Conditional Probabilities-Definitions and properties:

I- Conditional Probability

Definition 1: Let A and B be two events of the same random experiment such that event B occurred before event A. The probability of event A given that event B is realized is denoted by $P(A / B)$.

Example 1

The table at right shows the distribution of 100 grade 12 students in a secondary school.

	H	LS	ES	Total
Boys	15	12	20	47
Girls	10	18	25	53
Total	25	30	45	100

One student is selected randomly from the school.

Consider the events:

H: "The selected student is an H student"
L: "The selected student is an LS student"
E: "The selected student is an ES student"
B: "The selected student is a boy"

Part A :

Calculate the probabilities $P(H / B)$ $P(L / B)$, $P(B / E)$, $P(\overline{B} / E)$, $P(E / \overline{B})$, $P(B / B)$.

Part B:

1) Calculate the probability of selecting a boy knowing that he is in the LS section.
2) Calculate the probability of selecting an H student given that it is a girl.
3) The selected student is in the ES section. Calculate the probability of being a boy.
4) Knowing that the selected student is a girl, calculate the probability of being in the ES section.
5) Calculate the probability of selecting an LS girl.
6) Calculate the probability of selecting a boy or an ES student.

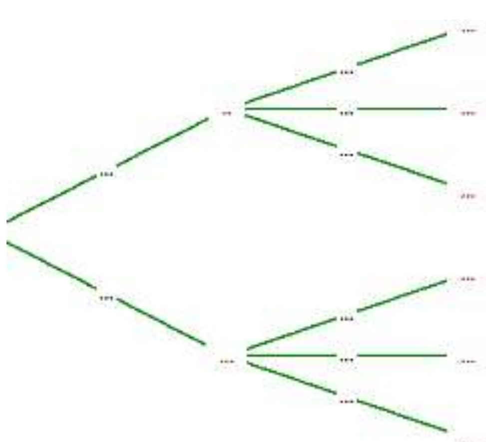
Part C: (Properties)

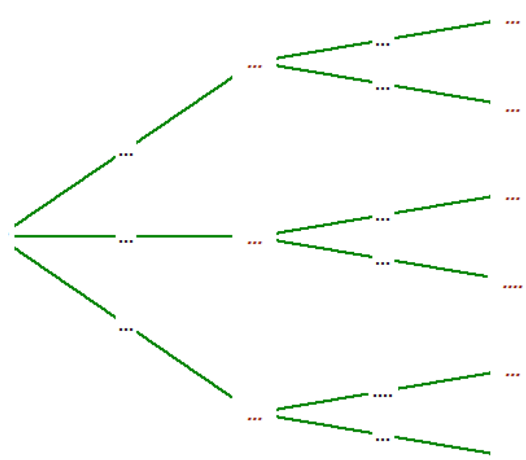
1) Calculate $P(L)$, $P(L \cap B)$, $P(B)$.
2) Calculate $P(L / B)$ and $P(B / L)$
3) What relation exists among $P(L / B)$, $P(B)$, and $P(L \cap B)$?
4) What relation exists among $P(B / L)$, $P(L)$, and $P(B \cap L)$?

Part D : (rules of total probability)

1) Calculate $P(B)$, $P(B \cap H)$, $P(B \cap L)$, and $P(B \cap E)$.
2) What relation exists among $P(B)$, $P(B \cap H)$, $P(B \cap L)$, and $P(B \cap E)$?
3) Calculate $P(L)$, $P(L \cap B)$, $P(L \cap \overline{B})$.
4) What relation exists among $P(L)$, $P(L \cap B)$, $P(L \cap \overline{B})$?

Part E : (Tree diagram)





Independent Events

Definition : Let A and B be two events of the same random experiment.
A and B are independent if and only if $P(A / B) = P(A)$ OR $P(A \cap B) = P(A) \times P(B)$

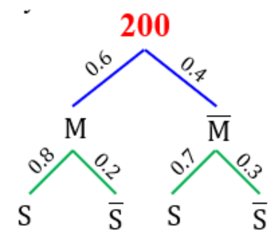
Example 1 : A and B are two events such that $P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.2$.
Are A and B independent? Justify.

Application 1: E and F are two events such that $P(E) = 0.5$, $P(F) = 0.25$, and $P(E / F) = 0.25$.
Are E and F independent? Justify.

Exercise 1: In a factory, 60% of the employees are men. We know that: 80% of the men are single and 70% of the women are single. One employee is randomly selected and interviewed.

Consider the events: M: "The selected employee is a man" S: "The selected employee is single"

1) Calculate the probabilities $P(M)$ and $P(S/M)$ and prove that $P(S \cap M) = 0.48$.
2) Calculate $P(S \cap \overline{M})$ and show that $P(S) = 0.76$.
3) Are the two events M and S independent? Justify your answer.
4) .The selected employee is not single. Calculate the probability of being a woman



Firas, the owner of the factory, knows that there are 200 employees in the factory.

On the LABOUR DAY, Firas decides to select, randomly and successively without replacement, a group of three employees to give them one-month-salary as BONUS at the end of May.

1) Calculate the probability of selecting exactly two women.
2) Calculate the probability of selecting exactly two men.
3) Knowing that the probability of selecting three women is $\frac{2054}{32835}$,

find, without calculations, the probability of selecting three men.