ExaMath Groups	Mathematics Exam Class: LS	Prepared by: Hanaa Sleiman Edited by: H. Ahmad
Number of questions: 3	Sample 05 – year 2023 Duration: 1½ hours	Name: N°:

- *This exam consists of three problems inscribed on two pages.*
- *The use of a non-programmable calculator is allowed.*

I- (4 points)

Use the given to tell whether each of the following statements is true or false with justification:

1) **Given:** Given the equation (E): $\ln(1-2x)+\ln(1+2x)=2\ln 4$.

Statement: (E) admits one real solution.

2) **Given:** Given the limit $L=\lim\limits_{x\to 0^+}\dfrac{\ln(1+x^2)}{x}$.

Statement: The value of L is 0.

3) **Given:** Let f be the function defined over $]0\,;+\infty[$ by $f(x)=(\ln x)^2-2\ln x$.

Statement: The curve of f admits an inflection point.

4) **Given:** An urn contains 3 red balls and 5 black balls. We draw successively and with replacement 2 balls from the urn.

Statement: The probability of drawing two balls of different colors is $\dfrac{15}{64}$.

II- (5 points)

The results will be rounded up 10^{-3} if necessary.

Studies show that certain species of floral bacteria can enhance pollen germination. In order to study the effect of bacteria on the increase of the number of pollen germs in the flowers: Dahlias and Harebells, a laboratory detects the presence of bacteria called Acinetobacter in these flowers and obtains the following results:

- 70% of the flowers are Dahlias, among which 30% are affected by this bacteria.
- 60% of Harebells are affected by the bacteria.

We choose randomly a flower. Consider the following events:

D : "The flower is a Dahlia";

B : "The flower is affected by the bacteria".

1) Calculate $P(D\cap B)$ and verify that $P(\bar{D}\cap B)=0{,}18$. Deduce the probability that the flower is affected by the bacteria.

2) We know that the flower is not affected by bacteria, what's the probability that it's a Dahlia?

3) There are 500 flowers, and students of a certain school want to plant these flowers in a public garden in order to promote pollen germination, so they decide to make bouquets of 3 flowers.

A student chooses one of these bouquets at random.

a) What is the probability that the bouquet contains 2 affected Dahlias and one affected Harebells?

b) Each Dahlia costs 5\$ and each Harebells costs 3\$. An additional cost of 0.5\$ is added if the flower is affected. What is the probability of paying 12.5\$ for a bouquet?

III- (11 points)

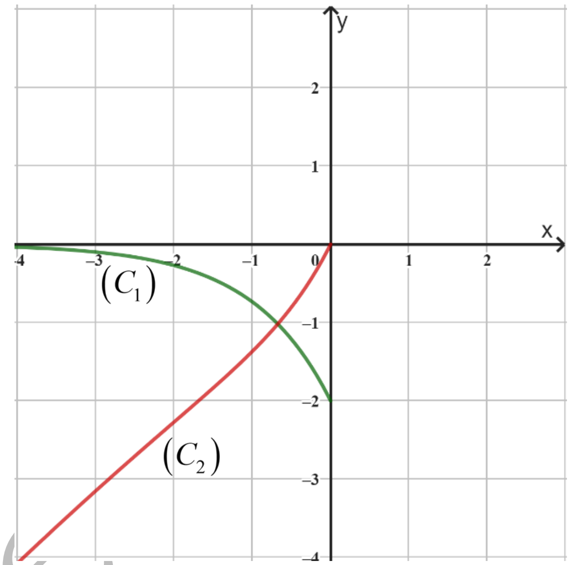
Given the function f defined over $]-\infty;0]$ by $f(x)=(x-3)e^x+\frac{1}{2}x^2$.

Denote by (C) the representative curve of f in an orthonormal system $(O\,;\vec{i}\,;\vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $f(0)$.

2) Given the 2 functions h and g defined over $]-\infty;0]$ by $h(x)=-2e^x$ and $g(x)=x(e^x+1)$.

In the opposite figure (C_1) and (C_2) are respectively the representative curves of h and g in an orthonormal system.



a) Justify that $f'(x)=h(x)+g(x)$, and deduce using the curves (C_1) and (C_2) that f is strictly decreasing over $]-\infty;0]$.

b) Draw the table of variations of the function f over $]-\infty;0]$.

3) Show that the curve (C) of the function f intersects the x-axis at a unique point of abscissa β. Verify that $-1.5<\beta<-1.4$.

4) Trace (C) (take $\beta\approx-1.45$).

5) Let φ be the function defined over $]-\infty;0]$ by $\varphi(x)=|f(x)|$.

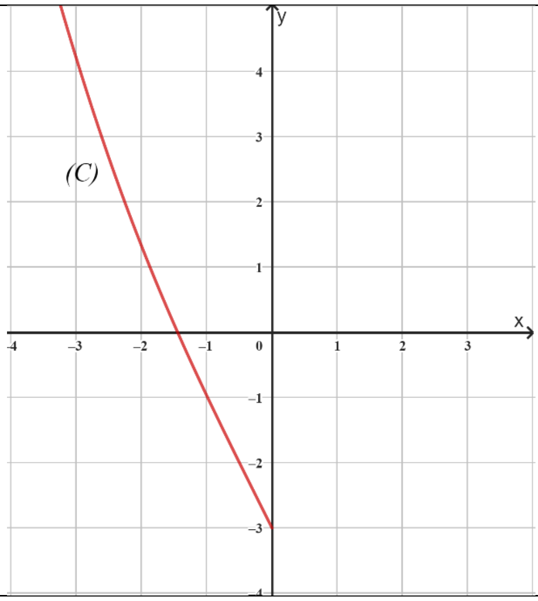
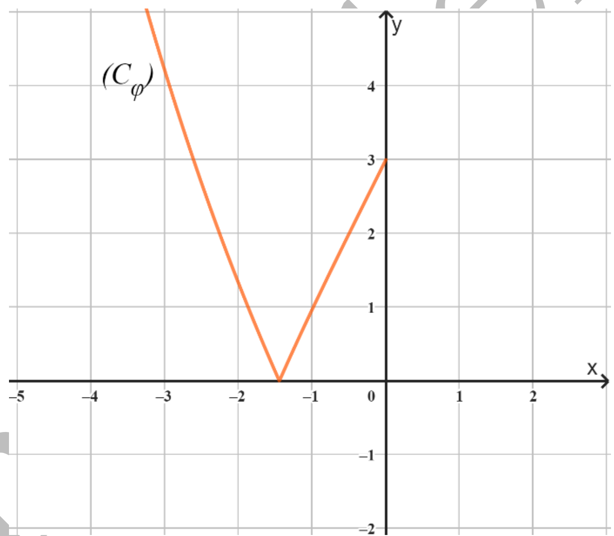
a) Explain how to draw the curve (C_φ) of the function φ using the curve (C) of f and plot (C_φ) in a new coordinate system.

b) Let m be a real parameter. Determine graphically according to the values of m, the number of solutions of the equation $|f(x)|=e^m$.

<table>
<tr><th>QI</th><th>Answers</th><th>4 pts</th></tr>
<tr><td>1)</td><td>The equation exists if $\begin{cases}1-2x>0\\1+2x>0\end{cases}$ so if $\begin{cases}x<\dfrac{1}{2}\\x>-\dfrac{1}{2}\end{cases}$ therefore if $x\in\left]-\dfrac{1}{2}\;;\;\dfrac{1}{2}\right[$.
(E): $\ln(1-2x)+\ln(1+2x)=2\ln 4$ is equivalent to: $\ln\left[(1-2x)(1+2x)\right]=\ln 4^2=\ln 16$;
$(1-2x)(1+2x)=16$; $1-4x^2=16$; then $x^2=-\dfrac{15}{4}$ then (E) has no real root.
The statement is false.</td><td>1</td></tr>
<tr><td>2)</td><td>$L=\lim\limits_{x\to 0^+}\dfrac{\ln(1+x^2)}{x}\overset{HR}{=}\lim\limits_{x\to 0^+}\dfrac{\frac{2x}{1+x^2}}{1}=\lim\limits_{x\to 0^+}\dfrac{2x}{1+x^2}=0$;
The statement is true.</td><td>1</td></tr>
<tr><td>3)</td><td>f is differentiable two times over $]0\;;\;+\infty[$;
$f'(x)=2\ln x\times\dfrac{1}{x}-2\times\dfrac{1}{x}=\dfrac{2\ln x-2}{x}$;
$f''(x)=\dfrac{\frac{2}{x}\times x-1\times(2\ln x-2)}{x^2}=\dfrac{4-2\ln x}{x^2}$ has the same sign as $4-2\ln x$ since $x^2>0$ for every $x\in]0\;;\;+\infty[$;
$f''(x)=0$ if $4-2\ln x=0$ so if $\ln x=2$ so if $x=e^2$;
Sign of $f''(x)$:
$\begin{array}{c|ccccc} x & 0 & & e^2 & & +\infty \\ \hline f''(x) & \| & + & 0 & - & \end{array}$
$f''(x)$ vanishes at $x=e^2$ and changes the sign, then the curve of f admits an inflection point.
The statement is true.</td><td>1</td></tr>
<tr><td>4)</td><td>The probability of drawing two balls of different colors is: $\dfrac{3^1\times 5^1}{8^2}\times\dfrac{2!}{1!\times 1!}=\dfrac{15}{32}$.
The statement is false.</td><td>1</td></tr>
</table>

QII	Answers	5 pts
1)	$P(D\cap B)=P(B/D)\times P(D)=0.3\times 0.7=0.21$; $P(\overline{D}\cap B)=P(B/\overline{D})\times P(\overline{D})=0.6\times 0.3=0.18$; Using the total probability formula: $P(B)=P(D\cap B)+P(\overline{D}\cap B)=0.21+0.18=0.39$.	2
2)	$P(D/\overline{B})=\dfrac{P(D\cap\overline{B})}{P(\overline{B})}=\dfrac{P(\overline{B}/D)\times P(D)}{1-P(B)}=\dfrac{0.7\times 0.7}{1-0.39}\approx 0.803$.	1
3) a)	The number of Dahlias is $\dfrac{70}{100}\times 500=350$ and that of Harebells is 150. The number of affected Dahlias is $\dfrac{30}{100}\times 350=105$ and the number of affected Harebells is $\dfrac{60}{100}\times 150=90$; The probability that the bouquet contains 2 affected Dahlias and one affected Harebells is $\dfrac{C_{105}^{2}\times C_{90}^{1}}{C_{500}^{3}}\approx 0.024$.	1
3) b)	To pay a sum of 12.5$ for a bouquet, the bouquet must contain 1 affected Dahlia and 2 affected Harebells, the probability is then $\dfrac{C_{105}^{1}\times C_{90}^{2}}{C_{500}^{3}}\approx 0.02$.	1

QIII	Answers	11 pts
1)	$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\left(xe^{x}-3e^{x}+\frac{1}{2}x^{2}\right)=0-0+\infty=+\infty$; $f(0)=(0-3)e^{0}+\frac{1}{2}(0)^{2}=-3$.	1
2) a)	$f'(x)=e^{x}+e^{x}(x-3)+x=e^{x}(x-2)+x=xe^{x}-2e^{x}+x=x(e^{x}+1)-2e^{x}$ $=h(x)+g(x)$; Over $]-\infty;0]$, (C_1) is strictly below the x-axis, so $h(x)<0$ if $x\in]-\infty;0]$; Over $]-\infty;0]$, (C_2) is below the x-axis, so $g(x)\le 0$ if $x\in]-\infty;0]$; So $h(x)+g(x)<0$ for every $x\in]-\infty;0]$, so $f'(x)<0$ over $]-\infty;0]$ then f is strictly decreasing over $]-\infty;0]$.	2
2) b)	Table of variations of the function f over $]-\infty;0]$: $\begin{array}{c\|ccc} x & -\infty & & 0 \\ \hline f'(x) & & - & \\ \hline f(x) & +\infty & \searrow & -3 \end{array}$	2
3)	The function f is continuous over $]-\infty;0]$, strictly decreasing and changes the sign only once then the curve (C) of f cuts the x-axis into a unique point of abscissa β. In addition, $f(-1.4)\approx -0.105<0$, and $f(-1.5)\approx 0.12>0$, so $-1.5<\beta<-1.4$	1

4)		**2**
5) a)	$\varphi(x)=\lvert f(x)\rvert=\begin{cases} f(x) & \text{if } x\ge 0 \\ -f(x) & \text{if } x<0 \end{cases}=\begin{cases} f(x) & \text{if } x\in\left]-\infty;\beta\right] \\ -f(x) & \text{if } x\in\left]\beta;0\right] \end{cases}$, so (C_φ) is the union of the part of (C) over $\left]-\infty;\beta\right]$ with the symmetric with respect to the x-axis of the part of (C) over $\left]\beta;0\right]$. 	**2**
5) b)	$\lvert f(x)\rvert=e^m$ is equivalent to $\varphi(x)=e^m$; If $e^m>3$, then $m>\ln 3$, the equation admits only one solution; If $0<e^m\le 3$, then $m\le\ln 3$ the equation admits two solutions.	**1**