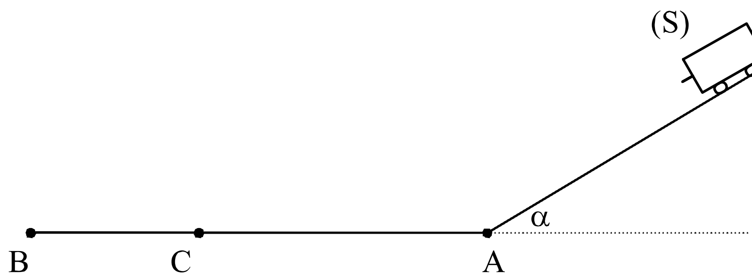
Energy

1- Conservation and non conservation of mechanical energy

A trolley (S), of small dimensions and of a mass m = 300 g, is released without speed, from the top O of an inclined plane OA (OA = 40 cm) making with the horizontal an angle $\alpha = 30^0$.

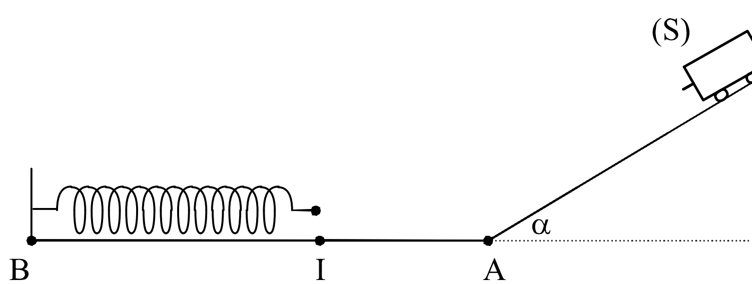
We neglect resistive forces from the rolling of the trolley during its displacement from O towards A.

The reference of the gravitational potential energy is the horizontal plane passing through A.
Given g = 10 m/s^2.



(S) surpasses the point A on the horizontal support AB and stops at the point C under the action of a resistive force of a constant value f = 3 N.

1) Calculate the mechanical energy of the system [(S) ; Earth ; support], when (S) leaves the point O.
2) Deduce the velocity of (S) at the point A.
3) Calculate the variation of the mechanical energy of the system [(S) ; Earth ; support] when (S) passes from A to C. Deduce the distance AC.
4) The system [(S) ; Earth ; support ; air] is energetically isolated . Calculate the variation of the internal energy of the system [(S) ; Earth ; support ; air] when (S) passes from A to C. Interpret the result.
5) We do the preceding experiment again, releasing the trolley without initial velocity from O, but on AB we place a spring of un-joined turns BI (IA = 10 cm) having a spring constant k = 20 N/m as shown in the figure below. The resistive force on (S) doesn't change on AB.



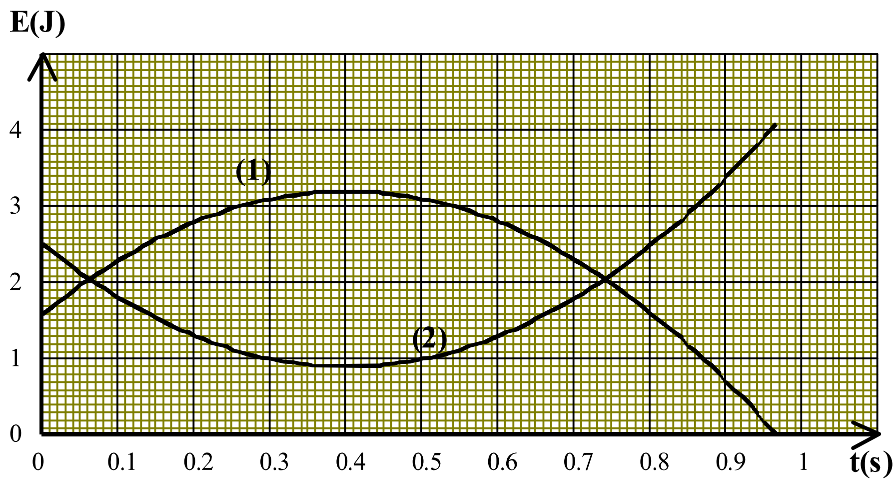
- **a)** Calculate the speed of (S) at I.
- **b)** Deduce the value of the maximum compression x_m of the spring (Establish a second order equation in x_m).

[Correction ▼]

2- Evolution of different forms of energy as functions of time

A ball (S) of mass m = 200 g is launched, at the instant $t_0 = 0$ s, in space, from a point O at a height h from the ground, with a speed $\vec{V}_0$. The reference of gravitational potential energy is horizontal level of the ground. Given $g = 10\ m/s^2$.

The graphs (1) and (2) in the figure above represent, as a function of time, the kinetic and the potential energy of the system [Earth ; (S)].



1) Curve (1) corresponds to the gravitational potential energy. Justify.
2) Verify, at three instants of your choice, the conservation of the mechanical energy of the system [Earth; (S)].
3) Represent, in the preceding reference, The graph of the mechanical energy of the system [Earth ; (S)].
4) Using the graph calculate :
   a) The speed V_0 and the height h.
   b) The instant of collision of (S) with the ground and its speed at this instant.

Correction ▼

3- Calculating the force of friction

We consider the horizontal elastic pendulum, on the axis x'Ox, formed of a perfectly elastic spring of constant K and a solid (S) of mass m = 250 g as indicated in **figure 1**.

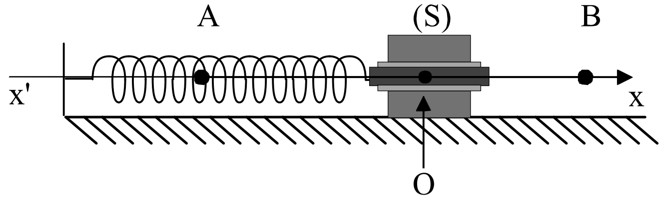


Figure 1

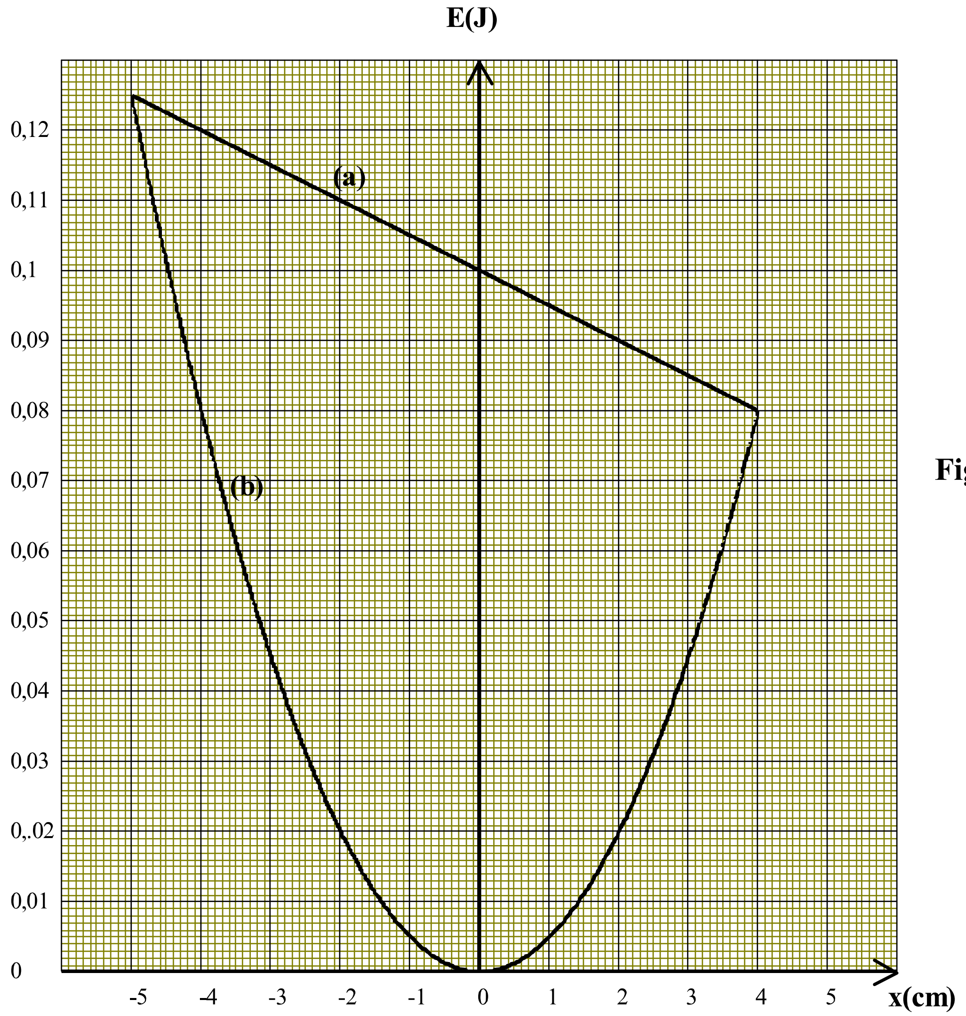


Figure 2

When (S) is at O, the spring is neither compressed nor elongated. We displace (S), along x'Ox, to a point A of abscissa $x_A = -5$ cm and then we release it at the instant $t_0 = 0$, without speed, (S) moves in the positive direction towards the point B of abscissa $x_B = +4$ cm at which the spring reaches it maximum.

An appropriate device records, as (S) moves from the point A to the point B, the variation of the mechanical energy E_m and that of the potential energy E_P of the system (Pendulum ; Earth) as a function of the abscissa x of the center of mass (S) as shown in **figure 2.**
The zero level of the potential energy is the horizontal plane passing through x'Ox.

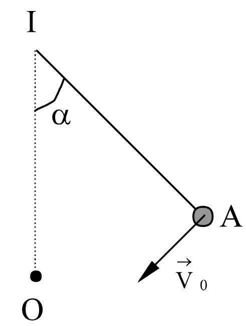
1) Give the expression of the potential energy E_P.
2) Which of the graphs (a) or (b) represents that of the potential energy of the oscillator? Justify.
3) The mechanical energy of the system (Pendulum; Earth) is not conserved. Justify.
4) Using the graphs (a) and (b), calculate the speed V of (S) when it passes through the point of abscissa x = + 2 cm and the value of K.

5) a) Using the graph shows that: $\frac{dE_m}{dx} = \mu$ where μ whose value and unit are to be determined.

b) Deduce the value of the force of friction between (S) and the support?

Correction ▼

4- Oscillation of a pendulum (*)



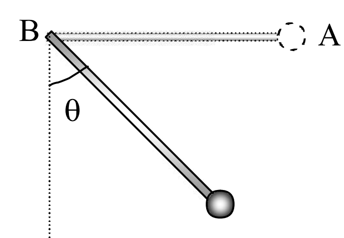
(P) is a simple pendulum of length L = 80 cm and of mass m = 50 g. (P) is shifted from its equilibrium position by an angular abscissa $\alpha = 45^0$, then released with an initial velocity V_0 = 2 m/s as indicated in the figure.
The horizontal plane passing through O is taken as the zero reference of the gravitational potential energy. Given g = 10 m/s^2.

1) The mechanical energy of the system [Earth ; pendulum] is conserved ; calculate the velocity of the mass m when it passes through O.
2) Determine the maximum elongation α_1 of the pendulum.
3) Determine the value of α at which the potential energy of the system [Earth, pendulum] is equal to its kinetic energy.
4) The pendulum (P) oscillates. We fix at O a horizontal, plane, and rough piece of carton. Each time the mass m passes over the carton the maximum amplitude decreases by 5 % from its previous value.

- **a)** Due to cause is this decrease in the amplitude took place ? Is the mechanical energy conserved?
- **b)** The initial amplitude (the first amplitude) of the pendulum is α_1. Verify that the nth amplitude has the expression: $\alpha_n = (0.95)^{n-1}\alpha_1$.
- **c)** Calculate the variation of the mechanical energy of the system [Earth ; pendulum] between the 1st and the 20th amplitude.

Correction ▼

5- A solid in rotation (*)



Consider a homogeneous and rigid rod AB of length L = 60 cm and of mass M = 600 g. At A we attach an object assumed a particle of mass m = 60g, the system (rod – m) forms a pendulum which can rotate without friction around a horizontal axis passing through B.

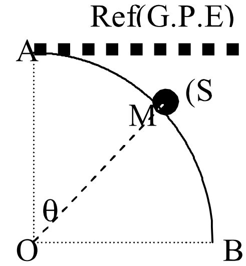
The moment of inertial of the rod with respect to the axis of rotation is : $I_{rod} = \frac{1}{3}M.L^2$.

1) Verify that the moment of inertia of the pendulum is $I = 0.0936\,\mathrm{Kgm^2}$.
2) Knowing that G is the center of mass of the pendulum. Verify that $BG \approx 0.327$ m.
3) The pendulum initially in the horizontal position, is released without initial speed at the instant t_0 = 0.
The horizontal plane passing through B is taken as the zero level of gravitational potential energy.

- **a)** Calculate the mechanical energy of the system [Earth ; pendulum ; support] at the instant t_0.
- **b)** Write the expression of the mechanical energy of the system [Earth ; pendulum ; support] as a function of the angular speed of the pendulum θ' and of the angle θ between the position of the pendulum and the vertical at a certain instant t.
- **c)** Determine the position occupied by (P) where the angular speed of the pendulum is 3 rad/s?

Correction ▼

6- Application on the variation of mechanical energy



A dense solid (S) assumed a particle , of mass m = 50 g, is released from a point A with out initial velocity, on a spherical portion of center O. (S) describes then, in the vertical plane, a circular trajectory of radius R = 20 cm as shown in the adjacent figure. At a given instant (S) takes the position of a point M on the arc AB such that: $A\hat{O}M = \theta$.
Given: g = 10 m/s^2 and the horizontal plane passing through A is taken as the zero level of gravitational potential energy.

1) Calculate, as a function of m, g, R and θ, the gravitational potential energy of the system [Earth ; (S)].
2) The solid (S) leaves its circular trajectory at $\theta = 70^0$ with a speed V = 1 m/s.
   a) Calculate the variation of the mechanical energy of the system [Earth; (S)] between $\theta = 0$ and $\theta = 70^0$. What is this variation due to?
   b) Calculate the value of the force of friction, supposed constant, between (S) and its trajectory.

Correction ▼

7- Mechanical energy of a system

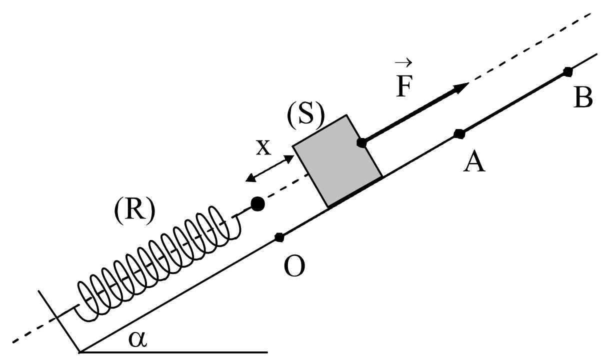


Figure (a)

In the figure (a) we consider:
A solid (S) of mass m = 100 g joined to upper end of elastic spring (R) of un-joined turns, negligible mass, and stiffness k rests on an plane inclined by an angle α with respect to the horizontal.

The forces of friction are negligible in the entire problem.

The axis of the spring is taken along the line of greatest slope of the inclined plane. (S) is at O and the spring is neither compressed nor elongated. We pull (S) with a constant force $\vec{F}$ parallel to the axis of the spring. When (S) reached the point A of the inclined plane, (S) climbs the plane towards the point B, the highest point in the trajectory.

In the figure (b) we represent the graph of the mechanical energy of the system [Earth ; (S) ; spring], when (S) climbs from O towards B, as a function of the distance x between O and (S). The zero level of gravitational potential energy is taken to be the horizontal plane passing through O.
Given g = 10 m/s^2.

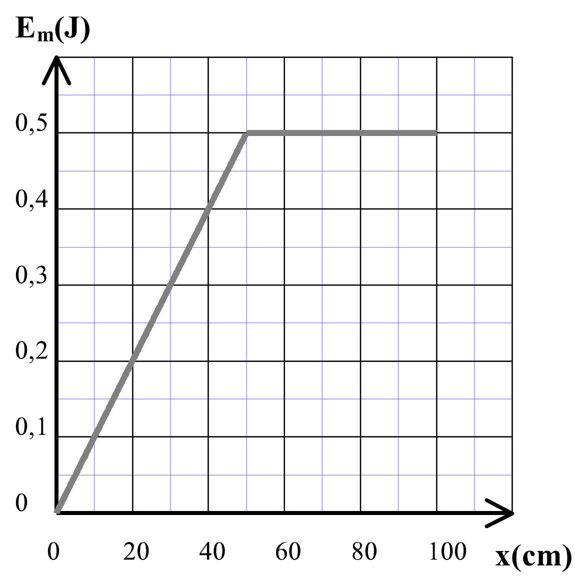


Figure (b)

1) Using the graph, answer the following questions :
   a) Is the mechanical energy of the system [Earth ; (S) ; spring] conserved ? Why ?
   b) Calculate the distances OA and AB.
   c) Verify that : $\alpha = 30^0$.
   d) Knowing that : $\Delta E_m = W\left(\vec{F}\right)$. Deduce the value F of $\vec{F}$.

2) At the point B, (S) rebounds back and re-descends compressing the spring to a maximum distance x_m = 12 cm.
   a) What is the mechanical energy of the system [Earth ; (S) ; spring] in this phase ?
   b) Calculate the value of k.

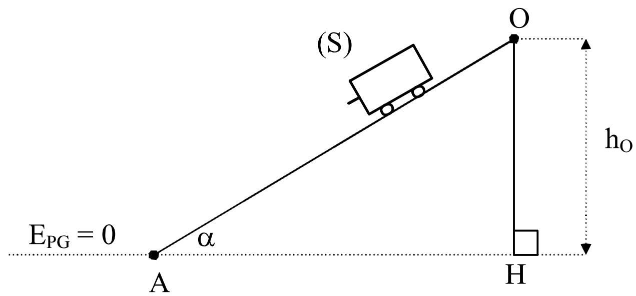
Correction ▼

Corrections

1- Conservation and non conservation of mechanical energy

1) The mechanical energy of the system [(S) ; Earth ; support] is the sum of the kinetic energy E_K and the gravitation potential energy E_{PG}.
When (S) leaves the point O, the mechanical energy of the system:

$$E_{mO} = E_{KO} + E_{PGO} = \tfrac{1}{2}mV_O^2 + mgh_O$$



With : $V_O = 0$ m/s (released from rest from O) and $h_O = OH$, in the triangle OHA , right at H, we have :

$$\sin\alpha = \frac{OH}{OA} = \frac{h_O}{OA} \Rightarrow h_O = OA\sin\alpha .$$

Therefore : $E_{mO} = mg.OA\sin\alpha$

Numerically: $E_{mO} = 0.3\times10\times0.4\times\sin 30^0 = 0.6\text{ J}$. $\boxed{E_{mO} = 0.6\text{ J}}$.

2) On the rail [OA], (S) displaces freely and without speed hence the mechanical energy of the system [(S) ; Earth ; support] is conserved. Hence $E_{mA} = E_{mO} = 0.6\text{ J}$.

On the other hand : $E_{mA} = E_{KA} + \underbrace{E_{PGA}}_{=0} = \tfrac{1}{2}mV_A^2 \Rightarrow V_A = \sqrt{\frac{2E_{mA}}{m}} = \sqrt{\frac{2\times0.6}{0.3}} = 2\text{ m/s}$.

The speed of (S) when it passes through the point A : $\boxed{V_A = 2\text{ m/s}}$.

3) The variation of the mechanical energy of the system as (S) passes from A to C is :

$$\underset{A\to C}{\Delta E_m} = E_{mC} - E_{mA}$$

With : $E_{mC} = \underbrace{E_{KC}}_{\substack{=0\\ \text{zero}\\ \text{speed}}} + \underbrace{E_{PGC}}_{=0} = 0\text{ J}$ and $E_{mA} = 0.6\text{ J}$. Hence : $\boxed{\underset{A\to C}{\Delta E_m} = -0.6\text{ J}}$.

Between A and C, (S) moves freely and without friction hence, the variation of the mechanical energy if the system is equal to the work of the force of friction :

$$\underset{A\to C}{\Delta E_m} = \underset{A\to C}{W(\vec{f})} = -f\times AC \Rightarrow AC = -\frac{\underset{A\to C}{\Delta E_m}}{f} = -\frac{-0.6}{3} = 0.2\text{ m}$$. Then $\boxed{AC = 20\text{ cm}}$.

4) The total energy E of the system [(S) ; Earth ; support ; air] is the sum of its mechanical energy E_m and its internal energy U :
$E = E_m + U$ $\Delta E = \Delta\Delta_m + \Delta\Delta \Rightarrow \Delta U = -\Delta E_m$

The system is energetically isolated, hence the total energy is conserved and its variation is zero $(\Delta E = 0)$.
Using the previous expression: $\underset{A\to C}{\Delta E} = \underset{A\to C}{\Delta E_m} + \underset{A\to C}{\Delta U} \Rightarrow 0 = \underset{A\to C}{\Delta E_m} + \underset{A\to C}{\Delta U} \Rightarrow \underset{A\to C}{\Delta U} = -\underset{A\to C}{\Delta E_m} = +0.6\text{ J}$.

The variation of the internal energy is : $\boxed{\underset{A\to C}{\Delta U} = +0.6\text{ J}}$.

Interpretation : $\underset{A\to C}{\Delta U} = +0.6\text{ J} > 0 \Rightarrow$ The internal energy of the system increases, hence the system will warm up.

5) a) Between A and I, (S) moves freely and under the action of the force of friction hence, the variation of the mechanical energy of the system is equal to the work of the frictional force:

$$\underset{A\to I}{\Delta E_m} = \underset{A\to I}{W(\vec{f})} \Rightarrow \underbrace{E_{mI}}_{E_{KI}+\underbrace{E_{PGI}}_{=0}} - \underbrace{E_{mA}}_{=0,6} = -f.AI \Rightarrow E_{KI} - 0.6 = -3\times0.1 \Rightarrow E_{KI} = 0.3\text{ J} .$$

$$E_{KI} = \tfrac{1}{2}mV_I^2 \Rightarrow V_I = \sqrt{\frac{2E_{KI}}{m}} = \sqrt{\frac{2\times0.3}{0.3}} = \sqrt{2}\text{ m/s} = 1.414\text{ m/s}.\ \boxed{V_I = 1.414\text{ m/s.}}$$

b) Consider the system [(S) ; (R) ; Earth ; support], the mechanical energy of the system is the sum of its energies: Kinetic energy E_K, gravitational potential energy E_{PG} and elastic potential energy E_{Pe}.
Suppose that the spring is compressed to its maximum when (S) reaches a point M between I and B, under these conditions the speed of (S) is null $(E_{KM} = 0\,J)$, $E_{PeM} = \frac{1}{2}kx_m^2$ and it is evident that E_{PGM} = 0 J. The mechanical energy of the system has the expression : $E_{mM} = \frac{1}{2}kx_m^2 = 10x_m^2$.

(S) moves freely and with friction between I and M hence, the variation of the mechanical energy of the system is equal to the work of the frictional force:

$$\underset{I \to M}{\Delta E_m} = \underset{I \to M}{W(\vec{f})} \Rightarrow E_{mM} - \underbrace{E_{mI}}_{E_{KI}=0.3J} = -f \times \underbrace{IM}_{x_m} \Rightarrow 10x_m^2 - 0.3 = -3x_m \Rightarrow 10x_m^2 + 3x_m - 0.3 = 0$$

The acceptable solution of this second degree equation is : $\boxed{x_m = 0.0791\,m = 7.91\,cm}$.

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2- Evolution of different forms of energy as functions of time

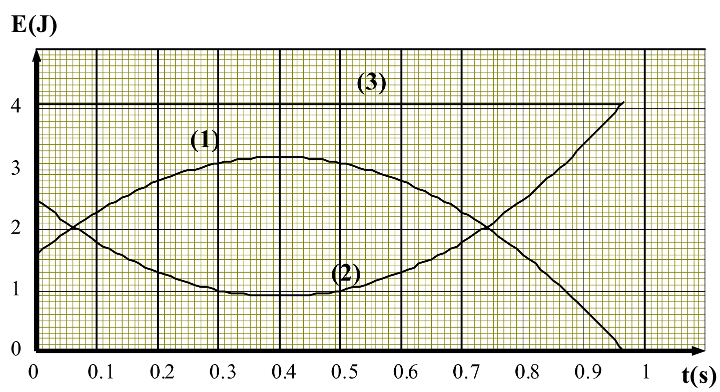
1) The gravitational potential energy: E_{PG} = mgh where h is the height of (S) with respect to the ground. After a certain time h decreases and is becomes zero when (S) reaches the ground, the same happens for the gravitational potential energy and this is verified by curve (1). Hence curve (1) corresponds to the gravitational potential energy.

2) We summarize the values of the chosen energies in the table below :

Time : t(s)	0	0.4	0.74
Gravitational potential energy : E_{PG} (J)	1.6	3.2	2.05
Kinetic energy : E_K (J)	2.5	0.9	2.05
Mechanical energy : $E_m = E_K + E_{PG}$ in (J)	4.1	4.1	4.1

A conservation of the mechanical energy of the system [Earth ; (S)] is found at the instants: t_0 = 0 s ; t_1 = 0.4 s and t_2 = 0.74 s.

3) The graph of the mechanical energy is a horizontal straight line of ordinate 4.1 J. This is curve (3).



4) a) At t_0 = 0 s, we have : E_{K0} = 2. 5 J and E_{PGP0} = 1.6 J.

$$E_{K0} = \frac{1}{2}mV_0^2 \Rightarrow V_0 = \sqrt{\frac{2E_{K0}}{m}} = \sqrt{\frac{2\times 2.5}{0.2}} = 5\,m/s\,.\ \boxed{V_0 = 5\,m/s}\ .$$

$$E_{PG0} = +mgh \Rightarrow h = \frac{E_{PG0}}{mg} = \frac{1.6}{0.2\times 10} = 0.8\,m\,.\boxed{h = 80\,cm}\ .$$

c) At the point of impact with the ground E_{PG} = 0 J, from the graph, the time at the instant of impact corresponds to the abscissa of the point of intersection of curve (1) with the time axis, it is then :
$\boxed{t = 0.97\ s}$.

At the time of impact, the kinetic energy of (S) : $E_K = 4.1\,J \Rightarrow V = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2\times 4.1}{0.2}} = 6.403\ m/s$.

The speed of (S) at the moment of : $\boxed{V = 6.403\ m/s}$.

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3- Calculating the force of friction

1) The potential energy of the system (Pendulum ; Earth) is the sum of the :

- Gravitational potential energy : E_{PG} = 0 J (since the center of inertia of (S) is on the zero reference of gravitational potential energy) ;
- Elastic potential energy : $E_{Pe} = \frac{1}{2}Kx^2$.

$$\boxed{E_P = E_{PG} + E_{Pe} = \tfrac{1}{2}Kx^2}$$

2) E_P is a second degree function of x, hence it curve is in the form of a parabola. This is the case of the curve (b). Curve (b) corresponds to E_P.

3) Curve (a) representing the mechanical energy is a decreasing straight line, hence the mechanical energy of the system (Pendulum, Earth) is not conserved.

4) At a point of abscissa : x = +2 cm, we have : E_P = 0.02 J and E_m = 0.09 J. (from curves (a) and (b))

Hence : $E_K = E_m - E_P$ = 0.09 – 0.02 = 0.07 J$\Rightarrow V = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2\times 0.07}{0.25}} \cong 0.75\ m/s$.

Hence : $\boxed{V \cong 0.75\ m/s}$.

5) a) the graph of the mechanical energy as a function of x is straight hence the derivative of E_m with respect to x represents the slope of this line :

The slope of a line is a constant number hence : $\frac{dE_m}{dx} = \mu =$ constant.

On the other hand the slope pf the line is defined by : $\mu = \frac{\Delta E_m}{\Delta x}$

$$\Rightarrow \mu = \frac{E_{mB} - E_{mA}}{x_B - x_A} = \frac{(0.08 - 0.125)\ \overbrace{J}^{N.m}}{[4-(-5)]\times 10^{-2}\,m} = -0.5\ N \Rightarrow \boxed{\mu = -0.5\ N}$$.

b) In the presence of friction, we can write : $\underset{A\to B}{\Delta E_m} = \underset{A\to B}{W(\vec{f})} = -f.AB = -f.\Delta x$

On the other hand : $\mu = \frac{\Delta E_m}{\Delta x} \Rightarrow \Delta E_m = \mu.\Delta x$,

We can deduce that : $-f = \mu \Rightarrow f = -\mu = 0.5\ N$. $\boxed{f = 0.5\ N}$.

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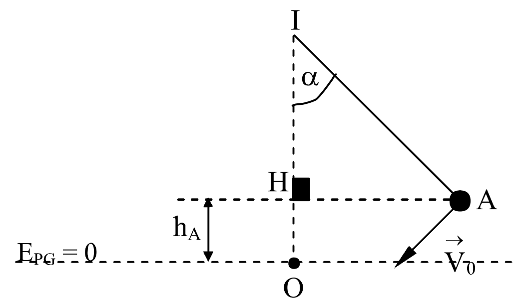
4- Oscillation of a pendulum

1) At the instant of launching the mass is at the point A: $E_{mA} = E_{KA} + E_{PGA} = \frac{1}{2}mV_0^2 + mg.h_A$

With: h_A = OH = IO − IH = L − Lcosα = L(1 − cosα).



Hence: $E_{mA} = \frac{1}{2}mV_0^2 + mg.L(1 - \cos\alpha)$

Numerically :
$E_{mA} = \frac{1}{2}0.05 \times 2^2 + 0.05 \times 10 \times 0.8(1 - \cos 60^0) = 0.3\ J$

The mechanical energy of the system [Earth; (P)] is conserved : $E_{mO} = E_{mA} = 0.3\ J$.

On the other hand : $E_{mO} = E_{KO} + E_{PGO} = \frac{1}{2}mV_1^2 + 0 = \frac{1}{2}mV_1^2 \Rightarrow V_1 = \sqrt{\frac{2E_{mO}}{m}} = \sqrt{\frac{2 \times 0.3}{0.05}} = 3.46\ m/s$.

The speed of m when it passes by the point O : $\boxed{V_1 = 3.46\ m/s}$.

2) When the pendulum reaches its maximum position, the velcoity of m becomes zero and it is the same as its kinetic energy. For maximum displacement:

$$E_m = E_K + E_{PG} = 0 + mg.L(1 - \cos\alpha_1) \Rightarrow 1 - \cos\alpha_1 = \frac{E_m}{mg.L} \Rightarrow \cos\alpha_1 = 1 - \frac{E_m}{mg.L}$$

Using the conservation of the mechanical energy of the system [Earth; (P)]

$$\cos\alpha_1 = 1 - \frac{0.3}{0.05 \times 10 \times 0.8} = 0.25 \Rightarrow \boxed{\alpha_1 = 75.5^0}$$

3) The general expression of the gravitational potential energy of the system [Earth ; (P)] is : $E_{PG} = mg.L(1 - \cos\alpha)$

When : $E_K = E_{PG}$, hence : $E_m = 2\ E_{PG} \Rightarrow E_m = 2mg.L(1 - \cos\alpha) \Rightarrow \cos\alpha = 1 - \frac{E_m}{2mg.L}$

then : $\cos\alpha = 1 - \frac{0.3}{2 \times 0.05 \times 10 \times 0.8} = 0.625 \Rightarrow \boxed{\alpha = 51.3^0}$.

4) a) This decrease in the amplitude is due to friction between the mass m and the carton. Hence the mechanical energy is no more conserved.
b) The first amplitude: α_1
The second amplitude: $\alpha_2 = \alpha_1 - \frac{5}{100}\alpha_1 = (1 - \frac{5}{100})\alpha_1 = 0.95\alpha_1$

The third amplitude: $\alpha_3 = \alpha_2 - \frac{5}{100}\alpha_2 = (1 - \frac{5}{100})\alpha_2 = 0.95\alpha_2 = 0.95(0.95\alpha_1) = 0.95^2\alpha_1$

The fourth amplitude: $\alpha_4 = \alpha_3 - \frac{5}{100}\alpha_3 = (1 - \frac{5}{100})\alpha_3 = 0.95\alpha_3 = 0.95(0.95^2\alpha_3) = 0.95^3\alpha_1$

And so on …
The n^{th} oscillation has an amplitude: $\alpha_n = 0.95^{n-1}\alpha_1$

c) At the end of each oscillation the kinetic energy is zero hence : $E_{m(n^{th})} = mg.L(1 - \cos\alpha_n)$

After the first oscillation: $E_{m1} = mg.L(1 - \cos\alpha_1)$

After the 20^{th} oscillation: $E_{m20} = mg.L(1 - \cos\alpha_{20})$

The variation in the mechanical energy:

$$\Delta E_m = E_{m20} - E_{m1} = mg.L[(1 - \cos\alpha_{20}) - (1 - \cos\alpha_1)] = mgL(\cos\alpha_1 - \cos\alpha_{20})$$

$\alpha_1 = 75.5^0$ and $\alpha_{20} = 0.95^{20-1} \times 75.5^0 = 28.5^0$;

Hence : $\Delta E_m = 0.05 \times 10 \times 0.8(\cos 75.5^0 - \cos 28.5^0) = -0.251\ J$. $\boxed{\Delta E_m = -0.251\ J}$.

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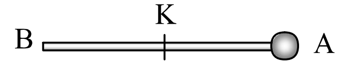
5- A solid in rotation

1) The pendulum is formed of two elements: the rod and the small mass m at A.

The moment of inertia of the pendulum is: $I = I_{rod} + I_m = \frac{1}{3}ML^2 + m\underbrace{\overline{OA}}_{=L}{}^2 = [\frac{1}{3}M + m]L^2$

Numerically: $I = (\frac{1}{3}0.6 + 0.06)0.6^2 = 0.0936\,\text{kg.m}^2$. $\boxed{I = 0.0936\,\text{kg.m}^2}$.

2) Let K be the center of inertia of the rod (K is the midpoint of [AB]). The position of the center of inertia G is determined by the relation : $\overrightarrow{BG} = \dfrac{M\,\overrightarrow{BK} + m\,\overrightarrow{BA}}{M+m}$

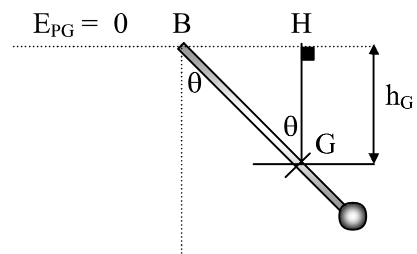


Projecting this relation along the vector $\overrightarrow{BA}$: $BG = \dfrac{M.BK + mBA}{M+m} = \dfrac{M\frac{L}{2} + mL}{M+m} = \dfrac{0.6\times0.3 + 0.06\times0.6}{0.6+0.06} = \dfrac{3.6}{11}\text{m} = 0.327\,\text{m}$.

$\boxed{BG = 32.7\,\text{cm}}$.

3) a) At $t_0 = 0$, the pendulum is horizontal and its center of inertia is in the plane of reference, hence its gravitational potential energy is zero and since the pendulum leaves without initial speed, hence its kinetic energy is zero. The mechanical energy at $t_0 = 0$ is zero.

$\boxed{E_{m(t_0=0)} = 0\,J}$



b) The kinetic energy of a solid in rotation: $E_K = \frac{1}{2}I\theta'^2$;

the gravitational potential energy of a solid with dimensions :

$E_{PG} = -(M+m)gh_G$; with $h_G = GH = BG\cos\theta$.

Hence: $E_{PG} = -(M+m)g.BG\cos\theta$

The mechanical energy of the system [Earth ; pendulum ;

support] $E_m = \frac{1}{2}I\theta'^2 - (M+m)g.BG\cos\theta = \frac{1}{2}0.0963\theta'^2 - (0.6+0.06)\times10\times\dfrac{3.6}{11}\cos\theta$

hence : $\boxed{E_m = 0.04815.\theta'^2 - 2.16.\cos\theta}$.

c) The mechanical energy of the system is conserved, hence : $E_m = E_{m(t=0)} = 0$ with $\theta' = 3\,\text{rad/s}$

$\Rightarrow 0{,}04815\times3^2 - 2.16\times\cos\theta = 0 \Rightarrow \cos\theta = 0.20 \Rightarrow \boxed{\theta = 78.42^0}$.

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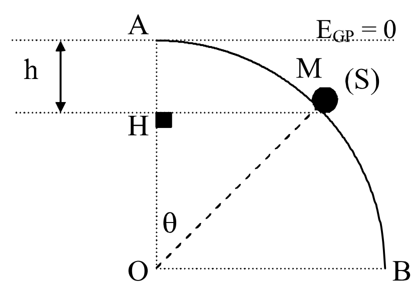
6- Application on the variation of mechanical energy

1) The gravitational potential energy of the system [Earth ; (S)] :

$E_{PG} = -mg.h$ with geometry of the figure gives: $h = AH = OA - OH$.

In the triangle OHM, right at M, we have:

$\cos\theta = \dfrac{OH}{OM} = \dfrac{OH}{R} \Rightarrow OH = R\cos\theta$ then: $h = R - R\cos\theta = R(1-\cos\theta)$



Hence: $\boxed{E_{PG} = -mg.R(1-\cos\theta)}$.

2) a) The variation of the mechanical energy :

$\underset{\theta=0\to\theta=70^0}{\Delta E_m} = E_{m(\theta=70^0)} - E_{m(\theta=0^0)}$ [for $\theta = 0^0$, (S) is at A and for $\theta = 70^0$, (S) is at M]

3)

$E_{mA} = E_{m(\theta=0^0)} = E_{C(\theta=0^0)} + E_{PP(\theta=0^0)} = 0 + 0 = 0\,J$.

$E_{mM} = E_{m(\theta=70^0)} = E_{C(\theta=70^0)} + E_{PP(\theta=70^0)} = \frac{1}{2}mV^2 - mgR(1-\cos\theta)$

$E_{m(\theta=70^0)} = \frac{1}{2}0.05\times1^2 - 0.05\times10\times0.2(1-\cos 70^0) = -0.0408\text{ J}$. $\boxed{\Delta E_{m\ \theta=0\to\theta=70^0} = -0.0408\text{ J}}$.

This variation of the mechanical energy is due to the existence of the force of friction between (S) and the trajectory.

b) In the presence of friction: $\Delta E_{m\ \theta=0\to\theta=70^0} = W(\vec{f})_{\theta=0\to\theta=70^0}$ .Or : $W(\vec{f})_{\theta=0\to\theta=70^0} = -f.\ell$ where ℓ is the length of the arc AM expressed in meters, hence : $\ell = R\theta_{rad}$. The equivalent to the angle 70^0 in radian is : $70\times\frac{\pi}{180} = \frac{7\pi}{18}$ rad . Hence :

$$f = -\frac{\Delta E_{m\ \theta=0\to\theta=70^0}}{\ell} = -\frac{\Delta E_{m\ \theta=0\to\theta=70^0}}{R\theta_{rad}} = -\frac{-0.0408}{0.2\times\frac{7\pi}{18}} = 0.167\text{ N} .$$

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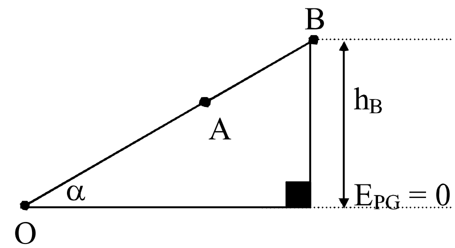
7- Mechanical energy of a system

1) a) The mechanical energy of the system [Earth ; (S) ; spring] is not conserved, since for $x\in[0\ ;\ 50\text{ cm}]$, the graph of the mechanical is a decreasing segment of line.

b) The force $\vec{F}$ doesn't exist between A and B, (S) moves freely and without friction, its mechanical energy is conserved. From the graph if $x\in[50\text{ cm}\ ;\ 100\text{ cm}]$, the graph of the mechanical is a horizontal segment which corresponds to its conservation hence : AB = 100 – 50 = 50 cm.

The first portion of the of the graph: $x\in[0\ ;\ 50\text{ cm}]$ corresponds to the existence of $\vec{F}$ between O and A, hence: OA = 50 cm .

c) The velocity of (S) at the point B which is the highest point of the trajectory is zero, hence the mechanical energy of the system, when is at the point B, is a gravitational potential energy:



(S)

$E_{mB} = E_{PGB} = mg.h_B = mg.OB\sin\alpha$;

From the graph : $E_{mB\ x=100\text{ cm}}$ = 0.5 J ;

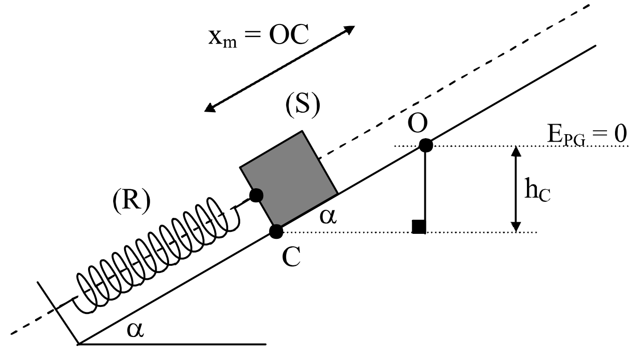
$$\sin\alpha = \frac{E_{mB}}{mg.OB} = \frac{0{,}5}{0.1\times10\times(0.5+0.5)} = 0.5 \quad\Rightarrow \boxed{\alpha = 30^0} .$$

d) The force $\vec{F}$ exists between O and A, hence using the variation of the mechanical energy between O and A :

$$\Delta E_{m\ O\to A} = W(\vec{F})_{O\to A} \Rightarrow E_{mA\ x=50\text{ cm}} - E_{mO\ x=0} = F.OA \Rightarrow 0.5 - 0 = 0.5F \Rightarrow \boxed{F = 1\text{ N}}$$

2) a) From the point B, at which the mechanical energy of the system is 0,5 J, (S) descends freely and without friction, hence the mechanical energy remains conserved and equal to 0.5 J.

b) Let C be the point reached by (S) where the spring is compressed to the maximum, at this point the speed of (S) is zero.



$E_{mC} = E_{KC} + E_{PGC} + E_{PeC}$

$E_{mC} = 0 - mgh_C + \frac{1}{2}kx_m^2$

With: h_C = OC.sinα = x_m.sinα,

Hence: $E_{mC} = -mgx_m\sin\alpha + \frac{1}{2}kx_m^2$

On the other hand: $E_{mC} = 0.5$ J (Conservation of mechanical energy)

$$0.5 = -0.1\times10\times0.12\times\sin 30^0 + \frac{1}{2}k\times0.12^2 \Rightarrow 0.5 = -0.06 + 0.0072k \Rightarrow \boxed{k = 77.8\text{ N/m}}.$$

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