Basic concepts of diffraction of light.

A monochromatic beam of light of wavelength 750 nm is incident normally on a vertical thin slit of width 0.5 mm. A vertical screen is placed 100 cm away from the slit.

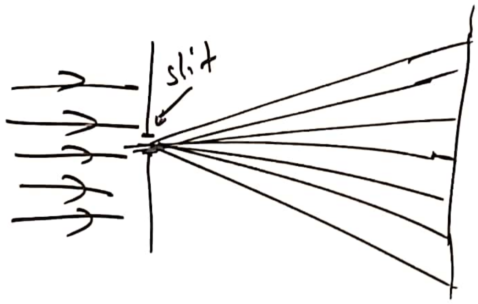
1.
   - **1.1.** Draw a diagram showing the shape of the beam of light emerging from the slit.
   - **1.2.** Describe what you would observe on the screen.
   - **1.3.** The above phenomenon confirms a certain aspect of light. Name this aspect.
2. Draw a rough diagram that shows the variation of the intensity I of the diffracted light on the screen, as a function of sine (8 is the angle of diffraction of a point in the diffraction pattern).
3. Determine the linear width of the central fringe. Deduce the angular width of the central fringe.
4. Determine the positions of the centers of the third dark and the third bright fringes in the positive side of the central bright fringe.
5. M is a point on a screen at distance x=7.5 mm from the center O of the central fringe. Determine, whether M is on the center of a bright, dark or neither a dark nor on a bright.
6. Deduce whether we can isolate a ray of light by replacing the slit by a hole of very small opening. Why?
7. Describe what you would observe on the screen if the slit is replaced by another one of width 2 cm. Illustrate your answer by drawing a figure.
8. We place the whole set-up of part in water of index of refraction n_{water}. We obtain a new diffraction pattern. The new width of the central fringe is 2.25 mm.
   - **8.1.** Calculate the wavelength λ′ of the laser light in water.
   - **8.2.** Determine the relation among λ, λ′ and n_{water}.
   - **8.3.** Deduce the value of n_{water}.
9. The width of the slit is now 0.5 mm and the radiation used belongs to the visible spectrum.

   (Wavelength of the visible spectrum: (Violet) 400 nm ≤ λ ≤ 800 nm (Red). D=1m.
   - **9.1.** Specify the color of the center of the central fringe.
   - **9.2.** Determine the wavelengths of the radiations that give dark fringes at a point M 6 mm away from the center of the central fringe.

$\lambda = 750\,\eta m$ $a = 0.5\,mm = 0.5\times10^{-3}\,m$

$= 750\times10^{-9}\,m$. $D = 100\,cm \times 10^{-2} = 1\,m$.

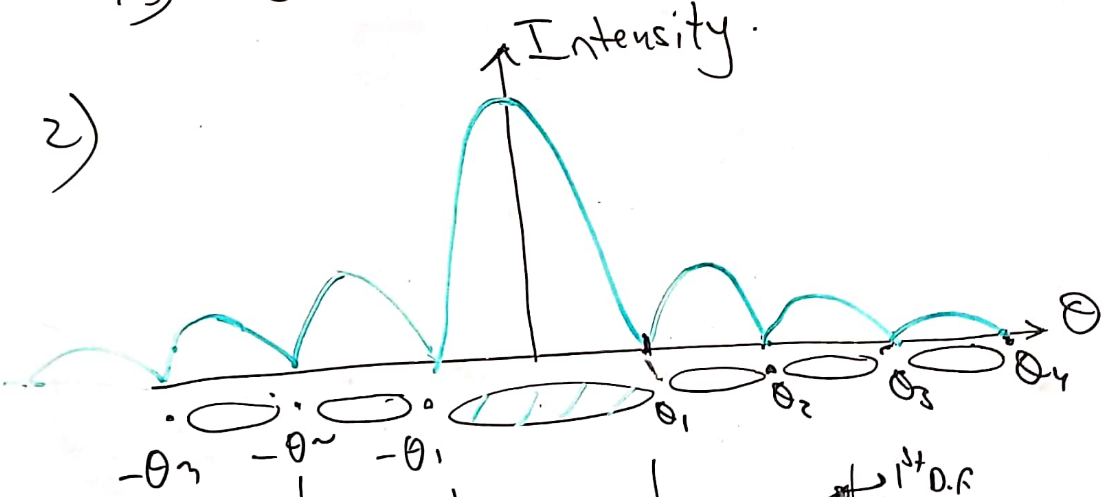
1) 1.1)



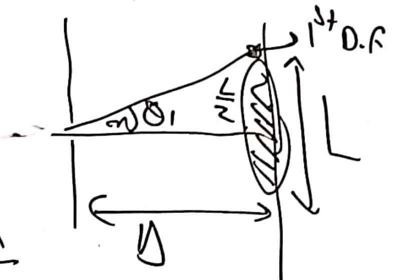
1.2) Alternating B.f and D.f on both sides of C.B.f. directed horizontally perpendicular to the axis of the slit where the width of C.B.f is double the width of any other B.f and more intense.

1.3) Wave aspect of light.

2)



3) $\tan\theta_1 = \dfrac{\frac{L}{2}}{D} = \dfrac{L}{2D}$



$\theta_1 = \dfrac{L}{2D}$ $\theta = \dfrac{n\lambda}{a}$

$\theta_1 = \dfrac{1\lambda}{a}$

$\dfrac{\lambda}{a} = \dfrac{L}{2D}$

CamScanner

$$\Rightarrow \boxed{L = \frac{2\lambda D}{a}}$$

but $L = \alpha D$

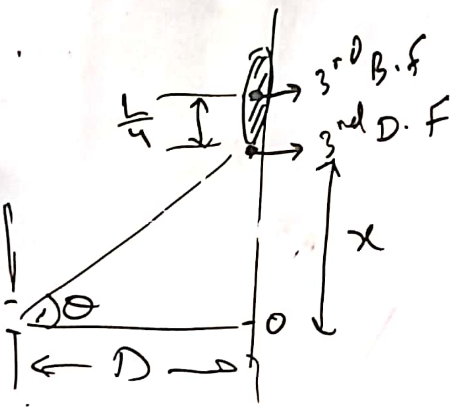
$$\alpha = \frac{L}{D} = \frac{\frac{2\lambda D}{a}}{D} \Rightarrow$$

$$\boxed{\alpha = \frac{2\lambda}{a}}$$

$$L = \frac{2 \times 750 \times 10^{-9} \times 1}{0.5 \times 10^{-3}} = 3 \times 10^{-3}\ m$$

$$\alpha = 3 \times 10^{-3}\ rd.$$

4) $x = ?$



$$\tan\theta = \frac{x}{D}$$

$x = D\theta \qquad \theta = \frac{n\lambda}{a}$

$x = \frac{n\lambda D}{a}$ for D.f.

for 3rd D.f $\quad n = 3.$

$$x_3 = \frac{3\lambda D}{a} = \frac{3(750 \times 10^{-9})(1)}{0.5 \times 10^{-3}} = 4.5 \times 10^{-3}\ m.$$

for 3rd B.f:

$$x_3' = x_3 + \frac{L}{4} = 4.5 \times 10^{-3} + \frac{3 \times 10^{-3}}{4} = 5.25 \times 10^{-3}\ m$$

6) N

6.1) ra

6.2) (

Ans)

5) $x = 7.5\,mm = 7.5 \times 10^{-3}\,m$

if M is the center of a D.F

then $x = \frac{n \lambda D}{a}$

$$n = \frac{x a}{\lambda D} = \frac{7.5 \times 10^{-3} \times 0.5 \times 10^{-3}}{750 \times 10^{-9} \times 1}$$

$n = 5$ is a natural integer

$\Rightarrow$ True M is the center of the 5^{th} D.F.

6) NO, we cannot isolate a light

6.1) ray due to diffraction phenomenon.

6.2) when can we isolate the light ray in this case. write the condition.

Ans) when the diameter of the hole ($d = a$) is greater than 1 mm.

$a \geq 1\,mm.$

$\Rightarrow$ No Diffraction

Rectilinear propagation of light

$\times 10^{-3}\,m.$

$- 5.25 \times 10^{-3}\,m$

7) $a = 2$

No di

$\Rightarrow$ we

8) $L' = 2.$

8.1) L'

$\lambda' =$

8.2) n

8

4

9) (violet)

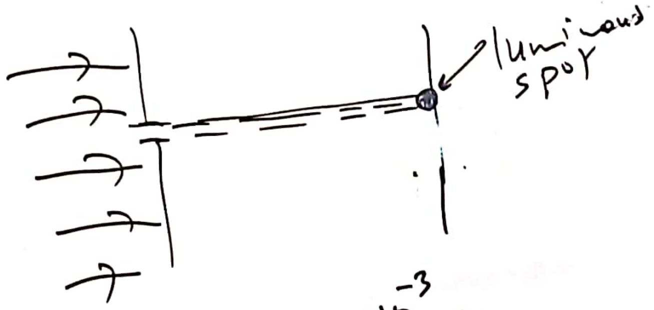
9.1)

9.2)

7) $a = 2cm = 2\times10 = 20mm > 1mm.$

No diffraction $\Rightarrow$ Rectilinear propagation of light.

$\Rightarrow$ we will observe luminous spot.



8) $L' = 2.25\ mm = 2.25\times10^{-3}\ m.$

8.1) $L' = \frac{2\lambda' D}{a} \Rightarrow \lambda' = \frac{L' a}{2D}$

$\lambda' = \frac{2.25\times10^{-3}\times0.5\times10^{-3}}{2(1)} = 562.5\times10^{-9} m.$

8.2) $n_{water} = \frac{C}{V} = \frac{\lambda \cancel{f}}{\lambda' \cancel{f'}}$ $\quad f = f'$

$n_{water} = \frac{\lambda}{\lambda'}$

8.3) $n_{water} = \frac{750\times10^{-9}}{562.5\times10^{-9}} = 1.3$

9) $400nm \leq \lambda \leq 800nm$ (red)
(violet)

9.1) white, due to superposition of all colors.

9.2) $x = 6mm = 6\times10^{-3} m.$

it

on.

e

hole ($d=a$)

in of light

$n_m = \frac{C}{V}$ $\quad > 0$

$f = f' = cst.$

$\lambda' = \frac{\lambda}{n_m}$ $\searrow$

$L' = \frac{L}{n_m}$ $\searrow$

$\alpha' = \frac{\alpha}{n_m}$ $\searrow$

$L_n' = \frac{L_n}{n_m}$ $\searrow$

$\alpha_n' = \frac{\alpha_n}{n_m}$ $\searrow$

$V = \frac{C}{n_m}$ $\searrow$

$\theta' = \frac{\theta}{n_m}$ $\searrow$

no. of Fringes $\nearrow$
(Narrower pattern)

$n_m = \frac{\lambda}{\lambda'} = \frac{L}{L'} = \frac{\alpha}{\alpha'} = \frac{\theta}{\theta'} = \frac{\alpha_n}{\alpha_n'} = \frac{L_n}{L_n'}$

$L \nearrow \lambda \nearrow \alpha \nearrow$

no. of fringes $\searrow$

Broader (wider) pattern.

q.2) $X = 6$ mm. $400\eta m \leq \lambda \leq 800\eta m$

$$X = \frac{n\lambda D}{a} \Rightarrow \lambda = \frac{xa}{nD}$$

$$\Rightarrow \lambda = \frac{6\times10^{-3}\times0.5\times10^{-3}}{n(1)}$$

$$\lambda = \frac{3\times10^{-6}}{n} \text{ in meters}$$

$$\Rightarrow \frac{400\times10^{-9}}{1} \leq \frac{3\times10^{-6}}{n} \leq \frac{800\times10^{-9}}{1} \qquad \times10^{9}$$

$$\frac{400}{1} \leq \frac{3000}{n} \leq \frac{800}{1}$$

$$\frac{1}{800} \leq \frac{n}{3000} \leq \frac{1}{400} \qquad \times(3000)$$

$$3.75 \leq n \leq 7.5$$

$n = 4, 5, 6, 7.$

$n_1 = 4 \quad \lambda_1 = \frac{3\times10^{-6}}{4} = 750\times10^{-9} m = 750\eta m.$

$n_2 = 5 \quad \lambda_2 = \frac{3\times10^{-6}}{5} = 600\times10^{-9} m = 600\eta m$

$n_3 = 6 \quad \lambda_3 = \frac{3\times10^{-6}}{6} = 500\times10^{-9} m = 500\eta m$

$n_4 = 7 \quad \lambda_4 = \frac{3\times10^{-6}}{7} = 428.57\times10^{-9} m = 428.57\eta m$

Exercise 3 (7 pts) **Aspects of light**

You can choose one of the following Parts (I or II)

I) Diffraction of light

Diffraction means surface scattering that essentially creates new wave fronts that propagate in the same direction as the original incident light. Light is bent due to obstacles of small dimensions such as water droplets in clouds or narrow web of spider.

In this exercise, we want to find the width "a" of a spider web string, for that a source of green light of wavelength $\lambda = 520$ nm in vacuum, illuminates the string of a spider web found in an old closet. The wall of the closet is at a distance D =1.6 m from the string. We observe green vertical bands on the wall.

1. Due to what the diffraction of visible light?
2. What is the status of the web string (vertical or horizontal ....) ?
3. What aspect of light this phenomenon show evidence of?
4. We count 1 wide band in the middle and 3 smaller bands along each side of this wide band. The total angular width of the observed pattern is $\alpha = 2.08 \times 10^{-2}$ rd.
   - 4.1) Write the expression of α' the angular width of the wide band interms of λ and a .
   - 4.2) deduce the expression of α interms of λ and a .
   - 4.3) show that a = 0.2 mm .
5. To verify the value of a we measure the width of the pattern and find that its width is L= 33.28 mm .
   - 5.1) Determine the expression of the linear width L interms of λ ,D and a.
   - 5.2) Calculate a .Is this result agrees with that in part 4.3? .

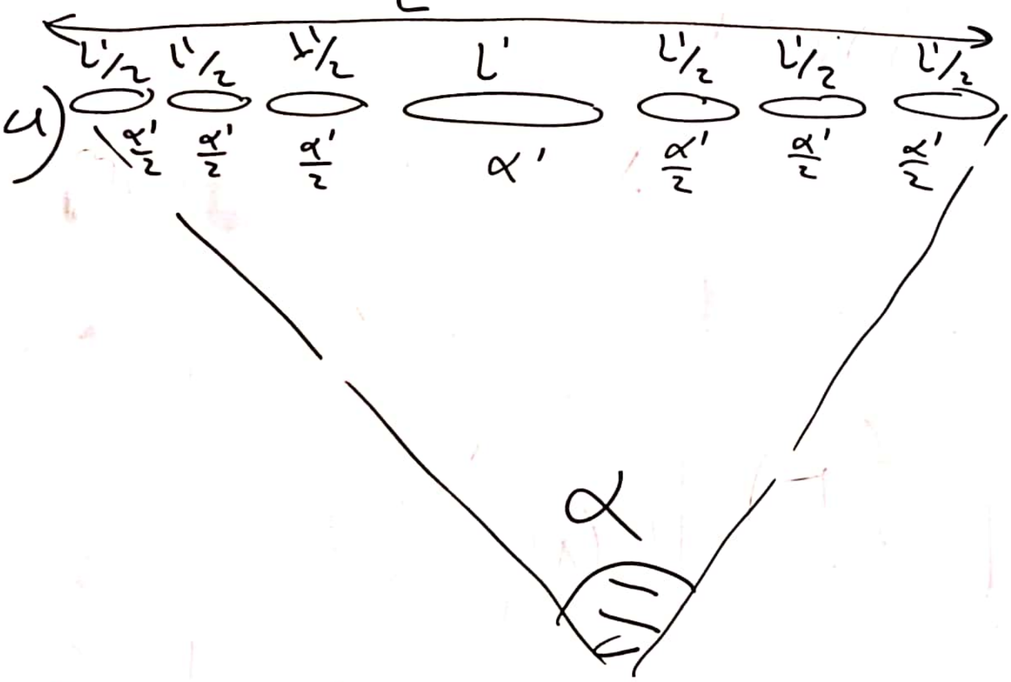
$D = 1.6m$ $\lambda = 520\,nm = 520 \times 10^{-9}\,m.$

(1) 1) Due to deflection of light when crosses or hit abstacles of dimension very close to their wave length where $\boxed{a \leq 1\,mm.}$

($\frac{1}{2}$) 2) horizontal string

($\frac{1}{2}$) 3) wave aspect

4)



$\alpha = 2.08 \times 10^{-2}$ rd.

4.1) $\alpha' = \frac{2\lambda}{a}$

$(\frac{1}{2})$

4.2) $\alpha = \frac{\alpha'}{2} \times 3 + \alpha' + \frac{\alpha'}{2} \times 3$

(1) $\alpha = 4\alpha' = \frac{8\lambda}{a}$

4.3) $\alpha = \frac{8\lambda}{a}$

$a = \frac{8\lambda}{\alpha} = \frac{8 \times 520 \times 10^{-9}}{2.08 \times 10^{-2}}$

(1) $a = 2 \times 10^{-4} m = 0.2 \, mm.$

5) $L = 33.28 \, mm.$

5.1) 1st) $L = \alpha \times D$

$L = \frac{8\lambda}{a} \times D$

$L = \frac{8\lambda D}{a}$

2nd) $L = \frac{L'}{2} \times 3 + L' + \frac{L'}{2} \times 3$

$L = 4L'$ but $L' = \alpha' D = \frac{2\lambda D}{a}$

$L = 4\left(\frac{2\lambda D}{a}\right) = \frac{8\lambda D}{a}$

5.2) $L = \frac{8\lambda D}{a} \Rightarrow a = \frac{8\lambda D}{L}$

(1) $a = \frac{8 \times 520 \times 10^{-9} \times 1.6}{33.28 \times 10^{-3}} = 2 \times 10^{-4}$ m

$a = 0.2$ mm

This result agrees with the result in part (4.3).

$\left(\frac{1}{2}\right)$