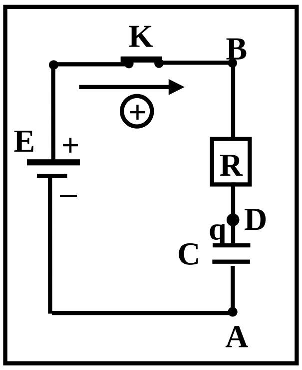
Charging of a capacitor:



Consider the adjacent circuit consist of:

- an ideal battery of electromotive force E;
- a resistor of resistance R;
- a capacitor of capacitance C, initially uncharged;
- a switch K.

Switch K is closed at $t_0 = 0$,

At an instant t, the capacitor is charged by q and the circuit carries a current i.

1) Name the phenomenon that takes place.

2) Redraw the circuit showing the direction of the current.

3) An oscilloscope displays the voltage u_{BD} across the resistor on channel Y_1, and the voltage $u_{DA} = u_C$ across the capacitor on channel Y_2, the « INV » button of channel Y_2 being pressed. Why did we push in the knob «Inv»?

4) Redraw the circuit showing the connections of the oscilloscope displaying the voltages u_C and u_R.

5) Write the expression of the current i in terms of q.

6) Deduce the expression of i in terms of the capacitance C and the voltage u_C.

7) Derive the differential equation that governs the variation of the voltage, $u_{DA} = u_C$, across the capacitor.

8) Verify that the solution of this differential equation is $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$.

Or The solution of the obtained differential equation is of the form: $u_C = E\,(1 - e^{\frac{-t}{\tau}})$, where τ is constant. Determine the expression of τ in terms of R and C.

Or The solution of this differential equation is: $u_C = D\left(1 - e^{-\frac{t}{\tau}}\right)$. Determine the expressions of the constants D and τ in terms of E, R and C.

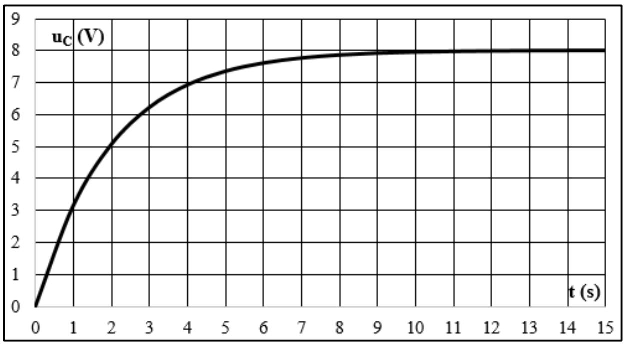
Or The solution of this differential equation has the form of: $u_C = A + B\,e^{Dt}$. Determine the constants A, B and D in terms of E, R and C.

9) Determine, at the instant t = 0, the expression of the voltage u_C in terms of E.

10) Determine, at the instant $t = \tau$, the expression of the voltage u_C in terms of E.

11) Verify that the capacitor becomes practically fully charged at t = 5 RC.

12) Referring to the graph of $u_C = f(t)$ of the adjacent document: Determine the value of τ.



13) Deduce the value of the value of the capacitance if the resistance R = 200 Ω.

14) determine $\frac{du_C}{dt}\Big|_{t=0}$

From the differential equation of u_C

Or from the solution $u_C = E(1-e^{-\frac{t}{\tau}})$.

Or Graphiclally

15) Determine the expression of the current i as a function of time t.

16) Determine, at the instant t = 0, the value of the current i.

or Determine the maximum value of the current I.

17) Determine, at the instant $t = \tau$, the expression of the current i in terms of E and R. deduce its value.

18) Deduce the value of the current i in steady state.

19) Sktetch i as function of t.

20) Determine $\frac{di}{dt}\Big|_{t=0}$

from the solution i

Or Graphiclally

21) Determine the expression of the voltage across the resistor u_R as a function of time t.

Or if $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$. Deduce the expression of the voltage across the resistor u_R as a function of time t.

22) Determine, at the instant t = 0, the expression of the voltage across the resistor u_R.

or Determine the maximum value of the current u_R.

23) Determine, at the instant $t = \tau$, the expression of u_R in terms of E.

24) Deduce the value of the u_R in steady state.

25) Sktetch u_R as function of t.

26) Determine $\frac{duR}{dt}\Big|_{t=0}$

from the solution u_R

Or Graphiclally

27) If $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$. Determine the expression of the charge q as a function of time t.

28) Determine, at the instant $t = \tau$, the expression of q in terms of E and C.

29) Deduce the value of the u_R in steady state.

or Determine the maximum value of q.

30) Sktetch q as function of t.

31) Determine $\frac{dq}{dt}\Big|_{t=0}$

from the solution q

Or Graphiclally

32) Derive the differential equation that governs the variation of the current i as function of time.

33) Derive the differential equation that governs the variation of the voltage u_R as function of time.

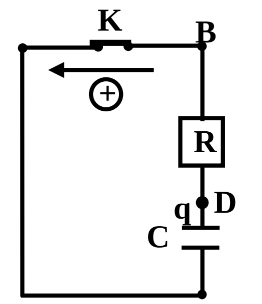
34) Derive the differential equation that governs the variation of the q as function of time.

35) A capacitor of capacitance 5000 μF is charged under a voltage of 12 V. Calculate the accumulated charge and the energy stored in this capacitor.

Discharging of a capacitor:

Consider the fully charged capacitor in the adjacent circuit, switch K is closed at new $t_0 = 0$,

At an instant t, the capacitor is charged by q and the circuit carries a current i.



Doc. 5

36) Name the phenomenon that takes place.

37) Write the expression of the current i in terms of q.

38) Deduce the expression of i in terms of the capacitance C and the voltage u_C.

39) Derive the differential equation that governs the variation of the voltage, $u_{DA} = u_C$, across the capacitor.

40) Verify that the solution of this differential equation is $u_C = Ee^{-\frac{t}{RC}}$.

Or The solution of the obtained differential equation is of the form: $u_C = E\, e^{-\frac{t}{\tau}}$,where τ is constant. Determine the expression of τ in terms of R and C.

41) Determine, at the instant t = 0, the expression of the voltage u_C in terms of E.

42) Determine, at the instant t = τ, the expression of the voltage u_C in terms of E.

43) Verify that the capacitor becomes practically fully discharged at t = 5 RC.

44) Sketch u_C

45) determine $\frac{du_C}{dt}\Big|_{t=0}$ Graphiclally

46) Determine the expression of the current i as a function of time t.

47) Determine, at the instant t = 0, the value of the current i.

or Determine the maximum value of the current I.

48) Determine, at the instant t = τ, the expression of the current i in terms of E and R.

49) Deduce the value of the current i in steady state.

50) Sktetch i as function of t.

51) Determine $\frac{di}{dt}\Big|_{t=0}$

from the solution i

Or Graphiclally

52) Determine the expression of the voltage across the resistor u_R as a function of time t.

Or if $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$. Deduce the expression of the voltage across the resistor u_R as a function of time t.

53) Determine, at the instant t = 0, the expression of the voltage across the resistor u_R.

or Determine the maximum value of the current u_R.

54) Determine, at the instant t = τ, the expression of u_R in terms of E.

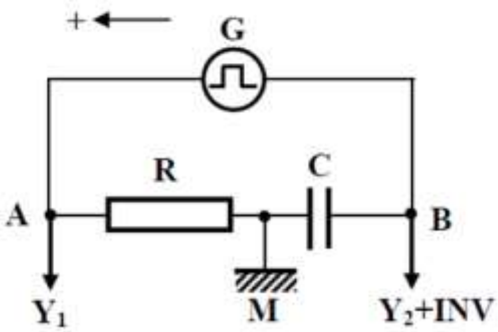
55) Deduce the value of the u_R in steady state.

56) Sktetch u_R as function of t.

57) Determine $\frac{du_R}{dt}\Big|_{t=0}$

from the solution u_R

Or Graphiclally



58) If $u_C = Ee^{-\frac{t}{RC}}$. Determine the expression of the charge q as a function of time t.

59) Determine, at the instant t = τ, the expression of q in terms of E and C.

60) Deduce the value of the u_R in steady state.

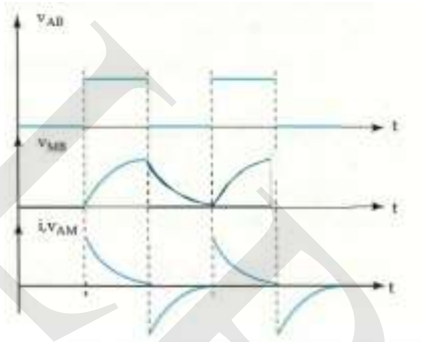
61) Sktetch q as function of t.

62) Determine $\frac{dq}{dt}\Big|_{t=0}$ Graphiclally

63) Derive the differential equation that governs the variation of the current i as function of time.

64) Derive the differential equation that governs the variation of the voltage u_R as function of time.

65) Derive the differential equation that governs the variation of the q as function of time.



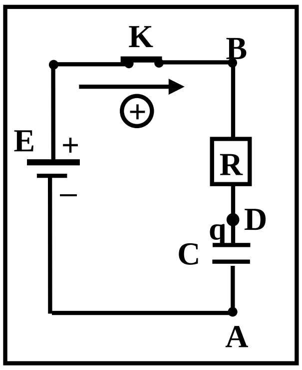
RC series circuit under square signal voltage

Consider the electric circuit, includes a resistor of resistance R, a capacitor of capacitance C, and a low frequency generator (LFG) delivering square signal voltage. Such that; Square signal: It is a periodic signal which takes a constant maximum value during one half-period, and is zero in the second half.

66) The waveform of uAM represents the "image" of the current i. Why?

67) During discharging the curve u_R or i is negative. Why?

Charging of a capacitor:



Consider the adjacent circuit consist of:

- an ideal battery of electromotive force E;
- a resistor of resistance R;
- a capacitor of capacitance C, initially uncharged;
- a switch K.

Switch K is closed at $t_0 = 0$,

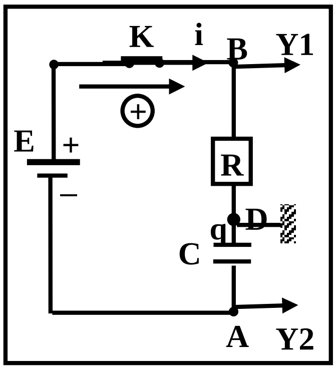
At an instant t, the capacitor is charged by q and the circuit carries a current i.

1) Name the phenomenon that takes place.

Ans: Charging of a capacitor

2) Redraw the circuit showing the direction of the current.

Ans: on the figure



3) An oscilloscope displays the voltage u_{BD} across the resistor on channel Y_1, and the voltage $u_{DA} = u_C$ across the capacitor on channel Y_2, the « INV » button of channel Y_2 being pressed. Why did we push in the knob «Inv»?

Ans: To display u_{DA} and not u_{AD}

4) Redraw the circuit showing the connections of the oscilloscope displaying the voltages u_C and u_R.

Ans: on the figure

5) Write the expression of the current i in terms of q.

Ans: $i = \dfrac{dq}{dt}$

6) Deduce the expression of i in terms of the capacitance C and the voltage u_C.

Ans: $q = Cu_C$ so $i = C\dfrac{du_C}{dt}$

7) Derive the differential equation that governs the variation of the voltage, $u_{DA} = u_C$, across the capacitor.

Ans: Apply law of addition of voltage $u_{BA} = u_{BD} + u_{DA}$, then $E = Ri + u_C$ where $i = \frac{dq}{dt} = C\frac{du_C}{dt}$

Then , $E = RC\frac{du_C}{dt} + u_C$

8) Verify that the solution of this differential equation is $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$.

Ans: $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$, then $\dfrac{du_C}{dt} = \dfrac{E}{RC}e^{\frac{-t}{RC}}$

$$RC\frac{du_C}{dt} + u_C = RC\frac{E}{RC}e^{\frac{-t}{RC}} + E\left(1 - e^{\frac{-t}{RC}}\right) = E\left(e^{\frac{-t}{RC}}\right) + E - E\left(e^{\frac{-t}{RC}}\right) = E \text{ verified}$$

Or The solution of the obtained differential equation is of the form: $u_C = E\,(1 - e^{\frac{-t}{\tau}})$, where τ is constant. Determine the expression of τ in terms of R and C.

Ans: $u_C = E\,(1 - e^{\frac{-t}{\tau}}) = E - E\,e^{-\frac{t}{\tau}}$, then $\dfrac{du_C}{dt} = \dfrac{E}{\tau}\, e^{\frac{-t}{\tau}}$

Replacing u_C and $\dfrac{du_C}{dt}$ by their expressions in the differential equation, gives:

$E = RC\,\dfrac{E}{\tau}\, e^{\frac{-t}{\tau}} + E - E\,e^{\frac{-t}{\tau}}$, then $E\,e^{\frac{-t}{\tau}}\,(\dfrac{RC}{\tau} - 1) = 0$

$E\,e^{\frac{-t}{\tau}} = 0$ is rejected Then, $(\dfrac{RC}{\tau} - 1) = 0$; therefore, $\tau = RC$

Or The solution of this differential equation is: $u_C = D\left(1-e^{-\frac{t}{\tau}}\right)$. Determine the expressions of the constants D and τ in terms of E, R and C.

Ans: $u_C = D\left(1-e^{-\frac{t}{\tau}}\right)$ then $\frac{du_C}{dt} = -D\left(-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}} = \frac{D}{\tau}e^{-\frac{t}{\tau}}$

Replace u_C and $\frac{du_C}{dt}$ by their expressions in the differential equation.

We get: $RC\frac{D}{\tau}e^{-\frac{t}{\tau}} + D - De^{-\frac{t}{\tau}} = E \quad \forall t$

$D(\frac{RC}{\tau}-1)e^{-\frac{t}{\tau}} + D - E = 0 \quad \forall t$

Comparing, we get: D-E=0 then D=E

$\left(\frac{RC}{\tau}-1\right) = 0$ then $\tau = RC$

Or The solution of this differential equation has the form of: $u_C = A + B\,e^{Dt}$. Determine the constants A, B and D in terms of E, R and C.

Ans: $u_C = A + B\,e^{D\,t}$, so $\frac{d\,u_C}{d\,t}$= BD e^{Dt}, substitute in the differential equation

BD e^{Dt}+$\frac{A + B\,e^{D\,t}}{R\;C}$ =$\frac{E}{R\,C}$, then RC B D e^{Dt} + A + Be^{Dt} = E

Be^{Dt} (RC D + 1) + A = E.

A = E ; and Be^{Dt} (RC D + 1) = 0. But Be^{Dt}= 0 is rejected,

then (R C D + 1) = 0 , thus D = - $\frac{1}{R\,C}$.

At t_o = 0 , u_C = 0= A + B $e^{D\,t}$, so 0 = A + B , then B = - A, therefore B = - E .

9) Determine, at the instant t = 0, the expression of the voltage u_C in terms of E.

Ans: At t = 0 ; $u_C = E\left(1-e^{0}\right) = E\left(1-1\right) = 0$

10) Determine, at the instant t = τ, the expression of the voltage u_C in terms of E.

Ans: At t = τ ; $u_C = E\left(1-e^{-\frac{\tau}{\tau}}\right) = E\left(1-e^{-1}\right) \approx 0{,}63E$

11) Verify that the capacitor becomes practically fully charged at t = 5 RC.

At t = 5 RC: u_C = E (1 – $e^{\frac{-5\,RC}{R\,C}}$) = E (1 - e^{-5}), then u_C = 0.99 E. Therefore, the capacitor becomes practically fully charged at t = 5 R C.

12) Referring to the graph of u_C = f(t) of the adjacent document: Determine the value of τ.

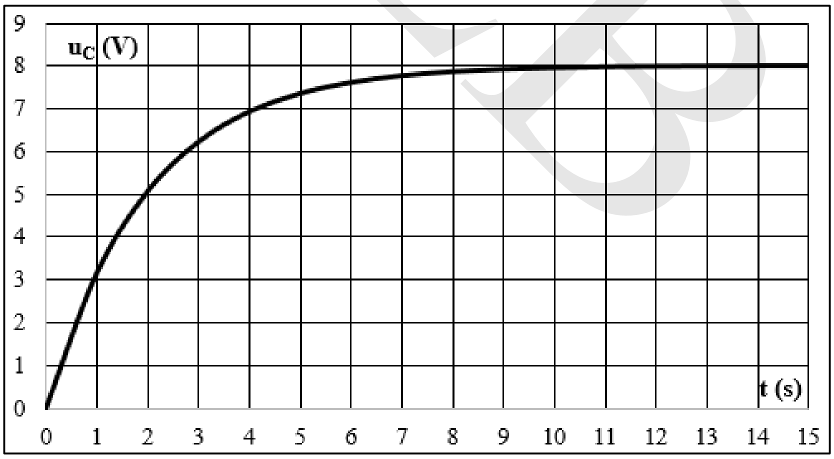
Ans: At t = τ ;

u_C = 0.63E = 0.63 x 8 = 5.04 V ≈ 5 V

from the graph we get : τ = 2 s

13) Deduce the value of the value of the capacitance if the resistance R = 200 Ω.

Ans: τ= RC then C= τ/R = $\frac{2}{200}$ = 10^{-2} F



14) determine $\frac{du_C}{dt}\Big|_{t=0}$

From the differential equation of u_C

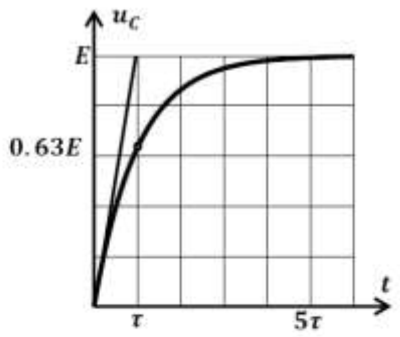
Ans: $\frac{du_C}{dt} + \frac{u_C}{RC} = \frac{E}{RC}$ at t= 0, $u_C = 0$ so $\frac{du_C}{dt}\Big|_{t=0} = \frac{E}{RC}$

Or from the solution $u_C = E(1-e^{-\frac{t}{\tau}})$.

Ans: $u_C = E(1-e^{-\frac{t}{\tau}})$ then $\frac{du_C}{dt} = \frac{E}{\tau}e^{-\frac{t}{\tau}}$ So $\frac{du_C}{dt}\Big|_{t=0} = \frac{E}{\tau}e^{0} = \frac{E}{RC}$

Or Graphiclally

Ans: $\frac{du_C}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= \frac{E}{\tau} = \frac{E}{RC}$



15) Determine the expression of the current i as a function of time t.

Ans: $i = C\frac{du_C}{dt} = C\frac{E}{\tau}e^{-\frac{t}{\tau}} = C\frac{E}{RC}e^{-\frac{t}{\tau}} = \frac{E}{R}e^{-\frac{t}{\tau}}$

16) Determine, at the instant t = 0, the value of the current i.

or Determine the maximum value of the current I.

Ans: At t = 0 ; $i = \frac{E}{R}e^{0} = \frac{E}{R} = \frac{8}{200} = 0.04$ A then I = 0.04 A

17) Determine, at the instant t = τ, the expression of the current i in terms of E and R. deduce its value.

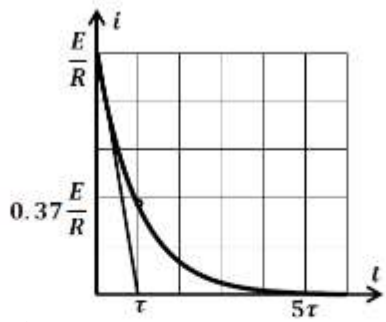
Ans: At t = τ ; $i = \frac{E}{R}e^{-\tau/\tau} = \frac{E}{R}e^{-1} = 0.367\frac{E}{R}$ then i = 0.0147 A

18) Deduce the value of the current i in steady state.

Ans: at steady state: t = ∞ ; $i = \frac{E}{R}e^{-\frac{\infty}{\tau}} = \frac{E}{R}\times 0 = 0$ A

19) Sktetch i as function of t.

Ans: figure



20) Determine $\frac{di}{dt}\Big|_{t=0}$

from the solution i

Ans: $i = I_m e^{-\frac{t}{\tau}}$ then $\frac{di}{dt} = -\frac{I_m}{\tau}e^{-\frac{t}{\tau}}$ So $\frac{di}{dt}\Big|_{t=0} = -\frac{I_m}{\tau}e^{0} = -\frac{I_m}{RC} = -\frac{E}{R^2C}$

Or Graphiclally

Ans: $\frac{di}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= -\frac{E/R}{\tau} = -\frac{E}{R^2C}$

21) Determine the expression of the voltage across the resistor u_R as a function of time t.

Ans: $u_R = Ri = R\frac{E}{R}e^{-\frac{t}{RC}} = Ee^{-\frac{t}{RC}}$

Or if $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$. Deduce the expression of the voltage across the resistor u_R as a function of time t.

Ans: $u_R + u_C = E$ then $u_R = E - u_C = E - E\left(1 - e^{\frac{-t}{RC}}\right) = Ee^{-\frac{t}{RC}}$

22) Determine, at the instant t = 0, the expression of the voltage across the resistor u_R.

or Determine the maximum value of the current u_R.

Ans: At t = 0 ; $u_R = Ee^{-\frac{t}{RC}} = Ee^{0} = E$

23) Determine, at the instant t = τ, the expression of u_R in terms of E.

Ans: At t = τ; $u_R = Ee^{-1} = 0.367$ E

24) Deduce the value of the u_R in steady state.

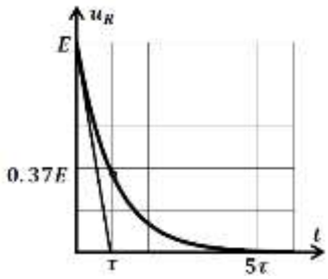
Ans: at steady state: t = ∞ ; $u_R = Ee^{-\frac{\infty}{RC}} = E x 0 = 0$ V

25) Sktetch u_R as function of t.

Ans: figure

26) Determine $\frac{du_R}{dt}\Big|_{t=0}$

from the solution u_R



Ans: $u_R = Ee^{-\frac{t}{\tau}}$ then $\frac{du_R}{dt} = -\frac{E}{\tau}e^{-\frac{t}{\tau}}$ So $\frac{du_C}{dt}\Big|_{t=0} = -\frac{E}{\tau}e^{0} = -\frac{E}{RC}$

Or Graphiclally

Ans: $\frac{du_R}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= -\frac{E}{\tau} = -\frac{E}{RC}$

27) If $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$. Determine the expression of the charge q as a function of time t.

Ans: $q = Cu_C = CE(1-e^{-\frac{t}{RC}})$

28) Determine, at the instant t = τ, the expression of q in terms of E and C.

Ans: At t = τ; $q = Cu_C = CE(1-e^{-1}) = 0.63\ CE$

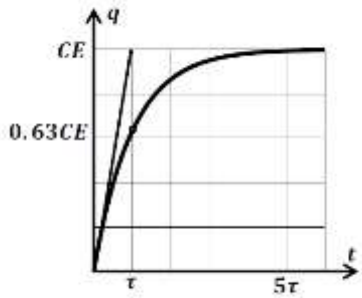
29) Deduce the value of the u_R in steady state.

or Determine the maximum value of q.

Ans: at steady state: t = ∞ ; $q = CE(1-e^{-\infty}) = CE(1-0) = CE$ then $Q_m = CE$.

30) Sktetch q as function of t.

Ans: figure



31) Determine $\frac{dq}{dt}\Big|_{t=0}$

from the solution q

Ans: $q = Q_m(1-e^{-\frac{t}{\tau}})$ then $\frac{dq}{dt} = \frac{Q_m}{\tau}e^{-\frac{t}{\tau}}$ So $\frac{dq}{dt}\Big|_{t=0} = \frac{Q_m}{\tau}e^{0} = \frac{Q_m}{RC} = \frac{E}{R}$

Or Graphiclally

Ans: $\frac{dq}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= \frac{CE}{\tau} = \frac{E}{R}$

32) Derive the differential equation that governs the variation of the current i as function of time.

Ans: Apply Law of addition of voltages: $u_R + u_C = u_G$ where $u_R = Ri$ and $q = Cu_C$

$Ri + \frac{q}{C} = E$ where $i = \frac{dq}{dt}$ Derive with respect to time $R\frac{di}{dt} + \frac{1}{C}\frac{dq}{dt} = 0$

$R\frac{di}{dt} + \frac{i}{C} = 0$ then $\frac{di}{dt} + \frac{i}{RC} = 0$

33) Derive the differential equation that governs the variation of the voltage u_R as function of time.

Ans: Law of addition of voltages: $u_R + u_C = u_G$ where $q = Cu_C$

$u_R + \frac{q}{C} = E$ where $i = \frac{dq}{dt}$ Derive with respect to time

$\frac{du_R}{dt} + \frac{1}{C}\frac{dq}{dt} = 0$ then $\frac{du_R}{dt} + \frac{i}{C} = 0$ then $\frac{du_R}{dt} + \frac{Ri}{RC} = 0$ so $\frac{du_R}{dt} + \frac{u_R}{RC} = 0$

34) Derive the differential equation that governs the variation of the q as function of time.

Ans: Law of addition of voltages: $u_R + u_C = u_G$

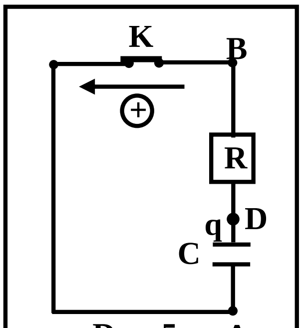
$Ri + u_C = E$ Where $i = \frac{dq}{dt}$ and $q = Cu_C$ then $R\frac{dq}{dt} + \frac{q}{C} = E$ so $\frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}$

35) A capacitor of capacitance 5000 μF is charged under a voltage of 12 V. Calculate the accumulated charge and the energy stored in this capacitor.

Ans: Accumulated charge: Q = CV = 5000x 0^{-6}x12 = 6 x 10^{-2} C.

Stored energy: W = 1/2 QV = 1/2x(6x10^{-2})x12 = 36x10^{-2} J.

Discharging of a capacitor:



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Consider the fully charged capacitor in the adjacent circuit, switch K is closed at new $t_0 = 0$,

At an instant t, the capacitor is charged by q and the circuit carries a current i.

36) Name the phenomenon that takes place.

Ans: discharging of a capacitor

37) Write the expression of the current i in terms of q.

Ans: $i = -\frac{dq}{dt}$

38) Deduce the expression of i in terms of the capacitance C and the voltage u_C.

Ans: $q = Cu_C$ so $i = -C\frac{du_C}{dt}$

39) Derive the differential equation that governs the variation of the voltage, $u_{DA} = u_C$, across the capacitor.

Ans: Apply law of addition of voltage $u_{BA} = u_{BD} + u_{DA}$, then $0 = -Ri + u_C$ where $i = -C\frac{du_C}{dt}$

Then , $RC\frac{du_C}{dt} + u_C = 0$

40) Verify that the solution of this differential equation is $u_C = Ee^{-\frac{t}{RC}}$.

Ans: $u_C = Ee^{-\frac{t}{RC}}$ then $\frac{du_C}{dt} = -\frac{E}{RC}e^{\frac{-t}{RC}}$

$RC\frac{du_C}{dt} + u_C = -RC\frac{E}{RC}e^{\frac{-t}{RC}} + Ee^{-\frac{t}{RC}} = -E\left(e^{\frac{-t}{RC}}\right) + E\left(e^{\frac{-t}{RC}}\right) = 0$ verified

Or The solution of the obtained differential equation is of the form: $u_C = E\,e^{-\frac{t}{\tau}}$,where τ is constant. Determine the expression of τ in terms of R and C.

Ans: $u_C = E\,e^{-\frac{t}{\tau}}$, then $\frac{du_C}{dt} = -\frac{E}{\tau}e^{\frac{-t}{\tau}}$

Replacing u_C and $\frac{du_C}{dt}$ by their expressions in the differential equation, gives:

$-RC\frac{E}{\tau}e^{\frac{-t}{\tau}} + E\,e^{\frac{-t}{\tau}} = 0$, then $E e^{\frac{-t}{\tau}}\left(\frac{RC}{\tau} - 1\right) = 0$

$E e^{\frac{-t}{\tau}} = 0$ is rejected Then, $\left(\frac{RC}{\tau} - 1\right) = 0$; therefore, $\tau = RC$

41) Determine, at the instant t = 0, the expression of the voltage u_C in terms of E.

Ans: At t = 0 ; $u_C = E(e^0) = E(1) = E$

42) Determine, at the instant t = τ, the expression of the voltage u_C in terms of E.

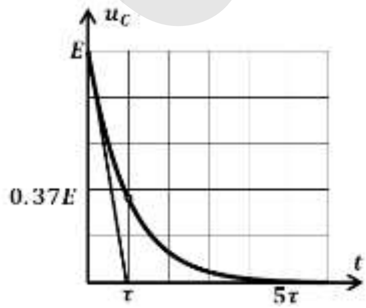
Ans: At t = τ ; $u_C = E\left(e^{-\frac{\tau}{\tau}}\right) = E(e^{-1}) \approx 0{,}37E$

43) Verify that the capacitor becomes practically fully discharged at t = 5 RC.

At t = 5 RC: $u_C = E\left(e^{\frac{-5RC}{RC}}\right) = E(e^{-5})$, then $u_C = 0.006$ E. Therefore, the capacitor becomes practically fully discharged at t = 5 R C.

44) Sketch u_C

Ans: the figure



45) determine $\frac{du_C}{dt}\Big|_{t=0}$ Graphiclally

Ans: $\frac{du_C}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= -\frac{E}{\tau} = -\frac{E}{RC}$

46) Determine the expression of the current i as a function of time t.

Ans: $i = -C\dfrac{du_C}{dt} = -C\dfrac{E}{\tau}e^{-\frac{t}{\tau}} = -C\dfrac{E}{RC}e^{-\frac{t}{\tau}} = -\dfrac{E}{R}e^{-\frac{t}{\tau}}$

47) Determine, at the instant t = 0, the value of the current i.
or Determine the maximum value of the current I.

Ans: At t = 0 ; i = $\frac{E}{R}e^{0} = \frac{E}{R}$

48) Determine, at the instant t = τ, the expression of the current i in terms of E and R.

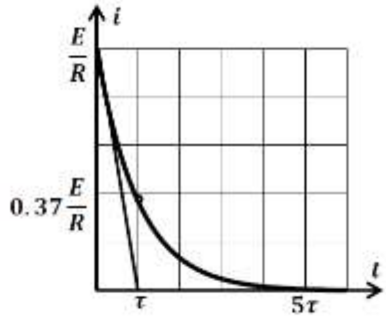
Ans: At t = τ ; i = $\frac{E}{R}e^{-\tau/\tau} = \frac{E}{R}e^{-1} = 0.367\frac{E}{R}$

49) Deduce the value of the current i in steady state.

Ans: at steady state: t = ∞ ; $i = \dfrac{E}{R}e^{-\frac{\infty}{\tau}} = \dfrac{E}{R}\times 0 = 0$ A

50) Sktetch i as function of t.

Ans: figure



51) Determine $\frac{di}{dt}\Big|_{t=0}$ from the solution i

Ans: $i = I_m e^{-\frac{t}{\tau}}$ then $\frac{di}{dt} = -\frac{I_m}{\tau}e^{-\frac{t}{\tau}}$ So $\frac{di}{dt}\Big|_{t=0} = -\frac{I_m}{\tau}e^{0} = -\frac{I_m}{RC} = -\frac{E}{R^2C}$

Or Graphiclally

Ans: $\frac{di}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= -\frac{E/R}{\tau} = -\frac{E}{R^2C}$

52) Determine the expression of the voltage across the resistor u_R as a function of time t.

Ans: $u_R = Ri = R\frac{E}{R}e^{-\frac{t}{RC}} = Ee^{-\frac{t}{RC}}$

Or if $u_C = E\left(1 - e^{\frac{-t}{RC}}\right)$. Deduce the expression of the voltage across the resistor u_R as a function of time t.

Ans: $u_R + u_C = E$ then $u_R = E - u_C = E - E\left(1 - e^{\frac{-t}{RC}}\right) = Ee^{-\frac{t}{RC}}$

53) Determine, at the instant t = 0, the expression of the voltage across the resistor u_R.
or Determine the maximum value of the current u_R.

Ans: At t = 0 ; $u_R = Ee^{-\frac{t}{RC}} = Ee^{0} = E$

54) Determine, at the instant t = τ, the expression of u_R in terms of E.

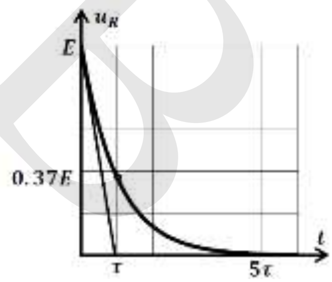
Ans: At t = τ; $u_R = Ee^{-1} = 0.367$ E

55) Deduce the value of the u_R in steady state.

Ans: at steady state: t = ∞ ; $u_R = Ee^{-\frac{\infty}{RC}} = Ex0 = 0$ V

56) Sktetch u_R as function of t.

Ans: figure



57) Determine $\frac{du_R}{dt}\Big|_{t=0}$ from the solution u_R

Ans: $u_R = Ee^{-\frac{t}{\tau}}$ then $\frac{du_R}{dt} = -\frac{E}{\tau}e^{-\frac{t}{\tau}}$ So $\frac{du_C}{dt}\Big|_{t=0} = -\frac{E}{\tau}e^{0} = -\frac{E}{RC}$

Or Graphiclally

Ans: $\frac{du_R}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= -\frac{E}{\tau} = -\frac{E}{RC}$

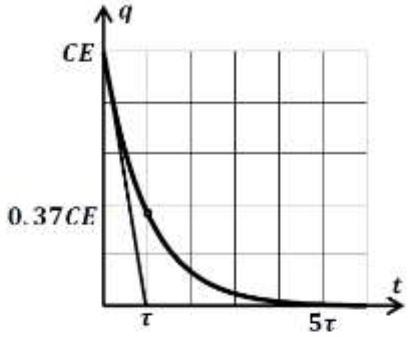
58) If $u_C = Ee^{-\frac{t}{RC}}$. Determine the expression of the charge q as a function of time t.

Ans: $q = Cu_C = CE(1 - e^{-\frac{t}{RC}})$

59) Determine, at the instant t = τ, the expression of q in terms of E and C.

Ans: At t = τ; $q = Cu_C = CE(e^{-1}) = 0.37$ CE

60) Deduce the value of the u_R in steady state.
Ans: at steady state: $t = \infty$; $q = CE(e^{-\infty}) = CE(0) = 0$ then Q_m
61) Sktetch q as function of t.
Ans: figure



62) Determine $\frac{dq}{dt}\Big|_{t=0}$ Graphiclally
Ans: $\frac{dq}{dt}\Big|_{t=0}$ = slope of the tangent to the curve at (t=0) = tan α $= -\frac{CE}{\tau} = -\frac{E}{R}$

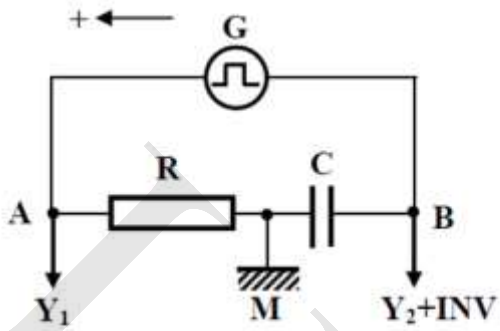
63) Derive the differential equation that governs the variation of the current i as function of time.
Ans: Law of addition of voltages: $u_{AB(R)} + u_{BA(C)} = 0$ where $u_{BA(C)} = -u_C$
$u_R - u_C = 0$ where $u_R = Ri$ and $q = Cu_C$ then $Ri - \frac{q}{C} = 0$
$Ri = \frac{q}{C}$ Derive with respect to time $R\frac{di}{dt} = \frac{1}{C}\frac{dq}{dt}$ where $i = -\frac{dq}{dt}$
$R\frac{di}{dt} + \frac{i}{C} = 0$ so $\frac{di}{dt} + \frac{i}{RC} = 0$

64) Derive the differential equation that governs the variation of the voltage u_R as function of time.
Ans: Law of addition of voltages: $u_{AB(R)} + u_{BA(C)} = 0$ where $u_{BA(C)} = -u_C$
$u_R - u_C = 0$ where $q = Cu_C$ then $u_R - \frac{q}{C} = 0$
$u_R = \frac{q}{C}$ Derive with respect to time $\frac{du_R}{dt} = \frac{1}{C}\frac{dq}{dt}$ Where $i = -\frac{dq}{dt}$
$\frac{du_R}{dt} + \frac{i}{C} = 0$ then $\frac{du_R}{dt} + \frac{Ri}{RC} = 0$ so $\frac{du_R}{dt} + \frac{u_R}{RC} = 0$

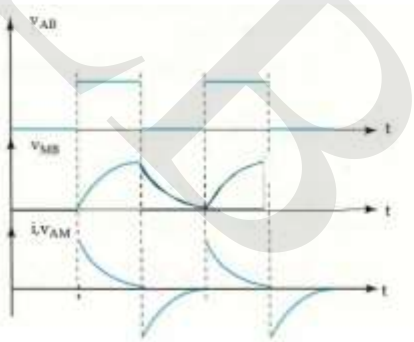
65) Derive the differential equation that governs the variation of the q as function of time.
Ans: Law of addition of voltages:
$u_{AB(R)} + u_{BA(C)} = 0$ where $u_R = Ri$ and $u_{BA(C)} = -u_C$ then $Ri - u_C = 0$ Where $i = -\frac{dq}{dt}$ and $q = Cu_C$
$R\frac{dq}{dt} - \frac{q}{C} = 0$ so $R\frac{dq}{dt} + \frac{q}{C} = 0$

RC series circuit under square signal voltage

Consider the electric circuit, includes a resistor of resistance R, a capacitor of capacitance C, and a low frequency generator (LFG) delivering square signal voltage. Such that; Square signal: It is a periodic signal which takes a constant maximum value during one half-period, and is zero in the second half.



66) The waveform of uAM represents the "image" of the current i. Why?
Ans: $u_{AM} = Ri$ then $i = u_{AM}/cte$ so the curve of u_{AM} represents the image of i.



67) During discharging the curve u_R or i is negative. Why?
Ans: During discharging:
$u_{AM(R)} + u_{MB(C)} = 0$
$u_{AM(R)} = -u_{MB(C)}$