AMJAD

MATHEMATICS DEPARTMENT
Final Exam

Class: GS

Date: 16-6- 2023

Duration: 3 hours

Name of the Student: ____

Instructions:

1. **Scientific calculators are allowed.**
2. **The exam consists of 6 pages (including this cover page) and 4 exercises.**
3. **If the figures in this exam are used for construction or other additional information then submit the question sheet with your answer sheet as well.**
4. **Full mark is 40.**
5. **Answer Problems I and II on a separate answer sheet for Mr Nabil and problems III and IV for Mr Fadi.**

I- (8 points)

Consider two urns U_1 and U_2.

U_1 contains four red balls and three green balls.

U_2 contains two red balls and one green ball.

A-

We draw at random a ball from U_1 and we put it in U_2, then we draw at random a ball from U_2.

Designate by X the number of red balls remaining in the urn U_2 after the two preceding draws.

1) Prove that the probability $P(X = 2)$ is equal to $\frac{9}{14}$.

2) Find the three values of X and determine the probability of each value of X.

B-

In this part, each red ball carries the number 1 and each green ball carries the number – 1.

We choose at random an urn then we draw simultaneously and at random two balls from the chosen urn.

Consider the following events:

E: « The chosen urn is U_1 »

F: « The sum of the numbers carried by the two drawn balls is equal to 0 ».

1) a- Calculate the probabilities $P(F/E)$ and $P(F/\overline{E})$.

   b- Deduce that $P(F) = \frac{13}{21}$.

2) Designate by G the event « The sum of the numbers carried by the two drawn balls is equal to – 2 ».

   Calculate P(G).

II- (12 points)

A- Let h be the function defined over $]0\,;+\infty[$ by $h(x)=\dfrac{e^x-1}{x}$.

Denote by (C) the representative curve of h in an orthonormal system $\left(O\,;\vec{i}\,;\vec{j}\right)$.

1) a- Verify that $h'(x)=\dfrac{(x-1)e^x+1}{x^2}$.

b- Let g be the function defined over $]0\,;+\infty[$ by $g(x)=(x-1)e^x$.

Set up the table of variations of g and deduce that $h'(x)>0$.

2) a- Calculate $\lim\limits_{x\to 0}h(x)$, $\lim\limits_{x\to+\infty}h(x)$ and $\lim\limits_{x\to+\infty}\dfrac{h(x)}{x}$.

b- Set up the table of variations of h.

3) a- Write an equation of (Δ), the tangent to (C) at the point with abscissa 1.

b- Draw (Δ) and (C).

B- Consider the function f defined over $]0\,;+\infty[$ by $f(x)=h(x)+\ln x$ and denote by (Γ) its representative curve in the same system as (C).

1) a- Calculate $\lim\limits_{x\to 0}f(x)$ and $\lim\limits_{x\to+\infty}f(x)$.

b- Set up the table of variations of the function f.

2) a- Prove that the equation $f(x)=0$ has a unique solution α and verify that $0.3<\alpha<0.4$

b- Compare $h(\alpha)$ and $h(1)$. Deduce that $\ln\alpha>1-e$.

3) a- Discuss, according to the values of x, the relative positions of (C) and (Γ).

b- Draw (Γ).

4) A is a point on (C) and B is a point on (Γ) such that A and B have the same abscissa x.

m is any real number such that $m>0$. If $AB=m$, prove that there exist two values of x whose product is independent of m.

III- (8 points)

In the complex plane $(O;\vec{u},\vec{v})$, consider the points A and B of respective affixes 1 and -1. To every point M of affix z associate its image M' of affix z' such that $z'=\dfrac{z-1}{z+1}$. $(z\neq -1)$

1) In the particular case when $z=-1+2i$, write z' in exponential form.

2) Find the set of points M such that $|z'|=1$.

3) Determine the values of z for which $z'=z$

4) a) Show that $(z'-1)(z+1)$ is a real number.

b) Determine the set of points M' when M moves on the circle (Γ) of center B and radius 2.

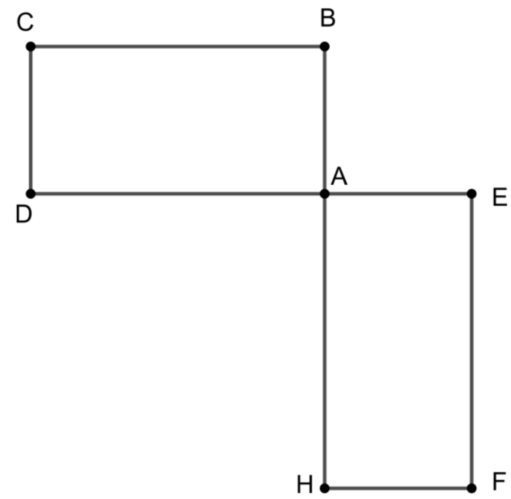
5) Let P be the point of affix $z_P=-2+i\sqrt{3}$.

a) Find the exponential form of the number z_p+1.

b) Show that P belongs to the circle (Γ).

c) Let Q be the point of affix $z_Q=-\bar{z}_P$, and let P' be the image of P. Show that A, P' and Q are collinear.

IV- (12 points)



In the adjacent figure:

- ABCD and AHFE are two congruent rectangles.
- D, A, and E are collinear.
- B ,A and H are collinear.
- AB=AE=1 and AD=AH=2.
- Denote by S the similitude that maps A onto B and D onto A.

1) Determine the angle of S and calculate its ratio .

2) Consider the rotation $R\left(A;\frac{\pi}{2}\right)$ and let I be the point of intersection of (BD) and (AF).

a) Determine R(D).

b) Construct L=R(B).

c) Prove that $\overrightarrow{FA}=\overrightarrow{HL}$, and that the two lines (AF) and (BD) are perpendicular.

3)a) Determine the image of the line (BD) and that of (AF) under S.

b) Deduce that I is the center of S.

4) Let G be the point so that AEGB is a square.

Denote by J the meeting point of (GD) and (AB).

a) Determine the nature of SoR.

b) Determine SoR(A).

c) Prove that J is the center of SoR.

5) The complex plane is referred to the direct orthonormal system $\left(A,\vec{u},\vec{v}\right)$ with $\vec{u}=\overrightarrow{AE}$ and $\vec{v}=\overrightarrow{AB}$.

a) Write the complex form of S. Deduce Z_I.

b) Determine the pre-image of G under S.

c) N is a variable point that moves on a circle (C') with center G and radius 1.

Prove that the pre-image of N under S moves on a circle (C) with center and radius are to be determined.

Name of student……………………………………….