Physics Guide

GRAVITY

Life & General Sciences

[illegible]
Grade 12

Book 2

Light & Atom

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100

Third Year Secondary
Scientific Section
Life & General Sciences (LS & GS)

Book 2
Light & Atom

Hassan Kamar

For My Little Angel

Second Edition
2020

Book 1

Book 1

Book 2*

Book 2*

Unit III

Light

Chapter 12

Diffraction of Light

LS – Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Diffraction of Light		1st(1)	-	-	-	-	-	2nd	-	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Diffraction of Light	-	-	-	-	-	-	-	-	-	1st(A)

Essentials

I-Definition

The diffraction is the bending in the direction of propagation of light when it meets a thin slit or a sharp edge (thin wire).

1. Rectilinear propagation

When a laser beam falls on a large hole, a spot:

- of same diameter as that of the hole is formed on the screen if the incident rays are parallel as shown in figure 1.
- similar dimension if the incidents rays are divergent as shown in figure 2.

This experiment is an evidence of rectilinear propagation of light.

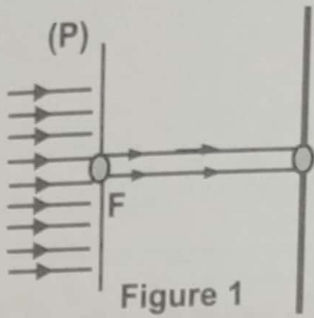


Figure 1

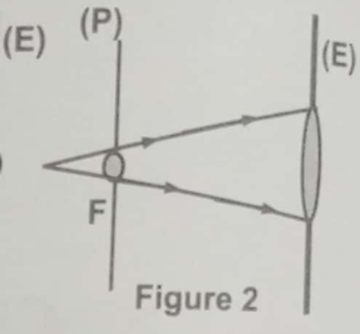


Figure 2

2. Diffraction of light by a rectangular slit

When a laser beam falls on a hole of small dimension, light deviates from its path without being reflected or refracted, we say that light undergoes diffraction phenomenon.

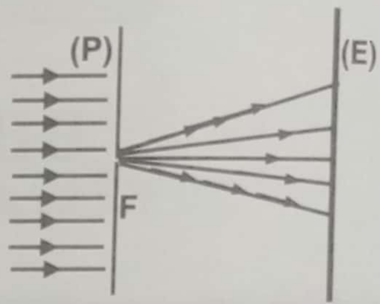


Figure 3

II-Pattern

1. Description

The diffraction phenomenon is an evidence of the wave aspect of light

In the zone of diffraction, and in a direction perpendicular to that of slit, we observe:

- a central bright fringe;
- alternate bright fringes separated by dark points;
- the linear width of the lateral fringes is half that of the central.

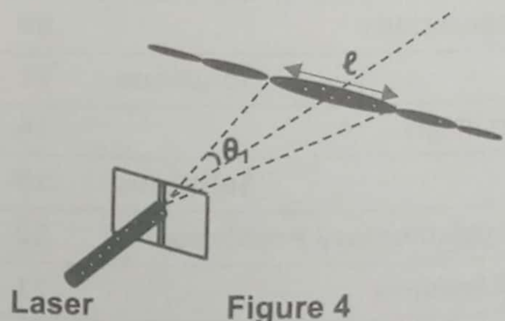


Figure 4

2. Theoretical study

Let ℓ be the linear width of the central bright, while α is its angular width, D is the distance slit-screen (figure 4).

The angular positions θ_n of dark points (zero intensity) are given by the equation $\sin\theta_n = n\dfrac{\lambda}{a}$.

- n is a non-zero integer $n=(\pm 1;\pm 2;...)$ characterizing the order (rank) of the dark point.
- a is the width of the slit.
- λ is the wavelength of the radiation used.

For small angles $\sin\theta_n \approx \theta_n = n\dfrac{\lambda}{a}$ & for the first dark, $n=1$, we get $\theta_1 = \dfrac{\lambda}{a}$.

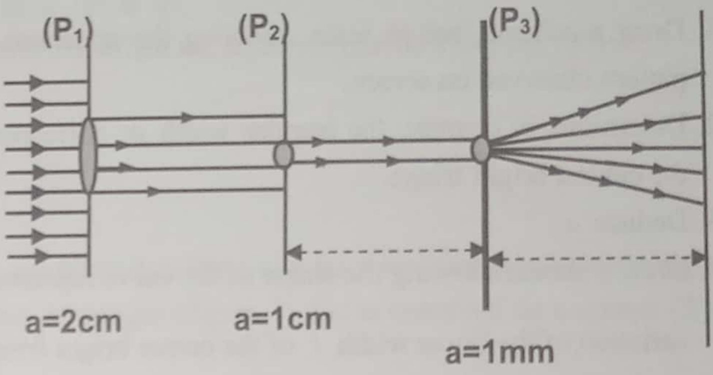
The angular width of the central fringe is $\alpha = 2\theta_1 = 2\frac{\lambda}{a}$.

✍ According to the geometry of figure 4: $\tan\theta_1 = \frac{\ell/2}{D} = \frac{\ell}{2D}$ and for small angles $\tan\theta_1 \approx \theta_1 = \frac{\ell}{2D}$

We get : $\alpha = \frac{\ell}{D}$; the linear width of the central bright is $\ell = \frac{2\lambda D}{a}$.

Note:

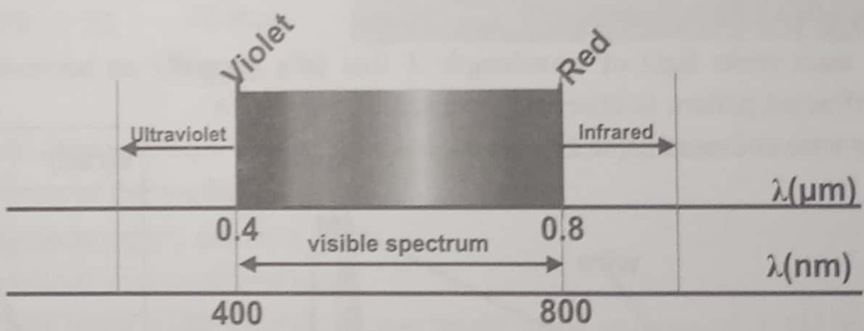
If we increase the diameter of the hole (slit), the linear width decreases. At a certain limit light tends to the rectilinear propagation for large values of a.
While if we decrease the diameter of the hole, the linear width of the central increases. This fact explains why it is impossible to isolate a ray of light (infinitesimal thin beam).



III-Visible Spectrum

The visible spectrum is the set of all the visible colors.

This spectrum is shown in the adjacent diagram.



Wavelength	Violet	Blue	Green	Yellow	Orange	Red
$\lambda(nm)$	400-424	424-491	491-575	575-585	585-610	610-750

The following table summarize few properties of electromagnetic waves, when it crosses from air to another medium (water).

	air	water
Index of refraction	$n = 1$	$n > 1$
Speed	$c = 3 \times 10^8 m/s$	$v = \frac{c}{n} < c$
Frequency (conserved)	υ	υ
Wavelength	λ	$\lambda' = \frac{\lambda}{n}$

Applications

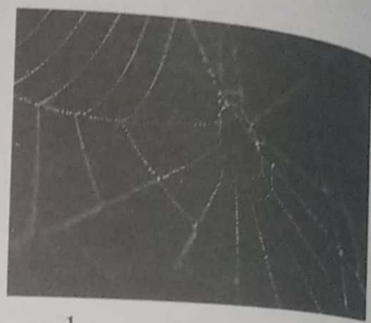
I- Diameter of a Wire by Diffraction

The Italian scientist Francesco Grimaldi (1618-1663) was the first to describe diffraction.

To measure the diameter « a » a wire in a spider's net, we use a laser whose wavelength $\lambda = 0.56\mu m$.

The light diffracts and the pattern obtained is observed on a screen placed $2m$ away from the wire.

The width of the central bright fringe is $\ell = 16\,cm$.



1. Draw a scheme, not to scale, showing the apparatus and the pattern observed on screen.
2. Determine, in degrees, the angular width α corresponding to the central bright fringe.
3. Deduce a.
4. Draw a sketch showing the shape of the curve representing the variation of the linear width ℓ of the center bright fringe in terms of $\frac{1}{a}$.

II- Wire and Diffraction

A laser emits light of wavelength λ that falls normally on horizontal wire of diameter a. The diffracted pattern is observed on a screen parallel to the wire and situated at $D = 2m$.

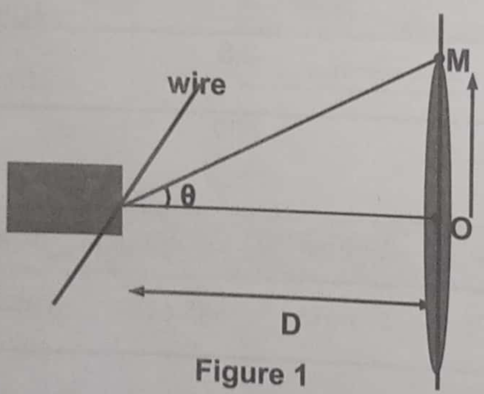


Figure 1

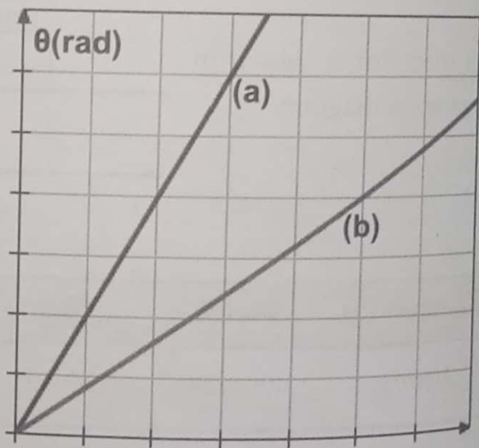


Figure 2

1. When the thin wire is illuminated by a laser, the light diffracts. Give the definition of diffraction phenomenon.
2. Determine the expression of the angular position θ of the first dark point obtained on the screen in terms λ & a (for small angles).
3. Specify whether the curves of figure 2 represent θ as a function of a ? or $\frac{1}{a}$?
4. The same apparatus is used to perform two experiments using blue and violet radiations. Specify the curve that correspond to each radiation.
5. If the first dark point is at $1.2cm$ from the point O while using a blue radiation whose wavelength is $\lambda = 500\ nm$. Determine a.

III- Laser and Slit

A thin horizontal rectangular slit (P) of width a, is illuminated by a laser source that emits light of wavelength $\lambda = 400\ nm$. The pattern obtained is observed on a screen (E) placed perpendicular to the laser beam at a distance D from the slit.

1. Give the name of the physical phenomenon that takes place on the slit.
2. Calculate the frequency of this radiation before reaching the slit and deduce its value as it exits.
3. What is the direction of the pattern observed?
4. Let ℓ be the linear width of the central bright.
   Determine the expression of ℓ in terms of λ, D & a.
5. The linear width of the central bright is doubled when the screen is moved away by $100\ cm$.
   Determine D.

IV- Diameter a Thin Wire

In order to find the diameter of a thin wire, we place it in front of a laser emitting a monochromatic radiation $(\lambda = 690\,nm)$. The pattern is formed of fringes (figure 1) that is observed on a screen (E) placed at a distance $D = 2m$ from the wire.

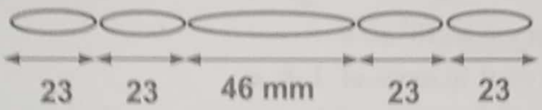


Figure 1

1. Give the name of the physical phenomenon which is observed during this experiment.
2. a) Give the physical significance of the symbol λ.
   b) Explain the meaning of monochromatic radiation.
3. Indicate whether the wire is placed vertical or horizontal.
4. Determine the expression of the linear width ℓ of the central bright fringe in terms of λ, D and a. Using the results of the experiment, calculate the diameter of the wire used.
5. We repeat, under the same conditions, but we use a laser emitting a blue radiation $(\lambda = 480\,nm)$.
   a) Calculate the new width ℓ' of the central fringe.
   b) Construct the new pattern observed on screen.

V- Diffraction

A laser source that emits a monochromatic light of wavelength $\lambda = 0.64\ \mu m$ is used to illuminate a thin wire of diameter d. A horizontal diffraction pattern is observed on a screen situated at a distance $D = 2m$.

1. Indicate whether the wire is placed vertical or horizontal.
2. The linear width of the central bright fringe obtained is $\ell = 1.2\,cm$.
   Determine the expression of ℓ in terms of λ, D & d.
3. Calculate the diameter of the wire.
4. If this wire is replaced by another one whose diameter is, double the previous one.
   Determine the linear width of the new central bright fringe obtained.

VI- Graphical Study

A laser beam of light, of wavelength in vacuum λ, falls normally on a vertical slit of width «a». The diffraction pattern is observed on a screen placed perpendicularly to the laser beam at a distance D from the slit (figure 1).

The variation of the angle of diffraction θ corresponding to the first dark point as a function of $\frac{1}{a}$ is shown in figure 2.

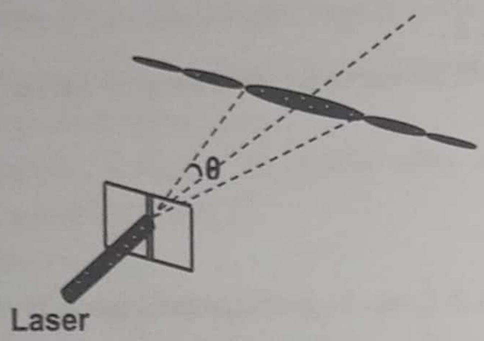


Figure 1

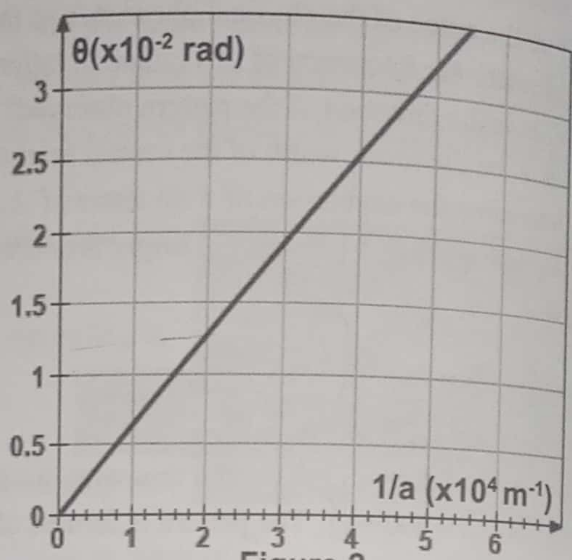


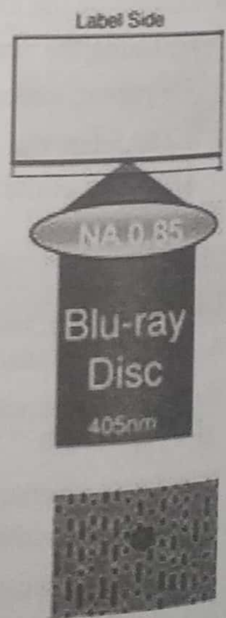
Figure 2

The speed of light in vacuum $c = 3\times10^{8}\, m/s$.

1. Determine the expression of θ in terms of λ & a.
2. Justify that the expression of θ is compatible with the curve shown in figure 2.
3. Determine the wavelength of the radiation used.
4. Deduce its frequency.

VII- The Laser in our Daily Life

CD, DVD and Blu-ray are very common in our daily life when we try to read and write data, movies, … their systems function using respectively radiations of wavelengths $\lambda_C = 780\, nm$, λ_D & $\lambda_B = 405\, nm$.



1. Give the domain of the visible spectrum to which λ_C & λ_B belong.
2. Give the expression of the linear width ℓ of the central bright fringe obtained through diffraction apparatus in terms of the wavelength λ, the distance slit screen D, and the diameter of the aperture d.
3. While using a « DVD » laser, we get $\ell_D = 4.8\, cm$.
   Calculate the DVD reader wavelength λ_D if the linear width of the central fringe obtained while using the same setup with Blu-ray is $\ell_B = 3\, cm$.
4. Compare the linear width of the central bright while using λ_B to that of the other sources.
5. A CD is constituted of *polycarbonate of optical quality whose index of refraction is* $n = 1.55$ for the luminous radiation used by the CD reader.
   a) Determine the expression of the new wavelength λ'_C in terms of λ_C & n.
   b) Calculate the value of λ'_C.

VIII- Particular Phenomenon

A laser beam of light, of wavelength in vacuum $\lambda = 0.5\mu m$, falls normally on a horizontal slit of width $a = 0.1mm$. The diffraction pattern is observed on a screen placed perpendicularly to the laser beam at a distance D from the slit (figure 1).

An appropriate device records the intensity of the light received from S on the screen (E) as a function of x.

The curve in figure 2 shows the intensity as a function of x.

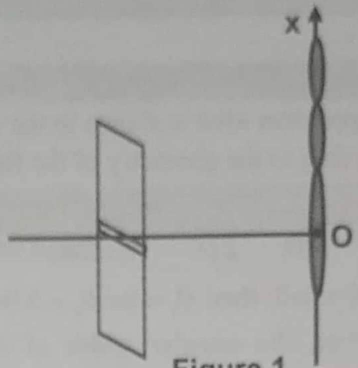


Figure 1

1. Specify the nature of the point of abscissas $-0.5cm$, 0 & $0.75cm$.
2. Indicate the position of the first and second dark point from the positive side
3. Refer to figure 2, give the linear width of the central bright fringe ℓ.
4. Give the expression of ℓ as a function of the given.
5. Deduce the value of D.

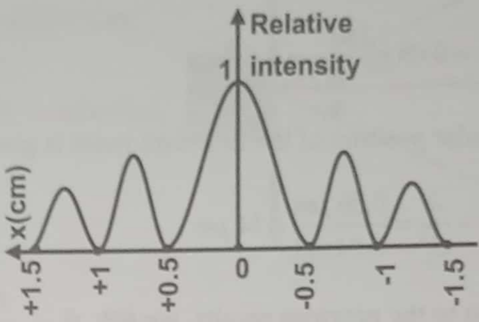
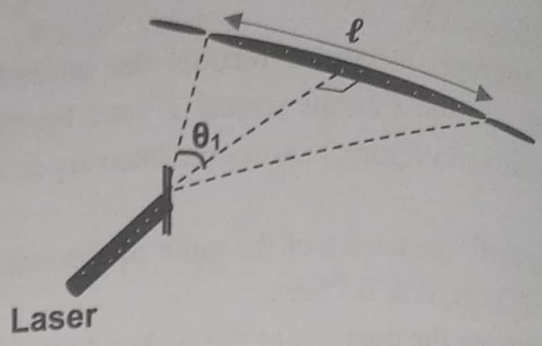


Figure 2

Solutions – Applications

I-

1. The apparatus used is shown in the adjacent figure.



2. According to the geometry of the figure:

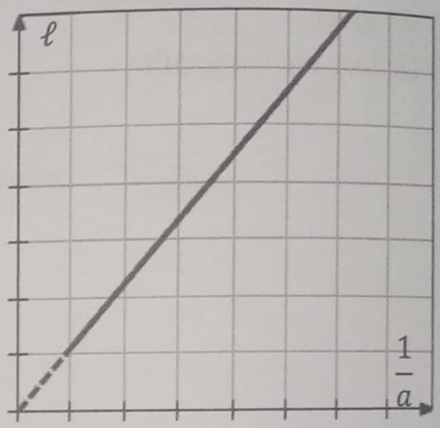
$\tan\theta_1 = \frac{\ell/2}{D} = \frac{\ell}{2D} = \frac{16\times10^{-2}\,m}{2\times 2m} = 0.04$, which is very small; then $\theta_1 \approx \tan\theta_1 = 0.04$ rad.

However, the angular width of the central is $\alpha = 2\theta_1 = 0.08$ rad;

Thus, $\alpha = 0.08\times\frac{180^0}{\pi} \approx 4.6^0$.

3. The angular position of the first dark point is given by $\sin\theta_1 = 1\times\frac{\lambda}{a} \approx \theta_1$ (small angles);

Then, $a = \frac{\lambda}{\theta_1} = \frac{0.56\,\mu m}{0.04\,\text{rad}} = 14\,\mu m$.



4. Referring to the previous results, we get: $\theta_1 = \frac{\ell}{2D} = \frac{\lambda}{a}$,

so $\ell = \frac{2\lambda D}{a}$; then ℓ is proportional to $\frac{1}{a}$.

(We can say that the expression of ℓ is of the form $\ell = \beta x$ where $x = \frac{1}{a}$ & $\beta = 2\lambda D$).

Thus, the graphical representation of ℓ as a function of $\frac{1}{a}$ is a straight line passing through origin.

II-

1. The diffraction of light is the sudden change in the direction of propagation of the wave when it meets a sharp edge or bending when it meets a thin obstacle.

2. The positions of the dark points are given by $\sin\theta_n = n\frac{\lambda}{a}$;

For the first dark point $n=1$ & for small angles $\sin\theta \approx \theta$, then $\theta = \frac{\lambda}{a}$.

3. The relation $\theta = \frac{\lambda}{a}$ shows that θ & a are inversely proportional.

$\theta = \lambda\times\frac{1}{a}$, then θ & $\frac{1}{a}$ are proportional; thus the graphical representation of θ in terms of $\frac{1}{a}$ should be a straight line passing through origin, which is confirmed referring to the curves.

4. The wavelength λ is the slope of the straight line representing θ in terms $\frac{1}{a}$.

However, slope$(a)>$ slope(b) (closer to the ordinate axis means greater slope).

And we have $\lambda_{blue} > \lambda_{violet}$; thus the straight line (a) corresponds to the blue radiation.

5. According to the geometry of the figure $\tan\theta = \frac{x}{D} \approx \theta$;

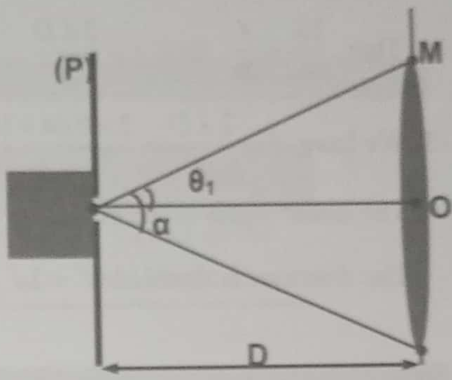
So, $\frac{x}{D} = \frac{\lambda}{a}$; then $a = \frac{\lambda \times D}{x} = \frac{500\times10^{-9}\,m \times 2m}{1.2\times10^{-2}\,m} \approx 8.3\times10^{-5}\,m = 83\,\mu m$.

III-

1. The appearance of this pattern is due to the diffraction of the light.
2. The frequency of the incident light is given by:

$\upsilon = \frac{c}{\lambda} = \frac{3\times10^{8}\,m/s}{400\times10^{-9}\,m} = 7.5\times10^{14}\,Hz$.

However, the frequency is conserved through diffraction, then $\upsilon_{exit} = \upsilon = 7.5\times10^{14}\,Hz$.



3. The pattern observed should be vertical, in a direction perpendicular to that of slit.
4. According to the geometry of the figure:

$\tan\theta_1 = \frac{\ell/2}{D}$; for small angles $\theta_1 = \frac{\ell/2}{D}$;

The positions of the dark points are given by $\sin\theta_n = n\frac{\lambda}{a}$,

For small angles and for the first dark $\theta_1 = \frac{\lambda}{a}$;

$\theta_1 = \frac{\lambda}{a} = \frac{\ell}{2D}$; thus $\ell = \frac{2\lambda D}{a}$.

5. The linear width of the central bright is proportional to the distance D, the central is doubled so $\ell' = 2\ell$; then the distance also should be doubled, $D' = 2D$ but $D' = D + 100\,cm$;

We get $2D = D + 100\,cm$, thus $D = 100\,cm$.

IV-

1. Diffraction of light.
2. a) λ stands for wavelength.

   b) Monochromatic is a radiation formed of a single frequency (color or wavelength).
3. The pattern is observed horizontally so the wire is placed vertically.
4. We know that $\ell = 2\frac{\lambda D}{a} \Rightarrow a = \frac{2\lambda D}{\ell} = \frac{2\times690\times10^{-9}\,m\times2m}{46\times10^{-3}\,m} = 6\times10^{-5}\,m = 60\,\mu m$.
5. a) $\ell' = 2\frac{\lambda' D}{a} = \frac{2\times480\times10^{-9}\,m\times2m}{6\times10^{-5}\,m}$;

   Then $\ell' = 32\times10^{-3}\,m = 32\,mm$.[1]

   b) Figure 2.

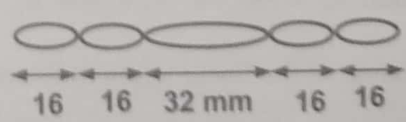
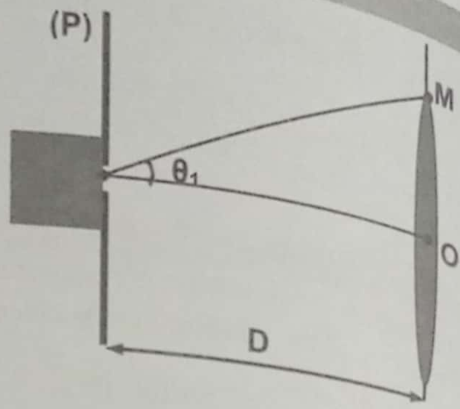


Figure 2

[1] We can say also that the diameter of the central is proportional to the wavelength $\frac{\ell'}{\ell} = \frac{\lambda'}{\lambda} \Rightarrow \ell' = \frac{\lambda'}{\lambda}\times\ell = \frac{480\,nm\times46\,mm}{690\,nm} = 32\,mm$.

V-

1. The diffracted pattern is horizontal then the wire should be vertical.
2. The position of the 1st dark point is given by:



$\sin\theta_1 = \frac{\lambda}{d}$, for small angles $\theta_1 = \frac{\lambda}{d}$;

According to the geometry of the figure:

$\tan\theta_1 = \frac{\ell/2}{D}$ & for small angles $\theta_1 = \frac{\ell/2}{D}$;

Then $\frac{2\lambda}{d} = \frac{\ell}{D}$, thus $\ell = \frac{2\lambda D}{d}$.

3. We have $d = \frac{2\lambda D}{\ell} = \frac{2\times0.64\times10^{-6}m\times2m}{1.2\times10^{-2}m} = 2.1\times10^{-4}m = 0.21\,mm$.
4. The linear width of the central is inversely proportional to the diameter of the wire used. The diameter is doubled $d' = 2d$, then $\ell' = \frac{\ell}{2} = \frac{1.2}{2} = 0.6\,cm = 6mm$.

VI-

1. The angular positions of the dark points are given by $\sin\theta_n = n\frac{\lambda}{a}$;

   For the 1st dark, $n=1$ & for small angles $\sin\theta \approx \theta$; we get $\theta = \frac{\lambda}{a}$.
2. We have $\theta = \frac{\lambda}{a} = \lambda \times \frac{1}{a}$.

   The expression of θ in terms of $\frac{1}{a}$ is of the form $y = mx$ (where $x = \frac{1}{a}$ & $m = \lambda$), then its graphical representation should be an increasing straight line passing through origin which is confirmed graphically.
3. The wavelength is the slope of the straight line: $\lambda = \frac{\Delta\theta}{\Delta\left(\frac{1}{a}\right)} = \frac{(2.5-0)\times10^{-2}}{(3.8-0)\times10^{4}m^{-1}} = 6.6\times10^{-7}m$
4. The frequency of the radiation $\upsilon = \frac{c}{\lambda} = \frac{3\times10^{8}m/s}{6.6\times10^{-7}m} = 4.5\times10^{14}Hz$.

VII-

1. The radiation $\lambda_C = 780\,nm$ belongs to the domain of red radiations; while $\lambda_B = 405\,nm$ to violet.
2. The expression of the linear width of the central bright is given by $\ell = 2\frac{\lambda D}{a}$.
3. The same setup is used, then the values of D & a are unchanged.

   Then $\frac{\ell_B}{\ell_D} = \frac{2\frac{\lambda_B D}{a}}{2\frac{\lambda_D D}{a}} = \frac{\lambda_B}{\lambda_D}$; thus $\lambda_D = \frac{\ell_D}{\ell_B}\lambda_B = \frac{4.8cm}{3cm}\times405\,nm = 648\,nm$.

4. The linear width of the central bright fringe is proportional to the wavelength. However $\lambda_B < \lambda_D < \lambda_C$, then $\ell_B < \ell_D < \ell_C$.

5. a) The frequency of this radiation is conserved $\upsilon = \frac{v}{\lambda'_C}$ & $\upsilon = \frac{c}{\lambda_C}$;

Then $\frac{v}{\lambda'_C} = \frac{c}{\lambda_C}$ & $v = \frac{c}{n}$; thus $\lambda'_C = \frac{\lambda_C}{n}$.

b) The wavelength in polycarbonate is $\lambda'_C = \frac{\lambda_C}{n} = \frac{780\ nm}{1.55} = 503\ nm$.

VIII-

1. Nature obtained at the specified points.

Position	$-0.5cm$	0	$0.75cm$
Nature	1st dark from the negative side	Center of central bright	Center of the first bright fringe

2. The first dark point at $0.5cm$, & the second dark at $1cm$.
3. The linear width of the central bright fringe is the distance between the 1st dark fringe from the positive side to the first dark from the negative side: $\ell = 0.5cm - (-0.5cm) = 1cm$.
4. We have $\ell = \frac{2\lambda D}{a}$.
5. Referring to the previous result, we get:

$$D = \frac{\ell \times a}{2 \times \lambda} = \frac{1 \times 10^{-2}m \times 0.1 \times 10^{-3}m}{2 \times 0.5 \times 10^{-6}m} = 1m.$$

Problems

I-Bac
Experimental Study

A CO_2 laser operates using radiations that lies between $9\ \mu m$ and $11.5\ \mu m$.

Part A
Laser property

1. What are, in nm, the wavelengths limits of the domain of visible radiations?
2. Does the CO_2 laser radiations belong to the ultraviolet, infrared or visible domain?

Part B
Observation

The preceding laser of wavelength λ is placed in front of slit of width is a (figure 1). Thus, on a screen placed at a distance D, we observe the pattern shown in figure below.

1. What is the name of the phenomenon thus observed?
2. What should be the dimension of the slit in order to observe the previous phenomenon?

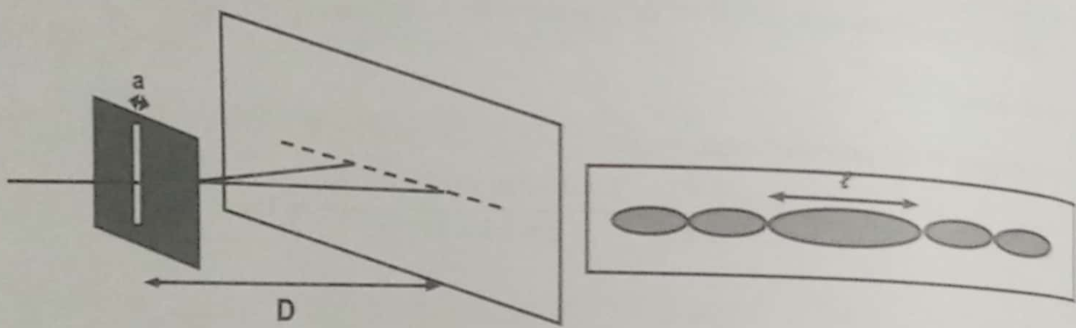


Figure 1

3. The linear width of the central ℓ is modified when λ, a or D are modified.
   In what follows five expressions of ℓ are proposed, where k a dimensionless constant.

i- $\ell = \frac{\lambda D a}{k}$	ii- $\ell = k\lambda\frac{D}{a}$	iii- $\ell = k\lambda\frac{D}{a^2}$	iv- $\ell = k a\frac{D}{\lambda}$	v- $\ell = k\lambda\frac{D^2}{a^2}$

   a) An experiment shows that ℓ is proportional to the wavelength.
   Basing on the previous statement, indicate with justification, which one of previous expressions should be eliminated.

   b) Using a dimensional study, show that *(ii)* & *(v)* are only valid.

Part C
Influence of the slit width

We modify the width of the slit a, while the other parameters as kept unchanged, and each time we measure the width of the central ℓ. The results obtained are listed in the following table, and using this data, we draw the curves shown in figures 2 & 3.

$a(\mu m)$	100	120	200	250	300	340
$\ell(mm)$	19	16	10	8	6.5	5.5

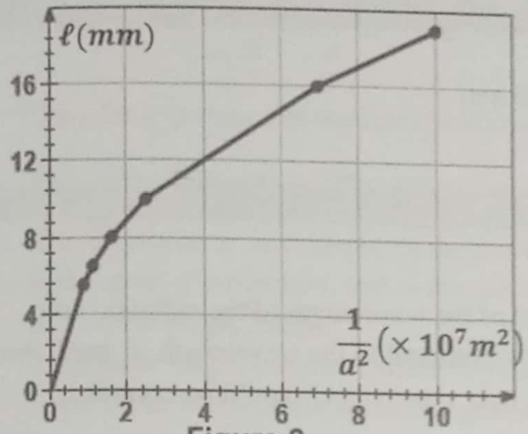


Figure 2

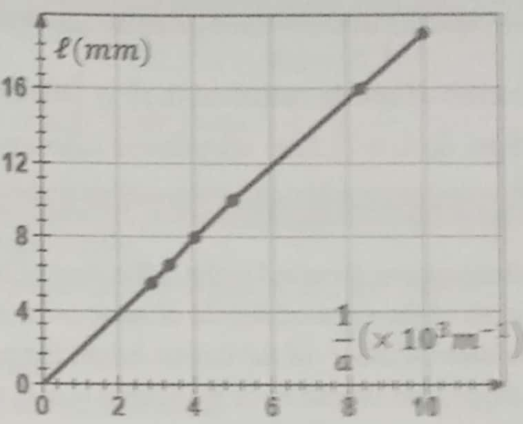


Figure 3

From the expressions previously listed in (Part B-3), indicate that (those) is (are) still valid(s).

Part D

Influence of D

We modify the distance D, while the other parameters are kept unchanged, and each time we measure the width of the central ℓ.

The results obtained are listed in the adjacent table.

$D(m)$	1	1.2	1.5	1.7
$\ell(mm)$	12.5	15	19	21

1. Plot the curve giving ℓ as a function of D.
   Taking as scale: on abscissa axis $1\,div \equiv 0.2\,m$
   & on ordinate axis $1div \equiv 4\,mm$.
2. Referring to the previous curve, write the relation $\ell = f(D)$.
3. Deduce the value of the integer k knowing that the previous measurements where obtained for $\lambda = 616\,nm$ and $a = 0.1\,mm$.

II- Polychromatic Beam

A laser emits light of wavelength $\lambda = 600\,nm$ falls normally on horizontal rectangular slit of width $a = 0.2\,mm$. The diffracted pattern is observed on a screen parallel to the slit and situated at $D = 2.5\,m$ from the slit.

1. What aspect of light does the diffraction phenomenon shows evidence of?
2. Describe the pattern observed.
3. Determine the position of the third dark point obtained on the screen.
4. If the source is replaced by a white light source that emits all the radiations of the visible spectrum.
   Determine the radiations that gives at this position a dark point.

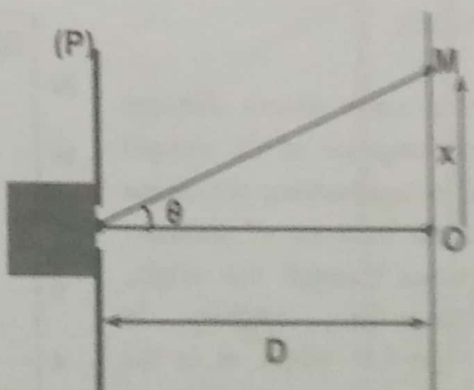


Figure 1

Solutions – Problems

Part A

1. The domain of visible radiations in air is $[400\ nm; 800\ nm]$.
2. $\lambda_1 = 9\mu m$ & $\lambda_2 = 11.5\mu m$ are infrared radiations.

Part B

1. The phenomenon observed is the diffraction of light.
2. The width of the slit used should of same order as that of the wavelength of the radiation used.
3. a) The linear width ℓ of the central bright fringe is proportional to the wavelength λ used, then its expression should be of the form $\ell = \alpha\lambda$; thus, only relation (iv) should be excluded.

   b) The dimensional study of the relations shows:

   (i) $[\ell] = \frac{[\lambda][D][a]}{[k]} = \frac{m \times m \times m}{1} = m^3$, which is not a length.

   (ii) $[\ell] = [k][\lambda]\frac{[D]}{[a]} = 1 \times m \times \frac{m}{m} = m$, it has the dimension of a length (possible expression).

   (iii) $[\ell] = [k][\lambda]\frac{[D]}{[a]^2} = 1 \times m \times \frac{m}{m^2} = \frac{m^2}{m^2} = 1$, which is dimensionless.

   (iv) Previously excluded.

   (v) $[\ell] = [k][\lambda]\frac{[D]^2}{[a]^2} = 1 \times m \times \frac{m^2}{m^2} = m$, it has the dimension of a length (possible expression).

   Thus, (ii) & (v) are only valid.

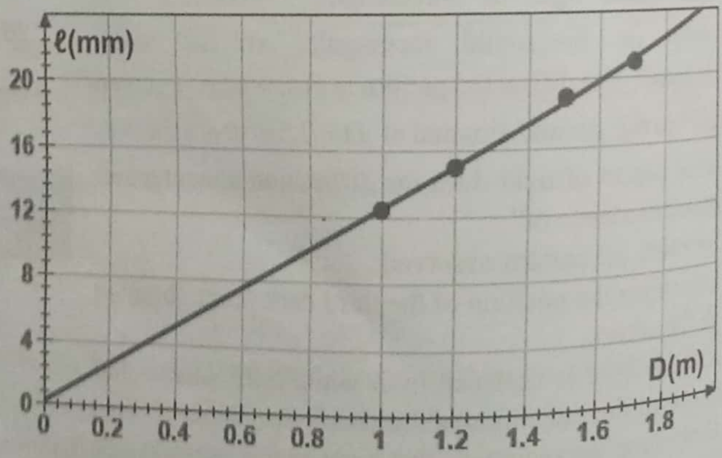
Part C

The curve of figure 3, shows that the linear width of the central bright fringe is proportional to the inverse of the width of the slit, then $\ell = \beta \times \frac{1}{a}$; where β is a constant.

Among the remaining possible relations, only (ii) satisfy this condition.

Part D

1. Graph.



2. The curve shows that the prolongation of the straight line representing the linear width in terms of distance, passes through the origin, then its equation is $\ell = m \times D$ where m is the slope, so $m = \frac{\Delta\ell}{\Delta D}$.

Thus, $m = \frac{\Delta \ell}{\Delta D} = \frac{(21-12.5)\times 10^{-3} m}{(1.7-1)m} = 0.0121$

3. We have $\ell = k \times \frac{\lambda D}{a} = \frac{k\lambda}{a} D$, with $m = \frac{k\lambda}{a}$; then $k = \frac{m \times a}{\lambda} = \frac{0.0121 \times 0.1 \times 10^{-3}}{616 \times 10^{-9}} \approx 2$.

Thus, the expression of the linear width of the central bright fringe is $\ell = 2\frac{\lambda D}{a}$.

II-

1. The diffraction is an evidence of the wave aspect of light.
2. In the zone of diffraction, and in the direction perpendicular to that of slit, we observe:
   - a central bright fringe;
   - alternate bright fringes separated by dark points;
   - the linear width of the lateral fringes is half that of the central.
3. The position of dark points are given by: $\sin\theta_n = n\frac{\lambda}{a}$ where $n = 3$ (3rd dark point)

   For small angles $\sin\theta_3 = 3\frac{\lambda}{a} \approx \theta_{3(rad)}$;

   According to the geometry of the figure $\tan\theta_3 = \frac{x_3}{D} \approx \theta_{3(rad)}$;

   Using the previous relations we get: $\frac{x_3}{D} = 3\frac{\lambda}{a}$;

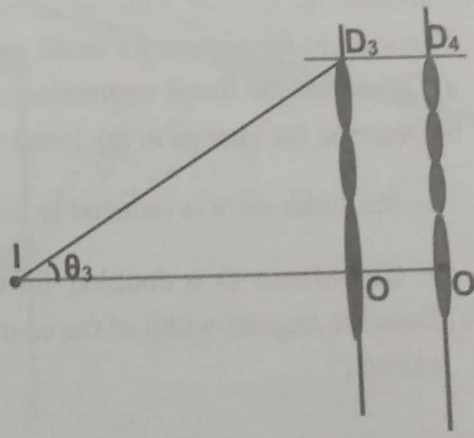
   Then $x_3 = 3\frac{\lambda D}{a} = \frac{3\times 600\times 10^{-9}\times 2.5}{0.2\times 10^{-3}} = 0.0225\,m = 22.5\,mm$.
4. The domain of visible radiations in air is $[400\,nm; 800\,nm]$.

   At the same point, using different radiations (for small angles) we get: $\tan\theta_n = \frac{x_3}{D}$;

   & $\sin\theta_n = n\frac{\lambda_n}{a} \approx \theta_{3(rad)}$, then $\lambda_n = \frac{a\,x_3}{n\,D} = \frac{0.2\times 10^{-3}\times 22.5\times 10^{-3}}{2.5\times n} = \frac{1.8\times 10^{-6}}{n}$

   But $400\times 10^{-9} \le \lambda_n \le 800\times 10^{-9}$; so $400\times 10^{-9} \le \frac{1.8\times 10^{-6}}{n} \le 800\times 10^{-9}$;

   We get $\frac{1.8}{0.8} \le n \le \frac{1.8}{0.4}$; so $2.25 \le n \le 4.5$ & n is an integer then $n = \{3; 4\}$; thus two radiations give a dark fringe at this position.



   For $n = 3$, $\lambda_3 = \frac{1.8\times 10^{-6}}{3} = 6\times 10^{-7}\,m = 600\,nm$.

   (previously given);

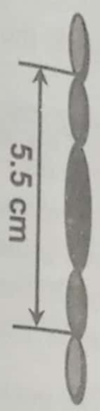
   For $n = 4$, $\lambda_4 = \frac{1.8\times 10^{-6}}{4} = 4.5\times 10^{-7}\,m = 450\,nm$.

   We conclude that at this point, the 3rd point corresponds to λ_3 will coincide with the fourth dark point for λ_4 as shown in the adjacent sketch.

Supplementary Problems

I-
Diffraction Pattern

A laser beam of light, of wavelength in vacuum λ, falls normally on a vertical slit of width $a = 0.2mm$. The diffraction pattern is observed on a screen placed perpendicularly to the laser beam at a distance $D = 4m$ from the slit and parallel to it. Let « ℓ » be the linear width of the central bright fringe.



1. Which aspect of light does this experiment illustrate?
2. Determine the expression of ℓ in terms of λ, D & a.
3. The adjacent sketch represents the pattern observed on the screen.
   a) Determine ℓ.
   b) Deduce λ.
4. If the laser is replaced by a white light.
   a) Specify the color obtained at the center of the central bright.
   b) Determine the wavelengths that give a black fringe $5cm$, from the center of the central.

Answer Key

3. a) $\ell = 2.75\,cm$.

4. b) 3 radiations giving black points.

II-Bac
Light Source

The monochromatic light emitted by a laser source, of wavelength λ, illuminates, under normal incidence, a very narrow slit F_1 of width $a = 0.8\,mm$ cut in an opaque screen (E_1). The phenomenon of diffraction is observed on a screen (E_2) parallel to (E_1), found at a distance $D = 6\,m$ from it. The central bright fringe on (E_2) has a linear width $\ell = 1\,cm$.

1. Draw the experimental setup used.
2. Describe the diffraction pattern observed.
3. Calculate the angular width of the central fringe and deduce the wavelength λ.
4. Variation of the pattern for small angles.
   a) Determine the literal expression of ℓ as a function of λ, D and a.
   b) Describe the change in the linear width ℓ, the wavelength λ kept unchanged if:
      i- the width « a » is reduced to its half value $a' = \frac{a}{2}$ at same D.
      ii- the distance D is doubled, for the same a.
   c) Does the angular width of the central bright fringe, in the two cases *i)* and *ii)* vary? Justify your answer.

Answer Key

3. $\lambda = 668\,nm$.

4. c) unchanged.

I-LS 2013 2[nd]
Applications of the Diffraction of Light

Part A

Measurement of the width of a slit

A laser beam of light, of wavelength in vacuum $\lambda = 632.8\,nm$, falls normally on a vertical slit of width «a». The diffraction pattern is observed on a screen placed perpendicularly to the laser beam at a distance $D = 1.5\,m$ from the slit.

Let «ℓ» be the linear width of the central fringe (Figure 1).

The angle of diffraction θ corresponding to a dark fringe of order n is given by $\sin\theta = n\dfrac{\lambda}{a}$ where $n = \pm1, \pm2, \pm3 \ldots$

For small angles, take $\tan\theta \approx \sin\theta \approx \theta$ in radian.

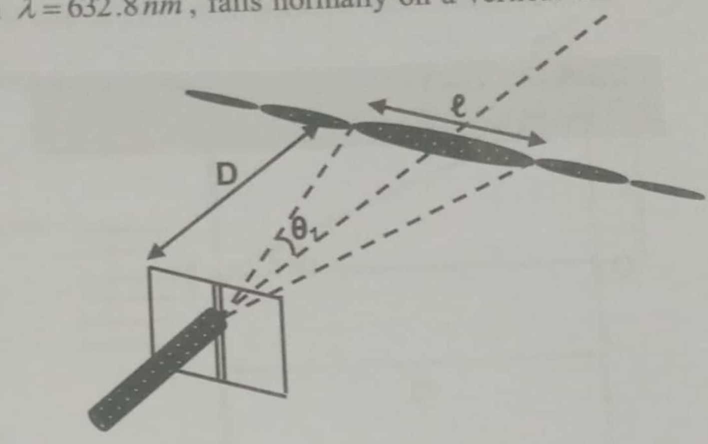


Figure 1

1. Describe the aspect of the diffraction pattern observed on the screen.
2. Write the relation among a, θ_1 and λ.
3. Establish the relation among a, λ, ℓ and D.
4. Knowing that $\ell = 6.3\,mm$, calculate the width «a» of the used slit.

Part B

Controlling the thickness of thin wire

A manufacturer of thin wires wishes to control the diameter of his product. He uses the same set-up mentioned in part (A) but he replaces the slit by a thin vertical wire. He observes on the screen the phenomenon of diffraction (figure 2).

For $D = 2.60\,m$, he obtains a central fringe of constant linear width $\ell_1 = 3.4\,mm$.

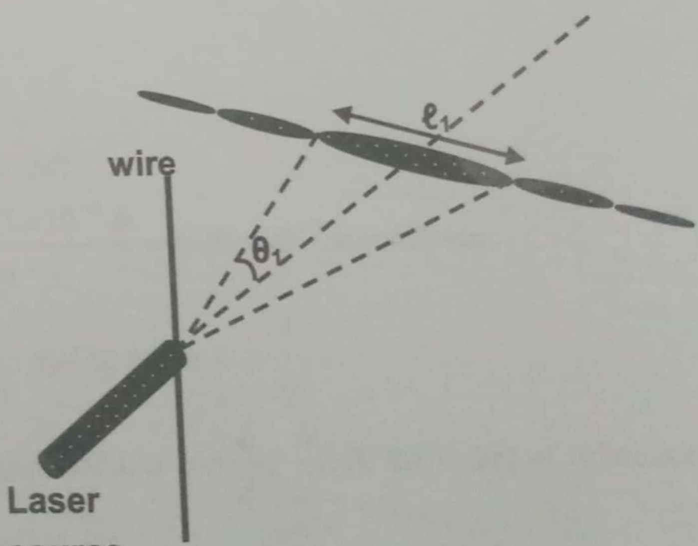


Figure 2

1. Calculate the value of the diameter «a_1» of the wire at the illuminated point.
2. The manufacturer illuminates the wire at different positions under the same precedent conditions. Specify the indicator that permits the manufacturer to check that the diameter of the wire is constant.

Part C

Measurement of the index of water

We place the whole set-up of part (A) in water of index of refraction n_{water}. We obtain a new diffraction pattern. We find that for $D = 1.5\,m$ and $a = 0.3\,mm$, the linear width of the central fringe is $\ell_2 = 4.7\,mm$.

1. Calculate the wavelength λ' of the laser light in water.

2. a) Determine the relation among λ, λ' and n_{water}.

   b) Deduce the value of n_{water}.

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See Page 45 – Part A

Solutions – Sessions

I-LS 2013 2nd

Part A

1. On the screen, the direction of the pattern of fringes is perpendicular to that of slit, we observe:
   - a central bright fringe;
   - alternate bright fringes separated by dark points;
   - the linear width of the central fringe is double that of any other lateral bright fringe.

2. For the first dark point $n = 1$, so $\sin\theta_1 = \frac{\lambda}{a}$ then

   $\theta_1 = \frac{\lambda}{a}$ (using the small angles approximation).

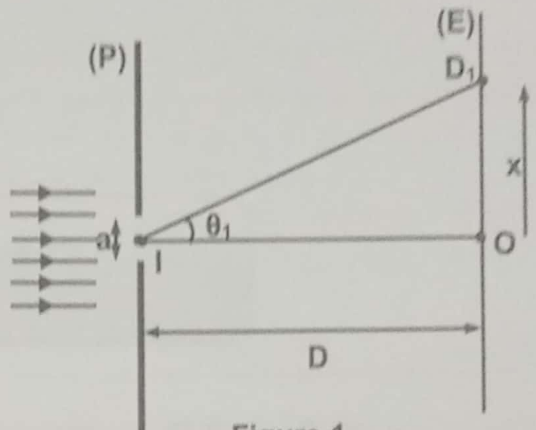


Figure 1

3. According to the geometry of the figure $\tan\theta_1 = \frac{\ell}{2D}$;

   However, for small angles $\tan\theta_1 \approx \theta_1 = \frac{\ell}{2D}$;

   Referring to the two expressions of θ_1 we get:

   $\theta_1 = \frac{\ell}{2D} = \frac{\lambda}{a}$, then $\ell = 2\frac{\lambda D}{a}$.

4. We have $\ell = 2\frac{\lambda D}{a}$, then $a = 2\frac{\lambda D}{\ell} = 2\frac{632.8\times10^{-9}\,m\times1.5m}{6.3\times10^{-3}\,m} = 3\times10^{-4}\,m = 0.3\,mm$.

Part B

1. When the wire of diameter « a_1 » is placed, the light diffracts:

   $$a_1 = 2\frac{\lambda D}{\ell_1} = 2\frac{632.8\times10^{-9}\times2.6}{3.4\times10^{-3}} = 9.7\times10^{-4}\,m = 0.97\,mm.$$

2. As long the linear width ℓ_1 of the central fringe remains unchanged, the diameter a_1 of the wire is uniform.

Part C

1. Referring to the relation established in part A we get:

   $$\ell_2 = 2\frac{\lambda' D}{a};\ \text{then}\ \lambda' = \frac{a\times\ell_2}{2D} = \frac{0.3\times10^{-3}\,m\times4.7\times10^{-3}\,m}{2\times1.5m} = 4.70\times10^{-7}\,m = 470\,nm.$$

2. a) The frequency υ of the radiation in air $\upsilon = \frac{c}{\lambda}$ and in water $\upsilon = \frac{v}{\lambda'}$;

   The frequency is conserved independent of the medium $\upsilon = \frac{c}{\lambda} = \frac{v}{\lambda'}$ & the index of refraction

   is given by $n_{water} = \frac{c}{v}$; then we get $\lambda' = \frac{\lambda}{n_{water}}$.

   b) Using the previous relation we get $n_{water} = \frac{\lambda}{\lambda'} = \frac{632.8nm}{470\,nm} = 1.346$.

Unit III

Light

Chapter 13

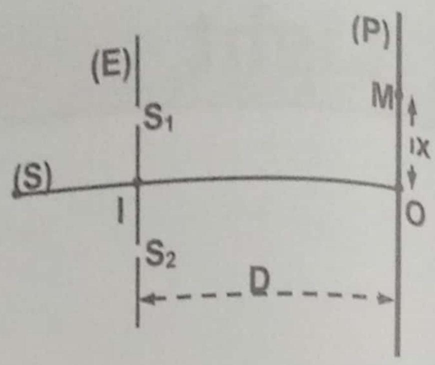
Interference of Light

LS - Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Interference of Light		1st(1)	-	-	-	2nd	-	-	-	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Interference of Light	-	-	2nd	-	-	-	-	1st(B)	-	1st(B)

Essentials

I-Definitions

- The interference is particular type of superposition studied using a system called Young's apparatus or Young's double slit (adjacent apparatus).
- Two sources are called synchronous if they have same frequency (color).
- Interference of light is due to the superposition of two synchronous and coherent light beams.
- Two synchronous sources are coherent if they keep a phase difference constant, which is ensured by taking a primary source that illuminates two secondary sources.



Two *independent* light sources could be synchronous but never coherent.

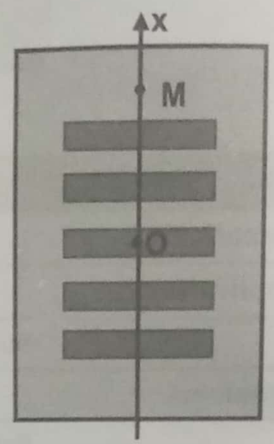
II-Young's Apparatus

1. Description

On a screen placed in the region of interference, parallel to the plane of slits, we observe:

- a central bright fringe.
- alternate, equidistant, straight bright and dark fringes.

The fringes obtained are parallel to the slits.

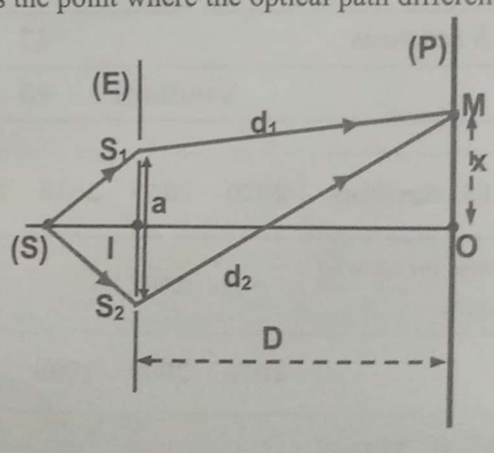


2. Optical path difference

The optical path difference between two light waves interfering at a point M is given by $\delta = (SS_2M) - (SS_1M)$, then $\delta = d_2 - d_1 = \frac{a\,x}{D}$

where $d_2 = S_2M$, $d_1 = S_1M$, $a = S_1S_2$, D is the distance (slit-screen) & $x = \overline{OM}$.

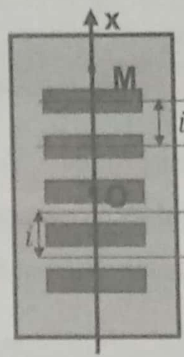
- The center of the central bright fringe is defined as the point where the optical path difference is zero $\delta = 0$.
  We have $\delta_O = (SS_2O) - (SS_1O) = 0$.
  O is the center of the central bright fringe.



- The bright fringes are due to a constructive interference (waves in phase), the optical path difference is $\delta = k\lambda$, and we get $x_k = k\frac{\lambda D}{a}$, k is whole number.
- The dark fringes are due to a destructive interference (waves opposite in phase), the optical path difference is $\delta = \left(k + \frac{1}{2}\right)\lambda$, and we get $x_k = \left(k + \frac{1}{2}\right)\frac{\lambda D}{a}$, k is whole number.

3. Interfringe distance

The interfringe i is the distance between the centers of two consecutives fringes of the same nature. Its expression in air is: $i = x_{k+1} - x_k = \dfrac{\lambda D}{a}$.



Note:

To identify the nature of the fringe obtained at a given point, we calculate the ratio $\dfrac{\delta}{\lambda}$; if we get:

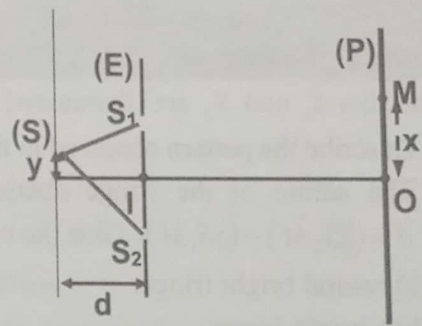
- whole number k, then it is the center of the bright fringe of rank k;
- half-number $k + \dfrac{1}{2}$, then it is the center of a dark fringe of rank $(k+1)$ if $k + \dfrac{1}{2} > 0$ & k if $k + \dfrac{1}{2} < 0$.

III-Particular Situations

The interference pattern is formed of: position of the central bright fringe & the interfringe distance.

- When (S) is displaced along the axis of $[S_1S_2]$, the pattern remains unchanged.

- When (S) is placed at a distance $d = SI$ then displaced perpendicular to the axis of $[S_1S_2]$ by an algebraic distance y, then the optical path difference becomes $\delta = (SS_2 - SS_1) + (S_2M - S_1M) = a\dfrac{y}{d} + a\dfrac{x}{D}$.



  In this case:

  the central bright fringe is displaced in a direction opposite to that of displacement of source;
  the interfringe distance will remains unchanged.

- When a thin plate of thickness e and whose index of refraction is n, is placed in front of one the slits S_1 or S_2, then the optical path difference becomes $\delta = a\dfrac{x}{D} \pm e(n-1)$. In this case:

  the central bright fringe is shifted to the side on which the plate is introduced;
  the interfringe distance will remains unchanged.

- When the whole is immersed in a medium of index n, the optical path difference becomes $\delta = na\dfrac{x}{D}$. In this case:

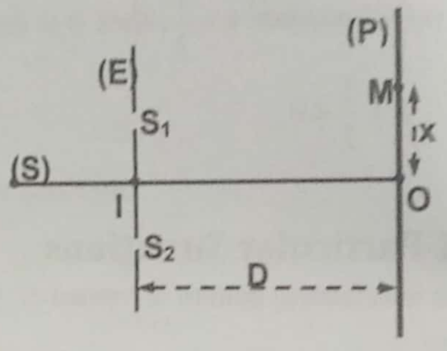
  the central bright fringe is still at O;

  the interfringe distance will decrease $i' = \dfrac{i}{n}$.

Applications

In all what follows, Young's slits apparatus is formed of two very thin slits S_1 and S_2, parallel and separated by a distance a, and a screen of observation (P) placed parallel to the plane of the slits at a distance D from this plane. Let M be a point on the screen defined by its abscissa $x = \overline{OM}$.

I- Pattern & Conditions



1. The slits S_1 and S_2 are illuminated respectively by a red and blue radiations. The interference is not observed, explain.
2. The slits S_1 and S_2 are illuminated by independent red radiations. The interference is not observed, explain.
3. We say that in order to observe the interference, the sources should be coherent. Give the physical meaning of this property and how it is ensured.
4. Draw the apparatus shown in the adjacent figure showing the region and zone of interference.

II- Nature of a Fringe

The slits S_1 and S_2 are illuminated by a same primary source.

1. Describe the pattern observed in the zone of interference.
2. The nature of the fringe obtained at a point M depends on the optical path difference $\delta = (SS_2M) - (SS_1M)$. Give the condition satisfied by δ so that M is at the center of:
   a) central bright fringe.
   b) a bright fringe.
   c) a dark fringe.
3. If the wavelength of the source is $\lambda = 680\ nm$. Calculate δ at the center of the:
   a) 2nd bright fringe from the positive side.
   b) 4th dark fringe from the positive side.
   c) 4th dark fringe from the negative side.

III- Value of Wavelength

The sources used in Young's apparatus S_1 and S_2 are coherent. The expression of the optical path difference is given by $\delta = S_2M - S_1M = \dfrac{a\,x}{D}$, where $a = 2mm$ & $D = 1.6m$.

1. Determine the expression of the abscissas of the center of the bright fringes.
2. Define the interfringe distance i.
3. Show that $i = \dfrac{\lambda D}{a}$.

4. Determine the expression of the abscissas of the center dark fringes and then the interfringe distance.
5. If the distance between the centers of four successive bright fringes is $1.8\,mm$.
   Show that the wavelength of the radiation used is $750\,nm$.
6. Determine the abscissa of the center of the 4^{th} dark fringe from the positive side.
7. Using the same apparatus but another wavelength λ', we notice that the center of the previous 4^{th} dark fringe is replaced by center of the 6^{th} bright. Determine λ'.

IV- Polychromatic Source

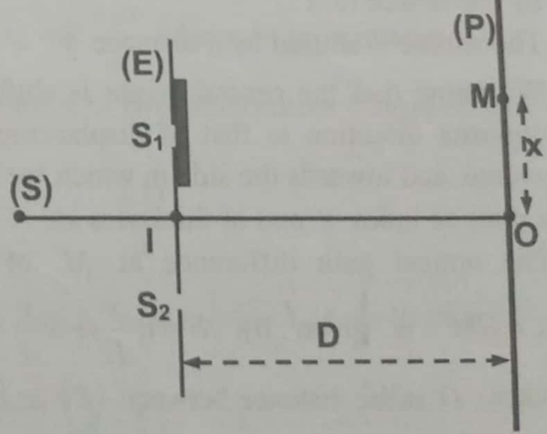
We use a white light source whose wavelengths $\lambda \in [0.4\,\mu m; 0.8\,\mu m]$. The expression of the optical path difference is given by $\delta = S_2M - S_1M = \frac{a\,x}{D}$, where $a = 1.5\,mm$ & $D = 1.8\,m$

1. Specify the color obtained at O.
2. Consider a point M of abscissa $x = 3\,mm$.
   a) Show that the wavelengths of the radiations that reach M in phase are given by $\lambda(\text{in } \mu\text{m}) = \frac{2.5}{k}$ with k being a non-zero positive integer.
   b) Determine the wavelengths of these radiations.
3. At the same point of abscissa $x = 3\,mm$.
   a) Show that the wavelengths of the radiations that reach M are opposite in phase if their wavelengths are $\lambda(\text{in } \mu\text{m}) = \frac{5}{2k+1}$, k being a non-zero positive integer.
   b) Determine the wavelengths of these radiations.

V- Pattern and Plates

The wavelength of the radiation emitted by the source is $\lambda = 680\,nm$.

A thin plate of thickness e and whose index of refraction $n = 1.52$ is placed in front of S_1 as shown in the adjacent diagram. The optical path difference at M is given by $\delta = a\frac{x}{D} - e(n-1)$.



1. Justify that the central bright fringe is no longer at O.
2. Determine expression of the abscissa of the new position of the center of the central bright fringe and indicate whether it is displaced upwards or downwards.
3. Determine the abscissas of the centers of the bright and dark fringes and the interfringe distance.
4. Give a description of the pattern observed on screen.
5. Determine e if the center of the central bright fringe occupies the position of the 3^{rd} bright fringe before placing the plate.
6. Specify the nature of the fringe obtained at O.

VI- Pattern and Position of Source

The source (S) is placed at a distance $d = 25\,cm$ from the plane (E) carrying two thin slits S_1 & S_2 separated by $a = 2mm$ acting as secondary sources. The wavelength of the radiation emitted by the source is $\lambda = 500\ nm$. The source is shifted upwards by a distance y and the pattern is obtained on a screen (P) parallel to (E) and at a distance D.

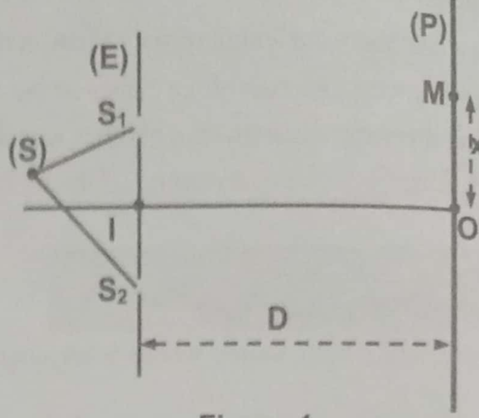


Figure 1

1. Justify that the central bright fringe is no longer at O.
2. Specify the direction of the displacement of the center of the central bright fringe.
3. The optical path difference at M is given by $\delta = a\dfrac{x}{D} + a\dfrac{y}{d}$.
   a) Determine the abscissas of the center of the dark fringes and the interfringe distance.
   b) We admit that this interfringe distance is the same for bright fringes, give a description of the pattern observed on screen.
   c) Determine y if the center of the new central bright fringe occupies the position that was taken by the center of the 2nd dark fringe before the displacement of the source.
4. We decrease the wavelength to a value λ', we notice that the center of the 5th bright fringe from the pattern when we are using $\lambda = 500\ nm$ is replaced by its successive while using λ'. Determine λ'.

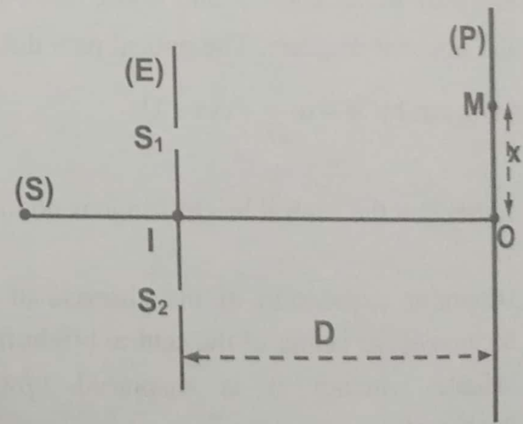
VII- Pattern and Source

The source (S) is placed at a distance d from the plane (E) of the slits separated by a distance $a = S_1S_2$. The wavelength of the radiation emitted by the source is λ.

The source is shifted by a distance y.

We know that the central bright is shifted in the opposite direction to that of displacement of the source, and towards the side in which we introduce a plate of index n and of thickness e.

The optical path difference at M of abscissa $x = OM$ is given by $\delta = a\dfrac{x}{D} + a\dfrac{y}{d} - e(n-1)$ where D is the distance between (E) & (P).

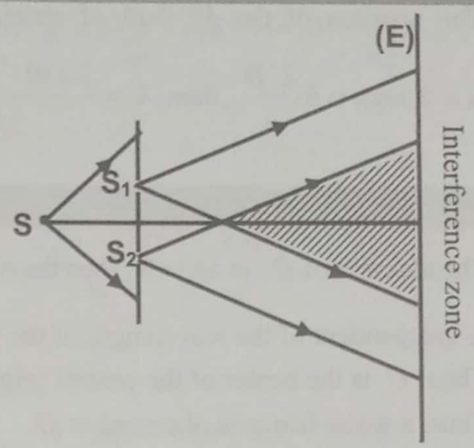


1. Determine the expression of y in terms of d, a, n & e so that the center of the central bright fringe remains at O.
2. Justify that the interfringe distance remains unchanged.

Solutions – Applications

I-

1. S_1 and S_2 have different frequencies, then they are neither coherent nor synchronous. Thus, the interference will not take place.
2. Independent red sources mean that these sources are synchronous but not coherent.
   Thus, the interference pattern is not observed.
3. The coherence of the sources means that their phase difference remains constant, which is ensured only if they are produced by the same primary source.
4. Diagram.



II-

1. In the zone of interference, we observe:
   - a central bright fringe at O.
   - alternate, equidistant, straight bright and dark fringes.
2. a) At the center of the central bright $\delta = 0$.
   b) At the center of a bright fringe $\delta = k\lambda$ where k is a whole integer.
   c) At the center of a dark fringe $\delta = \left(k + \frac{1}{2}\right)\lambda$ where k is a whole integer.
3. a) At the center of the 2nd bright, $k = 2$; so $\delta = 2\lambda = 2 \times 680 = 1360\ nm$.
   b) At the center of the 4th dark from positive side $k = 3$; so $\delta = 3.5\lambda = 3.5 \times 680 = 2380\ nm$.
   c) At the center of the 4th dark from negative side $k = -4$.
   So, $\delta = -3.5\lambda = -3.5 \times 680 = -2380\ nm$.

III-

1. For the centers of the bright fringes $\delta = \frac{a\,x_k}{D} = k\lambda$, so $x_k = k\frac{\lambda D}{a}$ where k is an integer.
2. The interfringe distance i is the distance between the centers of two consecutive fringes of same nature.
3. We have $i = x_{k+1} - x_k = (k+1)\frac{\lambda D}{a} - k\frac{\lambda D}{a} = \frac{\lambda D}{a}(k+1-k) = \frac{\lambda D}{a}$.
4. For the centers of the dark fringes $\delta = \frac{a\,x_k}{D} = \left(k + \frac{1}{2}\right)\lambda$, so $x_k = \left(k + \frac{1}{2}\right)\frac{\lambda D}{a}$; k is an integer.

   Then, $i = x_{k+1} - x_k = \left(k+1+\frac{1}{2}\right)\frac{\lambda D}{a} - \left(k + \frac{1}{2}\right)\frac{\lambda D}{a} = \frac{\lambda D}{a}\left(k+1+\frac{1}{2}-k-\frac{1}{2}\right) = \frac{\lambda D}{a}$.
5. The distance between the centers of four consecutive bright fringes is equal to three interfringe distance. So, $3i = 1.8\ mm$; then $i = 0.6\ mm$.

   However $i = \frac{\lambda D}{a}$, so $\lambda = \frac{i \times a}{D} = \frac{0.6 \times 10^{-3} \times 2 \times 10^{-3}}{1.6} = 7.5 \times 10^{-7}\ m = 750\ nm$.

6. The abscissas of the bright fringes are given by $x_k = \left(k+\frac{1}{2}\right)\frac{\lambda D}{a}$ & $i = \frac{\lambda D}{a}$, so $x_k = \left(k+\frac{1}{2}\right)i$;

At the center of the 4th dark, $k=3$, then $x_4 = \left(3+\frac{1}{2}\right)i = 3.5i = 2.1\,mm$.

7. The position of the 4th dark of abscissa $x = 2.1\,mm$ is occupied by the new 6th bright, so

$x = 2.1\,mm = 6\frac{\lambda' D}{a}$, then $\lambda' = \frac{2\times10^{-3}\,m\times2.1\times10^{-3}\,m}{6\times1.6m} = 4.375\times10^{-7}\,m = 437.5nm$.

IV-

1. The abscissa of O, is $x_O = 0$; then the optical path difference at this point is $\delta_O = \frac{a\,x_O}{D} = 0$ which is independent of the wavelength of the radiation used.

Then, O is the center of the central bright fringe for all the radiations of the visible spectrum;
Thus, a white fringe is observed at O.

2. a) The radiations are in phase, so $\delta = \frac{a\,x}{D} = k\,\lambda$, then $\lambda = \frac{a\,x}{k\,D}$.

Thus, $\lambda = \frac{1.5\times10^{-3}\times3\times10^{-3}}{1.8k} = \frac{2.5\times10^{-6}}{k} = \frac{2.5}{k}$ where k is a integer and λ in μm.

b) We have $\lambda \in [0.4\,\mu m; 0.8\,\mu m]$, so $0.4 \le \frac{2.5}{k} \le 0.8$, then $\frac{2.5}{0.8} \le k \le \frac{2.5}{0.4}$;

We get $3.125 \le k \le 6.25$; but k is an integer, then $k = \{4;5;6\}$.

Thus, three radiations that superpose at this point are in phase.

For $k=4$, $\lambda = 0.625\,\mu m$; for $k=5$, $\lambda = 0.5\,\mu m$; & for $k=6$, $\lambda = 0.42\,\mu m$.

3. a) We have $\delta = \frac{a\,x}{D} = \left(k+\frac{1}{2}\right)\lambda$, so $\lambda = \frac{2a\,x}{(2k+1)D} = \frac{2\times1.5\times10^{-3}\times3\times10^{-3}}{(2k+1)\times1.8} = \frac{5\times10^{-6}}{2k+1}$;

Then $\lambda = \frac{5}{2k+1}$ where k is a integer and λ in μm.

b) We have $\lambda \in [0.4\,\mu m; 0.8\,\mu m]$, so $0.4 \le \frac{5}{2k+1} \le 0.8$, then $\frac{1}{2}\left(\frac{5}{0.8}-1\right) \le k \le \frac{1}{2}\left(\frac{5}{0.4}-1\right)$;

We get $2.6 \le k \le 5.8$; but k is an integer, then $k = \{3;4;5\}$.

Thus, three radiations that superpose at this point are opposite in phase (center of dark fringe);

For $k=3$, $\lambda = 0.714\,\mu m$; for $k=4$, $\lambda = 0.556\,\mu m$; & for $k=5$, $\lambda = 0.455\,\mu m$.

V-

1. The optical path difference at O, of abscissa $x_O = 0$ is $\delta_O = a\frac{x_O}{D} - e(n-1) = -e(n-1) \ne 0$;

Then O is no longer the center of the central bright fringe.

2. Let O' be the new position of the central bright fringe, the optical path difference at this point is zero; so, $\delta_{O'} = a\frac{x_{O'}}{D} - e(n-1) = 0$; we get $x_{O'} = \frac{e(n-1)D}{a} > 0$; then the center of the central bright fringe is shifted upwards (to the side of which the plate is introduced).

3. The positions of the centers of bright fringes are given by: $\delta = a\dfrac{x_k}{D} - e(n-1) = k\lambda$;

$x_k = k\dfrac{\lambda D}{a} + \dfrac{e(n-1)D}{a}$ where k is an integer

The positions of the centers of dark fringes are given by: $\delta = a\dfrac{x_k}{D} - e(n-1) = \left(k + \dfrac{1}{2}\right)\lambda$;

$x_k = \left(k + \dfrac{1}{2}\right)\dfrac{\lambda D}{a} + \dfrac{e(n-1)D}{a}$ where k is an integer.

The interfringe distance for bright fringes:

$$i = x_{k+1} - x_k = \left[(k+1)\frac{\lambda D}{a} + \frac{e(n-1)D}{a}\right] - \left[k\frac{\lambda D}{a} + \frac{e(n-1)D}{a}\right] = \frac{\lambda D}{a};$$

The interfringe distance for dark fringes:

$$i = x_{k+1} - x_k = \left[\left(k+1+\frac{1}{2}\right)\frac{\lambda D}{a} + \frac{e(n-1)D}{a}\right] - \left[\left(k+\frac{1}{2}\right)\frac{\lambda D}{a} + \frac{e(n-1)D}{a}\right] = \frac{\lambda D}{a}.$$

Thus, the interfringe distance is not modified in the presence of the plate.

4. On the screen, we observe:
   - a central bright fringe above O;
   - alternate, equidistant, straight bright and dark fringes.

   The pattern is simply undergoing a translation.

5. The new position of the central bright fringe occupies the position of the 3rd bright fringe before introducing the plate.

   So, $x_{O'} = \dfrac{e(n-1)D}{a} = 3i = 3\dfrac{\lambda D}{a}$; then $e = \dfrac{3\lambda}{n-1} = \dfrac{3\times 680}{1.52-1} = 3923\ nm$.

6. The optical path difference at $O: \delta_O = a\dfrac{x_O}{D} - e(n-1) = -e(n-1) = -3923\,(1.52-1) = -2040\ nm$

   $\dfrac{\delta_O}{\lambda} = \dfrac{-2040\ nm}{680\ nm} = -3$ (whole number);

   Then O becomes the center of the third bright fringe from the negative side.

VI-

1. The optical path difference at O is $\delta_O = (SS_2O) - (SS_1O) = (SS_2 - SS_1) + (S_2O - S_1O)$;

   But O belongs to the perpendicular bisector of $[S_1S_2]$, so $S_2O - S_1O = 0$;

   Then, $\delta_O = SS_2 - SS_1 \neq 0$ (but $SS_2 > SS_1$).

2. Let O' be the new position of the central bright fringe, so the optical path difference at this point is zero: $\delta_{O'} = (SS_2O') - (SS_1O') = (SS_2 - SS_1) + (S_2O' - S_1O') = 0$;

   So, $SS_2 - SS_1 = S_1O' - S_2O'$; but $SS_2 - SS_1 > 0$ (the source is farther from S_2 than S_1);

   Then, $S_1O' > S_2O'$; thus the new central bright fringe is closer to S_2 than S_1 (the displacement of the central bright fringe is opposite to that of displacement of the source).

3. a) At the center of a dark fringe: $\delta = a\dfrac{x_k}{D} + a\dfrac{y}{d} = \left(k + \dfrac{1}{2}\right)\lambda$;

Then $x_k = \left(k+\frac{1}{2}\right)\frac{\lambda D}{a} - y\frac{D}{d}$.

The interfringe distance $i = x_{k+1} - x_k = \left(k+1+\frac{1}{2}\right)\frac{\lambda D}{a} - y\frac{D}{d} - \left(k+\frac{1}{2}\right)\frac{\lambda D}{a} + y\frac{D}{d}$;

Thus, $i = \frac{\lambda D}{a}$.

b) In the zone of interference, we observe:

- a central bright fringe O' below O.
- alternate, equidistant, straight bright and dark fringes.

The pattern is shifted downward but not modified.

c) The new position of the central bright is given by $\delta_{O'} = a\frac{x_0}{D} + a\frac{y}{d} = 0$; so $x_0 = -y\frac{D}{d}$.

The center of his central occupies the position of the 2nd dark, then $x_0 = -1.5i = -1.5\frac{\lambda D}{a}$;

Then, $-y\frac{D}{d} = -1.5\frac{\lambda D}{a}$; thus, $y = 1.5\frac{\lambda \times d}{a} = 1.5 \times \frac{500\times10^{-3}\,\mu m \times 25\times10^{-2}\,m}{2\times10^{-3}\,m} = 93.75\ \mu m$.

d) The center of the 5th bright fringe using λ is given by $x = x_5 = 5\frac{\lambda D}{a} - y\frac{D}{d}$;

Since the wavelength is decreased, the interfringe distance $\left(i = \frac{\lambda D}{a}\right)$ will decrease also; then the same point is replaced by the center of the $(5+1)^{th}$ bright while using λ'.

We get: $x = x_{k+1} = 6\frac{\lambda' D}{a} - y\frac{D}{d}$;

Then, $6\frac{\lambda' D}{a} - y\frac{D}{d} = 5\frac{\lambda D}{a} - y\frac{D}{d}$; thus $\lambda' = \frac{5}{6}\lambda = \frac{5}{6}\times 500 \approx 417\ nm$.

VII-

1. The center of the central bright fringe is at O, then $\delta_O = 0$ ($x_O = 0$).

$\delta_O = a\frac{x_O}{D} + a\frac{y}{d} - e(n-1) = 0$, so $0 + a\frac{y}{d} - e(n-1) = 0$; then $y = e(n-1)\frac{d}{a} > 0$.

The plate should be introduced from the same side to that of the displacement of the source.

2. The position of the centers of bright fringes are given by: $\delta = a\frac{x_k}{D} + a\frac{y}{d} - e(n-1) = k\lambda$.

$x_k = k\frac{\lambda D}{a} - y\frac{D}{d} + e(n-1)\frac{D}{a}$ where k is an integer.

The interfringe distance:

$i = x_{k+1} - x_k = (k+1)\frac{\lambda D}{a} - y\frac{D}{d} + e(n-1)\frac{D}{a} - k\frac{\lambda D}{a} + y\frac{D}{d} - e(n-1)\frac{D}{a}$;

Then, $i = \frac{\lambda D}{a}$ unchanged.

Problems

I-
Displacement of the Source

A source put at a point S, emitting a monochromatic radiation of wavelength λ in air, illuminates the two slits S_1 and S_2 that are separated by a distance a. The screen of observation is placed at a distance D from the plane of the slits.

The source S_0 placed at a distance $d = S_0I$ is displaced vertically up above the horizontal axis of symmetry IO, by the distance $y = \overline{S_0S}$.

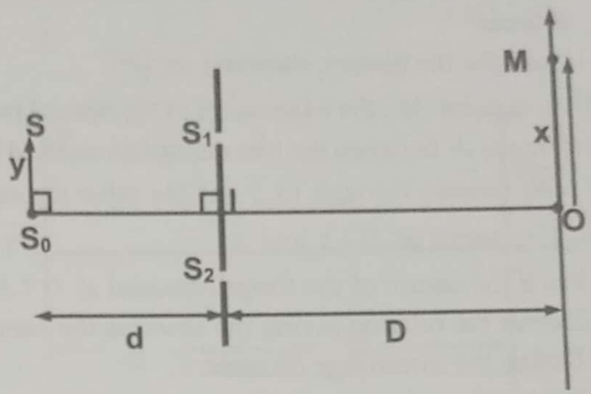


Figure 1

1. Justify that the center of the central bright fringe:
   a) is no longer at O.
   b) is displaced downwards.
2. At a point M of abscissa $x = \overline{OM}$, the optical path difference is given by the relation:
   $\delta = (SS_2M) - (SS_1M) = a\dfrac{x}{D} + a\dfrac{y}{d}$.
   a) Give the expression of δ, in terms of λ, so that M is at the center of bright fringe.
   b) Determine the expression giving the abscissas of the centers of bright fringes.
   c) Show that the interfringe distance for bright and dark fringes is $i = \dfrac{\lambda D}{a}$.
3. Give a qualitative description of the pattern observed on screen.
4. Given $\lambda = 650\ nm$, $D = 2\ m$, $d = 50\ cm$ & $a = 0.8\ mm$.
   a) Calculate the value of i.
   b) Determine y if the center of the central bright fringe is $1.6\ cm$ above O.
5. If the source S is replaced by a white light source emitting all the radiations in the visible spectrum $[400\ nm; 800\ nm]$.
   a) Determine the number of radiations that give at O a dark fringe.
   b) A spectrometer is placed at O, describe qualitatively the shape of the spectrum obtained.

II-
Central Bright Fringe

Young's slits apparatus is formed of two very thin slits (S_1) and (S_2), parallel and separated by a distance $a = 0.2\ mm$, and a screen of observation (P) placed parallel to the plane of the slits at a distance $D = 1\ m$ from this plane.

On the perpendicular to IO at point O and parallel to S_1S_2, a point M is defined by its abscissa $OM = x$.

Part A
Interference pattern

(S_1) and (S_2) are illuminated with a monochromatic radiation of wavelength λ issued from S that is placed at equal distances from (S_1) and (S_2).

1. a) Why (S_1) and (S_2) must be illuminated by the same primary source not by two independent sources?
   b) Describe the pattern observed on (P).

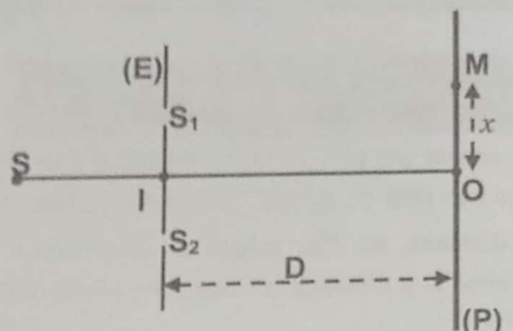


Figure 1

2. Give, at point M, the expression of the optical path difference δ between the two radiations emitted by S, one passing through (S_1) and the other through (S_2), in terms of D, x and a.
3. What is the nature of the fringe obtained at O? Justify your answer.
4. a) Derive the relation giving the abscissa the center of the kth dark fringe.
   b) Define the interfringe distance i.
   c) Derive the expression giving the expression of i as function of D, λ and a.
5. The center of the 6th dark fringe from the positive side is found at $x = 18\,mm$.
   Deduce the wavelength λ of the radiation used.

Part B
Effect of glass sheets

Consider a glass sheet (L_1) of thickness $e_1 = 5\,\mu m$ and of index of refraction $n_1 = 1.5$, placed behind (S_1) and the source (S) is replaced by a white light.

1. Does the light travel in glass faster or slower than in air? Justify.
2. The increase in the optical path S_1M due to the introduction of the sheet (L_1) is equal to $e_1(n_1 - 1)$
   a) Determine, in terms of the given, the new expression of the optical path difference δ'.
   b) Specify the new position $x_{O'}$ of the central bright fringe O' on screen and indicate, with justification the color obtained, at the considered point.
   c) We consider a new glass sheet (L_2) of thickness e_2 and of index of refraction $n_2 = 1.6$.
      i- Specify, with all the necessary explanations, behind which slit (S_1) or (S_2), this sheet should be placed in order to retrieve the new central bright fringe to O.
      ii- Determine the thickness e_2 of this sheet (L_2).

Part C
Fringes and index of refraction

The index of refraction n depends on the wavelength λ of the radiation used.

Color	Violet	Blue	Yellow	Red-Orange	Red
Wavelength $\lambda(nm)$	390	486	589	656	790
Index n	1.536	1.524	1.517	1.514	1.510

1. Plot the curve representing the change of the index n in terms of $\frac{1}{\lambda^2}$ using for scales: on abscissa axis: 1div $\equiv 1\times10^{12}\,m^{-2}$ & on ordinate axis: 1div $\equiv 0.0025$.

2. Justify that the expression of the index n can be written $n = A + \frac{B}{\lambda^2}$, where A & B are two constants to be determined to the nearest 10^{-3}.
3. Determine now the centrals that corresponds to the violet and red radiations while using the slit (L_1). Draw a conclusion concerning the consequence of this on the color of the central.

III-Engineering (2016/2017)
Interference

Young's double slit experiment is performed in air (Figure 1). These very fine slits, apart by $F_1F_2 = a = 1mm$, are illuminated by a monochromatic light of wavelength in air $\lambda = 589nm$, emitted by a fine source F placed at a distance $d = 20cm$ from the plane of the two slits.

The source F and the two slits are horizontal. The observation is made on a screen located at a distance $D = 100\ cm$ from the plane of the two slits.

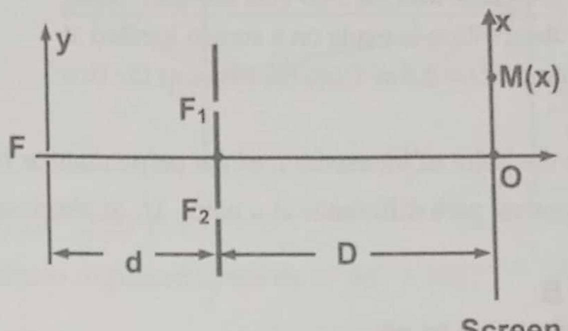


Figure 1

1. The optical path difference δ at a point M of abscissa x is given by: $\delta = F_2M - F_1M = a\frac{x}{D}$.
   a) M being the center of a bright fringe, determine the expression of the interfringe distance i and calculate its value.
   b) Specify the nature and the order of interference of the fringe whose center N is at $5.6mm$ from O.
2. The source F is moved by a distance z along the Fy axis. We notice that the center of the central fringe moves up and takes the place of the center of the tenth dark fringe.
   Specify the displacement direction of F and calculate this displacement.
3. F is returned to its initial position and we insert, in front of F_2, a small parallel plate of index $n = 1.5$ and of thickness e. We notice that the interference pattern is displaced by $b = 10mm$.
   Determine the value of e, knowing that the optical path difference at M is $\delta = a\frac{x}{D} + e(n-1)$.

IV-
Light in Different Mediums

Part A
Optical path

A & B are two points in a medium whose index of refraction is n, separated by a geometrical distance $AB = \ell$ as shown in the adjacent figure 1.

The optical path between these points denoted (AB) is the equivalent path traveled by the light in air during the same duration. Consequently, in air or vacuum the two paths are equivalent $(AB) = AB$, (optical=geometrical).

Show that $(AB) = n \times AB = n \times \ell$.

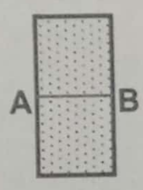


Figure 1

Experimental study

We use Young's double slit setup to perform an interference of light. These very fine slits (S_1) & (S_2) are separated by $S_1S_2 = a$, and illuminated by a monochromatic light of wavelength in air $\lambda = 580\,nm$, emitted by a primary source S equidistant from slits as shown in the adjacent figure 2.

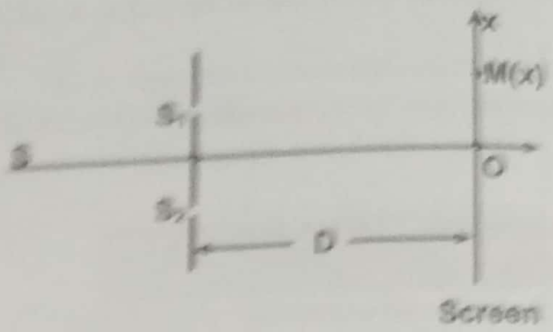


Figure 2

The source S and the two slits are horizontal. The observation is made on a screen located at a distance $D = 2.5\,m$ from the plane of the two slits.

O is the point of intersection of the perpendicular bisector of $[S_1S_2]$ with the screen.

The optical path difference at a point M of abscissa $x = OM$ is defined $\delta = (SS_2M) - (SS_1M)$.

Part B

Interference in air

1. Explain why the sources (S_1) & (S_2) are coherent.
2. Show that the optical path difference at O is zero.
3. Determine the expression of the optical path difference at M in terms of a, x & D.
4. If the distance between the centers of nine consecutive bright fringes is $5.8\,mm$. Show that $a = 2\,mm$.
5. Specify the nature of the fringe obtained at the point of abscissa $2.9\,mm$.
6. Determine the number of dark fringes whose centers extends over a segment of length $1.2\,cm$ centered at O.

Part C

Interference in medium of index n

The whole setup is immersed in water of index $n = 1.5$.

1. Show that the new optical path difference becomes $\delta' = n\,a\dfrac{x}{D}$.
2. Specify the position of the central bright fringe.
3. Justify that the new expression of the interfringe distance for bright and dark fringes is $i' = \dfrac{\lambda D}{n\,a}$.
4. Calculate the value of i'.
5. Give a qualitative description concerning the effect of the change in medium to the pattern observed on screen.

Solutions – Problems

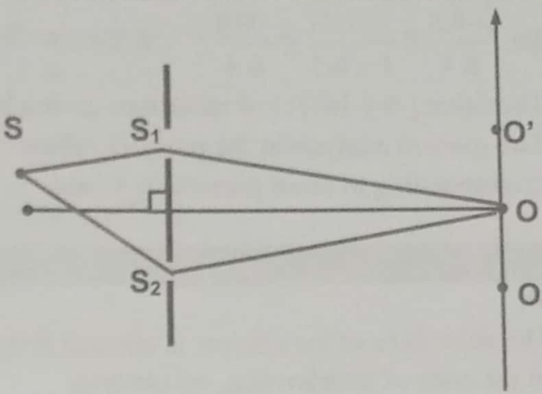
I-

1. a) The optical path difference at O:

$$\delta_O = (SS_2O) - (SS_1O)$$
$$= (SS_2 - SS_1) + (S_2O - S_1O)$$

But O belongs to the perpendicular bisector of $[S_1S_2]$, so $S_2O - S_1O = 0$;

Then $\delta_O = SS_2 - SS_1 \neq 0$, thus O is no longer center of the central bright fringe.



b) Let O' be the new position of the central bright fringe.

$$\delta_{O'} = (SS_2O') - (SS_1O') = (SS_2 - SS_1) + (S_2O' - S_1O') = 0;$$

So, $SS_2 - SS_1 = S_1O' - S_2O'$; but the source is displaced upwards so $SS_2 > SS_1$;

We get $S_1O' - S_2O' > 0$; so $S_1O' > S_2O'$;

Then O' is farther from S_1 than S_2, thus the center of the central bright fringe is displaced downwards.

2. a) The centers of the bright fringes $\delta = k\lambda$ where k is an integer.

b) We have $\delta = a\dfrac{x}{D} + a\dfrac{y}{d} = k\lambda$, so $x_k = k\dfrac{\lambda D}{a} - y\dfrac{D}{d}$.

c) The interfringe between bright fringes:

$$i_b = x_{k+1} - x_k = \left[(k+1)\frac{\lambda D}{a} - y\frac{D}{d}\right] - \left[k\frac{\lambda D}{a} - y\frac{D}{d}\right] = \frac{\lambda D}{a}.$$

The abscissas of the centers of dark fringes verify the relation $\delta = a\dfrac{x}{D} + a\dfrac{y}{d} = \left(k + \dfrac{1}{2}\right)\lambda$;

Then $x_k = \left(k + \dfrac{1}{2}\right)\dfrac{\lambda D}{a} - y\dfrac{D}{d}$.

For dark fringes: $i_d = x_{k+1} - x_k = \left[\left(k + 1 + \dfrac{1}{2}\right)\dfrac{\lambda D}{a} - y\dfrac{D}{d}\right] - \left[\left(k + \dfrac{1}{2}\right)\dfrac{\lambda D}{a} - y\dfrac{D}{d}\right] = \dfrac{\lambda D}{a}$.

3. The pattern obtained on screen is simply translated downwards due to the upwards translation of the source without a change in the interfringe distance.

4. a) The interfringe distance $i = \dfrac{\lambda D}{a} = 650 \times 10^{-9} \times \dfrac{2}{0.8 \times 10^{-3}} = 1.625 \times 10^{-3}\, m = 1.625\, mm$.

b) The position of the center of the central bright fringe verify the condition $\delta = a\dfrac{x_0}{D} + a\dfrac{y}{d} = 0$;

Then $y = -\dfrac{d}{D}x_0 = -\dfrac{0.5}{2} \times 1.6 \times 10^{-2} = -4 \times 10^{-3} = -4\, mm$.

5. a) The positions of the centers of the dark fringes are given by: $\delta = a\dfrac{x}{D} + a\dfrac{y}{d} = \left(k + \dfrac{1}{2}\right)\lambda$;

At O, $x=0$, then $\lambda = \frac{a\,y}{d(k+0.5)} = \frac{0.8\times 10^{-3}\times\left(-4\times 10^{-3}\right)}{0.5(k+0.5)} = \frac{-6.4\times 10^{-6}}{k+0.5}$.

In the visible domain $400\times 10^{-9} \le \frac{-6.4\times 10^{-6}}{k+0.5} \le 800\times 10^{-9}$;

So, $\frac{-0.8}{6.4} \le \frac{1}{k+0.5} \le \frac{-0.4}{6.4}$; $-16.5 \le k \le -8.5$ and k is an integer, thus $k = \{-9, \ldots; -16\}$;

Therefore $[-9-(-16)]+1=8$ radiations giving black spectral line at O.

b) The spectral analysis at the point O, shows a colored background intercepted by 8 black lines corresponding to those previously found.

II-

Part A

1. a) The coherence of the sources is ensured if they are produced by the same primary source.

b) In the zone of interference, we observe:

- a central bright fringe at O.
- alternate, equidistant, rectilinear bright and dark fringes.

2. The optical path difference at M is given by: $\delta = \frac{a\,x}{D}$.

3. At O, $x=0$; $\delta = \frac{a\,x_O}{D} = 0$; thus O is the center of the central bright fringe.

4. a) For the centers of dark fringes $\delta = \left(k+\frac{1}{2}\right)\lambda = \frac{a\,x_k}{D} \Rightarrow x_k = \left(k+\frac{1}{2}\right)\frac{\lambda D}{a}$.

b) The interfringe distance is the distance between the centers of two consecutive fringes of same nature.

c) $i = x_{k+1} - x_k = \left(k+1+\frac{1}{2}\right)\frac{\lambda D}{a} - \left(k+\frac{1}{2}\right)\frac{\lambda D}{a} = \frac{\lambda D}{a}\left(k+1+\frac{1}{2}-k-\frac{1}{2}\right) = \frac{\lambda D}{a}$.

5. The abscissa of the center of the 6th dark fringe is given by $x = 5.5i = 5.5\times\frac{\lambda D}{a}$;

Thus, $18\times 10^{-3} = 5.5\times\frac{\lambda\times 1}{2\times 10^{-4}}$; $\lambda = 6.5\times 10^{-7}\,m = 0.65\,\mu m$.

Part B

1. In a medium of index n, the speed becomes $v = \frac{c}{n} \prec c$. It moves slower.

2. a) $\delta' = (SS_2M) - (SS_1M) = \frac{a\,x}{D} - e_1(n_1 - 1)$.

b) At the new central $\delta' = 0$, so $\frac{a\,x_{O'}}{D} - e_1(n_1-1) = 0$.

Then $x_{O'} = \frac{e_1(n_1-1)D}{a} = \frac{5\times 10^{-6}\times(1.5-1)}{0.2\times 10^{-3}} = 0.0125\,m = 12.5\,mm$.

The new position of the central bright O', $x_{O'} = \frac{e_1(n_1-1)D}{a}$, is independent of the wavelength of the used. So this point corresponds to the center of all the central bright fringes (all the colors in the visible spectrum) whose combination creates at the considered point a white fringe.

c) The optical path due to the light crossing (S_1) (e_1, n_1) and reaches O is bigger than that crossing (S_2) to reach O.

Then $(S_1O) > (S_2O)$; to retrieve the central fringe to the point O, we should increase the path (S_2O), then the slit should be placed behind (S_2).

The new optical path difference at a point M is:

$\delta' = (SS_2M) - (SS_1M) = [S_2M + e_2(n_2 - 1)] - [S_1M + e_1(n_1 - 1)]$.

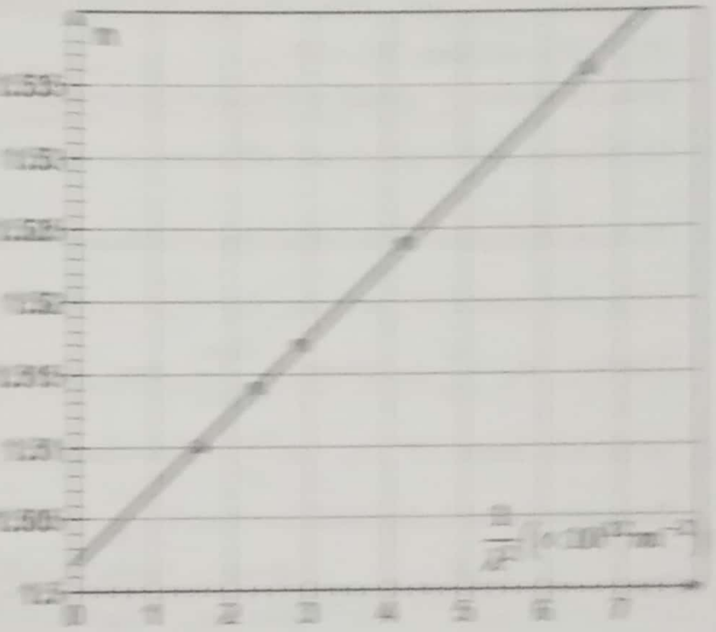
$\delta' = S_2M - S_1M + e_2(n_2 - 1) - e_1(n_1 - 1) = \frac{ax}{D} + e_2(n_2 - 1) - e_1(n_1 - 1)$.

The new central is at O again, so for $x = 0$, we have $\delta' = 0$.

Thus, $e_2 = \frac{e_1(n_1 - 1)}{n_2 - 1} = \frac{5\mu m(1.5 - 1)}{(1.6 - 1)} = 4.17\ \mu m$.

Part C

1. Graph.



2. The curve representing the change of the index n as a function of inverse of the square of the wavelength is represented by a straight line, then its equation is of the form $n = A + B\frac{1}{\lambda^2}$;

Where $B = \frac{\Delta n}{\Delta\left(\frac{1}{\lambda^2}\right)}$;

$$B = \frac{1.536 - 1.517}{\frac{1}{(390\times10^{-9})^2} - \frac{1}{(589\times10^{-9})^2}};$$

Thus $B = 5.130\times10^{-15}\ m^2$

& $A = n - B\frac{1}{\lambda^2} = 1.536 - 5.13\times10^{-15}\left(\frac{1}{(390\times10^{-9})^2}\right) = 1.502$.

Thus $n = 1.502 + \frac{5.13\times10^{-15}}{\lambda^2}$ (n unitless & λ in m).

3. The position of the centrals, for each color, are:

For violet $\delta'_v = \frac{a x_{ov}}{D} - e_1(n_v - 1) = 0$;

Then $x_{0violet} = \frac{e_1(n_v - 1)D}{a} = \frac{5\times10^{-6}(1.536 - 1)}{0.2\times10^{-3}} = 13.4\ mm$.

& similarly $x_{0red} = 12.75\ mm$.

The center of the central violet is not confounded with that of central red and this is the same for all other colors. Therefore, it is impossible to observe a central white fringe but partial centrals.

III-

1\. a) The abscissas of the centers of bright fringes are given by $\delta = a\frac{x_k}{D} = k\lambda$ where $k \in Z$, then $x_k = k\frac{\lambda D}{a}$.

The interfringe distance is the distance between the centers of two consecutive fringes of same nature $i = x_{k+1} - x_k = (k+1)\frac{\lambda D}{a} - k\frac{\lambda D}{a} = \frac{\lambda D}{a}$.

Thus, $i = \frac{\lambda D}{a} = \frac{589\,nm \times 1m}{1\times 10^{-3}m} = 589\times 10^{3}nm = 589\,\mu m$.

b) The nature of the fringe is defined by $\frac{\delta}{\lambda} = \frac{(ax/D)}{(ia/D} = \frac{x}{i} = \frac{5.6\times 10^{-3}}{589\times 10^{-6}} = 9.5$ (half number);

Then N is the center of the tenth dark fringe.

2\. Let O' be the position of the new central, $\delta_{O'} = (FF_2O') - (FF_1O') = 0$;

So, $\delta_{O'} = (FF_2 + F_2O') - (FF_1 + F_1O') = 0$; $FF_2 - FF_1 = F_1O' - F_2O'$;

The center of the new central occupies the position of the tenth dark fringe $F_1O' - F_2O' < 0$; so $FF_2 - FF_1 < 0$, then $FF_2 < FF_1$.

Thus, the source F should be displaced downwards.

The increase in the optical path difference due to the displacement of the source is $a\frac{y}{d}$;

The new expression of the optical path difference becomes $\delta = a\frac{x}{D} + a\frac{y}{d}$;

At the new central O', we have $\delta_{O'} = 0$ & $x_{O'} = 5.6\,mm$.

Then $y = -x_{O'} \times \frac{d}{D} = -5.6mm \times \frac{20cm}{100cm}$; thus, $y = -1.12mm$.

3\. At the new central O'', $\delta_{O''} = a\frac{x_{O''}}{D} + e(n-1) = 0$, so $x_{O''} = -e(n-1)\frac{D}{a} < 0$;

Then new central is displaced downwards $x_{O''} = -b < 0$ (to the same side of introduction of plate);

Thus, $-e(n-1)\frac{D}{a} = -b$, $e = \frac{b\times a}{D(n-1)} = \frac{1\times 10^{-3}m \times 10^{-3}m}{1m\times(1.5-1)} = 20\times 10^{-6}m = 20\,\mu m$.

IV-

Part A

The time needed to travel the distance AB is a medium of index of refraction n is $t = \frac{AB}{v}$;

The distance traveled in air within the same duration is $(AB) = c\times t = c\times\frac{AB}{v} = n\times AB$.

Part B

1\. The two sources (S_1) & (S_2) are illuminated by the same primary source, then they are coherent.

2\. The optical path difference at O is: $\delta_O = (SS_2O) - (SS_1O) = (SS_2 + S_2O) - (SS_1 + S_1O)$;

Then, $\delta_O = (SS_2 - SS_1) + (S_2O - S_1O)$;

But S & O belong to the perpendicular bisector of $[S_1S_2]$, then $SS_2 - SS_1 = 0$ & $S_2O - S_1O = 0$

So, $\delta_O = 0$; thus O is the center of the central bright fringe.

3. At any point M, we have $\delta = (SS_2M) - (SS_1M) = (SS_2 + S_2M) - (SS_1 + S_1M)$;

Then, $\delta = 0 + (S_2M - S_1M) = \frac{a\,x}{D}$.

4. If i is the interfringe distance, so $8i = 5.8\,mm$ (9 consecutive bright fringes), then $i = 0.725\,mm$;

But $i = \frac{\lambda D}{a}$; then $a = \frac{\lambda D}{i} = \frac{580 \times 10^{-9} \times 2.5}{0.725 \times 10^{-3}} = 2 \times 10^{-3}\,m = 2\,mm$.

5. The nature of the fringe is defined by $\frac{\delta}{\lambda} = \frac{a\,x}{\lambda D} = \frac{2 \times 10^{-3} \times 2.9 \times 10^{-3}}{580 \times 10^{-9} \times 2.5} = 4$ (whole number);

Then it is the center of the 4th bright fringe.

6. The segment is centered at O, so $-\frac{1.2 \times 10^{-2}}{2} \leq x_k \leq \frac{1.2 \times 10^{-2}}{2}$;

The abscissas of the centers of the dark fringes are given by $x_k = \left(k + \frac{1}{2}\right)i$.

So, $-0.006 \leq \left(k + \frac{1}{2}\right) \times 0.725 \times 10^{-3} \leq 0.006$; $\frac{-0.006}{0.725 \times 10^{-3}} - 0.5 \leq k \leq \frac{0.006}{0.725 \times 10^{-3}} - 0.5$;

Then $-8.7 \leq k \leq 7.8$ & k is an integer; so $k = \{-8, -7 ..., 7\}$;

Thus, the centers of 7-(-8)+1=16 dark fringes are present inside this segment.

Part C

1. At any point M, we have $\delta' = (SS_2M) - (SS_1M) = ((SS_2) + (S_2M)) - ((SS_1) + (S_1M))$;

Then, $\delta' = n(SS_2 - SS_1) + n(S_2M - S_1M) = 0 + n \times \frac{a\,x}{D} = na\frac{x}{D}$.[1]

2. At the center of the central bright fringe, the optical path difference is zero so, $\delta' = n \times \frac{a\,x}{D} = 0$;

Then $x = 0$, thus the central bright fringe remains at O.

3. For bright fringes, $\delta' = na\frac{x}{D} = k\,\lambda$; so $x_k = k\frac{\lambda D}{na}$;

The interfringe distance $i' = x_{k+1} - x_k = (k+1)\frac{\lambda D}{na} - k\frac{\lambda D}{na} = \frac{\lambda D}{na}(k + 1 - k) = \frac{\lambda D}{na}$.

For dark fringes, $\delta' = na\frac{x}{D} = \left(k + \frac{1}{2}\right)\lambda$; so $x_k = \left(k + \frac{1}{2}\right)\frac{\lambda D}{na}$;

The interfringe $i' = x_{k+1} - x_k = \left(k + \frac{1}{2} + 1\right)\frac{\lambda D}{na} - \left(k + \frac{1}{2}\right)\frac{\lambda D}{na} = \frac{\lambda D}{na}\left(k + \frac{1}{2} + 1 - k - \frac{1}{2}\right) = \frac{\lambda D}{na}$.

4. We have $i' = \frac{\lambda D}{na} = \frac{i}{n} = \frac{0.725}{1.5} = 0.483\,mm$.

5. The center of the central bright fringe remains invariant at O, while the fringes appears closer, (the interfringe is reduced).

---

[1] Let δ be the optical path difference in a medium of index n;
The time needed to travel this distance in the medium of index n is $t = \frac{\delta}{v}$ where $v = \frac{c}{n}$, then $t = \frac{\delta \times n}{c}$;
The optical path difference δ' is the equivalent path traveled in air during the same time is:
$\delta' = t \times c = \frac{\delta \times n}{c} \times c = n \times \delta = n\frac{ax}{D}$.

Supplementary Problems

Effect of Thin Glass Sheet

Young's slits apparatus is formed of two very thin slits S_1 and S_2, illuminated by a monochromatic radiation of wavelength λ, parallel and separated by a distance a, and a screen of observation (E) placed parallel to the plane of the slits at a distance $D = 1m$ from this plane.

A glass sheet of thickness $e = 5\mu m$ and of index of refraction n, is put now just behind the slit S_2.

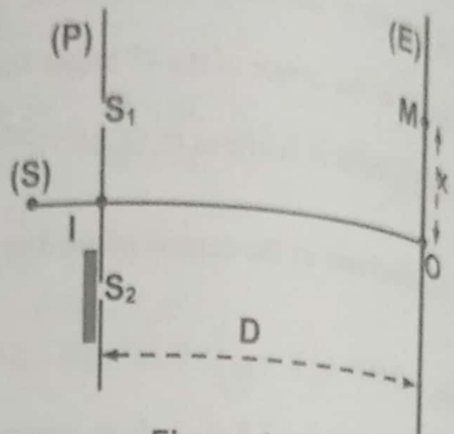


Figure 1

1. Justify that
   a) O is no longer the center of the central bright fringe.
   b) the center of the central bright fringe is displaced downwards.
2. We admit that the optical path difference at point M, of abscissa $x = OM$, is given by:
   $\delta = (SS_2M) - (SS_1M) = \frac{ax}{D} + e(n-1)$. The index of refraction is supposed constant.
   a) Verify that O is not the center of the central bright fringe.
   b) Determine the position of the center of the central bright fringes.
3. Determine the position of the center of the k^{th} bright fringe.
4. Justify that the interfringe distance for bright and dark fringes remains constant.
5. If the source is replaced by a white light emitting all the radiations in the visible spectrum. Show that the centers of all the central bright fringes coincide.
6. In reality the index of refraction n is related to the wavelength λ by the relation $n = 1.5 + \frac{B}{\lambda^2}$.

   Specify the effect of this relation on the central bright fringe.

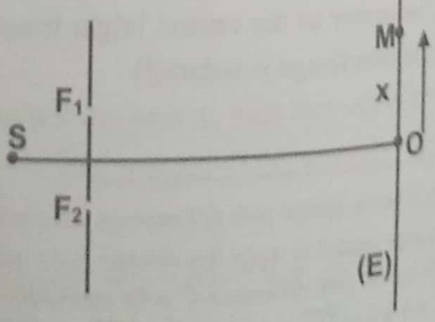
L-GS 2004 2nd
Interference of Light

Consider a source S of monochromatic light of wavelength λ and a glass plate of parallel faces of thickness e and of index $n = 1.5$.

The object of this exercise is to determine λ and e using Young's double slit apparatus.

Part A
Value of λ

Young's double slit apparatus is formed of two very thin and parallel slits F_1 and F_2 separated by a distance $a = 0.15\ mm$, and a screen of observation (E) placed parallel to the plane of the slits at a distance $D = 1.5m$ from this plane.



1. Upon illuminating F_1 by S and F_2 by another independent source S' synchronous with S, we do not observe a system of interference fringes. Why?
2. We illuminate F_1 and F_2 with S, placed equidistant from F_1 and F_2 we observe on (E) a system of interference fringes.
   a) Describe this system.
   b) At point O of the screen, equidistant from F_1 and F_2, we observe a bright fringe. Why?
   c) It can be shown that for a point M of (E), of abscissa $x = OM$, the optical path difference in air or in vacuum is given by: $\delta = F_2M - F_1M = \frac{ax}{D}$.
   Determine the expression of x_k corresponding to the k^{th} bright fringe and deduce the expression of the interfringe distance i.
3. We count 11 bright fringes over a distance $d = 5.6cm$. Determine the value of λ.

Part B

Value of e

Now, we place the glass plate just behind the slit F_1.

The optical path difference at point M becomes: $\delta' = \frac{ax}{D} - e(n-1)$.

1. Show that the interfringe distance i remains the same.
2. a) The central bright fringe is no longer at O. Why?
   b) The new position O' of the central bright fringe is the position that was originally occupied by the fifth dark fringe before introducing the plate. Determine the thickness e of the plate.

Answer Key

Part A 3. $\lambda = 0.56\ \mu m$

Part B 2. b) $e = 5.04\ \mu m$

III-LS & GS 2001 1st
Determination of the Wavelength of a Laser Light

Part A

Diffraction of light

The monochromatic light emitted by a laser source, of wavelength λ, illuminates, under normal incidence, a very narrow slit F_1 of width $a_1 = 0.1mm$ cut in an opaque screen (E_1). The phenomenon of diffraction is observed on a screen (E_2) parallel to (E_1), found at a distance $D = 4m$ from it as shown in figure 1. The central bright fringe on (E_2) has a linear width of $\ell = 5cm$.

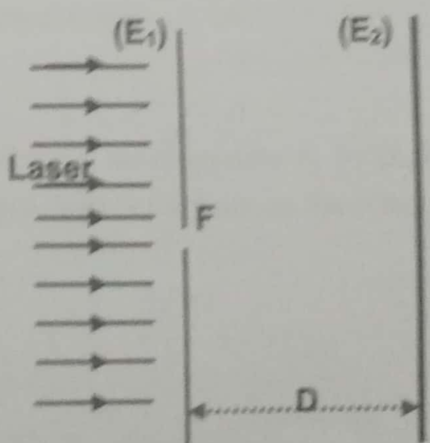


Figure 1

1. Describe the diffraction pattern observed on (E_2).
2. The phenomenon of diffraction shows evidence of a certain aspect of light. What is it?
3. Calculate the angular width of the central bright fringe.
4. Calculate the value of λ.

Part B

Interference of light

The positions of the laser source and the screen are not modified. A second slit F_2 identical to F_1 and parallel to is cut in (E_1) so that F_1 and F_2 are separated by a distance $a = 1mm$. We thus obtain the Young's slits apparatus as shown in figure 2.

We observe on (E_2) a system of interference fringes.

The distance between the center O of the central bright fringe and the fourth bright fringe is $1cm$.

The distance between the slit and screen is $D = 4m$.

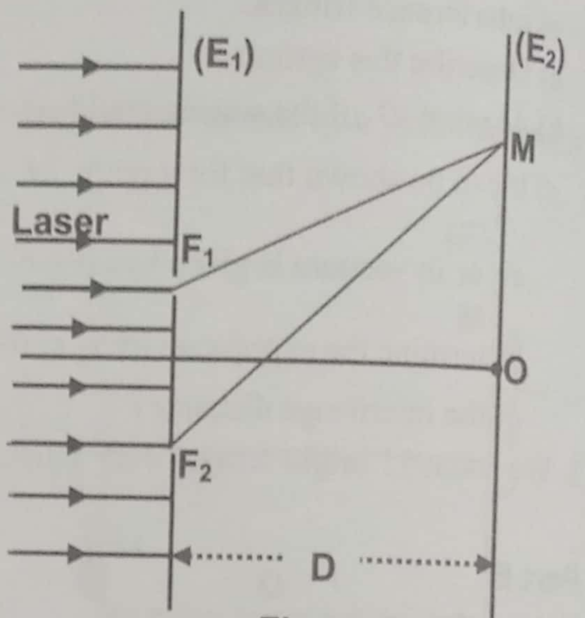


Figure 2

1. Due to what is the formation of the interference fringes?
2. Describe the aspect of the fringes observed on (E_2).
3. Consider a point M on (E_2) whose position is defined by its abscissa x relative to O.
   a) Write the expression of the optical path difference $\delta = F_2M - F_1M$ as a function of a, x and D.
   b) Deduce the expression giving the abscissas of the centers of the bright fringes.
   c) Calculate the wavelength λ.

Answer Key

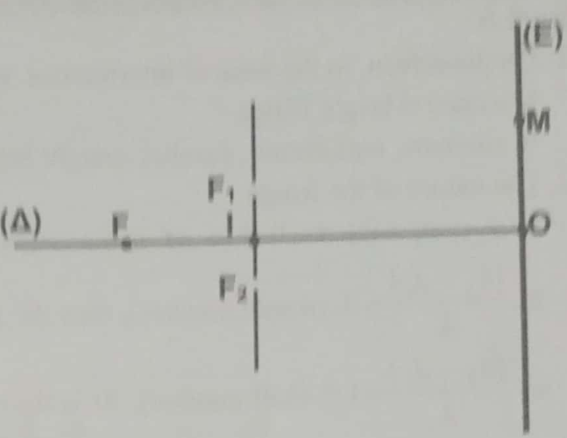
Part A 3. $\alpha = 0.0125\,(\text{rad})$ 4. $\lambda = 625\,nm$.

Part B 4. $k = 4$, $\lambda = \dfrac{x\,a}{k\,D} = 625\,nm$.

LS 2008 2nd
Interference of Light

Consider Young's experiment set-up that is formed of two very thin parallel slits F_1 and F_2, separated by a distance $a = 1\,mm$, and a screen of observation (E) placed parallel to the plane of the slits at a distance $D = 2\,m$ from the midpoint I of F_1F_2 and a thin slit F, equidistant from F_1 and F_2, situated on the straight line (Δ) whose intersection with (E) is the point O.

The object of this exercise is to study the interference pattern observed on the screen (E) in different situations.



Part A
First situation

The slit F is illuminated with a monochromatic light of wavelength $\lambda = 0.64\,\mu m$ in air.

1. Describe the interference pattern observed on (E).
2. Consider a point M on the screen at a distance d_1 from F_1 and d_2 from F_2.
   Specify the nature of the fringe thus formed at point M in each of the following cases:
   a) $d_2 - d_1 = 0$;
   b) $d_2 - d_1 = 1.28\,\mu m$;
   c) $d_2 - d_1 = 0.96\,\mu m$.
3. F is moved along (Δ). We observe that the interference fringes remain in their positions. Explain why.
4. F is moved perpendicularly to (Δ) to the side of F_2. We observe that the central fringe is displaced. In which direction and why?

Part B
Second situation

Now the slit F is illuminated with white light.

1. We observe at point O a white fringe. Justify.
2. Specify the color of the bright fringe that is the nearest to the central fringe.

Part C
Third situation

Consider two lamps (L_1) & (L_2) emitting radiations of same wavelength, we illuminate F_1 by (L_1) and F_2 by (L_2), we observe that the system of interference fringes does not appear on the screen (E). Why?

LS 2003 1st
See Page 62 – Part B

LS 2001 1st
See Page 46 – Part B

Part A

1. On the screen, in the zone of interference, we observe:
   - a central bright fringe.
   - alternate, equidistant, parallel straight bright and dark fringes.
2. The nature of the fringe:
   a) $d_2 - d_1 = 0$; $\delta = 0$ then M is the center of the central bright fringe.
   b) $\dfrac{(d_2 - d_1)}{\lambda} = 2$ (whole number), then M is the center of the 2nd bright fringe.
   c) $\dfrac{(d_2 - d_1)}{\lambda} = 1.5$ (half number), M is the center of the 2nd dark fringe.
3. At a point M the optical path difference:
   $\delta = (FF_2M) - (FF_1M) = (FF_2 + F_2M) - (FF_1 + F_1M) = (FF_2 - FF_1) + (F_2M - F_1M)$;
   But F belongs to the perpendicular bisector of $[F_1F_2]$, then FF_1 remains equal to FF_2 as long F along (Δ) independent of its position so $(FF_2 - FF_1) = 0$.
   Thus the optical path difference $\delta = (F_2M - F_1M) = \dfrac{ax}{D}$ does not vary thus the interfringe i and consequently the pattern obtained on screen is unchanged.
4. At O, the new optical path difference after displacement of source:
   $\delta_O = (F'F_2O) - (F'F_1O) = (F'F_2 + F_2O) - (F'F_1 + F_1O) = (F'F_2 - F'F_1) + (F_2O - F_1O)$
   But $(F_2O - F_1O) = 0$ & $(F'F_2 - F'F_1) \neq 0$ then $\delta_O \neq 0$;
   Therefore the central bright fringe is no longer at O.
   Let O' be the new central $\delta_{O'} = (F'F_1O') - (F'F_2O') = 0$, $F'F_2 - F'F_1 = F_1O' - F_2O'$.
   Since F is moved downwards then $F'F_2 < F'F_1 \Rightarrow F'F_2 - F'F_1 < 0$; using the above equation we get $F_2O' > F_1O'$. Then O' is above O.[1]

Part B

1. O is the center of the central fringe relative to all the colors, so at O all the colors of the visible spectrum combine together forming a white fringe that extends over the central that corresponds to the smallest interfringe which is associated to the violet fringe.
2. The 1st bright fringe relative to any color is located at $x = i = \dfrac{\lambda D}{a}$; the nearest bright fringe corresponds to the smallest value of x which is associated to the smallest wavelength (the violet). Thus the nearest bright fringe to the center O is the violet.

Part C

No, since the two sources are not coherent.

---

[1] If the source is displaced vertically then the central bright displaces in the opposite direction; If F is moved upwards then the central moves downwards and vice versa.

Unit III

Light

Chapter 14

Photoelectric Effect

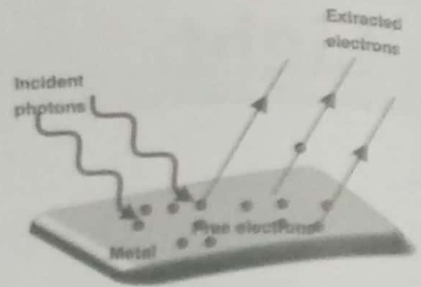
LS – Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Photoelectric Effect		$1^{st(2)}$	-	-	-	-	-	-	-	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Photoelectric Effect	2^{nd}	2^{nd}	-	-	-	-	-	$1^{st(A)}$	-	-

Essentials

I-Introduction

1. Definition

The photoelectric effect is the extraction of electrons from the surface of a metal when illuminated by a convenient radiation.



2. Photons

An electromagnetic radiation of frequency υ is made up of energy-particles called «photons», which is the corpuscular aspect of light.

A photon is chargeless, massless, travels in vacuum at the speed of light $c = 3\times10^8\ m/s$ & carries an energy proportional to its frequency υ given by $E_{ph} = h\upsilon = h\frac{c}{\lambda}$ where λ the wavelength of the associated radiation.

II-Einstein Relation (Conservation of Energy)

1. Definitions

The threshold frequency is the minimum frequency υ_0 of the radiation required to extract an electron from the surface of the metal.

The threshold wavelength is the maximum wavelength λ_0 of the radiation required to extract an electron from the surface of the metal.

The «work function» or extraction energy of a metal $W_0 = h\upsilon_0 = h\frac{c}{\lambda_0}$ is the minimum energy required to extract an electron from its surface.

2. Interpretation

When a photon carrying an energy E_{ph} hits a metal of work function W_0, it interacts (when possible) with one electron of the metal:

- if $E_{ph} < W_0$, (which is also equivalent to $\upsilon < \upsilon_0$ or $\lambda > \lambda_0$), then the photon will not interact and no extraction of electrons.
- if $E_{ph} = W_0$, then the photon is absorbed and an electron extracted without any kinetic energy.
- if $E_{ph} > W_0$, then the photon is absorbed and an electron is extracted with a certain kinetic energy. The maximum kinetic energy of the extracted electron is given by $E_{ph} = W_0 + KE_{max}$.

III-Quantum Efficiency η

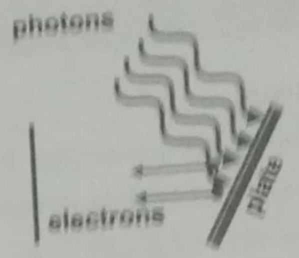
1. Power & incident photons

If P is the power received by the metal, then $P = n \times E_{ph}$ where:

- n is the number of incident photons per second.
- E_{ph} is the energy of a single photon.

2. Effective photons & emitted electrons

The photons that succeed, among those capable, of producing photoelectric emission (extraction of electrons) from the metal are called *effective photons.*



If the circuit (anode-cathode) is closed then the current that flows is $I = n_e \times e$, where:

- n_e is the number of electrons extracted per second which is also equal to the number of effective photons per second.
- e is the absolute charge of an electron (elementary charge $e = 1.6 \times 10^{-19}\,C$).

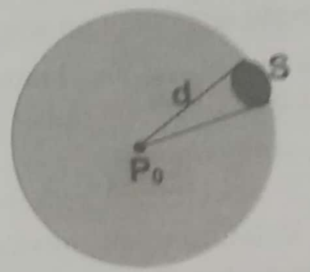
3. Quantum efficiency

The quantum efficiency is given by: $\eta = \dfrac{\text{number of effective photons /s}}{\text{total number of incident photons /s}}$;

or $\eta = \dfrac{\text{number of emitted electrons /s}}{\text{total number of incident photons /s}}$,

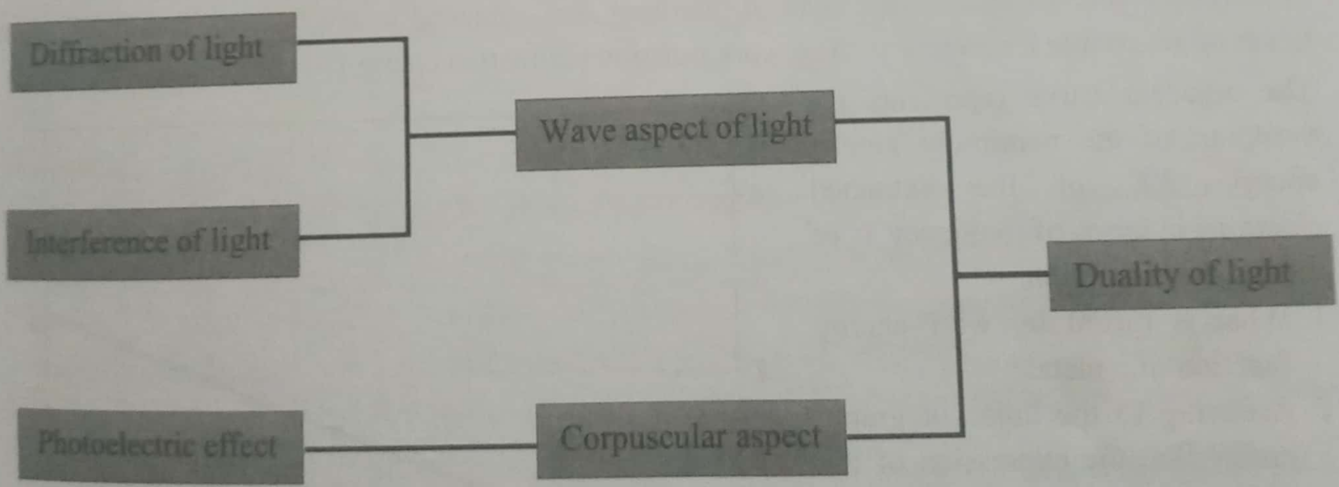
4. Point source

A point source delivering a power P_0, then the power p received by a metallic plate whose surface area is S placed at a distance d is given by: $\dfrac{P_0}{4\pi d^2} = \dfrac{p}{S}$.



IV-Duality of Light

The light has a double aspect that we call duality of light, but in each phenomenon one aspect is dominant as shown in the diagram below.



Applications

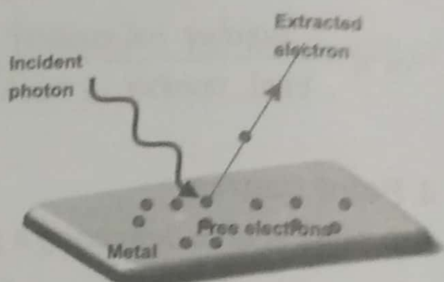
Given:

- Planck's constant: $h = 6.63 \times 10^{-34}\ J.s$;
- Speed of light in vacuum: $c = 3 \times 10^{8}\ m/s$;
- $1eV = 1.6 \times 10^{-19}\ J$;
- Mass of an electron: $m = 9.1 \times 10^{-31}\ kg$.

I- Photoelectric Emission

A metallic plate, covered with a layer of cesium, is illuminated with a monochromatic luminous beam of adjustable power. The work function (extraction energy) of cesium is $W_0 = 1.75\,eV$.

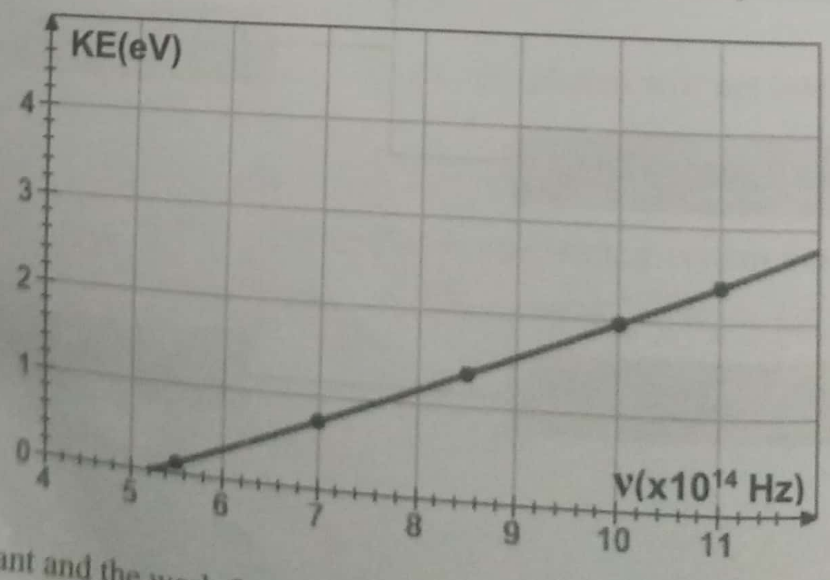
1. Define photoelectric emission.
2. What is meant by threshold wavelength? Calculate its value for cesium.
3. The metallic plate is illuminated by a power $P_0 = 50\,W$ emitting a radiation whose wavelength is $\lambda = 750\ nm$.
   a) Justify that the photoelectric emission will not take place.
   b) We keep increasing the power, but still no electrons are emitted.
      i- Specify, the aspect of light, which is contradicted by this result.
      ii- Interpret the previous observation referring to Einstein's hypothesis.
4. Determine the maximum kinetic energy of the electrons extracted if the incident photon carries an energy of $E_{ph} = 2.8\,eV$.



II- Planck's Constant

A metallic plate, covered with a layer of rubidium, is illuminated with a monochromatic luminous beam of adjustable frequency υ. The work function (extraction energy) of rubidium is W_0.

The adjacent curve represents the variation of the maximum kinetic energy KE of the extracted electrons in terms of frequency υ of the incident radiation.



1. What is meant by work-energy function of a metal?
2. Referring to the adjacent graph, justify that the expression of the kinetic energy can be written in the form $KE = a\upsilon + b$ where a & b are constants to be determined.
3. Deduce the values of Planck's constant and the work function of rubidium.
4. Describe what is likely to happen if this metal is illuminated by a radiation whose wavelength is $\lambda = 540\ nm$.

III- Source and Plate

Consider a point source emitting power $P_0 = 20W$, uniformly in all directions, a radiation of wavelength $\lambda = 550\ nm$.

1. Calculate, in eV, the energy carried by a photon.
2. A plate whose surface area $s = 20\ cm^2$ is placed at $5m$ from the source so that the rays received are normal to its plane.
   Determine:
   a) the power received by the plate.
   b) the number of incident photons per second received by the plate.

IV- Bonds and Photons

Hypochlorous acid $HClO$ is used worldwide to treat water in private pools and keep it clean. Under sunlight radiations the bond $Cl-O$ breaks (photolysis).

The energy required to break one mole is $251\ kJ$.

HOCL

1. Calculate, in eV, the energy of a photon required to break one bond between two atoms.
2. Determine the wavelength of the radiation needed.
3. Specify to which domain of the electromagnetic spectrum this radiation belong.

V- Photoelectric Effect

A metallic plate, covered with a layer of cesium, is illuminated with a polychromatic luminous beam formed of the following radiations whose wavelength are shown in the following table:

Radiation	(R_1)	(R_2)	(R_3)
Wavelength (nm)	400	600	800

The work function (extraction energy) of cesium is $W_0 = 1.88\ eV$.

A convenient apparatus (D) is used to detect the electrons emitted by the illuminated plate.

1. Define the term «the threshold wavelength of cesium» and then calculate its value.
2. Identify the radiations, which are capable of producing the photoelectric emission.
3. Specify the radiation which causes the extraction of the faster electrons.
4. Show that the expression of the maximum kinetic energy of the extracted electrons in terms of the wavelength of incident radiation is given by $KE_{max} = \dfrac{1243.125}{\lambda} - 1.88$ (λ in nm & KE in eV).
5. Deduce the maximum kinetic energies of the electrons extracted due to the efficient radiations.
6. Plot the graph representing the variation of the maximum kinetic energy of the electrons extracted in terms of wavelength using the result previously obtained.
   Take as scales: On ordinate axis 1div $= 0.2\ eV$.
   On abscissa axis 1div $= 100\ nm$.

VI- Photoelectric Emission

When a cesium plate of quantum efficiency η is illuminated by a radiation of wavelength $\lambda = 400\,nm$ and adjustable power P, electrons are extracted from the cathode and collected by the anode as shown in the diagram below (figure 1). (A) is an ammeter inserted in the circuit.

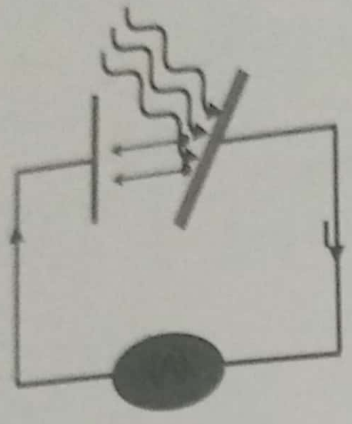


Figure 1

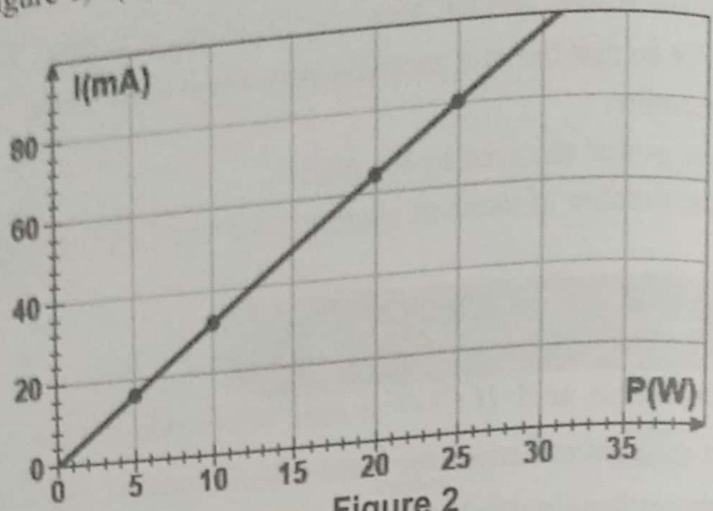


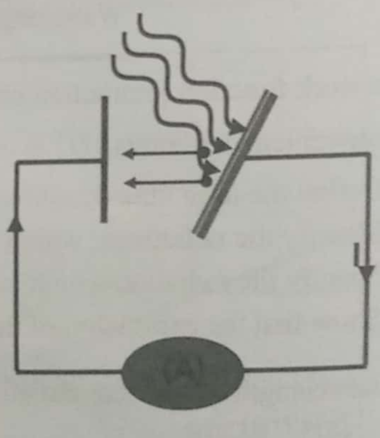
Figure 2

Let I be the current carried by the circuit.

1. What is meant by effective photons?
2. Recall the definition of the quantum efficiency.
3. Show that the current is related to the power by the relation $I = a\,P$ where a is a constant whose expression is to be determined in terms of η, e, h, c & λ.
4. A convenient apparatus is used to represent the variation of the current in terms of power and the curve obtained is shown in figure 2.
   a) Justify that the curve obtained is compatible with the equation previously derived.
   b) Deduce the value of a, then that of η.

VII- Characteristics of Photoelectric Emission

A metal of quantum efficiency 0.25% is illuminated by a monochromatic radiation of wavelength $\lambda = 360nm$ produced by a point source whose radiations falls normally on the surface of the cathode. Electrons are extracted from the metal and a current of $40\mu A$ is detected in the adjacent setup.



1. Determine the power received by the metallic plate.
2. We approach the source of light from the cathode so that the distance becomes half of its previous value. Describe what is likely to happen to:
   a) the quantum efficiency.
   b) the current.

Solutions - Applications

I-

1. The photoelectric emission is the extraction of electrons from a metal when illuminated by a convenient radiation.
2. The threshold wavelength is the maximum wavelength of an incident radiation capable of extracting an electron from the surface of a metal.

   The threshold wavelength of cesium is: $\lambda_0 = \frac{hc}{W_0} = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.75\times1.6\times10^{-19}} = 7.1\times10^{-7}\,m = 710\,nm$

3. a) We have $\lambda = 750\,nm > \lambda_0 = 710\,nm$, then the photoelectric emission will not take place.

   b) Effect of the power.

   i- The absence of electrons extracted contradicts the wave aspect of light, which confirms that increasing the power of the source, should increase the amplitude of the incident wave that become more energetic & at a certain level should be capable of extracting the electrons.

   ii- To interpret the photoelectric emission, Einstein suppose that the interaction takes place between one photon only and one electron, and the electron is not extracted unless the energy of the incident photon is greater than or equal to the work-energy function of the metal under study.

4. Einstein's relation: $KE_{max} = E_{ph} - W_0 = 2.8 - 1.75 = 1.05\,eV$.

II-

1. The work-energy function of a metal is the minimum energy required to extract an electron from its surface.
2. The curve representing the change of the kinetic energy of the extracted electrons in terms of the frequency of incident radiation is represented by a straight line, whose equation is $KE = a\upsilon + b$.

   Where $a = \frac{\Delta(KE)}{\Delta\upsilon} = \frac{(2-0)\times1.6\times10^{-19}\,J}{(10.1-5.3)\times10^{14}\,Hz} = 6.67\times10^{-34}\,J/Hz$;

   & $b = KE - a\upsilon = 0 - (6.67\times10^{-34})\times5.3\times10^{14} = -3.5351\times10^{-19}\,J$;

   Thus, $KE = 6.67\times10^{-34}\,\upsilon - 3.5351\times10^{-19}$ where υ in Hz & KE in J.

3. According to Einstein's relation, the energy of the incident photon is given by $E_{ph} = W_0 + KE$,

   then $KE = h\upsilon - W_0$, where W_0 is the ionization energy of the metal used.

   By comparison with the previous relation we get: $h = 6.67\times10^{-34}\,J.s$ & $W_0 = +3.5351\times10^{-19}\,J$.

4. Graphically, the threshold frequency $\upsilon_0 = 5.3\times10^{14}\,Hz$;

   The frequency of the radiation associated to the photon is $\upsilon = \frac{c}{\lambda} = \frac{3\times10^{8}}{540\times10^{-9}} = 5.6\times10^{14}\,Hz$;

   We have $\upsilon > \upsilon_0$, then electrons are extracted with a kinetic energy.

III-

1. The energy of a photon is $E_{ph} = h\frac{c}{\lambda} = 6.63\times10^{-34}\,J.s\times\frac{3\times10^{8}\,m/s}{550\times10^{-9}\,m} = 3.62\times10^{-19}\,J$;

So, $E_{ph} = \dfrac{3.62 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.2625\ eV$;

3. a) The power is proportional to the surface, so $\dfrac{P_{source}}{4\pi d^2} = \dfrac{P_{received}}{s_{plate}}$;

Then $P_{received} = P_{source} \dfrac{s_{plate}}{4\pi d^2}$; thus, $P_{received} = p = 20 \times \dfrac{20 \times 10^{-4}}{4\pi \times 5^2} = 1.27 \times 10^{-4}\ W$;

b) The number of incident photons per second is $n_{ph/s} = \dfrac{p}{E_{ph}} = \dfrac{1.27 \times 10^{-4}}{3.62 \times 10^{-19}} = 3.5 \times 10^{14}$.

IV-

1. $251 kJ$ is the energy required to break $N_A = 6.022 \times 10^{23}$ (Avogadro number) bonds.

Then the energy required to break one bond is: $E = \dfrac{E_t}{N_A} = \dfrac{251 \times 10^3}{6.022 \times 10^{23}} = 4.2 \times 10^{-19}\ J$;

We get, $E = \dfrac{4.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.625\ eV$.

2. The wavelength of the radiation associated to this photon is given by:

$\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{4.2 \times 10^{-19}} = 4.75 \times 10^{-7}\ m = 475\ nm$.

3. $\lambda = 475\ nm \in [400\ nm; 800\ nm]$, then this radiation is visible.

V-

1. The threshold wavelength is the maximum wavelength of the radiation capable of extracting an electron from the surface of cesium.

We have $W_0 = \dfrac{hc}{\lambda_0}$, so $\lambda_0 = \dfrac{hc}{W_0} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.88 \times 1.6 \times 10^{-19}} = 6.61 \times 10^{-7}\ m = 661\ nm$.

2. If the wavelength of the incident radiation is less than or equal to the threshold wavelength, then the photoelectric emission is possible.

Then, (R_1) & (R_2) are capable of extracting electrons.

3. The energy of a photon is inversely proportional to its wavelength, then the most energetic photon corresponds to the smallest wavelength.

Thus, (R_1) extracts the faster electrons.

4. According to the conservation of energy: $E_{ph} = W_0 + KE_{max}$ where KE_{max} is the kinetic energy of the extracted electrons.

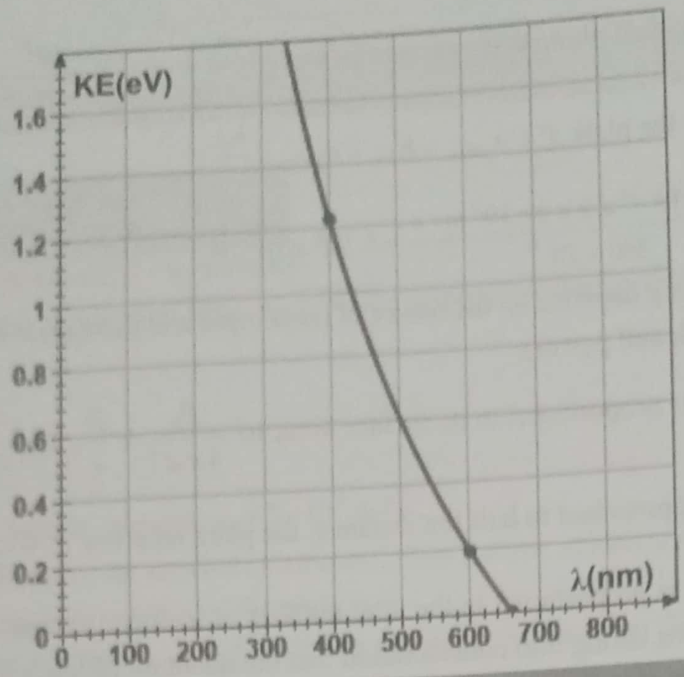
Then $KE_{max} = E_{ph} - W_0 = \dfrac{hc}{\lambda} - W_0$; $KE_{max} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda \times 10^{-9} \times 1.6 \times 10^{-19}} - 1.88$;

$KE_{max} = \dfrac{1243.125}{\lambda} - 1.88$ where λ in nm & KE in eV .

5. For (R_1), $KE_{1max} = \dfrac{1243.125}{400} - 1.88 = 1.23\ eV$.

For (R_2), $KE_{2max} = \dfrac{1243.125}{600} - 1.88 = 0.19\ eV$.

6. Graph.



VI-

1. The effective photons (when the photoelectric emission is possible) are the photons that succeed to extract electrons from the surface of the metal (which is equal to the number of extracted electrons).
2. The quantum efficiency is the rate of the number of effective photons per second to the total number of incident photons per second.
3. The number of electrons extracted per second is given by : $n_{e^-/s} = \frac{I}{e}$ where e is the absolute value of the charge of an electron.

The number of incident photons per second is given by: $n_{ph/s} = \frac{P}{E_{ph}} = \frac{P \times \lambda}{hc}$;

The quantum efficiency is given by $\eta = \frac{n_{e^-/s}}{n_{ph/s}}$, so $\frac{I}{e} = \eta \times \frac{P \times \lambda}{hc}$; then $I = \frac{\eta \times \lambda \times e}{hc} P$.

The expression of the current is of the form $I = aP$ where $a = \frac{\eta \times \lambda \times e}{hc}$.

4. a) The previous relation shows the current is proportional to the power of the incident, and then its graphical representation should be a straight line passing through origin, which is justified by the curve shown in figure 2.

b) The slope of the straight line is $a = \frac{\Delta I}{\Delta P} = \frac{(80-0) \times 10^{-3} A}{(25-0) W} = 3.2 \times 10^{-3}\ A/W$.

However $a = \frac{\eta \times \lambda \times e}{hc}$; so $\eta = \frac{a \times h \times c}{\lambda \times e} = \frac{3.2 \times 10^{-3} \times 6.63 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9} \times 1.6 \times 10^{-19}}$;

Then, $\eta = 9.945 \times 10^{-3} = 0.009945 = 0.9945\,\% \approx 1\%$.

VII-

1. The number of extracted electrons is : $n_{e^-/s} = \frac{I}{e} = \frac{40 \times 10^{-6}}{1.6 \times 10^{-19}} = 2.5 \times 10^{14}$ electrons/s;

The total number of incident photons per second is $n_{ph/s} = \frac{n_{e^-/s}}{\eta} = \frac{2.5 \times 10^{14}}{0.25 \times 10^{-2}} = 10^{17}$ photons/s;

The power received by the plate $P = n_{ph/s} \times E_{ph} = n_{ph/s} \times \frac{hc}{\lambda}$;

Then $P = 10^{17} \times \frac{6.63 \times 10^{-34} J.s \times 3 \times 10^{8} m/s}{360 \times 10^{-9}} = 5.525 \times 10^{-2} W$.

2. a) The quantum efficiency depends on the nature of metal used and independent from the incident radiation (wavelength and power).

b) The power received is proportional to its surface area; so $\frac{P_S}{4\pi d^2} = \frac{P}{s}$, then $P = \frac{s}{4\pi d^2} \times P_S$;

When the source is approached to half the distance, the plate receives $P_1 = \frac{s}{4\pi d_1^2} \times P_S = 4P$;

However, this power is proportional to the total number of incident photons and consequently the number of electrons taking into consideration that the quantum efficiency remains constant. Thus, the current is also quadrupled as proportional to the power received.

Problems

I- Photons and Vision

According to Planck-Einstein's hypothesis:

«An electromagnetic radiation of frequency υ, is composed of energy particles called photons».

Given:

- $1nm = 10^{-9} m$;
- $1eV = 1.6\times10^{-19} J$;
- The speed of light in vacuum: $c = 3\times10^{8} m/s$.

1. a) What aspect of light, the previous statement, does it show evidence of?
   b) State two physical properties of the photon.
   c) What is meant by duality of light? Give a physical phenomenon that is interpreted basing on each aspect.
2. To study the relationship between the energy E of a photon and its frequency υ, we measure the photon's energy of many electromagnetic radiations that are placed in the following table.

$\upsilon(\times 10^{14} Hz)$	0.3	3.75	5	7.5	15
$E(eV)$	0.124	1.56	2.1	3.11	6.21

   a) Plot the curve representing the photon's energy as a function of the frequency υ.
   Take as a scale:
   - on the abscissas axis: $1cm \equiv 1\times10^{14} Hz$;
   - on the ordinate axis: $1cm \equiv 0.5 eV$.

   b) Justify that the energy of a photon is proportional to its frequency υ.
   c) Determine, in SI units, the value of this constant of proportionality, called Planck's constant h.
   d) Knowing that the spectrum of visible radiations in vacuum extends in the interval: $400 nm \le \lambda \le 750 nm$.
   i- Determine the frequencies range of visible radiations.
   ii- Indicate to which domain the radiations mentioned in the previous table belong, the visible, ultraviolet or infrared spectrum.

In what follows we consider that the Planck's constant h is equal to: $h = 6.64\times10^{-34} J.s$.

3. To start up a visual excitation, 100 photons at least must reach the retina whose area is $1.5\times10^{-10} m^2$ during a duration of $0.1 s$. The retina's surface holds 20 rods.
   a) What is the number of photons needed to provoke an excitation of a rod during this duration?
   b) Photons
   i- Write the expression of a photon's energy in terms of h, c & λ_0.
   ii- Determine the power absorbed by a rod receiving a radiation whose wavelength $\lambda_0 = 550 nm$.
   c) Determine the minimum power of a luminous source emitting this same radiation uniformly in all directions of the space in order to be visible from $10 km$.

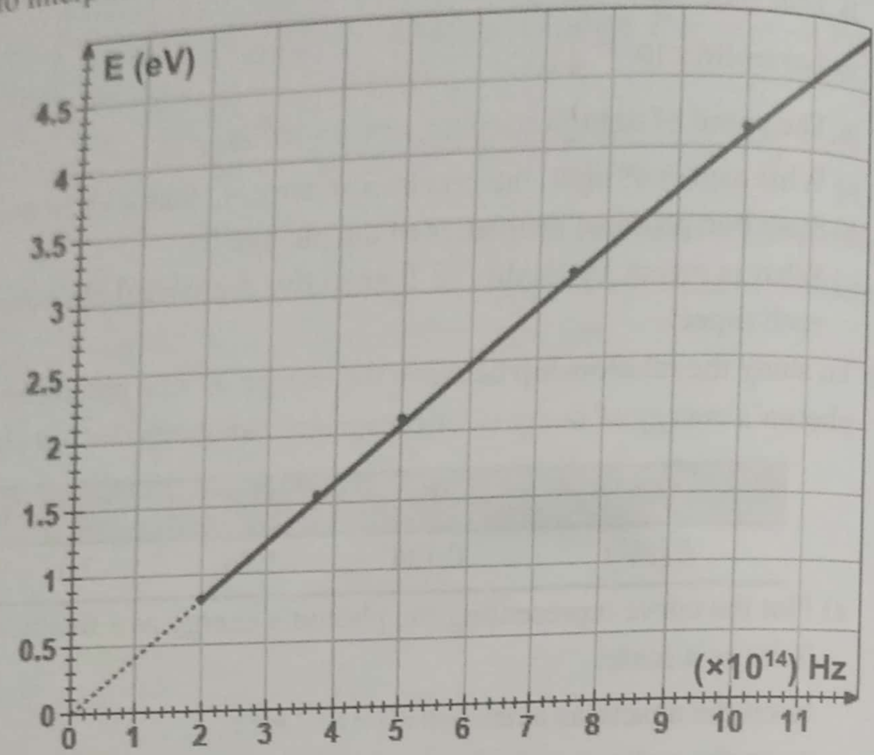
1\. a) The corpuscular aspect of light.

b) Photons are massless, chargeless & propagates in vacuum at the speed of light c.

c) It means that the light has a double aspect, which are complementary of each other:

- corpuscular aspect that is used to interpret the photoelectric effect.
- wave aspect that is used to interpret the diffraction & interference of light.

2\. a) Graph.



b) The graph representing the change of the photon's energy in terms of frequency has two properties:

- it is a straight line.
- the prolongation of this line passes through the origin.

Then, the energy of the photon is proportional to its frequency.

c) $E = f(\upsilon) = k\upsilon$ where k is called Planck's constant.

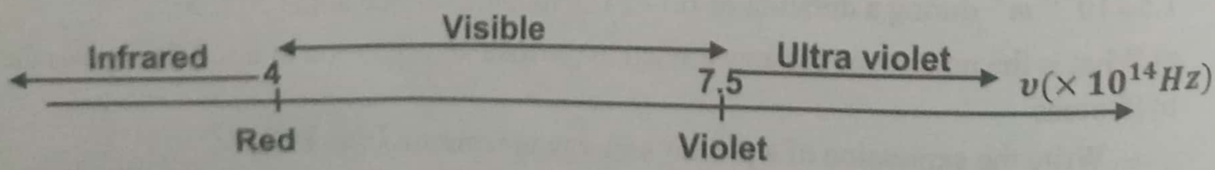
$$k = h = \frac{\Delta E}{\Delta \upsilon} = \frac{(6.21 - 1.56) \times 1.6 \times 10^{-19}\ J}{(15 - 3.75) \times 10^{14}\ Hz} = 6.6 \times 10^{-34}\ J.s\ .$$

d) Visible spectrum of light:

i- The visible spectrum extends $400\ nm \leq \lambda \leq 750\ nm$, but $\upsilon = \dfrac{c}{\lambda}$;

Then $400 \times 10^{-9}\ m \leq \dfrac{3 \times 10^8}{\upsilon} \leq 750 \times 10^{-9}\ m$;

$\dfrac{3 \times 10^8\ m/s}{750 \times 10^{-9}\ m} \leq \upsilon \leq \dfrac{3 \times 10^8\ m/s}{400 \times 10^{-9}\ m}$; thus $4 \times 10^{14}\ Hz \leq \upsilon_{\text{visible}} \leq 7.5 \times 10^{14}\ Hz$.



ii- The 1st and 2nd $\upsilon_1 = 0.3 \times 10^{14}\ Hz$ and $\upsilon_2 = 3.75 \times 10^{14}\ Hz$ are infrared radiations;

The 3rd and 4th $\upsilon_3 = 7.5 \times 10^{14}\ Hz$ and $\upsilon_4 = 15 \times 10^{14}\ Hz$ are visible radiations;

The 5th $\upsilon_5 = 15 \times 10^{14}\ Hz$ is an ultraviolet radiation.

3. a) Every rod must be the target of at least $\frac{100}{20}=5$ photons.

b) Photons and Power:

i- The energy of an incident photon is $E=h\frac{c}{\lambda_0}$.

ii- $E=h\frac{c}{\lambda_0}=6.64\times10^{-34}\frac{3\times10^{8}}{550\times10^{-9}}=3.6\times10^{-19}\,J$;

The power received by a rod is $P_{rod}=\frac{E_{rod}}{\Delta t}=\frac{5\times E}{\Delta t}=\frac{5\times3.6\times10^{-19}}{0.1}=1.8\times10^{-17}\,W$.

c) The retina is the target of at least 100 photons.

The minimum energy absorbed by the retina's unit of area is:

$I_{min}=\frac{P_{min}}{s_{retina}}=\frac{20\times P_{rod}}{1.5\times10^{-10}}=7.2\times10^{-7}\,W/m^2$.

This minimum power is due to a point source that delivers the power uniformly in all the directions assumed to be a sphere of radius $R=d=10\,km$.

$I_{min}=\frac{P_{min/S}}{S}\Rightarrow P_{min/S}=I_{min}\times4\pi\left(10\times10^{3}\right)^2=7.2\times10^{-7}\times4\pi\left(10\times10^{3}\right)^2\approx3033\,W$.

LS & GS 2003 1st
The Two Aspects of Light

To show evidence of the two aspects of light, we perform the two following experiments.

Part A

First experiment

Given:

- Planck's constant: $h = 6.6\times10^{-34}\ J.s$. $\quad 1eV = 1.6\times10^{-19}\ J$.
- Speed of light in vacuum: $c = 3\times10^{8}\ m/s$.

We cover a metallic plate by a thin layer of cesium whose threshold wavelength is $\lambda_0 = 670\ nm$.

Then we illuminate it with a monochromatic radiation of wavelength in vacuum is $\lambda = 480\ nm$.

A convenient apparatus is placed near the plate in order to detect the electrons emitted by the illuminated plate.

1. This emission of electrons by the plate shows evidence of an effect. What is that effect?
2. What does the term «threshold wavelength» represent?
3. Calculate in, J and eV, the extraction energy (work function) of the cesium layer.
4. What is the form of energy carried by an electron emitted by the plate? Give the maximum value of this energy.

Part B

Second experiment

The two thin slits of Young's apparatus, separated by a distance a, are illuminated with a laser light whose wavelength in vacuum is $\lambda = 480\ nm$.

The distance between the screen of observation and the plane of the slits is $D = 2\,m$.

1. Draw a diagram of the apparatus and show on it the region of the interference.
2. The conditions to obtain the phenomenon of interference on the screen are satisfied. Why?
3. Due to what is the phenomenon of interference?
4. a) Describe the aspect of the region of interference observed on the screen.

   b) We count 11 bright fringes. The distance between the centers of the farthest fringes is $l = 9.5\,mm$.

   What do we call the distance between the centers of two consecutive bright fringes? Calculate its value and deduce the value of a.

Part C

Conclusion

The two experiments show evidence of two aspects of light. Specify the aspect shown by each experiment.

Answer Key

Part A 3. $W_0 = 1.875\ eV$. $\quad$ 4. $1.1\times10^{-19}\ J$

Part B 4.b) $a = 1\,mm$.

I-LS 2010 2nd
Photoelectric Effect

A metallic plate, covered with a layer of cesium, is illuminated with a monochromatic luminous beam of wavelength $\lambda = 0.45\times10^{-6}\,m$ in vacuum.
The work function (extraction energy) of cesium is $W_0 = 1.88\,eV$.
A convenient apparatus (D) is used to detect the electrons emitted by the illuminated plate.

Given

- Planck's constant: $h = 6.6\times10^{-34}\,J.s$;
- Speed of light in vacuum: $c = 3\times10^{8}\,m/s$;
- $1eV = 1.6\times10^{-19}\,J$;
- Elementary charge: $e = 1.6\times10^{-19}\,C$.

1. What aspect of light does the phenomenon of photoelectric effect show evidence of?
2. Define the term «work function» of a metal.
3. The luminous beam illuminating the metallic plate is formed of photons.
   a) Energy of a photon
      i- Write down the expression of the energy E of a photon in terms of h, c & λ.
      ii- Calculate, in eV, the energy of an incident photon.
   b) (D) detects electrons emitted by the plate.
      Why do we have an emission of electrons by the plate?
   c) Calculate, in eV, the maximum kinetic energy of an emitted electron.
4. The luminous power P received by the plate is $10^{-3}\,W$, and the emitted electrons form a current $I = 5\,\mu A$.
   a) Calculate the number n of photons received by the plate in one second.
   b) Knowing that the current I is related to the number N of the electrons emitted per second and to the elementary charge e by the relation $I = N\times e$. Calculate N.
   c) Effective photons
      i- Calculate the quantum efficiency $r = \dfrac{N}{n}$.
      ii- Deduce that the number of effective photons in one second is relatively small.
   d) We increase the luminous power P received by the plate without changing the wavelength λ.
      Would the current increase or decrease? Why?

II-LS 2009 2nd
Photoelectric Effect

Given

- Speed of light in vacuum: $c = 3\times10^{8}\,m/s$;
- $1eV = 1.6\times10^{-19}\,J$.

Part A

The photoelectric effect was discovered by Hertz on 1887 . The experiment represented in figure 1 may show evidence of this effect. A zinc plate is fixed on the conducting rod of an electroscope.

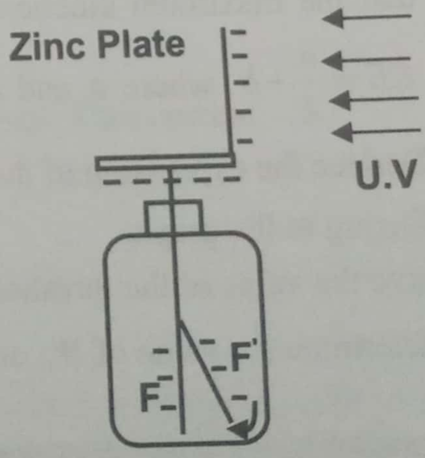


Figure 1

The whole set-up is charged negatively. If we illuminate the plate by a lamp emitting white light rich with ultraviolet radiations (U.V), the leaves F and F' of the electroscope approach each other rapidly.

1. Due to what is the approaching of the leaves?
2. The photoelectric effect shows evidence of an aspect of light.
   What is this aspect?

Part B

The experiments performed by Millikan towards 1915 , intended to determine the maximum kinetic energy KE of the electrons emitted by metallic plates when illuminated by monochromatic radiation of adjustable wavelength λ in vacuum.

In an experiment using a plate of cesium, a convenient apparatus allows us to measure the maximum kinetic energy KE of an emitted electron corresponding to the wavelength λ of the incident radiation.

The variation of KE as a function of λ is represented in the graph of figure 2.

The aim of this part is to determine the value of Planck's constant h and that of the extraction energy W_0 of cesium.

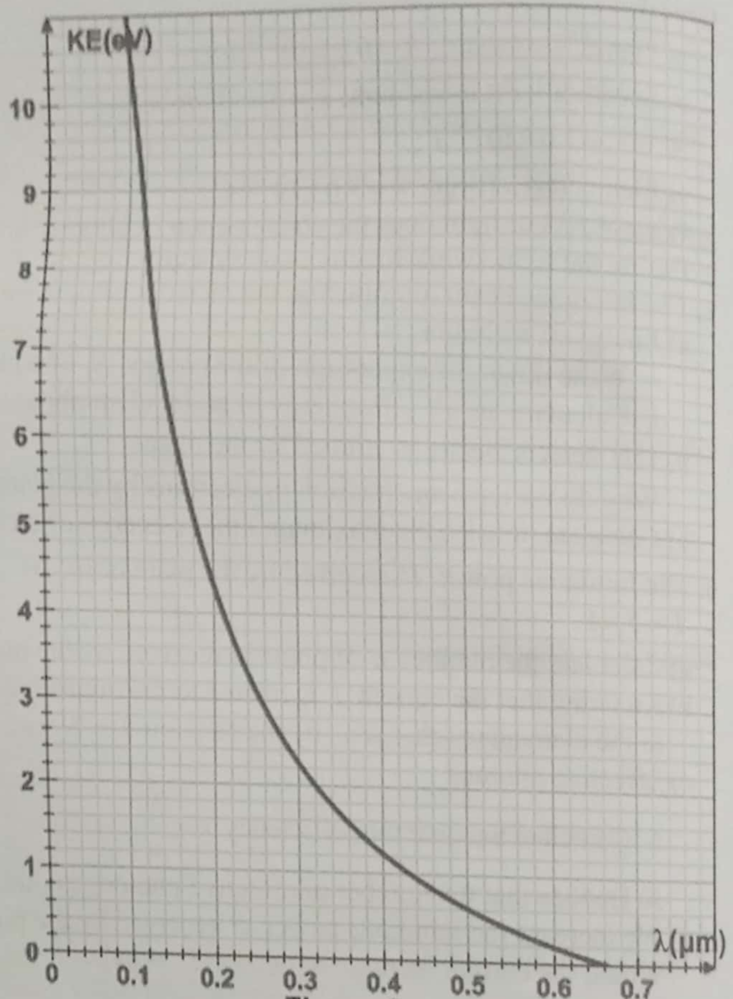


Figure 2

1. Write down the expression of the energy E of an incident photon, of wavelength λ in vacuum, in terms of λ, h & c.
2. a) Applying Einstein's relation about photoelectric effect, show that the maximum kinetic energy KE of an extracted electron may be written in the form $KE = \frac{a}{\lambda} + b$, where a and b are constants.

   b) Deduce the expression of the threshold wavelength λ_0 of cesium in terms of W_0, h & c.
3. Referring to the graph:

   a) give the value of the threshold wavelength λ_0 of cesium.

   b) determine the value of W_0 and that of h.

LS 2003 1st
See Page 62 – Part A

Solutions – Sessions

LS 2010 2nd

1. The corpuscular aspect of light.
2. The «extraction energy» or «Work energy function» is the minimum energy needed to extract an electron from the surface of the metal.
3. a) Energy of the photon

   i- The energy of a photon is given by: $E = h\dfrac{c}{\lambda}$.

   ii- $E = 6.6\times10^{-34}\,J.s\times\dfrac{3\times10^{8}\,m/s}{0.45\times10^{-6}\,m} = 4.4\times10^{-19}\,J$.

   Then $E = \dfrac{4.4\times10^{-19}}{1.6\times10^{-19}} = 2.75\,eV$.

   b) Since $E = 2.75\,eV > W_0 = 1.88\,eV$.

   c) According to the conservation of energy:
   $E = W_0 + KE_{max} \Rightarrow KE_{max} = E - W_0 = 2.75 - 1.88 = 0.87\,eV$.

4. a) The number n of photons received by the plate in one second is given by:

   $P = nE \Rightarrow n = \dfrac{P}{E} = \dfrac{10^{-3}\,W}{4.4\times10^{-19}\,J} = 2.27\times10^{15}$ photons $/s$.

   b) We have: $I = N\times e \Rightarrow N = \dfrac{I}{e} = \dfrac{5\times10^{-6}}{1.6\times10^{-19}} = 3.125\times10^{13}$ electrons $/s$.

   c) Effective photons

   i- The quantum efficiency: $r = \dfrac{N}{n} = \dfrac{3.125\times10^{13}}{2.27\times10^{15}} = 0.014 = 1.4\%$.

   ii- Since r is very small, then the number of extracted electrons is too small.

   d) The power is given by: $P = nE = nh\dfrac{c}{\lambda}$.

   The power is increased, but keeping the wavelength λ, will lead to an increase in the number of incident photons per second n.

   The quantum efficiency, of this metal, being constant and $N = rn$.
   Then the number of emitted electrons increases.
   However $I = Ne$, thus we will detect an increase in the intensity of the current.

   Second method:

   We have $I = Ne = r\times n\times e = r\times\dfrac{P}{E}\times e = r\times\dfrac{\lambda\times P}{hc}\times e$;

   However h, c & e are constant, r depends on the metal which is the same, and λ is not modified.
   Then the current increases when we increase the power of the source.

Part A

1. The plate carries an excess of electrons, when it is exposed to U.V radiations, electrons are extracted.
The decrease in the number of electrons carried by the leaves reduces the charge of the electroscope. Thus, the leaves approach from each other.
2. The photoelectric effect is an evidence of the corpuscular aspect of light.

Part B

1. The energy of a photon is given by $E = h\frac{c}{\lambda}$.
2. a) According to the conservation of energy: $E = KE_{max} + W_0 \Rightarrow KE_{max} = \frac{hc}{\lambda} - W_0$;

The expression of maximum the kinetic energy is of the form $KE_{max} = \frac{a}{\lambda} + b$;

Where $a = hc$ and $b = -W_0$.

b) For $\lambda = \lambda_0$, the electrons extracted (at rest) without kinetic energy so $KE = 0$;

Then $\frac{hc}{\lambda_0} - W_0 = 0$, thus $\lambda_0 = \frac{hc}{W_0}$.

3. a) We have for $\lambda = \lambda_0$; then $KE = 0$.
So it corresponds to the abscissa of the point of intersection with the abscissa axis, then $\lambda_0 = 0.66\,\mu m$.

b) We have $W_0 = \frac{hc}{\lambda_0} = \frac{3\times10^8}{0.66\times10^{-6}}h = 4.55\times10^{14}h$ .... (1)

Graphically we have, for $\lambda = 0.32\,\mu m = 0.32\times10^{-6}\,m$,

$KE = 2eV = 2\times1.6\times10^{-19} = 3.2\times10^{-19}\,J$;

But $E = W_0 + KE_{max} \Rightarrow \frac{3\times10^8}{0.32\times10^{-6}}h - W_0 = 3.2\times10^{-19}$;

$$9.375\times10^{14}h - W_0 = 3.2\times10^{-19} \ldots (2)$$

➢ *Replacing (1) in (2):*

$(9.375\times10^{14} - 4.55\times10^{14})h = 3.2\times10^{-19}$;

$$h = \frac{3.2\times10^{-19}}{(9.375\times10^{14} - 4.55\times10^{14})} = 6.63\times10^{-34}\,J.s;$$

➢ However $W_0 = 4.55\times10^{14}h = 4.55\times10^{14}\times6.63\times10^{-34} = 3.02\times10^{-19}\,J$.

Unit IV

Atom & Nucleus

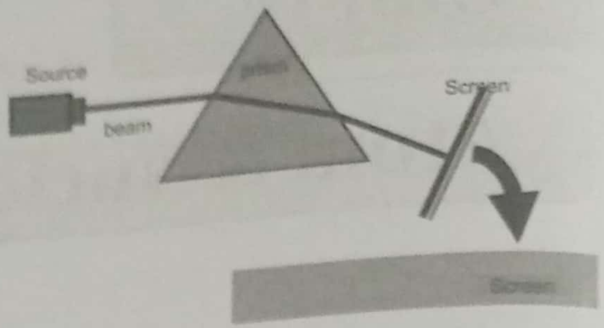
Chapter 15

Energy Levels of the Atom

LS – Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Energy Levels of the Atom		-	-	$2^{nd(A)}$	-	-	1^{st}	-	2^{nd}	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Energy Levels of the Atom	-	1^{st}	-	-	2^{nd}	-	1^{st}	$2^{nd\text{-}B}$	-	1^{st}

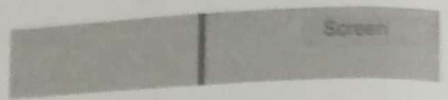
Essentials

I-Spectral Analysis



The spectral analysis is the decomposition of a beam into its constituents.
To analyze a beam, we use a prism and a screen. The light falls on the prism is divided into its constituents and the image is collected on a screen.

1. Monochromatic beam

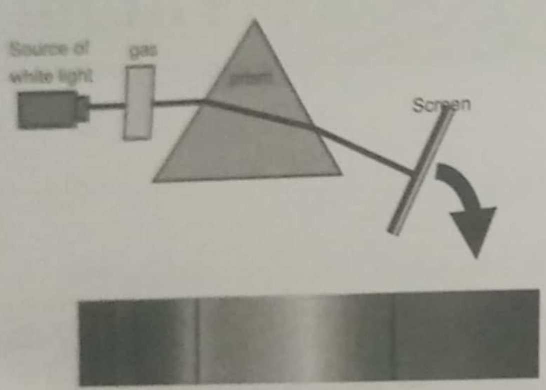


A monochromatic beam is formed of a single color (frequency). Its spectrum appears as a single luminous line.

2. White light

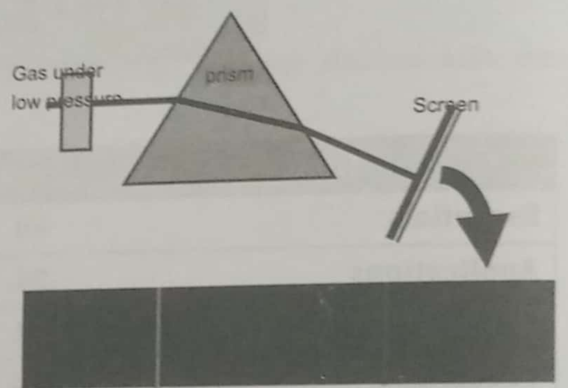
The white light is formed of a continuous spectrum that extends over the visible spectrum (all the colors appear).

3. Absorption spectrum of a gas



The absorption spectrum is formed by a colored background intercepted by black lines (discontinuous spectrum).

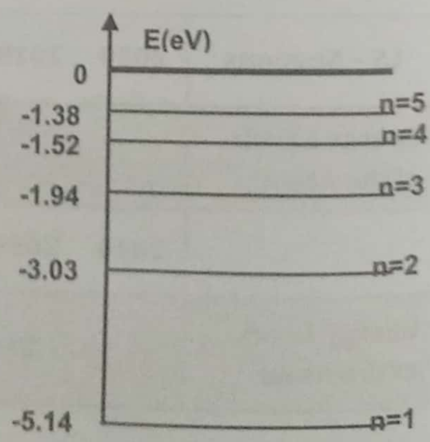
4. Emission spectrum of a gas



The emission spectrum is formed by a black background intercepted by a set of colored lines (discrete or discontinuous spectrum).

The radiations that are missing in the absorption spectrum appear in the emission spectrum, an atom absorbs the radiations that it emits.

II-Quantification of Energy Levels



The energy levels of an atom (shown in the adjacent figure) are quantified or quantized which means that:

- only a set of well-defined values is allowed.
- any intermediate state is forbidden.
- the distribution of energy levels is discrete or discontinuous.
- each atom is characterized by its spectrum (set of radiations).

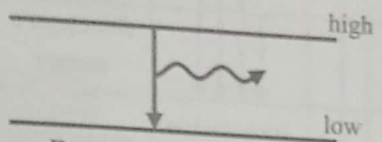
The energy level denoted:

- $n = 1$, is called ground or fundamental state;
- $n = 2$, 1st excited state;
- $n = \infty$, $E_\infty = 0$ is the ionized state.

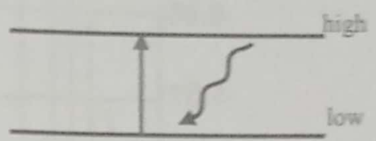
Note: For the hydrogen atom, the energy of these energy levels are given by $E_n(eV) = -\frac{13.6}{n^2}$ where n is a whole number.

III-Emission and Absorption of Photons

The energy of the photon emitted (absorbed) through the transition between two levels is: $E_{photon} = E_{high} - E_{low}$



Downwards transition and emission of a photon



Absorption of a photon and an upwards transition

IV-Maximum & Minimum

The wavelength corresponding to the transition from the level n, to p is $\lambda_{n,p} = \frac{hc}{E_n - E_p}$:

- the maximum wavelength (lowest frequency) needed to excite the atom corresponds to a transition verifying $\lambda_{max} = \frac{hc}{E_{p+1} - E_p}$; which is satisfied between the nearest energy level of the atom from the level p to $p+1$.
- while the minimum wavelength (largest frequency) needed to ionize the atom corresponds to the transition from the level p to the ionized state $\lambda_{min} = \frac{hc}{E_\infty - E_p}$.

V-Interaction Photon-Atom

A photon cannot cause the transition of an atom from an energy level E_p to a higher energy level E_n unless its energy **is exactly equal** the difference of the energies $(E_n - E_p)$ of the atom, and $E_{ph} = h\upsilon = h\frac{c}{\lambda} = E_n - E_p$.

If the atom is found in the level E_p and receives a photon, then the state of the atom is determined by: $E_{ph} + E_p < 0$ (inside energy levels of atom);

& if $E_{ph} + E_p = E_n$ (exists among the levels); then the photon is absorbed and the atom undergoes an upwards transition from the energy level p to n.

& if $E_{ph} + E_p \neq E_n$ (does not exist); then the photon is not absorbed and the atom remains in the energy level p.

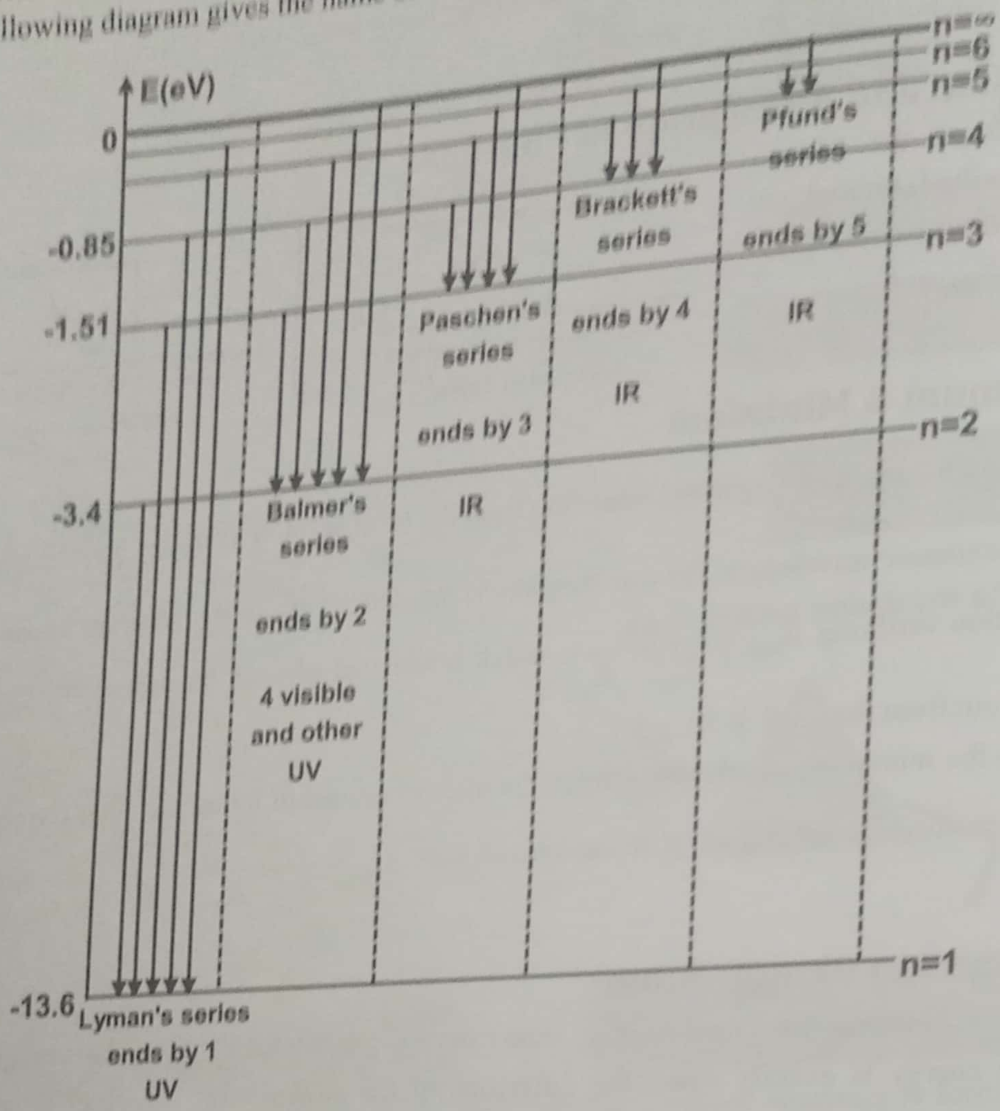
$E_{ph} + E_p = 0$; then the photon is absorbed, the atom is ionized and the electron extracted is at rest.

$E_{ph} + E_p > 0$; then the photon is absorbed, the atom is ionized and the electron is extracted carrying a kinetic energy: $KE = E_{ph} - W_{ion}$ where $W_{ion} = E_\infty - E_p$ is the ionization energy of the atom taken in level p.

VI-Hydrogen Atom Series

The radiations emitted by the hydrogen atom are grouped into series according to the energy level by which the transitions ends.

The following diagram gives the name of these series, and the corresponding domains it belongs.



VII-Interaction Electron-Atom

For an electron to cause a transition of an atom from an energy level E_p to a higher energy level E_n, its energy must be **at least equal** to the difference of the energies $E_n - E_p$ of the atom.

During one electron-atom collision, the atom absorbs, from the electron carrying the kinetic energy KE_i, an amount of energy enough to ensure a transition $\Delta E = E_n - E_p$ & the rest of the energy is carried by the electron as kinetic energy $KE_r = KE_i - \Delta E$.

For example:

If the atom is taken in the ground state, we calculate the sum $KE_i + E_1 < E_6$ (for example), then the atom will not overpass the 5[th] energy level, we have 4 possible transitions $1 \longrightarrow 2, \dots, 1 \longrightarrow 5$.

The electron that carries the smallest (largest) kinetic energy after interaction corresponds to the transition between the farthest (nearest) possible levels.

$(KE_r)_{min} = KE_i - \Delta E_{max} = KE_i - \Delta E_{1,5}$

& $(KE_r)_{max} = KE_i - \Delta E_{min} = KE_i - \Delta E_{1,2}$.

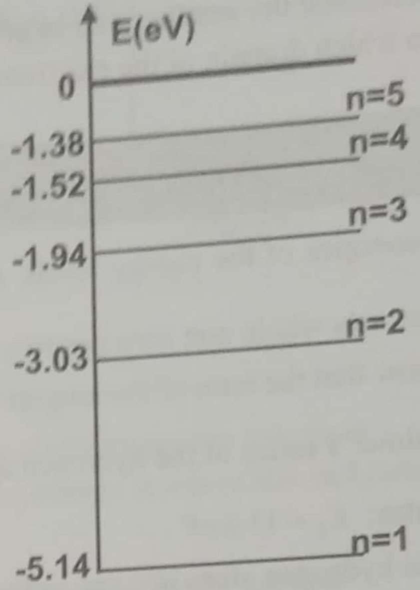
Applications

Given:

- $1 eV = 1.6\times10^{-19} J$.
- Planck's constant: $h = 6.63\times10^{-34} J.s$.
- Speed of light in vacuum: $c = 3\times10^{8} m/s$.
- Mass of an electron: $m = 9.1\times10^{-31} kg$.

I- Energy Levels of the Sodium Atom

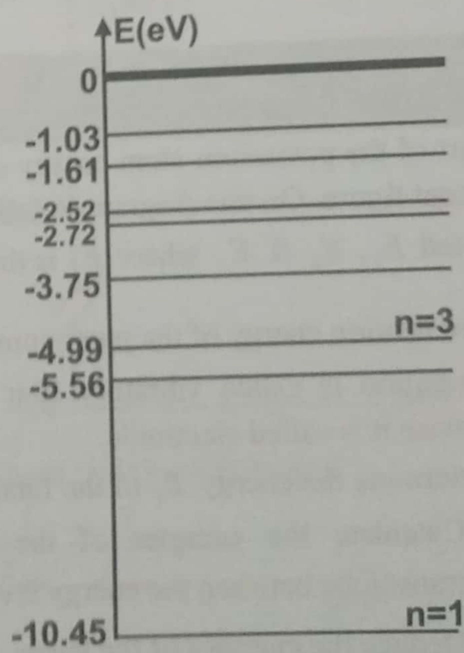
The adjacent figure shows a simplified diagram of the energy levels of a sodium atom.



1. Give the energy of the sodium atom in the:
   a) ground state.
   b) 3^{rd} excited state.
2. Determine, in nm, the wavelength of the radiation emitted or absorbed during the transition of the sodium atom from:
   a) ground state to the ionized state.
   b) the 1^{st} excited state to the ground state.
3. Specify to which domain of the electromagnetic spectrum each radiation belongs.
4. Two radiations of respective wavelengths λ_3 & λ_2 are emitted during the transition between the energy levels E_3 to E_1 & E_2 to E_1.
   Compare, without calculation, λ_3 & λ_2.

II- Interaction Photon-Atom

The adjacent diagram represents a part of the energy levels of the mercury atom. The atom is taken in the ground state.



1. Give the energy of the atom in the:
   a) 1^{st} excited state.
   b) ionized state.
2. The energy levels of the atom are quantified. Justify.
3. Determine the minimum frequency required:
   a) to excite this atom.
   b) to ionize this atom.
4. The atom receives a photon of wavelength $\lambda = 80\ nm$.
   a) Calculate, in eV, the energy carried by this photon.
   b) Show that the atom is ionized.
   c) Determine the speed of the electron extracted.
5. Show that the radiation emitted during the downward transition from $n = 7$ to $n = 4$ is visible.

III- Paschen Series of the Hydrogen Atom

The energies of the energy levels E_n of the hydrogen atom are given by the relation $E_n = -\frac{E_0}{n^2}$,

where n is whole non zero positive number, E_0 & E_n in eV where $E_0 = 13.6eV$.

In this application, we are interested to the Paschen series of the hydrogen, it is formed of the set radiations emitted when it undergoes a transition from a level $n > 3$ towards $n = 3$.

1. The values of E_n are quantized. Explain.
2. Determine the values of the energy of the hydrogen atom for $n = 1, 2, 3, 4, 5$ & ∞.
3. Represent the energy diagram of the hydrogen atom for the previous values.
4. Show that the expression of the wavelengths emitted, in Paschen series, are given by:

$\lambda = \lambda_P \frac{n^2}{n^2 - 9}$, where $\lambda_P = 823nm$.

5. Determine the smallest and largest wavelengths of this series.
6. To which domain of the electromagnetic spectrum does the radiations of Paschen series belong?

IV- Ratio of Wavelengths

The energies of the energy levels E_n of the hydrogen atom are given by the relation $E_n = -\frac{E_0}{n^2}$,

where n is whole non zero positive number E_0 & E_n in eV.

1. Show that the ratio of the longest wavelength emitted in the Lyman series to the longest one in the Balmer's series of the hydrogen atom is equal to $\frac{5}{27}$.

2. **Take:** $E_0 = 13.6eV$.

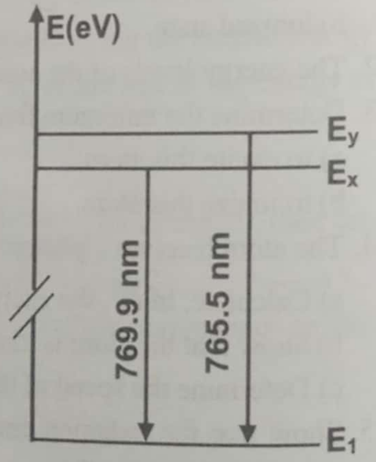
The hydrogen atom is taken in the ground state absorbs a photon whose associated wavelength is $\lambda = 95.2nm$.

a) Indicate the aspect of light that this study shows evidence.

b) Determine then the state of the hydrogen atom.

V- Potassium Doublet

A part of the potassium atom energy diagram, is represented in the adjacent figure. On this diagram we schematized three energy levels denoted E_1, E_x & E_y where E_1 is the fundamental.



The ionization energy of the potassium atom is $4.34eV$.

A transition is called vibratory if it emits an infrared radiation otherwise it is called electronic.

1. Determine the energy E_1 of the fundamental level.
2. a) Calculate the energies of the photons associated to the transitions between the energy levels E_1 & E_x and E_1 & E_y.

   b) Deduce the energies of the levels x & y.

3. Knowing that: $E_y - E_x = (E_y - E_1) - (E_x - E_1)$; deduce the wavelength of the radiation emitted when the atom undergoes a transition from the level y to the level x.
4. Identify the electronic and vibratory transitions.

VI- Interaction Electron-Atom

Figure 1, shows a simplified diagram of the energy levels of sodium atom.

1. Describe the absorption spectrum obtained when sodium is illuminated by white light.
2. Determine the maximum wavelength of the radiation capable of ionizing this atom.
3. The atom taken in the ground state receives an electron carrying a kinetic energy of $3.7eV$.
   a) Justify that this electron can excite the sodium atom.
   b) Show that the atom will not overpass the fourth energy level and then indicate the possible transitions.
   c) Specify the transition that corresponds to the atom absorbing the maximum energy and then deduce the energy of the electron after interaction.

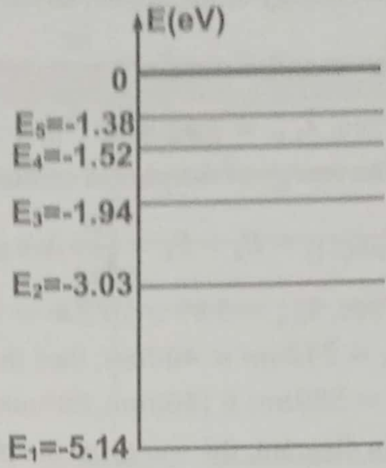
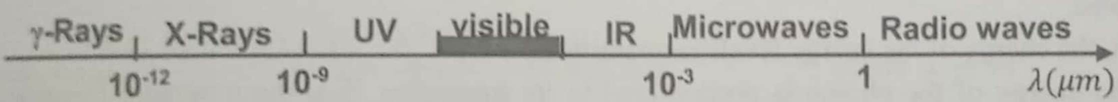


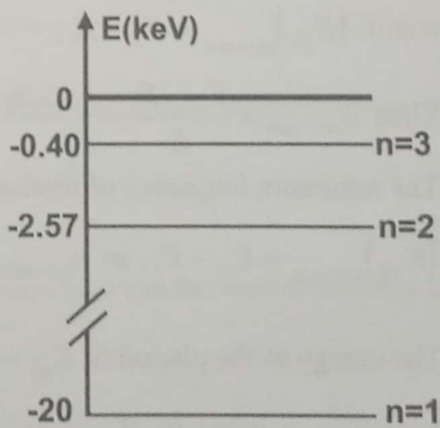
Figure 1

VII-Bac 2016 Molybdenum Atom

The axis below is a simplified representation (not to scale) of the electromagnetic spectrum.
The adjacent diagram represents a simplified representation of the energy levels of the molybdenum atom.



1. The atom is taken in the ground state receives a photon whose wavelength is $\lambda = 56\ pm$.
   a) Show that the energy of the incident photon is $22.19\ keV$.
   b) Justify that the atom is ionized.
   c) Deduce the kinetic energy of the extracted electron.
2. The atom is found in the 2nd excited state.
   a) Specify the three possible transitions.
   b) Determine the largest and smallest wavelengths λ_{min} & λ_{max} emitted during these transitions.



   c) To which domain of the electromagnetic spectrum does these radiations belongs?
   d) Show that the third radiation λ satisfy the relation $\frac{1}{\lambda} = \frac{1}{\lambda_{min}} - \frac{1}{\lambda_{max}}$.
   e) Deduce the value of λ.

1. a) In the ground state ($n = 1$), the energy of the sodium atom is $E_1 = -5.14\ eV$.
b) In the 2[nd] excited state ($n = 3$), the energy of the sodium atom is $E_3 = -1.94\ eV$.

2. a) The energy of the photon absorbed by the atom to undergo a transition from $n = 1$ to ∞ is:

$$E_{ph(1,\infty)} = E_\infty - E_1 = \frac{hc}{\lambda_{1,\infty}}, \text{we get } \lambda_{1,\infty} = \frac{hc}{E_\infty - E_1} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{[0-(-5.14] \times 1.6 \times 10^{-19}};$$

Then, $\lambda_{1,\infty} = 2.42 \times 10^{-7} m = 242nm$.

b) The energy of the photon emitted by the atom during the transition from $n = 1$ to $n = 2$ is:

$$E_{ph(2,1)} = E_2 - E_1 = \frac{hc}{\lambda_{2,1}}, \text{we get } \lambda_{2,1} = \frac{hc}{E_2 - E_1} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{[-3.03-(-5.14] \times 1.6 \times 10^{-19}};$$

Then, $\lambda_{2,1} = 5.89 \times 10^{-7} m = 589nm$.

3. $\lambda_{1,\infty} = 242nm < 400nm$, then this radiation is ultraviolet.
$\lambda_{2,1} = 589nm \in [400nm, 800nm]$, then this radiation is visible.

4. From diagram, the energy of the photon $E_{ph(3,1)}$ needed to ensure the transition from $n = 1$ into $n = 3$, is greater than that required from $n = 1$ into $n = 2$; $E_{ph(3,1)} > E_{ph(2,1)}$.
The energy of the photon is inversely proportional to its wavelength, then $\lambda_{3,1} < \lambda_{2,1}$.

1. a) The first excited state corresponds to $n = 2$, whose energy is $E_2 = -5.56\ eV$.
b) In the ionized state, $E_\infty = 0$.

2. The distribution of the energy levels is discrete, and then a set a well-defined values is only allowed.
Thus, the energy of the energy levels is quantified.

3. a) The energy of the photon is proportional to its frequency, then the minimum frequency of excitation is that required to reach the nearest energy level (the atom is taken in the ground state): $(E_{ph})_{exc-min} = E_2 - E_1$;

$$\text{Then } \upsilon_{exc-min} = \frac{E_2 - E_1}{h} = \frac{[-5.56-(-10.45)] \times 1.6 \times 10^{-19} J}{6.63 \times 10^{-34} J.s} = 1.18 \times 10^{15} Hz.$$

b) The minimum frequency of ionization is that required to reach the ionized state at rest:

$$(E_{ph})_{ion-min} = E_\infty - E_1, \text{ so } \upsilon_{ion-min} = \frac{E_\infty - E_1}{h} = \frac{[0-(-10.45)] \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 2.52 \times 10^{15} Hz.$$

4. a) The energy of the photon is $E_{ph} = h\frac{c}{\lambda} = 6.63 \times 10^{-34} J.s \times \frac{3 \times 10^8 m/s}{80 \times 10^{-9} m} = 2.49 \times 10^{-18} J$;

$$\text{Then, } E_{ph} = \frac{2.49 \times 10^{-18}}{1.6 \times 10^{-19}} = 15.56 eV.$$

b) The ionization energy is $W_{ion} = E_\infty - E_1 = 0 - (-10.45) eV = 10.45 eV$.
But $E_{ph} > W_{ion}$, then the atom is ionized and the electron is extracted with a kinetic energy.

c) The kinetic energy of the extracted electron $KE = E_{ph} - W_{ion} = 15.56 - 10.45 = 5.11 eV$.

However, $KE = \frac{1}{2} m_e v^2$; then $v = \sqrt{\frac{2 \times KE}{m_e}} = \sqrt{\frac{2 \times 5.11 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.34 \times 10^6 \ m/s$.

5. The energy emitted during the disexcitation $E_{ph} = E_7 - E_4 = -1.61\,eV - (-3.75)eV = 2.14\,eV$.

However $\lambda = \frac{hc}{E_{ph}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.14 \times 1.6 \times 10^{-19}} = 5.8 \times 10^{-7}\,m = 580\,nm$;

$\lambda = 580\,nm \in [400\,nm; 800\,nm]$, then this radiation is visible.

III-

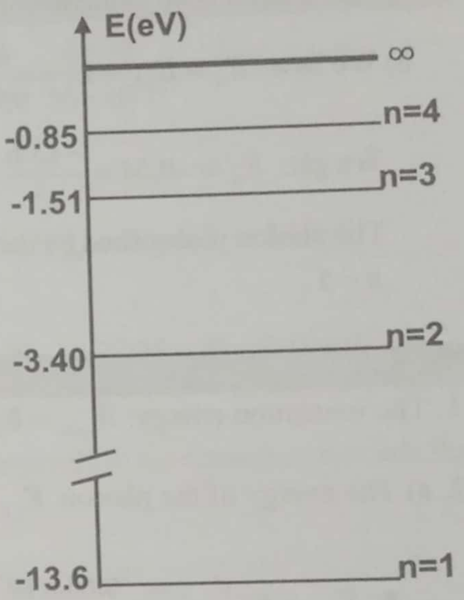
1. The energy of the energy levels of the hydrogen atom depends on a whole number; thus, it is quantified.
2. The energy of the energy levels of the hydrogen atom:

For $n = 1, E_1 = -\frac{13.6}{1^2} = -13.6\,eV$;

For $n = 2, E_2 = -\frac{13.6}{2^2} = -3.4\,eV; E_3 = -1.51\,eV$;

$E_4 = -0.85\,eV, E_5 = -0.544\,eV, \& \& E_\infty = 0\,eV$.

3. Adjacent diagram.



4. The energy of an emitted photon in the Paschen series

$$E_{ph} = E_n - E_3 = \frac{-13.6}{n^2} - \frac{-13.6}{3^2},$$

$$E_{ph} = 13.6\left(\frac{1}{9} - \frac{1}{n^2}\right) = \frac{13.6}{9}\left(\frac{n^2 - 9}{n^2}\right) (E_{ph} \text{ in } eV \ \& \ n > 3)$$

However $E_{ph} = \frac{hc}{\lambda}$, so $\frac{13.6}{9} \times 1.6 \times 10^{-19}\left(\frac{n^2 - 9}{n^2}\right) = 6.63 \times 10^{-34} \times \frac{3 \times 10^8}{\lambda}$;

Then $\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8 \times 9}{13.6 \times 1.6 \times 10^{-19}}\left(\frac{n^2}{n^2 - 9}\right) = 8.23 \times 10^{-7}\left(\frac{n^2}{n^2 - 9}\right)$ where λ in m,

Thus, $\lambda = 823\left(\frac{n^2}{n^2 - 9}\right)$ where λ in nm, $\lambda_P = 823\,nm$ and $n > 3$.

5. The smallest wavelength corresponds to the transition between the farthest possible levels $n = \infty$ towards $n = 3$, then $\lambda_{\min} = 823\left(\frac{n^2(1)}{n^2(1 - 9/n^2)}\right) = 823nm$.

In contrast, the largest wavelength corresponds to the transition between the nearest possible levels $n = 4$ towards $n = 3$, then $\lambda_{\max} = 823\frac{4^2}{4^2 - 9} = 1881\,nm$.

6. $\lambda_{\text{Paschen}} \in [823\,nm; 1881\,nm] > 800\,nm$; the radiations in the Paschen series belong to the infrared domain.

IV-

1. The longest wavelength in any series corresponds to the transition between the nearest possible energy levels. For Lyman's series it corresponds to the transition from $n = 2$ towards $n = 1$:

$E_{ph}=\frac{hc}{\lambda_{L\max}}=E_2-E_1$, then $\lambda_{L\max}=\frac{hc}{E_2-E_1}$; while in the Balmer's series corresponds to the transition from $n=3$ towards $n=2$: $E_{ph}=\frac{hc}{\lambda_{B\max}}=E_3-E_2$; then $\lambda_{B\max}=\frac{hc}{E_3-E_2}$;

$$\text{Thus, } \frac{\lambda_{L\max}}{\lambda_{B\max}}=\frac{\frac{hc}{E_2-E_1}}{\frac{hc}{E_3-E_2}}=\frac{E_3-E_2}{E_2-E_1}=\frac{-\frac{E_0}{3^2}-\left(-\frac{E_0}{2^2}\right)}{-\frac{E_0}{2^2}-\left(-\frac{E_0}{1^2}\right)}=\frac{\frac{1}{4}-\frac{1}{9}}{1-\frac{1}{4}}=\frac{5}{27}.$$

2. a) The photon is an evidence of the corpuscular aspect of light.

b) We have $E_n=E_{ph}+E_1=\frac{6.63\times10^{-34}\times3\times10^{8}}{95.2\times10^{-9}\times1.6\times10^{-19}}+\frac{-13.6}{1^2}=-0.54\,eV$;

We get: $E_n=-0.54=\frac{-13.6}{n^2}$; then, $n=\sqrt{\frac{13.6}{0.54}}\approx5$.

The photon is absorbed by the atom which undergoes an upwards transition from $n=1$, towards $n=5$.

V-

1. The ionization energy: $W_{ion}=E_\infty-E_1=0-E_1$, then $E_1=-W_{ion}=-4.34eV$.

2. a) The energy of the photon $E_{1,x}=h\frac{c}{\lambda_{1,x}}=\frac{6.63\times10^{-34}J.s\times3\times10^{8}m/s}{769.9\times10^{-9}m\times1.6\times10^{-19}(J/eV)}=1.61\,eV$;

& $E_{1,y}=h\frac{c}{\lambda_{1,y}}=\frac{6.63\times10^{-34}J.s\times3\times10^{8}m/s}{765.5\times10^{-9}m\times1.6\times10^{-19}(J/eV)}=1.62\,eV$.

b) However $E_{1,x}=E_x-E_1$, then $E_x=E_1+E_{1,x}=-4.34eV+1.61eV=-2.73eV$;

& $E_{1,y}=E_y-E_1$, then $E_y=E_1+E_{1,y}=-4.34eV+1.62eV=-2.72eV$.

Note: the levels x & y are very close from each other, and form a doublet.

3. We have $E_y-E_x=(E_y-E_1)-(E_x-E_1)$;

$\frac{hc}{\lambda_{y,x}}=\frac{hc}{\lambda_{y,1}}-\frac{hc}{\lambda_{x,1}}$; then $\frac{1}{\lambda_{y,x}}=\frac{1}{\lambda_{y,1}}-\frac{1}{\lambda_{x,1}}=\frac{1}{765.5}-\frac{1}{769.9}$; thus $\lambda_{y,x}=133945\ nm$.

4. The radiation $\lambda_{y,x}=133945nm>800nm$ (infrared) is associated to a vibratory transition; whereas the radiations $\lambda_{1,x}$ & $\lambda_{1,y}$ corresponds to electronic transitions.

VI-

1. The spectral analysis of the emerging beam is discontinuous. It is formed of colored background intercepted by a set of black lines corresponding to the photons absorbed by the atom.
2. The maximum wavelength needed to excite the atom corresponds to the less energetic photon, it leads to the excitation of the atom towards the ionized state and the electron extracted is at rest:

Then, $E_{ph}=E_\infty-E_1=\frac{hc}{\lambda_{\max}}$; $\lambda_{\max}=\frac{hc}{E_\infty-E_1}=\frac{6.63\times10^{-34}\times3\times10^{8}}{[0-(-5.14)]\times1.6\times10^{-19}}=2.41\times10^{-7}m$.

3. a) The minimum energy required to excite this atom $E_{\min}=E_2-E_1=-3.03-(-5.14)=2.11\,eV$.

$KE_i = 3.7\,eV > E_{\min} = 2.11\,eV$, then this atom is excited.

b) The state of the atom is defined by: $KE_i + E_1 = 3.7\,eV + (-5.14) = -1.44\,eV \in]E_4, E_5[$;

Then, this atom will not overpass the 4th energy level.

The possible transitions are: $1 \longrightarrow 2$, $1 \longrightarrow 3$ & $1 \longrightarrow 4$.

c) The atom absorbs maximum energy when it undergoes the farthest possible transition $1 \longrightarrow 4$

The maximum energy absorbed $\Delta E_{\max} = E_4 - E_1 = 3.62\,eV$;

The minimum energy carried by the electron $(KE_r)_{\min} = KE_i - \Delta E_{\max} = 3.7 - 3.62 = 0.08\,eV$.

VII-

1. a) The energy of the incident photon is $E_{ph} = h\dfrac{c}{\lambda} = 6.63 \times 10^{-34} \times \dfrac{3 \times 10^8}{56 \times 10^{-12}} = 3.55 \times 10^{-15}\,J$;

Then, $E_{ph} = \dfrac{3.55 \times 10^{-19}}{1.6 \times 10^{-19}} = 22187\,eV = 22.19\,keV$.

b) The ionization energy of the atom taken in the ground state is $W_{ion} = E_{\infty} - E_1 = 20\,keV$.

However $E_{ph} > W_{ion}$, then the atom is ionized.

c) The kinetic energy carried by the electron is $KE_{\max} = E_{ph} - W_{ion} = 22.19 - 20 = 2.19\,keV$.

2. a) The possible transitions are: $3 \longrightarrow 2$, $2 \longrightarrow 1$ & $3 \longrightarrow 1$.

b) The minimum wavelength (the most energetic photon) corresponds to the transition towards the farthest possible energy level $3 \longrightarrow 1$.

Then, $\lambda_{\min} = \dfrac{hc}{E_3 - E_1} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{(-0.4 - (-20)) \times 10^3 \times 1.6 \times 10^{-19}} = 6.3 \times 10^{-11}\,m$.

The maximum wavelength (the least energetic photon) corresponds to the transition towards the nearest possible energy level $3 \longrightarrow 2$.

Then, $\lambda_{\max} = \dfrac{hc}{E_3 - E_2} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{(-0.4 + 2.57) \times 10^3 \times 1.6 \times 10^{-19}} = 5.7 \times 10^{-10}\,m$.

c) $\lambda_{\min} = 6.3 \times 10^{-11}\,m$ & $\lambda_{\max} = 5.7 \times 10^{-10}\,m \in [10^{-9}; 10^{-12}]$; thus, these radiations are X-rays.

d) $\dfrac{1}{\lambda_{\min}} - \dfrac{1}{\lambda_{\max}} = \dfrac{1}{hc/(E_3 - E_1)} - \dfrac{1}{hc/(E_3 - E_2)} = \dfrac{E_3 - E_1}{hc} - \dfrac{E_3 - E_2}{hc} = \dfrac{E_3 - E_2}{hc} = \dfrac{1}{\lambda}$.

e) We have: $\dfrac{1}{\lambda} = \dfrac{1}{\lambda_{\min}} - \dfrac{1}{\lambda_{\max}} = \dfrac{1}{6.3 \times 10^{-11}} - \dfrac{1}{5.7 \times 10^{-10}}$; then $\lambda = 7.1 \times 10^{-11}\,m$.

Problems

1- Energy Diagram of the Hydrogen Atom

Given:

- $1eV = 1.6\times10^{-19}J$;
- Planck's constant: $h = 6.63\times10^{-34}J.s$;
- Speed of light in vacuum: $c = 3\times10^{8}m/s$.

Figure 1 represents a part of the energy diagram for the energy levels of the hydrogen atom.

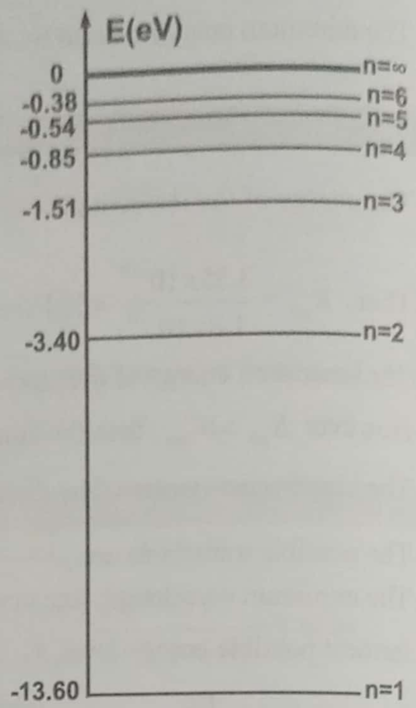


Figure 1

1. Give the value of the energy of the atom:
   a) taken in the fundamental state.
   b) in the ionized state?
2. Define the ionization energy of an atom & determine its value for the hydrogen atom.
3. Hydrogen atom has different series of radiations that result from the transitions between its energy levels.
   A part of the hydrogen's atom emission spectrum is schematized in figure 2, formed of its visible radiations and corresponds to the downwards transition from an energy level ranked $n > 2$ towards the energy level $n = 2$.
   a) Justify that the energy, in eV, of a radiation of wavelength λ is given by $E = \dfrac{1243}{\lambda}$ (λ in nm)

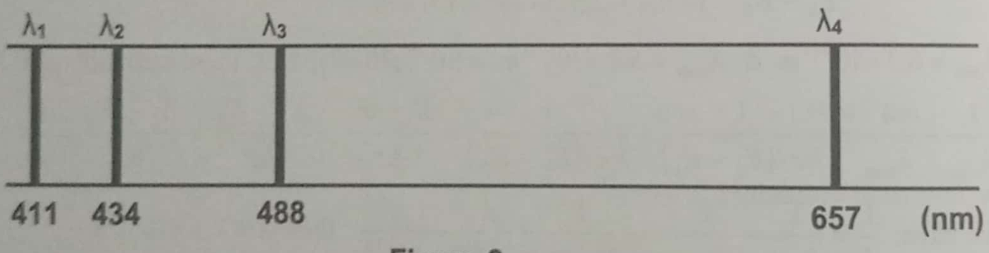


Figure 2

   b) Deduce:
      i- in eV, the energy of each of the 4 radiations emitted.
      ii- the corresponding transitions associated to the emission of each of these radiations.

In what follows, the hydrogen atom is taken in the ground state.

4. a) Determine, in eV, the energies of the photons absorbed through the transition from the ground state $n = 1$, towards the excited states $n = 3$ & $n = 4$.
   b) Deduce whether the hydrogen atom can absorb a photon of energy $12.3eV$.
5. An incident photon extracts an electron carrying a kinetic energy of $2.1eV$.
   Determine the wavelength of the radiation associated to this photon.

II-
Visible Spectrum and Hydrogen Atom[1]

Given:

- $1eV = 1.60\times10^{-19}\ J$;
- Planck's constant: $h = 6.63\times10^{-34}\ J.s$;
- Speed of light in vacuum: $c = 3\times10^{8}\ m/s$;
- The domain of visible radiations is: $[400\,nm ; 750\,nm]$.

The diagram represented in figure 1, is the spectrum of the hydrogen atom in the domain of visible radiations constituted of four lines denoted R_a, R_b, R_c & R_d of respective wavelengths in vacuum $\lambda_a = 657.12nm$, λ_b, λ_c & $\lambda_d = 410.70nm$.

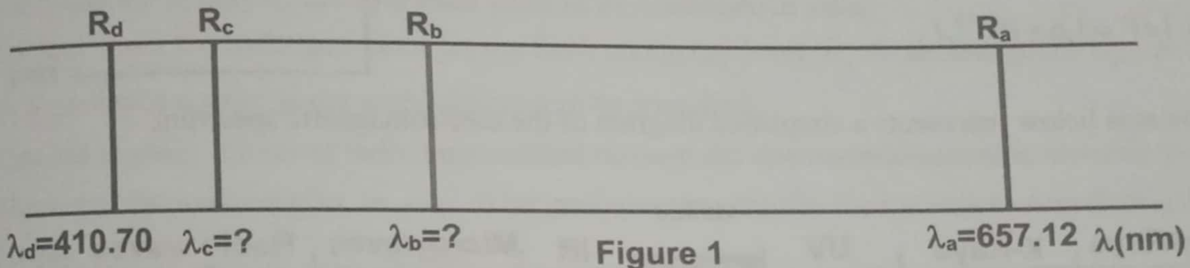


Figure 1

The energies of the different energy levels of the hydrogen atom are given by the relation:

$$E_n = -\frac{13.6}{n^2} \quad (\text{in } eV) \quad \text{where } n \text{ is a positive whole number.}$$

1. Calculate the value of the:
   a) minimum energy of the hydrogen atom.
   b) energy of the hydrogen atom in the second excited state.
2. Balmer's series is proper to the hydrogen atom and formed of the set of radiations emitted during the transitions from a level characterized by $n > 2$ to the energy level $n = 2$. The excited hydrogen atom delivers the excess of energy by emitting photons of wavelengths λ_n.
   a) Show that the wavelengths λ_n, in nm, satisfy the relation $\lambda_n = 366\dfrac{n^2}{n^2-4}$.
   b) Specify the values of n that correspond to the two previous radiations. Deduce the values of λ_b & λ_c.
3. We consider the transition of the hydrogen atom from $n = 2$ into $n = 1$.
   a) Determine the wavelength $\lambda_{2,1}$ of the radiation emitted during the transition.
   b) Specify whether this radiation is visible, infrared or ultraviolet.

In what follows, the hydrogen atom is taken in the ground state.

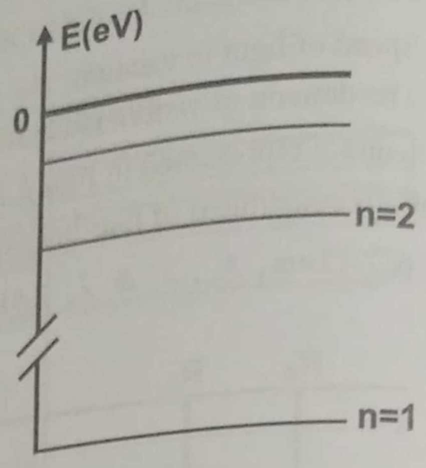
4. Specify whether a photon of energy $2.38\,eV$ is absorbed or not when received by the atom.
5. The hydrogen atom is hit by an electron of kinetic energy $12.5\,eV$.
   a) Justify that this electron can interact with the hydrogen atom.
   b) Deduce the possible transition that lead to the ejection of the electron carrying the minimum kinetic energy after interaction, and determine this energy.

[1] Johann Jacob Balmer (May 1, 1825 – March 12, 1898) was a Swiss mathematician

III-
The Hydrogen Spectrum

The quantified energy levels of the hydrogen atom are given by $E_n = -\frac{E_0}{n^2}$ where n is a non zero positive natural number and $E_0 = 13.6eV$.

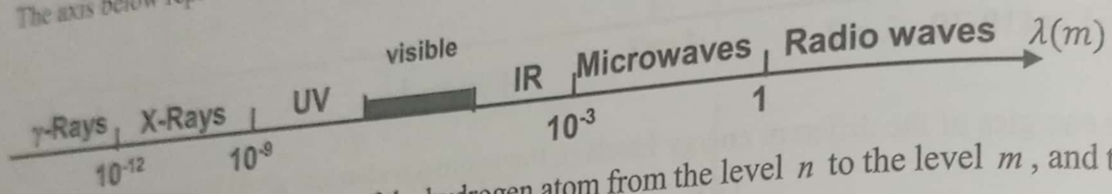
This atom can pass from an energy level n whose energy is E_n to another level m whose energy is E_m.



Given:

- Speed of light in vacuum: $c = 3\times10^8 m/s$;
- Planck's constant: $h = 6.62\times10^{-34} J.s$;
- $1eV = 1.6\times10^{-19} J$.

The axis below represents a simplified diagram of the electromagnetic spectrum.



1. Give the name of the transmission of the hydrogen atom from the level n to the level m, and then represent it on an energy diagram.
2. Describe briefly the spectrum obtained if $n > m$.
3. By convention, the energy of the hydrogen atom in the ionized state is considered zero. Use this convention to justify the sign $(-)$ of E_n.
4. We consider the transition of the hydrogen atom from the level n to the level m where $n > m$. Show that the expression of the wavelength λ that corresponds to the radiation emitted is given by the relation $\frac{1}{\lambda} = \frac{1}{\lambda_0}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where λ_0 is a constant whose value is to be determined in nm.
5. Deduce, the maximum wavelength λ_m of the radiation which is able to ionize the hydrogen atom taken in:
   a) ground state.
   b) 1st excited state.
6. Justify that the expression of the maximum wavelength of the radiation needed to ionize the atom taken in the energy level m is given by $\lambda_{max} = \lambda_0 \times m^2$.
7. We consider the four visible radiations of the hydrogen atom whose wavelengths are $\lambda_1 = 657\ nm$; $\lambda_2 = 486\ nm$; $\lambda_3 = 434\ nm$ & $\lambda_4 = 410\ nm$.
   a) To which range of the visible spectrum does each of these radiations belong?
   b) To which series does these radiations belong?
   c) We know that these radiations corresponds to the downward transition towards $m = 2$. Determine the level that corresponds to the radiation whose wavelength is $\lambda_3 = 434\ nm$, then justify those for the other radiations previously obtained.

IV- Balmer's Series

Given:

- $1eV = 1.60 \times 10^{-19} J$;
- Planck's constant: $h = 6.62 \times 10^{-34} J.s$;
- Speed of light in vacuum: $c = 3 \times 10^{8} m/s$.

The energy levels of the hydrogen atom are given by the relation $E_n = -\frac{13.6}{n^2}$, and n is whole non zero positive number and E_n in eV.

1. a) The energy of the energy levels of the hydrogen atom are quantified. Justify.
   b) Calculate the energy of the hydrogen atom in its fundamental state.
2. Show that when the hydrogen atom passes from an energy level E_q to another level E_p, p less than q, releases energy under a certain form to be specified.
3. We intend to study the set of radiations emitted through the downwards transition towards $p = 2$.
   a) Show that the wavelengths, in μm, of the radiations emitted by the hydrogen atom during these transitions are given by: $\lambda = \frac{0.365}{1 - \frac{4}{q^2}}$ where q is a whole number $q \geq 3$.
   b) Knowing that the wavelengths λ of the visible spectrum belongs to $[0.400\,\mu m; 0.750\,\mu m]$.
      i- Show that the emission spectrum of the hydrogen atom holds four visible rays for 4 different values of q whose values are to be determined.
      ii- Determine the wavelengths of the visible radiations in the Balmer's series.
4. The hydrogen atom being in the 1st excited state E_2 receives an electron carrying a kinetic energy of $2.9eV$.
   a) Specify the level that the atom cannot overpass.
   b) Indicate the transition that corresponds to the electron carrying the minimum kinetic energy after interaction and then calculate its value.
5. The population (number) of hydrogen atom N_n in the energy level E_n is given by $N_n = N_1 e^{\frac{(E_n - E_1)}{kT}}$ where T is the absolute temperature in Kelvin K, k is Boltzmann's constant $k = 1.38 \times 10^{-23} SI$ & N_1 is the population of atoms in the ground state.
   a) Specify the SI unit of the constant k.
   b) Complete the following table.

	T=3000K (Red or cold stars)	T=80000K (White stars)
$\frac{N_2}{N_1}$	....	...

   c) Specify the star whose spectrum is rich in the radiation that falls in the Balmer's series.

V-Bac
Distribution of Energy Levels

Given:

- $1eV = 1.60 \times 10^{-19} J$;
- Planck's constant: $h = 6.62 \times 10^{-34} J.s$;
- Speed of light in vacuum: $c = 3 \times 10^{8} m/s$;
- The domain of visible radiations is: $[400 nm; 750 nm]$.

The experiment performed by Franck and Hertz succeed to justify the quantification of energy levels, the schema of the experimental setup used is schematized in figure 1.

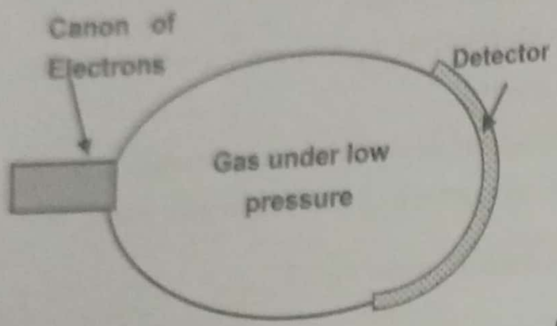


Figure 1

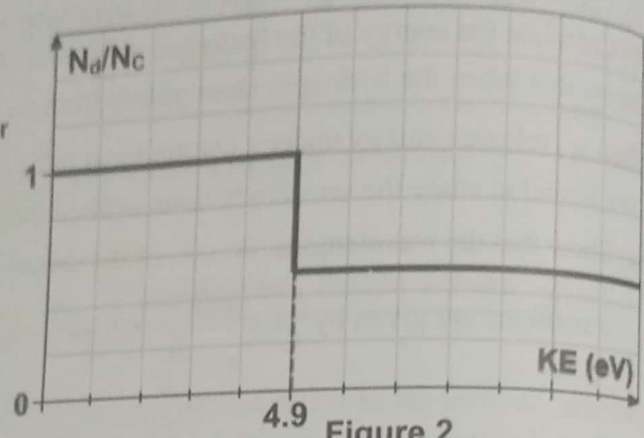


Figure 2

1. Indicate the role of the electrons emitted by the canon and that of the analyzer in this experiment.
2. If N_d represents the number of electrons received and counted by the detector per unit of time and N_C is the number of electrons released by canon.
   The results of the experimental study using a mercury gas allow us to trace the curve representing the variation of $\frac{N_d}{N_C}$ as a function of the electrons kinetic energy KE.
   Show that the curve represented in figure 2, justify that the energy levels of the mercury atom are quantified.
3. The diagram of figure 3, represents some energy levels of the mercury atom.
   a) Referring to the diagram, indicate the fundamental state of the mercury atom.
   b) The mercury atom, taken in the fundamental level, absorbs a photon whose energy is $5.45 eV$ and undergoes an upwards transition towards the 2nd excited state.
   Determine the value of E_3.

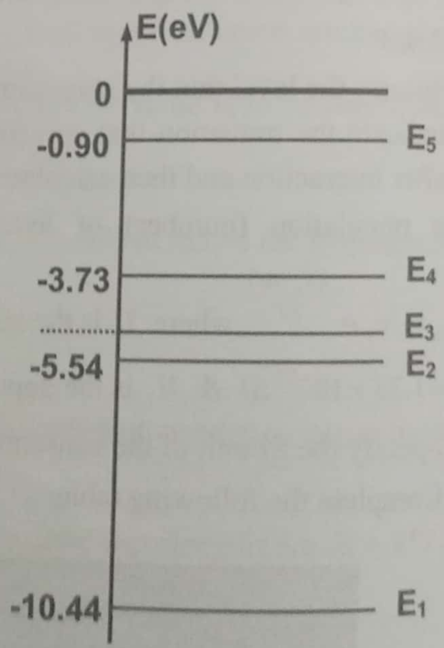


Figure 3

4. The mercury atom is in the 3rd excited level whose energy is E_4.

a) Calculate the wavelength $\lambda_{4,1}$ of the radiation when it undergoes a downward transition towards the fundamental state.

b) Indicate whether this radiation is visible or not.

5. The radiation of wavelength $\lambda = 438.6\,nm$ is emitted through a transition from an excited level E_p towards a lower level E_n.

Determine the levels p and n.

6. The mercury atom taken in the fundamental state receives successively two photons (a) and (b) of respective energies $10.00\,eV$ and $10.44\,eV$.

Indicate, with justification, which photon is capable of ionizing this atom.

VI-Engineering (2014/2015)
Mercury Atom

Given:

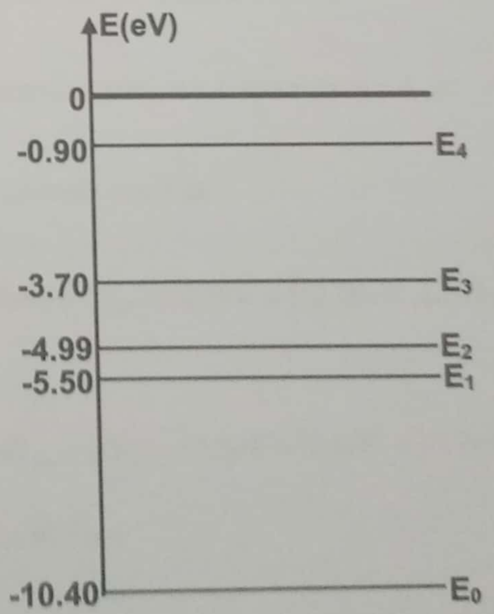
- mass of a mercury atom: $m_{Hg} = 3.34 \times 10^{-25}\,kg$;
- mass of an electron: $m_e = 9.1 \times 10^{-31}\,kg$;
- $c = 3.00 \times 10^{8}\,m/s$;
- $h = 6.63 \times 10^{-34}\,J.s$;
- $1eV = 1.60 \times 10^{-19}\,J$.

Part A
Emission and absorption of a photon

1. One of the visible radiations emitted by a mercury vapor lamp corresponds to the transition from the energy level E_4 to the energy level E_3.
Calculate the value of the corresponding wavelength $\lambda_{4/3}$.

2. Determine the value of the wavelength of the radiation that can be emitted by a mercury atom when it is taken initially in the energy level E_1.

3. A mercury atom is considered initially in the ground state E_0. This atom receives two photons of wavelengths $\lambda_1 = 253.7\,nm$ and $\lambda_2 = 589.0\,nm$.
Is there any interaction between the mercury atom and each of these two photons? Justify your answer.



Part B
Collision between an electron and a mercury atom

In 1914, Franck and Hertz (Nobel Prize 1925) made a surprising discovery by bombarding a mercury vapour, **the atoms being supposed at rest,** with electrons of adjustable kinetic energy KE of a few eV.

1. We consider the case where KE is less than a certain threshold, $E_S = 4.90eV$, and we suppose that the collision is perfectly elastic.

a) Show that the speed v_a of a mercury atom, after the collision, is given by $v_a = \frac{2m_e}{m_e + m_{Hg}}v$,

where v is the speed of the electron before the collision, the velocities being collinear.

b) Deduce that the electron, after the collision, keeps practically the same kinetic energy KE.

2. a) When KE reaches the value $KE = E_S = 4.90\,eV$; the electron, after the collision, loses practically all of its kinetic energy. Interpret this result.

b) For $E_S = 4.90\,eV < KE < 5.40\,eV$, the kinetic energy of some electrons, after the collision, diminishes precisely by $4.90\,eV$ of its initial value, while the other electrons keep their kinetic energy KE. Interpret this result.

c) What could happen to mercury atoms that undergo collision with electrons having the kinetic energy $KE = 6.00\,eV$?

Part C
Photoelectric effect

When a potassium photocathode receives successively two radiations emitted by the mercury vapor lamp, one of wavelength $\lambda_1 = 253.7\,nm$ and the other $\lambda_2 = 444.0\,nm$, we notice that the maximum kinetic energy of the ejected electrons are respectively $2.70\,eV$ and $0.60\,eV$.

1. a) Using Einstein's relation and these data, determine the value of the Planck's constant h.

b) Deduce the work function W_0 of the potassium photocathode.

2. Justify that the radiation due to the electronic transition $E_3 \longrightarrow E_2$ contribute to the emission of a photoelectron.

Solutions – Problems

I-

1. The energy of the atom in fundamental state is $E_1 = -13.6eV$ & in the ionized state $E_\infty = 0$.

2. The ionization energy W_{ion} is the minimum energy that should communicated to the atom in order to extract an electron from the influence of its nucleus.
$W_{ion} = E_\infty - E_1 = 0 - (-13.6eV) = 13.6eV$.

3. a) The energy of a photon is: $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{\lambda} = \frac{1.989\times10^{-25}}{\lambda}$ (λ in m & E in J)

Then $E = \frac{1.989\times10^{-25}}{1.6\times10^{-19}(\lambda\times10^{-9})} = \frac{1243}{\lambda}$ where λ in nm & & E in eV

b) Energies and transitions.

i- $E_{\lambda_1} = \frac{1243}{411} = 3.02eV$, $E_{\lambda_2} = \frac{1243}{434} = 2.86eV$, $E_{\lambda_3} = \frac{1243}{488} = 2.55eV$ & $E_{\lambda_4} = 1.89eV$.

ii- The final state is defined by: $E_{\lambda_1} + E_2 = 3.03 - 3.40 = -0.37eV = E_6$; then this radiation corresponds to the transition from $n=6$ towards $n=2$.

For the 2nd radiation, $E_{\lambda_2} + E_2 = 2.86 - 3.40 = -0.54\,eV = E_5$; corresponds to the transition from $n=5$ towards $n=2$.

Similarly $E_{\lambda_3} + E_2 = 2.55 - 3.40 = -0.85\,eV = E_4$, corresponds to the transition from $n=4$ towards $n=2$.

$E_{\lambda_4} + E_2 = 1.89 - 3.40 = -1.51\,eV = E_3$, corresponds to the transition from $n=3$ towards $n=2$.

4. a) The energy of the photon absorbed to ensure the transition from $n=1$ towards $n=4$ is:
$E_{ph(4.1)} = E_4 - E_1 = -0.85 - (-13.6) = 12.75\,eV$.
The energy absorbed to ensure the transition $n=1$ towards $n=3$ is:
$E_{ph(3.1)} = E_3 - E_1 = -1.51 - (-13.6) = 12.09\,eV$.

b) Since the energy levels are quantified, then the photon $E_{ph} = 12.3eV \in]12.09eV;12.75\,eV[$ cannot be absorbed.

5. The ionization energy $W_{ion} = 13.6eV$.
The energy of the incident photon is given by: $E_{ph} = W_{ion} + KE_e = 13.6eV + 2.1eV = 15.7eV$;

But $E_{ph} = \frac{hc}{\lambda}$, then $\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{15.7\times1.6\times10^{-19}} = 7.92\times10^{-8}\,m$.

II-

1. a) The minimum energy of the atom is in the fundamental state $E_1 = -13.6eV$.

b) In the 2nd excited state $n=3$, we get $E_3 = -\frac{13.6}{9} = -1.51\,eV$.

2. a) The energy of the photon emitted during a transition is given by:

$$E_{ph} = E_n - E_2 = -\frac{13.6}{n^2} - \left(\frac{-13.6}{2^2}\right) = 13.6\left(\frac{-1}{n^2} + \frac{1}{4}\right) = 13.6\left(\frac{n^2-4}{4n^2}\right) = \frac{13.6}{4}\left(\frac{n^2-4}{n^2}\right) \text{ in } eV$$

In J, $E_{ph} = \frac{13.6}{4}\left(\frac{n^2-4}{n^2}\right) \times 1.6 \times 10^{-19} = 5.44 \times 10^{-19}\left(\frac{n^2-4}{n^2}\right)$;

However $E_{ph} = h\frac{c}{\lambda_n} = 5.44 \times 10^{-19}\left(\frac{n^2-4}{n^2}\right)$;

Then $\lambda_n = \frac{6.63 \times 10^{-34} J.s \times 3 \times 10^8 m/s}{5.44 \times 10^{-19} J}\left(\frac{n^2}{n^2-4}\right) = 3.66 \times 10^{-7}\left(\frac{n^2}{n^2-4}\right)$ in m;

Thus, $\lambda_n = 3.66 \times 10^{-7}\left(\frac{n^2}{n^2-4}\right) \times 10^9 = 366\frac{n^2}{n^2-4}$ (λ_n in nm)

b) For the 1st radiation: $\lambda_a = 657.12 = 366\left(\frac{n^2}{n^2-4}\right) \Rightarrow \frac{n^2}{n^2-4} \approx 1.8$;

$n^2 = 1.8n^2 - 7.2 \Rightarrow 0.8n^2 = 7.2 \Rightarrow n^2 = 9$, then $n = 3$; thus, the radiation λ_a corresponds to the transition from $n = 3$ into $n = 2$.

- For the 4th radiation λ_d, we get : $\frac{n^2}{n^2-4} \approx 1.125 \Rightarrow n^2 = \frac{4 \times 1.125}{0.125} = 36$;

  So $n = 6$; thus, the radiation λ_d corresponds to the transition from $n = 6$ into $n = 2$.

- λ_b corresponds to the transition from $n = 4$ into $n = 2$; thus, $\lambda_b = 366 \times \frac{4^2}{4^2-4} = 488 nm$.

- λ_c corresponds to the transition from $n = 5$ into $n = 2$; thus, $\lambda_c = 366 \times \frac{5^2}{5^2-4} = 435.71 nm$

3. a) The energy of the emitted photon is: $E_{ph} = E_2 - E_1 = -\frac{13.6}{2^2} - \left(-\frac{13.6}{1^2}\right) = -3.4 + 13.6 = 10.2 eV$

$$\lambda_{2,1} = \frac{hc}{E_{ph}} = \frac{6.63 \times 10^{-34} J.s \times 3 \times 10^8 m/s}{10.2 \times 1.6 \times 10^{-19} J} = 1.22 \times 10^{-7} m = 122 nm.$$

b) $\lambda_{2,1} = 122 nm < 400 nm$, this radiation is ultraviolet.

4. If the photon is absorbed then $E_n = E_{ph} + E_1 = 2.38 + (-13.6) = -11.22 eV$;

$E_n = -11.22 = -\frac{13.6}{n^2}$; $n^2 = 1.2$, we get $n = 1.1$ which is not a whole number;

Then this photon is not absorbed and the atom remains in the ground (fundamental) state.

5. a) $KE_e + E_1 = 12.5 - 13.6 = -1.1 eV = -\frac{13.6}{n^2} \Rightarrow n = \sqrt{\frac{13.6}{1.1}} = 3.5$; [2]

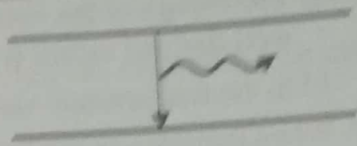
Then the atom will not overpass the third energy level.

b) The minimum kinetic energy carried by the electron after interaction corresponds to the farthest possible transition (from $n = 1$ into $n = 3$), then:

$KE_{min} = KE - \Delta E = KE - (E_3 - E_1) = 12.5 - (-1.51 + 13.6) = 0.41 eV$.

[2] The minimum energy required for interaction $\Delta E_{min} = E_2 - E_1 = 10.2 eV$; $KE_{e^-} > \Delta E_{min}$; so we have interaction.

1. The transmission from an energy level to another is called transition.



2. If $n > m$, we observe a set of discrete colored lines on a black background forming the emission spectrum of the atom under study.
3. Below the ionization state (∞), the electron remains linked to the nucleus so its energy is considered negative whereas when it is in the ionization state it becomes free (without kinetic energy) so its energy becomes zero.[3]
4. Through a transition from the level n into m a photon whose energy is released, such that:

$$E_{ph} = E_n - E_m = \left(-\frac{E_0}{n^2}\right) - \left(-\frac{E_0}{m^2}\right) = E_0\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \quad (\text{in } eV);$$

The wavelength of the emitted radiation λ is given by: $\frac{hc}{\lambda} = E_0\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$;

$$\frac{1}{\lambda} = \frac{E_0}{hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) = \frac{1}{\lambda_0}\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \text{ where } \frac{1}{\lambda_0} = \frac{E_0}{hc}; \text{ then } \lambda_0 = \frac{hc}{E_0}.$$

Thus, $\lambda_0 = \frac{6.62\times10^{-34}\,J.s \times 3\times10^{8}\,m/s}{13.6\times1.6\times10^{-19}\,J} = 9.13\times10^{-8}\,m = 91.3\,nm$.

5. a) The maximum wavelength corresponds to the transition from the ionized state ($n=\infty$ towards $m=1$), then $\frac{1}{\lambda_{1m}} = \frac{1}{\lambda_0}\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$. Thus $\lambda_{1m} = \lambda_0 = 91.3\,nm$.

b) If the atom is in the first excited state ($n=\infty$ towards $m=2$), then $\frac{1}{\lambda_{2m}} = \frac{1}{\lambda_0}\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$;

Thus, $\lambda_{2m} = 4\lambda_0 = 365.2\,nm$.

6. If the atom is taken in the energy level m & $n=\infty$, then $\frac{1}{\lambda_{max}} = \frac{1}{\lambda_0}\left(\frac{1}{m^2} - \frac{1}{\infty^2}\right)$;

Thus, $\lambda_{max} = m^2\lambda_0$.

7. $\lambda_1 = 657\,nm$; $\lambda_2 = 486\,nm$; $\lambda_3 = 434\,nm$ & $\lambda_4 = 410\,nm$.

a) These radiations belong to the interval $[400\,nm; 800\,nm]$, so they are visible.

$\lambda_1 = 657\,nm$ is red; $\lambda_2 = 486\,nm$ & $\lambda_3 = 434\,nm$ are blue; while $\lambda_4 = 410\,nm$ is violet.

b) These radiations belong to Balmer's series, that holds among others all the visible radiations emitted by the hydrogen atom.

c) We have $\frac{1}{\lambda} = \frac{1}{\lambda_0}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$, then $\frac{1}{n^2} = \frac{1}{4} - \frac{\lambda_0}{\lambda}$

For the 1st radiation $\lambda_1 = 657\,nm$; $\frac{1}{n^2} = \frac{1}{4} - \frac{91.3}{657}$; we get $n=3$;

Similarly for the 2nd radiation $\lambda_2 = 486\,nm$; we get $n=4$;

For the 3rd radiation $\lambda_3 = 434\,nm$; we get $n=5$;

For the 4th radiation $\lambda_4 = 410\,nm$; we get $n=6$.

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[3] By analogy with the gravitational potential energy:
- the ionized state of atom is analog for being on reference, its energy is zero.
- below ionized state (reference), the atom (system) needs energy to reach it, so it is negative.

IV-

1. a) The energy of the energy levels depends on n which is a whole number, and then only a set of well-defined values is only allowed. Thus, they are quantified.

b) In the fundamental state $n=1$, the energy $E_1=-13.6/1^2=-13.6\,eV$.

2. When the atom passes from a higher energy level to a lower one, its energy decreases. This energy appears as radiant energy carried by the emitted photon.

3. a) The energy of the photon emitted: $E_{ph}=E_h-E_\ell$, $h\dfrac{c}{\lambda}=\left(-\dfrac{13.6}{q^2}-\left(-\dfrac{13.6}{2^2}\right)\right)=\dfrac{13.6}{4}\left(1-\dfrac{4}{q^2}\right)$;

$$\lambda(m)=\frac{hc}{3.4\left(1-\frac{4}{q^2}\right)}=\frac{6.62\times10^{-34}\times3\times10^{8}}{3.4\times1.6\times10^{-19}\left(1-\frac{4}{q^2}\right)}=\frac{3.65\times10^{-7}}{1-\frac{4}{q^2}};$$

$$\lambda(\mu m)=\frac{3.65\times10^{-7}}{1-\frac{4}{q^2}}\times10^{6}=\frac{0.365}{1-\frac{4}{q^2}}\text{ with } q\geq3.$$

b) Visible radiations:

i- We have: $\lambda_{v}\leq\lambda\leq\lambda_R\Rightarrow0.4\leq\dfrac{0.365}{1-4/q^2}\leq0.75$;

By calculation we get $\dfrac{1}{4}\left(1-\dfrac{0.365}{0.4}\right)\leq\dfrac{1}{q^2}\leq\dfrac{1}{4}\left(1-\dfrac{0.365}{0.75}\right)\Rightarrow2.79\leq q\leq6.76$.

But $q\in N^*$ then $q\in\{3;4;5;6\}$; then 4 transitions are possible.

Thus, the emission spectrum of the hydrogen atom is formed of 4 lines in the visible domain.

ii- The respective wavelengths are $0.657\ \mu m$, $0.487\ \mu m$, $0.435\ \mu m$ & $0.411\ \mu m$.

4. a) The energy of the atom in the 1st excited state $E_2=-\dfrac{13.6}{2^2}=-3.4\,eV$.

$KE_e+E_2=2.9+(-3.4)=-0.5\,eV\geq-\dfrac{13.6}{n^2}$; then $n\leq\sqrt{\dfrac{13.6}{0.5}}\approx5.2$;

Then the atom will not overpass the energy level E_5.

b) The electron carries minimum kinetic energy if the atom absorbs the maximum possible energy; it corresponds to the transition between E_2 towards E_5;

The energy absorbed by the atom $\Delta E=E_5-E_2=-\dfrac{13.6}{5^2}-\left(-\dfrac{13.6}{2^2}\right)=2.86\,eV$.

The electron carries after interaction $KE_{after}=KE-\Delta E=2.9-2.86=0.04\,eV$.

5. a) The ratio $\dfrac{E_n-E_1}{kT}$ should be unit less, furthermore the energies are measured in J & T in K then k is measured in J/K.

b) For $T=3000\,K$, $\dfrac{N_2}{N_1}(\text{red stars})=e^{-\frac{(E_2-E_1)}{kT}}=e^{-\frac{(-3.4+13.6)\times1.6\times10^{-19}}{1.38\times10^{-23}\times3000}}=7.6\times10^{-18}$;

For $T=8000\,K$, $\dfrac{N_2}{N_1}(\text{white stars})=e^{-\frac{(E_2-E_1)}{kT}}=e^{-\frac{(-3.4+13.6)\times1.6\times10^{-19}}{1.38\times10^{-23}\times8000}}=1.2\times10^{-5}$;

c) $\frac{N_2}{N_1}$(white stars) >>>> $\frac{N_2}{N_1}$(red stars).

In white stars, there is a big population in the first excited state, so they are expected to be rich in radiations that fall in Balmer's series.

V-

1. The electrons emitted are by the canon excite the mercury atoms through collision.
The detector used to count the electrons that arrive with a specific kinetic energy after crossing the gas (with or without interaction).
2. The analysis of the curve shows that:
- as long the kinetic energy of the electrons emitted is $KE < 4.9\,eV$; we have $\frac{N_C}{N_d} = 1$, meaning that all the atoms are not excited. Thus, no atoms succeeded to absorb any energy.
- when $KE \geq 4.9\,eV$; we have $\frac{N_C}{N_d} < 1$ which indicates that the excited atoms, by collision with the electrons emitted by the canon, did absorb only the electrons whose energy is $4.9\,eV$. Then the energy of the mercury atom is quantified.

3. a) The fundamental state is the most stable state acquired by the atom, characterized also by the presence of the atom in the lower energy level. Then its energy is $E_1 = -10.44\,eV$.
b) We know that the energy of the photon absorbed must satisfy the relation: $E_{ph} = E_h - E_\ell$.
Then, $E_3 = E_{ph} + E_2 = 5.45 - 10.44 = -4.99\,eV$.

4. a) We have $E_{ph} = E_4 - E_1 \Rightarrow \lambda_{4,1} = \frac{hc}{E_4 - E_1} = 184.9 \times 10^{-9}\,m \approx 185\,nm$.
b) $\lambda_{4,1} \approx 185\,nm < [400\,nm; 800\,nm]$, then this radiation is not visible

5. The energy of this photon is $E_{ph} = \frac{hc}{\lambda} = 4.528 \times 10^{-19}\,J = 2.83\,eV$.
$E_{ph} = 2.83\,eV = E_5 - E_4$, then it corresponds to a transition from the level E_5 towards E_4.

6. The ionization energy is: $W_{ion} = E_\infty - E_1 = 10.44eV$.
$E_{ph}(a) = 10.00eV \prec W_{ion}$; so the photon (a) is not capable of ionizing this atom.
$E_{ph}(b) = 10.44eV = W_{ion}$; so the photon (b) is capable of ionizing this atom.

VI-

Part A

1. The energy of the emitted photon through the transition from E_4 to E_3 is $E_{ph} = E_4 - E_3$;
$E_{ph} = -0.90eV - (-3.70eV) = 2.80eV$;
The energy of the photon is: $E_{ph} = \frac{hc}{\lambda_{4,3}}$, then $\lambda_{4,3} = \frac{hc}{E_{ph}} = \frac{6.63 \times 10^{-34}\,J.s \times 3 \times 10^8\,m/s}{2.80 \times 1.6 \times 10^{-19}\,J}$;
Then $\lambda_{4,3} = 4.44 \times 10^{-7}\,m = 444nm$ (violet-indigo).
2. The only possible transition is from the energy level E_1 to E_0:
$E_{ph} = E_1 - E_0$; then $E_{ph} = -5.50eV - (-10.40eV) = 4.90eV$;

However, $E_{ph} = \frac{hc}{\lambda_{ph}}$, then $\lambda_{ph} = \frac{hc}{E_{ph}} = \frac{6.63\ldots}{4.9\times1.6\times10^{-19} J}$

Then $\lambda_{ph} = 2.537\times10^{-7} m = 253.7nm < 400nm$, which is an ultraviolet radiation.

3. For the photon of wavelength $\lambda_1 = 253.7nm$.

We know that the atom absorbs the photon which can emit, so photon is absorbed and the atom undergoes an upwards from the energy level E_0 to E_1.

$\lambda_1 = 253.7nm$ is the maximum wavelength which is able to excite this atom (it corresponds to the transition from E_0 to the nearest energy level E_1).

But $\lambda_2 > \lambda_1$, thus this photon is not absorbed and the atom remains in the ground state.

Part B

1. a) Conservation of linear momentum $\vec{P}_{\text{just before collision}} = \vec{P}_{\text{just after collision}}$; $m_e\vec{v} + \vec{0} = m_e\vec{v'} + m_{Hg}\vec{v_a}$

The velocities are collinear, then $m_e(v - v') = m_{Hg}v_a$ ....(1);

The collision is elastic, then the kinetic energy is conserved: $KE_{\text{just before collision}} = KE_{\text{just after collision}}$;

$\frac{1}{2}m_e v^2 = \frac{1}{2}m_e v'^2 + \frac{1}{2}m_{Hg}v_a^2$; we get $m_e(v - v')(v + v') = m_{Hg}v_a^2$ ....(2)

$\left(\frac{(2)}{(1)}\right)$: $\frac{m_e(v - v')(v + v')}{m_e(v - v')} = \frac{m_{Hg}v_a^2}{m_{Hg}v_a}$;

$\begin{cases} v + v' = v_a \\ v - v' = \frac{m_{Hg}}{m_e}v_a \end{cases}$, then $2v = \left(1 + \frac{m_{Hg}}{m_e}\right)v_a$; $v_a = \frac{2m_e}{m_e + m_{Hg}}v$.

b) The velocity of the electron after collision is $v' = v_a - v = \frac{(m_e - m_{Hg})}{(m_e + m_{Hg})}v$;

$$v' = \frac{(9.1\times10^{-31} - 3.34\times10^{-25})}{(9.1\times10^{-31} + 3.34\times10^{-25})}v = -0.99999455\,v \approx -v.$$

Therefore, the speed is almost unchanged (just a change in direction); thus, the kinetic energy of the electron is practically unchanged.

2. a) If $KE = E_S = 4.9eV$;

Referring to the diagram of energy levels $E_1 - E_0 = -5.50eV - (-10.40eV) = 4.9eV$.

Therefore, the kinetic energy of the incident electron $KE = E_S = E_1 - E_0 = 4.9eV$ represents the minimum energy required to excite this atom.

Thus, the atom absorbs practically all the kinetic energy of the incident electrons and undergoes an upwards transition from E_0 to E_1.

b) $E_S = 4.9eV < KE < 5.40eV$;

We have $E_1 - E_0 = 4.9eV$ & $E_2 - E_0 = -5eV - (-10.4eV) = 5.4eV$;

So the atom will not overpass E_1.

The mercury atom taken in the ground state will absorb enough energy to undergo the transition from E_0 to E_1, and after interaction the electron carries a kinetic energy of $KE' = KE - E_S = KE - 4.90$.

While the electrons which do not interact with the atom maintains their kinetic energy.

c) $KE = 6eV$, the final state of atom is $KE + E_0 = 6eV + (-10.40eV) = -4.4eV < E_3$;

Then, two transitions are possible from E_0 to E_1 or E_0 to E_2;

For the transition E_0 to E_1, the electron kinetic energy $KE' = KE - 4.90 = 6 - 4.90 = 1.1eV$;

Or it may undergoes an upward transition from E_0 to E_2, and the electron carries a kinetic energy of $KE'' = KE - 5.40 = 6 - 5.40 = 0.60eV$.

While the electrons which do not interact with the atom maintains their kinetic energy.

Part C

1. a) Referring to Einstein's relation $KE_{max} = E_{ph} - W_0 = h\frac{c}{\lambda} - W_0$;

Planck's constant is the slope of the straight line representing the evolution of the kinetic energy in terms of $\frac{c}{\lambda}$, so $h = \frac{\Delta(KE_{max})}{\Delta\left(\frac{c}{\lambda}\right)} = \frac{KE_{max2} - KE_{max1}}{\frac{c}{\lambda_2} - \frac{c}{\lambda_1}}$;

Then, $h = \dfrac{(2.7 - 0.6) \times 1.6 \times 10^{-19} J}{\dfrac{3 \times 10^8 m/s}{253.7 \times 10^{-9} m} - \dfrac{3 \times 10^8 m/s}{444 \times 10^{-9} m}} = 6.63 \times 10^{-34} J.s$.

b) We have $W_0 = E_{ph1} - KE_{max1} = 4.90eV - 2.70eV = 2.2eV$.

2. The energy of the photon emitted through the transition from E_3 to E_2 is:

$E'_{ph} = E_3 - E_2 = -3.70 - (-5.00) = 1.30eV < 2.2eV = W_0$.

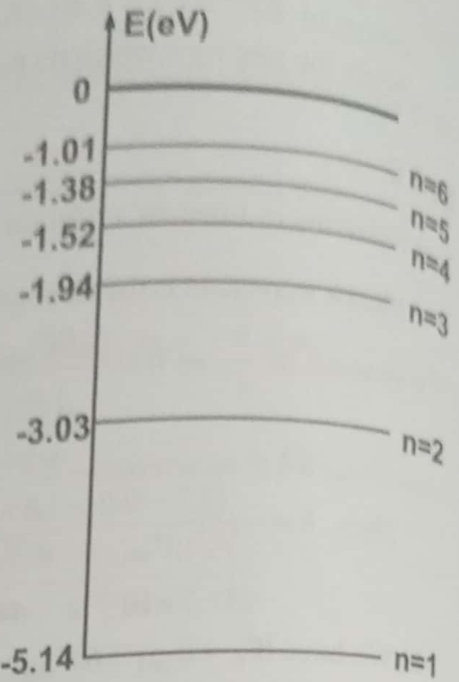
Thus, the photoelectron emission is not possible.

Supplementary Problems

Sodium Spectrum

The spectrum of the sodium atom reveals the presence of a yellow-orange ray. This ray corresponds to the transition from a level $n>1$ to the fundamental $n=1$.

The adjacent diagram is a simplified representation of the energy levels of the sodium atom.



Given:

- Speed of light in vacuum: $c=3\times10^{8}\,m/s$;
- Planck's constant: $h=6.63\times10^{-34}\,J.s$;
- $1eV=1.6\times10^{-19}\,J$.

1. Explain briefly, what is meant by the term «quantized energy» that is used to describe the distribution of levels shown in the adjacent diagram.
2. a) Calculate, in eV, the energy released during the transitions from the levels $n=2$ & $n=3$ to the fundamental state $n=1$.
   b) Calculate, to each of the preceding transitions, its corresponding wavelength & indicate the domain to which it belongs.
   c) Indicate the only possible transition that corresponds to the emission of the sodium yellow-orange radiation.
   d) Indicate, with justification, whether the spectrum obtained during these transitions corresponds to the emission or absorption spectrum.
3. The sodium atom taken in the ground sate.
   a) Define the ionization energy of an atom and calculate its value for the sodium.
   b) Indicate, with justification, whether a photon of energy of $4eV$ could be absorbed by the sodium atom.
4. The sodium atom taken in the excited state $n=3$, receives a photon carrying $4.2eV$.
   a) Show that the sodium atom could be ionized.
   b) Deduce, in joules, the maximum kinetic energy of the extracted electrons.
5. In fact, the sodium yellow-orange ray is constituted from a doublet due to the transitions from the energy levels E_2 & E_2' to the fundamental $(n=1)$, whose exact energy is taken as equal to $E_1=-5.139\,eV$.
   a) Calculate the energies E_2 & E_2' knowing that the frequencies of the radiations emitted during these transitions are $\upsilon_2=5.087\times10^{14}\,Hz$ & $\upsilon_2'=5.092\times10^{14}\,Hz$.
   b) Draw a simplified diagram showing E_2, E_2' & E_1 and their corresponding transitions.

Answer Key

2.b) $\lambda_{3,1}=388nm$. 4.b) $KE_{max}=2.26eV$ 5.a) $E_2'=-3.032\,eV$.

II- Hydrogen Atom & Wien's Law

The spectral analysis of the hydrogen atom is shown in figure 1, while figure 2 represents a set of its energy levels.

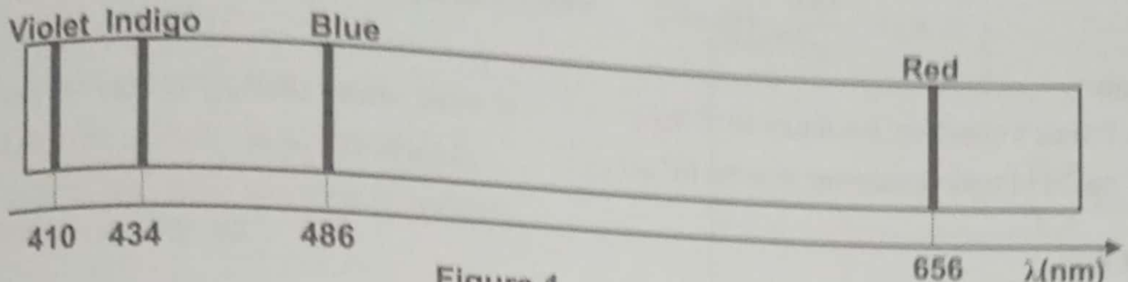


Figure 1

Given:

- Planck's constant: $h = 6.63 \times 10^{-34}\ J.s$;
- Speed of light in vacuum: $c = 3 \times 10^{8}\ m/s$;
- $1eV = 1.6 \times 10^{-19}\ J$.

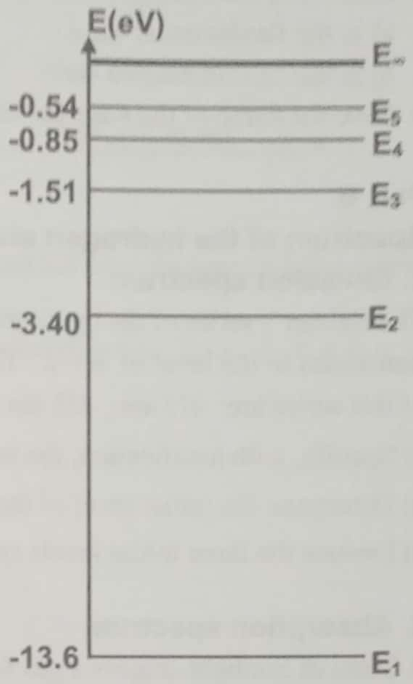


Figure 2

1. Indicate, with justification, whether the spectrum of figure 1 is:
   a) monochromatic or polychromatic.
   b) continuous or discontinuous.
   c) the emission or the absorption spectrum.
2. Explain the meaning of «quantified» by which we describe the energy levels of the hydrogen atom.
3. a) Indicate whether the hydrogen atom loses or gains energy when it passes from the level E_5 to the level E_2.
   b) Determine the wavelength of the radiation emitted during this transition and indicate its color.
4. Determine the transition that leads the hydrogen atom to the energy level E_2 emitting a blue radiation.
5. What is meant by the ionization energy? Calculate its value for the hydrogen atom.
6. Wien's law allows us to determine the surface temperature θ in ^0C of an object or star by identifying the radiation emitted with maximum intensity whose wavelength λ_{max} in nm. This law is given by $\theta = \dfrac{2.89 \times 10^{6}}{\lambda_{max}} - 273$.
   a) Determine the surface temperature of our Sun knowing the wavelength of maximum intensity emitted by our Sun is $4.8 \times 10^{-7}\ m$.
   b) The external temperature of Earth is $18^0 C$.
   Determine λ_{max} and then specify whether it is emitted by the hydrogen gas or not.
   c) Determine the surface temperature of a star if the wavelength of maximum intensity emitted by its surface is $6.56 \times 10^{-7}\ m$.

Answer Key

3.b) indigo. 4. blue ($n = 4$ into $n = 2$).

III-LS & GS 2004 1st

Energy Levels of the Hydrogen Atom

The energies of the different energy levels of the hydrogen atom are given by the relation:

$$E_n = -\frac{13.6}{n^2} \quad (\text{in } eV) \qquad \text{where } n \text{ is a positive whole number.}$$

Given:

- Planck's constant: $h = 6.63 \times 10^{-34} J.s$; $1eV = 1.6 \times 10^{-19} J$;
- Speed of light in vacuum: $c = 3 \times 10^8 m/s$; $1nm = 10^{-9} m$.

Part A

Energy of the hydrogen atom

1. The energies of the atom are quantized. Justify this using the expression of E_n.
2. Determine the energy of the hydrogen atom when it is:
   a) in the fundamental state.
   b) in the second excited state.
3. Give the name of the state for which the energy of the atom is zero.

Part B

Spectrum of the hydrogen atom

1. Emission spectrum

The Balmer's series of the hydrogen atom is the set of the radiations corresponding to the downward transitions to the level of $n = 2$. The values of the wavelengths in vacuum of the visible radiations of this series are: $411\,nm$; $435\,nm$; $487\,nm$; $658\,nm$.

a) Specify, with justification, the wavelength λ_1 of the visible radiation carrying the greatest energy.

b) Determine the initial level of the transition giving the radiation of wavelength λ_1.

c) Deduce the three initial levels corresponding to the emission of the other visible radiations.

2. Absorption spectrum

A beam of Sunlight crosses a gas formed mainly of hydrogen. The study of the absorption spectrum reveals the presence of dark spectral lines.

Give, with justification, the number of these lines and their corresponding wavelengths.

Part C

Interaction photon - hydrogen atom

1. We send on the hydrogen atom, being in the fundamental state, separately, two photons of respective energies $3.4eV$ and $10.2eV$.
   Specify, with justification, the photon that is absorbed.
2. A hydrogen atom found in its fundamental state absorbs a photon of energy 14.6 eV. The electron is thus ejected.
   a) Justify the ejection of the electron.
   b) Calculate, in eV, the kinetic energy of the ejected electron.

Answer Key

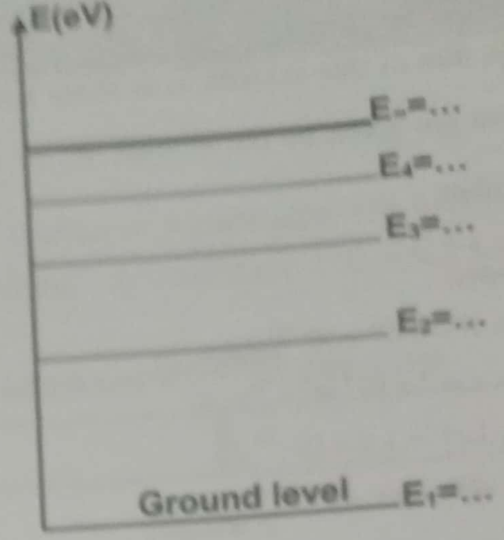
Part B 1.b) $n = 6$ towards $n = 2$.

Part C 1. Photon 3.4eV is not absorbed 2.b) 1eV

[illegible] 2003 1st
Energy Levels of the Hydrogen Atom

Given:
- Planck's constant: $h = 6.63 \times 10^{-34} J.s$;
- $1 eV = 1.6 \times 10^{-19} J$;
- Speed of light in vacuum: $c = 3 \times 10^{8} m/s$;
- Mass of an electron: $m = 9.1 \times 10^{-31} kg$;
- Limits of the visible spectrum in vacuum:
  $400 nm \le \lambda \le 800 nm$.

The quantized energy levels of the hydrogen atom are given by the formula: $E_n = -\frac{E_0}{n^2}$ (in eV) where $E_0 = 13.6 eV$ and n is a positive whole number.



Part A
Line spectrum

1. Explain briefly, what is meant by the term «quantized energy» and tell why the spectra (absorption or emission) of hydrogen are formed of lines.
2. Calculate the values of the energies corresponding to the energy level $n = 1, 2, 3, 4$ & $n = \infty$.
3. Redraw and complete the diagram.

Part B
Excitation of the hydrogen atom

The hydrogen atom is in its fundamental state.

1. Calculate the minimum energy of a photon that is able to:
   a) excite this atom.
   b) ionize this atom.
2. The hydrogen atom receives, separately, three photons of respective energies
   a) $11 eV$. b) $12.75 eV$. c) $16 eV$.
   Specify in each case the state of the atom. Justify.
3. The hydrogen atom being always in the fundamental state, receives now a photon of energy E. An electron of speed $v = 7 \times 10^{5} m/s$ is thus emitted. Calculate E.

Part C
Dis-excitation of the hydrogen atom

The hydrogen atom is now found now in the energy level $n = 3$.

1. Specify all possible transitions of the atom when it is dis-excited.
2. One of the emitted radiations is visible. Calculate its wavelength in vacuum.

Answer Key

Part B 1.a) $10.2 eV$ 2.b) an upwards transition towards $n = 4$
Part C 1. 3 possible transitions. 2. $\lambda_{3,2} = 658 nm$

I-LS 2014 1st
Hydrogen Atom

The aim of this exercise is to study Lyman series of the hydrogen atom. The energy levels of this atom are given by the relation $E_n = -\frac{E_0}{n^2}$, with $E_0 = 13.6eV$ and n is whole non zero positive number.

Given:

- $h = 6.62\times10^{-34}\,J.s$;
- $c = 3\times10^{8}\,m/s$;
- $1eV = 1.6\times10^{-19}\,J$;
- $400\,nm \le \lambda_{\text{visible}} \le 800\,nm$.

Part A

Energy levels of the hydrogen atom

1. a) Calculate the energy of the hydrogen atom when it is:
   i- in the fundamental state;
   ii- in the first excited state;
   iii- in the ionized state.
   b) The energy levels of the hydrogen atom are quantized. Justify.
2. This atom, taken in a given level E_p receives a photon of energy E and of wavelength λ in vacuum. Thus, the hydrogen atom passes to an energy level E_m such that $m > p$.
   a) Write the relation among E, E_p and E_m.
   b) Deduce the relation among E_0, p, m, h, c and λ.

Part B

The absorption «Lyman α» ray

Certain galaxies that are very far have in their center a very luminous nucleus called «quasar». The quasar spectrum contains emission and absorption spectrum rays. In the absorption series of Lyman, the atom passes from the fundamental state to an excited state of energy E_n by absorbing a photon of wavelength λ.

1. Determine the relation among h, c, λ, E_0 and n.
2. The wavelength of an absorption ray of the Lyman's series is given by the relation: $\frac{1}{\lambda} = R_H\left(1-\frac{1}{n^2}\right)$; R_H is the Rydberg constant.
   a) Show that $R_H = \frac{E_0}{hc}$.
   b) Deduce the value of R_H in the SI units.

3. a) Determine the longest wavelength of the absorption series of Lyman.
   b) Deduce to which of the following domains, do the rays of Lyman series belong to: visible, ultraviolet or infrared?

4. «Lyman α » of wavelength $\lambda_{\alpha} = 121.7 nm$, is one of the rays of the absorption spectrum of Lyman series. This ray permits us to detect gaseous clouds that surround a quasar.
Indicate the transition of the hydrogen atom that corresponds to the absorption of «Lyman α ».

II-LS 2012 2nd
Sodium Vapor Lamp

A sodium vapor lamp emits mainly a yellow light called doublet of wavelengths $589.0 nm$ and $589.6 nm$. Other wavelengths are also emitted, as those: $\lambda_1 = 330.3 nm$, $\lambda_2 = 568.8 nm$, $\lambda_3 = 615.4 nm$, $\lambda_4 = 819.5 nm$ and $\lambda_5 = 1138.2 nm$.

Figure 1 below shows only the yellow doublet of the emission spectrum of the sodium atom.

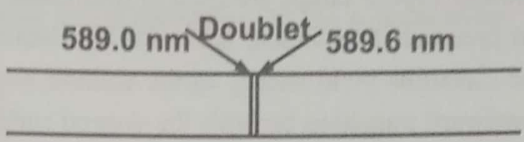


Figure 1

Given:

- Speed of light in vacuum: $c = 3 \times 10^{8} m/s$;
- Planck's constant: $h = 6.62 \times 10^{-34} J.s$;
- $1 eV = 1.6 \times 10^{-19} J$.

Part A
Spectrum analysis

1. To what range: visible, infrared or ultraviolet, does each of the radiations of the wavelengths λ_1, λ_2, λ_3, λ_4 and λ_5 belong?
2. Is the sodium vapor lamp a monochromatic or a polychromatic source of light? Justify your answer.
3. Consider the yellow radiation of wavelength $589.0 nm$. Show that the value of the energy of a photon corresponding to this radiation is approximately $2.11 eV$.

Part B
Energetic analysis of the diagram

Figure 2 shows a simplified diagram of the energy levels of a sodium atom.

1. a) One of these energy levels represents the ground state. Specify which one.
   b) What do we call each of the other shown levels?
2. a) Define the emission spectrum.
   b) Use the diagram of figure 2 to justify the discontinuity of the emission spectrum.
3. The emission of the yellow radiation of wavelength $589.0 nm$ is due to the transition of the sodium atom from an excited level E_n to the ground state.
Determine E_n.

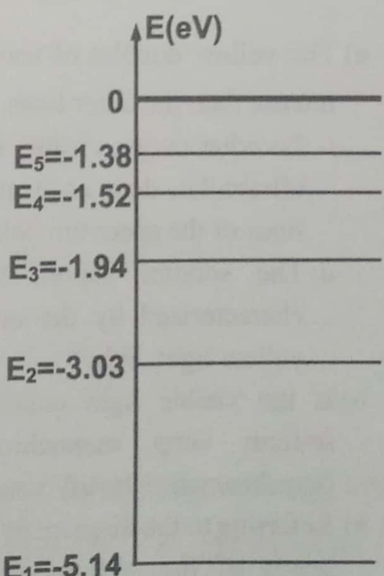


Figure 2

4. In fact, the energy level E_n is double. This double is constituted of two energy levels E_n and E'_n that are very close to each other. Compare, with justification, E_n and E'_n.
5. The sodium atom, being in an excited state E_x, receives a photon carrying an energy $1.51\,eV$ and passes to another excited state E_y; E_x and E_y exist on the diagram of figure 2.
   a) Determine the two levels E_x and E_y.
   b) Is the spectral line associated with the transition $x \to y$ an emission or absorption line? Justify your answer.

III-LS 2009 1st
Sodium Vapour Lamp

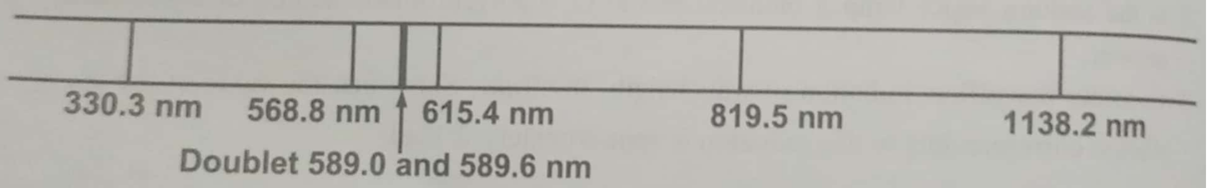
Sodium vapour lamps are used to illuminate roads. These lamps contain sodium vapour under very low pressure. This vapour is excited by a beam of electrons that cross the tube containing the vapour. The electrons yield energy to the sodium atoms which give back this received energy during their downward transition towards the ground state in the form of electromagnetic radiations.

Given:

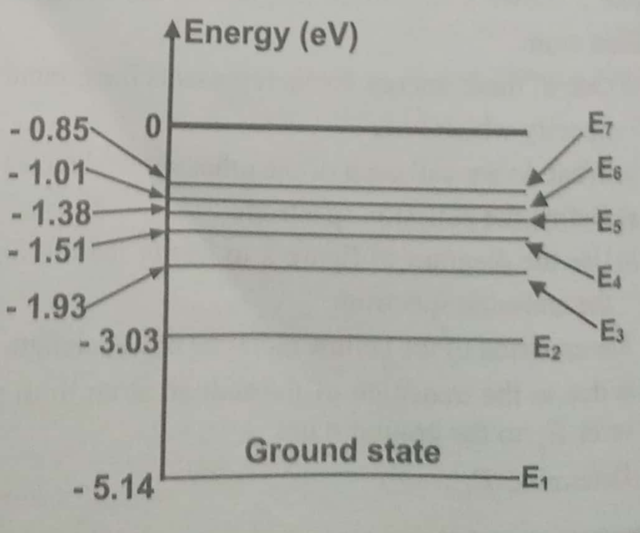
- $1e = 1.602 \times 10^{-19}\,C$;
- $h = 6.62 \times 10^{-34}\,J.s$;
- $c = 3 \times 10^{8}\,m/s$;
- $1\,nm = 10^{-9}\,m$.

1. What does each of the quantities h, c and e represent?
2. The analysis of the emission spectrum of a sodium vapour lamp shows the presence of lines of well-determined wavelengths λ.

   The figure below represents some of the lines of this spectrum.



   a) The yellow doublet of wavelengths, in vacuum, $\lambda_1 = 589.0\,nm$ and $\lambda_2 = 589.6\,nm$ is more intense than the other lines.
      i- To what range: visible, infrared or ultraviolet, does each of the other lines of the spectrum belong?
      ii- The sodium vapour lamps are characterized by the emission of yellow light. Why?
   b) Is the visible light emitted by the sodium lamp monochromatic or polychromatic? Justify your answer.
3. a) Referring to the diagram of the energy levels of the sodium atom in the adjacent figure:



i- Specify an indicator that justifies the discontinuity of the emission spectrum of the sodium vapour lamp.

ii- Verify that the emission of the line of wavelength λ_1 corresponds to the downward transition from the energy level E_2 to the ground state.

b) In fact, the energy level E_2 is double, *i.e.*, it is constituted of two energy levels that are very close to each other. Draw a diagram that shows the preceding downward transition as well as the downward transition corresponding to the emission of the radiation of wavelength λ_2.

4. The sodium atom, being in the ground state, is hit successively by the electrons (a) and (b) of respective kinetic energies $1.01\,eV$ and $3.03\,eV$.

a) Determine the electron that can interact with the sodium atom.

b) Specify the state of the sodium atom after each impact.

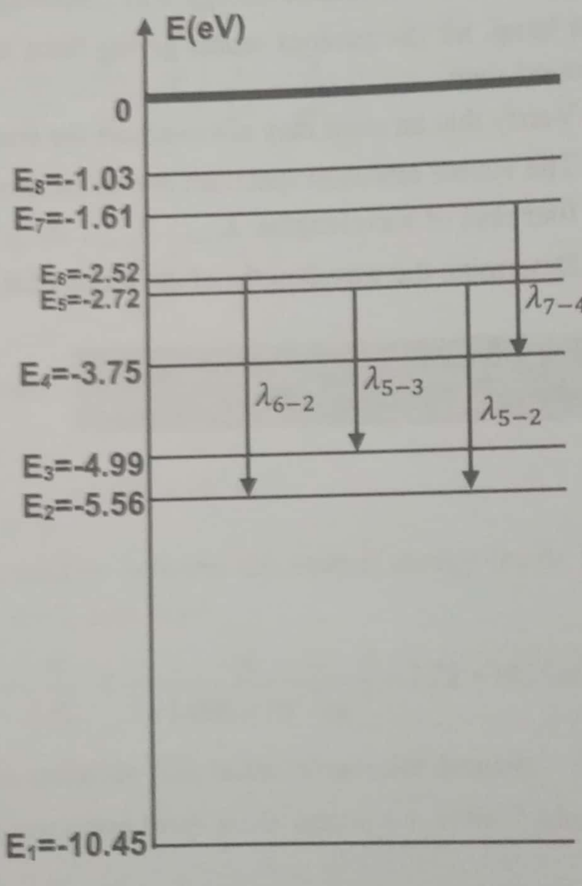
c) Deduce, after impact, the kinetic energy of the electron that interacts with the sodium atom.

L.S 2006 2nd

Emission Spectrum of a Mercury Vapour Lamp

The object of this exercise is to determine the visible emission spectrum of a mercury vapor lamp.

The adjacent diagram gives, in a simplified way, the energy level of the ground state, those of the excited states E_2, E_3, E_4, E_5, E_6, E_7, E_8 and the ionization energy level $E=0$ of the mercury atom.



Given

- Planck's constant: $h=6.62\times10^{-34}\,J.s$;
- Speed of light in vacuum: $c=3\times10^{8}\,m/s$;
- $1eV=1.6\times10^{-19}\,J$.

Part A

Quantization of the energy of the atom

1. The energy of the mercury atom is quantized. What is meant by «quantized energy»?

2. a) What is meant by «ionizing» an atom?

b) Calculate, in eV, the ionization energy of a mercury atom taken in the ground state.

3. Interaction photon-atom

A photon cannot cause the transition of an atom from an energy level E_p to a higher energy level E_n unless its energy is exactly the same as the difference of the energies (E_n-E_p) of the atom.

The mercury atom being in the ground state.

a) Determine the maximum wavelength of the wave associated to a photon capable of exciting the atom.

b) The mercury atom is hit with a photon of wavelength $\lambda_1 = 2.062 \times 10^{-7}\ m$.

i- Show that this photon cannot be absorbed.

ii- What is then the state of this atom?

c) The atom receives now a photon of wavelength λ_2. The atom is thus ionized and the extracted electron is at rest. Calculate λ_2.

Part B

Emission by a mercury vapor lamp

For an electron to cause a transition of an atom from an energy level E_p to a higher energy level E_n, its energy must be at least equal to the difference of the energies $(E_n - E_p)$ of the atom.

During one electron-atom collision, the atom absorbs, from the electron, an amount of energy enough to ensure a transition. The rest of the energy is carried by the electron as kinetic energy.

When the mercury vapor lamp is under a convenient voltage, an electric discharge takes place. Some electrons, each of kinetic energy $9\,eV$, moving in the vapor of mercury between the electrodes of the lamp, hit the gaseous atoms giving them energy. For that lamp, the atoms are initially in the ground state.

1. Verify that an atom may not overpass the energy level E_7.
2. The visible emission spectrum due to the downward transition of the mercury atom, is formed of four rays of wavelengths: $\lambda_{7\to4}$; $\lambda_{6\to2}$; $\lambda_{5\to2}$; $\lambda_{5\to3}$ **(refer to the diagram).**

   Determine the wavelengths of the limits of the visible spectrum of the mercury vapour lamp.

LS & GS 2003 2nd

See Page 191 – Part A

LS 2014 1st

Part A

1. a) Energy levels

i- In the fundamental state $n=1$, then $E_1=-\frac{13.6}{1^2}=-13.6eV$;

ii- In the first excited state $n=2$, then $E_2=-\frac{13.6}{2^2}=-3.4eV$;

iii- In the ionized state $n=\infty$, then $E_\infty=-\frac{13.6}{\infty^2}=0$.

b) The expression of the energies E_n depends on n which is a whole non-zero positive number, then only specific values are allowed.
Thus the energy of the atom is quantified.

2. a) $E=E_m-E_p$;

b) We have $E=\frac{hc}{\lambda}$; so $\frac{hc}{\lambda}=-\frac{E_0}{m^2}-\left(-\frac{E_0}{p^2}\right)=-\frac{E_0}{m^2}+\frac{E_0}{p^2}=E_0\left(\frac{1}{p^2}-\frac{1}{m^2}\right)$.

Part B

1. For Lyman series, we have $\frac{hc}{\lambda}=E_0\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$; then $\frac{1}{\lambda}=\frac{E_0}{hc}\left(1-\frac{1}{n^2}\right)$.

2. a) The relation is of the form $\frac{1}{\lambda}=R_H\left(1-\frac{1}{n^2}\right)$ where $R_H=\frac{E_0}{hc}$.

b) We have $R_H=\frac{E_0}{hc}=\frac{13.6\times1.6\times10^{-19}J}{6.62\times10^{-34}J.s\times3\times10^8m/s}=1.096\times10^7m^{-1}$.

3. a) The longest wavelength corresponds to the transition between the nearest energy levels into $n=1$, so it corresponds to the transition from $n=2$ into $n=1$.

So $\frac{1}{\lambda_{max}}=R_H\left(1-\frac{1}{2^2}\right)=R_H\left(\frac{3}{4}\right)$; then $\lambda_{max}=\frac{4}{3R_H}=\frac{4}{3\times1.096\times10^{-7}m^{-1}}=1.22\times10^{-7}m$

b) $\lambda_{max}=1.22\times10^{-7}m=122nm<400\ nm$; so this radiation falls in the ultraviolet domain.

4. $\lambda_a=121.7nm$, it corresponds to the downwards transition from $n=2$ into $n=1$ in the hydrogen atom.

LS 2012 2nd

Part A

1. $\lambda_1=330.3nm$ is an ultraviolet radiation.

$\lambda_2=568.8nm$ and $\lambda_3=615.4nm$ are visible radiations.

$\lambda_4=819.5\ nm$ and $\lambda_5=1138.2nm$ are infrared radiations.

2. The sodium vapor lamp is a polychromatic source, because it emits two radiations.

3. The energy of the photon is: $E_{ph} = h\frac{c}{\lambda} = 6.626\times10^{-34}\,J.s\times\frac{3\times10^{8}\,m/s}{589\times10^{-9}\,m} = 3.37\times10^{-19}\,J$.

Thus, $E_{ph} = \frac{3.37\times10^{-19}\,J}{1.6\times10^{-19}\,J/eV} = 2.11\,eV$.

Part B

1. a) The ground state is the level $E_1 = -5.14eV$.
   b) The levels E_2, E_3, E_4 and E_5 are called excited states.
   The level whose energy is 0 corresponds to the ionized state.
2. a) The emission spectrum is the set of spectral lines emitted by an atom during the downward transitions.
   b) To each electronic downward transition, between two energy levels, corresponds an emission line and since the energy levels diagram, of the sodium atom are discontinuous, then the spectral lines must be also discontinuous.
3. The energy of the level that corresponds to this transition is:
   $E_n = E_1 + E_{ph} = 2.11 + (-5.14) = -3.03eV = E_2$.
   This photon is due to the downward transition from the 1st excited level to the ground level.
4. The sodium atom emits the yellow doublet of wavelengths $\lambda = 589.0\,nm$ and $\lambda' = 589.6\,nm$.
   The energy of a photon $E_{ph} = h\frac{c}{\lambda}$ is inversely proportional to the wavelength, and since $\lambda < \lambda'$ then $E_{ph} > E_{ph'}$.
   But the transition is governed by the relation: $E_f = E_{ph} + E_i$; so the photon which has more energy corresponds to the transition to the higher level so $\lambda < \lambda'$ thus $E_n > E'_n$.[1]
5. a) The energy of the photon emitted due to this transition is: $E_{ph} = E_y - E_x = 1.51\,eV$.
   Comparing this difference to that between the levels shown in the diagram, we get:
   $E_4 - E_2 = 1.51\,eV$; thus, $y = 4$ and $x = 2$.
   b) The associated spectral line is an absorption line because the atom passes from one level to a higher energy level, so it absorbs energy.

III-LS 2009 1st

1. h: Planck's constant.
   c: speed of light in vacuum.
   e: elementary charge or charge of an electron.
2. a) Sodium spectrum.
   i- $330.3\,nm$ belongs to the ultraviolet domain.
   $568.8\,nm$, $589\,nm$ and $615.4\,nm$ belong to the visible domain.
   $819.5\,nm$ and $1138.2\,nm$ belong to the infrared domain.

---

[1] We can say also that the photon which has the largest wavelength corresponds to the transition between the nearest levels; $\lambda = \frac{hc}{E_{high} - E_{low}}$; and since $\lambda' > \lambda \Rightarrow E'_n < E_n$.

ii- The yellow radiation is present in the form of a doublet, which leads to the appearance of light (color) much more intense than the others.

b) It is polychromatic because it is made of several radiations of different frequencies.

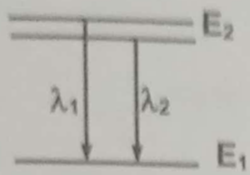
3. a) Sodium spectrum.

i- The discontinuity of the emission spectrum is justified by the discontinuous energy levels of the sodium atom.

ii- The energy of the incident photon is $E_{ph} = h\dfrac{c}{\lambda} = 3.37 \times 10^{-19} J = 2.11 eV$;

The corresponding energy transition: $E_1 + E_{ph} = -5.14 + 2.11 = -3.03 eV = E_2$

b) Schema.



4. a) The minimum energy needed to undergo an interaction is given by:

$E_{min} = E_2 - E_1 = -3.03eV - (-5.14eV) = 2.11eV$.

- $KE_{(a)} < E_{min}$; it is not possible to interact.
- $KE_{(b)} > E_{min}$; the interaction is possible.

b) For the electron (a): the atom remains in the ground state.

For the electron (b): $KE_{(b)} + E_1 = 3.03 + (-5.14) = -2.11\, eV < E_3$;

The atom will be in the first excited state E_2.

c) The incident electron (b), must ensure the energy required to the transition from $n = 2$ into $n = 1$ which is: $\Delta E = E_2 - E_1 = 2.11\, eV$;

While the rest of energy is carried by the reflected electron $KE_{ref} = KE_{(b)} - \Delta E = 0.92\, eV$.

IV-LS 2006 2nd

Part A

1. Quantized energy means that only specific values of energy are allowed.

2. a) Ionizing an atom is to giving it the minimum energy required to extract an electron.

b) The ionization energy is given by: $E_{ionization} = E_\infty - E_1 = 0 - (-10.45) = 10.45\, eV$.

3. a) The wavelength corresponding to the transition from the level n, to $n = 1$ is $\lambda = \dfrac{hc}{E_n - E_1}$.

The maximum wavelength corresponds to a transition to the nearest energy level:

$$\lambda_{max} = \frac{hc}{E_2 - E_1} = \frac{6.62 \times 10^{-34} J.s \times 3 \times 10^8 m/s}{(-5.56 - (-10.45)) \times 1.6 \times 10^{-19} J} = 2.54 \times 10^{-7}\, m.$$

b) Interaction.

i- For λ_1, the energy of the photon is:

$$E_{ph_1} = \frac{hc}{\lambda_1} = \frac{6.62 \times 10^{-34} J.s \times 3 \times 10^8 m/s}{2.062 \times 10^{-7} m} = 9.63 \times 10^{-19} J = 6.02\, eV;$$

The energy level of the atom must be $E_1 + E_{ph_1} = -4.43\,eV \neq E_n$; but this level does not exist in the energy diagram, so the photon is not absorbed.

ii- The atom remains in the ground state.

c) The extracted electron is at rest, then according to Einstein relation: $E_{ph} = W_{ion} + KE = W_{ion}$.

However $W_{ion} = E_\infty - E_1 = 10.45 \times 1.6 \times 10^{-19} J = \dfrac{hc}{\lambda_2}$, then $\lambda_2 = 1.188 \times 10^{-7} m$.

Part B

1. We have: $KE_e + E_1 = 9 + (-10.45) = -1.45\,eV < E_8$;

   The atom cannot then overpass the 7th energy level.

2. The energies emitted through these transitions are:

   $\Delta E_{6\to2} = E_6 - E_2 = 3.04\,eV$; $\Delta E_{5\to3} = E_5 - E_3 = 2.27\,eV$;

   $\Delta E_{7\to4} = E_7 - E_4 = 2.14\,eV$; and $\Delta E_{5\to2} = 2.84\,eV$.

   Then $\Delta E_{max} = \Delta E_{6\to2} = 3.04\,eV$ and $\Delta E_{min} = \Delta E_{7\to4} = 2.14\,eV$.

   The wavelength λ of the emitted radiation is given by: $\lambda = \dfrac{hc}{\Delta E}$;

   $$\lambda_{min} = \frac{hc}{\Delta E_{max}} = \frac{hc}{E_6 - E_2} = \frac{6.62 \times 10^{-34} J.s \times 3 \times 10^8 m/s}{3.04 \times 1.6 \times 10^{-19} J} = 4.08 \times 10^{-7} m = 408\,nm = \lambda_{6\to2}$$

   $$\lambda_{max} = \frac{hc}{\Delta E_{min}} = \frac{hc}{E_7 - E_4} = \frac{6.62 \times 10^{-34} J.s \times 3 \times 10^8 m/s}{2.14 \times 1.6 \times 10^{-19} J} = 5.8 \times 10^{-7} m = 580\,nm = \lambda_{7\to4}.$$

   The limits of the mercury visible spectrum $[408.3\,nm, 580\,nm]$

Unit IV

Atom & Nucleus

Chapter 16

The Nucleus

Sessions/Parts	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
The Nucleus		-	-	-	-	-	-	-	-	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
The Nucleus	-	-	-	-	1st(A)	-	2nd(A+B)	-	-	-

I-Nucleus

The nucleus of the atom ${}_Z^AX$ holds $\boldsymbol{A}$ nucleons.

The constituents of the nucleus are: $\boldsymbol{Z}$ protons and $\boldsymbol{N = (A - Z)}$ neutrons.

A nuclide is an element with a specific number of protons and neutrons.

Isotopes are nuclides having the same atomic number Z but different mass number A (isotopes differ only by the number of neutrons).

II-Stability of a Nucleus

1. Mass of an atom and nucleus

The mass of an atom $m_{\text{atom}} = \dfrac{\text{Molar Mass}}{N_A}$, where $N_A = 6.022 \times 10^{23}$ is Avogadro's number.

The mass of nucleus is $m_{\text{nucleus}} = m_{\text{atom}} - m_{\text{electrons}}$.

The mass of nucleus & elementary particles is measured in atomic mass unit u, where $1u = 1.66 \times 10^{-27} kg$.

2. Principle mass energy equivalence

According to the principle of mass energy equivalence, the mass and energy are two forms of each other: $E = \Delta m \times c^2$, where Δm is the loss of mass that is converted into energy.

We have $1u = 931.5 MeV/c^2$.

3. Binding energy for the nucleus of ${}_Z^AX$:

The mass of nucleons taken separately is the mass of Z protons each of mass m_p and $N = A - Z$ neutrons each of mass m_n is $m_{\text{nucleons}} = Zm_p + (A - Z)m_n$ is greater than that of nucleus m_X.

The mass defect of nucleus is the difference Δm between the sum of the masses of all nucleons taken separately and the mass of the nucleus: $\Delta m = Zm_p + (A - Z)m_n - m_X$.

Definition

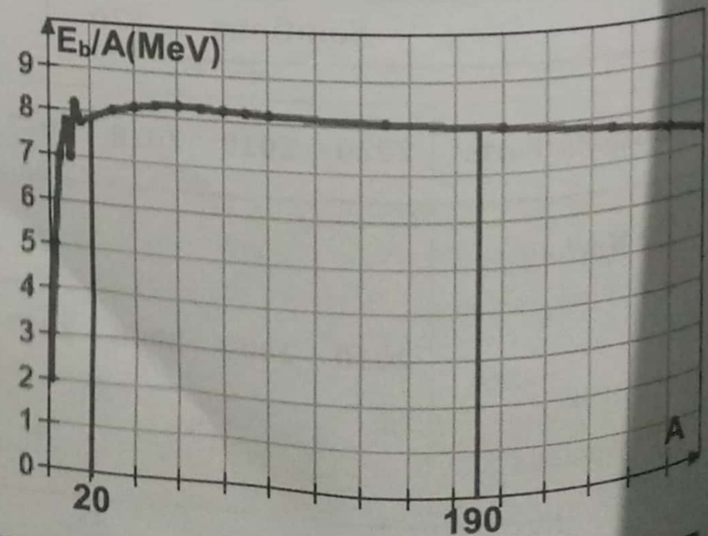
The binding energy of a nucleus is the minimal energy that should be given to the nucleus in order to break it up into its nucleons and given by: $E_b = \Delta m\, c^2$.

4. Binding energy per nucleon

The stability of a nucleus, is studied by the binding energy per nucleon (E_b / A):

- for $A < 20$ & $A > 190$, $\dfrac{E_b}{A} < 8MeV$; then the nucleus is less stable;
- for $20 < A < 190$, $\dfrac{E_b}{A} > 8MeV$; then the nucleus is more stable.

The adjacent curve called Aston's curve represents the distribution of nuclei.



The strong nuclear force (short range) maintains the cohesion of the nucleus as an entity.

III-Densities

1. Density of the atom

The density of the atom is given by: $\rho_{atom} = \frac{m_{atom}}{v_{atom}}$.

We suppose that the nucleus has a spherical volume of radius R, of volume $v_{nucleus} = \frac{4}{3}\pi R^3_{nucleus}$

2. Density of the nucleus

The volume of the nucleus is the sum of volume of A nucleons supposed identical.

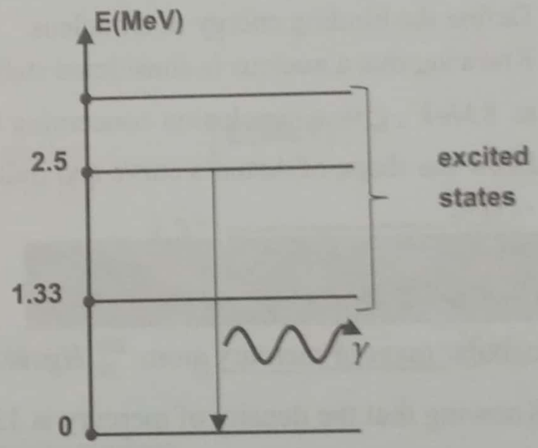
The radius of the nucleus is given by $R_{nucleus} = r_0 A^{\frac{1}{3}}$ where $r_0 = 1.2\ fm = 1.2 \times 10^{-15}\ m$ is the radius of a nucleon or hydrogen nucleus.

The density of the nucleus is $\rho_{nucleus} = \frac{m_{nucleus}}{v_{nucleus}}$ & the mass of nucleus is usually given, this density remains constant for all the nuclei $2.3 \times 10^{17}\ kg/m^3$ which is huge compared to that of atom.

IV-Energy Levels of the Nucleus

The nucleus possesses its proper set of energy levels, these energy levels are quantified.

If the nucleus is not subjected to any excitation (disturbance), it is found in a state called ground state denoted X (lowest state), and its energy is zero; otherwise it is found in an excited state denoted X^* (as shown in the adjacent diagram).



Equation of disexcitation

During the disexcitation of the nucleus (from excited to lower excited or from excited state to ground state), an electromagnetic wave is emitted called gamma radiation γ:

$$X^* \longrightarrow X + \gamma$$

- Gamma Radiation are very energy energetic, their energies are measured in MeV (and sometimes in keV).

- The energy of the radiation gamma emitted during the disexcitation (transition between level) is given by: $E_\gamma = E_{high} - E_{low}$.

- The wavelength λ (or frequency υ) of the radiation associated to this transition is given by: $E_\gamma = h\upsilon = h\frac{c}{\lambda}$ where h is Planck's constant & c is the speed of light in vacuum.

Applications

In what follows:

Planck's constant: $h = 6.63 \times 10^{-34}\ Js$;

- Avogadro's number: $6.022 \times 10^{23}\ mol^{-1}$;
- Mass of the proton: $m_p = 1.00728\ u$;
- Mass of the neutron: $m_n = 1.00866\ u$;
- $1u = 1.66 \times 10^{-27}\ kg$ & $1u = 931.5 MeV/c^2$.

I- Mercury Radionuclide

Mercury is widely used in the manufacturing of thermometers, its symbol Hg derives from its older Greek name *hydrargyrum*, and was coined by the combination of the two words hydror (water) & argyros (silver).

The mass of the mercury nucleus $^{200}_{80}Hg$ is $199.96833\,u$.

1. a) Give the constituents of the nucleus $^{200}_{80}Hg$.
   b) Show that the mass of the nucleus taken at rest is less than the sum of the masses of the nucleons taken separately at rest. Interpret the decrease in the mass.
2. Define the binding energy of a nucleus.
3. Knowing that a nucleus is considered stable when its binding energy per nucleon is larger or equal to $8 MeV$, give a conclusion concerning the stability of the nucleus $^{200}_{80}Hg$.
4. Draw the shape of Aston's curve and indicate on it the zone of stability.

II- Densities & Mercury

The molar mass of mercury atom $^{200}_{80}Hg$ is $200 g$ & the mass its nucleus is $199.96833\,u$.

1. Knowing that the density of mercury is $13546 kg/m^3$.
   Determine, in *nm*, the average radius of the mercury atom.
2. The radius of a nucleon is considered as equal to that of hydrogen nucleus $r_0 = 1.2 fm$.
   Calculate the density of the mercury nucleus.

III- Mass of a Nucleus

Wikipedia

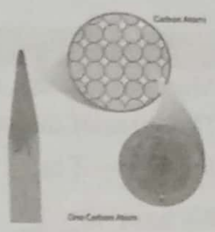
Lanthanum is a soft, ductile, silvery-white metallic chemical element with symbol La and atomic number 57. It tarnishes rapidly when exposed to air and is soft enough to be cut with a knife. It was first found by the Swedish chemist Carl Gustav Mosander in 1839.

If the binding energy per nucleon of the Lanthanum is $\frac{E_b}{A}\left(^{144}La\right) = 8.28 MeV/\text{nucleon}$.

Determine the mass of its nucleus in atomic mass unit u.

IV- Carbon Nucleus

The word «Carbon» comes from the Latin word «Carbo» meaning coal. There are three naturally occurring isotopes of carbon on Earth: 99% of the carbon is carbon-12, 1% is carbon-13 and carbon-14 occurs in trace amounts, *i.e.*, making up about 1 or 1.5 atoms per 10^{12} atoms of the carbon in the Atmosphere.



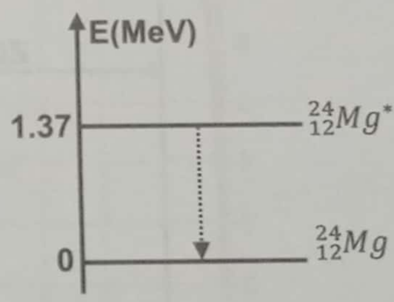
Given:

- The mass of ${}^{14}_{6}C$ nucleus is $14.00324u$.

1. Consider the isotope ${}^{14}_{6}C$.
   a) Indicate the constituents of its nucleus.
   b) Determine its binding energy per nucleon.
   c) Draw a conclusion concerning the stability of this nucleus.
2. If the radius of a nucleon is $r_0 = 1.2 \times 10^{-15}\,m$.
   a) Calculate the volume of the carbon nucleus.
   b) Deduce the density of its nucleus.

V- Magnesium Nucleus

The magnesium nucleus in the excited is denoted Mg^*
The magnesium nucleus is excited due to a certain nuclear process. A possible energy diagram of the energy levels of the nucleus is shown in the adjacent figure.



Given:

- The mass of magnesium nucleus is $m(Mg) = 23.97846\,u$.

1. Calculate the binding energy per nucleon of a magnesium nucleus.
2. Write the equation of the dis-excitation of the magnesium nucleus.
3. Deduce the energy of the radiation emitted and then its frequency.
4. Calculate the mass-equivalence, in u, of the radiation emitted.
5. Deduce the mass of the magnesium nucleus in its excited state.

VI- Xenon Nucleus

The binding energy per nucleon of xenon ${}^{139}_{54}Xe$ nucleus is $8.4MeV$.

1. Calculate the mass of xenon nucleus.
2. Knowing that the density of its nucleus is $2.3 \times 10^{17} kg/m^3$.
   Determine its radius.
3. Name the main forces of interaction that exist between the nucleons in a nucleus and specify the one that ensures its cohesion.

I-

1. a) Mercury nucleus contains 200 nucleons divided into 80 protons & 120 neutrons.

b) The mass of the nucleons taken separately and at rest is:

$m_{nucleons} = Zm_p + (A-Z)m_n = 80 \times 1.00728\,u + 120 \times 1.00866\,u = 201.6216\,u$;

We have $m_{nucleons} = 201.6216\,u > m_{nucleus} = 199.96833\,u$;

The mass defect is converted into binding energy in order to ensure the cohesion of the nucleus.

2. The binding energy is the minimum energy required to break up the nucleus into its nucleons.

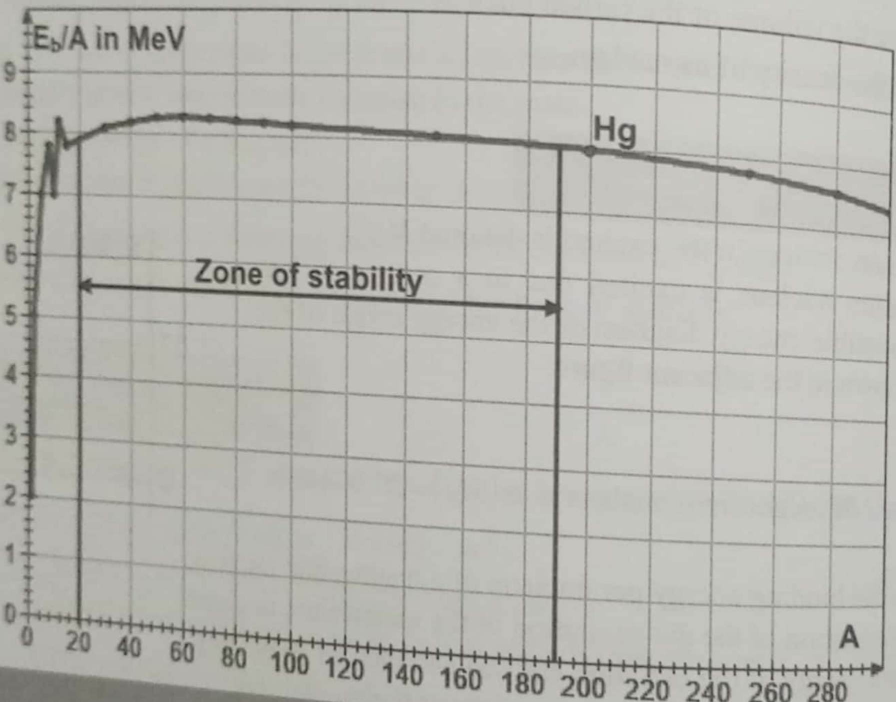
3. The mass defect is $\Delta m = 201.6216\,u - 199.96833\,u = 1.65327\,u$;

The binding energy of the nucleus is $E_b = \Delta m c^2 = 1.65327 \times 931.5 = 1540.02\,MeV$;

The binding energy per nucleon $\frac{E_b}{A} = \frac{1540.02\,MeV}{200} = 7.7\,MeV < 8\,MeV$;

Then the mercury nucleus is less stable.

4. Aston's curve:



II-

1. The density of the mercury atom is $\rho_{\text{mercury atom}} = \frac{m_{\text{mercury atom}}}{v_{\text{mercury atom}}}$;

The mass of mercury atom is $m_{\text{mercury atom}} = \frac{M_{\text{mercury atom}}}{N_A} = \frac{200 \times 10^{-3}\,kg/mo\ell}{6.022 \times 10^{23}\,mo\ell^{-1}}$;

Then $v_{\text{mercury atom}} = \frac{m_{\text{mercury atom}}}{\rho_{\text{mercury atom}}} = \frac{\frac{200 \times 10^{-3}\,kg/mo\ell}{6.022 \times 10^{23}\,mo\ell^{-1}}}{13456\,kg/m^3} = 2.47 \times 10^{-29}\,m^3$;

The atom has a spherical shape of radius R, then $v_{\text{mercury atom}} = 2.47 \times 10^{-29}\,m^3 = \frac{4}{3}\pi R^3$;

Then $R=\sqrt[3]{\frac{2.47\times10^{-29}\times3}{4\pi}}=1.81\times10^{-10}m=0.181nm$.

2. The density of the nucleus is $\rho_{\text{mercury nucleus}}=\frac{m_{\text{mercury nucleus}}}{v_{\text{mercury nucleus}}}=\frac{m_{\text{mercury nucleus}}}{A\times v_{\text{nucleon}}}$;

Then $\rho_{\text{mercury nucleus}}=\frac{199.96833\times1.66\times10^{-27}kg}{200\times\frac{4}{3}\times\pi\times\left(1.2\times10^{-15}m\right)^3}=2.3\times10^{17}kg/m^3$.

III-

We have $\frac{E_b}{A}\left({}^{144}La\right)=8.28MeV$ / nucleon ;

Then the binding energy of a nucleus is $E_b\left({}^{144}La\right)=144\times8.28MeV=1192.32$ MeV;

However $E_b\left({}^{144}La\right)=\frac{1192.32MeV}{931.5MeV/c^2}=1.28u\times c^2=\Delta m c^2$, then $\Delta m=1.28u$;

We have $\Delta m=m_{\text{nucleons}}-m_{\text{nucleus}}$, so $m_{\text{nucleus}}=m_{\text{nucleons}}-\Delta m=Zm_p+(A-Z)m_n-1.28$;

Then $m_{\text{nucleus}}=57\times1.00728u+87\times1.00866u-1.28u$;

Thus, $m_{\text{nucleus}}=145.16838u-1.28u=143.88838u$.

IV-

1. a) The carbon nucleus is formed of 6 protons and 8 neutrons.

b) The mass defect $\Delta m=Zm_p+(A-Z)m_n-m_{{}^{14}_{6}C}$;

$\Delta m=6\times1.00728u+(14-6)\times1.00866u-14.00324u=0.10972u$;

The binding energy $E_b=\Delta m c^2=0.10972\times931.5=102.2MeV$;

The binding energy per nucleon $\frac{E_b}{A}=\frac{102.2MeV}{14}=7.3MeV$.

c) $\frac{E_b}{A}=7.3MeV<8MeV$, then this nucleus is less stable.

2. a) The nucleus is formed of A nucleons, then $v_{\text{nucleus}}=A\times v_{\text{nucleon}}=A\times\frac{4}{3}\pi r_0^3$;

Thus, $v_{\text{nucleus}}=14\times\frac{4}{3}\pi\left(1.2\times10^{-15}\right)^3=1\times10^{-43}m^3$.

b) The density of the nucleus is:

$$\rho_{\text{nucleus}}=\frac{m_{\text{nucleus}}}{v_{\text{nucleus}}}=\frac{14.00324\times1.66\times10^{-27}kg}{1\times10^{-43}m^3}=2.3\times10^{17}kg/m^3.$$

V-

1. The mass defect $\Delta m=Zm_p+(A-Z)m_n-m_{{}^{24}_{12}Mg}$;

$\Delta m=12\times1.00728u+(24-12)\times1.00866u-23.97846u=0.21282u$;

The binding energy $E_b=\Delta m c^2=0.21282\times931.5MeV=198.2MeV$;

The binding energy per nucleon $\frac{E_b}{A} = \frac{198.2MeV}{24} = 8.26MeV$.

2. The equation of the disexcitation is: $Mg^* \longrightarrow Mg + \gamma$.

3. The energy of the radiation emitted is $E_\gamma = E_k - E_l = 1.37MeV - 0 = 1.37MeV$.

The frequency $\nu = \frac{E_\gamma}{h} = \frac{1.37\times10^6\times1.6\times10^{-19}J}{6.63\times10^{-34}J.s} = 3.3\times10^{20}Hz$.

4. Let $m_{e-\gamma}$ be the equivalent mass to the radiation emitted:

$E_\gamma = 1.37\ MeV = m_{e-\gamma}c^2$; so, $m_{e-\gamma} = 1.37\ MeV/c^2 = \frac{1.37}{931.5}u \approx 1.47\times10^{-3}u$.

5. $m_{Mg^*} = m_{Mg} + m_{e-\gamma} = 23.97846u + 1.47\times10^{-3}u = 23.97993u$.

1. The binding energy of the nucleus is given by: $E_b = 8.4\times139 = 1167.6\,MeV$;

We have $E_b = \Delta m c^2$, so $\Delta m = \frac{E_b}{c^2} = \frac{1167.6\,MeV}{c^2} = \frac{1167.6}{931.5}u \approx 1.25346\,u$;

However the mass defect is given by: $\Delta m = Z\,m_p + (A-Z)m_n - m_X$;

Then, $m_X = Z\,m_p + (A-Z)m_n - \Delta m = 54\times1.00728 + (139-54)\times1.00866 - 1.25346$;

Thus, $m_X = 138.87576\ u$.

2. The volume of the nucleus is given by: $v_{\text{nucleus}} = \frac{m_{\text{nucleus}}}{\rho_{\text{nucleus}}}$;

Then, $v_{\text{nucleus}} = \frac{138.87576\times1.66\times10^{-27}\,kg}{2.3\times10^{17}\,kg/m^3} \approx 10^{-42}\,m^3$;

However, the nucleus has a spherical shape, so $v_{\text{nucleus}} = \frac{4}{3}\pi R^3$ where R is the radius of the nucleus.

Thus, $R = \sqrt[3]{\frac{3\times v_{\text{nucleus}}}{4\pi}} = \sqrt[3]{\frac{3\times10^{-42}}{4\pi}} \approx 6.2\times10^{-15}\,m = 6.2\,fm$.

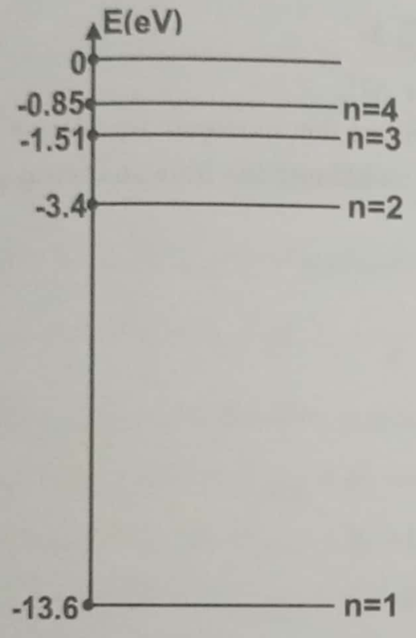
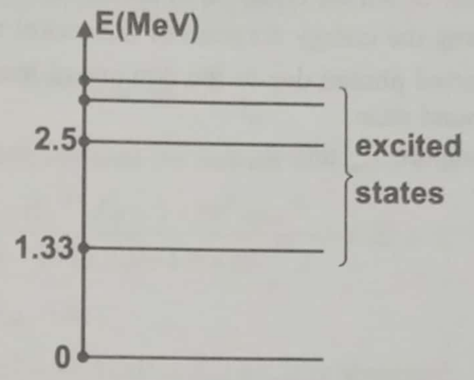
3. The stability of the nucleus is ensured by the strong nuclear force of short range between nucleons which overcomes the gravitational force and the repulsive electric force between the protons.

Problems

I-2013 (Saida)
Energy levels of the Hydrogen Atom and of the Nickel Nucleus

The aim of this exercise is to compare and study the energy levels of an atom and a nucleus.

Given:
- $h = 6.63 \times 10^{-34}\ J.s$;
- $c = 3 \times 10^{8}\ m.s^{-1}$;
- $1eV = 1.6 \times 10^{-19}\ J$.

Energy diagram of the hydrogen atom	Energy diagram of the nickel nucleus ^{60}Ni
	

Part A
Comparison
Referring to the two diagrams:
1. Name the state corresponding to the energy $E = 0$ in:
   a) the hydrogen atom.
   b) the nickel nucleus.
2. Show that the transitions of the nickel nucleus are much more energetic than those of the hydrogen atom.
3. Show that the energies of the hydrogen atom and that of the nickel nucleus are quantized.

Part B
Hydrogen atom
1. The hydrogen atom is in the ground level.
   Determine the minimum energy needed to ionize this atom.
2. The Lyman series of the hydrogen atom corresponds to a downward transition to the level $n = 1$.
   Determine the maximum wavelength λ_m of the emitted photons in this series.

3. We send, on a hydrogen atom taken in the ground state, separately, three photons a, b and c, whose energies are indicated in the table below.

Photon	a	b	c
Energy in eV	12.09	12.30	14.60

a) Specify the photons that are absorbed by the hydrogen atom.
b) Indicate the state of the hydrogen atom in each case.

Part C
Nickel 60 nucleus

The cobalt isotope $^{60}_{27}Co$, used for the treatment of certain kinds of cancer, is a β^- emitter (emission of an electron) according to the nuclear reaction $^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni^* + {}^{0}_{-1}e$.

The daughter nickel nucleus is found in an excited state $^{60}_{28}Ni^*$.

1. Write down the equation of the downward transition of Ni^*.
2. Using the energy diagram of the nickel nucleus, determine the maximum wavelength λ'_m of the emitted photon due to the downward transition of the nickel nucleus from an excited state to the ground state.
3. Compare λ'_m and λ_m.

Solutions – Problems

I-

Part A

1. a) For an atom, $E=0$ corresponds to the ionized state.
   b) For a nucleus, $E=0$ corresponds to the ground state.
2. For an atom, the energies of the levels are in the order of eV and for a nucleus, the energies of the levels are in the order of MeV.

   However $1MeV=10^6 eV$, then the nucleus transitions are more energetic than those of atoms.
3. The diagram shows that the distributions of these energy levels are discrete.

Part B

1. The minimum energy required to ionize an atom is the energy needed to reach the ionized state and the electron is extracted at rest: $E_{\min}=E_{\infty}-E_1=0-(-13.6)=13.6eV$.
2. The energy of the photon emitted is: $E_{ph}=E_n-E_1$ where $E_{ph-\min}=h\dfrac{c}{\lambda_m}$;

   The maximum wavelength corresponds to the transition towards the nearest level $n=2\Rightarrow n=1$;

   We have $h\dfrac{c}{\lambda_m}=E_2-E_1$, then $\lambda_m=\dfrac{hc}{E_2-E_1}=\dfrac{6.63\times10^{-34}J.s\times3\times10^{8}m.s^{-1}}{(-3.4-(-13.6))\times1.6\times10^{-19}J}=1.22\times10^{-7}m$.
3. a) The final state, when possible, is given by: $E_n=E_{ph}+E_1$;
   - For the photon (a): $E_{ph(a)}+E_1=12.09+(-13.6)=-1.51eV=E_3$; so, it is absorbed.
   - For the photon (b): $E_{ph(b)}+E_1=12.3+(-13.6)=-1.3eV\neq E_n$; so, it is not absorbed.
   - For the photon (c): $E_{ph(c)}+E_1=14.6+(-13.6)=1eV>0$; so, it is absorbed and an electron is extracted.

   b) For the photon (a), the atom undergoes a transition towards the 2nd excited state E_3.
   For the photon (b), the atom remains in the ground state.
   For the photon (c), the atom is ionized.

Part C

1. The equation of the dis-excitation ${}^{60}_{28}Ni^* \longrightarrow {}^{60}_{28}Ni+\gamma$.
2. The maximum wavelength corresponds (to the photon emitted carrying the smallest energy) to the transition between the nearest levels: $E_{ph_{\min}}=E_h-E_\ell=1.33-0=1.33\,MeV=h\dfrac{c}{\lambda'_m}$;

   $\lambda'_m=\dfrac{hc}{E_{ph_{\min}}}=\dfrac{6.63\times10^{-34}J.s\times3\times10^{8}m.s^{-1}}{1.33\times1.6\times10^{-13}J}=9.35\times10^{-13}m$.
3. Comparison of wavelength: $\dfrac{\lambda'_m}{\lambda_m}=7.6\times10^{-6}$; then $\lambda'_m <<<<<<< \lambda_m$.

Supplementary Problems

Given:

- Avogadro's number: $6.022 \times 10^{23}\ mol^{-1}$.
- Planck's constant: $h = 6.63 \times 10^{-34}\ J.s$
- Mass of the proton: $m_p = 1.00728\ u$.
- Mass of the neutron: $m_n = 1.00866\ u$.
- $1u = 1.66 \times 10^{-27}\ kg$.
- $1u = 931.5\ MeV/c^2$.
- $1eV = 1.6 \times 10^{-19}\ J$.

A- Aluminium

Aluminium or aluminum (in North American English) is a chemical element in the boron group with symbol Al and atomic number 13. It is a silvery-white, soft, nonmagnetic, ductile metal.
By mass, Aluminium makes up about 8% of the Earth's crust.
Aluminium has many known isotopes, with mass numbers range from 21 to 42, however $^{27}_{13}Al$ has a natural abundance above 99.9%.

Wikipedia

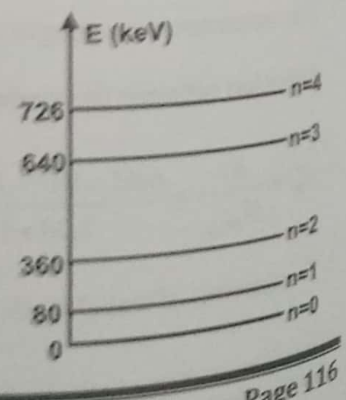
1. The density of aluminum is $2700\ kg/m^3$ and its molar mass is $27\ g/mol$.
   a) Calculate the mass of an aluminum atom.
   b) Deduce the volume of the atom.
2. Knowing that the radius of a nucleon is $r_0 = 1.2\ fm$.
   Determine the volume of its nucleus.
3. Compare the volume of the atom to that of its nucleus.
4. Knowing that the mass of its nucleus is $26.98153\ u$.
   Determine the binding energy per nucleon of the nucleus and then discuss its stability.

Answer Key

2. $1.95 \times 10^{-43}\ m^3$. 4. $\Delta m = 0.23435\ u$.

B- Xenon Nucleus

The adjacent figure represents an energy diagram of xenon nucleus (Xe-131).



1. The energy of the energy levels of the xenon nucleus is quantified. Justify.
2. What do we call the energy level labeled $n = 0$? $n = 2$?
3. Gamma radiations are emitted.
   Due to what is this emission?

4. Determine the frequencies $\upsilon_{4,3}$ & $\upsilon_{3,0}$ when the nucleus undergoes a transition from $n=4 \longrightarrow n=3$, and from $n=3 \longrightarrow n=0$.

5. Justify Ritz's relation between the frequencies:

$\upsilon_{4,0} = \upsilon_{4,3} + \upsilon_{3,0}$.

6. Deduce $\upsilon_{4,0}$.

Answer Key

4. $\upsilon_{4,3} = 2.07\times10^{19}\,Hz$ & $\upsilon_{3,0} = 1.54\times10^{20}\,Hz$.

5. We have $\upsilon_{4,3} + \upsilon_{3,0} = \dfrac{E_4 - E_3}{h} + \dfrac{E_3 - E_0}{h} = \dfrac{E_4 - E_0}{h} = \dfrac{h\upsilon_{4,0}}{h} = \upsilon_{4,0}$.

GS 2006 1st Part A
Atomic Nucleus

The object of this exercise is to compare the values of physical quantities characterizing the stability of different nuclei and to verify that, during nuclear reactions, certain nuclei are transformed into more stable nuclei with the liberation of energy.

Given:

- Mass of a neutron: $m_n = 1.0087\ u$;
- Mass of a proton: $m_p = 1.0073\ u$;
- $1u = 931.5\ MeV/c^2$.

Stability of atomic nuclei

Consider the table below that shows some physical quantities associated with certain nuclei.

Nucleus	$^{2}_{1}H$	$^{3}_{1}H$	$^{4}_{2}He$	$^{14}_{6}C$	$^{94}_{38}Sr$	$^{140}_{54}Xe$	$^{235}_{92}U$
Mass (u)	2.0136	3.0155	4.0015	14.0065	93.8945	139.89	234.9935
Binding energy E_b (MeV)	2.23	8.57	28.41	99.54	810.50	1164.7	…
Binding energy per nucleon $\frac{E_b}{A}$ (MeV/nucleon)	1.11	…	7.10	…	8.62	…	…

1. Define the binding energy of a nucleus.
2. Write the expression of the binding energy E_b of a nucleus $^{A}_{Z}X$ as a function of Z, A, m_p, m_n and m_X (the mass of the nucleus $^{A}_{Z}X$) and the speed of light in vacuum c.
3. Calculate, in MeV, the binding energy of the uranium 235 nucleus.
4. Complete the table by calculating the missing values of $\frac{E_b}{A}$.
5. Give the name of the most stable nucleus in the above table. Justify your answer.
6. Each of the considered nuclei in the table belongs to one of the three groups given by: $A<20$; $20<A<190$ & $A>190$.

Referring to the completed table, trace the shape of the curve representing the variation of $\frac{E_b}{A}$ as a function of A. Specify on the figure the three mentioned groups.

II-GS 2004 2nd/Part
Studying the Radionuclide $^{198}_{79}Au$

Given:

- Molar mass of $^{198}_{79}Au$: 198 g ;
- Avogadro's number: $6.022 \times 10^{23}\ mol^{-1}$;
- $1u = 1.66 \times 10^{-27}\ kg$;
- $1u = 931.5\ MeV/c^2$;
- Mass of the gold nucleus Au : $197.925\ u$;
- Mass of the proton: $m_p = 1.00728\ u$;
- Mass of the neutron: $m_n = 1.00866\ u$.

Part A
Comparison between the density of the gold nucleus and that of the gold atom

1. a) Calculate the mass of the gold atom $^{198}_{79}Au$.

   b) Compare the mass of the gold atom $^{198}_{79}Au$ with that of its nucleus.

2. The average radius of the gold atom is $r = 1.6 \times 10^{-10}\ m$. The average radius of a nucleon is $r_0 = 1.2 \times 10^{-15}\ m$. Compare the density of the gold atom with that of its nucleus. Give a conclusion about the distribution of mass in the atom.

Part B
Stability of the gold nucleus

1. a) Give the constituents of the nucleus $^{198}_{79}Au$.

   b) If the gold nucleus $^{198}_{79}Au$ is broken into its constituting nucleons, show that the sum of the masses of the nucleons taken separately at rest is greater than the mass of the nucleus taken at rest. Interpret this decrease in the mass.

2. Knowing that a nucleus is considered stable when its binding energy per nucleon is larger or equal to $8\ MeV$, give a conclusion about the stability of the nucleus $^{198}_{79}Au$.

I-

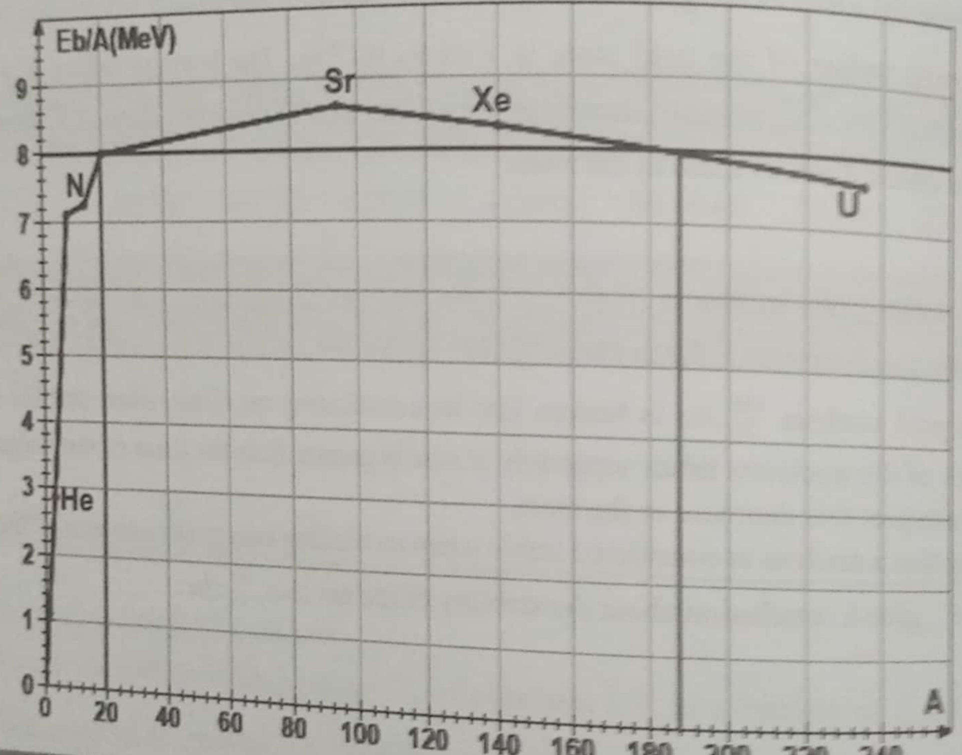
1. The binding energy of a nucleus is the minimum energy needed to break up the nucleus into its nucleons.
2. The binding energy is given by $E_b = \Delta m\, c^2$; where $\Delta m = \lfloor Z\, m_p + (A-Z) m_n - m_X \rfloor$
3. The mass defect is $\Delta m = 92\times 1.0073 + (235-92)\times 1.0087 - 234.9935 = 1.9222\ u$;

The binding energy $E_b = \left(1.9222\times 931.5\, MeV/c^2\right)\times c^2 = 1790.53\ MeV$.

4. Table.

$^{2}_{1}H$	$^{3}_{1}H$	$^{4}_{2}He$	$^{14}_{6}C$	$^{14}_{7}N$	$^{94}_{38}Sr$	$^{140}_{54}Xe$	$^{235}_{92}U$
1.11	2.86	7.10	7.11	7.25	8.62	8.32	7.62

5. The strontium is the most stable nucleus, since it has the largest binding energy per nucleon.
6. Curve is shown below (Aston's curve).



II-

Part A

1. a) The mass[1] of one gold atom is:

$$m_{atom} = \frac{\text{Molar mass}}{N_A} = \frac{M}{N_A} = \frac{198\ g/mo\ell}{6.022\times 10^{23}/mo\ell} = 3.288\times 10^{-22}\ g = 3.288\times 10^{-25}\ kg.$$

[1] The number of atoms N in a mass m formed of an element having a molar mass M is given by $N = \frac{m}{M} N_A$ where N_A is the Avogadro's number; the mass of an atom $(N=1)$ is $m_{atom} = \frac{M}{N_A}$.

b) The mass of gold nucleus is $m_{\text{nucleus}} = 197.925 \times 1.66 \times 10^{-27} = 3.286 \times 10^{-25}\,kg$;

$m_{\text{nucleus}} \approx m_{\text{atom}}$; thus, the majority of the mass of an atom is concentrated in the nucleus, in such a way that the mass of electrons can be considered as negligible compared to that of nucleus.

2. The density of gold atom is $\rho_{\text{atom}} = \dfrac{m_{\text{atom}}}{\dfrac{4}{3}\pi r^3} = \dfrac{3.288 \times 10^{-25}\,kg}{\dfrac{4}{3} \times \pi \times \left(1.6 \times 10^{-10}\,m\right)^3} = 19164\,kg/m^3$.

The density of the nucleus is $\rho_{\text{nucleus}} = \dfrac{m_{\text{nucleus}}}{v_{\text{nucleus}}} = \dfrac{m_{\text{nucleus}}}{\dfrac{4}{3}\pi r_0^3 \times A}$;

Then $\rho_{\text{nucleus}} = \dfrac{3.286 \times 10^{-25}\,kg}{\dfrac{4}{3} \times \pi \times \left(1.2 \times 10^{-15}\,m\right)^3 \times 198} = 2.3 \times 10^{17}\,kg/m^3$.

Then the density of the nucleus is huge compared to that of the atom.
Almost all the mass of the atom is concentrated in the nucleus whose volume is very small compared to that of the atom.

Part B

1. a) The nucleus of $^{198}_{79}Au$ is constituted of $Z = 79$ protons and $N = A - Z = 119$ neutrons.

b) Binding energy

- the mass of nucleons at rest is:
  $Z m_p + (A - Z) m_n = 79 \times 1.00728 + 119 \times 1.00866 = 199.60566\,u$
- the mass of the nucleus of $^{198}_{79}Au$ is $m_X = 197.925\,u$.

Thus, the mass of the gold nucleus is less than the sum of the mass of its nucleons.
The mass defect is due to the mass that is converted into binding energy, which ensures the cohesion of the nucleus as an entity.

2. The mass defect: $\Delta m = m_{\text{nucleons}} - m_{\text{nucleus}} = 199.60566\,u - 197.925\,u = 1.68066\,u$;

The binding energy is $E_b = \Delta m c^2 = 1.68066\,u \times c^2 = 1.68066 \times 931.5\,MeV = 1565.5\,MeV$;

The binding energy per nucleon: $\dfrac{E_b}{A} = \dfrac{1565.5\,MeV}{198} = 7.9\,MeV < 8\,MeV$.

Thus, the gold nucleus is less stable.

Unit IV

Atom & Nucleus

Chapter 17

Radioactivity

LS - Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Radioactivity		2nd	1st	1st	1st & 2nd	1st(B)	-	1st	1st	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Radioactivity	1st	-	1st	2nd	1st	1st	2nd	-	1st	1st

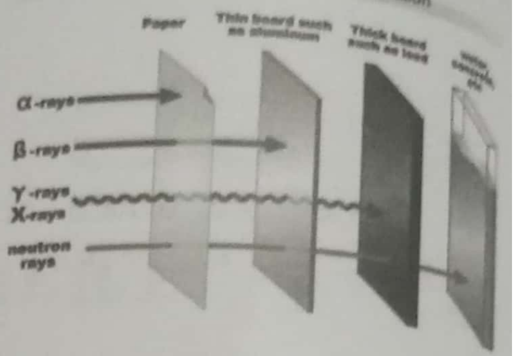
I-Radioactivity (Disintegration)

Radioactivity is a spontaneous nuclear process (natural) during which an unstable nucleus called *father* disintegrates into a more stable nucleus called ***daughter*** with a certain emission.

We have four types of emissions:

Type of decay		Nature	Identity	Symbol
Alpha	α	Particle	Helium nucleus	${}^{4}_{2}He$
Beta minus	β^-	Particle	Electron	${}^{0}_{-1}e$
Beta plus	β^+	Particle	Positron	${}^{0}_{+1}e$
Gamma	γ	Electromagnetic radiation		γ

Radiation types and the degree of penetration



II-Laws

All types of disintegrations and nuclear reactions are governed by laws of conservations.

1. Soddy's laws

Conservation of mass number $\sum A_{\text{before}} = \sum A_{\text{after}}$;

Conservation of atomic or charge number $\sum Z_{\text{before}} = \sum Z_{\text{after}}$;

2. Total energy

The total energy of a nucleus E_t is the sum of its rest energy $m_X c^2$ and its kinetic energy KE_X.

$$E_t(X) = E_{\text{rest}} + KE_X = m_X c^2 + KE_X$$

III-Emissions

1. Gamma

If the daughter nucleus is in an excited state, it drops into a lower energy level (dis-excited) emitting an electromagnetic radiation called gamma (γ), that:

- is carries an energy $E = h\upsilon$, where υ is the frequency of emitted radiation.
- is possesses huge penetrating power (many cm in lead).
- is propagates in vacuum at the speed of light $c = 3 \times 10^8 m/s$.

The equation of the dis-excitation is $Y^* \longrightarrow Y + \gamma$.

2. Alpha decay α

Alpha decays characterizes the heavy nuclei whose mass number $A > 200$.

A nucleus ${}^{A}_{Z}X$ is alpha emitter, the overall balanced equation is: ${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + \gamma$.

The energy liberated $E_\ell = \Delta m \times c^2$ where $\Delta m = m_{\text{before}} - m_{\text{after}}$ is the mass defect;

Conservation of energy: $E_\ell = KE_\alpha + E_\gamma$.

Note 1:
The kinetic energy of α particle is maximum, when the disintegration occurs without an emission of γ -radiation. We get $(KE_\alpha)_{max} = E_\ell$.

Note 2:
$KE_\alpha = E_\ell - E_\gamma$ is quantified due to the quantification of energy carried by gamma radiation.

Global conservation of energy

The equation of an alpha decay ${}_{Z}^{A}X \longrightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He + \gamma$.

$E_t(\text{before}) = E_t(\text{after})$; then $E_t(X) = E_t(Y) + E_t(\alpha) + E_\gamma$;

$m_X c^2 + KE_X = m_Y c^2 + KE_Y + m_\alpha c^2 + KE_\alpha + E_\gamma$;

We get: $m_X c^2 - m_Y c^2 - m_\alpha c^2 = KE_Y + KE_\alpha + E_\gamma - KE_X$.

So, $[m_X - (m_Y - m_\alpha)]c^2 = (KE_Y + KE_\alpha + E_\gamma) - KE_X$

This relation can be written : $[m_{before} - m_{after}]c^2 = E_{after} - E_{before}$;

The father nucleus is taken at rest $KE_X = 0$, the daughter nucleus is practically at rest KE_Y (negligible KE compared to that of the radiation.

Thus, the conservation of energy becomes: $E_\ell = \Delta m\, c^2 = KE_\alpha + E_\gamma$.

3. Beta minus decay β⁻

Beta minus β^- decay is the emission of an electron ${}_{-1}^{0}e$ which is always accompanied by an antineutrino ${}_{0}^{0}\bar{\upsilon}$.

The balanced equation is: ${}_{Z}^{A}X \longrightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + {}_{0}^{0}\bar{\upsilon} + \gamma$;

Characteristics of the antineutrino:

The antineutrino ${}_{0}^{0}\bar{\upsilon}$ has the properties:

- extremely small mass.
- speed equal to that of light.
- charge less.
- does not undergo ***any interaction*** with matter, which makes it is very ***difficult*** to detect.
- it has an enormous penetrating power; an antineutrino of $1 MeV$ can pass through a thickness of water of light-year, which about $10^{13} km$.

The emitted β^- particles, have the following properties:

- they are less ionizing than α particles.
- their penetrating power is few meters in air and about $7\,mm$ in aluminum.

β^- decay characterizes nuclei ***rich in neutrons*** according to the reaction ${}_{0}^{1}n \longrightarrow {}_{1}^{1}p + {}_{-1}^{0}e + {}_{0}^{0}\bar{\upsilon}$

Conservation of energy: $E_\ell = KE_{\beta^-} + E({}_{0}^{0}\bar{\upsilon}) + E_\gamma$.

4. Beta plus decay β^+

Beta plus decay β^+ is the emission of ***positron*** ${}_{+1}^{0}e$ (positive electron or antiparticle of electron and has a mass equal to that of electron but opposite charge) which is always accompanied by the emission of a ***neutrino*** ${}_{0}^{0}\upsilon$.

The balanced equation is: ${}_{Z}^{A}X \longrightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e + {}_{0}^{0}\upsilon + \gamma$.

This decay occurs in nuclei ***rich in protons*** according to the reaction: ${}_{1}^{1}p \longrightarrow {}_{+1}^{0}e + {}_{0}^{1}n + {}_{0}^{0}\upsilon$

Conservation of energy: $E_\ell = KE_{\beta^+} + E\left({}_{0}^{0}\upsilon\right) + E_\gamma$.

Note: quantification of the energy of the radiations

Type of decay	Conservation of Energy	Quantified Energy
Alpha	$E_\ell = KE_\alpha + E_\gamma$	KE_α
Beta minus	$E_\ell = KE_{\beta^-} + E_{{}_0^0\overline{\upsilon}} + E_\gamma$	$KE_{\beta^-} + E_{{}_0^0\overline{\upsilon}}$
Beta plus	$E_\ell = KE_{\beta^+} + E_{{}_0^0\upsilon} + E_\gamma$	$KE_{\beta^+} + E_{{}_0^0\upsilon}$

IV-Radioactive Properties

1. Activity

The activity of a radioactive sample A is the number of disintegrations per unit of time. If the unit of time is the second then the activity is measured in ***Becquerel*** (Bq).

2. Period T

The period (or half-life) is the duration after which the activity of a radioactive sample is reduced to its half. The decay constant (radioactive constant) λ is given by: $\lambda = \dfrac{\ell n(2)}{T}$.

3. Law of radioactive decay

If N_0 is the number of nuclei at $t=0$, and N is the number of nuclei remaining at the instant t, then: $N = N_0 e^{-\lambda t}$ (after a period $t = T$, we get $N = \dfrac{N_0}{2}$).

4. Activity and decays

The activity is defined as the number of disintegrations N_d per unit of time Δt.

So the average activity is $\overline{A} = \dfrac{N_d}{\Delta t}$ where $N_d = N_0 - N$.

Then $\overline{A} = \dfrac{N_0 - N}{\Delta t} = -\dfrac{N - N_0}{\Delta t} = -\dfrac{\Delta N}{\Delta t}$, the instantaneous activity is $A = -\dfrac{dN}{dt}$.

Thus $A = A_0 e^{-\lambda t}$ (law of radioactive decay relative to the activity) & $A = \lambda N$.

5. Number of nuclei N in a sample of mass m

$N = \frac{m}{M} N_A$ where M is the molar mass of the element & N_A is Avogadro's number.

Note: we can also use $N = \frac{m}{m_{\text{nucleus}}}$ where m is the mass of the sample.

6. Power transmitted

The power transmitted to an object (or absorbed by an organism) from a radioactive sample could be calculated by: $P = A \times E_{ab}$ where E_{ab} is the energy absorbed due to the disintegration of one nucleus & A is the activity of this sample measured in Bq (number of disintegration per second).

Note 1:

Nowadays dating fossils, rocks and other is performed using radioactive element.
The choice of the element depends on the range of age of the studied sample.

Couple	Period in years	Domain of age in years	Material
Carbon ^{14}C - Nitrogen ^{14}N	5570	Till 50000	Wood, bones…
Potassium ^{40}K - Argon ^{40}Ar	1.25×10^9	$10^{-5} ... 10^7$	Rocks, volcanos…
Uranium ^{238}U - Lead ^{206}Pb	4.5×10^9	$10^7 ... 10^9$	Rocks
Rubidium ^{87}Rb - Strontium ^{87}Sr	4.9×10^{10}	$10^7 ... 10^9$	Granite & volcanic rocks…

Note 2:

The use of any couple of radioelement to determine the age of a fossil… is accurate as long the age of the fossil used does not exceeds 10periods of the radioelement: $\Delta t_{\max} = 10T$

Applications

Given:

- Mass of the electron: $m(e) = 0.00055\ u$;
- Mass of the proton: $m_p = 1.00728\ u$;
- Mass of the neutron: $m_n = 1.00866\ u$;
- Planck's constant: $h = 6.63 \times 10^{-34}\ J.s$;
- Speed of light in vacuum: $c = 3 \times 10^8\ m/s$;
- Avogadro's number: $N_A = 6.022 \times 10^{23}\ /mo\ell$;
- $1u = 931.5\ MeV/c^2$;
- $1\ MeV = 1.6 \times 10^{-13}\ J$.

I-A
Radium

Radium's isotope 226 $\left({}^{226}_{88}Ra\right)$ is radioactive and alpha emitter, the daughter nucleus produced by this decay is the radon ${}^{A}_{Z}Rn$.

Given the masses of some nuclei:

- $m\left({}^{226}_{88}Ra\right) = 225.97905\ u$; $\quad m\left({}^{A}_{Z}Rn\right) = 221.97234\ u$; $\quad m(\alpha) = 4.00148\ u$.

1. Write the equation of this decay specifying the laws used.
2. Identify the particle emitted and give two properties for this type of emission.
3. Calculate the energy liberated by this decay.
4. Some disintegrations are accompanied by gamma radiation whose wavelength is $2.5\ pm$.
   a) Due to what is the emission of this radiation?
   b) Deduce the kinetic energy of the alpha particle emitted.

I-B
Dynamics of disintegration

The radium, in the disintegration of application I, is taken at rest, show that the ratio of the kinetic energies of radon to that of alpha is given by: $\dfrac{KE(Rn)}{KE_\alpha} = \dfrac{m(\alpha)}{m(Rn)}$.

Justify that the kinetic energy of radon is negligible compared to that carried by alpha.

II-
Sodium

Sodium 24 $\left({}^{24}_{11}Na\right)$ is radioactive, it decays into magnesium nucleus ${}^{24}_{12}Mg$.

Given the masses of some nuclei:

- $m\left({}^{24}_{11}Na\right) = 23.98493\ u$; $\quad m\left({}^{24}_{12}Mg\right) = 23.97846\ u$;

1. Write the complete equation of this decay specifying the laws used.
2. Identify the particle emitted.

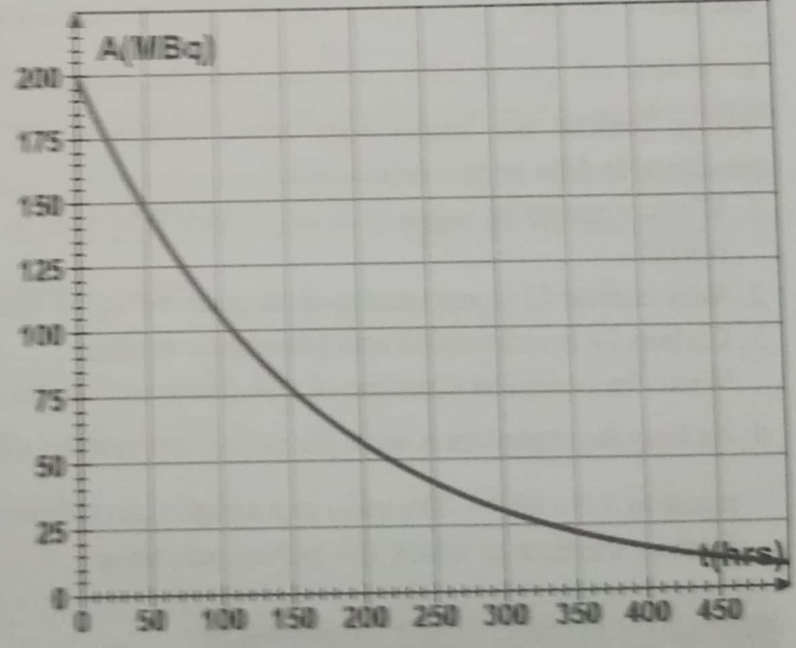
3. The previous emission of a particle is due to the disintegration of a nucleon inside the nucleus.
Write the equation of the reaction corresponding to this emission.
4. Calculate the energy liberated by this disintegration.
5. Determine the binding energy per nucleon of sodium and that of magnesium.
Compare the stability of these two nuclei.
6. The sodium nucleus is taken at rest, the daughter nucleus is obtained practically at rest in the ground state, and the kinetic energy carried by the particle emitted is $0.4\ MeV$.
Determine the energy carried by the antineutrino.

The Old Element

Some element such as samarium ^{149}Sm has a very long period, even larger than that of the universe.
A sample of 10g has an activity of $2.21 Bq$. The molar mass of samarium is $149\ g/mol$.
1. Define the activity of a radioactive sample.
2. Determine, in years, the period of this element.

Fluorine & Diagnosis

Fluorine18 $\left(^{18}_{9}F\right)$ is radioactive and β^{+} emitter and the daughter nucleus produced is an isotope of oxygen $^{?}_{Z}O$. To diagnose and monitor certain diseases such as lung cancer, breast cancer; we inject in patient's vein a solution of fluorodeoxyglucose (FDG) containing radioactive fluorine.
1. Write the equation of this disintegration specifying the laws used.
2. The binding energy of oxygen18 nucleus is $140\ MeV$ while that of Fluorine18 is $119\ MeV$.
Specify which nucleus is more stable.
3. The adjacent curve represents the evolution of the activity A of fluorine nuclei in terms of time t.
Referring to the curve:
a) give the initial activity A_0 of this sample.
b) determine the period of fluorine.
4. Deduce the initial number of fluorine nuclei injected.
5. The monitor of radioactive tracers is possible as long its activity remains larger than $5\ MBq$. Determine the duration over which the diagnosis is possible.



Age of a Rock

To determine the age of a volcanic rock, we use the technique of dating by the potassium 40.
Potassium 40 $\left(^{40}_{19}K\right)$ is radioactive and beta plus emitter, the daughter nucleus is argon $^{A}_{Z}Ar$.

Given masses of some nuclei:

- Potassium: $m\left({}^{40}_{19}K\right) = 39.9740\,u$.
- Argon: $m\left({}^{40}_{18}Ar\right) = 39.9640\,u$.
- Beta plus: $m\left({}^{0}_{1}\beta^{+}\right) = 0.00055\,u$.
- The period of potassium 40 is 1.27×10^{9} years.
- The molar mass of potassium and argon are supposed equal.

1. Write the equation of the disintegration of potassium 40.
2. Determine, in MeV, the energy liberated by the decay of one nucleus of potassium.
3. In what form does this energy liberated appear?
4. We suppose that at the instant of formation of the volcanic rock $t_0 = 0$, it contains only potassium and that all the argon is due to the disintegration of potassium. An analysis of volcanic rock shows that it contains $m_K = 2.6mg$ of potassium and $m_{Ar} = 0.004mg$ of argon.

   a) Show that the age of the rock is given by $t = \frac{T}{\ln 2} \ln\left(1 + \frac{m_{Ar}}{m_K}\right)$.

   b) Deduce its age.
5. Explain why it is not useful to use Carbon 14 whose half-life is 5570 years, to determine the age of this rock.

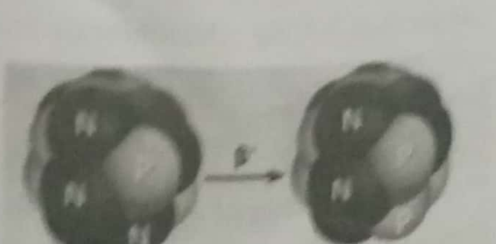
V-
Otzi the Iceman

"Otzi the Iceman" is a man found frozen by German tourists in glacier in the mountains of the Alpes and he seen today in a museum in Italy. We can estimate when he lives using carbon-14. The carbon has many isotopes ${}^{11}_{6}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$ & ${}^{14}_{6}C$, the abundance of ${}^{12}_{6}C$ is 98.9%, of ${}^{13}_{6}C$ is 1.1%, whereas the other isotopes ${}^{11}_{6}C$ & ${}^{14}_{6}C$ whose periods are respectively about 20min & 5700years are produced in labs or by cosmic rays.

1. What is meant by isotopes of an element? Specify by what they differ?
2. Why carbon 11 is not used to determine the age of Otzi man?
3. Carbon 14 is radioactive and beta minus emitter.
   Write the complete equation of this decay and identify the daughter nucleus.
4. As long the organism is alive the ratio of the number of nuclei ${}^{14}_{6}C$ to that of ${}^{12}_{6}C$ remains constant equal to 1.3×10^{-12}. The ratio in a sample taken from the frozen body is found 6.72×10^{-13}. Deduce the time at which this person was alive.

VI-
Power Absorbed

The molybdenum ${}^{99}_{42}Mo$ is β^{-} emitter, disintegrates into technetium (Tc) and its period is 5.8hours.

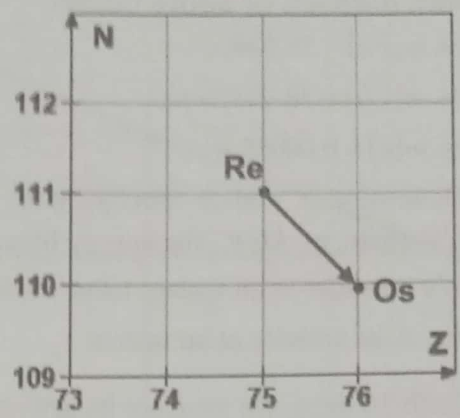


1. Write the equation of the decay of ${}^{99}_{42}Mo$.
2. 20g of molybdenum are placed in a container absorbing only β^{-} particles that carries each $0.4MeV$.

a) Calculate the number of electrons emitted during 3 hours.
b) Deduce the average power absorbed by the container during this duration.

VIII- Treatment with Rhenium

Radioactive elements are widely used in medical domain for treatment and diagnosis purposes. One of these applications, is the injection of the rhenium radioactive isotope $^{A}_{Z}Re$, that disintegrates giving the osmium daughter nucleus $^{A'}_{Z'}Os$, and used as a pain relieve for rheumatism through local injection on joints and for those carrying cancer liver.



- Molar mass of rhenium is $186\,g/mo\ell$.

1. Referring to the adjacent diagram (where N is the number of neutrons), determine Z, Z', A & A'.
2. Write the equation of the disintegration of rhenium, indicate the particle emitted.
3. The period of rhenium-186 is $T = 90$ days and the initial activity of the portion injected is $4\times10^{9}\,Bq$.
   a) Calculate, in s^{-1}, the radioactive constant of the element.
   b) Determine the number of nuclei N_0 present in the portion injected.
   c) Deduce the mass of the sample used.
4. The patient should not take another injection until the activity is reduced to 20% of its initial value. Determine the minimum number of days between two injections.

IX- Thorium in Sea Rocks

Thorium $^{230}_{90}Th$ in sea rocks is due to the decay of uranium $^{234}_{92}U$, for this we find these two nuclei in all the rocks but with different proportions that depends on its age. We suppose that at the instant of formation of the rock $t_0 = 0$, the rock contains only uranium 234. A study shows that the ratio of the number of thorium nuclei to that of uranium is 35%.

Given masses of some nuclei:

- $m(U) = 234.04083\ u$;
- $m_p = 1.00728\,u$;
- $m_n = 1.00866\ u$.

The half-life of uranium 234 is $T = 2.5\times10^{5}$ years.

1. Write down the equation of the disintegration of uranium 234 and identify the emitted particle.
2. Calculate the binding energy per nucleon of $^{234}_{92}U$.
3. Compare this energy to that of thorium.
4. Determine the relation between the number of nuclei of thorium, uranium present in the sample $N\left(^{230}_{90}Th\right)$, $N\left(^{234}_{92}U\right)$ with t & T.
5. Deduce the age of formation of this rock.
6. Calculate the value of the number of uranium nuclei to that of thorium after 10 periods. Conclude.

X-Bac
Radioactivity of Iron Isotope

Iron $^{59}_{26}Fe$ is radioactive and β^- emitter of radioactive period T. The daughter nucleus produced by this decay is an isotope of cobalt $^{A}_{Z}Co$.

Given masses of some nuclei:

- $m(Fe) = 58.93488\ u$;
- $m(Co) = 58.93320\ u$;
- $m(e) = 0.00055\ u$.

1. Determine Z and A specifying the laws used.
2. Calculate, in MeV, the energy liberated by this decay.
3. We consider at an instant taken as origin of time $t_0 = 0$ a mass $m_0 = 1mg$ of Fe & we designate by A its activity at an instant t.

   Each 10 days, we measure its activity we notice the ratio $\dfrac{A(t)}{A(t+10)} = 1.17$ remains constant.

   a) Determine the number of nuclei present in the sample at $t_0 = 0$.

   b) Write down the expression of the activity A as a function of t and the meaning of each term.

   c) Show that $T = 44.1$ days.

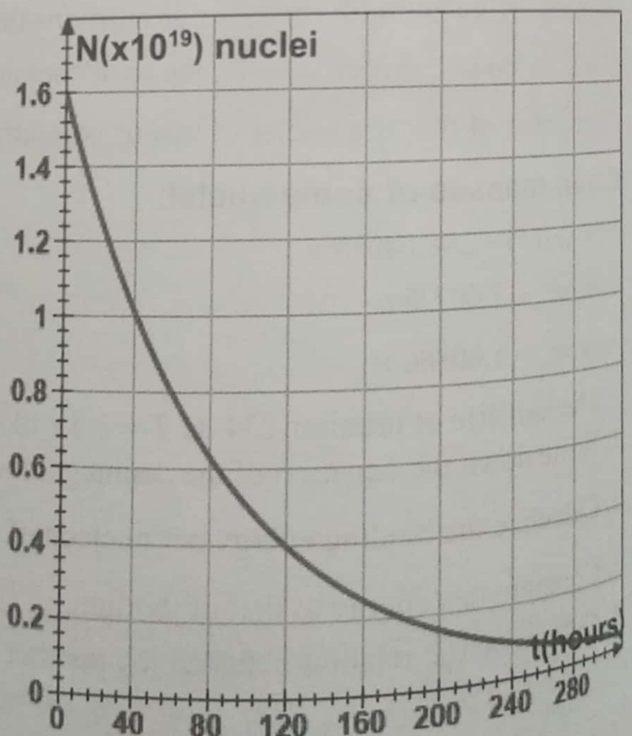
   d) Deduce, in Bq, the activity of the sample at $t_0 = 0$.

   e) Draw the graph representing the variation of the activity in terms of time showing T.

XI-
Bromine

Bromine $^{77}_{35}Br$ is radioactive and beta plus emitter, the daughter nucleus produced by this decay is the selenium $^{A}_{Z}Se$. A bromine 77 sample of mass m_0 taken at the instant $t_0 = 0$.

- Molar mass of bromine is $77 g/mol$.



1. Write the equation of this decay specifying the laws used.
2. The adjacent graph represent the variation of the number of bromine nuclei N present in the sample in terms of time.

   a) Give the number of nuclei N_0 present in the sample at $t_0 = 0$.

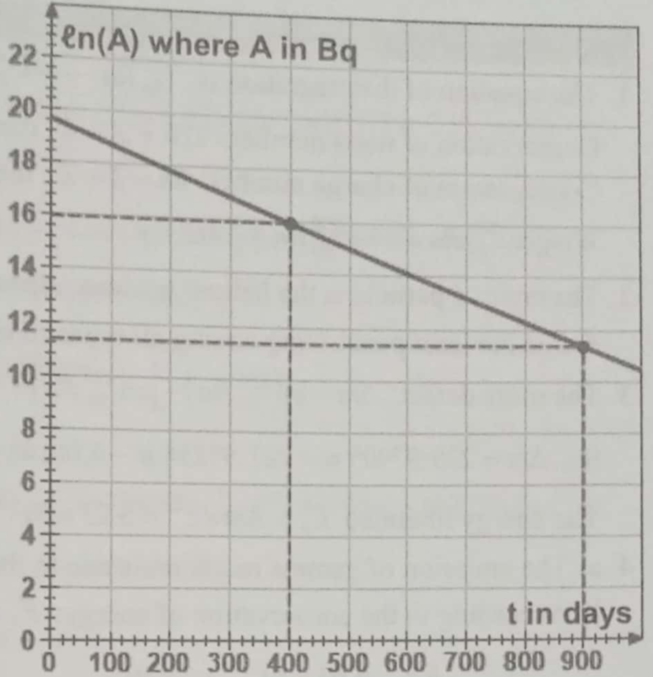
   b) Determine, using the graph, the period T of Bromine.

   c) Deduce m_0.
3. Show that the number of nuclei of selenium at an instant t is given by $N_{Se} = N_0\left(1 - e^{-\frac{\ln 2}{T}t}\right)$.
4. Deduce N_{Se} at $t = 160$ hours & $t = 280$ hours.
5. Copy the adjacent graph representing the variation of N_{Se} in terms of time.

XII-Graphical Study

The tungsten isotope $^{185}_{74}W$ is radioactive.

The study of the evolution of the natural logarithm of the activity $\ell n(A)$ of a tungsten sample in terms of time is shown in the adjacent figure.



The molar mass of tungsten $185\ g/mo\ell$.

1. Show that the expression of $\ell n(A)$ is compatible with the shape of the curve shown in the adjacent figure.
2. Define the activity and then the period of a radioactive sample.
3. Referring to the adjacent curve, determine:
   a) in Bq, the activity of the studied sample at the instant $t_0 = 0$.
   b) the radioactive constant.
4. Deduce:
   a) in days, the period of tungsten.
   b) the initial mass m_0 of the tungsten sample.

XIII-Engineering (2009/2010)
The Plutonium

The fifteen known isotopes of plutonium $\left({}_{94}Pu\right)$ are alpha emitter except the plutonium 241, which is beta emitter.

Plutonium has been used to make nuclear weapons (such as atomic bombs), in nuclear power plants to produce electricity, and as portable energy supply in space probes and vehicles.

Molar mass of plutonium 238: $M = 238\ g.mo\ell^{-1}$.

A heart pacemaker contains $m_0 = 130\ mg$ of the isotope 238 of plutonium Pu which is alpha emitter and whose half-life is $T = 87.8$ years that decays into a uranium nucleus U.

The energy released by each disintegration allows the pacemaker to produce electric energy.

1. a) Write the equation of disintegration of plutonium 238 nucleus, specifying the laws used.
   b) This reaction is accompanied by the emission of gamma radiation. Due to what is this emission?
2. a) Calculate the number N_0 of nuclei initially present in the pacemaker.
   b) Deduce the activity A_0 of this pacemaker.
   c) The pacemaker functions correctly as long as its activity remains higher or equal to $5.76\times10^{10}\ Bq$. Calculate the duration of its normal functioning under these conditions.
3. If we assume that all the nuclear energy is converted into electric energy and that one disintegration liberates $5.5\,MeV$.

   Calculate the average electric power furnished over the first five years.

I-A

1. The equation of disintegration is: ${}^{226}_{88}Ra \longrightarrow {}^{A}_{Z}Rn + {}^{4}_{2}He + \gamma$;
Conservation of mass number: $226 = A + 4$, then $A = 226 - 4 = 222$;
Conservation of charge number: $88 = 2 + Z$, then $Z = 88 - 2 = 86$;
We get ${}^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}He + \gamma$.
2. The emitted particle is the helium nucleus, alpha emissions takes place in heavy nuclei $(A > 200)$ & it is the least penetrating among all types of emissions.
3. The mass defect: $\Delta m = m\left({}^{226}_{88}Ra\right) - \left[m\left({}^{222}_{86}Rn\right) + m\left({}^{4}_{2}He\right)\right]$;
So, $\Delta m = 225.97905\,u - 221.97234\,u - 4.00148\,u = 5.23 \times 10^{-3}\,u$;
The energy liberated $E_\ell = \Delta m \times c^2 = 5.23 \times 10^{-3} \times 931.5 = 4.872\ MeV$.
4. a) The emission of gamma radiation is due to the dis-excitation of the daughter nucleus Rn.
b) According to the conservation of energy: $E_\ell = KE_\alpha + E_\gamma = 4.872\ MeV$;
But $E_\gamma = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2.5 \times 10^{-12}} = 7.956 \times 10^{-14}\,J = \dfrac{7.956 \times 10^{-14}}{1.6 \times 10^{-13}} = 0.497\ MeV$.
Then $KE_\alpha = E_\ell - E_\gamma = 4.872\ MeV - 0.497\ MeV = 4.375\ MeV$.

I-B

Just Before	**Ra at rest**	
Just Before	Rn	He

The ratio of kinetic energies $\dfrac{KE(Rn)}{KE_\alpha} = \dfrac{\frac{1}{2} m_{Rn} v_{Rn}^2}{\frac{1}{2} m_\alpha v_\alpha^2} = \dfrac{m_{Rn} v_{Rn}^2}{m_\alpha v_\alpha^2}$;

The linear momentum is conserved, $\overrightarrow{P_{Ra}} = \overrightarrow{P_{Rn}} + \overrightarrow{P_\alpha}$ (before and after disintegration);

But the father nucleus Ra is taken at rest; so, , $m_{Rn}\overrightarrow{v_{Rn}} + m_\alpha \overrightarrow{v_\alpha} = \vec{0}$; then $m_{Rn} v_{Rn} = m_\alpha v_\alpha$;

Thus, $\dfrac{KE(Rn)}{KE_\alpha} = \dfrac{m_\alpha (m_{Rn} v_{Rn})^2}{m_{Rn} (m_\alpha v_\alpha)^2} = \dfrac{m_\alpha}{m_{Rn}}$;

Using the numerical values, $\dfrac{KE(Rn)}{KE_\alpha} = \dfrac{m_\alpha}{m_{Rn}} = \dfrac{4.00148\,u}{221.97234\,u} = 0.018 = 1.8\%$;

$KE(Rn) = 1.8\% KE_\alpha$, then the kinetic energy of Rn is negligible compared to that carried by alpha particle.

II-

1. The equation of disintegration is: ${}^{24}_{11}Na \longrightarrow {}^{24}_{12}Mg + {}^{A}_{Z}X$;
Conservation of mass number: $24 = 24 + A$, then $A = 24 - 24 = 0$;
Conservation of charge number: $11 = 12 + Z$, then $Z = 11 - 12 = -1$;
We get: ${}^{24}_{11}Na \longrightarrow {}^{24}_{12}Mg + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon} + \gamma$.

2. The particle emitted is the electron ${}_{-1}^{0}e$.

3. The emission of the electron is due to the disintegration of a neutron: ${}_{0}^{1}n \longrightarrow {}_{1}^{1}p + {}_{-1}^{0}e + {}_{0}^{0}\bar{\upsilon}$.

4. The mass defect: $\Delta m = m\left({}_{11}^{24}Na\right) - \left[m\left({}_{12}^{24}Mg\right) + m\left({}_{-1}^{0}e\right)\right]$;

So, $\Delta m = 23.98493\ u - 23.97846\ u - 0.00055\ u = 5.92 \times 10^{-3}\ u$;

The energy liberated by one disintegration $E_{\ell} = \Delta m \times c^2 = 5.92 \times 10^{-3} \times 931.5 = 5.51\ MeV$.

5. For ${}_{11}^{24}Na$ nucleus, the mass defect:

$\Delta m = Zm_p + (A - Z)m_n - m_X = 11 \times (1.00728u) + 13 \times (1.00866u) - 23.98493u = 0.20773u$;

The binding energy $E_b = \Delta m c^2 = 0.20773 \times 931.5 = 193.5 MeV$;

The binding energy per nucleon $\dfrac{E_b}{A}\left({}_{11}^{24}Na\right) = 8.1\ MeV$.

For ${}_{12}^{24}Mg$ nucleus, the mass defect:

$\Delta m = Zm_p + (A - Z)m_n - m_X = 12(1.00728u) + 12(1.00866u) - 23.97846u = 0.21282u$;

The binding energy $E_b = \Delta m c^2 = 0.21282 \times 931.5 = 198.2 MeV$;

The binding energy per nucleon $\dfrac{E_b}{A}\left({}_{12}^{24}Mg\right) = 8.3\ MeV$.

The magnesium nucleus is more stable than that of sodium.

6. According to the conservation of energy : $E_{\ell} = KE_{\beta^-} + E\left({}_{0}^{0}\bar{\upsilon}\right) + E_{\gamma}$; but $E_{\gamma} = 0$, the daughter is obtained in ground state. Then $E\left({}_{0}^{0}\bar{\upsilon}\right) = E_{\ell} - KE_{\beta^-} = 5.51 - 0.4 = 5.11\ MeV$.

III-

1. The activity A of a radioactive sample is the number of disintegrations per unit of time.
   In SI units, the activity is measured in Becquerel (Bq).

2. We have $A = \lambda N$ where $N = \dfrac{m}{M}N_A$ & $\lambda = \dfrac{\ell n(2)}{T}$;

Then $T = \dfrac{\ell n(2)}{A} \times \dfrac{m}{M} \times N_A = \dfrac{\ell n(2)}{2.21} \times \dfrac{10\ g}{149\ g/mo\ell} \times 6.022 \times 10^{23} mo\ell^{-1} = 1.27 \times 10^{22} s$;

Thus, $T = \dfrac{1.27 \times 10^{22}}{60 \times 60 \times 24 \times 365} = 4.03 \times 10^{14}$ years.

IV-

1. The equation of disintegration is: ${}_{9}^{18}F \longrightarrow {}_{Z}^{A}O + {}_{+1}^{0}e + {}_{0}^{0}\upsilon + \gamma$;

Conservation of mass number: $18 = A + 0$, then $A = 18 - 0 = 18$;

Conservation of charge number: $9 = Z + 1$, then $Z = 9 - 1 = 8$;

We get: ${}_{9}^{18}F \longrightarrow {}_{8}^{18}O + {}_{+1}^{0}e + {}_{0}^{0}\upsilon + \gamma$.

2. The binding energies per nucleon $\dfrac{E_b\left({}_{9}^{18}F\right)}{A} = \dfrac{119}{18} = 6.61 MeV$ & $\dfrac{E_b\left({}_{8}^{18}O\right)}{A} = \dfrac{140}{18} = 7.77\ MeV$;

Then oxygen ${}_{8}^{18}O$ is more stable than fluorine ${}_{9}^{18}F$, which confirm the fact that the radioactivity is an evolution into a more stable state.

3. a) From graph, we have $A_0 = 200\ MBq$.

b) We know that at $t = T$ (after 1 period the activity is reduced to its half), $A = \frac{A_0}{2} = 100\ MBq$;

Then, $T = 110$ hours.

4. We have $A_0 = \lambda N_0$; $N_0 = \frac{A_0}{\lambda} = \frac{A_0 \times T}{\ell n(2)} = \frac{200 \times 10^6 \times 110 \times 60 \times 60}{\ell n(2)} = 1.14 \times 10^{14}$ fluorine nuclei.

5. Law of radioactive decay: $A = A_0 e^{-\lambda t} \geq 5\ MBq$;

Then $200 e^{-\lambda t} \geq 5$; $e^{-\lambda t} \geq \frac{1}{40}$, so $-\lambda t \geq -\ell n(40)$;

We get $t \leq \frac{T}{\ell n(2)} \times \ell n(40) = \frac{110}{\ell n(2)} \times \ell n(40)$; thus, $t \leq 585$ hours.

V-

1. The equation of the disintegration is: ${}^{40}_{19}K \longrightarrow {}^{A}_{Z}Ar + {}^{0}_{+1}e + {}^{0}_{0}\upsilon + \gamma$;

Conservation of mass number: $40 = 0 + A$, then $A = 40 - 0 = 40$;

Conservation of charge number: $19 = 1 + Z$, then $Z = 19 - 1 = 18$;

We get: ${}^{40}_{19}K \longrightarrow {}^{40}_{18}Ar + {}^{0}_{+1}e + {}^{0}_{0}\upsilon + \gamma$.

2. The mass defect: $\Delta m = m({}^{40}_{19}K) - [m({}^{40}_{18}Ar) + m({}^{0}_{1}e)]$;

So, $\Delta m = 39.9740\ u - 39.9640\ u - 0.00055\ u = 9.45 \times 10^{-3} u$;

The energy liberated $E_\ell = \Delta m \times c^2 = 9.45 \times 10^{-3} \times 931.5 MeV = 8.8 MeV$.

3. The energy liberated appears as kinetic energy carried by the positrons, and radiant energy carried by the radiations gamma (the energy carried by the neutrinos is usually not detected).

4. a) Let N_{0K} be the number of potassium nuclei at the instant of formation of the rock at $t_0 = 0$, N_K be the number of potassium nuclei at the instant t & N_{Ar} the number of argon nuclei.

We have $N_K = \frac{m_K}{M_K} \times N_A$ & $N_{Ar} = \frac{m_{Ar}}{M_{Ar}} \times N_A$; the number of potassium nuclei disintegrated is equal to the number of argon nuclei formed, then $N_{0K} = N_K + N_{Ar}$;

Law of radioactive decay: $N_K = N_{0K} e^{-\lambda t} = (N_K + N_{Ar}) e^{-\lambda t}$;

So, $e^{\lambda t} = \frac{N_K + N_{Ar}}{N_K} = 1 + \frac{N_{Ar}}{N_K}$; then $\lambda t = \ell n\left(1 + \frac{N_{Ar}}{N_K}\right)$ where $\lambda = \frac{\ell n(2)}{T}$;

Thus, $t = \frac{T}{\ell n(2)} \times \ell n\left(1 + \frac{\frac{m_{Ar}}{M_{Ar}} \times N_A}{\frac{m_K}{M_K} \times N_A}\right) = \frac{T}{\ell n(2)} \times \ell n\left(1 + \frac{m_{Ar}}{m_K}\right)$, (K & Ar have same molar mass);

b) The age of the rock is $t = \frac{1.27 \times 10^9}{\ell n(2)} \times \ell n\left(1 + \frac{0.004}{2.8}\right) = 2.62 \times 10^6$ years.

5. The half-life of carbon 14 is about 5570 years, which is very short compared to the age of the studied rock; after 2.62×10^6 years (which exceeds 10 periods) carbon-14 vanishes practically which make it impossible to determine the age of the sample.

VI-

1. Isotopes are nuclides having same atomic number but different mass number; isotopes differ only by their number of neutrons.
2. The period of carbon 11 is about 20 min, which is very short; then the number of nuclei will be practically zero within few hours and no trace left to date the organism over long duration.
3. The equation of disintegration: ${}^{14}_{6}C \longrightarrow {}^{A}_{Z}X + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$;

   Conservation of mass number: $14 = A + 0$, then $A = 14$;

   Conservation of charge number: $6 = Z - 1$, then $Z = 7$;

   The daughter nucleus is an isotope of nitrogen ${}^{14}_{7}N$.
4. Let N_0 be the initial number of nuclei at $t_0 = 0$, and N is the number of nuclei at the present instant t.

   Law of radioactive decay $N = N_0 e^{-\lambda t}$; but the number of carbon nuclei $N\left({}^{12}C\right)$ remains constant; $\frac{N}{N\left({}^{12}C\right)} = \frac{N_0}{N\left({}^{12}C\right)} e^{-\lambda t}$, $r = \frac{N}{N\left({}^{12}C\right)} = 6.72 \times 10^{-13}$ & $r = \frac{N_0}{N\left({}^{12}C\right)} = 1.3 \times 10^{-12}$;

   So, $e^{-\lambda t} = 0.5169$, then $t = -\frac{T}{\ell n(2)} \times \ell n(0.5169) = -\frac{5700}{\ell n(2)} \times \ell n(0.5169) \approx 5427$ years.

   Thus, Otzi man was alive 5427 years ago $(5427 - 2017 = 3410\ BC)$.

VII-

1. Equation of the disintegration ${}^{99}_{42}Mo \longrightarrow {}^{A}_{Z}Tc + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$;

   Conservation of mass number: $99 = A + 0$, then $A = 99$;

   Conservation of charge number: $42 = Z - 1$, then $Z = 43$;

   Thus, the equation of the disintegration becomes ${}^{99}_{42}Mo \longrightarrow {}^{99}_{43}Tc + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$.
2. a) The number of Mo nuclei in this mass is:

   $$N_0 = \frac{m_0}{M} \times N_A = \frac{20\ g}{99\ g/mo\ell} \times 6.022 \times 10^{23}\ mo\ell^{-1} = 1.22 \times 10^{23} \text{ nuclei.}$$

   The number of nuclei remaining after 3 hours is given by the law of radioactive decay:

   $$N = N_0 e^{-\lambda t} = 1.22 \times 10^{23}\ e^{-\frac{\ell n2}{5.8} \times 3} = 8.5 \times 10^{22} \text{ nuclei.}$$

   The number of emitted electrons N_e is equal to the number of disintegrated nuclei N_d:

   $$N_e = N_d = N_0 - N = 1.22 \times 10^{23} - 8.5 \times 10^{22} = 3.7 \times 10^{22} \text{ electrons.}$$

   b) The average power absorbed:

   $$P_{ab} = \frac{E_{ab}}{\Delta t} = \frac{N_e \times KE_{\beta^-}}{\Delta t} = \frac{3.7 \times 10^{22} \times 0.4 \times 1.6 \times 10^{-13}}{3 \times 60 \times 60} = 219259\ W.$$

VIII-

1. From diagram $Z = 75$ & $N = 111$, then $A = 75 + 111 = 186$.

   From diagram $Z' = 76$ & $N' = 110$, then $A' = 76 + 110 = 186$.
2. The equation of disintegration: ${}^{186}_{75}Re \longrightarrow {}^{186}_{76}Os + {}^{A}_{Z}X$;

   Conservation of mass number: $186 = 186 + A$, then $A = 0$;

Conservation of charge number: $75 = 76 + Z$, then $Z = -1$;

We get: ${}^{186}_{75}Re \longrightarrow {}^{186}_{76}Os + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon} + \gamma$.

The particle emitted ${}^{0}_{-1}e$ is the electron.

3. a) The radioactive constant $\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{90 \times 24 \times 60 \times 60} = 8.91 \times 10^{-8}\ s^{-1}$.

b) The number of nuclei is $N_0 = \frac{A_0}{\lambda} = \frac{4 \times 10^9}{8.91 \times 10^{-8}} = 4.49 \times 10^{16}$.

c) The mass of the sample injected is given by: $N_0 = \frac{m_0}{M} \times N_A$,

So $m_0 = \frac{N_0}{N_A} \times M = \frac{4.49 \times 10^{16}}{6.02 \times 10^{23}\ / mo\ell} \times 186\ g/mo\ell$; thus, $m_0 = 1.39 \times 10^{-5}\ g = 13.9\ \mu g$.

4. $A = 20\% A_0 = 0.2 A_0$; by referring to the law of radioactive decay: $A = A_0\, e^{-\lambda t}$;

We get $t = -\frac{T}{\ell n(2)} \times \ell n\left(\frac{A}{A_0}\right) = -\frac{90}{\ell n(2)} \times \ell n(0.2) \approx 209$ days.

IX-

1. The equation of disintegration is: ${}^{234}_{92}U \longrightarrow {}^{230}_{90}Th + {}^{A}_{Z}X + \gamma$;

Conservation of mass number: $A = 234 - 230 = 4$;

Conservation of charge number: $Z = 92 - 90 = 2$;

The emitted particle is the helium nucleus ${}^{4}_{2}He$.

2. The binding energy is given by $E_b = \Delta m c^2$, where Δm is the mass defect;

$\Delta m = Z m_p + (A - Z) m_n - m(U) = 92 \times 1.00728 + (234 - 92) \times 1.00866 - 234.04083 = 1.85865\ u$;

Then, $E_b = 1.85865 \times 931.5 = 1731.3\ MeV$;

The binding energy per nucleon is $\frac{E_b(U)}{A} = \frac{1731.3}{234} = 7.4\ MeV$.

3. The radioactivity is an evolution into a more stable state, then the binding energy per nucleon of thorium should be greater than that of uranium; thus, $\frac{E_b(Th)}{A} > 7.4\ MeV$.

4. Law of radioactive decay: $N\left({}^{234}_{92}U\right) = N_0\left({}^{234}_{92}U\right) e^{-\lambda t}$;

Where $N_0\left({}^{234}_{92}U\right)$ is the number of nuclei of uranium at the instant of formation of the rock;

But $N_0\left({}^{234}_{92}U\right) = N\left({}^{234}_{92}U\right) + N\left({}^{230}_{90}Th\right)$; the number of nuclei of uranium disintegrated is equal to the number of thorium formed; then, $N\left({}^{234}_{92}U\right) = \left[N\left({}^{234}_{92}U\right) + N\left({}^{230}_{90}Th\right)\right] e^{-\lambda t}$.

5. We have $N\left({}^{234}_{92}U\right) = 35\% N\left({}^{234}_{92}U\right) = 0.35 N\left({}^{234}_{92}U\right)$; so $N\left({}^{234}_{92}U\right) = \left[N\left({}^{234}_{92}U\right) + 0.35\, N\left({}^{234}_{92}U\right)\right] e^{-\lambda t}$;

We get, $1 = 1.35\, e^{-\lambda t}$; thus $t = -\frac{T}{\ell n(2)} \times \ell n\left(\frac{1}{1.35}\right) = \frac{2.5 \times 10^5}{\ell n(2)} \times \ell n(1.35) = 108240$ years.

6. The rate $\frac{N\left({}^{234}_{92}U\right)}{N\left({}^{230}_{90}Th\right)}=\frac{N_0\left({}^{234}_{92}U\right)e^{-\lambda t}}{N_0\left({}^{234}_{92}U\right)-N\left({}^{234}_{92}U\right)}=\frac{N_0\left({}^{234}_{92}U\right)e^{-\lambda t}}{N_0\left({}^{234}_{92}U\right)-N_0\left({}^{234}_{92}U\right)e^{-\lambda t}}=\frac{e^{-\lambda t}}{1-e^{-\lambda t}}$;

After 10 periods, $\lambda t=\lambda(10T)=\frac{\ell n(2)}{T}\times 10T=10\,\ell n(2)$;

Then, $\frac{N\left({}^{234}_{92}U\right)}{N\left({}^{230}_{90}Th\right)}=\frac{e^{-10\ell n2}}{1-e^{-10\ell n2}}=0.001=0.1\%$, the ratio will become very small, in such a way that the number of uranium nuclei is negligible compared to that of thorium.

Thus, the age becomes difficult to estimate using this technique.

X-

1. Equation of the disintegration: ${}^{59}_{26}Fe \longrightarrow {}^{A}_{Z}Co+{}^{0}_{-1}e+{}^{0}_{0}\bar{\upsilon}+\gamma$;

Conservation of mass number: $59=A+0$, then $A=59$;

Conservation of charge number: $26=Z-1$, then $Z=27$;

The equation of becomes: ${}^{59}_{26}Fe \longrightarrow {}^{59}_{27}Co+{}^{0}_{-1}e+{}^{0}_{0}\bar{\upsilon}+\gamma$.

2. The mass defect:

$\Delta m=m\left({}^{59}_{26}Fe\right)-\left[m\left({}^{A}_{Z}Co\right)+m\left({}^{0}_{-1}e\right)\right]=58.93488-58.93320-0.00055=1.13\times10^{-3}u$;

The energy liberated: $E_\ell=\Delta m c^2=1.13\times10^{-3}\times931.5=1.052595\ MeV$.

3. a) The number of nuclei is given by:

$N_0=\frac{m_0}{m_{\text{Fe-nucleus}}}=\frac{1\times10^{-6}kg}{58.93488\times1.66\times10^{-27}kg}=1.02\times10^{19}$ nuclei. (We suppose that the mass of the atom is equal to the mass of its nucleus).

b) Law of radioactive decay $A=A_0\,e^{-\lambda t}$ where A_0 is the activity at the instant taken as origin of time $t_0=0$ & λ is the radioactive constant.

c) We have: $\frac{A(t)}{A(t+10)}=\frac{A_0\,e^{-\lambda t}}{A_0\,e^{-\lambda(t+10)}}$;

$e^{10\lambda}=1.17$; so, $10\lambda=\ell n(1.17)$;

We get $T=\frac{10\,\ell n(2)}{\ell n(1.17)}\approx 44.1$ days.

d) The activity $A_0=\lambda N_0$;

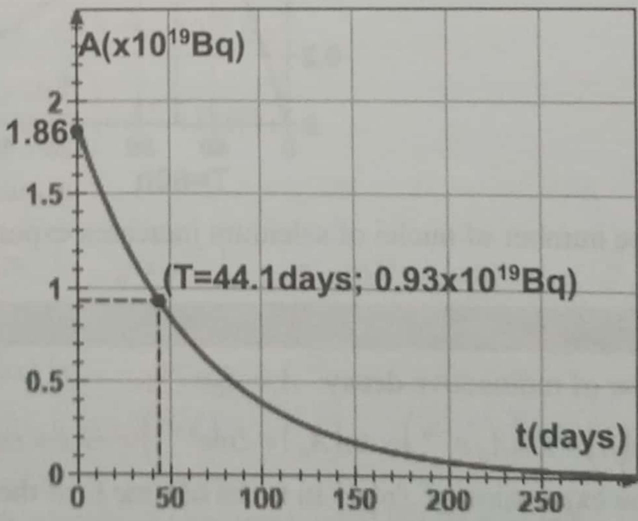
$A_0=\frac{\ell n(2)}{44.1\times24\times60\times60}\times1.02\times10^{19}$;

$A_0=1.86\times10^{12}\ Bq$.

e) The activity decreases exponentially.

On abscissa axis: 1div ≡ 50 days;

On ordinate axis: 1div ≡ $0.5\times10^{19}\ Bq$;



XI-

1. The equation of disintegration is: ${}^{77}_{35}Br \longrightarrow {}^{A}_{Z}Se+{}^{0}_{+1}e+{}^{0}_{0}\upsilon+\gamma$;

Conservation of mass number: $77 = A + 0$, then $A = 77$;

Conservation of charge number: $35 = Z + 1$, then $Z = 34$;

We get: ${}^{77}_{35}Br \longrightarrow {}^{77}_{34}Se + {}^{0}_{+1}e + {}^{0}_{0}\upsilon + \gamma$.

2. a) The number of nuclei at $t_0 = 0$ is $N_0 = 1.6 \times 10^{19}$ nuclei.

b) We know that $t = T$, $N = \frac{N_0}{2} = 0.8 \times 10^{19}$ nuclei, then $T = 60$ hours.

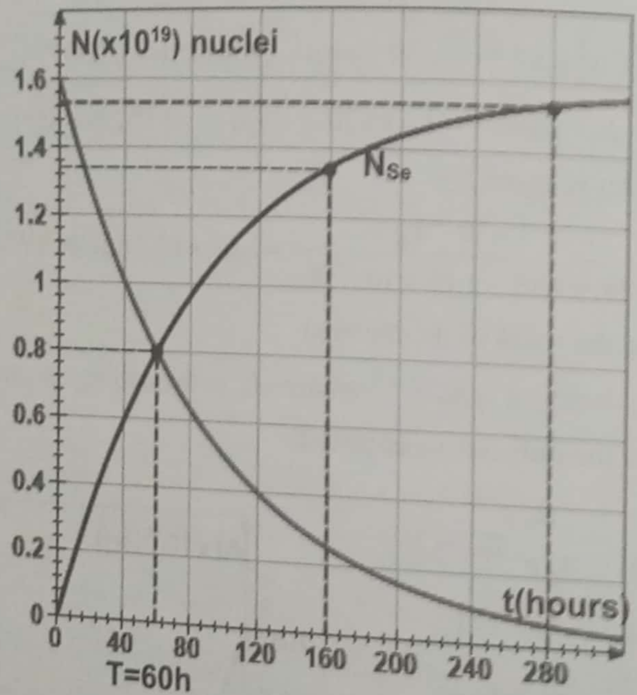
c) We have $N_0 = \frac{m_0}{M} \times N_A$; then, $m_0 = \frac{N_0 \times M}{N_A} = \frac{1.6 \times 10^{19} \times 77\ g/mol}{6.022 \times 10^{23}\ /mol} = 2.05 \times 10^{-3}\ g$.

3. The number of selenium nuclei formed is equal to the number of disintegrated nuclei of bromine.

$$N_{Se} = N_0 - N = N_0 - N_0 e^{-\lambda t};\ N_{Se} = N_0\left(1 - e^{-\lambda t}\right) = N_0\left(1 - e^{-\frac{\ell n2}{T}t}\right).$$

4. At $t = 160$ hours, $N_{Se} = 1.34 \times 10^{19}$ nuclei;

At $t = 280$ hours, $N_{Se} = 1.52 \times 10^{19}$ nuclei.



5. The number of nuclei of selenium increases exponentially.

XII-

1. Law of radioactive decay: $A = A_0 e^{-\lambda t}$;

$\ell n(A) = \ell n\left(A_0 e^{-\lambda t}\right) = \ell n(A_0) + \ell n\left(e^{-\lambda t}\right) = -\lambda t + \ell n(A_0)$;

The expression of $\ell n(A)$ in terms of time is of the form $\ell n(A) = at + b$ where $a = -\lambda < 0$;

Then its graphical representation should be a straight line decreasing and not passing through origin, which is confirmed graphically.

2. The activity of a radioactive sample is the number of decays per unit of time, and measured in SI units in Becquerel (Bq).

The period of a radioactive sample is the time needed so that the activity is reduced to its half.

3. a) From graph: $\ell n(A_0)=19.6$; then $A_0=e^{19.6}=3.25\times10^{8}\,Bq$.

b) The radioactive constant is related to the slope of the straight line:

$$-\lambda=\frac{\Delta\ell n(A)}{\Delta t}=\frac{(16-19.6)}{(400-0)\text{days}}=-9\times10^{-3}\text{day}^{-1}\text{; then } \lambda=9\times10^{-3}\text{day}^{-1}.$$

4. a) We have $T=\dfrac{\ell n(2)}{\lambda}=\dfrac{\ell n(2)}{9\times10^{-3}\text{day}^{-1}}=77\,\text{days}$.

b) We have $A_0=\lambda N_0$ & $N_0=\dfrac{m_0}{M}N_A$, then $m_0=\dfrac{A_0\times M}{\lambda\times N_A}$;

Thus, $m_0=\dfrac{3.25\times10^{8}\times60\times60\times24\ \text{decays.day}^{-1}\times185\ g.mo\ell^{-1}}{9\times10^{-3}\text{day}^{-1}\times6.022\times10^{23}mo\ell^{-1}}=9.6\times10^{-7}\,g=960\,ng$.

XIII-

1. a) $^{238}_{94}Pu$ is alpha emitter.

$^{238}_{94}Pu\longrightarrow{}^{A}_{Z}U+{}^{4}_{2}He+\gamma$;

Conservation of mass number: $238=A+4$, then $A=234$;

Conservation of charge number: $94=Z+2$, then $Z=92$;

We get: $^{238}_{94}Pu\longrightarrow{}^{234}_{92}U+{}^{4}_{2}He+\gamma$.

b) The emission of γ-rays is due to the dis-excitation of the daughter nucleus that undergoes a downward transition towards its ground state.

2. a) The number of nuclei in the sample is:

$$N_0=\frac{m_0}{M}\times N_A=\frac{130\times10^{-3}g}{238\ g.mo\ell^{-1}}\times6.022\times10^{23}mo\ell^{-1}=3.289\times10^{20}\ \text{nuclei.}$$

b) The activity at $t_0=0$ is:

$$A_0=\lambda N_0=\frac{\ell n(2)}{T}\times N_0=\frac{\ell n(2)}{87.8\times365\times24\times3600}\times3.289\times10^{20}=8.23\times10^{10}\ Bq.^{(1)}$$

c) Law of radioactive decay $A=A_0e^{-\lambda t}$; with $A\geq5.76\times10^{10}\,Bq$

$$\text{Then, } t\leq\frac{T}{\ell n(2)}\ell n\left(\frac{A_0}{A}\right)=\frac{87.8}{\ell n(2)}\times\ell n\left(\frac{8.23\times10^{10}}{5.76\times10^{10}}\right)=45.2\ \text{years.}$$

3. The number of nuclei disintegrated over 5 years is:

$$N_d=N_0-N=N_0\left(1-e^{-\frac{\ell n2}{T}\times5}\right)=3.289\times10^{20}\times\left(1-e^{-\frac{\ell n2}{87.8}\times5}\right)=1.27\times10^{19}\ \text{decays;}$$

The nuclear energy furnished over this duration is:

$E_{nu}=N_d\times E_t=1.27\times10^{19}\times5.5\times1.6\times10^{-13}=1.12\times10^{7}J$;

The average electric power: $P_e=P_{nu}=\dfrac{E_{nu}}{\Delta t}=\dfrac{1.12\times10^{7}J}{5\times365\times24\times60\times60\,s}=0.071\,W=71\,mW$.

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[1] The activity per year is: $A_0=\lambda N_0=\dfrac{\ell n2}{T}\times N_0=\dfrac{\ell n2}{87.8}\times3.289\times10^{10}=2.597\times10^{18}$ decays.

Problems

Given:

- Mass of the electron: $m(e) = 0.00055\ u$;
- Mass of the proton: $m_p = 1.00728\ u$;
- Mass of the neutron: $m_n = 1.00866\ u$;
- Planck's constant: $h = 6.63\times10^{-34}\ J.s$;
- Avogadro's number: $N_A = 6.022\times10^{23}\ /mo\ell$;
- Speed of light in vacuum: $c = 3\times10^{8}\ m/s$;
- $1u = 931.5\ MeV/c^2$;
- $1\ MeV = 1.6\times10^{-13}\ J$.

I- Dating by Chlorine

Chlorine isotope ${}^{36}_{17}C\ell$ is radioactive and formed in the higher atmosphere when argon ${}^{36}_{18}Ar$ is bombarded by a neutron. Later chlorine 36 decays spontaneously into argon 36.

Given mass of some nuclei

- Chlorine: $m(C\ell) = 35.96103\ u$;
- Argon: $m(Ar) = 35.95421\ u$;
- Electron: $m(e) = 0.00055\ u$.

Part A

Nuclear reactions

1. Write the nuclear reaction of formation of ${}^{36}_{17}C\ell$ and identify the emitted particle.
2. Write the equation of disintegration of ${}^{36}_{17}C\ell$.
3. Referring to the numbers of neutrons and protons, for each of the nuclei of chlorine and argon. Write the corresponding equation of decay of the nucleon.
4. Calculate, in J, the energy liberated by the decay of one nucleus of ${}^{36}_{17}C\ell$.

Part B

Dating by chlorine

Chlorine-36 whose period 308000 years, is renewed continuously in surface water. A sample taken from water surface gives an average activity of 12 μBq. However, in steady water trapped inside caves this activity decreases with time and is found to be 1.2 μBq in a sample identical to the previous.

1. Define the activity of a radioactive sample.
2. Give two factors on which the activity depends.
3. Show that the age t of this sample is given by $t = \dfrac{T}{\ell n(2)} \times \ell n(10)$.
4. Deduce the age of the trapped water.

II-
The Radithor

In all times, certain mixtures are considered as remedy against all pains.
In the beginning of the XXth, the Radithor, kind of «magical potion», was considered as a treatment for more than hundred diseases.

An American doctor has found with an antique dealer several bottles of Radithor. Although they are empty from about 10 years from their contents, the bottles are still radioactively dangerous. Each of them contains 1 micro curie $\left(1\mu C = 3.7\times10^{4}\ Bq\right)$ of radium 226 and radium 228.

Nucleus	Radium 226	Radium 228	Actinium 226	Radon 222	Helium 4
Symbol	${}^{226}_{88}Ra$	${}^{228}_{88}Ra$	${}^{226}_{89}Ac$	${}^{222}_{86}Rn$	${}^{4}_{2}He$

Nucleus	Radium 226	Radon 222	Helium 4
Mass in u	225.9770	221.9703	4.0015

Given

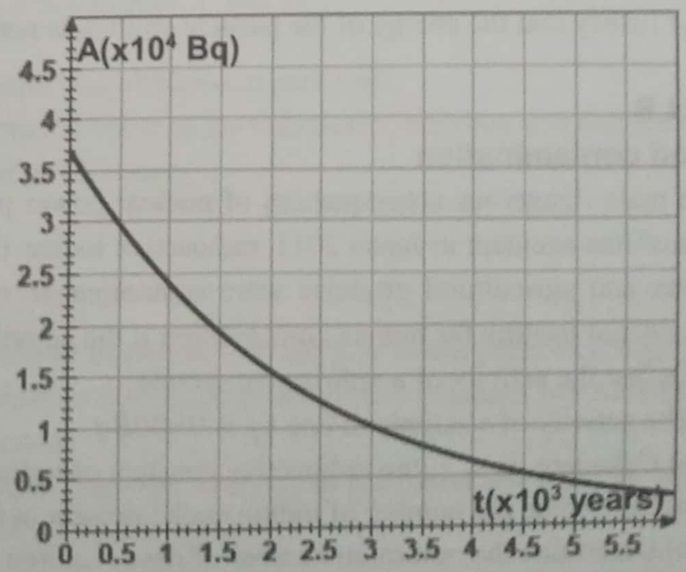
- $1u = 1.66054\times10^{-27}\ kg$;
- Speed of light in vacuum: $c = 3\times10^{8}\ m/s$;
- Avogadro's number: $N_A = 6.02\times10^{23}\ mol^{-1}$;
- Molar mass of ${}^{226}_{88}Ra$: $226\ g.mol^{-1}$.

1. The radium and the Mesothorium:
   On the label of the Radithor bottle is mentioned the presence of the Mesothorium, an ancient nomination of the radium 228. This «certified radioactive water» contains also the radium 226.
   a) The nuclei of radium 228 and radium 226 are isotopes. Explain.
   b) The radium 228 disintegrates to give the actinium Ac 228 isotope and a particle X.
      i- Write the equation of this disintegration. Identify X.
      ii- What is the type of this radioactivity?

In what follows, we neglect the presence of the radium 228 in the Radiator; we suppose that the radioactive activity of the bottle is due only to the presence of the radium isotope 226 which disintegrates according to the equation:

$${}^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^{4}_{2}He.$$

2. The adjacent curve represents the evolution of the activity as a function of time.



a) Write the law of radioactive decay relative to the activity A and the meaning of its terms.
b) Define the period T of a radioactive sample.
c) Determine graphically the period T.
d) Derive the relation between the period T and the radioactive constant λ.
Calculate λ in s^{-1}.

3. a) Referring to figure 2, determine the number of nuclei N_0 present in the bottle at $t = 0$.
b) Show that the mass of radium 226 present at $t = 0$ is $m_0 = 1.1\,\mu g$.

4. a) Calculate the energy liberated by the nuclear disintegration of a nucleus of radium 226.
b) Determine the total energy contained in the initial mass of this isotope.

III- Iodine

Iodine $^{131}_{53}I$ is radioactive, it decays giving the xenon nucleus $^{131}_{54}Xe$.

Given mass of some nuclei

- Iodine: $m(I) = 130.87868\ u$;
- Xenon: $m(Xe) = 130.87702\ u$;
- Electron: $m(e) = 0.00055\ u$;
- The period of iodine 131 is 8 days;
- Molar mass of iodine is $131\ g.mol^{-1}$.

Part A
Iodine emissions

1. Write the equation of this decay.
2. Identify the type of radioactivity.
3. Calculate the energy liberated by this disintegration.
4. Give two physical properties of the antineutrino that accompanies this emission.
5. Knowing that the kinetic energy carried by the particle is $0.4\,MeV$ & the wavelength of the radiation emitted is $\lambda = 3.45\ pm$.
   a) Calculate, in MeV, the energy carried by this radiation.
   b) Deduce the energy carried by the antineutrino.
   c) Justify that the energy of the particle emitted is not quantified.

Part B
Food contamination

The main dangerous consequences of nuclear power plant are accidents and leaks. As a result of Fukushima accident in Japan 2011, radioactive iodine 131 leaks and spreads.
Water and agricultural products were contaminated, one of these products is the spinach that is considered healthy for human consumption if the activity of one kg is less than $2000 Bq$.

1. Define the activity of a radioactive sample.
2. The activity of a sample of one kg is $9500 Bq$.
   a) Calculate, in s^{-1}, the radioactive constant of iodine 131.
   b) Determine the number of iodine nuclei present in the sample.
   c) Determine the minimum number of days required so that the sample becomes healthy.

IV-Bac
The Plutonium

Plutonium $^{241}_{94}Pu$ is not present in the nature, but obtained when the uranium $^{238}_{92}U$ nucleus is bombarded, in a nuclear reactor, by a certain number x of neutrons according to the reaction:

$$^{238}_{92}U + x\,^{1}_{0}n \longrightarrow {}^{241}_{94}Pu + y\,^{0}_{-1}e$$

Given the following nuclei masses

- $m\left(^{241}Pu\right) = 241.00514\ u$;
- $m\left(^{A}_{Z}Am\right) = 241.00357\ u$;

1. Determine the values of x and y, specifying the laws used.
2. Plutonium $^{241}_{94}Pu$ is a β^- emitter and it gives an americium $^{A}_{Z}Am$.
   a) Write the equation of disintegration of the plutonium nucleus.
   b) Studying the stability.
      i- Determine the binding energy per nucleon for each of the following nuclei $^{241}_{94}Pu$ and $^{A}_{Z}Am$
      ii- Deduce, with justification, which nuclei is more stable.
3. A plutonium ^{241}Pu sample that contains N_0 nuclei at $t_0 = 0$ and its activity is A_0. Let $A(t)$ be the activity of this radioactive sample at an instant t. Studying this radioactive sample we were able to calculate the ratio $\frac{A(t)}{A_0}$ and the results obtained are listed in the following table:

t(years)	0	3	6	9	12
$\frac{A(t)}{A_0}$	1.00	0.85	0.73	0.62	0.53

   a) Plot the curve representing $\ell n\left(\frac{A(t)}{A_0}\right)$ as a function of time, specifying the scales used.
   b) Graphical study.
      i- Write the expression of $\ell n\left(\frac{A(t)}{A_0}\right)$ as a function of λ and t.
      ii- Justify that this result is compatible with that obtained in part 3.a).
   c) Referring to the graphical study, calculate the value of the radioactive constant and deduce that the period T of this radioactive element is 13.2 years.

V-
Radioactivity of Polonium 210

Polonium 210 isotope is radioactive alpha emitter of period 138 days. A small mass of $0.05g$ can end the life a person whose mass is average of about $80kg$ within few weeks (Russian spy Alexander Litvinenko died three weeks after being poisoned).
It serves after ingestion as a poison, and passes from the stomach to the blood stream. The alpha projectile kills the cells and destroys blood red globules.

Mass of some nuclei

- $m(Po) = 209.98281\ u$;
- $m(Pb) = 205.97438\ u$;
- $m(\alpha) = 4.00258\ u$.

Part A

Properties of alpha decay

The decay of a radioactive element is accompanied by a certain emission α, β & γ. A particle α is ejected between two plates: P is charged positively whereas N is charged negatively. This experiment shows that the α particle is attracted by one of these plates.

1. Arrange in the increasing order of penetrating power the emissions α, β & γ.
2. Justify that the particle α possesses an electric charge.
3. Specify the plate that attracts α particles.
4. What happens to the radiations called γ when they pass between the plates?

Part B

Radioactive properties

The daughter nucleus produced by the decay of polonium $^{210}_{84}Po$ is an isotope of lead $^{A}_{Z}Pb$.

1. Determine Z and A specifying the laws used.
2. Calculate the energy liberated by this disintegration.
3. Some decays takes place without gamma emission while the others they are accompanied by the emission of radiations of wavelength $\lambda_1 = 1.8\ pm$.
   a) Determine the kinetic energy of the radiations alpha in both cases.
   b) Justify that the kinetic energy of these radiations is quantified.

Part C

Effect on health

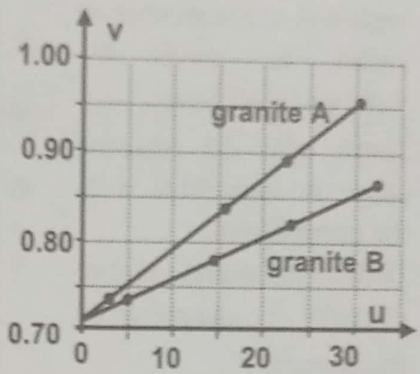
We suppose that whenever the polonium is inside the stomach, the body absorbs all the energy liberated.

1. Calculate the number N_0 of nuclei of polonium present in the sample of $0.05g$.
2. Show that the number of nuclei disintegrated, after t days, is given by $N_d = N_0\left(1 - e^{-\frac{t \times \ln 2}{138}}\right)$.
3. Determine the average power absorbed during 4 weeks.
4. Knowing that the maximum average nuclear power that a person may absorb without any risk is $2 \times 10^{-7} W$. Specify the physical state of this person.

VI-Engineering (2012/2013)
Rubidium-Strontium Dating

Some granitic rocks, during their crystallization, have detained an amount of rubidium $^{87}_{37}Rb$, a radioactive isotope of rubidium, of radioactive constant $\lambda = 1.42 \times 10^{-11}\ \text{year}^{-1}$, and another amount of strontium formed of stable isotopes $\left(^{87}_{38}Sr\right)$ and $\left(^{86}_{38}Sr\right)$. A $^{87}_{37}Rb$ nucleus decays into a $\left(^{87}_{38}Sr\right)$ nucleus.

1. Give, with justification, the type of the decay of a ${}^{87}_{37}Rb$ nucleus.
2. Calculate the radioactive half-life $t_{1/2} = T$ of the rubidium 87 sample.
3. $N({}^{87}_{37}Rb)$ and $N_0({}^{87}_{37}Rb)$ are respectively the number of rubidium atoms present at the current instant t and that of the atoms that were present at the instant $t_0 = 0$, instant of rock formation. Show that the number $N^*({}^{87}_{38}Sr)$ of strontium atoms formed from the instant t_0 until the instant t has the expression: $N^*({}^{87}_{38}Sr) = N({}^{87}_{37}Rb)(e^{\lambda t} - 1)$.
4. $N_0({}^{87}_{38}Sr)$ is the initial number of strontium-87 nuclei present in the sample. Give the expression $N({}^{87}_{38}Sr)$ of the total number of these nuclei present in the sample at the current instant t in terms of $N({}^{87}_{37}Rb)$, $N_0({}^{87}_{38}Sr)$, λ and t.
5. By measuring experimentally the ratios $u = \dfrac{N({}^{87}_{37}Rb)}{N({}^{86}_{38}Sr)}$ and $v = \dfrac{N({}^{87}_{38}Sr)}{N({}^{86}_{38}Sr)}$ in the minerals of two different granitic rocks (granite A, granite B), we obtain the adjacent two graphs.



a) Why was the ${}^{86}_{38}Sr$ isotope used as a reference?
b) Show that we can write: $v = au + b$, where $a = (e^{\lambda t} - 1)$.
c) Graphical study:
   i- Determine the value of a for each of the two granitic rocks.
   ii- Deduce the approximate age of each of the two rocks.
d) Why didn't we use the carbon-14 of half-life of 5730 years for dating this rock?

VII- Electrical and Nuclear Analogy

The aim of this problem is to justify an analogy between electric and nuclear physical quantities.

Part A
Study of an RC circuit

A generator delivering a constant voltage $E = 3.5V$ is connected across a resistor of resistance R connected in series with a capacitor of capacitance C taken neutral as shown in figure 1.

We intend to study the evolution of the voltage across the resistor u_R as a function of time.

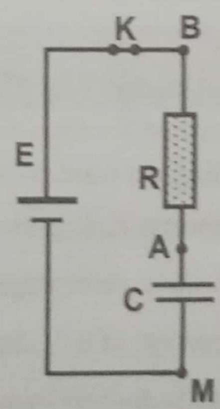


Figure 1

1. Redraw figure 1, showing the direction of current, the sign of the charges carried by the armatures and then the connections of the oscilloscope that allows us to display the voltage across the resistor $u_R = u_{BA}$.
2. Determine the value of $u_R = u_{BA}$ at $t = 0$.
3. Show that the differential equation satisfied by $u_R = u_{BA}$ is given by:

$$\frac{du_R}{dt} + \frac{1}{RC}u_R = 0$$

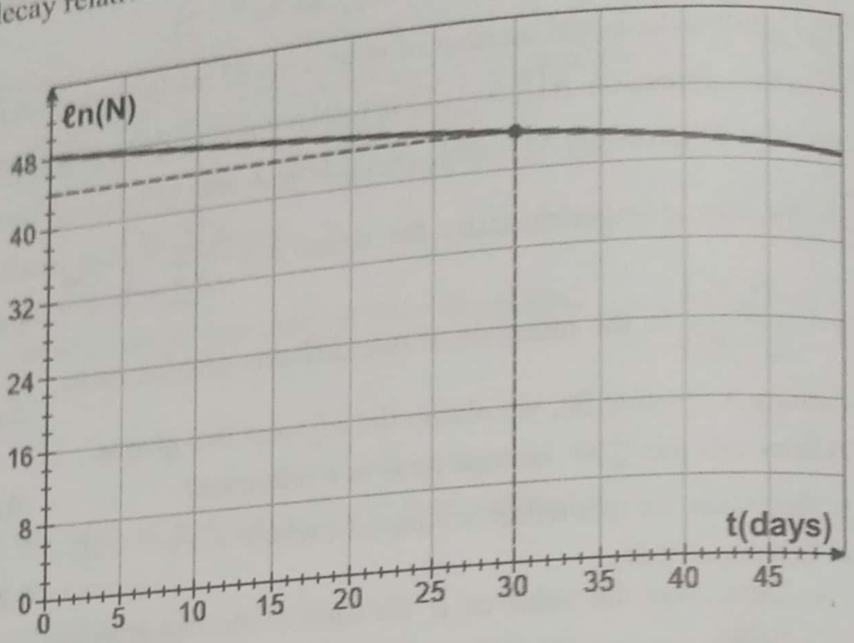
4. Verify that the voltage across the resistor can be written in the form $u_R = E e^{-\frac{t}{RC}}$.
5. Deduce the value of u_R in steady state.

Part B

Radioactivity

Bismuth-210 $_{83}^{210}Bi$ is radioactive and beta minus emitter, of period or half-life T. The daughter nucleus formed is an isotope of polonium $_{Z}^{A}Po$.

1. Write the equation of this disintegration specifying the laws used.
2. State the law of radioactive decay relative to the number of nuclei N specifying the meaning of each term.
3. Deduce that the number of nuclei satisfy the relation:
$$\frac{dN}{dt} + \lambda N = 0$$
4. Show that the natural logarithm of the number of nuclei is given by $\ell n(N) = \ell n(N_0) - \lambda t$.
5. Justify that the adjacent shape is compatible with the curve drawn.
6. Deduce the initial number of nuclei N_0 and period of the bismuth T.



Part C

Analogy

By comparing the results of the previous parts.

1. Match each of the physical quantities N, N_0 & λ with the convenient electrical physical quantity.
2. Plot the graph showing the variations as a function of time, showing the tangent at the origin of time, for:
   a) the voltage across the resistor $u_R(t)$.
   b) the number of non-disintegrated radioactive nuclei $N(t)$.

VIII-
Radioactivity of Potassium

Potassium K comes from the Latin word kalium, meaning alkalis and the Arabic word qali and from the English word potash, meaning pot ashes.

Potassium K is present in nature in three isotopes, two of them are stable $_{19}^{39}K$ and $_{19}^{41}K$ with respective percentages of 93.36% and 6.7%, while the third isotope $_{19}^{40}K$ is radioactive with a percentage of 0.012%. The radioactive isotope $_{19}^{40}K$ disintegrates along three possible modes:

- (1): by beta minus decay and the daughter nucleus formed is an isotope of calcium $_{Z}^{A}Ca$.
- (2): by beta plus decay and the daughter nucleus formed is an isotope of argon $_{Z_1}^{A_1}Ar$.
- (3): by electron capture and the daughter nucleus formed is an isotope of argon $_{Z_1}^{A_1}Ar$ obtained in an excited state emitting gamma radiation of energy 1.46 MeV.

Given masses of some nuclei and particles:

- $m(K) = 39.963998\ u$;
- $m(e) = 0.000546\ u$;
- The period of potassium: $T = 1.248 \times 10^{9}$ years ;
- Molar mass of potassium $^{40}_{19}K$ is: $40\ g/mo\ell$;
- Avogadro's number: $N_A = 6.022 \times 10^{23}\ /mo\ell$;
- $1u = 931.5\ MeV/c^2$.
- Planck's constant: $h = 6.63 \times 10^{-34}\ J.s$;
- The speed of light in vacuum is: $c = 3 \times 10^{8}\ m/s$.

Part A

Disintegrations

1. What is meant by isotopes?
2. Write the equation of the disintegration along mode (2), specifying the laws used.
3. For the decay of mode (3):
   a) Write the equation of the disintegration.
   b) Determine the wavelength of the radiation emitted.

Part B

Beta minus decay

1. Write the equation of disintegration of potassium along mode 1, specifying the laws used.
2. Knowing that the energy liberated by this decay is $1.33\ MeV$.
   a) In what form does the energy liberated appears?
   b) Deduce the mass defect corresponding to this decay.
   c) Determine the mass of the nucleus of $^{A}_{Z}Ca$.
3. The body of a moderate human contains in average $140\ g$ of potassium.
   Show that the activity of potassium is about $A_0 = 4455\ Bq$.

Part C

Attenuation of activity

The body tissues (bones & skin) absorbs radiations. The attenuation percentage could be simulated by the decay relation $\%A_{\beta^-} = e^{-\mu x}$ where $\%A_{\beta^-}$ is the percentage of attenuation for a beta minus emission after traveling a distance x & μ is a constant.

1. The attenuation percentage $\%A_{\beta^-}$ is reduced to 50% after traveling $4\ mm$ in human tissues.
   Show that $\mu = 173.3\ m^{-1}$.
2. Draw the curve representing the $\%A_{\beta^-}$ in terms of penetrating distance.
3. The attenuation percentage due to the radiations γ is simulated by: $\%A_\gamma = e^{-10\ell n2\, x}$ in SI units.
   a) Determine the distance needed so that the attenuation percentage is 50%.
   b) Draw on the same graph the curve representing the attenuation percentage of these radiations.
4. If the thickness of human tissues is $4\ cm$.
   Compare graphically the attenuation percentage of these two radiations on the skin.

I-

Part A

1. The equation of formation of chlorine $^{36}_{18}Ar + ^{1}_{0}n \longrightarrow ^{36}_{17}C\ell + ^{A}_{Z}X$;
   Conservation of mass number: $36+1=36+A$, then $A=1$;
   Conservation of charge number: $18+0=17+Z$, then $Z=1$;
   The emitted particle is the proton.
2. The equation of disintegration is: $^{36}_{17}C\ell \longrightarrow ^{36}_{18}Ar + ^{A}_{Z}X + \gamma$;
   Conservation of mass number: $36=36+A$, then $A=0$;
   Conservation of charge number: $17=18+Z$, then $Z=-1$;
   We get: $^{36}_{17}C\ell \longrightarrow ^{36}_{18}Ar + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon} + \gamma$.
3. $^{36}_{17}C\ell$ nucleus: 17 protons & 19 neutrons.
   $^{36}_{18}Ar$ nucleus: 18 protons & 18 neutrons.
   Then a neutron decays into a proton with the emission of an electron;
   The equation of disintegration of the neutron is: $^{1}_{0}n \longrightarrow ^{1}_{1}p + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon}$.
4. The mass defect: $\Delta m = m\left(^{36}_{17}C\ell\right) - \left[m\left(^{36}_{18}Ar\right) + m\left(^{0}_{-1}e\right)\right]$;
   $\Delta m = 35.96103 - (35.95421 + 0.00055) = 6.27\times10^{-3}u$;
   The energy liberated is $E_\ell = \Delta m c^2 = 6.27\times10^{-3}\times931.5 = 5.84\,MeV$;
   Thus, $E_\ell = 5.84\times1.6\times10^{-13} = 9.34\times10^{-13}\,J$.

Part B

1. The activity of a radioactive sample is the number of disintegrations per unit of time.
2. The activity depends on the number of nuclei (mass) of the sample studied and the period of the radioactive element.
3. Law of radioactive decay: $A = A_0 e^{-\lambda t}$.
   We get $t = -\dfrac{T}{\ell n(2)}\times \ell n\left(\dfrac{A}{A_0}\right) = -\dfrac{T}{\ell n(2)}\ell n\left(\dfrac{1.2}{12}\right) = \dfrac{T}{\ell n(2)}\ell n(10)$.
4. The water is trapped from 1 023 153 years (about million years).

II-

1. a) Isotopes are nuclides having same atomic number but different mass number.
   b) Type of decay
   i- $^{228}_{88}Ra \longrightarrow ^{228}_{89}Ac + ^{A}_{Z}X$;
   According to Soddy's laws: $A=0$ & $Z=-1$;
   $^{228}_{88}Ra \longrightarrow ^{228}_{89}Ac + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon} + \gamma$; the emitted particle is the electron.
   ii- This decay is a beta minus.
2. a) Law of radioactive decay $A = A_0 e^{-\lambda t}$.
   where A_0 is the activity of the radioactive sample at $t_0 = 0$;

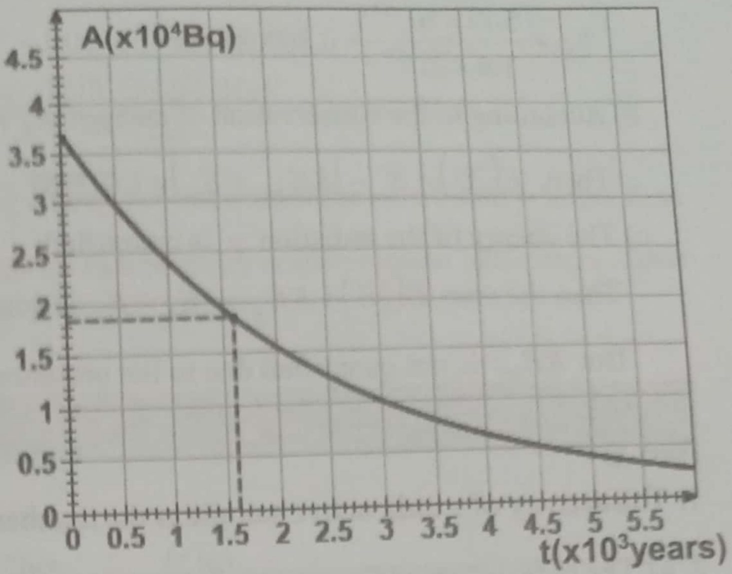
A is the activity of the radioactive sample at the instant t;

λ is the radioactive constant.

b) The period T of a radioactive sample is the duration after which the activity is reduced to its half.

c) At $t = T$; $A = \frac{A_0}{2} = \frac{37000}{2} = 18500\ Bq$;

Referring to the graph $T = 1600$ years.



d) At $t = T$; $A = \frac{A_0}{2}$;

Then $\frac{A_0}{2} = A_0\, e^{-\lambda T}$;

$e^{-\lambda T} = \frac{1}{2} \Rightarrow e^{\lambda T} = 2$;

we get $\lambda T = \ell n(2) \Rightarrow \lambda = \frac{\ell n(2)}{T}$.

So, $\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{1600 \times 365 \times 24 \times 3600}$; thus $\lambda = 1.37 \times 10^{-11}\ s^{-1}$.

3. a) We have $N_0 = \frac{A_0}{\lambda} = \frac{37000}{1.37 \times 10^{-11}} = 2.7 \times 10^{15}$ nuclei.

b) $N_0 = \frac{m_0}{M} \times N_A \Rightarrow m_0 = \frac{N_0 \times M}{N_A} = \frac{2.7 \times 10^{15} \times 226\ g.mo\ell^{-1}}{6.02 \times 10^{23}\ mo\ell^{-1}} = 1.01 \times 10^{-6}\ g = 1.01\mu g$.

4. a) The energy liberated $E_\ell = \Delta m c^2$;

The mass defect $\Delta m = m_{Ra} - (m_{Rn} + m_\alpha) = 225.9770 - (221.9703 + 4.0015) = 5.2 \times 10^{-3}\ u$;

$= 5.2 \times 10^{-3} \times 1.66504 \times 10^{-27}\ kg = 8.634808 \times 10^{-30}\ kg$.

Then $E_\ell = 8.634808 \times 10^{-30}\ kg \times (3 \times 10^{8}\ m.s^{-1})^2 = 7.77 \times 10^{-13}\ J$.

b) The energy liberated by one nucleus is $7.77 \times 10^{-13}\ J$, but the sample contains N_0 nuclei;

Then the energy contained in this sample is: $E = N_0 \times E_\ell = 2.7 \times 10^{15} \times 7.77 \times 10^{-13} = 2098\ J$.

III-

Part A

1. The equation of disintegration is: ${}^{131}_{53}I \longrightarrow {}^{131}_{54}Xe + {}^{A}_{Z}X + \gamma$;

Conservation of mass number: $131 = 131 + A$, then $A = 0$;

Conservation of charge number: $53 = 54 + Z$, then $Z = -1$;

We get: ${}^{131}_{53}I \longrightarrow {}^{131}_{54}Xe + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$.

2. The emitted particle is an electron, so it is a radioactivity beta minus.

3. The mass defect:

$\Delta m = m({}^{131}_{53}I) - [m({}^{131}_{54}Xe) + m({}^{0}_{-1}e)] = 130.87868 - (130.87702 + 0.00055) = 1.11 \times 10^{-3}\ u$;

The energy liberated is $E_\ell = \Delta m c^2 = 1.11 \times 10^{-3} \times 931.5 = 1.033\,965\ MeV$.

4. The antineutrino is charge less and it has an extremely small mass.

5. a) The energy of this radiation is $E_\gamma = h\frac{c}{\lambda} = 6.63\times10^{-34}\times\frac{3\times10^{8}}{3.45\times10^{-12}} = 5.77\times10^{-14}\ J$;

$E_\gamma = \frac{5.77\times10^{-14}}{1.6\times10^{-13}} = 0.360625\ MeV$.

b) According to the conservation of energy: $E_\ell = KE_\beta + E(^0_0\bar{\upsilon}) + E_\gamma$;

Then, $E(^0_0\bar{\upsilon}) = E_\ell - (KE_\beta + E_\gamma) = 1.033965 - (0.360625 + 0.4) = 0.27334\ MeV$.

c) The energy of the radiation γ is quantified;

Then the sum $E(^0_0\bar{\upsilon}) + KE_\beta = E_\ell - E_\gamma$ is quantified (constant minus quantified);

But KE_β is not quantified due to the presence $^0_0\bar{\upsilon}$ which can carry any possible energy.

Part B

1. The activity of a radioactive sample is the number of decays per unit of time.

2. a) The radioactive constant $\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{8\times24\times60\times60} = 1\times10^{-6}\,s^{-1}$.

b) We have $N = \frac{A}{\lambda} = \frac{9500}{10^{-6}} = 9.5\times10^{9}$ nuclei.

c) Law of radioactive decay $A = A_0\,e^{-\lambda t}$ & $A \le 2000\ Bq$, so $\frac{A}{A_0} = e^{-\lambda t} \le \frac{2000}{9500}$;

$-\lambda t \le \ell n\left(\frac{4}{19}\right)$; then $t \ge -\frac{1}{\lambda}\ell n\left(\frac{4}{19}\right) = -\frac{T}{\ell n(2)}\times\ell n\left(\frac{4}{19}\right) = -\frac{8}{\ell n(2)}\times\ell n\left(\frac{4}{19}\right) \approx 18$ days;

After 18 days the spinach becomes healthy for consumption.

IV-

1. We have $^{238}_{92}U + x\,^1_0n \longrightarrow {}^{241}_{94}Pu + y\,^0_{-1}e$.

- Conservation of mass number: $238 + x(1) = 241 + y(0)$, then $x = 3$;
- Conservation of atomic number: $92 + x(0) = 94 + y(-1)$, then $y = 2$;

Then the nuclear reaction $^{238}_{92}U + 3\,^1_0n \longrightarrow {}^{241}_{94}Pu + 2\,^0_{-1}e + 2\,^0_0\bar{\upsilon}$.

2. a) The equation of the disintegration is $^{241}_{94}Pu \longrightarrow {}^A_ZAm + {}^0_{-1}e + {}^0_0\bar{\upsilon} + \gamma$;

Laws of conservations: $A = 241$ & $Z = 95$.

Then $^{241}_{94}Pu \longrightarrow {}^{241}_{95}Am + {}^0_{-1}e + {}^0_0\bar{\upsilon} + \gamma$.

b) Study of stability:

i- The binding energy is given by: $E_b = \Delta m c^2 = [Zm_p + (A-Z)m_n - m_X]c^2$;

- For $^{241}_{94}Pu$:

$\Delta m = Zm_p + (A-Z)m_n - m_X = 94\times1.00728 + (241-94)\times1.00866 - 241.00514$;

Then, $E_b = 1.9522\ u\times c^2 = 1818.4743\ MeV$;

& per nucleon $\frac{E_b}{A} = \frac{1818.4743\ MeV}{241} = 7.5455\ MeV$.

For $^{241}_{95}Am$:

$\Delta m = Zm_p + (A-Z)m_n - m_X = 95\times 1.00728 + (241-95)1.00866 - 241.00357$;

Then, $E_b = 1.95239 \times 931.5\ MeV = 1818.651285\ MeV$;

& per nucleon $\frac{E_b}{A}(Am) = \frac{1818.651285\ MeV}{241} = 7.5462\ MeV$.

ii- We have $\frac{E_b}{A}(Am) > \frac{E_b}{A}(Pu)$; the binding energy per nucleon of the americium is larger than that of plutonium which justify the fact that the radioactivity is an evolution into a more stable state.

3. a) Complete the table

t(years)	0	3	6	9	12
$\ell n\left(\frac{A(t)}{A_0}\right)$	0	-0.16	-0.31	-0.48	-0.63

Graph.

b) Graphical study:

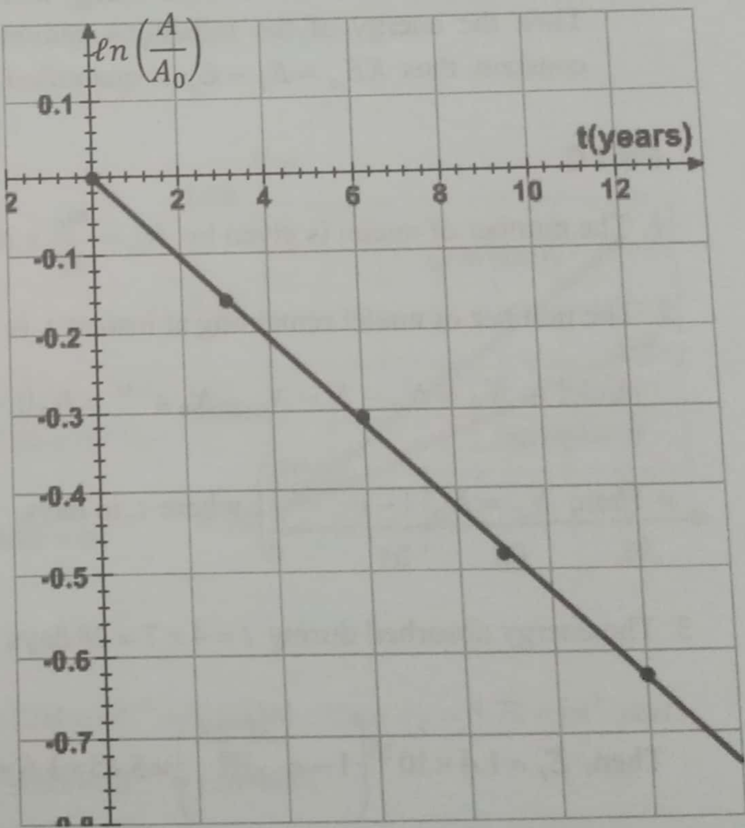
i- Law of radioactive decay: $A(t) = A_0\, e^{-\lambda t}$;

Then, $\ell n\left(\frac{A(t)}{A_0}\right) = \ell n\left(e^{-\lambda t}\right) = -\lambda t$.

ii- The expression of $\ell n\left(\frac{A(t)}{A_0}\right) = f(t)$ is of the form $y = at$, then its graphical representation should be :

- a straight line (time of degree 1).
- decreasing (slope negative $a = -\lambda$).
- passing through origin.

which are verified graphically.



c) The slope of this line is:

$$a = -\lambda = \frac{\ell n\left(\frac{A_2}{A_0}\right) - \ell n\left(\frac{A_1}{A_0}\right)}{t_2 - t_1} ;$$

We get $a = \frac{-0.31 - 0}{(6-0)\text{year}} = -0.05167$;

Then $\lambda = 0.05167\ \text{year}^{-1}$; thus the period $T = \frac{\ell n(2)}{\lambda} = 13.2$ years .

V-

Part A

1. Alpha is the least penetrating among all the radioactive emissions, followed by beta and gamma is the most penetrating.
2. Alpha radiation was attracted by one of the two plates, then it is charged.
3. Alpha is positively charged, so it will be attracted towards the negative plate.
4. The radiation gamma is neutral, so it will not deviate when it passes between the plates.

Part B

1. The equation of the disintegration: ${}^{210}_{84}Po \longrightarrow {}^{A}_{Z}Pb + {}^{4}_{2}He + \gamma$;

Conservation of mass number: $210 = A + 4$, then $A = 206$;

Conservation of charge number: $84 = Z + 2$, then $Z = 82$;

We get: ${}^{210}_{84}Po \longrightarrow {}^{206}_{82}Pb + {}^{4}_{2}He + \gamma$.

2. The mass defect: $\Delta m = m\left({}^{210}_{84}Po\right) - \left[m\left({}^{206}_{82}Pb\right) + m\left({}^{4}_{2}He\right)\right]$

$\Delta m = 209.98281\,u - 205.97438\,u - 4.00258\,u = 5.85 \times 10^{-3}\,u$;

The energy liberated is $E_\ell = \Delta m c^2 = 5.85 \times 10^{-3} \times 931.5 = 5.45\ MeV$.

3. a) According to the conservation of energy: $E_\ell = KE_\alpha + E_\gamma$;

- If the decay takes place without gamma emission, then $KE_{\alpha 1} = E_\ell = 5.45\ MeV$;
- The energy of gamma radiation is $E_\gamma = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.8 \times 10^{-12} \times 1.6 \times 10^{-13}} = 0.69\ MeV$,

Then, $KE_{\alpha 2} = E_\ell - E_\gamma = 5.45 - 0.69 = 4.76\ MeV$.

b) The gamma radiations are emitted during the dis-excitation of the daughter nucleus that possesses a discontinuous set of energy levels.

Then the energy of the radiations gamma is quantified and the energy liberated remains constant, thus $KE_\alpha = E_\ell - E_\gamma$ is quantified.

Part C

1. The number of nuclei is given by $N_0 = \dfrac{m_0}{M} \times N_A = \dfrac{0.05\ g}{210\ g.mo\ell^{-1}} \times 6.02 \times 10^{23}\ mo\ell^{-1} = 1.4 \times 10^{20}$.

2. The number of nuclei remaining at instant t is $N = N_0\, e^{-\lambda t}$; then the number of disintegrated nuclei is $N_d = N_0 - N = N_0 - N_0\, e^{-\lambda t} = N_0\left(1 - e^{-\lambda t}\right)$ & $\lambda = \dfrac{\ell n(2)}{T}$;

Then, $N_d = N_0\left(1 - e^{-\frac{t \times \ell n 2}{138}}\right)$ where t in days.

3. The energy absorbed during $t = 4 \times 7 = 28$ days, $E_t = N_d \times E_\ell = N_0\left(1 - e^{-\frac{t \times \ell n 2}{138}}\right) \times E_\ell$;

Then, $E_t = 1.4 \times 10^{20}\left(1 - e^{-\frac{28 \times \ell n 2}{138}}\right) \times 5.45 \times 1.6 \times 10^{-13} = 1.6 \times 10^{7}\ J$;

The average power absorbed is $P = \dfrac{E_t}{\Delta t} = \dfrac{1.6 \times 10^{7}}{4 \times 7 \times 24 \times 60 \times 60} = 6.6\ W$.

4. The power absorbed is very large compared to the threshold value. Then this dose is deadly.

VI-(2012/2013)

1. The equation of disintegration: ${}^{87}_{37}Rb \longrightarrow {}^{87}_{38}Sr + {}^{a}_{z}X$;

Applying the laws of conservation, we get $a = 0$ & $z = -1$;

So ${}^{a}_{z}X \equiv {}^{0}_{-1}e$, this is a beta minus decay β^-;

Thus, ${}^{87}_{37}Rb \longrightarrow {}^{87}_{38}Sr + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon} + \gamma$.

2. The period or half-life: $T = \dfrac{\ell n(2)}{\lambda} = \dfrac{\ell n(2)}{1.42\times10^{-11}} = 4.88\times10^{10}$ years.

3. We know that: $N\left({}^{87}_{37}Rb\right) = N_0\left({}^{87}_{37}Rb\right)e^{-\lambda t} \Rightarrow N_0\left({}^{87}_{37}Rb\right) = N\left({}^{87}_{37}Rb\right)e^{\lambda t}$.

Number of disintegrated nuclei de (Rb) = number of nuclei formed of $(Sr) = N^*\left({}^{87}_{38}Sr\right)$;

$N^*\left({}^{87}_{38}Sr\right) = N_0\left({}^{87}_{37}Rb\right) - N\left({}^{87}_{37}Rb\right) = N\left({}^{87}_{37}Rb\right)e^{\lambda t} - N\left({}^{87}_{37}Rb\right)$;

then $N^*\left({}^{87}_{38}Sr\right) = N\left({}^{87}_{37}Rb\right)\left(e^{\lambda t} - 1\right)$.

$t_0=0$	Type	t	
${}^{87}_{37}Rb$	Radioactive	$N*\left({}^{87}_{38}Sr\right)$	
${}^{87}_{38}Sr$ $N_0\left({}^{87}_{38}Sr\right)$	Stable	$N_0\left({}^{87}_{38}Sr\right)$	$N\left({}^{87}_{38}Sr\right)$
${}^{87}_{38}Sr$	Stable		

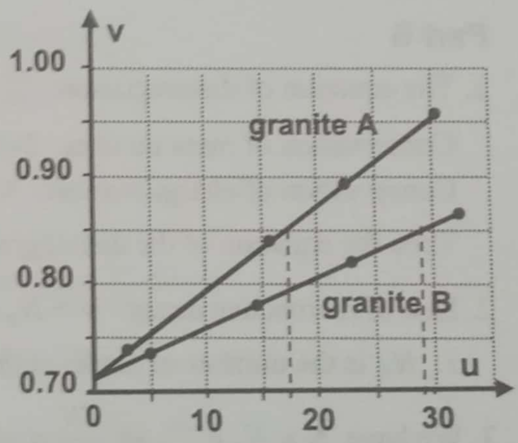
4. $N\left({}^{87}_{38}Sr\right) = N^*\left({}^{87}_{38}Sr\right) + N_0\left({}^{87}_{38}Sr\right)$; then $N\left({}^{87}_{38}Sr\right) = N\left({}^{87}_{37}Rb\right)\left(e^{\lambda t} - 1\right) + N_0\left({}^{87}_{38}Sr\right)$.

5. a) Since the isotope $\left({}^{86}_{38}Sr\right)$ is stable and its number does not vary over time.

b) $v = \dfrac{N\left({}^{87}_{38}Sr\right)}{N\left({}^{86}_{38}Sr\right)} = \dfrac{N\left({}^{87}_{37}Rb\right)\left(e^{\lambda t} - 1\right)}{N\left({}^{86}_{38}Sr\right)} + \dfrac{N_0\left({}^{87}_{38}Sr\right)}{N\left({}^{86}_{38}Sr\right)}$;

Thus, $v = au + b$, where: $a = \left(e^{\lambda t} - 1\right)$;

$b = \dfrac{N_0\left({}^{87}_{38}Sr\right)}{N\left({}^{86}_{38}Sr\right)}$ & $u = \dfrac{N\left({}^{87}_{37}Rb\right)}{N\left({}^{86}_{38}Sr\right)}$.



c) Graphical study:

i- For the granite A: $a_A = \dfrac{0.85 - 0.715}{17 - 0} = 7.94\times10^{-3}$;

For the granite B: $a_B = \dfrac{0.85 - 0.715}{29 - 0} = 4.65\times10^{-3}$.

ii- We have $a = e^{\lambda t} - 1$; $t = \dfrac{\ell n(a+1)}{\lambda}$.

For A: $e^{\lambda t_A} - 1 = 7.94\times10^{-3}$; $e^{\lambda t_A} = 1 + 7.94\times10^{-3} = 1.00794$; then $t_A = 5.77\times10^{8}$ years.

For B: $e^{\lambda t_B} - 1 = 4.65\times10^{-3}$; $e^{\lambda t_B} = 1 + 4.65\times10^{-3} = 1.00465$;

Then $t_B = 3.27\times10^{8}$ years.

d) The carbon 14 (as other isotope) is used to date samples whose ages do not exceed $10T$. Thus, the carbon 14 dates at the most 57000 years old.[1]

[1] To determine the age of a fossil we use the law of radioactive decay under one of its forms $A = A_0e^{-\lambda t}$ or $N = N_0e^{-\lambda t}$; by an electrical analogy, we know that the steady state is reached after $t = 5\tau = 5\times\dfrac{1}{\lambda}$ & $\lambda = \dfrac{\ell n2}{T}$; Then, $t = \dfrac{5}{\ell n2}T \approx 7.21T$. Therefore if the age of the fossil under study exceeds $7T$ then it is difficult to use this radioelement, but we use duration of 10periods as convention.

VII-

Part A

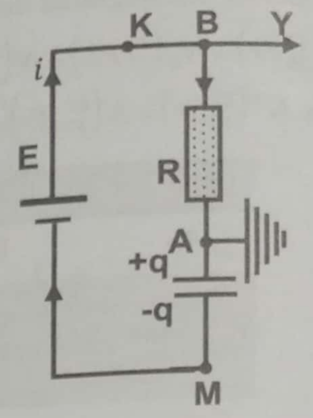
1. See the circuit.
2. The capacitor is taken neutral, then at $t=0$, we have $u_C=0$.

   Then $u_R=u_{BM}-u_C=E$.
3. Law of addition of voltages: $u_{BM}=u_{BA}+u_{AM}$, $E=Ri+u_C$;

   Then, $u_C=E-u_R$ & $i=\frac{u_R}{R}$.

   But $i=C\frac{du_C}{dt}=C\frac{d(E-u_R)}{dt}=-C\frac{du_R}{dt}$;

   We get $\frac{u_R}{R}=-C\frac{du_R}{dt}$; thus, $\frac{du_R}{dt}+\frac{1}{RC}u_R=0$.



4. We have $u_R=Ee^{-\frac{t}{RC}}$, so $\frac{du_R}{dt}=-\frac{E}{RC}e^{-\frac{t}{RC}}$;

   By substitution we get: $\frac{du_R}{dt}+\frac{1}{RC}u_R=-\frac{E}{RC}e^{-\frac{t}{RC}}+\frac{E}{RC}e^{-\frac{t}{RC}}=0$ (verified).

   Verifying the initial condition: at $t=0$, $u_R=Ee^0=E$.
5. In steady state, $t\longrightarrow+\infty$, then $u_R=0$.

Part B

1. The equation of disintegration: ${}^{210}_{83}Bi\longrightarrow{}^{A}_{Z}Po+{}^{0}_{-1}e+{}^{0}_{0}\overline{\upsilon}+\gamma$;

   Conservation of mass number: $210=A+0$, then $A=210$;

   Conservation of charge number: $83=Z-1$, then $Z=84$;

   Then the equation of the disintegration becomes: ${}^{210}_{83}Bi\longrightarrow{}^{210}_{84}Po+{}^{0}_{-1}e+{}^{0}_{0}\overline{\upsilon}+\gamma$.
2. Law of radioactive decay: $N=N_0e^{-\lambda t}$; where N is the number of nuclei remaining at an instant t, N_0 is the number of nuclei at the instant $t_0=0$, and λ is the radioactive constant.
3. We have $N=N_0e^{-\lambda t}$, so $\frac{dN}{dt}=-\lambda N_0e^{-\lambda t}=-\lambda N$, then $\frac{dN}{dt}+\lambda N=0$.
4. We have: $\ell n(N)=\ell n(N_0e^{-\lambda t})=\ell n(N_0)+\ell n(e^{-\lambda t})=\ell n(N_0)-\lambda t$.
5. The expression of $\ell n(N)$ in terms of time is of the form $\ell n(N)=at+b$ where $a=-\lambda<0$, then its graphical representation should be a decreasing straight line which is verified graphically.
6. From graph, at $t_0=0$, $\ell n(N_0)=48$; then $N_0=e^{48}=7\times10^{20}$ nuclei.

   However $-\lambda=\frac{\Delta\ell n(N)}{\Delta t}=\frac{(44-48)}{(30-0)\text{day}}=-0.13\text{ day}^{-1}$ & $T=\frac{\ell n(2)}{\lambda}=\frac{\ell n(2)}{0.13}=5.3$ days.

Part C

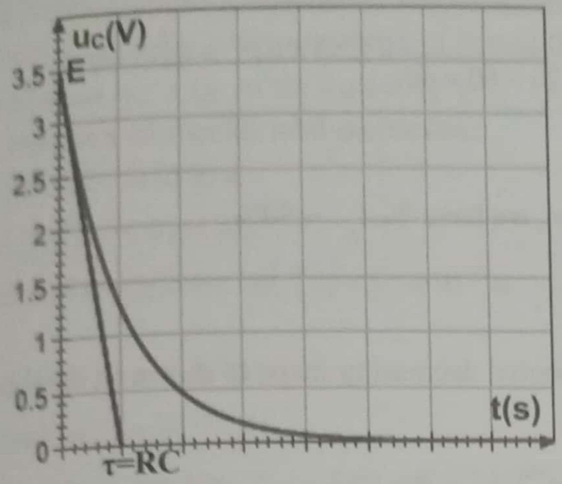
1. We have $\ell n(N)=\ell n(N_0)-\lambda t$;

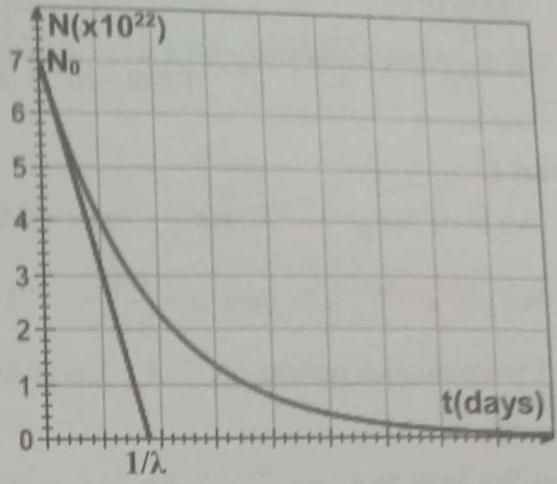
   From part A: $\ell n(u_R)=\ell n\left(Ee^{-\frac{t}{RC}}\right)=\ell n(E)-\frac{1}{RC}t$;

Comparing these two relations we get:

N_0 is analog to E; N is analog to u_R; & λ is analog to $\frac{1}{RC}$.

2. a) Graphs.





VIII-

Part A

1. Isotopes is a set of nuclides having same atomic number Z but different mass number A; isotopes differ only by the number of neutrons.

2. The equation of the disintegration: ${}^{40}_{19}K \longrightarrow {}^{A_1}_{Z_1}Ar + {}^{0}_{+1}e + {}^{0}_{0}\upsilon + \gamma$;

   Conservation of mass number: $40 = A_1 + 0$, then $A_1 = 40$;

   Conservation of charge number: $19 = Z_1 + 1$, then $Z_1 = 18$;

   Then ${}^{40}_{19}K \longrightarrow {}^{40}_{18}Ar + {}^{0}_{+1}e + {}^{0}_{0}\upsilon + \gamma$.

3. a) The equation of disintegration along mode (3): ${}^{40}_{19}K + {}^{0}_{-1}e \longrightarrow {}^{40}_{18}Ar + \gamma$.

   b) We have $E_\gamma = \frac{hc}{\lambda}$, then $\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.46\times1.6\times10^{-13}} = 8.5\times10^{-13}\,m$.

Part B

1. The equation of disintegration along mode (1): ${}^{40}_{19}K \longrightarrow {}^{A}_{Z}Ca + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon} + \gamma$;

   Conservation of mass number: $40 = A + 0$, then $A = 40$;

   Conservation of charge number: $19 = Z - 1$, then $Z = 20$;

   Then ${}^{40}_{19}K \longrightarrow {}^{40}_{20}Ca + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon} + \gamma$.

2. a) The energy liberated appears as kinetic energy carried by the emitted particles and radiant energy carried by gamma radiation.

   b) We know that the energy liberated is given by: $E_\ell = \Delta m c^2$;

   Then $\Delta m = \frac{E_\ell}{c^2} = \frac{1.33}{931.5} = 1.428\times10^{-3}\,u$;

   c) However the mass defect $\Delta m = m_K - m_{Ca} - m_e = 1.428\times10^{-3}\,u$.

   Then $m_{Ca} = 39.963998 - 0.000546 - 1.428\times10^{-3} = 39.962024\,u$.

3. The mass of radioactive isotope is: $m_K = \frac{0.012}{100} \times 140 = 0.0168\ g$;

The number of nuclei in this sample is: $N_K = \frac{m_K}{M} \times N_A = \frac{0.0168}{40} \times 6.022 \times 10^{23} = 2.53 \times 10^{20}$;

The activity:
$$A = \lambda N_K = \frac{\ell n(2)}{T} \times N_K = \frac{\ell n(2)}{1.248 \times 10^{9} \times 365 \times 24 \times 60 \times 60} \times 2.53 \times 10^{20} = 4455\ Bq.$$

Part C

1. We have $\%A_\beta = e^{-\mu x}$; for $x = 4\,mm = 0.004\,m$, we have $\%A_{\beta^-} = 50\%$;

We get $e^{\mu \times 0.004} = 2$, then $\mu = \frac{\ell n(2)}{0.004} = 173.3\,m^{-1}$.

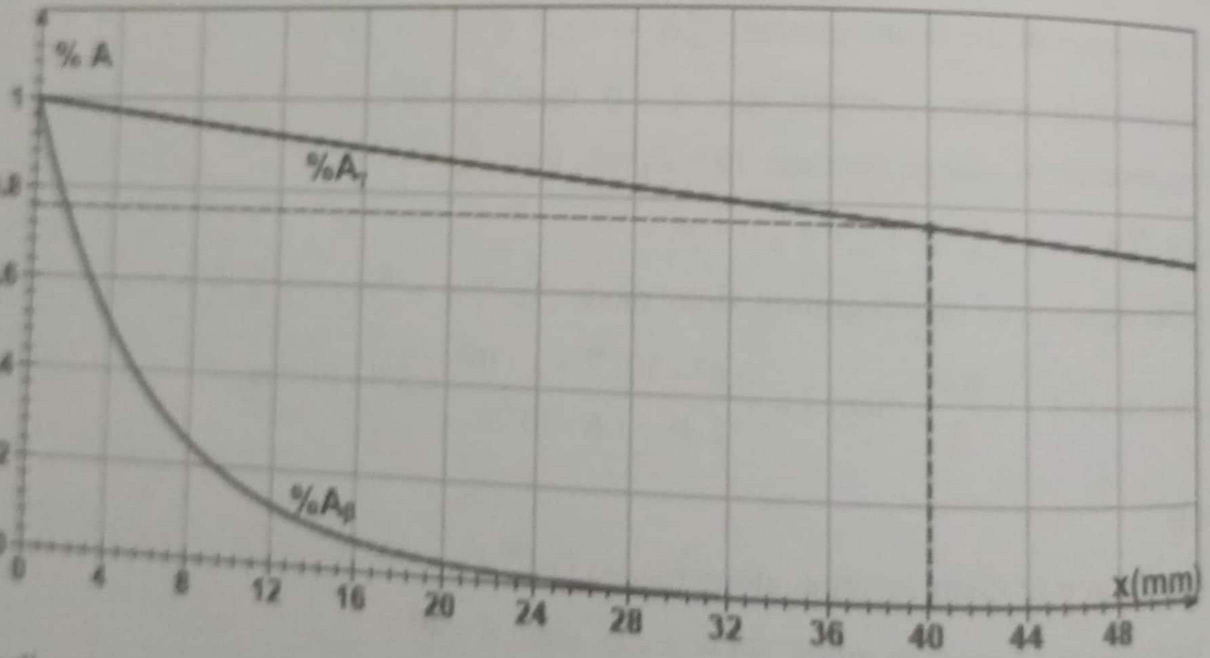
2. The attenuation percentage $\%A_\beta$ has an exponential decreasing shape as shown on the following graph.

3. a) We have $\%A_\gamma = e^{-10\ell n 2\, x}$, for $\%A_\gamma = 50\%$, $e^{10\ell n 2\, x} = 2$;

Then $x \times 10\,\ell n(2) = \ell n(2)$; thus $x = 0.1m = 10\,cm$.

After traveling $10\,cm$, the attenuation of the radiation gamma will be 50%.

b) The curve for the activity attenuation of gamma is shown on the graph below.



4. Graphically:

For $x = 4cm = 40\,mm$, the attenuation of the radiation beta minus tends to zero $\%A_{\beta-40mm} \approx 0$;

While the radiation gamma will decreases to become $\%A_{\gamma-40mm} \approx 75\%$.

Supplementary Problems

LS & GS 2001 1st
Energy liberated by the Disintegration of Cobalt

The cobalt isotope $^{60}_{27}Co$ is radioactive of period (half-life) $T = 5.3$ years. Consider a sample of this isotope of mass $m_0 = 1g$ at the instant $t_0 = 0$.

Given masses of nuclei and particles:

- $m\left(^{60}_{27}Co\right) = 59.9190\ u$;
- $m\left(^{60}_{28}Ni\right) = 59.9154\ u$;
- Mass of an electron: $m\left(^{0}_{-1}e\right) = 5.5\times10^{-4} u$.
- $1u = 931.5\ MeV/c^2$;
- $1\ MeV = 1.6\times10^{-13}\ J$;
- Avogadro's number: $N_A = 6.02\times10^{23}\ mo\ell^{-1}$;
- Molar mass of cobalt: $60\ g.mo\ell^{-1}$.

1. Determine the number of $^{60}_{27}Co$ nuclei that remain and the activity of this sample at the end of 10.6 years.
2. One of the disintegrations of $^{60}_{27}Co$ gives rise to the nickel isotope $^{60}_{28}Ni$.
   a) Write, with justification, the equation of disintegration of one cobalt nucleus $^{60}_{27}Co$. Identify the emitted particle.
   b) Calculate, in J, the energy liberated by the disintegration of $1g$ of cobalt $^{60}_{27}Co$.
   c) Knowing that the energy liberated from the complete combustion of $1g$ of coal is $30\ kJ$. Determine the mass of coal that would liberate the same amount of energy calculated in part b).

Answer Key

1. $A = 1.04\times10^{13}\ Bq$ 2.b) $E_t = 4.544\times10^{9}\ J$.

LS & GS 2002 1st
Radioactivity

Given masses of nuclei and particles:

- $m\left(^{131}_{53}I\right) = 130.87697\ u$; $m\left(^{A}_{Z}Xe\right) = 130.87538\ u$;
- Mass of an electron $m_{e^-} = 5.5\times10^{-4} u$.
- $1u = 931.5\ MeV/c^2$; Speed of light in vacuum: $c = 3\times10^{8}\ m/s$;
- Planck's constant: $h = 6.63\times10^{-34}\ J.s$; $1\ MeV = 1.6\times10^{-13}\ J$;

In order to detect a trouble in the functioning of the thyroid, we inject it with a sample of an iodine radionuclide $\left(^{131}_{53}I\right)$. This radionuclide has a period (half-life) of 8 days and it is a β^- emitter. The disintegration of the nuclide $\left(^{131}_{53}I\right)$ gives rise to a daughter nucleus $^{A}_{Z}Xe$ supposed at rest.

1. a) The disintegration of the nucleus $\left(^{131}_{53}I\right)$ is accompanied by the emission of a γ-radiation. Due to what is this emission?

b) Write the equation of the disintegration of $\left({}^{131}_{53}I\right)$ nucleus.

c) Calculate the decay constant of the radionuclide.
Deduce the number of the nuclei of the sample at the instant of injection knowing that the activity of the sample, at that instant, is $1.5\times10^{5}\ Bq$.

d) Calculate the number of the disintegrated nuclei at the end of 24 days.

2. a) Calculate the energy liberated by the disintegration of one nucleus of $\left({}^{131}_{53}I\right)$.

b) Calculate the energy of a γ-photon knowing that the associated wavelength is $3.55\times10^{-12}\ m$.

c) The energy of an antineutrino being $0.07\ MeV$, calculate the average kinetic energy of an emitted electron.

d) During the disintegration of the $\left({}^{131}_{53}I\right)$ nuclei, the thyroid, of mass $40g$, absorbs only the average kinetic energy of the emitted electrons and that of γ-photons.
Knowing that the dose absorbed by a body is the energy absorbed by a unit mass of this body, calculate, in J/kg, the absorbed dose by the thyroid during 24 days.

Answer Key

1.d) 1.3125×10^{11} nuclei

2.c) $KE(\beta^{-})=0.54876\ MeV$

2.d) $D=0.472\ J/kg$

II-GS 2003 1st
Radioactivity of Polonium 210

In order to study the radioactivity of polonium ${}^{210}_{84}Po$ which is an α emitter, we take a sample of polonium 210 containing N_0 nuclei at the instant $t_0=0$.

Given masses of nuclei and particles:

- Mass of a polonium 210 nucleus: $209.9828\ u$;
- Mass of a lead (Pb) nucleus: $205.9745\ u$;
- Mass of an α particle: $4.0015\ u$;
- $1u=1.66\times10^{-27}kg=931.5\ MeV/c^{2}$.

Part A
Determination of the half-life (period)

We measure, at successive instants, the number N of the remaining nuclei. We calculate the ratio N/N_0 and the result is tabulated as in the following table.

t in days	0	50	100	150	200	250	300
$\frac{N}{N_0}$	1	0.78	0.61	0.47	0.37	0.29	0.22
$-\ell n\left(\frac{N}{N_0}\right)$	0	0.25	...	...	...	1.24	...

1. Draw again the above table and complete it by calculating at each instant $-\ell n(N/N_0)$.
2. Trace the curve representing the variation $f(t)=-\ell n(N/N_0)$, as a function of time, using the scales: 1div on the abscissa represents 25 days & 1div on the ordinate represents 0.1
3. a) Knowing that $\ell n(N/N_0)=-\lambda t$, determine graphically the value of the radioactive constant λ of polonium 210.
   b) Deduce the half-life of polonium 210.

Part B
Activity of polonium 210

1. Define the activity of a radioactive sample.
2. Give the expression of the activity A_0 of the sample at the instant $t_0 = 0$, in terms of λ and N_0.

   Calculate its value for $N_0 = 5\times10^{18}$.
3. Give the expression, in terms of t, of the activity A of the sample.
4. Calculate the activity A:

   a) at the instant $t = 90$ days.

   b) when t increases indefinitely.

Part C
Energy liberated by the disintegration of polonium 210

1. The disintegration of a nucleus of polonium produces a daughter nucleus which is an isotope of lead ${}^{A}_{Z}Pb$. Determine A and Z.
2. Calculate, in MeV, the energy liberated by the disintegration of one nucleus of polonium 210.
3. The disintegration of a polonium nucleus may take place with or without emission of a photon. The energy of an emitted photon is $2.20\ MeV$. Knowing that the daughter nucleus has a negligible velocity, determine in each case the kinetic energy of the emitted α particle.
4. The sample is put in an aluminum container. Thus α particles are stopped by the container whereas the photons are not. Knowing that half of the disintegrations are accompanied by a γ-emission, determine the power transferred to the aluminum container at the instant $t = 90$ days.

Answer Key

Part A 3.b) $T = 139$ days.

Part B 2. $A_0 = 2.5\times10^{16}$ decays/day. 4.b) 0.

Part C 2. $E_\ell = 6.3342\,MeV$ 4. $0.153\,W$.

IV-LS 2004 2nd
The Carbon 14

The object of this exercise is to show evidence of some characteristic properties of the radioelement ${}^{14}_{6}C$ and to show the procedure followed to know the age of a wooden fossil.

Given:

- Mass of a proton: $m_p = 1.00728\ u$;
- Mass of a neutron: $m_n = 1.00866\ u$;
- Mass of a nucleus ${}^{14}_{6}C = 14.0065\ u$;
- Mass of a nucleus ${}^{14}_{7}N = 14.0031\ u$.
- $1u = 931.5\ MeV/c^2$;
- Molar mass of ${}^{14}_{6}C = 14 g.mo\ell^{-1}$;
- Avogadro's number $N_A = 6.02\times10^{23}\ mo\ell^{-1}$.

Part A
Formation of carbon 14

In the high atmosphere, the carbon ${}^{14}_{6}C$ is obtained by the impact of a nitrogen ${}^{14}_{7}N$, with a neutron.

1. Justify that the nuclides ${}^{12}_{6}C$ and ${}^{14}_{6}C$ are two isotopes.
2. Write the equation of the reaction of the formation of ${}^{14}_{6}C$.
3. Identify the emitted particle.

Part B
Disintegration of carbon 14

Carbon 14 is radioactive and is a β^- emitter. It disintegrates to give nitrogen $^{14}_{7}N$.
The emission of a β^- particle is due to the disintegration of a nucleon inside the nucleus.

1. The emission of a β^- particle is due to the disintegration corresponding to this emission.
   Write the equation of the reaction corresponding to this emission. $^{14}_{6}C$ and $^{14}_{7}N$.
2. Calculate the binding energy per nucleon of each of the nuclei $^{14}_{6}C$ and $^{14}_{7}N$.
3. In fact, a radioactive decay leads to a more stable state. Justify this statement taking into account the preceding results.
4. The activity of a substance containing carbon 14 is determined using a counter of β^- particles. A sample of wood containing $0.05\,g$ of carbon 14 of radioactive period $T = 5570$ years is exposed to the counter determine:
   a) the radioactive constant λ of carbon 14.
   b) the number of carbon 14 nuclei contained in this sample at the instant of exposure.
   c) the activity of the sample at the considered instant.

Part C
Age of wood fossil

We intend to determine the age of a wood fossil. We expose this piece to the counter β^-; it indicates 100 disintegrations in 5 minutes. Knowing that a piece of the same wood, freshly cut, gives 1000 disintegrations in 5minutes, determine the age of the wood fossil.

Answer Key

Part A 1. Proton.

Part B 2. Carbon $7.083\,MeV$, 4. c) $A = 2.68 \times 10^{17}$ nuclei/year

Part C 18503 years.

V-Bac 2010
The Americium 241 and Industrial Applications

One of the industrial applications of the americium 241 is the production of neutrons sources in the nuclear reactors in order to ignite the nuclear fission. On the other hand, some smoke detectors, are also equipped many industrial equipment, despite the recycling difficulties, use also the americium 241. The americium isotope 241 does not exist in the natural state. It is produced starting from plutonium 241 $\left(^{241}_{94}Pu\right)$ by a β disintegration.

In this exercise, we study these two applications: neutrons sources and smoke detectors.

- The americium 241 half-life: $T = 433$ years;
- The molar mass of the americium 241: $M\left(^{241}_{95}Am\right) = 241\,g.mo\ell^{-1}$;
- Avogadro number: $N_A = 6.02 \times 10^{23}\,mo\ell^{-1}$.

Part A
Obtaining of americium 241

Write the equation of the β disintegration giving americium 241 starting from plutonium 241.

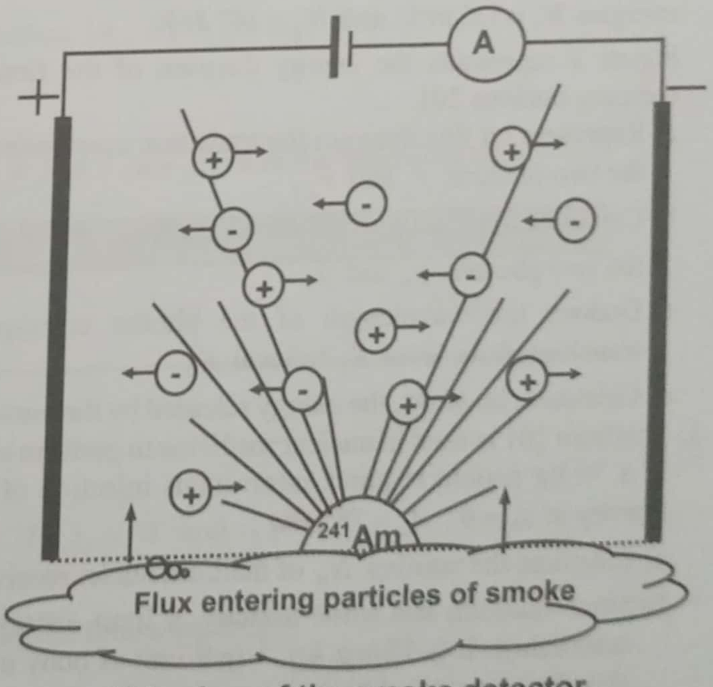
Part B
Disintegration of americium 241

1. During the disintegration of an americium 241 nucleus, the daughter nucleus obtained is neptunium 237 with another particle.

a) Write, using the given, the equation of this process.
b) What do we call this type of disintegration?
2. The neptunium nucleus is obtained in an excited state Np^*.
a) Write the equation of this dis-excitation.
b) What is the nature of the radiation emitted? Due to what is this emission?
c) Write the law of radioactive decay relative to the number of nuclei $N(t)$.
Indicate the meaning of all parameters in this equation? Then indicate their units in SI.
d) Name the parameters on which depends the number of disintegrations of a sample.
3. The activity $A(t)$ of a radioactive sample can be written $A(t) = \lambda N(t)$.
a) Deduce the law of activity decay.
b) What does an activity of a Becquerel represent?
c) We prepare starting from an americium 241 sample two secondary sources: one of mass m and another of mass $2m$. Do they have the same activity? Justify.

Part C
Industrial utilizations of the americium 241
1. Neutrons sources
The mixture beryllium - americium serves as a source of neutrons to trigger fission reactions.
The beryllium $^{9}_{4}Be$ reacts with the α particles emitted by the americium 241 to give a nucleus $^{A}_{Z}X$ and a neutron. Write the equation of this reaction then identify the nucleus $^{A}_{Z}X$.

2. Smoke detector
A smoke detector is constituted from a detection chamber in which is found two electrodes under a voltage and a source containing few tenths of milligrams of americium (**figure**).
The radiation α produced during the disintegration of the americium ionizes the molecules contained in the air of the detection chamber. The ions and the electrons obtained are attracted by the positive plate or negative according to the sign of their charge. The ammeter detects a current in the circuit.
When the smoke enters in the detection chamber, the ions and the electrons are fixed on the particles contained in the smoke. The modification in the value of the current triggers the alarm. In order to determine the americium mass contained in the detector, we measure the activity of the sample at an instant t_0. We find $A_0 = 2.1\times10^7\ Bq$.



Side view of the smoke detector

a) Calculate the number N_0 of nuclei presents at the instant of measurement.
b) Deduce the mass m_0 of the radioactive sample in grams.

Answer Key
Part A
Part B 2.c) Type, duration & mass 3.c) doubled
Part C 2.b) $m_0 = 0.12mg$.

VI-Engineering (2017/2018)

Use of Radioactive Nuclei in Medicine

Given

- $1u = 931.5\,MeV/c^2 = 1.66\times10^{-27}\,kg$; Speed of light in vacuum: $c = 3\times10^{8}\,m/s$;
- Planck's constant: $h = 6.63\times10^{-34}\,J.s$; $1eV = 1.6\times10^{-19}\,J$;
- Radioactive constant of thallium 201: $\lambda_{Tl} = 2.6\times10^{-6}\,s^{-1}$.

Particle or nucleus	Hg 201	proton	neutron	electron
Mass in u	200.970 032	1.00728	1.00866	0.00055

1. A thallium nucleus 201 is obtained by β^+ radioactive decay of a lead nucleus 201, itself obtained by bombarding a thallium 203 target with protons, according to the reaction:

$$^{203}_{81}T\ell + ^{1}_{1}H \longrightarrow ^{201}_{82}Pb + y\ ^{a}_{z}X \quad (1)$$

a) Identify the particle $\left(^{a}_{z}X\right)$ and calculate y.

b) Write the equation (2) of the decay of the lead nucleus 201 into a thallium nucleus 201. It will be assumed that the daughter nucleus is obtained in the ground state.

2. The thallium nucleus 201, having a binding energy per nucleon $E_b / A = 7.684\ MeV/$ nucleon, absorbs an electron and transforms into a mercury nucleus ($^{201}_{80}$Hg) according to the equation:

$$^{201}_{81}T\ell + ^{0}_{-1}e \longrightarrow ^{201}_{80}Hg + ^{0}_{0}\upsilon + \gamma \quad (3).$$

This nuclear reaction is accompanied by the emission of several photons, among which the photons γ_1 and γ_2 that are of respective energies $W_1 = 135\ keV$ and $W_2 = 167\ keV$.

Figure 1 represents the energy diagram of the first levels of the mercury nucleus 201.

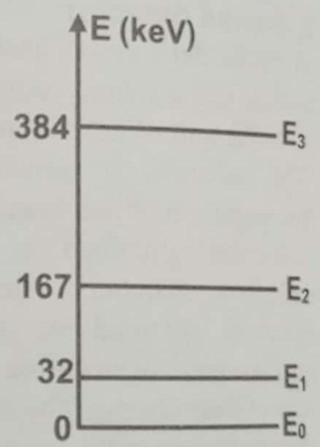


Figure 1

a) Represent on this diagram the transition corresponding to each of the two photons γ_1 and γ_2.

b) Calculate the values of the wavelengths λ_1 and λ_2, in vacuum, of the two photons γ_1 and γ_2.

c) Deduce the wavelength of the photon corresponding to the transition from level E_1 to level E_0.

d) Calculate, in MeV, the energy released by the nuclear reaction (3).

3. Thallium 201 is used in nuclear medicine to perform a diagnosis following heart pain. Examination of a 70 kg patient requires intravenous injection of a thallium chloride solution with an initial activity at $t_0 = 0$, $A_0 = 78\ MBq$.

a) Calculate the number N_0 of thallium nuclei received by this patient at the instant of injection.

b) Since thallium has some toxicity, a limit energy absorbed per unit of body mass has been established. It is $15 mg.kg^{-1}$ (per unit of body mass). Check by a calculation that the energy absorbed per unit of body mass injected to the patient is safe.

c) It is believed that the results of the examination can be used as long as the activity of thallium 201 is greater than $3\,MBq$.

Determine, in days, the time after which a new injection is necessary if we want to continue the examination to ensure the diagnosis.

Answer Key

2. b) $\lambda_1 = 9.19pm$ $\quad\lambda_1 = 7.44nm$ $\quad$ 2.d) $14.5MeV$.

3. b) $1.43\times10^{-7}\ mg/kg$ $\quad$ 3. c) 14.5 days.

LS 2013 1st
Dating by Carbon 14

The radioactive carbon isotope ${}^{14}_{6}C$ is a β^- emitter. In the atmosphere, ${}^{14}_{6}C$ exists with the carbon 12 in a constant ratio. When an organism is alive it absorbs carbon dioxide that comes indifferently from carbon 12 and carbon 14. Just after the death of an organism, this absorption stops and carbon 14, that it has, disintegrate with a half-life $T = 5700$ years.

In living organisms, the ratio of the number of carbon 14 atoms to that of the number of carbon 12 atoms is: $r_0 = \dfrac{\text{initial number of carbon 14 atoms}}{\text{number of carbon 12 atoms}} = \dfrac{N_0\left({}^{14}C\right)}{N'\left({}^{12}C\right)} = 10^{-12}$.

After the death of an organism by a time t, the ratio of the number of carbon 14 atoms to that of the number of carbon 12 atoms becomes: $r = \dfrac{\text{remaining number of carbon 14 atoms}}{\text{number of carbon 12 atoms}} = \dfrac{N\left({}^{14}C\right)}{N'\left({}^{12}C\right)}$.

1. The disintegration of ${}^{14}_{6}C$ is given by: ${}^{14}_{6}C \longrightarrow {}^{A}_{Z}N + \beta^- + {}^{0}_{0}\bar{\upsilon}$.

   Calculate Z and A, specifying the laws used.
2. Calculate, in year^{-1}, the radioactive constant λ of carbon 14.
3. Using, the law of radioactive decay of carbon 14, $N\left({}^{14}C\right) = N_0\left({}^{14}C\right) \times e^{-\lambda t}$.

   Show that $r = r_0\, e^{-\lambda t}$.
4. Measurements of $\dfrac{r}{r_0}$, for specimens a, b and c, are given in the following table:

Ratio	specimen a	specimen b	specimen c
$\dfrac{r}{r_0}$	0.914	0.843	0.984

   a) Specimen b is the oldest. Why?
   b) Determine the age of the specimen b.

5. a) Calculate the ratio $\dfrac{r}{r_0}$ for $t_0 = 0$, $t_1 = 2T$, $t_2 = 4T$ and $t_3 = 6T$.

   b) Trace then the curve $\dfrac{r}{r_0} = f(t)$ by taking the following scales:
   - On the abscissas axis: $1cm \equiv 2T$;
   - On the ordinate axis: $1cm \equiv \dfrac{r}{r_0} = 0.2$;

   c) To determine the date of death of a living organism, it is just enough to measure $\dfrac{r}{r_0}$.

   Explain why we cannot use the traced curve to determine the date of the death of an organism that died several millions years ago.

BLS 2012 1st

Iodine 131

The aim of this exercise is to show evidence of some characteristics of iodine 131.

Iodine 131 $\left({}^{131}_{53}I\right)$ is radioactive and is a β^- emitter. Its radioactive period (half-life) is 8 days.

Given

- $1u = 931.5\, MeV/c^2$; $1\, MeV = 1.6\times10^{-13}\, J$.

Masses of some nuclei:

- Iodine $\left({}^{131}_{53}I\right)$: $130.8770\, u$;
- Cesium $\left({}^{137}_{55}Cs\right)$: $136.8773\, u$;
- Xenon $\left({}^{131}_{54}Xe\right)$: $130.8754\, u$;
- Mass of an electron $m_e = 5.5\times10^{-4}\, u$.

Part A

Disintegration of iodine 131

1. Write down the equation of the disintegration of iodine 131 and identify the daughter nucleus.
2. The disintegration of iodine 131 nucleus, is often, accompanied with the emission of γ rays. Due to what is this emission?
3. Calculate the radioactive constant λ of iodine 131 in day^{-1} and in s^{-1}.
4. Show that the energy liberated by the disintegration of one nucleus of iodine 131 is: $E_{lib} = 1.56\times10^{-13}\, J$.

Part B

Application in medicine

During a medical examination of a thyroid gland of a patient, we inject this gland with a solution of iodine 131. The thyroid of this patient captures from this solution a number $N = 10^{11}$ of iodine nuclei.

1. Calculate, in Bq, the activity A corresponding to these N nuclei knowing that $A = \lambda N$.
2. Calculate, in J, the energy liberated by the disintegration of these N nuclei.
3. Deduce, in J/kg, the value of the dose absorbed by the thyroid gland knowing that its mass is $25\, g$.

Part C

Contamination

On the 26th of April 1986, an accident took place in the nuclear power plant of Chernobyl that provoked an explosion in one of the reactors. One of the many radioactive elements that were ejected to the atmosphere is the iodine 131. This element spread on the ground, absorbed by cows and contaminated their milk and then captured by the thyroid gland of consumers.

Every morning, a person drank a certain quantity of milk containing $N_0 = 2.6\times10^{16}$ nuclei of iodine 131.

We suppose that all these nuclei were captured by the thyroid of that person, and that the person drank the first quantity at the instant $t_0 = 0$.

1. Determine, in terms of N_0 and λ (expressed in day^{-1}), the number of iodine 131 nuclei that remained in the thyroid, at the instant:

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a) $t_1 = 1$ day, (just after drinking the 2nd quantity of milk);

b) $t_2 = 2$ days, (just after drinking the 3rd quantity of milk).

2. Deduce, at the instant $t_3 = 3$ days just after drinking the 4th quantity of milk that the number N_3 of the iodine 131 nuclei that remained in the thyroid is: $N_3 = N_0\left(1 + e^{-\lambda} + e^{-2\lambda} + e^{-3\lambda}\right)$ where λ is expressed in day^{-1}.
3. Serious troubles in the thyroid gland will take place if the activity of the iodine 131 exceeds $75 \times 10^9\ Bq$. Show that at the instant t_3, the person was in danger.

III-LS 2010 1st

The Radio-Isotope Polonium Po

Given

- $1u = 1.66 \times 10^{-27}\ kg = 931.5\ MeV / c^2$;
- $1\ MeV = 1.6 \times 10^{-13}\ J$;
- $h = 6.63 \times 10^{-34}\ J.s$;
- $c = 3 \times 10^8\ m/s$.

Mass of some nuclei

- $m(Po) = 209.9829\ u$;
- $m(Pb) = 205.9745\ u$;
- $m(\alpha) = 4.0026\ u$.

Part A

Decay of polonium 210

The polonium ${}^{210}_{84}Po$ is an α emitter. The daughter nucleus produced by this decay is the lead ${}^{A}_{Z}Pb$

1. Determine Z and A specifying the laws used.
2. Calculate, in MeV and in J, the energy liberated by this decay.
3. The nucleus ${}^{210}_{84}Po$ is initially at rest. We suppose that the daughter nucleus ${}^{A}_{Z}Pb$ is obtained at rest and in the fundamental state.
   Deduce the kinetic energy of the emitted α particle.
4. In general, the decay of ${}^{210}_{84}Po$ is accompanied by the emission of γ-radiation.
   a) Due to what is the emission of γ-radiation?
   b) The emitted γ-radiation has the wavelength $\lambda = 1.35 \times 10^{-12}\ m$ in vacuum.
   Using the conservation of total energy, determine the kinetic energy of the emitted α particle.

Part B

Radioactive period of polonium 210

The adjacent figure shows the curve representing the variations with time t of the number N of the nuclei present in the radioactive sample ${}^{210}_{84}Po$, this number being called N_0 at the instant $t_0 = 0$.
The same figure shows also the tangent to that curve at the instant $t_1 = 263$ days.

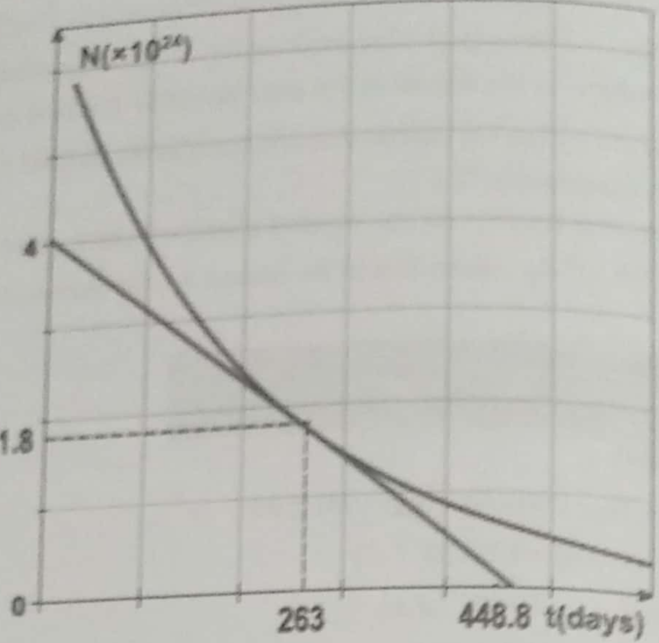
1. Write down the expression of N as a function of t and specify what does each term represent.

2. The activity of the radioactive sample is given by: $A = -\frac{dN}{dt}$.

a) Define the activity A.

b) Using the given on the figure above, determine the activity A of the sample at the instant $t_1 = 263$ days.

3. Deduce the value of the radioactive constant and the value of the half-life (period) of polonium 210.



N-LS 2006 1st

Determination of the Age of the Earth

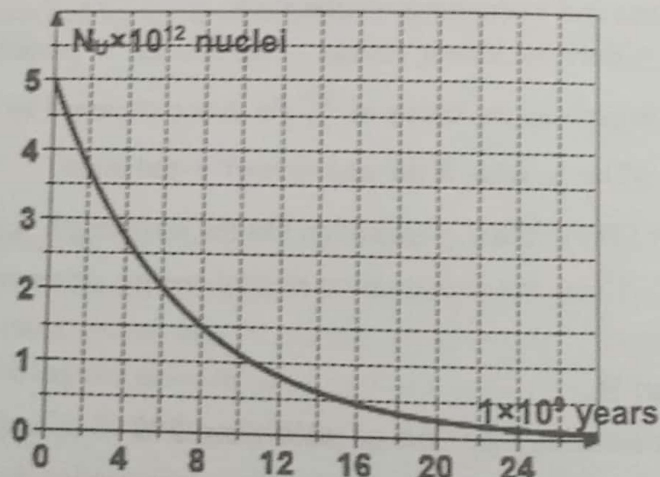
The object of this exercise is to determine the age of the Earth using the disintegration of a uranium 238 nucleus $\left({}^{238}_{92}U\right)$ into a lead 206 nucleus $\left({}^{206}_{82}Pb\right)$.

When we determine the number of lead 206 nuclei in a sample taken out from a rock that did not contain lead when it was formed, we can then determine its age that is the same as that of the Earth. The figure below represents the curve of the variation of the number N_u of uranium 238 nuclei as a function of time.

- 1 division on the axis of ordinates corresponds to 10^{12} nuclei.
- 1 division on the axis of abscissa corresponds to 10^9 years.

The equation of the disintegration of Uranium 238 into lead 206 is ${}^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + x\beta^- + y\alpha$.

1. Determine, specifying the laws used, the values of x and y.
2. Referring to the curve, indicate the number N_{0u} of uranium 238 nuclei existing in the sample at the date of its birth $t_0 = 0$.
3. Referring to the curve, determine the period (half-life) of uranium 238. Deduce the value of the radioactive constant λ of uranium 238.



4. a) Give, in terms of λ and t, the expression of the number N_u of uranium 238 nuclei remaining in the sample at instant t.

b) Calculate the number of uranium 238 nuclei remaining in the sample at instant $t_1 = 2\times10^9$ years.

c) Verify this result graphically.

5. The number of lead 206 nuclei existing in the sample at the instant of measurement (age of the Earth) is $N_{Pb} = 2.5 \times 10^{12}$ nuclei.

a) Give the relation among N_u, N_{0u} and N_{Pb}.

b) Calculate the number N_u of uranium nuclei remaining in the sample at the date of measurement.

c) Determine the age of the Earth.

V-LS 2007 2nd
Radioactivity of Cobalt

Cobalt $^{60}_{27}Co$ is a radioactive. The daughter nucleus $^{A}_{Z}Ni$ undergoes a downward transition to the ground state. The energy due to this downward transition is $E(\gamma) = 2.5060\ MeV$.

The β^- particle is emitted with a kinetic energy $KE(\beta^-) = 0.0010\ MeV$.

Numerical data

- Mass of an electron: $m_e = 5.486 \times 10^{-4} u$;
- Mass of the $^{60}_{27}Co$ nucleus: $59.91901\ u$;
- Mass of the $^{A}_{Z}Ni$ nucleus: $59.91544\ u$;
- $1\ u = 931.5\ MeV/c^2$;
- $1\ MeV = 1.6 \times 10^{-13}\ J$.

Part A
Study of the disintegration

1. Determine A and Z.
2. Calculate, in u, the mass defect Δm during this disintegration.
3. Deduce, in MeV, the energy E liberated by this disintegration.
4. During this disintegration, the daughter nucleus is practically obtained at rest.
   In what form of energy does E appear?
5. a) Deduce, from what preceded, that the electron emitted by the considered disintegration, is accompanied by a certain particle.
   b) Give the name of this particle.
   c) Give the charge number and the mass number of this particle.
   d) Deduce, in MeV, the energy of this particle.
6. Write down the global equation of this disintegration.

Part B
The use of cobalt 60

In medicine, we use a source of radioactive cobalt $^{60}_{27}Co$ of activity $A = 6 \times 10^{19}\ Bq$. The emitted β^- particles are absorbed by the living organism.

1. The energy of the particle mentioned in the question (A-5) is not absorbed by the living organism. Why?
2. Calculate, in watt, the power transferred to the organism.
3. This large power is used in radiotherapy. What is its effect?

VI-LS 2006 1st

Measurement of the Age of Earth

One of the questions that preoccupied man long ago since he started to explore the universe was the age of Earth. As from 1905, Rutherford proposed a measurement of the age of minerals through radioactivity. In 1956, Clair Paterson used the method (uranium - lead) to measure the age of a meteorite assuming that it originates from a planet that is formed approximately at the same time as that of Earth.

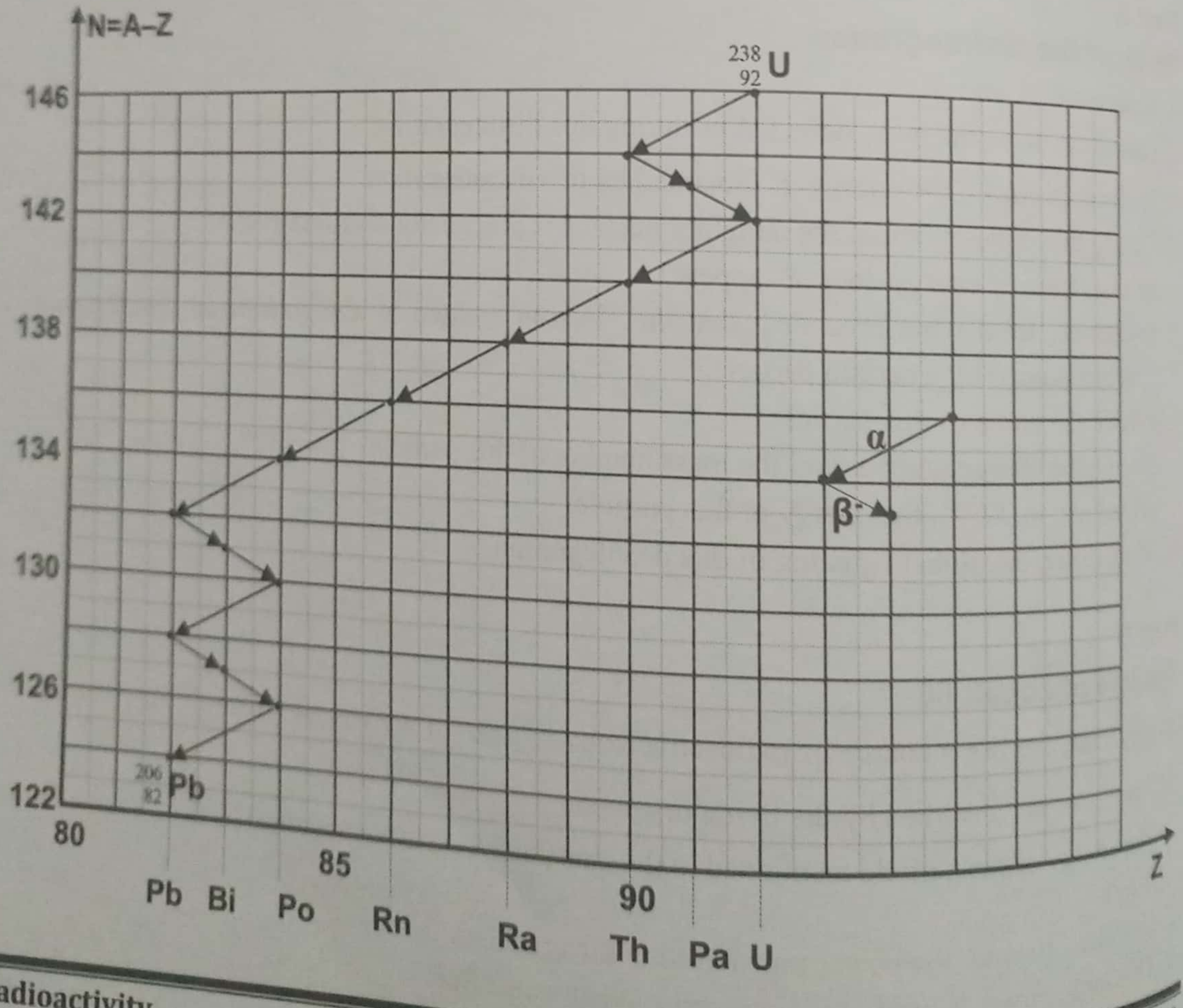
Part A

Radioactive family of uranium $^{238}_{92}U$

Uranium 238, of radioactive period $T = 4.5\times10^{9}\ y$ (year) is at the origin of a radioactive family leading finally to the stable lead isotope $^{206}_{82}Pb$. Each of these successive disintegrations is accompanied with the emission of an α particle or a β^- particle.

The diagram (Z, N) gives all the radioactive nuclei originating from the uranium $^{238}_{92}U$ leading to the stable isotope $^{206}_{82}Pb$ (**below**). The tables (**Next page**) give the radioactive period of each nuclide.

1. In the first disintegration, a uranium nucleus $^{238}_{92}U$ gives thorium nucleus $^{234}_{90}Th$ and a particle denoted by $^{A_1}_{Z_1}X$.
   a) Write the equation of this disintegration and calculate the values of A_1 and Z_1.
   b) Specify the type of radioactivity corresponding to this transformation.
2. In the second disintegration, the thorium nucleus $^{234}_{90}Th$ undergoes a β^- decay. The daughter nucleus is the protactinium $^{A_2}_{Z_2}Pa$. Calculate A_2 and Z_2.



3\. a) Referring to the diagram, tell how many α particles and how many β^- particles are emitted when the uranium nucleus ${}^{238}_{92}U$ is transformed into lead nucleus ${}^{206}_{82}Pb$.

b) Write the overall nuclear equation of the decay of uranium 238 into lead 206.

4\. Using the diagram (Z, N) of the figure and the tables, tell why after few billions of years we can neglect the presence of the intermediary nuclei among the products of the disintegration uranium-lead.

Nucleus	${}^{218}_{84}Po$	${}^{214}_{82}Pb$	${}^{214}_{83}Bi$	${}^{214}_{84}Po$	${}^{210}_{82}Pb$	${}^{210}_{83}Bi$	${}^{210}_{84}Po$
Radioactive period	3.1 min	27 min	20 min	$1.6\times10^{-4}\ s$	22 y	5d	138 d

Nucleus	${}^{238}_{92}U$	${}^{234}_{90}Th$	${}^{234}_{91}Pa$	${}^{234}_{92}U$	${}^{230}_{90}Th$	${}^{226}_{88}Ra$	${}^{222}_{86}Rn$
Radioactive period	$4.5\times10^{9}\ y$	24 d	6.7 h	$2.5\times10^{5}\ y$	$7.5\times10^{3}\ y$	$1.6\times10^{3}\ y$	3.8 d

Part B

The age of Earth

We have studied a sample of a meteorite whose age is equal to that of Earth. At the instant t, the sample studied contains 1g of uranium 238 and 0.88 g of lead 206. We suppose that at the instant of its formation $t_0 = 0$, the meteorite does not contain any atom of lead.

Numerical data

- molar mass of uranium $238\,g/mo\ell$;
- molar mass of lead $206\ g/mo\ell$;
- Avogadro's number $N_A = 6.02\times10^{23}\ mo\ell^{-1}$.

1\. Calculate, at the instant t:

a) the number of uranium 238 nuclei, denoted by $N_U(t)$, present now in the sample;

b) the number of lead 206 nuclei, denoted by $N_{Pb}(t)$, present now in the sample.

2\. Deduce the number of uranium 238 nuclei $N_U(0)$, present in the sample at the instant $t_0 = 0$.

3\. Give the expression of $N_U(t)$ as a function of $N_U(0)$, t, and T.

4\. Deduce the age of Earth at the instant.

VII-LS 2005 1st

Radioactivity

A physics laboratory is equipped with a radioactivity counter together with a source of radioactive cesium ${}^{137}_{55}Cs$ which is a β^- emitter.

The technical data sheet of the counter carries the following indications:

- nuclide: ${}^{137}_{55}Cs$;

- half-life: $T = 30$ years;
- activity of the source at the date of fabrication of the counter: $A_0 = 4.40 \times 10^5\ Bq$;
- energy of beta radiation: $0.514\ MeV$;
- energy of gamma radiation: $0.557\ MeV$;
- $1eV = 1.6 \times 10^{-19}\ J$;
- $1u = 931.5\ MeV/c^2$.

Masses of nuclei and particles

- $m(Cs) = 136.8773\ u$;
- $m(Ba) = 136.8756\ u$;
- $m(\text{electron}) = 5.5 \times 10^{-4}\ u$.

Part A

Energy liberated by a cesium nucleus

1. a) Write the equation of the decay of cesium 137, knowing that the daughter nucleus is ${}^{y}_{x}Ba$. Determine x and y.

   b) The barium ${}^{y}_{x}Ba$ obtained is in an excited state. Write the equation of the downward transition of the barium nucleus.

2. a) Calculate, in MeV, the energy E liberated during the disintegration of a cesium nucleus.

   b) Starting from the technical data sheet, deduce the energy carried by the antineutrino neglecting the kinetic energy of the barium nucleus.

Part B

Activity of cesium

1. At the beginning of the school year 2004, we measure, using the counter, the activity A of the source. We obtain the value $3.33 \times 10^5\ Bq$. Determine the year of fabrication of the counter equipped with its source knowing that $A = A_0 e^{-\lambda t}$, λ being the radioactive constant of cesium.
2. The activity of the source remains practically the same within one hour. Starting from the definition of the activity of a radioactive source, deduce the number n of disintegrations that cesium undergoes within one hour.

Part C

Consequences of using the cesium source

1. Having calculated the values of E and n, calculate, in J, the energy received by a student within one hour of experimental work in the laboratory knowing that the student absorbs 1% of the liberated nuclear energy.
2. Knowing that the maximum nuclear energy that a student may absorb within one hour without any risk is $1.2 \times 10^{-4}\ J$, verify that student is not subject to any danger.

Sessions – Solutions

LS 2013 1st

1. ${}^{14}_{6}C \longrightarrow {}^{A}_{Z}N + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon}$;

Conservation of mass number: $14 = A + 0$, then $A = 14$;

Conservation of atomic number: $6 = Z - 1$, then $Z = 7$;

Then the equation of the disintegration becomes: ${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon} + \gamma$.

2. The radioactive constant $\lambda = \frac{\ell n2}{T} = \frac{\ell n2}{5700} = 1.216 \times 10^{-4}$ year^{-1}.

3. We have $N({}^{14}C) = N_0({}^{14}C)e^{-\lambda t}$; dividing both sides by $N'({}^{12}_{6}C)$: $\frac{N({}^{14}C)}{N'({}^{12}C)} = \frac{N_0({}^{14}C)}{N'({}^{12}C)}e^{-\lambda t}$;

However $r = \frac{N({}^{14}C)}{N'({}^{12}C)}$ and $r_0 = \frac{N_0({}^{14}C)}{N'({}^{12}C)}$; then $r = r_0\, e^{-\lambda t}$.

4. a) The ratio $\frac{r}{r_0} = e^{-\lambda t}$ is a decreasing function so as t increases, $\frac{r}{r_0}$ decreases; thus, the smallest value of this ratio corresponds to the oldest specimen.

However $\frac{r}{r_0}(b) < \frac{r}{r_0}(a) < \frac{r}{r_0}(c)$; thus b is the oldest.

b) We have $\frac{r}{r_0} = e^{-\lambda t} = 0.843 \Rightarrow -\frac{\ell n(2)}{T} \times t = \ell n(0.843)$;

Then $t = -\frac{T}{\ell n(2)}\ell n(0.843) = 1406$ years.

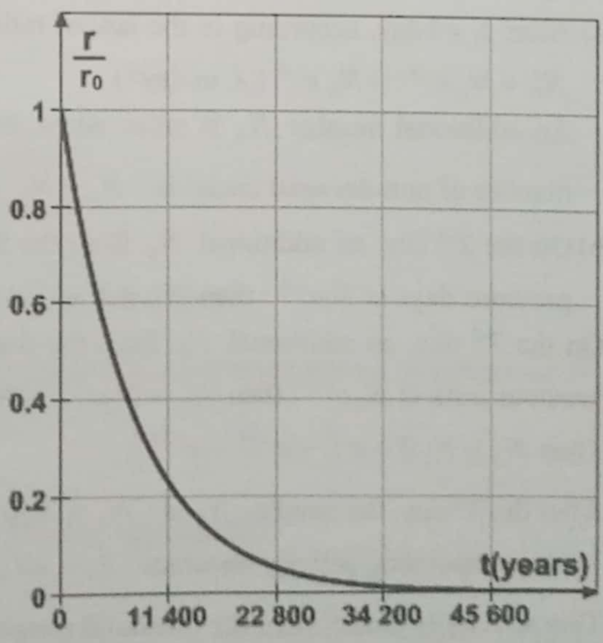
5. a) At $t_0 = 0$; $\frac{r}{r_0} = e^0 = 1$;

At $t_1 = 2T$; $\frac{r}{r_0} = e^{-2\ell n2} = 0.25$;

At $t_2 = 4T$; $\frac{r}{r_0} = e^{-4\ell n2} = 0.0625$;

At $t_3 = 6T$; $\frac{r}{r_0} = e^{-6\ell n2} = 0.015625$.

b) Graph.



c) The graphical analysis of this curve shows that after $t > 6T$, the curve tends to zero.

Thus after millions of years the ratio $\frac{r}{r_0}$ becomes zero so we cannot determine the age of such organism.

Part A

1. Equation of disintegration: ${}^{131}_{53}I \longrightarrow {}^{A}_{Z}X + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon} + \gamma$;
According to Soddy's laws $A = 131$ & $Z = 54$;
Then the daughter nucleus is the Xenon $\left({}^{131}_{54}Xe\right)$.
We get: ${}^{131}_{53}I \longrightarrow {}^{131}_{54}Xe + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon} + \gamma$.
2. The emission of the γ-rays is due to the dis-excitation of the daughter nucleus which drops into a lower energy level.
3. The radioactive constant $\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{8} = 0.087\ \text{day}^{-1}$;
$$\& \ \lambda = \frac{\ell n(2)}{T} = \frac{\ell n 2}{8 \times 24 \times 3600} \approx 10^{-6}\ s^{-1}.$$
4. The mass defect of this reaction is:
$\Delta m = m\left({}^{131}_{53}I\right) - \left[m\left({}^{131}_{54}Xe\right) + m\left({}^{0}_{-1}e\right)\right] = 130.8770 - (130.8754 + 0.00055) = 1.05 \times 10^{-3}\,u$;
The energy liberated is given by:
$$E = E_\ell = \Delta m c^2 = 1.05 \times 10^{-3} \times \frac{931.5 \times 1.6 \times 10^{-13}\,J}{c^2} \times c^2 = 1.56 \times 10^{-13}\,J.$$

Part B

1. The activity $A = \lambda N = 10^{-6} \times 10^{11} = 10^{5}\,Bq$.
2. The total energy liberated is $E_t = N \times E_\ell = 10^{11} \times 1.56 \times 10^{-13} = 1.56 \times 10^{-2}\,J$.
3. The dose is the energy absorbed by a unit of mass $D = \frac{E_t}{m} = \frac{1.56 \times 10^{-2}\,J}{25 \times 10^{-3}\,kg} = 0.624\,J/kg$.

Part C

1. a) After $t_1 = 1$ day, according to the law of radioactive decay the number of remaining nuclei is $N_0' = N_0 e^{-\lambda t} = N_0 e^{-\lambda}$ (λ in day^{-1}).
An additional number N_0 is taken when drinks the second quantity next morning. Thus the number of non-decayed nuclei is : $N_1 = N_0 + N_0 e^{-\lambda} = N_0\left(1 + e^{-\lambda}\right)$.
b) On the 2nd day, an additional N_0 from the third quantity and the number remaining from the previous days is $N_1 e^{-\lambda}$: then $N_2 = N_1 e^{-\lambda} + N_0 = N_0\left(1 + e^{-\lambda}\right)e^{-\lambda} + N_0 = N_0\left(1 + e^{-\lambda} + e^{-2\lambda}\right)$
2. On the 3rd day, an additional N_0 from the fourth quantity and the number remaining from the previous milk is $N_2 e^{-\lambda}$: then $N_3 = N_2 e^{-\lambda} + N_0 = N_0\left(1 + e^{-\lambda} + e^{-2\lambda}\right)e^{-\lambda} + N_0$;
Thus $N_3 = N_0\left(1 + e^{-\lambda} + e^{-2\lambda} + e^{-3\lambda}\right)$.
3. After the 3rd day, the number N_3 is : $N_3 = N_0\left(1 + e^{-0.087} + e^{-2\times0.087} + e^{-3\times0.087}\right) = 9.17 \times 10^{16}$ nuclei;
The corresponding activity becomes: $A_3 = \lambda N_3 = 10^{-6} \times 9.17 \times 10^{16} = 9.17 \times 10^{10}\,Bq$.
This activity is greater than the threshold danger activity $A_3 = 9.17 \times 10^{10}\,Bq > A_{th} = 7.5 \times 10^{10}\,Bq$
Thus, the person is in danger.

Part A

1. The equation of disintegration is: ${}^{210}_{84}Po \longrightarrow {}^{4}_{2}He + {}^{A}_{Z}Pb + \gamma$;

Conservation of mass number: $210 = 4 + A$, then $A = 210 - 4 = 206$;

Conservation of charge number: $84 = 2 + Z$, then $Z = 84 - 2 = 82$;

We get: ${}^{210}_{84}Po \longrightarrow {}^{4}_{2}He + {}^{206}_{82}Pb + \gamma$.

2. The mass defect of this reaction is:

$\Delta m = m\left({}^{210}_{84}Po\right) - \left[m\left({}^{4}_{2}He\right) + m\left({}^{206}_{82}Pb\right)\right] = 209.98294 - (4.0026 + 205.97454) = 0.0058\ u$;

The energy liberated is given by: $E_\ell = \Delta m c^2 = 0.0058 \times \dfrac{931.5 MeV}{c^2} \times c^2 = 5.4027\ MeV$.

$E_\ell = 5.4027 \times 1.6 \times 10^{-13} J = 8.64432 \times 10^{-13} J$.

3. According to the conservation of energy: $E_\ell = \left[KE_\alpha + KE_{Pb} + E_\gamma\right] - KE_{Po} = KE_\alpha$;

We have $KE_{Pb} = KE_{Po} = 0$ (at rest); and no γ-emission so $E_\gamma = 0$;

Thus $KE_\alpha = E_\ell = 5.4027\ MeV = 8.64432 \times 10^{-13} J$.

4. a) The emission of γ-ray is due to the dis-excitation of the daughter nucleus.

b) The energy of γ-ray is given by:

$$E_\gamma = h\frac{c}{\lambda} = 6.63 \times 10^{-34} J.s \times \frac{3 \times 10^{8} m/s}{1.35 \times 10^{-12} m} = 1.4733 \times 10^{-13} J = 0.92 MeV;$$

Global conservation of energy:

$m\left({}^{210}_{84}Po\right)c^2 + KE_{Po} = m_\alpha c^2 + KE_\alpha + m\left({}^{206}_{82}Pb\right)c^2 + KE_{Pb} + E_\gamma$.

But $KE_{Po} = KE_{Pb} = 0$ (at rest); so $m\left({}^{210}_{84}Po\right)c^2 = m_\alpha c^2 + KE_\alpha + m\left({}^{206}_{82}Pb\right)c^2 + E_\gamma$;

Then: $KE_\alpha = m\left({}^{210}_{84}Po\right)c^2 - \left[m_\alpha c^2 + m\left({}^{206}_{82}Pb\right)c^2\right] - E_\gamma$.

Thus: $KE_\alpha = E_\ell - E_\gamma = 5.4027 - 0.92 = 4.4827\ MeV$.

Part B

1. Law of radioactive decay: $N = N_0 e^{-\lambda t}$;

- N is the number of nuclei remaining at the instant t;
- N_0 is the number of nuclei at the instant $t_0 = 0$;
- λ is the radioactive constant.

2. a) The activity A is the number of disintegrations per unit of time.

b) We have $A|_{t_1} = -\left.\dfrac{dN}{dt}\right|_{t_1}$ is the slope of the tangent to the curve representing to variations N as a function of time of at the instant t_1.

$$A|_{t_1} = -\left.\frac{dN}{dt}\right|_{t_1} = -\frac{N_2 - N_1}{t_2 - t_1} = \frac{4.48 \times 10^{24}}{448.8} = 8.91 \times 10^{21}\text{ decays/day.}$$

3. We have: $A|_{t_1} = \lambda N|_{t_1} \Rightarrow \lambda = \dfrac{A|_{t_1}}{N|_{t_1}} = \dfrac{8.91 \times 10^{21}}{1.8 \times 10^{24}} = 0.00495\ \text{day}^{-1}; T = \dfrac{\ell n(2)}{\lambda} = \dfrac{\ell n(2)}{0.00945} \approx 140$ days.

V-LS 2008 1st

1. The equation of the disintegration is: ${}^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + x\ {}^{0}_{-1}e + y\ {}^{4}_{2}He$.

Conservation of mass number: $238 = 206 + 4y$, then $y = 8$ α decays;

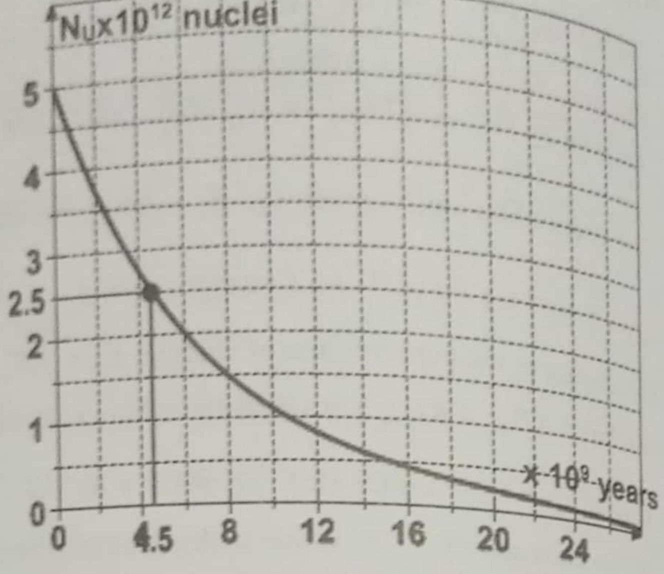
Conservation of charge number: $92 = 82 - x + 2y$, then $x = 6$ β^- decays;

We get: ${}^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 8\ {}^{4}_{2}He + 6\ {}^{0}_{-1}e + 6{}^{0}_{0}\overline{\upsilon}$.

2. The number of nuclei at $t_0 = 0$ is $N_{0u} = 5\times10^{12}$ nuclei.

3. After the half-life T, the number of nuclei is reduced to its half:

$N_u = \frac{N_{0u}}{2} = 2.5\times10^{12}$ nuclei.



On the graph, we find $T \approx 4.5\times10^{9}$ years.

The radioactive constant:

$\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{4.5\times10^{9}} = 1.54\times10^{-10}\,\text{year}^{-1}$

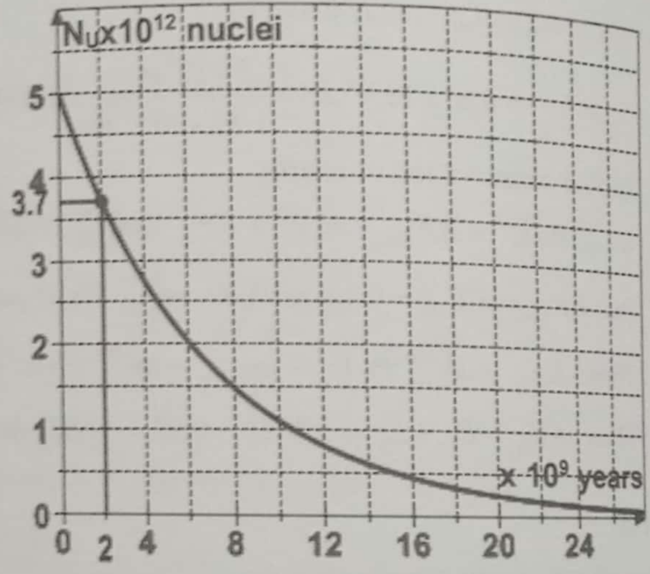
4. a) Law of radioactive decay: $N_u = N_{0u}\,e^{-\lambda t}$.

b) The number of nuclei remaining is:

$N_u = 5\times10^{12} e^{-1.54\times10^{-10}\times2\times10^{9}}$

$N_u = 3.675\times10^{12}$ nuclei.

c) On the graph 2×10^{9} years corresponds 3.7×10^{12} nuclei.



5. a) The disintegration of a uranium nucleus leads to the formation of a lead nucleus then:

$N_{0u} = (N_u)_{remained} + (N_u)_{disintegrated}$;

$N_{0u} = N_u + N_{Pb}$.

b) The number of uranium nuclei present at the considered instant is :

$N_u = N_{0u} - N_{Pb} = 5\times10^{12} - 2.5\times10^{12} = 2.5\times10^{12}$ nuclei.

$N_u = \frac{N_{0u}}{2}$; the age of the Earth is equal to the half-life of uranium 238 which is 4.5×10^{9} years.

V-LS 2007 2nd

Part A

1. The equation of the disintegration is: ${}^{60}_{27}Co \longrightarrow {}^{A}_{Z}Ni + {}^{0}_{-1}e + \gamma$;

Conservation of mass number: $60 = A + 0$, then $A = 60$;

Conservation of charge number: $27 = Z - 1$, then $Z = 28$;

We get: ${}^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni + {}^{0}_{-1}e + \gamma$.

2. The mass defect is $\Delta m = m_{before} - m_{after} = m(Co) - \left[m(Ni) + m(\beta^-)\right]$;

$\Delta m = 59.91901 - (59.91544 + 0.0005486) = 3.0214 \times 10^{-3} u$;

3. The energy liberated is $E_\ell = \Delta m c^2 = 3.0214 \times 10^{-3} u \times c^2$;

$$E_\ell = 3.0214 \times 10^{-3} \times 931.5 MeV = 2.8144341\ MeV.$$

4. The energy liberated appears as:
   - kinetic energy carried by the β^- particles;
   - radiant energy carried by γ-radiations.

5. a) $KE(\beta^-) + E(\gamma) = 0.0010\ MeV + 2.5060\ MeV = 2.507\ MeV$;

But $E_\ell = 2.8144341\ MeV > KE(\beta^-) + E(\gamma) = 2.507\ MeV$;

Then the missing energy is carried by a certain particle that accompanied β^-.

b) It is called antineutrino.

c) The antineutrino is a charge-less particle $(Z = 0)$ and it has an extremely small mass $(A = 0)$.

d) According to the principle of conservation of energy:

$$E({}_0^0\bar{\upsilon}) = E_\ell - [KE(\beta^-) + E(\gamma)] = 2.814434\ MeV - 2.507\ MeV = 0.3074341\ MeV.$$

6. The global equation of the disintegration is ${}_{27}^{60}Co \longrightarrow {}_{28}^{60}Ni + {}_{-1}^{0}e + {}_0^0\bar{\upsilon} + \gamma$.

Part B

1. The antineutrino is not absorbed by the living organism, since it does not undergo any interaction with matter (or it is very difficult to go an interaction with matter).

2. The energy that is absorbed in one second is the energy transferred by β^- particles.

The activity of a source is the number of the disintegrated nuclei N_d in one second and since each disintegration liberates a β^- particle; thus the number of particles in one second is the activity $A = 6 \times 10^{19}$ nuclei. The power transferred is $P = \frac{E_{ab}}{\Delta t} = \frac{N_d \times KE_{\beta^-}}{\Delta t} = \frac{N_d}{\Delta t} \times KE_{\beta^-}$;

But $A = \frac{N_d}{\Delta t}$; then $P = A \times KE_{\beta^-} = 6 \times 10^9 \times 0.0010 \times 1.6 \times 10^{-13} W = 9600\ W$.

3. This large power is used in radiotherapy in order to destroy cells infected by the cancer.

VI-LS 2006 1st

Part A

1. a) ${}_{92}^{238}U \longrightarrow {}_{90}^{234}Th + {}_{Z_1}^{A_1}X + \gamma$;

Conservation of mass number: $238 = 234 + A_1$, then $A_1 = 4$;

Conservation of charge number: $92 = 90 + Z_1$, then $Z_1 = 2$;

Then, the equation of the disintegration becomes ${}_{92}^{238}U \longrightarrow {}_{90}^{234}Th + {}_2^4X + \gamma$.

b) ${}_{Z_1}^{A_1}X$ is the helium nucleus, it is then an alpha (α) decay.

2. ${}_{90}^{234}Th \longrightarrow {}_{Z_2}^{A_2}Pa + {}_{-1}^{0}e + {}_0^0\bar{\upsilon} + \gamma$;

Conservation of mass number: $234 = A_2 + 0$, then $A_2 = 234$.

Conservation of charge number: $90 = Z_2 - 1$, then $Z_2 = 91$;

We get: ${}^{234}_{90}Th \longrightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$.

3. a) From diagram the long arrow corresponds to an α decay, whereas the short one corresponds to a β^- decay so we have 8α and $6\beta^-$.

b) The overall equation is ${}^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 8\,{}^{4}_{2}He + 6\,{}^{0}_{-1}e + 6\,{}^{0}_{0}\bar{\nu} + \gamma$.

4. The periods or half-life of all the intermediate nuclei being very small compared to the father (billions of years) and the final stable daughter, so they vanish practically.

Part B

1. a) The number of uranium nuclei is $N_U(t) = \dfrac{m(U)}{M} \times N_A$.

Then, $N_U(t) = \dfrac{1g}{238\,g.mo\ell^{-1}} \times 6.022 \times 10^{23}\,mo\ell^{-1} = 2.52941 \times 10^{21}$ nuclei.

b) The number of lead nuclei is $N_{Pb}(t) = \dfrac{m(Pb)}{M} \times N_A$.

Then, $N_{Pb}(t) = \dfrac{0.88g}{206\,g.mo\ell^{-1}} \times 6.022 \times 10^{23}\,mo\ell^{-1} = 2.57165 \times 10^{21}$ nuclei.

2. The number of lead Pb nuclei formed $N_{Pb}(t)$ is equal to the number of uranium disintegrated $N_{dU}(t)$, then the initial number of uranium nuclei is $N_u(0) = N_U(t) + N_{Pb}(t) = 5.10106 \times 10^{21}$ nuclei.

3. Law of radioactive decay $N_U(t) = N_U(0)e^{-\frac{\ell n2}{T}t}$.

4. We have $N_U(t) = N_U(0)e^{-\frac{\ell n2}{T}t}$; $e^{-\frac{\ell n2}{T}t} = \dfrac{N_U(t)}{N_U(0)}$;

Then the age of the meteorite is $t = -\dfrac{T}{\ell n(2)}\ell n\left[\dfrac{N_U(t)}{N_U(0)}\right] = 4.55 \times 10^9$ years.

VII-LS 2005 1st

Part A

1. a) The equation of this disintegration is ${}^{137}_{55}Cs \longrightarrow {}^{y}_{x}Ba + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$;

Conservation of mass number: $137 = y + 0$, then $y = 137$;

Conservation of charge number: $55 = x - 1$, then $x = 56$;

We get: ${}^{137}_{55}Cs \longrightarrow {}^{137}_{56}Ba + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$.

b) The equation of the dis-excitation ${}^{137}_{56}Ba^* \longrightarrow {}^{137}_{56}Ba + \gamma$.

2. a) The mass defect $\Delta m = m\left({}^{137}_{55}Cs\right) - \left[m\left({}^{137}_{56}Ba\right) + m\left({}^{0}_{-1}e\right)\right]$;

$$\Delta m = 136.8773u - \left(136.8756u + 5.5 \times 10^{-4}u\right) = 1.15 \times 10^{-3}u;$$

The energy liberated is given by: $E = \Delta m c^2 = 1.15 \times 10^{-3} \times 931.5 = 1.071225\ MeV$.

b) According to the conservation of energy $E = KE(\beta^-) + E({}^{0}_{0}\bar{\nu}) + E(\gamma)$;

The barium (daughter) nucleus is taken at rest;

Then $E({}^{0}_{0}\bar{\nu}) = 1.071225 - (0.557 + 0.514) = 2.25 \times 10^{-4}\ MeV$.

Part B

1. Law of radioactive decay $A = A_0 e^{-\lambda t}$, $e^{-\lambda t} = \dfrac{A}{A_0}$; so $e^{\lambda t} = \dfrac{A_0}{A}$;

Then $\lambda t = \ell n\left(\dfrac{A_0}{A}\right)$ & $\lambda = \dfrac{\ell n(2)}{T}$; thus $t = \dfrac{T}{\ell n(2)} \times \ell n\left(\dfrac{A_0}{A}\right) = \dfrac{30}{\ell n(2)} \times \ell n\left(\dfrac{4.4 \times 10^5}{3.33 \times 10^5}\right) \approx 12$ years.

The year of fabrication is $2004 - 12 = 1992$.

2. The activity is defined as the number of disintegrations n per unit of time Δt so $A = \dfrac{n}{\Delta t}$;

The number of disintegrations per hour is $n = A \times \Delta t = 3.33 \times 10^5 \times 3600 = 1.2 \times 10^9$ decays.

Part C

1. The energy liberated by the n disintegrations of cesium source is

$E_1 = E \times n = 1.071225 \times 1.6 \times 10^{-13} \times 1.2 \times 10^9 = 2.1 \times 10^{-4} J$;

The energy absorbed by the student during 1h is $E_2 = \left(\dfrac{1}{100}\right) E_1$;

Then $E_2 = 0.01 \times 2.1 \times 10^{-4} = 2.1 \times 10^{-6} J$.

2. The energy absorbed being less than the threshold energy $E_2 < 1.2 \times 10^{-4} J$, then the student is not subjected to any risk.

Unit IV

Atom & Nucleus

Chapter 18

Fission & Fusion

LS - Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Fission & Fusion		-	2nd	2nd(B)	-	-	-	-	-	2nd & 1st(A)
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Fission & Fusion	-	-	-	1st	-	-	-	2nd(B)	2nd	2nd

I-Provoked Nuclear Reactions

All nuclear reactions obey the laws of conservations of mass number A, the charge number Z and the total energy.

A provoked nuclear reaction requires the intervention of an external agent whereas a spontaneous nuclear reaction takes place without any external factor.

The global conservation can be written: $E_\ell = (m_{before} - m_{after})c^2 = (KE_{after} + E_\gamma) - KE_{before}$.

Types of provoked nuclear reactions

We have two types of provoked nuclear reactions: nuclear fission & nuclear fusion.

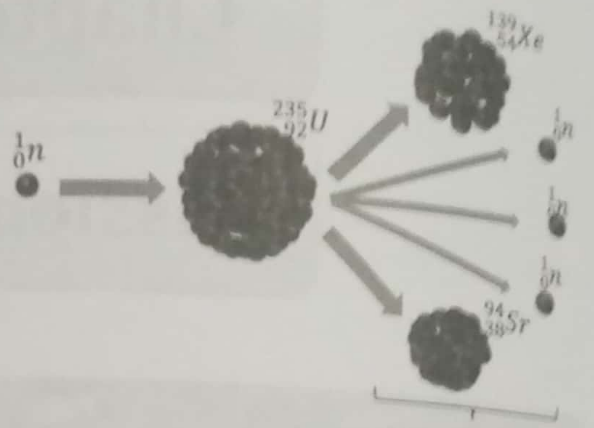
II-Nuclear Fission

It is a provoked nuclear reaction during which a **heavy** nucleus is divided into two or more **lighter** and more stable nuclei when bombarded by a thermal neutron.

Example: ${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{94}_{38}Sr + {}^{139}_{54}Xe + 3{}^{1}_{0}n$

The mass defect:

$$\Delta m = \left[m\left({}^{235}_{92}U\right) + m\left({}^{1}_{0}n\right)\right] - \left[m\left({}^{94}_{38}Sr\right) + m\left({}^{139}_{54}Xe\right) + 3m\left({}^{1}_{0}n\right)\right]$$



Fission products

Properties of nuclear fission

- The thermal neutron or slow neutron possesses a kinetic energy around $0.025eV$ (or for simplicity we may take it $0.1eV$).
  If the neutrons have a larger kinetic energy, we should reduce their speed using a moderator.
- The neutron is used since it is charge less so it is able to reach the nucleus.
- If more than two neutrons are emitted then this reaction is called chain reaction.
- Despite it is energetic and used in the production of electricity but it creates the problem of nuclear wastes that are dangerous and should be buried under ground for hundreds of years.

III-Nuclear Fusion

It is a reaction during which two **light** nuclei are combined together to form a **heavier** more stable nucleus.

Example: ${}^{3}_{1}H + {}^{2}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{0}n$.

Properties of nuclear fusion

- The nuclei used in nuclear fusion must have a high kinetic energy around $0.1MeV$, or it needs a high temperature around 10^8K to take place in order to overcome the force of repulsion between the positive nuclei.
- Despite it is not polluting but until now, the scientists did not succeed to transform it for domestic and only used as a weapon (H-bomb).
- The origin of radiant energy delivered by the sun is a nuclear fusion.

Applications

I- Fusion

The fusion nuclear energy is very promising, it is clean sustainable source of energy but it doesn't yet work on commercial scale. Scientists and engineers are developing what we call «Tokamak vessel» to perform fusion reactions where the hydrogen isotopes are in the plasma state. The vessel is surrounded by a set of complicated well engineered coils whose aim is to create a gap between the superheated plasma and the vessel.

One possible reaction is that between deuterium $_1^2$H & tritium $_1^3$H is as follows:

$$_1^2H +_1^3 H \longrightarrow _2^4He +_Z^A X .$$

Given the masses of some nuclei and particles:

- $m(_1^2\text{H})=2.01415\ u$;
- $m(_1^3\text{H})=3.01547\ u$;
- $m(_2^4\text{He})=4.00154\ u$;
- $m(_Z^AX)=1.00877\ u$; $1u=1.66\times10^{-27}\,kg$ & $c=3\times10^8\ m/s$.

1. Scientists have succeed to produce from fusion a power of 16 MW , but the power required to produce this amount is 24 MW of heat. Why this method is not useful to be reproduced?
2. What do we call the deuterium $_1^2$H & tritium $_1^3$H nuclei?
3. Determine A & Z specifying the laws used and then identify the particle emitted.
4. Calculate the energy liberated by the fusion of tritium with deuterium.
5. Consider a yearly consumption $50\ kg$ of tritium.
   a) Determine the nuclear power liberated by the fusion of this mass of tritium.
   b) Calculate the mass of deuterium needed during one year.
6. What are the conditions required by the nuclei undergoing fusion?

II-Engineering (2009/2010)
The Plutonium

The fifteen known isotopes of plutonium are alpha emitter except the plutonium 241, which is beta emitter; the isotopes 239 and 241 are also fissile. Plutonium 238 has commercial and military applications. Under the impact of a neutron, the plutonium 239 can undergo a fission nuclear reaction.

The equation of this reaction is written as: $_{94}^{239}Pu+_0^1n\longrightarrow _{52}^{135}Te+_Z^{102}Mo+y\,_0^1n$.

The adjacent table gives the binding energies of the three nuclei:

Nuclei	$_{94}^{239}Pu$	$_{52}^{135}Te$	$_Z^{102}Mo$
$E_b(MeV)$	1.79×10^3	1.12×10^3	8.64×10^2

1. Determine the values of y and Z.

2. a) Write the expression giving the binding energy E_b of a $^{A}_{Z}X$ in terms of the mass defect and c.

b) Give the expression of the mass of each of the three above nuclei in terms of its binding energy E_b, the mass m_n of a neutron, the mass m_p of a proton and c.

3. Determine, in MeV, the energy released by this reaction.

4. a) Calculate the binding energy per nucleon E_b / A for each of the three nuclei.

b) Draw the shape of the Aston's curve that gives the variations of E_b / A as a function of A.

c) Indicate the approximate location of the three nuclei on the curve.

III- Fusion in the Sun

In 1938, Hans Bethe proposed that the energy of the sun is due nuclear fusion. The hydrogen fuse into helium in the core of the Sun according to the reaction $x\ ^{1}_{1}H \longrightarrow ^{4}_{2}He + y\ ^{0}_{+1}e + y\ ^{0}_{0}\nu$.

The emitted neutrinos are detected by the Earth confirming the occurrence of the fusion reaction. The detection of neutrinos is difficult, we usually use detectors filled with water. Nevertheless, it remains a hard task; some neutrinos interact with electrons of water molecule and extract electrons.

Given the masses of some nuclei and particles:

- $m(^{1}_{1}H) = 1.00877\ u$;
- $m(^{0}_{-1}e) = m(^{0}_{+1}e) = 0.00055u$;
- $m(^{4}_{2}He) = 4.00154\ u$;
- $1u = 1.66 \times 10^{-27}\ kg$ & $c = 3 \times 10^{8}\ m/s$.

1. Define nuclear fusion.
2. Determine the values of x & y specifying the laws used.
3. Calculate, in Joule, the energy liberated by this reaction.
4. Give the physical properties of a neutrino.
5. Calculate the energy liberated by the fusion of 1 kg of hydrogen.
6. The fusion procedure in Sun is much more complicated. A possible reaction is the electron absorption of beryllium nucleus $^{7}_{4}Be$ leading to the formation of the nucleus $^{A}_{Z}X$ and emitting a neutrino. The nuclei and particles are at rest.

a) Write the reaction of formation of $^{A}_{Z}X$ specifying the laws used.

b) Knowing that the masses of nuclei $m(Be) = 7.018072\ u$ & $m(X) = 7.017654\ u$.
Applying the principle of conservation of global energy, determine the energy carried by the neutrino.

Solutions – Applications

1. The heat power 24 MW invested is greater than that produced 16 MW , which make it not useful to be used.

2. Deuterium ${}^{2}_{1}H$ & tritium ${}^{3}_{1}H$ nuclei are isotopes of hydrogen.

3. The nuclear reaction: ${}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + {}^{A}_{Z}X$;

Conservation of mass number: $2+3=4+A$, then $A=1$;

Conservation of charge number: $1+1=2+Z$, then $Z=0$;

The particle emitted is neutron ${}^{1}_{0}n$.

4. The mass defect: $\Delta m = \left[m\left({}^{2}_{1}H\right)+m\left({}^{3}_{1}H\right)\right]-\left[m\left({}^{4}_{2}He\right)+m\left({}^{1}_{0}n\right)\right]=0.01931\ u$.

The energy liberated is: $E_{\ell} = \Delta m c^2 = 0.01931 \times 1.66\times10^{-27}\times\left(3\times10^{8}\right)^2 = 2.9\times10^{-12}\ J$.

5. a) 1 nuclear fusion $\xrightarrow{\text{needs tritium}} 3.01547\times1.66\times10^{-27}\ kg \xrightarrow{\text{liberates}} 2.9\times10^{-12}\ J$;

$50\ kg \xrightarrow{\text{liberates}} E_{\text{nuclear}}$;

We get $E_{\text{nuclear}} = 2.9\times10^{16}\ J$.

The nuclear power delivered is $P_{\text{nuclear}} = \dfrac{E_{\text{nuclear}}}{\Delta t} = \dfrac{2.9\times10^{16}}{365\times24\times60\times60} = 9.2\times10^{8} W$.

b) 1 nuclear fusion $\xrightarrow{\text{needs tritium}} 3.01547\ u \xrightarrow{\text{\& needs deuterium}} 2.01415\ u$;

$50\ kg \xrightarrow{\text{needs}} m$; then $m = 33.4\ kg$ of deuterium.

6. The mixture of nuclei should be at high temperature of $10^{8} K$, in order to accelerate the nuclei undergoing nuclear fusion.

B

1. The reaction ${}^{239}_{94}Pu + {}^{1}_{0}n \longrightarrow {}^{135}_{52}Te + {}^{102}_{Z}Mo + y\,{}^{1}_{0}n$;

Conservation of mass number: $239+1=135+102+y$, then $y=3$;

Conservation of charge number: $94+0=52+Z+0$, then $Z=42$;

Then the nuclear reaction is ${}^{239}_{94}Pu + {}^{1}_{0}n \longrightarrow {}^{135}_{52}Te + {}^{102}_{42}Mo + 3\left({}^{1}_{0}n\right)$.

2. a) The binding energy of a nuclide ${}^{A}_{Z}X$ is given by: $E_b = \left[Z\,m_p + (A-Z)m_n - m_X\right]c^2$.

b) The mass of a nucleus ${}^{A}_{Z}X$ expressed in u is: $m_X = Z\,m_p + (A-Z)m_n - \dfrac{E_b(X)}{c^2}$.

For plutonium: $m\left({}^{239}_{94}Pu\right) = 94\,m_p + 145\,m_n - \dfrac{E_b(Pu)}{c^2}$;

$m\left({}^{135}_{52}Te\right) = 52\,m_p + 83\,m_n - \dfrac{E_b(Te)}{c^2}$ & $m\left({}^{102}_{42}Mo\right) = 42\,m_p + 60\,m_n - \dfrac{E_b(Mo)}{c^2}$;

3. The mass defect is: $\Delta m = \left[m\left({}^{239}_{94}Pu\right)+m\left({}^{1}_{0}n\right)\right]-\left[m\left({}^{135}_{52}Te\right)+m\left({}^{102}_{42}Mo\right)+3m\left({}^{1}_{0}n\right)\right]$;

$$\Delta m = \left[94m_p + 145m_n - \frac{E_b(Pu)}{c^2} + m_n\right] - \left[52m_p + 83m_n - \frac{E_b(Te)}{c^2} + 42m_p + 60m_n - \frac{E_b(Mo)}{c^2} + 3m_n\right]$$

Then $\Delta m = \frac{E_b(Te)}{c^2} + \frac{E_b(Mo)}{c^2} - \frac{E_b(Pu)}{c^2}$;

The energy liberated: $E_\ell = \Delta m c^2 = E_b(Te) + E_b(Mo) - E_b(Pu)$;

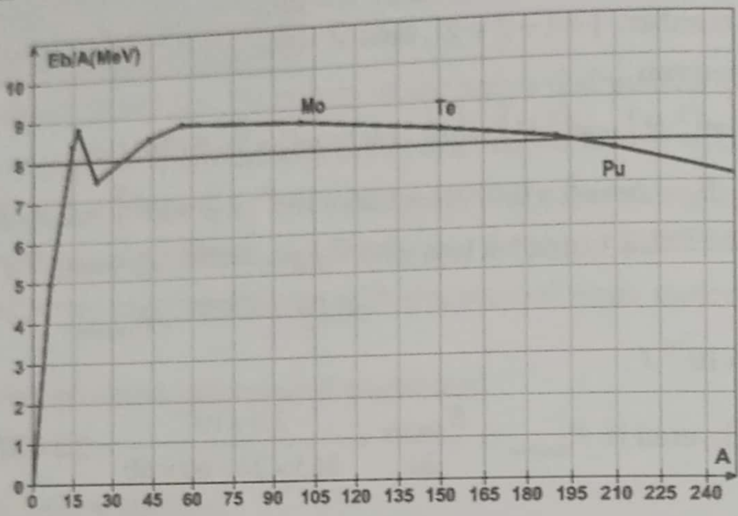
Thus, $E_\ell = 8.64 \times 10^2 + 1.12 \times 10^3 - 1.79 \times 10^3 = 194\ MeV$.

4. a) The binding energy per nucleon for each nucleus is:

$\frac{E_b(Pu)}{A} = \frac{1.79 \times 10^3}{239} = 7.49\ MeV$; $\frac{E_b(Te)}{A} = 8.29\ MeV$ & $\frac{E_b(Mo)}{A} = \frac{8.64 \times 10^2}{102} = 8.47\ MeV$.

b) Aston's curve.

c) The nuclei are located on the previous graph.



III-

1. The nuclear fusion is a provoked nuclear reaction during which two or more light nuclei fuse together to form a heavier nucleus accompanied by a certain emission.
2. We have $x\,{}_1^1\text{H} \longrightarrow {}_2^4\text{He} + y\,{}_{+1}^{0}e + y\,{}_0^0\upsilon$;

   Conservation of mass number: $x(1) = 4 + y(0) + y(0)$, then $x = 4$;

   Conservation of charge number: $4(1) = 2 + y\,(+1) + y\,(0)$, then $y = 2$;

   Then the nuclear reaction is: $4\,{}_1^1\text{H} \longrightarrow {}_2^4\text{He} + 2\,{}_{+1}^{0}e + 2\,{}_0^0\overline{\upsilon}$.
3. The mass defect due to this reaction is: $\Delta m = 4m\left({}_1^1\text{H}\right) - m\left({}_2^4\text{He}\right) - 2m\left({}_{+1}^{0}e\right) = 0.03244u$;

   The energy liberated:

   $E_\ell = \Delta m c^2 = 0.03244\ u \times c^2 = 0.03244 \times 1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2 = 4.85 \times 10^{-12}\ J$.
4. The neutrino is charge less and its mass is extremely small.
5. 1 nuclear fusion $\xrightarrow{\text{needs}} 4 \times 1.00877 \times 1.66 \times 10^{-27}\ kg \xrightarrow{\text{liberates}} 4.85 \times 10^{-12}\ J$;

   $1\,kg \xrightarrow{\text{liberates}} E_t$; then $E_t = 7.2 \times 10^{14}\ J$.
6. a) The nuclear reaction: ${}_4^7Be + {}_{-1}^{0}e \longrightarrow {}_Z^A\text{X} + {}_0^0\upsilon$;

   By laws of conservations we get: $A = 7$ & $Z = 3$.

   b) Each nucleus possesses a rest and kinetic energy;

   Global conservation of energy: $m(Be)c^2 + KE_{Be} + m(e) + KE_e = m(\text{X})c^2 + KE_X + E\left({}_0^0\upsilon\right)$;

   But the nuclei and particles are taken at rest, then $KE_{Be} = KE_e = KE_X = 0$;

   We get $E\left({}_0^0\upsilon\right) = \left[m(Be) + m(e) - m(\text{X})\right]c^2 = 9.68 \times 10^{-4}\ u \times c^2$;

   Thus, $E\left({}_0^0\upsilon\right) = 9.68 \times 10^{-4} \times 1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2 = 1.45 \times 10^{-13}\ J = 0.9\ MeV$.

Problems

Engineering (2007/2008)
The iodine 131

Iodine 131 is one of the gaseous effluents that may escape from a nuclear reactor functioning with enriched uranium. It is a β^- emitter of half-life $T = 8.05$ days, its daughter nucleus being the xenon (Xe). In fact, huge amounts of iodine 131 have been released during the accidents that occurred at Windscale (England) in 1957 $(1.4\times10^{15}\ Bq)$, at Three Mile Island (USA) in 1979 $(5.5\times10^{11}\ Bq)$ and at Chernobyl in 1986 $(5\times10^{17}\ Bq)$.

Part A
The fission of uranium 235

One possible fission reactions of uranium 235 yielding the iodine 131 is:

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{131}_{53}I + ^{73}_{30}Zn + ^{23}_{Z}F + y\,^{1}_{0}n.$$

1. Complete this equation.
2. Calculate the binding energy (E_b) for each nucleus.
3. a) Show that the energy liberated by the reaction can be written in the form:
   $E_l = E_b(I) + E_b(Zn) + E_b(F) - E_b(U)$.
   b) Calculate its value.

Nucleus	Binding energy per nucleon (MeV)
U 235	7.59
I 131	8.42
Zn 73	8.64
F 23	7.62

Part B
The disintegration of iodine 131

1. Write the disintegration reaction of the iodine 131.
2. a) Calculate the radioactive constant λ of the iodine.
   b) Deduce the duration at the end of which the activity of the gaseous effluents released during the accident occurred at Chernobyl becomes equal to the initial activity occurred during the accident at Three Mile Island.
3. Figure 3, shows the most possible disintegrations of iodine 131 into Xenon 131.
   a) Give the physical significance of Q.
   b) Verify that the maximum kinetic energy of the emitted β_2 is 333 keV.
   c) Deduce the maximum kinetic energy of each of β_1 and β_3.
4. a) Calculate the energy of the photon γ_3 which is one of the most probable.
   b) This photon hits a metallic plate. Justify that an electron is extracted from this metal.

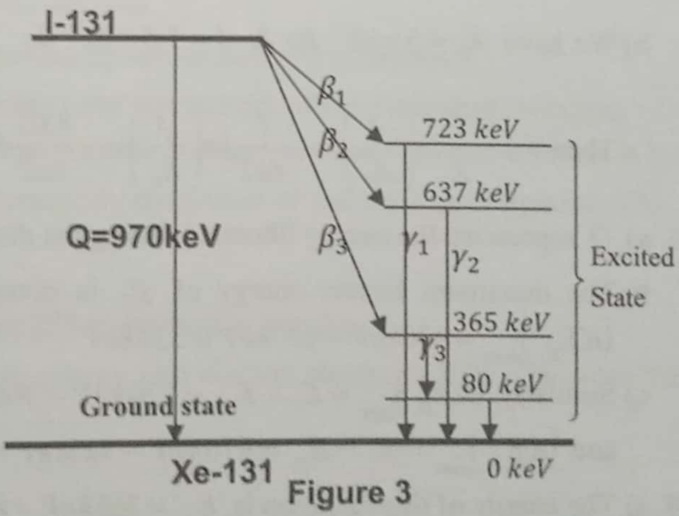


Figure 3

I-(2007/2008)

Part A

1. Nuclear reaction $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{131}_{53}I + ^{73}_{30}Zn + ^{23}_{Z}F + y\,^{1}_{0}n$.

Conservation of mass number: $235 + 1 = 131 + 73 + 23 + y$, then $y = 9$;

Conservation of charge number: $92 + 0 = 53 + 30 + Z$, then $Z = 9$;

The complete nuclear reaction $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{131}_{53}I + ^{73}_{30}Zn + ^{23}_{9}F + 9\,^{1}_{0}n$.

2. The binding energy: $E_b(X) = \left(\frac{E_b}{A}\right) \times A$; then $E_b\left(^{235}_{92}U\right) = 7.59 \times 235 = 1783.65\ MeV$;

$E_b\left(^{131}_{53}I\right) = 1103.02\ MeV$; $E_b\left(^{73}_{30}Zn\right) = 630.72\ MeV$ & $E_b\left(^{23}_{9}F\right) = 175.26\ MeV$.

3. a) The energy liberated $E_\ell = \Delta m c^2 = (m_{\text{before}} - m_{\text{after}})c^2$;

The binding energy of a nucleus is given by:

$E_b(X) = [Z m_p + (A-Z)m_n - m_X]c^2$; so $m_X = Zm_p + (A-Z)m_n - \frac{E_b(X)}{c^2}$;

$E_\ell = \left[\left[m\left(^{235}_{92}U\right) + m\left(^{1}_{0}n\right)\right] - \left[m\left(^{131}_{53}I\right) + m\left(^{73}_{30}Zn\right) + m\left(^{23}_{9}F\right) + 9\,m\left(^{1}_{0}n\right)\right]\right]c^2$;

Replacing the binding energies for the nuclei we get $E_\ell = E_b(I) + E_b(Zn) + E_b(F) - E_b(U)$

b) The energy liberated is $E_\ell = 1103.02 + 630.72 + 175.26 - 1783.65 = 125.35\ MeV$.

Part B

1. The equation of disintegration is $^{131}_{53}I \longrightarrow ^{A}_{Z}Xe + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon} + \gamma$;

Conservation of mass number: $131 = A + 0$, then $A = 131$;

Conservation of charge number: $53 = Z - 1$, then $Z = 54$;

The nuclear reaction: $^{131}_{53}I \longrightarrow ^{131}_{54}Xe + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon} + \gamma$.

2. a) The radioactive constant is given by $\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{8.05} = 0.086\ \text{day}^{-1}$.

b) We have $A_0 = 5 \times 10^{17}\ Bq$ & $A = 5.5 \times 10^{11}\ Bq$; but $A = A_0\, e^{-\lambda t}$, so $\frac{A}{A_0} = e^{-\lambda t}$;

Then $t = -\frac{1}{\lambda}\ell n\left(\frac{A}{A_0}\right) = -\frac{T}{\ell n 2}\ell n\left(\frac{A}{A_0}\right) = -\frac{8.05}{\ell n 2}\ell n\left(\frac{5.5 \times 10^{11}}{5 \times 10^{17}}\right) = 159.34$ days.

3. a) Q represents the energy liberated during the disintegration of $^{131}_{53}I$ into $^{131}_{52}Xe$.

b) The maximum kinetic energy of β_2 is obtained if it is emitted without an antineutrino $(KE_{\beta_2})_{\max} = 970keV - 637keV = 333keV$.

c) Similarly $(KE_{\beta_3})_{\max} = E_\ell - E_{\gamma_3} = 970keV - 365keV = 605keV$;

and $(KE_{\beta_1})_{\max} = E_\ell - E_{\gamma_1} = 970keV - 723keV = 247keV$.

4. a) The energy of this radiation is $E_{\gamma_3} = 365keV - 80keV = 285\ keV$.

b) An electron is extracted if the energy of the incident photon is greater than the work-energy or extraction energy of the metal under study.

LS & GS 2001 2nd

Controlled Nuclear Reaction

Chain reaction releases a considerable amount of energy. It may lead to an explosion if precautions were not taken. If this reaction is controlled inside a reactor, it may produce energy enough to function an electric power plant.

Given:

- $1u = 1.66\times10^{-27}\,kg = 931.5\,MeV/c^2$;
- Speed of light in vacuum: $c = 3\times10^8\,m/s$;
- Planck's constant: $h = 6.63\times10^{-34}\,J.s$;
- Avogadro's number: $N_A = 6.02\times10^{23}\,mol^{-1}$;
- Molar mass of uranium 235: $235\,g.mol^{-1}$;
- $1MeV = 1.6\times10^{-13}\,J$.

Part A

In a nuclear reactor of an atomic pile, the preparation of uranium 235, used as a fuel, takes place as follows.

1. The uranium $^{238}_{92}U$ captures a fast neutron and is transformed into a uranium nucleus $^{239}_{92}U$. Write the corresponding reaction.
2. The uranium nucleus 239 is radioactive; it is transformed into plutonium after two successive β^- disintegration according to the following reactions:

$$^{239}_{92}U \longrightarrow {}^{A}_{Z}Np + {}^{0}_{-1}e + \ldots\ldots \qquad \& \qquad {}^{A}_{Z}Np \longrightarrow {}^{A_1}_{Z_1}Pu + {}^{0}_{-1}e + \ldots\ldots$$

Complete these reactions and determine A, Z, A_1 & Z_1 specifying the laws used.

3. The radioactive plutonium nucleus (Pu) is an α emitter. The daughter nucleus is the uranium 235 isotope. Some α particles are ejected with a kinetic energy of $5.157\,MeV$ each and others with a kinetic energy of $5.144\,MeV$ each.
   a) Write the equation of disintegration of (Pu) nucleus.
   b) One of these α disintegrations is accompanied by the emission of γ radiation. Calculate the energy of this photon and deduce the wavelength of the associated radiation.
4. Uranium 235 is fissionable. During one of these possible fission reactions, the mass defect is $0.2\,u$. Calculate, in MeV and in J, the energy liberated by the fission of one nucleus of uranium 235.

Part B

In that atomic pile, a mass of $0.4\,kg$ of uranium 235 consumed in one day.

The efficiency of the transformation of nuclear energy into electric energy is 30%. Calculate the electric power of this pile.

Answer Key

Part A 3.b) $\lambda = 9.562\times10^{-11}\,m$ 4. $2.98\times10^{-11}\,J$

Part B 106MW

III-LS & GS 2002 2nd

Nuclear Fission

Given:

- $1u = 1.66 \times 10^{-27} kg = 931.5 MeV/c^2$;
- $1eV = 1.6 \times 10^{-19} J$.

Masses of some nuclei and particles:

- $m\left({}^{235}U\right) = 234.99342\ u$;
- $m\left({}^{A}I\right) = 138.89700\ u$;
- $m\left({}^{94}Y\right) = 93.89004\ u$;
- $m\left({}_{0}^{1}n\right) = 1.00866\ u$.

In a nuclear power station, the fissionable fuel is made up of 235 uranium nuclei. The nuclei that undergo a nuclear reaction must have been bombarded with a thermal neutron.

1. One of the possible reactions, that the uranium 235 undergoes, has the form of:

$${}_{0}^{1}n + {}_{92}^{235}U \longrightarrow {}_{53}^{A}I + {}_{Z}^{94}Y + 3{}_{0}^{1}n + \gamma.$$

a) The uranium 235 is fissionable. Why?

b) The nuclear reaction, that the uranium 235 nucleus undergoes, is said to be provoked. A provoked nuclear reaction is one of two types of nuclear reactions.
Name the other type and tell how it can be distinguish from the other.

c) Determine the values of A and Z specifying the supporting laws.

d) Calculate the energy liberated during the preceding reaction.
In what form does this liberated energy appear?

2. The nuclear power station converts 30% of the liberated energy into electrical energy.
Calculate the mass of the uranium 235 consumed by the power station during one day if the electric power it supplies is $6 \times 10^{8} W$.

Answer Key

1.d) $175.2 MeV$ 2. $2.4 kg$.

III-LS & GS 2003 2nd

The Isotope ${}_{3}^{7}Li$ of Lithium

As all the other chemical elements, the isotope ${}_{3}^{7}Li$ has properties that distinguish it from other chemical elements.

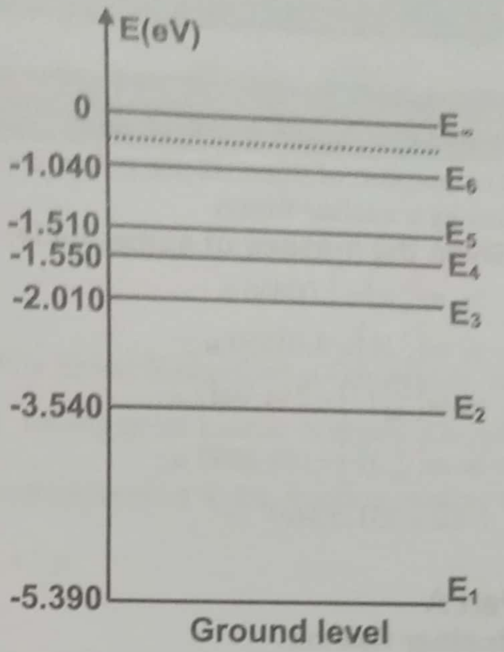
The object of this exercise is to show evidence of some properties of the isotope ${}_{3}^{7}Li$.

Given:

- $h = 6.62 \times 10^{-34} J.s$; $c = 3 \times 10^{8} m/s$;
- $1eV = 1.6 \times 10^{-19} J$; $1u = 931.5 MeV/c^2$;
- Mass of nucleus of lithium: $m(Li) = 7.01435\ u$;
- Mass of α particle: $m(\alpha) = 4.00150\ u$; Mass of a proton: $m_p = 1.00727\ u$.

Part A
Energy levels

The adjacent figure represents the energy levels of the lithium atom.



1. Calculate, in joule, the energy E_1 of the atom when it is in the ground state and E_6 when it is in the fifth state.
2. During the downward transition (disexcitation) from different energy levels to the ground level, the lithium atom emits some radiations.
   a) Calculate the highest and the lowest frequency of the emitted radiations.
   b) The corresponding emission spectrum is discontinuous. Why?
3. The lithium atom, being in the ground state, captures:
   a) a photon whose associated radiation has a wavelength of $\lambda = 319.9\,nm$.
   Show that the atom absorbs this photon. In what level would it be?
   b) a photon of energy $6.02\,eV$. An electron is thus liberated.
   Calculate, in eV, the kinetic energy of that electron.

Part B
Nuclear reaction

A nucleus ${}^{A}_{Z}X$, at rest, is bombarded by a proton (nucleus of the hydrogen atom ${}^{1}_{1}H$) carrying an energy of $0.65\,MeV$; we obtain two α particles.

1. Is this reaction spontaneous or provoked? Justify your answer.
2. Determine the values of Z and A by applying the convenient conservation laws. Identify the nucleus X.
3. Calculate the mass defect due to this reaction and deduce the corresponding energy liberated.
4. Knowing that the two obtained α particles have the same kinetic energy KE_α.
   Calculate KE_α.

Answer Key

Part A 3.a) the 5th   3.b) 0.63eV

Part B 3. 17.34MeV   4. 9MeV

I-LS 2011 2nd

Provoked Nuclear Reactions

The purpose of this exercise is to compare the energy liberated per nucleon by a nuclear fission to that by a nuclear fusion.

Given the masses of some nuclei in u

- $m\left({}_{0}^{1}n\right)=1.00866\ u$;
- $m\left({}_{1}^{3}H\right)=3.01550\ u$;
- $m\left({}_{92}^{235}U\right)=234.9942\ u$;
- $m\left({}_{54}^{A}Xe\right)=138.8892\ u$;

$m\left({}_{1}^{2}H\right)=2.01355\ u$;

$m\left({}_{2}^{4}He\right)=4.0015\ u$;

$m\left({}_{Z}^{94}Sr\right)=93.8945\ u$;

$1u=931.5\ MeV/c^2$

Part A
Nuclear fission

The fission of uranium 235 is used to produce energy.

1. The fission of a uranium 235 nucleus is produced by bombarding this nucleus by a slow neutron, called thermal, whose kinetic energy is around $0.025\ eV$. The equation of this reaction can be written: ${}_{92}^{235}U+{}_{0}^{1}n\longrightarrow {}_{Z}^{94}Sr+{}_{54}^{A}Xe+3{}_{0}^{1}n$;
   a) Calculate A and Z specifying the laws used.
   b) Show that the energy E liberated by the fission of a uranium nucleus is $179.947\ MeV$.
   c) Energy
      i- The number of nucleons participating in this reaction are 236. Why?
      ii- Calculate then E_1, the energy liberated per nucleon participating in this fission reaction.

2. Each neutron formed has an average kinetic energy $E_0=\dfrac{1}{100}E$.
   a) In this case, the neutrons obtained cannot, in general, realize the fission. Why?
   b) What should be done in order to realize this fission?

Part B
Nuclear fusion

Researches are currently performed in order to produce energy by nuclear fusion. The most accessible is the reaction between a deuterium nucleus ${}_{1}^{2}H$ and a tritium nucleus ${}_{1}^{3}H$.

1. The deuterium and the tritium are two isotopes of hydrogen. Write the symbol of the third isotope of the hydrogen.
2. Write the fusion reaction between a deuterium nucleus and a tritium nucleus knowing that this reaction liberates a neutron and a nucleus ${}_{Z}^{A}X$.
   Calculate Z and A and indicate the name of this nucleus ${}_{Z}^{A}X$.
3. Show that the energy liberated by this reaction is $E'=17.596\ MeV$.
4. Calculate E'_1, the energy liberated per nucleon participating in this reaction.

Part C
Conclusion

Compare E_1 and E'_1 then conclude.

II-LS 2011 1st
Nuclear Reactions and Dating

Given the masses of some nuclei in u

$m(\alpha)=4.00150\ u$; $m({}_{0}^{1}n)=1.00866\ u$; $m({}_{1}^{1}p)=1.00728\ u$;

$m({}_{7}^{14}N)=13.99924\ u$; $m({}_{6}^{14}C)=13.99995\ u$; $m({}_{8}^{17}O)=16.99473\ u$;

$1u=931.5\ MeV/c^2$.

Part A
Artificial reaction

The first provoked artificial reaction was performed in 1919 by Ernest Rutherford at Cambridge. He bombarded nitrogen nuclei $({}_{7}^{14}N)$ with α particles $({}_{2}^{4}He)$ having great kinetic energies. Oxygen nuclei $({}_{8}^{17}O)$ and protons $({}_{1}^{1}p)$ are obtained. The equation corresponding to the reaction relative to one nitrogen nucleus is written as: ${}_{2}^{4}He+{}_{7}^{14}N \longrightarrow {}_{8}^{17}O+x\,{}_{1}^{1}p$

1. Show that, specifying the used law, $x=1$.
2. a) Calculate the «mass before» and the «mass after» in this nuclear reaction.
   b) Deduce that this reaction needs an amount of energy to be performed.
3. We neglect the kinetic energy of the proton and those of nitrogen and oxygen nuclei. Show that, by applying the principle of conservation of the total energy, the kinetic energy of the α particle is equal to $1.183\ MeV$.

Part B
Natural reaction

A provoked reaction of nitrogen 14 occurs naturally. Indeed, when, in the upper atmosphere, a neutron of the cosmic radiation hits a nitrogen nucleus $({}_{7}^{14}N)$, a reaction takes place and produces a carbon nucleus $({}_{6}^{14}C)$, that is a radioactive isotope of the stable carbon nucleus $({}_{6}^{12}C)$.

The equation corresponding to this reaction is written as: ${}_{0}^{1}n+{}_{7}^{14}N \longrightarrow {}_{6}^{14}C+{}_{1}^{1}p$

1. Calculate the «mass before» and the «mass after» in this nuclear reaction.
2. Deduce that this reaction liberates energy.

Part C
Carbon dating

The plants absorb carbon dioxide from the atmosphere formed of both carbon 14 and carbon 12. The ratio of these two isotopes is the same in plants and in atmosphere. When the plant dies, it stops absorbing carbon dioxide. The carbon 14 existing in this plant disintegrates then without being compensated. The period (half-life) of carbon 14 is $T=5730$ years.

1. Calculate, in year^{-1}, the radioactive constant λ of carbon 14.
2. An analysis of a sample of wood (dead plant), found in an Egyptian tomb, shows that its activity is 750 disintegrations per minute whereas the activity of a plant of the same nature and of the same mass freshly cut is 1320 disintegrations per minute.
   Determine the age of the sample of wood found in the Egyptian tomb.

III-LS 2007 1st

Nuclear Fusion

Given the masses of the nuclei

- $m\left({}_1^2H\right) = 2.0134\ u$; $m\left({}_1^3H\right) = 3.0160\ u$;
- $m\left({}_2^4He\right) = 4.0015\ u$; $m\left({}_{92}^{235}U\right) = 235.12\ u$;
- $m\left({}_0^1n\right) = 1.0087\ u$;

$1u = 931.5\ MeV/c^2 = 1.66\times10^{-27}\ kg$; $1\ MeV = 1.6\times10^{-13}\ J$.

The combustion of 1 ton of fuel oil liberates an energy of $4.2\times10^{10}\ J$.

The controlled nuclear fusion, if this technique is well mastered, provides enormous energetic possibilities. Nowadays, all the studies, in research centers, focus on the fusion reaction between deuterium nucleus $\left({}_1^2H\right)$ and tritium nucleus $\left({}_1^3H\right)$ according to the following equation:

$${}_1^2H + {}_1^3H \longrightarrow {}_2^4He + {}_0^1n$$

The deuterium is abundant in nature; water is a huge reserve of this gas. The tritium is easily obtained by bombarding lithium (that exists in large quantities in minerals) by neutrons.

Part A

Advantages of the fusion of deuterium - tritium

1. Show that the mass defect in this reaction is $\Delta m = 0.0192\ u$.
2. Calculate, in MeV then in J , the energy liberated by this reaction.
3. Show that the energy liberated by the fusion of $1g$ of a mixture formed of equal numbers of deuterium nuclei and tritium nuclei is $3.42\times10^{11}\ J$.
4. Calculate, in J , the energy liberated by the combustion of $1g$ of fuel oil.
5. The fission of a uranium 235 nucleus gives, on the average, an energy of $200\ MeV$. Determine, in J , the energy liberated by the fission of $1g$ of uranium 235.
6. Give three reasons rendering the controlled fusion a source of energy better than that of fuel oil and nuclear fission.

Part B

Does the fusion reaction of deuterium - tritium take place in the Sun?

The two nuclei of deuterium and tritium repel each other. In order to fuse, they must collide with very high velocities, each of the two nuclei having, before collision, a kinetic energy whose minimum value is $KE = 0.35\ MeV$.

1. Why do the two deuterium and tritium nuclei repel?
2. The kinetic energy of a nucleus is proportional to the temperature T of the medium in which it exists: $KE = 1.3\times10^{-4}\ T$ (KE in eV and T in K). Calculate the minimum temperature T_1 of the medium convenient for the two nuclei to undergo fusion.
3. Such fusion reaction takes place in the core of certain stars. The temperature in the core of the Sun being $T_2 = 15\times10^6\ K$, show that this fusion reaction does not occur in the core of the Sun.

LS 2011 2nd

Part A

1. a) ${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{94}_{Z}Sr + {}^{A}_{54}Xe + 3{}^{1}_{0}n$;

Conservation of mass number: $235 + 1 = 94 + A + 3$, then $A = 236 - 97 = 139$;

Conservation of charge number: $92 + 0 = Z + 54 + 0$, then $Z = 92 - 54 = 38$;

The nuclear reaction is ${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{94}_{38}Sr + {}^{139}_{54}Xe + 3{}^{1}_{0}n$;

b) The mass defect: $\Delta m = [234.9942 + 1.00866] - [93.8945 + 138.8892 + 3 \times 1.00866] = 0.19318\ u$.

The energy liberated $E_\ell = \Delta m c^2 = 0.19318 \times \frac{931.5\ MeV}{c^2} \times c^2 = 179.94717\ MeV$.

c) Energy

i- We have 236 nucleons from the uranium nucleus and 1 neutron so the total is $235 + 1 = 236$ nucleons.

ii- The energy per nucleon is: $E_1 = \frac{E_\ell}{236} = \frac{179.94717}{236} = 0.762\ MeV$ / nucleon.

2. a) The energy of a neutron: $E_0 = \frac{1}{100}E = \frac{1}{100} \times 179.94717\ MeV = 1.7995\ MeV$.

$E_0 = 1.7995\ MeV > 0.025\ eV$.

b) These neutrons should be slow down (using moderators).

Part B

1. The third isotope is ${}^{1}_{1}H$.

2. The equation of this reaction is ${}^{3}_{1}H + {}^{2}_{1}H \longrightarrow {}^{A}_{Z}X + {}^{1}_{0}n$.

Conservation of mass number: $3 + 2 = A + 1$, then $A = 4$;

Conservation of charge number: $1 + 1 = Z + 0$, then $Z = 2$;

${}^{4}_{2}X$ is the Helium nucleus ${}^{4}_{2}He$.

Then the nuclear reaction becomes: ${}^{3}_{1}H + {}^{2}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{0}n$;

3. The mass defect: $\Delta m' = \left[m\left({}^{3}_{1}H\right) + m\left({}^{2}_{1}H\right)\right] - \left[m\left({}^{A}_{Z}X\right) + m\left({}^{1}_{0}n\right)\right]$;

$\Delta m' = [2.01355\,u + 3.0155\,u] - [4.0015\,u + 1.00866\,u] = 0.01889\ u$.

The energy liberated $E'_\ell = \Delta m c^2 = 0.01889 \times \frac{931.5\ MeV}{c^2} \times c^2 = 17.596035\ MeV$.

4. We have 2 nucleons from the deuterium ${}^{2}_{1}H$ and 3 from the tritium then the total is $2 + 3 = 5$ nucleons. The energy per nucleon is: $E'_1 = \frac{E'_\ell}{5} = \frac{17.596035\ MeV}{5} = 3.519207\ MeV$ / nucleon.

Part C

$E'_1 = 3.519207\ MeV > E_1$; then the nuclear fusion is more energetic, basing on the energy carried by nucleon, which made it more useful.

III-LS 2011 1st

Part A

1. ${}_{2}^{4}He + {}_{7}^{14}N \longrightarrow {}_{8}^{17}O + x\,{}_{1}^{1}p$;

Conservation of mass number $4+14=17+x$, then $x=18-17=1$;

The nuclear reaction ${}_{2}^{4}He + {}_{7}^{14}N \longrightarrow {}_{8}^{17}O + {}_{1}^{1}p$.

2. a) The mass before: $m_b = m({}_{2}^{4}He) + m({}_{7}^{14}N) = 4.00150\ u + 13.99924\ u = 18.00074\ u$;

The mass after: $m_a = m({}_{8}^{17}O) + m({}_{1}^{1}p) = 16.99473\ u + 1.00728\ u = 18.00201\ u$.

b) $m_b = 18.00074\ u < m_a = 18.00201\ u$;

The mass defect $\Delta m = m_b - m_a < 0$; thus, this nuclear reaction need energy to accomplish.

3. According to the conservation of global energy:

$m(He)c^2 + KE_\alpha + m({}_{7}^{14}N)c^2 + KE_N = m({}_{8}^{17}O)c^2 + KE_O + m({}_{1}^{1}p)c^2 + KE_p$;

But $KE_O = KE_p = KE_N = 0$ (at rest).

So, $E_\ell = [m(He) + m(N) - (m(O) + m(p))]c^2 = -KE_\alpha$.

The mass defect of this reaction is $\Delta m = m(He) + m(N) - [m(O) + m(p)] = 0.00217\ u$;

Then $+KE_\alpha = -E_\ell = -\Delta m c^2 = -(-0.00127\ u \times c^2) = 0.00127 \times 931.5\ MeV = 1.183\ MeV$.

Part B

1. The mass before: $m_b = m({}_{0}^{1}n) + m({}_{7}^{14}N) = 1.00866 + 13.99924 = 15.00790\ u$.

The mass after: $m_a = m({}_{6}^{14}C) + m({}_{1}^{1}H) = 13.99995 + 1.00728 = 15.00723\ u$.

2. $m_b = 15.00790\ u < m_a = 17.00723\ u \Rightarrow \Delta m = m_b - m_a > 0$; thus, this reaction liberates energy.

Part C

1. We know that $\lambda = \frac{\ell n2}{T} = \frac{\ell n2}{5730} = 1.21 \times 10^{-4}$ year^{-1}.

2. Law of radioactive decay $A = A_0 e^{-\lambda t}$; $-\lambda t = \ell n\left(\frac{A}{A_0}\right)$

Then $t = -\frac{1}{\lambda}\ell n\left(\frac{A}{A_0}\right) = -\frac{1}{1.21\times10^{-4}}\ell n\left(\frac{750}{1320}\right) \approx 4673$ years; its age is 4673 years.

III-LS 2007 1st

Part A

1. The mass defect is $\Delta m = [m({}_{1}^{2}H) + m({}_{1}^{3}H)] - [m({}_{2}^{4}He) + m({}_{0}^{1}n)] = 0.0192\ u$.

2. The energy liberated is: $E_\ell = \Delta m c^2 = 17.88\ MeV = 2.86 \times 10^{-12}\ J$.

3. The fusion deuterium-tritium requires a mass of:

$m = 2.0134 + 3.0160 = 5.0294\,u = 8.35 \times 10^{-24}\ g$ and liberates an energy of $E_\ell = 2.86 \times 10^{-12}\ J$

The energy liberated is proportional to the mass, $E' = \frac{2.86\times10^{-13}}{8.35\times10^{-24}} = 3.42\times10^{11}\ J$ (for $m' = 1g$).

4. The energy liberated by the combustion of $1g$ of fuel oil is: $\dfrac{42\times10^9}{1\times10^6}=4.2\times10^4\,J$.

5. The mass of uranium nucleus is: $m_U=235.12u=235.12\times1.66\times10^{-24}=3.9\times10^{-22}\,g$;

The energy liberated by $1g$ of uranium is proportional to its mass: $\dfrac{E_2}{m_2}=\dfrac{E_\ell}{m_U}$;

$$E_2=\frac{E_\ell}{m_U}\times m_2=\frac{3.2\times10^{-11}J}{3.9\times10^{-22}g}\times1g=8.2\times10^{10}J.$$

6. The advantage of nuclear fusion are:
- more energetic;
- not polluting;
- economical since the raw matter (Hydrogen) is abundant in nature so it does not cost much.

Part B

1. The nuclei repel each other because they are positively charged.
2. To obtain fusion $KE\geq0.35\,MeV$;

But $KE=1.3\times10^{-4}T$, then $T\geq\dfrac{0.35\times10^6}{1.3\times10^{-4}}=2.7\times10^9\,K$.

The minimum temperature $T_1=T_{\min}=2.7\times10^9\,K$.

3. The temperature in the core of the sun is smaller than the minimum required (180 times smaller).

$T_2=15\times10^6\,K<T_1=T_{\min}=2.7\times10^9\,K$.

So the fusion reaction of deuterium-tritium does not occur in the core of the Sun.

The electrical power is $\eta=\dfrac{P_e}{P_f};P_e=\eta P_f=0.3\times3.58\times10^8=1.06\times10^8\,W$.

Unit III

Light

GS- Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Diffraction of Light		-	-	-	2nd(B)	-	-	-	2nd(A)	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Diffraction of Light	-	-	-	2nd(A)	-	-	-	-	-	1st(B)

GS – Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Interference of Light		1st & 2nd(A)	1st	1st & 2nd	2nd(B)	2nd	-	-	2nd(B)	2nd
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Interference of Light	2nd	2nd	1st	-	1st	2nd	2nd	-	-	1st(B)

GS - Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Photoelectric Effect		2nd(C)	-	-	1st(B)	-	-	2nd(A)	-	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Photoelectric Effect	-	-	-	2nd	-	1st	-	1st	-	-

GS Sessions – Diffraction of Light

I-GS 2016 2nd
See Page 305 – Part B

II-GS 2012 2nd
See Page 201 – Part A

III-GS 2010 2nd
See Page 203 – Part A

IV-GS 2007 2nd
See Page 216 – Part A

V-GS 2001 1st
See Page 45 – Part B

GS 2012 2nd
Diffraction and Interference of Light

A laser source emits a monochromatic cylindrical beam of light of wavelength $\lambda = 640\ nm$ in air.

Part A
Diffraction

This beam falls normally on a vertical screen (P) having a horizontal slit F_1 of width a. The phenomenon of diffraction is observed on a screen (E) parallel to (P) and situated at a distance $D = 4m$ from (P).

Consider on (E) a point M so that M coincides with the second dark fringe counted from O, the center of the central bright fringe. $\widehat{OIM} = \theta$ (θ is very small) is the angle of diffraction corresponding to the second dark fringe (Figure 1).

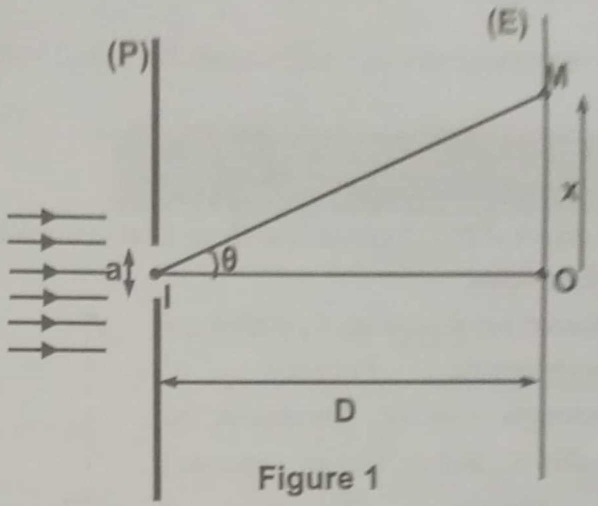


Figure 1

1. Write the expression of θ in terms of a and λ.
2. Determine the expression of $OM = x$ in terms of a, D and λ.
3. Determine the value of a if $OM = 1.28\ cm$.
4. We replace the slit F_1 by another slit F_1' of width 100 times larger than that of F_1. What do we observe on the screen (E)?

Part B
Interference

We cut in (P) another slit F_2 identical and parallel to F_1 so that the distance $F_1F_2 = a' = 1mm$. The laser beam falls normally on the two slits F_1 and F_2. We observe on (E) a system of interference fringes.

O' is the orthogonal projection of the midpoint I' of $[F_1F_2]$ on (E) (Figure 2).

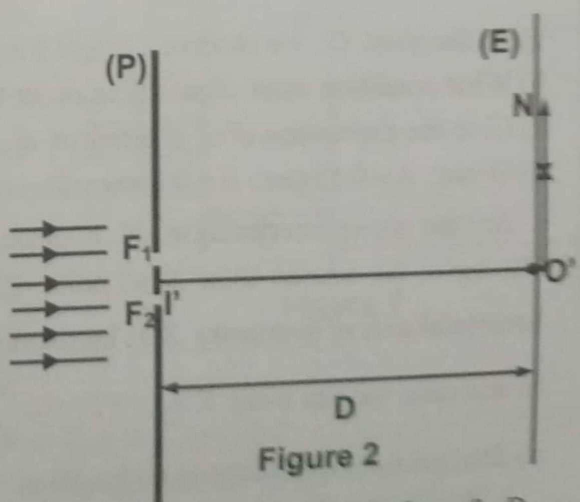


Figure 2

1. a) Due to what is the phenomenon of interference?
   b) Describe the fringes observed on (E).
2. Consider a point N on (E) so that $O'N = x$.
   a) Write the expression of the optical path difference $\delta = F_2N - F_1N$ in terms of a', x & D.
   b) If N is the center of a dark fringe of order k, write the expression of the optical path difference δ at N in terms of λ & k.

c) Deduce the expression of x in terms of a', D, k & λ.

d) Knowing that the interfringe distance i is the distance between the centers of two consecutive dark fringes, deduce then the expression of i in terms of λ, D & a'.

3. The whole set up of figure 2 is immersed in water of index of refraction n.

a) Medium and fringes.

i- The interfringe distance i varies and becomes i'. Why?

ii- Show that $i' = \frac{i}{n}$.

b) We move (E) parallel to (P) and away from it by a distance $d = \frac{4}{3}m$. We notice that the interfringe distance takes again the initial value i. Deduce then the value of n.

II-GS 2011 2nd
Interference of Light

The object of this exercise is to show how to use Young's double slit apparatus to measure very small displacements.

A source put at a point S, emitting a monochromatic radiation of wavelength λ in air, illuminates the two slits S_1 and S_2 that are separated by a distance a. The screen of observation is placed at a distance D from the plane of the slits.

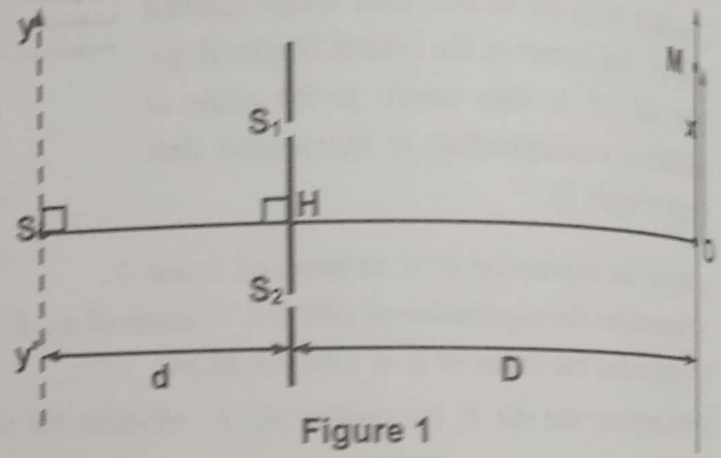


Figure 1

1. Describe the aspect of the interference fringes observed on the screen.

2. At a point M of abscissa $x = OM$, the optical path difference is given by the relation:

$$\delta = S_2M - S_1M = a\frac{x}{D}.$$

a) At the point O, we observe a bright fringe, called central bright fringe. Why?

b) What condition must δ satisfy in order to observe, at M, a dark fringe?

c) Give the expression of x in terms of a, D and λ, so that M is the center of a bright fringe.

d) Given: $\lambda = 0.55\ \mu m$; $a = 0.2\ mm$; $D = 1.5\ m$; $d = 10\ cm$. We take $x = 1.65\ cm$.

Are the waves interfering at M in phase or out of phase? Justify your answer.

3. We move the source from S to point S' vertically up on the axis $y'y$ perpendicular to the horizontal axis of symmetry SO, by the distance $b = SS'$.

In this case, we can write $S'S_2 - S'S_1 = a\frac{b}{d}$.

a) The central bright fringe is no longer at O but at point O'.

i- Justify this displacement.

ii- Specify, with justification, the direction of this displacement.

b) Determine the value of b, knowing that $OO' = 1cm$.

The object of this exercise is to show evidence of exploiting an optical phenomenon in the measurement of small displacements.

Part A
Diffraction

A laser beam illuminates, under normal incidence, a straight slit F, of width a, cut in an opaque screen (P). The light through F is received on a screen (E), parallel to (P) and found at $3m$ from (P) as shown in figure 1.

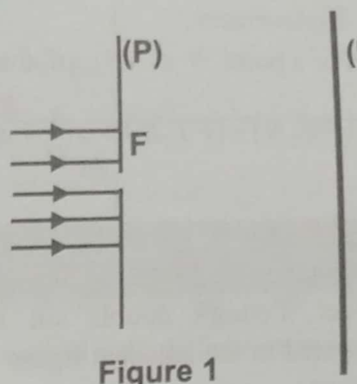


Figure 1

1. Describe what would be observed on (E) in the two following cases:
   a) $a = a_1 = 1cm$.
   b) $a = a_2 = 0.5mm$.
2. It is impossible to isolate a luminous ray by reducing the size of the slit. Why?
3. We use the slit of width $a_2 = 0.5mm$. The width of the central fringe of diffraction observed on (E) is $7.2mm$. Show that the wavelength of the light used is $\lambda = 600\,nm$.
4. We remove the screen (P). A hair of diameter d is stretched in the place of the slit F. We obtain on the screen a diffraction pattern. The measurement of the width of the central fringe of diffraction gives $12mm$. Determine the value of d.

Part B
Interference

In order to measure a small displacement of an apparatus, we fix the screen (P) on this apparatus.

In a screen (P'), we cut two parallel and very thin slits F_1 and F_2 separated by $1mm$.

We repeat the previous experiment by introducing (P') between (P) and (E).

(P) and (P') are parallel and are at a distance $D' = 1m$ from each other. The slit F, cut in (P), is equidistant from the two slits F_1 and F_2. The slit F is illuminated by the laser source of wavelength $\lambda = 600\,nm$. A phenomenon of interference is observed on the screen (E) located at a distance $D = 2m$ from (P').

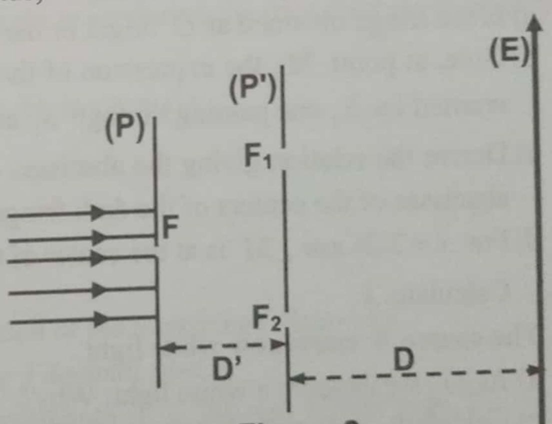


Figure 2

1. Show on a diagram the region where interference fringes may appear.
2. Specify, with justification, the position O, the center of the central fringe.
3. A point M on the screen is at a distance d_1 from F_1 and at a distance d_2 from F_2 such that: $d_2 = d_1 + 1500\,nm$.
   The point M is the center of the third dark fringe. Why?

4. We count on (E) 11 bright fringes. Calculate the distance d between the centers of the farthermost bright fringes.
5. We move the apparatus and hence the slit F a distance z to the side of F_2 normal to the perpendicular bisector of F_1F_2, the new position of F being denoted by F'. We observe that the central fringe occupies now the position that was occupied by the third bright fringe.
   a) Explain why the central fringe is displaced on the screen and determine the direction of this displacement.
   b) For a point N of (E), of abscissa x with respect to O, we can write:
   $(F'F_2N)-(F'F_1N)=a\frac{x}{D}+a\frac{z}{D'}$. Calculate the value of z.

V-GS 2009 2nd
Interference of Light

Consider Young's double slit apparatus that is represented in the adjacent figure 1. S_1 and S_2 are separated by a distance $a=1\,mm$.
The planes (P) and (E) are at a distance $D=1\,m$.
I is the midpoint of $[S_1S_2]$ and O the orthogonal projection of I on (P).
On the perpendicular to IO at point O and parallel to S_1S_2, a point M is defined by its abscissa $OM=x$.

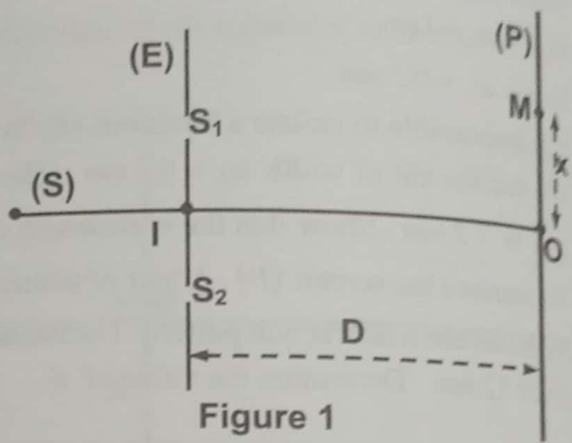


Figure 1

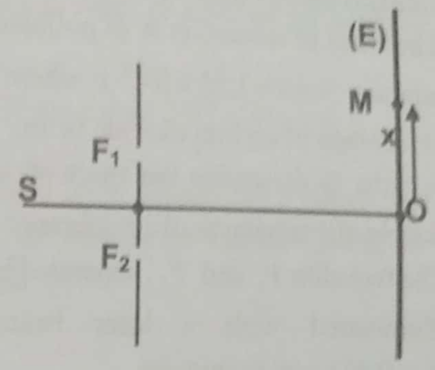
1. S_1 and S_2, illuminated by two lamps, emit synchronous radiations. Do we observe interference fringes on the screen? Why?
2. S_1 and S_2 are illuminated by a point source S put on IO. S emits a monochromatic radiation of wavelength λ in vacuum (or in air).
   a) Is the fringe obtained at O bright or dark? Why?
   b) Give, at point M, the expression of the optical path difference δ between the two radiations emitted by S, one passing through S_1 and the other through S_2, in terms of D, x and a.
   c) Derive the relation giving the abscissas of the centers of the bright fringes and that giving the abscissas of the centers of the dark fringes.
   d) For $x=2.24\,mm$, M is at the center of the fourth bright fringe (bright fringe of order 4). Calculate λ.
3. The source S emits now white light.
   a) At O, we observe a white light. Why?
   b) Calculate the wavelengths of the visible radiations that give at M, of abscissa $OM=x=2.24\,mm$, dark fringes. **Visible spectrum:** $0.400\,\mu m \le \lambda \le 0.800\,\mu m$.

V-GS 2008 1st
Index of Refraction of a Piece of Glass

Consider a glass sheet of thickness $e=5\,\mu m$ and of index of refraction n, and a source S of white light having a filter so that Young's apparatus receives monochromatic light of wavelength λ in air of adjustable value. The object of this exercise is to study how the index n varies with λ.

Part A
Light interference – Interfringe distance

Young's slits apparatus is formed of two very thin slits F_1 and F_2, parallel and separated by a distance $a = 0.1 mm$, and a screen of observation (E) placed parallel to the plane of the slits at a distance $D = 1 m$ from this plane.



1. F_1 and F_2 are illuminated with a monochromatic radiation of wavelength λ issued from S that is placed at equal distances from F_1 and F_2.
   a) F_1 and F_2 must have two basic properties for the phenomenon of interference to be observed. What are they?
   b) Describe the system of fringes observed on (E).
   c) At the point O of the screen, equidistant from F_1 and F_2, we observe a bright fringe. Why?
2. We admit that for a point M of (E), such that $OM = x$, the optical path difference in air or in vacuum is given by: $\delta = F_2M - F_1M = \dfrac{a\,x}{D}$.
   a) Determine the expression of x_k corresponding to the center of the k[th] bright fringe.
   b) Deduce the expression of the interfringe distance i in terms of λ, D and a.

Part B
Introducing the sheet

The glass sheet is put now just behind the slit F_1. c and v are the speeds of light in vacuum (and practically in air) and in the glass sheet respectively.

1. Light crosses the glass sheet of thickness e during a time interval τ. Give the expression of τ in terms of e and v.
2. Give the expression of the distance d, covered by light in air during the time interval τ, in terms of n and e.
3. Deduce that the new optical path difference at point M is given by:

$$\delta' = F_2M - F_1M = \frac{a\,x}{D} - e(n-1).$$

Part C
Measurement of n

N.B : *Introducing the sheet does not affect the expression of the interfringe distance i*
In this question the calculation of n must include 3 decimal places.

1. F_1 & F_2 are illuminated with a red radiation, of wavelength $\lambda_1 = 768\ nm$, issued from S. The center of the central fringe is formed at O', position that was occupied by the center of the 4[th] bright fringe in the absence of the sheet. Determine the value of n_1 the index of the sheet.
2. F_1 & F_2 are illuminated with a violet radiation of wavelength $\lambda_2 = 434\ nm$, issued from S. The center of the central fringe is now formed at O'', position that was occupied by the center of the 8[th] dark fringe in the absence of the sheet. Determine the value of n_2, the index of the sheet.
3. Can we consider the value of the index of refraction of a transparent medium without taking into account the radiation used? Why?

VI-GS 2006 1st

Index of Refraction of Atmospheric Air

The index of refraction of pure air is supposed to be equal to 1. Atmospheric air is not pure, it is polluted; it contains carbon dioxide.

The index of refraction n of polluted air is given by the relation: $n = 1 + 1.55 \times 10^{-4} y$ where $y\%$ represents the percentage of carbon dioxide in air.

In order to determine the value of y, we use Young's double-slit apparatus of interference.

The two slits F_1 and F_2, separated by a distance a, are illuminated with a laser beam of wavelength $\lambda = 0.633\,\mu m$ in pure air.

The beam falls normally on the plane (P) that contains the slits.

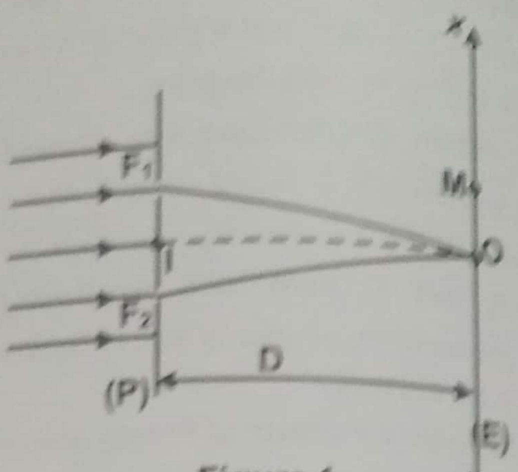


Figure 1

We observe interference fringes on a screen (E) parallel to (P) found at a distance D from this plane. Point O is the foot of the orthogonal projection of, the midpoint I of F_1F_2 on the plane (E) (figure 1).

Part A

Interference in pure air

Recall that for a point M of the screen where $OM = x$, the optical path difference $\delta = F_2M - F_1M$ is given by the relation $\delta = \dfrac{ax}{D}$.

1. O is the center of the central bright fringe. Why?
2. M is the center of the bright fringe of order k.
   a) Give the expression of δ in terms of k and λ.
   b) Deduce the expression of the interfringe distance i in terms of λ, D & a.
3. M is the point of (E) so that: $\delta = F_2M - F_1M = 1.266\ \mu m$.
   a) Specify the nature and the order of the fringe whose center is at M. Justify your answer.
   b) Express x in terms of i.

Part B

Interference in polluted air

We intend to measure the index of refraction n of air polluted with carbon dioxide. In Young's double-slit apparatus used, we consider that the beam issued from F_2 propagates in pure air whereas the beam issued from F_1 propagates a distance $\ell = 50\,cm$ in polluted air and the rest of its path in pure air (figure 2). We observe, in this case, that the system of interference fringes is displaced upwards.

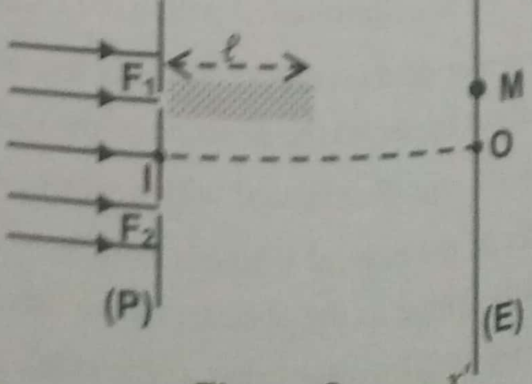


Figure 2

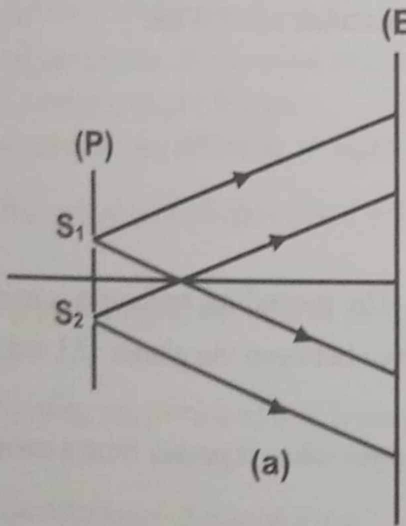
1. Knowing [...] F_1 in polluted air, give the expression of the time t that light needs to cover the distance ℓ in polluted air in terms of v and ℓ.
2. Knowing that c is the speed of light in pure air, determine the expression of the distance d that light issued from F_2 would cover in pure air during the same time t in terms of ℓ and n.
3. Give the expression of the increase in the optical path due to the passage in polluted air in terms of ℓ & n.
4. The new expression of the optical path difference is then $\delta' = F_2M - F_1M = \frac{ax}{D} - \ell(n-1)$.
   a) Knowing that the center of the central bright fringe is displaced up to occupy the position that was occupied by the center of the bright fringe of order 2, the interfringe distance being the same, determine the expression that gives n in terms of ℓ and λ.
   b) Show that the value of n is 1.0000025 .
5. a) The index n being given by: $n = 1 + 1.55 \times 10^{-6}\ y$. Calculate the value of y.
   b) Air polluted with carbon dioxide becomes harmful when $y \geq 0.5$.
   Is this polluted air harmful? Why?

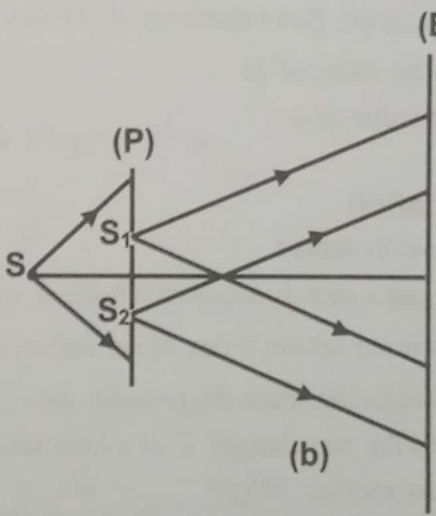
VII-GS 2005 2nd
Interference of Light

Part A

Conditions to obtain a phenomenon of interference

We are going to use Young's double slit and two identical lamps.
Consider each of the two set-ups (a) and (b) drawn below.





In the set-up (a), each of the slits S_1 and S_2 is illuminated by a lamp, the two lamps emit the same radiation. In the set-up (b), S_1 and S_2 are illuminated by a lamp placed at S next to a very narrow slit parallel to S_1 and S_2; the lamp emits the same preceding radiation.

1. The radiations emitted by the sources S_1 and S_2 in the two set-ups (a) and (b) have a common property. What is it?
2. One property differentiates the radiations issued from S_1 and S_2 in the set-ups (a) and (b) from those issued from S_1 and S_2 in the set-up (b). Specify this property.
3. The set-up (b) allows us to observe the phenomenon of interference. Why?

Part B

Interferences and measurement

We intend to perform a series of experiments about interference using Young's double slits apparatus. The slits are in a plane (P), separated by a distance a, and the pattern of interference is observed on a screen (E) found at a distance D from (P).

First situation

Interferences in air

Consider many light filters, each allowing the transmission of a specific monochromatic radiation of wavelength λ in air. We measure the distance $x = 5i$ along which five interfringe distances extend. The results obtained are tabulated as in the table below.

λ (nm)	470	490	530	560	610
$x = 5i$ (in mm)	11.75	12.30	13.00	14.30	15.25
i (in mm)					

1. a) Complete the table.

b) Interfringe.

i- Show that the expression of i as a function of λ is of the form $i = \alpha\lambda$ where α is a positive constant.

ii- Calculate α.

iii- Deduce the value of the ratio $\frac{D}{a}$.

2. We move (E) by 50 cm away from (P). We find that, for the radiation of wavelength $\lambda = 496$ nm in air, five interfringe distances extend over a distance of $18.6\ mm$.

Determine the value of D.

3. Deduce the value of a.

Second situation

Interferences in water

The radiation used now has a wavelength $\lambda = 520\ nm$ in air. The preceding apparatus is immersed completely in water whose index of refraction is n. The distance between the planes (E) and (P) is D and the distance between the two slits is a.

1. The value of the wavelength λ of a luminous radiation changes when it passes from a transparent medium into another. Why?
2. The interference fringes in water seem closer than in air. Why?
3. In water, five interfringe distances extend over a distance of $9.75\ mm$.

Determine the value of n.

Solutions

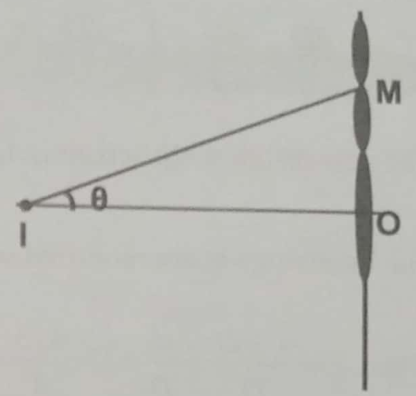
GS 2012 2nd

Part A

1. The dark points are defined by: $\sin\theta_k = k\frac{\lambda}{a}$;[1]

For the 2nd dark $k=2$, and for small angles $\sin\theta = \theta$;

Then $\theta = 2\frac{\lambda}{a}$.



2. According to the geometry of the figure in the triangle OIM; $\tan\theta = \frac{OM}{D} \approx \theta$ (small angles);

Then $\theta = \frac{x}{D}$; but $\theta = 2\frac{\lambda}{a} = \frac{x}{D}$; thus $x = 2\frac{\lambda D}{a}$.

3. According to the previous result $a = 2\frac{\lambda D}{x} = 2\frac{640\times10^{-9}\,m\times 4m}{1.28\times10^{-2}\,m} = 4\times10^{-4}\,m = 0.4\,mm$.

4. If the diameter of the slit is increased 100 times, the light will no longer diffracts, it undergoes rectilinear propagation. Then on the screen we will observe a bright spot of same dimensions as that of the slit.

Part B

1. a) The interference phenomenon is due to the superposition of the synchronous and coherent waves diffracted by F_1 & F_2.

b) On the screen, in the zone of interference, we observe:

- a central bright fringe;
- alternate, equidistant, straight bright and dark fringes.

2. a) The optical path difference $\delta = F_2N - F_1N = a'\frac{x}{D}$.

b) For dark fringes, the optical path difference is: $\delta = \left(k+\frac{1}{2}\right)\lambda$ where k is an integer.

c) By using the results of a & b we get: $\delta = a'\frac{x}{D} = \left(k+\frac{1}{2}\right)\lambda$; we get $x_k = \left(k+\frac{1}{2}\right)\frac{\lambda D}{a'}$.

d) The interfringe distance $i = x_{k+1} - x_k = \left(k+1+\frac{1}{2}\right)\frac{\lambda D}{a'} - \left(k+\frac{1}{2}\right)\frac{\lambda D}{a'}$;

We get $i = \frac{\lambda D}{a'}\left(k+1+\frac{1}{2}-k-\frac{1}{2}\right) = \frac{\lambda D}{a'}$.

3. a) Mediums:

i- In the new medium, the wavelength is modified; thus, the interfringe distance that depends on λ will be modified.

[1] The action verb «write» means we can give the result directly without justification $\theta = 2\frac{\lambda}{a}$.

ii- In the new medium, the interfringe distance $i' = \frac{\lambda' D}{a'}$

The speed $v = \lambda' f$ & $v = \frac{c}{n} \Rightarrow \lambda' = \frac{v}{f} = \frac{c}{nf}$;

But in air $c = \lambda f$ (the frequency remains conserved). $\lambda' = \frac{\lambda f}{nf} = \frac{\lambda}{n}$;

$i' = \frac{\lambda D}{na'} = \frac{\lambda D}{na'} = \frac{1}{n} \times \frac{\lambda D}{a'} = \frac{i}{n}$.

b) The new distance slit screen is $(d + D)$, the interfringe becomes $i'' = \frac{\lambda(d + D)}{na'}$;

The interfringe is not modified so $i'' = i$, we get $\frac{\lambda(d + D)}{na'} = \frac{\lambda D}{a'}$;

Then $n = \frac{d + D}{D} = \frac{d}{D} + 1 = \frac{4/3}{4} + 1 = 1 + \frac{4}{12} = 1.33$.

II-GS 2011 2nd

1. On the screen, in the zone of interference we observe:
   - a central bright fringe;
   - alternate, equidistant and straight bright and dark fringes.

2. a) O is the origin of abscissas $x_O = 0$;

So the optical path difference at this point : $\delta_O = S_2O - S_1O = a\frac{x_O}{D} = 0$.

Thus, O is the center of the central bright fringe.

b) M is the center of a dark fringe if $\delta = S_2M - S_1M = a\frac{x}{D} = \left(k + \frac{1}{2}\right)\lambda$.

c) M is the center of a bright fringe if $\delta = S_2M - S_1M = a\frac{x}{D} = k\lambda$; then $x = k\lambda\frac{D}{a}$.

d) For $x = 1.65\,cm$, the optical path difference is:

$\delta = a\frac{x}{D} = 0.2 \times 10^{-3}\,m\frac{1.65 \times 10^{-2}\,m}{1.5m} = 2.2 \times 10^{-6}\,m$;

$\frac{\delta}{\lambda} = \frac{2.2 \times 10^{-6}\,m}{0.55 \times 10^{-6}\,m} = 4$ (whole number); so this point is the center of a bright fringe.

Then the waves that interfere at this point are in phase.

3. a) Central fringe:

i- The new optical difference at O is $\delta_O = (S'S_2O) - (S'S_1O) = S'S_2 + S_2O - S'S_1 - S_1O$;

$= (S'S_2 - S'S_1) + (S_2O - S_1O) = S'S_2 - S'S_1 \neq 0$;

Thus, the center of the central bright fringe is no longer at O.

ii- O' is the new central fringe $\delta_{O'} = (S'S_2O') - (S'S_1O') = 0$;

$(S'S_2 + S_2O') - (S'S_1 + S_1O') = 0 \Rightarrow S'S_2 - S'S_1 = S_1O' - S_2O'$;

But according to the direction of displacement of the source $S'S_2 > S'S_1 \Rightarrow S_1O' > S_2O'$;

Then the new central O' is below O.

b) The new expression of the optical path difference is:

$$\delta_{O'} = (S'S_2O') - (S'S_1O') = (S'S_2 - S'S_1) + (S_2O' - S_1O') = a\frac{b}{d} + a\frac{x_{O'}}{D} = 0;$$

$$\frac{b}{d} = -\frac{x_{O'}}{D} \Rightarrow b = -\frac{d}{D}x_{O'} = -\frac{10\times10^{-2}\,m}{1.5m}\times(-1\times10^{-2}\,m);$$

Thus, $b \approx 6.67\times10^{-4}\,m$.

III-GS 2010 2nd

Part A

1. The centers of the dark fringes are defined by: $\sin\theta_k = k\frac{\lambda}{a}$;

For the 2nd dark $k=2$; and the angles are small so $\sin\theta = \theta$ then $\theta = 2\frac{\lambda}{a}$.

2. According to the geometry of the figure in the triangle OIM : $\tan\theta = \frac{OM}{D} \approx \theta$ (small angles);

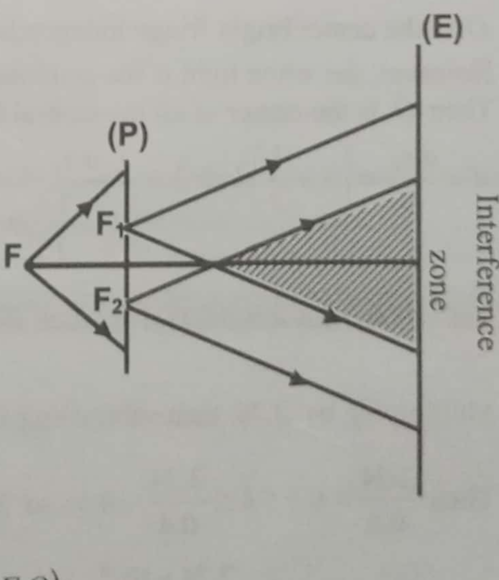
Then $\theta = \frac{x}{D}$; but $\theta = 2\frac{\lambda}{a} = \frac{x}{D} \Rightarrow x = 2\frac{\lambda D}{a}$.

3. According to the previous result: $a = 2\frac{\lambda D}{x} = 2\frac{640\times10^{-9}\,m\times4m}{1.28\times10^{-2}\,m} = 4\times10^{-4}\,m = 0.4\,mm$.

4. If the diameter of the slit is increased a 100 times, the diffraction phenomenon vanishes and the light propagates rectilinearly. Then on the screen, we will observe a bright spot having identical dimensions to that of the slit.

Part B

1. Schema.



2. The central fringe is characterized by:

$$\delta = (FF_2O) - (FF_1O)$$
$$= (FF_2 - FF_1) - (F_2O - F_1O) = 0$$

But $(FF_2 = FF_1) \Rightarrow F_2O = F_1O$, hence O is the intersection of the perpendicular bisector of F_1F_2 with the screen.

3. $\frac{d_2 - d_1}{\lambda} = \frac{1500\,nm}{600\,nm} = \frac{5}{2} = 2.5$ (Half-integer).

Then this point is the center of the 3rd dark.

4. $d = 10i = 10\lambda\frac{D}{a} = 12\,mm$.

5. a) The optical path difference at O is:

$$\delta_O = (F'F_2O) - (F'F_1O) = (F'F_2 - F'F_1) - (F_2O - F_1O);$$

But O is equidistant from F_1 & F_2 so, $F_2O = F_1O$ & $F'F_2 \neq F'F_1$;

We get $\delta_O = (F'F_2 - F'F_1) \neq 0$; thus, the path optical difference at O is not zero ($\delta_o \neq 0$) so the central fringe is no longer at O.

Let O' be the new position $\delta_{O'} = (F'F_2O') - (F'F_1O') = (F'F_2 - F'F_1) - (F_1O' - F_2O') = 0$,

$(F'F_2 - F'F_1) = (F_1O' - F_2O')$.

But F is moved downwards, then $F'F_2 - F'F_1 < 0 \Rightarrow F_2O' > F_1O'$;

Then the new central O' is displaced upwards.

b) If O' is the new central, then $\delta_{o'} = 0$ and consequently $a\dfrac{x}{D} + a\dfrac{z}{D'} = 0 \Rightarrow z = -x_{O'}\dfrac{D'}{D}$;

The bright fringe of order 3 is at the point of abscissa $x_{O'} = 3\lambda\dfrac{D}{a} = 3.6\times10^{-3}\,m = 3.6\,mm$.

Thus, $z = -1.8\,mm$.

IV-GS 2009 2nd

1. The interference is not observed, since the two sources are not coherent (independent sources).

2. a) The optical path difference at O is: $\delta_O = (SS_2O) - (SS_1O) = (SS_2 + S_2O) - (SS_1 + S_1O)$;

Then $\delta_O = (SS_2 - SS_1) + (S_2O - S_1O)$;

But S & O are on the perpendicular bisector of $[S_1S_2]$ then $SS_2 = SS_1$ & $S_2O = S_1O$;

$\delta_O = 0$, so O is the central bright.

b) The optical path difference at M is $\delta = a\dfrac{x}{D}$.

c) Bright fringes: $\delta = a\dfrac{x_k}{D} = k\lambda$, then $x_k = k\lambda\dfrac{D}{a}$ where $k \in Z$;

Dark fringes: $\delta = a\dfrac{x_k}{D} = \left(k + \dfrac{1}{2}\right)\lambda$, then $x_k = \left(k + \dfrac{1}{2}\right)\dfrac{\lambda D}{a}$ where $k \in Z$;

d) M is the center of the 4th bright then $k = 4$;

So $OM = x_4 = 4\dfrac{\lambda D}{a}$, then $\lambda = \dfrac{a x_4}{4D} = \dfrac{1\times10^{-3}\times2.24\times10^{-3}}{4} = 5.6\times10^{-7}\,m = 0.56\,\mu m$.

3. a) O is the center bright fringe independent of wavelength;

However, the white light is the combination of all the colors in the visible spectrum.

Then O is the center of all the central bright fringes whose combination form a white fringe.

b) $\delta = \dfrac{a x_k}{D} = \left(k + \dfrac{1}{2}\right)\lambda \Rightarrow \lambda = \dfrac{a x_k}{\left(k + \dfrac{1}{2}\right)D} = \dfrac{2.24\times10^{-6}}{k + 0.5}$ (λ in m)

But $0.400\,\mu m \le \lambda \le 0.800\,\mu m$; then $0.400 \le \dfrac{2.24}{k + 0.5} \le 0.800$;

Multiplying by 2.24 then subtracting 0.5 we get: $\dfrac{2.24}{0.8} - 0.5 \le k \le \dfrac{2.24}{0.4} - 0.5$

Then $\dfrac{2.24}{0.8} - 0.5 \le k \le \dfrac{2.24}{0.4} - 0.5$; so $2.3 \le k \le 5.1$ and $k \in Z$; thus $k = \{3, 4, 5\}$.

- For $k = 3$, $\lambda = \dfrac{2.24\times10^{-6}}{3 + 0.5} = 6.4\times10^{-7}\,m$;
- For $k = 4$, $\lambda = 4.98\times10^{-7}\,m$;
- For $k = 5$, $\lambda = 4.07\times10^{-7}\,m$;

k	3	4	5
$\lambda(\mu m)$	0.64	0.498	0.407

The spectral analysis shows at this point the visible spectrum intercepted by three black lines.

V-GS 2008 1st

Part A

1. a) F_1 & F_2 become synchronous and coherent.

b) On the screen, in the zone of interference we observe:

- a central bright fringe.
- alternate equidistant and straight bright and dark fringes.

c) We have $\delta_O = 0$, then the waves that superpose at O are in phase; thus, at O a bright fringe is formed.

2. a) For bright fringes: $\delta = k\lambda$ & $\delta = \dfrac{a\,x}{D}$, then $x_k = k\dfrac{\lambda D}{a}$.

b) The interfringe $i = x_{k+1} - x_k = (k+1)\dfrac{\lambda D}{a} - k\dfrac{\lambda D}{a} = \dfrac{\lambda D}{a}(k+1-k) = \dfrac{\lambda D}{a}$.

Part B

1. The time needed to cross the plate $\tau = \dfrac{e}{v}$.

2. The distance traveled in air during the same duration $d = c \times \tau = c \times \dfrac{e}{v} = n\,e$.

3. The increase in the optical path for the light crossing the plate is: $n\,e - e = e(n-1)$;

Then $\delta' = F_2M - (F_1M + e(n-1)) = F_2M - F_1M - e(n-1)$, we get $\delta' = \dfrac{a\,x}{D} - e(n-1)$.

Part C

1. The center of the central bright fringe $\delta' = 0 \Rightarrow \dfrac{a\,x_0}{D} - e(n_1 - 1) = 0$, we get $x_0 = \dfrac{e(n_1-1)D}{a}$.

But $x_0 = 4i_1 = 4\dfrac{\lambda_1 D}{a}$, then $n_1 = 1 + \dfrac{4\lambda_1}{e} = 1.614$.

2. We have $x_0 = 7.5 i_2 = 7.5\dfrac{\lambda_2 D}{a}$, then $n_2 = 1 + \dfrac{7.5\lambda_2}{e} = 1.651$.

3. The index of refraction of a transparent medium depends on the radiation used, because when the wavelength is modified $\lambda_1 \neq \lambda_2$; the index of refraction is also modified $n_1 \neq n_2$[2].

VI-GS 2006 1st

Part A

1. The abscissa of the point O is $x_O = 0$, then the optical path difference is $\delta_O = \dfrac{a\,x_O}{D} = 0$.

Thus the central bright fringe is at O.

2. a) For the bright fringes $\delta = k\lambda$ where $k \in Z$ (integer).

b) We know that $\delta = \dfrac{a\,x}{D} = k\lambda$, the abscissas of the center of bright fringes $x_k = k\dfrac{\lambda D}{a}$;

The interfringe i is the distance between two consecutives fringes of same nature:

[2] Yes: $\lambda_1 \neq \lambda_2$, we get $n_1 \neq n_2$; we can conclude that the index of refraction of the slit is independent of the wavelength of the radiation used.

$$i = x_{k+1} - x_k = (k+1)\frac{\lambda D}{a} - k\frac{\lambda D}{a} = \frac{\lambda D}{a}(k+1-k) = \frac{\lambda D}{a}.$$

3. a) At the considered point $\frac{\delta}{\lambda} = \frac{F_2M - F_1M}{\lambda} = \frac{1.266}{0.633} = 2$ (whole number).

Thus, M corresponds to the center of the 2nd bright fringe.

b) We have $x_k = k\frac{\lambda D}{a}$ & $i = \frac{\lambda D}{a}$, at the center of the 2nd bright $k = 2$, thus $x = 2i$.

Part B

1. The time needed to cover the distance ℓ is t in the medium, so $t = \ell / v$.
2. The distance covered by the light in pure air during this interval of time is: $d = ct = c\frac{\ell}{v}$ where $v = \frac{c}{n}$; then $d = n\ell$.
3. The increase in the optical path difference is $n\ell - \ell = \ell(n-1)$.
4. a) The new optical path difference is $\delta' = F_2M - F_1M = \frac{ax}{D} - \ell(n-1)$;[3]

   The new position O' of the central fringe is defined $\delta' = 0 \Rightarrow \frac{ax_{O'}}{D} = \ell(n-1)$;

   Then $x_{O'} = \frac{\ell(n-1)}{a}D$.

   But the central occupies the position of the 2nd bright from the positive side so $x_{O'} = 2i$;

   Then $x_{O'} = \frac{\ell(n-1)}{a}D = 2\frac{\lambda D}{a}$; $\ell(n-1) = 2\lambda$; thus $n = 1 + \frac{2\lambda}{\ell}$.

   b) The value of the index n is $n = \frac{2\lambda}{\ell} + 1 = 1.0000025$.
5. a) We have $n = 1 + 1.55\times10^{-6}\,y = 1.0000025$; then $y = 1.61$.

   b) $y = 1.61 \succ 0.5$, therefore the air of the room is harmful.

VII-GS 2005 2nd

Part A

1. The two sources are synchronous since they emit radiations having the same frequency.
2. In the set-up (a), the radiations emitted are only synchronous whereas in the set-up (b), the radiations are synchronous and coherent.
3. Since the radiations emitted by the two sources S_1 and S_2 are coherent, so we will observe the phenomenon of interference.

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[3] This result can be derived basing on the expression of the optical path difference :

$\delta' = (FF_2M) - (FF_1M) = (FF_2 - FF_1) + (F_2M - (F_1M))$, but $FF_2 = FF_1$;

Then $\delta' = (F_2M - (F_1M)) = F_2M - (F_1M + \ell(n-1)) = F_2M - F_1M - \ell(n-1)$;

Thus, $\delta' = \frac{ax}{D} - \ell(n-1)$.

Part B
First situation

1. a) Table

$\lambda(nm)$	470	496	520	580	610
$x = 5i$ (in mm)	11.75	12.40	13.00	14.50	15.25
i (in mm)	2.35	2.48	2.60	2.90	3.05

b) Interfringe:

i- Referring to the table we may write:

$$\frac{i}{\lambda} = \frac{2.35}{470\times10^{-6}} = \frac{2.48}{496\times10^{-6}} = \frac{2.60}{520\times10^{-6}} = \frac{2.90}{580\times10^{-6}} = \frac{3.05}{610\times10^{-6}};$$

Then $\frac{i}{\lambda} = 5000$.

ii- We have $i = \alpha\lambda$ we may conclude that $\alpha = 5000$.

iii- We know that the interfringe distance $i = \frac{\lambda D}{a}$, & $\frac{i}{\lambda} = \alpha$; then $\frac{D}{a} = \frac{i}{\lambda} = \alpha = 5000$.

2. For $\lambda = 496\ nm$, $i = 2.48 mm$ for a distance D, the interfringe distance is $i = \frac{\lambda D}{a}$;

$i' = \frac{18.6\,mm}{5} = 3.72\ mm$; for a distance $D' = D + 0.5$, so $i' = \frac{\lambda D'}{a} = \frac{\lambda(D+0.5)}{a}$;

$$\frac{i'}{i} = \frac{\frac{\lambda(D+0.5)}{a}}{\frac{\lambda D}{a}} = \frac{D+0.5}{D}, \text{ then } \frac{3.72}{2.48} = 1 + \frac{0.5}{D}\ ;\ 1.5 = 1 + \frac{0.5}{D};$$

Thus, $D = 1m$.

3. Since $\alpha = \frac{D}{a} = 5000$; we get $a = \frac{D}{\alpha} = \frac{1}{5000} = 2\times10^{-4}\,m = 0.2\ mm$.

Part B

Second situation

1. In air, we have $\lambda_{air} = \frac{c}{\upsilon}$ where c is the speed of light in air & υ is the frequency of the radiation.

In water, we have: $\lambda_{water} = \frac{v}{\upsilon}$ where v is the speed of light in water.

So: $\frac{\lambda_{water}}{\lambda_{air}} = \frac{v}{c} = \frac{1}{n}$ (knowing that $v = \frac{c}{n}$); then $\lambda_{water} < \lambda_{air}$

2. We know that i is proportional to λ; by passing from air into water, the wavelength decreases then the interfringe distance decreases also and the interference fringes seems closer.

3. In water, five interfringe distances extend over a distance of 9.75 mm .

$5i_{water} = 9.75$; then $i_{water} = 1.95\ mm$.

However $\frac{i_{water}}{i_{air}} = \frac{\lambda_{water}}{\lambda_{air}} = \frac{v}{c} = \frac{1}{n}$; thus $n = \frac{2.6}{1.95} = \frac{4}{3} = 1.33$.

LGS 2007 2nd
The Two Aspects of Light

Part A

Diffraction

A source of monochromatic radiation of wavelength λ in air illuminates under normal incidence a horizontal slit F of adjustable width « a » cut in an opaque screen (P). A screen of observation (E) is placed parallel to (P) at a distance $D=5m$ (Figure 1).

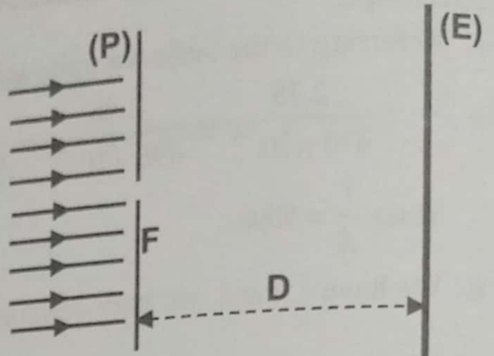


Figure 1

1. For λ, show on a diagram the shape of the luminous beam emerging from the slit in each of the two following cases:
   - width of the slit $a=2cm$.
   - width of the slit $a=0.4mm$.
2. The width of the slit is now kept at $0.4mm$ and the radiation used belongs to the visible spectrum. (Wavelength of the visible spectrum: $0.4\,\mu m \le \lambda \le 0.8\,\mu m$)
   a) Write, in this case, the expression giving the angular width of the central bright fringe in terms of λ & a.
   b) Show that the linear width of this central fringe is given by $\ell = \dfrac{2\lambda D}{a}$.
   c) Calculate the linear widths ℓ_{red} and ℓ_{violet}, when using successively a red radiation $(\lambda_{red}=0.8\,\mu m)$ and a violet radiation $(\lambda_{violet}=0.4\,\mu m)$.
   d) We illuminate the slit with white light. We observe over the linear width ℓ_{violet} white light. Justify.

Part B

Photoelectric effect

A source of wavelength $\lambda = 0.5\,\mu m$ in air illuminates separately two metallic plates, one made of cesium and the other of zinc. The table below gives, in eV, the values of the extraction energy W_0 (work function) for some metals.

Metal	Cesium	Rubidium	Potassium	Sodium	Zinc
$W_0(eV)$	1.89	2.13	2.15	2.27	4.31

Given:

- Planck's constant $h=6.6\times10^{-34}\,J.s$;
- Speed of light in vacuum $c=3\times10^{8}\,m/s$;
- $1eV=1.6\times10^{-19}\,J$.

1. Calculate, in J and in eV, the energy of an incident photon.

2. For what metal would photoelectric emission take place? Justify.
3. Calculate, in eV, the maximum kinetic energy of an emitted electron.
4. The cesium plate receives a monochromatic luminous beam of wavelength in air $\lambda = 0.5\ \mu m$, of power $P = 3978 \times 10^{-4} W$. The number of electrons emitted per second is then $n = 10^{18}$.
a) Calculate the number N of photons received by the plate in one second.
b) The quantum efficiency η of the plate is the ratio of the number of the electrons emitted per second to the number of photons received by the plate during the same time. Calculate η.

Part C

Duality wave-particle

The wave theory of light is used to interpret the phenomenon of diffraction. This theory is not able to interpret the photoelectric effect. Why?

I-GS 2005 1st

Photoelectric Effect

The experiments on photoelectric emission performed by Millikan around the year 1915, intended to determine the kinetic energy KE of the electrons emitted by metallic cylinders of potassium (K) and cesium (Cs) when these cylinders are illuminated by monochromatic radiation of adjustable frequency υ. The object of this exercise is to determine, performing similar experiments, Planck's constant (h), as well as the threshold frequency υ_0 of potassium and the extraction energy W_0 of potassium and that of cesium.

Part A

Properties of light

1. What aspect of light does the phenomenon of photoelectric effect show evidence of?
2. A monochromatic radiation is formed of photons. Give two characteristics of a photon.
3. For a given pure metal, the incident photons of a monochromatic radiation provoke photoelectric emission. Give the condition for this emission to take place.

Part B

Extraction energy

In a first experiment using potassium, a convenient apparatus is used to measure the kinetic energy KE of the electrons corresponding to frequency υ of the incident radiation. The obtained results are tabulated in the following table:

$\upsilon(Hz)$	$KE(eV)$
6×10^{14}	0.25
7×10^{14}	0.65
8×10^{14}	1.05
9×10^{14}	1.45
10×10^{14}	1.85

Given: $1eV = 1.6 \times 10^{-19} J$.

1. Using Einstein's relation about photoelectric effect, show that the kinetic energy of an extracted electron may be written in the form $KE = a\upsilon + b$.
2. a) Plot, on the graph paper, the curve representing the variation of the kinetic energy KE versus υ, using the following scale:

➢ on the axis of abscissas: $1cm$ represents a frequency of $10^{14}\ Hz$,

➢ on the axis of ordinates: $1cm$ represents a kinetic energy of $0.5\ eV$.

b) Using the graph, determine:

i- the value, in SI, of h , the Planck's constant.

ii- the threshold frequency υ_0 of potassium.

c) Deduce the value of the extraction energy W_0 of potassium.

Part C
Graphical reading

In a second experiment using cesium, we obtain the following values $KE = 1eV$ for $\upsilon = 7\times10^{14}\ Hz$

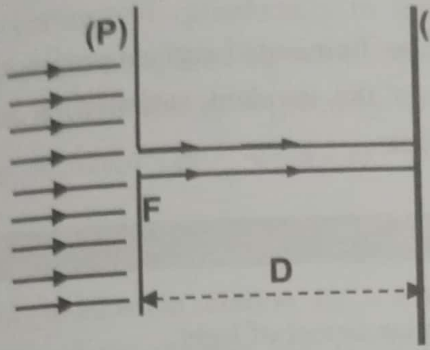
1. Plot, with justification, on the preceding system of axes, the graph of the variation of KE as a function of υ .
2. Deduce from this graph the extraction energy W_0' of cesium.

Solutions

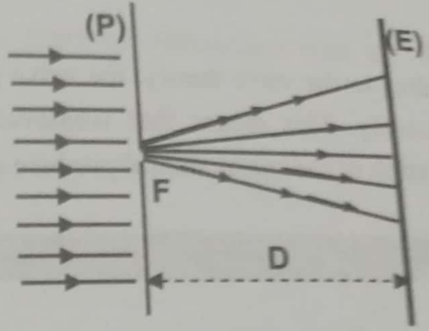
I-GS 2007 2nd

Part A

1. Schema

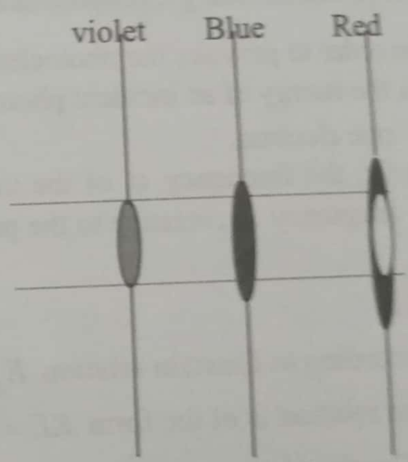


a=2cm



a=0,4 mm

2. a) The angular width α of the central is $\alpha = \dfrac{2\lambda}{a}$.



b) According to the geometry of the figure :

$\tan(\theta_1) = \dfrac{\ell}{2D} = \theta_1$ & $\alpha = 2\theta_1 = \dfrac{\ell}{D}$ (Figure);

Then $\ell = \dfrac{2\lambda D}{a}$.

c) $\ell_{red} = \dfrac{2\lambda_{red} D}{a} = \dfrac{2\times 0.8\times 10^{-6} m\times 5m}{0.4\times 10^{-3} m} = 0.02m = 2cm$.

& $\lambda_{red} = 2\lambda_{violet}$;

Then $\ell_{red} = 2\ell_{violet} \Rightarrow \ell_{violet} = 1cm$.

d) The linear width ℓ of the central fringe extends between $1cm \le \ell \le 2cm$.

The central violet being the smallest between all the centrals, all the central bright fringes superposed within $1cm$.

Thus, over the violet central bright fringe we will obtain the combination of all the colors leading to the appearance of white fringe.

Part B

1. The energy of the incident photon is : $E = h\dfrac{c}{\lambda} = \dfrac{6.63\times 10^{-34}\times 3\times 10^{8}}{0.5\times 10^{-6}} = 3.978\times 10^{-19} J$.

Then, $E = \dfrac{3.978\times 10^{-19} J}{1.6\times 10^{-19} J/eV} = 2.49\, eV$.

2. The photoelectric emission takes place if the energy of the incident photon is greater than or equal to the work energy function of the metal.

$E_{ph} = 2.49\, eV > W_0(Cs) = 1.89 eV$; then we have a photoelectric emission from the cesium.

$E_{ph} = 2.49\,eV < W_0(Zn) = 4.31\,eV$; then no photoelectric emission from zinc.

3. According to Einstein relation : $E = W_0 + KE_{max} \Rightarrow KE_{max} = E - W_0 = 2.49 - 1.89 = 0.6\,eV$

4. a) The number of photons received $P = NE \Rightarrow N = \dfrac{P}{E} = \dfrac{0.3978}{3.978\times10^{-19}} = 10^{18}$ photons /s.

b) The quantum efficiency: $\eta = \dfrac{n}{N} = \dfrac{10^{16}}{10^{18}} = 0.01 = 1\%$.

Part C

According to the wave theory, the wave gives energy to the illuminated surface progressively and continuously. This means that whatever the frequency of the incident radiation, a continuous illumination of the metal should produce photoelectric effect.

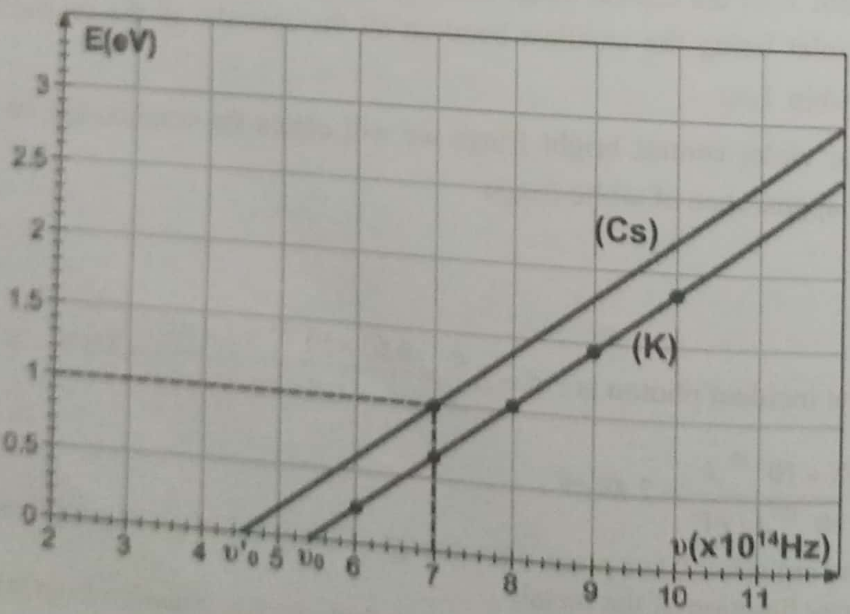
II-GS 2005 1st

Part A

1. The photoelectric effect shows evidence on the corpuscular aspect of light.
2. The properties of the photon are: massless; chargeless (electrically neutral); it travels in vacuum at the speed of $c = 3\times10^8\ m/s$;
(it carries an energy, proportional to its frequency, $E_{ph} = h\upsilon$)
3. In order to provoke the photoelectric emission:
   - the energy of an incident photon should be greater than or equal to the extraction energy W_0 of one electron.
   - Or, the frequency υ of the incident photon should be greater than or equal to the threshold frequency υ_0, relative to the pure metal.

Part B

1. According to Einstein relation $E_{ph} = W_0 + KE_{max}$ so $KE_{max} = h\upsilon - W_0$
This relation is of the form $KE = a\upsilon + b$ where $a = h$ and $b = -W_0$.
2. a) Graphical representation.



b) Graphical interpretation

i- Referring to relation derived in "1", h is the slope of the straight line representing $KE = f(\upsilon)$

$$h = \frac{\Delta(KE)}{\Delta\upsilon} = \frac{KE_2 - KE_1}{\upsilon_2 - \upsilon_1}\text{; so } h = \frac{(1.85 - 0.25)\times 1.6\times 10^{-19}\,J}{(10\times 10^{14} - 6\times 10^{14})Hz};$$

Then $h = 6.4\times 10^{-34}\,J.s$

ii- The threshold frequency υ_0, corresponds to an electron extracted without kinetic energy so for $\upsilon = \upsilon_0$, $KE = 0$.

It corresponds graphically to the abscissa of the point of intersection with the υ-axis (abscissa axis) then $\upsilon_0 = 5.4\times 10^{14}\,Hz$.

3. The extraction energy is given by: $W_0 = h\upsilon_0$.

Then, $W_0 = 6.4\times 10^{-34}\times 5.4\times 10^{14} = 3.456\times 10^{-19}\,J$.

Part C

1. Referring to Einstein relation $KE = h\upsilon - W_0'$.
The slope of the straight line is Planck's constant, which is unchanged, then from the point $(7\times 10^{14}\,Hz; 1eV)$ we construct a straight line parallel to that obtained in the first experiment.

2. The second straight line, intersects with the υ-axis at the point of abscissa $\upsilon_0' = 4.5\times 10^{14}\,Hz$.
Thus, $W_0' = h\upsilon_0' = 6.4\times 10^{-34}\times 4.5\times 10^{14} = 2.88\times 10^{-19}\,J$.

Unit IV

Atom & Nucleus

GS - Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Energy Levels of the Atom		2nd(B)	-	1st & 2nd	-	1st(A)	-	2nd	1st	1st
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Energy Levels of the Atom	1st	1st	-	1st & 2nd(A)	-	1st	1st	2nd(B)	-	1st

GS - Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Radioactivity		1st	2nd	-	-	1st(B)	1st(B)	-	-	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Radioactivity	-	2nd	1st(B) 2nd	-	2nd	2nd	2nd	1st	1st	1st

GS - Sessions	2020	2019	2018	2017	2016	2015	2014	2013	2012	2011
Fission & Fusion		2nd	1st	-	-	1st(A)	1st	1st	1st	-
	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Fission & Fusion	2nd 1st	-	1st	-	1st	1st	1st	2nd	2nd	1st

I-GS 2013 2nd
Photoelectric Effect

Given:

- $h = 6.62 \times 10^{-34} J.s$;
- $c = 3 \times 10^{8} m.s^{-1}$;
- $1eV = 1.6 \times 10^{-19} J$.

Part A
Emission of photoelectrons

Let W_0 be the minimum energy needed to extract an electron from the surface of a metal that covers the cathode of a photo cell and υ_0 the threshold frequency of this metal.

1. Define the threshold frequency υ_0.
2. Write the relation between W_0 and υ_0.
3. To determine W_0 and then the nature of the metal, we illuminate the cathode of the photo cell successively and separately each time with radiations of different frequencies and we determine the maximum kinetic energy KE_{max} of the emitted photoelectrons for each radiation of frequency υ. We obtain the results shown in table 1.

$\upsilon(\times 10^{14} Hz)$	5.5	6.2	6.9	7.5
$KE_{max}(eV)$	0.20	0.49	0.79	1.03

Table 1

a) Trace the curve representing the variations of KE_{max} in terms of υ.

Scale: on the horizontal axis: $1cm \equiv 10^{14} Hz$;
on the vertical axis: $1cm \equiv 0.20\, eV$.

b) Interpretation
   i- The obtained graph confirms with Einstein's relation concerning the photoelectric effect. Justify.
   ii- Name the physical constant that is represented by the slope of this graph.

c) Using the graph, determine the value of:
   i- this physical constant;
   ii- the threshold frequency υ_0.

d) Deduce the value of W_0.

e) Referring to table 2, indicate the nature of the metal used.

Metal	Cesium	Sodium	Potassium
$W_0(eV)$	2.07	2.28	2.30

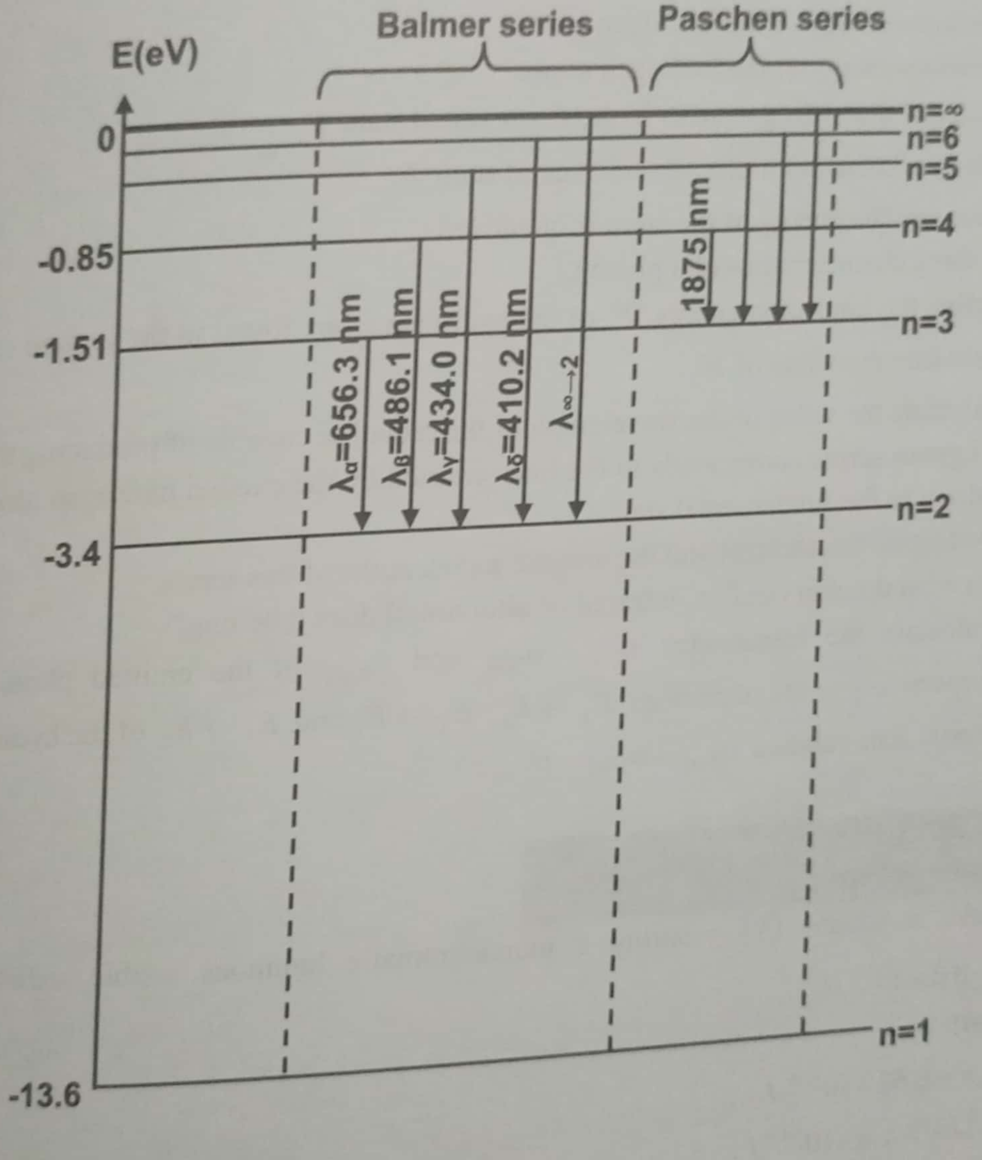
Table 2

Part B
The hydrogen atom

The spectral lines that constitute the hydrogen spectrum can be classified into many series; each series corresponds to the electronic transitions that lead to the same energy level. The figure below shows two of these series with the wavelengths of some emitted radiations.

The radiations emitted by the hydrogen gas lamp illuminates the cesium cathode of a photocell.

1. Consider the spectral line having the smallest wavelength of the Paschen series.
   a) To which transition does this line correspond?
   b) Deduce the energy of the corresponding emitted photon.
   c) Can the photons of the Paschen series extract photoelectrons from a cesium surface? Why?
2. Consider the spectral lines corresponding to the emitted radiations of wavelengths λ_α and λ_β of Balmer series.
   a) Referring to the energy diagram below, calculate the corresponding frequencies υ_α and υ_β.
   b) One of these two radiations can extract photoelectrons from the surface of cesium. Specify this radiation.



GS 2012 1st
Spectrum of the Hydrogen Atom

Rydberg found in 1885 an empirical formula that gives the wavelengths of the lines of Balmer series; other series are discovered after that date.

An atom in an excited state n, passes to a lower energy state m, emits electromagnetic rays of wavelengths λ, such that:

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right),\ \lambda \text{ in meter and } R = 1.097 \times 10^7 m^{-1}.$$

Given:

- Speed of light in vacuum: $c = 2.998 \times 10^8 \, m/s$;
- Planck's constant: $h = 6.626 \times 10^{-34} \, J.s$;
- $1eV = 1.60 \times 10^{-19} \, J$.

1. Show that the energy E_n of the hydrogen atom, corresponding to an energy level n, can be expressed as $E_n = -\frac{hcR}{n^2}$.

2. Deduce that the energy E_n, expressed in eV, may be written in the form $E_n = -\frac{13.6}{n^2}$.

3. Calculate the value of the:
   a) maximum energy of the hydrogen atom.
   b) minimum energy of the hydrogen atom.
   c) energy of the hydrogen atom in the first excited state E_2.
   d) energy of the atom in the second excited state E_3.
4. Deduce that the energy of the atom is quantized.
5. Give three characteristics of a photon.
6. a) Define the ionization energy W_i of the hydrogen atom, found in the ground state.
   b) Calculate the value of W_i.
   c) Calculate the value of the wavelength of the radiation capable of producing this ionization.
7. The Lyman series corresponds to the lines emitted by the excited hydrogen atom in a downward transition to the fundamental state.
   a) Determine the shortest and the longest wavelengths of this series.
   b) To what domain (visible, infrared or ultraviolet) does it belong?
8. a) Calculate the frequencies $\upsilon_{3\to1}$, $\upsilon_{2\to1}$ and $\upsilon_{3\to2}$ of the emitted photons corresponding respectively to the transitions $E_3 \to E_1$, $E_2 \to E_1$ and $E_3 \to E_2$ of the hydrogen atom.
   b) Verify Ritz relation $\upsilon_{3\to1} = \upsilon_{3\to2} + \upsilon_{2\to1}$.

III-GS 2011 1st
Aspects of Light

Consider a source (S) emitting a monochromatic luminous visible radiation of frequency $\upsilon = 6.163 \times 10^{14} \, Hz$.

Given:

- $h = 6.62 \times 10^{-34} \, J.s$;
- $1eV = 1.6 \times 10^{-19} \, J$;
- $c = 3 \times 10^8 \, m/s$.

First aspect of light

Part A

This source illuminates a very thin slit that is at a distance of $10 \, m$ from a screen. A pattern, extending over a large width, is observed on the screen.

1. Due to what phenomenon is the formation of this pattern?

2. Determine the width[1] of the slit knowing that the linear width of the central fringe is $40\ cm$.

Part B

Interference

The same source illuminates now the two slits of Young's double slit apparatus; these slits are vertical and are separated by a distance $a = 1\ mm$. A pattern is observed on a screen placed parallel to the plane of the slits at a distance $D = 2\ m$ from this plane.

1. Describe the observed pattern and calculate the interfringe distance i.

Part C

What aspect of light does the preceding experiments show evidence of?

Second aspect of light

Part A

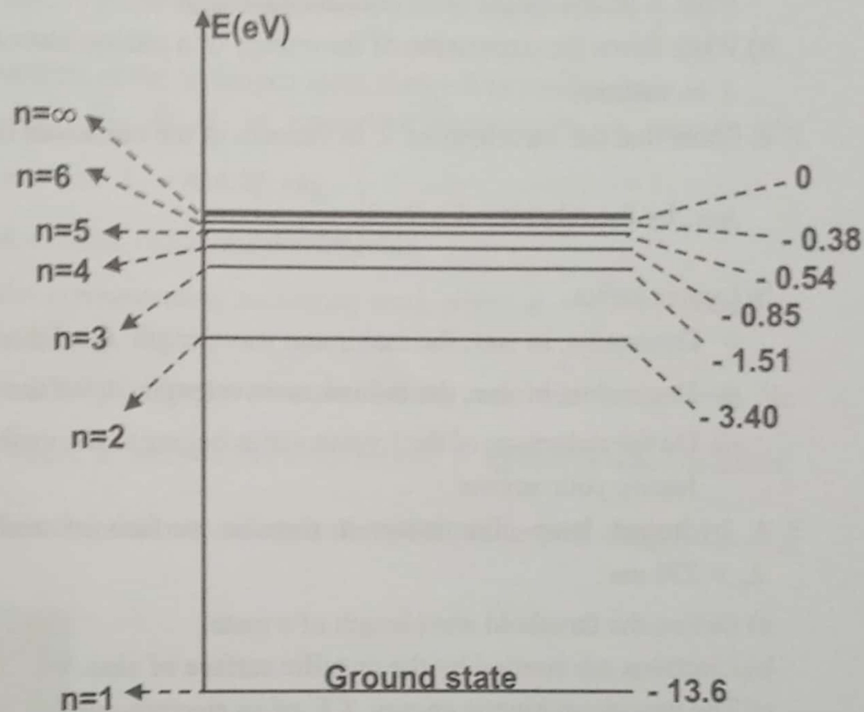
A luminous beam emitted by (S) falls on a cesium plate whose extraction energy is $W_0 = 1.89\ eV$.

1. a) Calculate the threshold frequency of cesium.
   b) Deduce that the plate will emit electrons.
2. Determine the maximum kinetic energy of an emitted electron.

Part B

The adjacent figure represents the energy diagram of a hydrogen atom.

The energy of the hydrogen atom is given by: $E_n = -\frac{13.6}{n^2}$ where E_n is expressed in eV and n is a non-zero positive integer.



1. A hydrogen atom, in its ground state, receives a photon from (S). This photon is not absorbed. Why?
2. The hydrogen atom, found in its first excited state, receives a photon from (S).
   This photon is absorbed and the atom thus passes to a new excited state.
   a) Determine this new excited state.
   b) The atom undergoes a downward transition. Specify the transition that may result in the emission of the visible radiation whose wavelength is the largest.

Part C

What aspect of light do the parts A and B show evidence of?

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[1] The expression of the width of the central should be justified with a geometrical clarification.

IV-GS 2010 1st
Energy Levels of the Hydrogen Atom

The energy levels of the hydrogen atom are given by the relation: $E_n = -\frac{13.6}{n^2}$ where E_n is expressed in eV and n is a non-zero whole number.

Given:

- Planck's constant: $h = 6.62 \times 10^{-34}\ J.s$;
- $1eV = 1.6 \times 10^{-19}\ J$;
- $1nm = 10^{-9}\ m$;
- Visible spectrum in vacuum: $400\ nm \leq \lambda \leq 750\ nm$;
- Speed of light in vacuum $c = 3 \times 10^{8}\ m/s$.

The Lyman series represents the set of radiations emitted by the hydrogen atom when it undergoes a downward transition from the level $n \geq 2$ to the ground state $n = 1$.

1. a) The energy of the hydrogen atom is said to be quantized.
   What is meant by the term «quantized energy»?
   b) Write down the expression of the energy of a photon associated with a radiation of wavelength λ in vacuum.
2. a) Show that the wavelengths λ in vacuum of the radiations of the Lyman series are expressed, in nm, by the relation: $\lambda = 91.3\left(\frac{n^2}{n^2-1}\right)$.
   b) Lyman series.
   i- Determine, in nm, the maximum wavelength λ_1 of the radiation of the Lyman series.
   ii- Determine, in nm, the minimum wavelength λ_2 of the radiation of the Lyman series.
   iii- Do the radiations of the Lyman series belong to the visible, ultraviolet or infrared spectrum? Justify your answer.
3. A hydrogen lamp illuminates a metallic surface of zinc whose threshold wavelength is $\lambda_0 = 270\ nm$.
   a) Define the threshold wavelength of a metal.
   b) Electrons are emitted by the metallic surface of zinc. Why?
   c) The maximum kinetic energy KE of an electron emitted by a radiation of the Lyman series is included between the values a and b: $a \leq KE \leq b$.
   Determine, in eV, the values of a and b.
   d) The maximum kinetic energy of these emitted electrons is quantized. Why?

V-GS 2009 1st
Energy Levels of the Hydrogen Atom

The energies of the various levels of the hydrogen atom are given by the relation: $E_n = -\frac{E_0}{n^2}$, where E_0 is a positive constant and n is a positive whole number.

Given

- $1e = 1.6\times10^{-19}\ C$;
- Planck's constant: $h = 6.62\times10^{-34}\ J.s$;
- Speed of light in vacuum: $c = 3\times10^{8}\ m/s$;
- $1nm = 10^{-9}\ m$.

1. a) The energy of the hydrogen atom is quantized. Explain the meaning of «*quantized energy*».
   b) Explain why the absorption or emission spectrum of hydrogen consists of lines.
2. A hydrogen atom, initially excited, undergoes a downward transition from the energy level E_2 to the energy level E_1. It then emits the radiation of wavelength in vacuum $\lambda_{2\to1} = 1.216\times10^{-7}\ m$.
   Determine, in eV, the value:
   a) of the constant E_0;
   b) of the ionization energy of the hydrogen atom taken in its ground state.
3. For hydrogen, we define several series that are named after the researchers who contributed in their study. Among these series we consider that of Balmer, which is characterized by the downward transitions from the energy level $E_p > E_2$ $(p>2)$ to the energy level $E_2 (n=2)$.
   To each transition $p \longrightarrow 2$ corresponds a line of wave $\lambda_{p\to2}$.
   a) Show that $\lambda_{p\to2}$, expressed in nm, is given by the relation: $\frac{1}{\lambda_{p\to2}} = 1.096\times10^{-2}\left(\frac{1}{4}-\frac{1}{p^2}\right)$.
   b) The analysis of the emission spectrum of the hydrogen atom shows four visible lines.
   We consider the three lines H_α, H_β & H_γ of respective wavelengths in vacuum are $\lambda_\alpha = 656.28\ nm$, $\lambda_\beta = 486.13\ nm$ and $\lambda_\gamma = 434.05\ nm$.
   Indicate to which transition each of these radiations corresponds.
   c) Show that the wavelengths of the corresponding radiations tend, when $p \longrightarrow \infty$, towards a limit λ_0 whose value is to be calculated.
4. Balmer, in 1885, knew only the lines of the hydrogen atom that belong to the visible spectrum. He wrote the formula: $\lambda = K\frac{p^2}{p^2-4}$, where K is a positive constant and p a positive whole number.
   Determine the value of K using the numerical values and compare its value with that of λ_0.

VI-GS 2007 2nd Tripoli
The Orion Nebula

The great Orion Nebula is composed of four very hot stars emitting ultraviolet radiation whose wavelength in vacuum is ***less than*** $91.2\,nm$, within a large «cloud» of interstellar gas formed mainly of hydrogen atoms.

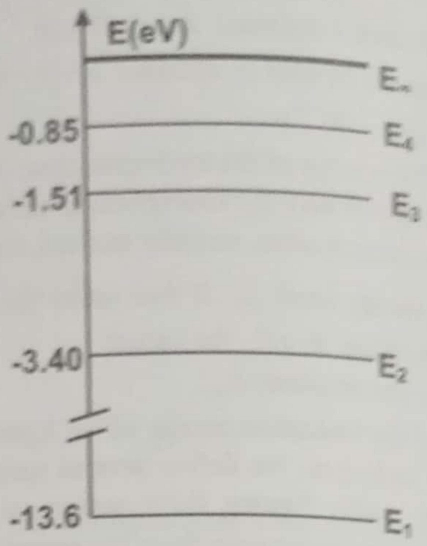
The diagram below represents some of the energy levels E_n of the hydrogen atom.

Given

- $1eV = 1.602\times10^{-19}\ J$;
- Planck's constant: $h = 6.626\times10^{-34}\ J.s$;
- Speed of light in vacuum: $c = 2.998\times10^{8}\ m/s$;
- Spectrum of rosy color: $640\ nm \le \lambda \le 680\ nm$; visible spectrum: $400\ nm \le \lambda \le 800\ nm$.

Part A

1. By convention, the energy of the hydrogen atom in the ionized state is considered zero.
   Use this convention to justify the sign (–) of E_n.
2. The hydrogen atom is in its fundamental (ground state).
   a) Show that the minimum energy needed to ionize this atom is equal to $E_i = 2.178 \times 10^{-18}\ J$.
   b) Calculate the wavelength λ_i of the wave associated to the photon whose energy is equal to E_i.
   c) Show that the light, emitted by the very hot stars in the Orion Nebula, can ionize the hydrogen atoms of the interstellar gas.
   Specify the dynamic state of these extracted electrons.



Part B

The interstellar gas in the Orion Nebula being ionized, some extracted electrons are captured by protons at rest (ionized hydrogen atoms) to form hydrogen atoms in an excited state. An excited hydrogen atom undergoes then a progressive downward transition.

1. Color of the Orion Nebula

Out of the possible transitions, we consider the transition of the atom from level 3 to level 2.

a) Calculate the wavelength, in vacuum, of the radiation corresponding to this transition.

b) This radiation is visible. Why?

c) Justify then the rosy color of the Nebula.

2. Maximum temperature on the surface of the Orion Nebula

The electron before it is captured by the hydrogen ion H^+ has a kinetic energy KE. The total energy of the system (ion + electron) : $E = 0 + KE$ is conserved.

When the atom undergoes a downward transition, after capturing the electron, it passes to an excited state characterized by its energy level E_n, by emitting a photon of frequency υ so that: $KE = E_n + h\upsilon$.

a) Show that for $n = 2$, we have : $\upsilon = \dfrac{KE}{h} + 8.22 \times 10^{14}$. ($\upsilon$ in Hz).

b) The average kinetic energy of the electrons is related to the temperature of the surface of the star by: $KE = \dfrac{3}{2}kT$, $(k = 1.38 \times 10^{-23}\ SI)$; and T is the temperature in Kelvin.

We notice that the smallest wavelength, in the emission band of rays of the Orion Nebula is $\lambda = 245\ nm$ in vacuum.

i- Show that this ray corresponds to KE_{max}. Calculate KE_{max}.

ii- Deduce the maximum value of T.

Ionization Energy

Given:

- $1eV = 1.6\times10^{-19}J$;
- Planck's constant: $h = 6.62\times10^{-34}J.s$;
- Speed of light in vacuum: $c = 3\times10^{8}m/s$.

The object of this exercise is to compare the ionization energy of the hydrogen atom with that of the helium ion He^{+} and that of the lithium ion Li^{2+} each having only one electron in the outermost shell.

The quantized energy levels of each are given by the expression $E_n = -\frac{E_0}{n^2}$ where E_0 is the ionization energy and n is a non-zero positive whole number.

Part A
Interpretation of the existence of spectral lines

1. Due to what is the presence of emission spectral lines of an atom or an ion?
2. Explain briefly the term «quantized energy levels».
3. Is a transition from an energy level m to another energy level p $(p < m)$ a result of an absorption or an emission of a photon? Why?

Part B
Atomic spectrum of hydrogen

For the hydrogen atom $E_0 = 13.6\ eV$.

1. A hydrogen atom, found in its ground state, interacts with a photon of energy $14\,eV$.
   a) Why?
   b) A particle is thus liberated. Give the name of this particle and calculate its kinetic energy.
2. a) Show that the expression of the wavelengths λ of the radiations emitted by the hydrogen atom is: $\frac{1}{\lambda} = R_1\left(\frac{1}{p^2} - \frac{1}{m^2}\right)$ where m and p are two positive whole numbers so that $m > p$ and R_1 is a positive constant to be determined in terms of E_0, h & c.
   b) Verify that $R_1 = 1.096\times10^{7}\ m^{-1}$.

Part C
Atomic spectrum of the helium ion He⁺

The spectrum of the ion He^{+} is formed, in addition to others, of two lines whose corresponding reciprocal wavelengths $\frac{1}{\lambda}$ are: $3.292\times10^{7}\ m^{-1}$; $3.901\times10^{7}\ m^{-1}$ respectively.

These lines correspond, respectively, to the transitions: $(m=2 \longrightarrow p=1)$ and $(m=3 \longrightarrow p=1)$.

1. a) Show that the values of $\frac{1}{\lambda}$ satisfy the relation $\frac{1}{\lambda} = R_2\left(\frac{1}{p^2} - \frac{1}{m^2}\right)$ where R_2 is a positive constant.
   b) Deduce that $R_2 = 4.389\times10^{7}\ m^{-1}$.

2. Determine a relation between R_2 and R_1.

Part D

Atomic spectrum of the lithium ion Li^{2+}

Also, the ion Li^{2+} may emit radiations whose wavelengths λ are given by: $\frac{1}{\lambda} = R_3\left(\frac{1}{p^2} - \frac{1}{m^2}\right)$ where m and p are two positive whole numbers so that $m > p$ and $R_3 = 9.860 \times 10^7 m^{-1}$.

Determine a relation between R_3 and R_1.

Part E

Charge number and ionization energy

The charge numbers Z of the elements hydrogen, helium and lithium are respectively 1, 2 and 3.

Compare the ionization energy of the hydrogen atom with that of He^+ ion and that of Li^{2+} ion.

Conclude.

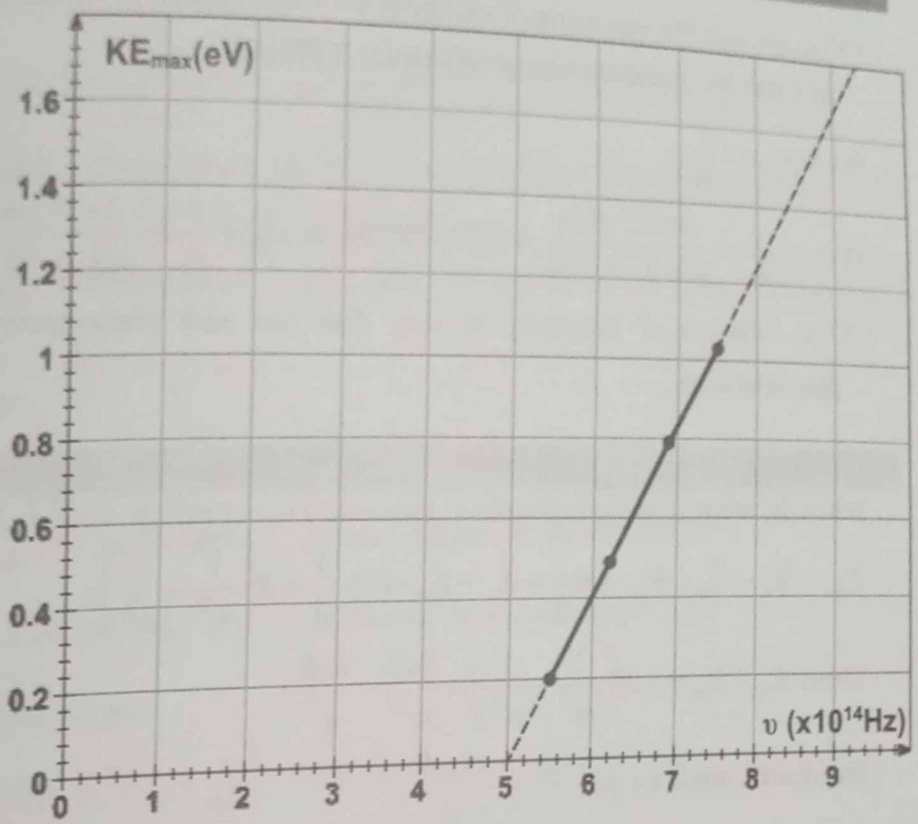
GS 2013 2nd

Part A

1. The threshold frequency υ_0 is the minimum frequency of the incident radiation required to extract an electron from the surface of a metal.

2. $W_0 = h\upsilon_0$.

3. a) Graph.



b) Graphical study:

i- According to Einstein relation $E_{ph} = W_0 + KE_{\max}$; $KE_{\max} = E_{ph} - W_0 = h\upsilon - h\upsilon_0$ is of the form $f(\upsilon) = a\upsilon + b$ so its graphical representation is a:

- straight line (the variable υ is of degree 1).
- increasing (the slope $a = h > 0$).
- not passing through origin $b = -W_0 \neq 0$.

ii- The slope $a = h$ is called Planck's constant.

c) Interpretation.

i- The slope $h = \dfrac{\Delta(KE_{\max})}{\Delta\upsilon} = \dfrac{(1.03 - 0.2)1.6\times10^{-19}\,J}{(7.5-5.5)\times10^{14}\,s^{-1}} = 6.64\times10^{-34}\,J.s$.

ii- Graphically, υ_0 is the abscissa of the point of intersection of the prolongation of the straight line with the abscissa axis $\upsilon_0 \approx 5\times10^{14}\,Hz$. [2]

d) The work function $W_0 = h\,\upsilon_0 = 6.62\times10^{-34}(J\,s)\times5\times10^{14}\,Hz = 3.32\times10^{-19}\,J$.

e) We have $W_0 = 3.32\times10^{-19}\,J = \dfrac{3.32\times10^{-19}}{1.6\times10^{-19}} = 2.075\,eV$;

The metal used is the Cesium.

---

[2] We have $KE_{\max} = h(\upsilon - \upsilon_0)$; then $\upsilon_0 = \upsilon - \dfrac{KE_{\max}}{h} = 5.5\times10^{14} - \dfrac{0.20\times1.6\times10^{-19}}{6.62\times10^{-34}} = 5.02\times10^{14}\,Hz$.

Pay attention that it is wrong to take the threshold frequency $5.5\times10^{14}Hz$, because for this particular frequency, the kinetic energy is not zero even if it is the first value listed on table.

Part B

1. a) The smallest wavelength of the Paschen series, corresponds to the photon emitted through to the farthest possible transition towards $n=3$.

Then it corresponds to the transition from ∞ to $n=3$.

b) The energy of the photon emitted is $E_{ph}=E_{\infty}-E_3=0-(-1.51)=1.51\,eV$.

c) No, because the maximum of energy of a photon emitted in Paschen series is $1.51\,eV$ which is less than the extraction energy of cesium $2.075\,eV$.

2. We have $\upsilon=\frac{c}{\lambda}$.

a) $\upsilon_\alpha=\frac{c}{\lambda_\alpha}=\frac{3\times10^8\,m/s}{656.3\times10^{-9}\,m}=4.57\times10^{14}\,Hz$ & $\upsilon_\beta=\frac{c}{\lambda_\beta}=\frac{3\times10^8\,m/s}{486.1\times10^{-9}\,m}=6.17\times10^{14}\,Hz$.

b) The radiation of frequency $\upsilon_\alpha<\upsilon_0$ does not emit photoelectrons; while $\upsilon_\beta>\upsilon_0$ emits photoelectrons.

II-GS 2012 1st

1. When the atom in an excited state n, passes to a lower energy state m, it emits a photon of energy:

$$E_{ph}=E_n-E_m;\ E_{ph}=h\frac{c}{\lambda}=E_n-E_m \text{ with } \frac{1}{\lambda}=R\left(\frac{1}{m^2}-\frac{1}{n^2}\right);$$

Then: $E_n-E_m=hcR\left(\frac{1}{m^2}-\frac{1}{n^2}\right)=\frac{hcR}{m^2}-\frac{hcR}{n^2}$;

By identification we get: $E_n=-\frac{hcR}{n^2}$.

2. The value of $hcR=6.626\times10^{-34}(J.s)\times2.998\times10^8(m/s)\times1.097\times10^7\,m^{-1}=2.179\times10^{-18}\,J$;

and in electron Volt: $hcR=\frac{2.179\times10^{-18}}{1.60\times10^{-19}}=13.6\,eV$;

Thus, $E_n=-\frac{hcR}{n^2}=-\frac{13.6}{n^2}$ (E_n in eV).

3. a) The maximum energy is when the atom is in the ionized state $n\to+\infty$; so $E_{max}=-\frac{13.6}{\infty}=0$.

b) The minimum energy is when it is in the ground state $n=1$, so $E_{min}=-\frac{13.6}{1^2}=-13.6\,eV$.

c) In the first excited state $n=2$, so $E_2=-\frac{13.6}{2^2}=-3.4\,eV$.

d) In the second excited state $n=3$, so $E_3=-\frac{13.6}{3^2}=-1.51\,eV$.

4. Since only specific values are allowed $(-13.6;-3.4;-1.51;.....)$ then the energy of the atom is quantified.

5. The characteristics of a photon are:
- massless (no mass);
- neutral (no charge);
- travels in vacuum at the speed of light;
- carrying an energy proportional to its frequency.

6. a) The ionization energy W_{ion} is the minimum energy required to extract the electron from the atom (without kinetic energy).

b) The ionization energy $W_{ion} = E_{\infty} - E_1 = 0 - (-13.6) = 13.6eV$.

c) The ionization energy is carried by the incident photon:[3]

$E_{ph} = W_{ion} = 13.6eV = 13.6 \times 1.6 \times 10^{-19} = 2.176 \times 10^{-18} J$;

However $E_{ph} = h\dfrac{c}{\lambda_i} \Rightarrow \lambda_i = \dfrac{hc}{E_{ph}} = \dfrac{6.626 \times 10^{-34} J.s \times 2.998 \times 10^{8} m/s}{2.176 \times 10^{-18} J} = 9.13 \times 10^{-8} m$.

7. a) The shortest wavelength corresponds to the transition between the farthest energy levels, from $n = \infty$, into $m = 1$; then: $\lambda_{min} = \lambda_i = 9.13 \times 10^{-8} m = 91.3nm$.

The longest wavelength corresponds to the transition between the nearest energy levels $n = 2$, into $m = 1$; then: $\dfrac{1}{\lambda_{max}} = R\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)$;

$$\lambda_{max} = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^{7} m^{-1}} = 1.22 \times 10^{-7} m = 122 nm .$$

b) $\lambda_{max} = 122nm$ & $\lambda_{min} = 91.3nm < 400\ nm$; so these radiations are ultraviolet.

8. a) $\upsilon_{3\to2} = \dfrac{c}{\lambda_{3\to2}} = 1.097 \times 10^{7} m^{-1}\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) \times 2.998 \times 10^{8} m/s = 4.58 \times 10^{14} Hz$;

By a similar calculation, we get:

$$\upsilon_{3\to1} = \frac{c}{\lambda_{3\to1}} = 1.097 \times 10^{7} m^{-1}\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \times 2.998 \times 10^{8} m/s = 2.93 \times 10^{14} Hz ;$$

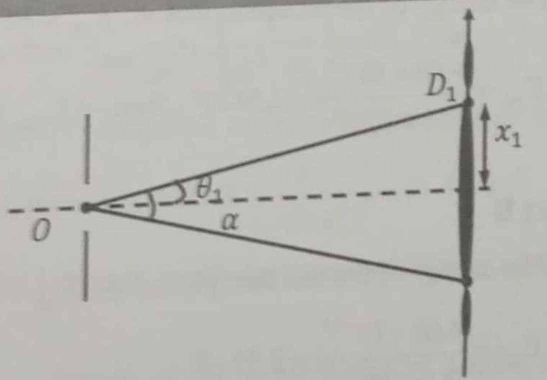
$$\upsilon_{2\to1} = \frac{c}{\lambda_{2\to1}} = 1.097 \times 10^{7} m^{-1}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \times 2.998 \times 10^{8} m/s = 2.47 \times 10^{15} Hz .$$

b) $\upsilon_{3\to2} + \upsilon_{2\to1} = 4.58 \times 10^{14} + 2.47 \times 10^{15} = 2.928 \times 10^{15} = \upsilon_{3\to1}$; thus Ritz's relation is verified.[4]

II-GS 2011 1st

First aspect

Part A



1. Diffraction of light.
2. The angular position (abscissa) of first dark point is defined by $\sin\theta_1 = 1\dfrac{\lambda}{a}$ and for small angles $\sin\theta_1 \approx \theta_1 = \dfrac{\lambda}{a}$.

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[3] We can use also the formula given $\dfrac{1}{\lambda} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$ where $n \to \infty$ & $m = 1$;

Then $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1} - 0\right) \Rightarrow \lambda = \dfrac{1}{R} = \dfrac{1}{1.097 \times 10^{7} m^{-1}} = 9.11 \times 10^{-8} m$;

[4] Ritz's relation: $E_3 - E_1 = h\upsilon_{3\to1}$; $(E_3 - E_2) + (E_2 - E_1) = h\upsilon_{3\to1} \Rightarrow h\upsilon_{3\to2} + h\upsilon_{2\to1} = h\upsilon_{3\to1}$;

Thus, $\upsilon_{3\to2} + \upsilon_{2\to1} = \upsilon_{3\to1}$. It can be derived also as a function of wavelength $\dfrac{1}{\lambda_{3\to1}} = \dfrac{1}{\lambda_{3\to2}} + \dfrac{1}{\lambda_{2\to1}}$

According to the geometry of the figure, where ℓ is the linear width of the central fringe and for small angles, also $\tan\theta_1 = \frac{\ell/2}{D} = \frac{\ell}{2D} \approx \theta_1$.

By equating the two previous results, we get $\theta_1 \approx \frac{\lambda}{a} = \frac{\ell}{2D} \Rightarrow \ell = 2\frac{\lambda D}{a}$.

The wavelength λ is given by $\lambda = \frac{c}{\upsilon} = \frac{3\times10^8}{6.163\times10^{14}} = 0.4868\ \mu m$.

The width of the slit used is $a = 2\frac{\lambda D}{\ell} = 24\,\mu m$

Part B

In the zone of interference, we observe:

- a central bright fringe at O;
- alternate, equidistant, straight bright and dark fringes.

The interfringe distance is given by: $i = \frac{\lambda D}{a} = \frac{0.4868\times10^{-6}\times2}{1\times10^{-3}} = 0.9736\ mm$.

Part C

The previous experiments (diffraction & interference) shows evidence the wave aspect of light.

Second aspect

Part A

1. a) The threshold frequency is given by:

$$W_0 = h\upsilon_0 \Rightarrow \upsilon_0 = \frac{W_0}{h} = \frac{1.89\times1.6\times10^{-19}\,J}{6.62\times10^{-34}\,J.s} = 4.568\times10^{14}\,Hz.$$

b) We have $\upsilon = 6.163\times10^{14}\,Hz > \upsilon_0$; then the photoelectric effect takes place.

2. According to the conservation of energy (Einstein relation):

$$E_{ph} = W_0 + KE_{max} \Rightarrow KE_{max} = E_{ph} - W_0.$$

The energy of the incident photon: $E_{ph} = h\upsilon = 6.62\times10^{-34}\,J.s\times6.163\times10^{14}\,Hz = 4.08\times10^{-19}\,J$;

$E_{ph} = \frac{4.08\times10^{-19}}{1.6\times10^{-19}} = 2.54\,eV$; then, $KE_{max} = E_{ph} - W_0 = 2.54 - 1.89 = 0.65\,eV$.

Part B

1. The energy of the incident photon is: $E_{ph} = h\upsilon = 6.62\times10^{-34}\times6.163\times10^{14} = 4.08\times10^{-19}\,J$.

$E_{ph} = \frac{4.08\times10^{-19}}{1.6\times10^{-19}} = 2.55\,eV$.

$E_{ph} + E_1 = 2.55eV + (-13.6eV) = -11.05\,eV \in \left]E_1; E_2\right[\neq E_n$. This photon is not absorbed.

2. a) The 1st excited state corresponds to $n=2$, then $E_{ph} + E_2 = 2.55 + (-3.4) = -0.85\,eV = E_4$. Then this photon is absorbed and the atom undergoes an upwards transition towards 3rd excited state.

b) The visible radiation corresponds to a transition towards $n=2$ (Balmer series). The atom is found in the state $n=4$, so two transitions are possible $3 \longrightarrow 2$ & $4 \longrightarrow 2$.

The radiation whose wavelength is the largest corresponds to a transition between the nearest energy levels $3 \longrightarrow 2$, so $E_{ph} = E_n - E_2 = h\frac{c}{\lambda}$.

Then, $\lambda_{max} = \frac{hc}{E_3 - E_2} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{[-1.51 - (-3.40)] \times 1.6 \times 10^{-19}} = 6.56 \times 10^{-7} m = 0.656\ \mu m$.

Part C

The previous experiments (photoelectric effect and emission of light) shows evidence of the corpuscular aspect of light.

IV-GS 2010 1st

1. a) The expression giving the energy of the energy levels depends on a whole number n, then they are quantified.

b) The energy of a photon $E = h\frac{c}{\lambda}$.

2. a) During a downwards transition towards $n = 1$ the energy of the photon emitted is :

$$E_{ph} = E_n - E_1 = \frac{-13.6}{n^2} + \frac{13.6}{1^2} = 13.6\left(\frac{n^2 - 1}{n^2}\right) = 13.6 \times 1.6 \times 10^{-19}\left(\frac{n^2 - 1}{n^2}\right)$$

$$E_{ph}(J) = 2.176 \times 10^{-18}\left(\frac{n^2 - 1}{n^2}\right) = h\frac{c}{\lambda};$$

$$\lambda = \frac{hc}{2.176 \times 10^{-18}}\left(\frac{n^2}{n^2 - 1}\right) = 9.13 \times 10^{-8}\left(\frac{n^2}{n^2 - 1}\right) \text{ where } \lambda \text{ in } m;$$

Then $\lambda = 9.13 \times 10^{-8}\left(\frac{n^2}{n^2 - 1}\right) \times 10^9 = 91.3\left(\frac{n^2}{n^2 - 1}\right)$ where λ in nm

b) Lyman series.

i- The maximum wavelength (less energetic photon) corresponds to the transition between the nearest energy levels (from $n = 2$ into $n = 1$), then $\lambda_{max} = 91.3\left(\frac{2^2}{2^2 - 1}\right) = 121.7\ nm$.

ii- The minimum wavelength (most energetic photon) corresponds to the transition between the farthest energy levels (from $n = \infty$ into $n = 1$), then $\lambda_{min} = 91.3\left(\frac{1}{1 - 1/n^2}\right) = 91.3\ nm$.[5]

iii- $\lambda_{Lyman} \in]91.3\ nm; 121.7\ nm[< 400\ nm$; then the Lyman's series spectrum belongs to ultra-violet domain.

3. a) The threshold wavelength λ_0 is the maximum wavelength of the incident photon needed to extract an electron from the surface of the metal.

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[5] It is practical to use some mathematical tools to find the limit $\lambda = 91.3\left(\frac{n^2}{n^2 - 1}\right)$ as $n \longrightarrow \infty$;

In fact $\lambda = 91.3\frac{n^2(1)}{n^2\left(1 - \frac{1}{n^2}\right)} = 91.3\frac{1}{1 - \frac{1}{n^2}}$ & as $n \longrightarrow \infty$; $\frac{1}{n^2} \longrightarrow 0$ so $1 - \frac{1}{n^2}$; then $\lambda \longrightarrow 91.3$.

b) For the hydrogen lamp $\lambda_{Lyman} \in]91.3\,nm; 121.7\,nm[< \lambda_0 = 270\,nm$.

c) According to the conservation of energy: $KE = E_{ph} - W_0 = h\frac{c}{\lambda} - h\frac{c}{\lambda_0}$.

$$KE_{max} = h\frac{c}{\lambda_{min}} - h\frac{c}{\lambda_0} = hc\left(\frac{1}{\lambda_{min}} - \frac{1}{\lambda_0}\right);$$

$$KE_{max} = 6.62\times10^{-34}\times3\times10^{8}\left(\frac{1}{91.3\times10^{-9}} - \frac{1}{270\times10^{-9}}\right) = 1.44\times10^{-19}\,J \approx 9eV = b;$$

$$KE_{min} = h\frac{c}{\lambda_{max}} - h\frac{c}{\lambda_0} = hc\left(\frac{1}{\lambda_{max}} - \frac{1}{\lambda_0}\right);$$

$$KE_{min} = 6.62\times10^{-34}\times3\times10^{8}\left(\frac{1}{121.7\times10^{-9}} - \frac{1}{270\times10^{-9}}\right) = 8.69\times10^{-19}\,J = 5.6eV = a;$$

Thus $a = 5.6eV \le KE \le 9eV = b$.[6]

d) Basing on Einstein's relation: $KE = E_{ph} - W_0$, but the energies of the photons are quantified (due to the transitions).

Then the obtained values of the kinetic energy are discontinuous so they are quantified.

V-GS 2009 1st

1. a) The energies of the energy levels hydrogen atom depends on n which is a whole number, and then it can take only well-defined values (discrete). Therefore, it is called quantized.

b) For an electronic transition $p \longrightarrow n$ the emitted photon (or absorbed) has a wavelength: $\lambda_{p,n} = \frac{hc}{E_p - E_n}$, but E_p and E_n are quantized then the difference $(E_p - E_n)$ is quantized too; which means that the $\lambda_{p,n}$ has a well determined values, which corresponds to a set of lines.

2. a) The energy of the photon emitted during this transition is:

$$E_{ph(2,1)} = h\frac{c}{\lambda_{2,1}} = \frac{6.62\times10^{-34}\times3\times10^{8}}{1.216\times10^{-7}} = 1.63\times10^{-18}\,J = \frac{1.63\times10^{-18}}{1.6\times10^{-19}} = 10.19eV.$$

But $E_{ph(2,1)} = E_2 - E_1 = -\frac{E_0}{2^2} - \left(-\frac{E_0}{1}\right) = -\frac{E_0}{4} + E_0 = \frac{3}{4}E_0;$

Then $E_0 = \frac{4}{3}E_{ph(2,1)} = \frac{4}{3}(10.19) = 13.58eV \approx 13.6eV$.

b) The ionization energy $W_0 = E_\infty - E_1 = 0 - (-E_0) = E_0 = 13.6eV$.

3. a) For a given transition we have: $\frac{hc}{\lambda_{p\to2}} = E_p - E_2 = -\frac{E_0}{p^2} + \frac{E_0}{4} = E_0\left(\frac{1}{4} - \frac{1}{p^2}\right);$

$$\frac{1}{\lambda_{p\to2}} = \frac{E_0}{hc}\left(\frac{1}{4} - \frac{1}{p^2}\right) = \frac{13.6\times1.6\times10^{-19}}{6.62\times10^{-34}\times3\times10^{8}}\left(\frac{1}{4} - \frac{1}{p^2}\right) = 1.096\times10^{7}\left(\frac{1}{4} - \frac{1}{p^2}\right);$$

Then $\frac{1}{\lambda_{p\to2}} = 1.096\times10^{-2}\left(\frac{1}{4} - \frac{1}{p^2}\right)$ where $\lambda_{p\to2}$ in nm.

[6] The same result can be derived basing on the framing of wavelengths $91.3\times10^{-9}\,m \le \lambda \le 121.7\times10^{-9}\,m$;

b) $\lambda_\alpha = 656.28\ nm \Rightarrow p = 3$, then it corresponds to the downward transition $3 \longrightarrow 2$

$\lambda_\beta : 4 \longrightarrow 2$ and $\lambda_\gamma : 5 \longrightarrow 2$.

c) When $p \longrightarrow \infty$, $\lambda \to \lambda_0 = \dfrac{4}{1.096 \times 10^{7}} = 364.96\ nm$.

For $\lambda_\alpha = 656.28\ nm \Rightarrow p = 3$; $K = \lambda \dfrac{p^2 - 4}{p^2} = 364.6\ nm \Rightarrow K \cong \lambda_0$.

GS 2007 2nd Tripoli

Part A

1. To ionize the hydrogen atom, taken in a state of energy E_n, it must be provided by an energy W such that: $E_n + W = 0$ or W is positive then E_n must be negative.

2. a) The minimal energy needed is that which corresponds to an electron at rest:

$E_i = E_\infty - E_1 = -E_1 = 13.6\,eV = 13.6 \times 1.6 \times 10^{-19} = 2.178 \times 10^{-18}\ J$.

b) We have: $\lambda_i = \dfrac{hc}{E_i} = \dfrac{6.626 \times 10^{-34} \times 2.998 \times 10^{8}}{2.178 \times 10^{-18}} = 91.24 \times 10^{-9}\ m = 91.24\ nm$.

c) Since the wavelength λ of the radiant light for the hot stars is smaller than λ_i.

Then $E > E_i$ and the hydrogen atoms of the interstellar gas are ionized and the electrons are extracted with a kinetic energy.

Part B

1. a) The wavelength $\lambda_{3,2}$ emitted through this transition is :

$$\lambda_{3,2} = \frac{hc}{E_3 - E_2} = \frac{6.626 \times 10^{-34} \times 2.998 \times 10^{8}}{[-3.4 - (-13.6)] \times 1.602 \times 10^{-19}} = 6.563 \times 10^{-7}\ m = 656.3\ nm.$$

b) Yes, it is visible since $400\,nm \le \lambda_{3,2} \le 750\,nm$.

c) The radiation belongs to the rosy domain $640\,nm \le \lambda_{3,2} \le 680\,nm$.

2. a) Conservation of energy : $KE = E_2 + h\upsilon \Rightarrow \upsilon = \dfrac{KE}{h} - \dfrac{E_2}{h} = \dfrac{KE}{h} - \dfrac{(-3.4) \times 1.602 \times 10^{-19}}{6.626 \times 10^{-34}}$.

Then, $\upsilon = \dfrac{KE}{h} + 8.22 \times 10^{14}$ where KE in J & υ in Hz.

b) Kinetic energy

i- The minimum wavelength λ_{min} corresponds to the maximum frequency $\upsilon_{max} = \dfrac{c}{\lambda_{min}}$;

So the kinetic energy is maximum $KE_{max} = E_2 + h\dfrac{c}{\lambda_{min}}$;

$KE_{max} = -3.4 \times 1.602 \times 10^{-19} + \dfrac{6.626 \times 10^{-34} \times 2.998 \times 10^{8}}{245 \times 10^{-9}}$; then, $KE_{max} = 2.66 \times 10^{-19}\ J$.

ii- We have $KE_{max} = \dfrac{3}{2} k T_{max}$, then $T_{max} = \dfrac{2KE_{max}}{3k} = \dfrac{2 \times 2.66 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}} = 12850\ K$.

Part A

1. The presence of a ray (spectral line) in an emission spectrum is due to the emission of one photon of specific wavelength by the excited atom (or ion) when it undergoes a downward transition.
2. The energy levels of an atom can only have specific and discontinuous values; that is why these levels are called quantized energy levels.
3. When an atom (or ion) passes from the level m to another p, $p<m$, it releases energy. This energy released is carried by the emitted photon.

Part B

1. a) The ionization energy of the hydrogen atom, being $13.6eV$, is smaller than $14\,eV$. The photon is then absorbed by the hydrogen atom.

   b) The photon being absorbed, the atom is ionized and it thus liberates an electron of kinetic energy: $KE = E_{ph} - E_0 = 14eV - 13.6eV = 0.4\,eV$.

2. a) When a hydrogen atom passes from a level m to a level p, it emits a photon of wavelength λ

$$E_m - E_p = -\frac{E_0}{m^2}+\frac{E_0}{p^2}=\frac{hc}{\lambda}\text{; then } \frac{1}{\lambda}=R_1\left(\frac{1}{p^2}-\frac{1}{m^2}\right) \text{ with } R_1=\frac{E_0}{hc}.$$

   b) $R_1 = \dfrac{E_0}{hc}=\dfrac{13.6\times1.6\times10^{-19}}{6.62\times10^{-34}\times3\times10^{8}}=1.096\times10^{7}\,m^{-1}$.

Part C

1. a) For $p=1$ and $m=2$, $\dfrac{3.292\times10^{7}}{\left(\frac{1}{1^2}-\frac{1}{2^2}\right)}=4.389\times10^{7}\,m^{-1}$;

   For $p=1$ and $m=3$, $\dfrac{3.292\times10^{7}}{\left(\frac{1}{1^2}-\frac{1}{3^2}\right)}=4.389\times10^{7}\,m^{-1}$;

   Then the physical quantity $\dfrac{1}{\lambda\left(1/p^2-1/m^2\right)}$ is the same for both transitions.

   b) We have $R_2 = 4.389\times10^{7}\,m^{-1}$.

2. $\dfrac{R_2}{R_1}=\dfrac{4.389\times10^{7}}{1.096\times10^{7}}=4$.

Part D

$$\frac{R_3}{R_1}=\frac{9.860\times10^{7}}{1.096\times10^{7}}=9.$$

Part E

We have $R=\dfrac{E_0}{hc}$, so $R_1=\dfrac{(E_0)_1}{hc}$; $R_2=\dfrac{(E_0)_2}{hc}=\dfrac{4E_0}{hc}=Z^2\dfrac{E_0}{hc}$ & $R_3=\dfrac{(E_0)_3}{hc}=\dfrac{9E_0}{hc}=Z^2\dfrac{E_0}{hc}$.

The increase in the constant R is proportional to Z^2.

GS 2008 2nd
Half-Life of Polonium 84

Polonium 210 nucleus $^{210}_{84}Po$ is an α emitter; and it is the only polonium isotope that exists in nature; it was found by Pierre Curie in 1898 in an ore. It is also obtained from the decay of a bismuth 210 nucleus $^{210}_{83}Bi$.

Given masses of nuclei and particles:

- $m(Bi) = 209.938445\ u$;
- $m(Po) = 209.936648\ u$;
- Mass of the electron: $m_e = 5.5 \times 10^{-4}\ u$;
- $1u = 931.5\ MeV/c^2 = 1.66 \times 10^{-27}\ kg$.

Here is a part of the periodic table of elements: $_{81}Th$; $_{82}Pb$; $_{83}Bi$; $_{84}Po$; $_{85}At$; $_{86}Rn$.

Part A
The polonium 210

1. a) Write down the equation of the decay of bismuth 210.
   b) Identify the emitted particle and specify the type of decay.
2. Calculate the energy liberated by this decay.
3. The decay of the bismuth 210 nucleus is accompanied with the emission of a γ-photon of energy $E(\gamma) = 0.96\ MeV$ and an antineutrino of energy $0.02\ MeV$. Knowing that the daughter nucleus is practically at rest, calculate the kinetic energy of the emitted particle.

Part B
Half-life of polonium 210

1. a) Write down the equation of the decay of polonium 210.
   b) Identify the daughter nucleus.
2. In order to determine the radioactive period T (half-life) of $^{210}_{84}Po$, we consider a sample of this isotope containing N_0 nuclei at the instant $t_0 = 0$.
   Let N be the number of the non-decayed nuclei at an instant t.
   a) Write down the expression of the law of radioactive decay.
   b) Determine the expression of $-\ln\left(\frac{N}{N_0}\right)$ as a function of t.
3. A counter allows to obtain the measurements that are tabulated in the following table:

t(days)	0	40	80	120	160	200	240
$\frac{N}{N_0}$	1	0.82	0.67	0.55	0.45	0.37	0.30
$-\ln\left(\frac{N}{N_0}\right)$	0	....	0.4	...	0.8	...	1.2

a) Complete the table.

b) Trace, on the graph paper, the curve giving the variation of $-\ell n\left(\frac{N}{N_0}\right)$ as a function of time.

Scale: 1div on the abscissa axis corresponds to 40 days.

1div on the ordinate axis corresponds to 0.2.

c) Is this curve in agreement with the expression found in the question (B-2, b)? Justify.

d) Graphical study:

i- Calculate the slope of the traced curve.

ii- What does this slope represent for the polonium 210 nucleus?

iii-Deduce the value of T.

II-GS 2008 2nd
The Radionuclide Potassium 40

The isotope of potassium $^{40}_{19}K$, is radioactive and is β^+ emitter; it decays to give the daughter nucleus argon $^{A}_{Z}Ar$. The object of this exercise is to study the decay of potassium 40.

Given masses of the nuclei and particles:

- $m\left(^{40}_{19}K\right) = 39.95355\ u$;
- $m\left(^{A}_{Z}Ar\right) = 39.95250\ u$;
- $m\left(^{0}_{1}e\right) = 5.5\times10^{-4}\ u$;
- m (neutrino) ≈ 0;

- Avogadro's number: $N_A = 6.02\times10^{23}\ mo\ell^{-1}$;
- $1u = 931.5 MeV/c^2$;
- Radioactive period of $^{40}_{19}K$: $T = 1.5\times10^9$ years;
- Molar mass of $^{40}_{19}K = 40 g.mo\ell^{-1}$;
- $1MeV = 1.6\times10^{-13}\ J$.

Part A
Energetic study of the decay of potassium 40
First situation
Energy liberated by one decay

1. Write down the equation of the decay of one potassium 40 nucleus and determine Z and A.
2. Calculate, in MeV, the energy E_1 liberated by this decay.
3. The daughter nucleus is supposed to be at rest. The energy carried by β^+ is, in general, smaller than E_1. Why?

Second situation
Energy received by a person

The mass, of potassium 40 at an instant t, in the body of an adult is, on the average, equal to $2.6\times10^{-3}\%$ of its mass. An adult person has a mass $M = 80\ kg$.

1. a) Calculate the mass m of potassium 40 contained in the body of that person at the instant t.
   b) Deduce the number of potassium 40 nuclei in the mass m at the instant t.
2. a) Calculate the radioactive constant λ of potassium 40.
   b) Deduce the value of the activity A of the mass m at the instant t.
3. Deduce, in J, the energy E liberated by the mass m per second.

Part B
Dating by potassium 40

Certain volcanic rocks contain potassium and part of it, is potassium 40. At the instant of its formation $(t_0 = 0)$, the number of nuclei of potassium 40 is N_0 in the volcanic rock and that of argon is zero. At the instant t, the rock contains respectively N_K and N_{Ar} nuclei of potassium 40 and of argon 40.

1. a) Write down the expression of N_K, that explains the law of radioactive decay, as a function of time.
   b) Deduce the expression of N_{Ar} as a function of time.
2. A geologist analyzes a volcanic rock. He notices that the number of argon 40 nuclei is twice less than the number of potassium 40 nuclei in this rock. Determine the age of this rock.

III-GS 2006 2nd
Radioactivity

The object of this exercise is to show evidence of some characteristics of a thorium nucleus 230 and its role in dating.

Given the masses of nuclei and particles:

- $m(\alpha) = 4.0015\ u$;
- $m({}^{A}_{88}Ra) = 225.9770\ u$;
- $m({}^{230}_{Z}Th) = 229.9836\ u$.

- $1eV = 1.6\times10^{-19}\ J$;
- $1u = 931.5\ MeV/c^2$;
- Planck's constant: $h = 6.63\times10^{-34}\ J.s$;
- Speed of light in vacuum: $c = 3\times10^{8}\ m.s^{-1}$;
- Avogadro's number: $N_A = 6.02\times10^{23}\ mo\ell^{-1}$.

Part A
Decay of thorium nucleus

The thorium nucleus ${}^{230}_{Z}Th$ is radioactive and is an α emitter. The daughter nucleus is an isotope of the radium ${}^{A}_{88}Ra$.

1. a) Write the equation of this decay and determine the values of A and Z.
   b) Determine the energy liberated by the decay of a thorium nucleus 230.
2. A decay of a thorium nucleus 230, at rest, takes place without the emission of γ-radiation. The daughter nucleus ${}^{A}_{88}Ra$ obtained has a speed almost zero.
   Determine the value of the kinetic energy KE_1 of the emitted α particle.
3. Another decay of a thorium nucleus 230 is accompanied with the emission of γ-radiation of wavelength $\lambda = 6\times10^{-12}\ m$ in vacuum.
   a) Calculate the energy of this radiation.
   b) Deduce the value of the kinetic energy KE_2 of the emitted α particle.
4. A sample of $1g$ of thorium 230, of activity $A_0 = 7.2\times10^{8}$ decays/s, is placed near a sheet of aluminum at the instant $t_0 = 0$. The α particles are stopped by the aluminum sheet whereas the photons are not absorbed.

a) Determine, in J, the energy W transferred to the aluminum sheet during the first second knowing that 50% of the decays are accompanied with γ-emission, and that the activity A_0 remains practically constant within this second.

b) Calculate the number of nuclei present in $1g$ of thorium 230.

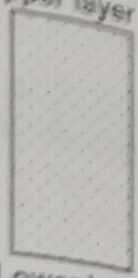
Deduce, in $year^{-1}$, the value of the radioactive constant λ of thorium 230.

Part B
Dating of marine sediments

Due to the phenomenon of erosion, a part of the rocks is driven into the oceans. Some of these rocks contain radioactive uranium 234 $\left({}^{234}_{92}U\right)$ which gives thorium 230.

The uranium 234 is soluble in seawater, whereas thorium is not, but it is accumulated at the bottom of the ocean with other sediments. We take a specimen, formed of a cylinder from the bottom of the ocean. This specimen has an upper layer that is just formed, and a lower layer that is formed a time t ago. We take a sample (a) from the upper part and another sample (b), of the same mass, from the lower part. It is found that the sample (a) produces 720decays/s and (b) 86.4decays/s. Determine t in years.

Upper layer



Lower layer

IV-GS 2005 2nd
The Technetium

Given masses of nuclei and particles:

- ${}^{99}_{42}Mo$: $98.88437\ u$;
- ${}^{99}_{43}Tc$: $98.88235\ u$;
- ${}^{0}_{-1}e$: $0.00055\ u$.

- $1u = 931.5 MeV/c^2 = 1.66\times10^{-27} kg$;
- Planck's constant: $h = 6.6\times10^{-34} J.s$;
- $1eV = 1.6\times10^{-19} J$;
- $c = 3\times10^{8} m/s$.

Part A
A bit of history

In 1937, Pierrier and Sègre obtained, for the first time, an isotope of technetium ${}^{99}_{43}Tc$ by bombarding the nuclei of molybdenum ${}^{98}_{42}Mo$ with an isotope of hydrogen ${}^{A}_{Z}H$ according to the following reaction:

$${}^{98}_{42}Mo + {}^{A}_{Z}H \longrightarrow {}^{99}_{43}Tc + {}^{1}_{0}n.$$

Determine Z and A specifying the law used.

Part B
Production of technetium 99 at the present time and its characteristic

The isotope ${}^{99}_{43}Tc$ is actually obtained in generator molybdenum/technetium, starting from the isotope ${}^{99}_{42}Mo$ of molybdenum. This molybdenum is β^- emitter.

1. Write the equation corresponding to the decay of ${}^{99}_{42}Mo$.
2. Determine, in MeV, the energy liberated by this decay.
3. Most of the technetium nuclei obtained are in an excited state $\left({}^{99}_{43}Tc^*\right)$.

a) Radiation

i- Complete the equation of the following downward transition ${}^{99}_{43}Tc^{*} \longrightarrow {}^{99}_{43}Tc + ...$

ii- Specify the nature of the emitted radiation.

b) The energy liberated by this transition, of value $0.14\,MeV$, is totally carried by the emitted radiation; the nuclei $\left({}^{99}_{43}Tc^{*}\right)$ and $\left({}^{99}_{43}Tc\right)$ are supposed to be at rest.

i- Determine, in u, the mass of the $\left({}^{99}_{43}Tc^{*}\right)$ nucleus.

ii- Calculate the wavelength of the emitted radiation.

Part C

Using of technetium in medicine

The isotope $\left({}^{99}_{43}Tc\right)$ is actually often used in medical imaging. The generator molybdenum/technetium is known in medicine, by the name «technetium cow». Also, the daily preparation of the medically needed technetium 99 of half-life $T_1 = 6$ hours, starting from its «parent» the molybdenum of half-life $T_2 = 67$ hours, allows a weekly supply.

1. Why is it preferable, in medical service that requires the used of technetium 99, to keep a reserve of molybdenum 99 and not a reserve of technetium 99?
2. Determine the number of technetium 99 nuclei obtained from a mass of $1g$ of molybdenum 99 at the end of 24 hours. Deduce the mass of these technetium nuclei.

V-GS 2004 2nd

Studying the Radionuclide ${}^{198}_{79}$Au

Given masses of nuclei and particles:

- Mass of the gold nucleus Au : $197.925\,u$;
- Mass of the mercury nucleus Hg : $197.923\,u$;
- Mass of the proton: $m_p = 1.00728\,u$;
- Mass of the neutron: $m_n = 1.00866\,u$;
- Mass of the electron: $5.50 \times 10^{-4}\,u$;

- Molar mass of ${}^{198}_{79}Au$: $198\,g$;
- Avogadro's number: $6.022 \times 10^{23}\,mo\ell^{-1}$;
- Speed of light in vacuum: $c = 3 \times 10^{8}\,m/s$;
- $1u = 931.5\,MeV/c^2 = 1.66 \times 10^{-27}\,kg$;
- $1eV = 1.6 \times 10^{-19}\,J$.

Part A

Comparison between the density of the gold nucleus and that of the gold atom

1. a) Calculate the mass of the gold atom ${}^{198}_{79}Au$.

b) Compare the mass of the gold atom ${}^{198}_{79}Au$ with that of its nucleus.

2. The average radius of the gold atom is $r = 16 \times 10^{-11}\,m$. The average radius of a nucleon is $r_0 = 12 \times 10^{-16}\,m$. Compare the density of the gold atom with that of its nucleus. Give a conclusion about the distribution of mass in the atom.

Part B

Stability of the gold nucleus

1. a) Give the constituents of the nucleus $^{198}_{79}Au$.

b) If the gold nucleus $^{198}_{79}Au$ is broken into its constituting nucleons, show that the sum of the masses of the nucleons taken separately at rest is greater than the mass of the nucleus taken at rest. Due to what is this increase in the mass?

2. Knowing that a nucleus is considered stable when its binding energy per nucleon is larger or equal to $8\,MeV$, give a conclusion about the stability of the nucleus $^{198}_{79}Au$.

Part C

Studying the disintegration of the gold nucleus $^{198}_{79}Au$

When the gold nucleus $^{198}_{79}Au$, at rest, disintegrates it gives a daughter nucleus (mercury nucleus $^{A}_{Z}Hg$) of negligible speed. We were able to detect the emission of a γ-photon of energy $0.412\,MeV$ and a β^- particle of kinetic energy $0.824\,MeV$.

1. Write the equation of this disintegration reaction and, specifying the laws used, determine A and Z.

2. a) Specify the physical nature of the γ-radiation.

b) Due to what is the emission of this γ-radiation?

3. a) Show, by applying the law of conservation of total energy, the existence of a new particle accompanying the emission of β^-.

b) Give the name of this particle.

c) Deduce its energy in MeV.

Solutions

I-GS 2009 2nd

Part A

1\. a) $^{210}_{83}Bi \longrightarrow ^{210}_{84}Po + ^{a}_{z}X$.

Conservation of mass number: $210 = 210 + a$, then $a = 0$;

Conservation of charge number: $83 = 84 + z$, then $z = -1$;

The equation of the disintegration $^{210}_{83}Bi \longrightarrow ^{210}_{84}Po + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon} + \gamma$.

b) The emitted particle is the electron $^{0}_{-1}e$;

The $^{210}_{83}Bi$ nucleus is a beta minus emitter.

2\. The mass defect : $\Delta m = m_{before} - m_{after} = m(Bi) - m(Po) - m(e)$;

$\Delta m = 209.938445 - (209.936648 + 5.5 \times 10^{-4}) = 1.247 \times 10^{-3}\,u$;

The energy liberated is : $E_\ell = \Delta m c^2 = 1.247 \times 10^{-3} \times 931.5 \dfrac{MeV}{c^2} \times c^2 = 1.16\,MeV$.

3\. According to the conservation of energy: $E_\ell = KE_{\beta^-} + E(^{0}_{0}\bar{\upsilon}) + E(\gamma)$;

Then $KE_{\beta^-} = E_\ell - (E(^{0}_{0}\bar{\upsilon}) + E_\gamma) = 1.16 - 0.96 - 0.02 = 0.18\,MeV$.

Part B

1\. a) Equation of the disintegration: $^{210}_{84}Po \longrightarrow ^{A}_{Z}X + ^{4}_{2}He$;

Conservation of charge number: $84 = Z + 2$, then $Z = 82$;

Conservation of mass number: $210 = A + 4$, then $A = 206$;

The equation of the disintegration $^{210}_{84}Po \longrightarrow ^{206}_{82}X + ^{4}_{2}He$.

b) $^{A}_{Z}X$ is an isotope of lead $^{206}_{82}Pb$.

2\. a) Law of radioactive decay $N = N_0\, e^{-\lambda t}$ where λ is the radioactive constant.

b) $-\ell n\left(\dfrac{N}{N_0}\right) = -\ell n\left(\dfrac{N_0\, e^{-\lambda t}}{N_0}\right) = -\ell n(e^{-\lambda t}) = \lambda t$.

3\. a) Missing values:

t(days)	0	40	80	120	160	200	240
$\dfrac{N}{N_0}$	1	0.82	0.67	0.55	0.45	0.37	0.30
$-\ell n\left(\dfrac{N}{N_0}\right)$	0	0.2	0.4	0.6	0.8	1	1.2

b) Graph.

c) The curve representing $-\ell n\left(\dfrac{N}{N_0}\right)$ as a function of time is:

- a straight line;
- passing through the origin.

Therefore, it is in agreement with the derived expression.

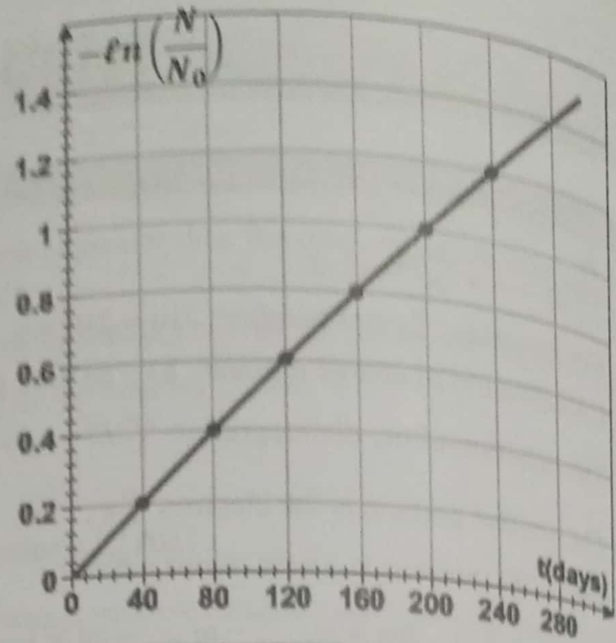
d) Graphical study:

i- The slope of the straight line is:

$\lambda = \frac{0.6-0}{(120-0)days} = 5\times10^{-3}\ day^{-1}$;

ii- The slope of this line is the radioactive constant λ of polonium 210.

iii- The period $T = \frac{\ell n(2)}{\lambda} = 138.6\,days$.



GS 2008 2nd

Part A
First situation

1. The equation of disintegration: ${}^{40}_{19}K \longrightarrow {}^{A}_{Z}Ar + {}^{0}_{+1}e + {}^{0}_{0}\upsilon + \gamma$.

   Conservation of mass number: $40 = A + 0$, then $A = 40$;

   Conservation of charge number: $19 = Z + 1$, then $Z = 18$;

   The equation of the disintegration ${}^{40}_{19}K \longrightarrow {}^{40}_{18}Ar + {}^{0}_{+1}e + {}^{0}_{0}\upsilon + \gamma$.

2. The mass defect is : $\Delta m = m\left({}^{40}_{19}K\right) - \left[m\left({}^{40}_{18}Ar\right) + m\left({}^{0}_{1}e\right)\right]$;

   $$= 39.95355 - (39.95250 + 0.00055) = 5\times10^{-4}u.$$

   The energy liberated by this decay is $E_1 = \Delta mc^2 = 5\times10^{-4}\times931.5MeV$; then $E_1 = 0.46575\ MeV$.

3. According to the conservation of energy $E_1 = E(\beta^+) + E\left({}^{0}_{0}\upsilon\right) + E(\gamma)$.

   Some of the energy liberated is carried by the neutrino and the gamma radiations.

Part A
Second situation

1. a) The mass of potassium in the human body $m = \frac{2.6\times10^{-3}}{100}\times80 = 2.08\times10^{-3}\ kg = 2.08\ g$.

   b) The number of potassium nuclei is $N = \frac{m}{M}N_A = \frac{2.08g}{40\ g/mo\ell}\times6.02\times10^{23}\ mo\ell^{-1} = 3.13\times10^{22}$.

2. a) The radioactive constant :

   - If T in years, then $\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{1.5\times10^{9}} = 4.62\times10^{-10}\ year^{-1}$;
   - If T in seconds, then $\lambda = \frac{\ell n(2)}{1.5\times10^{9}\times365\times24\times3600} = 1.47\times10^{-17}\ s^{-1}$.

   b) The activity A is given by $A = \lambda N = 1.47\times10^{-17}\times3.13\times10^{22} = 4.6\times10^{5}\ Bq$.

3. If N_d is the number of disintegrations per second then the energy liberated each second is:

   $E = \frac{N_d\times E_1}{\Delta t} = A\times E_1 = 4.6\times10^{5}\times0.46575 = 2.14\times10^{5}\ MeV = 3.43\times10^{-8}\ J$.

Part B

1. a) The number of potassium nuclei at an instant t is : $N_K = N_0 e^{-\lambda t}$.

b) We know that the number of potassium disintegrated is equal to the number of argon formed.

The number of argon nuclei at an instant t is: $N_{Ar} = N_0 - N_K = N_0\left(1-e^{-\lambda t}\right)$.

2. We have $N_{Ar} = \frac{N_K}{2}$; $N_0\left(1-e^{-\lambda t}\right) = \frac{N_0 e^{-\lambda t}}{2}$; so $2-2e^{-\lambda t} = e^{-\lambda t}$;

$e^{\lambda t} = \frac{3}{2}$; $\lambda t = \ell n\left(\frac{3}{2}\right)$; then $t = \frac{T}{\ell n(2)}\ell n\left(\frac{3}{2}\right) = 8.8\times10^{8}$ years.

III-GS 2006 2nd

Part A

1. a) The equation of the disintegration ${}^{230}_{Z}Th \longrightarrow {}^{4}_{2}He + {}^{A}_{88}Ra$.

Conservation of mass number: $230 = A+4$, then $A = 226$;

Conservation of charge number: $Z = 2+88$, then $Z = 90$;

The equation of this decay is ${}^{230}_{90}Th \longrightarrow {}^{4}_{2}He + {}^{226}_{88}Ra + \gamma$.

b) The mass defect is: $\Delta m = m\left({}^{230}_{Z}Th\right) - \left[m\left({}^{4}_{2}He\right) + m\left({}^{A}_{88}Ra\right)\right] = 5.1\times10^{-3}u$;

The energy liberated $E_\ell = \Delta m c^2 = 5.1\times10^{-3}\times931.5\left(MeV/c^2\right)\times c^2 = 4.75065\ MeV$.

2. Since the father nucleus and daughter nucleus are taken at rest then $KE_{Th} = KE_{Ra} = 0$;

By applying the conservation of total energy $E_\ell = KE_1(\alpha) + E_\gamma = 4.75065\ MeV$;

The disintegration takes place without the emission of γ radiation $KE_1(\alpha) = E_\ell = 4.75065\ MeV$

3. a) The energy carried by the γ-radiation is $E_\gamma = h\frac{c}{\lambda} = 3.315\times10^{-14}J = 0.21\ MeV$.

b) Law of conservation of energy: $E_\ell = \Delta m c^2 = KE_2(\alpha) + E_\gamma$;

$KE_2 = E - E_\gamma = 4.75065 - 0.21 = 4.54065\ MeV$.

4. a) The activity remains practically constant within this second the energy absorbed is then

$$W = \frac{A_0}{2}(KE_1) + \frac{A_0}{2}(KE_2) = \frac{7.2\times10^{8}}{2}(4.7065 + 4.54065)\times1.6\times10^{-13} = 5.35\times10^{-4}J.$$

b) Let N_0 be the number of nuclei present in $1g$:[1]

We have $N_0 = \frac{m}{M}N_A = \frac{1g}{230\,g.mo\ell^{-1}}\times6.02\times10^{23}mo\ell^{-1} = 2.62\times10^{21}$ nuclei.[2]

Then the radioactive constant is: $\lambda = \frac{A_0}{N_0} = \frac{7.2\times10^{8}}{2.62\times10^{21}} \approx 2.75\times10^{-13}s^{-1}$;

$\lambda = 2.75\times10^{-13}\times3600\times24\times365 = 8.67\times10^{-6}\,\text{year}^{-1}$

[1] We can also calculate the number of nuclei using the relation $N_0 = \frac{m_{sample}}{m_{nucleus}}$; but we need to know that $1u = 1.66\times10^{-27}kg$; then $N_0 = \frac{1\times10^{-3}}{229.9836\times1.66\times10^{-27}} = 2.62\times10^{21}$ nuclei.

[2] The molar mass of thorium 30 is $M = 230\,g/mo\ell$.

Part B

The sample (a) is taken from the upper part and the sample (b) from the lower part, both having the same mass so we may write that: $A_0 = 720\ Bq$ and $A = 86.4 Bq$.[3]

Law of radioactive decay $A = A_0 e^{-\lambda t}$; so $t = \frac{1}{\lambda}\ell n\left(\frac{A_0}{A}\right) = \frac{10^6}{8.67}\ell n\left(\frac{720}{86.4}\right) \approx 2.445 \times 10^5$ years.

IV-GS 2005 2nd

Part A

$^{98}_{42}Mo + ^{A}_{Z}H \longrightarrow ^{99}_{43}Tc + ^{1}_{0}n$

Conservation of mass number: $98 + A = 99 + 1$, then $A = 2$;

Conservation of charge number: $42 + Z = 43 + 0$, then $Z = 1$;

The nuclear reaction $^{98}_{42}Mo + ^{2}_{1}H \longrightarrow ^{99}_{43}Tc + ^{1}_{0}n$.

Part B

1. The equation of decay: $^{99}_{42}Mo \longrightarrow ^{99}_{43}Tc + ^{0}_{-1}e + ^{0}_{0}\bar{\upsilon}$.
2. The mass defect: $\Delta m = m\left(^{99}_{42}Mo\right) - \left[m\left(^{99}_{43}Tc\right) + m\left(^{0}_{-1}e\right)\right]$;

   $\Delta m = 98.88437\ u - (98.88235\ u + 0.00055\ u) = 1.47 \times 10^{-3}\ u$;

   The energy liberated: $E_\ell = \Delta m c^2 = 1.47 \times 10^{-3} \times 931.5\ MeV = 1.369305\ MeV$.
3. a) Radiation:

   i- $^{99}_{43}Tc^* \longrightarrow ^{99}_{43}Tc + \gamma$.

   ii- γ is an electromagnetic radiation.

   b) Energy:

   i- The energy liberated is given by:

   $$E'_\ell = \left[m(Tc^*) - m(Tc)\right]c^2 = E_\gamma;\ \text{so}\ \left[m(Tc^*) - m(Tc)\right] = \frac{E_\gamma}{c^2};$$

   Then, $m(Tc^*) = m(Tc) + \frac{E_\gamma}{c^2} = 98.88235\ u + \frac{0.14}{931.5}u = 98.88250\ u$.

   ii- The wavelength of the radiation: $\lambda = \frac{hc}{E_\gamma} = \frac{6.63 \times 10^{-34}\ J.s \times 3 \times 10^8\ m/s}{0.14 \times 1.6 \times 10^{-13}\ J} = 8.88 \times 10^{-12}\ m$.

Part C

1. Due to the larger period, it constitutes a reserve more durable.
2. Mass used:

   ➢ The number of Mo nuclei $N_0(Mo)$ in the sample:

   $$N_0(Mo) = \frac{m_0(Mo)}{m_{Mo}} = \frac{1 \times 10^{-3}}{98.88437 \times 1.66 \times 10^{-27}} = 6.0921 \times 10^{21}\ \text{nuclei.}$$

   ➢ After 24 hours, the number of nuclei of Mo becomes $N(Mo) = N_0\, e^{-\frac{\ell n2}{67}\times 24} = 4.7526 \times 10^{21}$.

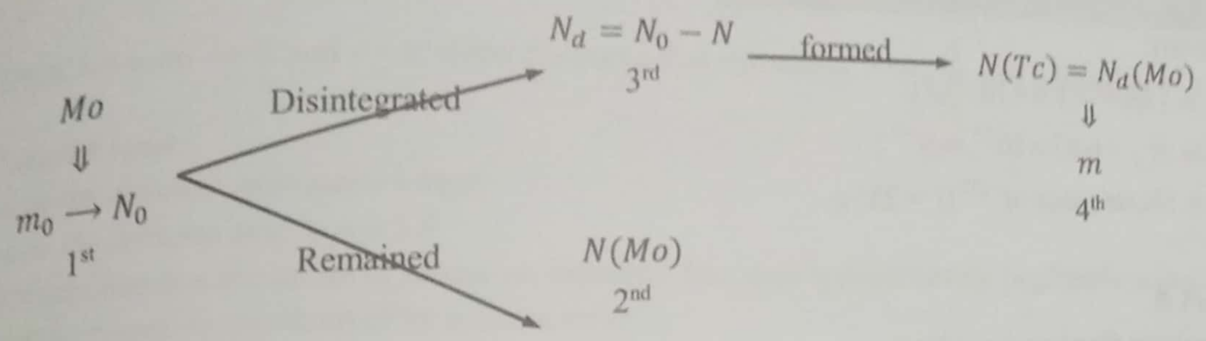
   ➢ The number of nuclei of Tc formed is equal of Mo disintegrated:

[3] The initial activity A_0 is always greater than the final A, $(A_0 > A)$.

$N(Tc) = N(Mo) = N_0 - N = 1.3395 \times 10^{21}$ nuclei.

The mass of Tc obtained is:

$m = 1.3395 \times 10^{21} \times 98.88235 \times 1.66 \times 10^{-27}\,kg = 0.00022\,kg = 0.22\,g$.



V-GS 2004 2nd

Part A + Part B **(see Page 114)**

Part C

1. The equation of disintegration is ${}^{198}_{79}Au \longrightarrow {}^{A}_{Z}Hg + {}^{0}_{-1}e + \gamma$

   Conservation of mass number: $198 = A + 0$, then $A = 198$;

   Conservation of charge number: $79 = Z - 1$, then $Z = 80$;

   The equation of the disintegration ${}^{198}_{79}Au \longrightarrow {}^{198}_{80}Hg + {}^{0}_{-1}e + \gamma$.

2. a) γ is an electromagnetic radiation.

   b) The emission of gamma radiations is due to the dis-excitation of the daughter nucleus ${}^{198}_{80}Hg$ which is formed in an excited state; it undergoes a downward transition into the ground state; emitting γ - rays.

3. a) Before the decay $E_{before} = KE + mc^2 = 0 + 197.925 \times 931.5 MeV = 184367.1375\,MeV$.

   After the decay $E_{after} = (m_{Hg}c^2 + KE_{Hg}) + (KE_e + m_e c^2) + E_\gamma$;

   $E_{after} = (197.923 \times 931.5 + 0) + (0.824 + 0.00055 \times 931.5) + 0.412 = 184367.02 MeV$

   $E_{before} \neq E_{after}$, then the total energy is not conserved.

   The energy defect is due to the emission of a certain particle accompanying this emission.

   b) The particle is called antineutrino.

   c) The energy carried by this particle is $E\left({}^{0}_{0}\overline{\upsilon}\right) = E_{before} - E_{after} = 0.1147\,MeV$.

I-GS 2014 1st
Nuclear Power Plant

Given:

- $1MeV = 1.6\times10^{-13}J$;
- $N_A = 6.02\times10^{23}\ mol^{-1}$;
- Molar mass of $^{235}U = 235g$.

Part A
Nuclear fission reaction

A nuclear reactor uses enriched uranium constituted of 3% of $^{235}_{92}U$ and 97% of $^{238}_{92}U$.

1. One of the possible nuclear reaction of uranium 235 is:

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{94}_{...}Sr + ^{140}_{54}Xe + ...^{1}_{0}n \quad (1)$$

a) Define the term fissile isotope.
b) Complete the equation of the reaction and specify the used laws.

2. The binding energy per nucleon of the nuclei of reaction (1) are given in the table below:

Nucleus	$^{235}_{92}U$	$^{140}_{54}Xe$	^{94}Sr
Binding energy per nucleon $\frac{E_B}{A}$	$7.5MeV$	$8.2MeV$	$8.5MeV$

Calculate the binding energy E_B of each nucleus.

3. a) Determine the expression of the mass of a nucleus $^{A}_{Z}X$ in terms of A, Z, m_p (mass of proton), m_n (mass of a neutron), E_B and c (speed of light in vacuum).
b) Show that the liberated energy by reaction (1) can be written $E_\ell = E_B(Sr) + E_B(Xe) - E_B(U)$
c) Calculate this energy in MeV.

4. In the core of the reactor, the fission of one uranium 235 nucleus liberates on average an energy of $200MeV$. 30% of this energy is transformed into electrical energy. A power plant furnishes an electric power of $1350\ MW$.
Determine, in kg, the daily consumption of $^{235}_{92}U$ in this power plant.

Part B
Danger of radioactivity

Iodine 131 is one of the emitted gases from a nuclear reactor. $^{131}_{53}I$ is a β^- emitter of half-life $T = 8$ days and the daughter nucleus is Xenon (Xe).

1. a) The disintegration of $^{131}_{53}I$ nucleus is usually accompanied with the emission of γ.
Write the equation of disintegration of iodine 131.
b) Indicate the cause of the emission of γ.

2. The iodine 131 causes serious problems because it has the ability to be fixed on the thyroid gland.

Let A_0 be the activity of a sample of iodine 131 at an instant $t_0 = 0$ and A is its activity at an instant t.

a) Calculate, in day^{-1}, the value of the decay constant λ of iodine 131.

b) Determine the expression of $-\ell n\left(\frac{A}{A_0}\right)$ in terms of λ and t.

c) Trace, between $t = 0$ and $t = 32$ days, the curve that represents $-\ell n\left(\frac{A}{A_0}\right)$ as a function of t.

Take the scale:

- on the abscissa axis $1cm \equiv 4$ days;
- on the ordinate axis $1cm \equiv 0.5$.

d) We suppose that the effect of iodine on organism becomes approximately negligible when its activity becomes one-tenth of its initial activity.

Determine from the traced curve the time at which there is no effect on organisms

GS 2013 1st
Nuclear Fission

The nuclear chain fission reaction, conveniently controlled in a nuclear power plant, can be a source of a huge amount of energy able to generate electric power.

Given masses of nuclei:

- ${}^{235}_{92}U$: 234.9934 u;
- ${}^{138}_{x}Ba$: 137.8742 u;
- ${}^{y}_{36}Kr$: 94.8871 u.

- molar mass of ${}^{235}U = 235\ g.mo\ell^{-1}$;
- Avogadro's number $N_A = 6.02 \times 10^{23}\ mo\ell^{-1}$;
- $m({}^{1}_{0}n) = 1.0087\ u$;
- $1u = 931.5\ MeV/c^2$;
- $1eV = 1.6 \times 10^{-19}\ J$.

Part A
Efficiency of a nuclear power plant

In the reactor of a nuclear power plant, we use natural uranium enriched with uranium 235. The nucleus of a uranium 235 captures a thermal neutron and is transformed into a nucleus of uranium 236 in an excited state.

The decay of this nucleus is accompanied by the emission of a photon γ of energy equal to $20MeV$.

1. a) Complete the following reaction: ${}^{236}_{92}U^* \longrightarrow .... + \gamma$.

b) Indicate the value of the excess of energy possessed by a uranium 236 nucleus in the excited state.

2. The obtained uranium nucleus, breaks instantaneously, producing two nuclides called fission fragments with the emission of some neutrons and γ photon, so the overall equation is:

$$ {}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{138}_{x}Ba + {}^{y}_{36}Kr + 3\,{}^{1}_{0}n + \gamma. $$

Determine:

a) x and y.

b) in MeV, the energy liberated by the fission of a uranium235nucleus.

c) the energy liberated by the fission of $1g$ of uranium 235.

d) the efficiency of the nuclear power plant, knowing that it provides an electric power of $800MW$ and consumes $2.8kg$ of uranium 235 per day.

Part B

Chain reaction

The kinetic energy of a neutron that may produce the fission of uranium 235 nucleus should be of the order of $0.04eV$.

We suppose that all the neutrons emitted by the fission reactions have the same kinetic energy.

1. The sum of the kinetic energies of the two fragments (Kr and Ba) is equal to $174MeV$ and the energy of the emitted γ photon is $E_\gamma = 20\ MeV$.

   a) Show, using the conservation of the total energy, that the kinetic energy of a neutron emitted by this fission is $2MeV$.

   b) Deduce that the emitted neutrons cannot produce fission reactions of uranium 235.

2. To produce a fission by an emitted neutron, it is necessary to slow it down by collisions with carbon 12 atoms in graphite rods. We suppose that each collision between a neutron and one carbon 12 atom is perfectly elastic and that the velocities before and after collision are collinear.

Take: $m\left({}_0^1n\right)=1u$ and $m\left({}^{12}C\right)=12\,u$.

   a) Let $\overrightarrow{v_0}$ be the velocity of one emitted neutron just before collision and $\overrightarrow{v_1}$ its velocity just after its first collision with a carbon 12 atom supposed initially at rest. Show that: $\left|\frac{v_1}{v_0}\right| = k = \frac{11}{13}$.

   b) Kinetic energy:

   i- Show that the ratio of the kinetic energies just after and just before the first collision of the emitted neutron is: $\frac{KE_1}{KE_0} = k^2$.

   ii- Determine the number of collisions needed for an emitted neutron with carbon 12 atoms, to slow down so that its kinetic energy is reduced to $0.04eV$.

III-GS 2012 2nd

Nuclear Reactor

Given:

- Atomic mass unit: $1u = 1.66\times10^{-27}\,kg = 931.5\,MeV/c^2$;
- $1MeV = 1.6\times10^{-13}\,J$;

Mass of particles in u:

- antineutrino ${}_0^0\bar{\nu} \approx 0$;
- electron ${}_{-1}^{0}e$: $5.5\times10^{-4}\,u$;
- neutron ${}_0^1n$: $1.0087\,u$.

Element	Molybdenum		Technetium		Tellurium
Nuclides	${}^{101}_{42}Mo$	${}^{102}_{42}Mo$	${}^{101}_{43}Tc$	${}^{102}_{43}Tc$	${}^{135}_{52}Te$
Mass (u)	100.9073	101.9103	100.9073	101.9092	134.9167

Element	Uranium		Neptunium	Plutonium
Nuclide	${}^{235}_{92}U$	${}^{238}_{92}U$	${}^{239}_{93}Np$	${}^{239}_{94}Pu$
Mass (u)	235.0439	238.0508	239.0533	239.0530

Read carefully the following text about fast neutrons, and answer the questions that follow

"...the basic substance used to obtain nuclear energy is the natural uranium which is mainly formed of the two isotopes: uranium 235 and uranium 238 ...

... The fast-neutron nuclear reactors (breeder reactors) use uranium 235 or plutonium239 (or the two at the same time) as fuel. In each reactor we put around the core which is constituted of uranium 235 $\left({}^{235}_{92}U\right)$, a cover made essentially of fertile uranium 238 $\left({}^{238}_{92}U\right)$. This cover can trap fast neutrons issued from the fission reactions of uranium 235 . These reactors transform more uranium 238 atoms into plutonium 239 .

Finally, in the very well studied fast neutrons reactors, the quantity of fissionable matter that is created exceeds notably the consumed quantity. For this reason, these reactors are called breeder reactors..."

Questions

1. a) What is meant by isotopes of an element?

   b) Give the composition of each of uranium 235 and uranium 238 nuclei.

2. In the reactor, the uranium 238 reacts with the fast neutrons according to the reaction:

$$ {}^{238}_{92}U + {}^{1}_{0}n \longrightarrow X \text{ (reaction 1)} $$

The obtained nucleus X is radioactive and after two successive β^- emissions, it is transformed into plutonium:

$$ X \longrightarrow {}^{0}_{-1}e + Y + {}^{0}_{0}\overline{\upsilon} \text{ (reaction 2);} $$

$$ Y \longrightarrow {}^{0}_{-1}e + {}^{....}_{94}Pu + {}^{0}_{0}\overline{\upsilon} \text{ (reaction 3).} $$

a) Identify X & Y.

b) Deduce the nuclear reaction that occurs between a $\left({}^{238}_{92}U\right)$ nucleus and a fast neutron that leads to the formation of a plutonium 239 (reaction 4).

3. The plutonium 239 $\left({}^{239}_{94}Pu\right)$ is fissile and can react with neutrons according to the reaction:

$$ {}^{239}_{94}Pu + {}^{1}_{0}n \longrightarrow B + {}^{135}_{52}Te + 3\left({}^{1}_{0}n\right) \text{ (reaction 5)} $$

a) Identify B.
b) Calculate, in MeV/c^2, the mass defect Δm in reaction 5.
c) Deduce, in MeV, the energy E liberated during the fission of a plutonium nucleus.
d) Determine, in joules, the energy liberated by the fission of one kilogram of plutonium.
4. From reactions 4 and 5, justify the definition of a breeder reactor given in the text.

IV-GS 2010 2nd
The Neutrons and the Nuclear Fission in a Reactor

Given masses of some nuclei and particles:

- $m\left({}^{235}_{92}U\right) = 234.99332\ u$;
- $m(I) = 138.89700\ u$;
- $m(Y) = 93.89014\ u$;
- $m_n = 1.00866\ u$;
- $1u = 931.5\ MeV/c^2$.

In a uranium 235 reactor, the fission of a nucleus $\left({}^{235}_{92}U\right)$ under the impact of a thermal neutron gives rise to different pairs of fragments with the emission of some neutrons. The most probable pairs of fragments have their mass numbers around 95 and 140. One of the typical fission reactions is the one which produces the iodine $\left({}^{A}_{53}I\right)$, the yttrium $\left({}^{94}_{Z}Y\right)$ and 3 neutrons.

Part A

Determine Z & A.

Part B

1. Show that the mass defect in this reaction is $\Delta m = 0.18886\ u$.
2. Determine, in MeV, the energy E liberated by this fission reaction.
3. Knowing that each neutron formed has an average kinetic energy $E_0 = 1\% E$. Calculate E_0.
4. For a neutron, produced by the fission reaction, to trigger a new nuclear fission of a uranium nucleus 235, it must have a low kinetic energy around $E_{th} = 0.025\ eV$ (thermal neutron). In order to reduce the kinetic energy of a produced neutron from E_0 to E_{th}, this neutron must undergo successive collisions with heavier nuclei at rest of mass $M = 2m_n$, called, «moderator» nuclei; these collisions are supposed elastic and all the velocities are collinear.
   a) Using the laws of conservation of linear momentum and kinetic energy, show that after each collision, the neutron rebounds with one third (1/3) of its initial speed.
   b) Determine, in terms of E_0, the expression of the kinetic energy E_1 of the neutron after the first collision.
   c) Deduce, in terms of E_0, the expression of the kinetic energy KE of the neutron after the k^{th} collision.
   d) Calculate the number k of collisions needed for the energy of a neutron to decrease from E_0 to $0.025\ eV$.

A Nuclear Reactor «The Breeder Reactor»

Read carefully the following selection:

"…..The nuclear reactors with fast neutrons use uranium 238 or plutonium 239 (or both at the same time) as fuel...The principle of a breeder reactor is to produce, starting from uranium 238 an amount of fissionable material that is equal or exceeds what the reactor consumes since the final result would be the consumption of uranium 238 only which is more abundant than the uranium 235….."

Given:

- Speed of light in vacuum: $c = 3\times10^{8}\,m/s$;
- Mass of neutron $\left({}_{0}^{1}n\right) = 1.0087\,u$;
- $1u = 1.66\times10^{-27}\,kg = 931.5\,MeV/c^{2}$;
- $1\,MeV = 1.6\times10^{-13}\,J$.

Element	Tellurium	Technetium	Molybdenum	Plutonium	Neptunium
Nuclide	${}_{52}^{135}Te$	${}_{43}^{102}Tc$	${}_{42}^{102}Mo$	${}_{94}^{239}Pu$	${}_{93}^{239}Np$
Mass (u)	134.9167	101.9092	101.9103	239.0530	239.0533

1. Draw from the selection an indicator showing that producing an equal amount of energy in a nuclear power plant, uranium 238 has an advantage over uranium 235.
2. In a breeder reactor, uranium 238 reacts with a fast neutron according to the following reaction:

$${}_{92}^{238}U + {}_{0}^{1}n \longrightarrow {}_{Z}^{A}X ……(1).$$

The nucleus ${}_{Z}^{A}X$ obtained is radioactive; it is transformed into fissionable plutonium according to the following equations:

$${}_{Z}^{A}X \longrightarrow {}_{-1}^{0}e + {}_{Z_1}^{A_1}X + {}_{0}^{0}\bar{\upsilon} …… (2).$$

$${}_{Z_1}^{A_1}X \longrightarrow {}_{-1}^{0}e + {}_{94}^{239}Pu + {}_{0}^{0}\bar{\upsilon} ……. (3).$$

a) Identify ${}_{Z}^{A}X$ and ${}_{Z_1}^{A_1}X$.

b) Write down the global (overall) balanced equation of the nuclear reaction between an uranium 238 nucleus and a neutron leading to plutonium 239. [This reaction is denoted as reaction (4)].

c) Specify for each of the preceding reactions whether it is spontaneous or provoked.

3. The plutonium ${}_{94}^{239}Pu$ may react with a neutron according to the following reaction:

$${}_{94}^{239}Pu + {}_{0}^{1}n \longrightarrow {}_{42}^{102}Mo + {}_{52}^{135}Te + 3\,{}_{0}^{1}n ………..(5)$$

a) Calculate, in MeV/c^{2}, the mass defect Δm in this reaction.

b) Deduce, in MeV, the amount of energy E liberated by the fission of one plutonium nucleus.

c) Calculate, in joules, the energy liberated by the fission of one kilogram of plutonium.

4. We suppose that each fission reaction produces 3 neutrons. Using the preceding reactions, show that the role of one of the three neutrons agrees with the statement of the selection: «... produce, starting from uranium 238 an amount of fissionable material that is equal or exceeds what the reactor consumes…».

VI-GS 2008 1st

Nuclear Fission

The object of this exercise is to show evidence of certain properties of nuclear fission.

Given the masses of nuclei and particles:

- $m\left({}^{235}U\right) = 234.964\ u$;
- $m\left({}^{92}Zr\right) = 91.872\ u$;
- $m\left({}^{142}Te\right) = 141.869\ u$;
- $m\left({}^{1}_{0}n\right) = 1.008\ u$;

$1u = 1.66 \times 10^{-27}\ kg$;

$c = 3 \times 10^{8}\ m/s$.

Part A

Energy of fission

One of the fission reactions of the uranium 235, in a nuclear power plant may be written in the form

$$ {}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{92}_{40}Zr + {}^{142}_{Z}Te + x\,{}^{1}_{0}n $$

1. Determine Z and x specifying the laws used.
2. Calculate the energy produced by the fission of one nucleus of uranium 235.
3. Determine the mass of uranium 235 used in the power plant during one year, knowing that its useful electric power is $900\ MW$, and that its efficiency is 30%.

Part B

Products of fission

Among the products of fission, we find, in the core of the reactor, the radioelement ${}^{137}_{55}Cs$ and ${}^{87}_{37}Rb$ of periods 30 years and 5×10^{11} years respectively.

These radioelements are placed in a pool called cooler. The nuclei ${}^{137}_{55}Cs$ and ${}^{87}_{37}Rb$ have the masses $137\ u$ and $87\ u$ respectively.

1. Suppose that $1g$ of each of the radioelements is introduced into the pool at the instant $t_0 = 0$.
   a) Calculate the number of nuclei of each of the radioelements at the instant $t_0 = 0$.
   b) Deduce, for each radioelement, the number of nuclei remaining after 3 years stay in the pool.
   c) Determine, for each radioelement, the number of decays per day at the moment of taking them out of the pool (3 years later).
2. Assuming that the danger of a radioelement on man depends on the radiations accumulated per day, which, of the two radioelements, is more dangerous? Justify.

Part C

Probability of fission

In a physics dictionary, we read that the probability of a nucleus ${}^{A}_{Z}X$ to become fissionable is proportional to the ratio $\frac{Z^2}{A}$, called the stability factor of a nucleus. This probability is no more zero when this ratio exceeds 35.

1. What do each of Z and A of the nuclide ${}^{A}_{Z}X$ represent?

2. Show that a nucleus must contain a number of neutrons N such that $N < \frac{Z(Z-35)}{35}$, so that the probability to become fissionable is not zero.

3. Determine the maximum number of nucleons that must be contained in a uranium nucleus, of $Z = 92$, so that the probability to undergo fission is not zero.

VIII-GS 2006 1st
Atomic Nucleus

The object of this exercise is to compare the values of physical quantities characterizing the stability of different nuclei and to verify that, during nuclear reactions, certain nuclei are transformed into more stable nuclei with the liberation of energy.

Part A (See Chapter – Nucleus)

Part B
Nuclear reactions and stability of the nuclei

Consider the following three nuclear reactions:

$${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon} \quad \text{(Reaction 1);}$$

$${}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + 2\,{}^{1}_{0}n \quad \text{(Reaction 2);}$$

$${}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n \quad \text{(Reaction 3).}$$

1. Indicate the type of each nuclear reaction (fission, radioactivity or fusion).

2. a) Show that each of the above nuclear reactions liberates energy.

b) Referring to the above table, verify that in each of these nuclear reactions, each of the produced nuclei is more stable than the initial nuclei.

VIII-GS 2005 1st
Fuel and a Power Plant

The object of this exercise is to compare the masses of different fuels used in power plants producing the same electric power. The power plant, of electric power $P = 3\times10^{9}W$, has an efficiency supposed to be 30 % whatever the nature of the fuel used be.

Part A
Energy furnished by the fuel

Calculate, in J, the energy furnished by the fuel during 1 day.

Given

- $1u = 931.5\,MeV/c^2 = 1.66\times10^{-27}\,kg$;
- $1MeV = 1.6\times10^{-13}\,J$;

Masses of nuclei and particles

- $m({}^{1}_{1}H) = 1.00728\,u$; $\quad m({}^{4}_{2}He) = 4.00150\,u$;
- $m({}^{4}_{2}X) = 0.00055\,u$; $\quad m({}^{235}U) = 235.04392\,u$.

Part B

First situation

Thermal power plant

The power plant uses fuel-oil. The combustion of $1 kg$ of this fuel-oil liberates $4.5\times10^{7}\ J$ of energy.

Calculate, in kg, the mass m_1 of fuel-oil consumed during $1 day$.

Second situation

Power plant using nuclear fission

In the power plant, we use uranium enriched with ^{235}U. One of the fission reactions is:

$$^{235}_{92}U+{}^{1}_{0}n\longrightarrow {}^{94}_{38}Sr+{}^{140}_{54}Xe+2\left({}^{1}_{0}n\right)$$

1. In order that fission reaction may take place, the neutron used must satisfy a condition. What is it?
2. The fission of uranium 235 nucleus liberates energy of 189 MeV.
   a) In what form does this energy appear?
   b) Calculate, in kg, the mass m_2 of uranium 235 necessary for the power plant to function during 1 day.

Third situation

Power plant using nuclear fusion

The thermonuclear fusion reaction has not yet been controlled. If such controlling becomes within reach, we may provoke reactions like those taking place in the Sun.

The balanced fusion reaction of hydrogen in the Sun may be written as:

$$4\,{}^{1}_{1}H\longrightarrow {}^{4}_{2}He+2\,{}^{A}_{Z}X.$$

1. Identify the particle ${}^{A}_{Z}X$ specifying the laws used.
2. What condition must be satisfied for this fusion to take place?
3. Determine, in J, the energy liberated in the formation of a helium nucleus.
4. Calculate, in kg, the mass m_3 of hydrogen necessary for the power plant to function 1day.

Part C

Suggest the mode that is the most convenient for the production of electric energy for a country. Justify your answer.

Solutions

I-GS 2014 1st

Part A

1\. a) An isotope is called fissile if it undergoes a nuclear fission under the impact of a thermal neutron.

b) ${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{94}_{Z}Sr + {}^{140}_{54}Xe + x\,{}^{1}_{0}n$;

Conservation of mass number: $235 + 1 = 94 + 140 + x(1)$, then $x = 2$;

Conservation of charge number: $92 + 0 = Z + 54$, then $Z = 38$;

The equation of the nuclear reaction is ${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{94}_{38}Sr + {}^{140}_{54}Xe + 2\,{}^{1}_{0}n$.

2\. The binding energies :

- For uranium: $E_B(U) = \left(\dfrac{E_B(U)}{A}\right) \times A = 7.5MeV \times 235 = 1762.5\,MeV$;
- For xenon: $E_B(Xe) = \left(\dfrac{E_B(Xe)}{A}\right) \times A = 8.2MeV \times 140 = 1148\ MeV$;
- For strontium: $E_B(Sr) = \left(\dfrac{E_B(Sr)}{A}\right) \times A = 8.5MeV \times 94 = 799\ MeV$.

3\. a) The binding energy of a nucleus ${}^{A}_{Z}X$ is given by : $E_B = \Delta m\, c^2$;

Where $\Delta m = Z\,m_p + (A - Z)m_n - m_X$ then $m_X = Z\,m_p + (A - Z)m_n - \dfrac{E_B(X)}{c^2}$;

b) The energy liberated is given by: $E_\ell = \Delta m\, c^2$; where $\Delta m = (m_n + m_U) - (m_{Sr} + m_{Xe} + 2m_n)$;

$$\Delta m = \left(m_n + 92m_p + 143m_n - \frac{E_B(U)}{c^2}\right) - \left(38m_p + 56m_n - \frac{E_B(Sr)}{c^2} + 54m_p + 86m_n - \frac{E_B(Xe)}{c^2} + 2m_n\right)$$

Then $\Delta m = \dfrac{E_B(Xe)}{c^2} + \dfrac{E_B(Sr)}{c^2} - \dfrac{E_B(U)}{c^2}$;

Thus, $E_\ell = \Delta m\, c^2 = \left(\dfrac{E_B(Xe)}{c^2} + \dfrac{E_B(Sr)}{c^2} - \dfrac{E_B(U)}{c^2}\right)c^2 = E_B(Xe) + E_B(Sr) - E_B(U)$;

c) $E_\ell = 799\,MeV + 1148\,MeV - 1762.5MeV = 184.5MeV$.

4\. The efficiency $\eta = \dfrac{P_{\text{electrical}}}{P_{\text{nuclear}}}$; so $P_{\text{nuclear}} = \dfrac{P_{\text{electrical}}}{\eta} = \dfrac{1350\ MW}{0.3} = 4500\ MW$;

The daily nuclear energy furnished $E_n = P_{\text{nuclear}} \times \Delta t = 4500 \times 10^6 \times 24 \times 3600 = 3.888 \times 10^{14}\,J$;

The number of uranium nuclei liberating this energy: $N = \dfrac{E_n}{E_\ell} = \dfrac{3.888 \times 10^{14}\,J}{200 \times 1.6 \times 10^{-13}\,J} = 1.215 \times 10^{25}$

The mass of these nuclei is given by:

$$m = \frac{N}{N_A} \times M = \frac{1.215 \times 10^{25}}{6.02 \times 10^{23}\,mo\ell^{-1}} \times 235 \times 10^{-3}\,kg.mo\ell^{-1} = 4.74\,kg.$$

Part B

1. a) $^{131}_{53}I \longrightarrow {}^{A}_{Z}Xe + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon} + \gamma$;

Conservation of mass number: $131 = A + 0$, then $A = 131$;

Conservation of charge number: $53 = Z - 1$, then $Z = 54$;

The equation of the disintegration is $^{131}_{53}I \longrightarrow {}^{131}_{54}Xe + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon} + \gamma$.

b) The emission of the radiation γ is due to the dis-excitation of the daughter nucleus of Xe.

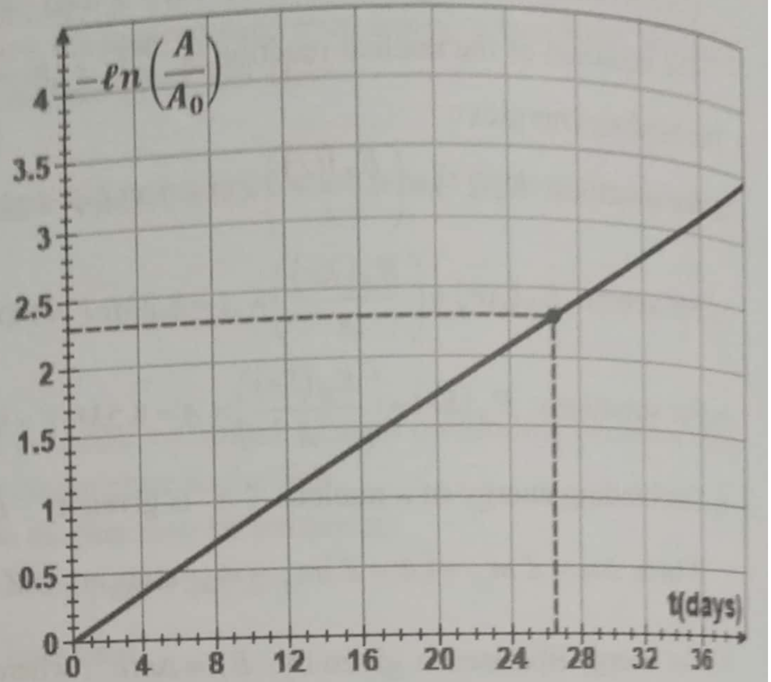
2. a) The radioactive constant $\lambda = \frac{\ell n(2)}{T} = \frac{\ell n(2)}{8} = 0.087$ day^{-1}.

b) Law of radioactive decay $A = A_0 e^{-\lambda t}$;

Then $-\ell n\left(\frac{A}{A_0}\right) = -\ell n\left(\frac{A_0 e^{-\lambda t}}{A_0}\right)$

$= -\ell n\left(e^{-\lambda t}\right) = \lambda t$.

c) The graphical representation of $-\ell n\left(\frac{A}{A_0}\right)$ in terms of t is a straight line passing through origin.



d) If $A = \frac{A_0}{10}$, so $\frac{A}{A_0} = 0.1$;

Then $-\ell n\left(\frac{A}{A_0}\right) = -\ell n(0.1) = 2.3$;

Referring to graph, for $t = 27$ days, the effect of iodine on organism becomes approximately negligible.

II-GS 2013 1st

Part A

1. a) $^{236}_{92}U^{*} \longrightarrow {}^{236}_{92}U + \gamma$.

b) The excess of energy is $20\ MeV$.

2. a) $^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{138}_{x}Ba + {}^{y}_{36}Kr + 3\,{}^{1}_{0}n + \gamma$;

Conservation of mass number: $1 + 235 = 138 + y + 3$, then $y = 95$;

Conservation of charge number: $0 + 92 = x + 36 + 0$, then $x = 56$;

Thus, the nuclear reaction $^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{138}_{56}Ba + {}^{95}_{36}Kr + 3\,{}^{1}_{0}n + \gamma$.

b) The mass defect $\Delta m = m_{before} - m_{after} = \left[m\left({}^{1}_{0}n\right) + m\left({}^{235}_{92}U\right)\right] - \left[m\left({}^{138}_{56}Ba\right) + m\left({}^{95}_{36}Kr\right) + 3m\left({}^{1}_{0}n\right)\right]$;

$= (1.0087 + 234.9934) - (137.8742 + 94.8871 + 3 \times 1.0087) = 0.2147\ u$;

The energy liberated $E_\ell = \Delta m c^2 = 0.2147 \times 931.5\ MeV \approx 200\ MeV$.

c) The number of uranium nuclei contained in $1g$ is:

$N = \frac{m}{M} \times N_A = \frac{1g}{235\,g.mo\ell^{-1}} \times 6.02 \times 10^{23}\ mo\ell^{-1} = 2.56 \times 10^{21}$ nuclei;

The energy liberated by $1g$ is $E = N E_\ell = 2.56 \times 10^{21} \times 200 \times 1.6 \times 10^{-13} = 8.19 \times 10^{10}\ J$.

d) The daily nuclear energy produced by $2.8\,kg$ is:

$E_d = 2800 \times E = 2800 \times 8.19 \times 10^{10} = 2.29 \times 10^{14}\,J$;

The daily electrical energy provided is $E_e = P_e \times \Delta t = 800 \times 10^6 \times 24 \times 3600 = 6.91 \times 10^{13}\,J$;

The efficiency of the power plant is $\eta = \dfrac{E_e}{E_d} = \dfrac{6.91 \times 10^{13}}{2.29 \times 10^{14}} \approx 0.3 = 30\%$.

Part B

1. a) We have ${}^1_0n + {}^{235}_{92}U \longrightarrow {}^{138}_{56}Ba + {}^{95}_{36}Kr + 3\,{}^1_0n + \gamma$;

The global conservation of energy:

$$m\left({}^1_0n\right)_{in} c^2 + KE\left({}^1_0n\right)_{in} + m(U)\,c^2 + KE(U)$$
$$= KE(Ba) + m(Ba)c^2 + KE(Kr) + m(Kr)c^2 + 3m\left({}^1_0n\right)_{em} c^2 + 3KE\left({}^1_0n\right)_{em} + E(\gamma)$$

We get, $\left[m\left({}^1_0n\right) + m(U)\right]c^2 - \left[m(Ba) + m(Kr) + 3\,m\left({}^1_0n\right)\right]c^2 =$

$$= \left[KE(Ba) + KE(Kr) + 3KE\left({}^1_0n\right)_{em} + E(\gamma)\right] - \left[KE\left({}^1_0n\right)_{in} + KE(U)\right]$$

Then $E_\ell = \left[KE(Ba) + KE(Kr) + 3KE\left({}^1_0n\right)_{em} + E(\gamma)\right] - \left[KE\left({}^1_0n\right)_{in} + KE(U)\right]$;

But $KE(Ba) + KE(Kr) = 174\,MeV$, $KE(U) = 0$ (at rest);

& $KE\left({}^1_0n\right)_{in} = 0.04eV = 4\times10^{-8}MeV$;

So, $200 = 174 + 3KE\left({}^1_0n\right)_{em} + 20 - 4\times10^{-8} \Rightarrow KE_{em} = \dfrac{\left(200 + 4\times10^{-8}\right) - (174+20)}{3} \approx 2MeV$.

b) The kinetic energy of the emitted electrons $2MeV$ is very large compared to that required to the occurrence of a fission reaction of $0.04eV$.

2. a) Conservation of linear momentum:

	neutron m_n	carbon m_c
Just before	$\overrightarrow{v_0}$	$\overrightarrow{0}$
Just after	$\overrightarrow{v_1}$	$\overrightarrow{v_2}$

$\overrightarrow{P}_{\text{just before collision}} = \overrightarrow{P}_{\text{just after collision}}$;

$m_n\overrightarrow{v_0} + m_C\overrightarrow{0} = m_n\overrightarrow{v_1} + m_C\overrightarrow{v_2}$;

The collision is collinear, then we may use algebraic expressions: $m_n(v_0 - v_1) = m_C v_2$ ....(1)

The collision is elastic, then the kinetic energy is conserved:

$$KE_{\text{just before collision}} = KE_{\text{just after collision}};$$

$\frac{1}{2}m_n v_0^2 + 0 = \frac{1}{2}m_n v_1^2 + \frac{1}{2}m_C v_2^2 \Rightarrow \frac{1}{2}m_n\left(v_0^2 - v_1^2\right) = \frac{1}{2}m_C v_2^2$;

$m_n\left(v_0^2 - v_1^2\right) = m_C v_2^2 \Rightarrow m_n(v_0 - v_1)(v_0 + v_1) = m_C v_2^2$ ....(2).

Dividing (2) by (1) we get: $v_0 + v_1 = v_2$ ....(3)

$\begin{cases} v_0 + v_1 = v_2 \\ v_0 - v_1 = \dfrac{m_C}{m_n}v_2 \end{cases}$; $\begin{cases} v_0 + v_1 = v_2 \\ v_0 - v_1 = 12\,v_2 \end{cases} \Rightarrow \begin{cases} -12v_0 - 12v_1 = -12v_2 \\ v_0 - v_1 = 12\,v_2 \end{cases}$

Then $-11v_0 - 13v_1 = 0 \Rightarrow \dfrac{v_1}{v_0} = -\dfrac{11}{13}$; thus, $\left|\dfrac{v_1}{v_0}\right| = \left|\dfrac{11}{13}\right| = k$.

b) Kinetic energy

i- $\frac{KE_1}{KE_0}=\frac{\frac{1}{2}m_n v_1^2}{\frac{1}{2}m_n v_0^2}=\left(\frac{v_1}{v_0}\right)^2=\left(\frac{11}{13}\right)^2=k^2.$

ii- The ratio of successive kinetic energies form a geometric sequence of common ratio k^2, then after n collisions, the nth kinetic energy becomes $KE_n=KE_0\times\left(k^2\right)^n=KE_0\,k^{2n}$;

$k^{2n}=\frac{KE_n}{KE_0}$; $2n\times\ell n(k)=\ell n\left(\frac{KE_n}{KE_0}\right)=\ell n\left(\frac{0.04}{2\times10^6}\right)=\ell n\left(2\times10^{-8}\right)$;

$n=\frac{1}{2}\times\frac{\ell n\left(2\times10^{-8}\right)}{\ell n\left(\frac{11}{13}\right)}\approx 53$ collisions.

III-GS 2012 2nd

1. a) Isotopes are nuclei having same atomic number Z but different mass number A.

b) For uranium 235 $\left({}^{235}_{92}U\right)$: 92protons & 143 neutrons;

For uranium 238 $\left({}^{238}_{92}U\right)$: 92protons & 146 neutrons.

2. a) Reaction 1: ${}^{238}_{92}U+{}^{1}_{0}n\longrightarrow{}^{A}_{Z}X$; according to Soddy's laws: $A=239$ & $Z=92$;

Thus, X is an isotope of uranium.

${}^{238}_{92}U+{}^{1}_{0}n\longrightarrow{}^{239}_{92}U$;

Reaction 2: ${}^{239}_{92}X\longrightarrow{}^{0}_{-1}e+{}^{A'}_{Z'}Y+{}^{0}_{0}\overline{\upsilon}$; according to Soddy's laws: $A'=239$ & $Z'=93$;

Thus, Y is an isotope of neptunium.

${}^{239}_{92}X\longrightarrow{}^{0}_{-1}e+{}^{239}_{93}Np+{}^{0}_{0}\overline{\upsilon}$

Reaction 3: ${}^{239}_{93}Np\longrightarrow{}^{0}_{-1}e+{}^{239}_{94}Pu+{}^{0}_{0}\overline{\upsilon}$ (reaction 3).

b) The addition of the reactions (1);(2)& (3) gives: ${}^{238}_{92}U+{}^{1}_{0}n\longrightarrow{}^{239}_{94}Pu+2\left({}^{0}_{-1}e\right)+2\left({}^{0}_{0}\overline{\upsilon}\right)$.

3. a) ${}^{239}_{94}Pu+{}^{1}_{0}n\longrightarrow{}^{A}_{Z}B+{}^{135}_{52}Te+3\left({}^{1}_{0}n\right)$ (reaction 5);

Conservation of mass number: $239+1=A+135+3$, then $A=102$;

Conservation of charge number: $94+0=Z+52+3$, then $Z=42$;

Thus, B is the molybdenum ${}^{102}_{42}Mo$.

b) The mass defect: $\Delta m=m_{before}-m_{after}=\left[m\left({}^{239}_{94}Pu\right)+m\left({}^{1}_{0}n\right)\right]-\left[m\left({}^{102}_{42}Mo\right)+m\left({}^{135}_{52}Te\right)+3m\left({}^{1}_{0}n\right)\right]$

$=(239.0533+1.0087)-(101.9103+134.9167+3\times1.0087)=0.2086\,u$;

$\Delta m=0.2086\,u=0.2086\times931.5MeV/c^2=194.3MeV/c^2$;

c) The energy liberated by the fission of one nucleus is :

$E_\ell=\Delta m c^2=194.3MeV/c^2\times c^2=194.3\,MeV$.

d) The number of plutonium nuclei in 1kg $N=\frac{m_{\text{sample}}}{m_{\text{nucleus}}}=\frac{1kg}{239.0533\times1.66\times10^{-27}kg}=2.52\times10^{24}$

The total energy liberated $E_t=N\times E_\ell=2.52\times10^{24}\times194.3\times1.6\times10^{-13}=7.83\times10^{13}J$.

5. Under the action of an incident neutron, a plutonium nucleus reacts according to the equation (5) and liberates three neutrons during its fission.

For these three neutrons:

- one is used to sustain the fission reaction of plutonium;
- the two others are available to react with the uranium 238 to create two new plutonium atoms.

For one fissile plutonium nucleus consumed, two fissile plutonium nuclei are created. This justifies the name of breeder reactor giving to such reactor.

N-GS 2010 2nd

Part A

$^{238}_{92}U + ^{1}_{0}n \longrightarrow ^{A}_{53}I + ^{94}_{Z}Y + 3^{1}_{0}n$

Conservation of mass number: $235 + 1 = A + 94 + 3$, then $A = 139$;

Conservation of charge number: $92 + 0 = 53 + Z + 0$, then $Z = 39$;

Thus, the nuclear reaction is $^{238}_{92}U + ^{1}_{0}n \longrightarrow ^{139}_{53}I + ^{94}_{39}Y + 3^{1}_{0}n$.

Part B

1. The mass defect: $\Delta m = m_{before} - m_{after} = \left[m\left(^{238}_{92}Pu\right) + m\left(^{1}_{0}n\right)\right] - \left[m\left(^{139}_{53}I\right) + m\left(^{94}_{39}Y\right) + 3m\left(^{1}_{0}n\right)\right]$;

We get $\Delta m = (234.99332 + 1.00866) - (138.89700 + 93.89014 + 3\times 1.00866) = 0.18886\ u$;

2. The energy liberated by the fission of nucleus $E_\ell = \Delta m c^2 = 0.18886 \times 931.5 MeV = 175.92 MeV$.

3. The energy carried by the neutron is $E_0 = 1\% E = 0.01 \times 175.92 = 1.7592\ MeV$.

4. a) Conservation of linear momentum: $\vec{P}_{\text{just before collision}} = \vec{P}_{\text{just after collision}}$;

$$m_n \vec{v_0} + \vec{0} = m_n \vec{v_1} + 2m_n \vec{v'};$$

The collision is collinear, then we may use algebraic expressions: $m_n v_0 = m_n v_1 + 2m_n v'$;

Then, $m_n(v_0 - v_1) = 2m_n v' \Rightarrow v_0 - v_1 = 2v'$ ....(1)

The collision is elastic, then kinetic energy is conserved:

$$KE_{\text{just before collision}} = KE_{\text{just after collision}};$$

$\frac{1}{2}m_n v_0^2 + 0 = \frac{1}{2}m_n v_1^2 + \frac{1}{2}2m_n v'^2$; we get $v_0^2 - v_1^2 = 2v'^2$.........(2)

Dividing (2) by (1) we get: $v_0 + v_1 = v'$ ....(3);

$(1) - 2\times(3): -v_0 - 3v_1 = 0$; then $v_1 = -\frac{1}{3}v_0$.

Thus, the neutron rebounds with one third (1/3) of its initial speed.

b) After the first collision, the kinetic energy of the neutron is:

$$E_1 = \frac{1}{2}m_n v_1^2 = \frac{1}{2}m_n\left(-\frac{1}{3}v_0\right)^2 = \frac{1}{9}\left(\frac{1}{2}m_n v_0^2\right) = \frac{1}{9}E_0;$$

c) After k collisions, the kinetic energy of the neutron becomes: $KE_k = \left(\frac{1}{9}\right)^k E_0$.

d) $KE_k = \left(\frac{1}{9}\right)^k E_0 = 0.025 eV$; then $0.025 eV = \frac{1}{9^k} 1.759 \times 10^6 eV$;

We get $9^{k} = \dfrac{1.759\times10^{8}}{0.025}$; thus, $k = \dfrac{\ell n\left(\dfrac{1.759\times10^{8}}{0.025}\right)}{\ell n(9)} \approx 9$ (whole number).

V-GS 2010 1st

1. *«... Starting from uranium 238 an amount of fissionable material that is equal or exceeds what the reactor consumes since the final result would be the consumption of uranium 238 only which is more abundant than the uranium 235.....»*

2. a) ${}^{238}_{92}U + {}^{1}_{0}n \longrightarrow {}^{A}_{Z}X$...... (1)

Conservation of mass number: $238 + 1 = A$, then $A = 239$;

Conservation of charge number: $92 + 0 = Z$, then $Z = 92$;

So, ${}^{A}_{Z}X = {}^{239}_{92}U$ is an isotope of uranium.

Thus, ${}^{238}_{92}U + {}^{1}_{0}n \longrightarrow {}^{239}_{92}U$..... (1)

${}^{A_1}_{Z_1}X \longrightarrow {}^{239}_{94}Pu + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon}$........ (3)

Conservation of mass number: $A_1 = 0 + 239$, then $A_1 = 239$;

Conservation of charge number: $Z_1 = -1 + 94 + 0$, then $Z_1 = 93$;

So, ${}^{A_1}_{Z_1}X = {}^{239}_{93}Np$ is the Neptunium.

Thus, ${}^{239}_{93}X \longrightarrow {}^{239}_{94}Pu + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon}$........ (3)

b) Adding the relations $(1) + (2) + (3)$:

${}^{238}_{92}U + {}^{1}_{0}n \longrightarrow {}^{A}_{Z}X$...... (1)

${}^{A}_{Z}X \longrightarrow {}^{0}_{-1}e + {}^{A_1}_{Z_1}X + {}^{0}_{0}\overline{\upsilon}$....... (2)

${}^{A_1}_{Z_1}X \longrightarrow {}^{239}_{94}Pu + {}^{0}_{-1}e + {}^{0}_{0}\overline{\upsilon}$........ (3)

${}^{238}_{92}U + {}^{1}_{0}n \longrightarrow {}^{239}_{94}Pu + 2\,{}^{0}_{-1}e + 2\,{}^{0}_{0}\overline{\upsilon}$..........(4)

c) (1) & (4) are provoked reaction; whereas (2) & (3) are spontaneous.

3. a) The mass defect:

$$\Delta m = m_{before} - m_{after} = \left[m\left({}^{239}_{94}Pu\right) + m\left({}^{1}_{0}n\right)\right] - \left[m\left({}^{102}_{42}Mo\right) + m\left({}^{135}_{52}Te\right) + 3m\left({}^{1}_{0}n\right)\right];$$

$$= 0.2086\,u = 0.2086\times931.5MeV/c^2 = 194.31MeV/c^2;$$

b) The energy liberated: $E_\ell = \Delta m c^2 = 194.31\left(MeV/c^2\right)\times c^2 = 194.31MeV$.

c) The mass of polonium 239 nucleus :

$m_{Po} = 239u = 239\times1.6605\times10^{-27}kg = 3.9686\times10^{-25}kg$;

The number of nuclei in $1kg$ is:

$N = \dfrac{m}{m_{Po}} = \dfrac{1kg}{3.9686\times10^{-25}kg} = 2.52\times10^{24}$ nuclei;

The total energy $E_t = N\times E_\ell = 2.52\times10^{24}\times194.31MeV = 4.9\times10^{26}MeV = 7.83\times10^{13}J$;

4. A neutron interacts with uranium 238 in order to form another nucleus of plutonium. This shows that the plutonium nuclei are in excess in the population of nuclei.

Part A

1. $^{235}_{92}U+^{1}_{0}n \longrightarrow ^{92}_{40}Zr+^{142}_{Z}Te+x\,^{1}_{0}n$;

Conservation of mass number: $235+1=92+142+x$, then $x=2$;

Conservation of charge number: $92=40+Z$, then $Z=52$;

The nuclear reaction $^{235}_{92}U+^{1}_{0}n \longrightarrow ^{92}_{40}Zr+^{142}_{52}Te+2\,^{1}_{0}n$.

2. The mass defect is : $\Delta m=\left[m\left(^{235}_{92}U\right)+m\left(^{1}_{0}n\right)\right]-\left[m\left(^{92}_{40}Zr\right)+m\left(^{142}_{Z}Te\right)+2\left(^{1}_{0}n\right)\right]$;

$\Delta m=234.964-91.872-141.869-1.008=0.215\,u=3.569\times10^{-28}\,kg$;

The energy liberated by this reaction $E=\Delta m c^2=3.569\times10^{-28}\times\left(3\times10^8\right)^2=3.21\times10^{-11}\,J$.

3. The power delivered by the uranium is $\eta=\dfrac{P_e}{P_{nu}}$, then $P_{nu}=\dfrac{900\times10^6}{0.3}=3\times10^9\,W$.

- The energy furnished by the uranium fuel in one year:

$E_{nu}=P_{nu}\times\Delta t=3\times10^9\times365\times24\times3600=9.46\times10^{16}\,J$.

- The number of nuclei undergoing fission required to produce this energy is:

$N=\dfrac{E_{nu}}{E_\ell}=\dfrac{9.46\times10^{16}}{3.21\times10^{-11}}=2.95\times10^{27}$ nuclei.

- The mass of these nuclei is $m=2.95\times10^{27}\times234.964\times1.66\times10^{-27}=1151\,kg$.

Part B

1. a) The number of cesium nuclei is:

$N_0(Cs)=\dfrac{m_0}{m_{\text{nucleus}}(Cs)}=\dfrac{1\times10^{-3}\,kg}{137\times1.66\times10^{-27}\,kg}=4.4\times10^{21}$ nuclei.

The number of rubidium nuclei is $N_0(Rb)=\dfrac{1\times10^{-3}\,kg}{87\times1.66\times10^{-27}\,kg}=6.9\times10^{21}$ nuclei.

b) The number of remaining nuclei of Cs is $N(Cs)=4.4\times10^{21}e^{-\frac{\ell n2}{30}\times3}=4.1\times10^{21}$ nuclei.

The number of rubidium nuclei will be practically unchanged $N(Rb)=N_0(Rb)$, because its period 5×10^{11} is very large compared to the duration.

c) The activity of Cs is $A_{Cs}=\lambda N=\dfrac{\ell n2}{T_{Cs}}N_{Cs}=\dfrac{\ell n2}{30\times365}\times4.1\times10^{21}=2.6\times10^{17}$ decays per day;

The activity of Rb is $A_{Rb}=\dfrac{\ell n2}{T_{Rb}}N_{Rb}=\dfrac{\ell n2}{5\times10^{11}\times365}\times6.9\times10^{21}=2.6\times10^{7}$ decays per day.

2. The cesium is more dangerous since its activity is very large compared to that of rubidium.

Part C

1. Z is the charge number & A is the mass number.

2. We have $\dfrac{Z^2}{A}>35$; $\dfrac{Z^2}{Z+N}>35$; so $Z(Z-35)>35\,N$; then $N<\dfrac{Z(Z-35)}{35}$.

3. If $\frac{Z^2}{A} > 35$, then $A < \frac{Z^2}{35} = \frac{(92)^2}{35} \approx 242$, so the maximum number of nucleons is $A_{max} = 241$.

VII-GS 2006 1st

See Page 118

Part B

1. ${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}e + {}^{0}_{0}\bar{\upsilon}$, spontaneous transformation, natural radioactivity.

${}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + 2\,{}^{1}_{0}n$, light nuclei combine to form a heavier nucleus, so it is nuclear fusion

${}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n$, under the impact of a neutron, a uranium nucleus is divided into two lighter nuclei so we have a nuclear fission.

2. a) The mass defect in each of the reactions.
   - Reaction (1): $\Delta m = 0.00285\ u > 0$; exo-energetic reaction, it liberates energy.
   - Reaction (2): $\Delta m = 0.0189 > 0$; exo-energetic reaction, it liberates energy.
   - Reaction (3): $\Delta m = 0.1983 > 0$; exo-energetic reaction, it liberates energy.

   b) A nucleus is more stable when its binding energy per nucleon is greater; referring to the table:
   - The nucleus of ${}^{14}_{7}N$ in the reaction (1) is more stable than carbon $\left({}^{14}_{6}C\right)$.
   - The nucleus ${}^{4}_{2}He$ in the reaction (2) is more stable than hydrogen isotopes ${}^{2}_{1}H$ and ${}^{3}_{1}H$.
   - The strontium and xenon nuclei ${}^{94}_{38}Sr$ and ${}^{140}_{54}Xe$ are more stable than uranium ${}^{235}_{92}U$.

VIII-GS 2005 1st

Part A

The power furnished by the fuel is: $\eta = \frac{P_{electrical}}{P_{fuel}}$, so $P_{fuel} = \frac{P_{electrical}}{\eta} = 10^{10}\,W$;

The energy daily furnished by the fuel (whatever its nature) is:

$E_{fuel} = P_{fuel} \times \Delta t = 10^{10} \times 24 \times 3600 = 8.64 \times 10^{14}\,J$.

Part B

First situation

The energy liberated by the fuel oil is proportional to the mass consumed:

$$m_1 = \frac{8.64 \times 10^{14}\,J}{4.5 \times 10^{7}\,J/kg} = 19.2 \times 10^{6}\,kg.$$

Second situation

1. The neutron must be thermal (a slow neutron carrying a kinetic energy of the order $0.1eV$).
2. a) The energy liberated appears as:
   - kinetic energy carried by the daughter nuclei and the neutrons.
   - radiant energy carried by the gamma radiations.

   b) According to the mass-energy equivalence $E = \Delta m\,c^2$, the energy liberated is proportional to the mass of uranium consumed:

$\frac{E}{m_2}=\frac{E_\ell}{m(U)}$, then $m_2=\frac{E}{E_\ell}m(U)=\frac{235.04392\times1.66\times10^{-27}kg\times8.64\times10^{14}J}{189\times1.6\times10^{-13}J}\approx11kg$.

Second method:

1 nucleus of U $\xrightarrow{\text{mass}}$ $235.04392\times1.66\times10^{-27}kg$ $\xrightarrow{\text{liberates}}$ $189\times1.6\times10^{-13}J$;

$m_2 \xrightarrow{\text{liberates}} 8.64\times10^{14}J$

Third situation

1. $4\,{}_1^1H \longrightarrow {}_2^4He+2\,{}_Z^AX$.

Conservation of mass number: $4\times1=4+2A$, then $A=0$;

Conservation of charge number: $4\times1=2+2Z$, then $Z=1$;

The emitted particle is the positron ${}_{+1}^{0}e$.

The nuclear reaction $4\,{}_1^1H \longrightarrow {}_2^4He+2\,{}_{+1}^{0}e+2{}_0^0\upsilon$.

2. The nuclei undergoing the fusion must have a high kinetic energy, of the order of $0.1MeV$ (and performed at high temperature 10^8K).

3. The mass defect is $\Delta m=4m({}_1^1H)-[m({}_2^4He)+2m({}_1^0e)]=0.02652\,u$;

The energy liberated by the nuclear fusion is: $E'_\ell=\Delta m c^2=0.02652\times931.5=24.7MeV$;

$$E'_\ell=24.7MeV=24.7\times1.6\times10^{-13}=3.95\times10^{-12}J.$$

4. The energy liberated is proportional mass of hydrogen consumed: $\frac{E}{m_3}=\frac{E'_\ell}{m(4{}_1^1H)}$;

So $m_3=\frac{E\times m(4{}_1^1H)}{E'_\ell}$; then $m_3=\frac{(4\times1.00728\times1.66\times10^{-27})kg\times8.64\times10^{14}J}{(2.47\times1.6\times10^{-13})J}=1.46\,kg$.

The mass of hydrogen needed is: $m_3=1.46\,kg$.

Part C

The nuclear fusion is the most convenient mode to produce energy because:

- it is a clean, green and sustainable source (no radioactive wastes nor polluting gases).
- it is economical, the hydrogen fuel is very abundant in nature.
- it is very energetic.

However, nuclear fusion is still uncontrolled yet.