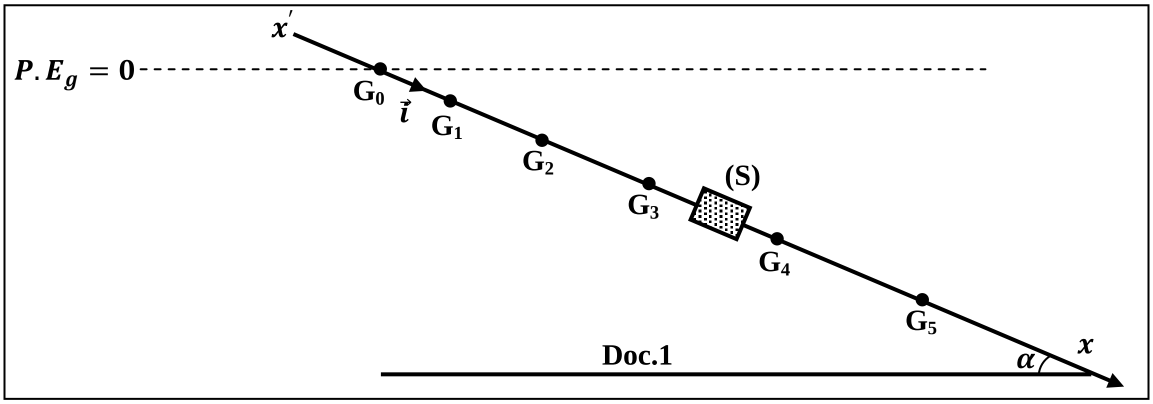
Shakib Irslan High School Physics Department	Physics Extra Sheet Linear Momentum	Academic Year: 2022-2023 Date:
Name: ……………………..		Class and Section: 12 LS

Exercise 1 (1st session 2006 LS): Verification of Newton's second law

A puck (S) of mass M = 100g and of center of mass G, may slide along an inclined track that makes an angle α with the horizontal so that $\sin\alpha = 0.40$. Thus G moves along an axis $x'x$ parallel to the track as shown in document 1. Take g = 10 m/s^2.



We release (S) without initial velocity at the instant $t_0 = 0$ and at the end of each interval of time $\tau = 50ms$, some positions $G_0, G_1, G_2, \ldots G_5$ of G are recorded at the instants $t_0 = 0$,t_1 , t_{2} , … t_5 respectively.

The values of the abscissa x of G $\left(x = \overline{G_0G}\right)$ are given in the table below.

t	0	τ	2τ	3τ	4τ	5τ
x (cm)	0	G_0G_1= 0.50	G_0G_2= 2.00	G_0G_3 = 4.50	G_0G_4 = 8.00	G_0G_5 =12.50

1- Verify that the speed of the puck at the instants $t_2 = 2\tau$ and $t_4 = 4\tau$ are $V_2 = 0.40$ m/s and $V_4 = 0.80$ m/s respectively.

2-

2.1- Calculate the mechanical energy of the system (puck-Earth) at the instants t_0, t_2 and t_4 knowing that the horizontal plane through G_0 is taken as a gravitational potential energy reference.

2.2- Why can we suppose that the puck moves without friction along the rail?

3- Determine the variation in the linear momentum $\Delta\vec{P} = \vec{P}_4 - \vec{P}_2$ of (S) during $\Delta t = t_4 - t_2$.

4-

4.1- Name the forces acting on (S) during its motion.

4.2- Show that the resultant $\sum\vec{F}$ of these forces may be written as $\sum\vec{F} = (Mg\sin\alpha)\vec{\imath}$.

5- Assuming that Δt is very small, $\frac{\Delta\vec{P}}{\Delta t}$ may be considered equal to $\frac{d\vec{P}}{dt}$. Show that Newton's second law is verified between the instants t_2 and t_4.

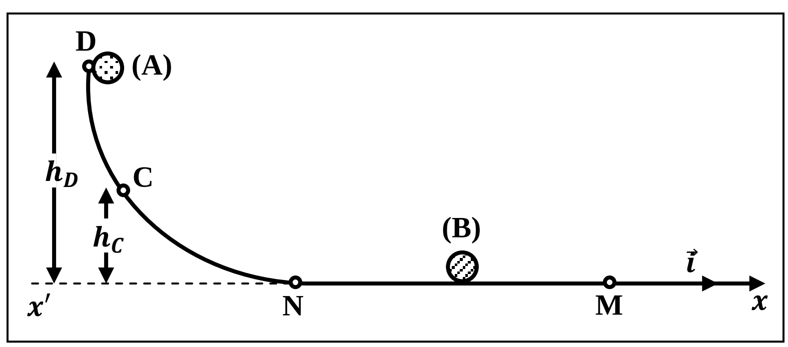
Exercise 2 (1st session 2016 LS): Nature of a collision

The aim of this exercise is to determine the nature of a collision between two objects.

For this aim, an object (A), considered as a particle, of mass m_A = 2kg, can slide without friction on a path situated in a vertical plane and formed of two parts: a circular part DN and a horizontal rectilinear part NM.

(A) is released, without initial velocity, from the point D situated at a height $h_D = 0.45m$ above the horizontal part NM (Fig.1).

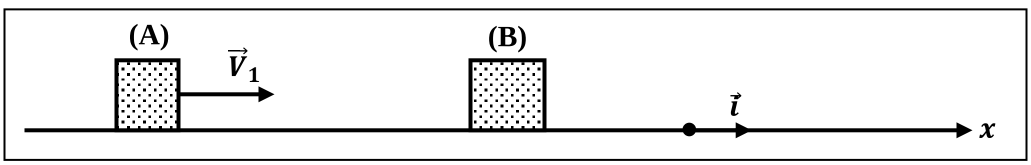
The horizontal plane passing through MN is taken as the reference level of gravitational potential energy. Take g = 10m/s^2.



1- Calculate the mechanical energy of the system [(A), Earth] at the point D.
2- Deduce the speed V_{1A} of (A) when it reaches the point N.
3- (A) reaches N and moves along NM with the same velocity $\vec{V}_{1A} = V_{1A}\vec{\imath}$. Another object (B), considered as a particle, of mass m_B = 4kg moves along the horizontal path from M toward N with the velocity $\vec{V}_{1B} = -1\vec{\imath}$ (V_{1B} in m/s).
   3.1- Determine the linear momentum $\vec{P}_s$ of the system [(A), (B)] before collision.
   3.2- Deduce the velocity $\vec{V}_G$ of the center of inertia G of the system [(A), (B)].
4- After collision, (A) rebounds and attains a maximum height $h_C = 0.27m$.
   4.1- Determine the mechanical energy of the system [(A), Earth] at the point C.
   4.2- Deduce the speed V_{2A} of (A) just after collision.
5- Determine, by applying the principle of the conservation of the linear momentum of the system [(A), (B)], the velocity V_{2B} of (B) just after collision.
6- Specify the nature of the collision.

Exercise 3 (1st session 2015 LS): Collision and interaction

In order to study the collision between two bodies, we consider a horizontal air table equipped with a launcher and two pucks (A) and (B) of respective masses $m_A = 0.4kg$ and $m_B = 0.6kg$.
(A), launched with the velocity $\vec{V}_1 = 0.5\vec{\imath}$, collides with (B) initially at rest.
(A) rebounds with the velocity $\vec{V}_2 = -0.1\vec{\imath}$ and (B) moves with the velocity $\vec{V}_3 = 0.4\vec{\imath}$ (V_1 V_2 and V_3 are expressed in m/s). Neglect all frictional forces.



A- Linear momentum
1-
   1.1- Determine the linear momentums:
      1.1.1- $\vec{P}_1$ and $\vec{P}_2$ of (A), before and after collision respectively;
      1.1.2- $\vec{P}_3$ of (B) after collision.
   1.2- Deduce the linear momentums $\vec{P}$ and $\vec{P}'$ of the system [(A), (B)] before and after collision respectively.
   1.3- Compare $\vec{P}$ and $\vec{P}'$. Conclude.
2-
   2.1- Name the external forces acting on the system [(A), (B)].
   2.2- Give the value of the resultant of these forces.
   2.3- Is this resultant compatible with the conclusion in question (1.3)? Why?

B- Type of collision
1- Determine the kinetic energy of the system [(A), (B)] before and after collision.
2- Deduce the type of the collision.

C- Principle of interaction
The duration of collision is $\Delta t = 0.04s$; we can consider that $\frac{\Delta\vec{P}}{\Delta t} \approx \frac{d\vec{P}}{dt}$.
1- Determine during Δt:
   1.1- the variations $\Delta\vec{P}_A$ and $\Delta\vec{P}_B$ in the linear momentums of the pucks (A) and (B) respectively;
   1.2- the forces $\vec{F}_{A/B}$ exerted by (A) on (B) and $\vec{F}_{B/A}$ exerted by (B) on (A).
2- Deduce that the principle of interaction is verified.

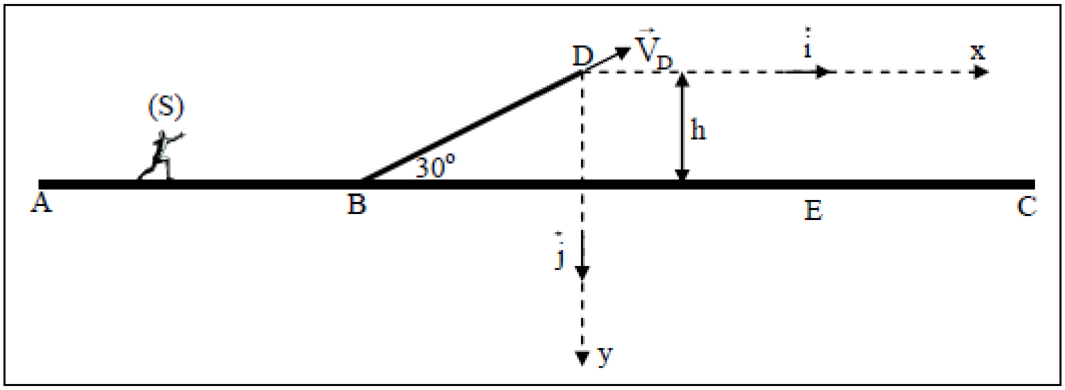
Exercise 4 (2nd session 2012 LS): Study of the motion of a skier

A skier (S), of mass m = 80 kg, is pulled by a boat using a rope parallel to the surface of water. He starts from point A at the instant $t_0 = 0$ without initial velocity.

The skier passes point B at the instant t = 60 s with a speed $V_B = 6$ m/s, then he releases the rope. He continues his motion along a board BD inclined by an angle of 30^o with respect to the horizontal surface of water.

Suppose that during the passage from AB to BD the speed at point B does not change.

The skier arrives point D, situated at an altitude h = 1.6 m from the water surface, with a velocity V_D, then he leaves the board at point D to hit the water surface at point E (see figure below).



Given:

- the skier is considered as a particle;
- on the path AB, the force of traction $\vec{F}$ exerted by the rope on the skier has a constant magnitude F and the whole forces of friction are equivalent to a single force $\vec{f}$ opposite to the displacement, of magnitude $f = 100N$;
- friction is negligible along the path BDE;
- after leaving point D the motion of the skier takes place in the vertical plane Dxy containing $\vec{V}_D$;
- the horizontal plane passing through AB is the reference level of the gravitational potential energy;
- $g = 10 m/s^2$.

A) Motion of the skier between A and B

1- What are the external forces acting on (S) along the path AB? Draw, not to scale, a diagram of these forces.

2- Applying Newton's second law $\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}$ on the skier, between the points A and B, express the acceleration a of the motion of the skier in terms of F, f and m

3- Determine the expression of the speed V of the skier in terms of F, f, m and the time t.

4- Deduce F.

B) Motion of the skier on the board BD

1- Why can we apply the principle of conservation of the mechanical of energy of system [(S), Earth] on the path BD?

2- Deduce that $V_D = 2$ m/s.

C) Motion of the skier between D and E

The skier leaves the board at point D, at an instant t_0, taken as a new origin of time.

1- Apply Newton's second law on the skier to show that, at an instant t, the vertical component P_y of the linear momentum of the skier is of the form: $P_y = 800t - 80$ (In SI unit).

2- Deduce the parametric equation $y(t)$ of the motion of the skier in the frame of reference Dxy.

3- Determine the duration taken by the skier to pass from D to E.

Exercise 5 (2nd session 2007 LS): Mechanical interaction

The object of this exercise is to study some physical quantities of a system whose parts are in mechanical interaction.
For that, we use two pucks (A) and (B), of respective masses $m_A = 100g$ and $m_B = 120g$, that may move without friction on a horizontal table. Each puck is surrounded by an elastic steel shock ring of negligible mass.
The two pucks are connected by a massless and inextensible taut thread thus compressing the steel shock rings. The system (S) thus formed is at rest. (Figure 1)

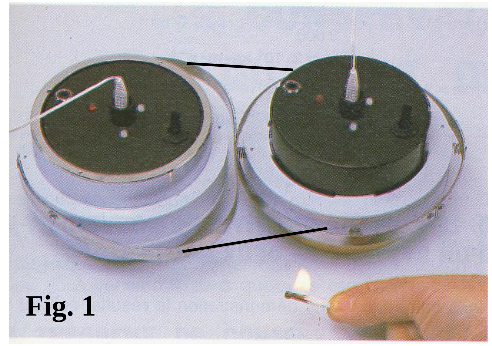


Fig. 1

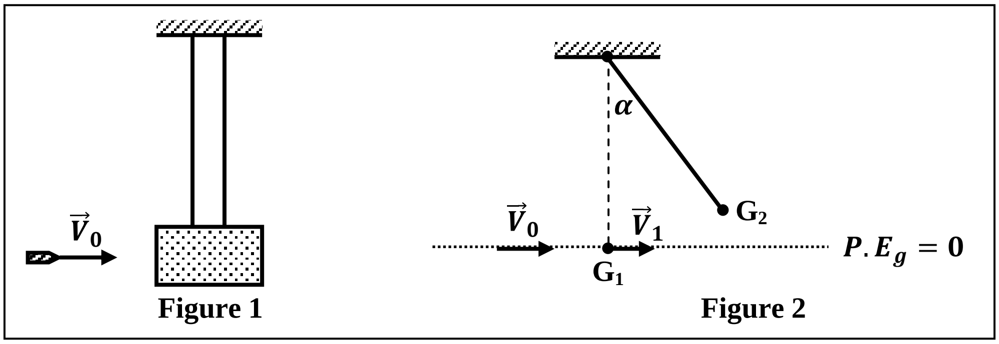
We burn the thread; the shock rings stretch and the pucks repel each other. The system (S) thus formed of the two pucks and the shock rings is said to "*explode*".
The positions of the center of mass of each puck are registered at successive instants separated by a constant time interval $\tau = 50$ ms.
Figure (2) represents, on the axis x'x, the dot-prints of the positions of the centers of masses G_A and G_B of the two pucks after the «explosion».

Fig. 2

1- Using the document of figure (2), show that , after explosion:
 1.1- The motion of each puck is uniform;
 1.2- The speeds of (A) and (B) are $V_A = 1.2m/s$ and $V_B = 1m/s$ respectively.
2- Verify the conservation of the linear momentum of the system (S) during explosion.
3- Applying Newton's second law $\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}$ on each puck and assuming that the time interval of the explosion $\Delta t = 0.05s$ is so small that $\frac{\Delta\vec{P}}{\Delta t}$ has the same value as $\frac{d\vec{P}}{dt}$,
 3.1- Determine the forces $\vec{F}_{A\to B}$ and $\vec{F}_{B\to A}$ exerted respectively by (A) on (B) and by (B) on (A).
 3.2- Verify the principle of interaction.
4- The system (S) possesses a certain energy before the explosion.
 4.1- Specify the part of (S) storing this energy.
 4.2- In what form is this energy stored?
 4.3- Determine the value of this energy.

Exercise 6 (2nd session 2004 LS): Determination of the speed of a bullet

A gun is used to shoot bullets, each of mass m = 20g, with a horizontal velocity $\vec{V}_0$ of value V_0.
In order to determine V_0, we consider a setup formed of a wooden block of mass M = 1kg, suspended from the ends of two inextensible sting of negligible mass and of the same length (figure 1).
This setup can be taken as a block of wood suspended from the free end a string of length $\ell = 1m$, initially at rest in the equilibrium position at G_1.
A bullet having the velocity V_0 hits the block and is embedded in at the level of the center of mass G of the block.
Just after impact, the system (block, bullet) moves with a horizontal velocity V_1. The pendulum thus attains a maximum angular deviation $\alpha = 37°$.
G_1 and G_2 are the respective positions of G in the equilibrium position and in the highest position.
Take the horizontal plane through G_1 as a gravitational potential energy reference (figure 2).
Neglect friction with air and take $g = 9.8m/s^2$.



1- During a collision, which one of the two physical quantities, the linear momentum or the kinetic energy of the system does not remain always conserved?

2- Determine the expression of the value of V_1 of the velocity $\vec{V}_1$ in terms of M, m and V_0.

3-

3.1- Determine, just after impact, the mechanical energy of the system (pendulum, Earth) in terms of V_0, M, and m.

3.2- Determine, in terms of M, m, g, ℓ and α, the mechanical energy of the system (pendulum, Earth) at point G_2.

3.3- Deduce the value of V_0.

4- Verify the answer of question (1).

<u>Exercise 7 (2nd session 2021 LS):</u> Studying the motion of an object

Consider:

- a rail AOB located in a vertical plane composed of two straight parts: a horizontal part AO and aninclined part OB making an angle $\alpha = 30^o$ with the horizontal;
- two objects (S_1) and (S_2) taken as particles of same mass m = 80g;
- a massless spring (R), of force constant k = 200 N/m and natural length ℓ_0, fixed from one of its ends to a support at A with the other end free.

Take:

- the horizontal plane containing O as the reference level forgravitational potential energy;
- $g = 10 m/s^2$.

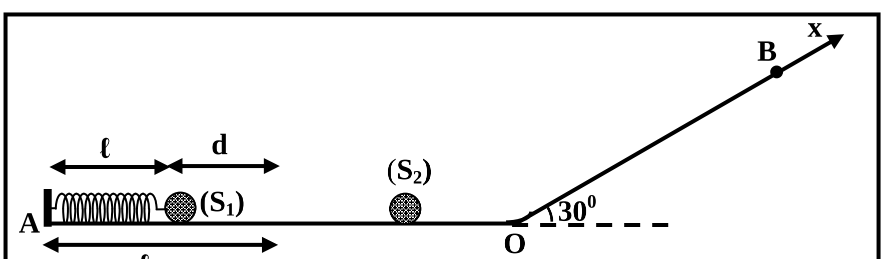
1- Launching particle (S_1)

In order to launch (S_1), it is placed against the free end of the spring, the spring is compressed by a distance d, and then the system [Spring - (S_1)] is released from rest (Doc.3).

When the spring returns to its natural length ℓ_0, (S_1) leaves the spring with a velocity $\vec{V}_1$ parallel to AO.

After launching, (S_1) moving with the velocity $\vec{V}_1$, collides head-on with (S_2) which is placed initially at rest on the rail AO.

Just after the collision, (S_1) stops and (S_2) moves with a velocity $\vec{V}_2$ parallel to AO and ofmagnitude $V_2 = 5 m/s$. (S_1) and (S_2) move without friction on the rail AO.



Doc. 3

1.1- Apply the law of conservation of linear momentum to show that the magnitude of $\vec{V}_1$ is $V_1 = 5 m/s$.

1.2- Deduce that the collision between (S_1) and (S_2) is elastic.

1.3- Determine the value of d.

2- Motion of (S_2) on the inclined part OB

At the instant $t_0 = 0$, (S_2) starts from O on the inclined part OB with a velocity $\vec{V_0} = V_0\vec{\imath} = 5\vec{\imath}\ (m/s)$, where $\vec{\imath}$ is the unit vector along the x-axis parallel to OB. On this part, (S_2) is submitted to a friction force $\vec{f}$ of constant magnitude f and of direction opposite to its motion.

2.1- Name the external forces acting on (S_2) during its motion along the track OB.

2.2- Show that the sum of the external forces acting on (S_2) during its upward motion along OB is:
$\sum\vec{F} = -(f + mg\sin\alpha)\vec{\imath}$.

2.3- The expression of the linear momentum of (S_2) during its upward motion along OB is:
$\vec{P} = (-0.9t + 0.4)\vec{\imath}\ (SI)$. Knowing that $\frac{d\vec{P}}{dt} = \sum\vec{F}$, determine f.

<u>Exercise 8 (2nd session 2010 LS):</u> Resistive force on a car

A car of mass M = 1500kg moves on a straight horizontal road; its center of gravity G is moving on the axis$(O;\vec{\imath})$. The car is acted upon by the forces:
- its weight;
- the normal reaction of the road;
- a constant motive force $\vec{F_m} = F_m\vec{\imath}$ where F_m = 3500N;
- a resistive force $\vec{F_f} = -F_f\vec{\imath}$.

In order to determine F_f , we measure the speed V of the car at different instants, separated by equal time intervals each being τ = 1s.

A- Value of F_f between the instants $t_0 = 0$ and $t_5 = 5$s

The results of the obtained recordings are tabulated as follows:

Instant	$t_0 = 0$	$t_1 = \tau$	$t_2 = 2\tau$	$t_3 = 3\tau$	$t_4 = 4\tau$	$t_5 = 5\tau$
Position	O	G_1	G_2	G_3	G_4	G_5
V [m/s]	0	2	4	6	8	10

1- Using the scale below, draw the curve representing the variation of the speed V as a function of time.
- 1cm on the axis of abscissas represents 1s;
- 1cm on the axis of ordinates represents 2m/s.

2- Show that the relation between the velocity $\vec{V} = V\vec{\imath}$ at a time t has the form $\vec{V} = bt\vec{\imath}$ where b is a constant.

3-

3.1- the constant b is a characteristic physical quantity of motion. Give its name.

3.2- Calculate its value.

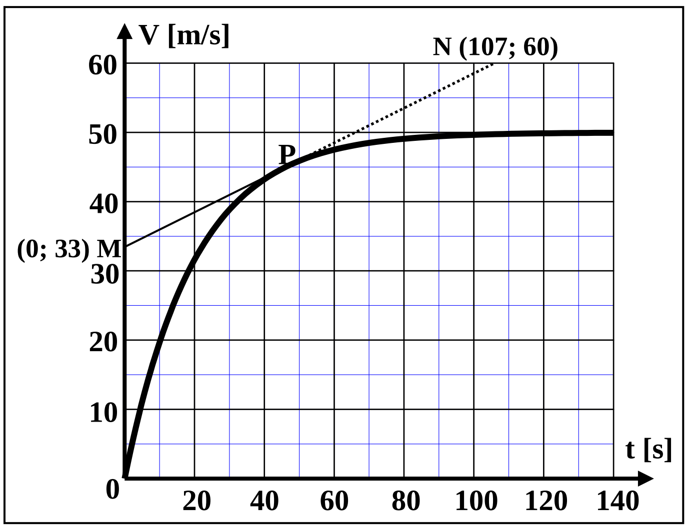
4- Applying Newton's second law,

4.1- show that F_f is constant between $t_0 = 0$ and $t_5 = 5$s;

4.2- calculate the value F_f of $\vec{F_f}$.

B- Variation of F_f between the instants $t_5 = 5$s and t = 140s

In reality, the measurement of the speed between the instants $t_0 = 0$ and t = 140s allows us to plot the graph of the adjacent figure.



1- Show that the part of this graph between the instants $t_0 = 0$ and $t_5 = 5$s is in agreement with the graph of part A.

2- We draw the tangent MN to the curve at the point P at the instant t_P where $V_P = 45m/s$.

2.1- Determine the value of the acceleration at the instant t_P.

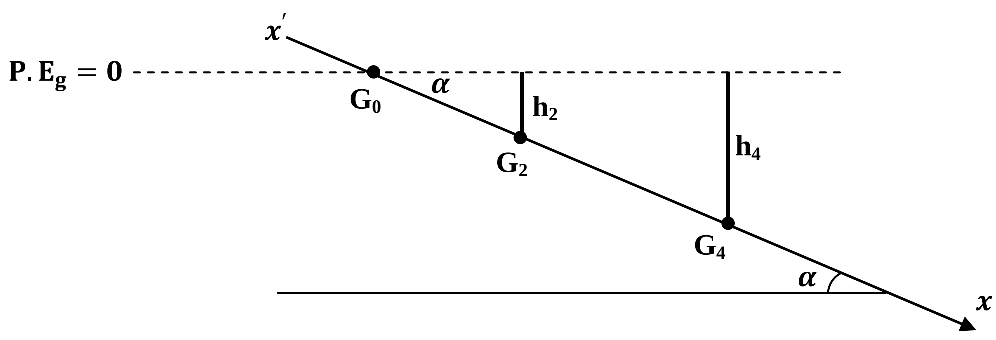
2.2- Deduce the value of F_f at the instant t_P.

3- Starting from the instant 100s, V attains a limiting value of $V_\ell = 50m/s$. Calculate then the value of F_f.

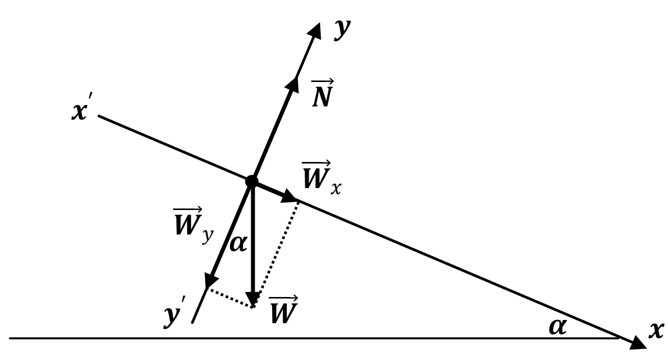
4- Indicate the time interval during which F_f increases.

Exercise 1:

Part	Answer	Mark
1	$V_2 = \frac{G_1G_3}{2\tau} = \frac{G_0G_3 - G_0G_1}{2\tau} = \frac{x_3 - x_1}{2\tau} = \frac{(4.5-0.5)\times 10^{-2}}{0.1} = 0.4m/s$ $V_4 = \frac{G_3G_5}{2\tau} = \frac{G_0G_5 - G_0G_3}{2\tau} = \frac{x_5 - x_3}{2\tau} = \frac{(12.5-4.5)\times 10^{-2}}{0.1} = 0.8m/s$	
2	$M.E = K.E + P.E_g;$ $M.E_0 = K.E_0 + P.E_{g\,0} = 0 + 0 = 0$ with $V_0 = 0$ and $h_0 = 0$ $M.E_2 = K.E_2 + P.E_{g\,2} = \frac{1}{2}MV_2^2 - Mgh_2;$	



Part	Answer	Mark
	$h_2 = G_0G_2 \times \sin\alpha = 2 \times 0.4 = 0.8cm = 0.008m \Longrightarrow M.E_2 = 0J.$ $M.E_4 = K.E_4 + P.E_{g\,4} = \frac{1}{2}MV_4^2 - Mgh_4;$ $h_4 = G_0G_4 \times \sin\alpha = 8 \times 0.4 = 3.2cm = 0.032m \Longrightarrow M.E_4 = 0J.$	
3.1	$M.E_0 = M.E_2 = M.E_4 \Longrightarrow$ The mechanical energy is conserved during motion $\Longrightarrow$ No friction.	
3.2	$\Delta\vec{P} = \vec{P}_4 - \vec{P}_2 = M\vec{V}_4 - M\vec{V}_2 = M\left(\vec{V}_4 - \vec{V}_2\right) = 0.1(0.8\vec{\imath} - 0.4\vec{\imath}) = 0.04\vec{\imath}$	
4.1	The forces acting on (S): The weight $\vec{W}$ of (S) and the normal reaction $\vec{N}$ of the path.	
4.2	$\sum\vec{F} = \vec{W} + \vec{N} = \vec{W}_x + \vec{W}_y + \vec{N}$ where: $\vec{W}_x = Mg\sin\alpha\,\vec{\imath}$, $\vec{W}_y = -Mg\cos\alpha\,\vec{\jmath}$ and $\vec{N} = N\vec{\jmath}$ $\vec{W}_y + \vec{N} = \vec{0}$ (no motion along $y'y$) $\Longrightarrow \sum\vec{F} = \vec{W}_1 = Mg\sin\alpha\,\vec{\imath}$	

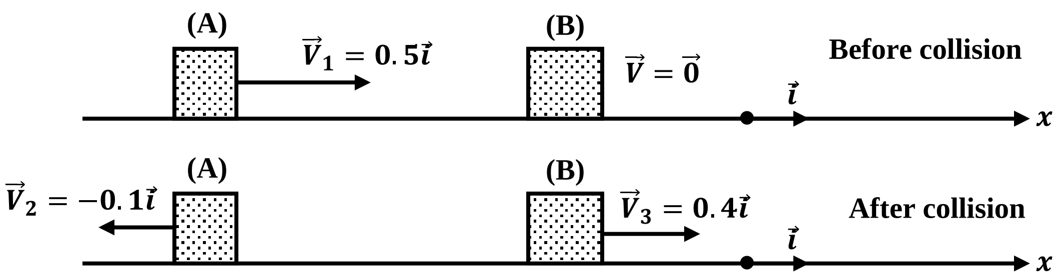
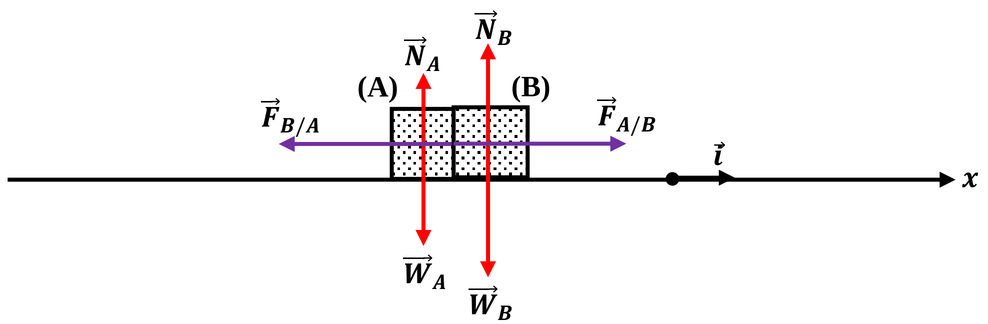


Part	Answer	Mark
5	The 2nd Law of is given by: $\sum\vec{F} = \frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t}$ We have: $\sum\vec{F} = Mg\sin\alpha\,\vec{\imath} = 0.4\vec{\imath}$ and $\frac{\Delta\vec{P}}{\Delta t} = \frac{0.04\vec{\imath}}{0.1} = 0.4\vec{\imath}$ $\Longrightarrow$ The 2nd law of Newton is thus verified.	

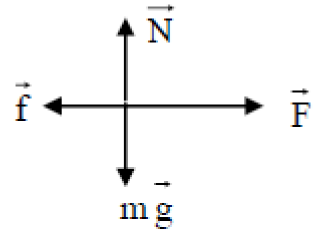
Exercise 2:

Part	Answer	Mark
1	$M.E_D = K.E_D + G.P.E_D = 0 + m_A g h_D = 9J$	
2	No friction $\Longrightarrow$ the mechanical energy of the system [(A); Earth] is conserved: $M.E_D = M.E_N \Longrightarrow K.E_D + G.P.E_D = K.E_N + G.P.E_N$ $0 + m_A g h_D = \frac{1}{2} m_A V_{A1}^2 + 0 \Longrightarrow V_{A1} = \sqrt{2gh_D} = \sqrt{2 \times 10 \times 0.45} = 3m/s$	
3.1	Linear momentum of the system [(A), (B)] before collision: $\vec{P}_S = \vec{P}_A + \vec{P}_B = m_A \vec{V}_{A1} + m_B \vec{V}_{B1} = 2 \times 3\vec{\imath} + 4 \times (-\vec{\imath}) = 2\vec{\imath}\ [kgm/s]$	
3.2	$\vec{P}_S = \vec{P}_G = (m_A + m_B)\vec{V}_G \Longrightarrow \vec{V}_G = \frac{\vec{P}_S}{m_A + m_B} = \frac{2\vec{\imath}}{2+4} = \frac{1}{3}\vec{\imath} = 0.33\vec{\imath}\ [m/s]$	
4.1	$M.E_C = K.E_C + G.P.E_C = 0 + m_A g h_C = 2 \times 10 \times 0.27 = 5.4\ J$	
4.2	Conservation of the mechanical energy of the system [(A), Earth]: $0 + m_A g h_C = \frac{1}{2} m V_{A2}^2 + 0 \Longrightarrow V_{A2} = \sqrt{2gh_C} = \sqrt{2 \times 10 \times 0.27} = 2.33m/s$	
5	Conservation of the linear momentum of the system [(A), (B)]: $\vec{P}_S = \vec{P}_S' \Longrightarrow m_A \vec{V}_{A1} + m_B \vec{V}_{B1} = m_A \vec{V}_{A2} + m_B \vec{V}_{B2}$ $2(3\vec{\imath}) + 4(-\vec{\imath}) = 2(-2.33\vec{\imath}) + 4\vec{V}_{B2} \Longrightarrow \vec{V}_{B2} = 1.66\vec{\imath}\ [m/s]$	
6	The kinetic energy of the system [(A), (B)] before collision: $K.E_{B.C} = K.E_A + K.E_B = \frac{1}{2} m_A V_{A1}^2 + \frac{1}{2} m_B V_{B1}^2 = \frac{1}{2}(2)(3)^2 + \frac{1}{2}(4)(1)^2 = 11J$ The kinetic energy of the system [(A), (B)] before collision: $K.E_{AC} = K.E_A' + K.E_B' = \frac{1}{2} m_A V_{A2}^2 + \frac{1}{2} m_B V_{B2}^2 = \frac{1}{2}(2)(2.33)^2 + \frac{1}{2}(4)(1.66)^2 = 11J$ $K.E_{BC} = K.E_{AC}$ (the kinetic energy of the system [(A); (B)] is conserved Therefore, the collision is elastic	

Exercise 3:

Part	Answer	Mark
		
A.1.1.1	$\vec{P}_1 = m_A\vec{V}_1 = 0.4 \times (0.5\vec{i}) = 0.2\vec{i}\ (kgm/s)$ $\vec{P}_2 = m_A\vec{V}_2 = 0.4 \times (-0.1\vec{i}) = -0.04\vec{i}\ (kgm/s)$	
A.1.1.2	$\vec{P}_3 = m_B\vec{V}_3 = 0.6 \times (0.4\vec{i}) = 0.24\vec{i}$	
A.1.2	$\vec{P} = \vec{P}_1 + \vec{0} = 0.2\vec{i}$ $\vec{P}' = \vec{P}_2 + \vec{P}_3 = -0.04\vec{i} + 0.24\vec{i} = 0.2\vec{i}$	
A.1.3	$\vec{P} = \vec{P}'$ Conclusion: the linear momentum of the system [(A), (B)] is conserved during collision.	
A.2.1	The external forces acting on the system [(A); (B)] are: The weight $m_A\vec{g}$ and the normal reaction of the air table $\vec{N}_A$. The weight $m_B\vec{g}$ and the normal reaction of the air table $\vec{N}_B$.	
		
A.2.2	We have: $\sum \vec{F}_{ext} = m_A\vec{g} + \vec{N}_A + m_B\vec{g} + \vec{N}_B = \vec{0}$ The sum of the external forces acting on the system (A, B) is thus zero.	
A.2.3	Yes, Since the system [(A), (B)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant$	
B.1	$K.E_{before} = \frac{1}{2}m_A V_1^2 + 0 = 0.05J$ $K.E_{after} = \frac{1}{2}m_A V_2^2 + \frac{1}{2}m_B V_3^2 = 0.05J$	
B.2	$K.E_{before} = K.E_{after} \Longrightarrow$ The collision is elastic.	
C.1.1	$\Delta\vec{P}_A = \vec{P}_2 - \vec{P}_1 = -0.24\vec{i}$ $\Delta\vec{P}_B = \vec{P}_3 - \vec{0} = 0.24\vec{i}$	
C.1.2	$\sum \vec{F}_{ext/A} = \frac{d\vec{P}_A}{dt}$ $\vec{F}_{B/A} + \vec{W}_A + \vec{N}_A = \frac{\Delta\vec{P}_A}{\Delta t} \Longrightarrow \vec{F}_{B/A} = \frac{\Delta\vec{P}_A}{\Delta t} = \frac{-0.24\vec{i}}{0.04} = -6\vec{i}\ (N)$ with $\vec{W}_A + \vec{N}_A = \vec{0}$ $\sum \vec{F}_{ext/B} = \frac{d\vec{P}_B}{dt}$ $\vec{F}_{A/B} + \vec{W}_B + \vec{N}_B = \frac{d\vec{P}_A}{dt} \Longrightarrow \vec{F}_{A/B} = \frac{\Delta\vec{P}_B}{\Delta t} = \frac{0.24\vec{i}}{0.04} = 6\vec{i}\ (N)$	
C.2	$\vec{F}_{B/A} = -\vec{F}_{A/B} \Longrightarrow$ The principle of interaction is thus verified.	

Exercise 4:

Part	Answer	Mark
A.1	The forces acting on (S) are: the weight $m\vec{g}$, the normal reaction of the water skateboard $\vec{N}$, $\vec{F}$ and $\vec{f}$  	
A.2	$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext} = m\vec{g} + \vec{N} + \vec{F} + \vec{f}$ project along the direction of motion $\Longrightarrow$ $\frac{dP}{dt} = F - f \Longrightarrow ma = F - f \Longrightarrow a = \frac{F-f}{m}$	
A.3	$V = \int a dt = at + V_0 (V_0 = 0)$ then $V = \left(\frac{F-f}{m}\right)t.$	
A.4	$V = V_B = 6m/s$ for t = 60s $\Longrightarrow 6 = \left(\frac{F-100}{80}\right)60 \Longrightarrow F = 108N$	
B.1	Since friction is negligible between B and D.	
B.2	$ME_B = ME_D \Longrightarrow \frac{1}{2}mV_B^2 + 0 = \frac{1}{2}mV_D^2 + mgh$ $\Longrightarrow \frac{1}{2}(80)(36) = \frac{1}{2}(80)V_D^2 + 80 \times 10 \times 1.6 \Longrightarrow V_D = 2m/s$	
C.1	$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext} = mg\vec{j} \Longrightarrow \frac{dP_y}{dt} = mg \Longrightarrow P_y = mgt + P_{0y}$ $P_{0y} = mV_{0y} = m(-V_D \sin 30°) = -80 \times 2 \times \frac{1}{2} = -80kgm/s$ $\Longrightarrow P_y = 800t - 80$	
C.2	$V_y = \frac{P_y}{m} = 10t - 1 \Longrightarrow y = 5t^2 - t + y_0 = 5t^2 - t\ (y_0 = 0).$	
C.3	$1.6 = 5t^2 - t \Longrightarrow 5t^2 - t - 1.6 = 0 \Longrightarrow \Delta = 1 + 32 = 33$ $t = \frac{1 \pm \sqrt{33}}{10} \Longrightarrow t = \frac{1+\sqrt{33}}{10} = 0.67s$	

Exercise 5:

Part	Answer	Mark
1.1	The distances covered during equal time intervals are equal.	
1.2	$V_B = \frac{d}{\Delta t} = \frac{d}{4\tau} = \frac{0.2}{4\times0.05} = 1m/s$ and $V_A = \frac{d}{\Delta t} = \frac{d}{4\tau} = \frac{0.24}{4\times0.05} = 1.2m/s$	
2	$\vec{P}_{before} = \vec{0}$; $\vec{P}_{after} = m_A\vec{V}_A + m_B\vec{V}_B = 0.1(-1.2\vec{i}) + 0.12(\vec{i}) = \vec{0}$ $\Longrightarrow \vec{P}_{before} = \vec{P}_{after}$ (The linear momentum is conserved for the system formed of the two pucks)	
3.1	Newton 2nd Law applied on A gives: $\frac{d\vec{P}_A}{dt} = m_A\vec{g} + \vec{N}_A + \vec{F}_{B\to A} = \vec{F}_{B\to A} = \frac{0.1(-1.2-0)\vec{i}}{0.05} = -2.4\vec{i}$ Newton 2nd Law applied on B gives: $\frac{d\vec{P}_B}{dt} = m_B\vec{g} + \vec{N}_B + \vec{F}_{A\to B} = \vec{F}_{A\to B} = \frac{0.12(1-0)\vec{i}}{0.05} = 2.4\vec{i}$	
3.2	$\vec{F}_{B\to A} = -\vec{F}_{A\to B}$	
4.1	The deformed elastic shock ring.	
4.2	Elastic potential energy.	
4.3	The mechanical energy of the system is conserved because the system is isolated (The system does not exchange energy with the surroundings); (Elastic potential energy is transformed into kinetic energy): $M.E_{before} = M.E_{after}$ $K.E_{before} + E.P.E_{before} = K.E_{after} + E.P.E_{after}$ $0 + E.P.E_{before} = \frac{1}{2}m_AV_A^2 + \frac{1}{2}m_BV_B^2 + 0 \Longrightarrow E.P.E_{before} = 0.132J$	

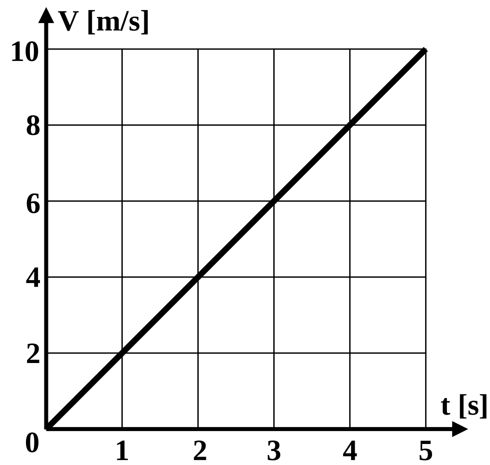
Exercise 6:

Part	Answer	Mark
1	The kinetic energy of the system [M; m].	
2	During collision, the system [M; m] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}_S}{dt} = \vec{0} \Longrightarrow \vec{P}_s = constant.$ Apply the principle of conservation of linear momentum: $\vec{P}_b = \vec{P}_a \Longrightarrow m\vec{V}_0 = (m+M)\vec{V}_1$ The velocities are collinear; then, the above equation can be written in its algebraic form: $mV_0 = (m+M)V_1 \Longrightarrow V_1 = \frac{m}{M+m}V_0$	
3.1	$M.E_1 = K.E_1 + P.E_{g1} = \frac{1}{2}(M+m)V_1^2 + 0 = \frac{1}{2}(M+m)V_1^2$ $M.E_1 = \frac{1}{2}(M+m)\left(\frac{m}{m+M}V_0\right)^2 = \frac{1}{2}\frac{m^2V_0^2}{(M+m)}$	
3.2	$M.E_2 = K.E_2 + P.E_{g2} = 0 + (M+m)gh_2$ with $h_2 = \ell - \ell\cos\theta = \ell(1-\cos\alpha)$ $M.E_2 = (M+m)g\ell(1-\cos\alpha)$	
3.3	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $M.E_1 = M.E_2 \Longrightarrow \frac{1}{2}\frac{m^2V_0^2}{(M+m)} = (M+m)g\ell(1-\cos\alpha)$ $V_0^2 = \frac{2(M+m)^2g\ell(1-\cos\alpha)}{m^2} \Longrightarrow V_0 = \frac{(M+m)}{m}\sqrt{2g\ell(1-\cos\alpha)}$ $V_0 = \frac{(1+0.02)}{0.02}\sqrt{2\times 9.8\times 1\times(1-\cos 37°)} = 101.3m/s$	
4	$K.E_b = \frac{1}{2}mV_0^2 = \frac{1}{2}\times 0.02\times 101.3^2 = 102.6J$ $K.E_a = \frac{1}{2}(M+m)V_1^2 = \frac{1}{2}\frac{m^2V_0^2}{(M+m)} = \frac{0.02^2\times 101.3^2}{2(1+0.02)} = 2J$ $K.E_b \neq K.E_a \Longrightarrow$ the kinetic energy is not conserved.	

Exercise 7:

Part	Answer	Mark
1.1	$\vec{P}_{J.B.C} = \vec{P}_{J.A.C} \Longrightarrow m\vec{V}_1 + \vec{0} = \vec{0} + m\vec{V}_2 \Longrightarrow \vec{V}_1 = \vec{V}_2$ Then, $V_1 = 5m/s$	
1.2	System [(S_1), (S_2)] The collision is elastic if $KE_{system\ (before)} = KE_{system\ (after)}$ $KE_{(before)} = KE_{(S_1)} + KE_{(S_2)} = \frac{1}{2}mV_1^2 + 0 = \frac{1}{2}\times 0.08\times 5^2 + 0 = 1J$ $KE_{(after)} = KE_{(S_1)} + KE_{(S_2)} = 0 + \frac{1}{2}mV_2^2 = 0 + \frac{1}{2}\times 0.08\times 5^2 = 1J$ Therefore, the collision is elastic.	
1.3	Apply the law of conservation of mechanical energy of the system [Oscillator-Earth]: $ME_{(R)\text{ is compressed by d}} = ME_{(R)\text{ is in its initial length}}$, $(KE + GPE + EPE)_{(R)\text{ is compressed by d}} = (KE + GPE + EPE)_{(R)\text{ is in its initial length}}$ $0 + \frac{1}{2}kd^2 + 0 = \frac{1}{2}mV_1^2 + 0 + 0 \Longrightarrow \frac{1}{2}\times 200\times d^2 = \frac{1}{2}\times 0.08\times 5^2$ then d = 0.1m = 10cm	
2.1	The forces acting on (S_2) on OB are: $m\vec{g}$: its weight, $\vec{N}$: Normal reaction and $\vec{f}$: friction	
2.2	$\sum \vec{F} = m\vec{g} + \vec{N} + \vec{f}$, Component along the direction $\overrightarrow{Ox}$: $\sum \vec{F} = -mg\sin\alpha\,\vec{i} + 0\vec{i} - f\vec{i} = -(f + mg\sin\alpha)\vec{i}$ Or: $\sum \vec{F} = m\vec{g} + \vec{N} + \vec{f} = -mg\sin\alpha\,\vec{i} + mg\cos\alpha\,\vec{j} - N\vec{j} - f\vec{i}$ But: $mg\cos\alpha\,\vec{j} - N\vec{j} = 0$, then, $\sum \vec{F} = -(f + mg\sin\alpha)\vec{i}$	
2.3	$\frac{d\vec{P}}{dt} = \sum \vec{F}$ $-0.9\vec{i} = -(f + mg\sin\alpha)\vec{i}$ $-0.9 = -f - 0.08\times 10\times 0.5$ Therefore, f = 0.5 N	

Exercise 8:

Part	Answer	Mark
A.1		
A.2	The graph is a straight line passing through the origin, in agreement with the function $\vec{V} = bt\vec{i}$ where b is a constant	
A.3.1	b the acceleration of the motion.	
A.3.2	$a = \frac{\Delta V}{\Delta t} = \frac{10-0}{5-0} = 2m/s^2.$	
A.4.1	$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = M\frac{d\vec{V}}{dt}.$ $\vec{N} + \vec{W} + \vec{F}_m + \vec{F}_f = m\frac{d\vec{V}}{dt}$ Projection along the direction of motion: $F_m - F_f = M\frac{dV}{dt} \Longrightarrow F_f = F_m - Mb$ $F_m = constant$, M = constant and b = constant $\Longrightarrow F_f = constant.$	
A.4.2	$F_f = F_m - Mb = 3500 - 1500 \times 2 = 500N.$	
B.1	For V < 10m/s, the part of the curve is a straight line.	
B.2.1	$a = \frac{dV}{dt} = slope\ of\ the\ tangent.$ $a = \frac{60-33}{107-0} = 0.25m/s^2.$	
B.2.2	$F_f = F_m - Mb = 3500 - 1500 \times 0.25 = 3125N.$	
B.3	$a = 0 \Longrightarrow F_f = F_m = 3500N.$	
B.4	5s < t < 100s.	