Class: Ls - GS	NBHS	ثانوية الدكتور نزيه البزري رقم 1499- الجنوب – صيدا.
	Math : Exponential function	

I) Consider the function f defined over IR by $f(x)=\frac{1}{2}e^{2x}-4e^{x}+3$.

(C) its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$.

1) a) Show that (d) : $y = 3$ is an asymptote to (C).
   b) Calculate the coordinates of point B the intersection of (C) and (d).
2) Show that $f'(x)=e^{x}(e^{x}-4)$. Set up the table of variations of f.
3) Draw (C) and (d) in the same system.
4) Calculate the area of the region bounded by (C), (x'x) and the lines x = 0 and x = 1.
5) Study graphically, according to the values of the parameter m, the number of solutions of the equation $e^{2x}-8e^{x}=2m-6$.

II) Consider the function f defined over IR by $f(x)=(2x-3)e^{x-2}+4$.

(C) its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.
2) Show that $f'(x)=(2x-1)e^{x-2}$. Set up the table of variations of f.
3) Determine the equation of the tangent (T) to (C) at the point of abscissa 2.
4) Draw (C) and (T).
5) Prove that line (l): $y = 4x$ cuts the curve (C) in two points α and β.
   Verify that: $0.9<\alpha<1$ and $2.9<\beta<3$.
6) a) Show that $F(x)=(2x-5)e^{x-2}+4x$ is an anti-derivative of f.
   b) Calculate, in terms of α and β the area of the region bounded by (C) and (l).

III) Consider the function f defined over IR by $f(x)=x+\frac{2e^{x}}{1+e^{x}}$.

(C) its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.
2) Prove that (D): $y = x$ is an asymptote to (C) at $-\infty$ and (D'): $y = x + 2$ is an asymptote to (C) at $+\infty$.
3) Study the relative position of (C) with respect to (D) and (D').
4) Study the variations of f and set up its table of variations.
5) Write the equation of the tangent (T) to (C) at the point of abscissa zero.
6) Show that I(0, 1) is the center of symmetry of (C).
7) Draw (C), (D), (D') and (T).
8) Calculate the area of the region bounded by (C), (D) and the two lines $x = 1$ and $x = e$.

IV) Consider the function f defined over IR by $f(x)=(2x+1)e^{-2x}$.

(C) its representative curve in an orthonormal system $(O,\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.
2) Find $f'(x)$ and set up its table of variations.
3) Find the points of intersection of (C) and (x'x).
4) Study the sign of f.

5) Show that $f''(x) = 4(2x-1)e^{-2x}$ and find the coordinates of B the inflection point of f.
6) Find the equation of the tangent (T) to (C) at B.
7) Let $g(x) = f(x) - \left(\frac{-2}{e}x + \frac{3}{e}\right)$.
   a) Find $g'(x)$ and $g''(x)$.
   b) Study the sign of $g''(x)$ and deduce the sense of variation of $g'(x)$.
   c) Find the sign of g' and deduce the sense of variations of g.
   d) Deduce the relative position of (C) and (T).
   e) Draw (C) and (T).
8) Calculate the area of the region bounded (C), x-axis and the two lines x = -1 and x = 1.

V) Consider the function f defined over $]0\,;+\infty[$ by $f(x) = 2x - 2 + \frac{1}{e^x - 1}$. Designate by (C) its representative curve in an orthonormal system $(O\,;\vec{i}\,;\vec{j})$.

1) a- Determine $\lim\limits_{x\to 0} f(x)$. Deduce an asymptote to (C).
   b- Determine $\lim\limits_{x\to +\infty} f(x)$ and show that the line (d) of equation $y = 2x - 2$ is an asymptote to (C).
   c- What is the relative position of (C) and (d) ?
2) a- Show that $f'(x) = \frac{(e^x - 2)(2e^x - 1)}{(e^x - 1)^2}$.
   b- Complete the adjacent table of variations of f.

x	0	ln2	$+\infty$
f '(x)		0	
f(x)			

3) Draw (d) and (C).
4) Verify that $\frac{1}{e^x - 1} = \frac{e^x}{e^x - 1} - 1$. Calculate the area of the region bounded by (C), line (d) and the two lines x = ln2 and x = ln3.
5) Consider the function g defined over $]0\,;+\infty[$ by $g(x) = \ln(f(x))$.
   a- Calculate $\lim\limits_{x\to 0} g(x)$ and $\lim\limits_{x\to +\infty} g(x)$.
   b- Set up the table of variations of g.
   c- Show that the equation $g(x) = 0$ admits two real distinct roots.

VI) A) Let h be a function defined over IR by: $h(x) = e^x - x - 1$.
Designate by (C) its representative curve in an orthonormal system.

1) a- Determine $\lim\limits_{x\to +\infty} h(x)$.
   b- Determine $\lim\limits_{x\to -\infty} h(x)$ and show that the line (d) of equation $y = -x - 1$ is an asymptote to (C).
2) a- Calculate $h'(x)$ and set up the table of variations of h.
   b- Draw (d) and (C).
   c- Deduce that $e^x \geq x + 1$ for every x.

B) Let f be the function defined by: $f(x) = \frac{e^x}{e^x - x}$. Designate by (C') its representative curve in an orthonormal system.

1) Show that the function f is defined over IR.
2) Determine the asymptotes of (C').
3) Verify that $f'(x) = \frac{(1-x)e^x}{(e^x - x)^2}$. Set up the table of variations of f.

4) a- Write the equation of tangent (T) to (C') at point E of abscissa 0.

b- Verify that $f(x)-x-1=\dfrac{x\left(x+1-e^{x}\right)}{e^{x}-x}$.

c- Study, according to the values of x, the position of (C') with respect to (T).

d- Draw (T) and (C').

VII) A) Consider the function h defined over $[0;+\infty[$ by $h(x)=(x-2)e^{x}-1$

1) Study the variation of h and setup its table of variations.

2) Show that *h(x)=0* admits a unique solution α over $]2.12;2.13[$. Deduce that $e^{\alpha}=\dfrac{1}{\alpha-2}$.

3) Study the sign of $h(x)$ over $[0,+\infty[$.

B) In this part take $\alpha=2.125$. Consider the function f defined over $[0,+\infty[$ by: $f(x)=\dfrac{e^{x}+1}{e^{x}+x}$.

Designate by (C) its curve in an orthonormal system.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$. Deduce the asymptote of (C).

2) Show that $f(\alpha)=\dfrac{1}{\alpha-1}$.

3) Show that $f'(x)=\dfrac{h(x)}{\left(e^{x}+x\right)^{2}}$. Set up the table of variations of *f*.

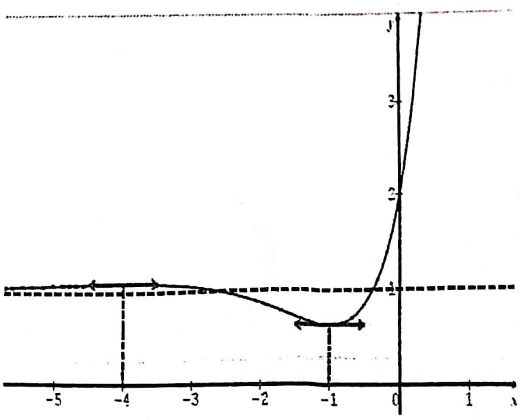
4) Write the equation of the line (T) tangent to (C) at a point of abscissa 0.

5) Draw (C) and (T).

6) Calculate the area of the region bounded by (C) $(x'Ox)$ and $x=e$.

VIII) Consider the function f defined on IR by $f(x)=x+(x+1)e^{-x}$ and let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) **a)** Calculate the limits of f at the boundaries of its domain. Calculate $f(-2)$.
   b) Prove that the straight line (d): $y=x$ is an asymptote to (C) at $+\infty$.
   c) Study the relative position of (C) & (d).
2) Prove that $f'(x)=1-xe^{-x}$ and $f''(x)=(x-1)e^{-x}$.
3) **a)** Setup the table of variations of f'.
   b) Deduce that f is an increasing function and setup the table of variations of f.
4) Prove that the curve (C) admits a point of inflection I whose coordinates are to be determined.
5) Prove that the equation *f(x) = 0* admits a unique root $\alpha\in\left]-0.7,-0.6\right[$.
6) Determine the coordinates of the point A on (C) where the tangent (T) to (C) at A is parallel to (d).
7) Draw (d), (T) & (C).
8) Determine a primitive of $(x+1)e^{-x}$. Deduce the area of the domain limited by (C), $(x'x)$ and the straight line (δ): $y=-x+1$.



IX) The plane is referred to an orthonormal system $(O;\vec{i},\vec{j})$
The adjacent curve is the graph of the function f defined over R.

1) Using the graph answer the following:
   a) Determine: f '(-1), f '(-4) and f(0).
   b) Determine: $\lim\limits_{x\to+\infty} f(x)$; $\lim\limits_{x\to-\infty} f(x)$ et $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}$.
   c) Study according to the values of x the sign f '(x).

2) Suppose that $f(x)=(ax^2+bx+c)e^x+r$, where: a, b, c and r are constants.

a) Justify that $f(x)=(x^2+3x+1)e^x+1$.

b) Calculate f(-1) and f(-4). Set up the table of variations of f f.

3) Let (γ) be the representative curve of the function g defined as $g(x)=(x^2+x)e^x+x$.

Let (d)be the straight line of equation $y=x$.

a) Study according to the values of x the position between (d) and (γ).

b) Show that $\lim\limits_{x\to-\infty} g(x)=-\infty$. Deduce that (d) is an asymptote to (γ).

c) Calculate $\lim\limits_{x\to+\infty} g(x)$ and $\lim\limits_{x\to+\infty} \frac{g(x)}{x}$.

d) Dresser le tableau de variations de g. Draw (T), (d) and (γ) in the same system.

X) A) The adjacent figure represents the curve (l) of a function G defined and differentiable over IR where $G(x)=\int g(x)dx$.

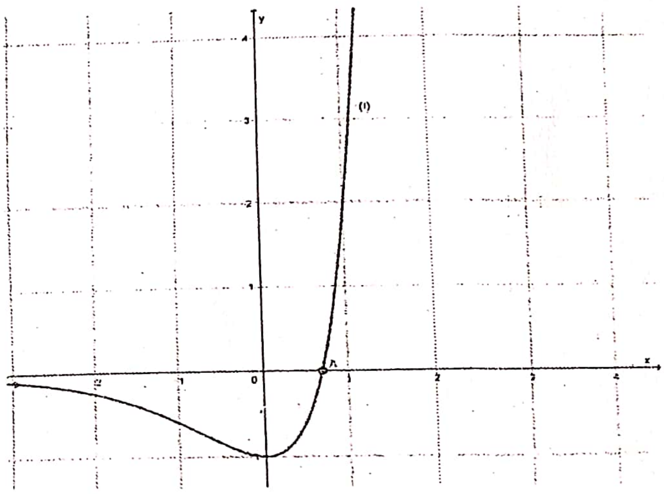
Given $A(\ln 2;\ 0)$

1) Find G(0) and g(0).

2) Using the adjacent figure, study the sign of the function g, then complete the following tables of signs.

x	$-\infty$	0	$+\infty$
$g(x)$			

x	$-\infty$	$\ln 2$	$+\infty$
$G(x)$			



3) a) Given that: $G(x)=ae^{2x}+be^x$ where a and b are two real numbers.

Show that a = 1 and b = -2.

b) Deduce that $g(x)=2e^{2x}-2e^x$.

4) Calculate the area of the region bounded by (C''), the representative curve of g, the x-axis and the two lines of equations x = 0 and x = ln2.

B) Consider the function f defined over $]\ln 2;+\infty[$ by $f(x)=\ln(e^{2x}-2e^x)$. Denote by (C) its representative curve in a direct orthonormal system.

1) Calculate the limits of f at the boundaries of its domain. Deduce an equation of an asymptote to (C).

2) Show that $f'(x)=\dfrac{g(x)}{G(x)}$. Set up the table of variations of f.

3) Find the coordinates of the point of intersection of (C) with the axis of abscissas.

4) Draw (C). (1 unit = 2cm).