg(z) = \frac{2\pi}{3}$	$\arg(z) = \frac{\pi}{6}$
2	For $n \in \mathbb{N}$, consider: $A_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right)$ The complex number defined by $z = A_2(A_1A_3)^2$ is:	Purely imaginary	Real negative	Real positive
3	The magnitude of an earthquake is given by: $R = \frac{1}{\ln 10}\ln\left(\frac{a}{T}\right) + B$ where T is the period in SI. Which expression is that of T?	$T = \frac{a}{10}e^{R-B}$	$T = \frac{10^{B-R}}{a}$	$T = a \times 10^{B-R}$
4	For $x > y > 0$, consider the below equation: $\ln\left(\frac{x-y}{3}\right) = \frac{1}{2}(\ln x + \ln y)$ Then $x^2 + y^2$ is equal to:	$2(x-y) - 3xy$	$9xy$	$11xy$
5	The points A and B of respective affixes z_A and z_B are defined such that: $Re\left(\frac{z_A - z_O}{z_B - z_O}\right) = 1$ What can we say about the triangle OAB?	OAB is a right triangle angled at O.	OAB is an isosceles triangle with principal vertex O.	OAB is a right isosceles triangle angled at O.

Exercise – 2 – (3 points)

Consider two urns U and V such that:

- Urn U contains 3 tokens labelled from 1 to 3.
- Urn V contains 4 black balls and 6 white balls.

A game consists of drawing a token from the urn U then drawing simultaneously and at random as many balls as it's indicated by the chosen token.

Consider the following events: A_i: "The token drawn from the urn U is labelled i"

B: "All the balls drawn from the urn V are black".

1. (a) Determine $P(B/A_1)$ and prove that $P(B/A_2) = \frac{2}{15}$.

   (b) Deduce that $P(B) = \frac{17}{90}$.

2. Knowing that all the balls drawn from the urn V are black, what is the probability that the chosen token is labelled 3?
3. Let the event C: "Exactly one of the drawn balls from the urn V is black".

   Prove that $P(A_3 \cap C) = \frac{1}{6}$.

4. We add a 4^{th} token labelled 4 to the urn U.

   The tokens are drawn one after one without replacement.

   A match occurs if the token numbered i is the i-th token chosen.

   Consider the event M_i: "A match occurs on the i-th draw" $(i = 1; 2; 3; 4)$

   Show that $P(M_i) = \frac{1}{4}$ for each i.

Exercise – 3 – (3 points)

In the complex plane referred to a direct orthonormal system $(O; \vec{u}; \vec{v})$, consider the points A and B of affixes 1 and -1 respectively. Consider the map f that associates, for every point M distinct than B, of affix z, the point M' of affix z' such that $z' = \frac{z-1}{z+1}$.

1. Verify that if $z = i$, then the complex number z' is purely imaginary.
2. (a) Verify that if $z = x + iy$, then $Im(z') = \frac{2y}{(x+1)^2 + y^2}$.

   (b) Deduce the locus of the point M if z' is real.

3. (a) For all complex numbers $z \neq -1$, prove that $(z' - 1)(z + 1) = -2$.

   (b) Deduce that $|z' - 1| = \frac{2}{|z+1|}$ and $\arg(z' - 1) = (2k + 1)\pi - \arg(z + 1)$.

(c) Prove that if M belongs to the circle (C) of center B and radius 2, then the point M' belongs to the circle (C') of center A and radius 1.

4. Let F be the point of affix $z_F = -2 + i\sqrt{3}$.
Write $z_F + 1$ in the trigonometric form and deduce that the point F belongs to (C).

Exercise – 4 – (7.5 points)

Let f be the function defined over $\mathbb{R}$ by $f(x) = 2\left(\frac{e^{4x}-1}{e^{4x}+1}\right)$ and we denote by (C) its representative curve in an orthonormal system of origin O.

1. **(a)** Calculate $\lim\limits_{x\to-\infty} f(x)$ and prove that $\lim\limits_{x\to+\infty} f(x) = 2$.

   (b) Deduce that (C) admits two horizontal asymptotes (d) and (d').
2. Solve the equation $f(x) = 0$ and verify that $f(x) > 0$ for all $x > 0$.
3. **(a)** Prove that $f'(x) = \frac{16e^{4x}}{(e^{4x}+1)^2}$ and verify that $f'(x) + [f(x)]^2 = 4$.

   (b) Set up the table of variations of f over $\mathbb{R}$.
4. **(a)** Prove that the origin O is a center of symmetry of (C).

   (b) Write an equation of the tangent (T) to (C) at the point O.
5. **(a)** Prove that $f''(x) = -64e^{4x}\frac{e^{4x}-1}{(e^{4x}+1)^3}$.

   (b) Deduce that the origin O is the inflection point of (C) and verify that $f''(x) < 0$ for all $x > 0$.
6. Draw (d), (d'), (T) and (C).
7. Prove that the equation $f(x) = 1$ admits a unique solution $\alpha = \frac{1}{4}\ln 3$.
8. Consider the function g defined by $g(x) = 1 + \ln[f(x) - 1]$ and denote by (G) is its representative curve in the same orthonormal system of origin O.

   Answer by **True** or **False**:

   (a) The function g is defined over $\left]\frac{1}{4}\ln 3\,;\ +\infty\right[$.

   (b) The derivative of the function g is given by $g'(x) = \frac{(2-f(x))(2+f(x))}{f(x)-1}$.

   (c) The table of variations of the function $f - g$ is given by:

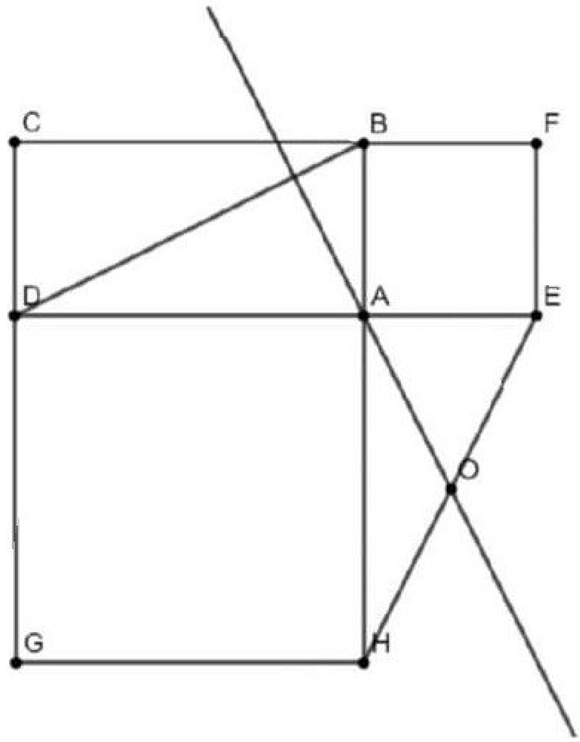
x	$\frac{1}{4}\ln 3$		$+\infty$
$(f-g)'(x)$		$-$	
$(f-g)(x)$	$+\infty$	$\searrow$	1

   The curve (C) of f is above the curve of (G), that of g, for all $x \in \left]\frac{1}{4}\ln 3\,;\ +\infty\right[$.

Exercise – 5 – (4 points)

In the figure below, consider:

- ABCD is a direct rectangle,
- AEFB and ADGH are two direct squares,
- O is the midpoint of [EH] and (AO) is perpendicular to (BD).



Part A

S is the similitude that maps A onto B and D onto A.

Let AB = 1 and AD = λ ($\lambda > 0$).

1. Determine, in terms of λ, the ratio and an angle of S.
2. Determine the images (d) and (d') of the lines (BD) and (AO) respectively by S.
3. Prove that the point K, intersection of (BD) and (AO), is the center of S.

Part B

The plane is referred to an orthonormal system $(A\,;\overrightarrow{AE};\overrightarrow{AB})$ such that K$\left(\frac{-\lambda}{\lambda^2+1};\frac{\lambda^2}{\lambda^2+1}\right)$.

1. Verify that $z' = \frac{i}{\lambda}z + i$ is the complex form of S.
2. Let $h = SoS$.
   (a) Determine, in terms of λ, the characteristics of the transformation h.
   (b) Determine $h(D)$ and the image of the line (AD) by h.
   (c) Determine the value of λ so that $\overrightarrow{KD} = 4\overrightarrow{KB}$.