Grade 12 GS **Exponential Functions (Extra sheet)** **2023-2024**

Exercise 1

In the table below only one of the proposed answers to each question is correct , choose the correct one with justification.

N⁰	Questions	Proposed answers		
		a	**b**	**c**
1	$e^{\frac{1}{2}ln9} \times e^{-\ln\frac{1}{3}} =$	e^3	9	$e^{\frac{3}{2}}$
2	Consider the function f given by $f(x) = ln\left(\frac{e^x}{e^x - 2}\right)$. The domain of definition of f is	$]\ln 2, +\infty[$	$]0, +\infty[$	$]-\infty, +\infty[$
3	For all real numbers x, $\frac{e^{-x}}{e^{-x}+2}$ is equal to	$\frac{1}{3}$	$\frac{1}{1+2e^x}$	$\frac{-e^x}{-e^x+2}$
4	For all real numbers x, $ln(e^x + 2) - x$ is equal to	$\ln\left(\frac{e^x+2}{x}\right)$	$\ln 2$	$\ln\left(\frac{e^x+2}{e^x}\right)$
5	The equation $e^{2x} + 2e^x - 1 = 0$ has in the set $\mathbb{R}$	2 distinct roots	No roots	Only one root
6	$\lim\limits_{x\to+\infty} \frac{\ln(e^x+1)}{x} =$	0	e	1
7	$\lim\limits_{x\to-\infty} (x + e^{-x}) =$	0	$-\infty$	$+\infty$

Exercise 2

Let f be the function defined on $\mathbb{R}$ as $f(x) = e^{2x} + (x-3)e^x$.

Denote by (C) the representative curve of f in an orthonormal system (O ; i ; j).

1) Determine $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to+\infty} \frac{f(x)}{x}$
2) Determine $\lim\limits_{x\to-\infty} f(x)$ and deduce an asymptote to (C).

3) Let g be the function defined on $\mathbb{R}$ as $g(x) = x - 2 + 2e^x$.
   a- Set up the table of variations of the function g.
   b- Calculate $g(0)$ then deduce, according to the values of x, the sign of $g(x)$.
4) Verify that $f'(x) = e^x g(x)$ and set up the table of variations of the function f.
5) Show that the equation $f(x) = 0$ has, on , a unique root α. Verify that $0.7 < \alpha < 0.8$.
6) Draw the curve (C).

Exercise 3

Consider the function f defined on $\mathbb{R}$ as $f(x) = 2 - (x+2)e^{-x}$.

Denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$

1) a- Determine $\lim_{x \to -\infty} f(x)$.

   b-Determine $\lim_{x \to +\infty} f(x)$. Deduce an asymptote (d) to (C).

2) a- Calculate $f'(x)$, and set up the table of variations of f.

   b-Show that the equation $f(x) = 0$ has two roots α and 0.Verify that $-1.6 < \alpha < -1.5$

3) a- Show that (C) has an inflection point whose coordinates are to be determined.

   b- Write an equation of (Δ), the tangent to (C) at its inflection point.

4) Let (d') be the line with equation $y = -x$.

   a-Verify that $f(x) + x = (x+2)(1 - e^{-x})$

   b- Study, according to the values of x, the relative positions of (d') and (C).

5) Draw (d), (Δ), (d') and (C).

6) Let g be the function defined as $g(x) = ln(-x - f(x))$.

   Determine the domain of definition of g.

Exercise 4

Part A

Let $g(x) = e^{2x} - 2e^{x} + 3$, and let (G) be its curve in $(O ; \vec{i}, \vec{j})$. Show that For all x in $\mathbb{R}$, $g(x) > 0$.

Part B

Let f be the function defined over $\mathbb{R}$by $f(x) = \ln(e^{2x} - 2e^{x} + 3)$ and (C) be its curve.

1) Determine $\lim_{x \to -\infty} f(x)$. Deduce an asymptote (d) to (C).
2) Discuss according to the values of x , the position of (C) with respect to (d).
3)Determine $\lim_{x \to +\infty} f(x)$. Deduce that $(d') : y = 2x$ is an oblique asymptote to (C).
4)Determine the coordinates of the intersection points of (d') and (C).
5)Calculate $f'(x)$ and set up the table of variation of f.
6) Trace (C).

Exercise 5

Consider the function f defined on $\mathbb{R}$ as $f(x) = (x + 2)e^{-x} + 1$.

Denote by (C) the representative curve of f in an orthonormal system (O;i , j).

1) a-Determine $\lim_{x \to +\infty} f(x)$. Deduce an asymptote (d) to (C).

   b- Find the coordinates of the point of intersection of (C) and (d).

2) Determine $\lim_{x \to -\infty} f(x)$ and calculate $f(-2.5)$.

3) Verify that $f'(x) = -(x + 1)e^{-x}$ and set up the table of variations of f.

4) a- Show that the equation $f(x) = 0$ has a unique root α on $\mathbb{R}$.

   b- Verify that $-2.2 < \alpha < -2.1$.

5) a- Prove that the point $I\ (0\ , 3)$ is the point of inflection of the curve (C).

   b- Determine an equation of (T), the tangent to (C) at I.

   c- The table below is the table of variations of the function g defined as $g(x) = (x + 2)e^{-x} + x - 2$

x	$-\infty$	0	$+\infty$
$g(x)$	$-\infty$	0	$+\infty$

Deduce, according to the values of x, the relative positions of (C) and (T).

6) Draw (d), (T) and (C).

7) Let k be the function given by $k(x) = \frac{x}{ln(-x-2)}$.

Denote by (C') the representative curve of k in an orthonormal system(O;i , j).

a- Determine the domain of definition of k.

b- Show that $k'(\alpha) = \frac{\alpha + 1}{\alpha^2 + 2\alpha}$.

c- Show that the tangent to (C') at the point with abscissa α intersects the $y - axis$ at the point $W(0\ ;\ \frac{1}{\alpha+2})$.

Exercise 6

Part A

Consider the function h defined on IR by: $h(x) = e^{2x} + 2e^{x} - 2$.

1) Solve the equation $h(x) = 0$.
2) Calculate $\lim_{x \to -\infty} h(x)$ *and* $\lim_{x \to +\infty} h(x)$.
3) a- Set up the table of variations of h.

   b- Draw the representative curve (H) of h in an orthonormal system.

Part B

Let g be the function defined on IR by $g(x) = \frac{e^{2x}+2}{e^{x}+1}$

and let f be a function defined by $f(x) = \ln\,[g(x)]$

Designate by (C) the representative curve of f in the plane referred to a new orthonormal system (O;i , j). ; (graphical unit: 2 cm).

1) a- Show that f is defined for every real number x.

   b- Calculate $\lim_{x \to -\infty} f(x)$ and deduce an asymptote (d) of (C).

2) a- Show that $f(x) = x + \ln(\frac{1+2e^{-2x}}{1+e^{-x}})$

   b-Calculate $\lim_{x \to +\infty} f(x)$.and prove that the line (d') of equation $y = x$ is an asymptote to (C).

   c-Study, according to the values of x, the relative positions of (C) and (d ').

3) a- Prove that $g'(x) = \frac{e^{x}(h(x))}{(e^{x}+1)^2}$

   b-Show that $f\,'(x)$ *and* $h(x)$ have the same sign and set up the table of variations of f.

   c-Find the abscissa of the point on the curve (C) at which the tangent to (C) is parallel to (d').

4) Draw $(d), (d')$ *and* (C).

Exercise 7

Part A

Consider the function f defined on IR by $f(x) = e^{2x} - 4e^{x} + 3$

Designate by (C) its representative curve in an orthonormal system (O; i , j) .

1) a- Determine $\lim_{x \to -\infty} f(x)$, $\lim_{x \to +\infty} f(x)$ *and* $\lim_{x \to +\infty} \frac{f(x)}{x}$.

   b- Solve the equation $f(x) = 0$.

2) Calculate $f'(x)$ and set up the table of variations of f.
3) Show that O is a point of inflection of (C) .
4) Write an equation of the tangent (T) at O to (C).
5) Let h be the function defined on IR by $h(x) = f(x) + 2x$.

   a- Show that $h'(x) \geq 0$ for every real number x.

   b- Deduce, according to the values of x, the relative positions of (C) and (T).

6) Draw (T) and (C).

Part B

Let g be the function given by $g(x) = \ln[f(x)]$

Designate by (Ω) its representative curve in an orthonormal system.

1) Justify that the domain of definition of g is $]-\infty, 0[\cup]\ln 3, +\infty[$
2) Determine $\lim_{x \to -\infty} g(x)$. Deduce an asymptote (D) of (Ω) .
3) Show that the line (d) of equation $y = 2x$ is asymptote to (Ω) at $+\infty$.
4) Determine the coordinates of the points of intersection of (Ω) with (d) and (D).
5) Set up the table of variations of g .
6) Draw (Ω).

Exercise 8

Part A

Let h be the function defined over]0;+∞[by $h(x) = \frac{e^x - 1}{x}$. Denote by (C) curve of h in an (O ; i , j).

1)a-Verify that $h'(x) = \frac{(x-1)e^x + 1}{x^2}$.

b-Let g be the function defined over]0;+∞[by $g(x) = (x - 1)e^x$

Set up the table of variations of g and deduce that h'(x) > 0.

2)a- Calculate $\lim_{x \to 0^+} h(x)$, $\lim_{x \to +\infty} h(x)$ and $\lim_{x \to +\infty} \frac{h(x)}{x}$.

b- Set up the table of variations of h .

3)a- Write an equation of (Δ) , the tangent to (C) at the point with abscissa 1.

b-Draw (Δ) and (C).

Part B

Consider the function f defined over]0 ; +∞[by $f(x) = h(x) + \ln x$ and denote by (Γ) be its curve.

1) a- Calculate $\lim_{x \to 0} f(x)$, $\lim_{x \to +\infty} f(x)$.

   b- Set up the table of variations of the function f .

2) a- Prove that the equation $f(x) = 0$ has a unique solution α and that $0.3 < \alpha < 0.4$.

   b- Compare $h(\alpha)$ and $h(1)$. Deduce that $\ln \alpha > 1 - e$.

3) a- Discuss, according to the values of x, the relative positions of (C) and (Γ).

   b- Draw (Γ) .

4) A is a point on (C) and B is a point on (Γ) such that A and B have the same abscissa x .
   m is any real number such that m > 0. If AB = m, prove that there exist two values of x whose product is independent of m .

Exercise 9

Part A

Consider the function g defined on $\mathbb{R}$ as $g(x) = (2x-1)e^{2x} + 1$.

1) Calculate $g'(x)$ and set up the table of variation of g .
   (It is not required to find the limits of g at $-\infty$ and $+\infty$).
2) Deduce the sign of $g(x)$.

Part B

Let f be the function defined, on $\mathbb{R}$ as $\begin{cases} f(x) = \frac{e^{2x}-1}{x} & for\ x \neq 0 \\ 2 & for\ \ x = 0 \end{cases}$

Let (C) be its representative curve in an orthonormal system (O ; i , j).

1) Find $\lim_{x\to 0} f(x)$ and deduce that f is continuous at 0 .
2) Find $\lim_{x\to -\infty} f(x)$,deduce an asymptote to (C).
3) Find $\lim_{x\to +\infty} f(x)$ and $\lim_{x\to +\infty} \frac{f(x)}{x}$.
4) a) $\lim_{x\to 0} \frac{f(x)-2}{x}$
   b)Deduce that the line (T) with equation $y = 2x + 2$ is tangent to (C) at the point with abscissa 0.

5) Verify that $f'(x) = \frac{g(x)^2}{x^2}$ $for\ x\ \neq 0$ and set up the table of variations of f.

6) Plot (T) and (C).

Grade 12 GS **Logarithm Functions (Solution of Extra sheet)** **2023-2024**

Exercise 1

1) $e^{\frac{1}{2}ln9} \times e^{-\ln\frac{1}{3}} = e^{\ln 9^{\frac{1}{2}}} \times e^{\ln 3} = \sqrt{9} \times 3 = 9$ Answer: b

2) $f(x) = ln\,(\frac{e^x}{e^x-2})$. $\frac{e^x}{e^x-2} > 0$

Roots : $\frac{e^x}{e^x-2} = 0$ $then\ e^x = 0$ $impossible$ $e^x - 2 \neq 0$ $then\ x \neq \ln 2$

x	$-\infty$ ln 2 $+\infty$
$\frac{e^x}{e^x-2}$	$-$ ‖ $+$

$x \in]\ln 2 , +\infty[$ Answer b.

3) **1st method**

$\frac{e^{-x}}{e^{-x}+2} = \frac{1}{1+2e^x}$ cross multiplication

$e^{-x} + 2 = e^{-x}(1 + 2e^x)$

$e^{-x} + 2 = e^{-x} + 2$ Answer b

2nd method

$$\frac{e^{-x}}{e^{-x}+2} = \frac{\frac{1}{e^x}}{\frac{1}{e^x}+2} = \frac{\frac{1}{e^x}}{\frac{1+2e^x}{e^x}} = \frac{1}{1+2e^x}$$

4) $ln(e^x + 2) - x = ln(e^x + 2) - lne^x = ln(\frac{e^x+2}{e^x})$ answer c

5) $e^{2x} + 2e^x - 1 = 0$

$(e^x)^2 + 2e^x - 1 = 0$

$e^x = \frac{-2+\sqrt{8}}{2}$ $accepted\ then\ x = \ln\frac{-2+\sqrt{8}}{2}$ $e^x = \frac{-2-\sqrt{8}}{2}$ $rejected$

since $e^x > 0\ for\ any\ x$ One root Answer c

6) $\lim\limits_{x\to+\infty} \frac{\ln(e^x+1)}{x} = \frac{\ln +\infty}{+\infty} = \frac{+\infty}{+\infty}$

$\lim\limits_{x\to+\infty} \frac{[\ln(e^x+1)]'}{(x)'} = \lim\limits_{x\to+\infty} \frac{\frac{e^x}{e^x+1}}{1} = \lim\limits_{x\to+\infty} \frac{e^x}{e^x+1} = \frac{+\infty}{+\infty}$ $\lim\limits_{x\to+\infty} \frac{(e^x)'}{(e^x+1)'} = \lim\limits_{x\to+\infty} \frac{e^x}{e^x} = 1$ Answer c

7) $\lim\limits_{x\to-\infty}(x + e^{-x}) = -\infty + e^{-(-\infty)} = -\infty + \infty$

$\lim\limits_{x\to-\infty}(x + e^{-x}) = \lim\limits_{x\to-\infty}(x + \frac{1}{e^x}) = \lim\limits_{x\to-\infty} \frac{x\,e^x+1}{e^x} = \frac{0+1}{e^{-\infty}} = \frac{1}{0^+} = +\infty$ since $\lim\limits_{x\to-\infty} xe^x = 0$

Answer c

Exercise 2

1) $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} e^{2x} + (x-3)e^{x} = e^{+\infty} + (+\infty)e^{+\infty} = +\infty$

$\lim\limits_{x\to+\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \frac{e^{2x}+(x-3)e^{x}}{x} = \frac{+\infty}{+\infty}$

$\lim\limits_{x\to+\infty} \frac{(e^{2x}+(x-3)e^{x})'}{(x)'} = \lim\limits_{x\to+\infty} \frac{2e^{2x}+e^{x}+(x-3)e^{x}}{1} = +\infty$

asymptotic direction parallel to y-axis

2) $\operatorname{Lim}\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} e^{2x} + (x-3)e^{x} = \lim\limits_{x\to-\infty} e^{2x} + xe^{x} - 3e^{x}$

$= e^{-\infty} + 0 - 3e^{-\infty} = 0 \quad y = 0 \quad \textit{Horizontal asymptote}$ since $\lim\limits_{x\to-\infty} xe^{x} = 0$

3) a-$g'(x) = 1 + 2e^{x} \quad g'(x) > 0$

x	$-\infty$	$+\infty$
$g'(x)$	$+$	
$g(x)$	0 ↗	$+\infty$

b- $g(0) = 0$

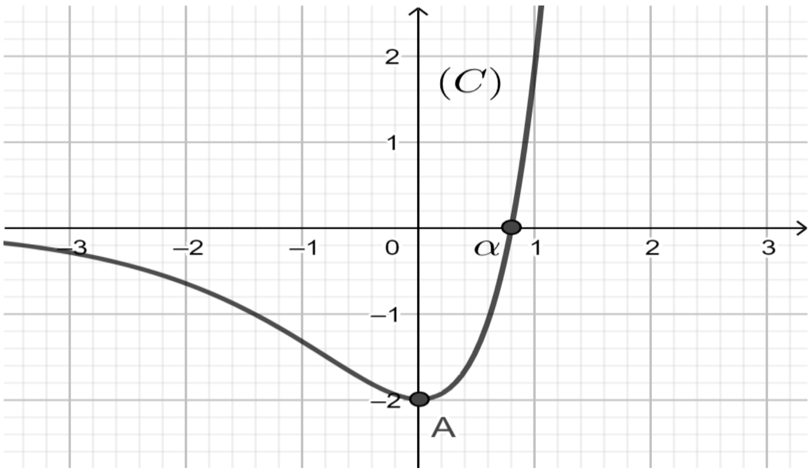
x	$-\infty$	0	$+\infty$
$g(x)$	$-$	0	$+$

4) $f'(x) = 2e^{2x} + e^{x} + (x-3)e^{x} = e^{x}(2e^{x} + 1 + x - 3) = e^{x}(2e^{x} + x - 2) = e^{x}g(x)$.

x	$-\infty$	0	$+\infty$
$f'(x)$	$-$	0	$+$
$f(x)$	0 ↘	-2	↗ $+\infty$

5) f is continuous
f is increasing
$f(0.7) = -0.6 \qquad f(0.7) < 0 \qquad and \qquad f(0.8) = 0.08 \qquad f(0.8) > 0$
Then the equation $f(x) = 0$ has, on , a unique root α. Verify that $0.7 < \alpha < 0.8$.

6)



Exercise 3

1) a- $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} 2-(x+2)e^{-x} = 2-(-\infty+2)e^{+\infty} = +\infty$

$\lim\limits_{x\to-\infty} \frac{f(x)}{x} = +\infty$ *asymptoic directio parallel* $y-axis$

b- $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} 2-(x+2)e^{-x} = 2-(+\infty+2)e^{-\infty} = -\infty(0)$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{(x+2)'}{(e^x)'} = \lim_{x\to+\infty} \frac{1}{e^x} = \frac{1}{+\infty} = 0$$

$\lim\limits_{x\to+\infty} f(x) = 2-0 = 2$ $\qquad$ (d): $y=2$ *horizontal asymptote*

2) a- $f'(x) = -\big(e^{-x}+(x+2)(-e^{x}) = -(e^{-x}(1-x-2)\big) = -e^{x}(-x-1) = e^{-x}(x+1).$

$f'(x)=0$ *then* $x=-1$

x	$-\infty$	-1	$+\infty$
$f'(x)$	$-$	0	$+$
$f(x)$	$+\infty$ ↘	$2-e$	↗ 2

b- f *is* … … ….

The equation $f(x) = 0$ has two roots α and 0.Verify that $-1.6<\alpha<-1.5$

3) a- $f''(x) = -e^{-x}(x+1)+e^{-x} = e^{-x}(-x-1+1) = -xe^{-x} = 0$

$x=0$ *then* $y=f(0)=0$ $(0,0)$

x	$-\infty$	0	$+\infty$
$f''(x)$	$+$	0	$-$

b- $(\Delta): y = ax + b$

$a = f'(0) = e^0(0+1) = 1$

$(\Delta): y = ax + b$

$0 = 1(0) + b \quad then \;\; b = 0$

$y = x$

4) a- $f(x) + x = (x+2)(1 - e^{-x})$

$2 - (x+2)e^{-x} + x = x - xe^{-x} + 2 - 2e^{-x}$

$2 - xe^{-x} - 2e^{-x} + x = x - xe^{-x} + 2 - 2e^{-x}$

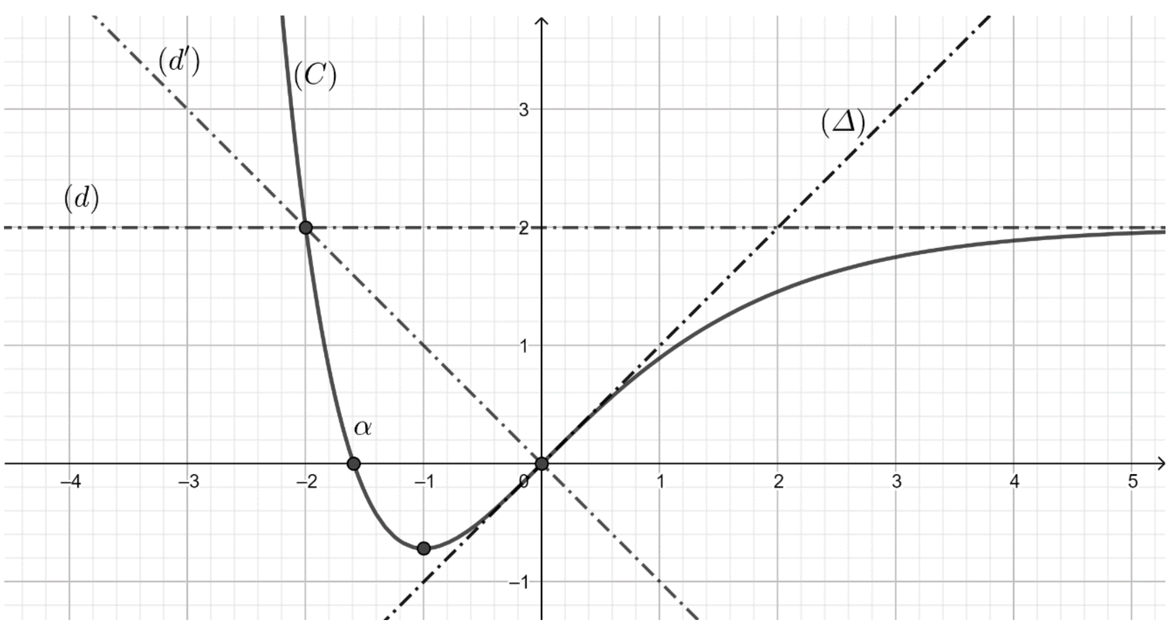
b- $f(x) - y_d = f(x) - (-x) = f(x) + x = (x+2)(1 - e^{-x}) = 0$

Then $x = -2 \quad and \;\; 1 - e^{-x} = 0 \;\; then \; e^{-x} = 1 \;\; then - x = \ln 1 = 0 \;\; then \; x = 0$

x	$-\infty$		-2		0		$+\infty$
$f(x) - y_d$		$+$	0	$-$	0	$+$	
$position$	(C) is above (d')			(C) is below (d')		(C) is above (d')	

(C) ∩ (d') at $(-2{,}2)$ (C) ∩ (d') at $(0{,}0)$

5)



6) $-x - f(x) > 0 \qquad \times(-1)$

$x + f(x) < 0$

$(x+2)(1 - e^{-x}) < 0$

Roots : $x+2=0$ $then\ x=-2$ $\quad 1-e^{-x}=0$ $then\ e^{-x}=1$ $then -x=0$ $then\ x=0$

x	$-\infty$		-2		0		$+\infty$
$(x+2)(1-e^{-x})$		$+$	0	$-$	0	$+$	

$x \in]-2{,}0[$

<u>Exercise 4</u>

Part A

$g'(x)=2e^{2x}-2e^{x}=0$ $\quad then \quad e^{x}=0\ \ rejected \quad or\ \ e^{x}=1\ \ then\ \ x=0$

x	$-\infty$		0		$+\infty$
$g'(x)$		$-$	0	$+$	
$g(x)$		$\searrow$	2	$\nearrow$	

min $g(x)=2$ $\qquad g(x)>0$

Part B

1) $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\ln(e^{-\infty}-2e^{-\infty}+3)=\ln 3$ $\qquad$ (d) : $y=\ln 3$ $\quad horizontal\ asymptote$

2) $f(x)-y_d=\ln(e^{2x}-2e^{x}+3)-\ln 3=\ln\left(\frac{e^{2x}-2e^{x}+3}{3}\right)=0$

Then $e^{\ln\left(\frac{e^{2x}-2e^{x}+3}{3}\right)}=e^{0}$ $\quad then \quad \frac{e^{2x}-2e^{x}+3}{3}=1$ $\quad then\ \ e^{2x}-2e^{x}+3=3$

Then $e^{2x}-2e^{x}=0$ $\quad then\ e^{x}=0$ $\quad rejected \quad and \quad e^{x}=2$ $\quad then\ \ x=\ln 2$

x	$-\infty$		$ln2$		$+\infty$
$f(x)-y_d$		$-$	0	$+$	
$position$		(C) is below (d)		(C) is above (d)	

(C) ∩ (d) at $(ln2, ln3)$

3) $\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\ln(e^{2x}-2e^{x}+3)=\ln(+\infty-\infty)$

$\lim\limits_{x\to+\infty}\ln(e^{x}(e^{x}-2)+3))=\lim\limits_{x\to+\infty}\ln\big((+\infty)(+\infty)+3\big)=+\infty$

$\lim\limits_{x\to+\infty} f(x)-y_{(d')}=\lim\limits_{x\to+\infty}\ln(e^{2x}-2e^{x}+3)-2x=\lim\limits_{x\to+\infty}\ln(e^{2x}-2e^{x}+3)-\ln e^{2x}$

$\lim\limits_{x\to+\infty}\ln\frac{e^{2x}-2e^{x}+3}{e^{2x}}=\lim\limits_{x\to+\infty}\frac{+\infty}{+\infty}$

$\lim\limits_{x\to+\infty}\ln\frac{(e^{2x}-2e^{x}+3)\prime}{(e^{2x})\prime}=\lim\limits_{x\to+\infty}\ln\frac{2e^{2x}-2e^{x}}{2e^{2x}}=\lim\limits_{x\to+\infty}\ln\frac{2e^{x}(e^{x}-2)}{2e^{2x}}=\lim\limits_{x\to+\infty}\ln\frac{(e^{x}-2)}{e^{x}}=\lim\limits_{x\to+\infty}\ln\frac{(e^{x}-2)\prime}{(e^{x})\prime}$

$=\lim\limits_{x\to+\infty}\ln\frac{e^{x}}{e^{x}}==\ln 1=0$

4) $f(x) - 2x = \ln\frac{e^{2x}-2e^{x}+3}{e^{2x}} = 0$ *then* $\frac{e^{2x}-2e^{x}+3}{e^{2x}} = 1$ *then* $e^{2x} - 2e^{x} + 3 = e^{2x}$

Then $2e^{x} = 3$ *then* $e^{x} = \frac{3}{2}$ *then* $x = \ln\frac{3}{2}$ *and* $y = 2x = 2ln\left(\frac{3}{2}\right)$ $(ln\frac{3}{2}, 2\ln\frac{3}{2})$

x	$-\infty$	$\ln\frac{3}{2}$	$+\infty$
$f(x) - y_{d'}$	$+$	0	$-$
position	(C) is above (d)		(C) is below (d)

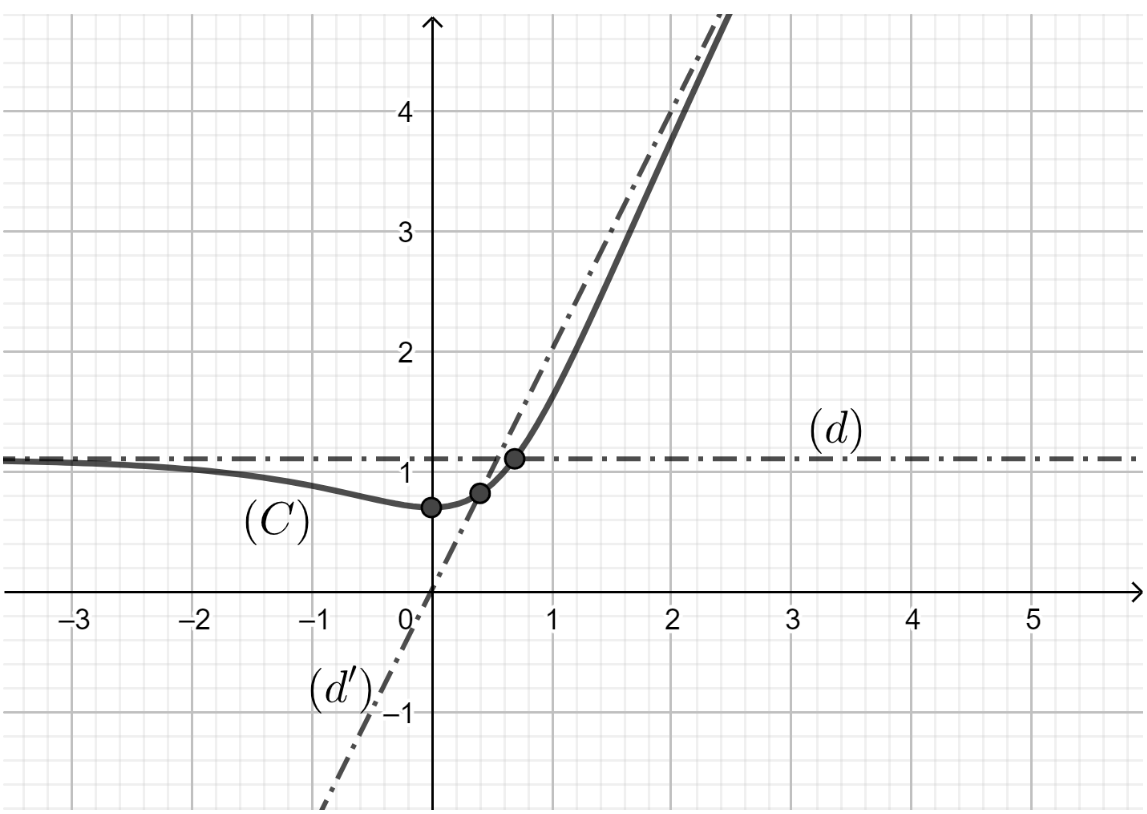
(C) ∩ (d) at $(ln\frac{3}{2}, 2\ln\frac{3}{2})$

5) $f'(x) = \frac{2e^{2x}-2e^{x}}{e^{2x}-2e^{x}+3} = 0$ *then* $2e^{2x} - 2e^{x} = 0$

then $e^{x} = 0$ *rejected and* $e^{x} = 1$ *then* $x = 0$

x	$-\infty$		0		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$	$\ln 3$	$\searrow$	$ln2$	$\nearrow$	$+\infty$

6)



Exercise 5

1) a- $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (x + 2)e^{-x} + 1 = (+\infty)(e^{-\infty}) + 1 = (+\infty)(0)$

$\lim_{x\to+\infty} \frac{(x+2)\prime}{(e^{x})\prime} + 1 = \lim_{x\to+\infty} \frac{1}{e^{x}} + 1 = \frac{1}{+\infty} + 1 = 1.$ (d) : y =1 horizontal asymptote

b- $f(x) - 1 = (x + 2)e^{-x} + 1 - 1 = (x + 2)e^{-x} = 0 \quad then \quad x = -2$

x	$-\infty$	-2	$+\infty$
$f(x) - y_d$	$-$	0	$+$
$position$	(C) is below (d)		(C) is above (d)

(C) ∩ (d) at $(-2,1)$

2) $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (x + 2)e^{-x} + 1 = (-\infty)(e^{+\infty}) + 1 = -\infty$

$f(-2.5) = (-2.5 + 2)e^{2.5} + 1 = 0.5e^{2.5} + 1 = -5.09$ $(-2.5, -5.09)$

3) $f(x) = (x + 2)e^{-x} + 1$

$f'(x) = e^{-x} + (x+2)(-e^{-x}) = e^{-x}\big(1 - (x+2)\big) = e^{-x}(1 - x - 2) = -(x + 1)e^{-x} = 0$

Then $x = -1$

x	$-\infty$		-1		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$	$-\infty$	$\nearrow$	$e+1$	$\searrow$	1

4) a- $f(x) is$ ……….. $so\ f(x) = 0$ has a unique root α on ℝ.

b- Verify that $-2.2 < \alpha < -2.1$.

5) a- $f''(x) = 0$ …………. $table\ of\ sign$ …….. I $(0\,,3)$ is the point of inflection of the curve (C).

b- (T) to (C) at I (T) : $y = -x + 3$

c-

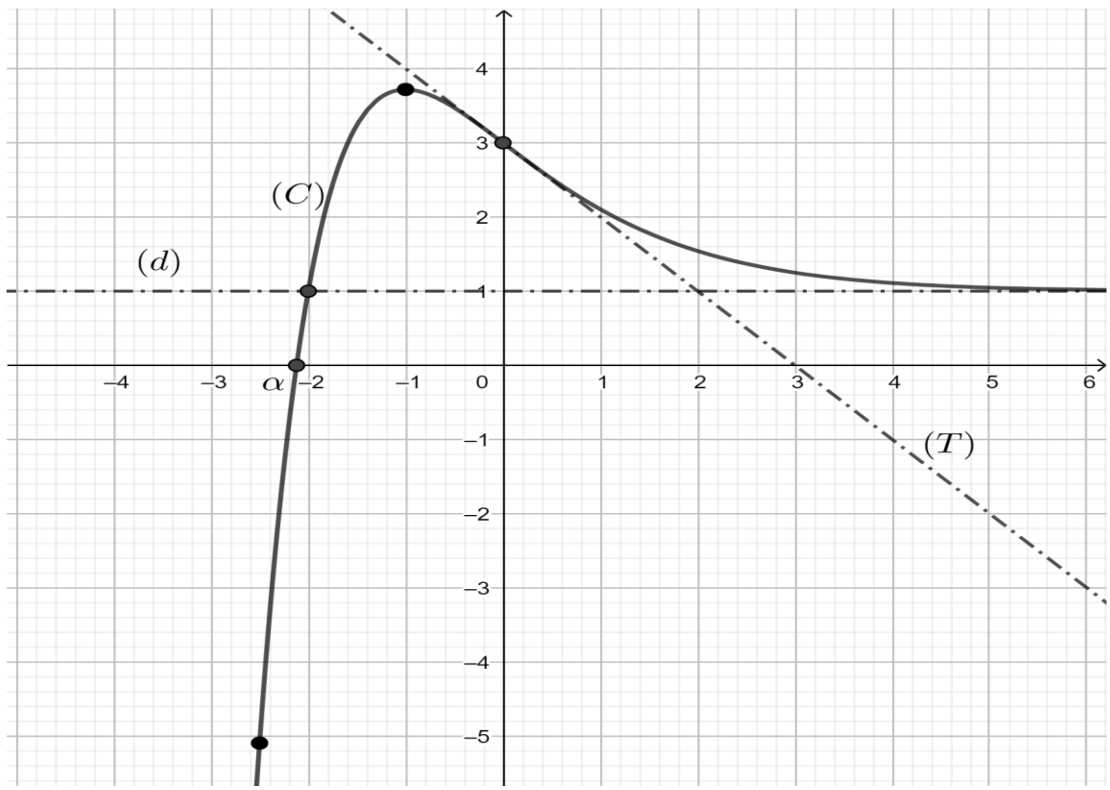
$$f(x) - y_T = (x + 2)e^{-x} + 1 — x + 3 = (x + 2)e^{-x} + 1 + x - 3$$

$$= (x + 2)e^{-x} + x - 2 = g(x)$$

x	$-\infty$	0	$+\infty$
$f(x) - y_T$	$-$	0	$+$
$position$	(C) is below (d)		(C) is above (d)

(C) ∩ (T) at (0,3)

6)



7)

a- $-x - 2 > 0$ $then$ $-x > 2$ $then$ $x < -2$

$ln(-x - 2) \neq 0$ $then - x - 2 \neq 1$ $then$ $-x \neq 3$ $then$ $x \neq -3$

$x \in]-\infty, -3[\cup] - 3, -2[$

b- $f(x) = 0$ has a unique root α on $\mathbb{R}$. Then $f(\alpha) = 0$ $then$ $(\alpha + 2)e^{-\alpha} + 1 = 0$

then $(\alpha + 2)e^{-\alpha} = -1$ $then$ $(\alpha + 2) = \frac{-1}{e^{-\alpha}}$ $then$ $(\alpha + 2) = -e^{\alpha}$ $then - \alpha - 2 = e^{\alpha}$

$$k'(x) = \frac{\ln(-x-2) - x\left(-\frac{1}{-x-2}\right)}{\ln^2(-x-2)} = \frac{\ln(-x-2) + \left(\frac{x}{-x-2}\right)}{\ln^2(-x-2)}$$

$$k'(\alpha)=\frac{\ln(-\alpha-2)+\left(\frac{\alpha}{-\alpha-2}\right)}{\ln^2(-\alpha-2)}=\frac{\ln(e^{\alpha})+\left(\frac{\alpha}{-\alpha-2}\right)}{\ln^2(e^{\alpha})}=\frac{\alpha+\left(\frac{\alpha}{-\alpha-2}\right)}{(\alpha)^2}=\frac{\frac{\alpha(-\alpha-2)}{-\alpha-2}+\left(\frac{\alpha}{-\alpha-2}\right)}{\alpha^2}=\frac{\frac{-\alpha^2-2\alpha+\alpha}{-\alpha-2}}{\alpha^2}$$

$$=\frac{\frac{-\alpha^2-\alpha}{-\alpha-2}}{\alpha^2}=\frac{\frac{\alpha^2+\alpha}{\alpha+2}}{\alpha^2}=\frac{\alpha^2+\alpha}{\alpha^2(\alpha+2)}=\frac{\alpha(\alpha+1)}{\alpha^2(\alpha+2)}=\frac{(\alpha+1)}{\alpha(\alpha+2)}=\frac{\alpha\ +1}{\alpha^2\ +\ 2\alpha}\ .$$

c- Let us find the equation of the tangent to (C') at the point with abscissa α

$$x=\alpha \quad then\ y=k(\alpha)=\frac{\alpha}{ln(-\alpha-2)}=\frac{\alpha}{ln(e^{\alpha})}=\frac{\alpha}{\alpha}=1$$

$$y=ax+b \quad with\ a=k'(\alpha)=\frac{\alpha+1}{\alpha^2+2\alpha}$$

$$y=ax+b$$

$$1=\frac{\alpha+1}{\alpha^2+2\alpha}(\alpha)+b \quad then\ b=1-\frac{\alpha+1}{\alpha^2+2\alpha}(\alpha)=1-\frac{\alpha^2+\alpha}{\alpha^2+2\alpha}=\frac{\alpha^2+2\alpha-\alpha^2-\alpha}{\alpha^2+2\alpha}=\frac{\alpha}{\alpha(\alpha+2)}=\frac{1}{\alpha+2}$$

$$y=\frac{\alpha+1}{\alpha^2+2\alpha}x+\frac{1}{\alpha+2}$$

$$W\left(0\,;\frac{1}{\alpha+2}\right)\in y-axis \quad since\ x=0$$

Let substitute the coordinates of this point in the tangent

$$\frac{1}{\alpha+2}=\frac{\alpha\ +1}{\alpha^2\ +\ 2\alpha}(0)+\frac{1}{\alpha+2}$$

$$\frac{1}{\alpha+2}=\frac{1}{\alpha+2}$$

<u>Exercise 6</u>

Part A

1) $x=-1-\sqrt{3} \qquad x=-1+\sqrt{3}$
2) $\lim\limits_{x\to-\infty}h(x)=-2 \qquad y=-2\ \ H.A. \qquad \lim\limits_{x\to+\infty}h(x)=+\infty$
3) a- $h'(x)=2e^{2x}+2e^{x} \qquad h'(x)>0$

x	$-\infty$	$+\infty$
$h'(x)$	+	
$h(x)$	-2 ↗	$+\infty$

b- Draw the representative curve (H) of h in an orthonormal system.

Part B

1) a- $\frac{e^{2x}+2}{e^x+1} > 0$ $since\ e^u > 0\ therefore$ f is defined for every real number x.

b- $\lim\limits_{x\to-\infty} f(x) = ln2$ (d) : $y = ln2$ $H.A.$

2) a- $f(x) = x + \ln\left(\frac{1+2e^{-2x}}{1+e^{-x}}\right) = 1 + \ln\left(\frac{1+\frac{2}{e^{2x}}}{1+\frac{1}{e^x}}\right) = 1 + \ln\left(\frac{\frac{e^{2x}+2}{e^{2x}}}{\frac{e^x+1}{e^x}}\right)$

$= \ln e^x + \ln\frac{e^x(e^{2x}+2)}{e^{2x}(e^x+1)} = \ln e^x + \ln\frac{(e^{2x}+2)}{e^x(e^x+1)} = \ln [e^x\ (\frac{(e^{2x}+2)}{e^x(e^x+1)}]$

$= \ln\frac{(e^{2x}+2)}{(e^x+1)} = \ln g(x).$

b- $\lim\limits_{x\to+\infty} f(x) = +\infty.$

$\lim\limits_{x\to+\infty} f(x) - x = \lim\limits_{x\to+\infty} x + \ln\left(\frac{1+2e^{-2x}}{1+e^{-x}}\right) - x = \lim\limits_{x\to+\infty} \ln\left(\frac{1+2e^{-2x}}{1+e^{-x}}\right) = \ln 1 = 0$

a- $f(x) - x = \ln\left(\frac{1+2e^{-2x}}{1+e^{-x}}\right) = 0$

$\frac{1+2e^{-2x}}{1+e^{-x}} = 1\quad then\ \ 1 + 2e^{-2x} = 1 + e^{-x}\quad then\ \ 2e^{-2x} - e^{-x} = 0$

Then $e^{-x} = 0\ \ rejected\ \ and\ e^{-x} = \frac{1}{2}\ \ then - x = \ln\frac{1}{2}\ \ then\ x = ln2$

…….

5) a- $g'(x) = \frac{e^x(h(x))}{(e^x+1)^2}$

b- $f\ (x) = \ln g(x) = \ln u\qquad f'(x) = \frac{u'}{u} = \frac{g'(x)}{g(x)}$

$g(x) > 0\quad since\ \ e^u > 0\quad so\ f'(x)\ \ has\ the\ same\ sign\ as\ g'(x)$

The sign of $g'(x)\ \ depends\ on\ h(x)\quad since\ e^u > 0$

Therefore $f'(x)\ \ and\ h(x)\quad have\ the\ same\ sign.$

c- tangent to (C) is parallel to (d').

$Slope\ tangent = slope\ (d')$

$f'(x) = 1$

$\frac{g'(x)}{g(x)} = 1\ then\ g'(x) = g(x)$

$\frac{e^x(e^{2x}+2e^x-2)}{(e^x+1)^2} = \frac{e^{2x}+2}{e^x+1}\qquad \times (e^x+1)\ both\ sides$

$\frac{e^x(e^{2x}+2e^x-2)}{e^x+1} = \frac{e^{2x}+2}{1}$

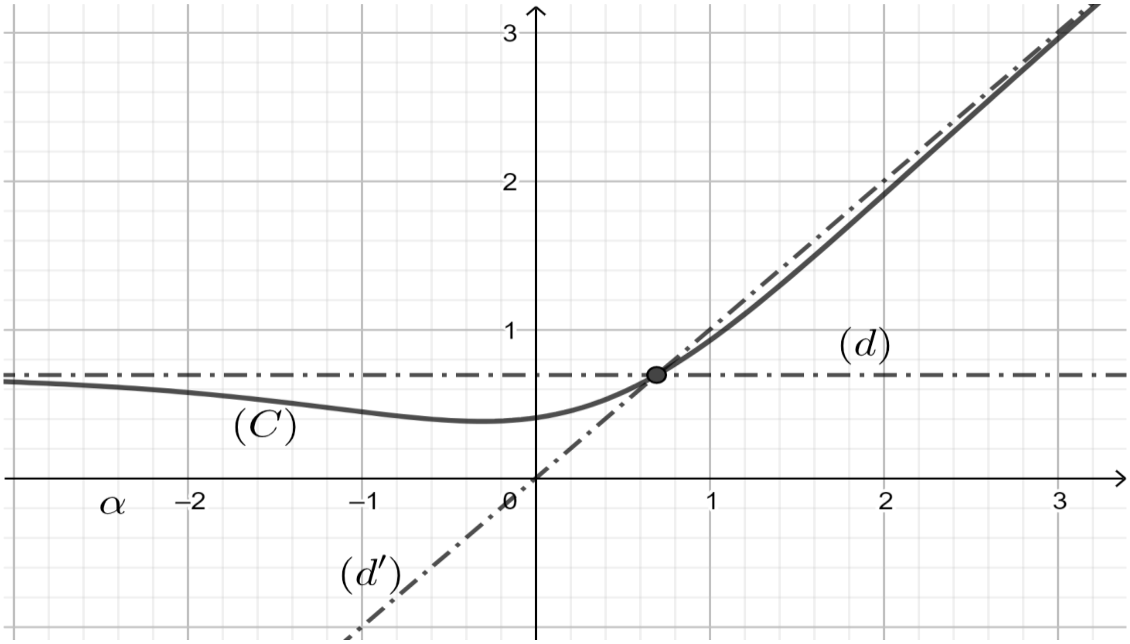
$e^{3x} + 2e^{2x} - 2e^x = e^{3x} + 2e^x + e^{2x} + 2$

$e^{2x} - 4e^x - 2 = 0$

$e^x = 2 + \sqrt{6}\qquad then\ \ x = \ln(2+\sqrt{6})\ or\ e^x = 2 - \sqrt{6}\quad rejected$

$y = f(\ln(2+\sqrt{6}) = \cdots..$

6)



Exercise 7

Part A

1) a- $\lim_{x\to-\infty} f(x) = 3 \quad y = 3$, $\lim_{x\to+\infty} f(x) = +\infty$

$\lim_{x\to+\infty} \frac{f(x)}{x} = +\infty \quad asymptotic\ direction\ parallel\ y - axis.$

b- $f(x) = 0 \quad then$ … ….

2) $f'(x) = 2e^{2x} - 4e^{x} = 0$ and set up the table of variations of f.

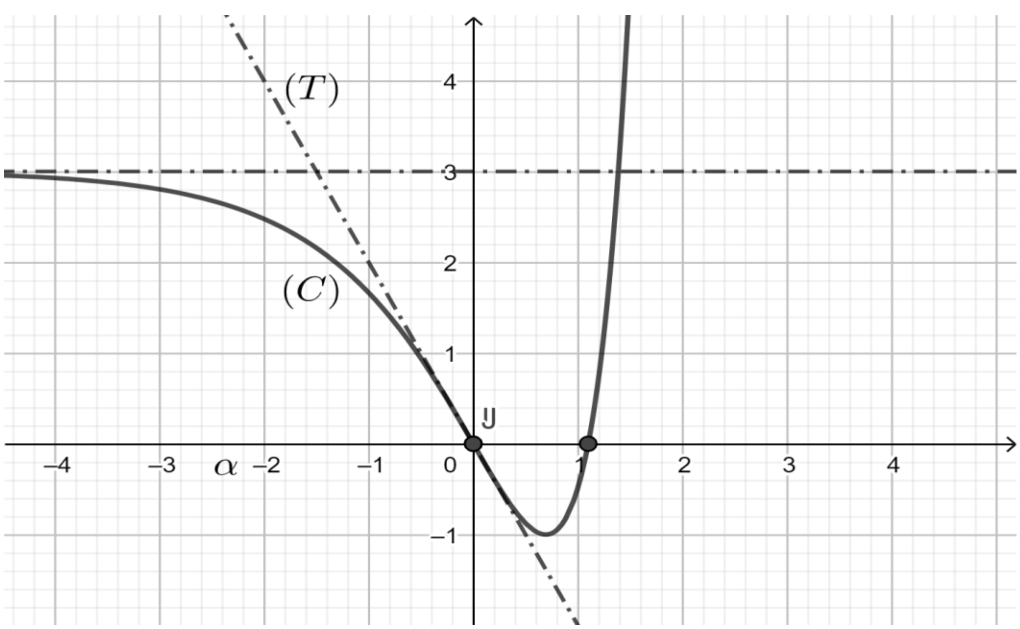
3) $f''(x) = 0$ … …. O is a point of inflection of (C) .

4)The tangent (T) at O to (C) : $y = -2x$

5) a-$h'(x) = 2e^{2x} - 2e^{x} + 2 \quad and\ set\ up\ the\ table\ of\ sign$ … …

b- Deduce, according to the values of x, the relative positions of (C) and (T).

6)



Part B

1) $f(x) > 0$ *then* $e^{2x} - 4e^{x} + 3 > 0$

Set up the table of sign ……

Then the domain of definition of g is $]-\infty, 0[\cup]\ln 3, +\infty[$

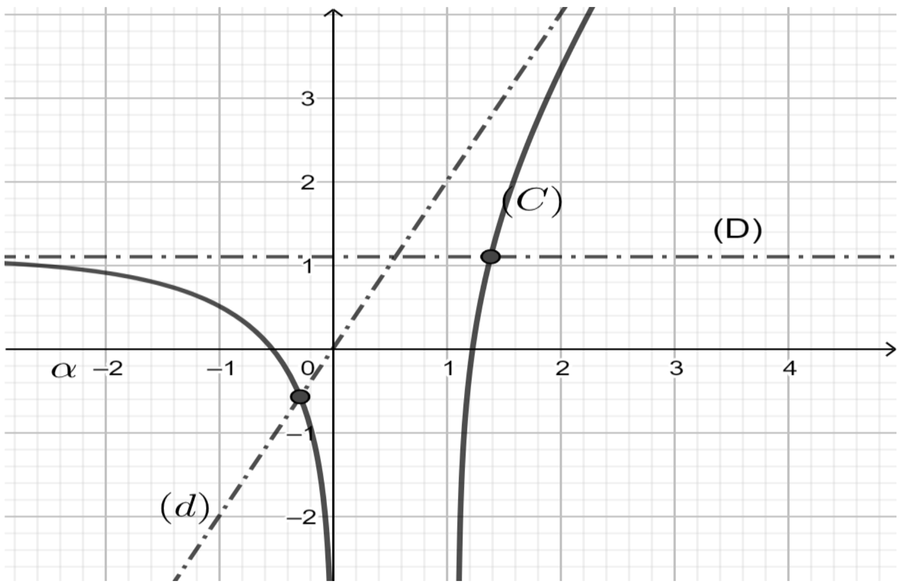
2) $\lim\limits_{x\to-\infty} g(x)$. Deduce an asymptote (D) of (Ω) .

3) Show that the line (d) of equation $y = 2x$ is asymptote to (Ω) at $+\infty$.

4) Determine the coordinates of the points of intersection of (Ω) with (d) and (D).

5) Set up the table of variations of g .

6)



Exercise 8

Part A

1)a-Verify that $h'(x) = \frac{(x-1)e^{x}+1}{x^2}$.

b-Let g be the function defined over]0;+∞[by $g(x) = (x-1)e^{x}$

Set up the table of variations of g and deduce that $h'(x) > 0$.

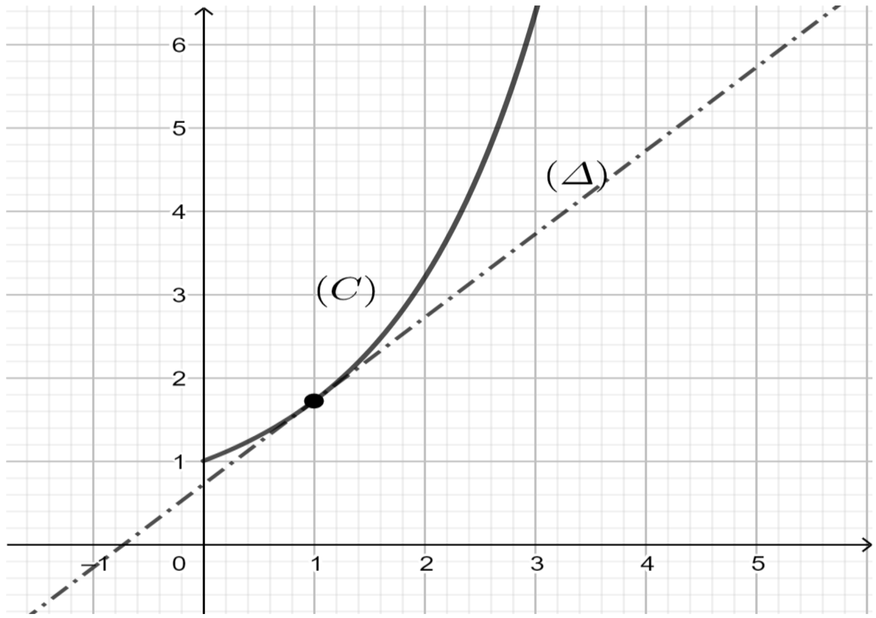
2)a- Calculate $\lim\limits_{x\to0^+} h(x)$, $\lim\limits_{x\to+\infty} h(x)$ *and* $\lim\limits_{x\to+\infty} \frac{h(x)}{x}$.

b-

x	0		$+\infty$
$h'(x)$	‖	+	
$h(x)$	‖ $-\infty$	↗	$+\infty$

3)a- (Δ) , the tangent to (C) at the point with abscissa 1 : $y = x + 0.72$

b-



Part B

1) a- $\lim_{x\to 0} f(x)$, $\lim_{x\to +\infty} f(x)$.

   b- $f'(x) = h'(x) + \frac{1}{x} = \frac{(x-1)e^x+1}{x^2} + \frac{1}{x} = \cdots$ ....

2) a- The equation $f(x) = 0$ has a unique solution α and that $0.3 < \alpha < 0.4$.

   b- $\alpha < 1$ *and h is increasing then*

   $h(\alpha) < h(1)$

   $f(x) = 0$ has a unique solution α

   $Then\ f(\alpha) = 0\ \ then\ \ h(\alpha) + \ln\alpha = 0\ \ then\ h(\alpha) = -\ln\alpha$

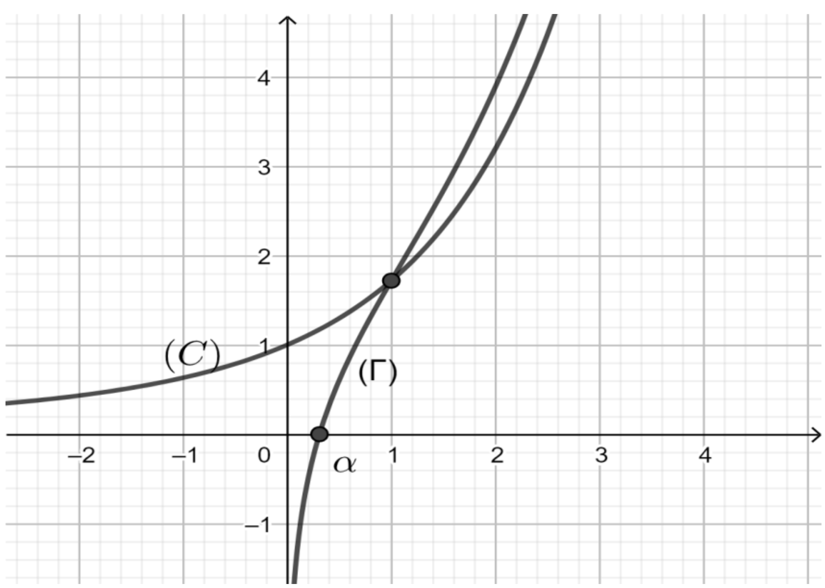
   $h(\alpha) < h(1)$

   $-\ln\alpha < e - 1 \qquad \times(-1)$

   $\ln\alpha > 1 - e$

3) a- Discuss, according to the values of x, the relative positions of (C) and (Γ).

   b-



4) $A\big(x, h(x)\big) \quad B(x, f(x))$

$$AB = m$$

$$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = m$$

$$\sqrt{(x - x)^2 + (f(x) - h(x))^2} = m$$

$$\sqrt{(h(x) + \ln x - h(x))^2} = m$$

$$\sqrt{(\ln x)^2} = m$$

$$|\ln x\,| = m$$

$$\ln x_1 = m \qquad \ln x_2 = -m$$

$$x_1 = e^m \qquad x_2 = e^{-m} \qquad x_1\ x_2 = e^m\ e^{-m} = e^{m-m} = e^0 = 1$$

<u>Exercise 9</u>

Part A

$g(x) = (2x - 1)e^{2x} + 1$.

1) $g'(x)$ and set up the table of variation of g .

2)The sign of $g(x)$.

Part B

1) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{e^{2x}-1}{x} = \frac{0}{0}$

$$\lim_{x\to 0^+} \frac{(e^{2x} - 1)'}{(x)'} = \lim_{x\to 0^+} \frac{2e^{2x}}{1} = 2$$

$$\lim_{x\to 0^-} f(x) = 2$$

$f(0) = 2$ *so* f is continuous at 0 .

2) $\lim\limits_{x\to -\infty} f(x) = 0 \qquad (d) : y = 0$

3) $\lim\limits_{x\to +\infty} f(x) = +\infty$ *and* $\lim\limits_{x\to +\infty} \frac{f(x)}{x} = +\infty$ *asymptotic direction parallel to* $y - axis$.

4) a) $\lim\limits_{x\to 0} \frac{f(x)-2}{x} = \frac{f(0)-2}{0} = \frac{0}{0} \qquad \lim\limits_{x\to 0} \frac{f'(x)}{1} = f'(0) = \cdots$ .....

b)The line (T) with equation $y = 2x + 2$ is tangent to (C) at the point with abscissa 0.

5) $f'(x) = \frac{g(x)^2}{x^2}$ *for* $x \neq 0$ and set up the table of variations of f.

6)