Chapter 2

Linear momentum

Summary

Linear momentum

1. Linear momentum of a particle

The linear momentum of a particle of mass m moves with velocity $\vec{V}$ is:

$$\vec{P} = m\vec{V}$$

$\vec{P}$ and $\vec{V}$ are vectors having the same line of action and same direction.
The magnitude of linear momentum of a particle of mass m with speed V is

$$P = mV$$

In S.I; m in kg and V in m/s so the unit of the magnitude of linear momentum P is kg.m/s.
Note: Linear momentum is vector quantity $\vec{P}$ where P is magnitude of linear momentum.

2. Linear momentum of a system of particles

The linear momentum $\vec{P}$ of the system is the sum of the linear momenta $m_i\vec{v}_i$ of its particles:

$$\vec{P}_{sys} = \sum \vec{p} = \sum m_i\vec{v}_i$$

Linear momentum of the center of mass of a system of particles

$$\vec{P}_G = \vec{P}_{sys}$$

3. Newton's second law

The general expression of Newton's second law:

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}$$

The time derivative of the linear momentum of a system of particles, in an inertial frame, is equal to the resultant of all external forces applied on this system.

Note: (Important)

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} \approx \frac{\Delta\vec{P}}{\Delta t}$$

(≈ means approximately equal to, or almost equal to)

If: Δt is very small e.g. Δt =0.05 s Or P = at + b Linear momentum is linear in time .

Theorem of the center of inertia

In an inertial frame, the sum of external forces applied on a system of constant mass is equal to the product of the mass of this system by the acceleration of its center of inertia.

$$\sum \vec{F}_{ext} = M\vec{a}_G$$

4. The conservation of Linear momentum

a) Isolated (or mechanically isolated) system

A system is said to be isolated if the sum of the external applied forces on the system is zero.

$$\sum \vec{F}\,ext = \vec{0}$$

b)Statement of the law of conservation of linear momentum

In the case of an isolated system:

$$\sum \vec{F}\ ext = \vec{0} \text{ But } \frac{d\vec{P}}{dt} = \sum \vec{F}\ ext$$

$$\text{Then } \frac{d\vec{P}}{dt} = \vec{0} \text{ So } \vec{P} = \text{constant at any time.}$$

Therefore the law of conservation of linear momentum could be stated as follows:

The linear momentum of an isolated system remains constant as time varies.

c) Collisions and explosions

During collision (or explosion) external forces are neglected with respect to impact forces (internal forces). Then the system can be treated as an isolated system. So, the linear momentum is conserved.

d) Types of collisions

In general two types of collisions can be distinguished:

Elastic and non elastic.

By definition, during an elastic collision, **the kinetic energy of the system composed of both bodies is conserved.**

$$\mathbf{Eki = Ekf}$$

In the case of a non-elastic collision, a part of the kinetic energy of the system converts into heat or deforms the shape of the system. **The kinetic energy of the system, in this case, is not conserved.**

$$\mathbf{Eki \neq Ekf}$$

5. Example of an elastic collision with collinear velocities

Consider the collision between solids of respective masses m1 and m2 and collinear velocities $\vec{v}1$ and $\vec{v}2$. Just after collision, the velocities of both solids become $\overrightarrow{v'}1$ and $\overrightarrow{v'}2$ respectively. The conservation of linear momentum of the system composed of both solids gives:

$m1\vec{v}1 + m2\vec{v}2 = m1\overrightarrow{v'}1 + m2\overrightarrow{v'}2$

Since the velocities are collinear, this vector relation can be written in algebraic form as follows:

$m1v1 + m2v2 = m1v'1 + m2v'2$ ... (1)

So $m_1 (v_1 - v'_1) = m_2 (v'_2 - v_2)$ ... (2)

The collision is elastic and the kinetic energy of the system is conserved, so:

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2$

$m_1 (v_1^2 - v'^2_1) = m_2 (v'^2_2 - v_2^2)$

So $m_1 (v_1 - v'_1)(v_1 + v'_1) = m_2 (v'_2 - v_2)(v'_2 + v_2)$ … (3)

$\frac{3}{2}$: $\frac{m_1 (v_1 - v'1)(v_1 + v'1)}{m_1 (v_1 - v'1)} = \frac{m_2 (v'2 - v_2)(v'2 + v_2)}{m_2 (v'2 - v_2)}$

$v_1 + v'_1 = v'_2 + v_2$

$v_1 - v_2 = - v'_1 + v'_2 \ldots$ (4)

Where $m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$ ... (1)

multiply (4) by m_1

$m_1v_1 - m_1v_2 = - m_1v'_1 + m_1v'_2$ and add to (1)

So $2m_1v_1 + (m_2 - m_1)v_2 = (m_1 + m_2)v'_2$

$$v'_2 = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

Substitute in (4) so

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

Case 1

If $m_1 = m_2$, we get $v'_1 = v_2$ and $v'_2 = v_1$, The bodies exchange their velocities.

Proof:

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 = \frac{0}{m_1 + m_2} v_1 + \frac{2m_2}{2m_2} v_2 = v_2$$

$$v'_2 = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2 = \frac{2m_1}{2m_1} v_1 + \frac{0}{m_1 + m_2} v_2 = v_1$$

Case 2

If the second body was initially at rest, then $v_2 = 0$ So

$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1$ and $v'_2 = \frac{2m_1}{m_1 + m_2} v_1$

If $m_1 > m_2$

So two solids move in direction of $\vec{v}1$

If $m_1 < m_2$

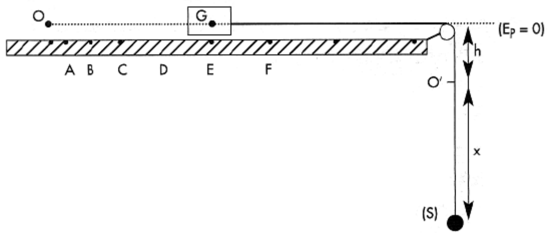
So $\vec{v}'1$ is opposite sign of $\vec{v}1$ while $\vec{v}'2$ have same sign as $\vec{v}1$

If $m_1 = m_2$

So $v'_1 = 0$ (stops) and $v'_2 = v_1$ ($\vec{v}'2$ have same sign as $\vec{v}1$)

Exercises

Exercise 1 (LS 2001-2-ex1) Verification of Newton's second



In order to verify Newton's second law related to the dynamics of a solid in translation, we consider a puck of center of inertia G and of mass M = 200 g, a horizontal air table, a solid (S) of mass m = 50 g, an inextensible string and a pulley of negligible mass. We build the set up represented in the adjacent figure. The part of the wire on the side of the puck is taut horizontally and the other part to the side of (S) is vertical. The horizontal plane passing through G is taken as the gravitational potential energy reference. At the instant t = 0, G is at O and the center of mass of (S) is at O', at a distance h below the reference. We release (S) without initial velocity, and, at the same time, the positions of G are recorded at successive instants separated by a constant time interval τ = 50 ms. At the instant t, G acquires a velocity $\vec{V}$ and (S) is found at a distance x below O'.
Neglect all frictions and take g = 10 m/s^2.

A- 1) Give the expression of the mechanical energy of the system (puck, string, (S), Earth) in terms of M, m, x, h, V and g. This energy is conserved. Why?
2) Deduce the expression of the acceleration of (S) in terms of g, m and M and calculate its value.
3) Draw a diagram showing the forces acting on the puck and determine, using the relation $\sum \vec{F} = M\vec{a}$ the force $\vec{T}$ exerted by the string on the puck.

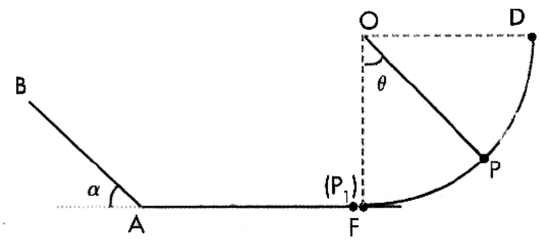
B- By means of a convenient method, we determine the speed V of the puck. The results are tabulated as shown below:

Point	A	B	C	D	E
t in ms	50	100	150	200	250
V in cm/s	10	20	30	40	50

Determine, using the table, the linear momentums $\vec{P}_B$ and $\vec{P}_D$ at D and determine the ratio $\frac{\Delta\vec{P}}{\Delta t} = \frac{\vec{P}_D - \vec{P}_B}{\Delta t}$.

C- Compare $\frac{\Delta\vec{P}}{\Delta t}$ and $\vec{T}$. Is Newton's second law thus verified? Justify.

Exercise 2 (LS 2002-1-ex1) Conservation and non-conservation of the mechanical energy



Consider a material system (S) formed of an inextensible and mass less string of length l = 0.45 m, having one of its ends O fixed while the other end carries a particle (P) of mass m = 0.1 kg. Take g = 10 m/s^2.
1) (S) is shifted from its equilibrium position by θ_m = 90°, while the string is under tension, and then released without initial velocity. Take the horizontal plane containing FA as a gravitational potential energy reference for the system [(S), Earth]. We neglect friction on the axis through O and air resistance.
a) Calculate the initial mechanical energy of the system [(S), Earth] when (P) was at D.
b) Determine the expression of the mechanical energy of the system [(S), Earth] in terms of l, m, g, V and θ, where V is the speed of (P) when the string passes through a position making an angle θ with the vertical.

c) Determine the value of θ, ($0° < \theta < 90°$), for which the kinetic energy of (P) is equal to the gravitational potential energy of the system [(S), Earth].

d) Calculate the magnitude V_o of the velocity $\vec{V_o}$ of (P) as it passes through its equilibrium position.

2) Upon passing through the equilibrium position, the string is cut, and (P) enters in a head-on collision with a stationary particle (P_1) of mass ml= 0.2 kg. As a result, (P_1) is projected with a velocity $\vec{V_1}$ of magnitude V_1 = 2m/s. Determine the magnitude V of the velocity $\vec{V}$ of (P) right after impact knowing that $\vec{V_o}$, $\vec{V_1}$, and $\vec{V}$ are collinear. Is the collision elastic? Justify your answer.

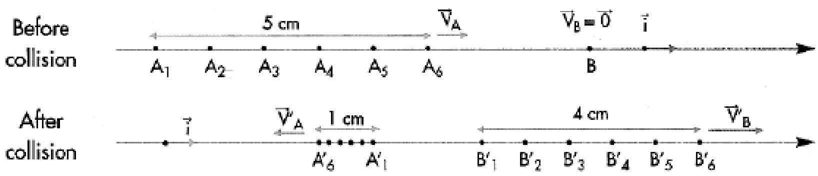
3) (P_1), being projected with a speed V_1 = 2m/s, moves along the frictionless horizontal track FA, and rises at A with the speed V_1, along the line of greatest slope of the inclined plane AB that makes an angle $\alpha = 30°$ with the horizontal.

a) Suppose now that the friction along AB is negligible. Determine the position of the point M at which (P_1) turns back.

b) In fact, AB is not frictionless; (P1) reaches a point N and turns back, where AN = 20 cm. Calculate the variation in the mechanical energy of the system [(P_1), Earth] between A and N, and then deduce the magnitude of the force of friction (assumed constant) along AN.

Exercise 3 (LS 2002-2-ex1) Collision and the laws of conservation

In order to study the collision between two bodies, we use a horizontal air table that is equipped with a launcher and two pucks (A) and (B) of respective masses m_A = 0.2 kg and m_B = 0.3 kg.



(A), thrown with the velocity $\vec{V_A} = V_A\vec{\imath}$, enters in a head-on collision with (B), initially at rest. (A) rebounds with the velocity $\overrightarrow{V'_A} = V'_A\vec{\imath}$, and (B) is projected with the velocity $\overrightarrow{V'_B} = V'_B\vec{\imath}$. The figure below shows, in real dimensions, a part of the dot-prints, that register the positions of the centers of masses of (A) and (B), obtained when the time interval separating two successive dots is τ = 20 ms.

A) Law related to the linear momentum

I) 1) Show, using the above dot-prints, that the velocities V_A, V'_A and V'_B are constant and calculate the algebraic values $\vec{V_A}$, $\overrightarrow{V'_A}$ and $\overrightarrow{V'_B}$.

2) Determine the linear momentums $\vec{P_A}$ and $\overrightarrow{P'_A}$ of the puck (A), before and after collision respectively and that $\overrightarrow{P'_B}$ of the puck (B) after collision.

3) Deduce the linear momentums, $\vec{P}$ and $\overrightarrow{P'}$, of the center of mass of the system [(A) and (B)] before and after collision respectively.

4) Compare $\vec{P}$ and $\overrightarrow{P'}$ then conclude.

II) 1) Name the forces acting on the system [(A), (B)].

2) What is the value of the resultant of these forces?

3) This result agrees with the conclusion of (I - 4). Why?

B) Law related to the kinetic energy

1) Calculate the kinetic energy of the system [(A), (B)] before and after collision.

2) Deduce the nature of this collision.

Exercise 4 (LS 2004-2-ex1) Determination of the speed of a bullet

A gun is used to shoot bullets, each of mass $m = 20g$, with a horizontal velocity $\vec{V_0}$ of value V_0. In

order to determine V_0, we consider a setup formed of a wooden block of mass $M = 1kg$, suspended from the ends of two inextensible sting of negligible mass and of the same length (figure 1).
This setup can be taken as a block of wood suspended from the free end a string of length $t = 1m$, initially at rest in the equilibrium position at G_1.
A bullet having the velocity $\vec{V}_0$ hits the block and is embedded in at the level of the center of mass G of the block. Just after impact, the system (block, bullet) moves with a horizontal velocity $\vec{V}_1$. The pendulum thus attains a maximum angular deviation $\alpha = 37^o$. G_1 and G_2 are the respective positions of G in the equilibium position and in the highest position. Take the horizontal plane through G_1 as a gravitational potential energy reference (figure 2).
Neglect friction with air and take $g = 9.8\ m/s^2$.

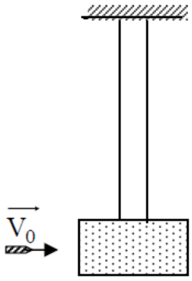


Figure 1

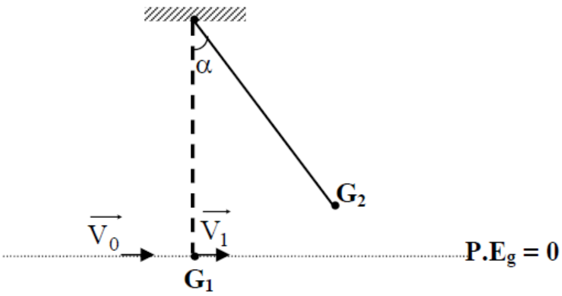


Figure 2

1) During a collision, which one of the two physical quantities, the linear momentum or the kinetic energy of the system does not remain always conserved?
2) Determine the expression of the value of V_1 ofthe velocity $\vec{V_1}$ in terms of M, m and V_0.
3) a) Determine, just after impact, the mechanical energy of the system (pendulum, Earth) in terms of V_0, M, and m.
b) Determine, in terms of M, m, g, ℓ and α, the mechanical energy of the system (pendulum, Earth) at point G_2.
c) Deduce the value of V_0.
4) Verify the answer of question (1).

Exercise 5 (LS 2006-1-ex1) Verification of Newton's second law

A puck (S) of mass M = 100 g and of center of mass G, may slide along an inclined track that makes an angle a with the horizontal so that sinα = 0.40. Thus G moves along an axis x'x parallel to the track as shown in figure (1).
Take $g = 10\ m/s^2$.

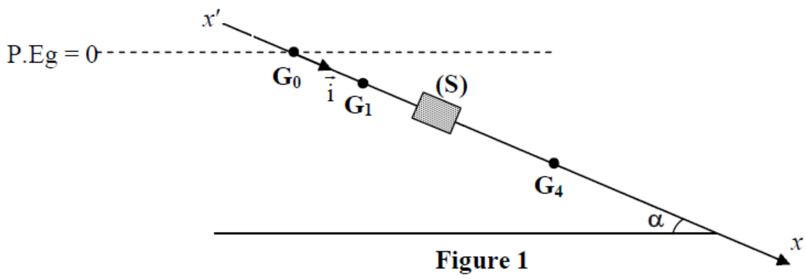


Figure 1

We release (S) without initial velocity at the instant $t_o = 0$ and at the end of each interval of time t = 50 ms, some positions G_o, G_1, G_2, …G_5 of G are recorded at the instants $t_o = 0$, t_1, t_2, … t_5 respectively.
The values of the abscissa x of G ($x = \overline{G_oG}$) are given in the table below.

t	0	τ	2τ	3τ	4τ	5τ
x (cm)	0	$G_0G_1 = 0.50$	$G_0G_2 = 2.00$	$G_0G_3 = 4.50$	$G_0G_4 = 8.00$	$G_0G_5 = 12.50$

1) Verify that the speed of the puck at the instants $t_2 = 2t$ and $t_4 = 4t$ are $V_2 = 0.40$ m/s and $V_4 = 0.80$ m/s respectively.

2) a) Calculate the mechanical energy of the system (puck-Earth) at the instants t_0, t_2 and t_4 knowing that the horizontal plane through G_0 is taken as a gravitational potential energy reference.

b) Why can we suppose that the puck moves without friction along the rail?

3) Determine the variation in the linear momentum $\overrightarrow{\Delta P} = \overrightarrow{P_4} - \overrightarrow{P_2}$ of (S) during $\Delta t = t_4 - t_2$.

4) a) Name the forces acting on (S) during its motion.

b) Show that the resultant $\sum \vec{F}$ of these forces may be written as $\sum \vec{F} = (Mgsin\alpha)\vec{\imath}$.

5) Assuming that Δt is very small, $\frac{\Delta \vec{P}}{\Delta t}$ may be considered equal to $\frac{d\vec{P}}{dt}$. Show that Newton's second law is verified between the instants t_2 and t_4.

Exercise 6 (LS 2007-2-ex1) Mechanical interaction

The object of this exercise is to study some physical quantities of a system whose parts are in mechanical interaction.

For that, we use two pucks (A) and (B), of respective masses $m_A = 100g$ and $m_B = 120g$, that may move without friction on a horizontal table. Each puck is surrounded by an elastic steel shock ring of negligible mass. The two pucks are connected by a massless and inextensible taut thread thus compressing the steel shock rings. The system (S) thus formed is at rest. (Figure 1)

We burn the thread; the shock rings stretch and the pucks repel each other. The system (S) thus formed of the two pucks and the shock rings is said to "explode".

The positions of the center of mass of each puck are registered at successive instants separated by a constant time interval $\tau = 50$ ms.

Figure (2) represents, on the axis $x'x$, the dot-prints of the positions of the centers of masses GA and GB of the two pucks after the "explosion".

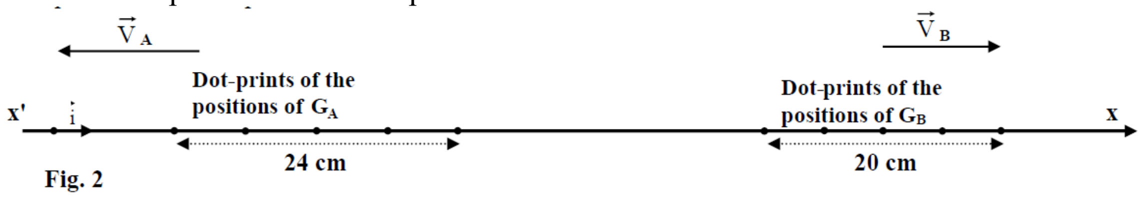


Fig. 2

1) Using the document of figure (2), show that, after explosion:

a) The motion of each puck is uniform;

b) The speeds of (A) and (B) are $V_A = 1.2 m/s$ and $V_B = 1 m/s$ respectively.

2) Verify the conservation of the linear momentum of the system (S) during explosion.

3) Applying Newton's second law $\frac{d\vec{P}}{dt} = \Sigma\vec{F}_{ext}$ on each puck and assuming that the time interval of the explosion $\Delta t = 0.05s$ is so small that $\frac{\Delta\vec{P}}{\Delta t}$ has the same value as $\frac{d\vec{P}}{dt}$.

a) Determine the forces $\vec{F}_{A\to B}$ and $\vec{F}_{B\to A}$ exerted respectively by (A) on (B) and by (B) on (A).

b) Verify the principle of interaction.

4) The system (S) possesses a certain energy before the explosion.

a) Specify the part of (S) storing this energy.

b) In what form is this energy stored?

c) Determine the value of this energy.

Exercise 7 (LS 2010-2-ex3) Resistive force on a car

A car of mass $M = 1500$ kg moves on a straight horizontal road; its center of gravity G is moving on the axis $(O, \vec{i})$.
The car is acted upon by the forces:
- its weight;
- the normal reaction of the road;
- a constant motive force $\vec{F}_m = F_m\vec{i}$ where $F_m = 3500N$;
- a resistive force $\vec{F}_f = -F_f\vec{i}$.

In order to determine F_f, we measure the speed V of the car at different instants, separated by equal time intervals each being $\tau = 1s$.

A- Value of $\vec{F}_f$ between the instants $t_0 = 0$ and $t_5 = 5s$

The results of the obtained recordings are tabulated as follows:

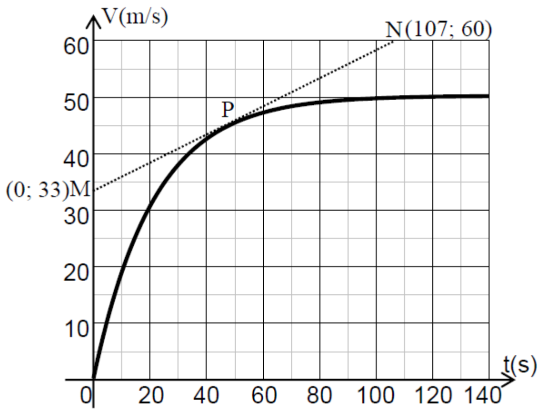
Instant	$t_0 = 0$	$t_1 = \tau$	$t_2 = 2\tau$	$t_3 = 3\tau$	$t_4 = 4\tau$	$t_5 = 5\tau$
Position	O	G_1	G_2	G_3	G_4	G_5
V(m/s)	0	2	4	6	8	10

1) Using the scale below, draw the curve representing the variation of the speed V as a function of time.
   - 1 cm on the axis of abscissas represents 1 s;
   - 1 cm on the axis of ordinates represents 1 m/s.

2) Show that the relation between the velocity $\vec{V} = V\vec{i}$ at a time t has the form $\vec{V} = bt\vec{i}$ where b is a constant.
3) a) the constant b is a characteristic physical quantity of motion. Give its name.
   b) Calculate its value.
4) Applying Newton's second law,
   a) show that F_f is constant between $t_0 = 0$ and $t_5 = 5s$;
   b) calculate the value F_f of $\vec{F}_f$.

B- Variation of F_f between the instants $t_5 = 5s$ and $t = 140s$

In reality, the measurement of the speed between the instants $t_0 = 0$ and $t = 140s$ allows us to plot the graph of the adjacent figure.

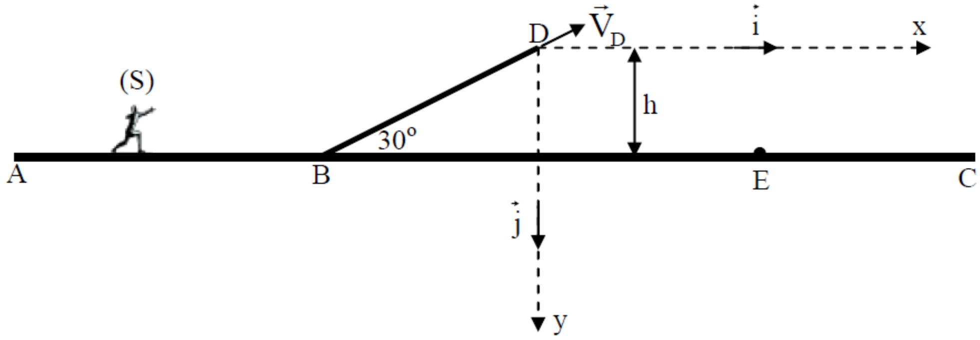


1) Show that the part of this graph between the instants $t_0 = 0$ and $t_5 = 5s$ is in agreement with the graph of part A.
2) We draw the tangent MN to the curve at the point P at the instant t_P where $V_P = 45m/s$.
   a) Determine the value of the acceleration at the instant t_P.
   b) Deduce the value of F_f at the instant t_P.
3) Starting from the instant 100s, V attains a limiting value of $V_t = 50m/s$. Calculate then the value of F_f.
4) Indicate the time interval during which F_f Increases.

Exercise 8 (LS 2012-2-ex1) Study of the motion of a skier

A skier (S), of mass $m = 80kg$, is pulled by a boat using a rope parallel to the surface of water. He starts from point A at the instant $t_0 = 0$ without initial velocity. The skier passes point B at the instant $t = 60s$ with a speed $V_B = 6m/s$, then he releases the rope. He continues his motion along a board BD inclined by an angle of 30^o with respect to the horizontal surface ofwater. Suppose that

during the passage from AB to BD the speed at point B does not change.
The skier arrives point D, situated at an altitude $h = 1.6m$ from the water surface, with a velocity , then he leaves the board at point D to hit the water surface at point E (see figure below).



Given:
- the skier is considered as a particle;
- on the path AB, the force of traction $\vec{F}$ exerted by the rope on the skier has a constant magnitude F and the whole forces of friction are equivalent to a single force $\vec{f}$ opposite to the displacement, of magnitude $f = 100N$;
- friction is negligible along the path BDE;
- after leaving point D the motion of the skier takes place in the vertical plane Dxy containing $\overrightarrow{V_D}$;
- the horizontal plane passing through AB is the reference level of the gravitational potential energy;
- $g = 10m/s^2$.

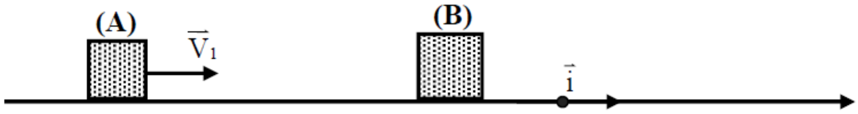
A- Motion of the skier between A and *B*
1) What are the external forces acting on (S) along the path AB? Draw, not to scale, a diagram of these forces.
2) Applying Newton's second law $\frac{d\vec{P}}{dt} = \Sigma\vec{F}_{ext}$ on the skier, between the points A and B, express the acceleration a of the motion of the skier in terms of F, f and m.
3) Determine the expression of the speed V of the skier in terms of F, f, m and the time t.
4) Deduce F.

B- Motion of the skier on the board BD
1) Why can we apply the principle of conservation of the mechanical of energy of system [(S), Earth] on the path BD?
2) Deduce that $V_D = 2m/s$.

C- Motion of the skier between *D* and *E*
The skier leaves the board at point D, at an instant t_0, taken as a new origin of time.
1) Apply Newton's second law on the skier to show that, at an instant t, the vertical component P_y of the linear momentum of the skier is of the form: $P_y = 800t - 80$ (In SI unit).
2) Deduce the parametric equation $y(t)$ of the motion of the skier in the frame of reference Dxy.
3) Determine the duration taken by the skier to pass from D to E.

Exercise 9 (LS 2015-1-ex1) Collision and interaction

In order to study the collision between two bodies, we consider a horizontal air table equipped with a launcher and two pucks (A) and (B) of respective masses $m_A = 0.4$ kg and $m_B = 0.6kg$. (A), launched with the velocity $\vec{V} = 0.5\vec{i}$, collides with (B) initially at rest. (A) rebounds with the velocity $\vec{V} = -0.1\vec{i}$ and (B) moves with the velocity $\vec{V} = 0.4\vec{i}$ (V_1, V_2 and V_3 are expressed in m/s). Neglect all frictional forces.



A- Linear momentum

1) a) Determine the linear momentums:

i) $\vec{P}$ and $\vec{P}$ of (A), before and after collision respectively;

ii) $\vec{P}$ of (B) after collision.

b) Deduce the linear momentums $\vec{P}$ and $\vec{P'}$ ofthe system $[(A),(B)]$ before and after collision respectively.

c) Compare $\vec{P}$ and $\vec{P'}$. Conclude.

2) a) Name the external forces acting on the system $[(A),(B)]$.

b) Give the value of the resultant of these forces.

c) Is this resultant compatible with the conclusion in question (l-c)? Why?

B- Type of collision

1) Determine the kinetic energy of the system $[(A),(B)]$ before and after collision. 2) Deduce the type of the collision.

C- Principle of interaction

The duration of collision is $\Delta t = 0.04s$; we can consider that $\frac{\Delta\vec{P}}{\Delta t} \approx \frac{d\vec{P}}{dt}$.

1) Determine during Δt:

a) the variations $\Delta\vec{P}_A$ and $\Delta\vec{P}_B$ in the linear momentums of the pucks (A) and (B) respectively;

b) the forces $\vec{F}_{A/B}$ exerted by (A) on (B) and $\vec{F}_{B/A}$ exerted by (B) on (A).

2) Deduce that the principle of interaction is verified.

Exercise 10 (LS 2016-2-ex2) Nature of a collision

The aim of this exercise is to determine the nature of a collision between two objects. For this aim, an object (A), considered as a particle, of mass $m_A = 2kg$, can slide without friction on a path situated in a vertical plane and formed of two parts: a circular part DN and a horizontal rectilinear part NM. (A) is released, without initial velocity, from the point D situated at a height $h_D = 0.45m$ above the horizontal part NM (Fig.1).

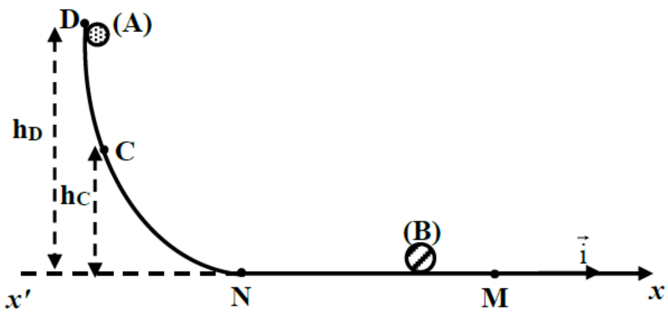


Fig. 1

The horizontal plane passing through MN is taken as the reference level of gravitational potential energy.

Take $g = 10m/s^2$.

1) Calculate the mechanical energy of the system [(A), Earth] at the point D.

2) Deduce the speed V_{1A} of (A) when it reaches the point N.

3) (A) reaches N and moves along NM with the same velocity $\vec{V}_{1A} = V_{1A}\vec{i}$. Another object (B), considered as a particle, of mass $m_B = 4$ kg moves along the horizontal path from M toward N with the velocity $\vec{V}_{1B} = -1\vec{i}$ (V_{1B} in m/s).

a) Determine the linear momentum $\vec{P}_S$ of the system $[(A),(B)]$ before collision.

b) Deduce the velocity $\vec{V}_G$ of the center of inertia G of the system $[(A),(B)]$.

4) After collision, (A) rebounds and attains a maximum height $h_C = 0.27m$.

a) Determine the mechanical energy of the system [(A), Earth] at the point C.

b) Deduce the speed V_{2A} of (A) just after collision.

5) Determine, by applying the principle of the conservation of the linear momentum of the system $[(A),(B)]$, the velocity $\vec{V}_{2B}$ of (B) just after collision.

6) Specify the nature of the collision.

Exercise 11 (LS 2021-2-ex2) Studying the motion of an object

Consider:

- a rail AOB located in a vertical plane composed of two straight parts: a horizontal part AO and an inclined part OB making an angle $\alpha = 30^0$ with the horizontal;
- two objects (S_1) and (S_2) taken as particles of same mass m = 80 g;
- a massless spring (R), of force constant k = 200 N/m and natural length ℓ_0, fixed from one of its ends to a support at A with the other end free.

Take:

- the horizontal plane containing O as the reference level for gravitational potential energy;
- $g = 10\ m/s^2$.

1) Launching particle (S_1)

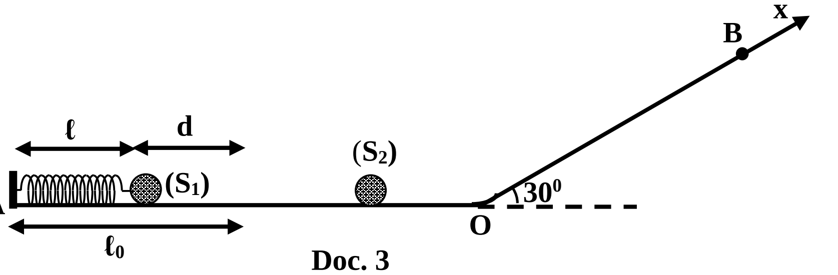
In order to launch (S_1), it is placed against the free end of the spring, the spring is compressed by a distance d, and then the system [Spring - (S_1)] is released from rest (Doc.3).

When the spring returns to its natural length ℓ_0, (S_1) leaves the spring with a velocity $\vec{V}_1$ parallel to AO .

After launching, (S_1) moving with the velocity $\vec{V}_1$, collides head-on with (S_2) which is placed initially at rest on the rail AO.

Just after the collision, (S_1) stops and (S_2) moves with a velocity $\vec{V}_2$ parallel to AO and of magnitude $V_2 = 5$ m/s.

(S_1) and (S_2) move without friction on the rail AO.



Doc. 3

1.1) Apply the law of conservation of linear momentum to show that the magnitude of $\vec{V}_1$ is $V_1 = 5$ m/s.

1.2) Deduce that the collision between (S_1) and (S_2) is elastic.

1.3) Determine the value of d.

2) Motion of (S_2) on the inclined part OB

At the instant $t_0 = 0$, (S_2) starts from O on the inclined part OB with a velocity $\vec{V}_0 = V_0\,\vec{i} = 5\,\vec{i}$ (m/s), where $\vec{i}$ is the unit vector along the x-axis parallel to OB. On this part, (S_2) is submitted to a friction force $\vec{f}$ of constant magnitude f and of direction opposite to its motion.

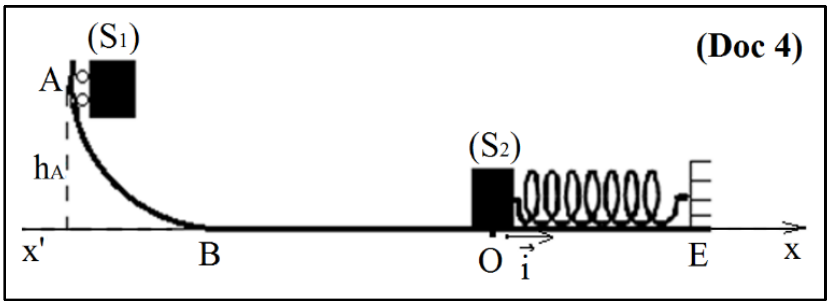
2.1) Name the external forces acting on (S_2) during its motion along the track OB.

2.2) Show that the sum of the external forces acting on (S_2) during its upward motion along OB is: $\Sigma\vec{F} = -(f + mg\sin\alpha)\,\vec{i}$.

2.3) The expression of the linear momentum of (S_2) during its upward motion along OB is:
$\vec{P} = (-0.9\,t + 0.4)\,\vec{i}$ (SI).
Knowing that $\frac{d\vec{P}}{dt} = \Sigma\vec{F}$, determine f.

Exercise 12 (CRDP LS Samples LS 2-ex2) Energies and collision

A particle (S_1), of mass m_1 = 200 g, is released from rest at the point A on a track ABOE, found in a vertical plane, as shown in the adjacent document (Doc 4).
The part AB, very smooth, along which we can neglect the force of friction, has the shape of a circular arc of radius h_A, and the part BO, a rough part, along which the force of friction $\vec{f}$ is supposed constant, is a rectilinear and horizontal path with BO = 1 m.



The particle (S_1) reaches the point B with the speed v_{1B} = 4 m/s, then it covers the track BO to reach the point O with the speed v_{1O} = 2 m/s.
At O, (S_1) enters into a head-on collision with a particle (S_2), of mass m_2 = 400 g, initially at rest and connected to the end of a horizontal spring of stiffness k = 100 N/m whose other end is fixed at E.
Take the horizontal plane containing BO as a gravitational potential energy reference level.
Take g = 10 m/s^2.

1) Conservation and non-conservation of the mechanical energy.

1-1) Applying the principle of conservation of the mechanical energy of the system [(S_1), Earth], determine h_A.

1-2) Determine the work done by the force of friction $\vec{f}$ along BO.

1-3) Deduce the magnitude f of the force of friction $\vec{f}$ along BO.

2) Elastic collision.
The collision between the particles (S_1) and (S_2) is perfectly elastic. All the velocities, before and after the collision, are along the horizontal axis x'Ox.

2-1) Determine the speed v'_{1O} of (S_1) and v'_{2O} of (S_2) just after the collision.

2-2) Neglecting the force of friction between (S_2) and the track, just after the collision, calculate the maximum compression x_m = OD of the spring.

2-3) In fact, the force of friction $\vec{f'}$ between (S_2) and the track, just after the collision, is not negligible and the maximum compression of the spring is x'_m = OD' = 6.4 cm.

2-3-1) Determine the decrease in the mechanical energy of the system [(S_2), Earth, spring], between O and D'.

2-3-2) In what form of energy does this decrease appear?

Solutions

Exercise 1 (LS 2001-2-ex1)

I) 1) The expression of mechanical energy of the system

$M.E = KE + P.E_g$

$= \frac{1}{2}(M+m)v^2 - mg(h+x)$

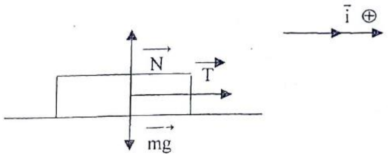
This energy is conserved because there is no friction.

2) M.E is constant so $\frac{dM.E}{dt} = 0$ then $2x\,\frac{1}{2}(M+m)v\,a - mgv = 0$

$(M+m)v\,a - mgv = 0$

$a = \frac{mgv}{(M+m)v} = \frac{500}{250} = 2 m/s^2$

Weight $m\vec{g}$



Normal reaction $\vec{N}$.

Tension of the string $\vec{T}$.

Apply Newton's second law

$\Sigma\vec{F} = M\vec{a}$

$M\vec{g} + \vec{N} + \vec{T} = M\vec{a}$

Project along the trajectory

$T = Ma = 200 \times 10^{-3} \times 2 = 0.4$ N So $\vec{T} = 0.4\,\vec{i}$

II)

Point	A	B	C	D	E
T in ms	50	100	150	200	250
V in cm/s	10	20	30	40	50

By using the table

The linear momentum $\vec{P}_B = M\vec{V}_B$

$P_B = MV_B = 200 \times 10^{-3} \times 20 \times 10^{-2} = 0.04$ kg.m/s

Linear momentum $\vec{P}_D = M\vec{V}_D$

$P_D = MV_D = 200 \times 10^{-3} \times 40 \times 10^{-2} = 0.08$ kg.m/s

$P_D - P_B = 0.08 - 0.04 = 0.04$ kg.m/s

$\Delta t = t_D - t_B = 200 - 100 = 100$ ms = 0..1 s

$\frac{\Delta P}{\Delta t} = \frac{PD - PB}{tD - tB} = \frac{0.04}{0.1} = 0.4$ kgm/s.

III) by compare $\frac{\Delta\vec{P}}{\Delta t}$ and $\vec{T}$ so $\frac{\Delta\vec{P}}{\Delta t} = \vec{T} = 0.4\,\vec{i}$

Yes Newton's second Law is verified

$\Sigma\vec{F} = \frac{\Delta\vec{P}}{\Delta t}$

Exercise 2 (LS 2002-1-ex1)

1) a) At D: $KE = 0J$ then $v = 0m/s$.

$P.E_g = mgl = 0.1 \times 10 \times 0.45 = 0.45J$.

$M.E = KE + P.E_g = 0.45J$.

b) $M.E = KE + P.E_g = \frac{1}{2}mv^2 + mgh$; and $h = 1 - 1\cos\theta$.

$M.E = \frac{1}{2}mv^2 + mgl(1 - \cos\theta)$.

c) M.E of the system [(S) , Terre] is conserved because friction is neglected.

$M.E = M.E_D = 0.45$ J.

$P.E_g = K.E = \frac{M.E}{2} = 0.45J \Rightarrow P.E_g = mgl(1 - \cos\theta) = 0.45 = \theta = 60^o$.

d) $M.E = M.E_F = 0.45J$; $P.E_{gF} = 0$.

$K.E = \frac{1}{2}mV_o^2 = 0.45 \Rightarrow V_o = 3m/s.$

2) During collision, the linear momentum of the system (P, P_1) is conserved:

$m\vec{V_0} = m\vec{V} + m_1\vec{V_1}$.

$\vec{V_o}, \vec{V}$ and $\vec{V_1}$ are collinear: $mV_o = mV + m_1V_1 \Rightarrow V = \frac{mV_o - m_1V_1}{m_1} = -1m/s.$

$K.E_i$ of the system before collision: $K.E_i = \frac{1}{2}mV_o^2 = 0.45$ J.

$K.E_f$ of the system before collision: $K.E_f = \frac{1}{2}mV^2 + m_1V_1^2 = 0.45$ J.

$K.E_i = K.E_f \Rightarrow$ the collision is elastic.

3) a) At A, $P.E_{gA} = 0J \Rightarrow M.E_A = K.E_A = \frac{1}{2}m_1V_A^2 = 0.4$ J.

M.E of the system [(S) , Terre] is conserved because friction is neglected,

$M.E_A = M.E_M$. At M, $E_{cM} = 0J \Rightarrow E_{mM} = E_{pM} = m_1gAM \sin\alpha = 0.4$

$\Rightarrow AM = 0.4m.$

b) At N, $K.E_c = 0J \Rightarrow M.E_N = P.E_{gN} = m_1gAN \sin\alpha = 0.2J$

$\Delta E_m = E_{mN} - E_{mA} = -0.20J.$

$\Delta E_m = W_{\vec{f}} = \vec{f}.\overrightarrow{AN} = -f \times \text{AN} \Rightarrow f = \frac{-\Delta E_m}{AN} = \frac{0.2}{0.2} = 1N.$

Exercise 3 (LS 2002-2-ex1)

A-I-1) the distances covered by the puck before and after collision during successive and equal intervals of time are equal.

Since velocities are collinear or held by the same axis x-axis then these velocities are constants.

$V_\text{A} = \frac{A_1A_6}{5\tau} = \frac{5\times10^{-2}}{5\text{x}20\times10^{-3}} = 0.5\ m/s.$

$V'_\text{A} = \frac{A'_1A'_6}{5\tau} = \frac{-1\times10^{-2}}{5\text{x}20\times10^{-3}} = -0.1\ m/s.$

$V'_\text{B} = \frac{B'_1B'_6}{5\tau} = \frac{4\times10^{-2}}{5\text{x}20\times10^{-3}} = 0.4\ m/s.$

2) Linear momentums:

$\vec{P}_\text{A} = m_\text{A}\vec{V}_\text{A} = 0.2 \times(0.5\ \vec{i}\ ) = 0.1\ \vec{i}$ (kg m/s).

$\overrightarrow{P'}_\text{A} = m_\text{A}\overrightarrow{V'}_\text{A} = 0.2 \times(-0.1\ \vec{i}\ ) = -0.02\ \vec{i}$ (kg m/s).

$\overrightarrow{P'}_\text{B} = m_\text{B}\overrightarrow{V'}_\text{B} = 0.3 \times(0.4\ \vec{i}\ ) = 0.12\ \vec{i}$ (kg m/s).

3) The linear momentum of the system $\vec{P}$ before collision:

$\vec{P} = \vec{P}_\text{A} + \vec{P}_\text{B} = 0.1\ \vec{i} + \vec{0} = 0.1\ \vec{i}$

The linear momentum of the system $\vec{P}$ before collision:

$\overrightarrow{P'} = \overrightarrow{P'}_\text{A} + \overrightarrow{P'}_\text{B} = -0.2\ \vec{i} + 0.12\ \vec{i} = 0.1\ \vec{i}$

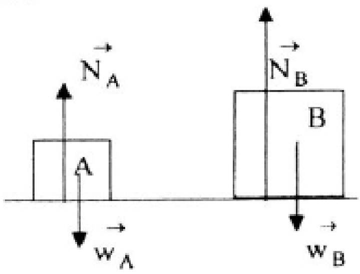
4) Comparing $\vec{P}$ and $\overrightarrow{P'}$

$\overrightarrow{P'} = \vec{P} = 0.1\ \vec{i}$ so $\vec{P}$ = constant

Conclusion: Linear momentum is conserved

II- 1) Forces acting on the system

Weight $\vec{W}$ ($\vec{W}_\text{A} + \vec{W}_\text{B}$) and the normal reaction $\vec{N}$ ($\vec{N}_\text{A} + \vec{N}_\text{B}$)



2) The resultant of the forces

$\Sigma\vec{F} = \vec{W} + \vec{N} = \overrightarrow{W_\text{A}} + \overrightarrow{W_\text{B}} + \vec{N}_\text{A} + \vec{N}_\text{B}$

$\Sigma\vec{F} = \vec{0} + \vec{0} = \vec{0}$

3) In part (1-4) $\overrightarrow{P'} = \vec{P}$ so $\Delta\vec{P} = \vec{O}$

$\frac{\Delta\vec{P}}{\Delta t} = \Sigma\vec{F} = \vec{O}$ which is Newton's 2nd Law.

B- 1) $K.E_{before} = \frac{1}{2}m_A V_A^2 + \frac{1}{2}m_B V_B^2 = \frac{1}{2}(0.2)(0.5)^2 + 0 = 0.025$ J

$E_{after} = \frac{1}{2}m_A V_A^2 + \frac{1}{2}m_B V_B^2 = \frac{1}{2}(0.2)(-0.1)^2 + \frac{1}{2}(0.3)(0.4)^2 = 0.025$ J

2) Nature of the collision $K.E_{before} = E_{after}$ So collision is elastic.

Exercise 4 (LS 2004-2-ex1)

1) The kinetic energy of the system (bullet, block).

2) $\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$.

$m\overrightarrow{V_0} = (M+m)\overrightarrow{V_1}$; Thus: $V_1 = \frac{mV_0}{(M+m)}$.

3) a) $M.E = P.E_g + K.E$.

$M.E = 0 + K.E = \frac{1}{2}(M+m)V_1^2$

$M.E = \frac{1}{2}(M+m)\left[\frac{mV_0}{(M+m)}\right]^2 = \frac{1}{2}\frac{m^2V_0^2}{(M+m)}$.

b) $M.E = (M+m)gh$

$h = l - lcos\alpha = l(1 - \cos\alpha)$.

Thus: $M.E = (M+m)gl(1-\cos\alpha)$.

c) The friction is neglected and the M.E of (pendulum, Earth) is conserved.

$\frac{1}{2}\frac{m^2V_0^2}{(M+m)} = (M+m)gl(l - \cos\alpha)$.

$V_0 = \frac{M+m}{\text{m}}\sqrt{2gl(1-\cos\alpha)}$.

$V_0 = 101.3\ m/s$.

4) $K.E_{before} = \frac{1}{2}mV_0^2$

$K.E_{before} = 102.6J$

$K.E_{after} = \frac{1}{2}(M+m){V_1}^2 = \frac{1}{2}\frac{m^2V_0^2}{(M+m)}$.

$E_{after} = 2J$

$K.E_{before}$ is ≠ from $K.E_{after}$.

Exercise 5 (LS 2006-1-ex1)

1) $V_2 = \frac{G_1G_3}{2\tau} = \frac{G_0G_3 - G_0G_1}{2\tau} = \frac{(4.5-0.5)\times10^{-2}}{0.1} = 0.4m/s$.

$V_4 = \frac{G_3G_5}{2\tau} = \frac{G_0G_5 - G_0G_3}{2\tau} = \frac{(12.5-4.5)\times10^{-2}}{0.1} = 0.8m/s$.

2) a) $M.E = K.E + P.E_g$;

$M.E_0 = K.E_0 + PE_{g0} = 0 + 0 = 0$

$M.E_2 = K.E_2 + PE_{g2} = \frac{1}{2}M(V_2)^2 - Mgh_2$;

$h_2 = G_0G_2 \times \sin\alpha = 2 \times 0.4 = 0.8$ cm $= 0.008m \Rightarrow M.E_2 = 0$ J.

$M.E_4 = K.E_4 + PE_{g4} = \frac{1}{2}M(V_4)^2 - Mgh_4$;

$h_4 = G_0G_4 \times \sin\alpha = 8 \times 0.4 = 3.2$ cm $= 0.032m \Rightarrow M.E_4 = 0$ J.

b) $M.E_0 = M.E_2 = M.E_4 \Rightarrow$ the mechanical energy is conserved during motion $\Rightarrow$ No friction.

3) $\Delta\vec{P} = \overrightarrow{P_4} - \overrightarrow{P_2} = M(V_4\vec{\imath} - V_2\vec{\imath}) = 0.04\vec{\imath}$.

4) a) The forces acting on (S):

The weight $\overrightarrow{W}$ of (S) and the normal reaction $\vec{N}$ of the path.

b) $\Sigma\vec{F} = \vec{W} + \vec{N} = \overrightarrow{W_1} + \overrightarrow{W_2} + \vec{N}$

where: $\overrightarrow{W_1} = Mg\sin\alpha\vec{\imath}$, $\overrightarrow{W_2} = -Mg\cos\alpha\vec{\jmath}$,

$\vec{N} = N\vec{\jmath}$; $\overrightarrow{W_2} + \vec{N} = \vec{0}$ (no motion along $y'y$)

$\Rightarrow \Sigma\vec{F} = \overrightarrow{W_1} = mg\sin\alpha\vec{\imath}$.

5) The 2^{nd} Law of is given by: $\Sigma\vec{F} = \frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t}$.

We have: $\Sigma\vec{F} = mg\sin\alpha\vec{\imath} = 0.4\vec{\imath}$ and $\frac{\Delta\vec{P}}{\Delta t} = \frac{0.04\vec{\imath}}{0.1} = 0.4\vec{\imath}$.

2^{nd} Law of Newton is thus verified.

Exercise 6 (LS 2007-2-ex1)

1) a) The distances covered during equal time intervals are equal.

b) $V_B = \frac{d}{t} = \frac{d}{4\tau} = \frac{0.2}{4\times0.05} = 1m/s$.

$V_A = \frac{0.24}{4\times0.05} = 1.2m/s$.

2) $\vec{P}_{before} = \vec{0}$; $\vec{P}_{after} = m_A\vec{V}_A + m_B\vec{V}_B = 0.1(-1.2\vec{\imath}) + 0.12(\vec{\imath})$

$\Rightarrow \vec{P}_{before} = \vec{P}_{after}$; The linear momentum is conserved for the system formed of the two pucks.

3) a) Newton 2^{nd} Law applied on A gives :

$\frac{\overrightarrow{dP}}{dt} = \overrightarrow{m_A}g + \overrightarrow{N_A} + \vec{F}_{B\to A} = \vec{F}_{B\to A} = \frac{0.1(-1.2-0)\vec{\imath}}{0.05} - 2.4\vec{\imath}$.

$\frac{\overrightarrow{dP}}{dt} = \overrightarrow{m_B}g + N_B + \vec{F}_{A\to B} = \vec{F}_{A\to B} = \frac{0.12(1-0)\vec{\imath}}{0.05} = 2.4\vec{\imath}$.

b) $\vec{F}_{B\to A} = -\vec{F}_{A\to B}$.

4) a) The deformed elastic shock ring.

b) Elastic potential energy.

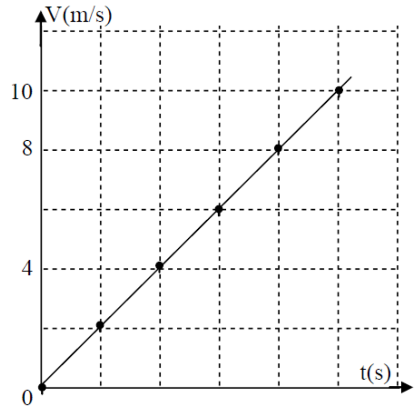
c) The mechanical energy of the system is conserved because the system is isolated (The system does not exchange energy with the surroundings) ; (Elastic potential energy is transformed into kinetic energy):

$M.E = K.E + P.E_{el} = M.E_{before} = M.E_{after} = 0 + P.E_{el} = K.E + 0$

$\Rightarrow P.E_{el} = \frac{1}{2}m_AV_A^2 + \frac{1}{2}m_BV_B^2 = 0.132J$.

Exercise 7 (LS 2010-2-ex3)

A- 1)



2) The graph is a straight line passing through the origin, in agreement with the function $\vec{V} = bt\vec{\imath}$ where b is a constant.

3) a) b the acceleration of the motion.

b) $b = \frac{\Delta V}{\Delta t} = \frac{10-0}{5} = 2m/s^2$.

4) a) $\sum \vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt} \Rightarrow \frac{d\vec{P}}{dt} = M\vec{g} + \vec{R} + \vec{F}_{\text{M}} + \vec{F}_F$. Projection along the horizontal:

$M\frac{dV}{dt} = F_m - F_f \Rightarrow Mb = F_m - F_f$; $F_m =$ const.

$M =$ const. and $b =$ constant $\Rightarrow F_f =$ constant.

b) $\Rightarrow F_f = F_m - mb,\ F_f = 3500 - 1500 \times 2 = 500N$.

B- 1) For $V < 10m/s$, the part ofthe curve is a straight line.

2) a) $a = \frac{dV}{dt}$ is the slope of the tangent.

$a = \frac{60-33}{107-0} = 0.25m/s^2$.

b) $F_f = 3500 - 1500 \times 0.25 = 3125$ N.

3) $a = 0 \Rightarrow F_f = F_m = 3500N$.

4) $5s < t < 100s$.

Exercise 8 (LS 2012-2-ex1)

A- 1) The forces acting on (S) are: the weight mg, the normal reaction of the surface of water $\vec{N}$, $\vec{F}$ and $\vec{f}$.

2) $\frac{\overrightarrow{dP}}{dt} = \sum \vec{F}_{ext} = \overrightarrow{mg} + \overline{N} + \vec{F} + \vec{f}$ project along the direction of motion

$\Rightarrow \frac{dP}{dt} = F - f \Rightarrow ma = F - f \Rightarrow a = \frac{F-f}{m}$.

3) $V =$ primitive of $a = at + V_o (V_o = 0)$ then $V = \left(\frac{F-f}{m}\right) t$.

4) $V = V_B = 6m/s$ for $t = 60s \Rightarrow 6 = \left(\frac{F-100}{80}\right) 60 = F = 108N$.

B- 1) Since friction is negligible between B and D.

2) $ME_B = ME_D \Rightarrow \frac{1}{2}m(V_B)^2 + 0 = \frac{1}{2}m(V_D)^2 + mgh$.

$\Rightarrow \frac{1}{2}(80)(36) = \frac{1}{2}(80)(V_D)^2 + 80 \times 10 \times 1.6 \Rightarrow V_D = 2m/s$.

C- 1) $\frac{\overrightarrow{dP}}{dt} = \sum \vec{F}_{ext} = mg\vec{j} \Rightarrow \frac{dP_y}{dt} = mg \Rightarrow P_y = mgt + P_{0y}$

$P_{0y} = mV_{0y} = m(-V_D \sin 30^o) = -80 \times 2 \times \frac{1}{2} = -80$

$\Rightarrow P_y = 800t - 80$.

2) $V_y = \frac{\mathrm{P_y}}{\mathrm{m}} = 10t - 1 \Rightarrow y = 5t^2 - t + y_0 = 5t^2 - t\ (y_0 = 0)$.

3) $1.6 = 5t^2 - t \Rightarrow 5t^2 - t + 1.6 \Rightarrow 0 = \Delta = 1 + 32 = 33$

$t = \frac{1 \pm \sqrt{33}}{10} \Rightarrow t = \frac{1+\sqrt{33}}{10} = 0.67s$.

Exercise 9 (LS 2015-1-ex1)

A.1.a.i $\vec{P}_1 = m_A\vec{V}_1 = 0.4 \times (0.5\,\vec{\imath}\,) = 0.2\,\vec{\imath}$ (kg m/s).

$\vec{P}_2 = m_A\vec{V}_2 = 0.4 \times (-\,0.1\,\vec{\imath}\,) = -\,0.04\,\vec{\imath}$ (kg m/s).

A.1.a.ii $\vec{P}_3 = m_B\,V_3 = 0.6 \times (0.4\,\vec{\imath}) = 0.24\,\vec{\imath}$.

A.1.b $\vec{P} = \vec{P}_1 + 0 = 0.2\,\vec{\imath}$.

$\vec{P}\,' = \vec{P}_2 + \vec{P}_3 = -\,0.04\,\vec{\imath} + 0.24\,\vec{\imath} = 0.2\,\vec{\imath}$.

A.1.c $\vec{P} = \vec{P}\,'$.

Conclusion: the linear momentum of the system [(A), (B)] is conserved during collision.

A.2.a The external forces acting on the system are:

The weight $m_A\vec{g}$ and the normal reaction of the air table $\vec{N}_A$.

The weight $m_B\vec{g}$ and the normal reaction of the air table $\vec{N}_B$.

A.2.b We have : $m_A\vec{g} + \vec{N}_A + m_B\vec{g} + \vec{N}_B = \vec{0}$

The sum of the external forces acting on the system (A, B) is thus zero.
A.2.c Yes, Since the system [(A),(B)] is isolated.
B.1 $KE_{before} = ½ m_A(V_1)^2 + 0 = 0.05$ J.
$KE_{after} = ½ m_A(V_2)^2 + ½ m_B(V_3)^2 = 0.05$ J.
B.2 $KE_{before} = KE_{after} \Rightarrow$ collision is elastic.
C.1.a $\Delta\vec{P}_A = \vec{P}_2 - \vec{P}_1 = -0.24\ \vec{i}.$
$\Delta\vec{P}_B = \vec{P}_3 = 0.24\ \vec{i}.$
C.1.b $\frac{\Delta\vec{P}_A}{\Delta t} = \vec{F}_{B/A} = \frac{-0.24\ \vec{i}}{0.04} = -6\ \vec{i}$. $\frac{\Delta\vec{P}_B}{\Delta t} = \vec{F}_{A/B} = \frac{0.24\ \vec{i}}{0.04} = 6\ \vec{i}$ (N)
C.2 $\vec{F}_{B/A} = -\vec{F}_{A/B}$ the principle of [interaction] is thus verified.

Exercise 10 (LS 2016-2-ex2)

1) $ME_{(D)} = KE_{(D)} + PE_{g(D)} = 0 + mAgh_D = 9J.$
2) No friction ⇒ mechanical energy of the system [(A), Earth] is conserved:
$ME_{(D)} = ME_{(N)}$; $0 + mAgh_D = \frac{1}{2}mAV_{1A}^2 \Rightarrow V_{1A}^2 = 2gh_D \Rightarrow V_{1A} = 3m/s.$
3) a) Linear momentum of the system $[(A),(B)]$ before collision:
$\vec{P}_S = mA\vec{V}_{1A} + m_B\vec{V}_{1B} = (2 \times 3\vec{i}) + [4 \times (-1\vec{i})] = 2\vec{i}\,(kg\ m/s).$
b) $\vec{P}_S = \vec{P}_G = (m_A + m_B)\vec{V}_G = 2\vec{i} = 6.$
$\vec{V}_G \Rightarrow \vec{V}_G = \frac{1}{3}\vec{i} = 0.33\,\vec{i}\,(m/s).$
4) a) $ME_{(C)} = KE_{(C)} + PE_{g(C)} = 0 + \text{mAghc} = 2 \times 10 \times 0.27 = 5.4$ J.
b) Conservation of the mechanical energy of the system [(A), Earth]
$0 + \text{mAghc} = \frac{1}{2}m_AV_{2A}^2 \Rightarrow V_{2A}^2 = 2gh_C \Rightarrow V_{2A} = \sqrt{54} = 2.323m/s.$
5) Conservation of the linear momentum of the system $[(A),(B)]$:
$m_A\vec{V}_{2A} + m_B\vec{V}_{2B} = 2\vec{i}\,(m/s).$
$2 \times (-2.33\vec{i}) + 4\vec{V}_{2B} = 2\vec{i} = (-2.33\vec{i}) + 2\vec{V}_{2B} = \vec{i}$
$\Rightarrow 2\vec{V}_{2B} = \vec{i} + 2.323\vec{i} = 3.323\vec{i} \Rightarrow \vec{V}_{2B} = 1.66\vec{i}\,(m/s).$
6) The kinetic energy of the system $[(A),(B)]$
$KE_{before} = \frac{1}{2}mAV_{1A}^2 + \frac{1}{2}m_BV_{1B}^2 = 11J.$
$KE_{after} = \frac{1}{2}mAV_{2A}^2 + \frac{1}{2}m_BV_{2B}^2 = 5.4 + \frac{1}{2} \times 4 \times (1.66)^2 = 5.4 + 5.58$
$= 10.91J \approx 11J \Rightarrow$ the collision is elastic.

Exercise 11 (LS 2021-2-ex2)

1.1 $\vec{P}_{J.B.C} = \vec{P}_{J.A.C}$
$m\vec{V}_1 + \vec{0} = \vec{0} + m\vec{V}_2$, $\vec{V}_1 = \vec{V}_2$
then, $V_1 = 5$ m/s
1.2 System $[(S_1), (S_1)]$
The collision is elastic if $KE_{system(before)} = KE_{system(after)}$
$KE_{(before)} = KE_{(S1)} + KE_{(S2)} = \frac{1}{2}mV_1^2 + 0 = \frac{1}{2} \times 0.08 \times 5^2 + 0 = 1$ J
$KE_{(after)} = KE_{(S1)} + KE_{(S2)} = 0 + \frac{1}{2}mV_2^2 = 0 + \frac{1}{2} \times 0.08 \times 5^2 = 1$ J
Therefore, the collision is elastic.
1.3 Apply the law of conservation of mechanical energy of the system [Oscillator- Earth]
$ME_{(R)\text{ is compressed by d}} = ME_{(R)\text{ is in its initial length}}$,
$(KE + GPE + EPE)_{(R)\text{ is compressed by d}} = (KE + GPE + EPE)_{(R)\text{ is in its initial length}}$
$0 + \frac{1}{2}kd^2 + 0 = \frac{1}{2}mV_1^2 + 0 + 0,$

$\frac{1}{2} \times 200 \times d^2 = \frac{1}{2} \times 0.08 \times 5^2$ then d = 0.1 m = 10 cm

2.1 The forces acting on (S_2) on OB are:

$m\vec{g}$: its weight,

$\vec{N}$: Normal reaction

$\vec{f}$: friction

2.2 $\Sigma\vec{F} = m\vec{g} + \vec{N} + \vec{f}$,

Component along the direction $\overrightarrow{Ox}$: $\Sigma\vec{F} = -mg\sin\alpha\,\vec{i} + 0\,\vec{i} - f\,\vec{i}$

$\Sigma\vec{F} = -(f + mg\sin\alpha)\vec{i}$

Or : $\Sigma\vec{F} = m\vec{g} + \vec{N} + \vec{f} = -mg\sin\alpha\,\vec{i} + mg\cos\alpha\,\vec{j} - N\,\vec{j} - f\,\vec{i}$

But : $mg\cos\alpha\,\vec{j} - N\,\vec{j} = 0$, then, $\Sigma\vec{F} = -(f + mg\sin\alpha)\vec{i}$

2.3 $\frac{d\vec{P}}{dt} = \Sigma\vec{F}$,

$-0.9\,\vec{i} = -(f + mg\sin\alpha)\vec{i}$

$-0.9 = -f - 0.08 \times 10 \times 0.5$

Therefore, f = 0.5 N

Exercise 12 (CRDP LS Samples LS 2-ex2)

1-1 ME(A) = ME(B)

$PE_g(A) + KE(A) = PE_g(B) + KE(B)$

$m_1gh_A + 0 = 0 + ½m_1(v_{1B})^2$

$$h_A = \frac{½(v_{1B})^2}{g}$$

$$h_A = \frac{½(4)^2}{10}$$

$h_A = 0.8$ m

1-2 Explanation:

$ME(O) - ME(B) = W(\vec{f})_{B\to O}$

$PE_g(O) + KE(O) - PE_g(B) - KE(B) = W(\vec{f})_{B\to O}$

$0 + ½m_1(v_{1O})^2 - 0 - ½m_1(v_{1B})^2 = W(\vec{f})_{B\to O}$

$W(\vec{f})_{B\to O} = ½ \times 0.2 \times (2)^2 - 0 - ½ \times 0.2 \times (4)^2$

$W(\vec{f}))_{B\to O} = -1.2$ J

1-3 $W(\vec{f})_{B\to O} = \vec{f}.\overrightarrow{BO} = -f \times BO$

$f = -\frac{W(\vec{f})_{B\to O}}{BO}$

$f = -\frac{-1.2}{1} = 1.2$ N

2-1 During the collision, the linear momentum of the system [(S_1),(S_2)] is conserved:

$\vec{p}_{before} = \vec{p}_{after}$

In algebraic values along the positive direction:

$m_1v_{1O} + 0 = m_1v'_{1O} + m_2v'_{2O}$

$m_1(v_{1O} - v'_{1O}) = m_2v'_{2O}$ (equation 1)

The collision being elastic, then the kinetic energy of the system is conserved:

$KE_{before} = KE_{after}$

$½m_1(v_{1O})^2 + 0 = ½m_1(v'_{1O})^2 + ½m_2(v'_{2O})^2$

$m_1(v_{1O} - v'_{1O})(v_{1O} + v'_{1O}) = m_2(v'_{2O})^2$ (equation 2)

Using both equations, (equation 2) and (equation 1), we get:

$v_{1O} + v'_{1O} = v'_{2O}$ (equation 3)

Using the equations, (equation 1) and (equation 3), we get :

$v'_{1O} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1O}$

Which gives: $v'_{1O} = -2/3 = -0.67$ m/s

then replace in (equation 3), we get: $v'_{2O} = 4/3 = 1.33$ m/s.

2-2 The mechanical energy of the system [(S_2), spring, Earth] is conserved.

ME(O) = ME(D)

$PE_g(O) + PE_e(O) + KE(O) = PE_g(D) + PE_e(D) + KE(D)$

$0 + 0 + \frac{1}{2}m_2(v'_{2O})^2 = 0 + \frac{1}{2}k(x_m)^2 + 0$

$m_2(v'_{2O})^2 = k(x_m)^2$

$x_m = (v'_{2O})\sqrt{\frac{m_2}{k}}$

$x_m = \frac{4}{3}\sqrt{\frac{0.4}{100}}$

x_m= OD = 0.084 m = 8.4 cm

2-3-1 The decrease in the mechanical energy of the system [(S_2), Earth, spring] is equal to:

$|\Delta ME| = \frac{1}{2}m_2(v'_{2O})^2 - \frac{1}{2}k(x'_m)^2 = \frac{1}{2} \times 0.4 \times (4/3)^2 - \frac{1}{2} \times 100 \times (0.064)^2 = 0.15$ J

2-3-2 This decrease appears in the form of thermal energy (heat).