2 Exercises and problems

N° 1
Charging of a capacitor « 1 »

A capacitor, of capacitance $C = 470\ \mu F$, is connected across a battery ($E = 6$ V).

1) Calculate the charge and the electric energy stored in capacitor at end of charging.
2) Determine the intensity of the current in the circuit at the end of charging.

N° 2
Charging of a capacitor « 2 »

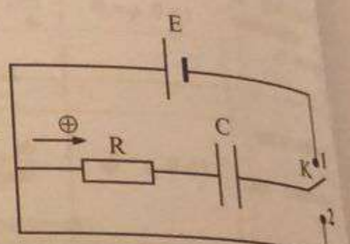
A capacitor, of capacitance $C = 100\ \mu F$, is placed in series with a resistor of resistance $R = 1\ k\Omega$, across the terminals of the battery delivering a constant voltage $E = 12$ V. At $t_0 = 0$ we close the switch.

1) Define the time constant τ of the circuit and calculate its value.
2) Find, in the steady state, the charge Q of the capacitor.
3) Calculate, at instant at $t = \tau$, the charge of the capacitor. Deduce, at the same instant, the voltage across the capacitor and the resistor and the intensity of current in the circuit.

N° 3
Charging and discharging of a capacitor (1)

In the electric circuit of the adjacent figure, the capacitor passes through two phases : Phase of charging and phase of discharging.

We represent, in figures (1) and (2) below, the curves, as a function time, of the voltage u_C across the terminals C and of the intensity i of current corresponding to phases of charging and discharging.



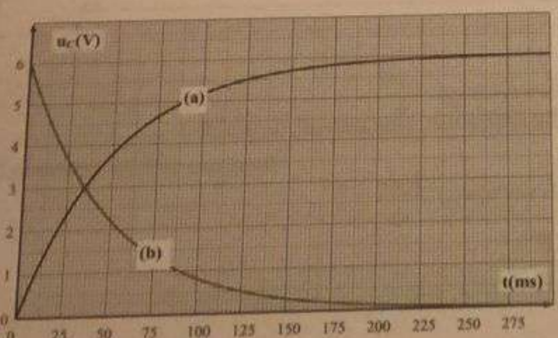


Figure (1)

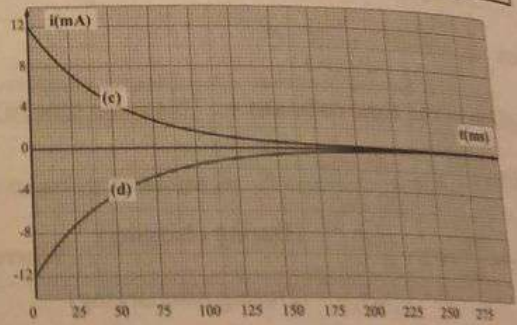
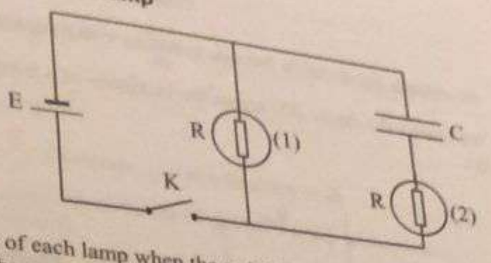


Figure (2)

1) On which position must we put the switch K in order to charge the capacitor?
2) a) Specify the curves which correspond to the phase of charging of the capacitor.
b) Extract the values of u_C and i at the beginning and at the end of charging.
c) Deduce E.
3) Extract the values of u_C and i at the beginning and at the end of discharging.
4) Give the significance of the negative sign of i in curve (d).
5) Verify that the time constant τ of the (R-C) circuit has the same values in the two phases and on any curves in figures (1) and (2).

N° 4
Aspect of lighting of a lamp

We consider two identical lamps, each carrying a resistor of resistance $R = 50\ \Omega$, a capacitor of capacitance $C = 500$ mF initially neutral, a battery of electromotive force $E = 12$ V and of negligible internal resistance and a switch K.
Using the preceding dipoles we connect the circuit represented in the adjacent figure.



1) At $t = 0$, we close K.
a) Write, with justification, the aspect of brightness of each lamp when the switch is closed.
b) Calculate, in the steady state, the intensities of the current in the different branches of the circuit and the electric energy stored in the capacitor.
c) Represent, in the same system of axes, the curves of the intensities of the currents i_1 and i_2 circulating in the lamps (1) and (2) respectively and indicate on each curve two particular points.
2) The steady phase is attained, at a new instant taken as an origin of time, we open the switch.
a) Describe, with justification, the aspect of each lamp.
b) Represent the curve of the intensity of the current i circulating in the lamps. Indicate on the curve two particular points.

N° 5
Differential equation of charging

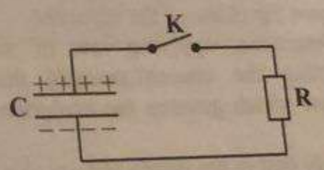
A capacitor, of capacitance $C = 100\ \mu F$, is placed in series with a resistor, of resistance $R = 1\ k\Omega$, across the terminals of a battery delivering a constant voltage $E = 12$ V. At $t_0 = 0$ we close the circuit.

1) Determine, as a function of C, R and E, the expression of the differential equation in terms of the charge q of the capacitor.

2) The solution of the differential equation is under the form : $q = \alpha e^{-\frac{t}{\tau}} + \beta$. Determine α, τ and β as a function of C, R and E.
3) Deduce the intensity i of the current in the circuit as function of time.
4) Calculate the instant when the intensity of the current becomes 10 mA.

N° 6
Differential equation of discharging

We consider the set up in the adjacent figure.



The switch K is open. The capacitor is initially charged under a constant voltage $E = 12$ V. We close K_2 at the instant $t_0 = 0$.

Given : $C = 500\ \mu F$; $R = 2\ k\Omega$ and $u_{AB} = u_C$.

1) Indicate on a figure the direction of the current in the circuit.
2) Determine the following differential equation : $\frac{du_C}{dt} + \frac{u_C}{RC} = 0$ (The positive direction is that of current).
3) Show that the solution of the preceding differential equation is under the form : $u_C = Ae^{-Bt}$ where A and B are constants to be determined.
4) Calculate the instant when the capacitor stores half of its initial energy.

N° 7
Characteristics of an electric dipole

An electric dipole (D) is formed, in series, with a capacitor, of capacitance C, and a resistor of resistance R.

We branch the dipole (D) across the terminals of a battery delivering a constant voltage E as shown in figure (a).

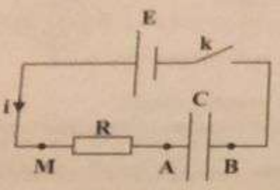
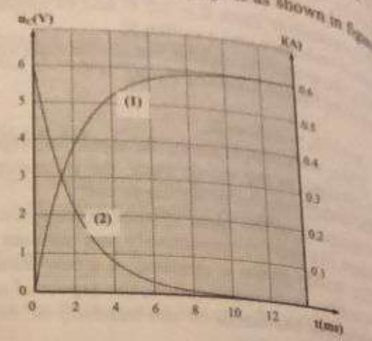


Figure (a)

Figure (b)

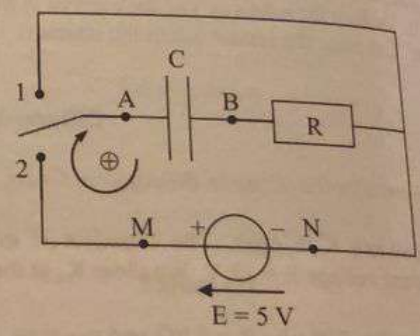


At $t_0 = 0$, we close the switch.

1) Name the phenomenon which takes place in the circuit.
2) Determine the differential equation that governs the variation of the voltage $u_C = u_{AB}$ as a function of time.
3) Verify that the solution of the preceding differential equation is : $u_C = E(1-e^{-\frac{t}{RC}})$.
4) Determine the intensity i of the current as function of time.
5) Deduce the expressions of u_C at the instants $t = \tau$ and $t \to +\infty$ and that of i at the instant $t_0 = 0$.
6) In the figure (b) we represent the graphs of u_C and i as a function of time.
a) Associate, with justification, to u_C and i the corresponding curve.
b) Deduce the values of E, R and C.

N° 8
Charging and discharging of a capacitor (2)

We want to study the discharging of a capacitor, initially neutral, of capacitance C = 60 μF, across a resistor of resistance R = 10 kΩ. We use, for this aim, the setup of the adjacent figure.



I – We close the switch on position 2.
1) Name the phase of the capacitor.
2) Determine, applying law of addition of voltage and respecting the chosen positive direction, the differential equation which governs the evolution of u_{AB} as a function of time.
3) Verify that in the steady state we have : $u_{AB} = E$.
4) Calculate the minimum duration, measured from the instant of closing the switch, in order to attain the steady state.
5) Calculate, in the steady state, the electric energy stored in the capacitor.

II – After several dozens of seconds, of closing the switch on position 2, we close the switch on position 1 at the time t = 0.
1) The discharging of the capacitor starts at the time t = 0. What can we say, using a voltmeter, if at this time, the charging of the capacitor is completed under the voltage E?

2) Verify that :

a) The differential equation of the discharging of the capacitor is : $\alpha \frac{du_{AB}}{dt} + u_{AB} = 0$ where α is a positive constant that we must identify.

b) The solution of the differential equation is : $u_{AB}(t) = Ee^{-\frac{t}{\alpha}}$.

3) We define the duration $t_{1/2}$ such that : $u_{AB}(t_{1/2}) = \frac{E}{2}$. Calculate $t_{1/2}$ as a function of α.

4) a) Trace, in a reference, the graph of u_{AB} as a function of time and its tangent at $t = 0$.
Scale : 1 cm ↔ 0.5 s (abscissa) and 1 cm ↔ 1 V (ordinate).

b) Trace, in the preceding reference, the graph of u_{AB} as a function of time when we replace the resistor by another one of resistance $R' = 20$ kΩ, and when the generator delivers a voltage $E' = 2.5$ V.

N° 9

Evaluating physical characteristics of a dipole in a circuit

We consider the circuit of figure (1) :

- C is a capacitor initially neutral;
- D_1 and D_2 are two resistors of respective resistances R_1 and R_2 ;
- The battery delivers a constant voltage U_0.

We suppose : $R = R_1 + R_2$.

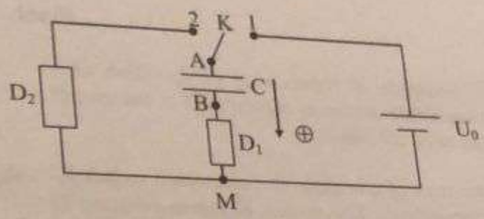


Figure (1)

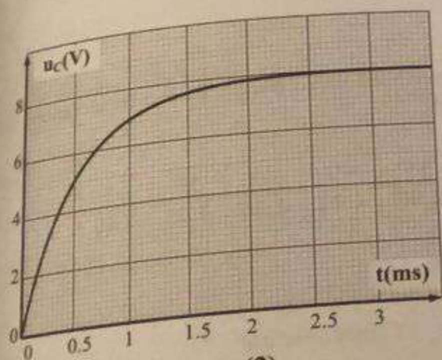


Figure (2)

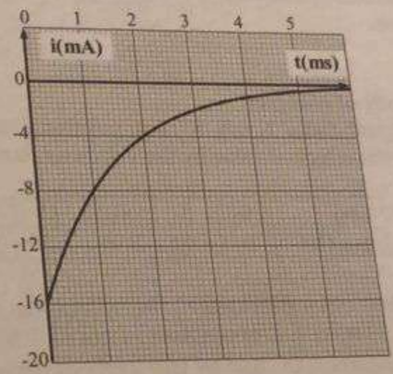


Figure (3)

I – Charging of the capacitor

At $t_0 = 0$, we close the switch K on 1.

An appropriate device permits us to record the voltage $u_{AB} = u_C$ of the capacitor as a function of time as indicated in figure (2).

1) Verify that : $i = C\frac{du_C}{dt}$.

2) Determine the expression of the differential equation (E_1) of u_C as a function of time.

3) The solution of the differential equation (E_1) is under the form: $u_C = Ae^{-\frac{t}{\tau_1}} + B$. Calculate A, τ_1 and B as a function of R_1, C and U_0.

4) Verify that τ_1 is time.
5) Using the graph in figure 2, extract U_0 and τ_1.

II – Discharging of the capacitor

We open K and we close it to 2 at an instant taken as a new origin of time.
An appropriate device permits us to record the intensity of the current, in the circuit, as a function of time indicated in figure (3).

1) a) Show that : $u_C = -Ri$

b) Deduce the differential equation (E_2) which governs i as a function of time.

c) Find, as a function of R and U_0, the intensity of the current at instant $t_0 = 0$.

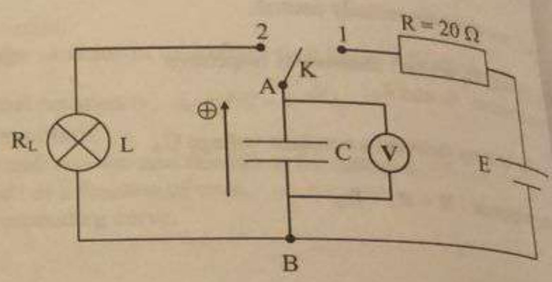
d) The solution of the differential equation (E_2) is under the form : $i = \alpha e^{-\frac{t}{\tau}}$. Calculate α and τ as a function of R, C and U_0.

2) Using the graph in figure 3, find R and τ.
3) Deduce the values of C, R_1 and R_2.

N° 10
Flash of a lamp

The principle of functioning of the flash of a photographic device is represented in the circuit of the adjacent figure.

At the start of charging, the electronic switch K is in position 1. The switch K leaves position 1 and is fixed in position 2 for approximately **10 ms**, then automatically K retains its precious position.



A – Study of charging

We suppose that at $t_0 = 0$, K is on position 1 and the capacitor is without any charge. In the table below we indicate the effective value by the voltmeter during each five seconds.

t (s)	0	5	10	15	20	25	30	35	40	45	50	55
u_{AB} (V)	0	1.85	2.95	3.58	3.96	4.2	4.31	4.40	4.44	4.46	4.48	4.49

1) Represent, on a graph paper, the voltage u_{AB} as a function of time.
   Scale : 1 cm ↔ 5 s for abscissa ; 1 cm ↔ 1 V for ordinate.
2) Extract, graphically and by two methods, the charging constant τ_C of the capacitor.
3) Deduce the capacitance C of the capacitor.
4) Calculate the electric energy stored in the capacitor, at the end of charging.

B – Study of discharging

The capacitor is totally charged under the voltage of the battery. At a new origin of time, K is on position 2.

We suppose that : $u_{AB} = u$ and the lamp is assimilated to a resistor of resistance R_L.

1) a) Determine, as a function of R_L, C, u and $\frac{du}{dt}$, the differential equation which governs the evolution of the voltage of the capacitor as a function of time.

a) The solution of the preceding equation is under the form $u = \alpha e^{-\beta t}$. Calculate, as a function of E, R, and C the constants of α and β.
After **10 ms** from the start of discharging, K is opened and the voltage across the capacitor is 2 V.
b) Calculate, at the moment of opening K, the stored energy.
c) Deduce the average electric power consumed by the lamp.
d) Why does the lamp emit an intense flash of light ?
e) Calculate the resistance of the lamp.

N° 11

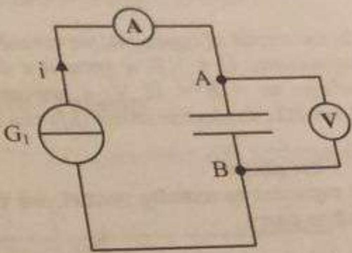
Calculating the capacitance and verifying some expressions

The aim of this exercise is to evaluate the capacitance C of a capacitor to verify the expression $\tau = RC$ and $i = C\frac{du_{AB}}{dt}$ in an (R-C) series circuit.

To achieve this objective we have :
A capacitor of capacitance C.
A resistor of resistance $R = 100\ \Omega$.
An analogue voltmeter.
An analogue ammeter.
A generator G_1 delivering a constant current of intensity I_0.
A generator G_2 delivering a constant voltage U_0.

A – Calculating the capacitance C

We connect the circuit in the adjacent figure where each 10 seconds we give the values recorded by the multimeters in the table below.

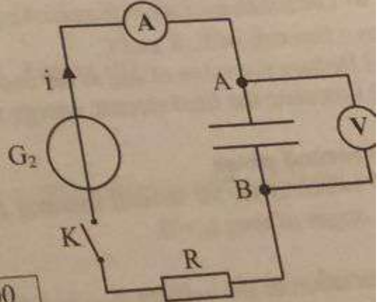


t(s)	0	10	20	30	40	50
i (mA)	50	50	50	50	50	50
u_{AB} (V)	0	0.5	1	1.5	2	2.5

1) Extract the value of I_0.
2) Verify that the charge of the capacitor at an instant t : $q_A = I_0 t$.
3) Show that : $C = \frac{I_0 t}{u_{AB}}$. Calculate C.

B – Verification of the expressions $\tau = RC$ and $i = C\frac{du_{AB}}{dt}$

We connect the adjacent figure where every 10 seconds we record the values indicated by the multimeters in the table below :



t (s)	0	10	20	30	40	50	60	70	80
u_{AB}(V)	0	1.14	2.17	3.11	3.95	4.72	5.41	6.0	6.6
i(mA)	120	108.6	98.25	89	80.43	72.8	65.8	59.6	54

t (s)	90	100	150	200	300	400	500	600	700
u_{AB}(V)	7.12	7.6	9.3	10.4	11.4	11.8	11.9	11.97	11.99
i(mA)	48.8	44.14	26.8	16.24	6	2.2	0.8	0.3	0.11

1) Represent u_{AB} as a function of time.
2) Extract the value U_0 of the voltage of the generator.

3) Let τ be the time constant of the circuit. We define τ to be the time at which the voltage across the capacitor becomes 63% of U_0.

a) Find graphically the value of τ.

b) Using another graphical method find the value of τ.

c) Verify the value of τ using its expression.

4) For small intervals of time with respect to τ, we can write : $\frac{du_{AB}}{dt} = \frac{\Delta u_{AB}}{\Delta t}$.

a) Verify that the unit, in SI, of the expression $C\frac{\Delta u_{AB}}{\Delta t}$ is the ampere.

b) Fill in the table below.

t (s)	0	10	20	30	40	50	60
u_{AB}(V)	0	1.14	2.17	3.11	3.95	4.72	5.41
$C\frac{du_{AB}}{dt}$ (A)		108.5×10^{-3}		89×10^{-3}		73×10^{-3}	
i(mA)	120	108.6	98.25	89	80,43	72.8	65.8

Give a conclusion.

N° 12
Electric energy transformed in mechanical energy

In the circuit of figure (a), we consider a capacitor (C), of capacitance C = 1 F, a generator delivering a constant voltage $u_G = E = 12$ V, a resistor (D), of resistance $R = 10\Omega$, an electric motor (M) and a switch k.

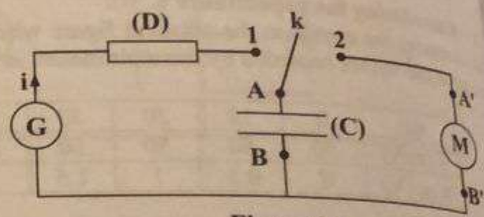


Figure (a)

A – First phase

The capacitor is initially neutral, we close at $t_0 = 0$, the switch to plot 1.

1) Name the phase of the capacitor.

2) Show that the intensity of the current at an instant t is given by : $i = C\frac{du_{AB}}{dt}$.

3) a) Determine the differential equation which governs the evolution of u_{AB} as a function of time.

b) The solution of the differential equation is under the form : $u_{AB} = a.e^{-b.t} + c$. Find the constants a, b, and c as a function of E, R and C.

c) Deduce the value of u_{AB} at the end of this phase.

4) Calculate the final electric energy stored in the capacitor.

B – Second phase

The switch is set for several minutes on plot 1 then it passes automatically to plot 2, at an instant taken as new origin of time $t_0 = 0$.

The variation of the voltage $u_{A'B'}$ as a function of the intensity i of current which traverses the motor is given in figure (b) in the next page.

1) Justify the direction of the electric current in the circuit as shown in figure (c) in the next page.

2) Extract the intensity of the current in the circuit at $t_0 = 0$.

3) a) Determine the equation giving $u_{A'B'}$ as a function of i.

b) Verify that the differential equation which governs the evolution of u_{AB} as a function of time is :

$$2\frac{du_{AB}}{dt} + u_{AB} = 6$$

c) Verify that the solution of the preceding differential equation is : $u_{AB} = 6(e^{-0.5t} + 1)$.

d) Calculate when $t \to +\infty$ the value of u_{AB}. Is the capacitor completely discharges? Why ?

e) Calculate the electric energy W supplied by the capacitor to the motor.

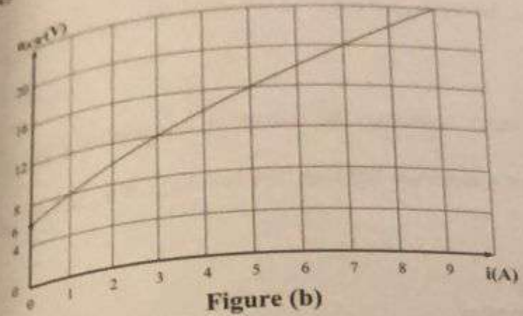


Figure (b)

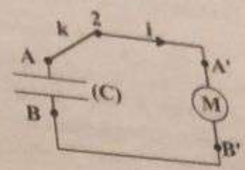


Figure (c)

4) The motor is used to lift, from rest to a height « h » and then bring back to rest, a body of mass $m = 1\,kg$. Knowing that 10 % of W is transformed by the motor into mechanical energy. Calculate h.
Given $g = 10\ m/s^2$.

N° 13

The capacitor is a temporary storing device of energy

In the circuit of figure (a), we consider a capacitor (C), of capacitance $C = 1$ F, a generator (G), a resistor (D), of resistance R, and a switch k.

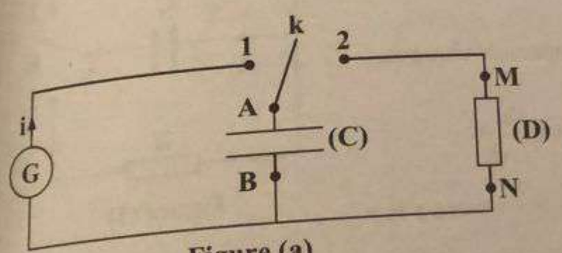


Figure (a)

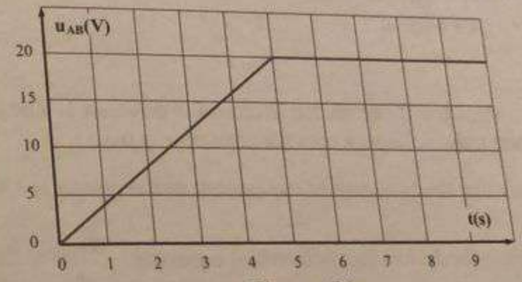


Figure (b)

A – First phase

The capacitor is initially neutral, we close at $t_0 = 0$, the switch to plot 1. The variation of the voltage u_{AB} of the capacitor, as a function of time, is represented in figure (b).

1) Name the phase of the capacitor.
2) Show that the intensity of the current i at an instant t is given by : $i = C\frac{du_{AB}}{dt}$.
3) Deduce the values of i during this phase.
4) At what instant is this phase over? Why ?
5) Calculate the electric energy stored in the capacitor.

B – Second phase

The switch passes, at an instant taken as an origin of time, automatically from 1 to 2.

1) Specify the direction of the electric current i in the resistor (D).
2) To what form of energy is the stored electric energy in the capacitor transformed to ? Why ?
3) Show that : $u_{MN} = -RC\frac{du_{AB}}{dt}$.
4) Write the differential equation which describes the variation of u_{AB} as a function of time.
5) The solution of the differential equation is under the form : $u_{AB} = a.e^{-bt}$. Determine the constants a and b as a function of U_0, R and C.
6) Calculate at $t = \frac{1}{b}$, the value of u_{AB}.
7) Represent as a function of time and in the same system, the curve of u_{AB} corresponding to $R = R_1 = 10\,\Omega$ and $R = R_2 = 50\,\Omega$.
8) The resistor (D) represents the filament of a lamp, of temporary lighting in a building. Why should we choose a lamp of resistance R_2 and not R_1 ?

N⁰ 14
The pacemaker

The aim of this exercise is to discover the Pacemaker (role and function), before realizing this objective, we first study the charging and discharging of a capacitor.

A – Charging and discharging of a capacitor

Consider the circuit of figure (1), composed of:

- A battery of electromotive force E and internal resistance r ;
- A capacitor of capacitance C ;
- A resistor of resistance R ;
- A switch K.

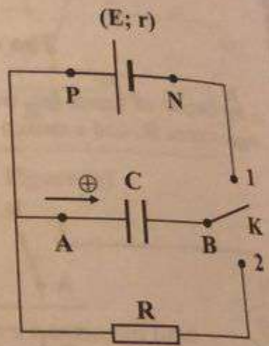


Figure (1)

1) At instant $t_0 = 0$, the switch is closed to position 1. The capacitor charges. A instant t, the voltage across the capacitor is u_{AB}.

a) Verify that the intensity of the current in the circuit is : $i = C\frac{du_{AB}}{dt}$.

b) Establish the following differential equation : $\frac{du_{AB}}{dt} + \frac{u_{AB}}{\tau} = \frac{E}{\tau}$ were τ is a characteristic to be determined as a function of r and C and give its unit.

c) The solution of the differential equation is of the form : $u_{AB} = a(1 - e^{-bt})$. Determine a and b as a function of E, r and C.

d) i – Trace the curve of u_{AB} as a function of time.

ii – Specify on the curve the ordinates of the particular points at instants $t_1 = \tau$ and $t_2 = 5\tau$.

2) The capacitor is completely charged. A new instant $t'_0 = 0$, taken as an origin of time when the switch is closed to position 2. The capacitor discharges.

a) Establish the following differential equation : $\frac{du_{AB}}{dt} + \frac{u_{AB}}{\tau'} = 0$. Determine τ' as a function of R and C.

b) The solution of the differential equation is of the form : $u_{AB} = a'e^{-b't}$. Determine a' and b' as a function of E, R and C.

c) i – Trace the curve of u_{AB} as a function of time.

ii – Specify on the curve the ordinates of the particular points at instants $t'_1 = \tau'$ and $t'_2 = 5\tau'$.

B – Discovering the Pacemaker

• The human heart beats 60 to 80 times per minute through a natural stimulator: the sinus node. When it no longer fulfills its role properly, surgery today is used to implant in the chest **[Figure (2)]** a device called a **pacemaker [Figure (3)]**.

Figure (2)

The **pacemaker** will force the heart muscle to beat regularly by sending small electrical impulses through electrodes called probes.

A pacemaker can be modeled by the circuit of **Figure(1)**. When charging is complete, **K** switches to position **2**, the capacitor discharges through the resistor of resistance **R** to a limiting value U_{limit}. At this instant, the circuit sends via sensors connected to the terminals of the resistor, a pulse to the heart, one then obtains a beat. When the operation is complete, **K** switches back to position **1**. The process starts again ...

Figure (3)

Internet site

Given : $R = 2\times10^6\ \Omega$ and $r = 1\ \Omega$.

1) Extract, from the preceding text, the role of a pacemaker.
2) Explain why the charging of the capacitor is much faster than its discharging.
3) In **figure (4)**, we represent, during the functioning of a **pacemaker** the graph showing the evolution of the voltage u_{AB}, between the terminals of the capacitor, as a function of time.
a) i – Specify, in the interval [0 ; 1,6 s[, the phase of the capacitor corresponding to each of the branches (GH), (IH) and (IJ) of the graph.
ii – Extract the duration T between two successive pulses.
iii – Deduce the number of pulses per minute.
iv – Is this result compatible with the normal cardiac frequency ? Why ?

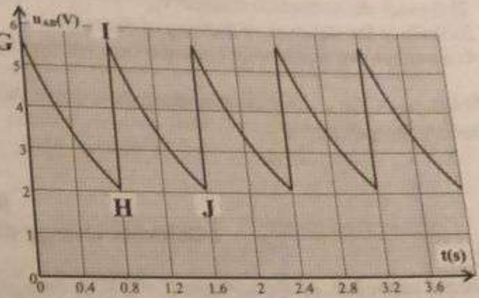
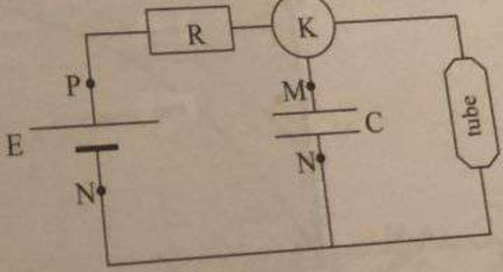


Figure (4)

4) a) Extract, from **figure (4)**, the values of E and U_{limit}.
b) Show that : $C \approx 0.4\ \mu F$.
c) Calculate the approximate duration of charging of the capacitor. Interpret the vertical direction of [HI].
5) Determine the electric energy, supplied by the capacitor, necessary to produce a heartbeat.

N° 15
Lighting of a neon tube

The neon discharge tube is an electric dipole which lights if the voltage across its terminals attains a value V_a, the tube remains lit for voltages smaller than V_a and goes off when the voltage across it becomes equal to a value V_e called voltage of extinction. ($V_e < V_a$).



This dipole acts as an open switch when turned off and as a resistor of resistance R' when it is lit.

In order to study the functioning of this dipole we consider the following circuit where K is an auto[...] switch used to connect and disconnect the generator in well-determined intervals of time.

A – The tube is off. ($R = 100\ k\Omega$; $C = 10\ \mu F$).

1) Name the phenomenon observed in the capacitor.

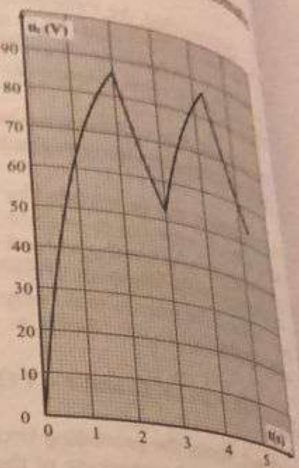
2) Establish the differential equation which describes the variation of the voltage $u_C = u_{MN}$ across the terminals of the capacitor.

3) The solution of this equation is $u_C = A(1 - e^{-\frac{t}{\tau}})$. Determine the constants A and τ. Calculate the value of τ.

4) Verify that, if the tube is not connected to the circuit, the maximum voltage attained by u_C is E.

5) After what interval of time is this maximum voltage attained ?

6) The adjacent figure represents u_C as a function of time during the charging and the discharging of the capacitor. Determine, by the aid of the graph, the value of E.



B – Tube Lit.

1) Extract, graphically, the value V_a where the tube lights up.

2) Extract the extinction voltage V_e.

3) Deduce the duration of lighting of the tube.

4) Establish the differential equation in u_C during the discharging.

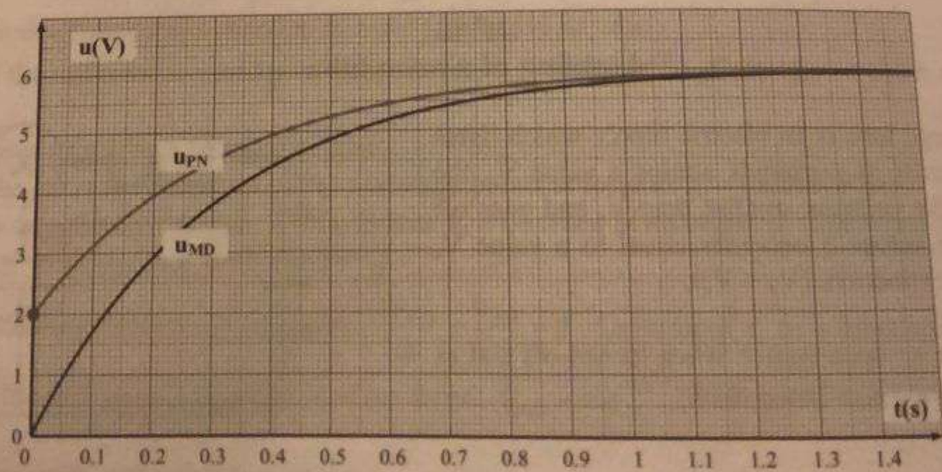
5) Let $u_C = V_a e^{-\frac{t}{\tau'}}$. We choose the instant where the tube lights as an origin of time. Calculate the value of τ'. Deduce the value of R'.

N° 16
Calorific energy dissipated by a battery

A neutral capacitor, of capacitance C = 1 F, is charged by a battery of e.m.f. E and of internal resistance r. An ammeter, of resistance r' is branched in series, with the capacitor and a voltmeter, of very high resistance, is branched in parallel across the capacitor.

At the time $t_0 = 0$, we close the switch, the capacitor starts charging.

A convenient device, branched in the circuit, represents the variation of the voltages u_{PN} and u_{MD} across the battery and the capacitor respectively as shown in the figure below.



1) Make a figure showing the circuit.

2) How could the ammeter be considered as an indicator to indicate that the charging phase of the capacitor has terminated ?

3) a) Find, a relation between E, r, r', the intensity i of current, and u_{MD}.

b) Determine the relation between i, C, and $\frac{du_{MD}}{dt}$.

c) Deduce the differential equation which governs the evolution of u_{MD} as a function of time.

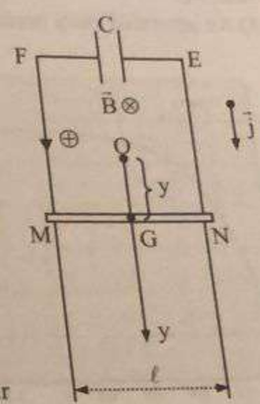
d) Show that the solution of this differential equation is : $u_{MD} = A\left(1 - e^{-\frac{t}{B}}\right)$ where A and B are constants to be determined.

4) Find u_{PN} as a function of E, r, r', C and t.

5) Choosing particular times and by the aid of the graph, find E ; r and r'.

6) The calorific energy dissipated by joule's effect by the battery is given by : $W = \int_0^{+\infty} ri^2 dt$. Calculate W.

N° 17
A rod is a generator of current of constant intensity



A rectilinear rod, of length ℓ, of mass m, is launched from point O without speed at time $t_0 = 0$, is in downward translational motion along two vertical rails remaining parallel to a horizontal direction and closing the rectangular circuit (ENMF) which contains a capacitor of capacitance C initially neutral. The whole setup is placed in a uniform magnetic field and perpendicular to the plane of the rails.

We neglect friction and the resistance in the circuit.

We designate by $y = \overline{OG}$ the ordinate of the center of inertia G of the rod and by $\vec{V} = V.\vec{j}$ its speed at the time t and by g the gravitational field strength.

1) a) Verify that the rod is the seat of an i.e.m.f. such that : $u_{NM} = B\ell V$.

b) Specify the direction of current, of intensity i, which appears in the circuit.

2) a) the rod is under the action of two forces. Name these forces and give their literal expressions.

b) Applying Newton's 2nd law, show that the acceleration of the rod is : $a = g - \frac{B\ell}{m}i$.

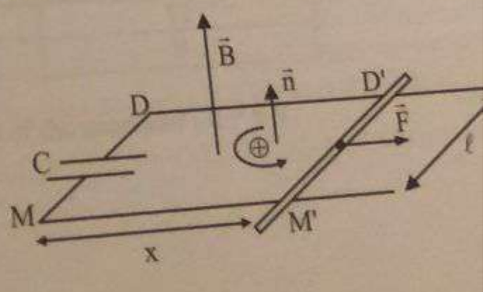
3) a) Applying on the circuit MNEF the law of addition of voltage, find the charge q_E of the armature E of the capacitor as a function of C, B, ℓ and V.

b) Deduce i as a function of a, C, B and ℓ.

4) a) Show that the motion of G is uniformly accelerated rectilinear translational motion.

b) Deduce that the rod is a generator of current of constant intensity : $I = \frac{m}{m + B^2\ell^2 C} B\ell Cg$.

N° 18
The capacitor moves a rod



A capacitor, of capacitance C = 1 F, is charged by a voltage of 6 V then branched across two horizontal conducting rails, separated by a distance $\ell = 10\,cm$ and situated in the same horizontal plane. The whole setup is placed in a uniform and vertical magnetic field $\vec{B}$ and of constant magnitude B. At the time t = 0, we place, perpendicular to the ra[...] a rectilinear and conducting rod, of mass m = 10 g and of resistance R, the rod starts to move fr[...] parallel to itself under the action of an electromagnetic force $\vec{F}$ as shown in the above figure.

We neglect the resistance of the rails and friction.

1) a) Specify the direction of current i in the circuit.

b) Give, as a function of i, B and ℓ, the magnitude F of $\vec{F}$.

2) Two physical phenomena appear in the circuit. Name these phenomena.

3) a) Calculate, as a function of B, ℓ and x, the magnetic flux traversing the surface MM'D'D.

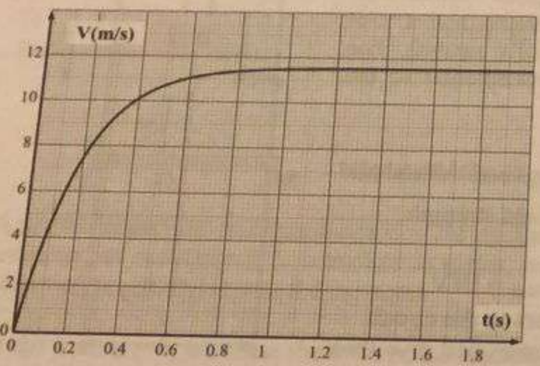
b) Deduce that the i.e.m.f. which appears in the circuit : $e = -B\ell V$ where V is the speed at time t.

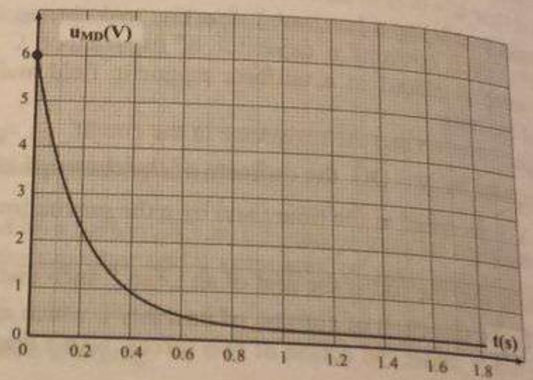
4) a) Applying law of addition of voltage, show that : $u_{MD} + RC\frac{du_{MD}}{dt} - B\ell V = 0$.

b) Show, applying Newton's 2nd law on the rod, that : $\frac{dV}{dt} = -\frac{CB\ell}{m}\frac{du_{MD}}{dt}$. Deduce V as a function of C, B, ℓ, m, u_{MD}, and U_0.

c) Determine the differential equation which governs the evolution of u_{MD} as a function of time. Deduce its solution.

5) An advanced study permits to represent the graphs of V and of u_{MD} as a function of time.





a) The graphs show that the speed of the rod and the voltage across the capacitor reach limiting values. Identify these limiting values.

b) The capacitor does not discharge totally. Justify.

c) By the aid of the preceding equations , find the values of B, m and R.

d) Calculate the electric energy liberated by the capacitor and the variation of the kinetic energy of the rod. Making the energetic diagram of the circuit, calculate the energy dissipated by joule's effect.