SAMPLE TEST 1
GRADE 12 LS-GS
MECHANICS

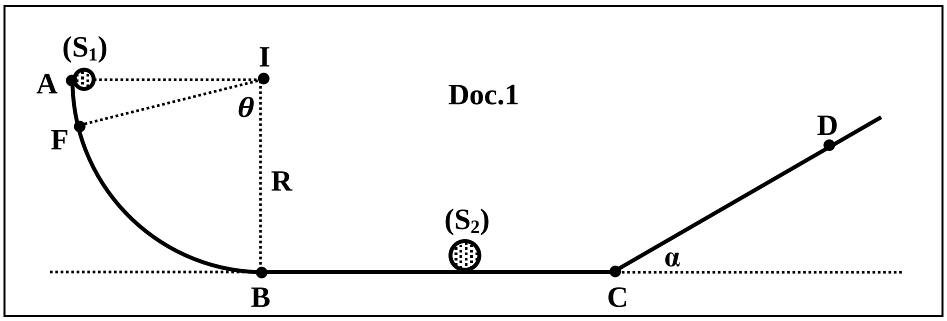
EXERCISE 1

CONSERVATION AND NON-CONSERVATION OF MECHANICAL ENERGY

Consider a track ABCD situated in a vertical plane and formed of three rails. AB is a circular rail of center I and radius R = 0.2m, BC is a horizontal rail and CD is a rough inclined plane making an angle $\alpha = 30^o$ with the horizontal.

Given:

- The force of friction is neglected along ABC.
- The horizontal plane passing through BC is taken as a gravitational potential energy reference.
- Take $g = 10m/s^2$.
- CD = 20cm.



A solid (S_1), taken as a particle of mass $m_1 = 100g$, is released without initial velocity from point A. (S_1) passes by point F with a speed $V_F = 1m/s$ where IF makes an angle θ with the vertical plane containing IB.

1- Apply the principle of conservation of mechanical energy to determine:

 1.1- the value of θ,

 1.2- the velocity V_1 of (S_1) as it passes by point B.

2- The solid (S_1), moving with a velocity V_1, enters in a perfectly elastic head-on collision with a solid (S_2), taken as a particle of mass m_2, and rests on BC. Just after collision, the velocities acquired by (S_1) and (S_2) are V_1' and $V_2' = 2m/s$ respectively.

 2.1- Show that $V_2' = \frac{2m_1V_1}{m_1+m_2}$.

 2.2- Calculate m_2.

3- The solid (S_2) reaches point C with a velocity V_2' and moves along the inclined plane where its stops at point D.

 3.1- Show that the solid (S_2) is subjected to a force of friction $\vec{f}$ along CD.

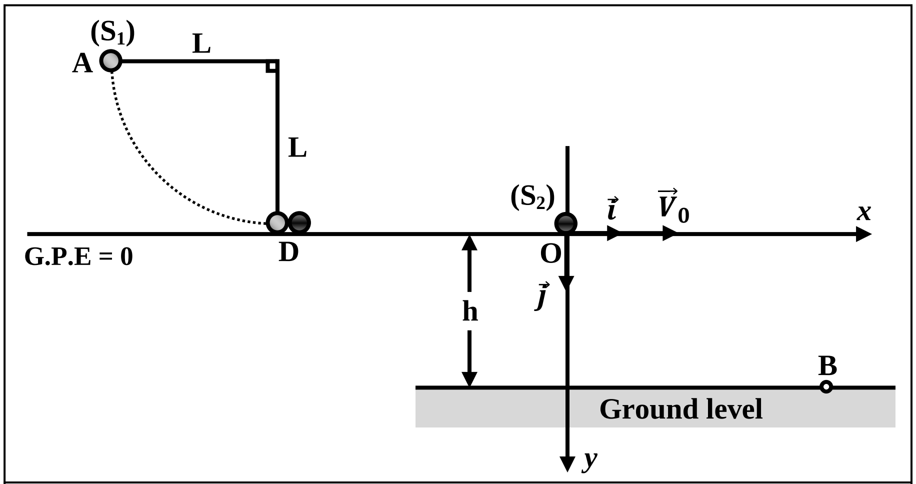
 3.2- Determine the magnitude f of the force of friction $\vec{f}$ along CD.

EXERCISE 2

COLLISION AND PROJECTILE

The system represented in document-1 consists of a simple pendulum (P) formed of a massless and inextensible rope of length L= 180cm that holds at one of its extremities a small sphere (S_1) of mass M = 100g.
The pendulum is shifted horizontally from its equilibrium position, and then released from point A without initial velocity. When (S_1) reaches its equilibrium position at point D, it enters in a perfectly elastic head-on collision with another small sphere (S_2), of mass m = 200g, placed on a horizontal rough table. The sphere (S_2) continues its motion along the table. (S_2) reaches point O with a velocity $\vec{V_0} = V_0\vec{i}$ and then falls freely describing a parabolic trajectory in the space reference system $(O; \vec{i}; \vec{j})$ before it hits the ground at point B situated at a height h = 1.25m below O.
The force of friction between the table and (S_2) opposes its motion and is assumed constant of magnitude f = 0.7N.
Take:

- the horizontal plane passing through DO as a gravitational potential energy reference,
- $g = 10 m/s^2$.



Document-1

1- Apply the principle of conservation of mechanical energy to show that the speed of the sphere (S_1) at point D before the collision with (S_2) is $V_1 = 6 m/s$.

2- Show that the algebraic value of the velocity of (S_2) just after collision is $V'_2 = 4 m/s$.

3- Determine the speed of (S_2) at point O knowing that DO = 100cm.

4- Choose the instant when (S_2) leaves point O as an origin of time $t_0 = 0$.

4.1- Show that the horizontal and the vertical components of the linear momentum $\vec{P}$ of (S_2) at an instant t are given by:

$$\vec{P} \left| \begin{array}{l} P_x = 0.6 \\ P_y = 2t \end{array} \right. \quad [SI]$$

4.1- Knowing that (S_2) reaches point B at the instant t = 0.5s, Determine the speed of the (S_2) at point B.

4.2- Verify the result obtained in (4.1) by applying the principle of conservation of mechanical energy.

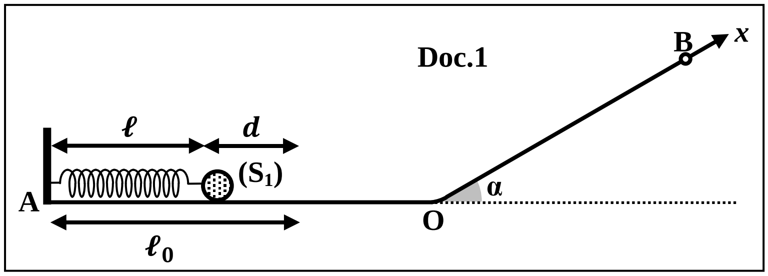
EXERCISE 3

STUDYING THE MOTION OF A PARTICLE

Consider:
- a rail AOB situated in a vertical plane formed of two straight parts: a horizontal part AO and an inclined part OB making an angle $\alpha = 30^o$ with the horizontal;
- a solid (S) taken as a particle of mass m = 80g;
- a spring (R), of neglibgible mass, force constant k = 200N/m and natural length ℓ_0, fixed from one of its ends to a support at A with the other end free.

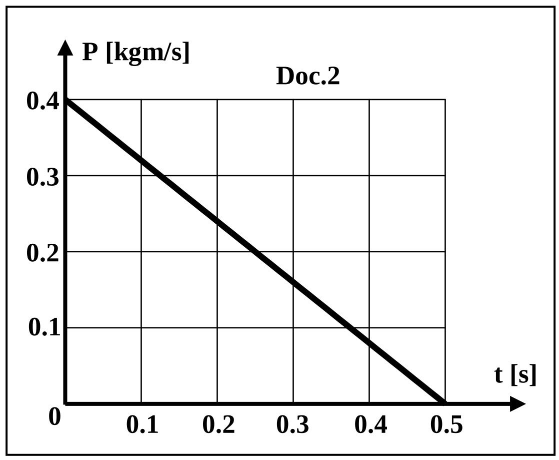
Take:
- the horizontal plane containing O as the reference level for gravitational potential energy;
- $g = 10m/s^2$.

In order to launch (S), it is placed against the free end of the spring, the spring is compressed by a distance d, and then the system [(R); (S)] is released from rest as shown in document 1.
When the spring returns to its natural length ℓ_0, (S) leaves the spring with a velocity $\vec{V}_0$ of magnitude V_0 and parallel to AO.
The force of friction between AO and (S) is neglected.



1- Apply the principle of conservation of mechanical energy to determine the relation between k, m, V_0 and d.

2- At the instant $t_0 = 0$, (S) starts from O on the inclined part OB with a velocity $\vec{V}_0 = V_0\vec{\imath}$, where $\vec{\imath}$ is the unit vector along the x-axis parallel to OB. On this part, (S) is submitted to a friction force $\vec{f}$ of constant magnitude f and of direction opposite to its motion.
The graph of document 2 represents the variation, as a function of time, of the algebriac value P of (S) during its upward motion along OB.



2.1- Show that $P = -0.8t + 0.4$ $[SI]$.
2.2- Determine the value of V_0; then deduce that of d.
2.3- Name and represent the external forces acting on (S) during its motion along the track OB.
2.4- Show that the sum of the external forces acting on (S) during its upward motion along OB is:
$\sum \vec{F}_{ext} = (-f - mg\sin\alpha)\vec{\imath}$.
2.5- Apply Newton's 2^{nd} law to determine f.
2.6- Determine the distance OB knowing that (S) stops at B.