2 Exercises and problems

N° 1
Linear momentum of a system

Two particles (a) and (b), of respective masses $m_1 = 0.1$ kg and $m_2 = 0.3$ kg, move with respective spe $v_1 = 2$ m/s and $v_2 = 1$ m/s. Determine the vectors of linear momenta of the particles (a) and (b) and the lin momentum of their center of inertia G in each of the following cases :

1) The velocity vectors of the particles are parallel and of the same direction.
2) The velocity vectors of the particles are parallel and of opposite directions.
3) The velocity vectors of the particles are perpendicular.

N° 2
An effect of a force

A particle, of mass m = 100 g, moves in a uniform and rectilinear motion with a speed $v_1 = 3$ m/s. At a gi instant the direction of motion of the particle is deviated by an angle of 120^0 , under the action of a force. The speed remains constant.

1) Calculate the magnitudes of the linear momentum vectors $\vec{p}_1$ and $\vec{p}_2$ of the particle before and after change of the direction of motion.
2) Represent, on a figure, the preceding linear momentum vectors and their variation vector $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$ the scale: 1 cm ↔ 0.1 kgm/s.
3) Calculate the magnitude of the vector $\Delta\vec{p}$.
4) The change in the direction of the motion takes 0.2 s. Calculate the average value of the force which make this variation.

N° 3
Motion of a solid on a horizontal and rectilinear path

A solid (S), of mass m = 250 g, is launched with a speed $V_0 = 6$ m/s, at $t_0 = 0$, on a rectilinear and horizon path.
The frictional force supposed constant between (S) and the support and of magnitude f = 2 N.

1) Represent on a figure the forces acting on (S).
2) Applying Newton's second law, write the expression of the linear momentum of (S) at given instant t.
3) Calculate the duration of motion of (S).

N° 4
Motion of a solid on an inclined path

A solid (S) of mass m = 0.4 kg, is launched, without speed at an instant $t_0 = 0$, slides on a rectilinear tra inclined by an angle $\alpha = 30^0$ with respect to the horizontal.

The force of friction is supposed constant and of magnitude f = 1.2 N. Given g = 10 m/s^2.

1) Represent on a figure the forces acting on (S).
2) Applying Newton's second law, write the expression of the linear momentum of (S) at a given time t.
3) Find the instant when the speed of (S) is 6 m/s .

N° 5
Graphical study and Newton's second law

A chariot of mass m = 100 g leaves from rest and moves upward on the line of greatest slope of an inclined plane by an angle $\alpha = 30°$ with respect to the horizontal, by a rope enrolled on the periphery of a pulley. The other extremity of the string carries a certain mass as shown in [figure (a)].

We neglect frictional force and we give $g = 10\ m/s^2$.

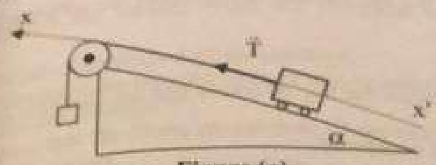


Figure (a)

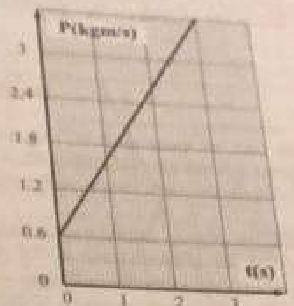


Figure (b)

1) Represent, on a figure, the forces acting on the chariot.

2) Applying Newton's second law, calculate the tension T in the rope as a function of m, g, α and $\frac{dP}{dt}$ where P is the linear momentum of the chariot at an instant t.

3) Given the graph of the linear momentum of the chariot as a function of time in the figure (b).

a) Extract :
- The speed of the chariot at t = 0.
- The instant of departure of the chariot.

b) Deduce, using the graph, the value of T.

N° 6
Studying the motion of a chariot

On an air table, inclined by an angle $\alpha = 30^0$ with respect to the horizontal, we study the motion of a chariot, of mass M = 1.2 kg, sliding along the line of greatest slope of a bench. An appropriate device represents to the scale ½ the different successive positions G_i of the center of inertia G of the chariot during each interval of time equal to $\tau = 50$ ms (see the figure below). Given $g = 10\ m/s^2$.

1) Show that the linear momentum vector when it passes through the point G_i is $\vec{P}_i = M\frac{G_{i-1}G_{i+1}}{2\tau}.\vec{i}$ where $G_{i+1}G_{i-1}$ represents the distance between the points G_{i+1} and G_{i-1}.

2) Knowing that at $t_0 = 0$ the chariot is at G_0 and at t_i it is at G_i. Complete the following table.

$[t_i ; t_{i+1}]$	$[\tau ; 2\tau]$	$[2\tau ; 3\tau]$	$[3\tau ; 4\tau]$	$[4\tau ; 5\tau]$
$\frac{\Delta\vec{P}}{\Delta t} = \frac{\vec{P}_{i+1} - \vec{P}_i}{t_{i+1} - t_i}$				

3) Is the chariot a pseudo-isolated system ? You can use the relation : $\frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t}$.

4) Specify the nature of the motion of the chariot?

5) Show the existence of a force of friction whose magnitude is to be calculated.

N° 7

Studying a collision between two objects

A body (A), of mass m = 0.1 kg, moving on a rectilinear and horizontal track x'x with a speed v = 2 … undergoes a head on collision with a body (B), of mass M = 0.2 kg, initially at rest. After the collision … (B) moves on the same track with a speed V = 1.2 m/s.

1) Which variable is conserved during the collision ?

2) Applying the conservation of the preceding variable, determine the line of action and the direction of … velocity $\vec{v}'$ of (A) after the collision.

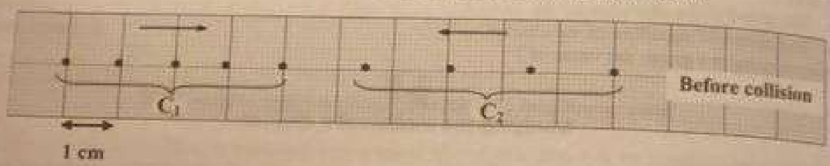
3) Verify that the kinetic energy of the system [(A) ;(B)] is not conserved.

4) Is the collision elastic? Under what form does the variation in the kinetic energy of the sys… [(A) ;(B)] appear?

N° 8

Collision between two pucks

Two pucks C_1 and C_2 regulated each to a time constant τ = 100 ms, are launched along the same line … horizontal air table. The pucks undergo a head-on collision. The registrations show the positions of the cen… of inertia of C_1 and C_2 before the collision and to a **real scale is given** in the figure below.



The mass of C_1 is : m_1 = 400 g and that of C_2 is m_2 = 100 g.

1) Calculate the speeds V_1 and V_2 of C_1 and C_2 respectively before the collision.
2) Find the linear momentum of the system (C_1 ; C_2) before the collision.
3) Deduce the speed of the center of inertia G of the system before the collision.
4) After the collision C_1 and C_2 form one body which moves with a speed V. Calculate V.
5) Is the collision elastic? Justify.

N° 9

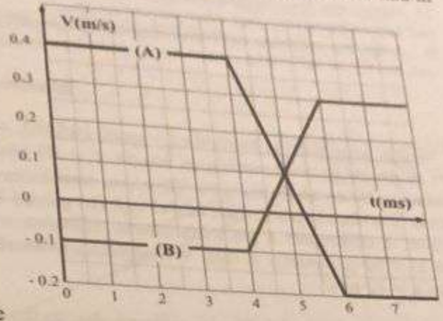
Separation of the elements of an isolated system

A man, of mass m = 60 kg, rides a chariot, of mass M = 100 kg. The system (man – chariot), suppo… isolated, moves with a speed V = 8 m/s on a horizontal and rectilinear track. At a given instant, the m… leaves the chariot with a horizontal speed (v = 2 m/s) in a direction opposite to the initial displacement of t… system (man – chariot).

Calculate the speed of the chariot just after the man leaves the chariot.

N° 10
Studying the collision from the graph

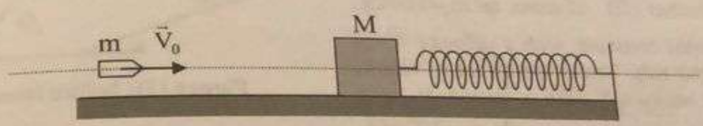
Two bodies (A) and (B), of respective masses $m_A = 0.2$ kg and $m_B = 0.3$ kg, moving without friction and in opposite directions along a horizontal axis $\overrightarrow{x'x}$ of unit vector $\vec{i}$, undergo a collision.
In the adjacent figure we represent the graphs of the algebraic values of the speeds of (A) and (B) as a function of time on the axis $\overrightarrow{x'x}$. We neglect friction.



1) Extract the instants when the collosion starts and ends.
2) Deduce the duration of the collision.
3) Determine the velocity vectors of (A) and (B) before and after the collision ?
4) Show that the linear momentum of the system [(A) ; (B)] is conserved.
5) Is the collision elastic ? Justify.
6) a) Determine, using the graph, the vectors of the forces of interaction between (A) and (B) during the collision.
b) Is the principle of interaction verified ? Justify

N° 11
Measuring the speed of a bullet

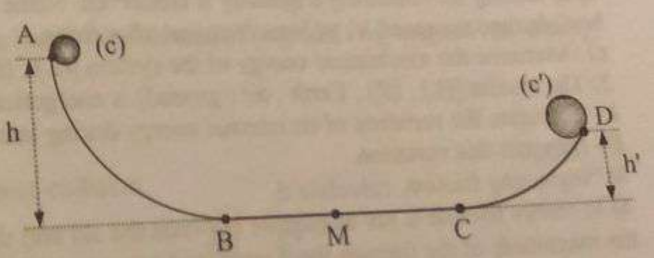
A bullet, of mass m = 9.5 g, moving with a horizontal velocity $\vec{V}_0$, undergoes a collision with a solid of mass M = 5.4 kg, attached to a spring, of constant K = 1000 N/m, as shown in the figure below. After the collision, the bullet and solid form one body and the spring is compressed by a maximum distance d_m = 15 cm. We neglect frictional forces.



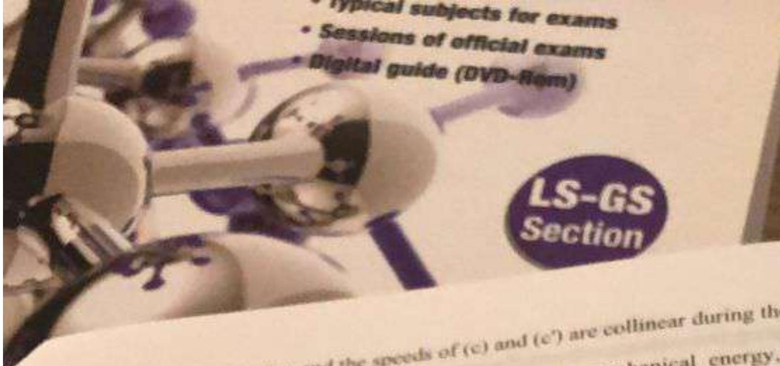
1) Calculate the speed V of the system (m ; M) just after the collision.
2) Deduce, in km/h, the speed V_0.
3) a) The collision is not elastic. Justify, by calculation, this note.
b) In what form of energy doest the variation in the kinetic energy of the system [m; M] appear ?

N° 12
Elastic collision « 1 »

Two small solids (c) and (c'), of respective masses m=40g and m'=80g, are found respectivelyy at the tops A and D of a track (A-B-C-D) situated in a vertical plane as shown in the adjacent figure (h = 125 cm ; h'= 20cm).



We launch (c) and (c') without speed, they descend and eneter in a perfectly elastic collision at a point M on [BC].

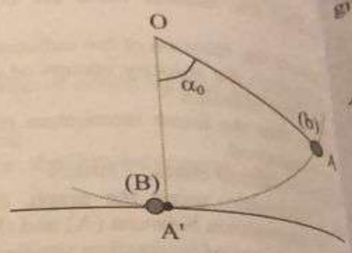


We neglect friction and the speeds of (c) and (c') are collinear during the choc. Take : $g = 10\ m/s^2$.

1) Calculate, applying the conservation of the mechanical energy, the speeds v and v' of (c) and (c') respectively just before the collision.
2) Applying the suitable laws of conservation of physical quantities, determine the speeds V and V' of (c) and (c') respectively just after the collision.
3) After the collision (c) moves up along the part (A-B) and attains a maximum height h_{max}. Calculate h

N° 13
Elastic collision «2»

A simple pendulum is formed of a small ball (b), of mass $m = 100$ g and of an inextensible string of length $\ell = 90$ cm. We displace the pendulum by an angle $\alpha_0 = 60°$ from its equilibrium position and we release it without speed. Just as the ball (b) passes by its equilibrium position it undergoes a perfectly elastic collision with a ball (B), of mass $M = 50$ g, initially at rest. We neglect frictional forces.
Given $g = 10\ m/s^2$.



The velocities of (b) and (B) are collinear during the collision.

1) Calculate the speed of (b) just before the collision.
2) Find the speeds of (b) and (B) just after the collision.
3) Determine the amplitude of the oscillations of the pendulum after the collision.

N° 14
Recoil of a missle launcher

A missle launcher (L), initially at rest, of mass $M = 5000$ kg, lays on a ski **[Figure (1)]**, holds horizontally a rochet (R) of mass m $m = 100$kg, supposing it remains constant, with a velocity $\vec{V}$ of magnitude $V = 350$ m/s. Under the effect of recoil, (L) is displaced backward with a speed V_1 and moves up a plane, inclined by an angle $\alpha = 10^0$ with respect to the horizontal, by a distance « d » after it compresses by $X_m = 1.26$m a spring of stiffness $K = 10^5$ N/m **[Figure (2)]**.

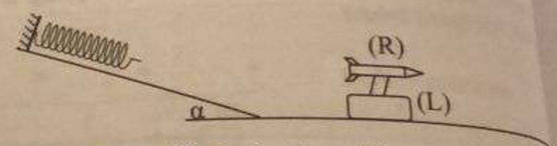


Figure (1): before launching the rocket

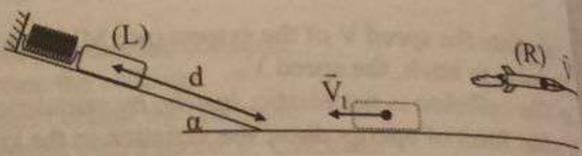


Figure (2): after launching the rocket

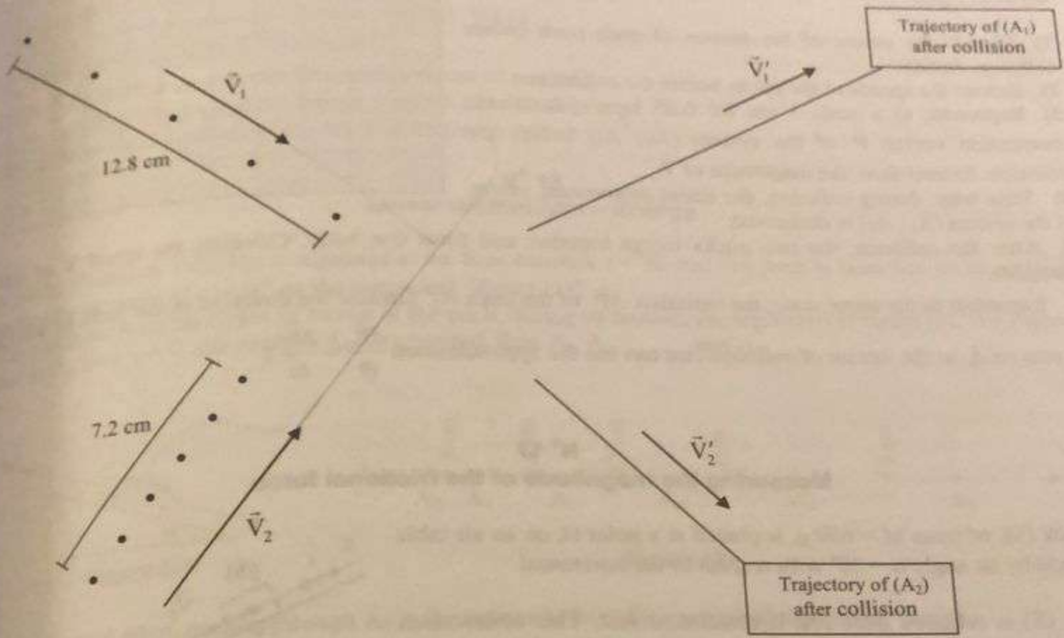
The reference level for gravitational potential energy s the horizontal ground. We neglect the height etween (R) and the reference level for gravitational otential energy. Take : $g = 10\ m/s^2$.

a) During the launching a quantity is conserved. Name this quantity.
Calculate the speed V_1 of launching just after firing.
Determine the mechanical energy of the system [(L) ; (R) ; Earth; air ; ground] just after firing.
The system [(L) ; (R) ; Earth ; air ; ground] is energetically isolated.
Calculate the variation of its internal energy during launching.
nterpret this variation.
eglecting friction, calculate d.
reality friction is not negligible between the ski and the inclined plane and in this case d = 3 m. Calcu
agnitude of the friction force supposed constant (the compression of the spring remains $X_m = 1.26$ m)

N° 15

Verification of the principle of interaction starting from a collision

Two moving pucks (A_1) and (A_2), regulated at the same time constant $\tau = 40$ ms, of respective masses $m_1 = 0.3$ kg and $m_2 = 0.5$ kg, are launched, on a horizontal airtable, with respective velocities $\vec{V}_1$ and $\vec{V}_2$, enter in a collision, and acquire the respective velocities $\vec{V}'_1$ and $\vec{V}'_2$ after the collision.
The registrations of the positions of the pucks before collision and their trajectoires after the collision are given in the figure below.

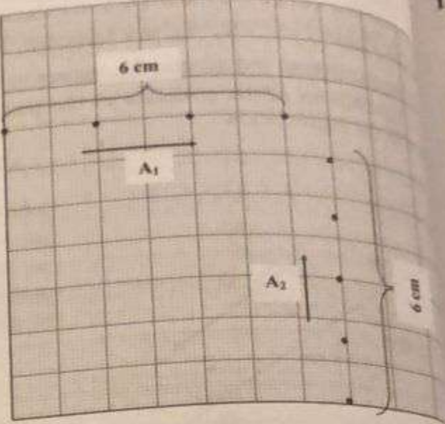


1) a) Determine the nature of motion of each puck before collision.
b) Calculate the magnitudes of the velocity vectors $\vec{V}_1$ and $\vec{V}_2$.
c) Represent, to the scale : 1 cm $\leftrightarrow$ 0.1 kgm/s, the linear momentum vectors of the two pucks before the collision and their resultant $\vec{P}$.
2) a) Applying the conservation of linear momentum, represent the linear momentum vectors of the pucks after collision.
b) Deduce the magnitudes of $\vec{V}'_1$ and $\vec{V}'_2$.
c) Is the collision elastic ? Justify.
3) We designate by $\Delta\vec{P}_1$ and $\Delta\vec{P}_2$ the variations of the linear momentums of (A_1) and (A_2) respectively during the colllision.
a) Represent $\Delta\vec{P}_1$ and $\Delta\vec{P}_2$.
b) Compare : $\frac{\Delta\vec{P}_1}{\Delta t}$ and $\frac{\Delta\vec{P}_2}{\Delta t}$ where Δt is the duration of collision.
c) This experiment verifies the principle of interaction. Justify. Take : $\frac{\Delta\vec{P}}{\Delta t} = \frac{d\vec{P}}{dt}$.

N° 16
Experimental study of a collision

Two pucks A_1 and A_2 of respective masses $m_1 = 250$ g and $m_2 = 200$ g are launched, towards each other, on a horizontal air table.
Each puck is regulated to a time constant $\tau = 20$ ms.

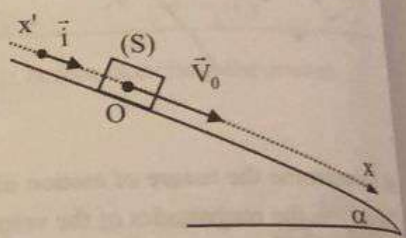
The recordings of the successive positions of the centers of mass of A_1 and A_2 before the collision are represented in the adjacent figure.



1) Specify the nature of the motion of each puck before collision. Justify.
2) Extract the speeds of the pucks before the collision.
3) Represent, to a scale 1 cm for 0.05 kgm/s, the linear momentum vector $\vec{P}$ of the system (A_1 ; A_2) before the collision. Extract then, the magnitude of $\vec{P}$.
4) State why, during collision, the linear momentum vector of the system (A_1 ; A_2) is conserved.
5) After the collision, the two pucks merge together and form one body. Calculate the speed V of ensemble.
6) Represent to the same scale, the variation $\Delta\vec{P}_1$ of the puck A_1. Deduce the direction of the force by which A_2 acts on A_1 at the instant of collision (we can use the approximation $\frac{d\vec{P}_1}{dt} = \frac{\Delta\vec{P}_1}{\Delta t}$).

N° 17
Measuring the magnitude of the frictional force

A puck (S), of mass $M = 600$ g, is placed at a point O, on an air table inclined by an angle $\alpha = 30^0$ with respect to the horizontal.

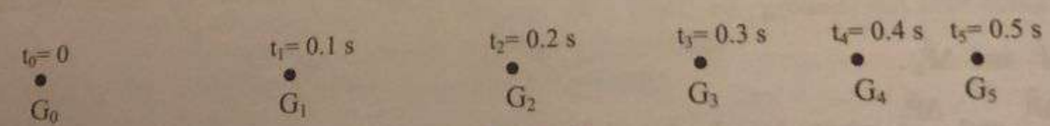


When (S) is released from rest it remains at rest. This observation proves the existence of a frictional force between (S) and the table.

Given $g = 10$ m/s^2.

At an instant $t_0 = 0$ taken as an origin of time, we launch (S) with a velocity $\vec{V}_0 = V_0.\vec{i}$ where $\vec{i}$ is the unit vector of the axis x'Ox parallel to the line of greatest slope of the table.

An appropriate device records the successive positions G_i of the center of mass of (S) during equal time intervals $\tau = 0.1$ s, as shown in the figure below.



Scale 1/2

1) Complete the table below:

t(s)	0	0.1	0.2	0.3	0.4	0.5
V(m/s)						
P(kgm/s)						

V and P are the speed and the linear momentum of (S) respectively at a time t.

2) Draw the graph of P as a function of time.
Scale: ordinate: 1 cm $\leftrightarrow$ 0.1 kgm/s and abscissa: 1 cm $\leftrightarrow$ 0.05 s.
3) The slope β of the previous curve is constant. Why?
4) Calculate the value of β. Interpret $|\beta|$.
5) Find the acceleration of (S). Deduce the nature of motion.
6) Extract the value V_o and the instant at which (S) will stop.
7) Calculate the magnitude of the force of friction.

N° 18
Linear momentum – Energy

A puck, of mass $m = 0.2$ kg, is regulated at the time constant $\tau = 50$ ms. The puck is launched on an air table, incline by an angle of $\alpha = 30^0$ on the horizontal [figure (1)].
The positions A_i of the center of inertia of the puch, during its motion, are registered in figure (2). We suppose that at the time $t_0 = 0$, the position A_0 is registered, then A_1, A_2,.........and A_5.

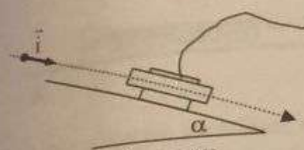


Figure (1)

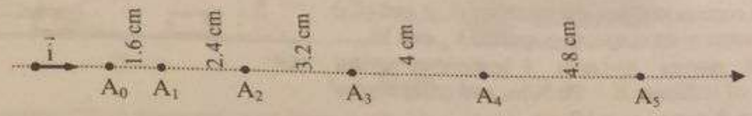


Figure (2)

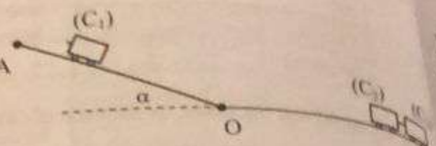
The horizontal plane passing through A_0 is the reference level for gravitational potential energy.

1) a) Complete the table below :

Time : t	τ	2τ	3τ	4τ
Points	A_1	A_2	A_3	A_4
Speed : V (m/s)	0.4		0.72	
Linear momentum of the puck : P (kgm/s)	0.08		0.144	

b) Represent the graph of P as a function of time.
Scale : 1 cm $\leftrightarrow$ 25 ms (abscissa) and 1 cm $\leftrightarrow$ 0.04 kg.m/s (ordinate)
c) Extract the magnitude of the velocity $\vec{V}_0$ at point A_0.
2) Find the resultant $\sum \vec{F}_{ext}$ of the forces acting on the puck.
3) Show that the puck is under the action of a friction force whose magnitude f is to be determined.
Take : $g = 10$ m/s^2.
4) a) Calculate the mechanical energies of the system [Earth ; puck] at the points A_0 and A_3.
b) Find again by energetic methode the value of f.

N° 19
Separation of masses after an elastic collision



Consider three chariots (C_1), (C_2) and (C_3), of respective masses $m_1 = m_2 = 150$ g and $m_3 = 100$ g. (C_2) and (C_3) are attached by a string which conserves the compression of a spring placed between them, the system $[(C_2) ; (C_3)]$ is at rest on a horizontal track OX.

(C_1) is at rest at the top A of an inclined track AO (AO = 90 cm) making an angle $\alpha = 30^0$ with the hori…

We release (C_1), at $t_0 = 0$, (C_1) without speed, it reaches the rail OX and undergoes a perfectly elastic co… between the system $[(C_2) ; (C_3)]$. Given $g = 10$ m/s². We neglect the forces which resist the motion … chariots.

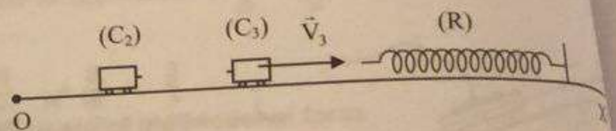
1) Applying the conservation of mechanical energy, calculate the speed V_1 of (C_1) when it reaches the … O.

2) Applying Newton's second law « $\sum \vec{F} = \frac{d\vec{P}}{dt}$ », determine, as a function of time, the linear momentu… (C_1) when it is between O and A.

3) At what instant does (C_1) reach O ?

4) The chariot (C_1) continues its motion on OX and enters into collision with the system $[(C_2) ; (C_3)]$ … speed V_1. Calculate the speeds V_1' and V' of (C_1) and of the system $[(C_2) ; (C_3)]$ respectively after collisio…

5) The system $[(C_2) ; (C_3)]$ moves with a speed V' on the track OX. At a given instant, the rope is cut and the chariots (C_2) and (C_3) separate with respective speeds V_2 and V_3. (C_3) moves towards a horizontal spring (R), of stiffness K = 90 N/m, and compresses it to a distance $x_0 = 10$ cm.



a) Applying the conservation of mechanical energy, calculate the value of V_3.

b) Find V_2.

c) Is the kinetic energy of the system $[(C_1) ; (C_2)]$ conserved ? Interpret.

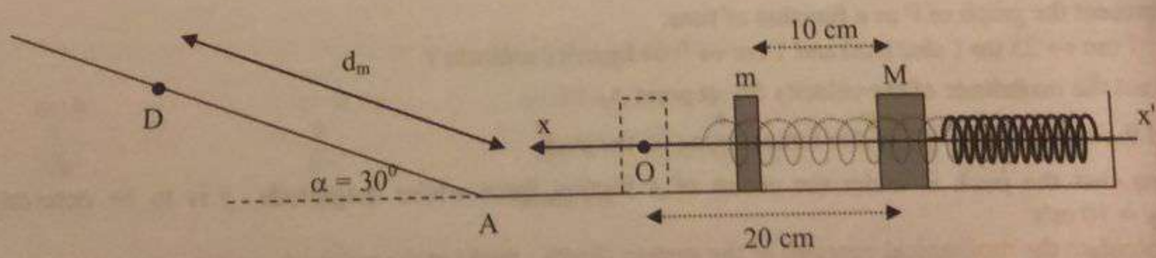
N° 20
Studying a collision

An elastic horizontal pendulum is formed of a solid of mass M = 150 g and a perfectly elastic spring of unjoi… turns and of stiffness K = 25 N/m.

When M is at O, the spring is neither compressed nor elongated.

We compress the spring displacing the mass M by a distance of 20 cm.

We place at 10 cm in front of M a mass m = 50 g as shown in the figure below.



The pendulum is compressed and released without initial speed, the spring elongates and M undergoes a collision with the mass m initially at rest.

The zero reference of gravitational potential energy is the horizontal plane passing through the center of inertia of M. We neglect the frictional forces. Given $g = 10\ m/s^2$.

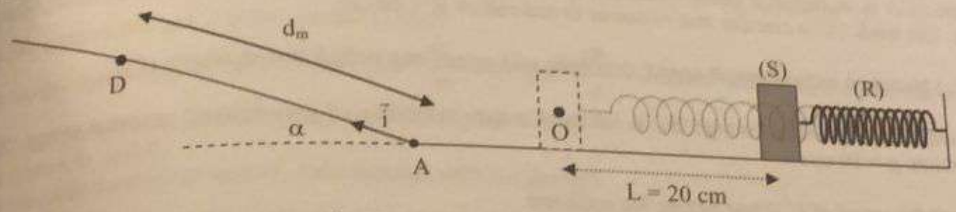
1) a) Verify that the elastic potential energy of the spring at the instant of collision : 0.125 J.
b) Applying the conservation of mechanical energy, calculate the speed V of M just before the collision.

2) After the collision m climbs, starting from point A, the line of greatest slope of an inclined plane making an angle $\alpha = 30^0$ with respect to the horizontal, and turns back at a point D where $AD = d_m = 90$ cm.
a) Show that the speed of just after the collision is $v = 3$ m/s.
b) Deduce the speed V' of M just after the collision. We suppose that the velocity vectors, during the collision are collinear.
c) Verify that the collision is not elastic.

N° 21

Calculating the magnitude of a force of friction by two methods

We consider a perfectly elastic spring (R), which is mass less, of unjoined turns, and of stiffness $K = 75$ N/m. We compress the spring to a distance $L = 20$ cm and we place in front of it a solid (S) of mass $M = 300$ g as shown in the figure below.



At a given instant we release the spring (R) it elongates and takes its free length when (S) reaches O. The zero reference of gravitational potential energy is the horizontal plane passing through the center of inertia of (S). Given $g = 10\ m/s^2$.

I – Studying the launching of (S)

1) Calculate the mechanical energy of the system [(S) ; (R) ; Earth ; support] at the instant of launching of (S).
2) Frictional forces are negligible. Find the speed of (S) when it passes through O.

II –The force of friction between the inclined plane and (S) is not negligible.

At O, the solid (S) leaves the spring and climbs, starting from the point A, a line of greatest slope of an inclined plane by an angle $\alpha = 30^0$ with respect to the horizontal, and reaches a point D where $AD = d_m = 56\,cm$ at which it rebounds. The duration of the climb $\tau = 354 ms$.

A – First method : Energetic Method

1) Calculate the variation of the mechanical energy of the system [(S) ; Earth ; support], when (S) passes from A to D. To what form of energy is this variation transformed to ?
2) Deduce the magnitude f of the force of friction supposed constant between the inclined plane and (S).

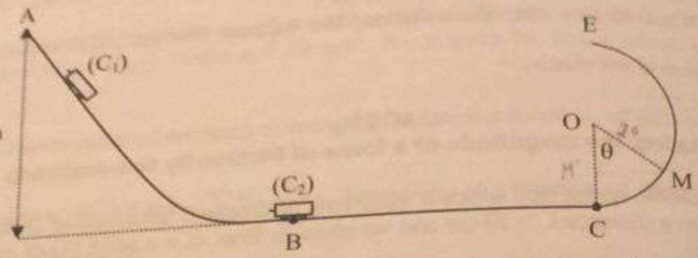
B – Second method: Dynamic method

1) Let $\sum \vec{F}$ be the resultant of the external forces acting on (S). Show that $\sum \vec{F} = -\beta.\vec{i}$ where β is a constant to be determined as a function of M, g, α and the magnitude f of the force of friction supposed constant.

2) Applying Newton's second law, determine the expression of the linear momentum P of (S) at ... knowing that at $t_0 = 0$, the solid (S) is at point A.
3) Deduce f.

N° 22
Elastic Collision

Two wagons (C_1) and (C_2), of respective masses $m_1 = 50$ g and $m_2 = 150$ g, rest on a track ABCE ... the vertical plane as shown in the figure below.



The wagon (C_1) is at rest at a point A at a height h = 80 cm from the horizontal track BC where ... immobile. The track CE is circular and of center O and radius R = 20 cm.

Wagon (C_1) launched without initial speed, descends and enters into collision with wagon (C_2).

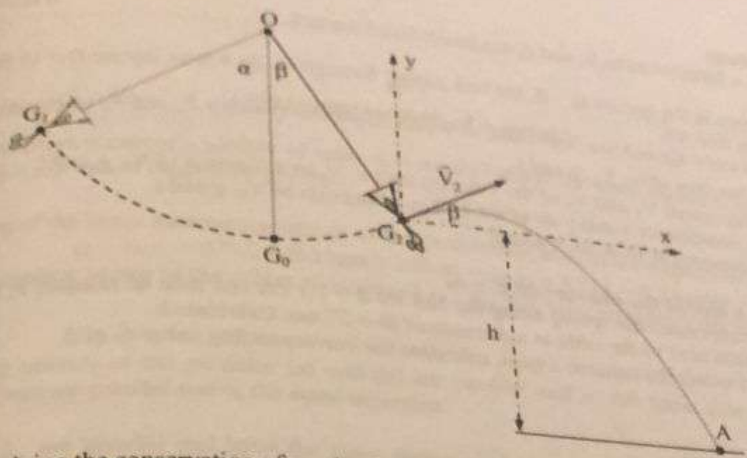
The horizontal plane passing through BC is taken as a zero reference of gravitational potential energy ... neglect friction. $g = 10\ m/s^2$.

1) Calculate the speed of (C_1) just before the collision.
2) The collision between (C_1) and (C_2) is perfectly elastic and the velocities are collinear.
a) Name the variable which are conserved during the collision.
b) Determine the speeds of the wagons just after the collision.
3) After the collision, wagon (C_2) moves and climbs a circular track reaching a point M such ... $\widehat{COM} = \theta = 60^0$ and then it rebounds.
a) Show that the wagon (C_2) is under a force of friction.
b) Knowing that this force of friction exists only on the circular part of the rail. Calculate its value supposed constant.
c) Represent the graph of the mechanical energy of the system [Earth; (C_2)], when (C_2) passes from B after the collision, as a function of the distance "d" covered by (C_2). Given BC = 50 cm.
Scale : Ordinate: 1 cm ↔ 0.1 J and abscissa: 1 cm ↔ 10 cm.

N° 23
An acrobat

An acrobat, of mass M = 80 kg, leaves from rest on a swing making an angle $\alpha = 60^0$ with the vertical passing by its equilibrium position G_0 and when it attains, at the date $t_0 = 0$, the position G_2, corresponding to $\beta = 30^0$, with the velocity $\vec{V}_2$, he leaves the swing and leaves in a free fall to reach the ground at point A with the velocity $\vec{V}_A$ (see the figure on the following page). We neglect friction.

Given : $g = 10\ m/s^2$, $OG_1 = OG_2 = L = 6$ m, h = 4 m.



1) Calculate, applying the conservation of mechanical energy, the magnitudes of the velocities $\vec{V}_2$ and $\vec{V}_A$.

2) a) Determine the components V_{2x} and V_{2y} of velocity $\vec{V}_2$ in the system (xG_2y).

b) Deduce the components P_{2x} and P_{2y} of linear momentum vector $\vec{P}_2$ of the acrobat at G_2.

3) We suppose that at $t_0 = 0$ the acrobat is at G_2.

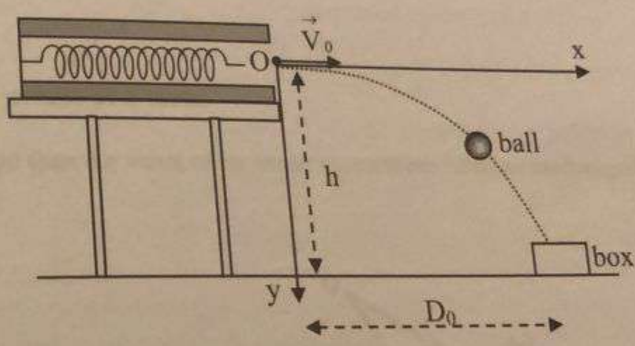
a) Verify, applying Newton's 2nd law, that : $\frac{dP_x}{dt} = 0$ and $\frac{dP_y}{dt} = -800\,N$ with P_x and P_y are the components of linear momentum vector $\vec{P}$ of the acrobat at a time $t > 0$.

b) Deduce P_x and P_y.

4) Calculate the time of impact of the acrobat with the ground.

N° 24
Studying a game

Two children are playing a game in which a spring is situated on a horizontal table. A ball (of mass $m = 20$ g) is placed in front of the spring. Once released, the spring elongates and takes its free length and launches the ball from O with a speed $\vec{V}_0$.



The child achieves the goal of getting the ball into the box at a distance $D_0 = 1.7$ m from the table (see the figure).

We designate by :

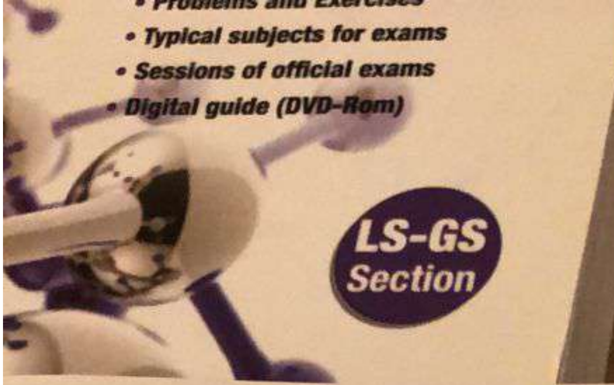
$h = 44$ cm : the height of the table with respect to the ground,

k : stiffness of the spring,

d : the compression of the spring.

D : the abscissa of the impact point of the ball with the ground.

The zero level of gravitational potential energy is the level of the table. We neglect friction. Take g = 10 m



3

I – Theoretical study

1) Calculate, as a function of m, k, and d, the speed V_0 of the ball.

2) We suppose that at the instant $t_0 = 0$, the ball passes through O. At a later instant $(t > 0)$ the ball is in air and undergoing free fall.

a) Applying Newton's second law, calculate, at a time t, the components P_x and P_y of the linear momen $\vec{P}$ of the ball as a function of m, V_0, g and t.

b) Deduce the components V_x and V_y of the velocity vector $\vec{V}$ as a function of V_0, g and t.

c) Determine the coordinates x and y de of the ball as a function of V_0, g and t.

d) Find a relation between h, D, d, m, k and g.

II – Calculation of k and of the convenient $d = d_0$

1) The first child compresses the spring using the ball by d = 1.1 cm and then he releases it, the ball doe enter the box and it falls next to the table at a distance of D = 27 cm. Calculate k.

2) If the second child wants the achieve a goal, calculate the corresponding value d_0 of d.