Capacitor

. Recall

$i = \dfrac{dq}{dt}$ $\qquad$ $U_C = \dfrac{q}{C}$

$\rightarrow q = C.U_C$

then $i = \dfrac{d[C.U_C]}{dt} = C\dfrac{dU_C}{dt} = i$

→ Charging of capacitor

applying law of addition of voltages:

$E = U_C + U_R$

at $t=0$: $U_C = 0 \rightarrow E = RI_0$
$\rightarrow I = \dfrac{E}{R}$

$U_C = \dfrac{q}{C} = \int \dfrac{i}{c}\, dt$

$t \neq 0$: $U_C \nearrow$ and $U_R \searrow$
then $E = U_C + U_R$

$t = \infty = 5\tau$: $U_R = 0$
$\Downarrow \rightarrow E = U_C$

($U_R = Ri = 0$ then $i = 0$)

steady phase: $Q = CE = cst \rightarrow \dfrac{dq}{dt} = 0 \rightarrow i = 0$ $\qquad$ ($i = \dfrac{dq}{dt}$)

Note: $i = 0$ then resistor acts as connecting wire.

→ D.E of charging

* in U_C:

Law of addition of voltages

$E = U_C + U_R$

$E = U_C + Ri$ where $i = C\frac{dU_C}{dt}$

$\div RC \left(E = U_C + RC\frac{dU_C}{dt} \right)$

$$\frac{dU_C}{dt} + \frac{U_C}{RC} = \frac{E}{RC}$$

* in q:

Law of addition...

$E = U_C + Ri$

$\div R \left(E = \frac{q}{C} + R\frac{dq}{dt} \right)$ where $i = \frac{dq}{dt}$ and $U_C = \frac{q}{C}$

$$\rightarrow \frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}$$

→ Solution of D.E

$U_C = E(1 - e^{-t/\tau})$ or $q = Q_{max}(1 - e^{-t/\tau})$ given

$\tau = RC$ $Q_{max} = CE$

- we substitute the solution in the D.E

* to get formula of current i:

$U_C = E(1 - e^{-t/\tau})$

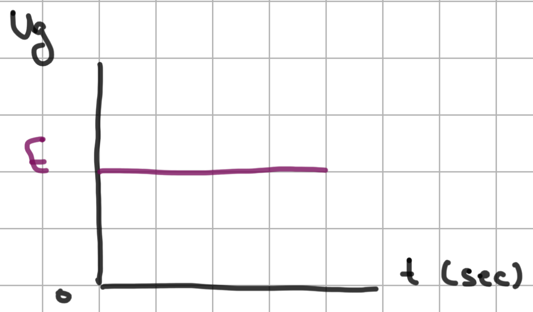
[same for $q = CE(1 - e^{-t/RC})$]

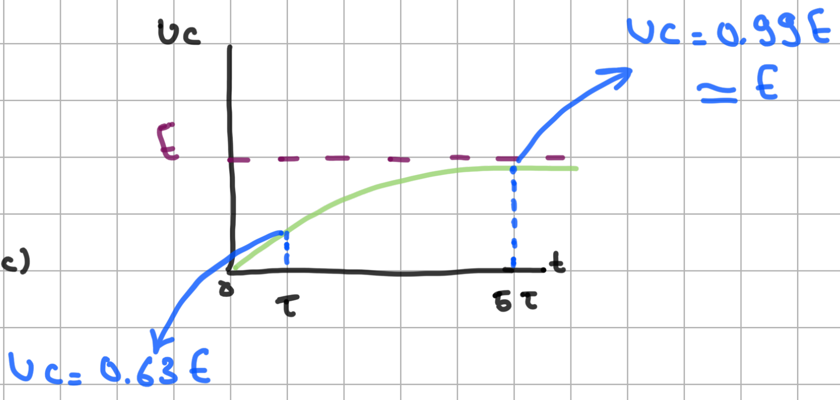
$i = C\frac{dU_C}{dt} \rightarrow i = C\frac{d}{dt}\left[E(1 - e^{-t/\tau})\right]$

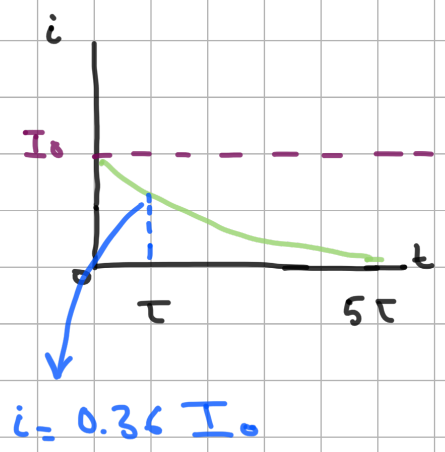
$= CE\left[0 + \frac{1}{RC}e^{-t/RC}\right]$

$= \frac{\cancel{C}E}{R\cancel{C}}e^{-t/RC} = \frac{E}{R}e^{-t/RC} = \boxed{I e^{-t/RC}} = i$

→ Shapes of Charging







→ Discharging of Capacitor

now capacitor acts as generator
(ysni i bytl3 mn l positive armature of C → 3ks l etgeh li ega fi bl charging)

Law of addition of voltages

$$E = U_C + U_R$$
$$0 = U_C + Ri$$

$t=0$: $U_C = E$ $\quad$ $U_R = 0$

$t \neq 0$: $U_C \downarrow$ and $U_R \nearrow$

t_∞ : $U_C = 0$ $\quad$ $U_R = E$

→ D.F of Discharging

* in U_C :

Law of ...

$0 = U_C + Ri$

$\left(0 = U_C + RC\frac{dU_C}{dt}\right) \div RC$

$$\frac{dU_C}{dt} + \frac{U_C}{RC} = 0$$

* in q:

$0 = U_C + Ri$

$\div R\left(0 = \frac{q}{C} + R\frac{dq}{dt}\right)$

$$\frac{dq}{dt} + \frac{q}{RC} = 0$$

→ solutions

$U_C = E\,e^{-t/\tau}$

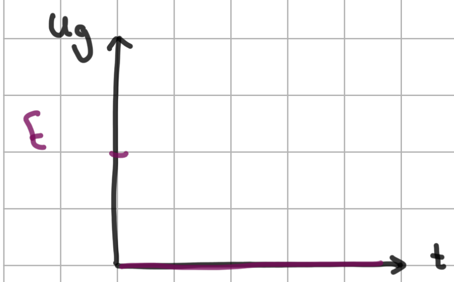
→ $i = \frac{C\,dU_C}{dt}$

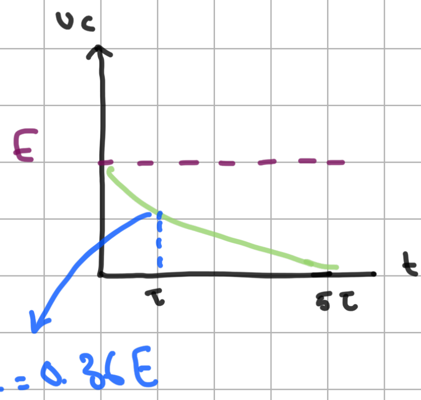
$= C\frac{d}{dt}\left[E\,e^{-t/RC}\right]$

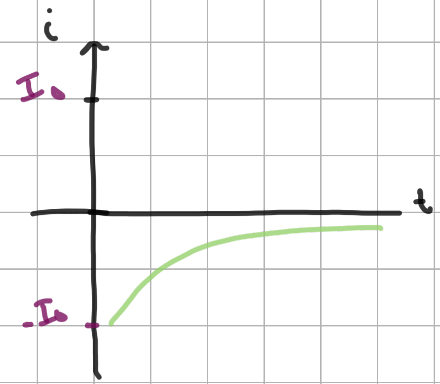
$= C\,E \times \frac{-1}{RC}\,e^{-t/RC}$

$= -\frac{C}{R}\,e^{-t/RC} \quad = \boxed{-I_0\,e^{-t/RC}}$

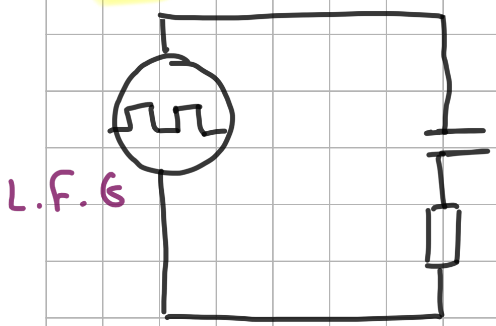
→ Shapes of Discharging

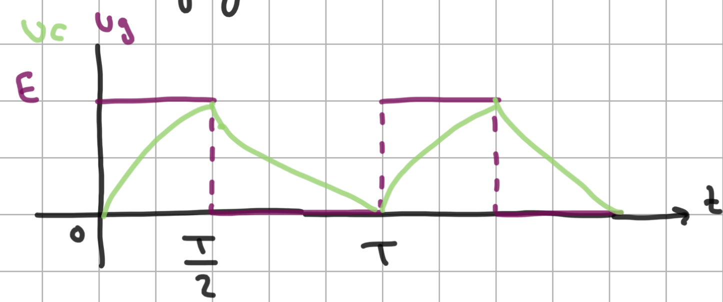


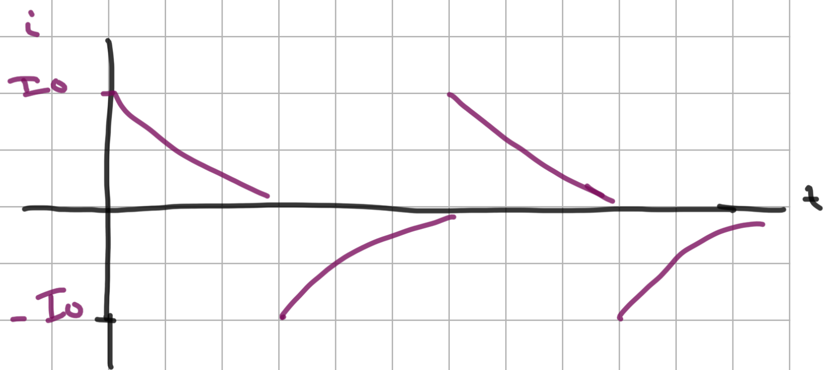




Note: btwn charging + discharging







→ Energ in Capacitor

$W = \frac{1}{2} QU$ or $\frac{1}{2} CU^2$ or $\frac{Q^2}{2C}$