Shakib Irslan High School Physics Department	PHYSICS EXTRA SHEET 3 LINEAR MOMENTUM	**Academic Year:** 2023-2024 **Date:**
Name: ……………………..		**Class and Section:** 12 LS

Exercise 1:

The position-time equation of a particle (M), of mass m = 0.5kg, moving in the frame reference system $(O;\vec{i})$ is:

$$\boldsymbol{x = 2t^2 - 4t + 1\ [SI]}$$

1- Determine, at the instant t, the expressions of the velocity, acceleration and the linear momentum of (M).

2- Calculate the net external force acting on (S).

Exercise 2:

The position vector of a particle (P) of mass m = 2kg and moving in the space reference system $(O;\vec{i};\vec{j})$ is:

$$\boldsymbol{\vec{r} = \left(5\sqrt{3}t\right)\vec{i} + (-5t^2 + 5t)\vec{j}\ [SI]}$$

Neglect air resistance and take g = 10m/s^2.

1- Determine the equation of the trajectory described by (P). Deduce its shape.

2- Determine, at an instant t, the:

2.1- Velocity of (P),

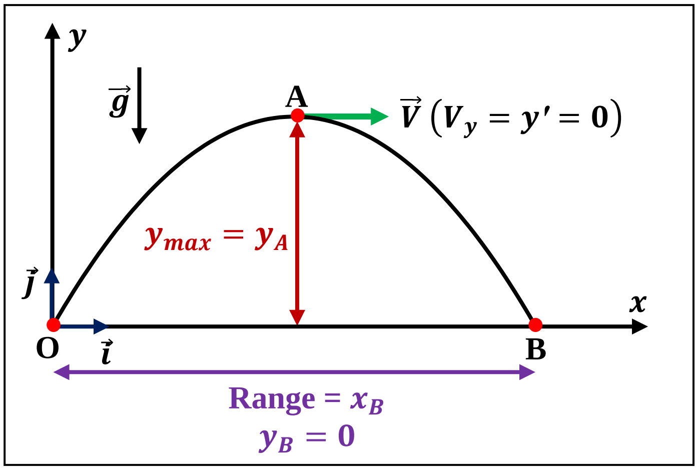
2.2- acceleration of (P),

2.3- linear momentum of (P).

3- Show that $\sum \vec{F}_{ext} = m\vec{a} = \frac{d\vec{P}}{dt}$.

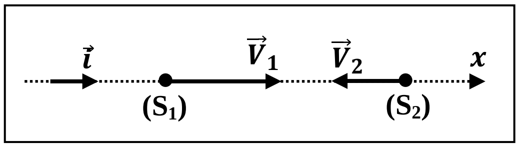
4- Determine the instant t_1 when (P) reaches its highest point A of its trajectory.

5- Calculate the range of (P).



Exercise 3:

Two particles (S_1) and (S_2) of respective masses $m_1 = 2kg$ and $m_2 = 3kg$ move towards each other with respective velocities of magnitudes $V_1 = 6m/s$ and $V_2 = 4m/s$ as shown in the adjacent document.



1- Write the velocity vectors $\vec{V}_1$ and $\vec{V}_2$ of (S_1) and (S_2) respectively.

2- Determine the linear momenta $\vec{P}_1$ and $\vec{P}_2$ of (S_1) and (S_2) respectively. Deduce the linear momentum $\vec{P}_S$ of the system [(S_1); (S_2)].

3- Specify whether the center of mass of the system [(S_1); (S_2)] moves. Justify.

Exercise 4:

In an inertial frame of reference $(O;\ \vec{i};\ \vec{j})$, the coordinates and masses of two particles (S_1) and (S_2) are given as a function of time t as follows:

$$(S_1)\left|\begin{array}{l} x_1 = 2t^2 + 5 \\ y_1 = t - 1 \\ m_1 = 1kg \end{array}\right. \qquad (S_2)\left|\begin{array}{l} x_2 = 2t + 4 \\ y_2 = t^2 + t \\ m_2 = 2kg \end{array}\right. \qquad [SI]$$

Denote by G the center of mass of the system (S) = [(S_1); (S_2)].

1- Determine $\vec{P}_1$ and $\vec{P}_2$ the respective linear momenta of (S_1) and (S_2) at any instant t.

2- Calculate $\vec{P}_S$ the linear momentum of the system (S) at any instant t.

3- Determine $\vec{r}_G$ the position vector of (S) at any instant t.

4- Calculate $\vec{P}_G$ the linear momentum of G at any instant t. Draw out a conclusion.

5- Determine $\vec{a}_1$ and $\vec{a}_2$ the respective acceleration vectors of (S_1) and (S_2) at any time t. Deduce $\vec{F}_1$ and $\vec{F}_2$ the net forces acting on (S_1) and (S_2) respectively.

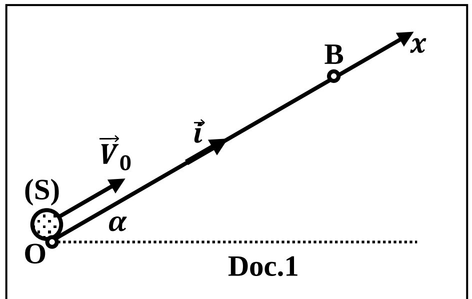
6- Verify that $\sum \vec{F}_{ext\,/(s)} = \frac{d\vec{P}}{dt}$.

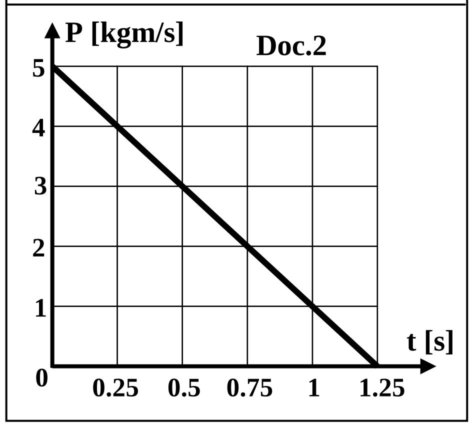
7- Specify whether the system (S) is isolated or not.

Exercise 5:

A solid (S), considered as a particle of mass m = 0.5kg, can slide on an inclined plane making an angle $\alpha = 30^o$ with respect to the horizontal.

At the instant $t_0 = 0s$, (S) is launched from point O with a velocity $\vec{V}_0 = V_0\vec{\imath}$ along a line Ox of greatest slope. Thus, (S) moves along the axis Ox of unit vector $\vec{\imath}$ as shown in document 1.





At an instant, the position of (S) relative to O is given by its abscissa x and the algebraic value of its velocity is $V = \frac{dx}{dt}$.

The force of friction $\vec{f}$ between the inclined plane and (S) opposes its motion and is assumed constant of magnitude f.

Take $g = 10m/s^2$.

Document 2 represents the variation of the algebraic value of the linear momentum P of (S) as a function of time.

1- Referring to document 2, determine the value of V_0.

2- Show that $P = -4t + 5\ (SI)$; then, deduce the expression of V as a function of time.

3- Name and represent the external forces acting on (S).

4- Show that the resultant of these forces may be written as:

$$\sum \vec{F}_{ext} = (-mg\sin\alpha - f)\vec{\imath}$$

5- Deduce the value of f.

6- Determine the distance OB knowing that (S) stops at point B.

Exercise 6:

A vessel at rest at the origin of an *xy* coordinate system explodes into three pieces. Just after the explosion, one piece, of mass *m*, moves with velocity $(-30m/s)\vec{\imath}$ and a second piece, also of mass *m*, moves with velocity $(-30m/s)\vec{\jmath}$. The third piece has mass 3*m*.

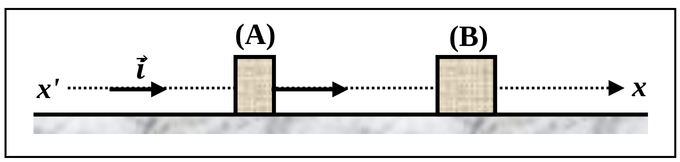
Just after the explosion, what are the (a) magnitude and (b) direction of the velocity of the third piece?

Exercise 7:

A rifle (R), of mass $m_1 = 4kg$, and initially at rest fires a bullet (B), of mass $m_2 = 5g$, with a velocity of $V_2 = 500m/s$. What is the velocity V_1 acquired by the rifle?

Exercise 8:

A solid (A), of mass $m_1 = 0.4kg$ and moving along a horizontal plane with a constant velocity $\vec{V}_1 = V_1\vec{\imath}$ $(V_1 = 5m/s)$, enters in an elastic head-on collision with a solid (B) of mass $m_2 = 0.6kg$ and initially at rest.



Neglect all frictional forces.

1- Show, just after collision, that the expressions of the respective velocities of (A) and (B) are given by:

$$V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1 \text{ and } V_2' = \frac{2m_1}{m_1 + m_2}V_1$$

2- Calculate the numerical values of V_1' and V_2'.

3- The duration of collision is $\Delta t = 0.02s$; we can consider that $\frac{\Delta\vec{P}}{\Delta t} \approx \frac{d\vec{P}}{dt}$.

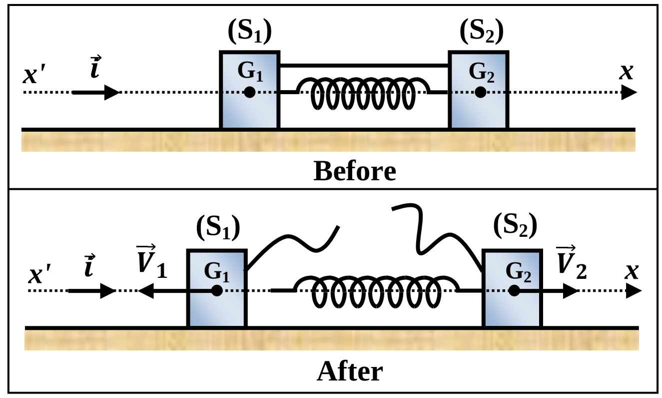
3.1- Determine during Δt:

3.1.1- the variations $\Delta\vec{P}_A$ and $\Delta\vec{P}_B$ in the linear momentums of (A) and (B) respectively;

3.1.2- the forces $\vec{F}_{A/B}$ exerted by (A) on (B) and $\vec{F}_{B/A}$ exerted by (B) on (A).

3.2- Deduce that the principle of interaction is verified.

Exercise 9:



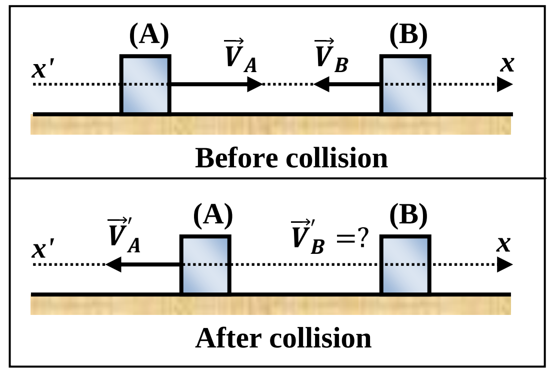
Two blocks (S_1) and (S_2), of respective masses m_1 = 2kg and m_2 = 3kg, are placed on a frictionless horizontal surface. A light spring is attached to (S_2), and the blocks are pushed together with the spring between them. A cord initially holding the blocks together is burned; after that happens, (S_2) moves to the right with a velocity $\vec{V_2} = 4\vec{i}\ (m/s)$. The x-axis is taken as a reference level for gravitational potential energy.

1- Determine the velocity $\vec{V_1}$ of (S_1).

2- Find the system's original elastic potential energy.

3- Is the original energy in the spring or in the cord? Explain.

Exercise 10:



A block (A) of mass m_A = 1.6kg and moving with a velocity $\vec{V_A} = 5\vec{i}\ (m/s)$ enters in a head-on collision with another block (B) of mass m_B = 2.4kg and moving with a velocity $\vec{V_B} = -2\vec{i}\ (m/s)$. After collision, the velocity of block (A) is $\vec{V_A'} = -3.4\vec{i}\ (m/s)$.

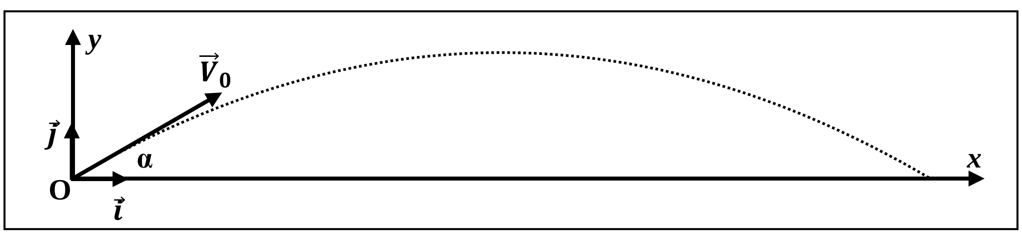
1- Determine the velocity $\vec{V_B'}$ of block (B) after collision.

2- Specify whether the collision between block (A) and block (B) is elastic or not.

3- Determine the force $\vec{F}_{A/B}$ exerted by block (A) on block (B) during collision knowing that it lasts for 100ms.

Exercise 11:

At the instant t_0 = 0s, a solid (S), considered as a particle of mass m = 500g, is launched from point O with an initial velocity vector $\vec{V_0}$ of magnitude $V_0 = 8m/s$ and making an angle α = 30° with the horizontal as shown in the document below.
Neglect air resistance and take g = 10m/s^2.



1- Determine, at the instant t_0 = 0s, the algebraic value of the horizontal and vertical components P_{0x} and P_{0y} of the initial linear momentum $\vec{P_0}$ of (S).

2- Apply Newton's second law to show that the expression of the linear momentum of (S) at an instant t is:

$$\vec{P} = 2\sqrt{3}\vec{i} + (-5t + 2)\vec{j}\ (SI)$$

3- Deduce the parametric equations $x(t)$ and $y(t)$ of (S) in the space reference system $(O; \vec{i}; \vec{j})$.

4- Determine the range and the maximum height reached by (S).

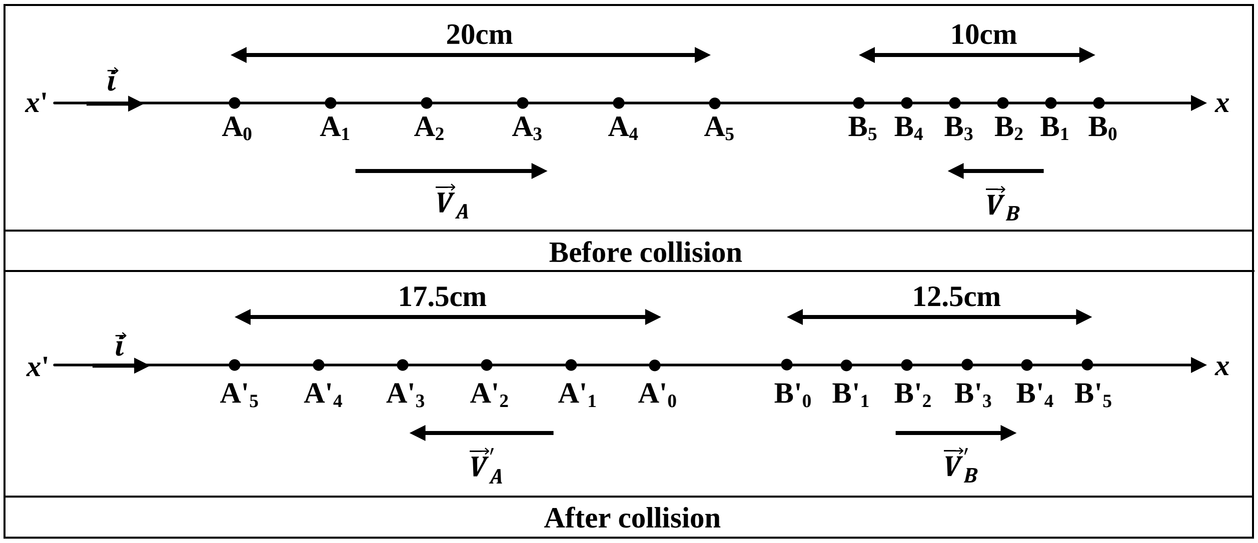
Exercise 12:

The aim of this exercise is to study the collision between two bodies and to verify the principle of interaction. To do this, we use a horizontal air table that is equipped with a launcher and two pucks (A) and (B) of respective masses m_A = 150g and m_B = 250g.

Puck (A), moving with a velocity $\vec{V}_A = V_A\vec{i}$, enters in a head-on collision with puck (B) moving with a velocity $\vec{V}_B = V_B\vec{i}$.

Just after collision, pucks (A) and (B) move with the respective velocities $\vec{V}_A' = V_A'\vec{i}$ and $\vec{V}_B' = V_B'\vec{i}$.

The document below shows, in real dimensions, a part of the dot-prints, that register the positions of the centers of mass of pucks (A) and (B), obtained when the time interval separating two successive dots is τ = 20ms.



Part I: collision between (A) and (B)

1- Show, using the above dot-prints, that the velocities $\vec{V}_A$, $\vec{V}_B$, $\vec{V}_A'$ and $\vec{V}_B'$ are constant and calculate their algebraic values V_A, V_B, V_A' and V_B'.

2- Calculate the linear momenta $\vec{P}$ and $\vec{P}'$ of the system [(A); (B)] before and after collision respectively,

3- Compare $\vec{P}$ and $\vec{P}'$. Conclude.

4-

- **4.1-** Name the external forces acting on the system [(A); (B)].
- **4.2-** What is the value of the resultant of these forces?
- **4.3-** This result agrees with the conclusion of part 3. Why?

5- Specify the nature of the collision between (A) and (B).

Part II: interaction between (A) and (B)

The collision between (A) and (B) lasts for Δt = 100ms.

1- Determine, during collision, the average force $\vec{F}_{A/B}$ exerted by (A) on (B) and the average force $\vec{F}_{B/A}$ exerted by (B) on (A).

2- Show that the principle of interaction is verified.

Exercise 13:

A block (B), of mass m = 1.5kg and center of inertia G, is placed on a horizontal rough plane. At the instant t_0 = 0s, (B) is launched from point O with a horizontal velocity $\vec{V_0} = V_0\vec{\imath}$. Thus, (B) moves along the axis Ox of unit vector $\vec{\imath}$ as shown in document 1.

At an instant, the position of (B) relative to O is given by its abscissa x and the algebraic value of its velocity is $v = \frac{dx}{dt}$.

The force of friction $\vec{f}$ between the plane and (B) opposes its motion and is assumed constant of magnitude f.

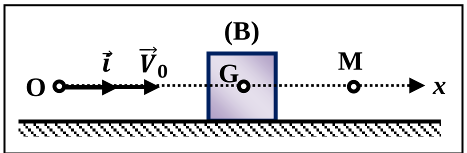
The graph of document 2 shows the variation of the algebraic value of the linear momentum P of (B) as a function of time.

1- Determine V_0.

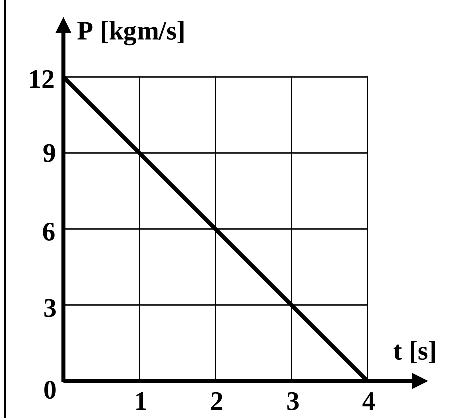
2- Name and represent the external forces acting on (B).

3- Apply Newton's 2nd law to determine f.

4- Apply the work-kinetic energy theorem to determine the distance OM knowing that (B) stops at point M.



Doc.1



Doc.2

Exercise 1:

Part	Answer key
1	$V = \frac{dx}{dt} = x' = 4t - 4 \Longrightarrow \vec{V} = V\vec{\imath} = (4t-4)\vec{\imath}$. $a = \frac{dV}{dt} = V' = 4m/s^2 \Longrightarrow \vec{a} = a\vec{\imath} = 4\vec{\imath}$. $P = mV = 2t - 2 \Longrightarrow \vec{P} = V\vec{\imath} = (2t-2)\vec{\imath}$.
2	$\sum F_{ext} = ma = 2N \Longrightarrow \sum \vec{F}_{ext} = 2\vec{\imath}$ or $\sum F_{ext} = \frac{dP}{dt} = 2 \Longrightarrow \sum \vec{F}_{ext} = 2\vec{\imath}$.

Exercise 2:

Part	Answer key
1	The parametric equations of motion of (P) are: $\begin{cases} x = 5\sqrt{3}t \ldots (1) \\ y = -5t^2 + 5t \ldots (2) \end{cases}$ From (1): $t = \frac{x}{5\sqrt{3}}$. Replace in (2): $y = -5\left(\frac{x}{5\sqrt{3}}\right)^2 + 5\left(\frac{x}{5\sqrt{3}}\right) = -\frac{1}{15}x^2 + \frac{1}{\sqrt{3}}x$ (Parabolic).
2.1	$\vec{V} = \frac{d\vec{r}}{dt} = \vec{r}\,' = 5\sqrt{3}\vec{\imath} + (-10t + 5)\vec{\jmath}$.
2.2	$\vec{a} = \frac{d\vec{V}}{dt} = \vec{V}' = -10\vec{\jmath}$.
2.3	$\vec{P} = m\vec{V} = 10\sqrt{3}\vec{\imath} + (-20t + 10)\vec{\jmath}$.
3	The only force acting on a projectile is its weight $\left(\vec{W} = m\vec{g}\right)$. $\sum \vec{F}_{ext} = m\vec{g} = 2 \times (-10\vec{\jmath}) = -20\vec{\jmath}$, $m\vec{a} = 2 \times (-10\vec{\jmath}) = -20\vec{\jmath}$ and $\frac{d\vec{P}}{dt} = -20\vec{\jmath}$.
4	$V_y = 0 \Longrightarrow -10t_1 + 5 = 0 \Longrightarrow t_1 = 0.5s$.
5	$y = 0 \Longrightarrow -\frac{1}{15}x^2 + \frac{1}{\sqrt{3}}x = 0 \Longrightarrow x\left(-\frac{1}{15}x + \frac{1}{\sqrt{3}}\right) = 0$. $x = 0$ (launching position) and $-\frac{1}{15}x + \frac{1}{\sqrt{3}} = 0 \Longrightarrow x = 5\sqrt{3}m$.

Exercise 3:

Part	Answer key
1	$\vec{V}_1 = 6\vec{\imath}$ and $\vec{V}_2 = -4\vec{\imath}$.
2	$\vec{P}_1 = m_1\vec{V}_1 = 12\vec{\imath}$ and $\vec{P}_2 = m_2\vec{V}_2 = -12\vec{\imath}$. Then, $\vec{P}_S = \vec{P}_1 + \vec{P}_2 = \vec{0}$.
3	$\vec{P}_S = \vec{P}_G = M\vec{V}_G = \vec{0} \Longrightarrow \vec{V}_G = \vec{0}$ (G at rest).

Exercise 4:

Part	Answer key	
1	$\vec{r}_1 = x_1\vec{\imath} + y_1\vec{\jmath} = (2t^2 + 5)\vec{\imath} + (t-1)\vec{\jmath}\ [m]$. $\vec{V}_1 = \frac{d\vec{r}_1}{dt} = \vec{r}_1' = 4t\vec{\imath} + \vec{\jmath}\ [m/s]$. $\vec{P}_1 = m_1\vec{V}_1 = 4t\vec{\imath} + \vec{\jmath}\ [kgm/s]$.	$\vec{r}_2 = x_2\vec{\imath} + y_2\vec{\jmath} = (2t + 4)\vec{\imath} + (t^2 + t)\vec{\jmath}\ [m]$. $\vec{V}_2 = \frac{d\vec{r}_2}{dt} = \vec{r}_2' = 2\vec{\imath} + (2t+1)\vec{\jmath}\ [m/s]$. $\vec{P}_2 = m_2\vec{V}_2 = 4\vec{\imath} + (4t+2)\vec{\jmath}\ [kgm/s]$.
2	$\vec{P}_S = \vec{P}_1 + \vec{P}_2 = (4t+4)\vec{\imath} + (4t+3)\vec{\jmath}\ [kgm/s]$.	
3	$X_G = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1)(2t^2+5)+(2)(2t+4)}{3} = \frac{2t^2+4t+13}{3}\ [m]$. $Y_G = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{(1)(t-1)+(2)(t^2+t)}{3} = \frac{2t^2+3t-1}{3}\ [m]$. $\vec{r}_G = X_G\vec{\imath} + Y_G\vec{\jmath} = \left(\frac{2t^2+4t+13}{3}\right)\vec{\imath} + \left(\frac{2t^2+3t-1}{3}\right)\vec{\jmath}\ [m]$.	
4	$\vec{V}_G = \frac{d\vec{r}_G}{dt} = \vec{r}_G' = \left(\frac{4t+4}{3}\right)\vec{\imath} + \left(\frac{4t+3}{3}\right)\vec{\jmath}\ [m/s]$. $\vec{P}_G = (m_1 + m_2)\vec{V}_G = (4t+4)\vec{\imath} + (4t+3)\vec{\jmath}\ [kgm/s]$. Therefore, $\vec{P}_S = \vec{P}_G$.	
5	$\vec{a}_1 = \frac{d\vec{V}_1}{dt} = \vec{V}_1' = 4\vec{\imath}\ [m/s^2]$. $\vec{a}_2 = \frac{d\vec{V}_2}{dt} = \vec{V}_2' = 2\vec{\jmath}\ [m/s^2]$.	$\vec{F}_1 = m_1\vec{a}_1 = 4\vec{\imath}\ [N]$. $\vec{F}_2 = m_2\vec{a}_2 = 4\vec{\jmath}\ [N]$.
6	$\sum \vec{F}_{ext\,/(S)} = \vec{F}_1 + \vec{F}_2 = 4\vec{\imath} + 4\vec{\jmath}\ [N]$ and $\frac{d\vec{P}}{dt} = 4\vec{\imath} + 4\vec{\jmath}\ [N]$.	
7	The system (S) is not isolated since $\sum \vec{F}_{ext\,/(S)} \neq \vec{0}$.	

Exercise 5:

Part	Answer key
1	$V_0 = \frac{P_0}{m} = \frac{5}{0.5} = 10m/s.$
2	The general equation of a straight line: $P = kt + P_0$. $k = slope = \frac{\Delta P}{\Delta t} = \frac{0-5}{1.25-0} = -4kgm/s^2$ and $P_0 = 5kgm/s$ at t_0 = 0s. Therefore, $P = -4t + 5$. $V = \frac{P}{m} = \frac{-4t+5}{0.5} = -8t + 10.$
3	Name and represent.
4	$\sum \vec{F}_{ext} = \vec{W} + \vec{N} + \vec{f} = \vec{W_x} + \vec{W_y} + \vec{N} + \vec{f}$ with $\vec{W_y} + \vec{N} = \vec{0}$. Then, $\sum \vec{F}_{ext} = -mg\sin\alpha\,\vec{i} - f\vec{i} = (-mg\sin\alpha - f)\vec{i}$.
5	By applying Newton's 2nd law: $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$. Projection along x-axis: $-mg\sin\alpha - f = \frac{dP}{dt}$. $-0.5 \times 10 \times \sin 30° - f = -4 \Longrightarrow f = 1.5N$.
6	$x = \int V dt = -4t^2 + 10t + x_0$ with $x_0 = 0m$. $x = -4(1.25)^2 + 10(1.25) = 6.25m$.

Exercise 6:

Part	Answer key
	During Explosion, the vessel is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant.$ Law of conservation of linear momentum: $\vec{P_b} = \vec{P_a}$. $\vec{P} = \vec{P_1} + \vec{P_2} + \vec{P_3}$. $M\vec{V} = m_1\vec{V_1} + m_2\vec{V_2} + m_3\vec{V_3}$ with $m_1 = m_2 = m$ and $m_3 = 3m$. $\vec{0} = m\vec{V_1} + m\vec{V_2} + 3m\vec{V_3} \Longrightarrow \vec{0} = -30\vec{i} - 30\vec{j} + 3\vec{V_3} \Longrightarrow \vec{V_3} = 10\vec{i} + 10\vec{j}\ [m/s]$. $V_3 = \lVert\vec{V_3}\rVert = \sqrt{V_{3x}^2 + V_{3y}^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2}m/s.$ $tan\,\alpha = \frac{V_{3y}}{V_{3x}} = \frac{10}{10} = 1 \Longrightarrow \alpha = 45°.$

Exercise 7:

Part	Answer key
	During firing, the system [(R); (B)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant.$ Law of conservation of linear momentum: $\vec{P_b} = \vec{P_a}$. $\vec{P} = \vec{P_1} + \vec{P_2}$. $M\vec{V} = m_1\vec{V_1} + m_2\vec{V_2}$. $\vec{0} = m_1\vec{V_1} + m_2\vec{V_2}$. $m_1V_1 + m_2V_2 = 0 \Longrightarrow V_1 = -\frac{m_2}{m_1}V_2 = -\frac{5\times10^{-3}}{4} \times 500 = -0.625m/s.$

Exercise 8:

Part	Answer key
1	During collision, the system [(A); (B)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant.$ Principle of conservation of linear momentum: $\vec{P}_i = \vec{P}_f \Longrightarrow m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$. The collision is head on; then, the above expression can be written in its algebraic form: $m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$ with $V_2 = 0$. $m_1(V_1 - V_1') = m_2V_2'$ ... (1). The collision is elastic; then, the kinetic energy is conserved: $K.E_i = K.E_f$. $\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2 \Longrightarrow m_1\left(V_1^2 - V_1'^2\right) = m_2V_2'^2$. $m_1(V_1 + V_1')(V_1 - V_1') = m_2V_2'^2$ ... (2). Divide (2) by (1): $V_1 + V_1' = V_2'$ ... (3). Replace (3) in (1): $m_1(V_1 - V_1') = m_2(V_1 + V_1') \Longrightarrow V_1' = \frac{m_1-m_2}{m_1+m_2}V_1$ ... (4). Replace (4) in (3): $V_2' = V_1 + \frac{m_1-m_2}{m_1+m_2}V_1 = \left(1 + \frac{m_1-m_2}{m_1+m_2}\right)V_1 = \left(\frac{m_1+m_2+m_1-m_2}{m_1+m_2}\right)V_1 = \frac{2m_1}{m_1+m_2}V_1$.
2	$V_1 = \frac{0.4-0.6}{0.4+0.6} \times 5 = -1m/s.$ $V_2 = \frac{2\times0.4}{0.4+0.6} \times 5 = 4m/s.$
3.1.1	$\Delta\vec{P}_A = \vec{P}_1' - \vec{P}_1 = m_1(\vec{V}_1' - \vec{V}_1) = 0.4 \times (-\vec{\imath} - 5\vec{\imath}) = -2.4\vec{\imath}\ [kgm/s]$. $\Delta\vec{P}_B = \vec{P}_2' - \vec{P}_2 = m_2(\vec{V}_2' - \vec{V}_2) = 0.6 \times (4\vec{\imath} - \vec{0}) = 2.4\vec{\imath}\ [kgm/s]$.
3.1.2	By applying Newton's 2nd law on (A): $\sum \vec{F}_{ext/A} = \frac{d\vec{P}_A}{dt} \Longrightarrow \vec{W}_A + \vec{N}_A + \vec{F}_{B/A} = \frac{\Delta\vec{P}_A}{\Delta t}$ with $\vec{N}_A + \vec{W}_A = \vec{0}$. $\vec{F}_{B/A} = \frac{-2.4\vec{\imath}}{0.2} = -12\vec{\imath}$. By applying Newton's 2nd law on (B): $\sum \vec{F}_{ext/B} = \frac{d\vec{P}_B}{dt} \Longrightarrow \vec{W}_B + \vec{N}_B + \vec{F}_{A/B} = \frac{\Delta\vec{P}_B}{\Delta t}$ with $\vec{N}_B + \vec{W}_B = \vec{0}$. $\vec{F}_{A/B} = \frac{2.4\vec{\imath}}{0.2} = 12\vec{\imath}$.
3.2	$\vec{F}_{A/B} = -\vec{F}_{B/A}$.

Exercise 9:

Part	Answer key
1	The system is isolated, then $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant$. Principle of conservation of linear momentum: $\vec{P}_i = \vec{P}_f$. $\vec{0} = m_1\vec{V}_1 + m_2\vec{V}_2 \Longrightarrow \vec{V}_1 = -\frac{m_2\vec{V}_2}{m_1} = -\frac{(3)(4\vec{i})}{2} = -6\vec{i}\ (m/s)$.
2	The non-conservative force (friction) is neglected; then, the mechanical energy is conserved. $ME_i = ME_f$. $KE_i + GPE_i + EPE_i = KE_f + GPE_f + EPE_f$. $0 + 0 + EPE_i = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 + 0 + 0$. $EPE_i = \frac{1}{2} \times 2 \times 6^2 + \frac{1}{2} \times 3 \times 4^2 = 60J$.
3	The original energy is in the spring. A force had to be exerted over a displacement to compress the spring, transferring energy into it by work. The cord exerts force, but over no displacement.

Exercise 10:

Part	Answer key
1	During collision, the system [(A); (B)] is isolated. $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant.$ Principle of conservation of linear momentum: $\vec{P}_{bc} = \vec{P}_{ac}$. $m_A\vec{V}_A + m_B\vec{V}_B = m_A\vec{V}_A' + m_B\vec{V}_B'$. $(1.6)(5\vec{\imath}) + (2.4)(-2\vec{\imath}) = (1.6)(-3.4\vec{\imath}) + 2.4\vec{V}_B' \Longrightarrow \vec{V}_B' = 3.6\vec{\imath}\ (m/s)$.
2	$KE_{bc} = \frac{1}{2}m_AV_A^2 + \frac{1}{2}m_BV_B^2 = \frac{1}{2}\times 1.6 \times 5^2 + \frac{1}{2}\times 2.4 \times 2^2 = 24.8J.$ $KE_{ac} = \frac{1}{2}m_AV_A'^2 + \frac{1}{2}m_BV_B'^2 = \frac{1}{2}\times 1.6 \times 3.4^2 + \frac{1}{2}\times 2.4 \times 3.6^2 = 24.8J.$ $KE_{bc} = KE_{ac} \Longrightarrow$ The kinetic energy of the energy [(A); (B)] is conserved. Therefore, the collision is elastic.
3	Isolate (B): $\sum \vec{F}_{ext/B} = \frac{d\vec{P}_B}{dt} \Longrightarrow m_B\vec{g} + \vec{N}_B + \vec{F}_{A/B} = \frac{d\vec{P}_B}{dt}$. $m_B\vec{g} + \vec{N}_B = \vec{0}$ and $\frac{d\vec{P}_B}{dt} = \frac{\Delta\vec{P}_B}{\Delta t}$. $\vec{F}_{A/B} = \frac{\Delta\vec{P}_B}{\Delta t} = \frac{m_B\vec{V}_B' - m_B\vec{V}_B}{\Delta t} = \frac{m_B(\vec{V}_B' - \vec{V}_B)}{\Delta t}$. $\vec{F}_{A/B} = \frac{2.4(3.6\vec{\imath}+2\vec{\imath})}{0.1} = 134.4\vec{\imath}\ (N)$.

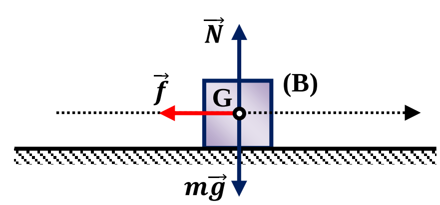
Exercise 11:

Part	Answer key
1	$P_{0x} = mV_{0x} = mV_0\cos\alpha = 0.5 \times 8 \times \cos 30° = 2\sqrt{3}kgm/s.$ $P_{0y} = mV_{0y} = mV_0\sin\alpha = 0.5 \times 8 \times \sin 30° = 2kgm/s.$
2	The only force acting on (S) is weight $\overrightarrow{W} = m\vec{g} = -mg\vec{\jmath}$. Apply Newton's 2nd law: $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} \Longrightarrow -mg\vec{\jmath} = \frac{dP_x}{dt}\vec{\imath} + \frac{dP_y}{dt}\vec{\jmath}$. $\frac{dP_x}{dt} = 0 \Longrightarrow P_x = constant = P_{0x} = 2\sqrt{3}kgm/s.$ $\frac{dP_y}{dt} = -mg \Longrightarrow P_y = -mgt + P_{0y} = -5t + 2\ (SI).$ Therefore, $\vec{P} = P_x\vec{\imath} + P_y\vec{\jmath} = 2\sqrt{3}\vec{\imath} + (-5t+2)\vec{\jmath}\ (SI)$.
3	$V_x = \frac{P_x}{m} = \frac{2\sqrt{3}}{0.5} = 4\sqrt{3}m/s.$ $V_y = \frac{P_y}{m} = \frac{-5t+2}{0.5} = -10t + 4\ (SI).$ $x = \int V_x dt = 4\sqrt{3}t + x_0 = 4\sqrt{3}t\ (SI)$ with $x_0 = 0$. $y = \int V_y dt = -\frac{10t^2}{2} + 4t + y_0 = -5t^2 + 4t\ (SI)$ with $y_0 = 0$.
4	$V_y = 0 \Longrightarrow -10t + 4 = 0 \Longrightarrow t = \frac{4}{10} = 0.4s.$ $y_m = -5 \times 0.4^2 + 4 \times 0.4 = 0.8m.$ $y = 0 \Longrightarrow -5t^2 + 4t = 0 \Longrightarrow t(-5t+4) = 0.$ $t = 0s$ (launching point) and $-5t + 4 = 0 \Longrightarrow t = \frac{4}{5} = 0.8s.$ $x = 4\sqrt{3}(0.8) = 3.2\sqrt{3}m = 5.54m.$

Exercise 12:

Part	Answer key
I.1	The distances covered by the pucks before and after the collision during successive and equal intervals of time (τ) are equal. Since $\vec{V}_A$, $\vec{V}_B$, $\vec{V}_A'$ and $\vec{V}_B'$ are collinear or held by the same axis $(O;\vec{\imath})$ then these velocities are constant. $V_A=\frac{A_0A_5}{5\tau}=\frac{20\times10^{-2}}{5\times20\times10^{-3}}=2m/s$, $V_B=-\frac{B_0B_5}{5\tau}=-\frac{10\times10^{-2}}{5\times20\times10^{-3}}=-1m/s.$ $V_A'=-\frac{A_0'A_5'}{5\tau}=-\frac{17.5\times10^{-2}}{5\times20\times10^{-3}}=-1.75m/s$, $V_B'=\frac{B_0'B_5'}{5\tau}=\frac{12.5\times10^{-2}}{5\times20\times10^{-3}}=1.25m/s.$
I.2	$\vec{P}=m_A\vec{V}_A+m_B\vec{V}_B=(0.15)(2\vec{\imath})+(0.25)(-1\vec{\imath})=0.05\vec{\imath}\,(kgm/s).$ $\vec{P}'=m_A\vec{V}_A'+m_B\vec{V}_B'=(0.15)(-1.75\vec{\imath})+(0.25)(1.25\vec{\imath})=0.05\vec{\imath}\,(kgm/s).$
I.3	$\vec{P}=\vec{P}'=0.05\vec{\imath}\Longrightarrow\vec{P}=constant.$ The linear momentum of the system [(A); (B)] is conserved.
I.4.1	The external forces acting of the system [(A); (B)] are: $\vec{W}_A$ and $\vec{W}_B$: Weights of (A) and (B) respectively. $\vec{N}_A$ and $\vec{N}_B$: Normal reaction of support forces acting on (A) and (B) respectively.
I.4.2	$\sum\vec{F}_{ext}=\vec{W}_A+\vec{N}_A+\vec{W}_B+\vec{N}_B=\vec{0}.$
I.4.3	$\Delta\vec{P}=\vec{P}'-\vec{P}=\vec{0}$ and $\sum\vec{F}_{ext}=\vec{0}.$ $\sum\vec{F}_{ext}=\frac{\Delta\vec{P}}{\Delta t}=\vec{0}.$
I.5	$KE=\frac{1}{2}m_AV_A^2+\frac{1}{2}m_BV_B^2=\frac{1}{2}(0.15)(2)^2+\frac{1}{2}(0.25)(1)^2=0.425J.$ $KE'=\frac{1}{2}m_AV_A'^2+\frac{1}{2}m_BV_B'^2=\frac{1}{2}(0.15)(1.75)^2+\frac{1}{2}(0.25)(1.25)^2=0.425J.$ $KE=KE'\Longrightarrow$ the kinetic energy is conserved and the collision is elastic.
II.1	Isolate (A): $\sum\vec{F}_{ext/A}=\vec{N}_A+\vec{W}_A+\vec{F}_{B/A}=\frac{\Delta\vec{P}_A}{\Delta t}$ with $\vec{N}_A+\vec{W}_A=\vec{0}.$ $\vec{F}_{B/A}=\frac{m_A\vec{V}_A'-m_A\vec{V}_A}{\Delta t}=\frac{m_A(\vec{V}_A'-\vec{V}_A)}{\Delta t}=\frac{(0.15)(-1.75\vec{\imath}-2\vec{\imath})}{100\times10^{-3}}=-5.625\vec{\imath}\,(N).$ Isolate (B): $\sum\vec{F}_{ext/B}=\vec{N}_B+\vec{W}_B+\vec{F}_{A/B}=\frac{\Delta\vec{P}_B}{\Delta t}$ with $\vec{N}_B+\vec{W}_B=\vec{0}.$ $\vec{F}_{A/B}=\frac{m_B\vec{V}_B'-m_B\vec{V}_B}{\Delta t}=\frac{m_B(\vec{V}_B'-\vec{V}_B)}{\Delta t}=\frac{(0.25)(1.25\vec{\imath}+1\vec{\imath})}{100\times10^{-3}}=5.625\vec{\imath}\,(N).$
II.2	$\vec{F}_{A/B}=-\vec{F}_{B/A}\Longrightarrow$ The principle of interaction is verified.

Exercise 13:

Part	Answer key	
1	$V_0=\frac{P_0}{m}=\frac{12}{1.5}=8m/s.$	
2	The external forces acting on (B) are weight $m\vec{g}$, normal reaction of a support $\vec{N}$ and friction $\vec{f}$.	
3	$\sum\vec{F}_{ext}=\frac{d\vec{P}}{dt}\Longrightarrow m\vec{g}+\vec{N}+\vec{f}=\frac{d\vec{P}}{dt}.$ Projection along x-axis: $-f=\frac{dP}{dt}\Longrightarrow f=-\frac{dP}{dt}.$ But P is linear, then $\frac{dP}{dt}=\frac{\Delta P}{\Delta t}=\frac{0-12}{4-0}=-3kgm/s^2.$ Therefore, $f=3N.$	
4	$\Delta KE=\sum W_{ext}\Longrightarrow KE_M-ME_O=W_{m\vec{g}}+W_{\vec{N}}+W_{\vec{f}}.$ $0-\frac{1}{2}mV_0^2=0+0-f\times OM\Longrightarrow-\frac{1}{2}(1.5)(8)^2=-3\times OM\Longrightarrow OM=16m.$	