Exercise 1:

Solve the following:

1) Ln (x+1) + Ln (x+2) = ln (x + 3)

2) ln ($\frac{x+1}{x+3}$) = 1

3) $(lnx)^2$ – 3lnx + 2 = 0

4) ln (x-2) – ln(x-3) $\leq$ ln2

5) $\begin{cases} x + y = 9 \\ lnx + lny = 1 \end{cases}$

1) Domain:

x > - 1 , x > -2 , x > -3

Number line:]-1, +∞ [

Ln[(x+1)(x+2)] = Ln (x + 3)

$Ln\ (x^2 + x + 2x + 2) = Ln\ (x + 3)$

x^2+3x + 2 = x + 3

x^2 +2x -1 = 0

x' = $-1 + \sqrt{2}$ (acc.) and x'' = -1 - $\sqrt{2}$ (rej.)

2) Domain:

solving x +1 = 0

x = -1

Cndn: x + 3 $\neq$ 0

x $\neq$ -3

x	-∞	−3		-1	+∞
x +3	-		+	0	+
x +1	-		-		+
	+		-	0	+

$D_f =]-\infty, -3[\cup]-1, +\infty[$

$\frac{x+1}{x+3} = e^1$

x + 1 = x.e + 3e

x – xe = 3e – 1

x (1- e) = 3e – 1 ---> $x = \frac{3e-1}{1-e}$

3) Domain: x > 0

Let t = lnx

$t^2 - 3t + 2 = 0$

t = 2 and t = 1

Then, lnx = 2 lnx = 1

x = e^2 x = e

(both accepted)

Teacher Mohammad

Exercise 1:

Solve the following:

4) $\ln(x-2) - \ln(x-3) \leq \ln 2$

5) $\begin{cases} x + y = 9 \\ lnx + lny = 1 \end{cases}$

4) Domain:

$x > 2$, $x > 3$

Number line:]3 , +∞ [

$$ln\frac{x-2}{x-3} \leq ln\ 2$$

$$\frac{x-2}{x-3} - 2 \leq 0$$

$$\frac{x-2-2(x-3)}{x-3} \leq 0$$

$$\frac{-x+4}{x-3} \leq 0$$ **---> table**

x	-∞		3		4		+∞
-x+4		+	‖	+	0	-	
x -3		-	‖	+		+	
		-	‖	+	0	-	

S =] -∞,3[∪ [4, +∞ [

Number line: domain + sol

Solution: [4, +∞ [

5) x + y = 9

ln(xy) = 1 *[change lnx + lny to ln(xy)]*

$xy = e^1$

Then S = 9 and P = e

$x^2 - 9x + e = 0$ **---> calculator**

x' = 8.69 (accepted) and y= 0.31 (accepted)

Exercise 2:

Find the following limits

1) $\lim\limits_{x\to 0}\left(\frac{\ln(1+x)}{x}\right)$

2) $\lim\limits_{x\to +\infty}\left(\frac{x\ln x}{x+1}\right)$

3) $\lim\limits_{x\to +\infty} \ln\left(\frac{x-1}{x+2}\right)$

1) $\lim\limits_{x\to 0}\left(\frac{\ln(1+x)}{x}\right) = \frac{\ln 1}{0} = \frac{0}{0}$ *[Ind. form]*

L' H R:

$$\lim_{x\to 0}\left(\frac{\ln(1+x)}{x}\right) = \lim_{x\to 0}\left(\frac{\frac{1}{1+x}}{1}\right)$$

$$= \frac{1}{1+0} = 1$$

2) $\lim\limits_{x\to +\infty}\left(\frac{x\ln x}{x+1}\right) = \frac{+\infty}{+\infty}$ *[Ind. form]*

L' H R:

$$\lim_{x\to +\infty}\left(\frac{x\ln x}{x+1}\right) = \lim_{x\to +\infty}\left(\frac{\ln x+1}{1}\right)$$

$$= +\infty$$

$(x\ln x)' = u'v + v'u$

$= 1\ln x + x.\frac{1}{x} = \ln x + 1$

3) $\lim\limits_{x\to +\infty} \ln\left(\frac{x-1}{x+2}\right) = \ln\frac{+\infty}{+\infty}$ *[Ind. form]*

$$\lim_{x\to +\infty} \ln\left(\frac{x-1}{x+2}\right) = \ln\left(\lim_{x\to +\infty}\frac{x-1}{x+2}\right)$$

Using H.R

$$\ln\left(\lim_{x\to +\infty}\frac{1}{1}\right) = \ln 1$$

$$= 0$$

Teacher Mohammad

Exercise 3

Part A:

Let g be the function defined over]0; + ∞ [as $\mathbf{h(x) = x^2 - \ln x + 2}$.

1) a) Find $\lim\limits_{x \to 0} h(x)$ and $\lim\limits_{x \to +\infty} h(x)$.

b) Set up the table of variations of h, deduce that h (x) > 0.

Part B

f is the function defined over]0;+ ∞[as $\mathbf{f(x) = x - \frac{1 - \ln x}{x}}$, (C) is the graph of f.

1) a- Find $\lim\limits_{x \to 0} f(x)$ and $\lim\limits_{x \to +\infty} f(x)$.

b-Prove that the line (Δ) with equation y = x is an asymptote to (C).

c- Study the relative position between (C) and (Δ).

2) a- Prove that f'(x) $= \frac{h(x)}{x}$.

b- Set up the table of variation of f(x).

c- Find the equation of tangent (T) at point B of abscissa 1.

d- Calculate f $(\frac{1}{2})$, f (1), then plot (Δ), (C) and (T).

Test 1_ Solution

A] h(x) = x^2 −ln x + 2.

a) $\lim\limits_{x\to 0} h(x)$ = 0 -Ln 0 +2

= +∞

$\lim\limits_{x\to +\infty} h(x)$ = +∞ - ∞ + 2

$\lim\limits_{x\to +\infty} x\left(x - \frac{Lnx}{x} + \frac{2}{x}\right)$ = +∞ (+∞ - 0 + 0) **Ind. form**

= +∞

b) h'(x) = 2x - $\frac{1}{x}$

= $\frac{2x^2-1}{x}$

$2x^2 - 1 = 0$

$x^2 = ½$

x = $\sqrt{\frac{1}{2}}$

x	0		$\sqrt{\frac{1}{2}}$		+∞
h'(x)		-	0	+	
h(x)	+∞	↘	2.84	↗	+∞

Min h(x) = 2.48 > 0

So, h(x) > 0 for any x ∈ $]0, +\infty[$

B] f(x) = x − $\frac{1-\ln x}{x}$

a) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \left(x - \frac{1}{x} - \frac{\ln x}{x}\right)$

= + ∞ - 0 – 0

= +∞

➢ $\lim\limits_{x\to 0} f(x)$ = 0 - $\frac{1-ln0}{0}$

= - $\frac{1-(-\infty)}{0}$

= - ∞

So, x= 0 is V.A at - ∞

Test 1_ Solution

B] f(x) = x – $\frac{1-\ln x}{x}$

b) y = x is O.A ?

$$\lim_{x\to+\infty}[f(x)-(x)] = \lim_{x\to+\infty} -\frac{1-\ln x}{x}$$

$$= \lim_{x\to+\infty}\frac{1}{x} = 0$$

y= x is O.A at + ∞

c) Relative position:

f(x) $-(x)$ = $-\frac{1-\ln x}{x}$

1 - lnx = 0

Lnx = 1

x = e^1

x	0		e		+∞
$-\frac{1-\ln x}{x}$		-		+	
		(C) below (d)		(C) Above (d)	

(C) Intersects (d) in (e, e)

f(x) = x – $\frac{1-\ln x}{x}$

2)a) $f'(x)$ =1 – ($\frac{u'v-v'u}{v^2}$)

= 1- $\frac{-1-1-\ln x}{x^2}$

$= \frac{x^2+2-\ln x}{x^2}$ = $\frac{h(x)}{x^2}$ > 0

[$h(x) > 0$ from part A and $x^2 > 0$]

b) table:

x	0		+∞
f'(x)		+	
f(x)	−∞	↗	+∞

Teacher Mohammad

Test 1_ Solution

$f(x) = x - \frac{1-\ln x}{x}$

c) Tangent (D) to (C) at point B with x= 1

(D): $y - y_B = f'(x_B)\,(x - x_B)$

$f'(1) = \frac{1+2-\ln 1}{1} = 3$

$y - 0 = 3\,(x - 1)$

$f(1) = 1 - \frac{1-0}{1} = 0$

(T): y = 3x - 3

d) Plot

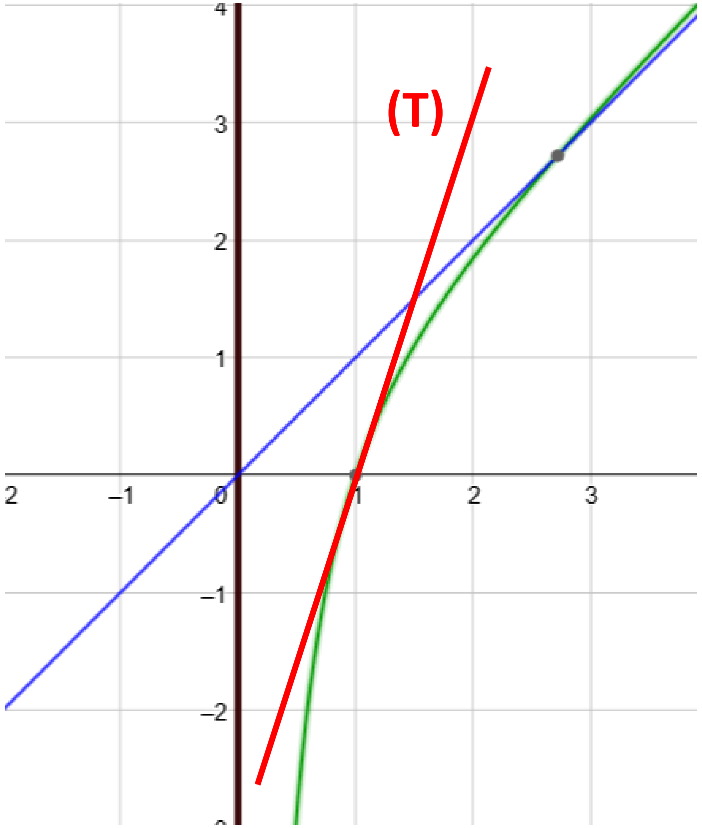
Y = x O.A

(e, e) int. point

X = 0 V.A

(T): y =3x – 3

At B(1,0)



3) Find the area bounded by (C), (d) and the 2 lines x = 1 and x= e.

$A = \int_1^e y_{(d)} - f(x)\,dx$

$= \int_1^e x - \left(x - \frac{1-\ln x}{x}\right) dx$

$= \int_1^e \frac{1-\ln x}{x}\,dx$

$U = 1 - \ln x$ $\quad u' = -\frac{1}{x}$ or $du = -\frac{1}{x}\,dx$

$= -\int_1^e u\,du$

$= -\left[\frac{(1-\ln x)^2}{2}\right]_1^e$

$= -\left[\frac{(1-\ln e)^2}{2} - \frac{(1-\ln 1)^2}{2}\right]$

$= -(0 - ½) = ½\ u^2$

Teacher Mohammad