Academic year: 2023-2024 **Date:** 18-12-2023 **Teacher's name:** H. Fneish **CRDP:** 1528 **Class and section:** 12 LS & GS En **Student's name:**	**PHYSICS EXAM** **MECHANICS**	ثانوية الأمير شكيب أرسلان الرسمية المختلطة **Mark:** /20 **Duration:** 80 minutes

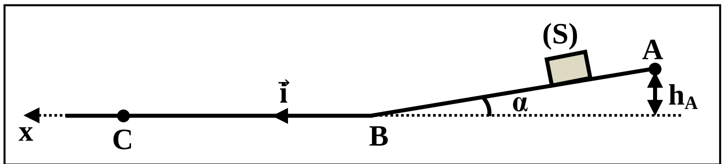
This exam is formed of two obligatory exercises in two pages
The use of non-programmable calculator is recommended

Exercise 1 (8 points)

Determination of the force of friction

A block (S), considered as a particle, of mass m = 100g, can slide on path ABC situated in a vertical plane. This path is formed of two parts:

- AB is straight and inclined by an angle α with respect to the horizontal ($\sin\alpha = 0.1$);
- BC is straight and horizontal.



Doc.1

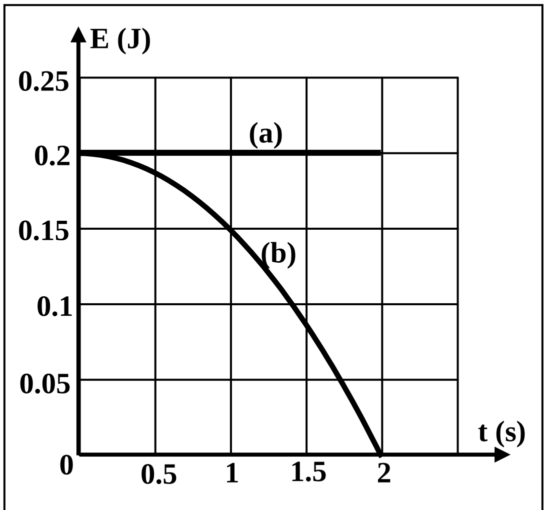
At instant $t_0 = 0$, the block (S) is released without initial velocity from point A, situated at a height h_A above the horizontal x-axis, confounded with BC, and of unit vector $\vec{i}$ (Doc. 1).
Along part AB, the motion of (S) takes place without friction, and along part BC, (S) is subjected to a force of friction $\vec{f}$ supposed constant and parallel to the displacement.
The aim of this exercise is to determine the magnitude f of the force of friction $\vec{f}$.

Take:

- the horizontal plane containing the x-axis as the reference level for gravitational potential energy;
- $g = 10 m/s^2$.

1) Motion of (S) between A and B

The block (S) slides without friction along part AB and reaches B at t = 2s.
The two curves (a) and (b) shown in document 2 represent the gravitational potential energy and the mechanical energy of the system [(S), Earth] as functions of time, during the motion of (S) between A and B.



Doc. 2

1.1) Indicate for each curve the appropriate energy. Justify.

1.2) Using document 2:

1.2.1) determine the distance AB covered by (S) along the inclined plane;

1.2.2) show that the speed of (S) at B is $V_B = 2 m/s$.

2) Motion of (S) between B and C

At t = 2s, the block (S) reaches B and continues its motion along part BC and stops at C at t = 4s.

2.1) Determine the linear momenta of (S), « $\vec{P}_B$ » at B and « $\vec{P}_C$ » at C.

2.2) Deduce the variation $\Delta\vec{P}$ of the linear momentum of (S) between B and C.

2.3) Show that the sum of the external forces exerted on (S) between B and C is $\sum\vec{F}_{ext} = -f\vec{i}$.

2.4) Determine the magnitude f of $\vec{f}$, knowing that $\Delta\vec{P} \cong \sum\vec{F}_{ext} . \Delta t$, where Δt is the duration of the motion between B and C.

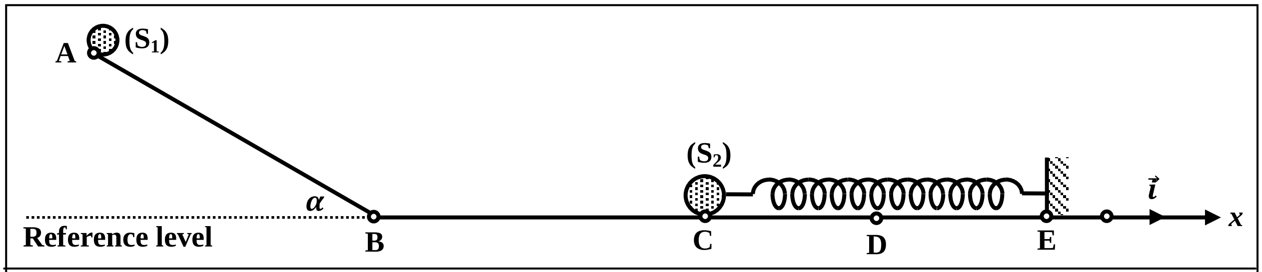
Exercise 2 (10 points)

Energy and linear momentum of a system

A frictionless track ABE, situated in a vertical plane, is formed of two parts. The first part AB is an inclined plane of length 1.6m and making an angle $\alpha = 30^{o}$ with the horizontal, the other part BE is a horizontal plane. A solid (S_1), considered as a particle of mass m_1 = 2kg, is released without initial velocity from point A as shown in document 3.

The horizontal plane passing through BE is taken as a gravitational potential energy reference.

Take $g = 10 m/s^2$.



Doc.3

1- Show that the expression of the speed of (S_1) at point B is $V_1 = \sqrt{2gAB\sin\alpha}$. Calculate its value.

2- As (S_1) reaches point C with a velocity $\vec{V}_1 = V_1\vec{i}$, it enters in a perfectly elastic head-on collision with a stationary solid (S_2) considered as a particle of mass m_2 = 3kg. (S_2) is connected to the free end of an un-stretched horizontal spring (R) of negligible mass and stiffness k = 100N/m. The other end of the spring is fixed to a support at point E.

 2.1- Determine, just after collision, the velocities $\vec{V}'_1$ and $\vec{V}'_2$ of (S_1) and (S_2) respectively.

 2.2- After collision, (S_2) compresses (R) until it stops at point D. Determine the maximum compression x_m = CD of the spring.

Exercise 3 (2 points)

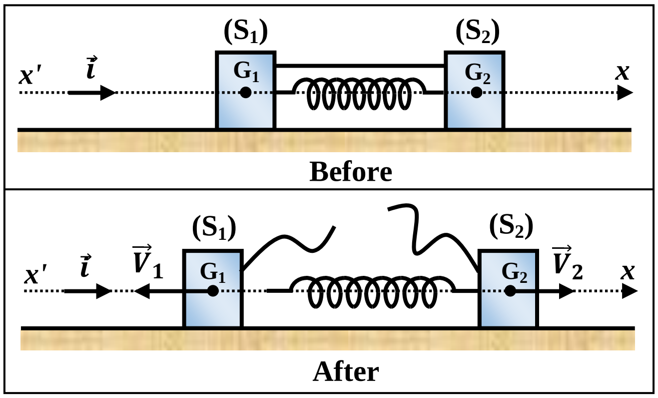
Launching of two solids

Two blocks (S_1) and (S_2), of respective masses m_1 = 2kg and m_2 = 3kg, are placed on a frictionless horizontal surface. A light spring is attached to (S_2), and the blocks are pushed together with the spring between them. A cord initially holding the blocks together is burned; after that happens, (S_2) moves to the right with a velocity $\vec{V}_2 = 4\vec{\imath}\,(m/s)$.

The x-axis is taken as a reference level for gravitational potential energy.

1- Determine the velocity $\vec{V}_1$ of (S_1).

2- Find the system's original elastic potential energy.



Exercise 1:

Part	Answer	Mark
1.1	Curve (a) corresponds to ME. Since no friction therefore ME = constant. Curve (b) corresponds to GPE, since as height decreases GPE decreases.	**1.5**
1.2.1	At A: $GPE_A = 0.2J$. But $GPE_A = mgh_A = mg(AB\sin\alpha)$. So $0.2 = 0.1 \times 10 \times AB \times 0.1$, we get: AB = 2m.	**1.5**
1.2.2	$ME_B = KE_B + GPE_B$. $0.2 = \frac{1}{2} \times 0.1 \times V_B^2 + 0$, we get $V_B = 2m/s$.	**1.5**
2.1	$\vec{P}_B = m\vec{V}_B$, so $\vec{P}_B = 0.2\vec{i}$; $\vec{P}_C = m\vec{V}_C = \vec{0}$ (kgm/s).	**1**
2.2	$\Delta\vec{P} = \vec{P}_C - \vec{P}_B$, so $\Delta\vec{P} = \vec{0} - 0.2\vec{i} = -0.2\vec{i}$ (kgm/s).	**1**
2.3	$\sum \vec{F}_{ext} = m\vec{g} + \vec{N} + \vec{f}$. $m\vec{g} + \vec{N} = \vec{0}$. So, $\sum \vec{F}_{ext} = -f\vec{i}$.	**1**
2.4	$\Delta\vec{P} = \sum \vec{F}_{ext} \, .\Delta t$, so $-0.2\vec{i} = -f\vec{i} \times 2$, we get $f = 0.1N$.	**0.5**

Exercise 2:

Part	Answer key	Mark
1	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $M.E_A = M.E_B \Longrightarrow G.P.E_A + K.E_A = G.P.E_B + K.E_B$. $\frac{1}{2}mV_A^2 + mgh_A = \frac{1}{2}mV_1^2 + mgh_B$ with $h_A = L\sin\alpha$, $V_A = 0$ and $h_B = 0$. $0 + mgL\sin\alpha = \frac{1}{2}mV_1^2 + 0$ $V_1^2 = 2gL\sin\alpha$. $V_1 = \sqrt{2gL\sin\alpha} = \sqrt{2 \times 10 \times 1.6 \times 0.5} = 4m/s$.	**2**
2.1	During collision, the system (S) = [(S_1); (S_2)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}_S}{dt} = \vec{0} \Longrightarrow \vec{P}_S = constant$. Principle of conservation of linear momentum: $\vec{P}_{bc} = \vec{P}_{ac} \Longrightarrow m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$. The collision is head on; then, the above expression can be written in its algebraic form: $m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$ with $V_2 = 0$. $m_1(V_1 - V_1') = m_2V_2'$ ... (1). The collision is elastic; then, the kinetic energy is conserved: $K.E_i = K.E_f$. $\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2 \Longrightarrow m_1\left(V_1^2 - V_1'^2\right) = m_2V_2'^2$. $m_1(V_1 + V_1')(V_1 - V_1') = m_2V_2'^2$ ... (2). Divide (2) by (1): $V_1 + V_1' = V_2'$ ... (3). Replace (3) in (1): $m_1(V_1 - V_1') = m_2(V_1 + V_1') \Longrightarrow V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1$. $V_1' = \frac{2-3}{2+3} \times 4 = -0.8m/s$ (the minus sign indicates that (S_1) rebounds back). Using equation (3): $V_2' = V_1 + V_1' = 4 - 0.8 = 3.2m/s$.	**6**
2.2	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $M.E_C = M.E_D$. $K.E_C + G.P.E_C + E.P.E_C = K.E_D + G.P.E_D + E.P.E_D$. $\frac{1}{2}m_2V_2'^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_m^2$. $x_m = \sqrt{\frac{m_2}{k}}V_2' = \sqrt{\frac{3}{10}} \times 3.2 = 0.55m = 55cm$.	**2**

Exercise 3:

Part	Answer key	Mark
1	The system is isolated, then $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = \text{constant}.$ Principle of conservation of linear momentum: $\vec{P}_i = \vec{P}_f$. $\vec{0} = m_1\vec{V}_1 + m_2\vec{V}_2 \Longrightarrow \vec{V}_1 = -\frac{m_2\vec{V}_2}{m_1} = -\frac{(3)(4\vec{i})}{2} = -6\vec{i}\ (m/s).$	**1**
2	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $ME_i = ME_f$. $KE_i + GPE_i + EPE_i = KE_f + GPE_f + EPE_f$. $0 + 0 + EPE_i = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 + 0 + 0.$ $EPE_i = \frac{1}{2} \times 2 \times 6^2 + \frac{1}{2} \times 3 \times 4^2 = 60J.$	**1**