The Islamic Institution for Education and Teaching **Al-Mahdi High Schools(Hadat)**	**In His Name**	Academic year: 2022-2023 Date : / 1 / 2023
	Extra Sheet Math	
Grade : 12 GS/LS	Note: ———	
Name: ……..		Probability

E1. A jeweler has, in his safe, **30** identical boxes each containing either a **necklace** or a **watch** or a **bracelet**, made of either **gold** or **platinum**. These articles are distributed as shown in the following table:

	Necklace	**Watch**	**Bracelet**
Platinum	5	2	6
Gold	3	6	8

A- A box is chosen at random from this safe.

1) What is the probability of obtaining a necklace?

2) What is the probability of obtaining a gold necklace?

3) What is the probability of obtaining a necklace knowing that it is made of gold?

B- A customer wants to buy 3 gifts. Suppose that he selects simultaneously 3 boxes from this safe.

1) Prove that the probability that this customer obtains **two** gold articles and **one** platinum article is $\frac{442}{1015}$.

2) Each platinum article is sold for 2 million LL, and each gold article is sold for 1.2 million LL. Let X be the random variable that is equal to the sum paid by the customer to buy any 3 articles chosen at random.

a- Determine the four possible values of X.

b- Determine the probability distribution corresponding to this random variable.

c- Calculate the expected value E(X). What does the number obtained represent?

E2. In a shop there are 1000 leather wallets, of which some are defective.

These wallets were manufactured by three factories α, β and γ according to the following table :

	Factory α	Factory β	Factory γ
Number of wallets	200	350	450
Percentage of defective wallets	5%	4%	2%

A wallet is chosen at random from these 1000 wallets, and consider the following events :

A : « The chosen wallet was produced by the factory α».

B : « The chosen wallet was produced by the factory β».

C : « The chosen wallet was produced by the factory γ ».

D : « The chosen wallet is defective ».

1) a- Prove that the probability $P(D \cap A)$ is equal to $\frac{1}{100}$.

b- Calculate the following probabilities: $P(D \cap B)$, $P(D \cap C)$ and $P(D)$.

2) Knowing that the chosen wallet is not defective, what is the probability that it was manufactured by the factory α ?

3) A wallet is sold for 50 000LL if manufactured by the factory α, for 60 000LL if manufactured by the factory β and for 80 000LL if manufactured by the factory γ .The price of any defective wallet is reduced by 30 % .Designate by X the random variable that is equal to the final price of the wallet that was randomly chosen. Find the six values of X and determine the probability distribution of X.

E3. An urn contains **ten** balls: **five** white, **two** red and **three** green balls.

1) **Three** balls are drawn, simultaneously and randomly, from this urn. Calculate the probability of each of the following events:

A : « **the three drawn balls have the same colour** »

B : « **at least one of the three drawn balls is red** »

2) **Two** balls are drawn randomly and successively from the given urn in the following manner :

-If the first ball drawn is white, then it is replaced back in the urn after which a second ball is drawn.
-But if the first ball is not white then it is kept outside the urn after which a second ball is drawn.
Designate by X the random variable that is equal to the number of times a white ball is drawn.

a- Show that $P(X=1) = \frac{19}{36}$.

b-Determine the probability distribution of X.

E4. In order to ensure that the cars in a given city are functioning well, a certain company is inspecting all the cars in this city. It is known that 20 % of these cars are under guarantee.

Among the cars under guarantee, the probability that a car has a defect is $\frac{1}{100}$.

Among the cars not under guarantee, the probability that a car has a defect is $\frac{1}{10}$.

1- Calculate the probability of each of the following events:

A : « The inspected car is under guarantee and has a defect » .

D : « The inspected car has a defect » .

2- Prove that the probability that an inspected car is under guarantee knowing that it has a defect is equal to $\frac{1}{41}$.

3- The car inspection is for free if the car is under guarantee; it costs 50 000 LL if the car is not under guarantee and does not have a defect; it costs 150 000 LL if the car is not under guarantee and has a defect .

Denote by X the random variable that is equal to the cost of inspection of a car .

a) What are the possible values of X ?

b) Determine the probability distribution of X and calculate the expected value of X .

4- The company inspects an average of 50 cars per day. Estimate the daily inspection cost collected by this company.

E5.

A- Two fair dice, each having six faces numbered from 1 to 6, are rolled. Let S designate the sum of the numbers on the two appearing faces, and consider the following events:

H : « the sum S is greater than or equal to 8 »,

C : « the sum S is less than or equal to 5 »,

E : « the sum S is equal to 6 or 7 ».

Show that the probability $P(H)=\frac{5}{12}$ and calculate P(C) and P(E).

B- 1) At a " Kermes " organized at the end of the school year, a student is in charge of a " stand " at which the following game is proposed: The player rolls two fair dice each having six faces numbered from 1 to 6.

- If he gets a sum greater than or equal to 8, then he draws randomly one ticket from a bag that contains 30 tickets out of which 20 win.
- If he gets a sum less than or equal to 5, then he draws randomly one ticket from another bag that contains 30 tickets out of which 10 win.
- But if the player gets a sum equal to 6 or 7, then he chooses randomly one of the two bags and draws a ticket at random from the chosen bag.

Designate by G the event: « the player wins a prize ».

a- Calculate the probability that the player draws a winning ticket knowing that he got a sum greater than or equal to 8. Deduce $P(H\cap G)$.

b- Show that $P(E\cap G) = \frac{11}{72}$.

c- Calculate $P(C\cap G)$; then deduce the probability P (G) that the player draws a winning ticket.

2) The school administration announced that "everybody wins". To achieve this they decided to give to every player who draws a winning ticket the amount of 5000 LL, and to every player who draws a non-winning ticket the amount of:

- 3000 LL if he realizes the event H and does not win,
- 2000 LL if he realizes the event C and does not win,
- 1000 LL if he realizes the event E and does not win.

Let X designates the random variable equal to the amount paid by the administration to a player.

Verify that $P(X = 3000) = \frac{5}{36}$ and determine the probability distribution for X.

E6. A factory produces watches. Each watch is tested before being approved for selling. If the test is positive, that is if the watch functions properly, then the watch is approved for selling. But if the test is negative then the watch is repaired after which it is tested again. If its second test is positive then it is approved for selling, but if the test is negative then the watch is destroyed. It is known that:

for **80%** of the watches produced, the first test is positive;

for **60%** of the **repaired** watches, the second test is positive.

One watch is chosen randomly from the production.

1) Prove that the probability for this watch to be destroyed is 0.08.

2) Determine the probability that this watch is approved for selling.

3) The cost of production of a watch is 40 000LL with an additional cost of 10 000LL if it needs to be repaired. Each watch is sold for 70 000LL.

Let X be the random variable that is equal to the profit achieved by the factory upon selling a watch.

a- Verify that the three possible values of X are: –50 000 , 20 000 and 30 000.
b- Determine the probability distribution for X.
c- Calculate the expected value E(X).
d- Suppose that the daily production is 50 watches. Estimate the daily profit for this factory.

E7. In a school, each student of the GS and LS sections practices only one sport. The students are distributed as shown in the following table:

	Football	**Basketball**	**Tennis**
LS	**1**	**6**	**3**
GS	**4**	**4**	**2**

The name of each student is written on a separate card, where all the 20 cards used are identical.

A- The cards carrying the names of the LS students are placed in a box B1 and those carrying the names of the GS students are placed in another box B2.

The school principal chooses at random a box and then draws, randomly and simultaneously, two cards from the chosen box.

Consider the following events:

E : The chosen box is B1

S : The two drawn cards carry the names of two students who practice the same sport.

1) a- Show that the probability P(S/E)=2/5 and deduce $P(E \cap S)$.

b- Prove that p(S) = 31/90

2) Knowing that the two selected cards carry the names of two students who practice different sports, what is the probability that these two students are in the LS section?

B- Assume, in this part, that the 20 cards carrying the names of the students are placed together in one box B. Three cards are drawn simultaneously and at random from this box.

1) Prove that the probability that the three drawn cards carry the names of three students, who practice the same sport, is 7/57.

2) Let X be the random variable equal to the number of sports practiced by the three students whose names are written on the three drawn cards. Determine the probability distribution of X.

E8. Consider two urns U and V :

U contains **three** balls numbered 0 and **two** balls numbered 1 .

V contains **five** balls numbered 1 to 5 .

A - One ball is drawn randomly from each urn.

Designate by X the random variable that is equal to the product of the two numbers that are marked on the two drawn balls.

1) Prove that P(X = 0) is equal to $\frac{3}{5}$.

2) Determine the probability distribution of X.

B – In this part, the 10 balls that were in urns U and V are all placed in one urn W .

Two balls are drawn, simultaneously and at random, from this urn W.

1) What is the number of possible draws of these 2 balls?

2) Let q designate the product of the too numbers that are marked on the two drawn balls. .

a – Show that the probability P(q = 0) is equal to $\frac{8}{15}$.

b – Calculate the probability P(q < 4) .