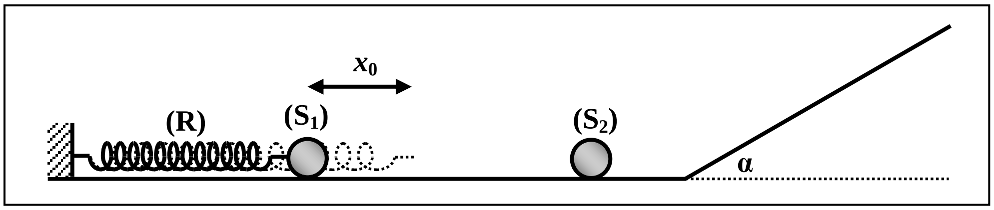
Shakib Irslan High School *Physics Department*	**PHYSICS EXTRA SHEET 4** **LINEAR MOMENTUM**	**Academic Year:** 2023-2024 **Date:** 11-12-2023
Name: ………………………		**Class and Section:** 12 LS & GS

Exercise 1:

A solid (S_1), taken as a particle of mass m_1 = 0.5kg, is connected to the free end of a spring (R) of negligible mass and stiffness k = 200N/m. The other end of (R) is fixed to a support. (R) and (S_1) are placed on the horizontal part of a frictionless track. The inclined part of the track makes an angle α = 30° with the horizontal. The horizontal plane containing the horizontal part of the track is taken as a gravitational potential energy reference. Take g = 10m/s^2.

(S_1) is shifted from its equilibrium position by x_0 = 20cm and then released from rest at the instant t_0 = 0s.



1- Determine the speed V_1 of (S_1) when it leaves the spring.

2- (S_1) enters in a perfectly elastic head-on collision with a stationary particle (S_2) of mass m_2 = 1.5kg.

2.1- Determine, just after collision, the algebraic value V_1' and V_2' of the velocities of (S_1) and (S_2) respectively.

2.2- After collision, (S_2) moves up the inclined rail. Determine the maximum distance covered by (S_2).

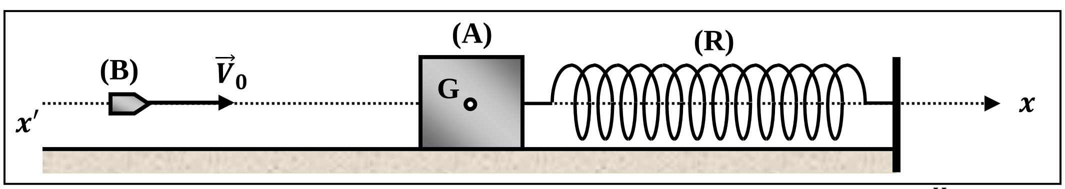
2.3- (S_1) compresses the spring by a distance x_m where it stops. Determine x_m.

Exercise 2:

A bullet (B), of mass m = 20g, moving with a horizontal velocity $\vec{V}_0 = V_0\vec{i}$, enters in a head-on collision with a solid (A), of center of inertia G and mass M = 5.6kg, and rests on a smooth horizontal plane. (A) is connected to the free end of an un-stretched spring (R) of negligible mass and stiffness k = 1000N/m. The other end of (A) is fixed to a vertical support as shown in the document below.

Just after collision, (B) and (A) from one body (S) and acquire a velocity $\vec{V} = V\vec{i}$.

The horizontal plane passing G is taken as a gravitational potential energy reference.



1- Show that the expression of the algebraic velocity of (S) just after collision is $V = \frac{mV_0}{m+M}$.

2- Determine, by applying the principle of conservation of mechanical energy, the value of V knowing that the spring undergoes a maximum compression of $x_m = 15cm$.

3- Deduce the value of V_0.

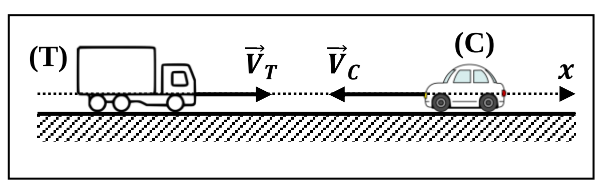
4-

4.1- Show that the collision between (B) and (A) is not elastic.

4.2- In what form does the kinetic energy lost appear?

Exercise 3:

A truck (T), of mass $m_T = 3000kg$, moves with a speed of 54km/h on a horizontal and rectilinear road. A car (C), of mass $m_C = 1200kg$, moves with a speed of 72km/h in the opposite direction. (T) and (C) enter in a head-on collision. (C) rebounds with a speed of 36km/h.



1- Determine the speed of the truck after collision.

2- Is the collision elastic? Deduce the energy lost during collision.

3- The collision lasts for 10ms. Calculate the average force exerted by the truck on the car.

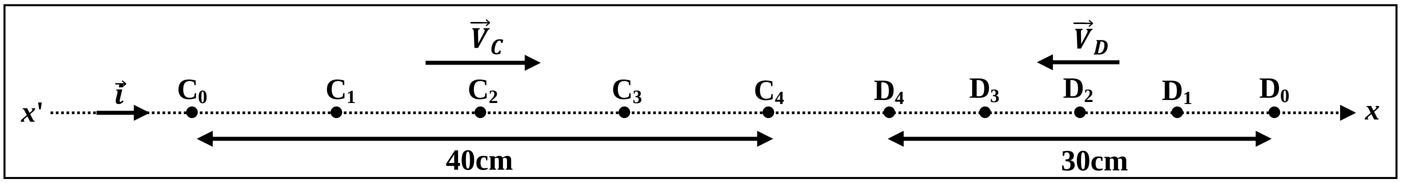
Exercise 4:

Consider a horizontal air table and two pucks (C) and (D) of respective masses m_C = 100g and m_D = 300g.

Puck (C), moving with a velocity $\vec{V}_C$, enters in a head-on collision with puck (D) moving in the opposite direction with a velocity $\vec{V}_D$.

The positions of the center of mass of each puck are registered at successive instants separated by a constant time interval τ = 50ms.

The document below represents, on the axis $x'x$, the dot-prints of the positions of the centers of masses of the two pucks before collision.



1- Show, referring to the above document, that the motion of the two pucks is uniform rectilinear.

2- Calculate, before collision, the magnitudes V_C and V_D of the velocities of (C) and (D) respectively.

3- After collision, (C) rebounds with a velocity $\vec{V}'_C = -3.25\vec{i}\ (m/s)$. Determine the velocity $\vec{V}'_D$ of (D) after collision.

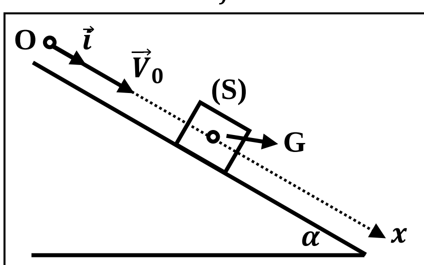
4- Specify the nature of collision between (C) and (D).

Exercise 5:

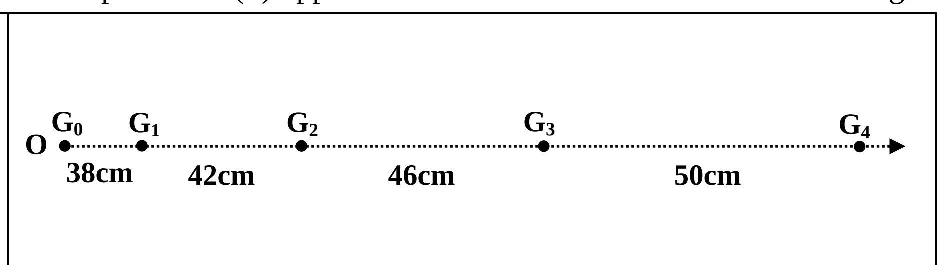
A solid (S), of center of inertia G and mass m = 1.25kg, is launched at the instant t_0 = 0s with an initial velocity vector $\vec{V}_0 = V_0\vec{i}$ from the top O of a rough inclined plane that makes an angle $\alpha = 30°$ with the horizontal. Thus, G moves along an axis $x'Ox$ parallel to the inclined plane as shown in document 1.

Document 2 represents the registrations of G during a constant time interval τ = 100ms.

The force of friction $\vec{f}$ between the inclined plane and (S) opposes its motion and assumed constant of magnitude f.



Doc.1



Doc.2

1- Complete the table below:

Dot	G_0	G_1	G_2	G_3	G_4
Instant: t [ms]	0	τ	2τ	3τ	4τ
Position: $x = \overline{OG}$ [cm]	0		80		176
Speed: V [m/s]	V_0		4.4		5.2
Linear momentum: P [kgm/s]	P_0		5.5		6.5

2- Trace the graph that represents the variation of P as function of time t.
Scale: horizontal axis: 1div → 100ms; Vertical axis: 1div → 1kgm/s.

3- Find the equation of the obtained graph.

4- Deduce the values of P_0 and V_0.

5-

5.1- Name and represent the external forces acting on (S).

5.2- Show that the sum of external forces acting on (S) is $\sum \vec{F}_{ext} = (mg\sin\alpha - f)\vec{i}$.

5.3- Deduce the value of f.

Exercise 1:

Part	Answer key	Mark
1	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $M.E_i = M.E_f \Longrightarrow K.E_i + G.P.E_i + E.P.E_i = K.E_f + G.P.E_f + E.P.E_f.$ $0 + 0 + \frac{1}{2}kx_0^2 = \frac{1}{2}m_1V_1^2 + 0 + 0 \Longrightarrow V_1 = \sqrt{\frac{k}{m_1}}x_0 = \sqrt{\frac{200}{0.5}} \times 0.2 = 4m/s.$	
2.1	During collision, the system [(S_1); (S_2)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant.$ The linear momentum is conserved: $\vec{P}_i = \vec{P}_f \Longrightarrow m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'.$ The collision is head on; then, the above expression can be written in its algebraic form. $m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$ with $V_2 = 0.$ $\boldsymbol{m_1(V_1 - V_1') = m_2V_2' \ldots (1).}$ The collision is elastic; then, the kinetic energy is conserved: $K.E_i = K.E_f.$ $\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2 \Longrightarrow m_1\left(V_1^2 - V_1'^2\right) = m_2V_2'^2.$ $\boldsymbol{m_1(V_1 + V_1')(V_1 - V_1') = m_2V_2'^2 \ldots (2).}$ Divide (2) by (1): $\boldsymbol{V_1 + V_1' = V_2' \ldots (3).}$ Replace (3) in (1): $m_1(V_1 - V_1') = m_2(V_1 + V_1') \Longrightarrow V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1.$ $V_1' = \frac{0.5 - 1.5}{0.5 + 1.5} \times 4 = -2m/s$ (the minus sign indicates that (S_1) rebounds back). Using equation (3): $V_2' = V_1 + V_1' = 4 - 2 = 2m/s.$	
2.2	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $M.E_i = M.E_f \Longrightarrow K.E_i + G.P.E_i = K.E_f + G.P.E_f.$ $\frac{1}{2}m_2V_2'^2 + 0 = 0 + m_2gd\sin\alpha \Longrightarrow d = \frac{V_2'^2}{2g\sin\alpha} = \frac{4}{2\times10\times0.5} = 0.4m = 40cm.$	
2.3	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $M.E_i = M.E_f \Longrightarrow K.E_i + G.P.E_i + E.P.E_i = K.E_f + G.P.E_f + E.P.E_f.$ $\frac{1}{2}m_1V_1'^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_m^2 \Longrightarrow x_m = \sqrt{\frac{m_1}{k}}V_1' = \sqrt{\frac{0.5}{200}} \times 2 = 0.1m.$	

Exercise 2:

Part	Answer key	Mark
1	During collision, the system (S) = [(A); (B)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}_S}{dt} = \vec{0} \Longrightarrow \vec{P}_S = constant.$ Law of conservation of linear momentum: $\vec{P}_{bc} = \vec{P}_{ac} \Longrightarrow m\vec{V}_0 + \vec{0} = (m + M)\vec{V}.$ $mV_0 = (m + M)V \Longrightarrow V = \frac{mV_0}{m+M}.$	
2	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $M.E_i = M.E_f \Longrightarrow K.E_i + G.P.E_i + E.P.E_i = K.E_f + G.P.E_f + E.P.E_f.$ $\frac{1}{2}(m + M)V^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_m^2 \Longrightarrow V = \sqrt{\frac{k}{m+M}}x_m = \sqrt{\frac{1000}{5.6+0.02}} \times 0.15 = 2m/s.$	
3	$V_0 = \frac{m+M}{m}V = \frac{0.02+5.6}{0.02} \times 2 = 562m/s.$	
4.1	$K.E_{bc} = \frac{1}{2}mV_0^2 = \frac{1}{2}(0.02)(562)^2 = 3{,}158.44J.$ $K.E_{ac} = \frac{1}{2}(m + M)V^2 = \frac{1}{2}(5.62)(2)^2 = 11.24J.$ $K.E_b \neq K.E_a \Longrightarrow$ The kinetic energy of the system (S) is not conserved.	
4.2	Heat or thermal energy	

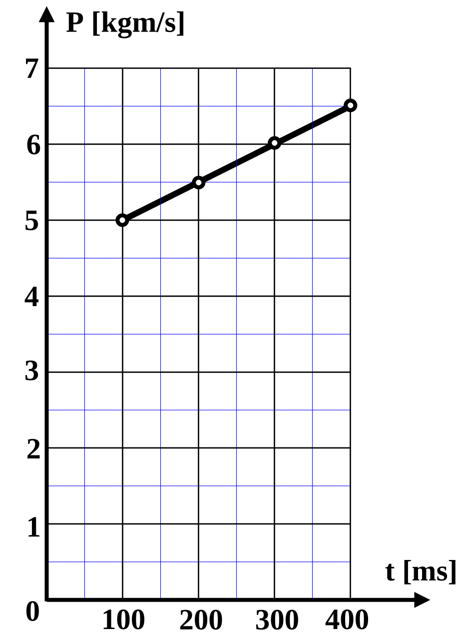
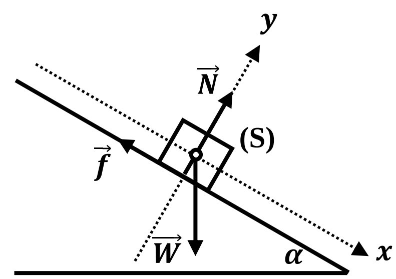
Exercise 3:

Part	Answer key	Mark
1	Before collision, the velocity of (T) is $V_T = 54km/h = 15m/s$ and that of (C) is $V_C = -72km/h = -20m/s \left(km/h \xrightarrow{\div 3.6} m/s\right)$. After collision, the velocity of (T) is V_T' and that of (C) is $V_C' = 36km/h = 10m/s$. During collision, the system [(T); (C)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}_s}{dt} = \vec{0} \Longrightarrow \vec{P}_S = constant$. Principle of conservation of linear momentum: $\vec{P}_b = \vec{P}_a$. $m_T\vec{V}_T + m_C\vec{V}_C = m_T\vec{V}_T' + m_C\vec{V}_C'$. The collision is head-on and the velocities are collinear: $m_TV_T + m_CV_C = m_TV_T' + m_CV_C'$. $(3000)(15) + (1200)(-20) = (3000)V_T' + (1200)(10) \Longrightarrow V_T' = 3m/s$.	
2	$K.E_b = \frac{1}{2}m_TV_T^2 + \frac{1}{2}m_CV_C^2 = \frac{1}{2}(3000)(15)^2 + \frac{1}{2}(1200)(20)^2 = 577{,}500J$. $K.E_a = \frac{1}{2}m_TV_T'^2 + \frac{1}{2}m_CV_C'^2 = \frac{1}{2}(3000)(3)^2 + \frac{1}{2}(1200)(10)^2 = 73{,}500J$. $K.E_b \neq K.E_a \Longrightarrow$ The kinetic energy of the system [(T); (C)] is not conserved; then, collision is not elastic. $Energy\ lost = \lvert\Delta K.E\rvert = 577{,}500J - 73{,}500J = 504{,}000J$.	
3	$\vec{F}_{T/C} = \frac{\Delta\vec{P}_c}{\Delta t} = \frac{\vec{P}_c' - \vec{P}_C}{\Delta t} = \frac{m_C\vec{V}_C' - m_C\vec{V}_C}{\Delta t} = \frac{m_C(\vec{V}_C' - \vec{V}_C)}{\Delta t} = \frac{(1200)(10\vec{\imath} + 20\vec{\imath})}{0.01} = 3{,}600{,}000\vec{\imath}\ [N]$.	

Exercise 4:

Part	Answer key	Mark
1	The dots are collinear and the distance separating two consecutive dots is constant.	
2	$V_C = \frac{C_0C_4}{4\tau} = \frac{40\times10^{-2}}{4\times50\times10^{-3}} = 2m/s$. $V_D = \frac{D_0D_4}{4\tau} = \frac{30\times10^{-2}}{4\times50\times10^{-3}} = 1.5m/s$.	
3	During collision, the system [(C); (D)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}_S}{dt} = \vec{0} \Longrightarrow \vec{P}_S = constant$. Principle of conservation of linear momentum: $\vec{P}_{S(bc)} = \vec{P}_{S(ac)}$. $m_C\vec{V}_C + m_D\vec{V}_D = m_C\vec{V}_C' + m_D\vec{V}_D'$. $(0.1)(2\vec{\imath}) + (0.3)(-1.5\vec{\imath}) = (0.1)(-3.25\vec{\imath}) + 0.3\vec{V}_D' \Longrightarrow \vec{V}_D' = -0.25\vec{\imath}\ (m/s)$.	
4	$K.E_{S(bc)} = \frac{1}{2}m_CV_C^2 + \frac{1}{2}m_DV_D^2 = \frac{1}{2}(0.1)(2)^2 + \frac{1}{2}(0.3)(1.5)^2 = 0.5375J$. $K.E_{S(ac)} = \frac{1}{2}m_CV_C'^2 + \frac{1}{2}m_DV_D'^2 = \frac{1}{2}(0.1)(3.25)^2 + \frac{1}{2}(0.3)(0.25)^2 = 0.5375J$. $K.E_{S(bc)} = K.E_{S(ac)} \Longrightarrow$ The kinetic energy of the system [(C); (D)] is conserved. Therefore, the collision is elastic.	

Exercise 5:

Part	Answer key	Mark
1	$x_1 = \overline{OG}_1 = 38cm.$ $x_3 = \overline{OG}_3 = 38cm + 42cm + 46cm = 126cm.$ $V_1 = \frac{x_2 - x_0}{2\tau} = \frac{(80-0)\times10^{-2}}{2\times100\times10^{-3}} = 4m/s.$ $V_3 = \frac{x_4 - x_2}{2\tau} = \frac{(176-80)\times10^{-2}}{2\times100\times10^{-3}} = 4.8m/s.$ $P_1 = mV_1 = 1.25 \times 4 = 5kgm/s.$ $P_3 = mV_3 = 1.25 \times 4.8 = 6kgm/s.$	
2		
3	The general equation of a straight line is: $P = kt + b.$ $k = \frac{\Delta P}{\Delta t} = \frac{6.5-5}{0.4-0.1} = 5kgm/s^2.$ $P = 5t + b.$ For $t = 100ms = 0.1s; P = 5kgm/s.$ $5 = 5 \times 0.1 + b \Longrightarrow b = 4.5kgm/s.$ Therefore, $P = 5t + 4.5.$	
4	$P_0 = b = 4.5kgm/s.$ $V_0 = \frac{P_0}{m} = \frac{4.5}{1.25} = 3.6m/s.$	
5.1	The external forces acting on (S): $\vec{W}$: Weight. $\vec{N}$: Normal reaction of a support. $\vec{f}$: Friction. 	
5.2	$\sum \vec{F}_{ext} = \vec{W} + \vec{N} + \vec{f} = \vec{W}_x + \vec{W}_y + \vec{N} + \vec{f} = mg\sin\alpha\,\vec{i} - f\vec{i} = (mg\sin\alpha - f)\vec{i}.$ $\vec{W}_y + \vec{N} = \vec{0}$ (no motion along the y-axis).	
5.2	$\vec{P} = P\vec{i} \Longrightarrow \frac{d\vec{P}}{dt} = \frac{dP}{dt}\vec{i}.$ $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{dP}{dt}\vec{i} \Longrightarrow (mg\sin\alpha - f)\vec{i} = \frac{dP}{dt}\vec{i}.$ $mg\sin\alpha - f = \frac{dP}{dt} \Longrightarrow 1.25 \times 10 \times \sin 30° - f = 5 \Longrightarrow f = 1.25N.$	