In His Name		
Rihab Zahraa High School Secondary Education Grade 12 GS	**Final Math exam**	Name: ________________ Date: 07 / May / 2024 Duration: 3 hours Score : 40 points

I-(6 pts) Choose the correct answer with justification, by showing all the steps of calculation.

	Answers / Questions	A	B	C
1)	f(x) = $\begin{cases} x^2(\ln(x)-1)+2x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$	f is continuous at 0	f isn't continuous at 0	f isn't differentiable at 0
2)	Let f(x) be an odd, continuous function over ℝ and such that $\int_{-2}^{5} f(x)dx$ = 6, then $\int_{-5}^{-2} f(x)dx$ =	$\int_{-5}^{-2} f(x)dx$ = 0	$\int_{-5}^{-2} f(x)dx$ = -6	$\int_{-5}^{-2} f(x)dx$ = 6
3)	A and B are 2 events of univers Ω such that P(B) = 0.15 and P(A∪ B) $= 0.25$ *then* $P(\bar{A}/\bar{B})$=	$\frac{15}{17}$	$\frac{15}{18}$	$\frac{15}{19}$
4)	For all x > 0, $\lim\limits_{x\to 1} \frac{\int_1^{\sqrt{x}} 2e^{t^2}dt}{x-1}$ =	e	e^2	1
5)	If z = -2$\left(\sin\left(\frac{\pi}{3}\right) + i\cos\left(\frac{\pi}{3}\right)\right)$. Then arg($\bar{z}$) =	$-\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$
6)	The similitude S that transforms S(A) = B and S(C) = D with Z_A = 1 – i ; Z_B = 2 – i ; Z_C = i ; Z_D = - 1. has as ratio k and angle θ:	K = $\sqrt{2}$ $\theta = -\frac{\pi}{4}$	K = $\sqrt{2}$ $\theta = \frac{\pi}{4}$	K = 2 $\theta = -\frac{\pi}{4}$

II-(9 pts) the parts A and B are independent.

Consider in the complex plane of direct orthonormal system $(O\,;\,\vec{U},\vec{V})$ the points B , C & D with respective affixes 1 +3i , 8 - 4i and 2+2i.

Part A :

1. Calculate $\frac{z_B - z_D}{z_B - z_C}$. Deduce that B, C and D are collinear.
2. Calculate $\frac{z_D}{z_B - z_C}$.
3. Deduce that D is the orthogonal projection of O to (BC).

Part B : Consider in the complex plane of direct orthonormal system $(O\,;\,\vec{U},\vec{V})$ the points A, B & C with respective affixes i ,-2i and – i. for every M of affix Z we associate a point M' of affix Z' such that $Z' = \frac{-2iz}{z-i}$. Z ≠ i.

1. In this part only, we suppose that $z' = 1+i$, write Z in algebraic form.

2. a) Show that $(z' + 2i)(Z - i) = 2$

b) Deduce that, if M moves on a circle of center A and radius 1 then M' moves on a circle to be determined.

3. a) Show that $z' + i = \dfrac{-i(z+i)}{z-i}$.

b) Show that $(\vec{u}, \overrightarrow{CM'}) = (\overrightarrow{MA}, \overrightarrow{MC}) - \frac{\pi}{2}\ [2\pi]$

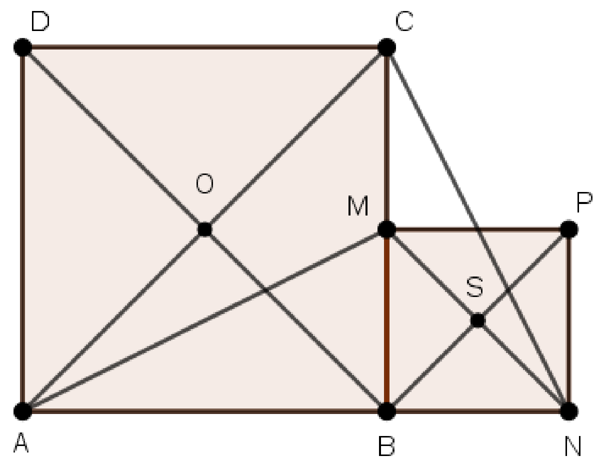
4. Let $z = x + iy\ ;\ z' = x' + iy'$.

a) Verify that $x' = \dfrac{2x}{x^2 + (y-1)^2}$ and $y' = \dfrac{-2x^2 - 2y^2 + 2y}{x^2 + (y-1)^2}$.

b) show that if M describe on the axis (y'y) except the point O then M' moves on a line to be determined.

<u>III-(8 pts)</u> Given:

- ABCD is a direct square of center O with AB = 4 and $(\overrightarrow{AB}\ ;\ \overrightarrow{AD}) = \frac{\pi}{2}\ [2\pi]$
- BNPM is a direct square of center S with BN = 2 and $(\overrightarrow{BN}\ ;\ \overrightarrow{BM}) = \frac{\pi}{2}\ [2\pi]$
- M is the midpoint of [BC].



A- Let R be the rotation of center B and angle $\frac{\pi}{2}$.

1) Determine: R(P).

2) Prove that: AM = CN and (AM) is perpendicular to (CN).

3) The lines (AM) and (BP) intersect at L, the lines (CN) and (BD) intersect at K.

a- Find the image of line (BD).

b- Prove that: R(K) = L.

4) Let E be the symmetric of C with respect to B.
Denote by F= R(M).
Prove that F is the orthocenter of triangle AME.

5) Let **t** be the translation of vector $\frac{1}{2}\overrightarrow{AB}$.

a- Determine $t\ oR(N)$ and $t\ oR(P)$. Determine the nature of $t\ oR$.

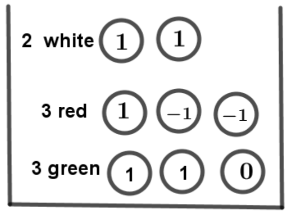
b- Prove that S is the center of $t\ oR$.

B- The plane is referred to a direct orthonormal system $(A, \vec{u}, \vec{v})$ with $\vec{U} = \frac{1}{4}\overrightarrow{AB}$ and $\vec{V} = \frac{1}{4}\overrightarrow{AD}$.

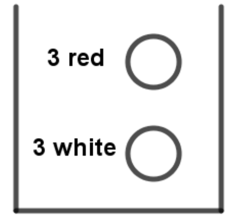
1) Write the complex form of R and $t\ oR$.

2) (C) is the circle of equation: $(x-2)^2 + (y+1)^2 = 9$. Denote by $(C') = R((C))$

a- Write an equation of (C').



urn U



urn V

IV-(5 pts)

An urn U contains 8 balls of which

- Two white balls numbered 1 and 1.
- Three red balls numbered 1, – 1 and – 1.
- Three green balls numbered 1, 1 and 0

An urn V contains six balls of which three red and three white.

A game starts as one ball is randomly selected from urn U.

- If the selected ball carries number 0 or 1, then two balls are selected randomly and successively without replacement from urn V.
- If the selected ball carries number – 1, we put it in the urn V after which three balls are selected randomly and simultaneously from urn V.

Consider two events :

M : the selected ball carries number 0 or 1.

N : At least one red ball is selected from V.

1) Show that $P(N/M) = \frac{4}{5}$ and deduce P(N∩M).
2) Calculate P(N ∩ $\bar{M}$).
3) Show that P(N) = $\frac{59}{70}$.
4) Given the event E : there is a no white ball left in the urn V. Calculate P(E).
5) Knowing that the selected ball from U carries number 1. What is the probability the selected balls from V are different color ?

V-(12 pts)

A-

Let h be the function defined on IR as h(x) = e^x – x - 1. Denote by (C) its representative curve in an orthonormal system.

1) a- Determine $\lim\limits_{x \to +\infty} f(x)$.

b- Determine $\lim\limits_{x \to -\infty} f(x)$ and show that the line (d) of equation y = – x – 1 is an asymptote to (C).

2) a- Calculate $h'(x)$ and set up the table of variations of h.

b- Draw (C) and (d).

c- Deduce that $e^x \geq x$ + 1for all $x \in$ IR.

3) a) Calculate the area of the domain $\mathfrak{D}$ bounded between (C), x-axis, y-axis and x = 2.

b) Deduce area of the domain limited by (C), y-axis and y = $e^2 - 3$.

B- Let f be the function defined as f(x) = $\frac{e^x}{e^x - x}$. Denote by (C') its representative curve in another orthonormal system.

1) Show that f is defined over IR.

2) Determine the asymptotes to (C').

3) Verify that $f'(x) = \frac{(1-x)e^x}{(e^x-x)^2}$ and set up the table of variations of f.

4) **a-** Write an equation of (T), the tangent to (C') at the point E with abscissa 0.

b- Verify that $f(x) - x - 1 = \frac{x(x+1-e^x)}{e^x - x}$

c- Study, according to the values of x, the relative positions of (C'). with respect to (T).

d- Draw (C') and (T).

C - For all natural numbers n, define the sequence (U_n) as $U_n = \int_0^n f(x)dx$

1) Show that the sequence (U_n) is increasing.
2) **a-** For $x \geq 0$, verify that $f(x) \geq 1$.

b- Is the sequence (U_n) convergent? Justify.

Show your best.

Mahmoud Chaalan.