Logarithmic Function

I. Definition:

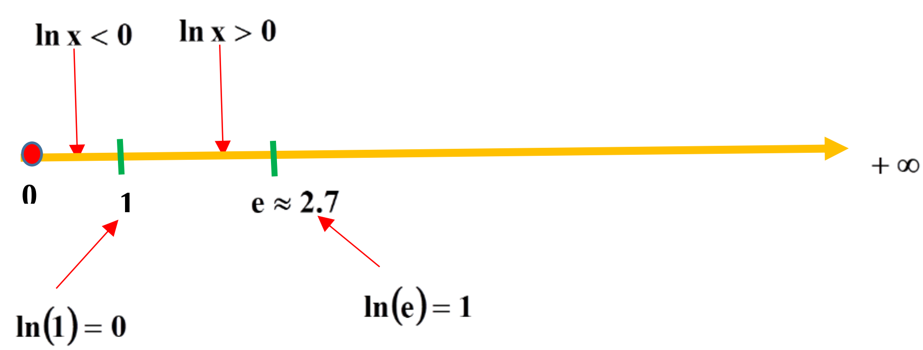
The natural logarithmic function $f(x)$, is the primitive of the function $g(x)=\dfrac{1}{x}$ defined over $]0,+\infty[$. It's defined by $f(x)=\ln(x)$, and it's called **Napierian logarithm** also.

II. Sign of $\ln(x)$:

Don't Forget, the logarithmic function $f(x)=\ln(x)$ defined over $]0,+\infty[$.

More over :

- $\ln(x)<0$ for $x\in\,]0,1[$.
- $\ln(x)>0$ for $x\in\,]1,+\infty[$.
- $\ln(1)=0$ and $\ln(e)=1$



III. Properties:

Complete the following tables then, **observe** the properties below:

Table 1:

a	b	$\ln(a)$	$\ln(b)$		$\ln(a\times b)$	$\ln(a)+\ln(b)$		$\ln\left(\frac{a}{b}\right)$	$\ln(a)-\ln(b)$
3	4								
8	0.5								
0.2	0.3								

Table 2:

a	$\ln(a)$	n		$n\times\ln(a)$	$\ln(a^n)$
2		3			
4		2			
0.1		4			

- Some of these properties can be observed from the results the we get from the above tables:

1) $\ln(a\times b)=\ln(a)+\ln(b)$, where $a>0$ and $b>0$.
2) $\ln\left(\frac{a}{b}\right)=\ln(a)-\ln(b)$, where $a>0$ and $b>0$.
3) $\ln\left(\frac{1}{a}\right)=-\ln(a)$, where $a>0$.
4) $\ln(a^n)=n\ln(a)$, where $a>0$ and n is an integer.
5) $\ln(a^r)=r\ln(a)$, where $a>0$ and r is rational
6) $\ln(a\times b)=\ln|a|+\ln|b|$, where a and b have the same sign.
7) $\ln\left(\frac{a}{b}\right)=\ln|a|-\ln|b|$, where a and b have the same sign.

IV. Limits:

We admits the following limits:

- $\lim\limits_{x\to+\infty} \ln x=+\infty$
- $\lim\limits_{x\to+\infty} \frac{\ln x}{x}=0$
- $\lim\limits_{x\to0^+} \ln x=-\infty$
- $\lim\limits_{x\to0^+} x\ln x=0$

V. Equations and In equations:

The function ln(x) defined over $]0,+\infty[$ is continuous and strictly increasing, then the following hold:

$$\left.\begin{array}{l} \ln a=\ln b \Leftrightarrow a=b \\ \ln a>\ln b \Leftrightarrow a>b \\ \ln a<\ln b \Leftrightarrow a<b \end{array}\right\} \text{Where } a>0 \text{ and } b>0$$

- $\ln x=a \Leftrightarrow x=e^a$, where $a\in \mathbb{R}$

VI. Derivatives:

- If $f(x)=\ln x\ (x>0)$ then, $f'(x)=(\ln x)'=\frac{1}{x}$
- In general, if u is a differentiable and strictly positive real function then, $(\ln u)'=\frac{u'}{u}$

VII. Study of ln(x):

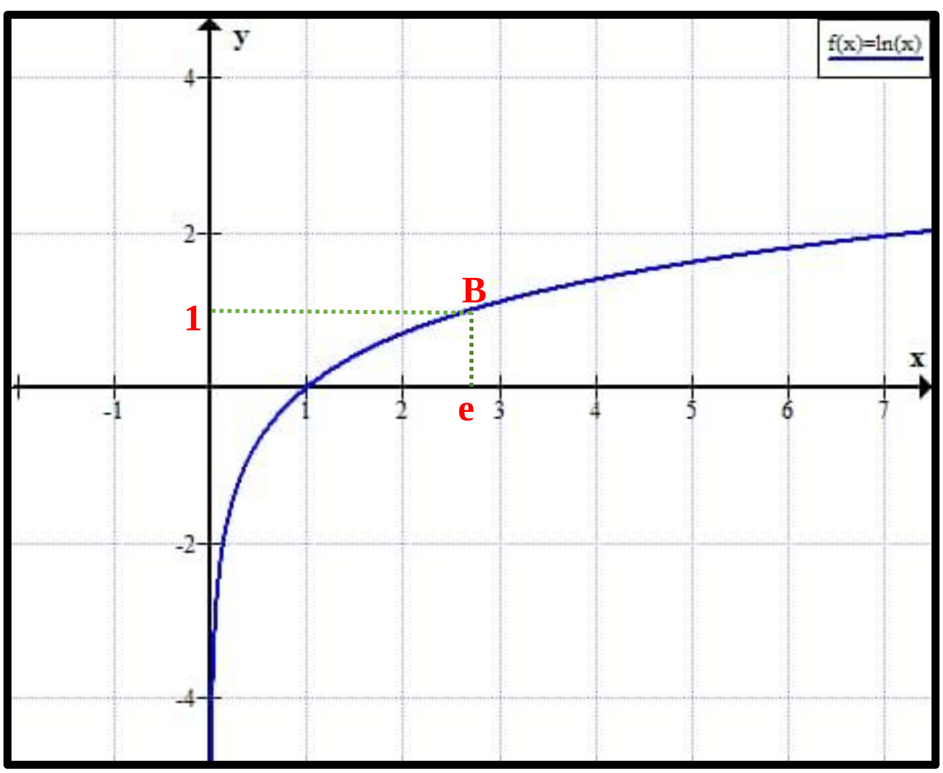
Consider the function $f(x)=\ln(x)$ defined over $]0,+\infty[$.and designate by (C) be its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$.

- $f(x)=\ln(x) \rightarrow f'(x)=\frac{1}{x}>0$ for every $x \in D_f$. So, f is strictly increasing over the interval $]0,+\infty[$.
- $\lim\limits_{x \to 0^+} \ln x = -\infty$ and $\lim\limits_{x \to +\infty} \ln x = +\infty$.

So, the table of variation of the function $f(x)=\ln(x)$ is given below.

x	0		$+\infty$
$f'(x)$		$+$	
$f(x)$	$-\infty$	$\nearrow$	$+\infty$

- The adjacent graph represent (C), representative curve of $f(x)=\ln(x)$ in an orthonormal system $(O,\vec{i},\vec{j})$.



Questions

Question 1:

In the table below only one among the proposed answers to each question is correct is correct .Write down the number of each question and give with justification, the corresponding answer.

N°	Questions	Answers		
		a	b	C
1	Let f be a function defined by : $f(x)=\frac{\ln(x-2)}{x}$. The domain of definition of f is:	$]0,+\infty[$	$]2,+\infty[$	$]0,2[\cup]2,+\infty[$
2	The solution of the equation $\ln(x-2)=\ln(-x+4)$ is:	1	2	3
3	$\ln e^2-2\ln\sqrt{2}-\ln e^{-3}=$	3	4	$\sqrt{2}$
4	$\lim\limits_{x\to+\infty}\frac{1+\ln(x)}{\ln(x)}=$	0	1	$+\infty$
5	The solution(s) of the equation : $\ln x-\frac{1}{\ln x}=0$ is (are):	$S=\{0,e\}$	$S=\{e\}$	$S=\{e,e^{-1}\}$
6	$\ln(\sqrt{7}-2)+\ln(\sqrt{7}+2)=$	$\ln 7$	$\ln 5$	$\ln 3$
7	$\lim\limits_{x\to1}\frac{\ln(1-x^2)}{\ln(x^2-4x+3)}=$	0	1	$\ln 2$

Question 2:

Determine the domain of definition of each of the following functions:

1. $f(x) = \ln(3-4x)$
2. $f(x) = \ln(x^2-1)$
3. $f(x) = \ln(x-2)+\ln(x+2)$
4. $f(x) = \ln\left(\dfrac{1-x}{1+x}\right)$
5. $f(x) = \sqrt{\ln(x)}$
6. $f(x) = \dfrac{\ln x+1}{x}$
7. $f(x) = \dfrac{x+2}{\ln x-1}$

Question 3:

Write each of the following in the simplest form:

1. $\ln\left(e^2\sqrt{e}\right)$; $\ln e^4 + \ln e^2$
2. $\ln 16 - 2\ln 8$; $\sqrt{\ln e^9}$; $\dfrac{1}{\ln\sqrt{\dfrac{1}{e}}}$
3. $\ln\left(3-2\sqrt{2}\right)+\ln\left(3+2\sqrt{2}\right)$
4. $\ln\left(\dfrac{1}{e}\right)^3 - \left(\ln\dfrac{1}{e}\right)^3$; $\ln(3e)+\ln\left(\dfrac{1}{3}e^2\right)$

Question 4: Evaluate each of the following limits:

1	$\lim\limits_{x\to1}(x-1)\ln(x^2-1)$	6	$\lim\limits_{x\to1}\ln\left(\dfrac{x+1}{x-1}\right)$
2	$\lim\limits_{x\to-\infty}\ln\left(1+\dfrac{1}{x^2}\right)$	7	$\lim\limits_{x\to e}\dfrac{\ln x-1}{x-e}$
3	$\lim\limits_{\substack{x\to4 \\ x>4}}\ln\left(1-\dfrac{4}{x}\right)$	8	$\lim\limits_{x\to0}\dfrac{\ln x}{x+\ln x}$
4	$\lim\limits_{x\to0}\dfrac{5-4\ln x}{x}$	9	$\lim\limits_{x\to+\infty}\left(x+\dfrac{lx+1}{x}\right)$
5	$\lim\limits_{x\to+\infty}\dfrac{x-1}{\ln x-1}$	10	$\lim\limits_{x\to0}\left(\ln^2 x-\ln x+2\right)$

Question 5:

Solve each of the following equations:

1. $\ln x + 4 = 0$
2. $\ln(x+2) = \ln(8-2x)$
3. $\ln(2x+1) + \ln(x-1) = \ln 2$
4. $\ln(x-1) - \ln(2x-3) = 0$
5. $\ln x + \ln(x-2) = \ln(2x-3)$
6. $\ln(2x-3) + 2\ln(x-2) = \ln(-2x^2 + 13x - 12)$
7. $(\ln x)^2 - 3\ln x - 4 = 0$
8. $\ln^2 x - 3\ln x + 2 = 0$

Question 6:

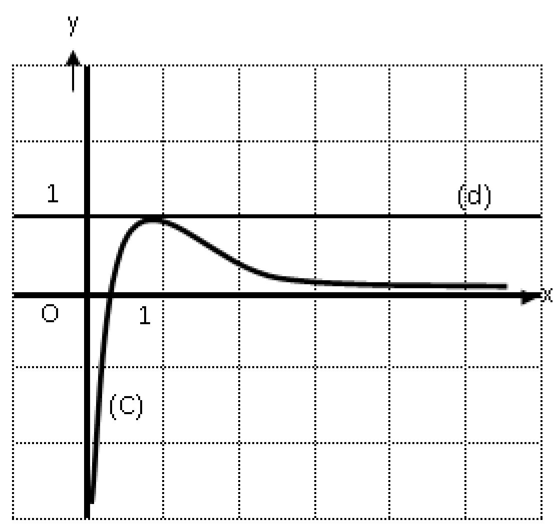
Solve each of the following Inequations:

1. $2\ln x \geq 7$
2. $\ln(2x-3) > 0$
3. $2\ln x - 6 \leq 0$
4. $\ln(2x+1) + \ln(x-1) \leq \ln 2$
5. $(\ln x - e)(2\ln x - 4) < 0$
6. $\ln(x-1) + \ln(x+1) \geq \ln 3$

Question 7:

Shown in the adjacent orthonormal system, the representative curve (C) of a function f that is defined on $]0 ; +\infty[$.

Indication : the line (d) of equation $y = 1$ is tangent to the curve (C) at the point $(1;1)$.



1) Determine $f(1)$ and $f'(1)$ and set up the table of variations of f.

2) The function f is expressed by $f(x) = \dfrac{a + b(\ln x)}{x}$, prove that $a = b = 1$.

3) Determine the abscissa of the point of intersection of (C) with the axis of abscissas, and solve the inequality $f(x) > 0$.