Haret Hreik official high school For boys

Class: Gr12-LS (B)

Name:



Scholar year: 2021 – 2022 **Subject: Mathematics** Date: 6-04-2022

Extra sheet 4

Ex1

Solve in IR

1)
$$e^{-2x+3} = e^{x+6}$$

2)
$$e^{x+1} = 1$$

3)
$$-2e^x + 5 = 1$$

1)
$$e^{-2x+3} = e^{x+6}$$
 2) $e^{x+1} = 1$ 3) $-2e^x + 5 = 1$ 4) $(2x-1)e^{x-1} = 0$

5)
$$3e^{2x} - e^x = 0$$

$$6) e^{2x} + 4e^x + 3 = 0$$

5)
$$3e^{2x} - e^x = 0$$
 6) $e^{2x} + 4e^x + 3 = 0$ **7**) $\ln(e^x + 2) = 2\ln 2$ **8**) $\frac{2}{e^{x+1}} = e^x$

8)
$$\frac{2}{e^{x}+1} = e^{2}$$

Ex2

Calculate the derivative of each of the following functions

1)
$$f(x) = 2x - 1 + 2xe^x$$

2)
$$f(x) = (2x + 1)e^x$$

3)
$$f(x) = \frac{2e^x}{e^x + 1}$$

4)
$$f(x) = \frac{x - e^x}{1 - e^x}$$

Ex3

Calculate the following limits

1)
$$\lim_{x \to +\infty} (x - 2 - 3e^x)$$
 2) $\lim_{x \to +\infty} \left(\frac{e^x - 1}{e^x + 1}\right)$ 3) $\lim_{x \to -\infty} \left(\frac{e^x + 1}{x}\right)$

2)
$$\lim_{x \to +\infty} \left(\frac{e^{x}-1}{e^{x}+1} \right)$$

3)
$$\lim_{x\to-\infty} \left(\frac{e^{x+1}}{x}\right)$$

4)
$$\lim_{x\to 0^+} \left(\frac{e^x}{e^x-1}\right)$$

5)
$$\lim_{x \to -\infty} (x+1)e^{-x}$$

5)
$$\lim_{x \to -\infty} (x+1)e^{-x}$$
 6) $\lim_{x \to +\infty} (2x+1)e^{-x}$

Ex4

Consider the function f defined over IR by $f(x) = (x - 1)e^x$ and designate by (C) its representative curve in an orthonormal system $(0; \vec{1}, \vec{1})$.

- 1) Calculate $\lim_{x \to \infty} f(x)$ and $\lim_{x \to \infty} f(x)$. Deduce an asymptote to (C).
- 2) Determine the points of intersection of (C) with the axes of coordinates.
- 3) Calculate f'(x) and set up the table of variations of f.
- **4**) Draw (C).

Ex5

3)

Consider the function f defined over IR by $f(x) = x + 1 + e^x$ and designate by (C) its representative curve in an orthonormal system $(0; \vec{1}, \vec{j})$.

- 1) Calculate $\lim_{x \to a} f(x)$.
- Determine the coordinates of the point of intersection of (C) and the y-axis. 2)
- a) Calculate $\lim_{x \to -\infty} f(x)$ and deduce that (d): y = x + 1 is an asymptote to (C) at $-\infty$.
 - **b)** Show that (d) is below (C).
- 4) Calculate f'(x) and set up the table of variations of f.
- 5) Write an equation of the tangent (T) to (C) at its point of abscissa 0.
- 6) Verify that f(x) = 0 admits over IR a unique root α and that $-1.3 < \alpha < -1.2$
- **7**) Draw (T) and (C).

Ex6

The plane is referred to an orthonormal system $(0; \vec{i}, \vec{j})$. (Unit: 2 cm).

Part A

Let g be the function defined over \mathbb{R} by $g(x) = 1 + (-x + 1)e^{-x}$.

- 1) Set up the table of variation of g
- 2) Deduce that g(x) > 0 for every value of x.

Part B

Consider the function f defined over \mathbb{R} by: $f(x) = x + 1 + xe^{-x}$.

Designate by (C) the representative curve of f

1) Calculate $\lim_{x \to -\infty} f(x)$ and f(-1).

2)

- a) Calculate $\lim_{x\to +\infty} f(x)$
- **b**) Show that the straight line (D) of equation y = x + 1 is an asymptote to (C) at $+\infty$.
- c) Study according to the values of x the relative position of (D) and (C).
- 3) Verify that f'(x) = g(x), and set up the table of variation of f.
- 4) Show that the tangent (T) to (C) at A(1; 2 + $\frac{1}{e}$) is parallel to (D).
- 5) Show that the equation f(x) = 0 admits over IR a unique solution α and then verify that $-0.5 < \alpha < -0.4$
- **6**) Draw (D), (T) and (C).

Ex7

The plane is referred to an orthonormal system $(0; \vec{i}, \vec{j})$.

Part-A

Let h be the function defined over IR by $h(x) = (2 - x)e^{x} - 2$. Designate by (H) its representative curve.

- 1) Calculate $\lim_{x \to -\infty} h(x)$ and $\lim_{x \to +\infty} h(x)$, deduce an asymptote to (H).
- 2) Calculate h'(x) and set up the table of variations of h.
- 3) The equation h(x) = 0 admits over IR two roots 0 and α , show that 1.5 < α < 1.6. Deduce the sign of h(x).

Part-B

Consider the function f defined over IR by $f(x) = \frac{e^x - 2}{e^x - 2x}$ and designate by (C) its representative curve.

- 1) Calculate $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to +\infty} f(x)$, deduce the asymptotes to (C).
- 2) Determine the coordinates of the points of intersection of (C) and its asymptotes.
- 3) Show that $f(\alpha) = \frac{1}{\alpha 1}$.

4)

- a) Show that $f'(x) = \frac{2h(x)}{(e^x 2x)^2}$ and deduce the sign of f'.
- **b)** Set up the table of variations of f.(**Take** $\alpha = 1.6$)
- 5) Trace (C) and its asymptotes.