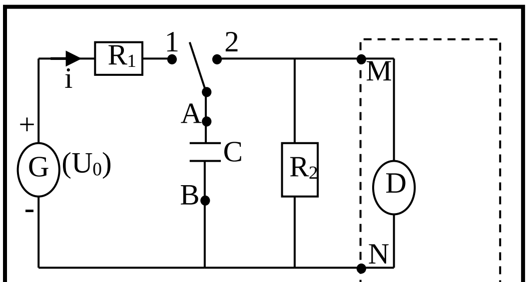
Simulation of the timing of a car's light

The circuit of the document (1) simulates the timing of the light a car .This circuit comprises:

- a generator delivering between its terminals a constant voltage U_0= 12V;
- a capacitor initially uncharged and of capacitance C = 470μF
- two resistors R_1 and R_2 where R_1 and R_2 have respectively the resistances R_1=100Ω and R_2 = 1000Ω ;
- a small electronic circuit (D) which commands a lamp that remains lie as long as the potential difference u_{MN} is higher than 4.5V. We suppose that the circuit (D) does not influence the circuit of the capacitor .
- Connecting wires and a switch K which :
  - ✓ when the door of the car is opened , K passes in position (1) .
  - ✓ when the door is closed K passes in position (2) .



Doc. 1

1-Charge of the capacitor

We open the door of the car at the instant $t_0 = 0$, that corresponds to putting the switch K on position (1) at $t_0 = 0$. At the instant t, the voltage across the capacitor is $u_c = u_{AB}$ and the circuit is traversed by a current i .

1.1) Using the law of addition of voltages , derive the differential equation describing the variation of the voltage u_c in terms of time .

1.2) The solution of this differential equation is of the form $u_C = A + B\, e^{-\frac{t}{\tau}}$ where A , B and τ are constants . Determine A , B and τ and specify the significance of τ .

1.3) Deduce the minimum duration at the end of which the capacitor will be supposed practically completely charged

2- Discharging the capacitor

We close the door of the car at an instant $t_0 = 0$, considering as new origin of time, which that corresponds to putting the switch K on position (2) at $t_0 = 0$.

2.1) Draw a figure of the new circuit showing the real direction of the discharging current i .

2.2) Show that the differential equation that governs the variations of u_c in terms of time is of the form :

$$\frac{du_C}{dt} + \frac{u_C}{R_2C} = 0$$

2.3)

2.3.1) What is the initial value of u_c .

2.3.2) $u_C = D + F\, e^{-\alpha t}$ is a solution of this differential equation .

2.3.2.1) Determine the numerical values of the constants : D , F and α .

2.3.2.2) Draw the shape of the graph representing the variations of u_C in terms of time .

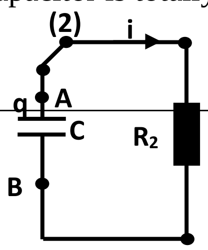
2.3.2.3) Show on the graph the point of abscissa $t = \frac{1}{\alpha}$.

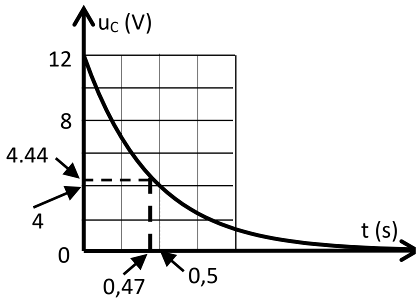
2.4)

2.4.1) Deduce from the graph an approximate value of the duration , during which the lamp remains glowing after closing the door of car .

2.4.2) Determine this duration by calculation .

2.4.3) Will this duration be sufficient if we want that lamp shines between 5 and 10 seconds ?

Part	First exercise	Points
1.1	$+\frac{1}{R_1C}u_C = \frac{E}{R_1C}$.	0.5
2.2	At $t_0 = 0$, $u_C = 0 \Rightarrow A + B = 0$; $\Rightarrow$ **A = – B** $\Rightarrow u_C = A - Ae^{-\frac{t}{\tau}}$ $\Rightarrow \frac{A}{\tau}e^{-\frac{t}{\tau}} - \frac{A}{R_1C}e^{-\frac{t}{\tau}} + \frac{A}{R_1C} = \frac{E}{R_1C}$ $\Rightarrow$**A = E** and $\frac{1}{\tau} = \frac{1}{R_1C}$ $\Rightarrow$ **$\tau = R_1C$** τ time constant is the time needed for u_C = 63%of its maximum value E.	1
3.3	Capacitor is totally charged $t = 5\tau = 5\times100\times470\times10^{-6} = 0.235$ s. 	0.5
2.1		0.5
2.2	$\frac{du_C}{dt}$	1
2.3.1	At $t_0 = 0$, $u_C = E = 12$ V.	0.25
2.3.2.1	$\Rightarrow -\alpha F e^{-\alpha t} + \frac{1}{R_2C}D + \frac{1}{R_2C}F e^{-\alpha t} = 0$ $\Rightarrow D = 0$;	1.5

2.3.2.2	$\frac{1}{\alpha}$= 0.47 s u_C = 0.37×12 = 4.44 V. 	**1**
2.4.1	At u_{min} = 4 V, t = 0.5 s	**0.25**
2.4.2.1	$4 = 12\ e^{-2.127t} \Rightarrow \ell n(\frac{4}{12}) = -\ 2.127\ t = -\ 1.098 \Rightarrow t = 0.52$ s.	**0.75**
2.4.3	Since 10 is greater than 2.35 s (5τ).	**0.25**