Summary: Exponential

Domain of definition:

- If $f(x) = e^x$, then f is defined for every real number x, therefore $\boldsymbol{D_f} = \mathbb{R} =]-\infty; +\infty[$.
  Note that: $e^x > 0$ for every real number x ; $e^u > 0$ for every u.

Particular points:

$\boldsymbol{e^0 = 1}$; $\boldsymbol{e^1 = e}$.

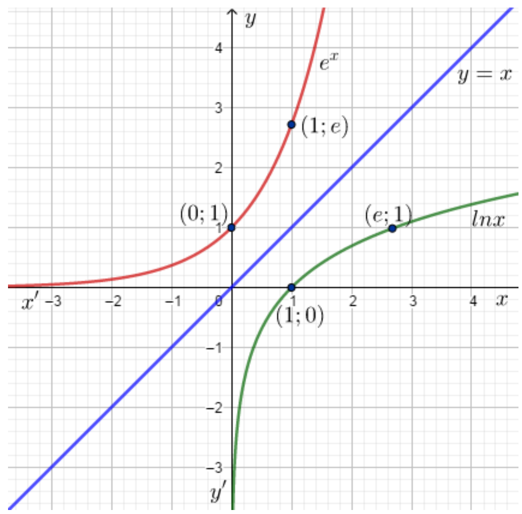
Limits:

$\lim\limits_{x\to-\infty} e^x = 0^+$; $\lim\limits_{x\to+\infty} e^x = +\infty$; $\lim\limits_{x\to+\infty} \frac{x^\alpha}{e^x} = 0^+$; $\lim\limits_{x\to+\infty} \frac{e^x}{x^\alpha} = +\infty$; $\lim\limits_{x\to-\infty} x^\alpha e^x = 0^-$.
Note that x is weaker than e^x.

Derivative:

- If $f(x) = e^x$; $\boldsymbol{f'(x) = (e^x)' = e^x > 0}$ for every real x,
  then f is **strictly increasing** over $\mathbb{R}$.

x	$-\infty$ $\qquad$ $+\infty$
$f'(x)$	$+$
$f(x)$	$0 \nearrow +\infty$



- If $f(x) = e^u$, then $f'(x) = u'e^u$, for example:
  $(e^{ax+b})' = ae^{ax+b}$; $(e^{-x})' = -e^{-x}$; $\left(e^{x^2}\right)' = 2xe^{x^2}$.

Properties:

$\boldsymbol{e^x \times e^y = e^{x+y}}$; $\boldsymbol{\frac{e^x}{e^y} = e^{x-y}}$; $\boldsymbol{(e^x)^y = e^{x.y}}$; $\boldsymbol{e^{-x} = \frac{1}{e^x}}$; $e^{\ln x} = x \ (x > 0)$; $\ln e^x = x \ (x \in \mathbb{R})$.

Equation and inequation:

- $\boldsymbol{e^x = e^y}$ is equivalent to $x = y$ for every x and y.
- $\boldsymbol{e^x > e^y}$ is equivalent to $x > y$ for every x and y.
- $\boldsymbol{e^x < e^y}$ is equivalent to $x < y$ for every x and y.
- $\boldsymbol{e^x = a}$ is equivalent to $x = \ln a$ where $a > 0$.
  Note that: $\boldsymbol{e^x = a}$ where $a \leq 0$ is impossible equation and has no solution.
- $\boldsymbol{e^x > a}$ is equivalent to $x > \ln a$ where $a > 0$.
- $\boldsymbol{e^x < a}$ is equivalent to $x < \ln a$ where $a > 0$.

Antiderivative and exponential function:

$\int e^x\,dx = e^x + C$; $\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$; $\int u'e^u\,dx = e^u + C$ (by change of variable).