Ghobeiry 3rd Official Secondary School for girls **Integrals** **Class: LS**

Mathematics Department

A: Recall:

Anti-derivative (primitive,indefinite integral) :

<u>1-Definition</u>: Let f be a continuous function over an interval I then if f'(x) = g(x) we say that f is an anti-derivative of g and we write : $\int g(x)dx = \int f'(x)dx = f(x) + C$ where C is an arbitrary constant.

<u>2-Rules:</u>

$\int ax^n dx = a\frac{x^{n+1}}{n+1} + C$ where a is a constant and $n \neq -1$	$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$
$\int \frac{1}{\cos^2(ax)}dx = \frac{1}{a}\tan(ax) + C$ where tan(ax) is defined	$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + C$

Techniques

(Change of the variable Or Substitution) $\int u' \times u^n dx = \frac{u^{n+1}}{n+1} + C, with\ n \neq -1$	(Integration by parts)-Not required For LS $\int u.v'dx = u.v - \int v.u'dx$ or $\int u\ dv = u.v - \int v\ du$

B: 1-Definition:(Definite integral)

Let f be a continuous function over an interval E; a and b are two real numbers of E.
Let F and G be anti-derivatives (primitives) of f over E;
G (x) = F(x) + k where k is a constant, so G(b) – G(a) = F(b) – F(a) , and this difference is said to be integral from a to b of the function f .

We write : $\int_a^b f(x)dx = G(x)\Big|_a^b = G(b) - G(a)$ (where a is the lower bound and b is the upper one and G is the anti-derivative of f)

2-**Main properties for integration**:

Table 1

f being a continuous function over an interval E ; the real numbers a , b and c are in E. Let F be one of the anti-derivatives of f over E .			
Properties	1	$\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(z)dz$	The variables x , t , z are said to be mute.
	2	$\int_a^a f(x)dx = 0$ and $\int_a^b f(x)dx = -\int_b^a f(x)dx$	
	3	$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$	Chasles' relation
	4	$\int_a^b f(x)dx$ takes the same sign of the product of (b – a) and f(x) when f(x)**keeps the samesign between a and b**.	

Table 2

Properties	6	f is continuous over an interval E of center 0 ; a is an element of E such that a > 0.	f is odd	$\int_{-a}^{a} f(x)dx = 0$
	7		f is even	$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$
	8	f is continuous over R, and a is a real number.	f is periodic of period T	$\int_{a}^{a+T} f(x)dx = \int_{0}^{T} f(x)dx$
				$\int_{T}^{a+T} f(x)dx = \int_{0}^{a} f(x)dx$
	9	f is continuous over an interval [u(x) ; v(x)] with u and v are differentiable functions.	$\phi(x) = \int_{u(x)}^{v(x)} f(t)dt$	$\phi'(x) = v'(x)f(v(x)) - u'(x)\, f(u(x))$

Table 3

Property	10	f and g are two functions that are continuous over the same interval E. a and b are two real numbers in E	α and β are two real numbers.	$\int_{a}^{b} [\alpha\, f(x) + \beta\, g(x)]dx = \alpha \int_{a}^{b} f(x)dx + \beta \int_{a}^{b} g(x)dx$

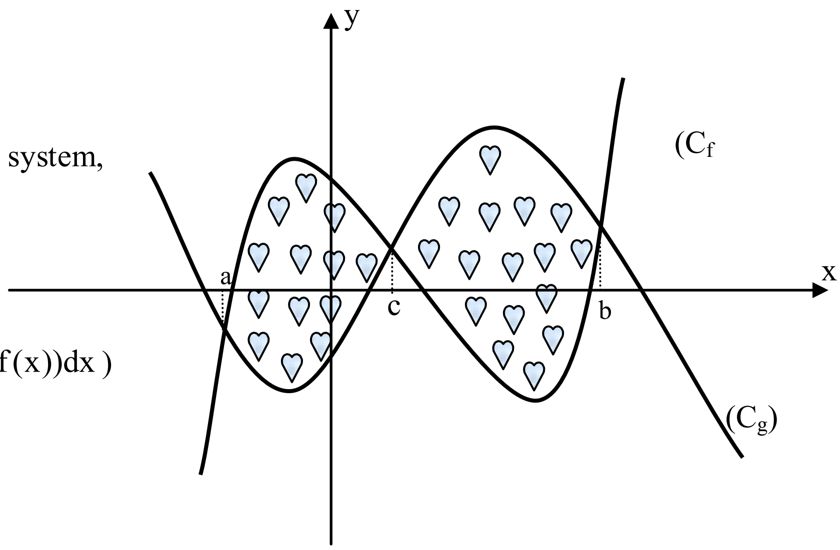
4-**Rules of calculation of an area:**

a-**Calculation of the area**:

The area A of the shaded region, in an orthogonal system, is given by the formula:

$$A = A'\, u^2 \quad \text{with} \quad A' = \int_{a}^{b} (Y_{greater} - Y_{smaller})dx =$$

$$\int_{a}^{b} |\, f(x) - g(x)\, |\, dx = \int_{a}^{c} (f(x) - g(x))dx + \int_{c}^{b} (g(x) - f(x))dx\,)$$



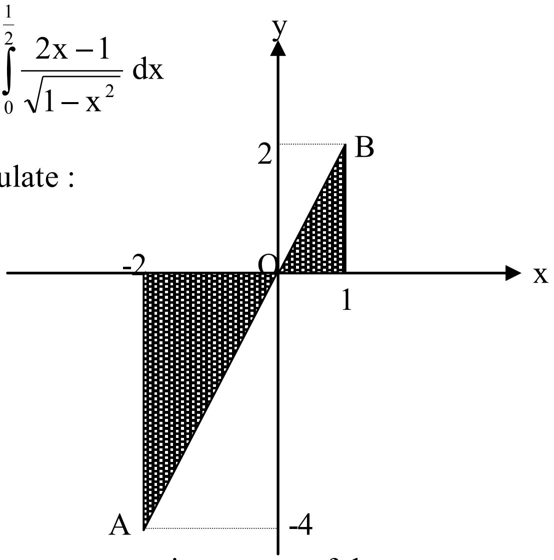
Test your knowledge

I-Calculate the following integrals:

J= $\int_{-1}^{2}(x^2-1)dx$, K = $\int_{-1}^{2}\frac{2x-1}{(x^2-x+3)^4}dx$, N = $\int_{-1}^{2}|x^2-1|\,dx$; P = $\int_{0}^{\frac{1}{2}}\frac{2x-1}{\sqrt{1-x^2}}dx$

II-Given the following figure in a direct orthonormal system calculate :

I = $\int_{-2}^{1}f(x)dx$



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III-

In the plane of an orthonormal system ,consider (C) and (C') as the representative curves of the two continuous functions f and g over the interval [a,b].

Let D be the region limited by (C) ,(C') and the two straight lines x=a and x= b. Calculate the area A of the domain D in each of the following:

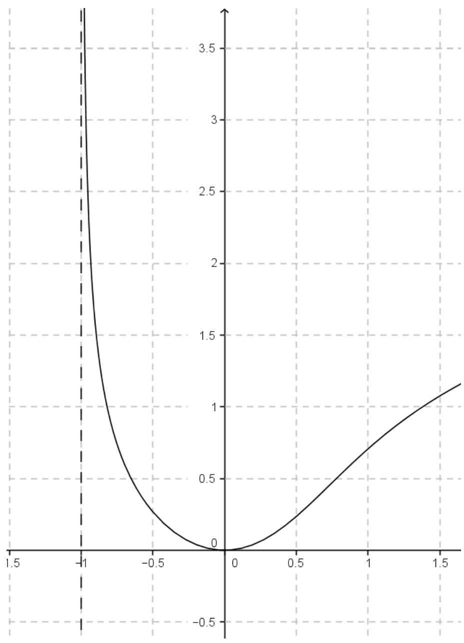
1- a=-1 ,b=2 ; $f(x) = -x^2+2x$, $g(x) = x-2$.
2- a = -2 ,b = 3 ; $f(x) = -x^2+2x$, $g(x) = x-2$.
3- a = -4 ,b = 2 ; $f(x) = x^2+2x-3$, $g(x) = 0$.

IV-

Let (C) be the representative curve of the function f defined over] -1; +∞ [by

$f(x) = \frac{x^2}{\sqrt{1+x^3}}$ in an orthonormal system $(O\,;\vec{i},\vec{j})$.

1) Set up the table of variations of f.
2) Calculate the area $A(\lambda)$ of the domain limited by (C) ,$x'ox$, $y'oy$, and the line of equation x = λ, where $-1<\lambda<0$.
3) Deduce $\lim_{\lambda\to -1}A(\lambda)$.



V-

In the adjacent figure Given the curves of two functions f and f'.

1- Using the graph determine which curve is that of f and which one is that of f'.
2- Calculate $f((1)$ and $f(2)$.
3- Calculate the area of the domain limited by the curve (Ω), (x'ox) , and the two lines of equations x = 1 and x = 2