

Physics Pro

Secondary Education

LS

2022-2023



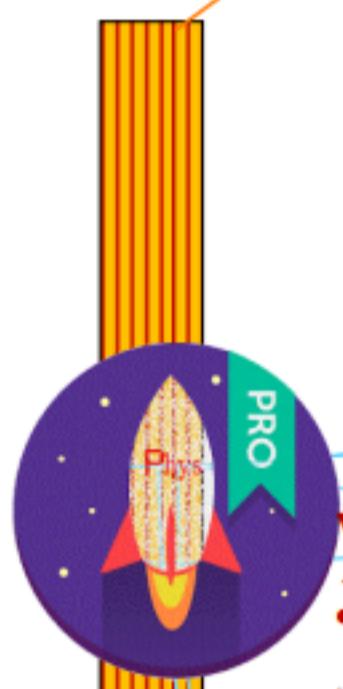
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Third Year+
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Mechanical energy

Chap.1



Kinetic form KE

particle

$$KE = \frac{1}{2} m v^2$$

Application 1 fig.1

1

2

General system

$$KE = \sum \frac{1}{2} m_i v_i^2$$

Gravitational Potential Energy PEg

Particle

$$PE_g = mgz_m$$

Application 2 fig.2

Extended object

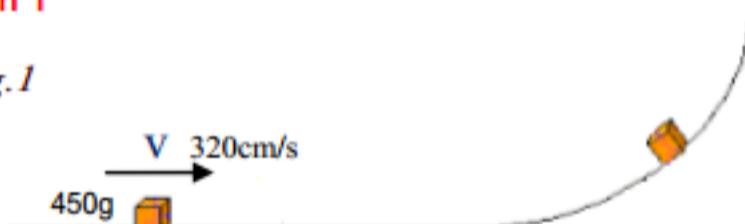
$$PE_g = mgz_G$$

4

center of mass of object

Application 1

fig.1



Application 2

reference $PE_g = 0$

A

fig.1

fig.2

B

$h_1 = 20\text{cm}$
(S)
particle

C

$h_1 = 0,44\text{m}$

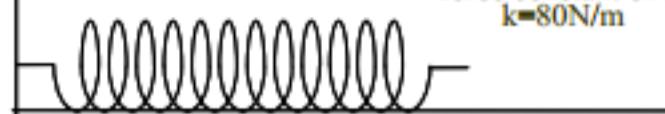
Elastic Potential Energy PEE

Application 3

Natural length of the spring is 7cm

force constant of the spring $k = 80\text{N/m}$

fig.3



a

Length of the spring is now 9cm

b

Length of the spring is now 5cm

c

$$PE_e = \frac{1}{2} k \Delta l^2$$

Application 3 fig.3

Spring

5

Mechanical energy of a system

$$ME = E + PE_g + PE_e$$

at the same instant t

isolated system ?

What is meant by «isolated system»?
No exchange of energy between
this system and its surrounding.

9

See exercise 2

conservation
of ME

For an isolated system

In case where (without friction)

$$E_1 = ME_2$$

10

See exercise 3

Non
conservation of ME

For an isolated system

In case where (friction exists)

$$ME_2 - ME_1 = - \Delta U$$

11

with ΔU (is the variation of
internal energy of the system).



We can write :

$$ME_2 - ME_1 = W_{\vec{f}} \quad \text{where : } \vec{f} \text{ (friction forces).}$$

$W_{\vec{f}} = - f.d$; where d : displacement.
 f (constant) and d (rectilinear).

Exercises

Energy

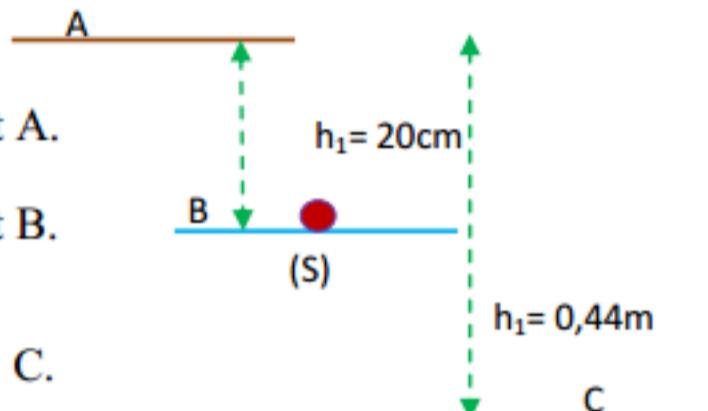
Exercises



Exercise 1 :

A ball (S) has a mass $m=300\text{g}$, is placed at level B as indicated in the figure below. Determine the gravitational potential energy E_{pg} of the system (S ; earth) in each of the following cases :

1. The reference level for gravitational potential energy is at A.
Ans. - 0,4J.
2. The reference level for gravitational potential energy is at B.
Ans. 0J.
3. The reference level for gravitational potential energy is at C.
(Take $g = 10\text{m/s}^2$). Ans. 0,48 J.



Exercise 2 :

Conservation of E_m

A small spherical particle, considered as a particle of mass $m=300\text{g}$,



Initially at rest at point A.

When we release this particle, it moves down the track without initial speed, continues its motion without friction to reach the point C with a velocity \vec{V}_C .

Apply the principle of conservation of mechanical energy to determine the speed V_C at point C.

Answer : $V_C=3,74\text{m/s}$.

Exercise 3 :

Non-conservation of E_m

Take the same given of the above application, but in this example we suppose that the friction is negligible on the part BC, while it has a constant value on AB such that the average magnitude of \vec{f} is $f = 0.4\text{N}$.



- 1) Determine the new value of V'_C and compare its value to the previous one V_C (without friction).

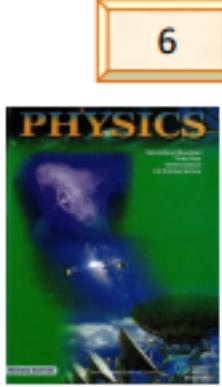
Answer : $V'_C=3,33\text{m/s}$

- 2) Where is the lost mechanical energy ? Deduce the variation of the internal energy.

Exercise 3:

A parachutist (S), of total mass 100 kg, jumps from an airplane flying at an altitude of 1000 m with a speed of 270 km/h with respect to the ground.

(S) reaches the ground with a speed of 18 km/h.



- a) Is the mechanical energy of the system ((S), Earth) conserved during the jump? Interpret.

- b) Calculate the thermal energy produced.

Exercise 4:

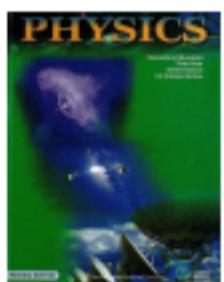
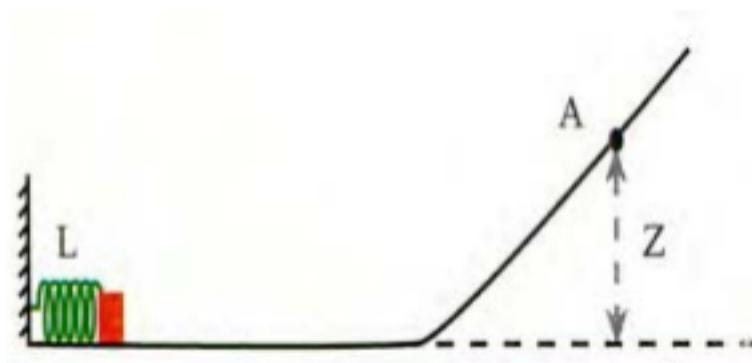
To launch a solid of mass m on an inclined slide, we use the apparatus shown in figure 1.

The natural length of the spring is L_0 . Before launching, the spring is compressed to a length L .

After launching, the center of mass of the solid reaches the point A of altitude Z with a velocity v . Neglect friction.

- a) Determine the relation among L_0 , L , m , v , Z , g , and the constant k of the spring.

- b) Calculate Z if the maximum altitude reached is 10 cm. Take $m=600\text{g}$, $L = 25\text{ cm}$ and $L_0=20\text{cm}$.

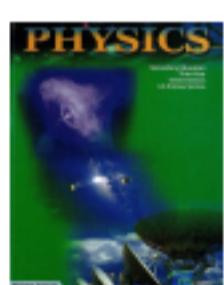
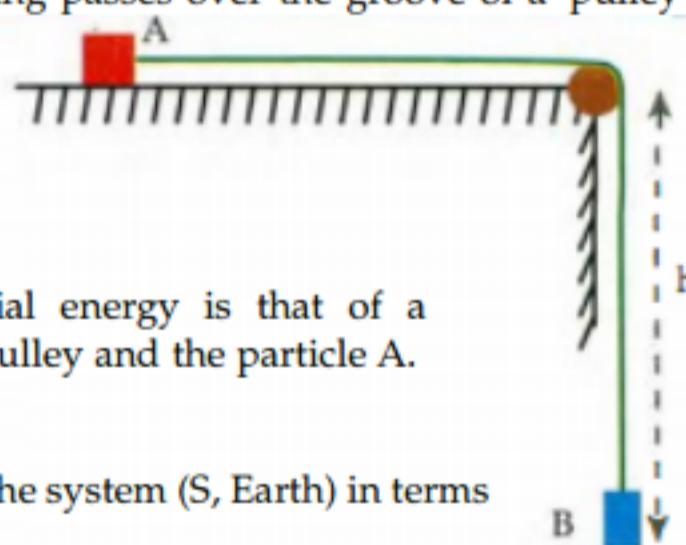


Exercise 5:

Two particles A and B, of respective masses m_A and m_B , are connected together by means of an inextensible string of negligible mass. The string passes over the groove of a pulley of negligible mass as shown in the figure

At instant t_0 , the system S (pulley, mass, string) is left without initial velocity, with particle B being at a distance h from the axis of the pulley.

The reference level of gravitational potential energy is that of a horizontal plane containing the axis of the pulley and the particle A.



- a) Calculate, at t_0 , the mechanical energy of the system (S, Earth) in terms of m_A , m_B , g , h .

- b) Calculate the mechanical energy of the system (S, Earth) after covering a distance x .

- c) Calculate, applying the principle of the conservation of mechanical energy, the velocity of A or B in terms of x , m_A , m_B and g .

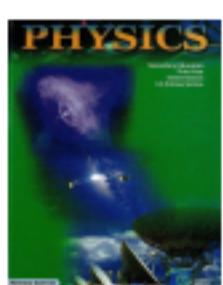
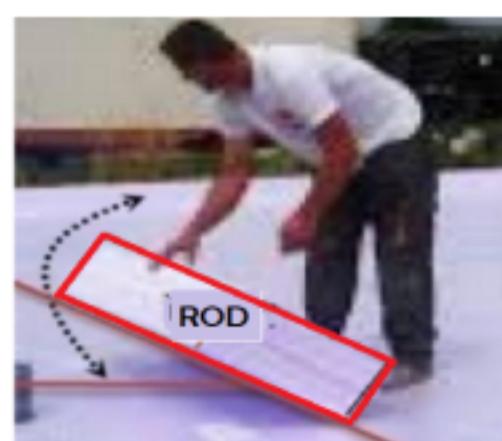
Deduce the acceleration of the motion.

Exercise 5:

A homogeneous rod, of mass $M=20\text{kg}$, of length $L = 1\text{m}$, and of negligible cross-section, rests initially on the horizontal ground chosen as a reference level for the gravitational potential energy.

The rod is rotated about one end until it takes a vertical position.

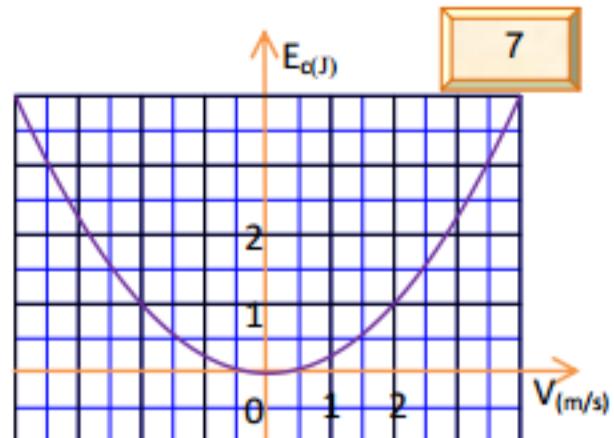
Calculate the variation of the mechanical energy of the system (Rod, Earth).



Exercise 5-b :

A body moves in translational motion with a speed V . The graph of the kinetic energy as function of V is given in the figure to the right. Deduce the mass of this body.

ANS. 0,5Kg.

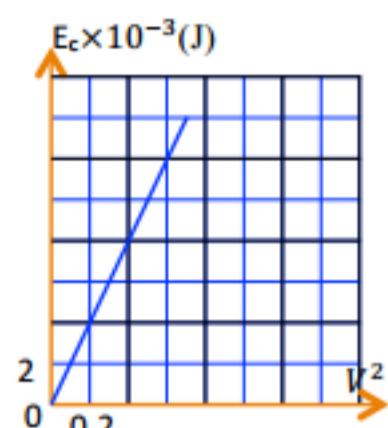
**Exercise 5-c :**

A body of mass m moves in translational motion with a speed V .

The graph of the kinetic energy as function of V^2 is shown in the figure to the right.

- 1) Justify the shape of the graph obtained.
- 2) Deduce the mass of this object.

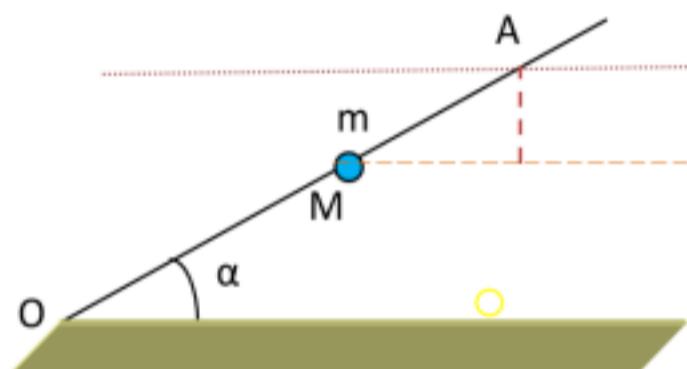
ANS. 40g.

**Exercise 5-d :**

The horizontal level passing through A is the reference level for gravitational potential energy E_{pg} .

See figure.

Calculate the gravitational potential energy E_{pg} of the system (Particle ; Earth).



Given: $m_{\text{particle}} = 50\text{g}$; $OA = 55\text{cm}$; $OM = 30\text{cm}$ and $\alpha = 40^\circ$.

Ans. $E_{pg} = -0,08\text{J}$.

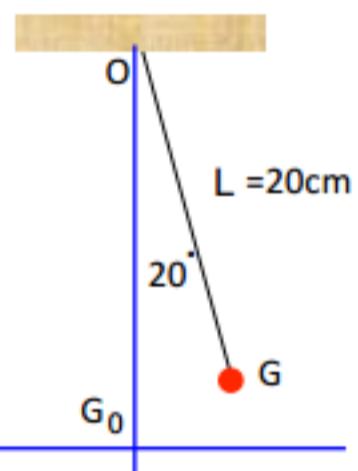
Exercise 5-e :

A ball of mass 120 g is attached to a string as shown in the figure.

Take the horizontal level passing through G_0 as reference level for gravitational potential energy E_{pg} .

Calculate the gravitational potential energy E_{pg} of the system (Particle ; Earth), when the pendulum is at the position G indicated in the figure.

Ans. $14,47\text{mJ}$

**Exercise 6****Determination
of the value of a force of friction**

PHYSICS
LS 2005-1

A solid (S) of mass $m = 200\text{ g}$ is free to move on a track AOB lying in a vertical plane. This rail is formed of two parts: the first one AO is straight and horizontal and the other OB is straight and inclined by an angle α with respect to the horizontal ($\sin \alpha = 0.1$). Along the part AO, (S) moves without friction, and along the part OB, (S) is acted upon by a force of friction \vec{f} that is assumed constant and parallel to the path.

The object of this exercise is to determine the magnitude f of the force \vec{f} of friction.

A- Launching the solid

In order to launch this solid on the part AO, we use a spring of constant $k = 320\text{ N/m}$ and of free length ℓ_0 ; one end of the spring is fixed at A to a support. We compress the spring by x_0 ; we place the solid next to the free end of the spring and then we release them. When the spring attains its free length ℓ_0 , the solid leaves the spring with the speed $V_0 = 8\text{ m/s}$; it thus slides along the horizontal part and then rises up at O the inclined part OB.



- 1) Determine the value of x_0 .
2) The solid reaches O with the speed $V_0 = 8 \text{ m/s}$. Justify.

B- Motion of the solid along the inclined part OB

(S) moves, at time $t_0 = 0$, up the inclined part OB with the speed V_0 at the instant $t_0 = 0$. A convenient apparatus is used to trace, as a function of time, the curves representing the variations of the kinetic energy K.E of the solid and the gravitational potential energy P.E_g of the system (solid - Earth).

These curves are represented in the adjacent figure between the instants $t_0 = 0$ and $t_4 = 4 \text{ s}$ according to the scale:

- 1 division on the time axis corresponds to 1 s
- 1 division on the energy axis corresponds to 1 J.

The horizontal line through point O is taken as a gravitational potential energy reference. Take $g = 10 \text{ m/s}^2$.

1) The curve (I) represents the variation of the kinetic energy K.E of (S) as a function of time. Why?

2) Using the data:

- specify the form of the energy of the system at the instant $t_4 = 4 \text{ s}$. Justify your answer.
- determine the maximum distance covered by the solid along the part OB.
- complete the table with the values of the mechanical energy M.E of the system at each instant t.

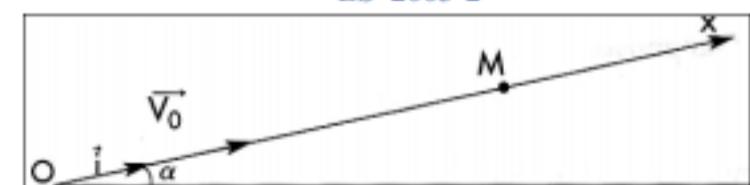
t (s)	0	1	2	3	4
M.E.(J)		5			

- justify the existence of a force of friction \vec{f} .
- calculate the variation in the mechanical energy of the system between the instants $t_0 = 0$ and $t_4 = 4 \text{ s}$.
- determine the f.

Exercise

Graphical study of energy exchange

PHYSICS
LS 2003-2



Consider an inclined plane that makes an angle α with the horizontal ($\sin \alpha = 0.2$) and a marble (B) of mass $m = 100 \text{ g}$, taken as a particle. We intend to study the energy exchange between the system (marble, Earth) and the surroundings.

To do that, the marble (B) is given, at the instant $t_0 = 0$, the velocity $\vec{V}_0 = V_0 \vec{i}$ along the line of greatest slope OX. Given $V_0 = 10 \text{ m.s}^{-1}$ and $g = 10 \text{ m/s}^2$.

The horizontal line through point O is taken as the gravitational potential energy reference.

A- The forces of gravity and of friction are supposed negligible.

- Determine the value of the mechanical energy M.E of the system (marble, Earth).
- At the instant t, the marble passes through a point M of abscissa OM = x. Determine, as a function of x, the expression of the gravitational potential energy P.E_g of the system (marble, Earth) when the marble passes through M.

3-a) Trace, on the same system of axes, the curves representing the variations of the energies M.E and P.E_g as a function of x.

- Scale:** - on the axis of abscissas: 1 cm represents 1 m;
- on the axis of energy: 1 cm represents 0.2 J.

- b) Determine, using the graph, the speed of the marble for x = 3 m.
c) Determine, using the graph, the value of x_m of x for which the speed of (B) is zero.

B-1. In reality, the speed of the marble becomes zero at a point of abscissa x = 3 m. The forces of friction are no longer negligible. Calculate then the work done by the forces of friction between x = 0 and x = 3 m.

2. The system (marble, Earth) thus exchanges energy with its surroundings. In what form and by how much?

Exercise 8

Acceleration of a particle

PHYSICS

LS 2011-2

The object of this exercise is to determine the expression of the magnitude of the acceleration of a particle using two methods. The apparatus used is formed of two particles (S₁) and (S₂) of respective masses m₁ and m₂, fixed at the extremities of an inextensible string passing over the groove of a pulley. (S₁), (S₂), the string and the pulley form a mechanical system (S).

The string and the pulley have negligible mass.

(S₁) may move on the line of greatest slope AB of an inclined plane that makes an angle α with the horizontal AC and (S₂) hangs vertically.

At rest, (S₁) is found at point O at a height h₁ above AC and (S₂) is found at O' at a height h₂ (adjacent figure).

At the instant t₀ = 0, we release the system (S) from rest. (S₁) ascends on AB and (S₂) descends vertically. At an instant t, the position of (S₁) is defined by its abscissa x = OS₁ on an axis x'Ox confounded with AB, directed from A to B.

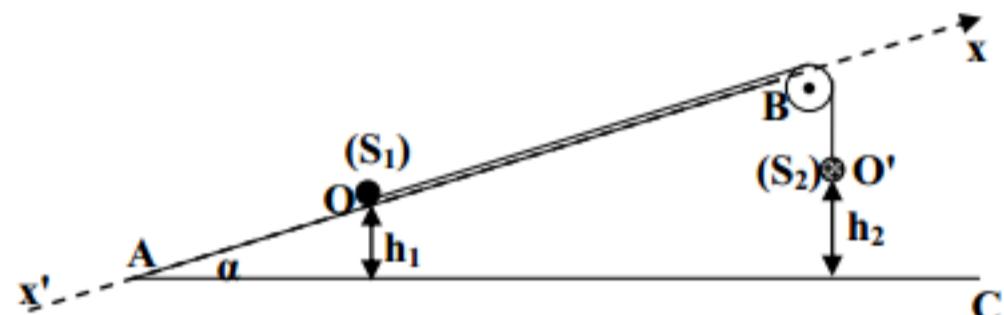
Take the horizontal plane containing AC as a gravitational potential energy reference.
Neglect all the forces of friction.

1) Energetic method

- Write down, at the instant t₀ = 0, the expression of the mechanical energy of the system [(S), Earth] in terms of m₁, m₂, h₁, h₂ and g.
- At the instant t, the abscissa of (S₁) is x and the algebraic value of its velocity is v. Determine, at that instant t, the expression of the mechanical energy of the system [(S), Earth] in terms of m₁, m₂, h₁, h₂, x, v, α and g.
- Applying the principle of conservation of mechanical energy, show that :

$$v^2 = \frac{2(m_2 - m_1 \sin \alpha)gx}{(m_1 + m_2)}.$$

- Deduce the expression of the value a of the acceleration of (S₁).



2) Dynamical method

- Redraw a diagram of the figure and show, on it, the external forces acting on (S₁) and on (S₂). (The tension in the string acting on (S₁) is denoted by T₁ of magnitude T₁ and that acting on (S₂) is denoted by T₂ of magnitude T₂).
- Applying the theorem of the center of mass $\sum \vec{F}_{\text{ext}} = m \vec{a}$, on each particle, determine the expressions of T₁ and T₂ in terms of m₁, m₂, g, α and a.
- Knowing that T₁ = T₂, deduce the expression of a.

Linear momentum

Chap.2



In case of a **particle** or a **solid** in translation motion

$$\vec{P} = m\vec{V}$$

1

In case of a system of **many particles**

$$\overrightarrow{P}_{(S)} = \sum \vec{P}_i$$

$$= \sum m_i \vec{v}_i$$

2

Consider G, the center of mass of the system :

$$\overrightarrow{P}_{(S)} = M\vec{V}_G$$

3

$$\sum \overrightarrow{F}_{ext} = \frac{d\vec{p}}{dt}$$

4

Remark :in case where $\sum \overrightarrow{F}_{ext} = \text{const}$ we can take $\frac{d\vec{p}}{dt} = \frac{\Delta \vec{p}}{\Delta t}$

Observations we face :



5

Collision with coupling



6

Elastic collision:
 $KE_{bef\ coll} = KE_{aft\ coll}$

Inelastic collision:
Decreasing of KE



7

Explosion of an object

Conservation of \vec{P}

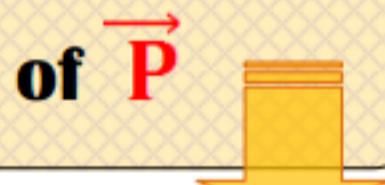


$$\sum \overrightarrow{F}_{ext} = \vec{0}$$

8

$$\overrightarrow{P}_{before} = \overrightarrow{P}_{after}$$

Non-conservation of \vec{P}



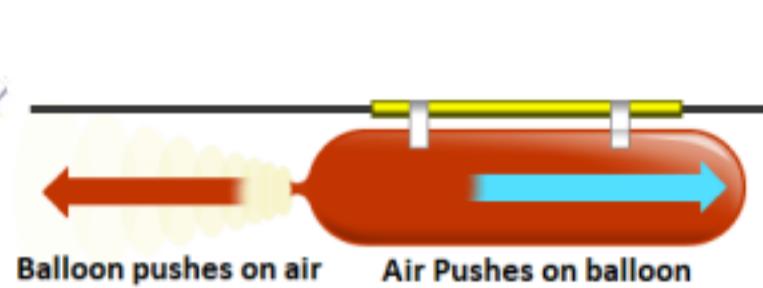
$$\sum \overrightarrow{F}_{ext} = \text{const} \neq \vec{0}$$

9

$$\overrightarrow{P}_{after} - \overrightarrow{P}_{before} = \sum \overrightarrow{F}_{ext} \times \Delta t$$

Application of the theorem $\sum \overrightarrow{F_{ext}} = \frac{d\vec{p}}{dt}$; where $\sum \overrightarrow{F_{ext}} = \vec{0}$

Conservation of the linear momentum



1

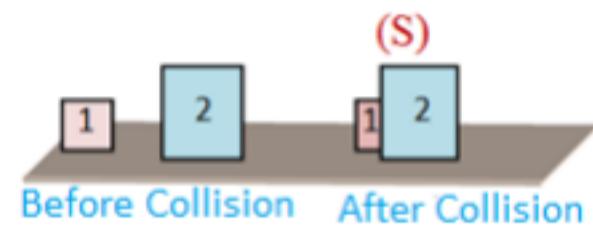
1-n-elastic Collision: theoretical Study

In the adjacent figure, (1) and (2) bind together and form one object after the collision.

System (object 1; object 2) is isolated.

$$\overrightarrow{P_{before}} = m_1 \overrightarrow{v_1} + m_2 \overrightarrow{v_2} \text{ and } \overrightarrow{P_{after}} = (m_1 + m_2) \vec{V} ;$$

Then the conservation of linear momentum can be written as:



$$\overrightarrow{P_{before}} = \overrightarrow{P_{after}} ; \quad \text{finally ; } \vec{V} = \frac{m_1 \overrightarrow{v_1} + m_2 \overrightarrow{v_2}}{(m_1 + m_2)}$$

2

2-elastic Collision: theoretical study

- An object B₁ hits elastically another stationary object B₂, the last object is set in motion.
- If the two objects are on the same line when they enter into collision, (front collision) the first object may stop.



Verification

Consider the isolated system (object 1; object 2) during the collision (since $\sum \overrightarrow{F_{ext}} = \vec{0}$), then linear momentum is **conserved**.

$$\overrightarrow{P_{before}} = m_1 \overrightarrow{v_1} + \vec{0} \text{ and } \overrightarrow{P_{after}} = m_1 \overrightarrow{v'_1} + m_2 \overrightarrow{v'_2} \Rightarrow m_1 \overrightarrow{v_1} = m_1 \overrightarrow{v'_1} + m_2 \overrightarrow{v'_2} \quad (1)$$

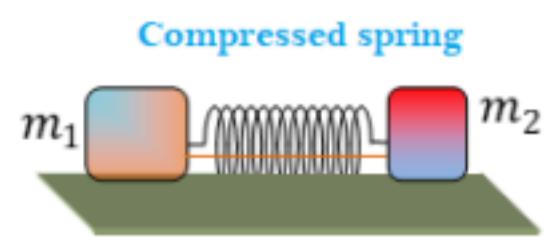
$$\text{Elastic Collision} \Rightarrow KE_{before} = KE_{after} \Rightarrow \frac{1}{2} m_1 v_1^2 + 0 = \frac{1}{2} m_1 v'_1^2 + \frac{1}{2} m_2 v'_2^2 \quad (2)$$

From the above equation we can deduce that : speed of object (1) after the collision is:

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \text{ and that of object (2) after the collision is } v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

3

Exploding system: Theoretical study



The two objects (m_1 ; m_2) are bound together around a compressed spring taut by a string.

The system (spring, m_1 , m_2) being at rest; we burn the string at an instant t .

- $\overrightarrow{P_{before}} = \vec{0}$ and $\overrightarrow{P_{after}} = m_1 \vec{v}_1 + m_2 \vec{v}_2$.
- The system is isolated since $\sum \overrightarrow{F_{ext}}$ is null during the explosion.
- Therefore, there is a conservation of the linear momentum, written as:
- $\overrightarrow{P_{before}} = \overrightarrow{P_{after}}$; finally; $\vec{v}_2 = -\frac{m_1}{m_2} \vec{v}_1$

2.4- Application of the theorem $\sum \overrightarrow{F_{ext}} = \frac{d\overrightarrow{p}}{dt}$ where $\sum \overrightarrow{F_{ext}} \neq \vec{0}$

Non-conservation of the linear momentum

In the case where the given system is not isolated, $\sum \overrightarrow{F_{ext}} \neq \vec{0}$.

We will focus only on the special case where $\sum \overrightarrow{F_{ext}} = \text{constant}$.

In this specific case:

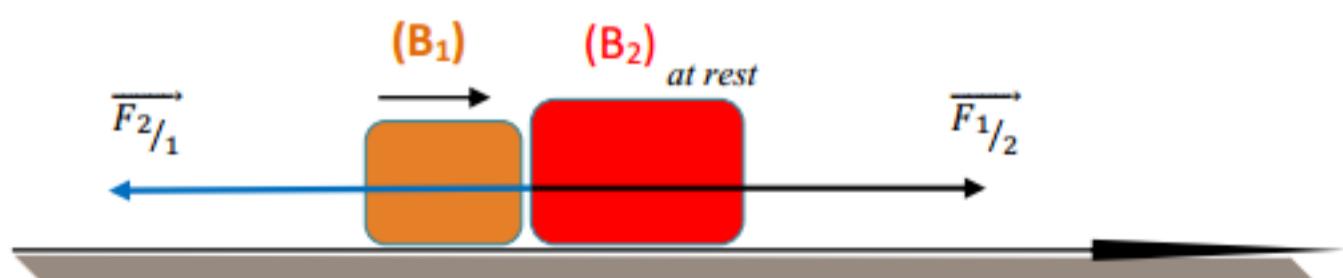
$$\overrightarrow{P_{after}} - \overrightarrow{P_{before}} = \sum \overrightarrow{F_{ext}} \times \Delta t . \text{ (see application in the question 3)}$$

Question 3: Determine the reaction forces $\overrightarrow{F_{1/2}}$ exerted by the object (B_1) on the object (B_2). And the opposite force $\overrightarrow{F_{2/1}}$ exerted by (B_2) on (B_1) during the elastic collision between the two objects knowing that this collision lasts for 0.3 sec.

Given: the speeds just before collision: $v_1=2\text{m/s}$; $v_2=0$ and the speeds just after collision are $v'_1=-\frac{2}{3}\text{m/s}$ et $v'_2=\frac{4}{3}\text{m/s}$.

Fig

During the
collision



$$m_1 = 100\text{g} \text{ and } m_2 = 200\text{g}$$

Answer:

$$\vec{v}'_2 = +\frac{8}{9}\vec{t} \text{ et } \overrightarrow{F_{2/1}} = -\frac{8}{9}\vec{t} . \text{ Note that:}$$

$$\overrightarrow{F_{1/2}} = -\overrightarrow{F_{2/1}} \quad (\text{verification of Newton's 3rd law}) .$$

Exercises

Linear momentum

Exercises



Exercise 1:

Skaters and collision.

A skater of mass 60kg is at rest. Another skater of mass 40kg rushes into his hand with a speed 12 km/h . After holding his hand, the two skaters thus form one body (S) moving with a velocity \vec{V} .

1) Calculate V. *Answe. $4,8\text{km/h}$*

2) Calculate the kinetic energy of (S) after collision. *Answe. Before $222,22\text{J}$; after 88.88J*

3) Specify the nature of this collision.



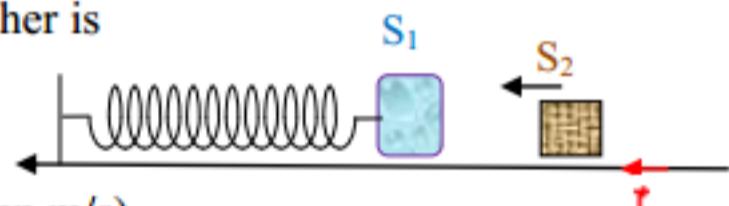
Exercise 2 :

Compression of spring

Given a spring of negligible mass and stiffness $k=10 \text{ N/m}$. The spring rests horizontally with one of its extremities is fixed to a vertical support while the other is attached to a solid (S_1) of mass $M_1 = 300 \text{ g}$,

The solid (S_1) being at rest, is hit by another solid

(S_2) of mass $M_2 = 100 \text{ g}$, moving with a velocity $\vec{V}_2 = 1,0\vec{i} \text{ (en m/s)}$.



After collision (S_2) sticks to (S_1) thus forming one body. We neglect frictional forces.

1) Calculate the velocity of the system just after collision.

2) Determine the value of the maximum compression of the spring.

Exercise 3 : Collision and conservation of mechanical energy

A solid (S_1) of mass $m_1 = 50 \text{ g}$, is launched without initial speed from point A. The solid slides on an inclined plane making an angle $\alpha = 30^\circ$ with the horizontal (see figure).

After covering distance $AB=1\text{m}$, (S_1) reaches the horizontal plane where it continues sliding before striking a solid (S_2) of mass $m_2 = 200 \text{ g}$, initially at rest

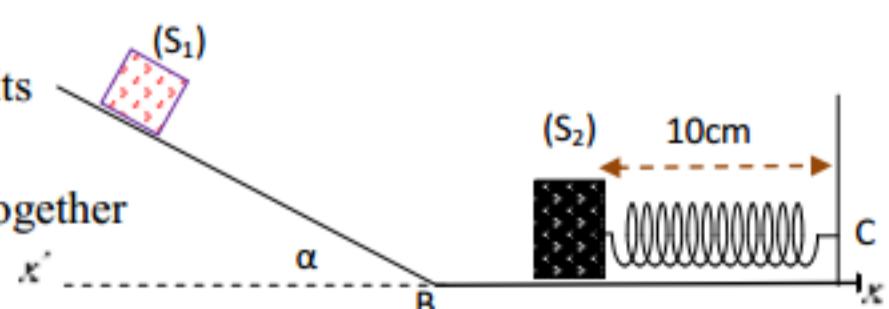
We suppose frictional forces are negligible.

1) Determine the speed v_1 of (S_1) just before it hits (S_2).

2) At the moment of impact (S_1) and (S_2) stick together to form a single body (S).

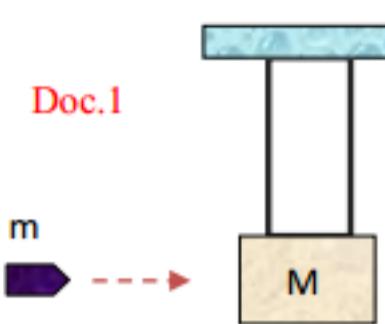
Calculate the speed of (S) just after impact.

3) Knowing that the length of the spring becomes 7 cm after maximum compression, calculate the stiffness of the spring.



Exercise 4-a :**Ballistic Pendulum**

A projectile of mass m moving at a speed $V_1 = 100 \text{ m/s}$, hits a bag filled with sand of mass $M = 90 \text{ kg}$. The bag is suspended vertically from two wires of negligible masses. **Doc.1**
The projectile thus embedded in the sand and the whole system ($m+M$) goes up to a maximum height h . (see doc.2).



1) Calculate the speed of the system just after collision.

2) Neglecting all frictional forces, determine h .

3) This setup can be used to determine the speed of a projectile.

Explain how.

Exercise 4-b : Formula $\overline{P}_{\text{before}} = \overline{P}_{\text{after}}$ in case of recoil of a gun

A gun of mass $m_1 = 1,5 \text{ kg}$ initially at rest is placed horizontally.

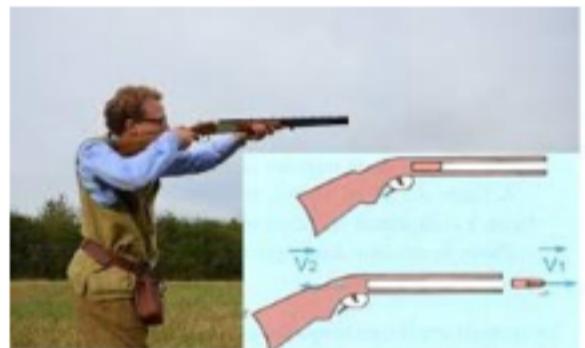
The gun launches a bullet of mass $m_2 = 15 \text{ g}$ with a velocity \vec{V}_2 of magnitude $V_2 = 750 \text{ m/s}$.

Let \vec{V}_1 be the velocity of the gun just after firing.

1. Show that \vec{V}_1 cannot be null and its sense is opposite to \vec{V}_2

2. Determine \vec{V}_1 . *Ans. 7,5 \text{ m/s}*

3. Explain why the recoil will be less sensitive if the gun is well supported.

**Exercise 4-c :****Explosion**

An object of mass m , initially at rest on a horizontal axis

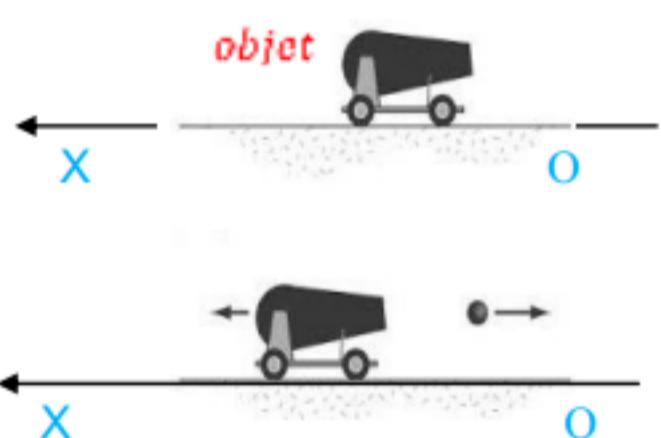
$\overrightarrow{x'ox}$, explodes into two pieces.

Just after explosion one piece of mass $m/3$ moves on

$\overrightarrow{x'ox}$, with a speed 5 m/s .

Determine the velocity vector of the second piece

Just after explosion.

**Exercise 4-d :****Principle of interaction**

Determine $\overrightarrow{F_{1/2}}$ and $\overrightarrow{F_{2/1}}$ the forces of interaction exerted by (B_1) on (B_2) and by (B_2) on (B_1) during the elastic collision

Of the two blocks knowing that it lasts 0.3 s.

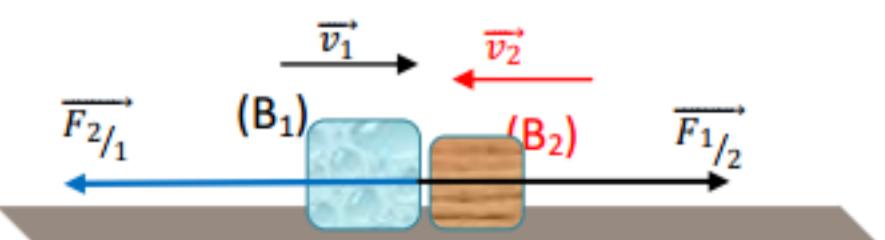
Given: speed of the two blocks before collision: $v_1 = 2 \text{ m/s}$; $v_2 = 0$ and their speed after collision

$$v'_1 = -\frac{2}{3} \text{ m/s} \text{ and } v'_2 = \frac{4}{3} \text{ m/s} .$$

Figure

During collision

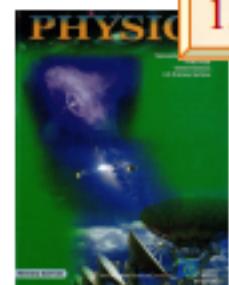
$$\text{Answer : } \overrightarrow{F_{1/2}} = +\frac{8}{9} \vec{t} \text{ et } \overrightarrow{F_{2/1}} = -\frac{8}{9} \vec{t} .$$



$$m_1 = 100 \text{ g} \text{ et } m_2 = 200 \text{ g}$$

Note : $\overrightarrow{F_{1/2}} = -\overrightarrow{F_{2/1}}$ (verification of principle of interaction (Newton's 3rd law))

Application in the case where $\sum \vec{F}_{ext} = \vec{const} \neq \vec{0}$



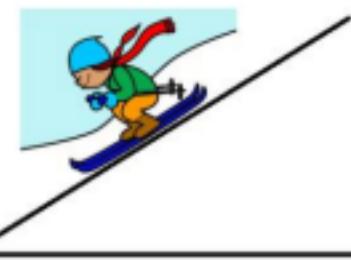
conservation of linear momentum

Exercise 4-e :

Theorem of the center of inertia

A skier of mass 80 kg descends along a line of greatest slope of an inclined plane making an angle α with the horizontal such that $\sin\alpha = 0.1$.

He started with a speed 15 m/s at the top of the track, and his speed becomes 18 m/s after covering a distance 100 m.



- Determine $\Delta \vec{P}$, the variation of the linear momentum of the skier.

Answer 240 kg.m/s

- 2.1 Show that the skier is subjected to a force of friction \vec{f} supposed constant.

2.2 Calculate the value of \vec{f} . **Answer** 40.40 N **Answer** ME not conserved

- Determine $\sum \vec{F}$ the sum of external forces acting on the skier.

Answer

- The duration of the distance covered by the skier is $\Delta t = 6.06$ s, verify that: $\sum \vec{F} \times \Delta t = \Delta \vec{P}$.

Exercise 5

Verification of Newton's second law

PHYSICS

IS - IS - 2006-1

A puck (S) of mass $M = 100$ g and of center of mass G, may slide along an inclined track that makes an angle α with the horizontal so that $\sin\alpha = 0.40$. Thus G moves along an axis $x'x$ parallel to the track as shown in figure (1). Take $g = 10$ m/s².

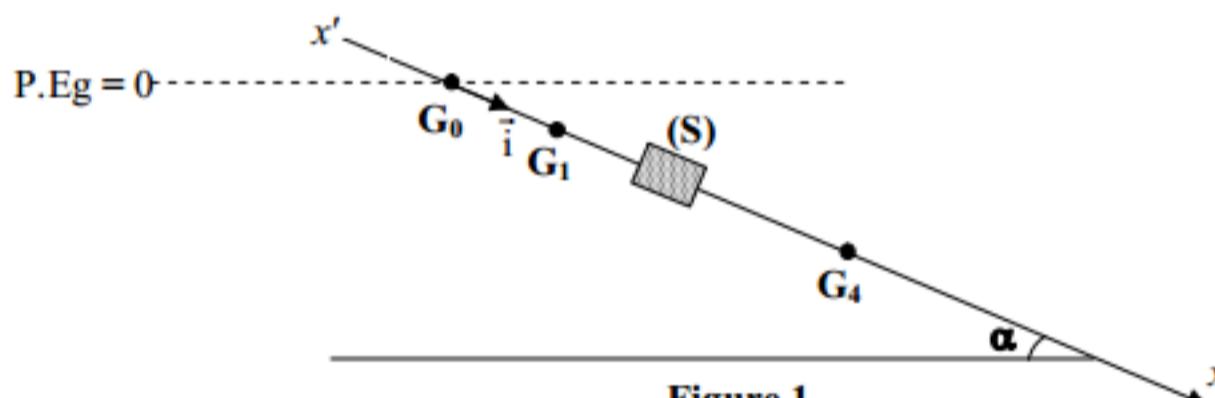


Figure 1

We release (S) without initial velocity at the instant $t_0 = 0$ and at the end of each interval of time $\tau = 50$ ms, some positions $G_0, G_1, G_2, \dots, G_5$ of G are recorded at the instants $t_0 = 0, t_1, t_2, \dots, t_5$ respectively.

The values of the abscissa x of G ($x = \overrightarrow{G_0 G}$) are given in the table below.

t	0	τ	2τ	3τ	4τ	5τ
x (cm)	0	$G_0 G_1 = 0.50$	$G_0 G_2 = 2.00$	$G_0 G_3 = 4.50$	$G_0 G_4 = 8.00$	$G_0 G_5 = 12.50$

1) Verify that the speed of the puck at the instants $t_2 = 2\tau$ and $t_4 = 4\tau$ are $V_2 = 0.40$ m/s and $V_4 = 0.80$ m/s respectively.

2) a) Calculate the mechanical energy of the system (puck-Earth) at the instants t_0, t_2 and t_4 knowing that the horizontal plane through G_0 is taken as a gravitational potential energy reference.

b) Why can we suppose that the puck moves without friction along the rail?

3) Determine the variation in the linear momentum $\Delta \vec{P} = \vec{P}_4 - \vec{P}_2$ of (S) during $\Delta t = t_4 - t_2$.

4) a) Name the forces acting on (S) during its motion.

b) Show that the resultant $\Sigma \vec{F}$ of these forces may be written as $\Sigma \vec{F} = (Mg \sin\alpha) \vec{i}$.

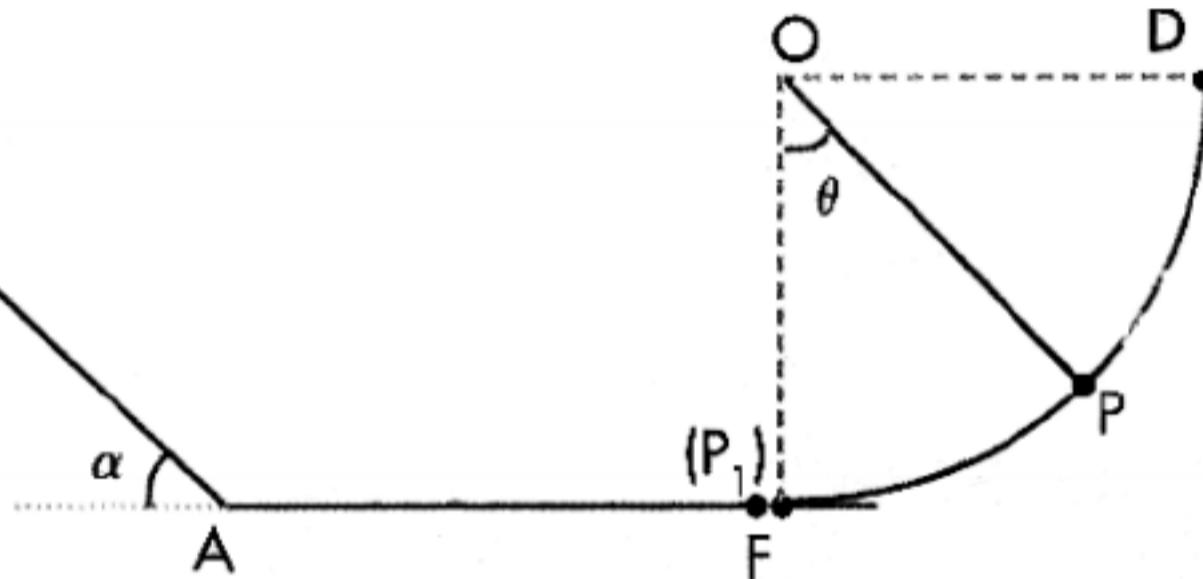
- 5) Assuming that Δt is very small, $\frac{\Delta \vec{P}}{\Delta t}$ may be considered equal to $\frac{d\vec{P}}{dt}$. Show that Newton's

Exercise 6

Conservation and non conservation of the mechanical energy

PHYSICS
GS - LS - 2002-1

Consider a mechanical system (S) formed of an inextensible string of length $l = 0.45$ m, having one of its ends O fixed while the other end carries a particle (P) of mass $m = 0.1$ kg. Take $g = 10$ m/s 2 .



1) (S) is shifted from its equilibrium position by $\theta_m = 90^\circ$, while the string is under tension, and then released without initial velocity.

Take the horizontal plane containing FA as a gravitational potential energy reference for the system [(S), Earth]. We neglect friction on the axis through O and air resistance.

- Calculate the initial mechanical energy of the system [(S), Earth] when (P) was at D.
- Determine the expression of the mechanical energy of the system [(S), Earth] in terms of l , m , g , V and θ , where V is the speed of (P) when the string passes through a position making an angle θ with the vertical.
- Determine the value of θ , ($0^\circ < \theta < 90^\circ$), for which the kinetic energy of (P) is equal to the gravitational potential energy of the system [(S), Earth].
- Calculate the magnitude V_0 of the velocity \vec{V}_0 of (P) as it passes through its equilibrium position.

2) Upon passing through the equilibrium position, the string is cut, and (P) enters in a head-on collision with a stationary particle (P_1) of mass $m_1 = 0.2$ kg. As a result, (P_1) is projected with a velocity \vec{V}_1 of magnitude $V_1 = 2$ m/s. Determine the magnitude V of the velocity \vec{V} of (P) right after impact knowing that \vec{V}_0 , \vec{V}_1 , and \vec{V} are collinear.

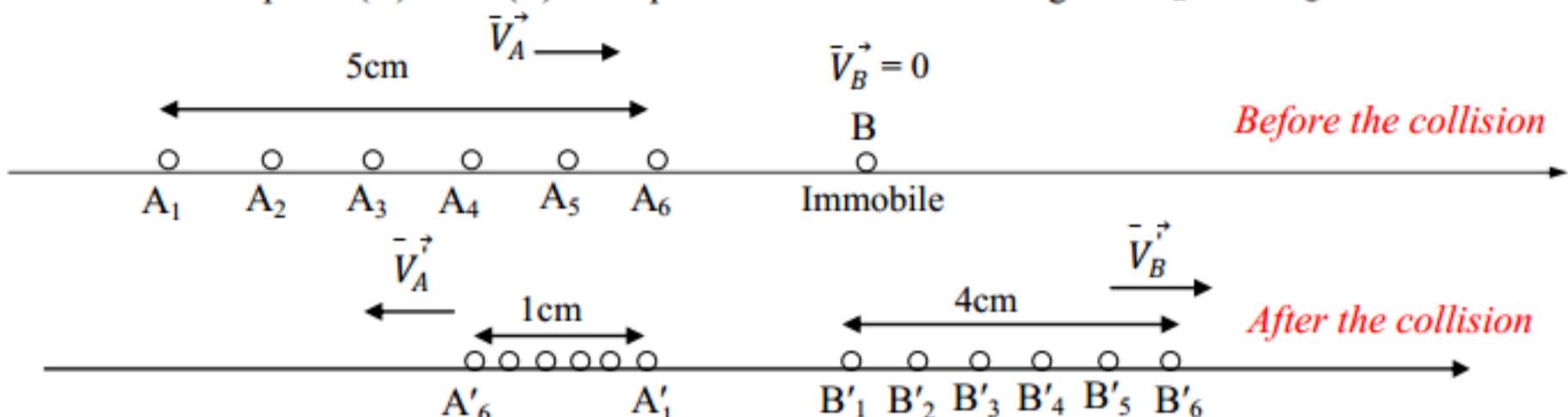
Is the collision elastic? Justify your answer.

3) (P_1), being projected with a speed $V_1 = 2$ m/s, moves along the frictionless horizontal track FA, and rises at A with the same speed V_1 , along the line of greatest slope of the inclined plane AB that makes an angle $\alpha = 30^\circ$ with the horizontal.

- Suppose now that the friction along AB is negligible. Determine the position of the point M at which (P_1) turns back.
- In fact, AB is not frictionless; (P_1) reaches a point N and turns back, where $AN = 20$ cm. Calculate the variation in the mechanical energy of the system [(P_1) , Earth] between A and N, and then deduce the magnitude of the force of friction (assumed constant) along AN.

Exercise 7 :**Collision and the laws of conservation**

In order to study the collision between two bodies, we use a horizontal air table that is equipped with a launcher and two pucks (A) and (B) of respective masses $m_A = 0.2 \text{ kg}$ and $m_B = 0.3 \text{ kg}$.



(A), thrown with the velocity $\vec{V}_A = V_A \hat{i}$, enters in a head-on collision with (B), initially at rest. (A) rebounds with the velocity $\vec{V}'_A = V'_A \hat{i}$, and (B) is projected with the velocity $\vec{V}'_B = V'_B \hat{i}$.

The figure below shows, in real dimensions, a part of the dot-prints, that register the positions of the centers of masses of (A) and (B), obtained when the time interval separating two successive dots is $\tau = 20 \text{ ms}$.

A) Law related to the linear momentum

- I) 1) Show, using the above dot-prints, that the velocities V_A, V'_A and V'_B are constant and calculate the algebraic values \bar{V}_A, \bar{V}'_A and \bar{V}'_B .
 - 2) Determine the linear momentums \bar{P}_A and \bar{P}'_A of the puck (A), before and after collision respectively and that \bar{P}'_B of the puck (B) after collision.
 - 3) Deduce the linear momentums, \bar{P} and \bar{P}' , of the center of mass of the system [(A) and (B)] before and after collision respectively.
 - 4) Compare \bar{P} and \bar{P}' then conclude.
- II) 1) Name the forces acting on the system [(A), (B)].
 - 2) What is the value of the resultant of these forces?
 - 3) This result agrees with the conclusion of (I - 4). Why?

B) Law related to the kinetic energy

- 1) Calculate the kinetic energy of the system [(A), (B)] before and after collision.
- 2) Deduce the nature of this collision.

Exercise 8:**Mechanical interaction**

The object of this exercise is to study some physical quantities of a system whose parts are in mechanical interaction

For that, we use two pucks (A) and (B), of respective masses $m_A = 100\text{g}$ and $m_B = 120\text{g}$, that may move without friction on a horizontal table.

Each puck is surrounded by an elastic steel shock ring of negligible mass. The two pucks are connected by a massless and inextensible taut thread thus compressing the steel shock rings. The system (S) thus formed is at rest. (Figure 1)

We burn the thread; the shock rings stretch and the pucks repel each other. The system (S) thus formed of the two pucks and the shock rings is said to "explode".

The positions of the center of mass of each puck are registered at successive instants separated by a constant time interval $\tau = 50 \text{ ms}$.

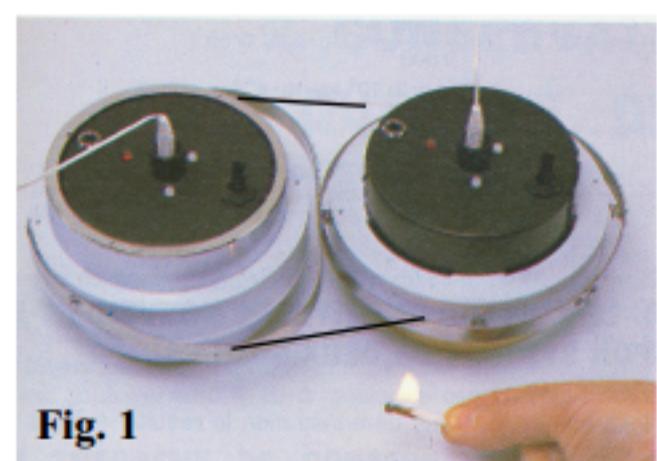


Fig. 1

Figure (2) represents, on the axis $x'x$, the dot-prints of the positions of the centers of masses G_A and G_B of the two pucks after «explosion» .

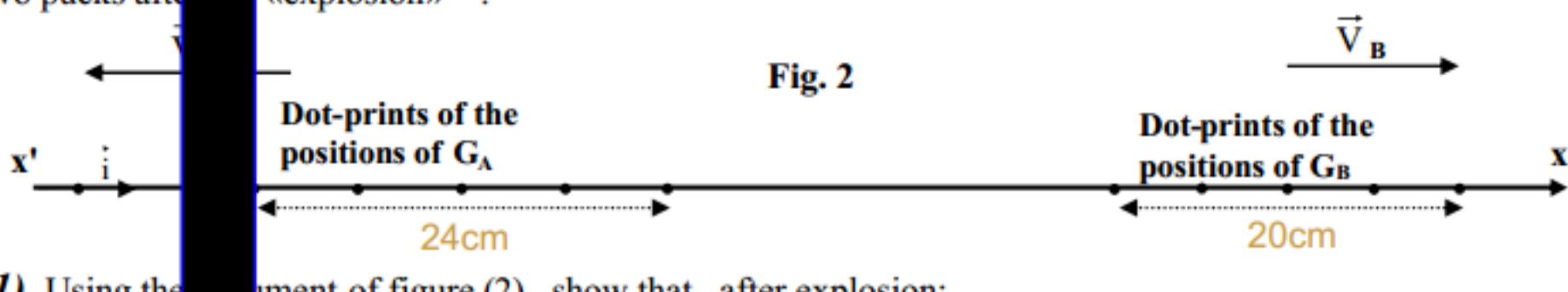


Fig. 2

- 1) Using the measurement of figure (2), show that, after explosion:
 - a- The motion of each puck is uniform;
 - b- The speeds of (A) and (B) are $V_A = 1.2 \text{ m/s}$ and $V_B = 1 \text{ m/s}$ respectively.
- 2) Verify the conservation of the linear momentum of the system (S) during explosion.
- 3) Applying Newton's second law $\frac{d\vec{P}}{dt} = \sum \vec{F}_{\text{ext}}$ on each puck and assuming that the time interval of the explosion $\Delta t = 0.05 \text{ s}$ is so small that $\frac{\Delta \vec{P}}{\Delta t}$ has the same value as $\frac{d\vec{P}}{dt}$,
 - a- Determine the forces $\vec{F}_{A \rightarrow B}$ and $\vec{F}_{B \rightarrow A}$ exerted respectively by (A) on (B) and by (B) on (A).
 - b- Verify the principle of interaction.
- 4) The system (S) possesses a certain energy before the explosion.
 - a- Identify the part of (S) storing this energy.
 - b- What form is this energy stored?
 - c- Determine the value of this energy

Exercise

A particle of mass $m_1 = 200 \text{ g}$, is released from rest at the point A on a track ABOE, found in the adjacent document (Doc 4). The part AB is very smooth, along which we can neglect the force of friction, has the shape of a circular arc of radius h_A , and the part BO is rough part, along which the force of friction \vec{f} is supposed constant, is a rectilinear path with $BO = 1 \text{ m}$.

The particle reaches the point B with the speed $v_{1B} = 4 \text{ m/s}$, then it covers the track BO to reach the point O with the speed $v_{1O} = 2 \text{ m/s}$.

At O, (S_1) enters into a head-on collision with a particle (S_2), of mass $m_2 = 400 \text{ g}$, initially at rest and connected to one end of a horizontal spring of stiffness $k = 100 \text{ N/m}$ whose other end is fixed at E. Take the horizontal plane containing BO as a gravitational potential energy reference level.

Take $g = 10 \text{ m/s}^2$

- 1) Conservation and non-conservation of the mechanical energy.
 - 1-1) According to the principle of conservation of the mechanical energy of the system [(S_1) , Earth], determine h_A .
 - 1-2) Determine the work done by the force of friction \vec{f} along BO.
 - 1-3) Determine the magnitude f of the force of friction \vec{f} along BO.
- 2) Elastic collision.

The collision between the particles (S_1) and (S_2) is perfectly elastic. All the velocities, before and after the collision, are along the horizontal axis $x'ox$.

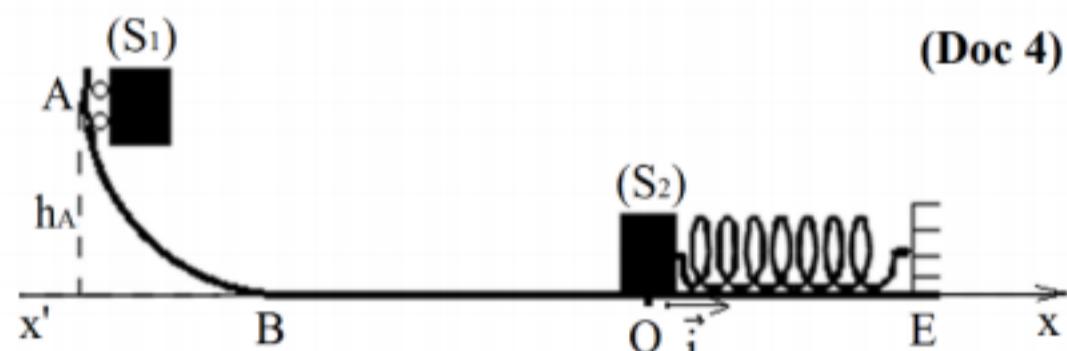
 - 2-1) Determine the speed v'_{1O} of (S_1) and v'_{2O} of (S_2) just after the collision.

Energies and collision

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2016-2017



(Doc 4)

- 2-2) Neglecting the force of friction between (S_2) and the track, just after the collision, calculate the maximum compression $x_m = OD$ of the spring.
- 2-3) In fact, the force of friction \vec{f} between (S_2) and the track, just after the collision, is not negligible and the maximum compression of the spring is $x'_m = OD' = 6.4 \text{ cm}$.
- 2-3-1) Determine the decrease in the mechanical energy of the system [(S_2) , Earth, spring], between O and D'.
- 2-3-2) In what form of energy does this decrease appear?

Exercise 10:**Study of the motion of a solid****PHYSICS
LS 2021-2**

Consider :

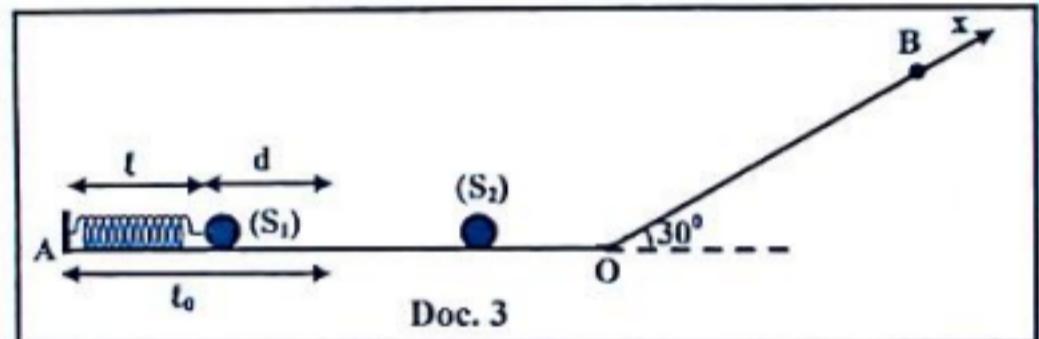
- a rail AOB located in a vertical plane composed of two straight parts: a horizontal part AO and an inclined part OB making an angle $\alpha = 30^\circ$ with the horizontal;
- two objects S_1 and S_2 assimilated to particles, and of the same mass $m = 80 \text{ g}$
- a spring (R) of negligible mass, of force constant $k = 200 \text{ N/m}$ and of natural length l_0 , attached by one of its two ends to a fixed support A and the other end is free.

Take :

- The horizontal plane containing O as the level of reference of the potential energy of gravity;
- $g = 10 \text{ m/s}^2$.

I. Launching particle (S_1)

To launch (S_1), we place it against the free end of the spring, we compress (R) by a distance d , and then we release the system [spring, (S_1)] without initial speed (Doc. 3).



When the string returns to the natural length l_0 , (S_1) leaves the spring with a velocity \vec{V}_1 , parallel to AO.

After launch, (S_1) moving at velocity \vec{V}_1 , collides head-on with (S_2) initially at rest on the AO rail. Just after the collision, (S_1) stops and (S_2) moves with a velocity \vec{V}_2 parallel to AO and of value $V_2 = 5 \text{ m/s}$.

(S_1) and (S_2) move without friction on the AO part of the rail.

- 1-1) Applying the law of conservation of momentum during collision, show that the value of \vec{V}_1 is $V_1 = 5 \text{ m/s}$.
- 1-2) Deduce that the collision between (S_1) and (S_2) is elastic.
- 1-3) Determine the value of d .

II. Movement of (S_2) on the inclined part

At the instant $t_0 = 0$, (S_2) approaches the inclined plane OB at O with a velocity

$$\vec{V}_0 = V_0 \vec{t} = 5\vec{t} \text{ (m/s)}, \text{ with } \vec{t} \text{ the unit vector of the } x'x \text{ axis parallel to the OB part of the rail.}$$

On this part, (S_2) is submitted to the action of a friction force \vec{f} , parallel to OB, in the direction opposite to the displacement and of constant value f .

- 2-1) Name the external forces acting on (S_2) along the path OB,
- 2-2) Show that the sum of the external forces exerted on (S_2), during its upward movement on OB is : $\sum \vec{F} = - (f + mgsina) \vec{t}$.
- 2-3) The expression of the linear momentum of (S_2) during its upward movement on OB is :

$$\vec{P} = (-0.9t + 0.4) \vec{t} \text{ (SI). Knowing that } \frac{d\vec{P}}{dt} = \sum \vec{F}, \text{ determine } f.$$

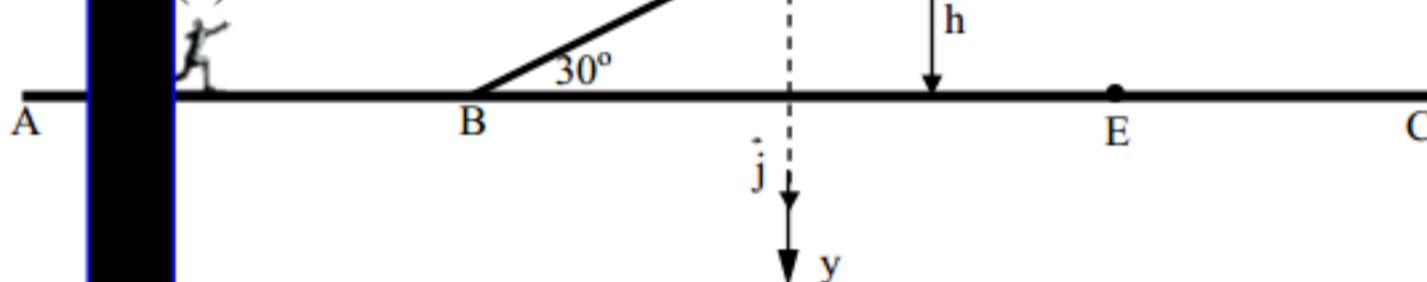
Exercise 11:**Study of the motion of a skier****PHYSICS**

LS 2012-2

A skier (S), of mass $m=80\text{kg}$, is pulled by a boat using a rope parallel to the surface of water. He starts from point A with an initial velocity $\vec{V}_0 = 0$ at the instant $t_0 = 0$ without initial velocity. The skier passes point B at the instant $t = 60\text{s}$ with a speed $V_B = 6\text{m/s}$, then he releases the rope. He continues his motion along a board BD inclined by an angle of 30° with respect to the horizontal surface of water. Suppose that during the passage from AB to BD the speed at point B does not change. The skier arrives at point D, situated at an altitude $h = 1.6\text{m}$ from the water surface, with a velocity \vec{V}_D , then he leaves the board at point D to hit the water surface at point E (see figure below).

Given:

- ❖ the skier (S) is considered as a particle;



- ❖ on the path AB, the force of traction \vec{F} exerted by the rope on the skier has a constant magnitude and the whole forces of friction are equivalent to a single force \vec{f} opposite to the displacement, of magnitude $f = 100\text{N}$;
- ❖ friction is negligible along the path BDE;
- ❖ after leaving point D the motion of the skier takes place in the vertical plane Dxy containing \vec{V}_D ;
- ❖ the horizontal plane passing through AB is the reference level of the gravitational potential energy;

A – Motion of the skier between A and B

- 1) What are the external forces acting on (S) along the path AB? Draw, not to scale, a diagram of these forces.
- 2) Applying Newton's second law $\frac{d\vec{P}}{dt} = \Sigma \vec{F}_{\text{ext}}$ on the skier, between the points A and B, express the acceleration a of the motion of the skier in terms of F , f and m .
- 3) Determine the expression of the speed V of the skier in terms of F , f , m and the time t .
- 4) Deduce

B – Motion of the skier on the board BD

- 1) Why can we apply the principle of conservation of the mechanical energy of system [(S), Earth] on the skier on the board BD?
- 2) Deduce that $V_D = 2\text{m/s}$.

C – Motion of the skier between D and E

The skier leaves the board at point D, at an instant t_0 , taken as a new origin of time.

- 1) Apply Newton's second law on the skier to show that, at an instant t , the vertical component P_y of the linear momentum of the skier is of the form: $P_y = 800t - 80$ (In SI unit).
- 2) Deduce the parametric equation $y(t)$ of the motion of the skier in the frame of reference Dxy.
- 3) Determine the duration taken by the skier to pass from D to E.

Exercise 12:**Determination of the speed of a bullet****PHYSICS**
LS 2004-2

A gun is used to shoot bullets, each of mass $m = 20\text{g}$, with a horizontal velocity \vec{V}_0 of value V_0 .

In order to determine V_0 , we consider a setup formed of a wooden block of mass $M = 1\text{kg}$, suspended from the end of two inextensible strings of negligible mass and of the same length (figure 1). This setup can be taken as a block of wood suspended from the free end a string of length $\ell = 1\text{m}$, initially at the equilibrium position at G_1 .

A bullet having the velocity V_0 hits the block and is embedded in it at the level of the center of mass G of the block.

Just after impact, the system (block, bullet) moves with a horizontal velocity \vec{V}_1 .

The pendulum thus attains a maximum angular deviation $\alpha = 37^\circ$.

G_1 and G_2 are the respective positions of G in the equilibrium position and in the highest position.

Take the horizontal plane through G_1 as a gravitational potential energy reference (figure 2).

Neglect friction with air and take $g = 9,8 \text{ m/s}^2$.

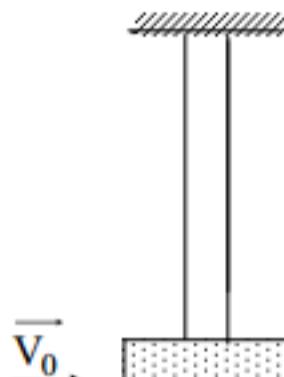


Figure 1

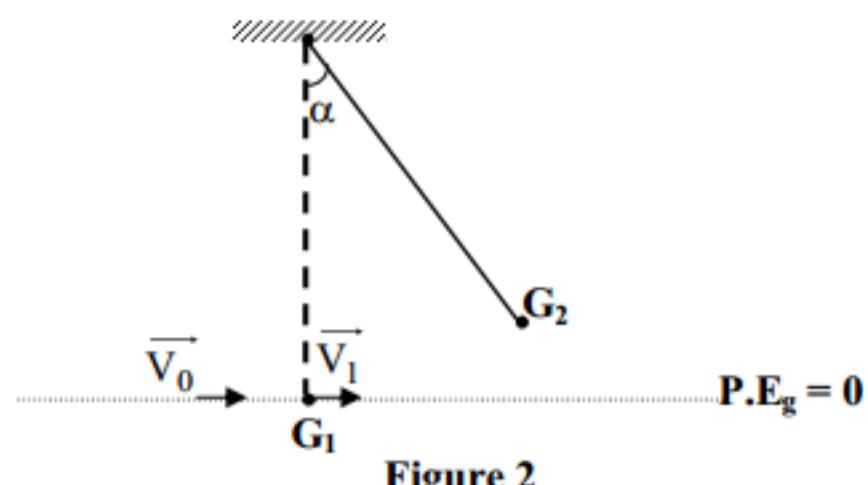


Figure 2

- During a collision, which one of the two physical quantities, the linear momentum or the kinetic energy of the system does not remain always conserved ?
- Determine the expression of the value of V_1 of the velocity \vec{V}_1 in terms of M , m and V_0 .
- a) Determine, just after impact, the mechanical energy of the system (pendulum, Earth) in terms of V_0 , M , and m .
b) Determine, in terms of M , m , g , ℓ and α , the mechanical energy of the system (pendulum, Earth) at point G_2 .
c) Deduce the value of V_0 .
- Verify the answer of question (1).

PHYSICS
IS 2016-2

Exercise 13:

Nature of a collision

The aim of this exercise is to determine the nature of a collision between two objects.

For this aim, an object (A), considered as a particle, of mass $m_A = 2 \text{ kg}$, can slide without friction on a path situated in a vertical plane and formed of two parts: a circular part DN and a horizontal rectilinear part NM.

(A) is released, without initial velocity, from the point D situated at a height $h_D = 0.45 \text{ m}$ above the horizontal part NM (Fig.1).

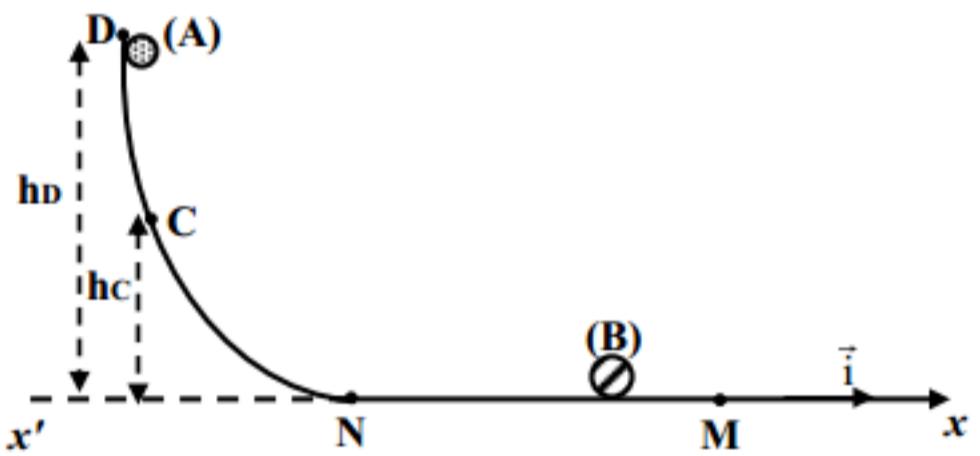


Fig. 1

The horizontal plane passing through MN is taken as the reference level of gravitational potential energy.

- Calculate the mechanical energy of the system [(A), Earth] at the point D.
- Deduce the speed V_{1A} of (A) when it reaches the point N.
- (A) reaches N and moves along NM with the same velocity $\vec{V}_{1A} = V_{1A} \vec{i}$. Another object (B), considered as a particle, of mass $m_B = 4 \text{ kg}$ moves along the horizontal path from M toward N with the velocity $\vec{V}_{1B} = -1 \vec{i}$ (V_{1B} in m/s).
 - Determine the linear momentum \vec{P}_s of the system [(A), (B)] before collision.
 - Deduce the velocity \vec{V}_G of the center of inertia G of the system [(A), (B)].

- 4) After collision, (A) rebounds and attains a maximum height $h_c = 0.27 \text{ m}$.
- Determine the mechanical energy of the system [(A), Earth] at the point C.
 - Deduce the speed V_{2A} of (A) just after collision.
- 5) Determine, by applying the principle of the conservation of the linear momentum of the system [(A), (B)], the velocity \vec{V}_{2B} of (B) just after collision.
- 6) Specify the nature of the collision.

PHYSICS

LS 2015-1

Exercise 14 :

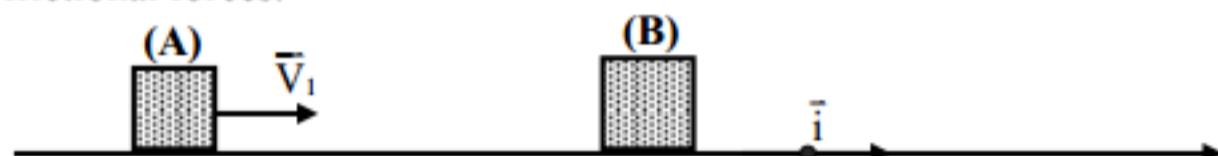
Collision and interaction

In order to study the collision between two bodies, we consider a horizontal air table equipped with a launcher and two pucks (A) and (B) of respective masses $m_A = 0.4 \text{ kg}$ and $m_B = 0.6 \text{ kg}$.

(A), launched with the velocity $\vec{V}_1 = 0.5 \hat{i}$, collides with (B) initially at rest.

(A) rebounds with the velocity $\vec{V}_2 = -0.1 \hat{i}$ and (B) moves with the velocity $\vec{V}_3 = 0.4 \hat{i}$ (V_1 , V_2 and V_3 are expressed in m/s). Neglect all frictional forces.

A – Linear momentum



- 1) a) Determine the linear momentums:

- \bar{P}_1 and \bar{P}_2 of (A), before and after collision respectively;
- \bar{P}_3 of (B) after collision.

- b) Deduce the linear momentums \bar{P} and \bar{P}' of the system [(A), (B)] before and after collision respectively.

- c) Compare \bar{P} and \bar{P}' . Conclude.

- 2) a) Name the external forces acting on the system [(A), (B)].

- b) Give the value of the resultant of these forces.

- c) Is this resultant compatible with the conclusion in question (1- c)? Why?

B – Type of collision

- 1) Determine the kinetic energy of the system [(A), (B)] before and after collision.

- 2) Deduce the type of the collision.

C – Principle of interaction

The duration of collision is $\Delta t = 0.04 \text{ s}$; we can consider that $\frac{\Delta \bar{P}}{\Delta t} \approx \frac{d\bar{P}}{dt}$.

- 1) Determine during Δt :

- the variations $\Delta \bar{P}_A$ and $\Delta \bar{P}_B$ in the linear momentums of the pucks (A) and (B) respectively;
- the forces $\vec{F}_{A/B}$ exerted by (A) on (B) and $\vec{F}_{B/A}$ exerted by (B) on (A).

- 2) Deduce that the principle of interaction is verified.

Exercise 15 :

Verification of the principle of interaction

PHYSICS

LS 2022-1

The aim of this exercise is to verify the principle of interaction between two blocks.

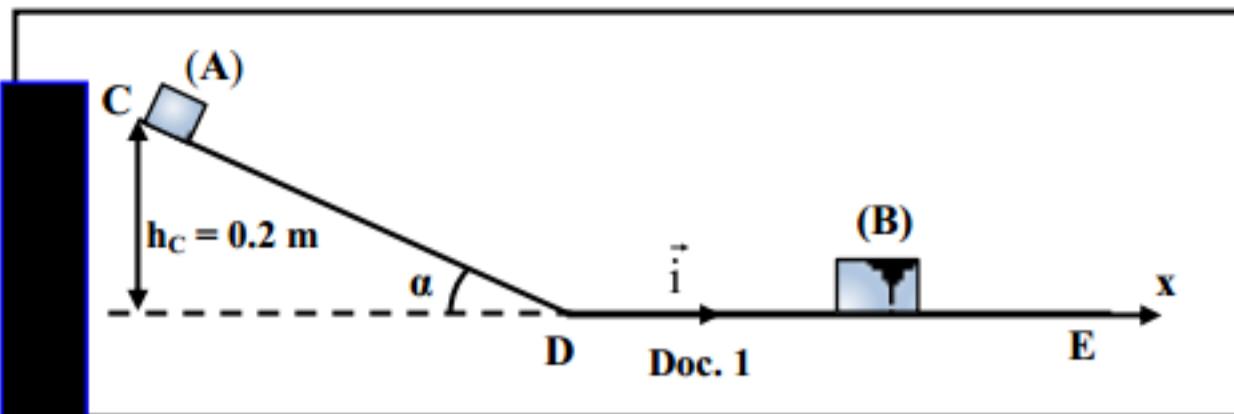
For this purpose, we consider two blocks (A) and (B) considered as particles of respective masses $m_A = 200 \text{ g}$ and $m_B = 800 \text{ g}$.

(A) and (B) can move without friction on a track CDE lying in a vertical plane.

This track is formed of two parts: the first one CD is straight and inclined by an angle α with respect to the horizontal and the second one DE is straight and horizontal.

Block (A) is released without initial velocity from point C situated at a height $h_C = 0.2 \text{ m}$ above a horizontal x-axis, confounded with DE, of unit vector \hat{i} (Doc. 1).

Take: • the horizontal plane containing the x-axis as a reference level for gravitational potential energy;
• $g = 10 \text{ m/s}^2$.



- 1) The mechanical energy of the system [(A), Track, Earth] is conserved between C and D. Why?
- 2) Deduce that the speed of (A) at point D is $V_A = 2 \text{ m/s}$.
- 3) (A) continues its motion with a velocity $\vec{V}_A = 2 \hat{i} \text{ (m/s)}$ along track DE until it makes a head-on elastic collision with (B) initially at rest.
- Show that the final velocities of (A) and (B) right after the collision are $\vec{V}'_A = -1.2 \hat{i} \text{ (m/s)}$ and $\vec{V}'_B = 0.8 \hat{i} \text{ (m/s)}$ respectively.
- 4) The duration of the collision is $\Delta t = 0.1 \text{ s}$, so $\frac{\Delta \vec{P}}{\Delta t} \approx \frac{d\vec{P}}{dt}$.
Apply, during time Δt , Newton's second law:
- 4.1) on (B) to determine the force $\vec{F}_{A/B}$ exerted by (A) on (B);
4.2) on (A) to determine the force $\vec{F}_{B/A}$ exerted by (B) on (A).
- 5) Deduce that the principle of interaction is verified.

Exercise

De termination of the stiffness of a spring

PHYSICS
LS 218-1

In order to determine the stiffness k of a massless spring (R), we consider:

- a track MN found in a vertical plane ;
- a massless ring (R) of horizontal axis and stiffness k having one end fixed to a support (A); the other end is connected to an object (S₁) considered as a particle of mass $m_1 = 0.2 \text{ kg}$;
- an object (S₂) considered as a particle of mass $m_2 = 0.3 \text{ kg}$ placed at the origin O of a horizontal x-axis of unit vector \hat{i} (Doc. 1).

Neglect all the forces of friction.

Take:

- the horizontal plane passing through NP as a reference level for gravitational potential energy;
- $g = 10 \text{ m/s}^2$

1- Collision between (S₁) and (S₂)

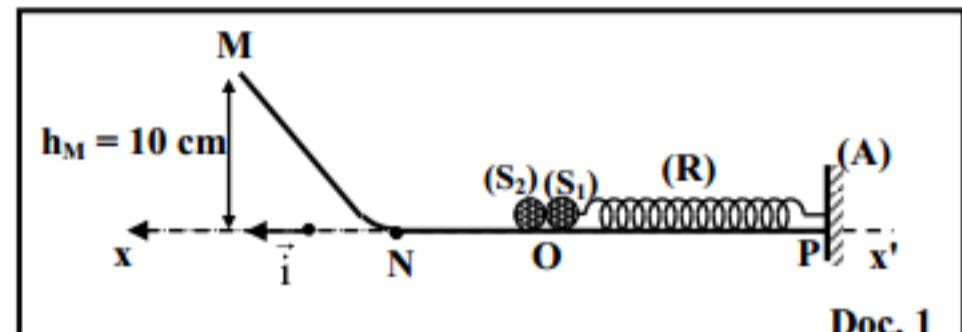
At equilibrium, (S₁) coincides with O. (S₁) is shifted from O to the right by a certain distance and it is released from rest. (S₁) reaches O with a velocity $\vec{V}_1 = 2 \hat{i} \text{ (m/s)}$, and enters into a head-on collision with (S₂) initially at rest. Just after collision, (S₁) rebounds with a velocity $\vec{V}'_1 = -0.4 \hat{i} \text{ (m/s)}$ and (S₂) moves to the left with a velocity $\vec{V}'_2 = V'_2 \hat{i}$.

- 1-1) Applying the principle of conservation of linear momentum for the system [(S₁), (S₂)], show that $V'_2 = 1.6 \text{ m/s}$.

- 1-2) Specify whether this collision is elastic or not.

2- Motion of (S₂) after collision

Just after collision, (S₂) moves along the horizontal track PN with the speed V'_2 and then continues its motion along the inclined plane MN. (S₂) leaves the inclined plane at M with a speed V_M . The height of M above the reference level is $h_M = 10 \text{ cm}$. Determine the speed V_M of (S₂) at point M.

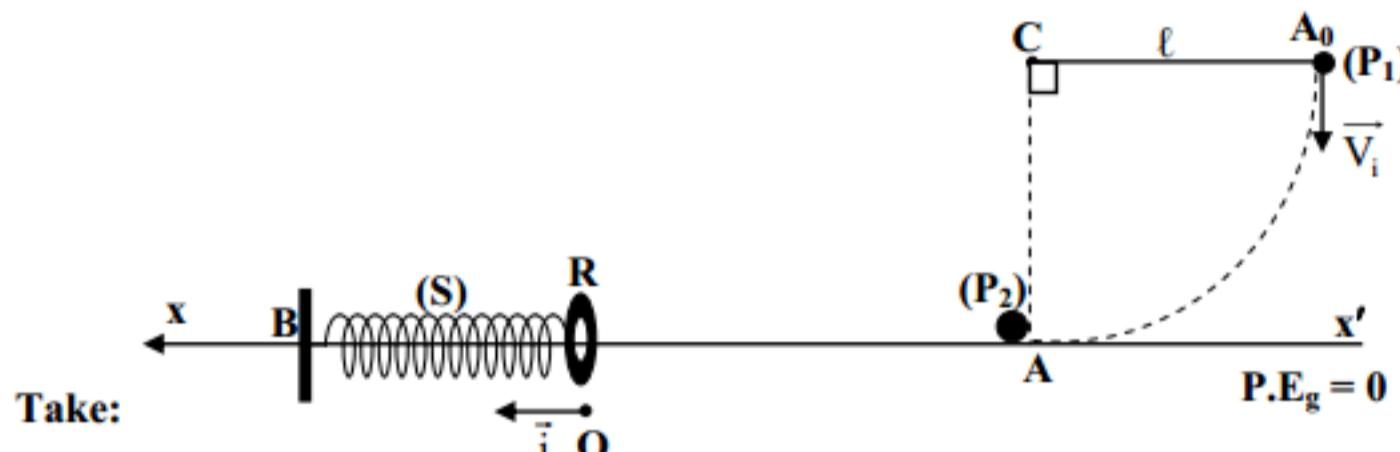


Doc. 1

A pendulum is formed of a massless and inextensible string of length $\ell = 1.8 \text{ m}$, having one of its ends C fixed to a support while the other end carries a particle (P_1) of mass $m_1 = 200 \text{ g}$.

The pendulum is stretched horizontally. The particle (P_1) at A_0 is then launched vertically downward with a velocity \vec{V}_i of magnitude $V_i = 8 \text{ m/s}$.

At the lowest position A, (P_1) enters in a head-on perfectly elastic collision with another particle (P_2) of mass $m_2 = 300 \text{ g}$ initially at rest. Neglect all frictional forces.



- the horizontal plane passing through A as a gravitational potential energy reference; • $g = 10 \text{ m/s}^2$.
- 1) a) Calculate the mechanical energy of the system [pendulum, Earth] at the instant of launching (P_1) at A_0 .
b) Determine the magnitude V_1 of the velocity \vec{V}_1 of (P_1) just before colliding with (P_2).
 - 2) a) Name the physical quantities that are conserved during this collision.
b) Show that the magnitude V'_2 of the velocity \vec{V}'_2 of (P_2), just after collision, is 8 m/s.

Exercise 18 :

Determination of the mass of a block and stiffness of a spring

Consider two blocks, (A) of unknown mass m_A and (B) of mass $m_B = 0.8 \text{ kg}$, and a spring (R) of negligible mass and of stiffness k. The aim of this exercise is to determine m_A and k.

Neglect all the forces of friction and take $g = 10 \text{ m/s}^2$.

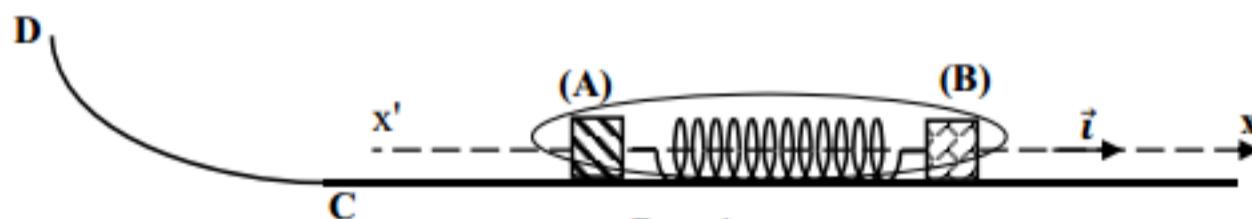
1- First experiment: Determination of m_A

The spring is placed on a horizontal track. The spring is compressed between (A) and (B) by means of a light string (Document 1).

The center of mass of (A) and that of (B) belong to the same horizontal plane which is taken as a reference level for gravitational potential energy.

The x-axis extends positively to the right.

We burn the string, (A) and (B) are ejected in opposite directions.



1-1) Name the external forces acting on the system [(A), (B) and (R)].

1-2) Deduce that the linear momentum of the system [(A), (B) and (R)] is conserved during the motion of (A) and (B) on the horizontal track.

1-3) The velocity of the center of mass of block (B) just after ejection is $\vec{V}_B = 0.75 \vec{i} \text{ (m/s)}$.

1-3-1) Determine the linear momentum \vec{P}_A of block (A).

1-3-2) Deduce in terms of m_A the velocity \vec{V}_A of the center of mass of (A) just after ejection.

1-4) Block (A) continues its motion and reaches a curvilinear path CD situated in the vertical plane (Document 1). The maximum height attained by the center of mass of (A) above the reference level is $h_{\max} = 5 \text{ cm}$.

1-4-1) Apply the principle of conservation of mechanical energy to the system [(A), Earth] to determine the magnitude \vec{V}_A of \vec{V}_A .

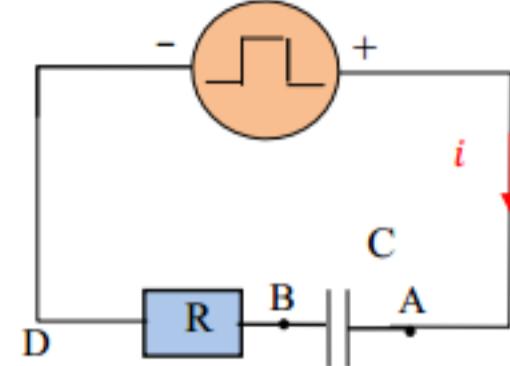
1-4-2) Deduce the value of the mass m_A .



Charging phase

$$i = \frac{dq}{dt} \text{ and } q = C u_c$$

We write then $i = C \frac{duc}{dt}$



Application



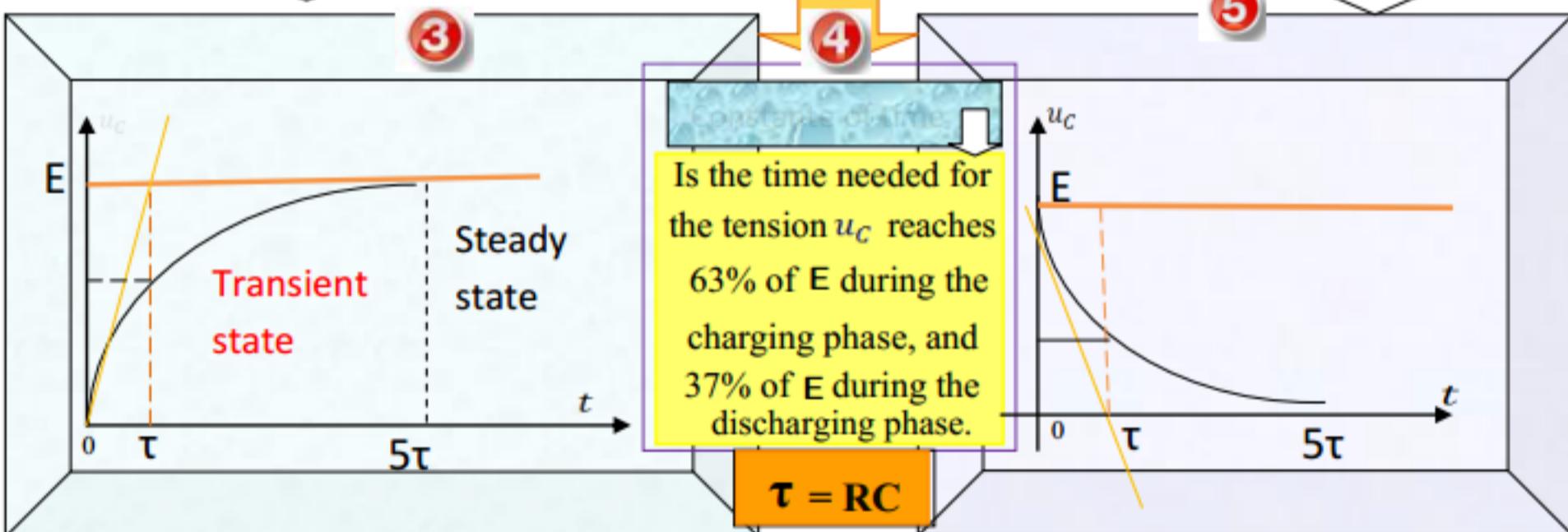
Discharging phase

$q \downarrow$ because the tension $u_c \downarrow$

We write then

$$i = -\frac{dq}{dt} > 0$$

Properties



Derivative with respect to time of the function : Ae^{-bt}

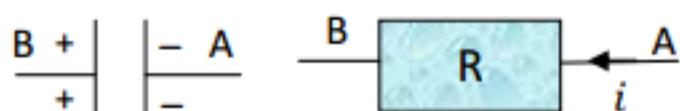
$$\frac{d}{dt}(Ae^{-bt}) = -Abe^{-bt}$$

⑥

Energy stored in a capacitor

- It is an electric form.
- Its value is $W = \frac{1}{2}Cu_c^2$ at an instant t .
- Its maximum value is $W_0 = \frac{1}{2}CE^2$
(Unit : J in international system.)

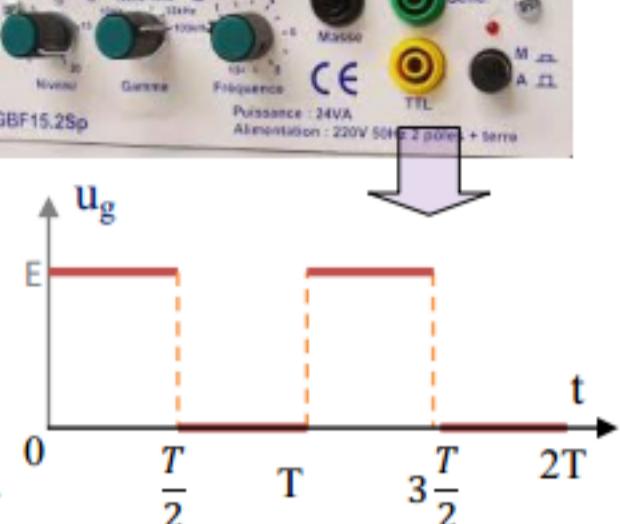
Remarks:



$$u_{BA} = u_c ; u_{AB} = Ri$$

L.F.G. Mode: square signal

A L.F.G. (Low frequency generator) delivers in the electric circuit a **periodic** voltage (Mode: square signal).



Complete: the voltage passes briefly from $u_g = E$ to a constant value $u_g = 0$.

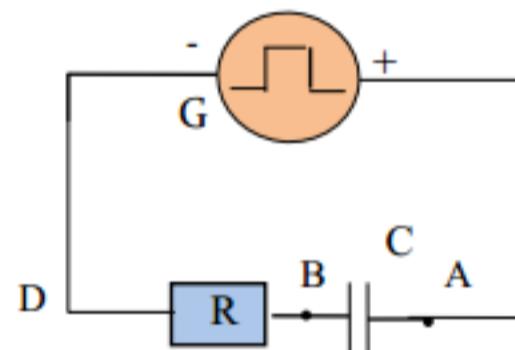
Then from the value 0 to the value $u_g = \dots$ (Alternatively) .

Question for $0 < t < 2T$ [see the above figure]; what interval of time, the L.F.G is equivalent to an identical battery?

Question for what is the L.F.G identical to, in the interval $\frac{T}{2} < t < T$?

Electric phases of a capacitor

circuit



Charging phase

Case when the value of u_g passes briefly from 0 to E →

- i.e. in the interval $0 < t < \frac{T}{2}$

The voltage u_C across the terminals of C was 0 at $t = 0$.

This voltage increases **progressively**, during an interval of time called charging phase. $u_C \nearrow$ for $0 < t < \frac{T}{2}$.

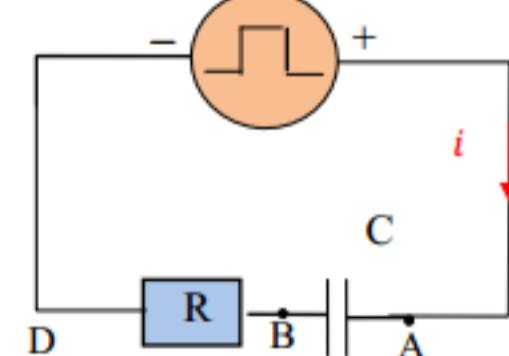
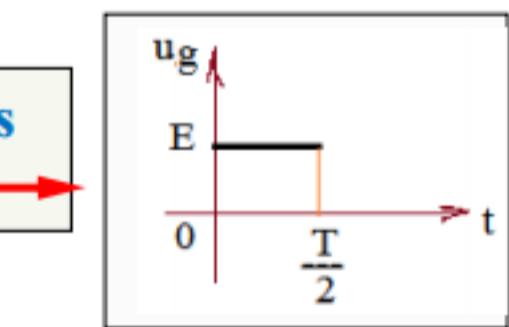
During this interval of time, the electric charge q of the armature A increases.

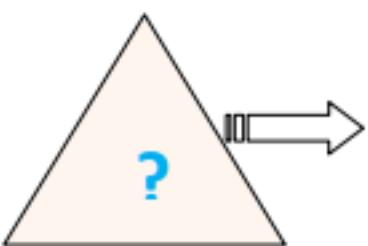
Then, the relationship between the electric current i and the charge q is written as: $i = \frac{dq}{dt}$.

Finally, The **equation of addition of voltage**, applied across the terminal of the dipole RC, is written:

$$\mathbf{u}_{AB} + \mathbf{u}_{BD} = E \quad (1) \Rightarrow u_C + Ri = E. \text{ by substituting } i = \frac{dq}{dt} = C \frac{du_C}{dt} \text{ in this equation, we can write now : } u_C + RC \frac{du_C}{dt} = E \quad (2)$$

This is the differential equation that governs the evolution of the voltage u_C across the terminals of the capacitor during the charging phase.





Verify that: $u_C = E(1 - e^{-\frac{t}{\tau}})$ is a solution of the equation (2), with $\tau = RC$.

We should verify that $u_C = E - E e^{-\frac{t}{\tau}}$ is the solution of the differential equation (2).

then we find its derivative $\frac{du_C}{dt}$.

$$\frac{du_C}{dt} = 0 - E \left(-\frac{1}{\tau} e^{-\frac{t}{\tau}}\right) = \frac{E}{\tau} e^{-\frac{t}{\tau}}.$$

And we substitute in (2), we obtain

$$\text{at : } E - E e^{-\frac{t}{\tau}} + RC \frac{E}{\tau} e^{-\frac{t}{\tau}} = E;$$

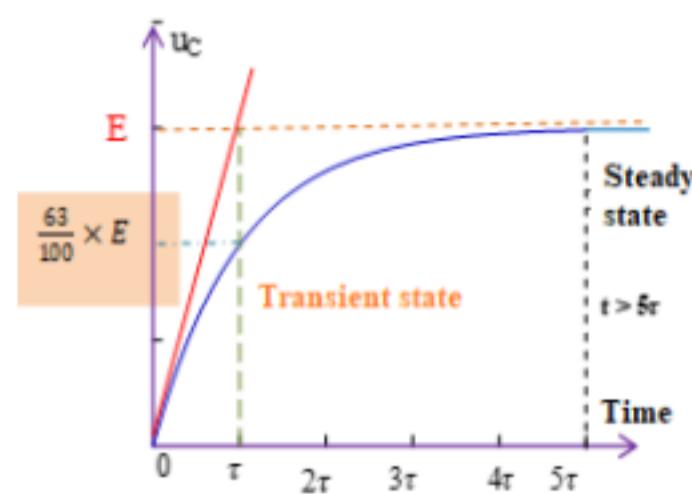
$$E e^{-\frac{t}{\tau}} \left(\frac{RC}{\tau} - 1\right) = 0 \Rightarrow \frac{RC}{\tau} - 1 = 0$$

$$\text{Generally ; } \tau = R_{eq} C$$

with R'_{eq} :

is the equivalent resistance

Definition:



Formulas

During the charging phase :

- The time constant : $\tau = R_{eq} C$
- $u_C = 0,63E$ for $t = \tau$

During the charging phase:

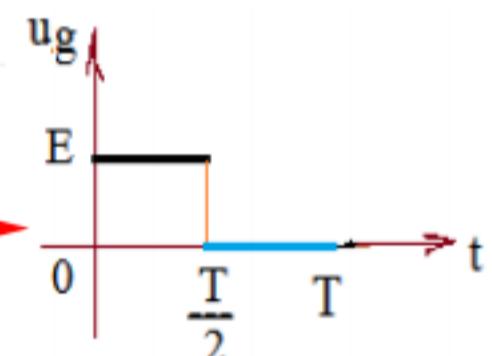
The time constant τ of an RC circuit is the time interval after which the voltage of the capacitor reaches 63% of the value of E.

And for $t \approx 5\tau$, u_C is approximately equal to E .

Discharging phase

Case when the value of u_g passes briefly from E to 0

- i.e. in the interval $\frac{T}{2} < t < T$



The voltage u_C across the terminals of C was equal to E at the instant $t = \frac{T}{2}$.

Attention: We can use this instant as a new origin of time ($t = 0$) at the beginning of this phase, called phase of discharging of the capacitor.

Even more, pay attention to the new direction of current i (real direction > 0) the electric current is now delivered from the capacitor to the circuit ; So, $q \downarrow$, then $\frac{dq}{dt} < 0$.

In this case, we can write :

$$i = -\frac{dq}{dt} = -C \frac{du_C}{dt}$$

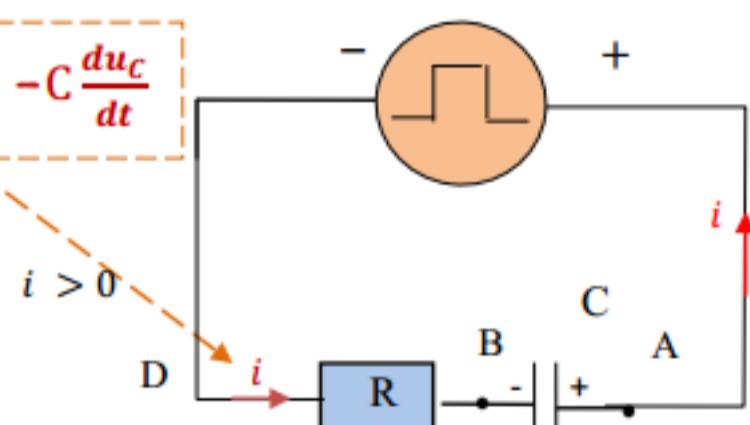
The law of addition of voltage:

$$u_{AD} = u_{AB} + u_{BD} \quad (1)$$

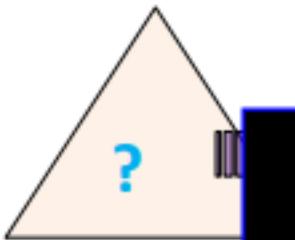
with $u_{AD} = u_g = 0$. i.e. $0 = u_C - Ri$ with $u_C = u_{AB}$.

Finally: $0 = u_C + RC \frac{du_C}{dt}$ by replacing $i = -C \frac{du_C}{dt}$.

We obtain: (1) $\Rightarrow u_C + RC \frac{du_C}{dt} = 0 \quad (2)$.



This is the differential equation that governs the evolution of the voltage u_C across the terminals of the capacitor in the discharging phase.



Verify that : $u_C = E e^{-\frac{t}{\tau}}$ is a solution of the equation (2) , with $\tau = RC$.

We need to verify that $u_C = E e^{-\frac{t}{\tau}}$ is a solution of the differential equation (2).

Then we find the derivative $\frac{du_C}{dt}$.

$$\frac{du_C}{dt} = E \left(-\frac{1}{\tau} \right) = -\frac{E}{\tau} e^{-\frac{t}{\tau}}.$$

And then substitute in (2) ,

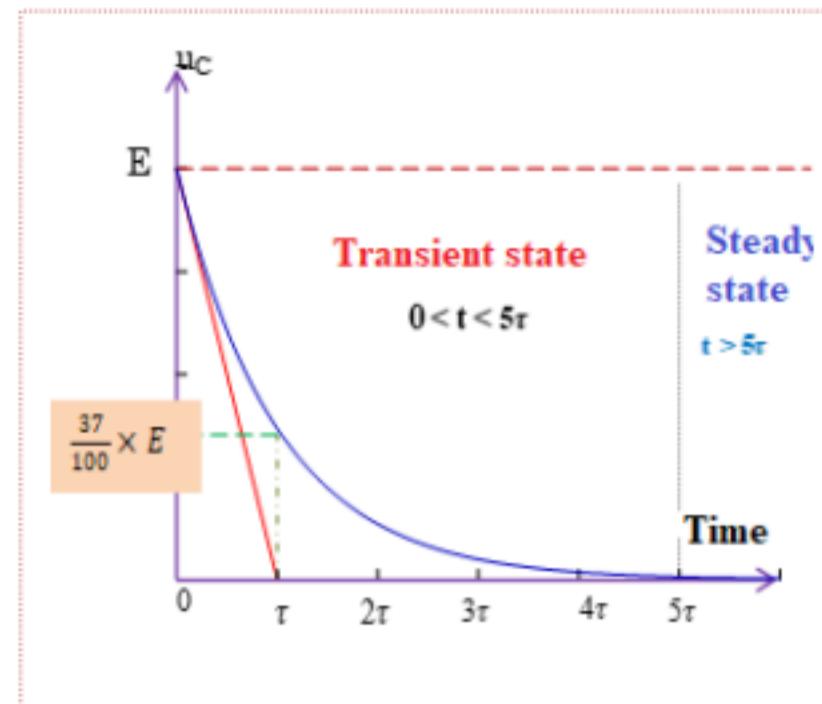
$$We find that -E e^{-\frac{t}{\tau}} + R C e^{-\frac{t}{\tau}} = 0 ;$$

\Rightarrow

$$E e^{-\frac{t}{\tau}} \left(\frac{RC}{\tau} - 1 \right) = 0 \Rightarrow \frac{RC}{\tau} - 1 = 0 \\ Generally \quad = R'_{eq} C$$

with R'_{eq}
is the equivalent resistance

Definition : 



Formulas

During the discharging phase :

- The time constant: $\tau = R'_{eq} C$

- $u_c = 0,37E$ for $t = \tau$

During the discharging phase

- The time constant τ of an RC circuit is the duration after which the remaining voltage of the capacitor is 37 % of the value of E .
and; for $t \approx 5\tau$, u_C is approximatlly = 0

RC dipole



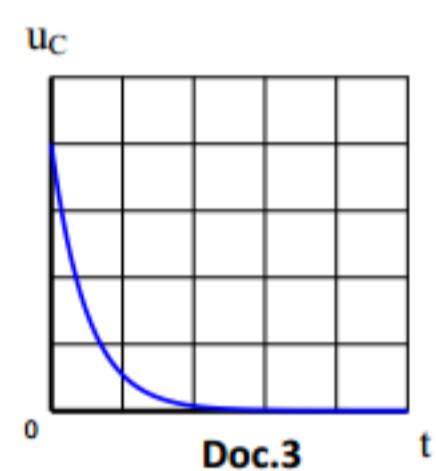
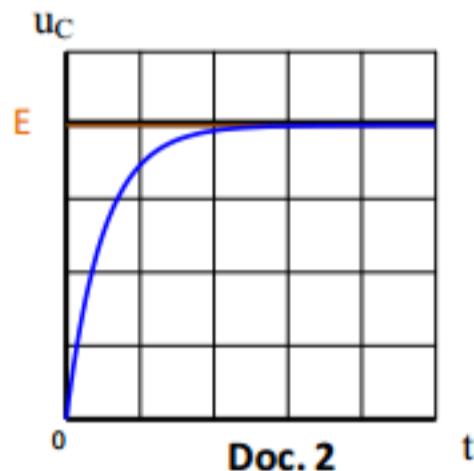
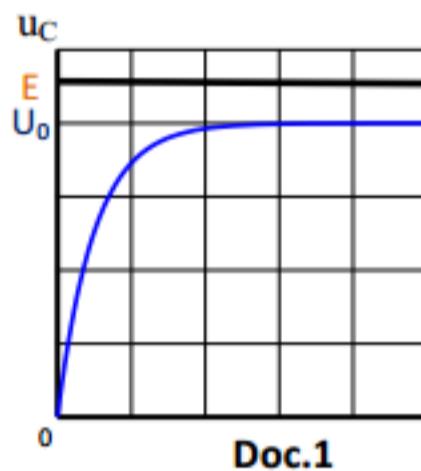
Chap.
10-A

Exercises

Exercise 1 -a :

Consider the diagram of the adjacent circuit, with $R \neq 0$.

We close K; choose with justification the document corresponding to this situation.



Exercise 2 -b :

During the charging phase of a capacitor, the differential equation of u_C during the time is:

$$\frac{du_C}{dt} + \frac{1}{RC} u_C = \frac{E}{RC}. \quad \text{The solution of this equation can be written as: } u_C = A(1 - e^{-\frac{t}{\tau}}).$$

Verify that $A=E$ and $\tau = RC$

Exercise 3 -c :

During the charging phase of a capacitor, the differential equation of u_C during the time is:

$$\frac{du_C}{dt} + \frac{1}{RC} u_C = \frac{E}{RC}.$$

The solution of this equation can be written as: $u_C = A - B e^{-\frac{t}{\tau}}$.

Determine A, B and τ .

Exercise 4 -d :

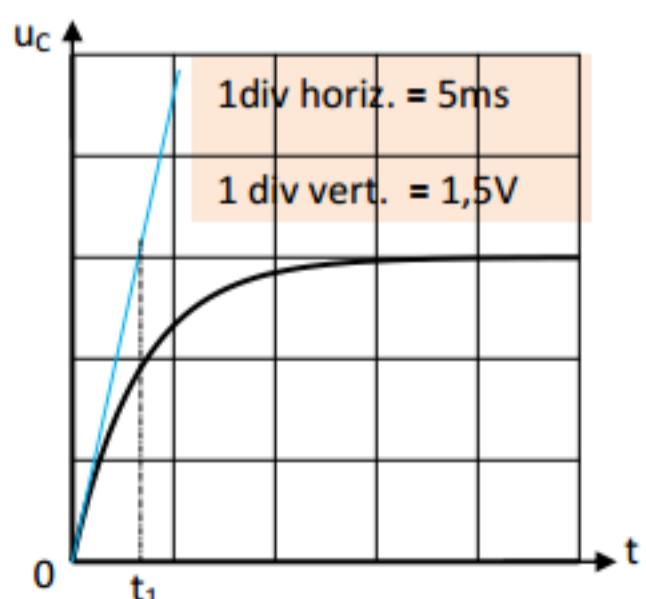
Given: $u_C = A + B e^{-\lambda t}$ is the solution of the differential equation $\frac{du_C}{dt} + \frac{1}{RC} u_C = \frac{E}{RC}$

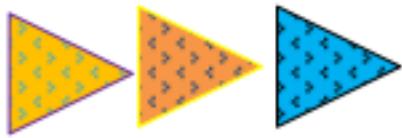
Determine A, B and λ .

Exercise 5 -e :

During the phase of charging of the capacitor, the curve u_C has the shape given in the adjacent document.

- 1) Use the method of tangent at the origin to determine the time Constant of the dipole RC.
- 2) Knowing that the dry cell is ideal and characterized by $\text{emf} = E$
 - 2.1- Determine E.
 - 2.2- Calculate the voltage across the resistor R at the instant t





Visit

Exercise 2

Charging and discharging a capacitor

The aim of this exercise is to study the charging and the discharging of a capacitor.

For this purpose, we set up the circuit of document 3 that includes:

- an ideal battery of electromotive force $E = 10 \text{ V}$;
- two resistors of resistances $R_1 = R_2 = 4 \text{ k}\Omega$;
- a capacitor of capacitance C ;
- a switch K .

1) Charging the capacitor

The switch K is initially at position (0) and the capacitor is uncharged.

At the instant $t_0 = 0$, K is turned to position (1) and the charging process of the capacitor starts.

At an instant t , plate B of the capacitor carries a charge q and the circuit carries a current i .

An appropriate device allows us to display the voltage $u_{AB} = u_{R_1}$ across the resistor and the voltage $u_{BD} = u_C$ across the capacitor.

Curves (a) and (b) of document 4 show these voltages as functions of time.

1.1) Curve (a) represents u_{R_1} and curve (b) represents u_C . Justify.

1.2) The time constant of this circuit is given by $\tau_1 = R_1 C$.

1.2.1) Using document 4, determine the value of τ_1 .

1.2.2) Deduce the value of C .

1.3) Calculate the time « t_1 » needed by the capacitor to practically become completely charged.

2) Discharging the capacitor

The capacitor is completely charged. At an instant taken as a new initial time $t_0 = 0$, the switch K is turned to position (2), and the capacitor starts discharging through the resistors of resistances R_1 and R_2 . At an instant t the circuit carries a current i (Doc. 5).

2.1) Show, using the law of addition of voltages, that the differential equation which governs u_C is:

$$RC \frac{du_C}{dt} + u_C = 0 \text{ where } R = R_1 + R_2.$$

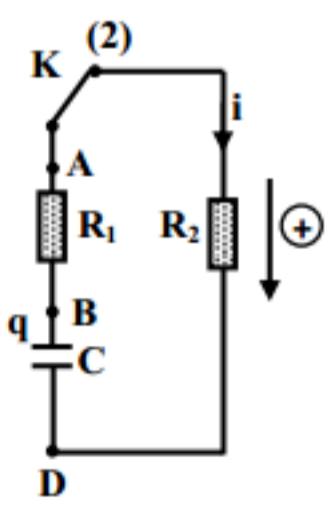
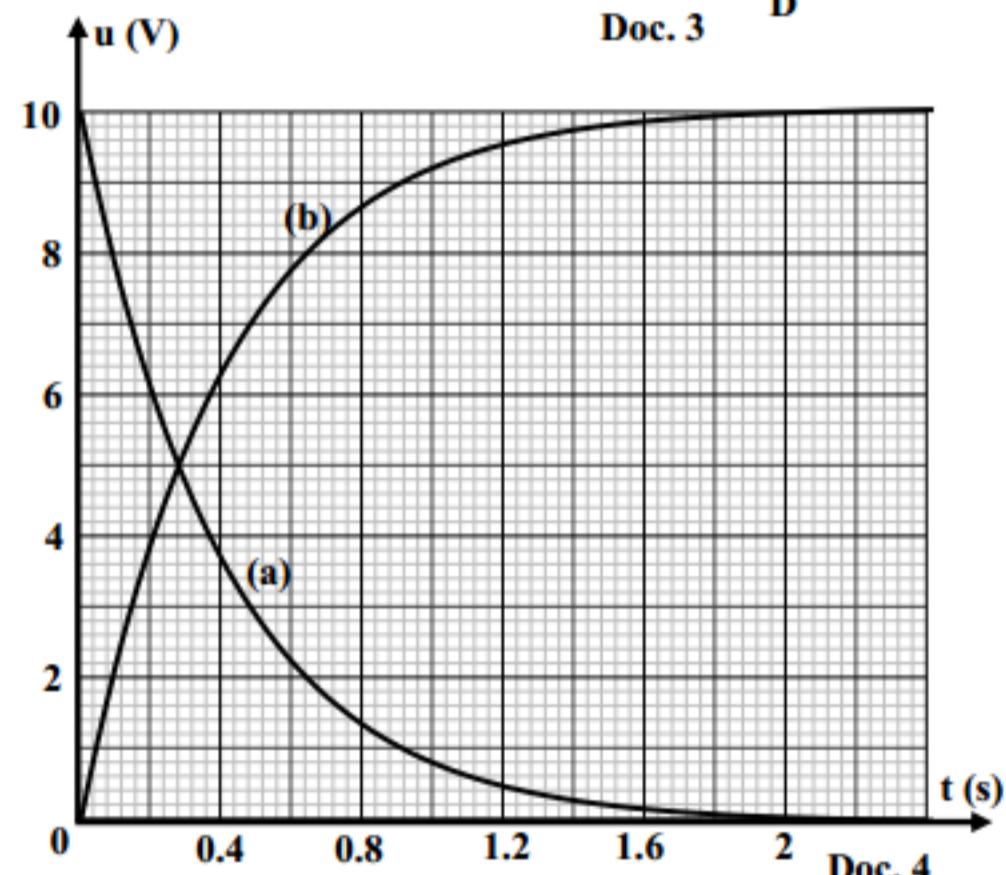
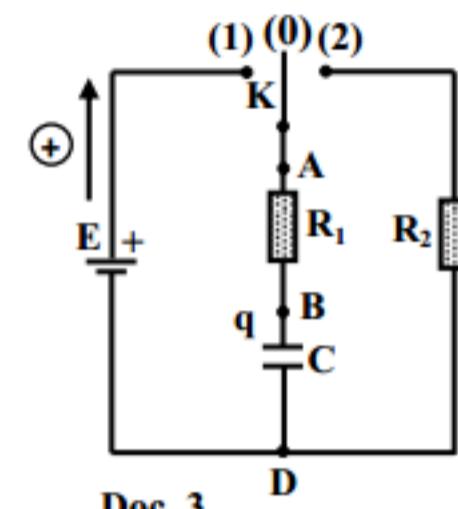
2.2) The solution of this differential equation is of the form: $u_C = E e^{\frac{-t}{\tau_2}}$ where τ_2 is the time constant of the circuit of document 5.

Determine the expression of τ_2 in terms of R and C .

2.3) Verify that the time needed by the capacitor to practically become completely discharged is $t_2 = 5 \tau_2$.

3) Duration of charging and discharging the capacitor

Show, without calculation, that « t_2 » is greater than « t_1 ».



Doc. 5

Exercise 3

Charging of a capacitor

PHYSICS

LS 2020*

The aim of this exercise is to determine the capacitance C of a capacitor. For this aim we set-up the series circuit of document 3 that includes:

- an ideal battery (G) of emf E ;
- a resistor (R) of resistance $R = 1 \text{ k}\Omega$;
- a capacitor (C), initially uncharged, of capacitance C ;
- a switch (K).

We close the switch K at the instant $t_0 = 0$, and the charging process starts. At an instant t , the capacitor (D) carries a charge q and the circuit carries a current i .

An oscilloscope, conveniently connected, allows to display the voltage $u_{\text{AM}} = u_R$ across the resistor.

- 1) Redraw the circuit of document 3 and show on it the connections of the oscilloscope.
- 2) Establish the differential equation that governs the variation of the voltage $u_{DF} = u_C$.

- 3) Show that $u_C = E \left(1 - e^{-\frac{t}{RC}} \right)$ is a solution of the established differential equation.

- 4) Deduce the expression of u_R in terms of E , R , C and t .

- 5) Document 4 shows u_R as a function of time.

- 5-1)** Show that the shape of the curve is in agreement with the exponential function of u_R .

- 5-2)** Spin the value of E .

- 6) The time constant τ of the $(R-C)$ series circuit is given by $\tau = RC$. Choose, among the four statements below, the two statements that describe correctly the charging phase of the capacitor. Justify your answer.

- Statement 1)** τ is the time during which the voltage across the resistor is 37% of its maximum value.

- Statement 2)** τ is the time during which the voltage across the resistor attains its maximum value.

- Statement 3)** τ is a physical quantity that permits to slow down the establishment of the steady state.

- Statement 4)** τ is the time during which the voltage across the capacitor will be equal to that across the resistor.

- 7) Using document 4, determine the value of τ .
- 8) Deduce the value of C .

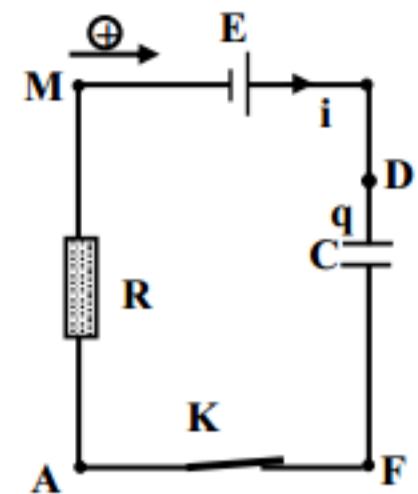
Exercise

Capacitance of a capacitor

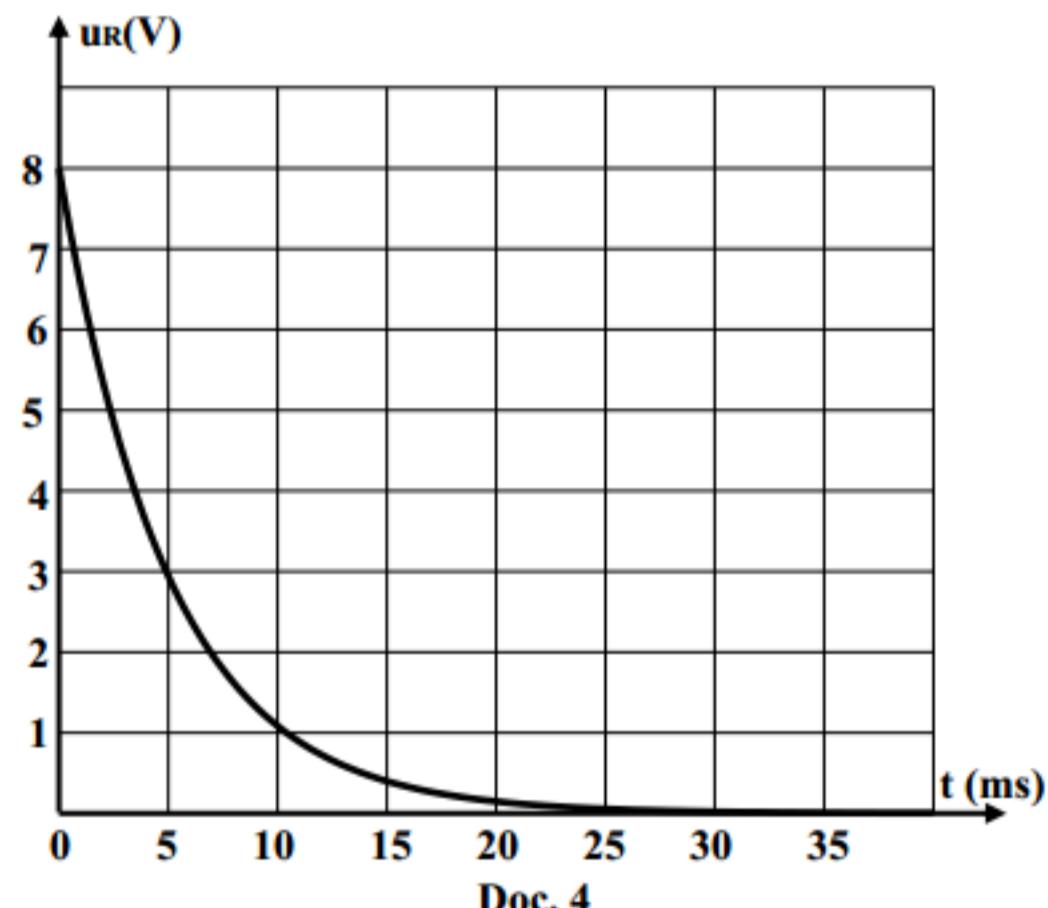
The aim of this exercise is to determine the capacitance C of a capacitor. We set-up the series circuit of document 4.

This circuit includes:

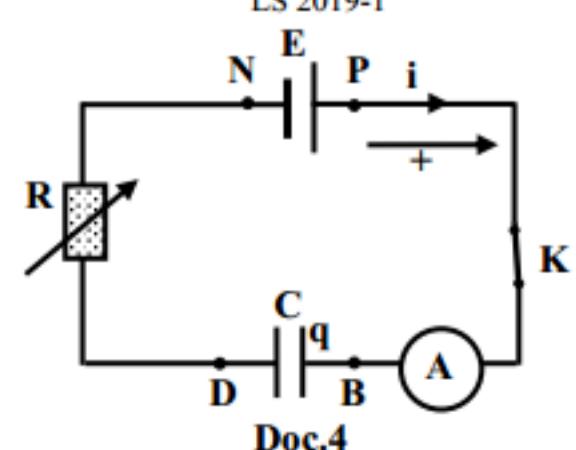
- an ideal battery of electromotive force $E = 10 \text{ V}$;
- a rheostat (R) of resistance R ;
- a capacitor (C) of capacitance C ;
- an ammeter (A) of negligible resistance;
- a switch K .



Doc. 3



Doc. 4



Doc.4

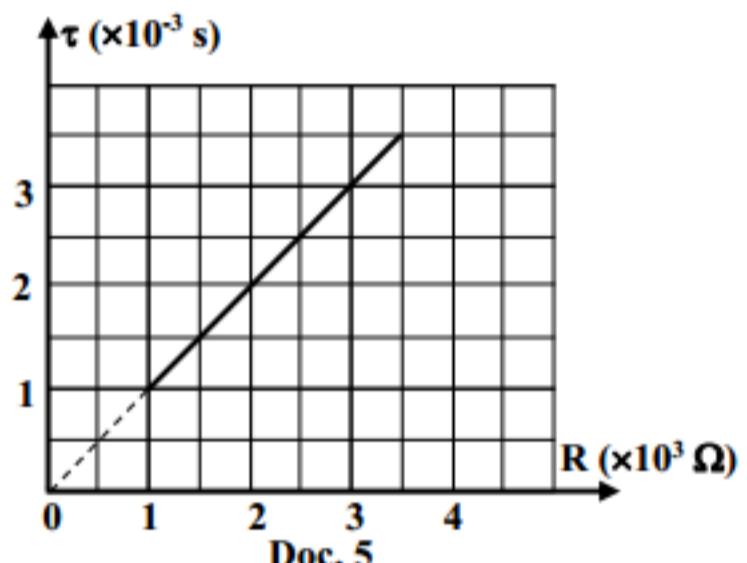
Initially the capacitor is uncharged. We close the switch K at the instant $t_0 = 0$.

At an instant t , plate B of the capacitor carries a charge q and the circuit carries a current i as shown in document 4.

- 1) Write the expression of i in terms of C and u_C , where $u_C = u_{BD}$ is the voltage across the capacitor.
- 2) Establish the differential equation that governs the variation of u_C .
- 3) The solution of this differential equation is of the form: $u_C = a + b e^{-\frac{t}{\tau}}$. Determine the expressions of the constants a , b and τ in terms of E , R and C .
- 4) Deduce that the expression of the current is: $i = \frac{E}{R} e^{\frac{-t}{RC}}$.
- 5) The ammeter (A) indicates a value $I_0 = 5 \text{ mA}$ at $t_0 = 0$. Deduce the value of R .
- 6) Write the expression of $u_R = u_{DN}$ in terms of E , R , C and t .
- 7) At an instant $t = t_1$, the voltage across the capacitor is $u_C = u_R$.

7-1) Show that $t_1 = R C \ln 2$.

7-2) Calculate the value of C knowing that $t_1 = 1.4 \text{ ms}$.



- 8) In order to verify the value of C , we vary the value of R . Document 5 represents τ as a function of R .
 - 8-1) Show that the shape of the curve in document 5 is in agreement with the expression of τ obtained in part 3.
 - 8-2) Using the curve of document 5, determine again the value of C .

Exercise 5 :

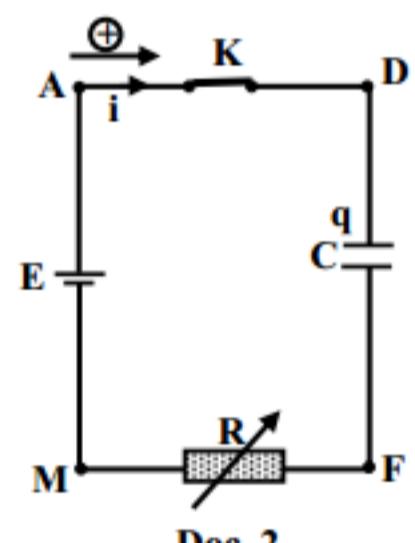
Effect of the resistance on the charging of a capacitor

PHYSICS
LS 2018-2

The aim of this exercise is to study the effect of the resistance of a resistor on the charging of a capacitor.

For this aim, we set-up the circuit of document 2 that includes:

- A capacitor, initially uncharged, of capacitance $C = 4 \mu\text{F}$;
- a resistor of adjustable resistance R ;
- an ideal battery of voltage $u_{AM} = E$;
- a switch K .



We close the switch at $t_0 = 0$, and the charging process starts.

1. Theoretical study

- 1.1) Derive the differential equation that describes the variation of the voltage $u_{DF} = u_C$ during the charging of the capacitor.
- 1.2) The solution of this differential equation has the form of: $u_C = A + B e^{Dt}$. Determine the constants A , B and D in terms of E , R and C .
- 1.3) Verify that the capacitor becomes practically fully charged at $t = 5 RC$.
- 1.4) Indicate the effect of the resistance of the resistor on the duration of the charging of the capacitor.

2. Experimental study

We adjust R to two different values R_1 and R_2 ; an appropriate device allows to trace, for each value of R, the voltage u_C as a function of time (Doc. 3).

- curve (a) corresponds to $R = R_1$.
- curve (b) corresponds to $R = R_2$.

2.1) Using the curves of document 3:

- specify the value of E;
- specify, without calculation, whether the value of R_2 is: equal to, greater than, or less than the value of R_1 ;
- determine the values of R_1 and R_2 .

2.2) The capacitor is fully charged, the electric energy stored in the capacitor is W_C .

- Is the value of W_C affected by the resistance of the resistor? Justify.
- Deduce the value of W_C .

Exercise 6 :

Measurement of the speed of a bullet

PHYSICS

LS 2006-1

In order to measure the speed of a bullet, a convenient setup is used. The principle of functioning of this setup is based on the charging of a capacitor.

A- Study of the charging of a capacitor

We are going to study the charging of a capacitor using a series circuit formed of a resistor of resistance R, a switch K and a capacitor of capacitance C initially neutral across the terminals of a generator of constant emf E and of negligible internal resistance (figure 1)

The switch K is closed at the instant $t_0 = 0$. The capacitor starts to charge. At the instant t, the circuit carries a current i and the armature A of the capacitor carries the charge q.

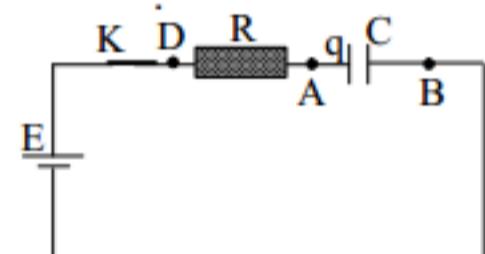


Figure 1

- Applying the law of addition of voltages, determine the differential equation that describes the variation of the voltage $u_C = u_{AB}$ across the capacitor as a function of time.
- a) Verify that $u_C = E(1 - e^{-\frac{t}{\tau}})$ is the solution of the differential equation where $\tau = RC$.
 - What does the time interval τ represent?
- After what time would the steady state be practically attained?

B – Measurement of the speed of a bullet

The setup used to measure the speed V of a bullet is represented in figure 2. $E = 100 \text{ V}$; $R = 1000 \Omega$; $C = 4 \mu\text{F}$; $L = 1 \text{ m}$.

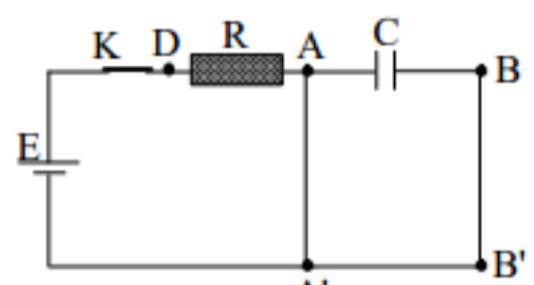


Figure 2

AA' and BB' are two thin parallel connecting wires lying in a vertical plane and are of negligible resistance. AA' and BB' are separated by a distance L.

The capacitor being neutral, the switch K is closed

- a) The potential difference between A and A' is zero. Why?
b) The charging of the capacitor did not start. Why?
- K being closed, we shoot the bullet normally at AA' and BB' with a speed V (fig.3).

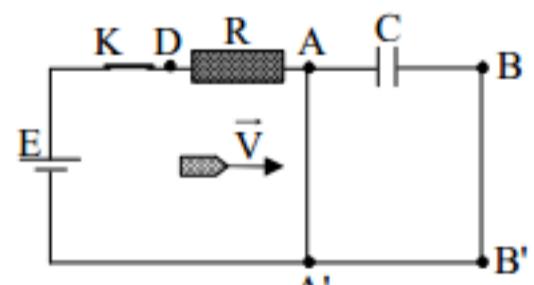


Figure 3

At the instant $t_0 = 0$, the bullet cuts the wire AA' and the capacitor starts to charge (fig.4).

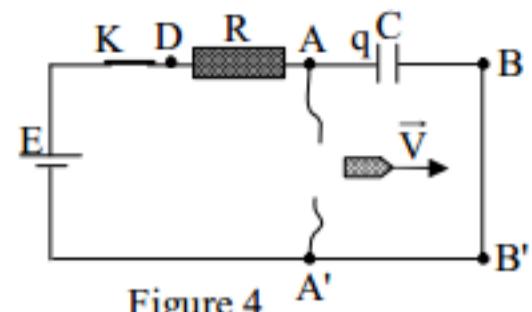
The bullet continues its motion which is considered uniform rectilinear of the same speed V.

At the instant t = 0, the bullet cuts the wire BB' and the phenomenon of charging starts. The voltage across the capacitor is then 45.7 V

a) Taking into consideration the study in part A, determine the time interval t taken by the bullet to cover the distance L.

b) Calculate

3) In order to measure precisely the value of V, the distance L between AA' and BB' must not exceed maximum value L_{\max} . Determine the value of L_{\max} .



PHYSICS
LS 2008-1

Exercise

Role of a capacitor in a circuit

The object of this exercise is to study the role of a capacitor in an electric circuit in two different cases. ($g = 10 \text{ m/s}^2$)

A- Variation of the current in a circuit

We connect two circuits whose diagrams are represented in the diagram below; the two identical lamps L_1 and L_2 are fed respectively with two identical generators G_1 and G_2 each of constant voltage E , the component (D) being a capacitor that is initially uncharged (Fig.1).

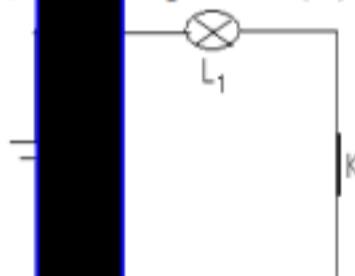
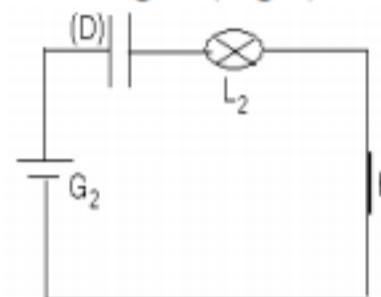


Figure 1



We close the two switches simultaneously at the instant $t_0 = 0$. we notice initially that L_1 and L_2 glow with the same brightness, but the brightness of the lamp L_2 decreases progressively and finally its light goes out, L_1 keeping its same brightness.

a) What can we say about the voltage across each of the lamps at the instant $t_0 = 0$? Justify.

b) i) How does the voltage across L_2 vary starting from the instant $t_0 = 0$?

ii) Deduce the value of the voltage across the capacitor when the light of L_2 goes out, the value of the voltage across the capacitor.

iii) Indicate the role of the capacitor in the variation of the current in an RC circuit fed by a DC voltage during the charging phase.

B- Energy stored in a capacitor

1- Qualitative study

Consider the experiment whose diagram is represented in figure (3), where (M) is a motor to which an body of mass m is suspended, a capacitor of large capacitance, G an ideal generator of constant voltage E and K₁ and K₂ are two switches.

In the first step of the experiment, we open K₂, and we close K₁.

In the second step of the experiment, we open K₁ and we close K₂. We observe that the body rises.

Explain what happens in each step of the experiment and tell why the body rises

2- Quantitative study

The capacitor has as a capacitance $C = 1 \text{ F}$, the body has a mass $m = 500 \text{ g}$ and the e.m.f of the generator is $E = 3 \text{ V}$

a- Calculate the energy initially stored in the capacitor.

b- Calculate the height rised by the body neglecting all energy losses.

c- What type of energy transfer did take place?

d- In fact the body rises 83 cm. Why?

e- Deduce the role of the capacitor in the previous circuit.

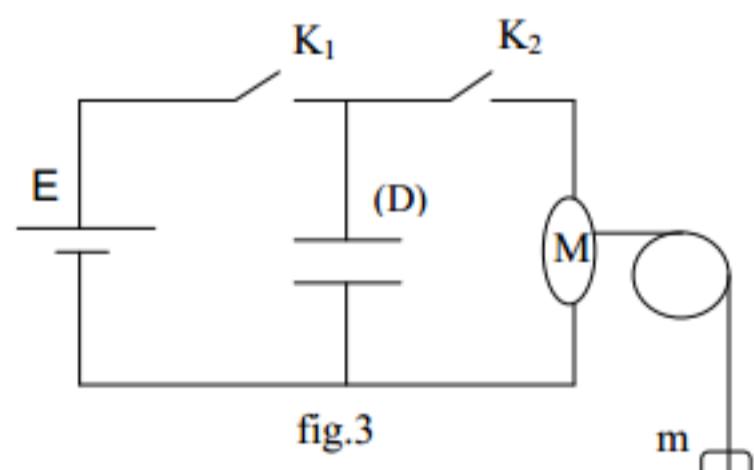


fig.3

Exercise 8 :**Brightness of a lamp**

The aim of this exercise is to study the brightness of a lamp in two experiments.

For this purpose, consider:

- an ideal battery of electromotive force $E = 9 \text{ V}$;
- a lamp L acting as a resistor of resistance $R = 10 \Omega$;
- a capacitor of capacitance $C = 0.1 \text{ F}$;
- a switch K.

Given that the brightness of the lamp increases with the increase of the current it carries and vice-versa.

1) First experiment: charging the capacitor

We connect the capacitor, initially uncharged, in series with the lamp and switch K across the battery (Doc. 4).

Switch K is closed at $t_0 = 0$, and the capacitor starts charging.

- 1.1)** Show that the differential equation that governs the variation of the

$$\text{voltage, } u_{DA} = u_C, \text{ across the capacitor is: } E = RC \frac{du_C}{dt} + u_C.$$

- 1.2)** The solution of the obtained differential equation is of the form:

$$u_C = E \left(1 - e^{-\frac{t}{\tau}}\right), \text{ where } \tau \text{ is constant.}$$

- 1.2.1)** Determine the expression of τ in terms of R and C.

- 1.2.2)** Calculate τ .

- 1.3)** Deduce that the expression of the charge current is $i = 0.9 \text{ e}^{-t} (\text{SI})$.

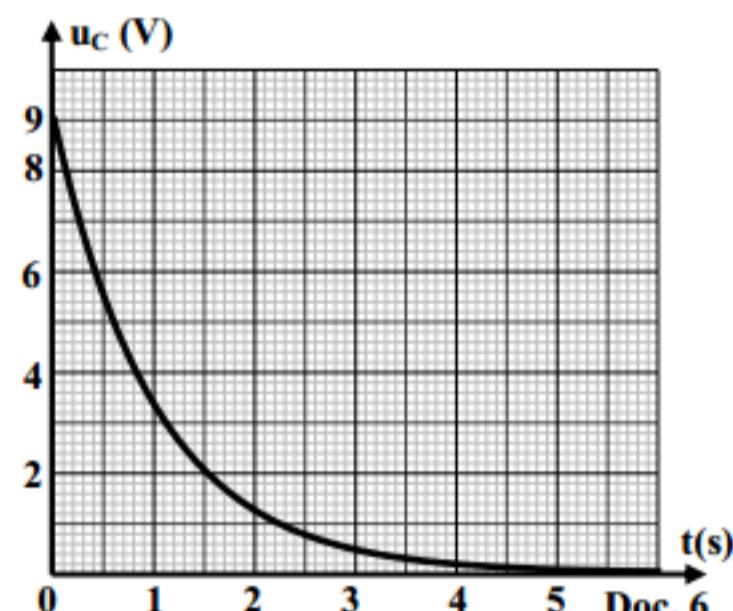
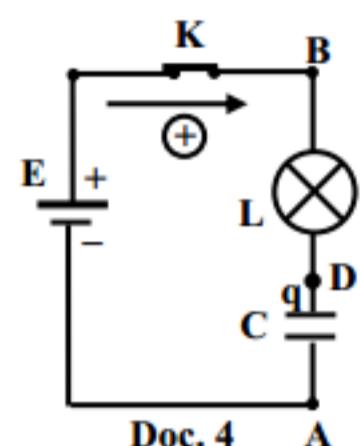
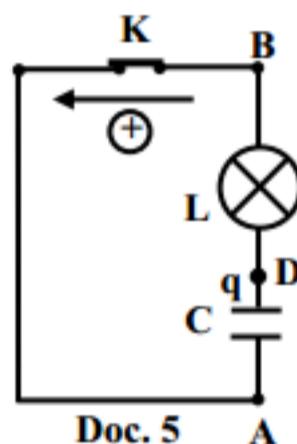
2) Second experiment: discharging the capacitor

The fully charged capacitor is connected in series with the lamp and switch K.

We close K at $t_0 = 0$ taken as a new initial time.

The capacitor discharges through the lamp (Doc. 5).

Document 6 shows the voltage $u_{DA} = u_C$ as a function of time.



- 2.1)** Use document 6 to determine the value of the time constant τ' of this RC circuit.

- 2.2)** Given that $u_C = E e^{-\frac{t}{\tau'}}$. Deduce the expression of the discharge current as a function of time.

3) Conclusion

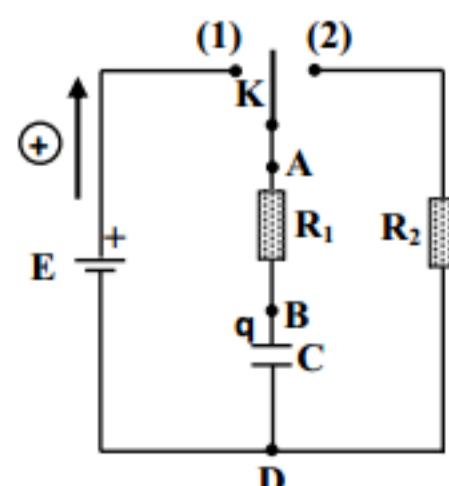
Using parts (1.3) and (2.2), describe the brightness of the lamp in the first and the second experiments during the time interval $[0, 5 \text{ s}]$. Justify your answer.

Exercise 9 :**Charging and discharging of a capacitor**

The aim of this exercise is to determine the capacitance of a capacitor by two different methods.

Consider the circuit represented in document 1. It is formed of an ideal generator that maintains across its terminals a constant voltage of value E , a capacitor of capacitance C , two resistors of resistances $R_1 = 10 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$ and a double switch K.

Doc. 1



1 – Charging the capacitor

The capacitor is initially neutral. At the instant $t_0 = 0$, we put K in position (1); the charging phenomenon of the capacitor starts.

1-1) Theoretical study

- 1-1-1)** Show that the differential equation that describes the

variation of the voltage $u_C = u_{BD}$ across the capacitor has the form: $E = R_1 C \frac{du_C}{dt} + u_C$.

- 1-1-2)** The solution of this differential equation has the form: $u_C = A(1 - e^{-\frac{t}{\tau_1}})$.

Determine the expressions of the constants A and τ_1 in terms of E, R_1 and C.

- 1-1-3)** Deduce that $u_C = E$ at the end of charging of the capacitor.

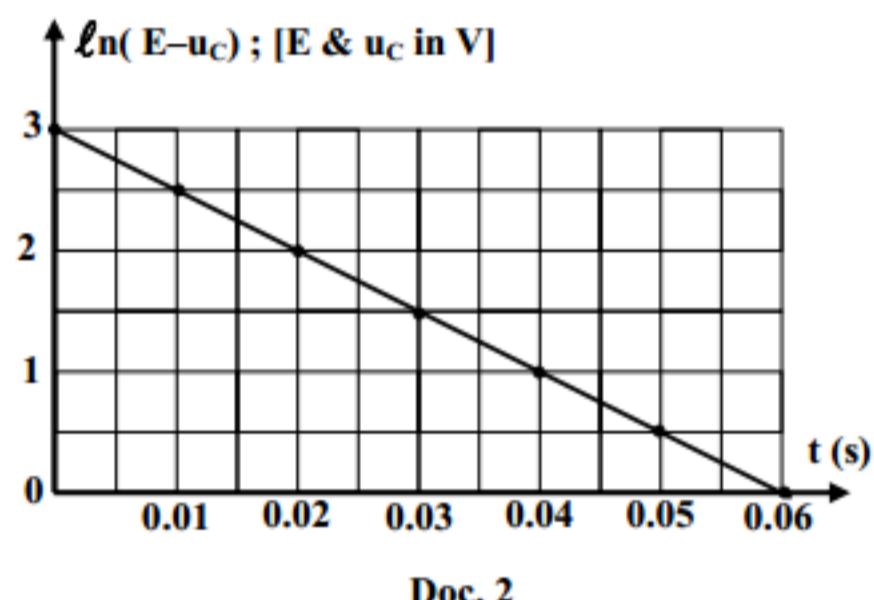
1-2) Experimental study

In order to determine the value of C, we use a convenient apparatus, which traces, during the charging of the capacitor, the curve representing $\ln(E - u_C) = f(t)$ (Doc. 2). [\ln is the natural logarithm]

- 1-2-1)** Determine, using the solution of the obtained differential equation, the expression of $\ln(E - u_C)$ in terms of E, R_1 , C and t.

- 1-2-2)** Show that the shape of the curve in document 2 is in agreement with the obtained expression of $\ln(E - u_C) = f(t)$.

- 1-2-3)** Using the curve of document 2, determine the values of E and C.



2 – Discharging the capacitor

The capacitor being fully charged. At an instant taken as a new origin of time $t_0 = 0$, the switch K is placed at position (2); thus the phenomenon of discharging of the capacitor starts (Doc. 3).

2-1) Theoretical study

- 2-1-1)** Show that the differential equation in the voltage $u_C = u_{BD}$ across the capacitor has the form: $u_C + \alpha \frac{du_C}{dt} = 0$; where α is a constant to be determined in terms of R_1 , R_2 and C.

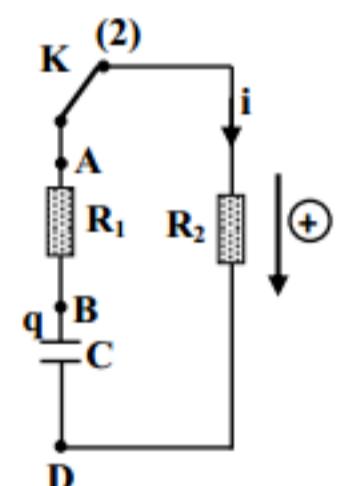
- 2-1-2)** The solution of this differential equation has the form: $u_C = E e^{-\frac{t}{\tau_2}}$ where τ_2 is constant. Show that $\tau_2 = \alpha$.

2-2) Experimental study

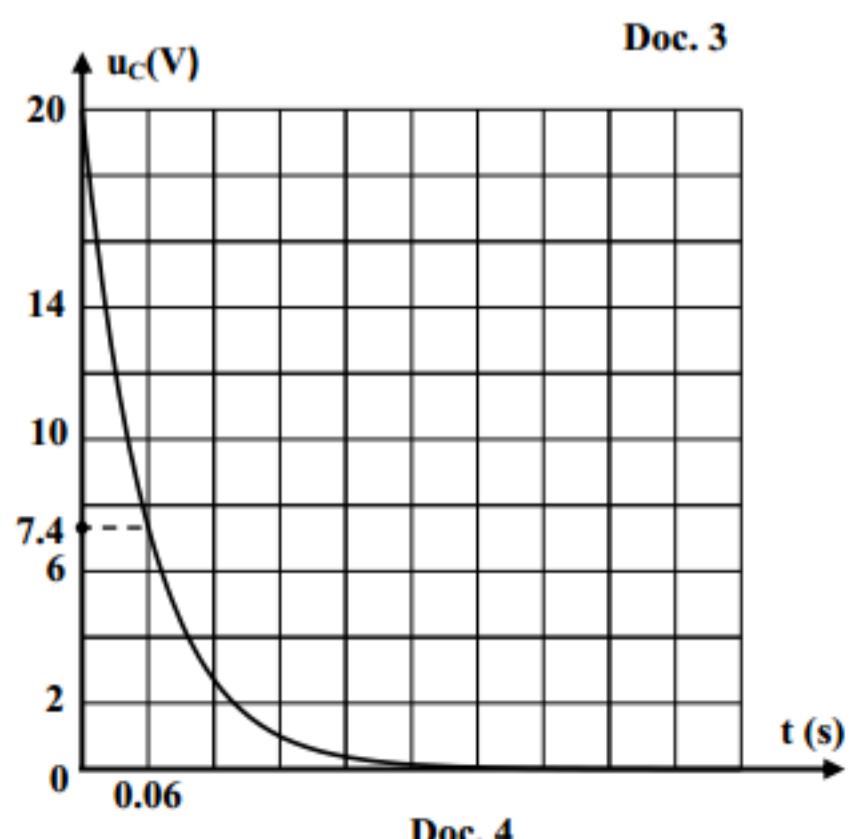
The variation of the voltage u_C across the capacitor as a function of time is represented in document 4.

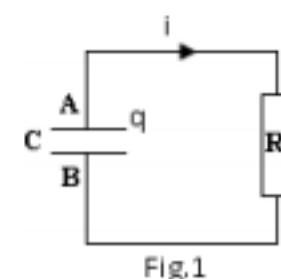
- 2-2-1)** Determine, using document 4, the value of the time constant τ_2 of the discharging circuit.

- 2-2-2)** Deduce the value of C.



Doc. 3





A capacitor of capacitance C is initially charged under a voltage E .

At $t_0 = 0$, we connect across the terminals of the capacitor a resistor of resistance $R = 1 \text{ k}\Omega$ (Fig. 1).

At an instant t , the armature A carries the charge $q > 0$ and the circuit carries a current i .

A – Theoretical study

- 1) Write the relation between i and q .
- 2) Show that the differential equation of the voltage $u_C = u_{AB}$ across the capacitor is $\frac{du_C}{dt} + \frac{1}{RC} u_C = 0$.
- 3) The solution of this differential equation is $u_C = D e^{-\frac{t}{\tau}}$. Determine the expressions of the constants D and τ in terms of E , R and C .
- 4) Show that after a time $t = \tau$, the voltage across the capacitor attains 37% of its maximum value E .

B – Determination of the capacitance C

In order to determine the value of C , we use a convenient apparatus, which traces, during the discharging of the capacitor, two curves representing $u_C = g(t)$ (Fig. 2) and $\ln(u_C) = f(t)$ (Fig. 3)

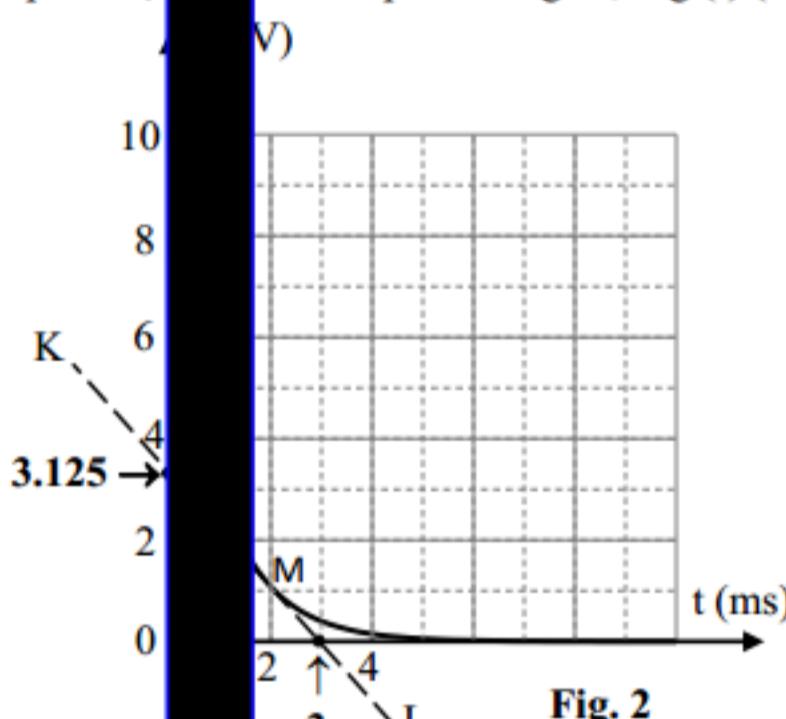


Fig. 2

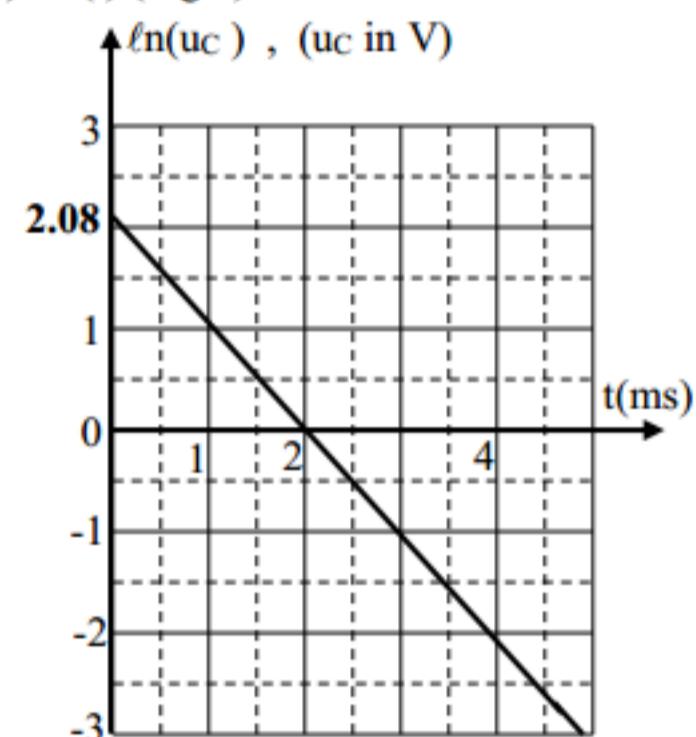


Fig. 3

We proceed in three following methods:

1) First method

Referring to the curve of figure 2:

- a) give the value of E ;
- b) using the result of part (A – 4) determine the value of τ and deduce the value of C .

2) Second method

The figure 2 shows also the tangent KL to the curve at point M (2 ms, 1 V).

- a) Referring to this figure determine the slope of the tangent at point M .
- b) Determine the value of C .

3) Third method

- a) Determine the expression of $\ln(u_C)$ in terms of E , R , C and t .
- b) Show that the shape of the curve in figure 3 is in agreement with the obtained expression of the function $\ln(u_C) = f(t)$.
- c) Referring to the curve of figure 3, determine again the values of E and C .

Exercise

Charging and discharging of a capacitor

The aim of this exercise is to determine, by two different methods, the value of the capacitance C of a capacitor. For this aim, we set-up the circuit of figure 1. This circuit is formed of an ideal generator delivering a constant voltage of value $E = 10 \text{ V}$, a capacitor of capacitance C , two identical resistors of resistances $R_1 = R_2 = 10 \text{ k}\Omega$ and a double switch K .

A – Charging the capacitor

The switch K is in the position (0) and the capacitor is neutral. At the instant $t_0 = 0$, we turn K to position (1) and the charging of the capacitor starts.

1) Theoretical study

- a) Applying the law of addition of voltages and taking the positive direction along the circuit as that of the current, show that the differential equation that describes the variation of the voltage

$$u_C = u_{BD} \text{ across the capacitor has the form: } E = R_1 C \frac{du_C}{dt} + u_C .$$

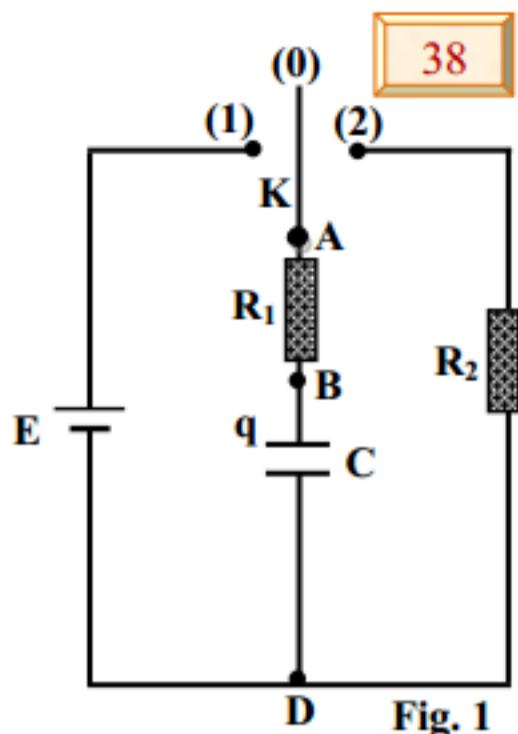


Fig. 1

- b) The solution of this differential equation has the form: $u_C = A(1 - e^{-\frac{t}{\tau_1}})$ where A and τ_1 are constants.
Show that $A = E$ and $\tau_1 = R_1 C$.
- c) Show that at the end of charging $u_C = E$.
- d) Show that the expression $u_{AB} = u_{R_1} = E e^{-\frac{t}{R_1 C}}$.
- e) Establish the expression of the natural logarithm of $u_{R_1} [\ln(u_{R_1})]$ as a function of time.

2) Graphical study

The variation of $\ln(u_{R_1})$ as a function of time is represented by figure 2.

- a) Justify that the shape of the obtained graph agrees with the expression of $\ln(u_{R_1})$ as a function of time.

- b) Deduce, using the graph, the value of the capacitance C.

B – Discharging the capacitor

The capacitor being fully charged, we turn the switch K to position (2). At an instant $t_0 = 0$, taken as a new origin of time, the discharging of the capacitor starts.

- 1) During discharging, the current circulates from B to A in the resistor of resistance R_1 . Justify.
- 2) Taking the positive direction along the circuit as that of the current, show that the differential equation in the voltage u_C across the capacitor has the form: $u_C + (R_1 + R_2) C \frac{du_C}{dt} = 0$.
- 3) The solution of the above differential equation has the form:
 $u_C = E e^{-\frac{t}{\tau_2}}$ where τ_2 is the time constant of the circuit during discharging. Show that $\tau_2 = (R_1 + R_2) C$.
- 4) The variation of the voltage u_C across the capacitor and the tangent to the curve $u_C = f(t)$ at the instant $t_0 = 0$, are represented in figure 3. Deduce, from this figure, the value of the capacitance C.

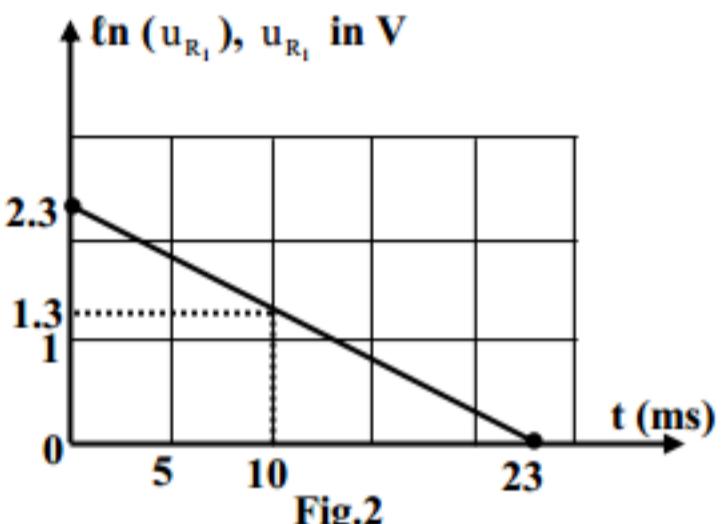


Fig. 2

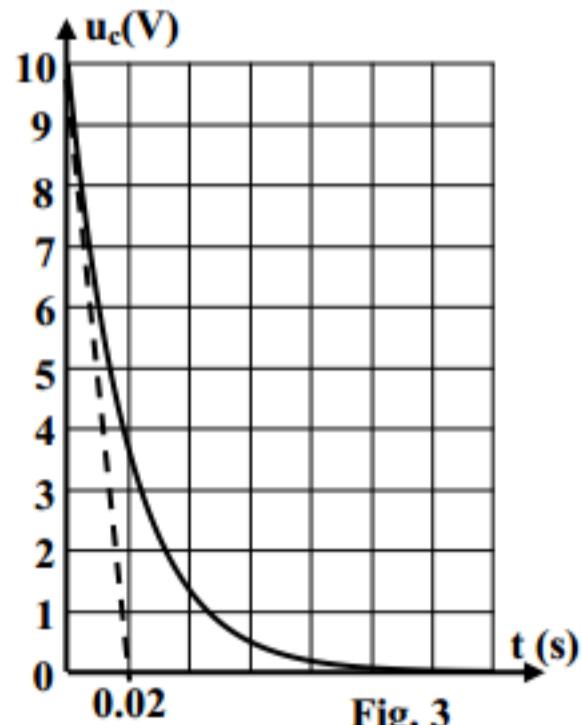


Fig. 3

Exercise 12 :**Determination of the characteristics of an electric component**

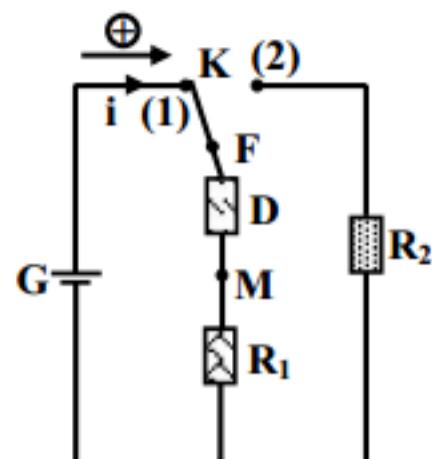
LS 2016-1

An electric component (D), of unknown nature, which may be a resistor or a pure coil of inductance L or a capacitor of capacitance C.

To determine the nature and the characteristic of (D) we consider the following:

- An ideal generator G of constant electromotive force (e.m.f) E;
- Two resistors of resistances $R_1 = 100 \Omega$ and $R_2 = 150 \Omega$;
- A double switch K.

We set up the circuit of figure 1.

**Fig.1****A – First Experiment**

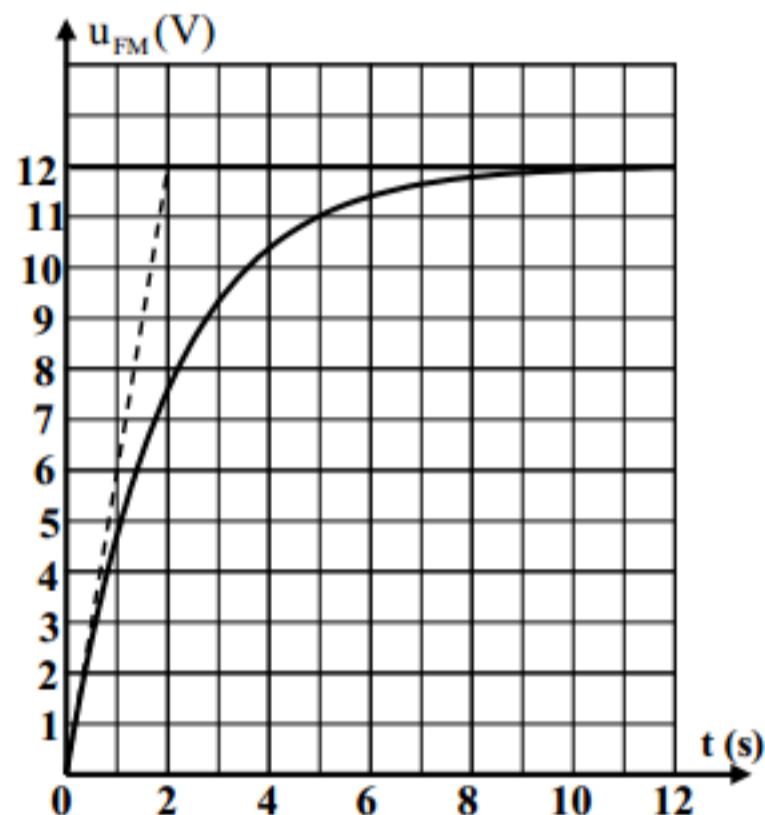
At an instant $t = 0$, the switch K is turned to position (1). Figure 2 shows the variation of the voltage u_{FM} across the terminals of the capacitor D as a function of time and the tangent to this curve at $t_0 = 2$ s.

- 1) The component (D) is a capacitor. Justify.
- 2) Indicate the value of the e.m.f E of the generator.
- 3) Calculate at $t_0 = 0$, the current carried by the circuit.
- 4) Derive the differential equation describing the variation of the voltage $u_C = u_{FM}$.
- 5) The solution of the differential equation has the form:

$$u_C = u_{FM} = A + B e^{-\frac{t}{\tau}}.$$

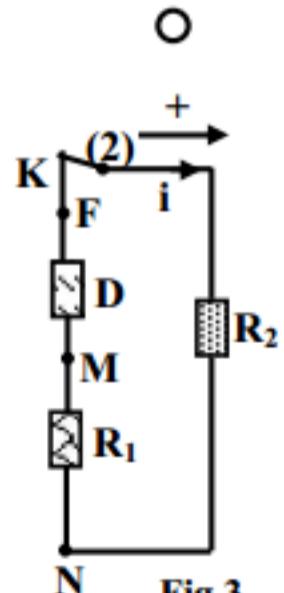
Determine the expressions of the constants A, B and τ in terms of C and E.

- 6) Determine graphically, the value of the time constant τ .
- 7) Deduce the value of C.

**Fig. 2****B – Second Experiment.**

During the charging of the capacitor and at an instant t_1 , we turn the switch K to the position (2) (see figure 3).

- 1) Name the phenomenon that takes place.
- 2) The resistor R_2 can support a maximum power of $P_{max} = 0.24$ W.
 - a) Calculate the maximum value of the current which can pass through R_2 without damaging it (the thermal power is given by the relation: $p = R i^2$).
 - b) Applying the law of addition of voltages, show that the maximum voltage across the terminals of the capacitor is $u_{FM} = 10$ V so that R_2 will not be damaged.
 - c) At the instant t_1 the current is maximum. Determine, graphically, the maximum duration $t_2 - t_1$ of the charging process of the capacitor so that the resistor R_2 will not be damaged.

**Fig.3**

Exercise 13**Determination of the resistance of a resistor**

We intend to determine the resistance R of a resistor (R). We thus connect up the circuit represented in figure (1) that is formed of an ideal generator of e.m.f $E = 5$ V, the resistor (R), an uncharged capacitor (C) of capacitance $C = 33 \mu\text{F}$ and a double switch (K).

A – Charging of the capacitor

- 1) We intend to charge the capacitor. To what position, 1 or 2, must then (K) be moved?
- 2) The circuit reaches a steady state after a certain time. Give then the value of the voltage u_{AB} across (C) and that of the voltage across (R).

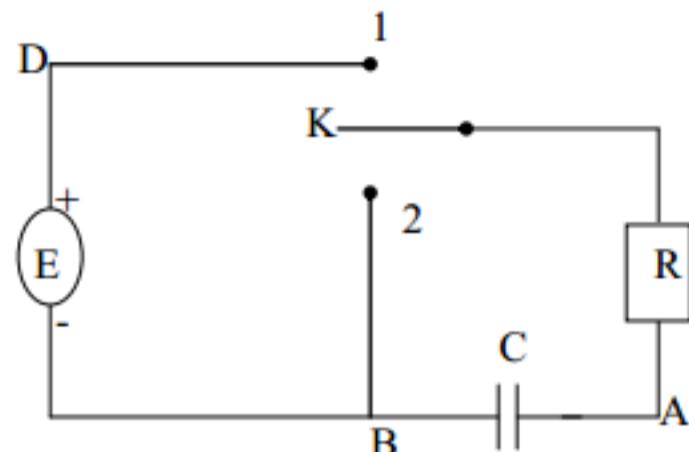


Fig. 1

B – Discharging of the capacitor

- 1) Draw a diagram of the circuit during the discharging of the capacitor and show on it the direction of the current it carries.
- 2) Derive the differential equation in $u_C = u_{AB}$ during the discharging.
- 3) The solution of this differential equation has the form :

$$u_C = E e^{-\frac{t}{\tau}} \quad (u_C \text{ in V}, t \text{ in s})$$

where τ is a constant.

- a) Determine the expression of τ in terms of R and C .
- b) Determine the value of u_C at the instant $t_1 = \tau$.
- c) Give, in terms of τ , the minimum duration needed at the end of which the capacitor is practically totally discharged.
- d) Derive the expression of $\ln u_C$, the natural logarithm of u_C , in terms of E , τ and t .
- e) The diagram of figure 2 represents the variation of $\ln u_C$ as a function of time.

Referring to the graph of figure 2, determine the value of R .

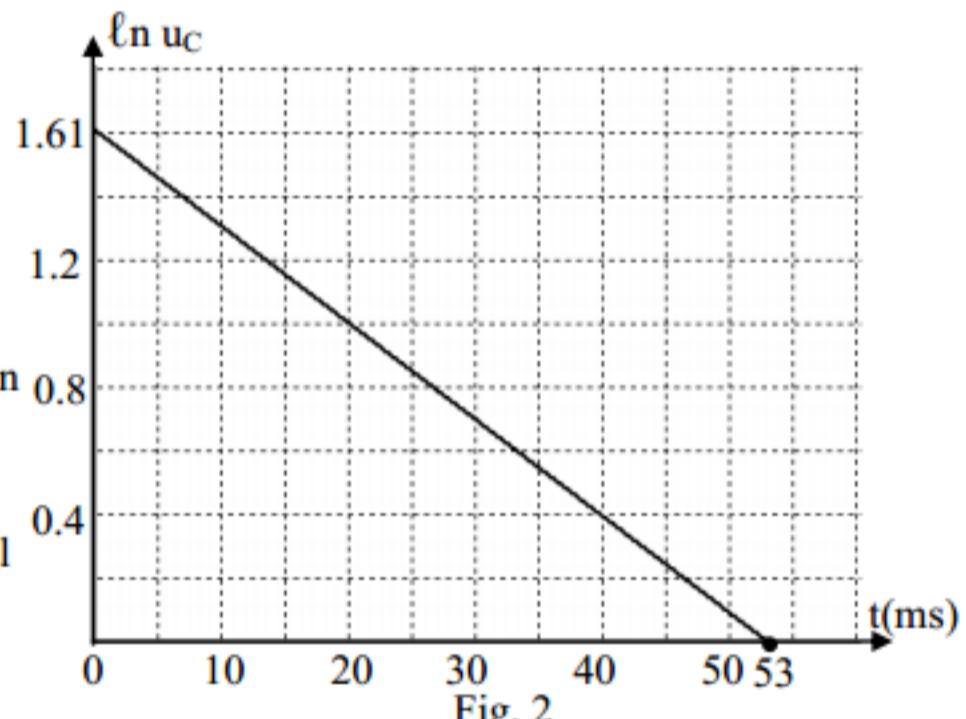


Fig. 2

Light: Diffraction

Chap.13



Monochromatic light

1

This source of light emits radiation of same frequency .

1

Phenomenon of diffraction of light

(Wave aspect)

When the light meets a sharp obstacle, or passes through a narrow hole or slit < 1mm it undergoes the diffraction.

2

Diffraction pattern

3

- We observe different fringes, bright and dark, situated on both sides of the central bright fringe that is the brightest one
- The fringes are collinear with a direction perpendicular to that of the slit.

Formulas

4

Dark fringes Positions

$$\theta_n = n \frac{\lambda}{a} \quad \text{for } \theta_n \text{ very small}$$

Case of a linear slit

5

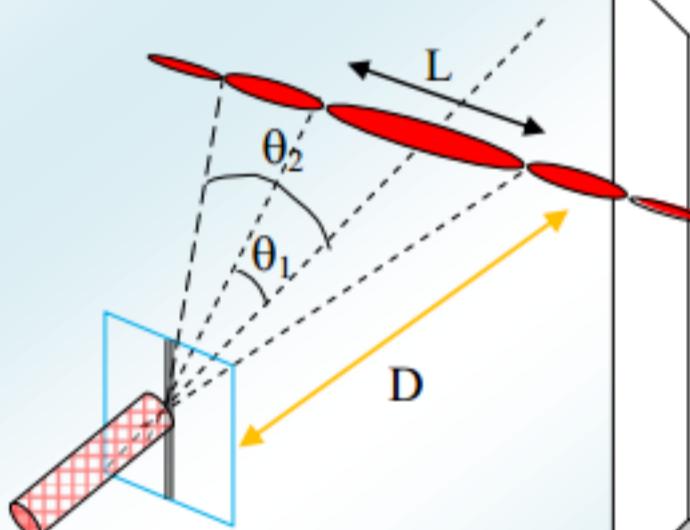
Central fringe

Angular width :

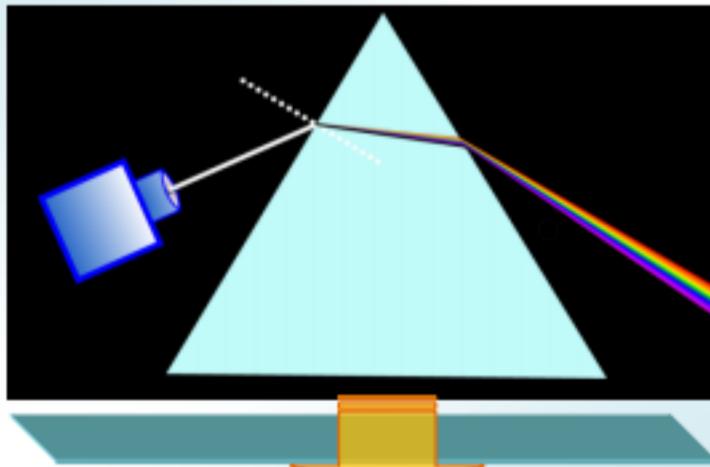
$$\alpha = 2 \frac{\lambda}{a}$$

Linear width :

$$L = 2 \frac{\lambda D}{a}$$

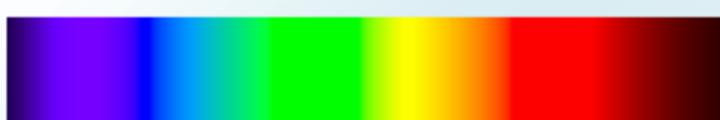


Scattering of poly-chromatic light by a prism



Continuous visible spectrum

6



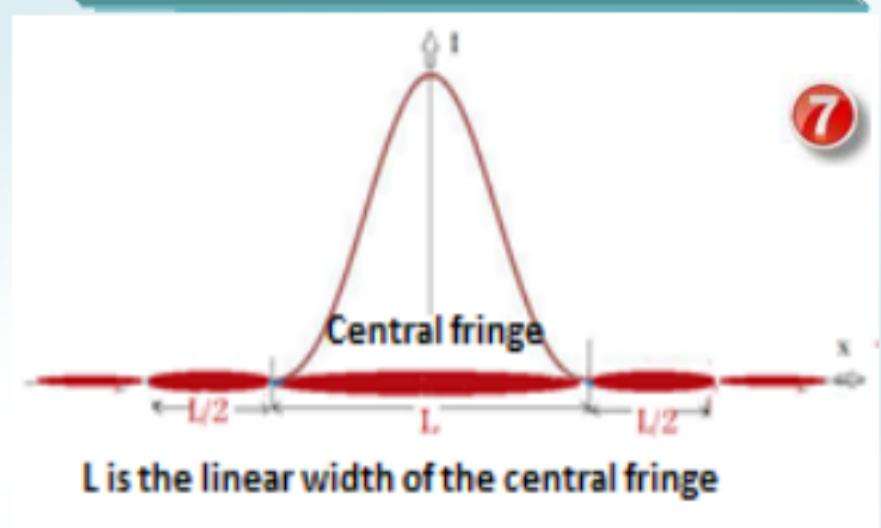
$$3,75 \cdot 10^{14} < f_{Hz} < 7,5 \cdot 10^{14}$$

or in Vacuum : $400 < \lambda_{nm} < 800$

violet red

Monochromatic

Radiations of same frequencies

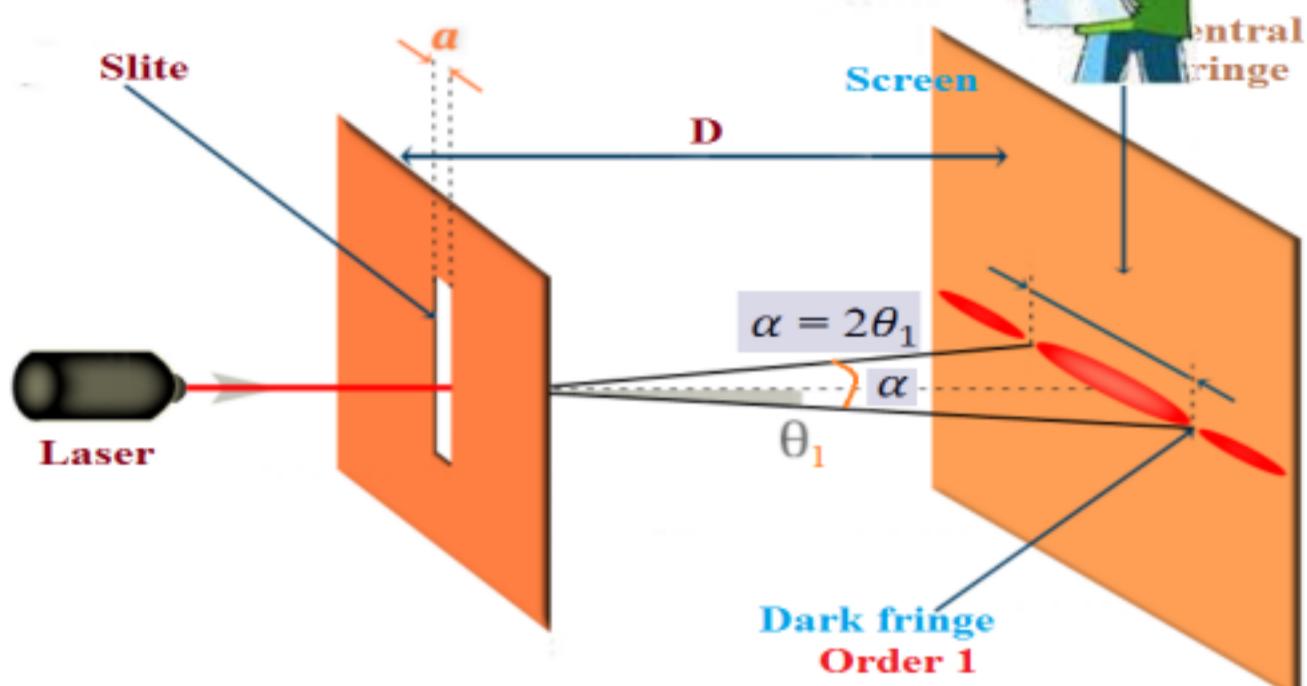


Diffraction

Chap.

13

Exercises



Exercise 1 :

Determination of the wavelength of a laser light

The monochromatic light emitted by a laser source, of wavelength λ , illuminates, under normal incidence, a very narrow slit F_1 of width $a_1 = 0.1$ mm cut in an opaque screen (E_1). The phenomenon of diffraction is observed on a screen (E_2) parallel to (E_1), found at a distance $D = 4$ m from it (fig. 1).

The central bright fringe on (E_2) has a linear width = 5 cm.

- 1) Describe the diffraction pattern observed on (E_2).
- 2) The phenomenon of diffraction shows evidence of a certain aspect of light. What is it?
- 3) Calculate the angular width of the central bright fringe.
- 4) Calculate the value of λ .

PHYSICS
GS-2001_1

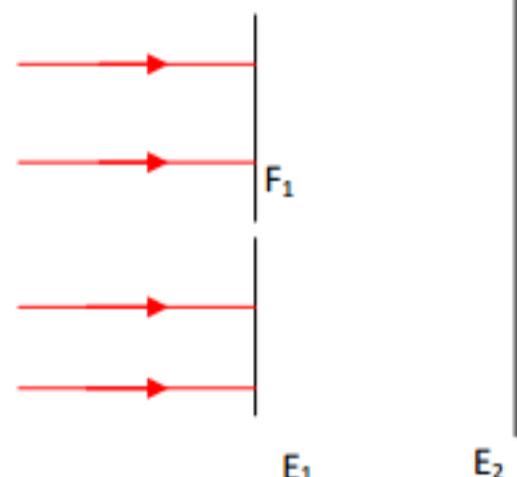


fig. 1

Exercise 2 :

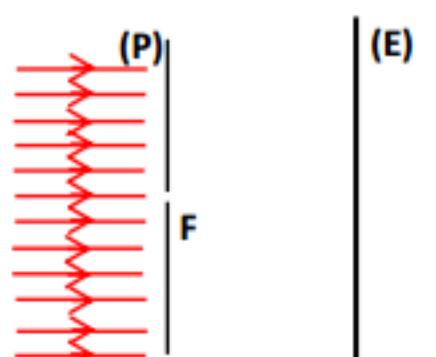
Sources and interference

A source of monochromatic radiation of wavelength λ in air illuminates under normal incidence a horizontal slit F of adjustable width a cut in an opaque screen (P). A screen of observation (E) is placed parallel to (P) at a distance $D = 5$ m (Fig.1).

- I) For $\lambda = 0.5 \mu\text{m}$, show on a diagram the shape of the luminous beam emerging from the slit in each of the two following cases :
 - width of the slit $a = 2 \text{ cm}$.
 - width of the slit $a = 0.4 \text{ mm}$.

- 2) Which phenomenon will take place when the value of a is in the order of 1 mm ?

PHYSICS
GS-2007_2

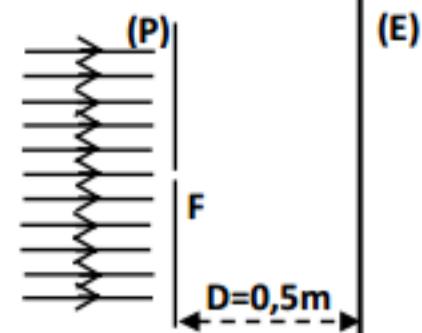


Exercise 3**Central fringe****PHYSICS**

GS-2007_2

The width of the slit is now kept at 0.4 mm and the radiation used belongs to the visible spectrum.
(wavelength of light in the visible spectrum : $0.4 \mu\text{m} \leq \lambda \leq 0.8 \mu\text{m}$)

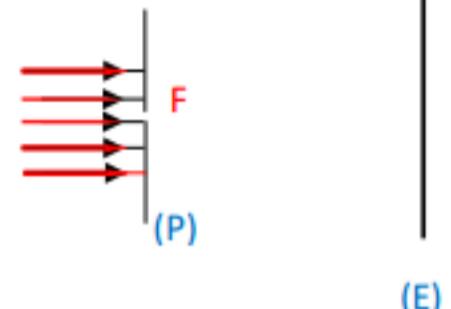
- 1)** Write, in this case, the expression giving angular width of the central bright fringe in terms of λ and a .
- 2)** Show that the linear width of this central fringe is given by : $L = \frac{2D\lambda}{a}$.
- 3)** Calculate the linear widths L_{red} and L_{violet} , when using successively a red radiation ($\lambda_{\text{red}} = 0.8 \mu\text{m}$) and a violet radiation ($\lambda_{\text{violet}} = 0.4 \mu\text{m}$).
- 4)** We illuminate the slit with white light. We observe over the linear width L_{violet} white light. Justify.

**Exercise 4****Diffraction****PHYSICS**

GS-2010_2

A laser beam illuminates, under normal incidence, a straight slit F, of width a , on an opaque screen (P). The light through F is received on a screen (E), parallel to (P) and found at 3 m from (P) (Fig.1).

- 1)** Describe what would be observed on (E) in the two following cases:
 - a)** $a = a_1 = 0.5 \text{ mm}$.
 - b)** $a = a_2 = 0.05 \text{ mm}$.
- 2)** It is impossible to isolate a luminous ray by reducing the size of the slit. Why?
- 3)** We use the slit of width $a_2 = 0.5 \text{ mm}$. The width of the central fringe of diffraction is 7.2 mm. Show that the wavelength of the light used is $\lambda = 600 \text{ nm}$.
- 4)** We remove the slit F and we replace it by a hair of diameter d on the screen (P). A hair of diameter d is stretched in the place of the slit F. We obtain on the screen (E) a diffraction pattern. The measurement of the width of the central fringe of diffraction gives 12 mm. Determine the value of d .

**Exercise 5****Aspect of light****PHYSICS**

GS-2011-1

Consider a source S emitting a monochromatic luminous visible radiation of frequency $\nu = 6.163 \times 10^{14} \text{ Hz}$.

Given: $c = 3 \times 10^8 \text{ m/s}$; $h = 6.62 \times 10^{-34} \text{ J}\cdot\text{s}$; $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$.

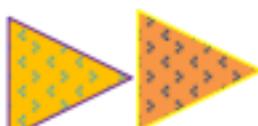
- A –** This source illuminates a very thin slit that is at a distance of 10 m from a screen. A pattern, extending over a large width, is observed on the screen.

- 1)** Due to what phenomenon is the formation of this pattern?
- 2)** Determine the width of the slit knowing that the linear width of the central fringe is 40 cm.

- B –** The same source illuminates now the two slits of Young's double slit apparatus, these slits are vertical and are separated by a distance $a = 1 \text{ mm}$. A pattern is observed on a screen placed parallel to the plane of the slits at a distance $D = 2 \text{ m}$ from this plane.

Describe the observed pattern and calculate the interfringe distance i .

- C –** What aspect of light do the two preceding experiments show evidence of ?

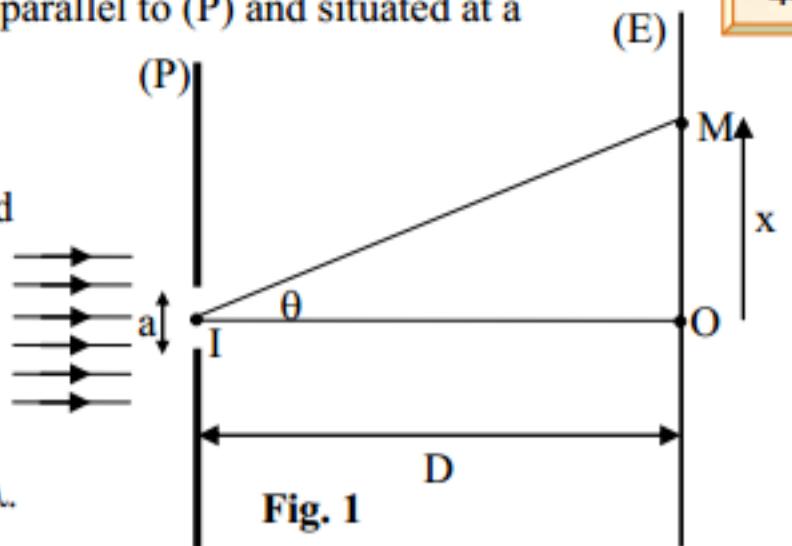


Problems

Visit [website physpro](http://physpro.com) **PHYSICS**
Diffraction GS-2012-2

A laser source emits a monochromatic cylindrical beam of light of wavelength $\lambda = 640 \text{ nm}$ in air. This beam falls normally on a vertical screen (P) having a horizontal slit F_1 of width a .

The phenomenon of diffraction is observed on a screen (E) parallel to (P) and situated at a distance $D = 4 \text{ m}$ from (P).



- 1) Write the expression of θ in terms of a and λ .
- 2) Determine the expression of $OM = x$ in terms of a , D and λ .
- 3) Determine the value of a if $OM = 1.28 \text{ cm}$.
- 4) We replace the slit F_1 by another slit F'_1 of width 100 times larger than that of F_1 . What do we observe on the screen (E)?

Problem 2:

Applications of the diffraction of light

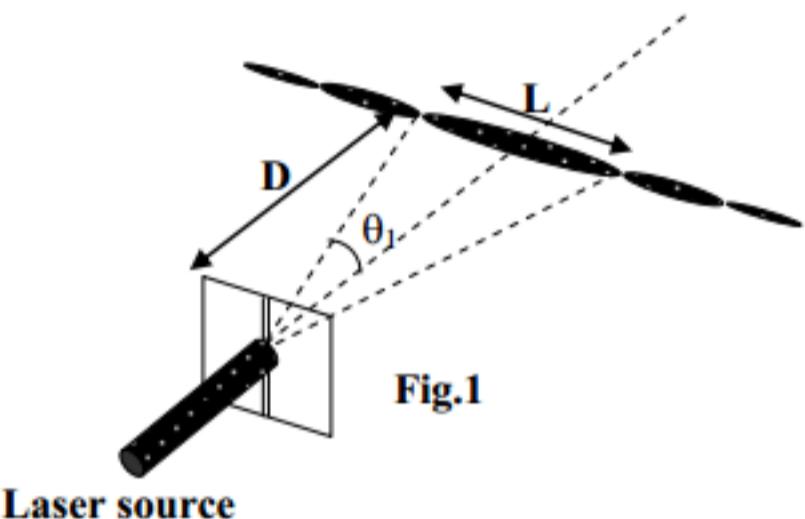
PHYSICS
LS 2013-2

A – Measurement of the width of a slit

A laser beam of light, of wavelength in vacuum $\lambda = 632.8 \text{ nm}$, falls normally on a vertical slit of width « a ». The diffraction pattern is observed on a screen placed perpendicularly to the laser beam at a distance $D = 1.5 \text{ m}$ from the slit.

Let « L » be the linear width of the central fringe (Fig. 1). The angle of diffraction θ corresponding to a dark fringe of order n is given by $\sin \theta = \frac{n\lambda}{a}$ where $n = \pm 1, \pm 2, \pm 3 \dots$

For small angles, take $\tan \theta \approx \sin \theta \approx \theta$ in radian.



- 1) Describe the aspect of the diffraction pattern observed on the screen.
- 2) Write the relation among a , θ_1 and λ .
- 3) Establish the relation among a , λ , L and D .
- 4) Knowing that $L = 6.3 \text{ mm}$, calculate the width « a » of the used slit.

B – Controlling the thickness of thin wire

A manufacturer of thin wires wishes to control the diameter of his product. He uses the same set-up mentioned in part (A) but he replaces the slit by a thin vertical wire. He observes on the screen the phenomenon of diffraction (figure 2).

For $D = 2.60 \text{ m}$, he obtains a central fringe of constant linear width $L_1 = 3.4 \text{ mm}$.

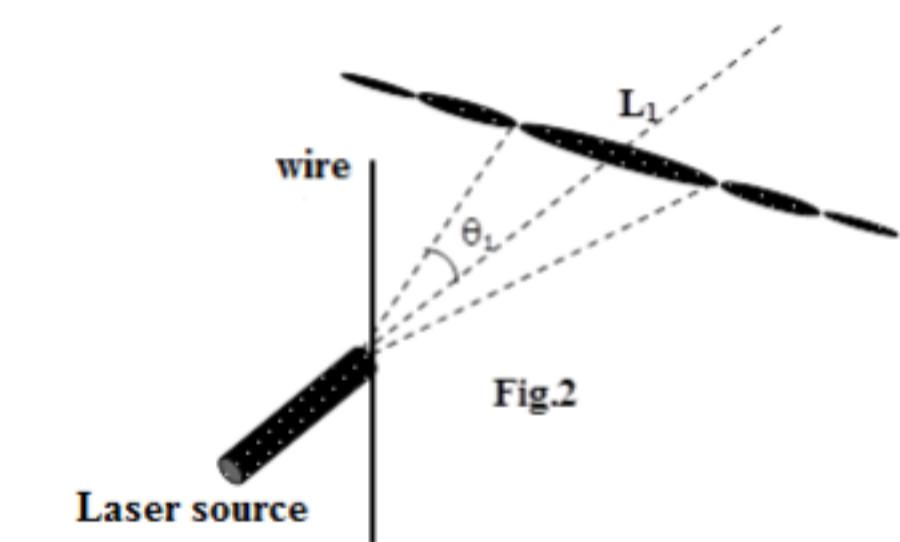
- 1) Calculate the value of the diameter « a_1 » of the wire at the illuminated point.
- 2) The manufacturer illuminates the wire at different positions under the same precedent conditions. Specify the indicator that permits the manufacturer to check that the diameter of the wire is constant.

C – Measurement of the index of water

We place the whole set-up of part (A) in water of index of refraction n_{water} . We obtain a new diffraction pattern.

We find that for $D = 1.5 \text{ m}$ and $a = 0.3 \text{ mm}$, the linear width of the central fringe is $L_2 = 4.7 \text{ mm}$.

- 1) Calculate the wavelength λ' of the laser light in water.
- 2) a) Determine the relation among λ , λ' and n_{water} .
- b) Deduce the value of n_{water} .



(Corpuscular aspect)

Aspects of light

(Corpuscular)

The light is formed of tiny particles called photons. These photons are of zero mass and zero electric charges.

Energy of the photon

$$E_{\text{pho}} = h\nu = h \frac{c}{\lambda}$$

with h : Planck's constant

Photoelectric effect**Definition :**

It's the emission of electrons from the surface of a metal when illuminated by a suitable radiation.

Definitions

④

Threshold wavelength

Def. : This is the maximum wavelength (of the radiation) below it, this radiation can extract electrons from the surface

condition of emission

$$\lambda < \text{or equal to } \lambda_s$$

⑤

Threshold frequency ν_s

Def. : This is the minimum frequency ν_s (of a radiation), above it, this radiation can extract electrons from the surface

condition of emission

$$\nu > \text{or equal to } \nu_s$$

⑥

Work function W_0

Def. : This is the minimum energy needed to extract an electron from the surface of a metal

$$W_0 = h \frac{c}{\lambda_0}$$

condition of emission

$$E_{\text{photon}} > \text{or equal to } W_0$$

KE_{max} of an extracted electron from the metal

$$KE = E_{\text{pho}} - W_0$$

We face three cases :

- 1) $E_{\text{pho}} < W_0$ → KE = 0 *no emission*
- 2) $E_{\text{pho}} = W_0$ → KE = 0
- 3) $E_{\text{pho}} > W_0$ → KE = $E_{\text{pho}} - W_0$

Source of radiation and power

$$P = N \cdot E_{\text{photon}} = N h \nu$$

with N : nb of emitted pho/sec

Effective photons and efficiency

$$r = \frac{N_{\text{pho effective}}}{N_{\text{pho received}}}$$

Note : Always $r < 1$

Photoelectric effect

Chap.

16

Exercises



Problem 1:

The two aspects of light

PHYSICS
GS-2003_1

We cover a metallic plate by a thin layer of cesium whose threshold wavelength is $\lambda_o = 670 \text{ nm}$.

Then we illuminate it with a monochromatic radiation of wavelength in vacuum $\lambda = 480 \text{ nm}$.

A convenient apparatus is placed near the plate in order to detect the electrons emitted by the illuminated plate.

- 1- This emission of electrons by the plate shows evidence of an effect. What is that effect?
- 2- What does the term "threshold wavelength" represent?
- 3- Calculate, in J and eV, the extraction energy (work function) of the cesium layer.
- 4- What is the form of energy carried by an electron emitted by the plate? Give the maximum value of this energy.

Given: Planck's constant: $h = 6.6 \times 10^{-34} \text{ J.s}$; speed of light in vacuum: $c = 3 \times 10^8 \text{ m/s}$; $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

Problem 2:

Photoelectric Effect

PHYSICS
GS 2005-1

The experiments on photoelectric emission performed by Millikan around the year 1915, intended to determine the kinetic energy K.E of the electrons emitted by metallic cylinders of potassium (K) and cesium (Cs) when these cylinders are illuminated by monochromatic radiation of adjustable frequency ν .

The object of this exercise is to determine, performing similar experiments, Planck's constant (h), as well as the threshold frequency ν_o of potassium and the extraction energy W_o of potassium and that of cesium.

- I - 1) What aspect of light does the phenomenon of photoelectric effect show evidence of ?
- 2) A monochromatic radiation is formed of photons. Give two characteristics of a photon.
- 3) For a given pure metal, the incident photons of a monochromatic radiation provoke photoelectric emission. Give the condition for this emission to take place.

K.E the electrons corresponding to frequency ν of the incident radiation.

The obtained results are tabulated in the following table:

Given : 1 eV = 1.6×10^{-19} J.

1- Using Einstein's relation about photoelectric effect, show that the kinetic energy of an extracted electron may be written in the form :

$$K.E = a\nu + b.$$

2- a) Plot , on graph paper, the curve

representing the variation of the kinetic energy K.E versus ν , using the following scale:

- on the axis of abscissas: 1 cm represents a frequency of 10^{14} Hz
- on the axis of ordinates: 1 cm represents a kinetic energy of 0.5 eV.

b) Using this graph, determine:

i) the value, in SI, of h , the Planck's constant.

ii) the threshold frequency ν_0 of potassium.

3- Deduce the value of the extraction energy W_0 of potassium.

III- In a second experiment using cesium, we obtain the following values: $K.E = 1$ eV for $\nu = 7 \times 10^{14}$ Hz.

1) Plot, with the same specification ,on the preceding system of axes, the graph of the variation of K.E as a function of ν .

2) Deduce from this graph the extraction energy W_0 of cesium.

ν (Hz)	E_c (eV)
6×10^{14}	0,25
7×10^{14}	0,65
8×10^{14}	1,05
9×10^{14}	1,45
10×10^{14}	1,85

Problem 3

Photoelectric effect

PHYSICS GS_2007_2

A source of wavelength $\lambda = 0.5 \mu\text{m}$ in air illuminates separately two metallic plates, one made of cesium and the other of zinc.

The table below gives, in eV, the values of the extraction energy W_0 (work function) for some metals.

M	Cesium	Rubidium	Potassium	Sodium	Zinc
W ₀	1.89	2.13	2.15	2.27	4.31

Given : $h = 6.62 \times 10^{-34}$ J.s ; $1 \text{ eV} = 1.6 \times 10^{-19}$ J ; $c = 3 \times 10^8$ m/s.

- 1) Calculate in J and in eV, the energy of an incident photon.
- 2) For which metal would photoelectric emission take place? Justify.
- 3) Calculate in eV the maximum kinetic energy of an emitted electron.
- 4) The cesium plate receives a monochromatic luminous beam of wavelength in air $\lambda = 0.5 \mu\text{m}$, of power 3978×10^{-4} W. The number of electrons emitted per second is then $n = 10^{16}$.
 - a) Calculate the number N of photons received by the plate in one second.
 - b) The quantum efficiency r of the plate is the ratio of the number of the electrons emitted per second to the number of photons received by the plate during the same time. Calculate r.

Duality wave-particle

The wave theory of light is used to interpret the phenomenon of diffraction. This theory is not able to interpret the photoelectric effect. Why?

Problem 4

Photoelectric Effect

PHYSICS LS_2009_2

A) The photoelectric effect was discovered by Hertz on 1887. The experiment represented in figure 1 may show evidence of this effect. A zinc plate is fixed on the conducting rod of an electroscope. The whole setup is charged negatively.

If we illuminate the plate by a lamp emitting white light rich with ultraviolet radiations (U.V), the leaves F and F' of the electroscope approach each other rapidly.

- 1) Due to what is the approaching of the leaves?
- 2) The photoelectric effect shows evidence of an aspect of light. What is this aspect?

B) The experiments performed by Millikan towards 1915, intended to determine the maximum kinetic energy K.E of the electrons emitted by metallic plates when Zinc plate illuminated by monochromatic radiation of adjustable wavelength λ in vacuum.

In an experiment using a plate of cesium, a convenient apparatus allows us to measure the maximum kinetic energy K.E of an emitted electron corresponding to the wavelength λ of the incident radiation. The variation of K.E as a function of λ is represented in the graph of figure 2.

The aim of this part is to determine the value of Planck's constant h and that of the extraction energy W_0 of cesium.

- 1) Write down the expression of the energy E of an incident photon, of wavelength λ in vacuum, in terms of λ , h and c .
- 2) a) Applying Einstein's relation about photoelectric effect, show that the maximum kinetic energy K.E of an extracted electron may be written in the form

$$K.E = \frac{a}{\lambda} + b, \text{ where } a \text{ and } b \text{ are constants.}$$

- b) Deduce the expression of the threshold wavelength λ_0 of cesium in terms of W_0 , h and c .

- 3) Referring to the graph:

- a) Give the value of the threshold wavelength λ_0 of cesium;
- b) Determine the value of W_0 and that of h .

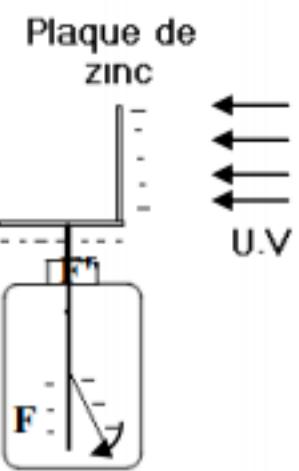


Fig.1

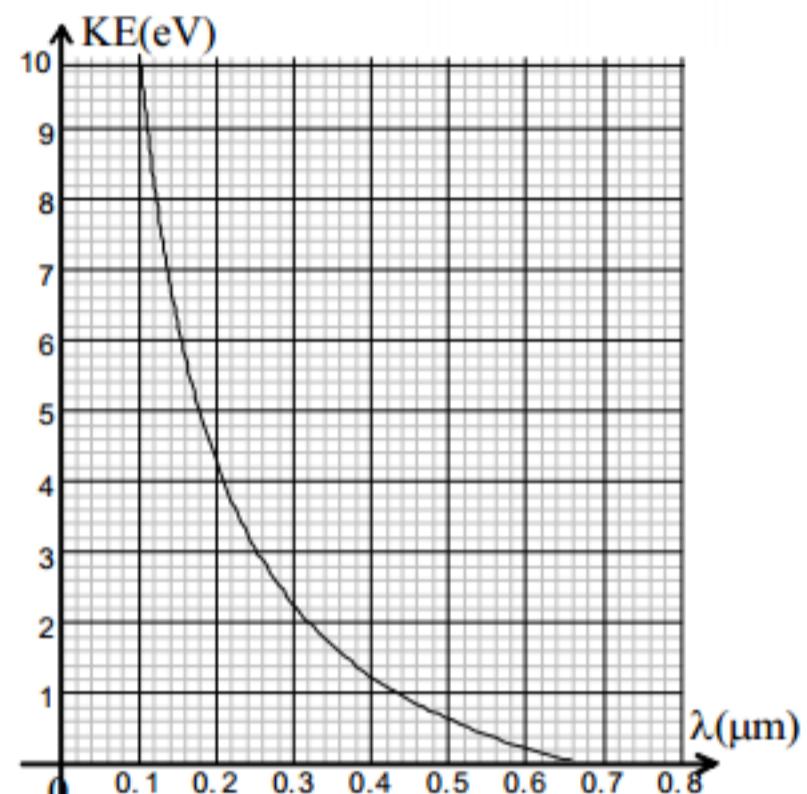


Fig.2

PHYSICS LS-2010-2

Problem 5 :

Photoelectric effect

A metallic plate, covered with a layer of cesium, is illuminated with a monochromatic luminous beam of wavelength $\lambda = 0.45 \times 10^{-6} \text{ m}$ in vacuum.

The work function (extraction energy) of cesium is $W_0 = 1.88 \text{ eV}$.

A convenient apparatus (D) is used to detect the electrons emitted by the illuminated plate.

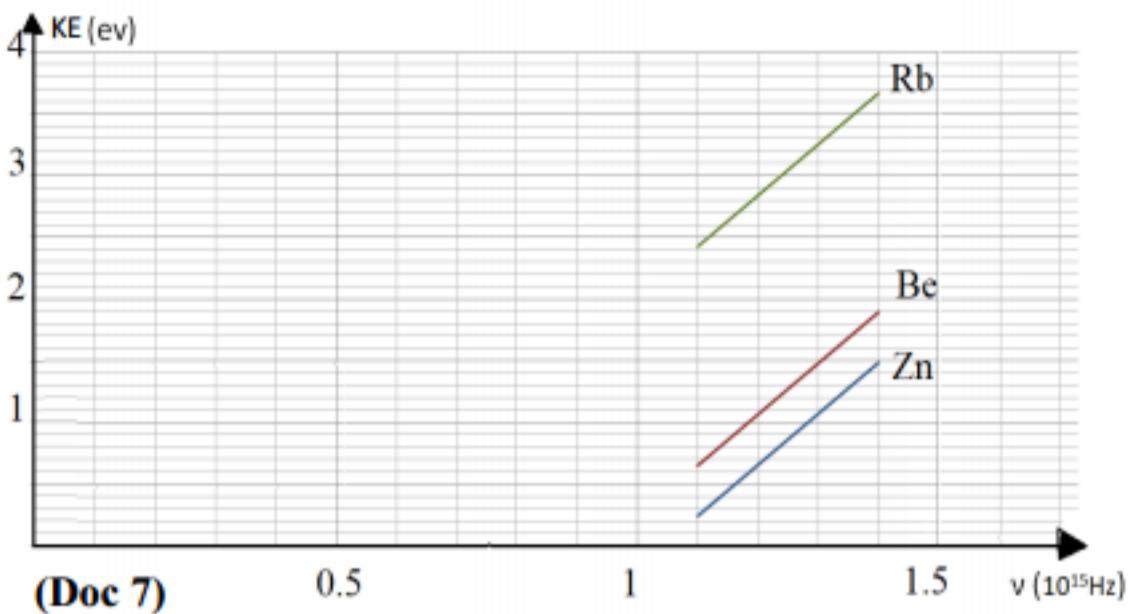
- 1) What aspect of light does the phenomenon of photoelectric effect show evidence of ?
- 2) Define the term "work function" of a metal.
- 3) The luminous beam illuminating the metallic plate is formed of photons.
 - a) i) Write down the expression of the energy E of a photon in terms of h , c and λ .
 - ii) Calculate, in eV, the energy of an incident photon.
 - b) (D) detects electrons emitted by the plate.
Why do we have an emission of electrons by the plate?
- c) Calculate, in eV, the maximum kinetic energy of an emitted electron.
- 4) The luminous power P received by the plate is 10^{-3} W , and the emitted electrons form a current
 - a) Calculate the number n of photons received by the plate in one second. $I = 5 \mu\text{A}$.
 - b) Knowing that the current I is related to the number N of the electrons emitted per second and to the elementary charge e by the relation: $I = N \times e$. Calculate N .
 - c) i) Calculate the quantum efficiency $r = \frac{N}{n}$.
 - ii) Deduce that the number of effective photons in one second is relatively small.

Problem 6:**Photoelectric effect**

An experimenter uses a mono-energetic electromagnetic radiation source of adjustable frequency ν to illuminate, respectively, three metal plates, one in Zinc (Zn), another one in Beryllium (Be) and a third one in Rubidium (Rb). The experimenter varies the frequency ν of the incident radiation and records, for each value of ν , the value of the maximum kinetic energy of an electron emitted by each of the three plates in the table (Doc 6). He obtains the graph giving $KE = f(\nu)$ for each of the three plates, these graphs being represented in (Doc 7). Take: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$; $c = 3 \times 10^8 \text{ m/s}$.

$\nu (10^{15} \text{ Hz})$	KE (eV)		
	Zn	Be	Rb
1.10	0.241	0.651	2.421
1.15	0.448	0.858	2.628
1.20	0.655	1.065	2.835
1.25	0.862	1.272	3.042
1.30	1.069	1.479	3.249
1.35	1.276	1.686	3.456
1.40	1.483	1.893	3.663

(Doc 6)



(Doc 7)

- 1) We notice that the photoelectric effect does not occur for some visible and infrared incident radiation regardless of the intensity of radiation and the duration of exposure. Why does this result defy the wave theory of light?
- 2) Indicate the aspect of light that the photoelectric effect phenomenon shows in evidence.
- 3) Interpret, based on Einstein's hypothesis relative to the photoelectric effect, the fact that the three graphs are parallel line segments.
- 4) Calculate, referring to (Doc 6), the value of Planck's constant.
- 5) Determine, referring to the graphs in (Doc 7), the threshold frequency of each of the metal plates.
- 6) Deduce the value of the extraction energy corresponding to each of the metal plates.
- 7) The experimenter illuminates each plate with an incident radiation of wavelength, in vacuum, 333 nm.
 - 7-1) Specify, for each plate, whether there is an emission of electrons or not.
 - 7-2) Calculate, in case we have an emission of electrons, the maximum kinetic energy of an emitted electron.
- d) We increase the luminous power P received by the plate without changing the wavelength λ . Would the current increase or decrease? Why?

Problem 7:**Photoelectric effect**

PHYSICS
GS 2016 - 1

A hydrogen lamp of power $P_S = 2\text{W}$, emits uniformly radiation in all directions in a homogeneous and non-absorbing medium. This lamp illuminates a potassium cathode C of a photoelectric cell of work function $W_0 = 2.20 \text{ eV}$ and of a surface area $s = 2\text{cm}^2$ placed at a distance $D = 1.25\text{m}$ from the lamp (figure 1).

- 1) Calculate the threshold wavelength of the potassium cathode.
- 2) Among the rays of Balmer series, specify the radiation that can produce photoelectric emission.
- 3) Using a filter we illuminate the cell by a blue light H_β of wavelength $\lambda = 486.13\text{nm}$. The generator G is adjusted so that the anode (A) captures all the emitted electrons by the cathode of quantum efficiency $r = 0.875\%$.
 - a) Show that the received power of the radiation P_0 of the cell is $2.04 \times 10^{-5}\text{W}$.
 - b) Determine the number N_0 of the incident photons received by the cathode C in one second.
 - c) Determine the current in the circuit.

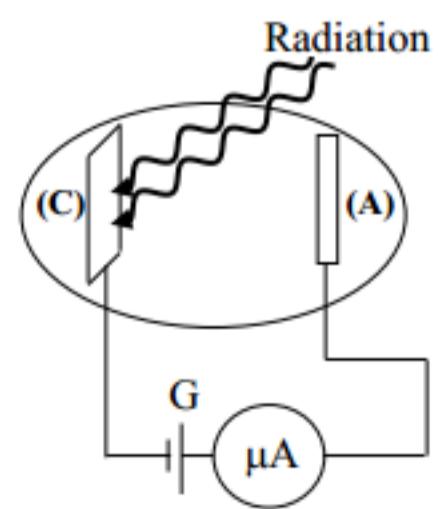
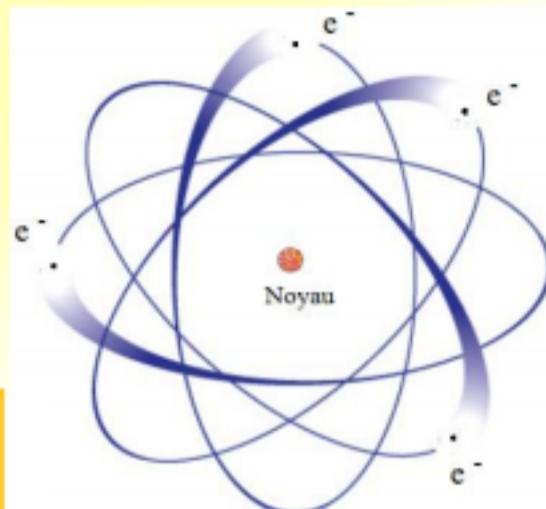


Figure 1

Bohr model

The energy of the atom is quantified

The values of energy are discrete, that means discontinuous values.



Energy of the photon and the wavelength

During a transition from a level m to p

$$E_{\text{pho}} = |E_m - E_p| = h \frac{c}{\lambda_{mp}}$$

$$\text{then } \lambda = hc/\Delta E$$

Hydrogen Atom**Balmer series**

$$E_n = -\frac{13.6}{n^2} \text{ ev}$$

Ionization state
Excited states

$n=1$ ground state

Emission spectrum

Transition $n > 2 \rightarrow n=2$

$400\text{nm} \leq \lambda \leq 800\text{nm}$ in vacuum

These are the visible rays

$$H_\alpha : 3 \rightarrow 2 ; H_\beta : 4 \rightarrow 2$$

$$H_\gamma : 5 \rightarrow 2 ; H_\delta : 6 \rightarrow 2$$

Rydberg formula

$$\frac{1}{\lambda_{n,2}} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

Transition:
 $n > 2 \rightarrow n=2$
light

Some definitions :

a- Fundamental state

b- Ionization state

c- Ionization energy

Answers

- a) Corresponding to the stable state, having minimum energy.
- b) This is the state corresponding to the level where the energy $E=0$ (ionized atom).
- c) This is the minimum energy needed to ionize the atom, taken in its ground state to be ionized.

Another spectral series of the hydrogen atom H

- **Lyman series:** $n > 1$ to $n = 1$ { U.V }
Ultraviolet : $\lambda < 400\text{nm}$.
- **Paschen series:** $n > 3$ to $n = 3$ { I.R. }
Infrared : $\lambda > 800\text{nm}$

Absorption spectrum

Black rays



Atom

Exercises



Chap.

17

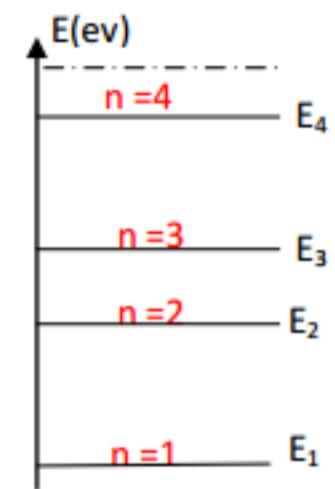
Test Your Knowledge

Given : Plank's constant $6.63 \times 10^{-34} \text{ J.s}$; speed of light in vacuum $3 \times 10^8 \text{ m/s}$.

Correct the false statements and answer the questions.

1) An atom, given in its fundamental state, absorbs a photon:

1. 1- The atom stays in its fundamental state.
1. 2- The fundamental state is associated to the level $n = 2$.
1. 3- The atom undergoes a deexcitation.
1. 4- The minimum value of the absorbed energy that can excite this atom is $E_2 - E_1$.
1. 5- A photon of energy $E_{ph} < E_2 - E_1$ is absorbed by this atom.
1. 6- A photon of energy $E > E_2 - E_1$ is necessarily absorbed by this atom.
1. 7- The true answer of part 1.6 verify the quantization of energy of the atom.
1. 8- The energy of the atom in the ionized state is $E = 0$.
1. 9- The ionization energy of an atom is E_1 .
1. 10- The transition from $n = 1$ to $n = 3$ needs the absorption of a photon of energy E_3 .



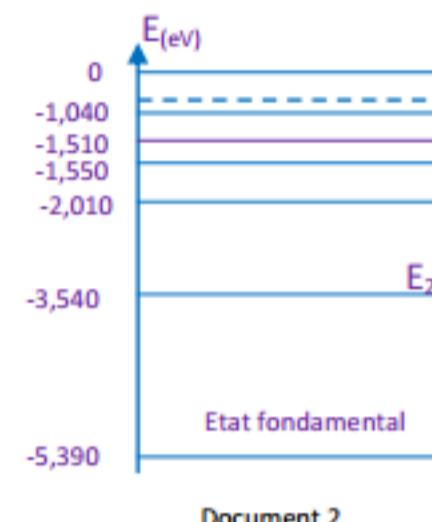
Document 1

2) Knowing that the specific energies of the H atom are given by $E_n = -\frac{13,6}{n^2}$ in eV ; and the H atom is given in its fundamental state.

2. 1- The energy of the H atom in the 2nd excited state is -3,4 eV.
2. 2- This atom needs at least an energy of 10.2 eV to be excited.
2. 3- If the atom undergoes the transition from $n = 4$ to $n = 1$, we say that it is excited to the excited level $n = 1$.
2. 4- Absorbing an energy equal to $E_4 - E_1$, this atom will be excited to the third excited level.
2. 5- If the atom undergoes the transition from $n = 4$ to $n = 1$, we say that it will be excited to the level $n = 1$.
2. 6- The H atom, passing from level $n = 4$ to $n = 1$, absorbs a photon.
2. 7- The H atom undergoes the transition from $n = 3$ to $n = 2$. The emitted radiation has a wavelength $\lambda_{32} = 120 \text{ nm}$.
2. 8- The H atom, when deexcited from level $n = 3$ to $n = 2$, emits a visible radiation.
2. 9- The H atom, given in its fundamental state, receives four photons of respective energies: 10 eV, 10.4 eV, 12.09 eV and 13.8 eV. Specify the energetic state of the atom after receiving each photon.
2. 10- An H atom, in its fundamental state, receives a particle (an electron for example) of kinetic energy 10.4 eV.
 - 2.10.1- What will be the new state of the atom?
 - 2.10.2- By comparing the result with the question 2.9, explain the difference between « particle » and « photon ».

- 3) The Rydberg formula is related to the hydrogen atoms only. It allows us to calculate the wavelength of the radiation that can be emitted or absorbed by the H atom.

- 3.1) Write the Rydberg formula corresponding to the transition from m to p with p > m.
- 3.2) Calculate the wavelength of the emitted radiation during the transition from n = 5 to n = 2.
- 3.3) Is the emitted radiation in the part 3.2 visible? What series is it associated to?
3. 1- Use the Rydberg formula to verify that the energy of the H atom at an energy level n is given by: $E_n = -\frac{13,6}{n^2}$.
3. 2- What transitions correspond to the Balmer series? how can we specify these transitions corresponding to the H atom?
3. 3- What series does the transitions from n > 1 to n = 1 correspond to?
What domain do these radiations belong to? (Visible; UV ou IR)?
3. 4- What transitions correspond to the Paschen series?
What domain do these radiations belong to? (Visible; UV ou IR)?
3. 5- An atom is characterized by its energy diagram given in the document 2.
The atom is in the energy level E₂.
Can this atom absorb a photon of energy 1.99 eV?
3. 6- The atom in the preceding question is deexcited from level n = 3 to a lower energy level. What are the possible deexcitation of this atom. Show them by an arrow.



Document 2

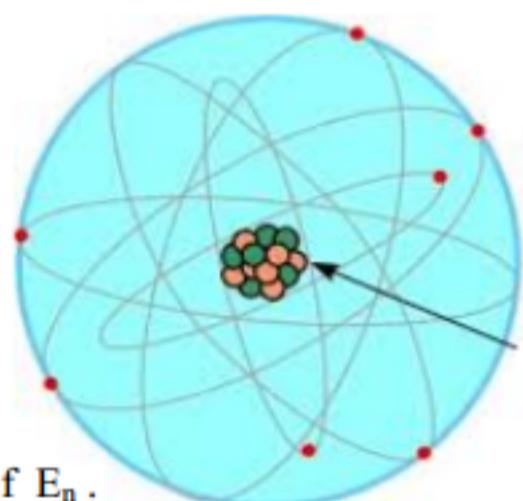
3. 7- Inversely, if the preceding atom undergoes an excitation from the fundamental state to the 3rd excited state. Show this transition by an arrow.

Exercise 1 : Atom and quantification of energy

The energies of the different energy levels of the hydrogen atom are given by the relation: $E_n = -\frac{13,6}{n^2}$ (in eV)
where n is a positive whole number.

Planck's constant : $h = 6,63 \times 10^{-34} \text{ J.s}$; $1 \text{ eV} = 1,60 \times 10^{-19} \text{ J}$;
Speed of light in vacuum : $c = 3 \times 10^8 \text{ m/s}$; $1 \text{ nm} = 10^{-9} \text{ m}$.

- 1) The energies of the atom are quantized. Justify this using the expression of E_n.
- 2) Determine the energy of the hydrogen atom when it is:
 - a) in the fundamental state .
 - b) in the second excited state.
- 3) Give the name of the state for which the energy of the atom is zero.

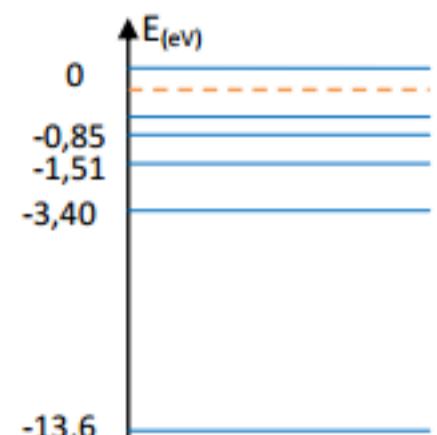


Exercise 2 : Excitation of atom by radiations (photons)

The hydrogen atom is in its fundamental state.

- 1) Calculate the minimum energy of a photon that is able to:
 - a) excite this atom;
 - b) ionize this atom.
- 2) The hydrogen atom receives, separately, three photons of respective energies:
 a) **11 eV** b) **12,75 eV** c) **16 eV**

Specify in each case the state of the atom. Justify.

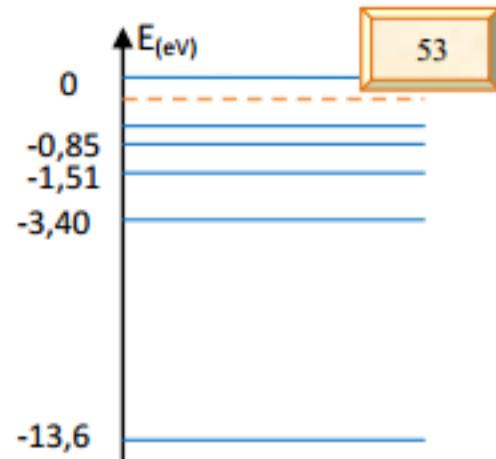


Exercise 3 :

Deexcitation of atom H

The hydrogen atom is found now in the energy level $n = 3$.

- 1) Specify all the possible transitions of the atom when it is dis-excited.
- 2) One of the emitted radiations is visible. Calculate its wavelength in vacuum.



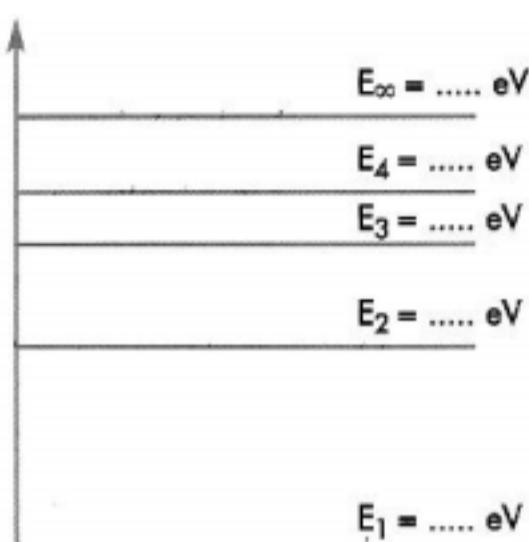
Exercise 4 :

Quantized energy

The quantized energy levels of the hydrogen atom are given by the formula:

$$E_n = \frac{E_0}{n^2} \text{ where } E_0 = 13,6 \text{ eV and } n \text{ is a whole number } \geq 1.$$

- 1) Explain briefly what is meant by the term "quantized energy" and tell why the spectra (absorption or emission) of hydrogen are formed of lines.
- 2) Calculate the values of the energies corresponding to the energy levels $n = 1, 2, 3, 4$ and $n = \infty$. Redraw and complete the adjacent diagram.



Exercise 5 :

Sodium atom

Document 1 represents some of the energy levels of the sodium atom.

Given: $h = 6.6 \times 10^{-34} \text{ J.s}$; $c = 3 \times 10^8 \text{ m/s}$;

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$; $1 \text{ u} = 931.5 \text{ MeV/c}^2$.

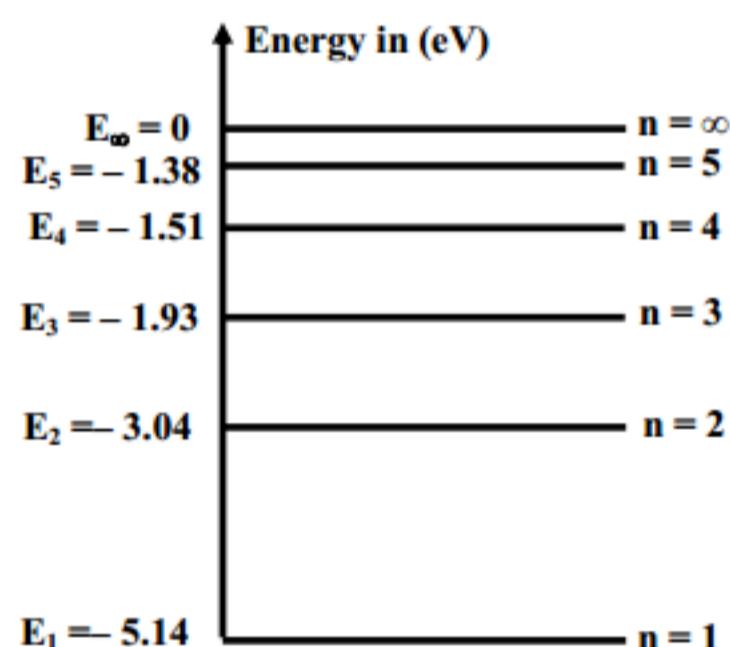
The aim of this exercise is to study the excitation and the de-excitation of the sodium atom.

Excitation of the sodium atom

Consider a sample of sodium atoms, initially in the ground state.

This sample is illuminated by white light that contains all the visible radiations: $0.4 \mu\text{m} \leq \lambda_{\text{visible}} \leq 0.8 \mu\text{m}$.

- 1-1) Using document 1, show that the energy of the sodium atom is quantized.
- 1-2) Determine, in eV, the maximum energy and the minimum energy of the photons in the white light.
- 1-3) Using document 1, show that white light is not capable to ionize the sodium atom.
- 1-4) Determine, in nm, the wavelength of the photon that excites the sodium atom to the first excited state.



Doc. 1



Problems

Visit website physprolb.com

Problem 1 :

Spectrum of the hydrogen atom

PHYSICS

A - Emission spectrum

The Balmer's series of the hydrogen atom is the set of the radiations corresponding to the downward transitions to the level of $n = 2$.

The values of the wavelengths in vacuum of the visible radiations of this series are :

411 nm ; 435 nm ; 487 nm ; 658 nm.

- a) Specify, with justification, the wavelength λ_1 of the visible radiation carrying the greatest energy.
- b) Determine the initial level of the transition giving the radiation of wavelength λ_1 .
- c) Deduce the three initial levels corresponding to the emission of the other visible radiations.

B - Absorption spectrum

A beam of ~~S~~ light crosses a gas formed mainly of hydrogen . The study of the absorption spectrum reveals the presence of dark spectral lines.

Give , with justification, the number of these lines and their corresponding wavelengths.

C - Interaction of photon-hydrogen atom

- 1) We send on a hydrogen atom , being in the fundamental state, separately, two photons of respective energies 3.4 eV and 10.2 eV .
Specify, with justification , the photon that is absorbed .
- 2) A hydrogen atom found in its fundamental state absorbs a photon of energy 14.6 eV.The electron is thus ejected.
 - a. Justify the direction of the electron.
 - b. Calculate, in eV, the kinetic energy of the ejected electron.

Problem 2

Ionization energy

PHYSICS GS- 2007-1

Given: 1 eV = 1.6×10^{-19} J ; Planck's constant $h = 6.62 \times 10^{-34}$ J.s; speed of light in vacuum $c = 3 \times 10^8$ m/s.

The object of this exercise is to compare the ionization energy of the hydrogen atom with that of the helium ion H^+ and that of the lithium ion Li^{2+} each having only one electron in the outermost shell.

The quantized energy levels of each is given by the expression $E_n = -\frac{E_0}{n^2}$ where E_0 is the ionization energy and n is a non-zero positive whole number.

I- Interpretation of the existence of spectral lines

- 1) Due to what is the presence of emission spectral lines of an atom or an ion?
- 2) Explain briefly the term "quantized energy levels".
- 3) Is a transition from an energy level m to another energy level p ($p < m$) a result of an absorption or an emission of a photon? Why?

II- Atomic spectrum of hydrogen

For the hydrogen atom $E_0 = 13.6$ eV.

- 1) A hydrogen atom, found in its ground state, interacts with a photon of energy 14 eV.

a) Why?

b) A particle is thus liberated. Give the name of this particle and calculate its kinetic energy.

- 2) *a)* Show that the expression of the wavelengths λ of the radiations emitted by the hydrogen atom is:

$$\frac{1}{\lambda} = R_1 \left(\frac{1}{m^2} - \frac{1}{p^2} \right) \text{ where } m \text{ and } p \text{ are two positive whole numbers so that } m > p \text{ and } R_1 \text{ is a positive constant to be determined in terms of } E_0, h \text{ and } c.$$

b) Verify that $R_1 = 1.096 \times 10^7 \text{ m}^{-1}$.

III- Atomic spectrum of the helium ion He^+

The spectrum of the helium ion He^+ is formed, in addition to others, of two lines whose corresponding

reciprocal wavelengths $\frac{1}{\lambda}$ are: $3.292 \times 10^7 \text{ m}^{-1}$; $3.901 \times 10^7 \text{ m}^{-1}$ respectively. These lines correspond,

respectively, to the transitions: ($m = 2 \rightarrow p = 1$) and ($m = 3 \rightarrow p = 1$).

- 1) *a)* Show that the values of $\frac{1}{\lambda}$ satisfy the relation $\frac{1}{\lambda} = R_2 \left(\frac{1}{p^2} - \frac{1}{m^2} \right)$ where R_2 is a positive constant.

b) Deduce that $R_2 = 4.389 \times 10^7 \text{ m}^{-1}$.

- 2) Find a relation between R_2 and R_1 .

Problem

Interaction radiation-matter

PHYSICS LS 2005 2

- I- At the beginning of the 1880's, Balmer identified, in the emission spectrum of hydrogen, the four visible rays denoted by H_α , H_β , H_γ and H_δ .

In 1913, Bohr elaborated a theory about the structure of the atom and showed that we can associate , to

the hydrogen atom, energy levels given by the formula:

$$E_n = -\frac{E_0}{n^2} \text{ where } E_0 \text{ is a positive constant expressed in eV and } n \text{ is a whole non-zero number.}$$

According to Bohr, each of the rays of Balmer series is characterized by its wavelength λ in air and the corresponding downward transition :

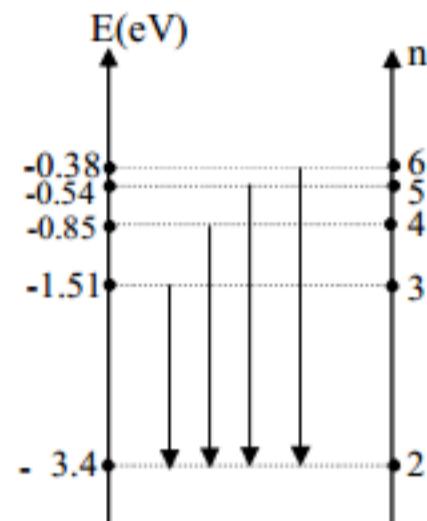
- $H_\alpha (\lambda_\alpha = 658 \text{ nm}; \text{ transition from } n=3 \text{ to } n=2);$
- $H_\beta (\lambda_\beta = 487 \text{ nm}; \text{ transition from } n=4 \text{ to } n=2);$
- $H_\gamma (\lambda_\gamma = 435 \text{ nm}; \text{ transition from } n=5 \text{ to } n=2);$
- $H_\delta (\lambda_\delta = 412 \text{ nm}; \text{ transition from } n=6 \text{ to } n=2).$

The corresponding energy diagram of this series is represented in the adjacent figure.

- 1) Determine, using the diagram, the value of E_0 in eV.

2) a - Emission spectrum

- i) Show, starting from the diagram, that the ray H_β corresponds to the emission of a photon of energy 2.55 eV.
- ii) Verify that the value of the wavelength of the ray H_β is around 487 nm



b - Absorption spectrum

In order to obtain the absorption spectrum of the hydrogen atom, we illuminate hydrogen gas with white light. What do we observe in the absorption spectrum?

- 3) The hydrogen atom, being in its first excited state ($n=2$), collides with a photon of energy 2.26 eV. This photon is not absorbed. Why?

II-A hydrogen lamp illuminates now a photosensitive metallic surface of threshold wavelength $\lambda_0 = 500 \text{ nm}$.

- 1) What are the visible radiations that may provoke photoelectric emission? Why?
- 2) a) Determine the radiation that is able to extract an electron having the highest possible kinetic energy KE.
b) Calculate then KE .

III- The atomic line spectra and the phenomenon of photoelectric effect show evidence of a characteristic concerning the energy of an electromagnetic wave and the energetic exchange between matter and electromagnetic waves. Specify this characteristic.

Given:

- speed of light in vacuum $c = 3 \times 10^8 \text{ m/s}$;
- Planck's constant $h = 6.63 \times 10^{-34} \text{ J.s}$;
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$;
- $1 \text{ nm} = 10^{-9} \text{ m}$.

Problem 4 :

Emission spectrum of a mercury vapor lamp

PHYSICS
LS 2006-2

The object of this exercise is to determine the visible emission spectrum of a mercury vapor lamp.

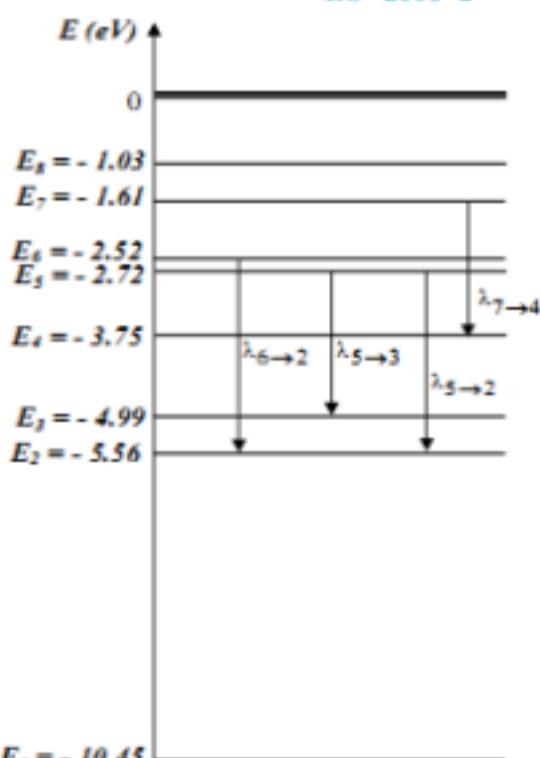
The adjacent diagram gives, in a simplified way, the energy level of the ground state, those of the excited states E_2 , E_3 , E_4 , E_5 , E_6 , E_7 , E_8 and the ionization energy level $E = 0$ of the mercury atom.

Given:

Planck's constant $h = 6.62 \times 10^{-34} \text{ J.s}$; speed of light in vacuum :
 $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$; $c = 3 \times 10^8 \text{ m/s}$;

I- Quantization of the energy of the atom

- 1) The energy of the mercury atom is quantized.
What is meant by "quantized energy"?
- 2) a) What is meant by « ionizing » an atom?
b) Calculate, in eV, the ionization energy of a mercury atom taken in the ground state.



3) Interaction photon-atom.

A photon cannot cause the transition of an atom from an energy level E_p to a higher energy level E_n unless its energy is exactly the same as the difference of the energies ($E_n - E_p$) of the atom.

The mercury atom being in the ground state.

a) Determine the maximum wavelength of the wave associated to a photon capable of exciting this atom.

b) The mercury atom is hit with a photon of wavelength $\lambda_1 = 2.062 \times 10^{-7}$ m.

i) Show that this photon cannot be absorbed.

ii) What is then the state of this atom?

c) The atom receives now a photon of wavelength λ_2 . The atom is thus ionized and the extracted electron is at rest.

Calculate λ_2 .

II- Emission by a mercury vapor lamp

For an electron to cause a transition of an atom from an energy level E_p to a higher energy level E_n , its energy must be at least equal to the difference of the energies ($E_n - E_p$) of the atom.

During one electron-atom collision, the atom absorbs, from the electron, an amount of energy enough to ensure a transition. The rest of the energy is carried by the electron as kinetic energy.

When the mercury vapor lamp is under a convenient voltage, an electric discharge takes place.

Some electrons, each of kinetic energy 9 eV, moving in the vapor of mercury between the electrodes of the lamp, hit the gaseous atoms giving them energy. For that lamp, the atoms are initially in the ground state.

1) Verify that an atom may not overpass the energy level E_7 .

2) The visible emission spectrum due to the downward transition of the mercury atom, is formed of four rays of wavelengths: $\lambda_{7 \rightarrow 4}$; $\lambda_{6 \rightarrow 2}$; $\lambda_{5 \rightarrow 2}$; $\lambda_{5 \rightarrow 3}$ (*refer to the diagram*).

Determine the wavelengths of the limits of the visible spectrum of the mercury vapor lamp.

Problem 5 :

Sodium vapor lamp

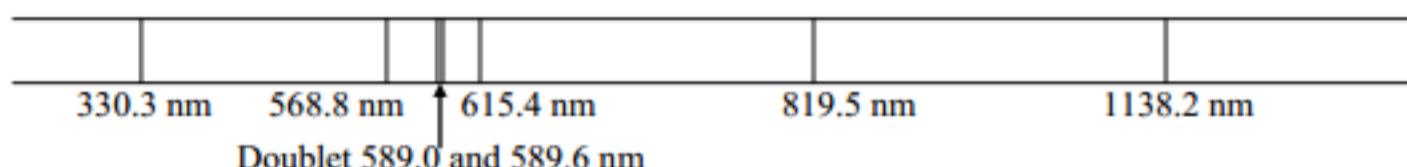
PHYSICS LS 2009-1

Sodium vapor lamps are used to illuminate roads. These lamps contain sodium vapor under very low pressure. This vapor is excited by a beam of electrons that cross the tube containing the vapor. The electrons yield energy to the sodium atoms which give back this received energy during their downward transition towards the ground state in the form of electromagnetic radiations

Given: $h = 6.62 \times 10^{-34}$ J.s; $c = 3 \times 10^8$ ms $^{-1}$; $e = 1.60 \times 10^{-19}$ C ; $1 \text{ nm} = 10^{-9}$ m.

1) What do each of the quantities h , c and e represent?

2) The analysis of the emission spectrum of a sodium vapor lamp shows the presence of lines of well-determined wavelengths λ . The figure below represents some of the lines of this spectrum.



a) The yellow doublet of wavelengths, in vacuum, $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm is more intense than the other lines.

i) To what range: visible, infrared or ultraviolet, does each of the other lines of the spectrum belong?

ii) The sodium vapor lamps are characterized by the emission of yellow light. Why?

b) Is the visible light emitted by the sodium lamp monochromatic or polychromatic? Justify your answer.

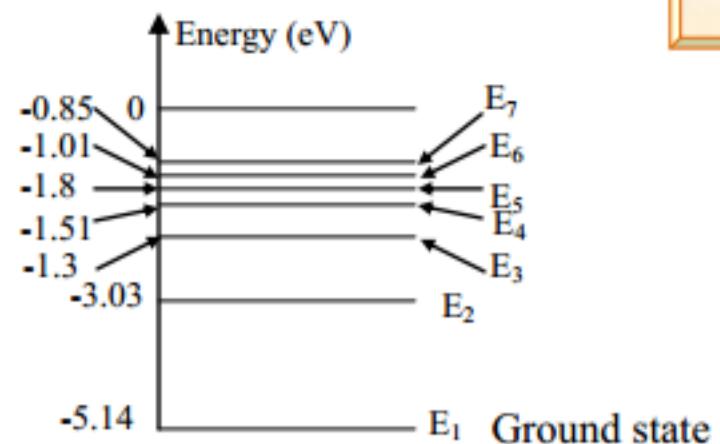
3) a) Referring to the diagram of the energy levels of the sodium atom in the adjacent figure:

- i) Specify an indicator that justifies the discontinuity of the emission spectrum of the sodium vapor lamp.

- ii) Verify that the emission of the line of wavelength λ_1 corresponds to the downward transition from the energy level E_2 to the ground state.

b) In fact, the energy level E_2 is double, i.e., it is constituted of two energy levels that are very close to each other. Draw a diagram that shows the preceding downward transition as well as the downward transition corresponding to the emission of the radiation of wavelength λ_2 .

- 4) The sodium atom, being in the ground state, is hit successively by the electrons (a) and (b) of respective kinetic energies 1.01 eV and 3.03 eV .



Atomic Nucleus A_Z^X

Chap. 18

Constants (nucleons) ①

Protons of positive charges.

Neutrons and do not have any charge.

Mass of nucleus $< \Sigma$ (masses of nucleons)

$$\Delta m = \{ Zm_p + (A-Z)m_n \} - m_X$$

Number of nucleons $\leftarrow A$ X
Number of protons $\leftarrow Z$

Isotopes ? ③

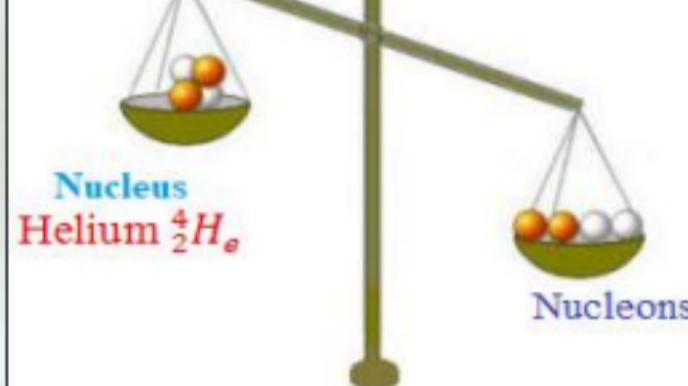
Are nuclei of the same element having the same charge number Z , but the number of nucleons A is different.

Properties

Definition ④

Binding energy

This is the minimum energy needed to break up the nucleus and separate all the nucleons



Definition ⑤

Binding energy per nucleon

This is the needed energy to extract a nucleon from the nucleus of an atom.

Binding Energy ⑥

$$E_b = \Delta m \times c^2$$

with

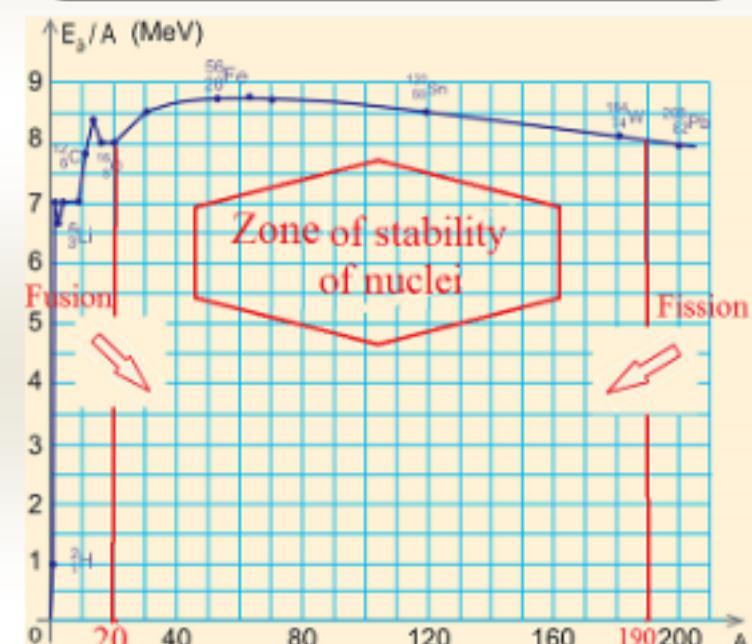
$$\Delta m = \{ Zm_p + (A-Z)m_n \} - m_X$$

$$1u = 931.5 \text{ MeV}/c^2$$

Stability of a nucleus

$$\frac{E_b}{A} \approx 8 \text{ MeV}$$

⑧



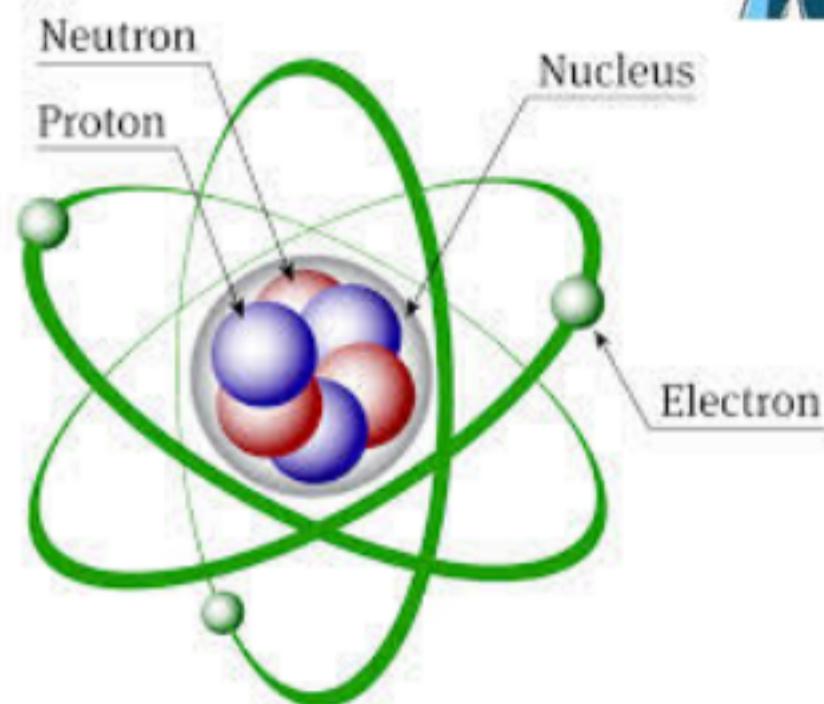
Nucleus

Exercises



Chap.

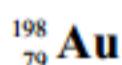
18



Exercise 1 :

Studying the radionuclide

PHYSICS
GS -2016-1



Given:

molar mass of $^{198}_{79} \text{Au}$: 198 g ;

mass of the electron: 5.50×10^{-4} u;

$1 \text{ u} = 931.5 \text{ MeV} / c^2 = 1.66 \times 10^{-27} \text{ kg}$;

mass of the gold nucleus Au : 197.925 u ;

mass of the proton $m_p = 1.00728 \text{ u}$;

Avogadro's number : $6.022 \times 10^{23} \text{ mol}^{-1}$;

speed of light in vacuum $c = 3 \times 10^8 \text{ m/s}$;

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$;

mass of the mercury nucleus Hg : 197.923 u;

mass of the neutron $m_n = 1.00866 \text{ u}$.

A- Comparison between the density of the gold nucleus and that of the gold atom

1) a) Calculate the mass of the gold atom $^{198}_{79} \text{Au}$.

b) Compare the mass of the gold atom $^{198}_{79} \text{Au}$ with that of its nucleus.

2) The average radius of the gold atom is $r = 16 \times 10^{-11} \text{ m}$. The average radius of a nucleon is $r_0 = 12 \times 10^{-16} \text{ m}$. Compare the density of the gold atom with that of its nucleus. Give a conclusion about the distribution of mass in the atom.

B- Stability of the gold nucleus

1. a) Give the constituents of the nucleus $^{198}_{79} \text{Au}$.

b) If the gold nucleus $^{198}_{79} \text{Au}$ is broken into its constituting nucleons, show that the sum of the masses of the nucleons taken separately at rest is greater than the mass of the nucleus taken at rest.

Due to what is this increase in the mass?

2. Knowing that a nucleus is considered stable when its binding energy per nucleon is larger or equal to 8 MeV, give a conclusion about the stability of the nucleus $^{198}_{79} \text{Au}$.

Exercise 2 :
Stability of atomic nuclei

Consider the table below that shows some physical quantities associated with certain nuclei.

Nucleus	${}^2_1\text{H}$	${}^3_1\text{H}$	${}^4_2\text{He}$	${}^{14}_6\text{C}$	${}^{14}_7\text{N}$	${}^{94}_{38}\text{Sr}$	${}^{140}_{54}\text{Xe}$	${}^{235}_{92}\text{U}$
Mass (u)	2.0136	3.0155	4.0015	14.0065	14.0031	93.8945	139.892	234.9935
Binding energy E _b (MeV)	2.23	8.57	28.41	99.54	101.44	810.50	1164.75	
Binding energy per nucleon $\frac{E_b}{A}$ (MeV/nucleon)	1.11		7.10		7.25	8.62		

Numerical data:

Mass of a neutron: m_n = 1.0087 u ; mass of a proton : m_p = 1.0073 u ;
mass of an electron : m_e = 0.00055 u ; 1 u = 931.5 MeV /c².

1) a) Define the binding energy of a nucleus.

b) Write the expression of the binding energy E_b of a nucleus ${}^A_Z\text{X}$ as a function of Z, A, m_p, m_n, m_X (the mass of the nucleus ${}^A_Z\text{X}$) and the speed of light in vacuum c.

c) Calculate , in MeV, the binding energy of the uranium 235 nucleus.

d) Complete the table by calculating the missing values of $\frac{E_b}{A}$.

e) Give the name of the most stable nucleus in the above table. Justify your answer.

2) Each of the considered nuclei in the table belongs to one of the three groups given by:

$$A < 20 ; \quad 20 < A < 190 ; \quad A > 190.$$

Referring to the completed table, trace the shape of the curve representing the variation of $\frac{E_b}{A}$ as a function of A . Specify on the figure the three mentioned groups.

Nuclear Reactions

Spontaneous

Without any
external
intervention

The Radioactivity / What does this mean ?

1

Answer: This is a spontaneous disintegration of a non-stable nucleus into another more stable nucleus .



The daughter nucleus resulting from this disintegration is more stable than the parent

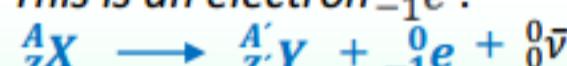
Types of disintegration

2

 α **Identification :**This is the helium nucleus ${}_{2}^{4}He$ 

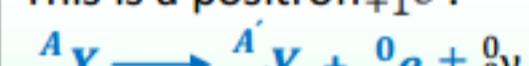
- This type of disintegration is the characteristic of **heavy nuclei having $A > 200$** .
- This particle is positively charged, the symbol of this particle is the helium ion: He^{2+}
- These particles are emitted with large speed $v \approx 2.10^7 \text{ m/s}$.
- Their penetrating power in matter is very weak
- They can hardly penetrate a sheet of paper.

3

 β^- **Identification :**This is an electron ${}_{-1}^0 e$.

- This type of disintegration is the characteristic of nuclei rich in neutrons.
- The particles β^- (betta -) are emitted with large speed $v \approx 2.7 \times 10^8 \text{ m/s}$.
- They are less ionizing than α particles but 100 times more penetrating.

4

 β^+ **Identification :**This is a positron ${}_{+1}^0 e$.

- This type of disintegration is characteristic of nuclei rich in protons.
- The particles β^+ (betta +) are emitted with large speed $v \approx 2.7 \times 10^8 \text{ m/s}$.
- They are less ionizing than α particles but 100 times more penetrating.

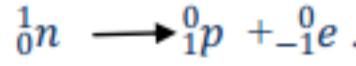
Properties

5

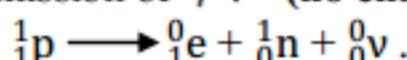
Radiations γ

- It's an electromagnetic wave.
- It's emitted by the daughter nucleus
- It has no mass and no charge.

- **Neutrino** ${}_{0}^0 \nu$
Particle accompanied with the emission of β^- . (**no charge**)



- **Antineutrino** ${}_{0}^0 \bar{\nu}$
Particle accompanied with the emission of β^+ . (**no charge**)



- **Soddy's laws** ⑧

conservation of mass number

$$\sum A_{\text{Before}} = \sum A_{\text{After}}$$

conservation of atomic number

$$\sum Z_{\text{Before}} = \sum Z_{\text{After}}$$

- **Liberated energy** ⑨

$$E_f = \Delta m c^2$$

with $\Delta m = m^{\text{bef}} - m^{\text{af}}$

mass defect.

$$\sum(mc^2 + Ec)^{before} = \sum(m'c^2 + E'c)^{After} + E_\gamma$$

10

Number of nuclei in a sample

Consider a sample containing a certain mass m of a radioactive element ${}^A_Z X$ at an instant t.

We can find the number of nuclei in this sample using the relation: $N = \frac{m}{M} N_A$

with : M (Molar mass) and N_A (Avogadro's number).

We can also write this formula : $N = \frac{m}{m_X} N_A$

with : m_X : mass of nuclei .

$$N = \frac{m}{m_X} N_A$$

$$N = \frac{m}{M} N_A$$

11



Question

A sample of radioactive Bismuth ${}^{212}_{83} Bi$, has a mass m=50mg .

given: the molar mass of ${}^{212} Bi$ is 212g and Avogadro's number is

$N_A = 6.022 \times 10^{23}$ nuclei/mole $^{-1}$.

Calculate the number of nuclei of ${}^{212}_{83} Bi$ in this sample.

Answer: The number of nuclei in the sample is: $N = \frac{m}{M} N_A = \frac{0.05}{212} \times 6.022 \times 10^{23} = 1.42 \times 10^{20}$ nuclei.

Radioactivity half-life (Period)

13

Definition :

What is the half-life T of a radioactive sample?

The half life T of a radioactive substance, is the duration in which the activity is reduced by one half.

The half life

T is a characteristic of each nucleus.

$$t_{1/2} = T \Rightarrow N = \frac{N_0}{2}$$

$$N_0 \xrightarrow{T} \frac{N_0}{2} \xrightarrow{T} \frac{N_0}{4} \xrightarrow{\dots} \xrightarrow{nT} N = \frac{N_0}{2^n}$$

Remaining nuclei after $t = nT$
Disintegrated nuclei : $N - N_0$

The period during which the activity is reduced only on the radioactive constant λ , then we obtain :

$$\lambda = \frac{\ln 2}{T}$$

$$T = \frac{\ln 2}{\lambda}$$

Law of decay :

14

$$N = N_0 e^{-\lambda t}$$

$$\text{with } \lambda = \frac{\ln 2}{T}$$

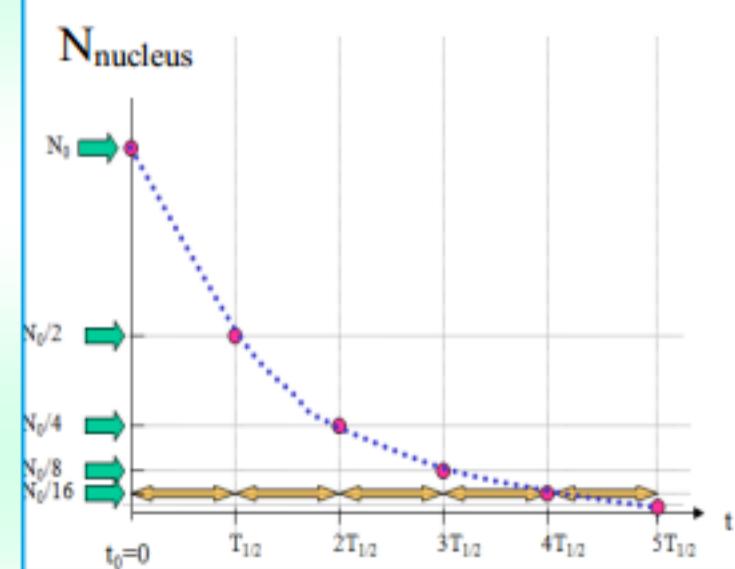
Activity

a sample

15

The activity A of a radioactive substance is the number of disintegrations per unit of time.
A : is expressed in Becquerel's (Bq) ;
sec. $1 \text{ Bq} = 1 \text{ disintegration/sec.}$

$$A = \lambda N \text{ at an instant t.}$$



$$A = -\frac{dN}{dt}$$

The minus sign is obtained since $dN < 0$ to obtain $A > 0$; then $A = A_0 e^{-\lambda t}$

17

Radioactivity



Chap.

Exercises

19



Exercise 1 :

Formation of carbon 14

In the high atmosphere the carbon isotope $^{14}_7\text{N}$ is obtained by the impact of a nitrogen $^{14}_6\text{C}$ with a neutron.

1. The nuclides $^{12}_6\text{C}$ and $^{14}_6\text{C}$ are two isotopes. Why?
2. Write the equation of the formation of $^{14}_6\text{C}$.
3. Identify the emitted particle.

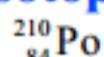
Studying the disintegration of the gold nucleus $^{198}_{79}\text{Au}$

When the gold nucleus $^{198}_{79}\text{Au}$, at rest, disintegrates it gives a daughter nucleus (mercury nucleus $^{197}_{80}\text{Hg}$) of negligible speed. We were able to detect the emission of a γ photon of energy 0.412 MeV and a β^- particle of kinetic energy 0.824 MeV.

1. Write the equation of this disintegration reaction and, specifying the laws used ,determine A and Z.
2. a) Specify the physical nature of the γ radiation.
b) Due to what is the emission of this γ radiation ?
3. a) Show , by applying the law of conservation of total energy, the existence of a new particle accompanying the emission of β^- .
b) Give the name of this particle.
c) Deduce its energy in MeV.

Exercise 3 :

The radio-isotope polonium



Given: $1\text{u} = 1.66 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2$; $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$;

Mass of some nuclei : $m(\text{Po}) = 209.9829 \text{ u}$; $m(\text{Pb}) = 205.9745 \text{ u}$; $m(\alpha) = 4.0026 \text{ u}$;

In general, the decay of $^{210}_{84}\text{Po}$ is accompanied by the emission of γ radiation.

- a) Due to what is the emission of γ radiation?
- b) The emitted γ radiation has the wavelength $\lambda = 1.35 \times 10^{-12} \text{ m}$ in vacuum.
- c) Using the conservation of total energy, determine the kinetic energy of the emitted α particle.

Exercise 4 :

Decay of polonium 210

The polonium $^{210}_{84}\text{Po}$ is an α emitter. The daughter nucleus produced by this decay is the lead $^{206}_{82}\text{Pb}$.

Given : $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$;

The liberated energy by this decay is $E = 5.4 \text{ MeV}$, and the the nucleus $^{210}_{84}\text{Po}$ is initially at rest.

We suppose that the daughter nucleus $^{206}_{82}\text{Pb}$ is obtained at rest and in the fundamental state.

Deduce the kinetic energy of the emitted α particle

Exercise 5 :**The radionuclide Potassium 40**

The isotope of potassium $^{40}_{19}\text{K}$, is radioactive and is β^+ emitter; it decays to give the daughter nucleus argon $^{40}_{18}\text{Ar}$.

The object of this exercise is to study the decay of potassium 40.

- Write down the equation of the decay of one potassium 40 nucleus and determine Z and A.
- The value of E, liberated energy by the decay , is 0,47Mev ; and the daughter nucleus is supposed to be at rest . The energy carried by β^+ is, in general, smaller than E . Why?

Exercise 6 :**Radioactivity of Cobalt**

Cobalt $^{60}_{27}\text{Co}$ is a β^- radioactive. The daughter nucleus $^{A}_{Z}\text{Ni}$ undergoes a downward transition to the ground state. The energy due to this downward transition is $E(\gamma) = 2.5060 \text{ MeV}$.

The β^- particle is emitted with a kinetic energy $K.E(\beta^-) = 0.0010 \text{ MeV}$.

Numerical data: mass of the $^{60}_{27}\text{Co}$ nucleus: 59.91901 u ; mass of the $^{A}_{Z}\text{Ni}$ nucleus : 59.91544 u ; mass of an electron : $5.486 \times 10^{-4} \text{ u}$; $1 \text{ u} = 931.5 \text{ MeV}/c^2$; $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$.

- Determine A and Z.
- Calculate, in u, the mass defect Δm during this disintegration.
- Deduce, in MeV, the energy E liberated by this disintegration.
- During this disintegration, the daughter nucleus is practically obtained at rest.
In what form of energy does E appear?

Exercise 7 :**Detection of a new particle**

When the gold nucleus $^{198}_{79}\text{Au}$, at rest, disintegrates it gives a daughter nucleus (mercury nucleus $^{80}_{80}\text{Hg}$) of negligible speed. We were able to detect the emission of a γ photon of energy 0.412 MeV and a β^- particle of kinetic energy 0.824 MeV.

- When the liberated energy by the decay is 1,351Mev , show applying the the law of conservation of total energy, the existence of a new particle accompanying the emission of β^- .
- Give the name of this particle.
- Deduce its energy in MeV.

Exercise 8 :**Detection of a new particle**

Cobalt $^{60}_{27}\text{Co}$ is a β^- radioactive. The daughter nucleus $^{A}_{Z}\text{Ni}$ undergoes a downward transition to the ground state. The energy due to this downward transition is $E(\gamma) = 2.5060 \text{ MeV}$.

The β^- particle is emitted with a kinetic energy $K.E(\beta^-) = 0.0010 \text{ MeV}$.

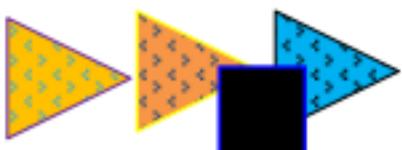
- Deduce, from what preceded , that the electron, emitted by the considered disintegration, is accompanied by a certain particle. We have $E = 2,8144\text{Mev}$,the liberated energy by the decay.
- Give the name of this particle.
- Give the charge number and the mass number of this particle.
- Deduce in MeV, the energy of this particle.
- Write down the global equation of this disintegration.

Exercise 9 :**The use of Cobalt**

In medicine, we use a source of radioactive cobalt $^{60}_{27}\text{Co}$ of activity $A = 6 \times 10^{19} \text{ Bq}$.

The emitted β^- particles are absorbed by the living organism.

- The energy of the particle mentioned in the question (d) is not absorbed by the living organism. Why?
- Calculate, in watt, the power transferred to the organism.
- This large power is used in radiotherapy .What is its effect?



Problems

Visit [website physprob.com](http://physprob.com)

Energy liberated by the disintegration of the cobalt

PHYSICS
LS 2001-1

Problem

${}_{Z}^{A}X$	${}_{27}^{60}\text{Co}$	${}_{28}^{60}\text{Ni}$	${}_{-1}^{0}\text{e}$
Masse (en u)	59,9190	59,9154	0,00055

- 1 u = 931,5 MeV/c².
 - Speed of light in vacuum: c = 3 x 10⁸ ms⁻¹
 - Planck's constant: h = 6,63 x 10⁻³⁴ J.s
 - Avogadro's constant: 6,02 x 10²³ mol⁻¹.
 - Molar mass of cobalt: 60 g.mol⁻¹.
- 1) Determining the remaining number of ${}_{27}^{60}\text{Co}$ nuclei and the activity of this sample at the end of 10.6 years.
- 2) One of the successive beta decays of the cobalt sample gives rise to the nickel isotope ${}_{28}^{60}\text{Ni}$.
- Write, with justification, the equation of the disintegration of one cobalt nucleus ${}_{27}^{60}\text{Co}$ and identify the emitted particle.
 - Calculate the energy, in MeV, the energy liberated by this disintegration.
 - Determine the energy liberated by the disintegration of 1 g of cobalt ${}_{27}^{60}\text{Co}$.
 - Knowing that the energy liberated from the complete combustion of 1 g of coal is 30 kJ, find the mass of coal that would liberate the same amount of energy calculated in part c).

Problem

Radioactivity

PHYSICS
GS 2002-1

Given the masses of the nuclei: m(${}_{53}^{131}\text{I}$) = 130,87697 u; m(${}_{54}^{131}\text{Xe}$) = 130,87538 u;

mass of an electron: m_e = 5,5 x 10⁻⁴ u;

1 u = 931,5 MeV/c²; 1 MeV = 1,6 x 10⁻¹³ J; h = 6,63 x 10⁻³⁴ J.s et c = 3x10⁸m/s

In order to determine the energy released during the disintegration of a radionuclide ${}_{53}^{131}\text{I}$, we consider a trouble in the functioning of the thyroid, we inject it with a sample of an iodine radionuclide ${}_{53}^{131}\text{I}$. This radionuclide has a period (half-life) of 8 days and it is a β^- emitter. The disintegration of the nuclide ${}_{53}^{131}\text{I}$ gives rise to a daughter nucleus ${}_{54}^{131}\text{Xe}$ supposed at rest.

- a) The disintegration of a nucleus of ${}_{53}^{131}\text{I}$ is accompanied by the emission of a γ radiation. Due to what is this emission?
 - b) Write the equation of the disintegration of ${}_{53}^{131}\text{I}$ nucleus.
 - c) Calculate the decay constant of the radionuclide. Deduce the number of the nuclei of the sample at the instant of injection, knowing that the activity of the sample, at that instant, is $1,5 \times 10^5$ Bq.
 - d) Calculate the number of the disintegrated nuclei at the end of 24 days.
- 2) a) Calculate the energy liberated by the disintegration of one nucleus of ${}_{53}^{131}\text{I}$.
- b) Calculate the energy of a γ photon knowing that the associated wavelength is $3,55 \times 10^{-12}$ m.
- c) The energy of an antineutrino being 0,07 MeV, calculate the average kinetic energy of an emitted electron.
- d) During the disintegration of the ${}_{53}^{131}\text{I}$ nuclei, the thyroid, of mass 40 g, absorbs only the average kinetic energy of the emitted electrons and that of γ photons. Knowing that the dose absorbed by a body is the energy absorbed by a unit mass of this body, calculate, in J/Kg, the absorbed dose by that thyroid during 24 days.

In order to study the radioactivity of polonium $^{210}_{84}\text{Po}$ which is an α emitter, we take a sample of polonium 210 containing N_0 nuclei at the instant $t_0 = 0$.

A- Determination of the half-life (period)

We measure, at successive instants, the number N of the remaining nuclei. We calculate the ratio N/N_0 and the result is tabulated as in the following table:

t (in days)	0	50	100	150	200	250	300
N / N_0	1	0.78	0.61	0.47	0.37	0.29	0.22
$-\ln(N / N_0)$	0	0.25				1.24	

- 1- Draw again the above table and complete it by calculating at each instant $-\ln(N/N_0)$.
- 2- Trace the curve representing the variation of $f(t) = -\ln(N/N_0)$, as a function of time, using the scale: 1 cm on the abscissa represents 25 days; 1 cm on the ordinate represents 0.1.
- 3- a- Knowing that $\ln(N/N_0) = -\lambda t$, determine graphically the value of the radioactive constant λ of polonium 210.
b- Deduce the half-life of polonium 210.

B- Activity of polonium 210

- 1- Define the activity of a radioactive sample.
- 2- Give the expression of the activity A_0 of the sample at the instant $t_0 = 0$, in terms of λ and N_0 . Calculate its value for $N_0 = 5 \times 10^{18}$.
- 3- Give the expression, in terms of t , of the activity A of the sample.
- 4- Calculate the activity A :
 - a- at the instant $t = 90$ days.
 - b- When t increases indefinitely.

C- Energy liberated by the disintegration of polonium 210

- 1- The disintegration of a nucleus of polonium produces a daughter nucleus which is an isotope of lead $^{A}_{Z}\text{Pb}$. Determine A and Z .
- 2- Calculate, in MeV, the energy liberated by the disintegration of one nucleus of polonium 210.
- 3- The disintegration of a polonium nucleus may take place with or without the emission of a photon. The energy of an emitted photon is 2.20 MeV. Knowing that the daughter nucleus has a negligible velocity, determine in each case the kinetic energy of the emitted α particle.
- 4- The sample is put in an aluminum container. Thus, the α particles are stopped by the container whereas the photons are not.

Knowing that half of the disintegrations are accompanied by a γ emission, determine the power transferred to the aluminum container at the instant $t = 90$ days. **Numerical data:**

Mass of a polonium 210 nucleus: 209.9828 u Mass of lead (Pb) nucleus: 205.9745 u

Mass of an α particle: 4.0015 u $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV/c}^2$.

Problem 4 :

The carbon 14

The object of this exercise is to show evidence of some characteristic properties of the radio element $^{14}_6\text{C}$ and to show the procedure followed to know the age of a wooden fossil.

Given :

mass of a proton : $m_p = 1,00728 \text{ u}$;

mass of a neutron : $m_n = 1,00866 \text{ u}$;

mass of a nucleus $^{14}_6\text{C} = 14,0065 \text{ u}$;

mass of a nucleus $^{14}_7\text{N} = 14,0031 \text{ u}$;

Avogadro's number : $N_A = 6,02 \times 10^{23} \text{ mol}^{-1}$.

Molar mass of $^{14}_6\text{C} = 14 \text{ g/mol}$;

A - Formation of carbon 14

In the high atmosphere the carbon isotope $^{14}_7\text{N}$ is obtained by the impact of a nitrogen $^{14}_7\text{N}$ with a neutron.

1. The nuclides $^{12}_6\text{C}$ and $^{14}_6\text{C}$ are two isotopes. Why?
2. Write the equation of the formation of $^{14}_6\text{C}$.
3. Identify the emitted particle.

B – Disintegration of carbon 14

Carbon 14 is radioactive β^- emitter. It disintegrates to give nitrogen $^{14}_7\text{N}$.

1. The emission of a β^- particle is due to the disintegration of a nucleon inside the nucleus.
2. Calculate the binding energy per nucleon of each of the nuclei $^{14}_6\text{C}$ and $^{14}_7\text{N}$.
3. In fact, a radioactive decay leads to a more stable state. Justify this statement taking into account the preceding results.
4. The activity of a substance containing carbon 14 is determined using a counter of β^- particles. A sample of wood containing 0.05g of carbon 14 of radioactive period $T = 5570$ years is exposed to the counter. Determine:
 - a) The radioactive constant λ of carbon 14.
 - b) The number of carbon 14 nuclei contained in this sample at the instant of exposure.
 - c) The activity of the sample at the considered instant.

C- Age of a wood fossil

We intend to determine the age of a piece of wood fossil. We expose this piece to the counter of β^- particles; it indicates 100 disintegration in 5 minutes. Knowing that a piece of the same wood, freshly cut, gives 1000 disintegrations in 5 minutes, determine the age of the wood fossil.

Problem 5:

The technetium 99

PHYSICS
GS 2005-2

A - A bit of history...

In 1937, Pierrier and Sègre obtained, for the first time, an isotope of technetium $^{99}_{43}\text{Tc}$ by bombarding the nuclei of molybdenum $^{98}_{42}\text{Mo}$ with an isotope of hydrogen ^A_ZH according to the following reaction :



Determine Z and A specifying the laws used.

B- Production of technetium 99 at the present time and its characteristic

The isotope $^{99}_{43}\text{Tc}$ is actually obtained in a generator molybdenum/technetium, starting from the isotope $^{99}_{42}\text{Mo}$ of molybdenum. This molybdenum is a β^- emitter.

- 1) Write the equation corresponding to the decay of $^{99}_{42}\text{Mo}$.
 - 2) Determine, in MeV, the energy liberated by this decay.
 - 3) Most of the technetium nuclei obtained are in an excited state $[^{99}_{43}\text{Tc}^*]$
- a- i) Complete the equation of the following downward transition: $^{99}_{43}\text{Tc}^* \longrightarrow ^{99}_{43}\text{Tc} + \dots$
- ii) Specify the nature of the emitted radiation.
- b- The energy liberated by this transition, of value 0.14 MeV, is totally carried by the emitted radiation; the nuclei $[^{99}_{43}\text{Tc}^*]$ and $^{99}_{43}\text{Tc}$ are supposed to be at rest.
- i) Determine, in u, the mass of the $^{99}_{43}\text{Tc}^*$ nucleus.
- ii) Calculate the wavelength of the emitted radiation.

C- Using technetium 99 in medicine

The isotope $^{99}_{43}\text{Tc}$ is actually often used in medical imaging. The generator molybdenum/technetium is known, in medicine, by the name “technetium cow”. Also, the daily preparation of the medically needed technetium 99, of half-life $T_1 = 6$ hours, starting from its “parent” the molybdenum of half-life $T_2 = 67$ hours, allows a weekly supply.

- 1) Why is it preferable, in medical service that requires the use of technetium 99, to keep a reserve of molybdenum 99 and not a reserve of technetium 99 ?
- 2) Determine the number of technetium 99 nuclei obtained from a mass of 1g of molybdenum 99 at the end of 24 hours. Deduce the mass of these technetium nuclei.

Given : Masses of nuclei and particles: $^{99}_{42}\text{Mo} = 98,88437 \text{ u}$; $^{99}_{43}\text{Tc} = 98,88235 \text{ u}$; $^0_1e = 55 \times 10^{-5} \text{ u}$.

1u = $931,5 \text{ MeV}/c^2 = 1,66 \times 10^{-27} \text{ kg}$;

Planck's constant: $h = 6,63 \times 10^{-34} \text{ J.s}$; $1\text{eV} = 1,6 \times 10^{-19} \text{ J}$; $c = 3 \times 10^8 \text{ m/s}$.

Problem 6

Radioactivity

PHYSICS
GS-2006-2

The object of this exercise is to show evidence of some characteristics of a thorium nucleus ^{230}Th and its role in dating.

Given : speed of light in vacuum : $c = 3 \times 10^8 \text{ ms}^{-1}$; $1\text{eV} = 1,6 \times 10^{-19} \text{ J}$;

Avoigadro's number : $N = 6,02 \times 10^{23} \text{ mol}^{-1}$;

Planck's constant : $h = 6,63 \times 10^{-34} \text{ J.s}$; $1\text{u} = 931,5 \text{ MeV}/c^2$;

masses of the nuclei : $m(^A_{88}\text{Ra}) = 225,9770 \text{ u}$; $m(^{230}_{90}\text{Th}) = 229,9836 \text{ u}$; $m(\alpha) = 4,0015 \text{ u}$.

A- Decay of a thorium nucleus 230

The thorium nucleus ($^{230}_{90}\text{Th}$) is radioactive and is an α emitter. The daughter nucleus is the isotope of the radon ($^{226}_{88}\text{Ra}$).

- 1- **a)** Write the equation of this decay and determine the values of A and Z.
b) Determine the energy liberated by the decay of a thorium nucleus 230.
- 2- A decay of a thorium nucleus 230 at rest, takes place without the emission of γ radiation. The daughter nucleus ($^{226}_{88}\text{Ra}$) obtained has a speed almost zero. Determine the value of the kinetic energy K.E₁ of the emitted α particle.
- 3- Another decay of a thorium nucleus 230 is accompanied with the emission of a γ radiation of wavelength $5 \times 10^{-12} \text{ m}$ in vacuum.
a) Calculate the energy of this radiation.
b) Determine the value of the kinetic energy K.E₂ of the emitted α particle.
- 4- A sample of thorium nucleus 230 of activity $A_0 = 7,2 \times 10^8 \text{ decays/s}$ is placed near a sheet of aluminum at instant $t_0 = 0$. The α particles are stopped by the aluminum sheet whereas the photons are not absorbed.
a) Determine in J, the energy W transferred to the aluminum sheet during the first second knowing that 50% of the decays are accompanied with γ emission, and that the activity A_0 remains practical constant within this second.
b) Calculate the number of nuclei present in 1g of thorium 230. Deduce, in year^{-1} , the value of the constant λ of thorium 230.

Problem 7

Measurement of the age of Earth

PHYSICS
LS-2006-1

One of the questions that preoccupied man long ago since he started to explore the universe was the age of the Earth. As from 1905, Rutherford proposed a measurement of the age of minerals through radioactivity.

In 1956, Clair Patterson used the method (uranium - lead) to measure the age of a meteorite originating from a planet that is formed approximately at the same time as that of Earth.

I – A radioactive family of uranium $^{238}_{92}\text{U}$

Uranium 238, with a radioactive period $T = 4.5 \times 10^9 \text{ y}$ (year) is at the origin of a radioactive family leading finally to the stable lead isotope $^{206}_{82}\text{Pb}$.

Each of these successive disintegrations is accompanied with the emission of an α particle or a β^- particle.

The diagram (page 4) gives all the radioactive nuclei originating from the uranium $^{238}_{92}\text{U}$ leading to the stable isotope $^{206}_{82}\text{Pb}$. (page 4)

The tables (page 4) give the radioactive period of each nuclide.

1) In the first disintegration, a uranium nucleus $^{238}_{92}\text{U}$ gives thorium nucleus $^{234}_{90}\text{Th}$ and a particle denoted by ${}_{Z_1}^{A_1}\text{X}$.

a) Write the equation of this disintegration and calculate the values of A_1 and Z_1 .

b) Specify the type of radioactivity corresponding to this transformation.

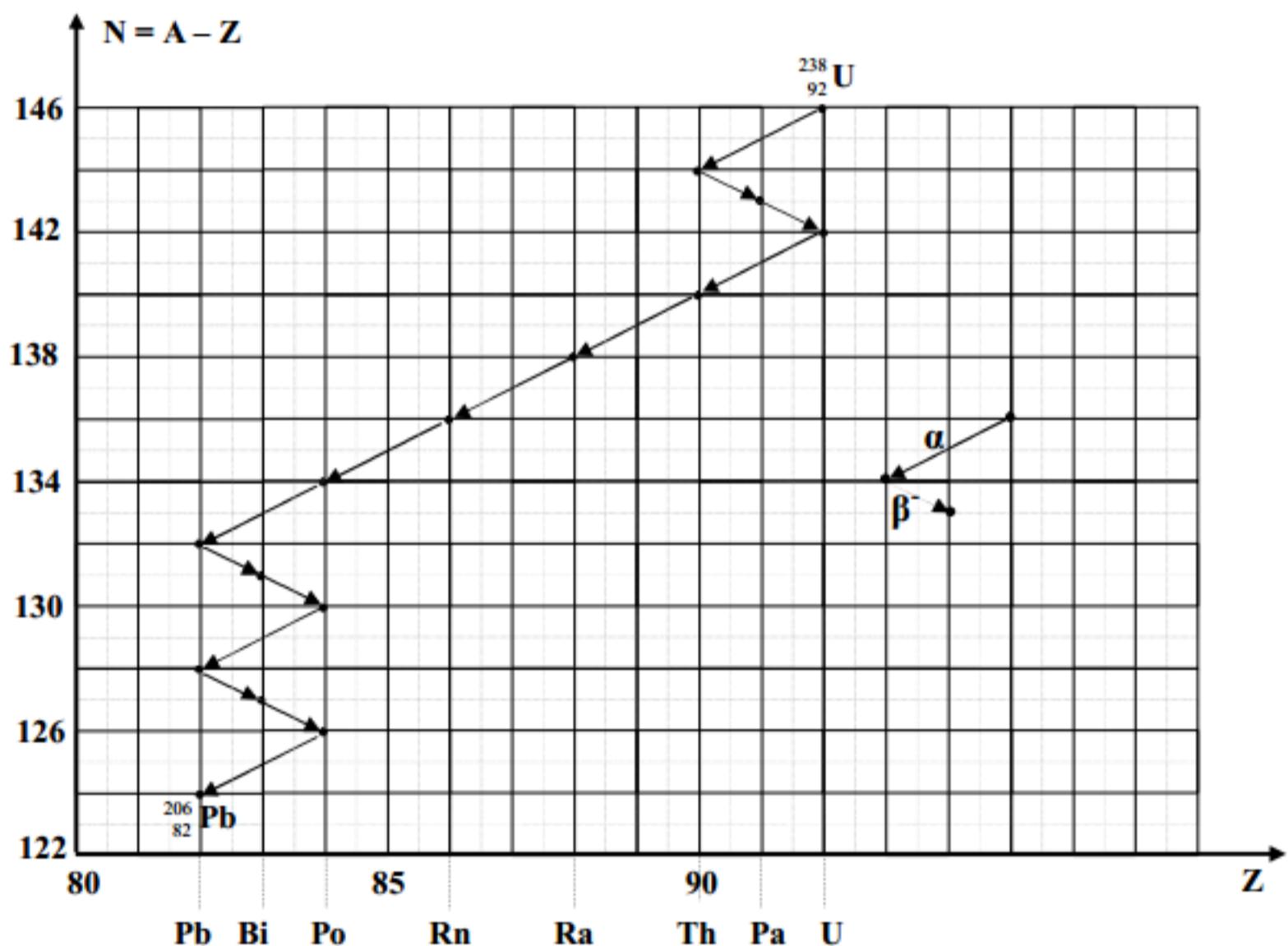
2) In the second disintegration, the thorium nucleus $^{234}_{90}\text{Th}$ undergoes a β^- decay.

The daughter nucleus is the protactinium ${}_{Z_2}^{A_2}\text{Pa}$. Calculate A_2 and Z_2 .

3) a) Referring to the diagram, tell how many α particles and how many β^- particles are emitted when the uranium nucleus $^{238}_{92}\text{U}$ is transformed into lead nucleus $^{206}_{82}\text{Pb}$.

b) Write the overall nuclear equation of the decay of uranium 238 into lead 206.

4) Using the diagram (Z,N) of the figure and the tables, tell why after few billions of years we can neglect the presence of the intermediary nuclei among the products of the disintegration uranium-lead.



Nucleus	$^{238}_{92}\text{U}$	$^{234}_{90}\text{Th}$	$^{234}_{91}\text{Pa}$	$^{234}_{92}\text{U}$	$^{230}_{90}\text{Th}$	$^{226}_{88}\text{Ra}$	$^{222}_{86}\text{Rn}$
Radioactive period	4.5×10^9 y	24 d	6.7 h	2.5×10^5 y	7.5×10^3 y	1.6×10^3 y	3.8 d

Nucleus	$^{218}_{84}\text{Po}$	$^{214}_{82}\text{Pb}$	$^{214}_{83}\text{Bi}$	$^{214}_{84}\text{Po}$	$^{210}_{82}\text{Pb}$	$^{210}_{83}\text{Bi}$	$^{210}_{84}\text{Po}$
Radioactive period	3.1 min	27 min	20 min	1.6×10^{-4} s	22 y	5 d	138 d

II- The age of Earth

We have studied a sample of a meteorite whose age is equal to that of Earth. At the instant t , the sample studied contains 1g of uranium 238 and 0.88g of lead 206.

We suppose that at the instant of its formation $t_0 = 0$, the meteorite does not contain any atom of lead.

Numerical data: molar mass of uranium: 238 g/mol; molar mass of lead: 206 g/mol;
Avogadro's number: $N_A = 6.02 \times 10^{23}$ /mol.

1) Calculate, at the instant t :

- the number of uranium 238 nuclei, denoted by $N_U(t)$, present now in the sample;
- the number of lead 206 nuclei, denoted by $N_{Pb}(t)$, present now in the sample.

2) Deduce the number of uranium 238 nuclei $N_U(0)$, present in the sample at the instant $t_0 = 0$.

3) Give the expression of $N_U(t)$ as a function of $N_U(0)$, t , and T .

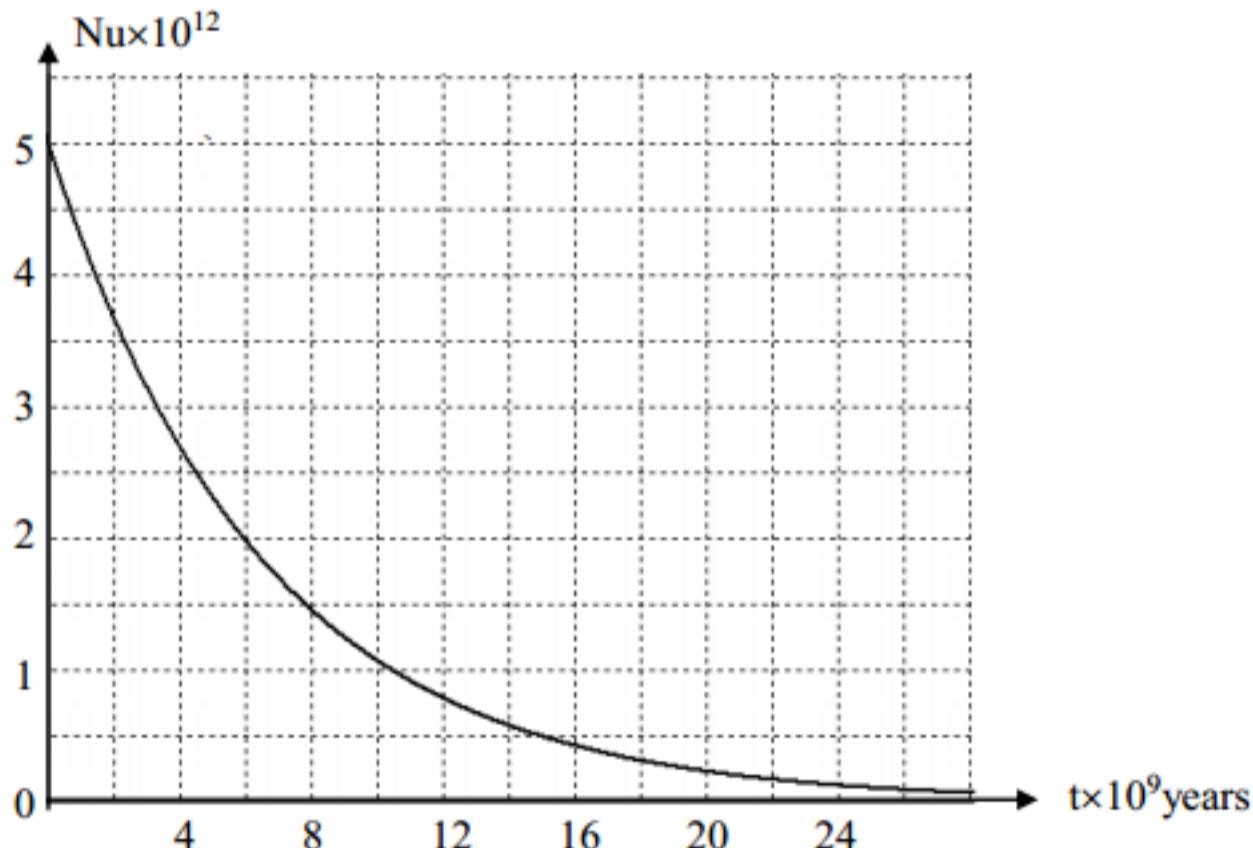
4) Deduce the age of Earth at the instant t .

Problem 8:

Determination of the age of the Earth

PHYSICS
LS-2008-1

The object of this exercise is to determine the age of the Earth using the disintegration of a uranium 238 nucleus ($^{238}_{92}U$) into a lead 206 nucleus ($^{206}_{82}Pb$).

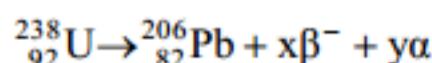


When we determine the number of lead 206 nuclei in a sample taken out from a rock that did not contain lead when it was formed, we can then determine its age that is the same as that of the Earth. The above figure represents the curve of the variation of the number N_u of uranium 238 nuclei as a function of time.

1 division on the axis of ordinates corresponds to 10^{12} nuclei.

1 division on the axis of abscissa corresponds to 10^9 years.

The equation of the disintegration of Uranium 238 into lead 206 is:



- Determine, specifying the laws used, the values of x and y .
- Referring to the curve, indicate the number N_{0u} of uranium 238 nuclei existing in the sample at the date of its birth $t_0 = 0$.
- Referring to the curve, determine the period (half-life) of uranium 238. Deduce the value of the radioactive constant λ of uranium 238.

4. a) Give, in terms of N_{0u} , λ and t , the expression of the number N_u of uranium 238 nuclei remaining in the sample at instant t .
 b) Calculate the number of uranium 238 nuclei remaining in the sample at instant $t_1 = 2 \times 10^9$ years:
 c) Verify the result graphically:
5. The number of lead 206 nuclei existing in the sample at the instant of measurement (age of the Earth) is 1.2×10^{12} nuclei.
 a) Give the relation among N_u , N_{0u} and N_{pb} .
 b) Calculate the number N_u of uranium nuclei remaining in the sample at the date of measurement.
 c) Determine the age of the Earth.

Problem 9

The radionuclide Potassium 40

PHYSICS
GS-2008-2

The isotope potassium $^{40}_{19}\text{K}$, is radioactive and is β^+ emitter; it decays to give the daughter nucleus argon $^{40}_{18}\text{Ar}$.

The object of this exercise is to study the decay of potassium 40.

masses of nuclei: $m(^{40}_{19}\text{K}) = 39.95355 \text{ u}$; $m(^{40}_{18}\text{Ar}) = 39.95250 \text{ u}$;

masses of particles: $m(^0_1\text{e}) = 5.5 \times 10^{-4} \text{ u}$; $m(\text{neutrino}) \approx 0$;

Avogadro's number: $N = 6.02 \times 10^{23} \text{ mol}^{-1}$; $1 \text{ u} = 931.5 \text{ MeV/c}^2$;

Radioactive half-life of $^{40}_{19}\text{K}$: $T = 1.5 \times 10^9$ years; molar mass of $^{40}_{19}\text{K} = 40 \text{ g mol}^{-1}$. $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$.

A – Energetics and study of the decay of potassium 40

1) Energy released by one decay

a) Write down the equation of the decay of one potassium 40 nucleus and determine Z and A.

b) Calculate, in MeV, the energy E_1 liberated by this decay.

c) The daughter nucleus is supposed to be at rest. The energy carried by β^+ is, in general, smaller than E_1 . Why?

2) Energy received by a person

The mass of potassium 40 at an instant t , in the body of an adult is, on the average, equal to 2.6×10^{-4} % of its mass.

An adult person has a mass $M = 80 \text{ kg}$.

a) i) Calculate the mass m of potassium 40 contained in the body of that person at the instant t .

ii) Determine the number of potassium 40 nuclei in the mass m at the instant t .

b) i) Calculate the radioactive constant λ of potassium 40.

ii) Determine the value of the activity A of the mass m at the instant t .

c) Deduce, in J, the energy E liberated by the mass m per second.

B – Dating by potassium 40

Certain volcanic rocks contain potassium and part of it, is potassium 40. At the instant of its formation ($t_0 = 0$), the number of nuclei of potassium 40 is N_0 in the volcanic rock and that of argon is zero. At the instant t , the rock contains respectively N_K and N_{Ar} nuclei of potassium 40 and of argon 40.

1) a) Write down the expression of N_K , that explains the law of radioactive decay, as a function of time.

b) Deduce the expression of N_{Ar} as a function of time.

2) A geologist analyzes a volcanic rock. He notices that the number of argon 40 nuclei is twice less than the number of potassium 40 nuclei in this rock. Determine the age of this rock.

Problem 10

Determination of the half-life of Polonium 210

PHYSICS
GS-2009-2

Polonium 210 nucleus ($^{210}_{84}\text{Po}$) is an α emitter, and it is the only polonium isotope that exists in nature; it was found by Marie Curie in 1898 in an ore. It is also obtained from the decay of a bismuth 210 nucleus ($^{210}_{83}\text{Bi}$). Masses of the nuclei: $m(\text{Bi}) = 209.938445 \text{ u}$; $m(\text{Po}) = 209.936648 \text{ u}$
mass of the electron : $m_e = 0.00055 \text{ u}$

$$1 \text{ u} = 931.5 \text{ MeV} / c^2 = 1.66 \times 10^{-27} \text{ kg.}$$

Here is a part of the periodic table of elements: $_{81}\text{Th}$; $_{82}\text{Pb}$; $_{83}\text{Bi}$; $_{84}\text{Po}$; $_{85}\text{At}$; $_{86}\text{Rn}$.

A –The polonium 210

- 1) a) Write down the equation of the decay of bismuth 210.
- b) Identify the emitted particle and specify the type of this decay.
- 2) Calculate the energy liberated by this decay.
- 3) The decay of the bismuth 210 nucleus is accompanied with the emission of a γ photon of energy $E(\gamma) = 0.96 \text{ MeV}$ and an antineutrino of energy 0.02 MeV . Knowing that the daughter nucleus is practically at rest, calculate the kinetic energy of the emitted particle.

B – Half-life of polonium 210

- 1) a) Write down the equation of the decay of polonium 210.
- b) Identify the daughter nucleus.
- 2) In order to determine the radioactive period T (half-life) of $^{210}_{84}\text{Po}$, we consider a sample of this isotope containing N_0 nuclei at the instant $t_0 = 0$. Let N be the number of the non-decayed nuclei at an instant t .
 - a) Write down the expression of the law of radioactive decay.
 - b) Determine the expression of $-\ln\left(\frac{N}{N_0}\right)$ as a function of t .
- 3) A counter allows to obtain the measurements that are tabulated in the following table :

t (days)	0	40	80	120	160	200	240
$\frac{N}{N_0}$	1	0.82	0.67	0.55	0.45	0.37	0.30
$-\ln\left(\frac{N}{N_0}\right)$	0		0.4		0.8		1.2

a) Complete the table.

b) Trace, on the graph paper, the curve giving the variation of $-\ln\left(\frac{N}{N_0}\right)$ as a function of time.

Scale: 1 cm on the abscissa axis corresponds to 40 days.

1 cm on the ordinate axis corresponds to 0.2.

- c) Is this curve in agreement with the expression found in the question (B – 2, b) ? Justify.
- d) i) Calculate the slope of the traced curve.
ii) What does this slope represent for the polonium 210 nucleus?
iii) Deduce the value of T .

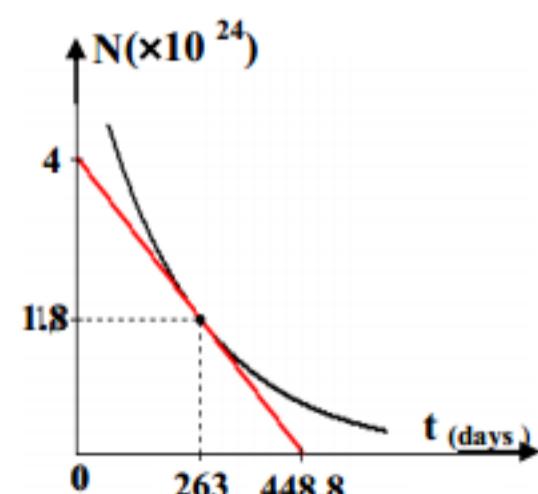
Problem 11:

Radioactive period of polonium 210

PHYSICS
LS-2010 -1

The adjacent figure shows the curve representing the variations with time t of the number N of the nuclei present in the radioactive sample $^{210}_{84}\text{Po}$, this number being called N_0 at the instant $t_0 = 0$. The same figure shows also the tangent to that curve at the instant $t_1 = 263$ days.

- 1) Write down the expression of N as a function of t and specify what each term represent.
- 2) The activity of the radioactive sample is given by: $A = -\frac{dN}{dt}$.
 - a) Define the activity A .
 - b) Using the given on the figure above, determine the activity A of the sample at the instant $t_1 = 263$ days.
- 3) Deduce the value of the radioactive constant and the value of the half-life (period) of polonium 210.



Problem 12**Nuclear reactions and dating**

$m(\alpha) = 4.0026 \text{ u}$; $m(^1_0 n) = 1.00866 \text{ u}$; $m(^1_1 p) = 1.00728 \text{ u}$; $m(^{14}_7 N) = 13.99924 \text{ u}$;
 $m(^{14}_6 C) = 12.00000 \text{ u}$; $m(^{17}_8 O) = 16.99473 \text{ u}$; $1 \text{ u} = 931.5 \text{ MeV}/c^2$.

A – Artificial reaction

The first provoked artificial reaction was performed in 1919 by Ernest Rutherford at Cambridge. He bombarded nitrogen nuclei ($^{14}_7 N$) with α particles ($^4_2 He$) having great kinetic energies. Oxygen nuclei ($^{17}_8 O$) and protons ($^1_1 p$) are obtained. The equation corresponding to the reaction relative to one nitrogen nucleus is written as: $^4_2 He + ^{14}_7 N \longrightarrow ^{17}_8 O + x ^1_1 p$

- 1) Show that, specifying the used law, $x = 1$.
- 2) a) Calculate the “mass before” and the “mass after” in this nuclear reaction.
 b) Deduce that this reaction needs an amount of energy to be performed.
- 3) We neglect the kinetic energy of the proton and those of nitrogen and oxygen nuclei. Show that, by applying the principle of conservation of the total energy, the kinetic energy of the α particle is equal to 1.183 MeV .

B – Natural reaction

A provoked reaction of nitrogen 14 occurs naturally. Indeed, when, in the upper atmosphere, a neutron of the cosmic radiation hits a nitrogen nucleus ($^{14}_7 N$), a reaction takes place and produces a carbon nucleus ($^{14}_6 C$), that is a radioactive isotope of the stable carbon nucleus ($^{12}_6 C$). The equation corresponding to this reaction is written as: $^1_0 n + ^{14}_7 N \longrightarrow ^{14}_6 C + ^1_1 p$

- 1) Calculate the “mass before” and the “mass after” in this nuclear reaction.
- 2) Deduce that this reaction liberates energy.

C – Carbon dating

The plants absorb carbon dioxide from the atmosphere formed of both carbon 14 and carbon 12. The ratio of these two isotopes is the same in plants and in atmosphere. When the plant dies, it stops absorbing carbon dioxide. The carbon 14 existing in this plant disintegrates then without being compensated. The period (half-life) of carbon 14 is $T = 5730$ years.

- 1) Calculate, per year $^{-1}$, the radioactive constant λ of carbon 14.
- 2) An analysis of a sample of wood (dead plant), found in an Egyptian tomb, shows that its activity is 750 disintegrations per minute whereas the activity of a plant of the same nature and of the same mass is 320 disintegrations per minute.
 Determine the age of the sample of wood found in the Egyptian tomb.

Problem**Iodine 131**

The aim of this exercise is to show evidence of some characteristics of iodine 131.

Iodine 131 ($^{131}_{53} I$) is radioactive and is a β^- emitter. Its radioactive period (half-life) is 8 days.

Given: Mass of an electron: $m_e = 5.5 \times 10^{-4} \text{ u}$; $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$; $1 \text{ u} = 931.5 \text{ MeV}/c^2$.

Element	Iodine ($^{131}_{53} I$)	Cesium($^{137}_{55} Cs$)	Xenon($^{131}_{54} Xe$)
Mass of nucleus	130.8770 u	136.8773 u	130.8754 u

A – Disintegration of iodine 131

- 1) Write down the equation of the disintegration of iodine 131 and identify the daughter nucleus.
- 2) The disintegration of iodine 131 nucleus, is often, accompanied with the emission of γ rays. Due to what is this emission?
- 3) Calculate the radioactive constant λ of iodine 131 in day^{-1} and in s^{-1} .
- 4) Show that the energy liberated by the disintegration of one nucleus of iodine 131 is $E_{lib} = 1.56 \times 10^{-13} \text{ J}$.

B – Application in medicine

During a medical examination of a thyroid gland of a patient, we inject this gland with a solution of iodine

131. The thyroid of this patient captures from this solution a number $N = 10^{11}$ of iodine nuclei.

- 1) Calculate, in Bq, the activity A corresponding to these N nuclei knowing that $A = \lambda N$.
- 2) Calculate, in J, the energy liberated by the disintegration of these N nuclei.
- 3) Deduce, in J/kg, the value of the dose absorbed by the thyroid gland knowing that its mass is 25 g.

C – Contamination

On the 26th of April 1986, an accident took place in the nuclear power plant of Chernobyl that provoked an explosion in one of the reactors. One of the many radioactive elements that were ejected to the atmosphere is the iodine 131. This element spread on the ground, absorbed by cows and contaminated their milk and then captured by the thyroid gland of consumers.

Every morning, a person drank a certain quantity of milk containing $N_0 = 2.6 \times 10^{16}$ nuclei of iodine 131.

We suppose that all these nuclei were captured by the thyroid of that person, and that the person drank the first quantity at the instant $t_0 = 0$.

- 1) Determine, in terms of N_0 and λ (expressed in day⁻¹), the number of iodine 131 nuclei that remained in the thyroid, at the instant:
 - a) $t_1 = 1$ day, (just after drinking the 2nd quantity of milk);
 - b) $t_2 = 2$ days, (just after drinking the 3rd quantity of milk).
- 2) Deduce, at the instant $t_3 = 3$ days just after drinking the 4th quantity of milk that the number N_3 of the iodine 131 nuclei that remained in the thyroid is: $N_3 = N_0 (1 + e^{-\lambda} + e^{-2\lambda} + e^{-3\lambda})$ where λ is expressed in day⁻¹.
- 3) Serious troubles in the thyroid gland will take place if the activity of the iodine 131 exceeds 75×10^9 Bq. Show that at the instant t_3 , the person was in danger.

Problem 14:

Dating by Carbon 14

PHYSICS
LS-2013-1

The radioactive carbon isotope $^{14}_6\text{C}$ is a β^- emitter. In the atmosphere, $^{14}_6\text{C}$ exists with the carbon 12 in a constant ratio.

When an organism is alive it absorbs carbon dioxide that comes indifferently from carbon 12 and carbon 14. Just after the death of an organism, this absorption stops and carbon 14, that it has, disintegrate with a half life $T = 5700$ years.

In living organisms, the ratio of the number of carbon 14 atoms to that of the number of carbon 12 atoms is: $r_0 = \frac{\text{initial number of carbon 14 atoms}}{\text{number of carbon 12 atoms}} = \frac{N_0(^{14}\text{C})}{N(^{12}\text{C})} = 10^{-12}$.

After the death of an organism by a time t , the ratio of the number of carbon 14 atoms to that of the

number of carbon 12 atoms becomes: $r = \frac{\text{remaining number of carbon 14 atoms}}{\text{number of carbon 12 atoms}} = \frac{N(^{14}\text{C})}{N(^{12}\text{C})}$.

1) The disintegration of $^{14}_6\text{C}$ is given by: $^{14}_6\text{C} \rightarrow {}_Z^A\text{N} + \beta^- + {}_0^0\bar{\nu}$.

Calculate Z and A, specifying the laws used.

2) Calculate, in year⁻¹, the radioactive constant λ of carbon 14.

3) Using, the law of radioactive decay of carbon 14, $N(^{14}\text{C}) = N_0(^{14}\text{C}) \times e^{-\lambda t}$.

Show that $r = r_0 e^{-\lambda t}$.

4) Measurements of $\frac{r}{r_0}$, for specimens a, b and c, are given in the following table:

ratio	specimen a	specimen b	specimen c
$\frac{r}{r_0}$	0.914	0.843	0.984

a) Specimen b is the oldest. Why?

b) Determine the age of specimen b.

5) a) Calculate the ratio $\frac{r}{r_0}$ for $t_0 = 0$, $t_1 = 2T$, $t_2 = 4T$ and $t_3 = 6T$.

b) Trace then the curve $\frac{r}{r_0} = f(t)$ by taking the following scales:

- On the abscissa axis: 1 cm $\rightarrow 2T$
- On the ordinate axis: 1 cm $\rightarrow \frac{r}{r_0} = 0.2$

c) To determine the date of death of a living organism, it is just enough to measure $\frac{r}{r_0}$.

Explain why we cannot use the traced curve to determine the date of the death of an organism that died several millions years ago.

Problem 15:

The radioactivity of cobalt-60

PHYSICS LS-2016-1

The cobalt isotope $^{60}_{27}\text{Co}$ is radioactive of a radioactive constant $\lambda = 4.146 \times 10^{-9} \text{ s}^{-1}$. Consider a sample of this isotope of mass $m_0 = 1 \text{ g}$ at the instant $t_0 = 0$.

Given:

- | Symbol | $^{60}_{27}\text{Co}$ | $^{60}_{28}\text{Ni}$ | ^A_ZX |
|-------------|-----------------------|-----------------------|----------------|
| Mass (in u) | 59.9190 | 59.9154 | 0.00055 |
- 1 year = 365 days.
 - 1u = 931.5 MeV/c² ; • Avogadro's number: $6.02 \times 10^{23} \text{ mol}^{-1}$; • Molar mass of cobalt: 60 g.mol⁻¹;

- Calculate, in years, the period of the cobalt- 60 nucleus.
- a) Determine, at $t_0 = 0$, the number of nuclei N_0 presented in 1 g of cobalt- 60.
b) Define the activity A of a radioactive sample.
c) Determine the activity of the cobalt sample at the instant $t = 15.9$ years.
- The disintegrations of $^{60}_{27}\text{Co}$ gives rise to a nickel isotope $^{60}_{28}\text{Ni}$ according to the following reaction:



- a) Calculate, specifying the laws used, A and Z.
b) Name the emitted particles.
c) Calculate, in MeV, the energy liberated by this disintegration.
d) Determine the energy liberated by the disintegration of 1g of cobalt- 60.
- Knowing that the energy liberated by the fission of 1 g of $^{235}_{92}\text{U}$ is 5.127×10^{23} Mev, calculate the mass of $^{235}_{92}\text{U}$ whose fission provides an energy equivalent to that liberated by the disintegration of 1 g of cobalt-60.

Problem 16:

Determination of the volume of the blood of a person by radioactivity

PHYSICS LS-2016-2

In order to determine the volume of the blood of a person, we use the radionuclide sodium $^{24}_{11}\text{Na}$.

- Given: • Planck's constant: $h = 6.63 \times 10^{-34} \text{ J.s}$; • Speed of light in vacuum: $c = 3 \times 10^8 \text{ m/s}$;
• Avogadro's number: $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$; • Molar mass of sodium 24: $M = 24 \text{ g}$; • $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$.
• Selection from the periodic table:

Element	Fluorine	Neon	Sodium	Magnesium	Aluminium
Nuclide	$^{19}_9\text{F}$	$^{20}_{10}\text{Ne}$	$^{23}_{11}\text{Na}$	$^{24}_{12}\text{Mg}$	$^{27}_{13}\text{Al}$

A - Sodium $^{24}_{11}\text{Na}$ is obtained by bombarding the sodium $^{23}_{11}\text{Na}$ by a neutron.

- Write the equation of this nuclear reaction.
- This reaction is provoked. Justify.

B - The sodium 24 is radioactive β^- emitter.

- 1) Write the equation of this disintegration.
- 2) Name the obtained daughter nucleus.
- 3) The disintegration of sodium 24 is accompanied by the emission of a dangerous radiation γ .
 a) Indicate the nature of this radiation.
 b) Indicate the cause of the emission of this radiation.
 c) One of the emitted photons has energy of 3 MeV. Calculate the wavelength of the corresponding radiation.

C - The radioactive decay constant of sodium 24 is $\lambda = 1.28 \times 10^{-5} \text{ s}^{-1}$.

- 1) At the instant $t_0 = 0$, we inject a solution containing $m_0 = 2.4 \times 10^{-4} \text{ g}$ of sodium 24 into the blood of a person. Calculate the number of nuclei N_0 of sodium 24 in the injected solution.
- 2) Calculate at the instant $t = 6$ hours, the number of sodium 24 nuclei remaining in the blood of the person.
- 3) Suppose that the sodium 24 is uniformly distributed in the blood of the person. At the instant $t = 6$ hours, 10 mL of blood taken from the person contains 9.03×10^{15} nuclei of sodium 24. Calculate the volume of the blood of the person.

Problem 1

Scintigraphy in medicine

PHYSICS
LS - 2018-1

The bones scintigraphy is a medical examination that permits to observe bones and articulations. The aim of this examination is to study a radioactive sample used in this scintigraphy.

This medical examination uses technetium-99 produced due to the disintegration of molybdenum-99 according to the following nuclear reaction: $^{99}_{42}\text{Mo} \rightarrow ^{99}_{43}\text{Tc} + ^A_Z\text{X} + \gamma$

The energy of the emitted gamma (γ) photon is 140 keV.

Given: $c = 3 \times 10^8 \text{ m.s}^{-1}$; $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$; Planck's constant $h = 6.6 \times 10^{-34} \text{ J.s}$.

- 1- Identify the emitted particle ^A_ZX , indicating the used laws.
- 2- The emitted particle ^A_ZX is always accompanied with the emission of another particle. Name this particle.
- 3- Indicate the cause of the emission of the gamma photon.
- 4- Calculate the wavelength of the emitted gamma photon.

5- Technetium-99 is a radioactive substance.

The graph document 2 represents the activity of a sample of technetium-99 as a function of time. Using document 2, show that the radioactive half-life of technetium-99 is $T = 6$ hrs.

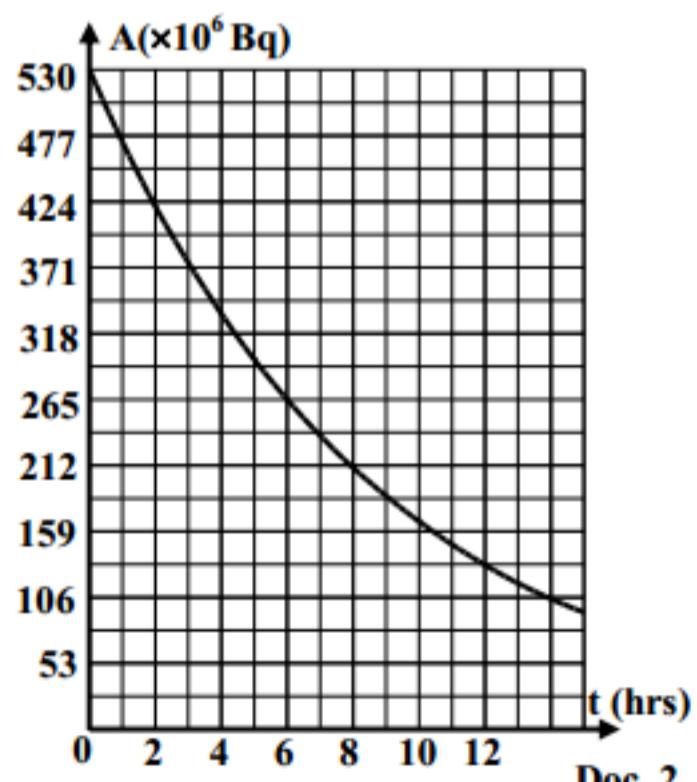
- 6- In a session of scintigraphy examination, a patient is injected at $t = 0$ by technetium-99 of activity $A_0 = 530 \times 10^6 \text{ Bq}$. At the end of the examination session, the activity of technetium in the body of the patient is 63% of its initial value.

6-1) Write at instant t , the expression of the activity A in term of A_0 , t and the decay constant λ .

6-2) Using the preceded expression, determine:

6-2-1) the duration of the examination session;

6-2-2) the ratio $\frac{A}{A_0}$ of technetium-99 at $t = 40$ hrs.



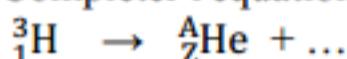
Le tritium ${}^3_1\text{H}$ est un isotope radioactif de l'hydrogène. Le tritium est produit dans la haute atmosphère par les rayonnements cosmiques et amené sur Terre par la pluie. Le tritium peut être utilisé pour déterminer l'âge des liquides contenant cet isotope d'hydrogène.

Dans cet exercice, on compte déterminer l'âge d'un liquide contenu dans une ancienne bouteille en utilisant l'activité du tritium.

1- Décroissance radioactive du tritium

Le tritium est un émetteur beta-moins (β^-). Il se désintègre pour donner un des isotopes de l'hélium sans émission de rayonnement gamma.

- 1-1)** Compléter l'équation de la décroissance du tritium et déterminer A et Z.



- 1-2)** Le noyau d'hélium est produit à l'état fondamental. Pourquoi ?

- 1-3)** Une particule X accompagne la désintégration ci-dessus afin de satisfaire à une certaine loi.
Nommer cette particule et cette loi.

2- Détermination de la période radioactive du tritium

On considère un échantillon de l'isotope radioactif du tritium ${}^3_1\text{H}$.

À un instant $t_0 = 0$, le nombre des noyaux présents dans cet échantillon est N_0 .

L'activité A de l'échantillon radioactif représente le nombre de désintégrations par unité de temps.

L'activité à un instant t est donnée par l'expression suivante : $A = -\frac{dN}{dt}$, où N est le nombre des noyaux restants (non désintégrés) à l'instant t.

- 2-1)** Montrer que l'équation différentielle du premier ordre qui décrit les variations de N est :

$$\frac{dN}{dt} + \lambda N = 0, \text{ avec } \lambda \text{ est la constante de décroissance radioactive de l'isotope radioactif.}$$

- 2-2)** Vérifier que $N = N_0 e^{-\frac{t}{\tau}}$ est une solution de l'équation

$$\text{différentielle ci-dessus, où } \tau = \frac{1}{\lambda}.$$

- 2-3)** Déduire que l'expression de l'activité est donnée par :

$$A = A_0 e^{-\frac{t}{\tau}}, \text{ où } A_0 \text{ est l'activité initiale de l'échantillon.}$$

- 2-4)** Calculer A en fonction de A_0 lorsque $t = \tau$.

- 2-5)** Le document 6 représente l'activité d'un échantillon de tritium en fonction du temps.

- 2-5-1)** Montrer que $\tau = 17,7$ ans.

- 2-5-2)** Déduire la période radioactive du tritium.

3 Détermination de l'âge d'un liquide

Une ancienne bouteille contient un certain liquide, elle est juste

ouverte (en 2018). On a trouvé que l'activité du tritium dans ce liquide est 10,4 % de l'activité initiale du même liquide fraîchement préparé. Déterminer l'année de production du liquide dans l'ancienne bouteille.

