ExaMath groups	**Mathematics Exam** **Section:** L.S.	**Prepared by:** Dr. A. Moussawi **Edited by:** H. Ahmad
Number of questions: 3	**Sample 02 - 2022** **Duration:** 90 min	**Name:** **N^o:**

- *This exam consists of three problems. It is inscribed on two pages, numbered 1 and 2.*
- *Use of a non-programmable calculator is permitted.*

I - (2 points)

In the table below, only one of the answers given to each question is correct.
Choose, **with justification**, the correct answer.

N^o	Question	Suggested answers A	B	C
1.	The inequality $\ln(x)+\ln(x+1)\geq\ln\left(x^2+1\right)$ is verified in:	$[1\,;+\infty[$	$]0\,;+\infty[$	$]1\,;+\infty[$
2.	A class contains 12 boys and 4 girls. If we choose three students from the class at random, the probability that all of them are girls is:	$\frac{1}{140}$	$\frac{11}{28}$	$\frac{17}{28}$
3.	Let f a function defined over $]0\,;+\infty[$ by: $f(x)=e^{\frac{1}{x}}$. The derivative of f is:	$e^{-\frac{1}{x^2}}$	$\dfrac{-e^{\frac{1}{x}}}{x^2}$	$-x^2e^{\frac{1}{x}}$
4.	$\lim\limits_{x\to-\infty}\dfrac{\ln(e^x+1)}{e^x}=$	1	0	$+\infty$

II - (5 points)

In a school of statistics, after studying the candidates' files, recruitment is done in two ways:

- 10% of candidates are selected on the basis of their application. These candidates must then pass an oral test, after which 60% of them are finally admitted to the school.
- Candidates who have not been selected on the basis of their applications take a written test after which 20% of them are admitted to the school.

Part A

A candidate is randomly chosen for this recruitment competition. Consider :

- D the event: « the candidate was selected on the basis of his application »;
- A the event: « the candidate was admitted to the school »;
- $\overline{D}$ and $\overline{A}$ the contrary events of the events D and A respectively.

1) Translate the situation by a weighted probability tree.
2) Calculate the probability that the candidate will be selected on the basis of his application and admitted to the school.
3) Show that the probability of the event A is equal to 0.24.
4) A candidate admitted to the school is chosen at random. What is the probability that his application was not selected?

Part B

We consider a sample of three candidates chosen at random, assimilating this choice to a drawing with replacement.

1) Calculate the probability that only one of the three randomly selected candidates will be admitted to the school.

2) Calculate the probability that at least two of the three randomly selected candidates will be admitted to this school.

III - (11 points)

Let f the function defined over $]0\,;\,e[\cup]e\,;\,+\infty[$ by $f(x)=\dfrac{1}{x(1-\ln x)}$.

Denote by $\left(C_f\right)$ its representative curve in an orthonormal system $\left(O\,;\,\vec{i}\,;\,\vec{j}\right)$.

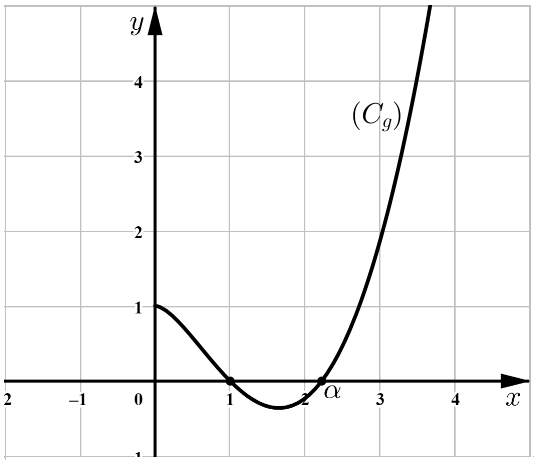
1) Calculate the limits: $\lim\limits_{x\to 0^+} f(x)$, $\lim\limits_{x\to e^-} f(x)$ and $\lim\limits_{x\to e^+} f(x)$. Deduce the existence of two asymptotes to $\left(C_f\right)$.

2) Calculate $\lim\limits_{x\to+\infty} f(x)$ and interpret graphically the result.

3) Show that for every $x\in\,]0\,;\,e[\cup]e;+\infty[$, $f'(x)=\dfrac{\ln x}{x^2(1-\ln x)^2}$.

4) Draw the table of variations of f.

5) Let g the function defined over $]0\,;\,+\infty[$ by $g(x)=1-x^2(1-\ln x)$.
   The curve $\left(C_g\right)$ below is the representative curve of the function g in an orthonormal system.



   The curve (C_g) intersects the x-axis at two points of abscissas 1 and α where α is a real number.

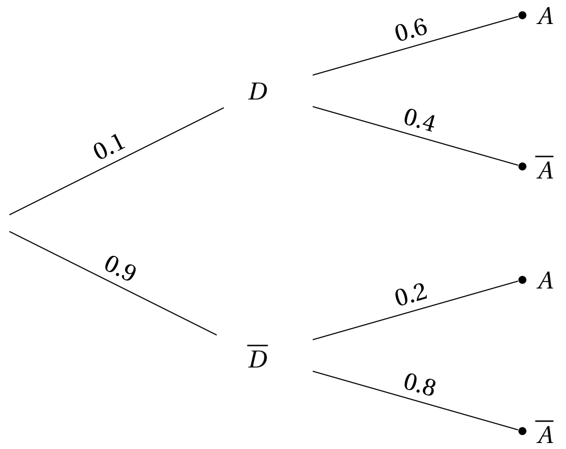
   a) Verify that $2.2<\alpha<2.3$.

   b) Show that for every $x\in\,]0\,;\,+\infty[$, $f(x)-x=\dfrac{g(x)}{x(1-\ln x)}$.

   c) Deduce the relative position of the curve $\left(C_f\right)$ and the line (d) of equation $y=x$.

6) Draw $\left(C_f\right)$ and (d).

QI	Answers	Grade
1.	The inequality is defined when $\begin{cases} x>0 \\ x+1>0 \\ x^2+4>0 \end{cases}$ so for $x>0$; $\ln(x)+\ln(x+1)\geq \ln\left(x^2+1\right)$; $\ln\left(x^2+x\right)\geq \ln\left(x^2+1\right)$; $x^2+x\geq x^2+1$; $x\geq 1$; The solution set is $[1\,;+\infty[$; The correct answer is **A**.	½
2.	$P(3\text{girls})=\dfrac{C_3^4}{C_3^{16}}=\dfrac{1}{140}$; The correct answer is **A**.	½
3.	$f'(x)=-\dfrac{1}{x^2}e^{\frac{1}{x}}$; The correct answer is **B**.	½
4.	$\lim\limits_{x\to-\infty}\dfrac{\ln\left(e^x+1\right)}{e^x}=\lim\limits_{x\to-\,gi}\dfrac{1}{e^x+1}=1$; The correct answer is **A**.	½

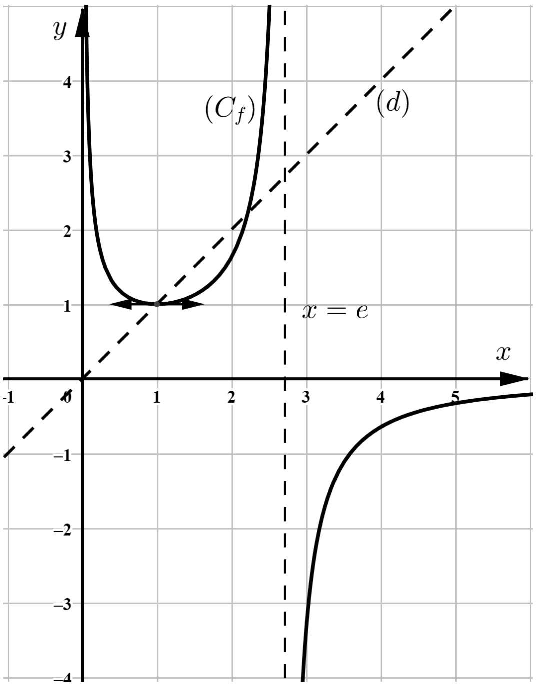
QII	Answers	Grade
A.1.		1
A.2.	$P(D\cap A)=P(D)\times P(A/D)=0.1\times 0.6=0.06$;	¾
A.3.	$P(A)=P(D\cap A)+P\left(\overline{D}\cap A\right)=0.06+0.18=0.24.$	1
A.4.	$P\left(\overline{D}/A\right)=\dfrac{P\left(A\cap\overline{D}\right)}{P(A)}=\dfrac{P\left(\overline{D}\cap A\right)}{P(A)}=\dfrac{0.18}{0.24}=\dfrac{18}{24}=\dfrac{3}{4}=0.75.$	¾
B.1.	Consider the event E: « only one of the three randomly selected candidates is admitted to the school »; $P(E)=P(A)\times P(\overline{A})\times P(\overline{A})\times\dfrac{3!}{2!\times 1!}=0.24\times(1-0.24)^2\times 3=0.42$	¾
B.2.	Consider the event F: « At least two of the three randomly selected candidates are admitted to this school »; $P(F)=P(A)\times P(A)\times P(\overline{A})\times\dfrac{3!}{2!\times 1!}+P(A)\times P(A)\times P(A)=0.15.$	¾

<table>
<tr><th>QIII</th><th>Answers</th><th>Grade</th></tr>
<tr><td>1.</td><td>

- $\lim\limits_{x\to 0^+} f(x) = +\infty$;
- $\lim\limits_{x\to e^-} f(x) = +\infty$;
- $\lim\limits_{x\to e^+} f(x) = -\infty$;

The lines with equations $x = 0$ and $x = e$ are two vertical asymptotes to $\left(C_f\right)$.</td><td>2</td></tr>
<tr><td>2.</td><td>$\lim\limits_{x\to +\infty} f(x) = 0$;
The line with equation $y = 0$ is horizontal asymptote to $\left(C_f\right)$ at $+\infty$.</td><td>1</td></tr>
<tr><td>3.</td><td>$f'(x) = \dfrac{\ln x}{x^2(1-\ln x)^2}$.</td><td>1</td></tr>
<tr><td>4.</td><td>Table of variations of f :
<table>
<tr><td>x</td><td>0</td><td></td><td>1</td><td></td><td>e</td><td></td><td>$+\infty$</td></tr>
<tr><td>$f'(x)$</td><td>||</td><td>$-$</td><td>0</td><td>$+$</td><td>||</td><td>$+$</td><td></td></tr>
<tr><td>$f(x)$</td><td>||</td><td>$+\infty \searrow$</td><td>1</td><td>$\nearrow +\infty$</td><td>||</td><td>$-\infty \nearrow$</td><td>0</td></tr>
</table></td><td>1½</td></tr>
<tr><td>5.a.</td><td>$g(2.2) \approx -0.02 < 0$ and $g(2.3) \approx 0.12 > 0$; so $2.2 < \alpha < 2.3$.</td><td>¾</td></tr>
<tr><td>5.b.</td><td>$f(x) - x = \dfrac{g(x)}{x(1-\ln x)}$.</td><td>¾</td></tr>
<tr><td>5.c.</td><td>$f(x) - x = \dfrac{g(x)}{x(1-\ln x)}$.
Table of sign of $f(x) - x$:
<table>
<tr><td>x</td><td>0</td><td></td><td>1</td><td></td><td>α</td><td></td><td>e</td><td></td><td>$+\infty$</td></tr>
<tr><td>$g(x)$</td><td>||</td><td>$+$</td><td>0</td><td>$-$</td><td>0</td><td>$+$</td><td>||</td><td>$+$</td><td></td></tr>
<tr><td>x</td><td>||</td><td>$+$</td><td></td><td>$+$</td><td></td><td>$+$</td><td>||</td><td>$+$</td><td></td></tr>
<tr><td>$1-\ln x$</td><td>||</td><td>$+$</td><td></td><td>$+$</td><td></td><td>$+$</td><td>||</td><td>$-$</td><td></td></tr>
<tr><td>$f(x)-x$</td><td>||</td><td>$+$</td><td>||</td><td>$-$</td><td>0</td><td>$+$</td><td>||</td><td>$-$</td><td></td></tr>
</table>

- (C_f) is above (d) if $x \in]0\,;1[\cup]\alpha\,;e[$;
- (C_f) is below (d) if $x \in]1\,;\alpha[\cup]e\,;+\infty[$;
- (C_f) intersects (d) at the points of coordinates $(1\,;1)$ and $(\alpha\,;\alpha)$.
</td><td>2</td></tr>
</table>

QIII	Answers	Grade
6.		**2**