First Exercise: (*6 points*) *Determination of the resistance of a resistor*

We intend to determine the resistance R of a resistor (R). We thus connect up the circuit represented in figure (1) that is formed of an ideal generator of e.m.f E = 5 V, the resistor (R), an uncharged capacitor (C) of capacitance C = 33 μF and a double switch (K).

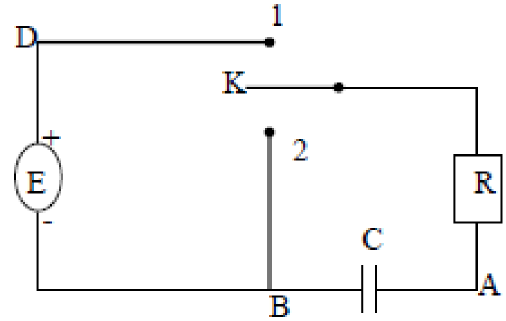


Fig. 1

A – Charging of the capacitor

1) We intend to charge the capacitor. To what position, 1 or 2, must then (K) be moved?
2) The circuit reaches a steady state after a certain time. Give then the value of the voltage u_{AB} across (C) and that of the voltage across (R).

B – Discharging of the capacitor

1) Draw a diagram of the circuit during the discharging of the capacitor and show on it the direction of the current it carries.
2) Derive the differential equation in $u_C = u_{AB}$ during the discharging.
3) The solution of this differential equation has the form :

$u_C = E\, e^{-\frac{t}{\tau}}$ (u_C in V , t in s)

where τ is a constant.

a) Determine the expression of τ in terms of R and C .

b) Determine the value of u_C at the instant $t_1 = \tau$.

c) Give, in terms of τ , the minimum duration needed at the end of which the capacitor is practically totally discharged.

d) Derive the expression of $\ell n\, u_C$, the natural logarithm of u_C , in terms of E , τ and t.

e) The diagram of figure 2 represents the variation of $\ell n\, u_C$ as a function of time .

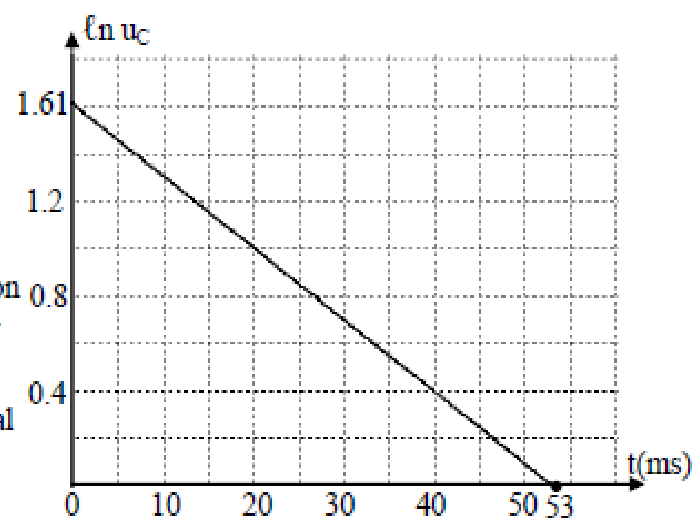


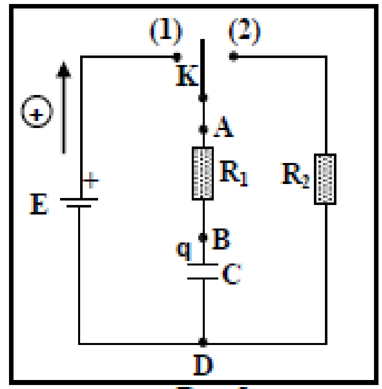
Fig. 2

Referring to the graph of figure 2, determine the value of R.

EX:2

Exercise 2: (7 points) **Charging and discharging of a capacitor**

The aim of this exercise is to determine the capacitance of a capacitor by two different methods.
Consider the circuit represented in document 1. It is formed of an ideal generator that maintains across its terminals a constant voltage of value E, a capacitor of capacitance C, two resistors of resistances $R_1 = 10\ \text{k}\Omega$, $R_2 = 20\ \text{k}\Omega$ and a double switch K.



Doc. 1

1 – Charging the capacitor

The capacitor is initially neutral. At the instant $t_0 = 0$, we put K in position (1); the charging phenomenon of the capacitor starts.

1-1) Theoretical study

1-1-1) Show that the differential equation that describes the variation of the voltage $u_C = u_{BD}$ across the capacitor has the form: $E = R_1 C\dfrac{du_C}{dt} + u_C$.

1-1-2) The solution of this differential equation has the form: $u_C = A(1 - e^{\frac{-t}{\tau_1}})$.
Determine the expressions of the constants A and τ_1 in terms of E, R_1 and C.

1-1-3) Deduce that $u_C = E$ at the end of charging of the capacitor.

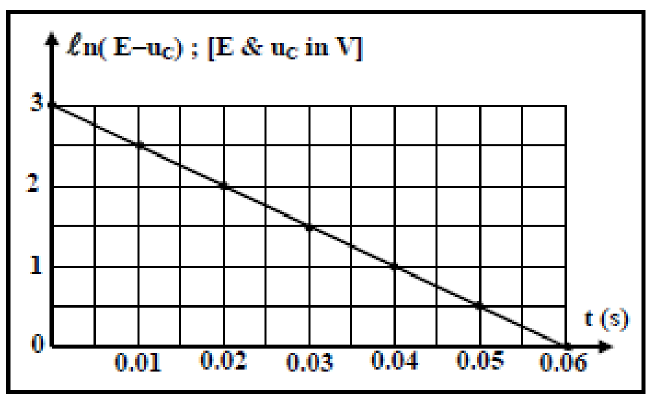
1-2) Experimental study

In order to determine the value of C, we use a convenient apparatus, which traces, during the charging of the capacitor, the curve representing $\ell n(E - u_C) = f(t)$ (Doc.2). [ℓn is the natural logarithm]

1-2-1) Determine, using the solution of the obtained differential equation, the expression of $\ell n\,(E - u_C)$ in terms of E, R_1, C and t.

1-2-2) Show that the shape of the curve in document 2 is in agreement with the obtained expression of $\ell n\,(E - u_C) = f(t)$.

1-2-3) Using the curve of document 2, determine the values of E and C.



Doc. 2

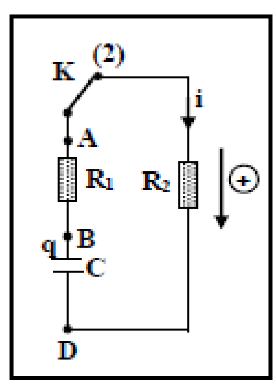
2 – Discharging the capacitor

The capacitor being fully charged. At an instant taken as a new origin of time $t_0 = 0$, the switch K is placed at position (2); thus the phenomenon of discharging of the capacitor starts (Doc. 3).

2-1) Theoretical study

2-1-1) Show that the differential equation in the voltage $u_C = u_{BD}$ across the capacitor has the form: $u_C + \alpha\dfrac{du_C}{dt} = 0$; where α is a constant to be determined in terms of R_1, R_2 and C.

2-1-2) The solution of this differential equation has the form: $u_C = E\,e^{\frac{-t}{\tau_2}}$ where τ_2 is constant. Show that $\tau_2 = \alpha$.



Doc. 3

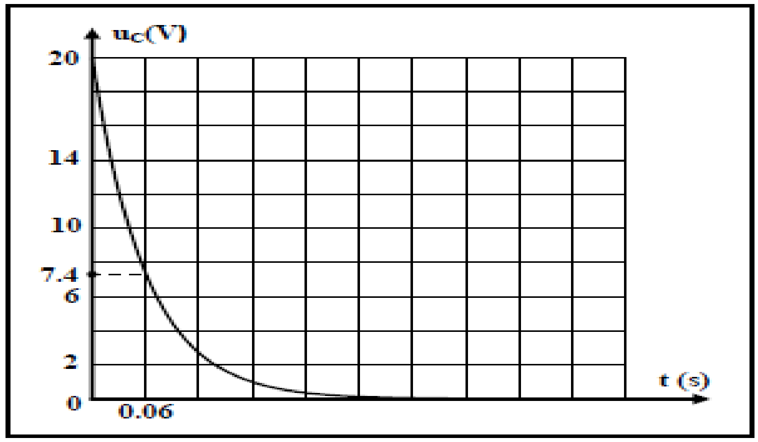
2-2) Experimental study

The variation of the voltage u_C across the capacitor as a function of time is represented in document 4.

2-2-1) Determine, using document 4, the value of the time constant τ_2 of the discharging circuit.

2-2-2) Deduce the value of C.

Doc. 3



Doc. 4

<u>Third exercise:</u> (7 points) Charging and discharging of a capacitor

The aim of this exercise is to determine, by two different methods, the value of the capacitance C of a capacitor. For this aim, we set-up the circuit of figure 1. This circuit is formed of an ideal generator delivering a constant voltage of value E = 10 V, a capacitor of capacitance C, two identical resistors of resistances $R_1 = R_2 = 10\ k\Omega$ and a double switch K.

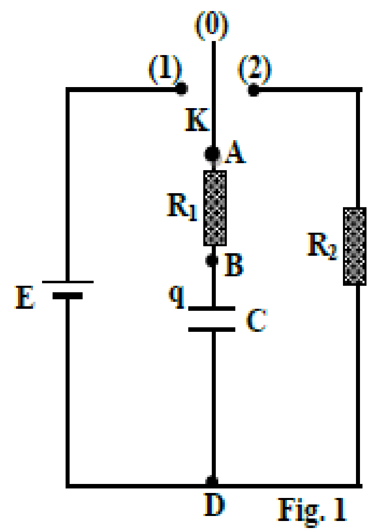


Fig. 1

A – Charging the capacitor

The switch K is in the position (0) and the capacitor is neutral. At the instant $t_0 = 0$, we turn K to position (1) and the charging of the capacitor starts.

1) Theoretical study

a) Applying the law of addition of voltages and taking the positive direction along the circuit as that of the current, show that the differential equation that describes the variation of the voltage $u_C = u_{BD}$ across the capacitor has the form: $E = R_1 C\dfrac{du_C}{dt} + u_C$.

b) The solution of this differential equation has the form: $u_C = A(1 - e^{\frac{-t}{\tau_1}})$ where A and τ_1 are constants. Show that $A = E$ and $\tau_1 = R_1C$.

c) Show that at the end of charging $u_C = E$.

d) Show that the expression $u_{AB} = u_{R_1} = E\ e^{\frac{-t}{R_1C}}$.

e) Establish the expression of the natural logarithm of u_{R_1} [$\ell n\,(u_{R_1})$] as a function of time.

2) Graphical study

The variation of $\ell n\,(u_{R_1})$ as a function of time is represented by figure 2.

a) Justify that the shape of the obtained graph agrees with the expression of $\ell n(u_{R_1})$ as a function of time.

b) Deduce, using the graph, the value of the capacitance C.

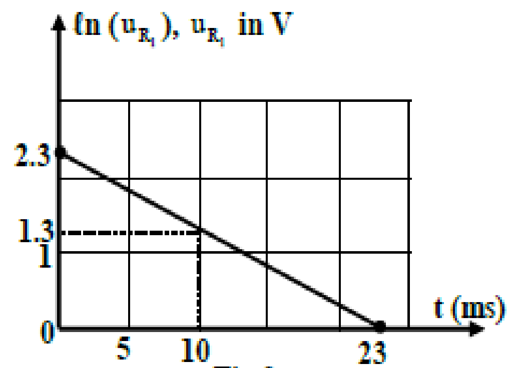


Fig.2

B – Discharging the capacitor

The capacitor being fully charged, we turn the switch K to position (2). At an instant $t_0 = 0$, taken as a new origin of time, the discharging of the capacitor starts.

1) During discharging, the current circulates from B to A in the resistor of resistance R_1. Justify.

2) Taking the positive direction along the circuit as that of the current, show that the differential equation in the voltage u_C across the capacitor has the form: $u_C + (R_1 + R_2)\, C\dfrac{du_C}{dt} = 0$.

3) The solution of the above differential equation has the form: $u_C = Ee^{\frac{-t}{\tau_2}}$ where τ_2 is the time constant of the circuit during discharging. Show that $\tau_2 = (R_1 + R_2)\, C$.

4) The variation of the voltage u_C across the capacitor and the tangent to the curve $u_C = f(t)$ at the instant $t_0 = 0$, are represented in figure 3. Deduce, from this figure, the value of the capacitance C.

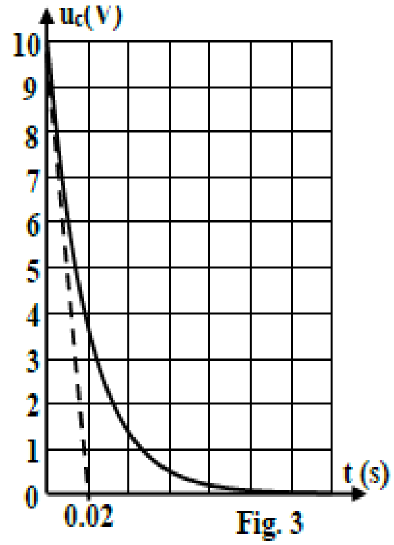
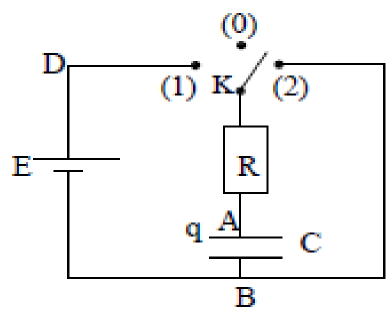


Fig. 3

EX:4

Discharging of a capacitor: The lightning

The electric circuit of the adjacent figure allows us to perform charging and discharging of a capacitor of capacitance C, through a resistor of resistance R. The used generator has a constant electromotive force E and is of negligible internal resistance.



A – Charging of the capacitor

The capacitor is initially uncharged and the switch K is in position (0).

1) To which position, (1) or (2), must we turn the switch K in order to charge the capacitor?

2) The variation of the voltage $u_C = u_{AB}$ across the terminals of the capacitor as a function of time is given by the expression: $u_C = E\,(1 - e^{\frac{-t}{RC}})$. Deduce the value of u_C in terms of E, at the end of the charging of the capacitor.

B – Discharging of the capacitor

The charging of the capacitor being completed, the switch K is again in position (0).

1) To which position must we turn the switch K in order to discharge the capacitor?

2) The instant $t_0 = 0$ corresponds to the starting of the discharging. At an instant t, the circuit carries a current i.

a) Draw the circuit of discharging and indicate on it the real direction of the current chosen as a positive direction.

b) i) In this case, the current is written as $i = -\dfrac{dq}{dt}$ and not as $i = +\dfrac{dq}{dt}$. Why?

ii) Show that the differential equation in i has the form: $i + RC\dfrac{di}{dt} = 0$.

c) Verify that $i = \dfrac{E}{R}\, e^{\frac{-t}{RC}}$ is the solution of this differential equation.

3) Trace the shape of the curve representing the variation of i as a function of time.

4) Give, in terms of R and C, the duration at the end of which the capacitor is practically completely discharged.

C – The lightning

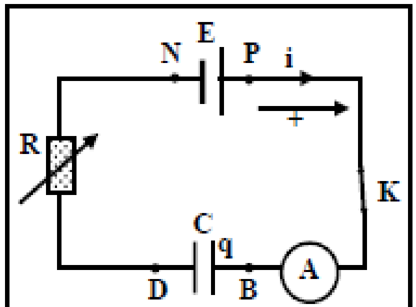
In a cloud, the collisions between the water particles give rise to positive and negative charges: The lower part of the cloud becomes negatively charged while its upper part positively charged. Simultaneously, the ground is charged positively by induction. A capacitor of capacitance $C = 10^{-10}$ F is thus formed having the ground as the positive armature, the lower part of the cloud as the negative armature and the air between them being the insulator. The voltage across its armatures is $E = 10^8$V. In certain conditions, the air between the armatures becomes a conductor of resistance $R = 5000\ \Omega$. We suppose that the lightning corresponds to the complete discharging of this capacitor through air.

1) Calculate the duration of the lightning.
2) Determine the maximum current due to the lightning.

EX:5

The aim of this exercise is to determine the capacitance C of a capacitor. We set-up the series circuit of document 4. This circuit includes:

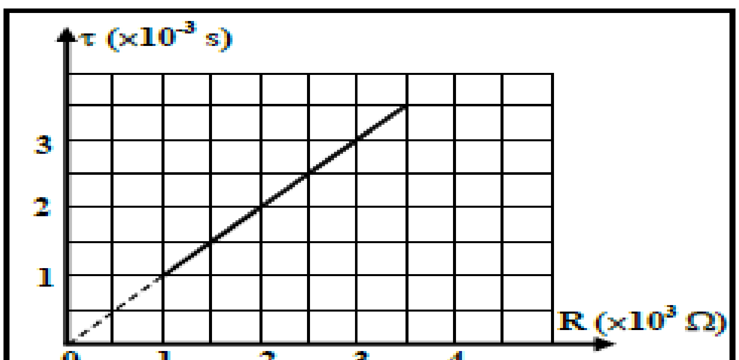
- an ideal battery of electromotive force $E = 10$ V;
- a rheostat of resistance R;
- a capacitor of capacitance C;
- an ammeter (A) of negligible resistance;
- a switch K.



Doc.4

Initially the capacitor is uncharged. We close the switch K at the instant $t_0 = 0$. At an instant t, plate B of the capacitor carries a charge q and the circuit carries a current i as shown in document 4.

1) Write the expression of i in terms of C and u_C, where $u_C = u_{BD}$ is the voltage across the capacitor.
2) Establish the differential equation that governs the variation of u_C.
3) The solution of this differential equation is of the form: $u_C = a + b\ e^{\frac{-t}{\tau}}$. Determine the expressions of the constants a, b and τ in terms of E, R and C.
4) Deduce that the expression of the current is: $i = \frac{E}{R}\ e^{\frac{-t}{RC}}$.
5) The ammeter (A) indicates a value $I_0 = 5$ mA at $t_0 = 0$. Deduce the value of R.
6) Write the expression of $u_R = u_{DN}$ in terms of E, R, C and t.
7) At an instant $t = t_1$, the voltage across the capacitor is $u_C = u_R$.
   7-1) Show that $t_1 = R\,C\,\ell n\,2$.
   7-2) Calculate the value of C knowing that $t_1 = 1.4$ ms.
8) In order to verify the value of C, we vary the value of R. Document 5 represents τ as a function of R.
   8-1) Show that the shape of the curve in document 5 is in agreement with the expression of τ obtained in part 3.
   8-2) Using the curve of document 5, determine again the value of C.



Doc. 5