وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام ٢٠٢٣ الاستثنائية الأربعاء في ٢٣ آب ٢٠٢٣
عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف	

[redacted]

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

N°	Questions	Proposed answers		
		a	**b**	**c**
1	For all real numbers $a > 0$, $\ln\left(\frac{e}{a}\right) + \ln(ae^2) =$	0	3	2
2	The solution set of the equation $\ln(x-1) + \ln(x+1) = 0$ is	$\{-\sqrt{2}\}$	$\{-\sqrt{2}, \sqrt{2}\}$	$\{\sqrt{2}\}$
3	$\lim\limits_{x\to+\infty} [\ln(1+2x) - \ln(1+x)] =$	$\ln 2$	2	0
4	The domain of definition of the function f given by $f(x) = \ln(1 - 2e^{-2x})$ is	$\left]-\infty, \frac{\ln 2}{2}\right]$	$[0, +\infty[$	$\left]\frac{\ln 2}{2}, +\infty\right[$

II- (6 points)

Part A

Consider two urns U and V.

- U contains two red balls holding each the number 0 and two green balls holding each the number 1.
- V contains three red balls holding each the number -1 and two green balls holding each the number 1.

A game consists of choosing randomly one of the two urns U and V and then selecting 2 balls simultaneously and randomly from the chosen urn.

Consider the following events:

U: "Urn U is chosen",

V: "Urn V is chosen",

S: "The two selected balls have the same color",

Z: "The sum of the numbers on the selected balls is zero".

1) Calculate the following probabilities: $P(S/U)$ and $P(S/V)$. Deduce that $P(S) = \frac{11}{30}$.
2) The two selected balls do not have the same color. Show that the probability that they are selected from urn U is $\frac{10}{19}$.
3) Calculate $P(Z)$.
4) Show that $P(S \cup Z) = \frac{2}{3}$.

Part B

All the balls from the two urns U and V are placed in one urn W.

Three balls are selected randomly and successively without replacement from W.

1) What is the number of possible selections of the three balls?
2) Calculate the probability that the product of the numbers on the three selected balls is zero.

III- (10 points)

Part A

Consider the function g defined on $\mathbb{R}$ as $g(x) = 1 + (x-1)e^{-x}$.

The table below is the table of variations of g.

x	$-\infty$		2		$+\infty$
$g'(x)$		$+$	0	$-$	
$g(x)$	$-\infty$	$\nearrow$	$1+e^{-2}$	$\searrow$	1

1) Calculate $g(0)$.
2) Show that for all $x \leq 0$, $g(x) \leq 0$ and for all $x \geq 0$, $g(x) \geq 0$.

Part B

Consider the function f defined on $\mathbb{R}$ as $f(x) = x(1 - e^{-x})$ and denote by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

Let (d) be the line with equation $y = x$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-1.5)$.
2) a) Determine $\lim\limits_{x \to +\infty} f(x)$.
   b) Show that the line (d) is an asymptote to (C) at $+\infty$.
   c) Study, according to the values of x, the position of (C) with respect to (d).
3) a) Show that $f'(x) = g(x)$.
   b) Set up the table of variations of f.
4) Show that (C) has an inflection point I whose coordinates are to be determined.
5) Draw (d) and (C).

Part C

Consider the function h defined over $[0 ; +\infty[$ as $h(x) = xe^{-x}$.

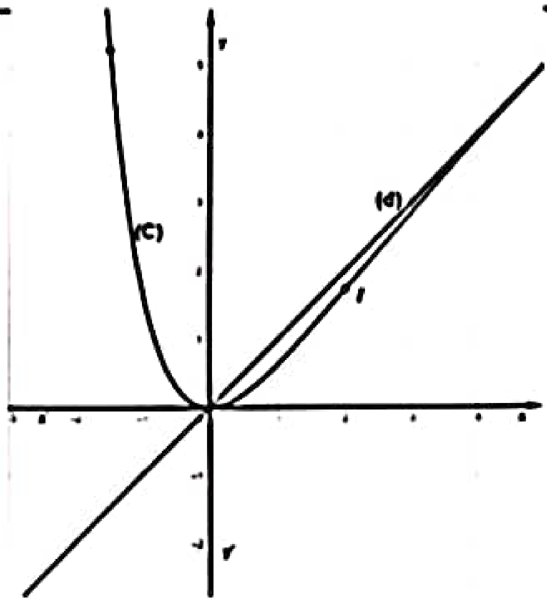
1) Set up the table of variations of h.
2) Let $M(x_M, f(x_M))$ and $N(f(x_M), x_M)$ are two variables points where $x_M > 0$.
   Determine the maximum length of segment [MN] as well as the corresponding position of M

أسس التصحيح

I	Answer key	6pts
1	$\ln e - \ln(a) + \ln e^2 + \ln(a) = 1 + 2\ln e = 3$ b	1.5
2	Conditions: $x - 1 > 0$ then $x > 1$ and $x + 1 > 0$ then $x > -1$ $\ln[(x-1)(x+1)] = 0$ then $x^2 - 1 = 1$ $x^2 = 2$ then $x = \sqrt{2}$ accepted or $x = -\sqrt{2}$ rejected c	1.5
3	$\lim\limits_{x\to+\infty} \ln\left(\frac{1+2x}{1+x}\right) = \lim\limits_{x\to+\infty} \ln\left(\frac{2x}{x}\right) = \ln(2)$ a	1.5
4	$1 - 2e^{-2x} > 0$ then $-2e^{-2x} > -1$ then $2e^{-2x} < 1$ $e^{-2x} < \frac{1}{2}$ then $-2x < -\ln 2$ then $x > \frac{\ln 2}{2}$ c	1.5

II	Answer key	9 pts
A1	$P(S/U) = \frac{C_2^2 + C_2^2}{C_4^2} = \frac{1}{3}$; $P(S/V) = \frac{C_3^2 + C_2^2}{C_5^2} = \frac{2}{5}$ $P(S) = P(S \cap U) + P(S \cap V) = \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{5} = \frac{11}{30}$	2
A2	$P(U/\bar{S}) = \frac{P(U \cap \bar{S})}{P(\bar{S})} = \frac{P(U) - P(U \cap S)}{1 - P(S)} = \frac{10}{19}$	1.5
A3	$P(Z) = \frac{1}{2} \times \frac{C_2^2}{C_4^2} + \frac{1}{2} \times \frac{C_2^1 \times C_3^1}{C_5^2} = \frac{23}{60}$	1.5
A4	$P(S \cup Z) = P(S) + P(Z) - P(S \cap Z) = \frac{11}{30} + \frac{23}{60} - \frac{1}{2} \times \frac{C_2^2}{C_4^2} = \frac{2}{3}$	1.5
B1	$A_9^3 = 504$	1
B2	<u>First method</u> P(product is 0) = 1 − P(product different from 0) = $1 - \frac{A_7^3}{A_9^3} = \frac{7}{12}$ <u>Second method</u> $P(\text{product is } 0) = \frac{A_2^1 \times A_7^2 + A_2^2 \times A_7^1}{A_9^3} \times \frac{3!}{2!} = \frac{7}{12}$	1.5

III	Answer key	15pts
A1	$g(0) = 0$	1
A2	<u>First method</u> If $x \in]-\infty, 0]$ then $g(x) \in]-\infty, 0]$ therefore $g(x) \le 0$ If $x \in [0, 2] \cup [2, +\infty[$ then $g(x) \in [0, 1 + e^{-2}] \cup]1, 1 + e^{-2}] = [0, 1 + e^{-2}]$ therefore $g(x) \ge 0$ <u>Second method</u> Over $]-\infty, 0]$, g is continuous and increasing from $-\infty$ to 0 then $g(x) \le 0$ Over $[0, +\infty[$, g is continuous and increasing from 0 to $1 + e^{-2} > 0$ then decreasing to $1 > 0$ thus $g(x) \ge 0$.	1.5

<table>
<tr><td>B1</td><td>$\lim_{x\to-\infty} f(x) = -\infty(1-e^{+\infty}) = +\infty$, f(-1.5) = 5.2</td><td>1</td></tr>
<tr><td>B2a</td><td>$\lim_{x\to+\infty} f(x) = +\infty(1-e^{-\infty}) = +\infty$</td><td>1</td></tr>
<tr><td>B2b</td><td>$\lim_{x\to+\infty}[f(x)-x] = \lim_{x\to+\infty} -x\,e^{-x} = \lim_{x\to+\infty}\frac{-x}{e^x} = \lim_{x\to+\infty}\frac{-1}{e^x} = 0$
Then (d): $y = x$ is an oblique asymptote to (C).</td><td>0.5</td></tr>
<tr><td>B2c</td><td>$f(x) - y_d = -xe^{-x}$
(C) is above (d) for all $x < 0$; (C) is below (d) for all $x > 0$; (C) intersects (d) at (0, 0)</td><td>1</td></tr>
<tr><td>B3a</td><td>$f'(x) = 1 - e^{-x} + xe^{-x} = 1 + (x-1)e^{-x} = g(x)$.
Then $f'(x)$ and g(x) have the same sign.</td><td>1</td></tr>
<tr><td>B3b</td><td><table><tr><td>x</td><td>$-\infty$</td><td></td><td>0</td><td></td><td>$+\infty$</td></tr><tr><td>$f'(x)$</td><td></td><td>$-$</td><td>0</td><td>$+$</td><td></td></tr><tr><td>$f(x)$</td><td>$+\infty$</td><td>↘</td><td>0</td><td>↗</td><td>$+\infty$</td></tr></table></td><td>1.5</td></tr>
<tr><td>B4</td><td>$f''(x) = g'(x)$
$f''(x)$ vanishes at $x = 2$ while changing its sign from positive to negative then (C) admits an inflection point $I(2, 2-2e^{-2})$.</td><td>1.5</td></tr>
<tr><td>B5</td><td></td><td>2</td></tr>
<tr><td>C1</td><td>$\lim_{x\to+\infty} h(x) = 0$; $h(0) = 0$
$h'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x}$
<table><tr><td>x</td><td>0</td><td></td><td>1</td><td></td><td>$+\infty$</td></tr><tr><td>$h'(x)$</td><td></td><td>$+$</td><td>0</td><td>$-$</td><td></td></tr><tr><td>$h(x)$</td><td>0</td><td>↗</td><td>e^{-1}</td><td>↘</td><td>0</td></tr></table></td><td>2</td></tr>
<tr><td>C2</td><td>$MN^2 = (x - xe^{-x} - x)^2 + (x - x + xe^{-x})^2 = 2x^2e^{-2x}$ then $MN = h(x)\sqrt{2}$
The length is maximum when h is maximum, from C1 that is $x = 1$
The maximum length is: $MN = \sqrt{2}\,e^{-1}$ then $M(1, 1-e^{-1})$</td><td>1</td></tr>
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