Exercise 1 Variation of Mechanical Energy

Consider an inclined plane that makes an angle $\alpha = 30^0$ with the horizontal plane. An object (S), supposed as a particle, of mass $m = 0.5$ kg is launched from the bottom O of the inclined plane, at the instant $t_0 = 0$, with a velocity $\vec{V}_0 = V_0\vec{i}$ along the line of the greatest slope (OB). Let A be a point of OB such that $OA = 5m$ (fig. 1). The position of (S), at the instant t, is given by $\overrightarrow{OM} = x\vec{i}$ where $x = f(t)$. The variation of the mechanical energy of the system [(S), Earth], as a function of x, is represented by the graph of figure 2.

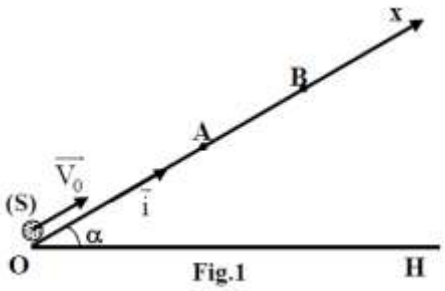

Fig.1

Take:

- The horizontal plane passing through OH as a gravitational potential energy reference;
- $g = 10ms^{-2}$.

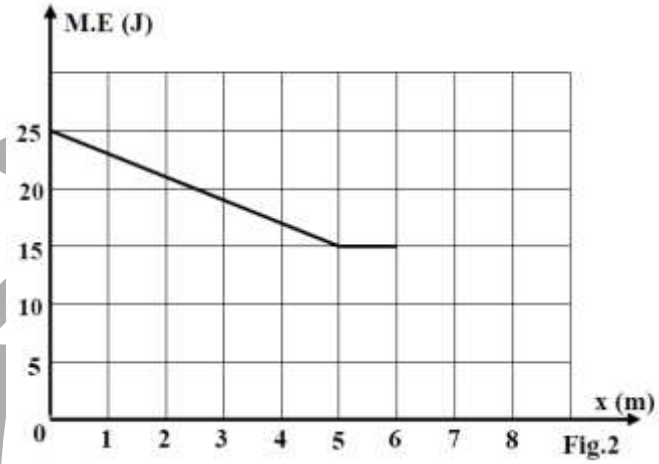

Fig.2

1) Using the graph of figure 2:

1-1) Show that (S) is submitted to a force of friction between the points of abscissas $x_0 = 0$ and $x_A = 5m$.

1-2) 1-2-1) Calculate the variation of the mechanical energy of the system [(S), Earth] between the instants of the passage of (S) through the points O and A;

1-2-2) Deduce the magnitude of the force of friction, supposed constant, between O and A.

1-3) Show, for $0 \leq x \leq 5m$, that the expression of the mechanical energy M.E of the system [(S), Earth] as function of x is M.E = – 2x + 25. (ME in J; x in m)

2) Determine, for $0 \leq x \leq 5m$, the gravitational potential energy $P.E_g$ of the system [(S), Earth] as function of x.

3) Determine, for $0 \leq x \leq 5m$, the kinetic energy KE of the system (S) as function of x.

4) Trace, on the same system of axes, the curves representing the variations of the energies M.E, KE and $P.E_g$ as a function of x for $0 \leq x \leq 5m$.

Scale: - on the axis of abscissas: 1 cm represents 1 m;
- on the axis of energy: 1 cm represents 5 J.

5) Let v be the speed of (S) when it passes through the point M of abscissa x so that $0 \leq x \leq 5m$.

5-1) Determine the relation between v and x.

5-2) Deduce that the algebraic value of the acceleration of (S) is $a = -9ms^{-2}$.

6) Determine the values of the speed of (S) at O and at A.

7) Specify if (S) is subjected to friction force between 5 m and 6 m.

8) Determine the speed of (S) at the point of abscissa x = 6 m.