I- **Consider the function f defined over the interval $]-5;2[$ $by\ f(x)=1+ln\left(\frac{x+5}{2-x}\right)$. Let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.**

1) **Calculate $\lim_{x\to-5} f(x)$ and $\lim_{x\to2} f(x)$. Deduce the asymptotes to (C) .**
2) **Show that f(x) is strictly increasing and setup the table of variations of f(x) .**
3) **Show that the equation f(x)=0 admits a unique root $\propto$ and verify that $-3.2<\propto<-3.1$.**
4) **Calculate f(0) and draw (C) .**
5) **a. Show that the function f has an inverse function g in the interval $]-5;2[$ and determine its domain of definition .**
   b. Set up the table of variations of g(x) .
   c. Calculate the expression g(x).
   d. Draw (G) the representative curve of g in the same system of (C) .
6) **Let $(\Delta): x=1$. The straight line (Δ) cuts the curve (G) in a point M.**
   a. Calculate the coordinates of M .
   b. Solve the inequality $g(x)>1$.

II-

A- Consider the function g defined over the interval $]0;+\infty[$ $by\ \ g(x)=-3x+3-6lnx$.

1. **Calculate $\lim_{x\to0} g(x)$ and $\lim_{x\to+\infty} g(x)$.**
2. **Set up the table of variations of g(x) .**
3. **Calculate g(1) and study the sign of g(x) .**

B- Consider the function f(x) defined over the interval $]0;+\infty[$ by $f(x)=1+3\frac{x+lnx}{x^2}$. Let (C) be the representative curve of f(x) in an orthonormal system $(O;\vec{i};\vec{j})$.

1. **Calculate $\lim_{x\to0} f(x)$. Deduce an asymptote to (C) .**
2. **a) Prove that $(d): y=1\ is\ an\ asymptote\ to\ (C)$.**
   b) Study the relative position between (C) and (d) .
3. **Show that $f'(x)=\frac{g(x)}{x^3}$ and set up the table of variations of f(x).**
4. **Show that the equation $f(x)=0$ admits a unique root $\propto$, and verify that $0.53<\propto<0.54$**
5. **a) Show that $\int\frac{x+lnx}{x^2}dx=lnx-\frac{lnx}{x}-\frac{1}{x}+C$.**
   b) Calculate the area $A(\alpha)$ of the region bounded between (C) , x-axis and the two lines $x=1\ and\ x=\propto$.
   c) Show that $A(\alpha)=\frac{\alpha^3+3\alpha^2-\alpha-3}{\alpha}$.
6. **The function f admits an inverse function $f^{-1}(x)$ over the interval $[1;+\infty[$.**

Draw (C') the curve of $f^{-1}(x)$ in the same system of (C) .

III- **Consider the function f(x) defined over the interval $]-4;+\infty[$ by $f(x) = -x^2 ln(x+4)$.**
Let (C) be the representative curve of f(x) in an orthonormal system $(O;\vec{i};\vec{j})$.

1. **Calculate $\lim\limits_{x\to-4} f(x)$ *and* $\lim\limits_{\to+\infty} f(x)$ .Deduce an asymptote to (C) .**
2. **Calculate the coordinates of A and B the points of intersection between (C) and the axis of abscissas .**
3. **The table below is the table of variations of $f'(x)$ the derivative of the function f(x).**

x	-4		α		-1.25		0		$+\infty$
$f''(x)$	‖	+		+	0	−		−	
$f'(x)$	‖	$-\infty$ ↗	0	↗	1.97	↘	0	↘	$-\infty$

4. **Set up the table of variations of f(x).**
5. **Show that f(x) admits a point of inflection whose coordinates are to be determined.**
6. **Suppose α =Draw (C) .**
7. **Let g(x) be the antiderivative of f(x).**
   a) **Set up the table of variations of g(x) .**
   b) **Show that $g(x) = \frac{1}{3}\left(\frac{x^3}{3} - 2x^2 + 16x\right) - \frac{1}{3}(x^3+64)ln(x+4)$ *when* $g(0) = \frac{-64}{3}ln4$.**

IV- **Consider the function f(x) defined over the interval $]-\infty;-2[\cup]1;+\infty[$ by $f(x) = ln(x^2+x-2)$. Let (C) be the representative curve of f(x) in an orthonormal system $(O;\vec{i};\vec{j})$.**

1) **Calculate $\lim\limits_{x\to-\infty} f(x)$ *and* $\lim\limits_{x\to+\infty} f(x)$.**
2) **Calculate $\lim\limits_{x\to-2} f(x)$ *and* $\lim\limits_{x\to1} f(x)$ .Deduce the asymptotes to (C) .**
3) **Calculate the coordinates of A and B the points of intersection between (C) and the axis of abscissas .**
4) **Show that f(x) is strictly increasing and set up the table of variations of f(x).**
5) **Draw (C) .**
6) **a) Show that f(x) admits an inverse function h(x) and determine the domain of definition of h(x).**
   b) Show that $h(x) = \frac{-1+\sqrt{9+4e^x}}{2}$.
   c) Draw (C') the representative curve of h(x) in the same system of (C) .
   d) Show that the equation f(x) = h(x) does not admit a solution.

V- Consider the function f defined over the interval $]0;+\infty[$ $by\ f(x)=x(lnx-1)^2$. Let (C) be its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1. Calculate $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.
2. Show that $f'(x)=(lnx-1)(lnx+1)$ and set up the table of variations of f(x).
3. Solve the equation $f(x)-x=0$ and study the relative position of (C) with respect to the line (d): $y=x$.
4. Draw (C) and (d) .
5. a) Show $F(x)=\frac{x^2}{4}(2ln^2x-6lnx+5)$ is the primitive of f(x) .

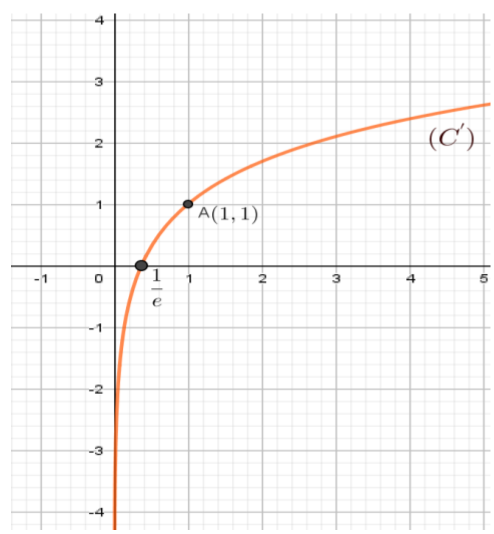
   b) Deduce the area of the region bounded between (C) , x-axis , and the two lines $x=e\ and\ x=e^2$.
6. a) Show that f(x) admits an inverse function $f^{-1}(x)$ and determine its domain of definition .

   b) (C') is the representative curve of $f^{-1}(x)$. Draw (C') in the same system of (C).

   c) Determine the area of the region bounded between $(C'), y-axis$, and the two lines $y=e$ and $y=e^2$.

   d) Let $E(e;0)$ and $F(0;e) be\ two\ points\ of\ (C) and\ (C') respectively$. Calculate the area between (C) , (C') $and\ the\ line\ (EF)$.

VI- The adjacent curve (C') is the representative curve of the derivative $f'(x)$ of a function f ,over the interval $]0;+\infty[$. Let (C) be the curve of f(x).



1. Write an equation of the tangent (T) to (C) at the point A(1 ; 1) .
2. Justify that f(x) does not admit a point of inflection .
3. Study the variations of f(x).
4. Suppose $f(x)=axlnx+b$ defined over the interval $]0;+\infty[$.

   a) Show that a = b = 1 .

   b) Set up the table of variations of f(x).

   c) Copy (C') and draw (C) and (T) in the same system .(1 unit = 2cm)
5. Calculate the area of the region bounded between (C') , x-axis and the two lines $x=\frac{1}{e}$ and $x=1$.
6. a) Show that the function f(x) admits an inverse $h(x)$ over the interval $\left[\frac{1}{e};+\infty\right[$ and determine its domain of definition.

b) Set up the table of variations of $h(x)$.

c) Solve the inequality $h(x) < x$.

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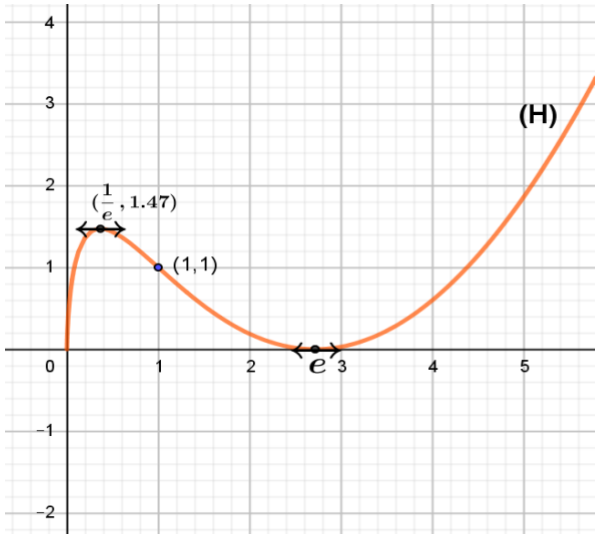
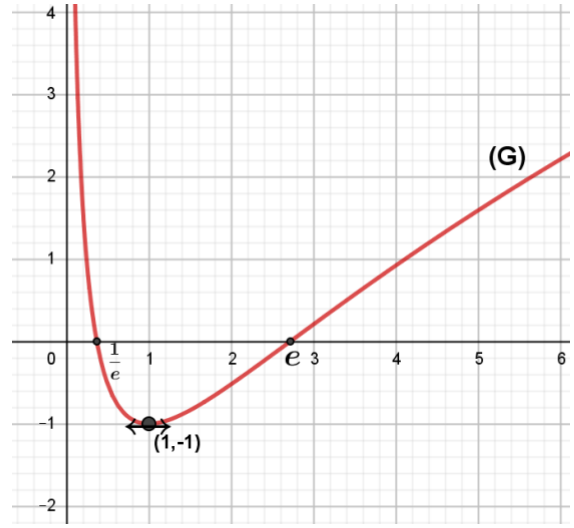
VII- The two curves (G) and (H) are the curves of the function $f(x)$ and its derivative $f'(x)$.





1. **Show that (H) is the representative curve of f(x).**
2. **Show that the curve (G) admits an inflection point of coordinates to be determined.**
3. **Set up the table of variations of the function f(x).**
4. **Write an equation of the tangent to (H) at the point of abscissa 1.**
5. **Calculate the area of the region bounded between (G) , x-axis .**
6. **Let $(x) = -f(x)$ *be a function defined over the interval* $]0;+\infty[$.**
   a) **Set up the table of variations of g(x)**
   b) **Draw the curve (C) the representative curve of g(x) .**

VIII-