Mokhtara Official Highschool . (313)		**Subject : Maths**
Classe: Grade 12 (GS)	**Exam -1-**	**Date : 21 /3 /2022.**
Prof. : Mohammad Ghor.	**/40**	**Duration : 4h.**

Exercise 1 : (5pts).

Answer by True or **false**, justify your answer:

1. The equation (E) : $\ln(x+3) + \ln(x+2) = \ln(x+11)$ is verified for $x = 1$.
2. z, z' and p are non zero complex numbers. If $z' \times \bar{z} = z$ and $p = \frac{1}{3} - i(\frac{2\sqrt{2}}{3})$, then $|z'| = |p|$.
3. $\lim\limits_{x\to 0^+} (\frac{1}{x\ln x - x}) = +\infty$.
4. If g is a continuous function over the interval [-2 ; 5] such that g (-2) = 3 and g(5) = -2 then the equation g(x) = -1 admits a unique root over [-2 ; 5].
5. $\int \frac{x^2 - x + 5}{x-1}dx = \frac{x^2}{2} + 5\ln|x-1| + C$. (where C : is a constant value).
6. If $z_1 + z_2 + z_3 = 0$ and $z_1 + z_2 = \frac{1}{8}(1+i)^4 - \frac{1}{2}i^9$, then $z = (z_3)^3 = \frac{1}{4} + \frac{i}{4}$
7. If $u(x) = e^{-x} \times (\ln(1+e^x))$, then $u'(x) = e^{-x} \times [\frac{e^x}{e^x+1} - \ln(1+e^x)]$.

Exercise 2 : (3pts).

Choose the correct answer with justification.

1. For all real numbers $x > 0$, $\ln(x + x^2) =$…..

   a) $\ln x + \ln(x + 1)$; b) $\ln(x^2) + \ln x$.

2. The real number $e^{-3\ln(\frac{1}{2})}$ is equal to …..

   a) $\frac{-1}{8}$; b) 8

3. $\lim\limits_{x\to+\infty}(x^2 - \ln x) = \ldots\ldots$ a) $+\infty$ b) $-\infty$.

4. If A ,B and C are three points of respective affixes z_A , z_B et z_C such that :

   $z_A - z_B = 4i(z_C - z_A)$ then :…….

   a)ABC is right isosceles at A ; b) (AB) and (AC) are perpendicular

5. Given f (x) = ln (-x) , x < 0. f ′(x) = …..

   a) $\frac{-1}{x}$; b) $\frac{1}{x}$.

6. If E ($\sqrt{3}$ - i) , F ($\sqrt{3}$ + i) and G(2i) , then OEFG is a ….

   a) Square ; b) Rhombus.

▶<u>Exercise 3:</u> (5½ pts).

Consider the complex plane of a direct orthonormal system $(O; \vec{u}, \vec{v})$.

Designate by A the point of affix $z_A = 1$, and the circle (C) of center A and radius 1.

A) Consider the points F , B and E of respective affixes 2 , ($1+e^{i\frac{\pi}{3}}$) et $(1+(z_B)^2)$

1. a) Prove that the point B belongs to (C).

   b) Determine the measure in radians of the oriented angle $(\vec{AF}; \vec{AB})$. Place the point B

2. a) Find the exponential form of $(z_B - z_A)$ and of $(z_E - z_A)$.

   b) Deduce that the points A , B and E are collinear and place the point E .

B) M(z) and M' (z') are points in the complex plane such that : $z' = 1 + z^2$.

1. For every $z \neq 0$ and $z \neq 1$, find a geometric interpretation of the argument of the complex number $\frac{z'-1}{z-1}$.

2. Deduce that if $\frac{z^2}{z-1}$ is real , then A, M and M′ are collinear.

3. Suppose that z = x +iy and z′ = x′ + iy′ where x, y, x′ and y′ are real numbers.

   a) Express x′ and y′ in terms of x and y.

   b) Find the set of points M such that z' is real.

   c) In the case where z' = 4 – 4i , write z in algebraic form.

d) In this part , suppose $z = 2 + i$:

i) Find the exponential form of z' .

ii) Find the values of the natural number n such that $(z')^n$ is real .

iii) Prove that M' moves on a circle (C') to be determined. .

Exercise 4: (4pts).

A) Given $E = \int_0^1 \frac{2x}{(x+1)(x+2)}dx$ and $F = \int_1^2 (x+1)\ln x dx$.

1. Without using the calculator.

a) Show that $\frac{2x}{(x+1)(x+2)} = \frac{-2}{x+1} + \frac{4}{x+2}$

b) Calculate E

c) Show that $\int (x+1)\ln x\, dx = \left(\frac{x^2}{2} + x\right)\ln x - \frac{x^2}{4} - x + C$

b) Verify that : $F = 4\ln 2 - \frac{7}{4}$.

2. Deduce that $2E + 3F - 8\ln 3 = -\frac{21}{4}$.

B) Let f be a function defined by : $f(x) = \int_2^{2x+5} t^3\, e^{4t+3}\, dt.$

1. Determine f '(x).
2. Solve the equation f '(x) = 0.

Exercise 5: (6pts).

Consider a triangle OAB right isosceles such that OA = OB and $\left(\overrightarrow{OA}, \overrightarrow{OB}\right) = \frac{\pi}{2}[2\pi]$

I, J, and K are respectively the midpoints of the segments [AB] , [OB], and [OA] respectively. Let **r** be the rotation of center I and angle $\frac{\pi}{2}$ and by **t** the translation of vector $\frac{1}{2}\overrightarrow{AB}$. Let **f = r ∘ *t*** and **g = t ∘ *r***

1) a) Determine f (K) , f(J) , and f (A).

b) Precise the nature and characteristic elements of f.

2) a) Determine g (J) , and g (O).

b) Precise the nature and characteristic elements of g.

3) Let h = g ∘ f^{-1}

a) Determine h(O) and find the nature of h.

b) M being any point in the plane, Let M_1 = f (M) and M_2 = g (M).

Show that the vector $\overrightarrow{M_1M_2}$ is equal to a fixed vector.

4) Consider the complex orthonormal system$(O\ ,\overrightarrow{OA},\ \overrightarrow{OB})$.

Find the complex forms of **r , t , f , and g.**

Exercise 6: (16½pts).

A) Consider the function f defined over IR by : f (x) = $e^x + 2e^{-x}$.

Designate by (C) its representative curve in an orthonormal system (O; $\vec{i}$, $\vec{j}$).

1 unit = 2cm.

1. Determine f ' (x) .
2. Calculate : $\lim\limits_{x\to+\infty} f(x)$., $\lim\limits_{x\to-\infty} f(x)$ et f ($\frac{1}{2}\ln 2$).
3. Draw the table of variations of f.
4. Trace (C).
5. Calculer the area of the domain limited by (C), (x´x) , (y´y) and the straight line x =2.

B) Consider the function g defined over IR by : g (x) = ln (f(x)). Designate by (C´) its representative curve in an orthonormal system (O; $\vec{i}$, $\vec{j}$).

1. a) Verify that : $e^x + 2e^{-x} = e^x(1+\frac{2}{e^{2x}})$.

   b) Prove that for every x $\in$ IR, : g(x) = x + ln (1 + $2e^{-2x}$).

   c) Calculate : $\lim\limits_{x\to+\infty} g(x)$..

   d) Prove that (d_1) : y = x is an asymptote to (C´) at $+\infty$.

e) Study the relative position of (C´) and (d_1).

2. a) Prove that for every x $\in$ IR, : g(x) = - x + ln (2 + e^{2x}).

   b) Calculate : $\lim\limits_{x \to -\infty} g(x)$..

   c) Prove that (d_2) : y = - x +ln2 is nan asymptote to (C´) at -∞.

   d) Study the relative position of (C´) and (d_2).

3. a) Calculate g' (x) .

   b) Prove that g (ln($\sqrt{2}$)) = $\frac{3}{2}\ln 2$.

   c) Draw the table of variations of g.

4. Trace (C´), (d_1) and (d_2) .

C) Let F be the function defined over] 0; +∞[by : F(x) = x.$(\ln x)^2$ – 2xlnx + 2x + 1.

1. Prove that F(x) is a primitive (antiderivative) of $(\ln x)^2$.
2. Deduce : $\int [\frac{1-2e^{-2x}}{1+2e^{-2x}} + 2(\ln x)^2]dx$.
3. Solve the equation : F(x) = 1 + x .

Good Work