V. Calculate the following integrals.

1) $\int_0^1 \frac{dx}{x+1}$ 2) $\int_2^3 (2x+1+\frac{1}{3x-1})\,dx$ 3) $\int_1^e \frac{dx}{x(1+\ln x)}$

4) $\int_{-1}^0 \frac{2dx}{9x-1}$ 5) $\int_1^e \frac{\ln x}{x}\,dx$ 6) $\int_1^2 \frac{(3x-1)dx}{3x^2-2x+5}$

7) $\int_0^1 \frac{2xdx}{x^2+2}$ 8) $\int_e^{e^2} \frac{dx}{x\ln x}$ 9) $\int_1^{3/2} \frac{(x-1)dx}{x^2-2x}$

VI. The plane refers to an orthonormal system $(o, \vec{i}, \vec{j})$. 1 unit: 2 cm.

In all the problem we have $x>0$.

1) Consider the function $g(x) = -x^2 + 1 - \ln x$

a) Calculate $\lim\limits_{x \to 0^+} g(x)$ and $\lim\limits_{x \to +\infty} g(x)$.

b) Set up the table of variations of g.

c) Calculate g(1). Deduce the sign of g(x).

2) Consider the function $f(x) = -\frac{1}{2}x + 1 + \frac{\ln x}{2x}$

a) calculate f'(x) and express f'(x) in terms of g(x)

b) Calculate $\lim\limits_{x \to 0^+} f(x)$; $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to +\infty} [f(x) - (-\frac{1}{2}x + 1)]$.

Deduce the asymptotes to the graph (C) of f.

c) set up the table of variations of f.

3) a) Study the relative position of (C) with respect to (d): $y = -\frac{x}{2} + 1$

b) Draw (d) and (C).

4) Prove that f admits on]0, 1] an inverse function f^{-1}. Specify the domain of definition of f^{-1} and draw (C') the representative curve of f^{-1} in the same system $(o, \vec{i}, \vec{j})$

5) Determine the coordinates of point of (C) where the tangent is parallel to (d).

Logarithmic Functions

I. Solve the following equalities and inequalities:

1) $\ln(2x-5) + \ln(x-6) = 2\ln 3$
2) $\ln(x+1) - \ln(x+3) = \ln 2$
3) $\frac{1}{2}\ln(x+6) - \ln(x+4) = 0$
4) $\ln(-x+4) = \ln\left(\frac{3x-19}{x-1}\right)$
5) $\ln(x+1) + \ln(x+2) = 0$
6) $\ln^2 x - \ln x - 2 = 0$
7) $\ln(x-2) - \ln(x-3) \leq \ln 2$
8) $(\ln x)^2 - 3\ln x + 2 \leq 0$
9) $\ln(x^2+1) = 1$
10) $\ln(x^2-ex) > 2 + \ln 2$

II. Solve the following systems:

a) $\begin{cases} \ln x + \ln y = 2\ln 5 \\ x + y = 26 \end{cases}$ b) $\begin{cases} 2\ln x + \ln y = 7 \\ 3\ln x - 5\ln y = 4 \end{cases}$ c) $\begin{cases} x^2 + y^2 = 18 \\ \ln x + \ln 2y = 2\ln 3 + \ln 2 \end{cases}$

d) $\begin{cases} \ln(xy) = 5 \\ \ln x . \ln y = 6 \end{cases}$ e) $\begin{cases} x - y = -2 \\ \ln 2x + \ln|y-2| = 0 \end{cases}$ f) $\begin{cases} \ln(2x-y) = 0 \\ \ln(x+3y) = 2\ln 2 \end{cases}$

III. Determine the domain of definition, the domain of differentiability and the derivative of each of the functions below.

1) a) $f(x) = \ln(2x+1)$ b) $g(x) = \ln(3-4x)$

2) a) $f(x) = x\ln x - x$ b) $g(x) = \ln(1+\sqrt{x})$

3) a) $f(x) = \ln(\ln x)$ b) $g(x) = \ln\left(\frac{x+1}{x-1}\right)$

4) a) $f(x) = \ln(x+1) - \ln(x-1)$ b) $g(x) = x\ln(x-1)$

5) a) $f(x) = \ln|x|$ b) $g(x) = \ln|\ln x|$

6) a) $f(x) = \ln|1-x^2|$ b) $f(x) = \ln(-x^2)$

IV. Study the variation of the following function and draw their graphs.

1) $f(x) = x + \ln x$ 2) $f(x) = \sqrt{x} - \ln x$ 3) $f(x) = \frac{1}{x} - \ln x$

4) $f(x) = x - x\ln x$ 5) $f(x) = \ln(1 + \frac{1}{x})$ 6) $f(x) = \frac{x}{\ln x}$

7) $f(x) = \ln|x+1|$ 8) $f(x) = \frac{\ln x}{x}$

Part A

Consider the function f defined over $]0\,;+\infty[$ as $f(x) = x + \frac{1+e^x}{1-e^x}$.

Denote by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

Let (D) be the line with equation $y = x - 1$.

1) Determine $\lim_{x\to 0^+} f(x)$. Deduce an asymptote to (C).
2) a) Determine $\lim_{x\to +\infty} f(x)$.

   b) Show that (D) is an asymptote to (C).

   c) Show that, for all $x \in]0\,;+\infty[$, (C) is below (D).
3) Show that $f'(x) = \frac{e^{2x}+1}{(1-e^x)^2}$ and set up the table of variations of f.
4) a) Show that (C) intersects the x-axis at a unique point A.

   b) Let α be the abscissa of A. Verify that $1.4 < \alpha < 1.6$.
5) Draw (D) and (C).

Part B

Consider the function h defined over $]-\infty\,;0[\cup]0\,;+\infty[$ as $h(x) = x + \frac{1+e^x}{1-e^x}$.

Denote by (C') the representative curve of h in the same orthonormal system $(O;\vec{i},\vec{j})$.

1) Show that the point O is the center of symmetry of (C').
2) Draw over $]-\infty\,;0[$ the curve (C') as well as its asymptote at $-\infty$.

Part C

Consider the function g defined over $]0\,;+\infty[$ as $g(x) = \ln[x - f(x)]$,

1) Show that, for all $x \in]0\,;+\infty[$, $g(x) > 0$.
2) Knowing that, for all $x \in]0\,;+\infty[$, $\ln x \leq x - 1$.

   Show that $\ln[(e^x - 1).g(x)] \leq 1$.

14) Part A:

Let g be the function defined over $]0,+\infty[$ by $g(x)=\frac{1}{2}x^2+1-\ln x$

1) a) Calculate $\lim_{x\to 0^+} g(x)$ and $\lim_{x\to +\infty} g(x)$.

b) Set up the table of variations of g.

2) Deduce the sign of g(x) over $]0,+\infty[$

Part B:

Given the function f defined over $]0,+\infty[$ by $f(x)=3-x-\frac{2\ln x}{x}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$ (1 unit= 2cm)

1) a) Calculate $\lim_{x\to 0^+} f(x)$ and give its graphical interpretation.

b) Determine $\lim_{x\to +\infty} f(x)$ and verify that the line (d) of equation $y=-x+3$ is an asymptote of (C).

c) Study according to the values of x, the relative position of (C) and (d).

2) Verify that $f'(x)=\frac{-2g(x)}{x^2}$ and set up the table of variations of f.

3) Show that the equation f(x)=0 admits a unique solution α and verify that $2.2<\alpha<2.3$.

4) Determine the coordinates of the point A of (C) where the tangent (T) to (C) at A is parallel to (d).

5) Construct (T), (d) and (C).

6) Calculate $\int_1^3 \frac{\ln x}{x}dx$ and deduce, in cm^2, the area of the domain limited by (C), (d) and the two vertical lines of equations x=1 and x=3.

7) Show that $g(\alpha)=\frac{2\alpha^2-3\alpha+2}{2}$

15) A- Consider the function g defined over $]0;+\infty[$ as $g(x)=x^2-2\ln x$.

1) Determine $\lim_{x\to 0} g(x)$ and $\lim_{x\to +\infty} g(x)$.

2) Set up the table of variations of g and deduce that $g(x)>0$.

B- Let f be the function defined over $]0;+\infty[$ as $f(x)=\frac{x}{2}+\frac{1+\ln x}{x}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Determine $\lim_{x\to 0} f(x)$ and deduce an asymptote to (C).

2) a- Determine $\lim_{x\to +\infty} f(x)$ and show that the line (Δ) with equation $y=\frac{x}{2}$ is an asymptote to (C).

b- Study, according to the values of x, the relative positions of (C) and (Δ).

3) Show that $f'(x)=\frac{g(x)}{2x^2}$ and set up the table of variations of f.

4) Calculate the coordinates of the point B on (C) where the tangent (T) is parallel to (Δ).

5) Show that the equation f(x) = 0 has a unique solution α, then verify that $0.34<\alpha<0.35$.

6) Plot (Δ), (T) and (C).

7) Let h be the function [illegible]fined over $]0;+\infty[$ as $h(x)=\frac{1+\ln x}{x}$.

a- Find an antiderivative H of h.

b- Deduce the measure of the area of the region bounded by (C), (Δ) and the lines with equations x = 1 and x = e.

16) Part A:

The table below is the table of variations of the function g *defined over* $]0,+\infty[$ *by* $g(x)=\ln x(a\ln x+b)$ where a *and* b are two integers.

x	0		e		$+\infty$
$g'(x)$		$-$	0	$+$	
$g(x)$	$+\infty$	$\searrow$	-1	$\nearrow$	$+\infty$

1) Prove that $a+b=-1$.
2) Determine $g'(x)$ in terms of a *and* b and verify that $2a+b=0$.
3) Deduce that $g(x)=\ln x\,(\ln x-2)$.
4) Solve the equation $g(x)=0$ and study the sign of $g(x)$ *over* $]0,+\infty[$.
5) The line (d_m) of equation $y=m$ cuts (C_g) (the curve of the function g) in two points A and B of abscissas α *and* β respectively where $\beta>\alpha>0$ *and* $m>-1$.
   a) Show that $\alpha\beta=e^2$.
   b) Determine m so that $AB=2e\sqrt{3}$.

Part B:

Let f be the function defined over $[0,+\infty[$ *by* $\begin{cases} f(x)=x(\ln x-2)^2 & for\ x>0 \\ f(0)=0 \end{cases}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1) a) Study the continuity and the derivability to the right of f at 0.
   b) Find $\lim\limits_{x\to+\infty} f(x)$ *and* $\lim\limits_{x\to+\infty}\frac{f(x)}{x}$. Give a graphical representation to the result.
2) For $x>0$, verify that $f'(x)=g(x)$ and set up the table of variations of f.
3) a) Show that (C) has a point of inflection I whose coordinates are to be determined.
   b) Write the equation of the tangent (T) to (C) at point I.
4) Draw (T) and (C).
5) The line (Δ) of equation $y=x$ intersects (C) at three points O, I and J. Calculate the coordinates of J.

17) A- Let g be the function defined over $]0;+\infty[$ by $g(x)=x+\ln x$.

1) Calculate $\lim\limits_{x\to0} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.
2) Set up the table of variations of g.
3) Prove that the equation $g(x)=0$ has a unique solution α and verify that $0.5<\alpha<0.6$.
4) Determine, according to the values of x, the sign of g(x).

B- Consider the function f defined over $]0;+\infty[$ by $f(x)=x(2\ln x+x-2)$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to0} f(x)$, $\lim\limits_{x\to+\infty} f(x)$ and determine f(e).
2) Prove that $f(\alpha)=-\alpha(\alpha+2)$.
3) Verify that $f'(x)=2g(x)$ and set up the table of variations of f.
4) Draw (C). (Take $\alpha=0.55$)
5) Use integration by parts to calculate $\int_{0.5}^{1} x\ln x\,dx$ and deduce the area of the region bounded by the curve (C), the axis of abscissas and the two lines with equations $x=0.5$ and $x=1$.
6) The curve (C) cuts the axis of abscissas at a point with abscissa 1.37. Designate by F an antiderivativ of f on $]0;+\infty[$; determine, according to the values of x the variations of F

24 Let f be the function defined, over $[0\,;+\infty[$, as $\begin{cases} f(x)=x(\ln x-1)^2 & \text{for } x>0 \\ f(0)=0 \end{cases}$

and let (C) be its representative curve in an orthonormal system $(O\,;\vec{i},\vec{j})$.

0) ~~We admit that~~ Show that f is continuous at $x=0$.

1) a- Determine $\lim\limits_{x\to 0}\dfrac{f(x)}{x}$. Give a graphical interpretation to the result.

b- Find $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}$.

2) For $x>0$, verify that $f'(x)=(\ln x)^2-1$ and set up the table of variations of f.

3) Show that (C) has a point of inflection I and write an equation of (T), the tangent at I to (C).

4) The line (Δ) with equation $y=x$ intersects (C) at three points O, I and J.

Calculate the coordinates of J.

5) Plot (T) and (C).

6) a- Show that the function F defined on $]0\,;+\infty[$ as $F(x)=\dfrac{x^2}{2}\left[(\ln x)^2-3\ln x+\dfrac{5}{2}\right]$

is an antiderivative of f.

b- Deduce the area of the region bounded by the x-axis, the tangent (T) and the curve (C).

~~7)~~ For all x in the interval $[e\,;+\infty[$, prove that f has an inverse function f^{-1} and plot the representative curve of f^{-1} in the same system as (C).

8) Let (d_m) be the line with equation $y=mx$ where $m>0$.

The line (d_m) intersects the curve (C) at three distinct points O, M and M′.

a-Calculate, in terms of m, the coordinates of the points M and M′.

b-Denote by P the point on (d_m) with abscissa $x=e$.

Prove that $\overrightarrow{OM}\cdot\overrightarrow{OM'}=OP^2$.

95)

The table at right is the table of variations of a function h defined, on $]0\,;+\infty[$, as $h(x) = a + b\frac{\ln x}{x}$, where a and b are two real numbers. Denote by (H) the representative curve of h in an orthonormal system $(O;\vec{i},\vec{j})$.

x	0		e		$+\infty$
$h'(x)$	‖	$-$	0	$+$	
$h(x)$	‖ $+\infty$	$\searrow$	$1-\frac{2}{e}$	$\nearrow$	1

Part A

1) a- Verify that $a = 1$ and $b = -2$.

   b- Deduce, for all $x > 0$, that $\ln x \le \frac{1}{e}x$.

2) Let f be an antiderivative of h.

   a- Show that $f''(x) = h'(x)$ and deduce that f has an inflection point I whose abscissa is $x = e$.

   b- Determine $f(x)$ so that the point I is on the line (d) with equation $y = x$.

Part B

Consider the function f defined, on $]0\,;+\infty[$, as $f(x) = x + 1 - (\ln x)^2$.

Denote by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$. (Unit: 2 cm)

Let (d) be the line with equation $y = x$.

1) a- Calculate $\lim\limits_{x\to 0^+} f(x)$ and give a graphical interpretation for the obtained result.

   b- Calculate $\lim\limits_{x\to +\infty} \frac{f(x)}{x}$, then deduce that (d) is an asymptotic direction to (C).

   c- Study the relative positions of (C) and (d).

2) a- Show that f is strictly increasing for all $x > 0$, and set up its table of variations.

   b- Determine the point A on (C), where the tangent (T) to (C) at A is parallel to (d).

3) a- Show that the equation $f(x) = 0$ has a unique solution α.

   b- Verify that $0{,}31 < \alpha < 0{,}32$.

   c- Prove that $\ln\alpha = -\sqrt{\alpha + 1}$.

4) Draw (T), (d), and (C).

5) a- Verify that $x\ln x - x$ is an antiderivative of $\ln x$.

   b- Use integration by parts to determine $\int (\ln x)^2 dx$.

   c- Calculate $A(\alpha)$ and in cm^2, the area of the domain bounded by the curve (C), the x-axis, and the two lines with equations $x = \alpha$ and $x = 1$.

   d- Justify that $A(\alpha) = 2\left(\alpha^2 + 4\alpha + 4\alpha\sqrt{\alpha+1} - 1\right)\ cm^2$.

Let g be the function defined, on $]-\infty,+\infty[$, by g(x)= $a+(x^2+b)e^{-x}$

A) Calculate a and b knowing that the curve of g passes through the origin and admits a horizontal asymptote of equation y=1 at $+\infty$.

B) Suppose that $g(x)=1+(x^2-1)e^{-x}$

a) Calculate $\lim\limits_{x\to-\infty} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.

b) Find g'(x) and set up the table of variations of g.

c) Show that the equation g(x)=0 has two roots 0 and α, *where* $-0.8<\alpha<-0.7$.

d) Deduce the sign of g(x).

$1 - 2^2 e^{-1}$

C) Consider the function f defined on $\mathbb{R}$, *by* f(x)= $x-(x+1)^2e^{-x}$. (C) is the representative curve of f in an orthonormal system $(O,\vec{i},\vec{j})$ with **1 unit=2 cm.**

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ *and* $\lim\limits_{x\to+\infty} f(x)$.

2) i) Show that the line (d); y=x is an oblique asymptote at $+\infty$.

ii) Study the relative position of (C) and (d).

3) Verify that f'(x)=g(x).

4) Set up the table of variations of f.

5) Show that (C) admits two tangents (T) and (T') of slope 1 whose equations are to be determined.

6) Verify that $f(\alpha)=\frac{\alpha^2+1}{\alpha-1}$.

7) Draw the curve (C) with (d), (T) and (T').

Bekaa Youth Education Center
Cycle III
Grade 12
Name:

Test: Math
Duration: 60 minutes
Mark: /80
Date: 28/10/2023

I) (2 points)

1) Solve the equation $(\ln x)^2 - \ln x - 12 = 0$
2) Find the domain of definition of the function $f(x) = \frac{\ln(x-2)}{1-\ln x}$

II) (8 points)

Part A:

Given the function g defined over $]0, +\infty[$ by $g(x) = x^2 - 3 + 2\ln x$.

1) a) Calculate $\lim\limits_{x\to 0^+} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.
   b) Set up the table of variations of g.
2) a) Show that the equation $g(x) = 0$ admits a unique solution α and verify that $1.4 < \alpha < 1.5$.
   b) Deduce the sign of $g(x)$ over $]0, +\infty[$.

Part B:

Given the function f defined over $]0, +\infty[$ by $f(x) = x - 2 + \frac{1-2\ln x}{x}$ and designate by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$ (1 unit= 2cm).

1) a) Calculate $\lim\limits_{x\to 0^+} f(x)$ and deduce an asymptote to (C).
   b) Calculate $\lim\limits_{x\to +\infty} f(x)$ and verify that the line (d) of equation $y = x - 2$ is an asymptote to (C).
   c) Study according to the values of x, the relative position of (C) and (d).
2) a) Show that $f'(x) = \frac{g(x)}{x^2}$.
   b) Verify that $f(x) = 2\alpha - 2 - \frac{2}{\alpha}$ and set up the table of variations of f.
3) Take $f(\alpha) = -0.5$ and construct (d) and (C).
4) Calculate in cm^2, the area of the domain limited by (C), (d) and the two vertical lines of equations $x = \frac{1}{e}$ and $x = 1$.

Good Wo

III) (8 points)

Part A: Let g be the function defined over $\left[\frac{1}{e^3}, +\infty\right[$ by $g(x) = x(1+\ln x)^2 - 1$

1) Solve the inequation: $\ln^2 x + 4\ln x + 3 \leq 0$
2) Calculate limit of $g(x)$ as x tends to $+\infty$.
3) Calculate $g'(x)$ then set up the table of variation of g.
4) Calculate $g(1)$ and deduce the sign of $g(x)$

Part B:

Let f be the function defined over $\left]\frac{1}{e}, +\infty\right[$ by $f(x) = x - 1 + \frac{1}{1+\ln x}$;and designate by (C) its representative curve in an orthonormal system.

1)
   a) Determine $\lim\limits_{x \to 1/e} f(x)$. Deduce an asymptote to (C).
   b) Determine $\lim\limits_{x \to +\infty} f(x)$ and show that the line (d) of equation $y = x - 1$ is an asymptote to (C).
   c) Study the relative position of (C) and (d).
2)
   a) Show that $f'(x) = \frac{g(x)}{x(1+\ln x)^2}$
   b) Set up the table of variation of f
   c) Draw (C)
3)
   a) Show that, for $x \in [1, +\infty[$, f has an inverse function f^{-1} and find the domain of definition of f^{-1}.
   b) Trace the curve *(C') of the function* f^{-1} in the same system above.

Part C:

Let h be the function defined over $\left]\frac{1}{e}, +\infty\right[$ by $h(x) = \ln(f(x))$.

1) Determine $\lim\limits_{x \to 1/e} h(x)$ and $\lim\limits_{x \to +\infty} h(x)$.
2) Set up the table of variation of h
3) Prove that the equation $h(x) = 0$ has a unique root α to be determined.

Bekaa Youth Education Center

Cycle III

Grade: 12

Name:

Test: Math

Duration:50 minutes

Mark: /80

Date: /11/2022

I)

The plane is referred to an orthonormal system $(O, \vec{i}, \vec{j})$.

Part A:

Let g be the function defined on $\mathbb{R}$ by $g(x) = 2 - xe^{-x+1}$

1) a) Calculate $\lim_{x \to -\infty} g(x)$ and $\lim_{x \to +\infty} g(x)$.

   b) Verify that $g'(x) = (x-1)e^{-x+1}$ and set up the table of variations of g.

2) Deduce the sign of g(x) on $\mathbb{R}$

Part B:

Consider the function f defined on $\mathbb{R}$ as $f(x) = 2x + (x+1)e^{-x+1}$ and denote by (C) its representative curve.

1) Determine $\lim_{x \to -\infty} f(x)$ and calculate f(-1.5).

2) a) Determine $\lim_{x \to +\infty} f(x)$ and show that the line (d) of equation y=2x is an asymptote to (C).

   b) Study, according to the values of x, the relative position of (C) and (d).

3) Verify that f '(x)=g(x) and set up the table of variations of f.

4) Show that the equation f(x)=0 has a unique solution α on $\mathbb{R}$ and verify that $-0.8 < \alpha < -0.7$

5) Determine the coordinates of the point E on (C) where the tangent (T) to (C) at E is parallel to (d).

6) Draw (d), (T) and (C).

7) a) Determine the real numbers a and b so that the function $x \to (ax+b)e^{-x+1}$ is an antiderivative (primitive) of the function $x \to (x+1)e^{-x+1}$.

   b) Deduce the area of the domain limited by (C), (d) and the two vertical lines of equations x=1 and x=2.

8) Show that $g(\alpha) = \frac{2(\alpha^2+\alpha+1)}{\alpha+1}$

Part C:

Consider the function h defined by $h(x) = \ln(f(x) - 2x)$ and denote by (H) its representative curve.

a) Find the domain of definition of the function h.

b) Find the abscissa of point A on (H) where the tangent to (H) is parallel to the x-axis.

Good Work!

2) Determine a, b and c such that $A(-1; 1)$ is a point of(C).

3) In what follows we suppose that $f(x)=e^{-x^2+1}$.

a) Prove that f admits on $[0; +\infty[$ an inverse function f^{-1}.

b) Calculate $\left(f^{-1}\right)'\left(\frac{1}{e}\right)$.

c) Verify that the point $B(1; -1)$ is a point of (C'), the graph of f^{-1}, and find the equation of the tangent to (C') at B.

IV) (8 pts)

Part A: Let g be the function defined on $]0; +\infty[$ by $g(x)=\ln x+\frac{2}{x}-1$; and (C) be its representative curve in an orthonormal system $\left(O;\vec{i};\vec{j}\right)$.

1)

a) Calculate $\lim\limits_{x\to+\infty} g(x)$.

b) Prove that the line of equation $x=0$ is an asymptote *of* (C).

2)

a) Set up the table of variations of g then deduce the sign of $g(x)$.

b) Prove that the curve (C) has a point of inflection I.

c) Draw (C).

3) Calculate the area of the region bounded by the curve (C), the x-axis and the two lines of equations $x=1$ and $x=e$.

Part B: Let f be the function defined over $]0; +\infty[$ by $f(x) = (x+2)\ln x - 2x$. Designate by (Γ) its representative curve in an orthonormal system $\left(O;\vec{i};\vec{j}\right)$.

1) Determine $\lim\limits_{x\to0^+} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Deduce an asymptote of (Γ).

2)

a) Calculate $f'(x)$ and show that $f'(x)$ and $g(x)$ have the same sign.

b) Set up the table of variations of f.

3) Prove that the equation $f(x) = 0$ admits a unique solution a, and verify that $3.6 < a < 3.7$.

4) Verify that the point $W(2; 4\ln 2 - 4)$ is the point of inflection of (Γ) and trace (Γ).

5)

a) Prove that f admits an inverse function f^{-1}.

b) Trace the curve (Γ) of the function f^{-1} in the same system as that of f.

Good Work

I) (3 pts)

Choose the correct answer with justification:

1) If $z=-\sqrt{3}e^{-i\pi/3}$ and $z'=\sqrt{3}e^{i\pi/3}$; then $\arg(z-z')=$	$\pi/3$	0	π	$2\pi/3$
2) If $z.z'=\lvert z'\rvert=2$; then $\left\lvert z'-\frac{1}{z}\right\rvert=$	1	2	$\lvert z'\rvert$	$\lvert z\rvert$
3) The domain of definition of the function $f(x)=\ln(\ln(x-1))$ is	$]0;+\infty[$	$]1;+\infty[$	$]2;+\infty[$	$]-\infty;+\infty[$

II) (4 $\frac{1}{2}$ pts)

In the complex plane referred to a direct orthonormal system $(O;\vec{u};\vec{v})$; consider the points A and B such that: $z_A=3+3i$ and $z_B=1-i$.

1)

a) Write z_A and z_B in the trigonometric form.

b) Deduce that $(z_A\times z_B)$ is real and $\frac{z_A}{z_B}$ is a pure imaginary number.

c) Find the nature of the triangle OAB.

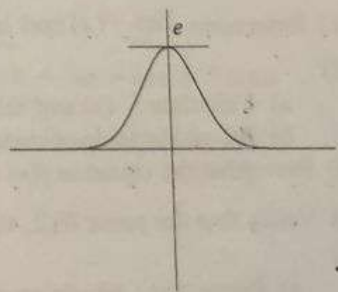
2) To every point M, of nonzero affix z, associate the point M' of affix z' such that $z'=\frac{\bar{z}}{z}-1$.

a) Find the point M such that $z'=\bar{z}-1$.

b) If $z=3e^{-i\pi/6}$, find an argument of z'.

c) Find the set of points M' when $|z|=|z-\bar{z}|$.

III) (4 $\frac{1}{2}$ pts)



The adjacent curve represents the graph (C) of a function defined on $]-\infty;+\infty[$ by $f(x)=e^{ax^2+bx+c}$, where a, b and c are three real numbers such that $a\neq 0$.

1) Determine $\lim\limits_{x\to\pm\infty} f(x)$ and set up the table of variation of f.

Please Turn The Page

1) a) Calculate the probabilities $P(A \cap R)$ and $P(\bar{A} \cap R)$.

   b) Verify that $P(R) = 0.47$.

2) Calculate $P(U)$.

3) Show that the probability that the interviewed member is still teaching or is working in another domain is 0.93.

4) The interviewed member does not teach at private schools.
   Calculate the probability that the member stayed in Lebanon.

5) The group consists of 500 teachers.

   a) Show that the number of teachers that teach at private schools is 235.

   b) Three members are interviewed from this group.
      Calculate the probability of interviewing at least two members who teach at private schools.

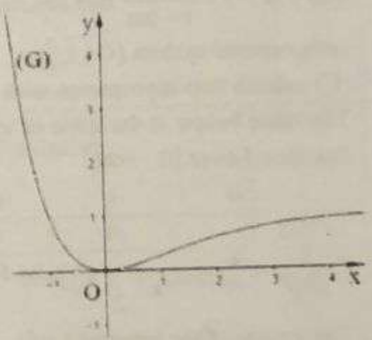
III- (10 points)

The plane is referred to an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

The adjacent curve (G) is the representative curve of a differentiable function g over $]-\infty; +\infty[$.

(G) is tangent to the x-axis at O.



1) Using the curve (G):

   a) Verify that $g(x) \geq 0$ for all real numbers x.

   b) The function g' is the derivative of g.
      Study, according to the values of x, the sign of g'.

2) Knowing that $g(x) = (ax + b)e^{-x} + 1$ where a and b are two real numbers, show that $a = b = -1$.

Part B

Consider the function f defined, on $\mathbb{R}$, as $f(x) = (x + 2)e^{-x} + x$.

Denote by (C) the representative curve of f.

Let (D) be the line with equation $y = x$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-2.5)$.

2) a) Determine $\lim\limits_{x \to +\infty} f(x)$.

   b) Show that the line (D) is an asymptote to (C).

   c) Study, according to the values of x, the position of (C) with respect to (D).

3) Show that $f'(x) = g(x)$ then set up the table of variations of f.

4) a) Show that the equation $f(x) = 0$ has, on $\mathbb{R}$, a unique root α.

   b) Verify that $-1.7 < \alpha < -1.6$.

5) a) Prove that (C) has an inflection point W whose coordinates are to be determined.

   b) Show that the line (T) with equation $y = 2$ is tangent to (C) at W.

6) Draw (T), (D) and (C).

7) Consider the function h defined over $]-2, 0[$ as $h(x) = \dfrac{\ln(x+2) - \ln(-x)}{x}$.

   Prove that $h(\alpha)$ is a natural number to be determined.

وزارة التربية والتعليم العالي المديرية العامة للتربية دائرة الامتحانات الرسمية	امتحانات الشهادة الثانوية العامة فرع: علوم الحياة	دورة العام 2023 العادية الاثنين 10 تموز 2023

عدد المسائل: ثلاث	مسابقة في مادة الرياضيات المدة: ساعة ونصف	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة أو اختزان المعلومات أو رسم البيانات
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة)

I- (4 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give, **with justification**, the answer that corresponds to it.

N°	Questions	Proposed answers a	Proposed answers b	Proposed answers c
1	The solution of the equation $2\ln(x) = \ln(25)$ is	5	$\frac{25}{2}$	-5
2	Consider the function f defined over $]e, +\infty[$ as $f(x) = x - 3 - \frac{3\ln x}{1 - \ln x}$ and denote by (C) its curve in an orthonormal system $(O; \vec{i}, \vec{j})$. (C) admits two asymptotes with equations	$x = 1$ and $y = x - 3$	$x = e$ and $y = x$	$x = -e$ and $y = x - 3$
3	The table below is the table of variations of a continuous function f over $[0, +\infty[$. $\begin{array}{c\|ccccccc} x & 0 & & 1 & & e & & +\infty \\ \hline f'(x) & & - & 0 & + & 0 & - & \\ \hline f(x) & 5 & \searrow & -2 & \nearrow & 3 & \searrow & -1 \end{array}$ The image of the interval $I = [1, +\infty[$ by f is	$[-2, 3]$	$[-2, -1[$	$]-1, 3]$
4	A code is a number formed of three digits using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The number of possible even codes greater than or equal to 300 is	280	350	500

II- (6 points)

During the financial crisis in Lebanon, a study on a group of teachers showed that:

- 30% currently work abroad out of which:
  - 40% teach at private schools only.
  - 20% teach at public schools only.
  - The remaining teachers started working in another domain.
- Out of those who stayed in Lebanon:
  - 50% teach at private schools only.
  - 40% teach at public schools only.
  - The remaining teachers retired and stopped working.

A member from the group is randomly interviewed. Consider the following events:

A: "The interviewed member currently work abroad"

R: "The interviewed member teaches at private schools only"

U: "The interviewed member teaches at public schools only"

D: "The interviewed member works in another domain"

N: "The interviewed member is retired and stopped working"

V- (7 points)

Part A

Consider the function g defined over $]-\infty\,;+\infty[$ as $g(x) = e^x - x$.

3) Determine $\lim\limits_{x\to-\infty} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.

4) a) Calculate $g'(x)$ and set up the table of variations of g.

   b) Deduce that $g(x) \geq 1$ for all $x \in]-\infty\,,+\infty[$.

Part B

Consider the function f given by $f(x) = \ln(e^x - x)$.

Denote by (C) its representative curve in an orthonormal system$(O\,;\vec{i},\vec{j})$.

Let (d) be the line with equation $y = x$.

9) Show that the domain of definition of f is $]-\infty\,,+\infty[$.

10) Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-6)$.

11) a) Determine $\lim\limits_{x\to+\infty} f(x)$.

   b) Show that the line (d) is an asymptote to (C) at $+\infty$.

   c) Study, according to the values of x, the position of (C) with respect to (d).

12) Calculate $f'(x)$ and set up the table of variations of f.

13) Determine the coordinates of the point K on (C) where the tangent is parallel to (d).

14) Let t be the function defined over $]-\infty\,,+\infty[$ as $t(x) = (-x+2)e^x - 1$.

   The following table is the table of variations of t:

x	$-\infty$		1		$+\infty$
t'(x)		+	0	−	
t(x)	−1	↗	e − 1	↘	$-\infty$

   a) Show that the equation $t(x) = 0$ has exactly two roots α and β on $\mathbb{R}$.

   b) Suppose that $\alpha < \beta$, verify that $-1.2 < \alpha < -1.1$ and that $1.8 < \beta < 1.9$

15) Knowing that $f''(x) = \dfrac{t(x)}{(e^x - x)^2}$, show that (C) has two inflection points whose abscissas are to be determined.

16) Draw (C) and (d).

Part C

Consider the function h given by $h(x) = \ln[(1 - g(x))\,t(x)]$.

Determine the domain of definition of h.

II- (3 points)

Consider two urns U and V.

- U contains 2 dice A and B:
  - A is a perfect die with 6 faces numbered 1, 2, 3, 4, 5 and 6.
  - B is a perfect die with 6 faces numbered 1, 1, 1, 2, 2 and 3.
- V contains: 2 red balls, 4 white balls and 3 black balls.

A game consists of choosing randomly a die from the urn U then throwing the die once.

- If the number on the upper face of the die is even, three balls are selected randomly and simultaneously from the urn V.
- If not, three balls are selected randomly and successively without replacement from the urn V.

Consider the following events:

A: "The chosen die is A",

B: "The chosen die is B",

O: "The number on the upper face of the die is odd",

S: "The three selected balls have the same color".

5) Calculate the probabilities $P(O / A)$, $P(O / B)$ and show that $P(O) = \frac{7}{12}$.

6) Calculate $P(S / O)$, $P(S / \overline{O})$, $P(O \cap S)$ and show that $P(S) = \frac{5}{84}$.

7) The three selected balls do not have the same color. Calculate the probability that the number on the upper face of the die is odd.

8) The game is repeated twice.

   The player wins if the three selected balls do not have the same color in each of the two games.

   Calculate the probability that the player does not win.

III- (3 points)

The complex plane is referred to a direct orthonormal system $(O ; \vec{u}, \vec{v})$.

Consider the points A, M and M' with affixes $z_A = i$, $z_M = z$ and $z_{M'} = z'$ such that $z' = \frac{z - i}{i\bar{z} - 1}$ with $z \neq i$.

5) Write z' in exponential form for $z = 2 + i$.

6) Let $z = x + iy$ and $z' = x' + iy'$ where x, y, x' and y' are real numbers with $x \neq 0$ or $y \neq 1$.

   c) Show that $x' = \frac{2x(y-1)}{x^2 + (y-1)^2}$ and $y' = \frac{(y-1)^2 - x^2}{x^2 + (y-1)^2}$.

   d) Calculate z' such that M varies on the line (d) with equation $y = x + 1$ deprived of point A.

7) a) For all $z \neq i$, show that $|z'| = 1$ and $\arg(z') = 2\arg(z - i) - \frac{\pi}{2} + 2k\pi$ where k is an integer.

   b) As the points M varies in the plane distinct from A, show that M' varies on a circle to be determined.

   c) Calculate z' in the case where $(\vec{u} ; \overrightarrow{AM}) = \frac{\pi}{2} + k\pi$ where k is an integer.

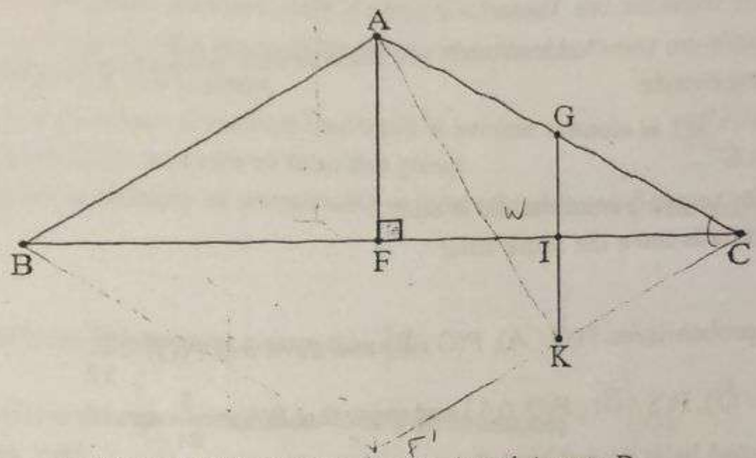
8) a) Express $z + z'$ in terms of z and $\bar{z}$.

   b) Show that if M varies on the circle (C) with center O and radius 1 deprived of A then [MM'] is a diameter of (C).

IV- (4 points)

In the figure below, we have:

- AFC is a direct semi-equilateral triangle.
- FC = 6 cm and $(\overrightarrow{AF}\,;\,\overrightarrow{AC}) = \frac{\pi}{3}\ [2\pi]$.
- G is the midpoint of [AC] and I is the midpoint of [FC].
- K is the symmetric of G with respect to I and B is the symmetric of C with respect to F.



Part A

Let S be the direct plane similitude that transforms G onto A and A onto B.

5) Show that the ratio of S is equal to 2 and that a measure of an angle of S is $\frac{\pi}{3}$.

6) a) Show that S(K) = C. Find S(I).

   b) Determine the point F′ the image of F by S.

7) Let h = S ∘ S ∘ S.

   a) Show that h is a negative dilation whose ratio is to be determined.

   b) Determine h(I).

8) Let W be the center of S.

   d) Show that W, B and C are collinear.

   e) Deduce that W, A and K are collinear. Construct W.

   f) Determine the image of the line (GK) by h.

Part B

The complex plane is referred to an orthonormal system $(F\,;\,\vec{u}\,,\,\vec{v})$ such that $\vec{u} = \frac{1}{3}\overrightarrow{FI}$.

3) Show that complex form of h is $z' = -8z + 18$.

4) Calculate the affix of point W.

III- (10 points)

Part A

Consider the function g defined on $\mathbb{R}$ as $g(x) = 1 + (x-1)e^{-x}$.

The table below is the table of variations of g.

x	$-\infty$		2		$+\infty$
$g'(x)$		$+$	0	$-$	
$g(x)$	$-\infty$	$\nearrow$	$1+e^{-2}$	$\searrow$	1

1) Calculate $g(0)$.

2) Show that for all $x \leq 0$, $g(x) \leq 0$ and for all $x \geq 0$, $g(x) \geq 0$.

Part B

Consider the function f defined on $\mathbb{R}$ as $f(x) = x(1-e^{-x})$ and denote by (C) its representative curve in an orthonormal system $(O ; \vec{i}, \vec{j})$.

Let (d) be the line with equation $y = x$.

1) Determine $\lim\limits_{x \to -\infty} f(x)$ and calculate $f(-1.5)$.

2) a) Determine $\lim\limits_{x \to +\infty} f(x)$.

   b) Show that the line (d) is an asymptote to (C) at $+\infty$.

   c) Study, according to the values of x, the position of (C) with respect to (d).

3) a) Show that $f'(x) = g(x)$.

   b) Set up the table of variations of f.

4) Show that (C) has an inflection point I whose coordinates are to be determined.

5) Draw (d) and (C).

Part C

Consider the function h defined over $[0 ; +\infty[$ as $h(x) = xe^{-x}$.

1) Set up the table of variations of h.

2) Let $M(x_M , f(x_M))$ and $N(f(x_M), x_M)$ are two variables points where $x_M > 0$.
   Determine the maximum length of segment [MN] as well as the corresponding position of M.

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I- (4 points)

In the table below, only one among the proposed answers to each question is correct.
Write the number of each question and give, with justification, the answer that corresponds to it.

N°	Questions	Proposed answers		
		a	b	c
1	For all real numbers $a > 0$, $\ln\left(\frac{e}{a}\right) + \ln(ae^2) =$	0	3	2
2	The solution set of the equation $\ln(x-1) + \ln(x+1) = 0$ is	$\{-\sqrt{2}\}$	$\{-\sqrt{2}, \sqrt{2}\}$	$\{\sqrt{2}\}$
3	$\lim\limits_{x \to +\infty} [\ln(1+2x) - \ln(1+x)] =$	$\ln 2$	2	0
4	The domain of definition of the function f given by $f(x) = \ln(1 - 2e^{-2x})$ is	$\left]-\infty, \frac{\ln 2}{2}\right]$	$[0, +\infty[$	$\left]\frac{\ln 2}{2}, +\infty\right[$

II- (6 points)

Part A

Consider two urns U and V.

- U contains two red balls holding each the number 0 and two green balls holding each the number 1.
- V contains three red balls holding each the number -1 and two green balls holding each the number 1.

A game consists of choosing randomly one of the two urns U and V and then selecting 2 balls simultaneously and randomly from the chosen urn.

Consider the following events:

U: "Urn U is chosen",
V: "Urn V is chosen",
S: "The two selected balls have the same color",
Z: "The sum of the numbers on the selected balls is zero".

1) Calculate the following probabilities: $P(S/U)$ and $P(S/V)$. Deduce that $P(S) = \frac{11}{30}$.
2) The two selected balls do not have the same color. Show that the probability that they are selected from urn U is $\frac{10}{19}$.
3) Calculate $P(Z)$.
4) Show that $P(S \cup Z) = \frac{2}{3}$.

Part B

All the balls from the two urns U and V are placed in one urn W.
Three balls are selected randomly and successively without replacement from W.

1) What is the number of possible selections of the three balls?
2) Calculate the probability that the product of the numbers on the three selected balls is zero.

26) A) Let g be the function defined over $\mathbb{R}$ by $g(x)=(1-x)e^{x}+1$

1) Calculate $\lim_{x\to+\infty} g(x)$ and $\lim_{x\to-\infty} g(x)$ then set up the table of variations of g.

2)a) Show that the equation g(x)=0 has a unique root α and verify that $1.2<\alpha<1.3$

b) Deduce the sign of g over $\mathbb{R}$.

B) Let f be the function defined over $\mathbb{R}$ by $f(x)=\frac{x}{e^{x}+1}$ and designate by (C) its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$. (1 unit: 2cm).

1) Calculate $\lim_{x\to+\infty} f(x)$. Deduce an asymptote to (C).

2) a) Calculate $\lim_{x\to-\infty} f(x)$ and verify that the line (Δ)of equation y=x is an asymptote to (C).

b) Study according to the values of x the relative position of (C) and (Δ).

3) a) Show that for every real x, $f(-x)+x=f(x)$.
b) Verify that $f(\alpha)=\alpha-1$ and $f(-\alpha)=-1$.

4) a) Verify that $f'(x)=\frac{g(x)}{(e^{x}+1)^2}$ and set up the table of variations of f.
b) Show that $f'(-\alpha)=1$ and write the equation of the tangent (T) to (C) at point A of abscissa $-\alpha$.

5) Take $\alpha=1.25$ and draw (Δ),(T) and (C).

---

25) **Part A**

Consider the function g defined on $\mathbb{R}$ as $g(x)=(-x+2)e^{-x}+1$.

1) Calculate g'(x).
2) Show that $g(x)>0$ for all $x\in\mathbb{R}$.

Part B

Consider the function f defined on $\mathbb{R}$ as $f(x)=x+2+(x-1)e^{-x}$.

Denote by (C) the representative curve of f.

Let (d) be the line with equation $y=x+2$.

1) Determine $\lim_{x\to-\infty} f(x)$ and calculate $f(-1)$.
2) a) Determine $\lim_{x\to+\infty} f(x)$.
   b) Show that (d) is an asymptote to (C).
   c) Study, according to the values of x, the position of (C) with respect to (d).
3) Show that $f'(x)$ and g(x) have the same sign then set up the table of variations of f.
4) Show that (C) has an inflection point W whose coordinates are to be determined.
5) a) Show that the equation $f(x)=-1$ has, on $\mathbb{R}$, a unique root β.
   b) Knowing that the equation $f(x)=0$ has, on $\mathbb{R}$, a unique root α, show that $\beta<\alpha$.
   c) Prove that $f\left(\frac{e^{-\beta}-3}{e^{-\beta}+1}\right)$ is an integer to be determined.
6) Draw (d) and (C). (Take $\alpha=-0.3$ and $\beta=-0.5$).

Part C

Consider the function h given by $h(x)=\ln[1+f(x)]-\ln[-f(x)]$.

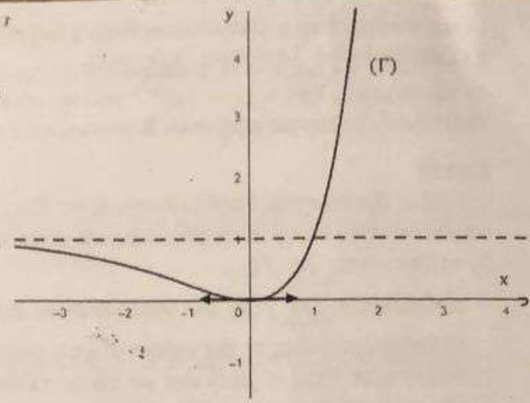
1) Determine the domain of definition of h.
2) Set up the table of variations of h

23)

Part A:

The adjacent curve (Γ) is the representative curve, in an orthonormal system, of the function g defined over $\mathbb{R}$ by: $g(x) = (ax + b)e^{x} + c$, where a, b and c are real numbers.

- The $x-axis$ is tangent to the curve (Γ) at the point of coordinates $(0;0)$.
- The line of equation $y = 1$ is an asymptote to (Γ) at $-\infty$.



1) Prove that $a = 1,\ b = -1$ and $c = 1$.
2) Use the curve (Γ) to determine the sign of $g(x)$ according to the values of x in $\mathbb{R}$.

Part B:

Let f be the function defined over $\mathbb{R}$ by $f(x) = (x-2)e^{x} + x$.
Designate by (C) the representative curve of f in an orthonormal system $(O\,;\vec{\imath}\,;\vec{\jmath}\,)$.

1) a) Calculate the limits of f $at +\infty$ and $at -\infty$.
   b) Show that the line (Δ) of equation $y = x$ is an asymptote to (C) at $-\infty$.
   c) Study the relative position of (C) with respect to (Δ).

2) Show that $f'(x) = g(x)$ and set up the table of variations of the function f.

3) a) Show that the equation $f(x) = 0$ admits a unique solution α in $\mathbb{R}$ and verify that: $1.6 < \alpha < 1.7$.
   ~~b) Verify that~~ $g(\alpha) = -\alpha - \frac{2}{\alpha-2}$

4) Show that there exist a point E of the curve (C) where the tangent (T) to (C) is parallel to the line (Δ). Calculate the coordinates of E and give an equation of the tangent (T).

5) Draw (Δ), (T) and (C).

6) a) Calculate $\int_0^1 (2-x)e^{x}\,dx$.
   b) Deduce the area of the domain limited by (C), the oblique asymptote (Δ) and the 2 vertical lines of equations $x = 0$ and $x = 1$.

24)

Part A:

Let g be the function defined over $\mathbb{R}$ by $g(x) = e^{x} + x + 2$. Designate by (Γ) its representative curve in an orthonormal system $(O, \vec{\imath}, \vec{\jmath})$.

1) a) Calculate $\lim\limits_{x\to+\infty} g(x)$.
   b) Calculate $\lim\limits_{x\to-\infty} g(x)$ and show that the straight line (Δ) of equation $y = x + 2$ is an asymptote to (Γ).
2) Calculate $g'(x)$ and set up the table of variations of g.
3) Show that the equation $g(x) = 0$ admits a unique solution $\alpha \in\,]-2.2, -2.1[$.
4) Draw (Γ) and (Δ).
5) Designate by $A(\alpha)$ the area of the domain limited by (Γ) and the axes of coordinates. Show that $A(\alpha) = \left(3 - \alpha - \frac{\alpha^2}{2}\right)$.

Part B:

Let f be the function defined over $\mathbb{R}$ by $f(x) = \frac{xe^{x}-1}{e^{x}+1}$ and designate by (C) its representative curve in an orthonormal system $(O, \vec{\imath}, \vec{\jmath})$.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) of equation $y = x$ is an asymptote to (C).
2) a) Show that $f(x) = \frac{x}{1+e^{-x}} - \frac{1}{1+e^{x}}$.
   b) Calculate $\lim\limits_{x\to-\infty} f(x)$ and deduce an asymptote to (C).
3) a) Verify that $f'(x) = \frac{e^{x}.g(x)}{(e^{x}+1)^2}$. Deduce according to the values of α the sign of $f'(x)$.
   b) Set up the table of variations of f.
4) Taking $\alpha = -2.2$.
   a) Determine the coordinates of the points of intersection of (C) with (d) and $y'oy$.
   b) Draw (C) (1 unit=2cm).

21) Given the function g defined over $\mathbb{R}$ by $g(x) = xe^{x-1} + 1$

1) Calculate $\lim\limits_{x\to+\infty} g(x)$ and $\lim\limits_{x\to-\infty} g(x)$.
2) Verify that $g'(x) = (x+1)e^{x-1}$ and set up the table of variations of g.
3) Deduce the sign of g(x) over $\mathbb{R}$.

Part B:

Consider the function f defined over $\mathbb{R}$ by $f(x) = (x-1)e^{x-1} + x$ and denote by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$. 1 unit = 2 cm.

1) a) Calculate $\lim\limits_{x\to+\infty} f(x)$.

   b) Calculate $\lim\limits_{x\to-\infty} f(x)$ and verify that the line (d) of equation y=x is an asymptote to (C).

   c) Study according to the values of x the relative position of (C) and (d).
2) Verify that $f'(x) = g(x)$ and set up the table of variations of f.
3) Show that the equation f(x)=0 has a unique root α and verify that $0.3 < \alpha < 0.4$.
4) Show that (C) has a point of inflection W to be determined.
5) Determine the coordinates of a point A on (C) where the tangent (T) to (C) at A is parallel to (d).
6) Draw (d), (T) and (C).

22) Consider the function f defined on $\mathbb{R}$ as $f(x) = (x+2)e^{-x} + 1$.

Denote by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a) Determine $\lim\limits_{x\to+\infty} f(x)$. Deduce an asymptote (d) to (C).

   b) Find the coordinates of the point of intersection of (C) and (d).
2) Determine $\lim\limits_{x\to-\infty} f(x)$ and calculate $f(-2.5)$.
3) Verify that $f'(x) = -(x+1)e^{-x}$ and set up the table of variations of f.
4) a) Show that the equation $f(x) = 0$ has a unique root α on $\mathbb{R}$.

   b) Verify that $-2.2 < \alpha < -2.1$.
5) a) Prove that the point I (0 , 3) is the point of inflection of the curve (C).

   b) Determine an equation of (T), the tangent to (C) at I.

   c) The table below is the table of variations of the function g defined as

   $g(x) = (x+2)e^{-x} + x - 2$.

x	$-\infty$	0	$+\infty$
g(x)	$-\infty$ $\nearrow$	0	$\nearrow$ $+\infty$

   Deduce, according to the values of x, the relative positions of (C) and (T).
6) Draw (d), (T) and (C).
7) Let k be the function given by $k(x) = \dfrac{x}{\ln(-x-2)}$.

   Denote by (C') the representative curve of k in an orthonormal system $(O;\vec{i},\vec{j})$.

   a) Determine the domain of definition of k.

   b) Show that $k'(\alpha) = \dfrac{\alpha+1}{\alpha^2 + 2\alpha}$.

   c) Show that the tangent to (C') at the point with abscissa α intersects the y-axis at the point $W\left(0 ; \dfrac{1}{\alpha+2}\right)$.

8) Consider the function f defined on $\mathbb{R}$ by $f(x)=\frac{e^x}{e^x+1}$. Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Deduce the asymptotes of the curve (C).
2) Calculate $f'(x)$ and set up the table of variations of f.
3) Show that $f''(x)=\frac{e^x(1-e^x)}{(1+e^x)^3}$. Prove that (C) has a point of inflection I to be determined.
4) Write an equation of the tangent (T) to (C) at the point I.
5) Draw (T) and (C).
6) The function f has on $\mathbb{R}$ an inverse function g.
   a- Draw the representative curve (G) of g in the given system.
   b- Verify that $g(x)=\ln\left(\frac{x}{1-x}\right)$.
   c- (G) and (C) intersect at a point with abscissa α. Calculate, in terms of α, the area of the region bounded by (C), (G) and the two axes of coordinates.

19) Consider the function f defined, on $]0;+\infty[$, by $f(x)=2x-2+\frac{1}{e^x-1}$ and designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{x\to0} f(x)$ and deduce an asymptote to (C).
   b- Determine $\lim\limits_{x\to+\infty} f(x)$ and show that the line (d) with equation $y=2x-2$ is an asymptote to (C).
   c- What is the position of (C) relative to (d)?
2) a- Show that $f'(x)=\frac{(e^x-2)(2e^x-1)}{(e^x-1)^2}$.
   b- Copy and complete the adjacent table of variations of f.

x	0	$\ln 2$	$+\infty$
$f'(x)$		0	
$f(x)$			

3) Draw (d) and (C).
4) Verify that $\frac{1}{e^x-1}=\frac{e^x}{e^x-1}-1$ and calculate the area of the region bounded by the curve (C), the line (d) and the two lines with equations $x=\ln 2$ and $x=\ln 3$.
5) Let g be the function defined over $]0;+\infty[$ by $g(x)=\ln(f(x))$
   a- Calculate $\lim\limits_{x\to0} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.
   b- Set up the table of variations of g.
   c- Prove that the equation $g(x)=0$ has two distinct roots.

XVI) Let f and g be two functions defined on R by $f(x) = 2x + (2x+1)e^{-2x}$ and $g(x) = 1 - 2xe^{-2x}$. Designate by (C) the representative curve of f in an orthonormal system $(O, \vec{i}, \vec{j})$. (unit= 2cm).

1) Calculate $g'(x)$ and prove that $g(x) > 0$ for all x in R.

2) a) Calculate $\lim_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = 2x$ is an asymptote to (C).

b) Study according to the values of x, the relative position of (C) and (d).

c) Calculate $\lim_{x \to -\infty} f(x)$

3) a) Verify that $f'(x) = 2g(x)$.

b) Set up the table of variations of f.

c) Prove that the point $W(\frac{1}{2}, 1 + \frac{2}{e})$ is a point of inflection of (C).

4) Determine the coordinates of the point I on (C) at which the tangent (D) to (C) is parallel to the asymptote (d).

5) Prove that the equation $f(x) = 0$ has a unique root α and verify that $-0.4 < \alpha < -0.3$

6) Draw the curve (C) and the lines (d) and (D).

7) Solve the inequality $e^{f(x)} < 1$

8) Calculate, in cm^2, the area of the region bounded by the curve (c), its asymptote (d) and the lines of equations $x = 0$ and $x = 1$

XVII) Consider the functions f and g, defined on $]0, +\infty[$ by $f(x) = 2x\ln x$ and $g(x) = e^{\frac{1}{2x}}$. Designate by (C) the representative curve of f and by (G) that of g, in an orthonormal system $(O, \vec{i}, \vec{j})$. (Unit = 2cm)

1) a) Calculate the limits of f at 0 and at $+\infty$ and specify f(e).

b) Set up the table of variations of f and draw (C).

2) Calculate the area of the domain bounded by the curve (C), the axis of abscissas and the lines of equations $x = 1$ and $x = \sqrt{e}$

3) a) Calculate the limits of g at 0 and at $+\infty$. Specify the asymptotes of (G).

b) Set up the table of variations of g and draw (G) (in the same system as (C)).

4) a) Prove that the function g admits, on $]0, +\infty[$, an inverse function g^{-1}.

b) Specify the domain of definition of g^{-1}, and determine $g^{-1}(x)$ in terms of x

5) The line of equation $y = 1$ cuts (C) at a point A of abscissa a, and the line of equation $y = x$ cuts (G) at a point B of abscissa b. Prove that $a = b$ and verify that $1.4 < a < 1.5$

c) Deduce the area of the region that is bounded by the curve (G), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$

XIV) Consider the function f that is defined, on $I =]1, +\infty[$ by $f(x) = x + 1 - \frac{3e^x}{e^x - e}$ and let (C) be its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

1) a) Prove that the line of equation $x = 1$ is an asymptote to (C).

b) Calculate $\lim_{x \to +\infty} f(x)$ and show that the line (d) of equation $y = x - 2$ is an asymptote to (C).

c) Determine the relative position of (C) and (d).

2) Prove that $f'(x) > 0$ for all values of x in I and set up the table of variations of f.

3) Prove that the equation $f(x) = 0$ has a unique root $\propto$ and verify that $2.6 < \propto < 2.7$

4) Draw the curve (C).

5) Designate by (D) the region that is bounded by (C), the line (d) and the lines of equations $x = 3$ and $x = 4$.

Calculate $\int_3^4 \frac{e^x dx}{e^x - e}$ and deduce the area of the region (D).

6) a) Prove that f, on the interval I, has an inverse function g.

b) Prove that the equation $f(x) = g(x)$ has no roots.

XV) Consider the function f defined over $]-\infty, 0[\cup]0, +\infty[$ by $f(x) = x - 1 - \frac{4}{e^x - 1}$

Designate by (C) the representative curve of f in an orthonormal system $(O, \vec{i}, \vec{j})$.

1) a) Show that the axis of ordinates is an asymptote to (C).

b) Calculate $\lim_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = x - 1$ is an asymptote to the curve (C).

c) Prove that the line (D) of equation $y = x + 3$ is an asymptote to (C) at $-\infty$

2) Prove that the point S(0;1) is a center of symmetry of (C).

3) a) Calculate $f'(x)$ and set up the table of variations of f.

b) Show that the equation $f(x) = 0$ has two roots α and β and verify that:

$1.7 < \alpha < 1.8 \text{ and } -3.2 < \beta < -3.1$

4) Draw (d), (D) and (C).

5) a) Prove that $f(x) = x + 3 - \frac{4e^x}{e^x - 1}$. b) Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations $x = 2$ and $x = 3$.

6) Let g be the inverse function of f on $]0, +\infty[$. Prove that the equation $f(x) = g(x)$ has no roots.

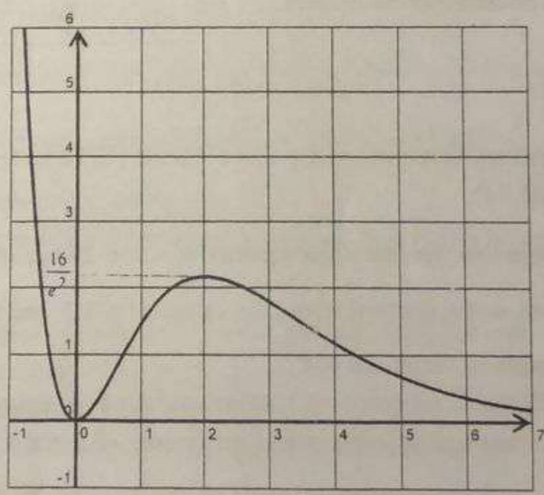
X)

Consider the function f defined on R by $f(x) = \frac{2e^x}{e^x+1} - 1$ and let (C) be its representative curve in a direct orthonormal system $(O, \vec{i}, \vec{j})$. (unit: 2cm)

1) Verify that $f(x) + f(-x) = 0$ and determine the center of symmetry of (C).
2) Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$, and deduce the asymptotes of (C).
3) Calculate $f'(x)$ and set up the table of variations of f.
4) Write an equation of the straight line (D) tangent to (C) at O.
5) Draw (D) and (C).
6) Calculate the area A of the domain bounded by (C) , the axis of abscissas and the lines of equations $x = 0$ and $x = 1$
7) Designate by f^{-1} the inverse function of f on R.
   a) Determine the domain of definition of f^{-1} and find $f^{-1}(x)$
   b) Draw the representative curve (G) of f^{-1}
   c) Calculate the area B of the domain bounded by (G) , the axis of ordinates and the lines of equations $y = 0$ and $y = 1$.

XI)

The graph (C) of a function f defined and differentiable over R, in an orthonormal system , is given below. We know that this function is of the form :
$f(x) = (ax^2 + bx + c)e^{-x}$. While choosing some indications on the graph determine coefficients a, b and c.



XII)

Consider the function f defined on R by $f(x) = x + (x+1)e^{-x}$, and let (C) be its representative curve.

1) a) Determine $\lim_{x \to +\infty} f(x)$ and verify that the line (d) of equation $y = x$ is an asymptote to (C).

b) Calculate the coordinates of the point of intersection E of the curve (C) with its asymptote (d).

c) Find the values of x for which the curve (C) is above the line (d).

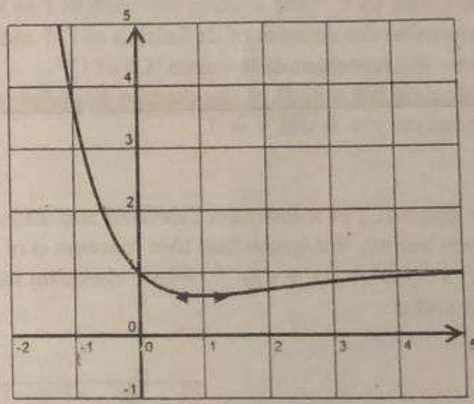
2) a) Determine $\lim_{x \to -\infty} f(x)$

b) Calculate f(-2) and give your answer in the decimal form , to the nearest 10^{-1}

3) the curve (G) shown below is the representative curve of f' , the derivative of f , in an orthonormal system.

a) Justify that the function f is strictly increasing.

b) Calculate the area of the domain bounded by the curve (G) , the axis of abscissas and the two lines of equations $x = 0$ *and* $x = 1$



4) a) Write an equation of the tangent (D) to (C) at The point A of abscissa 0.

b) Prove that the curve (C) admits an inflection point W of abscissa 1.

c) Set up the table of variations of f and draw the curve (C) and its tangent (D) , in an orthonormal system.

XIII)

Let f be the function that is defined on R by :$f(x) = x + 2 - e^{-x}$, and (C) be its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$.

1) a) Calculate $\lim_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = x + 2$ is an asymptote of (C).

b) Calculate $\lim_{x \to -\infty} f(x)$ and give, in the decimal form , the values of f(-1.5) and f(-2).

2) Calculate $f'(x)$ and set up the table of variations of f.

3) Write an equation of the line (T) that is tangent to (C) at the point A of abscissa 0.

4) Show that the equation $f(x) = 0$ has a unique root α and verify that $-0.5 < \alpha < -0.4$.

5) Draw (d) , (T) and (C).

6) Designate by g the inverse function of f , on R.

a) Draw, in the system $(O, \vec{i}, \vec{j})$ the curve (G) that represent g.

b) Designate by $A(\alpha)$ the area of the region that is bounded by the curve (C) , the axis of abscissas and the two lines of equations $x = \alpha$ and $x = 0$. Show that $A(\alpha) = -\frac{\alpha^2}{2} - 3\alpha - 1$ units of area.

V)

1) Determine the real numbers a and b such that the function $x \to (ax+b)e^x$ is an antiderivative (primitive) of the function $x \to (2x+1)e^x$

b) On no le book

VI)

The function f is defined by $f(x) = \begin{cases} x+1 & \text{for } x \leq 0 \\ e^x & \text{for } x > 0 \end{cases}$. Is f differentiable at 0.

VII)

Determine the domain of definition , the domain of differentiability and the derivative function of each of the following functions.

1) $f(x) = e^{2x+1}$ 2) $f(x) = e^{-x^2}$ 3) $f(x) = e^{\sqrt{x}}$ 4) $f(x) = e^{\frac{1}{x}}$

5) $f(x) = e^x \ln x$ 6) $f(x) = e^{x\ln x}$ 7) $f(x) = \frac{e^x - 1}{e^x+1}$ 8) $f(x) = \ln(e^x + e^{-x})$

VIII)

Study the variations and construct the graphs of the following functions.

1) $f(x) = \frac{e^x}{x}$ 2) $f(x) = \frac{e^x}{e^x-1}$ 3) $f(x) = \frac{1}{xe^x}$

5) $f(x) = xe^{-x}$ 6) $f(x) = \frac{xe^x}{x+1}$ 7) $f(x) = e^{1-x^2}$

IX)

Consider the function f defined on R by $f(x) = \frac{2e^{2x}}{e^{2x}+1}$ and designate by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$. (unit = 2cm).

1) Calculate the limits of f at $-\infty$ and at $+\infty$. Deduce the asymptotes to (C).
2) a) Verify that $f'(x) = \frac{4e^{2x}}{(e^{2x}+1)^2}$
   b) Set up the table of variations of f.
3) a) Verify that the point I(0,1) is a center of symmetry of (C).
   b) Write the equation of the tangent line (d) to (C) at I.
   c) Draw (d) and (C).
4) a) Prove that f admits on $[0, +\infty[$ an inverse function f^{-1}.
   b) Precise the domain of definition of f^{-1}
   c) Draw the representative curve (C') of f^{-1} in the same system $(O, \vec{i}, \vec{j})$.

Exponential functions

I)

Solve in R , the following equations:

1) $e^{3x} = e^{x}$ 2) $e^{\sqrt{x}} = 2$ 3) $e^{2x} = 0$ 4) $e^{x} = -1$

5) $e^{2x^2+3x-1} = 0$ 6) $e^{2x} + e^{x} - 2 = 0$ 7) $e^{2x} - 13 + 12e^{-x} = 0$

8) $e^{2x} - 3e^{x+1} + 2e^{2} = 0$ 9) $e^{3x} - 2e^{2x} + 3e^{x} - 2 = 0$ 10) $e^{x} + \frac{e}{e^{x}} = 1 + e$

II)

Solve in R, the following inequations:

1) $e^{2x-4} \geq 12$ 2) $1 \leq e^{x^2-x}$ 3) $e^{x+1} \geq \frac{3}{e^{x}}$ 4) $e^{1+\ln x} \geq 2$

5) $e^{x} - 4e^{-x} - 3 > 0$ 6) $e^{2(x+1)} + 3e^{x+2} < 4e^{2}$ 7) $e^{3x} - 5e^{x} + 4 \geq 0$

III)

Solve in $\mathbb{R}^2 = R \times R$ the following systems:

1) $\begin{cases} 2e^{x} - e^{y} = 15 \\ e^{x} + 2e^{y} = 40 \end{cases}$ 2) $\begin{cases} e^{x-y} = 3 \\ e^{x} - 2e^{y} = 1 \end{cases}$ 3) $\begin{cases} xy = \frac{1}{6} \\ e^{2x} \times e^{3y} = \frac{1}{e^{2}} \end{cases}$

4) $\begin{cases} e^{x} - 2e^{-y} = 1 \\ e^{x+y} = 3 \end{cases}$ 5) $\begin{cases} e^{x} \times e^{y} = 8 \\ e^{x} + e^{y} = 6 \end{cases}$ 6) $\begin{cases} x - 2y = 0 \\ (e^{x} + e^{y})(e^{x} - e^{y}) = 2 \end{cases}$

7) $\begin{cases} e^{x}e^{y} = e^{\frac{5}{2}} \\ \ln(x+1) + \ln(x+2) = 0 \end{cases}$ 8) $\begin{cases} e^{x+1}e^{y-2} = 2 \\ \ln x + \ln y = \ln(x-1) + \ln(y-1) \end{cases}$

IV)

Calculate the following anti derivatives:

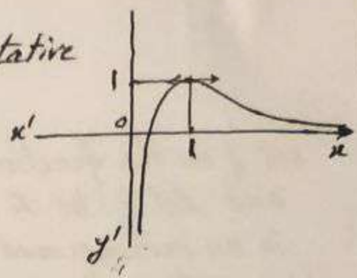
1) $\int e^{2x}dx$ 2) $\int e^{-x}dx$ 3) $\int e^{\frac{x+1}{2}}dx$ 4) $\int xe^{x^2+2}dx$

5) $\int (x+1)e^{x^2+2x}dx$ 6) $\int \frac{1}{x^2} e^{\frac{1}{x}}dx$ 7) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ 8) $\int \frac{e^{x}}{2e^{x}+1}dx$

9) $\int \cos x e^{\sin x}dx$ 10) $\int e^{x}e^{e^{x}}dx$ 11) $\int \frac{e^{\tan x}}{\cos^2 x}dx$ 12) $\int \frac{e^{2x} - 3e^{x} + 1}{e^{x}}dx$

(5pts)

III) **Part A** : the curve to the right is the representative curve of the function g defined over $]0,+\infty[$ by $g(x) = \frac{a + b\ln x}{x}$



1) Calculate $g(1)$
2) prove that $g'(x) = \frac{b - a - b\ln x}{x^2}$
3) prove that $a = b = 1$
4) Calculate $\lim_{x \to 0^+} g(x)$ and $\lim_{x \to +\infty} g(x)$

Part B : Let h be the function defined over $]0,+\infty[$ by $h(x) = x^2 - 2\ln x$

1) Calculate $\lim_{x \to 0^+} h(x)$ and $\lim_{x \to +\infty} h(x)$.
2) Find $h'(x)$ and set up the table of variations of h.
3) Deduce the sign of $h(x)$.

Part C : Let f be the function defined over $]0,+\infty[$ by $f(x) = \frac{x}{2} + g(x)$ and let (C) be its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$. (1 unit: 2cm)

1) prove that the axis of ordinates is an asymptote to (C)
2) Calculate $\lim_{x \to +\infty} f(x)$ and prove that the straight line (d) of equation $y = \frac{x}{2}$ is an asymptote to (C)
3) study, according to the values of x, the relative positions of (C) and (d)
4) prove that $f'(x) = \frac{h(x)}{2x^2}$ and set up the table of variations of f.
5) prove that the equation $f(x) = 0$ has a unique root α such that $0.34 < \alpha < 0.35$.
6) Write an equation of the tangent (T) to (C) at point B of abscissa 1.
7) Draw (T), (d) and (C)
8) prove that $f'(\alpha) = 1 + \frac{1}{\alpha^2}$
9) Calculate, in cm^2, the area of the domain limited by (C), (d) and the two lines of equations $x = \frac{1}{e}$ and $x = 1$

23) Let f be the function defined over $[0,+\infty[$ by $\begin{cases} f(x) = x(1+(\ln x)^2) & \text{for } x>0 \\ f(0)=0 \end{cases}$
and let (c) be its representative curve
in an orthonormal system $(o, \vec{i}, \vec{j})$. (unit = 3cm)

A) 1) a) study the continuity and the derivability to the right of f at 0.

b) Find $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to +\infty} \frac{f(x)}{x}$. Give a graphical interpretation to the result.

2) for $x>0$, verify that $f'(x) = (1+\ln x)^2$ and set up the table of variations of f.

3) a) Write the equation of the tangent (T) to (c) at point E of abscissa 1.

b) study, according to the values of x, the relative position of (c) and (T).

4) prove that the curve (c) has a point of inflection whose coordinates are to be determined

5) Draw (T) and (c)

6) a) show that the function F defined over $]0,+\infty[$ by
$F(x) = \frac{x^2}{2}\left[(\ln x)^2 - \ln x + \frac{1}{2}\right]$ is an antiderivative of $x(\ln x)^2$

b) Let $\alpha \in]0,1[$. Calculate in terms of α the area $I(\alpha)$ of the domain limited by (c), (T) and the 2 lines $x=\alpha$ and $x=1$

7) a) show that f has over $[0,+\infty[$ an inverse function g whose domain of definition is to be determined

b) draw (c') the representative curve of g in the same system as that of (c)

B) consider the sequence (U_n) defined by $\begin{cases} U_0 = \frac{2}{e} \\ U_{n+1} = g(U_n) & \text{for all natural number } n \end{cases}$

1) prove, by mathematical induction, that $0 < U_n \leq 1$

2) prove that (U_n) is decreasing

3) Deduce that (U_n) is convergent and determine its limit.

) Part A : Given the function g defined over $]-1,+\infty[$ by $g(x)=x^2+2x+4-2\ln(x+1)$

1) show that $g'(x)=\frac{2x(x+2)}{x+1}$
2) study the variations of g
3) Deduce that $g(x)>0$ for all $x\in]-1,+\infty[$

Part B : Consider the function f defined over $]-1,+\infty[$ by $f(x)=x-\frac{1-2\ln(x+1)}{x+1}$

Designate by (C) its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$.
(1 unit = 2cm)

1) a) Calculate $\lim_{x\to -1^+} f(x)$, and deduce an asymptote to (C).
   b) Calculate $\lim_{x\to+\infty} f(x)$ and verify that the line (d) of equation $y=x$ is an asymptote to (C)
   c) study according to the values of x the relative position of (C) and (d).
2) verify that $f'(x)=\frac{g(x)}{(x+1)^2}$ and set up the table of variations of f.
3) show that the equation $f(x)=0$ admits a unique root α and verify that $0.3<\alpha<0.4$
4) a) the line (T) of equation $y=x+\frac{2}{e\sqrt{e}}$ is tangent to (C) at the point of abscissa x_0. Determine x_0.
   b) Calculate $f(0)$ and Draw (d), (T) and (C).
5) Calculate, in cm^2, the area of the region limited by (C), (d) and the two straight lines of equations $x=e-1$ and $x=e^2-1$

) A- Let g be the function defined on $]0;+\infty[$ as $g(x)=x^3-1+2\ln x$.

1) Determine $\lim_{x\to0} g(x)$ and $\lim_{x\to+\infty} g(x)$.
2) Calculate $g'(x)$ then set up the table of variations of g.
3) Calculate $g(1)$ then deduce the sign of $g(x)$ according to the value of x.

B- Consider the function f defined on $]0;+\infty[$ as $f(x)=x-\frac{\ln x}{x^2}$ and denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$. Let (d) be the line with equation $y=x$.

1) Determine $\lim_{x\to0} f(x)$ and deduce an asymptote to (C).
2) a- Discuss, according to the values of x, the relative positions of (C) and (d).
   b- Determine $\lim_{x\to+\infty} f(x)$ and show that (d) is an asymptote to (C).
3) a- Verify that $f'(x)=\frac{g(x)}{x^3}$ and set up the table of variations of f.
   b- Determine the point E on (C) where the tangent (Δ) to (C) is parallel to (d).
   c- Plot (d), (Δ) and (C).
4) Let α be a real number greater than 1. Denote by $A(\alpha)$ the area of the region bounded by (C), (d) and the two lines with equations $x=1$ and $x=\alpha$.
   a- Verify that $\int\frac{\ln x}{x^2}dx=\frac{-1-\ln x}{x}+k$, where k is a real number.
   b- Express $A(\alpha)$ in terms of α.
   c- Using the graphic, show that $A(\alpha)<\frac{(\alpha-1)^2}{2}$

8) **part A**: Given the function g defined over $]0,+\infty[$ by $g(x) = 2x^2+1-\ln(2x)$

1) Calculate $\lim\limits_{x\to 0^+} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$
2) Calculate $g'(x)$ and set up the table of variations of g.
3) Deduce the sign of $g(x)$ over $]0,+\infty[$

Part B: Consider the function f defined over $]0,+\infty[$ by $f(x) = 2x + \frac{\ln(2x)}{x}$ and designate by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$. 1 unit = 2cm.

1) a) Calculate $\lim\limits_{x\to 0^+} f(x)$. Deduce an asymptote to (C).
   b) Calculate $\lim\limits_{x\to +\infty} f(x)$ and verify that the line (Δ) of equation $y = 2x$ is an asymptote to (C).
   c) Study according to the values of x the relative position of (C) and (Δ).
2) verify that $f'(x) = \frac{g(x)}{x^2}$ and set up the table of variations of f.
3) show that the equation $f(x)=0$ has a unique solution α and verify that $0.37 < \alpha < 0.38$
4) Determine the coordinates of point A of (C) where the tangent (T) to (C) at A is parallel to (Δ).
5) verify that $g(\alpha) = 4\alpha^2+1$.
6) Draw (Δ), (T) and (C).
7) Calculate, in cm^2, the area of the domain limited by (C), the line (Δ) and the two vertical lines of equations $x = \frac{1}{2}$ and $x = \frac{e}{2}$

) Let f and g be two functions defined on $]0;+\infty[$ by:

$$f(x) = \frac{x^2-1+\ln x}{x} \quad \text{and} \quad g(x) = x^2+2-\ln x.$$

Denote by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

A- 1) Calculate $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.

2) Calculate $g'(x)$ and set up the table of variations of g.

3) Determine the sign of g(x) on $]0;+\infty[$.

B- 1) a - Calculate $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to +\infty} f(x)$.

b - Show that the line (L) with equation $y = x$ is asymptote to (C) and study, according to the values of x, the relative positions of (C) and (L).

2) a - Show that $f'(x) = \frac{g(x)}{x^2}$

b - Set up the table of variations of f.

3) Show that the equation $f(x) = 3$ has a unique solution α belonging to $]2.9;3[$.

4) Draw (L) and (C).

5) Calculate the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 1$ and $x = e$.

XI. Consider the function f that is defined, on $]0;+\infty[$, by: $f(x)=(\ln x)^2+2\ln x-3$

Designate by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a) Calculate $\lim_{x\to+\infty} f(x)$.

b) Calculate $\lim_{x\to 0} f(x)$ and deduce an asymptote to (C).

2) Determine the abscissas of the points of intersection of (C) with the axis of abscissas.

3) a) Calculate $f'(x)$ and set up the table of variations of f.

b) Verify that $f''(x)=\frac{-2\ln x}{x^2}$; Show that (C) has a point of inflection I, and write an equation of the tangent (d) to (C) at the point I.

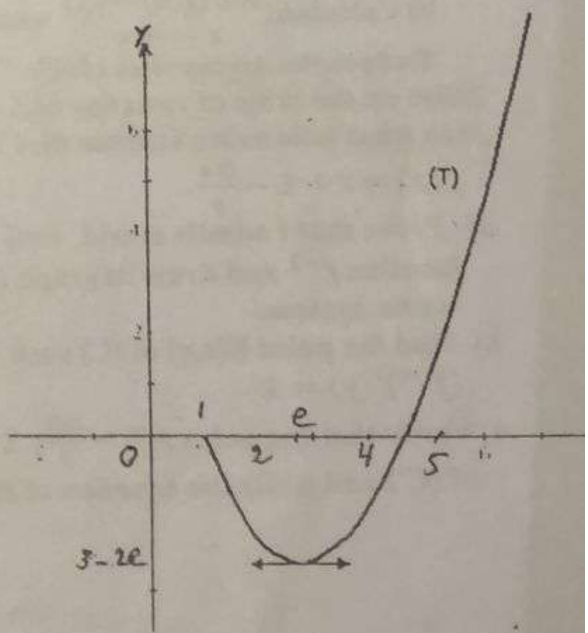
4) ~~Draw the line (d) and the curve (C).~~

5) a) Prove that the function f has, on $[1;+\infty[$, an inverse function g and determine the domain of definition of g.

b) Verify that the point A(5 ; e^2) belongs to (G), the representative curve of g, and write an equation of the tangent to the curve (G) at A.

6) Determine graphically, according to the values of the real number m, the number of roots of the equation $(\ln x)^2+2\ln x=m$.

7) The curve (T) shown below is the representative curve, on $[1;+\infty[$ of a function F, where F is a primitive (anti derivative) of the function f: Calculate the area of the region that is bounded by the curve (C), the axis of abscissas and the two lines of equations $x=1$ and $x=e$.



XII. Consider the function f defined over $I =]0, 2[$ by $f(x) = \ln\left(\frac{2-x}{x}\right)$.

And designate by (C) its representative curve in an orthonormal system $(o, \vec{i}, \vec{j})$. Unit: 2cm.

1) Determine the limits of f at the bounds of the interval I and deduce the asymptotes of (C).

2) a) show that the point W(1,0) is a center of symmetry for (C).

b) Verify that W is a point of inflection of (C) and write an equation of the tangent (d) to (C) at the point W.

3) Set up the table of variations of f and draw (d) and (C).

4) Prove that the equation $f(x) = x$ admits a unique solution α and show that: $0.6 < \alpha < 0.8$

5) a) Prove that f admits, on I, an inverse function g and verify that:

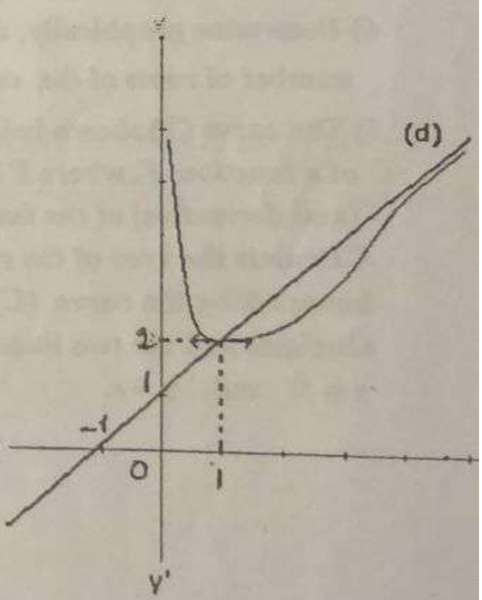
$$g(x) = \frac{2}{e^x+1}$$

b) Designate by (G) the representative curve of g. Prove that (G) admits a center of symmetry which is to be determined.

c) Prove that α is the unique solution of the equation $g(x) = x$.

XIII.

The adjacent curve (C) represents a function f defined Over $]0, +\infty[$ and (d) is the line of equation $y = px + q$



1) a) Verify that p = q = 1

b) Calculate $\lim\limits_{x \to +\infty} (f(x) - y_d)$ and $\lim\limits_{x \to 0} f(x)$.

Deduce the asymptotes of (C).

2) Set up the table of variation of f.

3) In what follows we suppose that :

$f(x) = x + 1 - \frac{\ln x}{x}$

a) Prove that f admits over $[1, +\infty[$ an inverse function f^{-1} and draw its graph (C') in the same system.

b) Find the point E(x,y) of (C) such that $(f^{-1})'(y) = 1$

4) Verify that the point $A(3 - \frac{\ln 2}{2}; 2)$ is a point of (C') and write the equation of the tangent to (C') at A.

<table>
<tr><td>10</td><td>The solution (S) of the equation
$\ln(3+2x-x^2)=0$ is (are)</td><td>$x=1+\sqrt{3}$</td><td>$x=1-\sqrt{3}$</td><td>$x=1\pm\sqrt{3}$</td></tr>
<tr><td>11</td><td>The equation $ln^2x+\ln x-6=0$ has two roots x_1 and x_2. The product x_1x_2 is equal to</td><td>-6</td><td>e^{-1}</td><td>e^{30}</td></tr>
<tr><td>12</td><td>The function g defined on $]-\infty,1]$ by $g(x)=\ln(1+\sqrt{1-x})$ is</td><td>A strictly increasing function</td><td>A strictly decreasing function</td><td>A negative function</td></tr>
<tr><td>13</td><td>The domain of definition of the function $f(x)=\frac{\ln x}{x-1}$ is</td><td>$]0,1[$</td><td>$]0,1[\cup]1,+\infty[$</td><td>$]0,+\infty[$</td></tr>
<tr><td>14</td><td>The inequality $\ln x<1$ is verified for</td><td>$x<0$</td><td>$0<x<e$</td><td>$x>e$</td></tr>
<tr><td>15</td><td>$\lim\limits_{x\to+\infty}\frac{(x-1)e^x}{x}=$</td><td>$+\infty$</td><td>$-\infty$</td><td>0</td></tr>
<tr><td>16</td><td>x is a real number.
The number of solutions of the equation
$e^{2x}-2e^x-3=0$ is</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>17</td><td>For all real numbers $a>0$.
$\ln(a\sqrt{e})+2\ln(e\sqrt{a})+\ln\left(\frac{1}{a^2}\right)=$</td><td>$\frac{5}{2}$</td><td>$4\ln a$</td><td>$\ln a$</td></tr>
<tr><td>18</td><td>Consider the function f defined and continuous on $\mathbb{R}$ as $f(x)=e^{-2x}$.
The image of the interval $[0,+\infty[$ by f is</td><td>$[0,1]$</td><td>$]0,1]$</td><td>$]0,+\infty[$</td></tr>
<tr><td>19</td><td>The solution of the equation 2ln x=ln 25 is</td><td>5</td><td>$\frac{25}{2}$</td><td>-5</td></tr>
<tr><td>20</td><td>Consider the function f defined over $]e,+\infty[$ as $f(x)=x-3-\frac{3\ln x}{1-\ln x}$ and denote by (C) its curve in an orthonormal system $(O;\vec{i},\vec{j})$.
(C) admits two asymptotes with equations</td><td>x=1
and
y=x-3</td><td>x=e
and
y=x</td><td>x=-e
and
y=x-3</td></tr>
<tr><td>21</td><td>The table below is the table of variations of a continuous function f over $[0,+\infty[$.
<table><tr><td>x</td><td>0</td><td></td><td>1</td><td></td><td>e</td><td></td><td>$+\infty$</td></tr><tr><td>f'(x)</td><td></td><td>-</td><td>0</td><td>+</td><td>0</td><td>-</td><td></td></tr><tr><td>f(x)</td><td>5</td><td>$\searrow$</td><td>-2</td><td>$\nearrow$</td><td>3</td><td>$\searrow$</td><td>-1</td></tr></table>The image of the interval $I=[1,+\infty[$ by f is</td><td>$[-2,3]$</td><td>$[-2,-1[$</td><td>$]-1,3]$</td></tr>
</table>

Logarithmic Functions

0

I) Simplify each of the following expressions

1) $\ln 27 - \ln 3 - \ln 2 + \ln 8$
2) $\ln\left(\frac{8}{27}\right) + 2\ln 4 + 3\ln 3$
3) $\ln e^2 - 2\ln\sqrt{e} - \ln e^{-3}$
4) $\ln\left(\frac{1}{e^2}\right) + 4\ln(e\sqrt{e})$
5) $\ln(2+\sqrt{3})^{30} + \ln(2-\sqrt{3})^{30}$
6) $\ln\sqrt{e^3} - 3 + 2\ln\sqrt{e^5}$
7) $\ln\sqrt{75} - \ln 15 + \ln\sqrt{135} - \ln\sqrt{27}$
8) $\ln\sqrt{2} + \ln 4 - \ln\frac{1}{8} + 2\ln\sqrt[3]{2}$
9) $\ln e\sqrt{e} - \frac{1}{2}\ln\frac{1}{e^2} + \ln e^{-3}$
10) $\ln 2 + \ln(2+\sqrt{2}) + \ln(2+\sqrt{2+\sqrt{2}}) + \ln(2-\sqrt{2+\sqrt{2}})$

II) Evaluate each of the following limits.

1) $\lim\limits_{x\to+\infty} \ln\left(\frac{2x+1}{x-1}\right)$
2) $\lim\limits_{x\to1} \frac{\ln x}{x-1}$
3) $\lim\limits_{x\to0} \frac{\ln(x+1)}{x}$
4) $\lim\limits_{x\to0^+} x(1-\ln x)$
5) $\lim\limits_{x\to0} \frac{\ln(1+2x)}{x^2}$
6) $\lim\limits_{x\to+\infty} \frac{1+\ln x}{\ln x}$
7) $\lim\limits_{x\to0}(ln^2 x - \ln x)$
8) $\lim\limits_{x\to e} \frac{\ln x - 1}{x-e}$
9) $\lim\limits_{x\to1} \frac{\ln(1-x^2)}{\ln(x^2-4x+3)}$
10) $\lim\limits_{x\to+\infty}(x^2 - 3\ln x)$
11) $\lim\limits_{x\to+\infty}\left(\frac{x+\ln x}{x} + 1\right)$
12) $\lim\limits_{x\to+\infty}\left(\frac{\ln x}{x} + \frac{x^2-1}{2x}\right)$
13) $\lim\limits_{x\to0^+} x\ln(1+\frac{1}{x})$

III) In the table below, only one of the answers given to each question is correct. Choose, with justification, the correct answer.

N	Question	Answers		
		a	b	c
1	The domain of definition of the function f defined by $f(x) = \ln(x^2-4)$ is	$]2,+\infty[$	$]-\infty,2[$	$]-\infty,-2[\cup]2,+\infty[$
2	$\lim\limits_{x\to+\infty} x\ln(1+\frac{1}{x}) =$	1	0	$+\infty$
3	For $x > 2$, the solution of the equation $\ln(x-2) = 2$ is	4	e^2+2	e^4
4	The domain of definition of the function f given by $f(x) = \frac{\ln(x+2)}{x-1}$ is	$[-2,+\infty[$	$]-2,1[\cup]1,+\infty[$	$]-\infty,1[\cup]1,+\infty[$
5	A solution of the equation $3(\ln x)^2 + 2\ln x - 5 = 0$ is	$x = e^{-\frac{5}{3}}$	$x = e^{-3}$	$x = e^5$
6	$\lim\limits_{x\to+\infty} \frac{x\ln x}{x+e} =$	$+\infty$	1	$\frac{1}{e}$
7	The solution of the inequality $(\ln x)^2 < -\ln x$ is	$]e^{-1},1[$	$]0,e^{-1}[\cup]1,+\infty[$	$]0,1[$
8	$\ln(x+2) > 0$ for $x \in$	$]-2,+\infty[$	$]-1,+\infty[$	$]-\infty,+\infty[$
9	Let $f(x) = x\ln x - x + 1$ then $f([1,e]) =$	$[0,1]$	$\{1\}$	$]-\infty,1[$

IX. Part A: Given the function g defined over $]0; +\infty[$ by $g(x) = -x^2 - 4 + 4\ln x$.

1) Study the variation of g.
2) Deduce that $g(x)<0$ for all $x \in]0; +\infty[$

Part B: Given the function f defined by $f(x) = 2 - x - \frac{4\ln x}{x}$ over $]0; +\infty[$, and designate by (C) its representative curve in an orthonormal system $(O, \vec{i}, \vec{j})$ 1unit=2cm.

1)
   a) Calculate $\lim_{x \to 0^+} f(x)$, and deduce an asymptote to (C).
   b) Calculate $\lim_{x \to +\infty} f(x)$ and verify that the line (d) of equation $y = -x+2$ is an asymptote to (C).
   c) Study according to the values of x the relative position of (C) and (d).
2) Verify that $f'(x)$ has the same sign as $g(x)$, and set up the table of variation of f.
3) Show that the equation $f(x)=0$ admits a unique solution α and verify that $1 < \alpha < \frac{3}{2}$.
4) Determine the coordinates of the point A where the tangent (T) at A to (C) is parallel to (d).
5) Construct (T), (d) and (C).
6) Calculate in cm^2, the area of the domain limited by (C), (d) and the two vertical lines $x=2$ and $x=e$.
7)
   a) Show that f admits on $]0; +\infty[$ an inverse function h whose domain is to be determined.
   b) Show that $f(x)=h(x)$ admits a unique root.

X. Consider the function f defined by $f(x) = \ln(\ln x)$.

1) Determine the domain I of definition of f.
2) Prove that f is strictly increasing.
3) Determine limits of f at the boundaries of I.
4) a) Show that there exists a unique real number x_0 belonging to the interval $]1, e[$ such that $f(x_0) = -1$
   b) Draw the curve of f in an orthonormal system.

VII. Let f be the function defined on $]0;+\infty[$, by $f(x)=\frac{1}{2}x+\frac{1+\ln x}{x}$ and (C) be its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$

1) Prove that the line of equation $x=0$ is an asymptote of (C).
2) a) Calculate $\lim\limits_{x\to+\infty} f(x)$ and verify that the line *(d)* of equation $y=\frac{1}{2}x$ is an asymptote of *(C)*.
   b) Determine the coordinates of E, the point of intersection of the line (d) with the curve (C)
3) Verify that $f'(x)=\frac{x^2-2\ln x}{2x^2}$
4) The adjacent table is the table of variations of the function h defined by $h(x)=x^2-2\ln x$

x	0		1		$+\infty$
$h'(x)$		$-$	0	$+$	
$h(x)$	$+\infty$	$\searrow$	1	$\nearrow$	$+\infty$

   a) Verify that f is strictly increasing on $]0;+\infty[$
   b) Consider, on the Curve (C), a point W of abscissa 1. Write an equation of the line (D) tangent to (C) at the point W.
5) Draw the curve (C) and the lines (d) and (D), and plot the points E and W.
6) Calculate the area of the region bounded by the curve (C), The asymptote (d) and the straight lines of equations $x=1$ and $x=e$.

VIII. Let f be the function defined, on $]0;+\infty[$ by $f(x)=x+2\frac{\ln x}{x}$. (C) its the representative curve of f in an orthonormal system $(o,\vec{i},\vec{j})$; unit: 2cm.

1) a – Calculate $\lim\limits_{x\to 0} f(x)$ and give its graphical interpretation.
   b – Determine $\lim\limits_{x\to+\infty} f(x)$ and verify that the line (d) of equation $y=x$ is an asymptote of (C).
   c – Study according to the values of x, the relative position of (C) and (d).
2) The table below is the table of variations of the function f', the derivative of f.

x	0		e		$e\sqrt{e}$		$+\infty$
$f''(x)$		$-$		$-$	0	$+$	
$f'(x)$	$+\infty$	$\searrow$	1	$\searrow$	$1-e^{-3}$	$\nearrow$	1

   a – Show that f is strictly increasing on its domain of definition, and set up its table of variations.
   b – Write an equation of the line (D) that is tangent to (C) at the point G of abscissa e.
   c – Prove that the curve(C) has a point of inflection L.
   d – Show that the equation $f(x)=0$ has a unique root α and verify that $0,75<\alpha<0,76$
3) Draw (D), (d) and (C).
4) Calculate, in cm^2, the area of the region bounded by the curve (C), the line (d) and the two lines of equations $x=1$ et $x=e$.