TRANSFORMATIONS

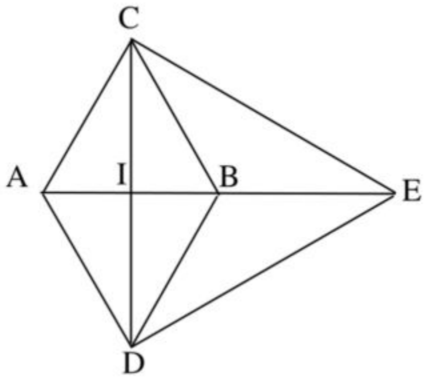
2004-2022

+SAMPLES

2004(1)

IV - (3points)

In the adjacent figure,
ABC , ADB and CDE are three direct equilateral triangles such that $(\vec{AB};\vec{AC}) = \frac{\pi}{3} \quad (2\pi)$.



Designate by I the midpoint of [AB] .

1) Show that AE = 2AB.

Let S **be the direct similitude, with center** W, **ratio** k **and angle** θ , **that transforms** A **onto** B **and** E **onto** D.

2) Determine k and verify that $\theta = \frac{-2\pi}{3} \quad (2\pi)$.

3) Designate by (T) the circle circumscribed about triangle ACE.
Prove that the image of (T), under S, is the circle (T ') of diameter [BD] and deduce that the image of point C under S is point J, the midpoint of [DE].

4) The complex plane is referred to a direct orthonormal system $(A ; \vec{u} , \vec{v})$ such that $\vec{u} = \vec{AI}$.

a- Determine the affixes of the points B , C , D and E.

b- Find the complex form of S and specify the affix of its center W.

5) Let S' be the direct similitude with center W, ratio 2 and angle $\frac{-\pi}{3}$.

a- Determine the nature and the elements of the transformation S'oS .

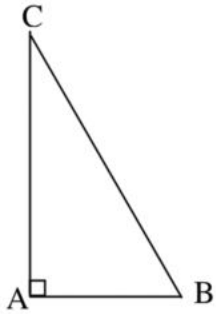
b- Calculate the affix of point A' , the image of A under S'oS .

2004(2)

III- (2 points)

In an oriented plane, consider a direct triangle ABC right angled at A, such that $AB = 2cm$ and $(\overrightarrow{BC}\,;\overrightarrow{BA}) = \frac{\pi}{3}\ (2\pi)$.

Let **S** be the direct similitude that transforms A onto B and B onto C.



1) Determine the ratio and the angle of **S**.

2) a- Construct the point C', the image of C under **S**.
(Give the steps of this construction).
b- Calculate the area of triangle BCC'.

3) Let O be the midpoint [AB] , and consider the direct orthonormal system $(O;\vec{u},\vec{v})$ such that $\vec{u} = \overrightarrow{OB}$.
a- Find the complex form of the similitude **S**.
b- Determine the affix of point W , the center of **S** .
c- Let $\mathbf{S}^{-1}$ be the inverse transformation of **S** . Give the complex form of $\mathbf{S}^{-1}$.

2005(1)

V– (3.5 points)

The complex plane is referred to a direct orthononormal system $(O\,;\vec{u},\vec{v})$.

Let A be the point of affix 2, and B be the point of affix 2i .

Designate by E the image of A under the rotation R with center O and angle $\frac{\pi}{3}$,

and by F the image of B under the transformation T that is defined by the complex form : $z' = (\frac{-1}{2} - i\frac{\sqrt{3}}{2})\,z$.

1) a- Determine the nature and the characteristic elements of T .

b- Prove that the four points A, B, E and F belong to the same circle with center O and whose radius is to be determined.

2) a- Prove that $\dfrac{z_E - z_A}{z_F - z_B}$ is real .

b- Verify that $\dfrac{z_F - z_A}{z_E - z_B} = -i$.

c- Deduce that AEBF is an isosceles trapezoid and that $(\overrightarrow{BE},\overrightarrow{AF}) = -\frac{\pi}{2}\ (2\pi)$.

3) Consider : the dilation (homothecy) h that transforms A onto F and E onto B ,

and the rotation r with angle $\frac{\pi}{2}$, that transforms B onto F .

a- Determine W, the center of h.

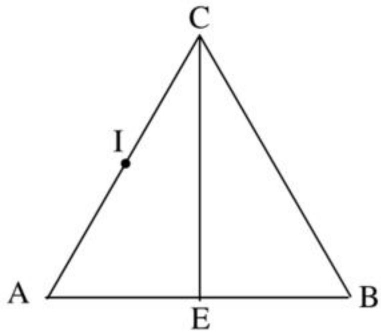
b- Prove that $h\circ r = r\circ h$.

c- Let $S = h\circ r$.

Determine the nature and the characteristic elements of S .

2005(2)

III- (3 points)



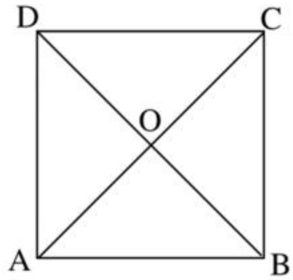
Given, in an oriented plane, a direct equilateral triangle ABC of side 4 cm .
Designate by E and I the mid points of [AB] and [AC] respectively.
Let S be the direct plane similitude that transforms A onto E and E onto C.

1) a- Determine the ratio and an angle of S.
   b- Construct the image under S of each of the straight lines (AC) and (EI), and deduce the image of I under S.

2) Suppose that the plane is referred to a direct orthonormal system $(A\,;\,\vec{u}\,,\vec{v}\,)$ where $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.
   a- Give the complex form of S.
   b- Find the affix of the point W, the center of S.
   c- Prove that W is a point on [AC] .
   d- Let J be the image of the point I under $S \circ S$; Compare WC and WJ.

2006(1)

III– (3 points)



Consider, in an oriented plane, the direct square ABCD with center O such that $(\overrightarrow{AB}, \overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$.

Let r be the rotation with center O and angle $\frac{\pi}{2}$ and h be the dilation (homothecy) with center C and ratio 2.
Designate by S the transformation $r \circ h$.

1) Determine the nature of S and specify its ratio and its angle.

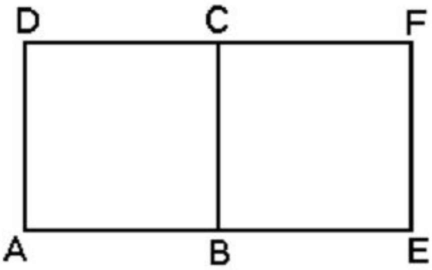
2) Designate by W the center of S.
   a- Show that S(C) = D and that S(O) = B.
   b- Construct the point W, specifying clearly the steps of this construction .

3) The plane is referred to an orthonormal system $(A\ ;\ \overrightarrow{AB}\ ,\ \overrightarrow{AD})$.
   a- Write the complex form of S and deduce the affix of the center W.
   b- Determine the image of the square ABCD under S .

2006(2)

III-(3 points)

Consider, in an oriented plane, the two direct squares ABCD and BEFC.
Let S be the direct plane similitude that transforms A onto E and E onto F.



1) a- Determine the ratio k and an angle α of S.
   b- Construct geometrically the center W of S.
   c- Find the point G that is the image of F under S.

2) Let h be the transformation that is defined by $h = S \circ S$.
   a- Determine the nature and the elements of h.
   b- Specify h(A), and express $\overrightarrow{WA}$ in terms of $\overrightarrow{WF}$.

3) The complex plane is referred to an orthonormal system $(A; \overrightarrow{AB}, \overrightarrow{AD})$.
   a- Determine the affixes of the points E, F and W.
   b- Find the complex form of S.
   c- Give the complex form of h and find the affix of h(E).

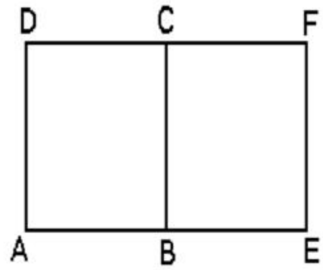
2007(1)

III- (3 points)

Consider, in an oriented plane, a direct rectangle AEFD such that: $(\overrightarrow{AE}, \overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$, $AE = 2\sqrt{2}$ and $AD = 2$.

Designate by B and C the midpoints of [AE] and [FD] respectively.

Let S be the direct plane similitude that transforms A onto C and E onto B.



1) a- Determine the ratio k and an angle α of S.
   b- Show that S(F) = E and deduce S(D).

2) Let W be the center of S and let h be the transformation defined by h = S o S.
   a- Determine the nature and the characteristic elements of h.
   b- Find h(D) and h(F) and construct the point W.

3) Designate by I the mid point of [BE].
   a- Prove that W, C and I are collinear.
   b- Express $\overrightarrow{WC}$ in terms of $\overrightarrow{WI}$.

4) The complex plane is referred to the orthonormal system $(A; \vec{u}, \vec{v})$ where $z_B = \sqrt{2}$ and $z_D = 2i$.
   a- Find the complex form of S.
   b- Determine the affix of W.

2007(2)

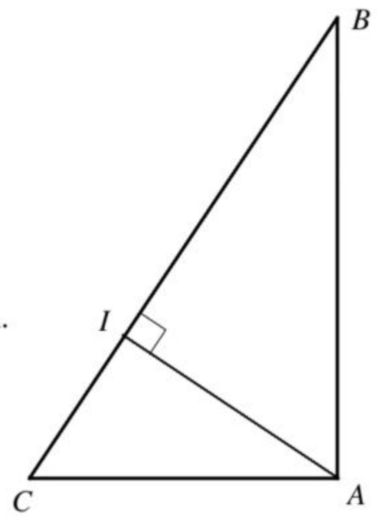
V – (3 points)

Given a triangle ABC such that $AB = 6$, $AC = 4$

and $(\overrightarrow{AB}\ ;\ \overrightarrow{AC}) = \frac{\pi}{2}\ (2\pi)$

Let I be the orthogonal projection of A on (BC).

1- Let h be the dilation of center I that transforms C onto B.
Construct the image (d) of the line (AC) under h.
Deduce the image D of A under h.



2- Let S be the similitude that transforms A onto B, and C onto A.
a) Determine the ratio and an angle of S.
b) Determine the image by S of each of the two straight lines (AI) and (CB). Deduce that I is the center of S.
c) Determine the image of (AB) by S.
Deduce that $S(B) = D$.

3- a) Determine the nature and the characteristic elements of $S \circ S$.
b) Prove that $S \circ S(A) = h(A)$.
c) Prove that $S \circ S = h$.

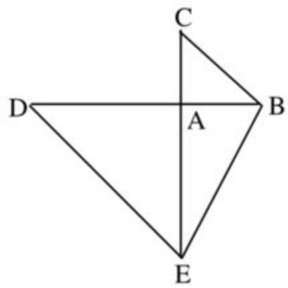
4- Let E be the mid point of [AC].
a) Determine the points F and G such that $F = S(E)$ and $G = S(F)$.
b) Show that the points E, I and G are collinear.

2008(1)

IV- (3 points)

In the figure below, ABC and ADE are right isosceles triangles such that AB=1 ; AD=2 and $(\overrightarrow{AB};\overrightarrow{AC})=(\overrightarrow{AD};\overrightarrow{AE})=\frac{\pi}{2}\ [2\pi]$.

Let O be the midpoint of $[BE]$.



1) Let r be the rotation with center A and angle $\frac{\pi}{2}$, and h be the dilation with center E and ratio 2.

   Let $f = r \circ h$.

   a- Prove that f is a similitude whose ratio and angle are to be determined.

   b- Determine f(O) and f(A) and deduce that (AO) is perpendicular to(CD).

2) Let s be the direct plane similitude that transforms A onto C and D onto A.

   a- Precise the ratio and an angle of s.

   b- Determine the image by s of the straight line (CD) and that of the straight line (AO).

   c- Deduce the center I of s.

3) Let $g = f \circ s$ and A' be the symmetric of A with respect to D.

   a- Prove that g is the symmetry with center D.

   b- Deduce that f(C)=A'.

4) The complex plane is referred to a direct orthonormal system $(A;\overrightarrow{AB};\overrightarrow{AC})$.

   a- Determine the affixes of the points O, C and A'.

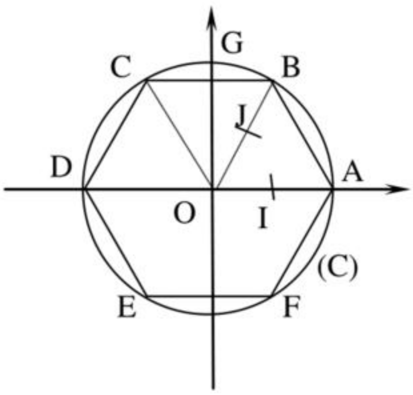
   b- Write the complex form of f and determine the affix of its center Ω.

2008(2)

V- (3 points)

In an oriented plane, given a direct regular hexagon ABCDEF of center O, such that: $(\overrightarrow{OA} ; \overrightarrow{OB}) = \frac{\pi}{3}\ (2\pi)$.

(C) is the circle circumscribed about this hexagon.
I and J are the midpoints of [OA] and [OB] respectively.



Let S be the similitude that transforms A onto B and B onto J.

1)a- Determine the ratio and an angle of S.
b-Prove that S(D) = A. Find S(O) and verify that S(C) = I.
c- Determine the image of the hexagon ABCDEF by S.

2) The circle (C') is the image of (C) by S. Determine the center and the ratio of each of the two dilations (homothecies) that transforms (C) onto (C').

3) G is the midpoint of the arc BC on the circle (C).
The plane is referred to the orthonormal system $(O ; \overrightarrow{OA}, \overrightarrow{OG})$.
a- Find the affix of each of the points B, C, E and F.
b- Write the complex form of S and deduce the affix of its center W.
c- H is the point of intersection of [AJ] and [BI]. Determine the point H' the image of H by S.

2009(1)

V- (3 points)

ABCD is a square of side 2 and of center O such that $(\overrightarrow{AB}, \overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$.

E and F are the midpoints of [AB] and [BC] respectively and G is the midpoint of [BF].
Let S be the direct plane similitude that transforms A onto B and D onto E.

1) Calculate an angle and the ratio of S.

2) Verify that $S(B) = F$, and determine $S(E)$.

3) Let $h = SoS$.
   a- Show that h is a dilation and precise its ratio.
   b- Prove that the center I of S is the point of intersection of (AF) and (DG).
   c- Determine the image by S of the square ABCD and deduce the nature of triangle OIC.

4) Let (A_n) be the sequence of points defined by: $A_o = A$ and $A_{n+1} = S(A_n)$ for all natural integers n.
   a- Let $L_n = A_nA_{n+1}$ for all n.
   Prove that (L_n) is a geometric sequence whose common ratio and first term are to be determined.
   Calculate $S_n = L_0 + L_1 + \dots\dots\dots + L_n$ and $\lim\limits_{n\to+\infty} S_n$.

   b- Calculate $\left(\overrightarrow{IA}, \overrightarrow{IA_n}\right)$ in terms of n and prove that if n is even then the points I, A and A_n are collinear.

2009(2)

III- (3 points)

In an oriented plane, consider the rectangle ABCD such that:

$\left(\overrightarrow{AB};\overrightarrow{AD}\right)=\frac{\pi}{2}(\bmod 2\pi)$, $AB = 4$ and $AD = 3$.

Let H be the orthogonal projection of A on (BD) and h be the dilation, of center H, that transforms D to B.

1) a- Determine the image of the straight line (AD) by h.

   b- Deduce the image E of point A by h. Plot E.

   c- Construct the point F image of B by h and the point G image of C by h, then determine the image of rectangle ABCD by h.

2) Let S be the direct similitude that transforms A onto B and D onto A.

   a- Determine an angle of S.

   b- Determine the image of the straight line (AH) by S and the image of the straight line (BD) by S.

   c- Deduce that H is the center of S.

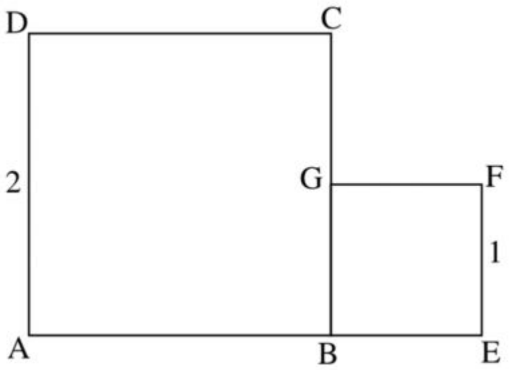
3) Show that $S(B)=E$ and deduce that $S\circ S(A)=h(A)$.

4) Show that $S\circ S=h$.

2010(1)

III- (3 points)

In an oriented plane, consider two direct squares ABCD and BEFG of sides 2 and 1 respectively.



Let S be the direct plane similitude, of ratio k and angle α, that transforms E onto A and G onto C.

A- 1) Verify that $k = 2$ and $\alpha = -\frac{\pi}{2}$ (2π).

2) Prove that the image of point F by S is B then deduce S(B).

3) Construct the center W of S.

B- The complex plane is referred to a direct orthonormal system $(B; \overrightarrow{BE}; \overrightarrow{BG})$.

1) Determine the affixes of the points A and E.

2) Write the complex form of S.

3) Determine the algebraic form of the affix of W.

4) Show that the points C, W and E are collinear.

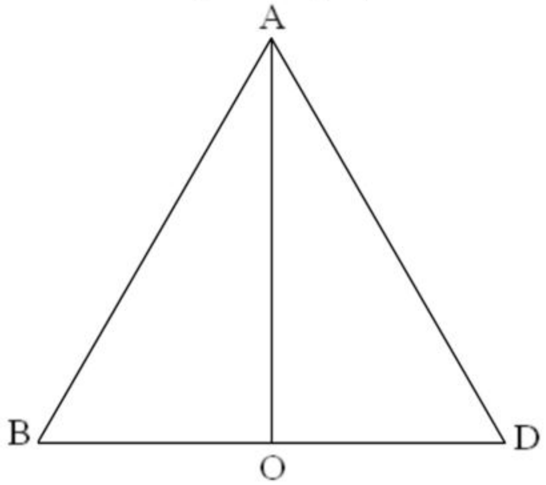
C- Let R be the rotation with center A and angle $\frac{\pi}{2}$.

Prove that RoS is a dilation whose center and ratio are to be determined.

2010(2)

V– (3 points)

In the figure below, ABD is an equilateral triangle with side 2 such that $(\overrightarrow{AB};\overrightarrow{AD}) = \frac{\pi}{3}\ (2\pi)$ and O is the midpoint of $[BD]$.



Let S be the direct similitude that transforms O onto D and D onto A.

1) Determine the image of B by S.

2) Determine the ratio and an angle of S.

3) a- Let A' be the image of the point A by S. Determine the nature of the triangle DAA' and deduce that the point A' is the symmetric of A with respect to B.

b- Let G be the centre of gravity of triangle AOD and G' be its image by S. Prove that $\overrightarrow{G'B} = \frac{2}{3}\overrightarrow{DO}$.

4) The plane is referred to the direct orthonormal system $(O;\ \vec{u},\ \vec{v})$ such that $\vec{u} = \overrightarrow{OD}$. Let L be the symmetric of B with respect to D and h be the dilation with center L and ratio $-\frac{1}{2}$.

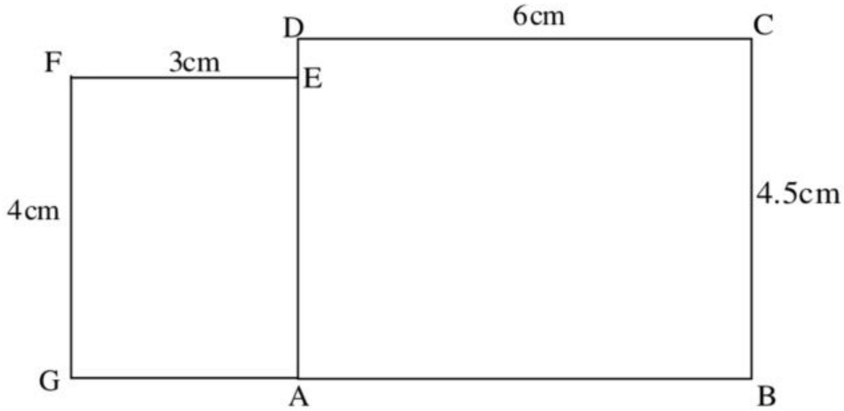
a- Determine the complex form of S, and deduce the affix of its center W.

b- Determine the complex form of h.

c- Determine the complex form of $S \circ h$. Verify that $S \circ h$ is a rotation and determine an angle of it.

2011(1)

V- (3 points)



In the figure above, ABCD and AEFG are two direct rectangles so that $(\overrightarrow{AB}, \overrightarrow{AD}) = \frac{\pi}{2} \pmod{2\pi}$.

S is the direct plane similitude that transforms B onto E and C onto F;

T is the translation with vector $\overrightarrow{EF}$;
f is the similitude defined by T o S.

1) a- Determine the ratio k and an angle α of S.

b- Determine the image of D by S.

c- Prove that A is the center of S.

2) a- Find f(B) and f(A).

b- Specify the ratio and an angle of the similitude f.

c- Construct the center W of f.

3) The complex plane is referred to a direct orthonormal system $(A\ ;\ \frac{1}{6}\overrightarrow{AB}, \frac{1}{4}\overrightarrow{AE})$.

a- Write the complex form of f.

b-Deduce the affix of point W.

4) Let F_1 be the image of F by S, and for any nonzero natural integer n, let F_{n+1} be the image of F_n by S.

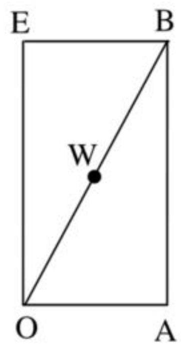
Determine the values of n so that A, F_1 and F_n are collinear.

2011(2)

V- (3 points)

Consider in an oriented plane the rectangle OABE such that OA = 2 and $\left(\overrightarrow{OA}, \overrightarrow{OB}\right) = \frac{\pi}{3}\ (2\pi)$.

Designate by (C) the circle with diameter [OB] and center W.



Let S be the direct plane similitude with center O, ratio $\sqrt{3}$ and angle $\frac{\pi}{3}$.

A-

1) Let A' be a point on the semi-straight line [OB) such that OA' = $2\sqrt{3}$.

   Prove that A' is the image of A under S.

2) a- Verify that the triangle OAW is equilateral.
   b- Determine the image under S of triangle OAW.
   c- Construct then the circle (C'), the image of (C) under S.

B-

The complex plane is referred to a direct orthonormal system $(O\,;\vec{u},\,\vec{v})$ such that:

$z_A = 2$ and $z_E = 2\sqrt{3}i$.

1) Write the complex form of S.

2) Find the affix of W and that of W' the image of W under S.

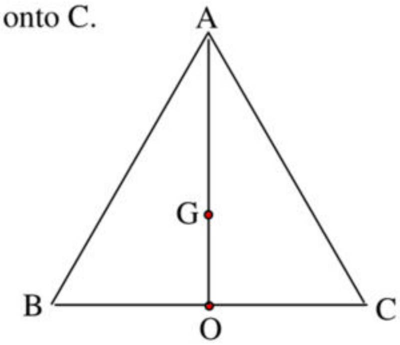
3) Let f be the plane transformation with complex form $z' = iz + 4 + 2i\sqrt{3}$.
   a- Show that f is a rotation whose angle and center H are to be determined.
   b- Verify that f(W') = W and determine $f \circ S(W)$.
   c- Determine the nature and the characteristic elements of $f \circ S$.

2012(1)

V- (3 points)

Consider a direct equilateral triangle ABC with center G. Let O be the midpoint of [BC] and F that of [AC].

Let S be the direct plane similitude that transforms A onto B and G onto C.



A-

1) a-Determine the ratio and an angle of S.

   b-Prove that F is the center of S.

2) a-Determine the line (d) , image of (AB) under S.

   b-Construct the point D image of B under S.

B- The complex plane is referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$ such that: $z_A = i\sqrt{3}$ and $z_C = 1$.

   a- Find the complex form of S.

   b- Determine the nature and characteristic elements of $S \circ S$.

   c- Express $\overrightarrow{FD}$ in terms of $\overrightarrow{FA}$.

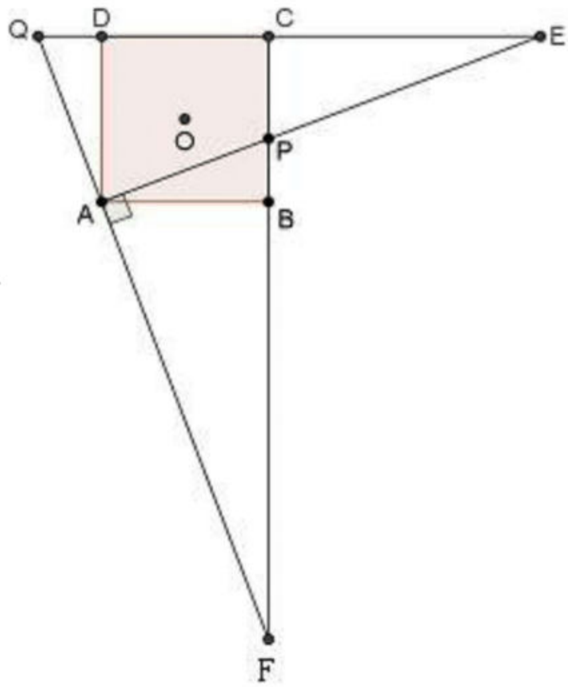
4) Denote by (E) the ellipse with center O and vertices B,C and G. (E') is the image of (E) under S.

   a- Determine the center and the focal axis of the ellipse (E').

   b- Calculate the area of the region bounded by (E') .

2012(2)

IV- (3 points)



In the figure to the right, ABCD is a square with side 1 and center O such that $\left(\overrightarrow{AB},\overrightarrow{AD}\right)=\frac{\pi}{2}(2\pi)$.

P is a point on the segment [BC] such that PB = t with $0 < t < 1$.
The line (AP) intersects the line (CD) at E.
The perpendicular to (AP) at A intersects (CB) at F and (CD) at Q.
Denote by M the midpoint of [FE] and by N that of [PQ].

1) Let r be the rotation with center A and angle $\frac{\pi}{2}$.

a- Determine, with justification, the image of (BC) under r.
b- Show that r(P)= Q and determine r(F).
c- Specify the nature of each of the triangles APQ and AFE.

2) Let s be the similitude with center A, ratio $\frac{1}{\sqrt{2}}$ and angle $\frac{\pi}{4}$.

a- Prove that s(P) = N; and determine s(F) and s(B).
b- Deduce that M, B, N and D are collinear.

3) a- Prove that $BF=\frac{1}{t}$.

b- Determine t so that the area of triangle AMN is equal to $\frac{5}{8}$.

4) The complex plane is referred to the system $\left(A;\overrightarrow{AB},\overrightarrow{AD}\right)$.

a- Write the complex form of s.

b- In the case where $t=\frac{1}{3}$, determine the affixes z_M and z_N of the points M and N and deduce that $\frac{z_M-1}{z_N-1}$ is a real number.

2013(1)

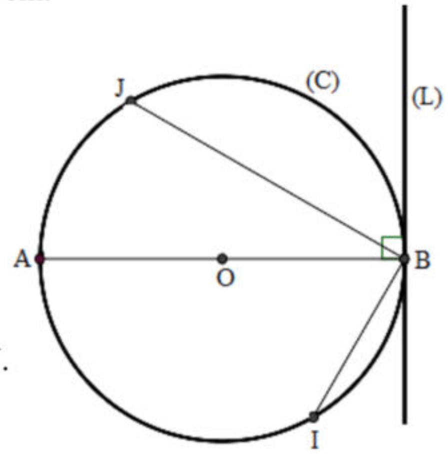
IV- (3 points)

In an oriented plane, consider a circle (C) with center O and radius 2 cm.

[AB] is a diameter of (C).

I and J are two points of (C) so that $\left(\overrightarrow{BI}, \overrightarrow{BA}\right) = -\frac{\pi}{3}\,[2\pi]$

and $\left(\overrightarrow{BA}, \overrightarrow{BJ}\right) = -\frac{\pi}{6}\,[2\pi]$.



The line (L) is tangent at B to the circle (C).

Consider the direct similitude S with center B that transforms I onto J.

1) Determine an angle of S and verify that its ratio k is $\sqrt{3}$.

2) a- Show that (AJ) is the image of the line (AI) under S.

b- Find the image of the line (AB) under S.

c- Deduce S (A) and then find S (J).

3) Let (C′) be the image of (C) under S. Determine (C′) and calculate its area.

4) Let $S_2 = S \text{ o } S$, $\quad S_3 = S \text{ o } S \text{ o } S$,……………, $S_n = \underbrace{S \text{ o } S \text{ o } S \text{ o} \ldots \text{o } S}_{\text{n times}}$.

n being a natural number ($n \geq 2$).

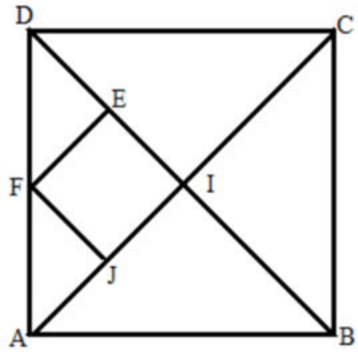
a- Verify that S o S is a dilation whose center and ratio are to be determined.

b- Determine, in terms of n, the ratio and an angle of the similitude S_n .

c- Find the values of n for which S_n is a dilation.

2013(2)

IV- (3 points)



ABCD is a direct square with center I.

Denote by E, F and J the respective midpoints of the segments [ID], [DA] and [AI].

Let S be the similitude with ratio k and an angle α that maps A onto J and B onto I.

1) Calculate k and determine a measure of α.

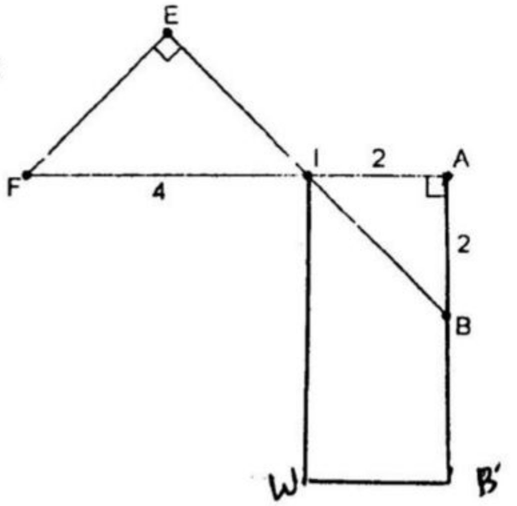
2) Prove that E is the image of C under S and determine S(D), S(I) and S(J).

3) a- Specify the nature of $S \circ S$, then calculate its ratio and a measure of its angle.
   b- Determine SoS(A) and SoS(B), then construct the point Ω, center of S.

4) The plane is referred to a direct orthonormal system $(A\,;\overrightarrow{AB}, \overrightarrow{AD})$.
   Determine the complex form of S, then deduce the coordinates of the point Ω.

2014(1)

III- (3 points)



In the adjacent figure, triangle AIB is right isosceles so that

$\left(\overrightarrow{AI}, \overrightarrow{AB}\right) = \frac{\pi}{2}(2\pi)$ and AI = AB = 2.

Triangle IEF is right at E and IF = 4.

Denote by I the intersection point of (EB) and (AF).

Let S be the similitude that maps F onto I and E onto A.

1) Show that I is the midpoint of [BE].

2) Show that $-\frac{\pi}{4}$ is an angle of S and $\frac{\sqrt{2}}{2}$ is the ratio of S.

3) a- Prove that S(I) = B.

   b- Let B' be the image of B under S.

   Prove that B is the midpoint of [AB'].

4) W is the point such that IAB'W is a rectangle.

   a- Prove that W is the center of S.

   b- Show that FEBW is a rectangle.

5) Consider the dilation h with ratio −2 that maps B onto F.

   a- Construct the point L such that L = h (W).

   b- Determine the center of h.

6) Consider the similitude Soh.

   Determine the ratio and an angle of Soh ,then find Soh(B).

7) The complex plane is referred to the direct orthonormal system $\left(I;\vec{u},\vec{v}\right)$ with $\vec{u} = \frac{1}{2}\overrightarrow{IA}$.

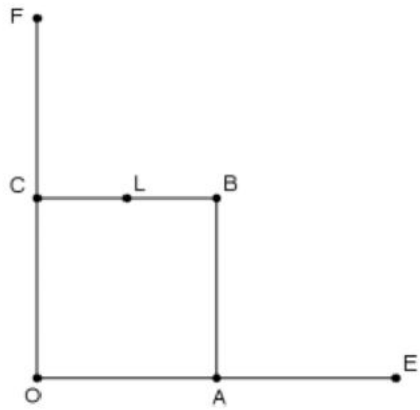
   Find the complex form of Soh, then deduce the affix of its center.

2015(1)

V- (3 points)

In the figure below, OABC is a direct square so that:

$OA = 2$, and $\left(\overrightarrow{OA};\overrightarrow{OC}\right) = \frac{\pi}{2}\ [2\pi]$.



Let E be the symmetric of O with respect to A , F the symmetric of O

with respect to Cand L the midpoint of segment [BC].

S is the similitude that maps O onto E and C onto O.

1) Calculate the ratio k and an angle α of S.

2) a- Determine the image of line (BC) under S .

b- Prove that the image of the line (OB) under S is the line (EF).

c- Determine S(B), then S(L).

3) The complex plane is referred to a direct orthonormal system $\left(O;\frac{1}{2}\overrightarrow{OA},\frac{1}{2}\overrightarrow{OC}\right)$.

a- Write the complex form of S.

b- Deduce the affix of the center I of S.

c- Prove that I is the intersection point of (OL) and (EC).

2015(2)

V- (3 points)

Consider a direct equilateral triangle ODA with side equal to 1.

Let R be the rotation with center O and angle $\frac{\pi}{2}$.

Denote by B = R (A), D'= R (D). Let C be the point so that D = R(C). (C is the pre-image of D)

1) a- Make a figure.
   b- Show that O is the midpoint of [CD'] and that BC= $\sqrt{3}$.

2) a- Justify that (AC) is perpendicular to (BD) and that AC = BD.
   b- Show that (AD) is parallel to (BC).

3) Denote by E the point of intersection of lines (AC) and (BD). Let h be the dilation with center E that transforms A onto C.
   a- Determine h (D).
   b- Calculate the ratio of h.

4) Let L be the midpoint of [AD] and F = h (L).
   Show that O, E, F and L are collinear.

5) Let R' be the rotation with center E and angle $-\frac{\pi}{2}$. Consider $S = h \circ R'$.
   a- Determine the nature of S and so its elements.
   b- Prove that S (A) = B.

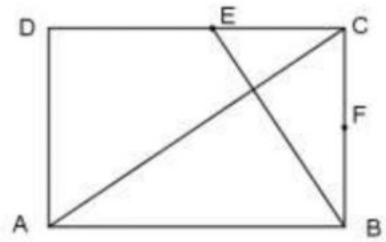
2016(1)

IV- (3 points)

ABCD is a direct rectangle such that AB = 3,
AD = 2 and $\left(\overrightarrow{AB},\overrightarrow{AD}\right)=\frac{\pi}{2}[2\pi]$.

F is the midpoint of segment [BC].

The perpendicular through B to the line (AC) intersects (DC) at E.



Let S be the similitude that maps A onto B and B onto F.

1) Determine the ratio (scale factor) k and an angle α of S.
2) Justify that the image of line (AC) under S is (BE).
3) Determine the image of (BC) under S. Deduce the point H image of C under S.
4) Determine the image of the rectangle ABCD under S.
5) The plane is referred to a direct orthonormal system $\left(A;\vec{u},\vec{v}\right)$ with $\vec{u}=\frac{1}{3}\overrightarrow{AB}$ and $\vec{v}=\frac{1}{2}\overrightarrow{AD}$.
   Write the complex form of S, then deduce the affix of its center W.

6) Let M be a point in the plane with affix $z=3\cos\theta+2i\sin\theta$ $\left(\text{with } 0<\theta<\frac{\pi}{2}\right)$.

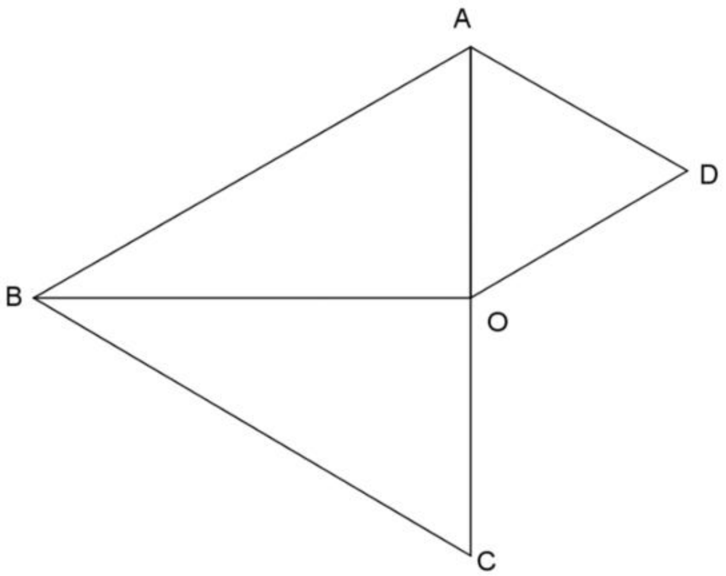
   a- Prove that M moves on an ellipse (Γ) with center A, having B and D as two of its vertices.

   b- Write an equation of (Γ') the image of the ellipse (Γ) under S.

2016(2)

V- (3 points)

In the following figure, ABC and AOD are two direct equilateral triangles where O is the midpoint of [AC].



Let S be the direct plane similitude that transforms B onto O and C onto D.

1)a- Determine the ratio k and an angle α of S.
  b- Verify that A is the center of S.

2) Consider the transformation R such that $R(B) = C$ and $R(C) = A$.
  a- Prove that R is a rotation and determine an angle of R.
  b- Determine the center G of R.

3) Let $h = S \circ R$.
  a- Determine h (B) and h(C).
  b- Determine the nature, the center and the ratio of h.

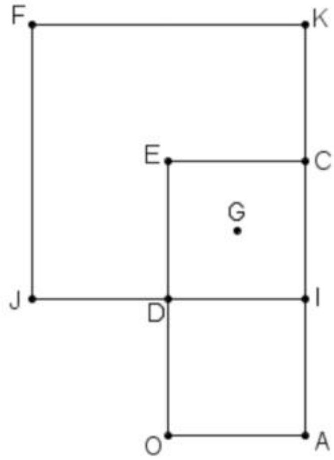
4) The plane is referred to the direct orthonormal system $(O; \vec{u}, \vec{v})$ such that $\overrightarrow{OA} = 2\vec{v}$.
  a- Determine the complex form of S.

  b- Consider the ellipse (E) with equation $\frac{x^2}{12} + \frac{y^2}{4} = 1$. Let (E′) be the image of (E) under S.
  Determine an equation of the focal axis of (E′).

2017(1)

V- (3 points)

In the figure to the right:



- DICE and JIKF are two direct squares with centers G and E respectively.
- A is the symmetric of C with respect to I.
- O is the symmetric of E with respect to D.

Let S be the direct plane similitude that maps A onto I and I onto E.

Part A

1) a- Show that the ratio of S is equal to $\sqrt{2}$ and that $\frac{\pi}{4}$ is an angle of S.
   b- Determine S(C).

2) a- $S \circ S$ is a similitude. Find an angle of $S \circ S$ and calculate its ratio.
   b- Find $S \circ S(A)$ and deduce that O is the center of S.

3) The two straight lines (OC) and (AD) intersect at L.
   Let $L' = S(L)$.
   Prove that the three points I, D and L′ are collinear.

Part B

The plane is referred to a direct orthonormal system $\left(O;\overrightarrow{OA},\overrightarrow{OD}\right)$.

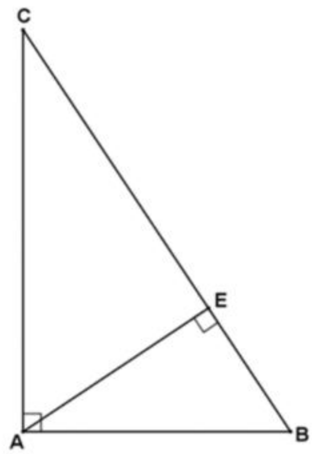
1) Write the complex form of S and determine the affix of G' such that $G' = S(G)$.

2) Let (T) be the ellipse with center I. The points O and G are two of its vertices.
   Denote by (T′) be the image of (T) under S. Write an equation of (T′).

2017(2)

V- (4 points)

In an oriented plane, consider a triangle ABC right angled at A such that AB = 4 , AC = 6 and $(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{\pi}{2}\ [2\pi]$.



Denote by E the orthogonal projection of point A on line (BC).

Let S be the direct plane similitude that maps B onto A and A onto C.

1) Calculate the ratio (scale factor) k of S and find a measure of angle α of S.
2) a- Determine the image of line (AE) under S and the image of line (BC) under S.

   b- Deduce that E is the center of S.
3) Let F = S(C).

   a- Prove that A, E and F are collinear.

   b- Show that (CF) is parallel to (AB).

   c- Construct F and calculate CF.
4) Denote by h the dilation that maps A onto B and with ratio $\frac{-1}{3}$.

   a- Determine $S \circ h(A)$.

   b- $S \circ h$ is a direct plane similitude.
   Determine its center, its ratio and a measure of its angle.
5) The complex plane is referred to a direct orthonormal system $(A; \vec{u}, \vec{v})$ with $\vec{u} = \frac{1}{4}\overrightarrow{AB}$ and $\vec{v} = \frac{1}{6}\overrightarrow{AC}$.

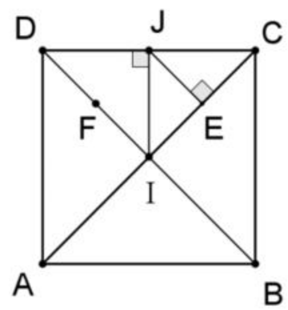
   a- Write the complex form of $S \circ h$.

   b- Calculate the affix of point $B' = S \circ h(B)$.

   c- Let (P) be the parabola with vertex A and focus B and (P′) be the image of (P) under $S \circ h$.
   Write an equation of (P′).

2018(1)

III- (3 points)



ABCD is a direct square with side 1 such that $\left(\overrightarrow{AB}, \overrightarrow{AD}\right) = \frac{\pi}{2} \quad [2\pi]$.

Denote by I, J, E and F the midpoints of the segments [AC], [CD], [IC] and [DI] respectively.

Consider the direct plane similitude S that transforms A onto I and C onto J.

1) Verify that the ratio k of S is equal to $\frac{\sqrt{2}}{4}$ and find an angle α of S.

2) a- Show that S(B) = E.

   b- Deduce the image of the square ABCD by S.

3) The plane is referred to the direct orthonormal system $\left(A; \overrightarrow{AB}, \overrightarrow{AD}\right)$.

   a- Determine the complex form of S.

   b- Deduce the affix of W, the center of S.

4) Let (P) be the parabola with focus A and directrix (BC) and (P') be the image of (P) by S.

   a- Show that D is on (P).

   b- Specify the tangent to (P') at F.

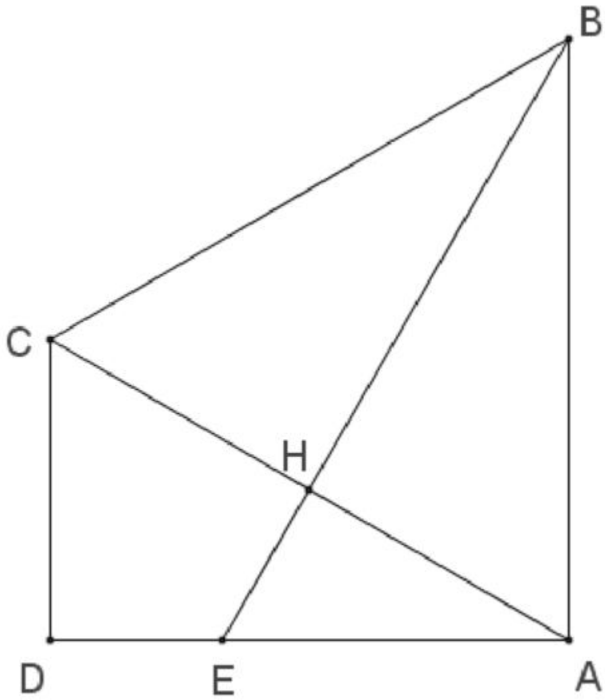
2018(2)

V- (3 points)

In the figure below:

- ABCD is a right trapezoid such that: $\left(\overrightarrow{AB};\overrightarrow{AD}\right)=\left(\overrightarrow{DA};\overrightarrow{DC}\right)=\frac{\pi}{2}\ \ [2\pi]$
- ABC is a direct equilateral triangle with side 2
- H is the midpoint of [AC]
- E is the point of intersection of the two lines (BH) and (AD).

Let S be the direct plane similitude that maps B onto A and A onto E.



1) a- Prove that $\frac{\sqrt{3}}{3}$ is the ratio (scale factor) of S. (You may use $\tan \widehat{EBA}$)

   b- Verify that $-\frac{\pi}{2}$ is an angle of S.

2) a- Verify that the image of (BE) under S is (AC), then determine the image of the line (AC) under S.

   b- Deduce that H is the center of S.

3) Let (Δ) be the line drawn through E and perpendicular to (AD).

   (Δ) intersects (AC) at F.

   The parallel through C to (AD) intersects (Δ) at L.

   Show that $S(E)=F$ and that $S(D)=L$.

4) Consider the direct plane similitude S' with center B, ratio $\frac{\sqrt{3}}{2}$ and an angle $\frac{\pi}{6}$.

   a- Determine the ratio and an angle of $S\circ S'$.

   b- Determine $S\circ S'(B)$.

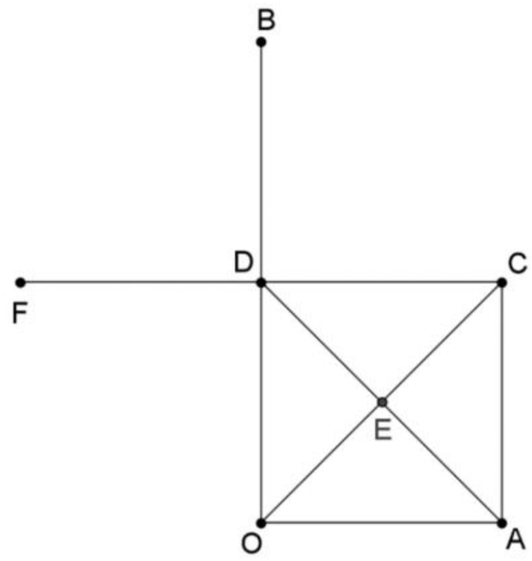
   c- Prove that E is the center of $S\circ S'$.

2019(1)

V- (3 points)

In the figure below,

- OACD is a direct square with center E and side 2.
- F is the symmetric of C with respect to D.
- B is the symmetric of O with respect to D.



Denote by S the direct plane similitude of center O that maps A onto B.

Part A

1) a- Calculate the ratio k and an angle α of S.

   b- Verify that S(E) = F.

   c- Show that the triangle OBF is right isosceles.

2) Consider the direct plane similitude $S'\left(E,\ 2, \frac{\pi}{2}\right)$ and the transformation $h = S \circ S'$.

   Denote by W the center of h. Show that $\overrightarrow{WF} = -4\,\overrightarrow{WE}$.

Part B

The plane is referred to a direct orthonormal system $\left(O;\ \vec{u}, \vec{v}\right)$ with $\vec{u} = \frac{1}{2}\overrightarrow{OA}$.

1) Show that the complex form of h is $z' = -4z + 2 + 6i$ and deduce the affix of W.
2) For all $n \in \mathbb{N}$, consider the numerical sequence (d_n) defined as $d_n = WE_n$ where $E_0 = E$ and $E_{n+1} = h(E_n)$.

   a- Verify that $d_0 = \frac{\sqrt{10}}{5}$.

   b- Show that (d_n) is a geometric sequence of common ratio 4.

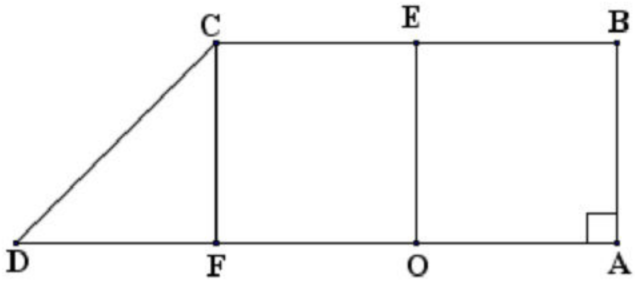
   c- Determine the number of points E_n such that $d_n < 2019$.

2019(2)

IV- (3 points)

In the adjacent figure:

- ABEO and OECF are two direct squares of side 1
- $\left(\overrightarrow{AB}, \overrightarrow{AO}\right) = \frac{\pi}{2} + 2k\pi$, where $k \in \mathbb{Z}$
- D is the symmetric of O with respect to F.



Let S be the direct plane similitude that transforms A onto C and B onto D.

1) a- Show that the ratio of S is equal to $\sqrt{2}$ and that an angle of S is $\frac{3\pi}{4}$.

   b- Show that O is the center of the similitude S.

   c- Determine S(E).

2) Let S^n be the transformation defined as $S^n = \underbrace{S \text{ o } S \text{ o } S \text{ o...o } S}_{\text{n times}}$, where n is a natural number with $n \geq 2$.

   a- Determine the value of n when the image of the square OABE under S^n is a square whose area is 16 and deduce, in this case, that S^n is a negative dilation.

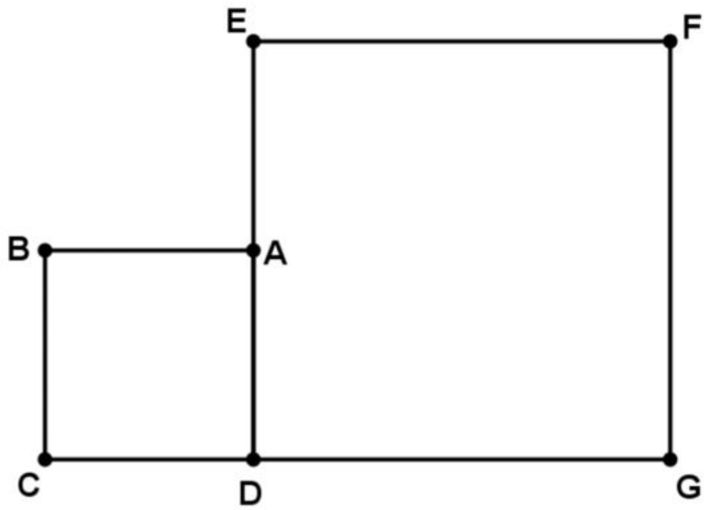
   b- Determine the smallest value of n so that S^n is a positive dilation.

2021(1)

IV- (4 points)

In the following figure,

- ABCD and EDGF are two direct squares.
- CD = 1 and DG = 2.



Denote by S the direct plane similitude with angle $\frac{\pi}{2}$ that maps B onto D and maps A onto E.

1) Calculate the ratio k of S and show that S(C) = G.

2) Denote by (T) and (T′) the circles with diameters [BD] and [AE] respectively.

(T) and (T′) intersect in two points W and A.

Show that W is the center of S.

3) **a-** Show that the image of line (BD) by S is the line (DF).

b- Determine the image of line (AD) by S.

c- Show that S(D) = F.

4) Let h be the transformation defined as $h = S \circ S$.

a- Determine the nature and the characteristic elements of h.

b- Determine h(B) and deduce that $\overrightarrow{WF} = -4\overrightarrow{WB}$.

5) The complex plane is referred to a direct orthonormal system $(C; \overrightarrow{CD}, \overrightarrow{CB})$.

a- Determine the complex form of h.

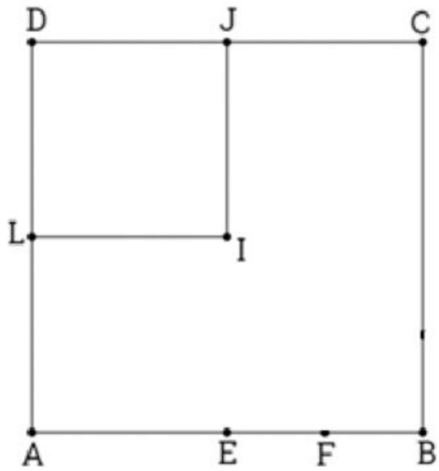
b- Calculate the affix of point W.

2021(2)

IV- (4 points)

In the figure below:

- ABCD is a direct square with center I and side 8.
- E is the midpoint of [AB].
- F is the midpoint of [EB].
- J is the midpoint of [DC].
- L is the midpoint of [DA].



1) Let S be the direct plane similitude that maps F onto I and maps B onto J.

a- Show that $k = 2$ and $\alpha = \frac{\pi}{2}$ are respectively the ratio and an angle of S.

b- Show that E is the center of S.

2) Let S' be the direct plane similitude with ratio $k' = 2$ and an angle $-\frac{\pi}{2}$ that maps I onto B.

a- Show that $S'(L) = C$.

b- Show that the image of (LD) by S' is the line (DC).

c- Determine the image of (IC) by S'.

d- Determine the image of A by S'.

3) Let $h = S \circ S'$.

a- Show that h is a dilation whose ratio is to be determined.

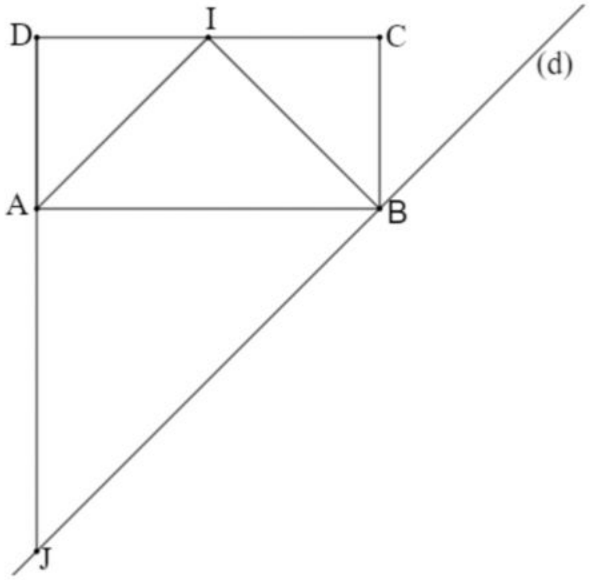
b- Verify that $h(I) = J$.

c- Let W be the center of h, prove that W is the center of gravity of triangle ABJ.

IV- (4 points) 2022(1)

In the figure below,

- ABCD is a direct rectangle.
- AB = 2 and BC = 1.
- I is the midpoint of [DC].
- (d) is the perpendicular to (BI) at B.
- J is the point of intersection of (d) and (AD).



Part A

Let S be the direct plane similitude with center B that transforms C onto A.

1) Calculate the ratio k of S and determine an angle α of S.
2) a) Show that the image of the line (CI) by S is the line (AD).

   b) Determine the image of (BI) by S and verify that S(I) = J.
3) Let (C) be the circle with diameter [BI].

   a) Determine (C') the image of (C) by S.

   b) Prove that the image of the line (AI) by S is tangent to (C').

Part B

Let R be the rotation with center I and angle $-\frac{\pi}{2}$ and $h = S \circ R$.

1) Determine h(I).
2) Prove that h is a dilation whose ratio is to be determined.

Part C

The plane is referred to the direct orthonormal system $(A\,;\vec{u}\,,\vec{v})$, with $z_B = 2$, $z_D = i$, and $z_J = -2i$.

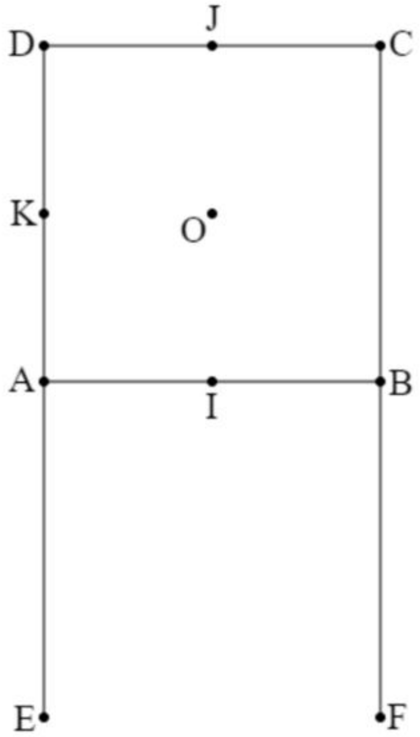
1) Determine the complex form of h.
2) Let W the center of h. Determine the affix of W.

2022(2)

IV- (4 points)

In the figure below:

- ABCD is a direct square with center O such that AB = 2.
- E is the symmetric of D with respect to A.
- F is the symmetric of C with respect to B.
- I is the midpoint of [AB], J is the midpoint of [CD] and K is the midpoint of [AD].



Let S be the direct plane similitude that transforms J onto A and transforms O onto B.

1) Calculate the ratio of S and verify that $\frac{\pi}{2}$ is an angle of S.

2) **a)** Show that S(D) = E.

   b) Determine S(K).

3) **a)** Show that I is not the center of S.

   b) Let W be the center of S.

   Show that I and W are the points of intersection of two circles whose diameters are to be determined.

   c) Construct the center W of S.

4) **a)** Determine the point G image of A by S.

   b) Show that $\overrightarrow{WG} = -4\,\overrightarrow{WJ}$.

5) The complex plane is referred to a direct orthonormal system $(I;\ \overrightarrow{IB}, \overrightarrow{IO})$.

   a) Determine the complex form of S.

   b) Determine the affix of the center W of S.

Sample 1

IV- (3 pts)

In the oriented plane , given a rectangle ABCD with center O so that $AB = 4cm$,

and $(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{\pi}{6}(2\pi)$.

Let E be the symmetric of A wit respect to D . Denote by S the similitude that maps E onto O and A onto B . .

1. Verify that the ratio of the similitude is $k = \frac{1}{2}$ and determine a measure of the angle α of S.
2. Determine the image of D under S . Show that C is the center of S .
3. Let I be the point of $[EO]$, distinct from E and O ; and (Γ) the circle with center I and passing through A . (Γ) intersects (AD) and (AB) respectively at M and P.

   a. Draw (Γ) and plot the points M and P .

   b. Justify that $C \in (\Gamma)$.
4. Let N be the orthogonal projection of C on (MP).

   a. Show that $(\overrightarrow{MP}, \overrightarrow{MC}) = \frac{\pi}{6}(2\pi)$.

   b. Deduce that $S(M) = N$.
5. Prove that B , N and D are colinear .
6. The plane is referred to an orthonormal direct system $(A, \vec{u}, \vec{v})$, with $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.

   a. Determine the affixes of the points B and C.

   b. Give the complex form of S.

Sample 2

IV- (3 points)

In the oriented plane, consider the right isosceles triangle ABC so that AB = AC = 4 cm and $(\overrightarrow{AB},\overrightarrow{AC}) = \frac{\pi}{2} + 2k\pi$ Denote by D the symmetric point of A with respect to B, O the midpoint of [CD].and (T) the circle with diameter [CD].

Denote by S the similitude that maps D onto B and B onto C.

1) Determine the ratio k and the angle α of S.

2) Let I be the center of S and h the transformation defined by $h = S o S$.

a) Show that h(I) = I and h(D) = C.

b) Deduce that I is a point on a circle (T) and that IC = 2ID.

c) Show that ID = 4.

d) Deduce that I is the 4th vertex of a rectangle and plot I.

3) Consider the orthonormal system $(A, \vec{U}, \vec{V})$, so that $\vec{U} = \frac{1}{4}\overrightarrow{AB}$, $\vec{V} = \frac{1}{4}\overrightarrow{AC}$.

Determine the complex form of S.

4) For all $n \in N$ consider the sequence of points (Dn) defined by $D_0=D$ and $D_{n+1}= S(D_n)$.

(U_n) is the sequence defined by U_n= Area of the triangle I D_nD_{n+1}

a) Calculate U_n in terms of n.

b) Calculate in terms of n, the product $P=U_0 \times U_1 \times \ldots\ldots\ldots \times U_n$.

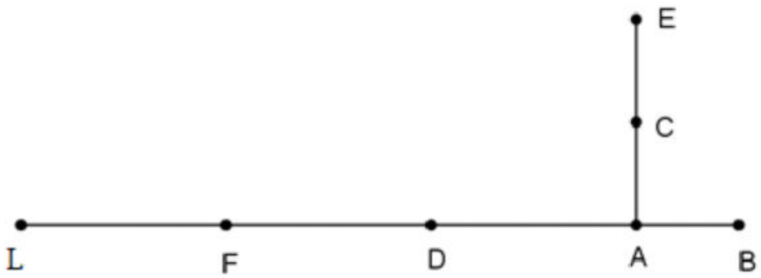
Sample 3

IV-(3 points)

In the next figure, (AE) and (BL) are two perpendicular lines so that:

AB = AC = 1, AE = AD = DF = FL = 2.

Let S be the direct similitude of the plane that maps A onto D and C onto F.



1) Determine the ratio and the angle of S.

2)

a- G is a point so that $\overrightarrow{DG} = \overrightarrow{AE}$, prove that S(B)=G.

b- Find S(E) .

3) Let H and K the respective midpoints of [BE] and [GL].The lines (AH) and (DK) intersect at I. The lines (AH) and (DG) intersect at O.

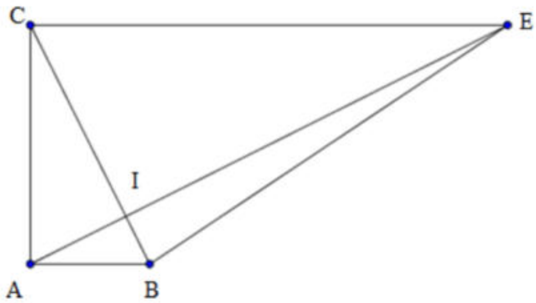
a- Prove that S(H)=K and S(D) = O.

b- Deduce that I is the center of S.

Sample 4

V- **(3 points)**

In the next figure , ABEC is a right trapezoid so that AB =1 , AC = 2 , and CE = 4 .
S is the similitude that maps A onto C and C onto E . (BC) intersects (AE) at I.



1) Calculate the scalar product $(\overrightarrow{BA}+\overrightarrow{AC}).(\overrightarrow{AC}+\overrightarrow{CE})$, deduce that (AE) is perpendicular to (BC)

2) Show that 2 is the scale factor of S and $-\frac{\pi}{2}$ is an angle of it .

3) a- Determine S(AE) and S(BC).
b- Deduce that I is the center of S .
c- Determine S(B).

4) G is the midpoint of [AB] and H is that of [EC] .

a- Prove that H = SoS(G) .

b- Express $\overrightarrow{IH}$ in terms of $\overrightarrow{IG}$.

5) F is the orthogonal projection of B on (EC) . h is the dilation with center F and scale factor $-\frac{1}{3}$

a-Determine an angle of hoS and so its scale factor .
b-Prove that C is the center of hoS.

6) a- The plane is referred to the direct orthonormal system $(A;\vec{u},\vec{v})$ with $\vec{u}=\overrightarrow{AB}$ and $\vec{v}=\frac{1}{2}\overrightarrow{AC}$.

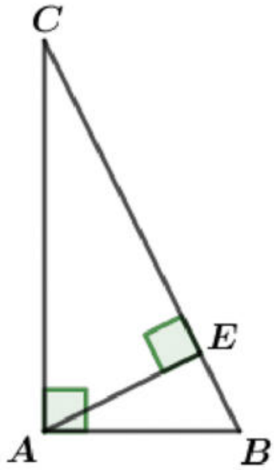
b- Find the complex form for S . Deduce z_I.

7) M is a variable point that moves o the curve (C)with equation : $y=\frac{2}{1+e^x}$, and M'= S(M).

M' moves on the curve (C') = S((C)).
a- Prove that the midpoint H of [CE] is on (C').
b- Write an equation of the tangent (T) to (C') at H .
c- Show that $y=2[1-\ln(\frac{4-x}{x})]$ is an equation of (C').

Sample 5

IV- (4 point)



In the next figure,

- Triangle ABC is right at A
- $AB = 2$, $AC = 4$
- $[AE]$ is an altitude in the triangle ABC
- Let S be the similitude that maps A onto B and E onto C

Part A.

1) Determine an angle of S and show that its scale factor is $\frac{5}{2}$.
2) Let $F = S(B)$ and $L = S(C)$.
   a-Construct F.
   b-Show that L is the meeting point of (CF) and (AB).
3) a-Construct $(d) = S(AF)$, then determine $S(d)$.
   b-Deduce that the center I of S is the meeting point of (d) and (AF).
4) Let h be the dilation that maps F onto A and B onto C.
   a-Determine the center J of h, then verify that the scale factor of h is $-\frac{4}{5}$.
   b-Construct $G = h(L)$.
5) a-Determine the nature of Soh.
   b-Show that C is the center of Soh.
   c-Deduce that E is the center of hoS.

Part B

The complex plane is referred to the system $(A;\vec{u},\vec{v})$ with $\vec{u} = \frac{1}{2}\overrightarrow{AB}$ and $\vec{v} = \frac{1}{4}\overrightarrow{AC}$.

1) a-Write the complex form of hoS.
   b-Deduce z_E.
2) Determine $hoS(C)$, then find z_G.
3) Determine the nature of the quadrilateral $LAGC$.

b-
c-