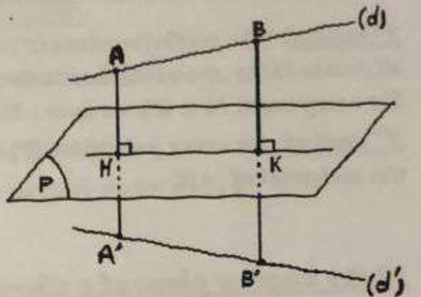
14-symmetry of a line with respect to a plane



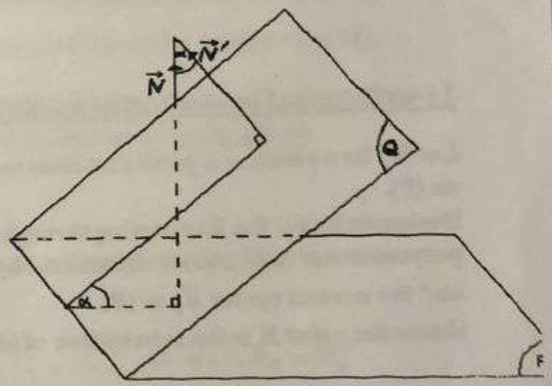
Let (d) be a line, (P) be a plane and (d') is the symmetric of (d) with respect to (P).
Let A and B be any two points on (d), A' and B' are the symmetric of A and B with respect to (P). Hence (d') becomes the line passing through A' and B'. that is (d') passes through A' and is of direction vector A'B'
Problems: N: 1) – 2) – 3) – 4) – 5) – 6)

15-angle of two lines

Let (d) and (d') be two lines of direction vectors $\vec{v}(a,b,c)$ and $\vec{v}'(a',b',c')$ respectively and let α be the acute angle of (d) and (d').

We have $\cos\alpha = |\cos(\vec{v},\vec{v'})| = \dfrac{|\vec{v}.\vec{v}'|}{\|\vec{v}\|\times\|\vec{v}'\|} = \dfrac{|aa'+bb'+cc'|}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}$

16-angle of two planes

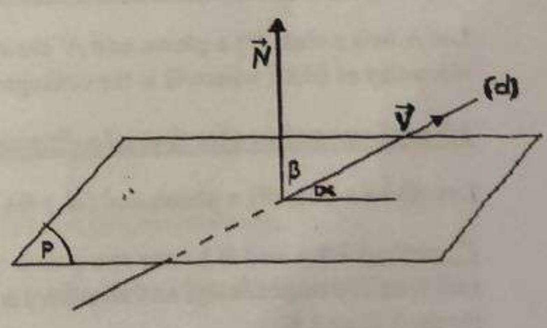


Let (P) and (Q) be two planes of equations $ux+vy+wz+r=0$ **and** $u'x+v'y+w'z+r'=0$ **respectively.** $\vec{N}$ (u, v, w) **is a normal vector to (P) and** $\vec{N}'(u', v', w')$ **is a normal vector to (Q).**

The acute angle α of these two planes is the acute angle between the normal vectors $\vec{N}$ and $\vec{N}'$.

$\cos\alpha = |\cos(\vec{N},\vec{N}')| = \dfrac{|\vec{N}.\vec{N}'|}{|\vec{N}|\times|\vec{N}'|}$

17-angle between a straight line and a plane



let (P) be a plane of normal vector $\vec{N}$ and (d) be a line of direction vector $\vec{v}$.
designate by α the acute angle that (d) makes with (P).

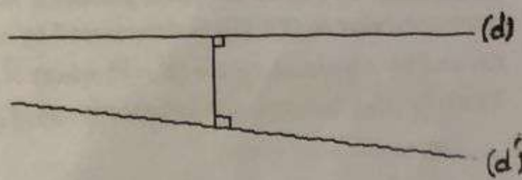
$\alpha = \frac{\pi}{2} - \beta$ **where** $\beta = (\vec{V},\vec{N})$

$\sin\alpha = \sin\left(\frac{\pi}{2} - \beta\right) = \cos\beta = |\cos(\vec{V},\vec{N})| = \left|\dfrac{|\vec{V}.\vec{N}|}{\|\vec{V}\|\times\|\vec{N}\|}\right|$ **and**

$\cos\alpha = \sqrt{1-\sin^2\alpha}$

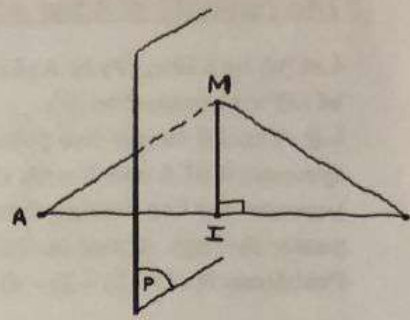
Problems: N: 7) – 8) – 9)

18-the common perpendicular:



The common perpendicular to two straight lines in space is a straight line meeting each one of them at a right angle
Problems: N: 10) – 11) ….. 23)

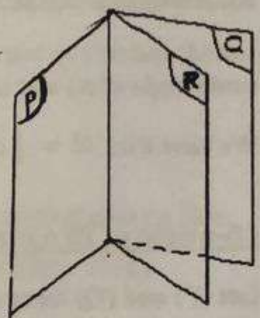
9-equation of the mediator plane of the segment $[AB]$



1st method: The mediator plane (P) of the segment $[AB]$ is the set of points M(x,y,z) equidistant to the two points A and B. that is for every point $M \in (P)$ we have : MA = MB

2nd method: for every point $M \in (P)$ we have $\overrightarrow{IM} . \overrightarrow{AB}=0$ where I is the midpoint of $[AB]$

10-the bisector plane of a dihedral

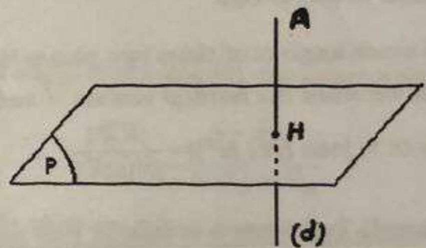


The bisector plane of a dihedral (P, Q) is the plane (R), a set of points M equidistant to (P) and (Q). That is for every $M \in (R)$ we have: $d(M \to (P)) = d(M \to (Q))$

remark: we have two bisector planes of the dihedral (P, Q), which are perpendicular.

11-orthogonal projection of a point on a plane

Let (P) be a plane, A a point that does not belong to (P), and H the orthogonal projection of A on (P).



Designate by (d) the line passing through A and perpendicular to (P), (d) is determined by the point (A) and the normal vector $\overrightarrow{N_p}$ to (P).

Hence the point H is the intersection of (d) and (P).

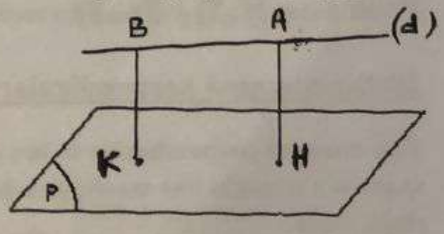
12-symmetry of a point with respect to a plane

Let A be a point, (P) a plane, and A' the symmetric of A with respect to (P). Then H is the midpoint of $[AA']$ where H is the orthogonal projection of A on (P).

13-orthogonal projection of a line on a plane

Let (d) be a line, (P) a plane, and (d_1) the orthogonal projection of (d) on (P)

1st method: let A and B be any two points on (d), H and K are the orthogonal projections of A and B on (P) respectively, and then (d_1) is the line passing through H and K.



2nd method: let (Q) be the plane passing through (d) and perpendicular to (P). (Q) is determined by a fixed point $A \in$ (d) and by a normal vector $(\vec{N} \wedge \vec{V})$ where $\vec{N} \perp (P)$ and $\vec{V} // (d)$.

The line (d_1) becomes the intersection of (P) and (Q).

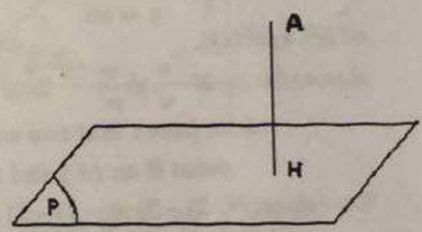
We get 3 equations of two unknowns' m and t.
To solve this system we solve a system of 2 equations of two unknowns if m and t satisfy the remaining equation then the two lines (d) and (d') are concurrent and we can determine the coordinates of the point of intersection in replacing m or t by its value in (d) or (d').
but if the obtained solution does not satisfy the remaining equation, then the two straight lines are skew. Remark 4
Exercises: N: 7) c) d) and e) – 8) d) – 9) a) and c) – 10) a)

5-relative position of a line and a plane

Let (P) be a plane of equation: ux+ vy+ wz +r = 0 and (d) a line of parametric equations:
$\begin{cases} x = at + x_0 \\ y = bt + y_0 \\ z = ct + z_0 \end{cases}$. ~~$\vec{N}$(u, v, w) is a normal vector of (P) and $\vec{v}$ (a, b, c) is a direction vector of (d)~~

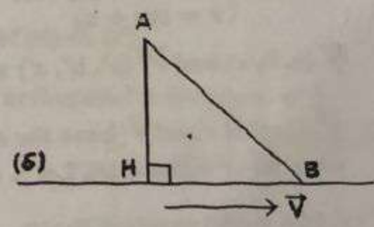
To find (d) ∩ (P), we substitute x, y, and z by their values in the equation of (P) hence:
$u(at+x_0) + v(bt+y_0) + w(ct+z_0) + r = 0$
$At=B$
If A=0 and B≠0 then 0t = B, no solution that is (d) ∩ (P)= ∅ and therefore (d)//(P)
if A=B=0 then 0t=0, we have infinity of solutions and in this case (d)⊏ (P)
if A≠0 for every B then $t=\frac{B}{A}$ we have one solution, that is (d)∩ (P) = $\{H\}$
With $x_H= a\left(\frac{B}{A}\right) +x_0$, $y_H= b\left(\frac{B}{A}\right) +y_0$, $z_H=c\left(\frac{B}{A}\right) +z_0$
Exercises: N: 11) a) c) e) and f) – 12) – 13) – 14) a) and c) – 15) a) – 16) b)

6-distance from a point to a plane



Let (P) plane of equation ux+ vy+ wz +r = 0 and A (x_0, y_0, z_0) be a point. The distance from A to (P) is given by the formula:
$$d(A \rightarrow (P)) = \frac{|ux_0+vy_0+wz_0+r|}{\sqrt{u^2+v^2+w^2}}$$
Exercise: N: 17) b) and c).

7-distance from a point to a line



1st method: Let (δ) be a line with parametric equations
$\begin{cases} x = at + x_0 \\ y = bt + y_0 \\ z = ct + z_0 \end{cases}$ and let A be any point. The distance from A to (δ) is given by the formula: $d(A \rightarrow \delta) = \frac{||\vec{v} \wedge \overrightarrow{AB}||}{||\vec{v}||}$ where v is a direction vector of (δ) and B is a fixed point of (δ).
Exercises: 18) a) and d) – 19) – 20) – 21) – 1)

8-distance between two parallel planes

The distance between two parallel planes is the distance from any point on one to the other plane.

Parallelism and orthogonality

In this chapter the space is referred to an orthonormal system $(o, \vec{i}, \vec{j}, \vec{k})$

1-Relative position of two planes:

Let (P): ux+vy+wz+r=0 and (Q): u'x+v'y+w'z+r'=0 be two given planes.

$\vec{N}_P$ (u,v,w) is a normal vector to (P) and $\vec{N}_Q$ (u',v',w') is a normal vector to (Q)

(P)//(Q) iff $\frac{u}{u'}=\frac{v}{v'}=\frac{w}{w'}\neq\frac{r}{r'}$

(P)≡(Q) iff $\frac{u}{u'}=\frac{v}{v'}=\frac{w}{w'}=\frac{r}{r'}$

(P) and(Q) intersect iff $\frac{u}{u'}\neq\frac{v}{v'}$ or $\frac{u}{u'}\neq\frac{w}{w'}$ or $\frac{v}{v'}\neq\frac{w}{w'}$

(P) ⊥ (Q) iff uu' + vv' + ww' = 0

2-cartesian equations of the intersection of two planes:

let (P): ux+ vy+ wz +r = 0 and (Q) : u'x+v'y+w'z+r'=0 be tow planes intersecting along a line (d) then the system $\begin{cases} ux + vy + wz + r = 0 \\ u'x + v'y + w'z + r' = 0 \end{cases}$ is a system of Cartesian equations of (d).

3-parametric equations of the intersection of two planes

Let (P): ux+ vy+ wz +r = 0 and (Q): u'x+v'y+w'z+r'=0 be two intersecting planes

if $\frac{u}{u'}\neq\frac{v}{v'}$ let z=m then $\begin{cases} ux + vy = -mw - r \\ u'x + v'y = -mw' - r' \end{cases}$

we obtained $\begin{cases} x = am + x_0 \\ y = bm + y_0 \\ z = m \end{cases}$ which are the parametric equations of the line (d), the intersection of (P) and (Q).

Remarks: 1- if $\frac{u}{u'}\neq\frac{w}{w'}$ let y=m 2- if $\frac{v}{v'}\neq\frac{w}{w'}$ Let x = m. 3- $\overrightarrow{N_P}\wedge\overrightarrow{N_Q}=\overrightarrow{V_d}$

4- to prove that two straight lines (d) and (d') are skew we take a point A on (d) and a point B on (d') and then we prove that: $\overrightarrow{AB}.(\overrightarrow{V_d}\wedge\overrightarrow{V_{d'}})\neq 0$

Exercises: N: 2) – 3) a) – 4) – 5) a) and d) – 6) a)

4-relative position of two lines:

Let (d): $\begin{cases} x = am + x_0 \\ y = bm + y_0 \\ z = cm + z_0 \end{cases}$ and (d'): $\begin{cases} x = a't + x'_0 \\ y = b't + y'_0 \\ z = c't + z'_0 \end{cases}$ be two given lines

$\vec{V}$(a, b, c) and $\vec{V'}$ (a', b', c') are direction vectors for (d) and (d') respectively.

1st case: if $\vec{V}$and $\vec{V}'$ have the same direction ($\vec{V}=k\vec{V'}$) the two lines (d) and (d') are either parallel or coinciding. Let A $(x_0, y_0, z_0)\in$ (d)

i) If A ∈ (d') then (d))≡(d')

ii)If A ∉ (d') then (d) // (d')

2nd case: if $\vec{V}$and $\vec{V}'$ do not have the same direction. Study (d) ∩ (d'): $\begin{cases} am + x_0 = a't + x_0 \\ bm + y_0 = b't + y_0 \\ cm + z_0 = c't + z_0 \end{cases}$