Transformation	Translation $t_{\tilde{v}}$	<b>Rotation R(w</b> ; α)	Dilation h(w; k)	Similitude S(w; k; α)
Definition	$t(M) = M' \Leftrightarrow \overrightarrow{MM'} = \overrightarrow{V}$	$R(M) = M' \Leftrightarrow \begin{cases} wM' = wM \\ (\overrightarrow{wM}; \overrightarrow{wM'}) = \alpha[2\pi] \end{cases}$	$h(M) = M' \Leftrightarrow \overrightarrow{wM'} = k.\overrightarrow{wM}$	$S(M) = M' \Leftrightarrow \begin{cases} wM' = k.wM \\ (\overrightarrow{wM}; \overrightarrow{wM'}) = \alpha[2\pi] \end{cases}$
Figure	A A' B'	A A	W. A. A.	w \alpha A
Characteristic Property	$\begin{vmatrix} A & \xrightarrow{t} & A' \\ B & \xrightarrow{t} & B' \end{vmatrix} \Leftrightarrow \overrightarrow{A'B'} = \overrightarrow{AB}$	$\begin{vmatrix} A \xrightarrow{R} A' \\ B \xrightarrow{R} B' \end{vmatrix} \Leftrightarrow \begin{cases} A'B' = AB \\ (\overline{AB}; \overline{A'B'}) = \alpha [2\pi] \end{cases}$	$\begin{vmatrix} A \xrightarrow{h} A' \\ B \xrightarrow{h} B' \end{vmatrix} \Leftrightarrow \overline{A'B'} = k.\overline{AB}$	$\begin{bmatrix} A & \xrightarrow{s} & A' \\ B & \xrightarrow{s} & B' \end{bmatrix} \Leftrightarrow \begin{cases} A'B' = k.AB \\ (\overline{AB}; \overline{A'B'}) = \alpha[2\pi] \end{cases}$
Complex form	$z' = az + b$ $a = 1 \text{ and } b = z_{\vec{v}}$	z' = az + b $a = e^{i\alpha}$ and $b = z_w (1-a)$	z' = az + b $a = k$ and $b = z_w (1-a)$	z' = az + b $a = ke^{i\alpha}$ and $b = z_w (1-a)$
Image of a line (D)	A line (D') parallel or confounded to (D)	A line (D') such that $(D; D') = \alpha$	A line (D') parallel or confounded to (D)	A line (D') such that $(D;D') = \alpha$
Image of a circle (C) of center I and radius R	A circle (C') $I' = t_{\tilde{v}}(I) \text{ and } R' = R$	A circle (C') $I' = R(I) \text{ and } R' = R$	A circle (C') $I' = h(I) \text{ and } R' =  k R$	A circle (C') $I' = S(I) \text{ and } R' = k.R$
Distance	Yes	Yes	No $(\times  \mathbf{k} )$	No (×k)
Midpoint	Yes	Yes	Yes	Yes
Collinearity of points	Yes	Yes	Yes	Yes
Oriented Angles	Yes	Yes	Yes	Yes
Area	Yes	Yes	No $(\times k^2)$	No $(\times k^2)$

## **Composite of two transformations:**

(k > 0 and k' > 0)

	$t_{ec{ ext{u}}}$	$r(I;\alpha)$	h(I; lpha)	$S(I;k;\alpha)$
$\mathbf{t}_{ec{\mathbf{v}}}$	$\vec{t_{u+v}}$	$r(w;\alpha)$	h(w;k)	$S(w;k;\alpha)$
$r(J;\beta)$	$r(w; \beta)$	$r(w;\alpha+\beta)$	$S(w;k;\beta)$	$S(w;k;\alpha+\beta)$
h(J;k')	h(w;k')	$S(w;k';\alpha)$	h(w;k.k')	$S(w;k.k';\alpha)$
$S(J;k';\beta)$	$S(w;k';\beta)$	$s(w;k';\alpha+\beta)$	$S(w;k.k';\beta)$	$S(w;k.k';\alpha+\beta)$

## Remark:

- $> \quad If \ k>0 \ , \ then \ r\big(I;\alpha\big) \circ h\big(J;k\big) = S\big(w\,;k\,;\alpha\big).$
- $> \ \, \text{If} \ \, k<0 \; , \, \text{then} \ \, r\big(I;\alpha\big)\!\circ\!h\big(J;k\big)\!=\!S\big(w\,;\!\big|k\big|;\alpha+\pi\big).$