



Physics/ Grade 12 LS

Unit one – Mechanics

Chapter 2 – Linear momentum

Teacher Mohamad Seif

2020 – 2021

Objectives



- 1 To determine position and velocity vector of a particle.**
- 2 To determine position of center of mass of system of particles.**
- 3 To identify types of motion.**
- 4 To determine Linear momentum of a particle**

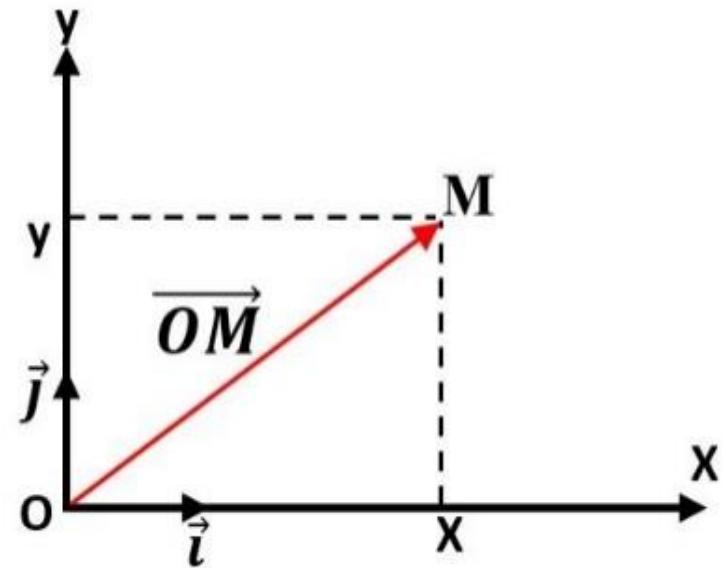
Position vector and Velocity vector .

Position vector: It is the vector that joins the origin O to the moving particle:

$$\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j}$$

Velocity Vector: The velocity vector is the derivative of the position vector w.r.t time:

$$\vec{v} = (x)' \vec{i} + (y)' \vec{j}$$





Position vector and Velocity vector .

Application:

Given the parametric equations $x = 2t$ and $y = 4t^2$ are the coordinates of point (M) at time t.

1. Find the position vector of (M) at instant t.
2. Find the velocity vector of the point M at instant t.

Solution:

1. $\overrightarrow{OM} = \vec{r} = x\vec{i} + y\vec{j}$

$$\overrightarrow{OM} = \vec{r} = 2t\vec{i} + 4t^2\vec{j}$$

2. The velocity vector is the derivative of position vector w.r.t time:

$$\vec{V} = 2\vec{i} + 8t\vec{j}$$

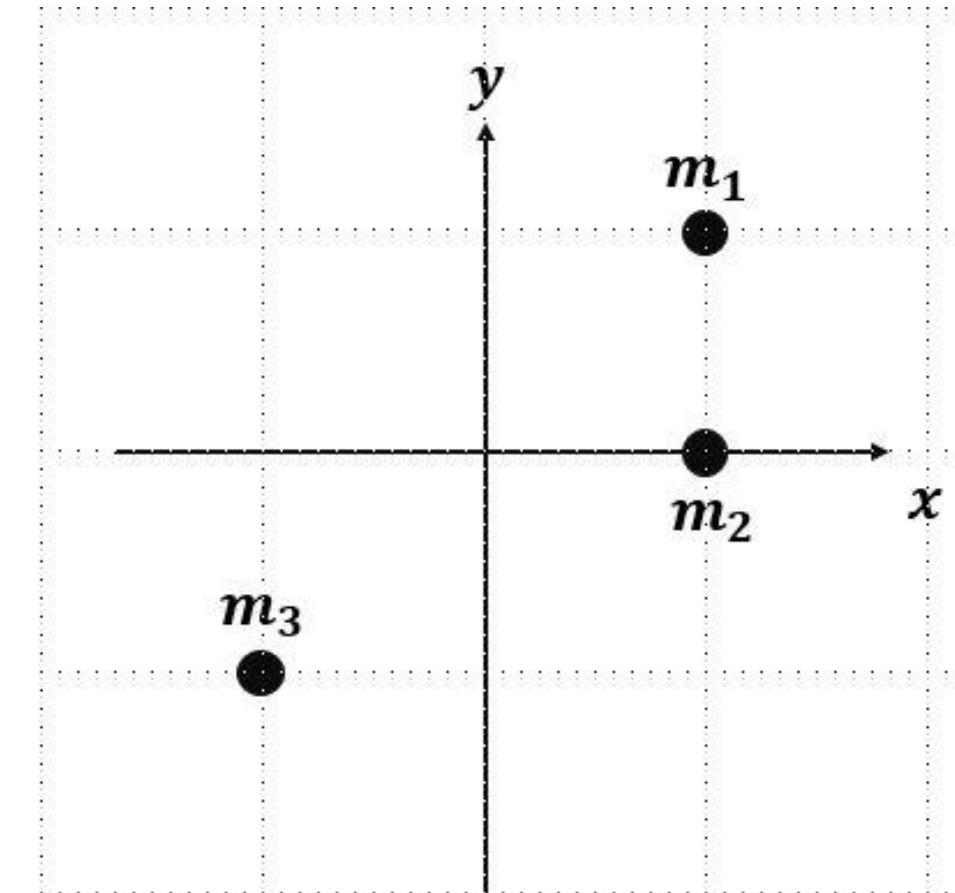
Position of the center of mass of a system of particles

Consider a system of particles as shown in the figure:

- Particle (1): of mass m_1 and a position vector \vec{r}_1
- Particle (2): of mass m_2 and a position vector \vec{r}_2
- Particle (3): of mass m_3 and a position vector \vec{r}_3

The position of the center of mass of the above system is:

$$\overrightarrow{OG} = \vec{r}_G = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$



Position of the center of mass of a system of particles

Application:

Consider a system formed of three particles as shown in the figure. Given $m_1 = 1\text{kg}$; $m_2 = 2\text{kg}$; $m_3 = 3\text{kg}$

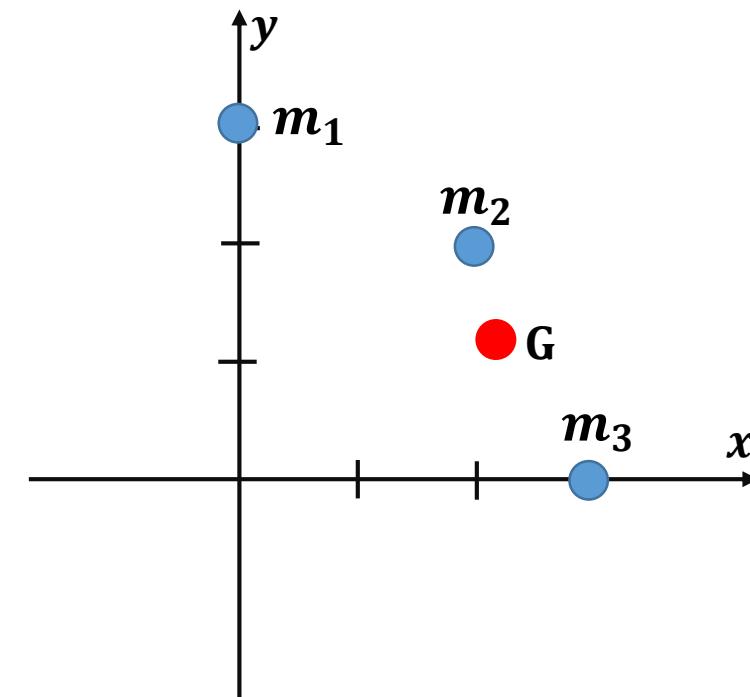
Find the position of the center of mass of the above system.

$$\overrightarrow{OG} = \vec{r}_G = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$

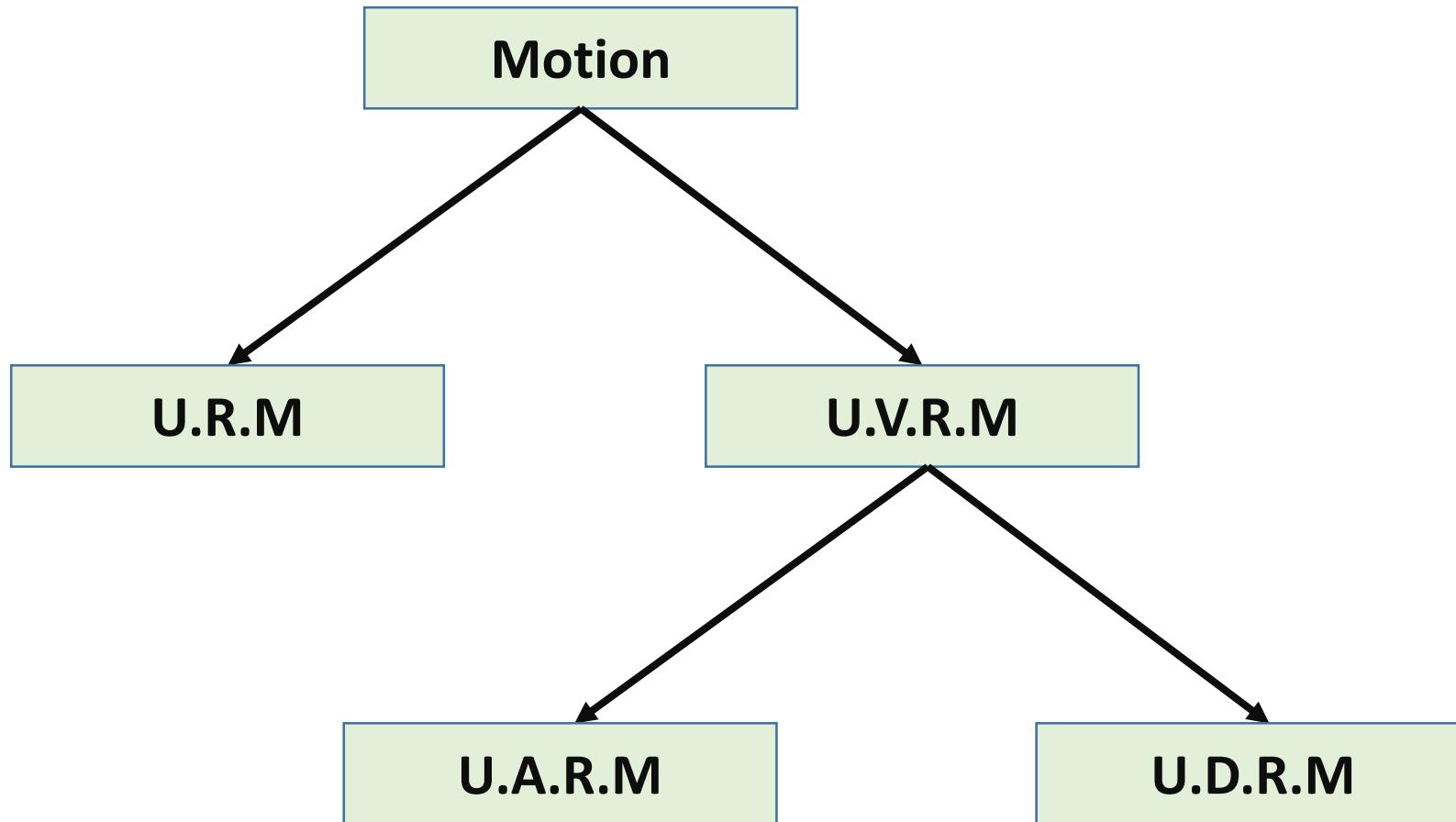
$$\overrightarrow{OG} = \vec{r}_G = \frac{1(3\vec{j}) + 2(2\vec{i} + 2\vec{j}) + 3(3\vec{i})}{(1 + 2 + 3)}$$

$$\overrightarrow{OG} = \vec{r}_G = \frac{3\vec{j} + 4\vec{i} + 4\vec{j} + 9\vec{i}}{6}$$

$$\overrightarrow{OG} = \vec{r}_G = \frac{13\vec{i} + 7\vec{j}}{6} = 2.2\vec{i} + 1.2\vec{j}$$

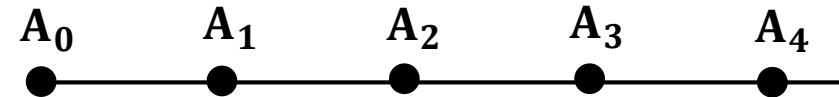


Types of motion.



Types of motion.

Uniform Rectilinear Motion (U.R.M): A motion is said to be U.R.M if the velocity is constant ($V = \text{cst}$), and the acceleration is zero ($a = 0$)



Uniformly Accelerated Rectilinear Motion (U.A.R.M): A motion is said to be U.A.R.M if the velocity increases with time ($V \neq \text{cst}$) and the acceleration is constant and positive ($a > 0$)

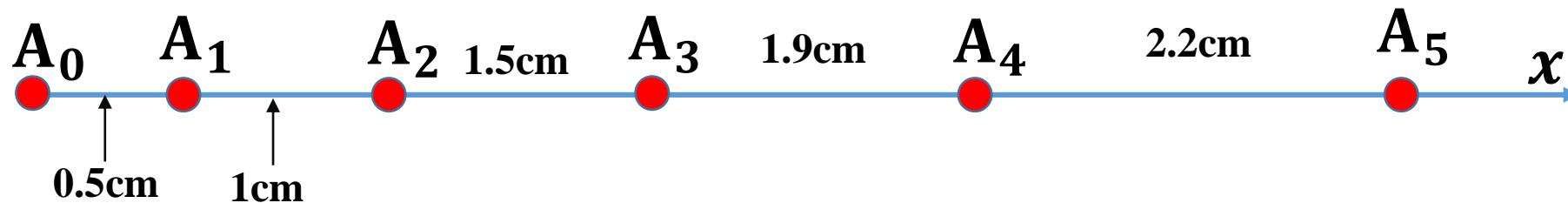


Uniformly Decelerated Rectilinear Motion (U.D.R.M): A motion is said to be U.D.R.M if the velocity decreases with time ($V \neq \text{cst}$) and the acceleration is constant and negative ($a < 0$)



Average velocity.

Consider a particle moves a straight line. A_0, A_1, A_2, A_3, A_4 and A_5 are the successive positions of the center of mass at the instants t_0, t_1, t_2, t_3, t_4 and t_5 . Let τ to be the time interval between any two successive positions.



The average velocity:

$$V_{1,3} = \frac{A_1 A_3}{2\tau} = \frac{(1 + 1.5) \times 10^{-2}}{2 \times 50 \times 10^{-3}} \rightarrow V_{1,3} = 0.25 \text{ m/s}$$

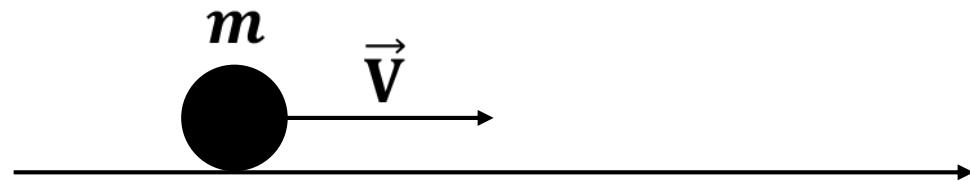
$$V_{0,5} = V_A = \frac{A_0 A_5}{5\tau} = \frac{(0.5 + 1 + 1.5 + 1.9 + 2.2) \times 10^{-2}}{5 \times 50 \times 10^{-3}} \rightarrow V_{0,5} = 0.71 \text{ m/s}$$

Concept of Linear Momentum of a particle.

Linear momentum:

Linear momentum is a **vector quantity** that depends on the mass m and the velocity vector (\vec{V}) of a particle in motion. Linear momentum is the product of mass with the velocity vector of the moving particle

$$\vec{P} = m \cdot \vec{V}$$



\vec{P} is of same direction as velocity

- m : mass of the particle, expressed in kg.
- \vec{V} : velocity vector of the particle, expressed in m/s.
- \vec{P} : Linear momentum of the particle, expressed in kg.m.s.

Concept of Linear Momentum of a particle.

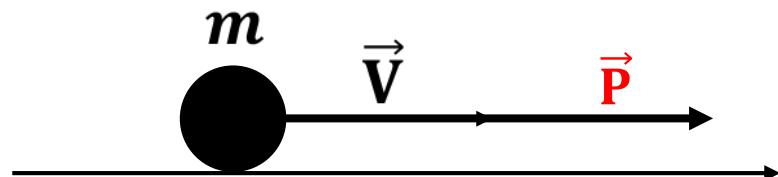
Application:

Consider a ball of mass 250g moving positively with a velocity of magnitude $V = 3\text{m/s}$

1. Calculate the linear momentum of the ball.
2. Draw on the figure the linear momentum vector without scale.

$$1. \vec{P} = m \times \vec{V} = 0.25 \times (+3) \rightarrow \vec{P} = \mathbf{0.75\text{kg.m/s}}$$

2. The linear momentum is of same direction as velocity vector.



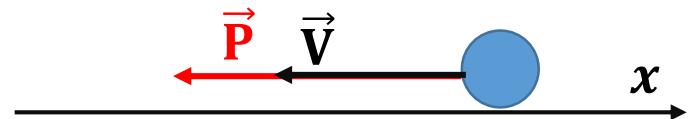
Concept of Linear Momentum of a particle.

Application:

Consider a ball of mass 1.5kg moving as shown in the figure with a velocity of magnitude $V = 5\text{m/s}$

1. Calculate the linear momentum of the ball.
2. Draw on the figure the linear momentum vector without scale.

1. $\vec{P} = m \times \vec{V} = 1.5 \times (-5) \rightarrow \vec{P} = -7.5\text{kg. m/s}$
2. The direction of linear momentum is same of the velocity vector.



I hope that you are safe and doing well





Physics/ Grade 12 LS

Unit one – Mechanics

Chapter 2 – Linear momentum

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Objectives



- 1 To define Linear momentum of a particle.**
- 2 To determine Linear momentum of system particles**
- 3 To determine Linear momentum of center of mass of system of particles.**

Linear Momentum of system of particles.

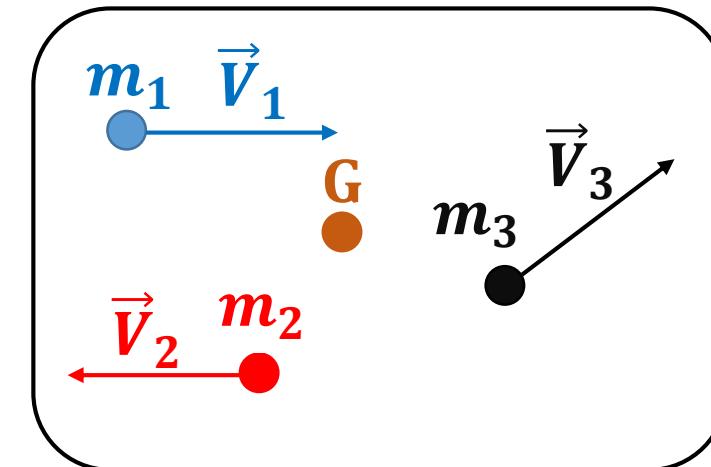
Consider a system of particles as shown in the figure.

The linear momentum of the system of particles is the **vector sum** of all the linear momentum of its particles.

$$\vec{P}_{\text{sys}} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \dots$$

Where:

- $\vec{P}_1 = m_1 \vec{V}_1$
- $\vec{P}_2 = m_2 \vec{V}_2$
- $\vec{P}_3 = m_3 \vec{V}_3$



Linear Momentum of the center of mass of a system of particles.

Consider the same system of particles as shown in the figure.

The **position vector** of the center of mass is:

$$\vec{r}_G = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$

$$M\vec{r}_G = m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3$$

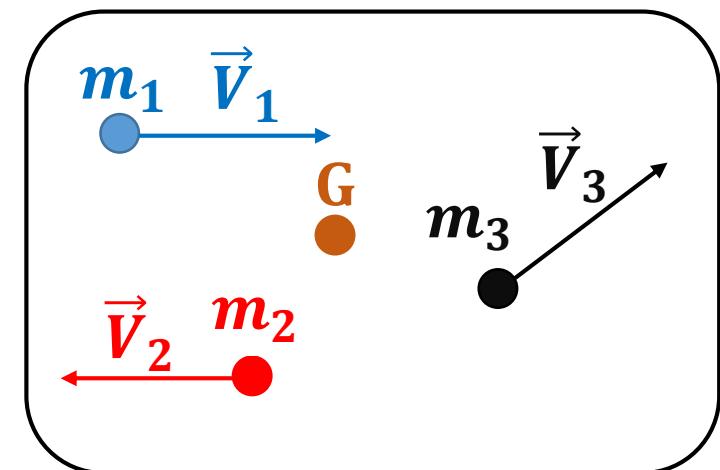
Differentiate the above equation w.r.t time:

$$M\vec{V}_G = m_1 \vec{V}_1 + m_2 \vec{V}_2 + m_3 \vec{V}_3$$

$$\vec{P}_G = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

$$\text{But } \vec{P}_{sys} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

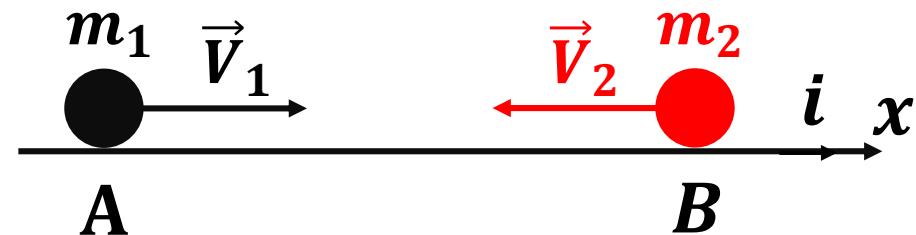
$$\boxed{\vec{P}_G = \vec{P}_{sys} = M\vec{V}_G}$$



Linear Momentum of the center of mass of a system of particles.

Consider a system of two balls A and B moving horizontally and in opposite directions. Ball (A) of mass $m_1 = 50\text{g}$ and moves with a velocity $\mathbf{V}_1 = 4\text{m/s}$, and ball (B) of mass $m_2 = 75\text{g}$ and moves with a velocity $\mathbf{V}_2 = 6\text{m/s}$.

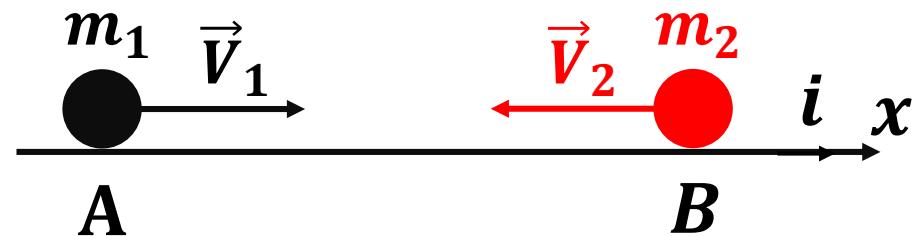
1. Determine the linear momentum of the system (A – B).
2. Deduce the velocity of the center of mass of the above system.



Linear Momentum of the center of mass of a system of particles.

Ball (A): $m_1 = 50g$ and $V_1 = 4m/s$.

Ball (B): $m_2 = 75g$ and $V_2 = 6m/s$.



Solution:

$$1. \quad \vec{P}_{\text{sys}} = \vec{P}_1 + \vec{P}_2 = m_1 \vec{V}_1 + m_2 \vec{V}_2$$

$$\vec{P}_{\text{sys}} = 0.005 \times (4) + 75 \times 10^{-3} \times (-6i)$$

$$\vec{P}_{\text{sys}} = -0.25i \text{ kg.m/s}$$

$$2. \quad \vec{P}_G = \vec{P}_{\text{sys}} = M \vec{V}_G \rightarrow \vec{V}_G = \frac{\vec{P}_{\text{sys}}}{M} = \frac{-0.25i}{(50+75) \times 10^{-3}}$$

$$\vec{V}_G = -2 \text{ m/s}$$

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Objectives



1

To apply the Newton's second law.



Newton's second law & Linear Momentum.

The Linear momentum of a system is given by:

$$\vec{P} = \mathbf{M}\vec{V}_G$$

derive w.r.t time: $\frac{d\vec{p}}{dt} = \mathbf{M}\vec{V}' \rightarrow \frac{d\vec{p}}{dt} = M\vec{a}$

According to Newton's second law: $\sum \vec{F}_{ex} = M\vec{a}$

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ex}$$



Can be applied for system or for particle.



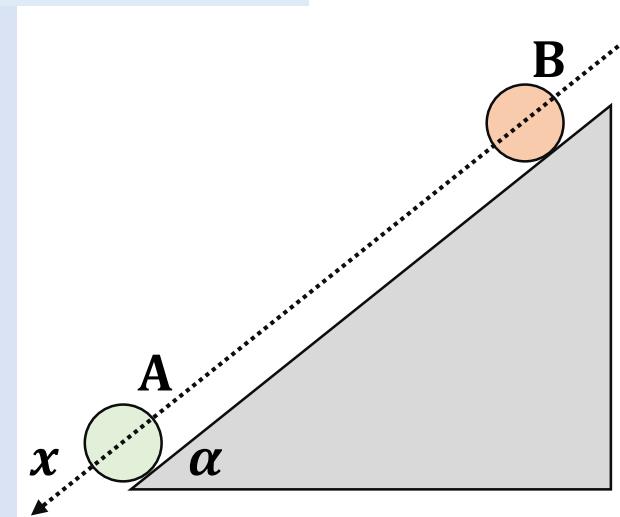
The sum of all external forces acting on a system of particles is equal to the time derivative of the linear momentum of the system.

Newton's second law & Linear Momentum.

Application: An object A of mass $m_A = 2\text{kg}$ is launched at $t_0 = 0$ with an initial velocity $V_0 = 10\text{m/s}$, from the bottom of an inclined plane making an angle $\alpha = 30^\circ$ with the horizontal. Another object B of mass $m_B = 3\text{kg}$ is released **from rest** from the top of the inclined plane at $t_0 = 0$ as shown in Doc1.

The magnitude of friction force exerted by the inclined on each object is $f = 5\text{N}$. The x-axis is parallel to the inclined and oriented positively down word. Take $g = 10\text{N/kg}$.

1. Apply newton's second law to determine the expression of the linear momentum of object B.
2. Apply newton's second law to determine the expression of the linear momentum of object A.
3. Calculate the linear momentum of the center of mass of the system [A -B].
4. Deducethe value of the velocity V_G of center of mass of the system [A -B].
5. Determine the value of the velocity V_G of the system at $t = 0.5\text{s}$.



Newton's second law & Linear Momentum.

$m_A = 2\text{kg}$; $V_0 = 10\text{m/s}$; $\alpha = 30^\circ$; $m_B = 3\text{kg}$ is released from rest; $f = 5\text{N}$

1. Apply newton's second law, determine the linear momentum of object B.

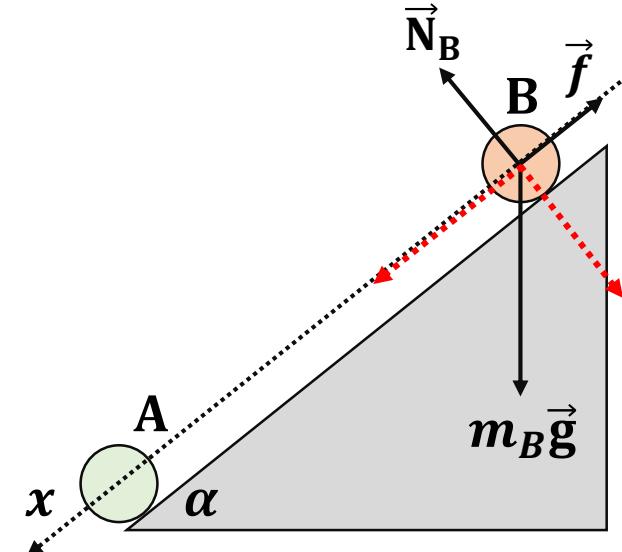
$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ex}; \text{ then } \frac{d\vec{P}}{dt} = m_B \vec{g} \sin \alpha + m_B \vec{g} \cos \alpha + \vec{N}_B + \vec{f}$$

Project along positive direction of motion:

$$\frac{dP_B}{dt} = m_B g \sin \alpha - f = 3 \times 10 \times \sin 30 - 5N$$

$$\frac{dP_B}{dt} = 10$$

$$dP_B = 10dt \rightarrow \int dP_B = \int 10dt$$



$$P_B = 10t + P_0$$

At $t_0 = 0$ B is at rest, then $V_0 = 0$; $P_0 = m \times V \rightarrow P_0 = 0$ then:

$$P_B = 10t$$

Newton's second law & Linear Momentum.

$m_A = 2\text{kg}$; $V_0 = 10\text{m/s}$; $\alpha = 30^\circ$; $m_B = 3\text{kg}$; B released from rest; $f = 5\text{N}$

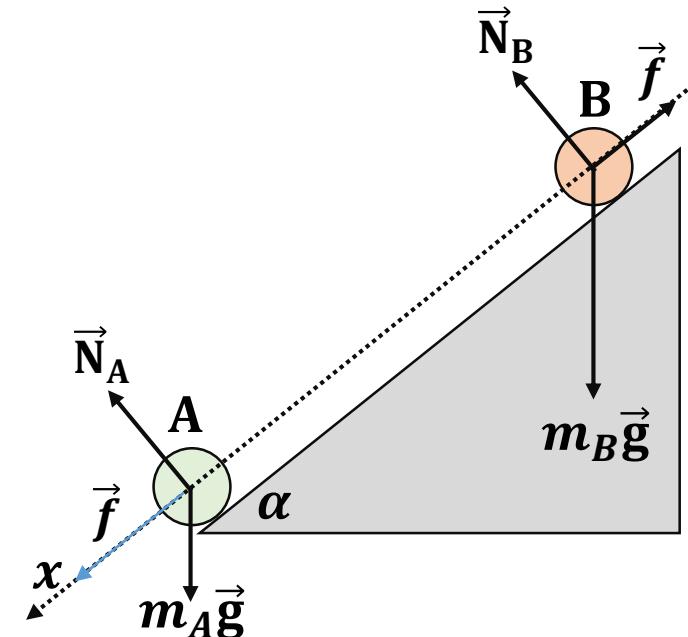
2. Apply newton's second law to determine the linear momentum of object A.

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{\text{ex}}; \text{ then } \frac{d\vec{P}}{dt} = m_A \vec{g} \sin \alpha + m_A \vec{g} \cos \alpha + \vec{N}_A + \vec{f}$$

Project along positive direction of motion: $\frac{dP_A}{dt} = m_A g \sin \alpha + f = 2 \times 10 \times \sin 30 + 5\text{N}$

$$\frac{dP_A}{dt} = 15 \rightarrow dP_A = 15dt \rightarrow \int dP_A = \int 15dt$$

$$P_A = 15t + P_0$$



At $t_0 = 0$; $V_0 = 10\text{m/s}$; $P_0 = m_A V_0 = 2 \times (-10) \rightarrow P_0 = -20\text{kg.m/s}$

$$P_A = 15t - 20$$

Newton's second law & Linear Momentum.

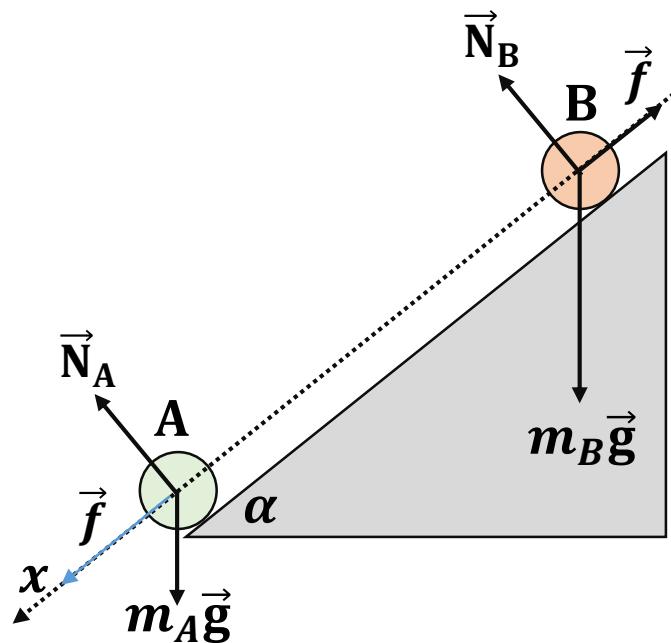
$m_A = 2\text{kg}$; $V_0 = 10\text{m/s}$; $\alpha = 30^\circ$; $m_B = 3\text{kg}$; B released from rest; $f = 5\text{N}$

3. Deduce the linear momentum of the center of mass of the system [A -B].

$$\vec{P}_G = \vec{P}_{sys} = \vec{P}_A + \vec{P}_B$$

$$\vec{P}_G = 15t - 20 + 10t$$

$$\vec{P}_G = 25t - 20$$



Newton's second law & Linear Momentum.

$m_A = 2\text{kg}$; $V_0 = 10\text{m/s}$; $\alpha = 30^\circ$; $m_B = 3\text{kg}$; B released from rest; $f = 5\text{N}$

4. Calculate the value of the velocity V_G of center of mass (G) of the system [A -B].

$$\begin{aligned}\vec{P}_G &= M\vec{V}_G \\ 25t - 20 &= (2 + 3)\vec{V}_G \\ \vec{V}_G &= \frac{25t - 20}{5} \rightarrow \vec{V}_G = 5t - 4\end{aligned}$$

5. Determine the value of the velocity V_G of the system at $t = 0.5s$.

$$\vec{V}_G = 5t - 4$$

For $t = 0.5s$; then:

$$\begin{aligned}\vec{V}_G &= 5(0.5) - 4 \\ \vec{V}_G &= -1.5\text{m/s}\end{aligned}$$



Newton's second law & Linear Momentum.

Application:

A solid of mass $m = 5\text{kg}$ move on a horizontal plane. It starts from rest at $t_0 = 0$ under the action of two forces: $\vec{F}_1 = 5\vec{i} + 15\vec{j}$ and $\vec{F}_2 = 10\vec{i} + 10\vec{j}$

1. Determine at any instant t , the linear momentum P of the solid.
2. Deduce the velocity of the center of mass of the solid in terms of time



Newton's second law & Linear Momentum.

Given: $m = 5\text{kg}$; starts from rest; $\vec{F}_1 = 5\vec{i} + 15\vec{j}$ and $\vec{F}_2 = 10\vec{i} + 10\vec{j}$

1. Determine at any instant t , the linear momentum P of the solid.

Apply newton's 2nd law:

$$\sum \vec{F}_{ex} = \frac{d\vec{P}}{dt} \rightarrow m\vec{g} + \vec{N} + \vec{F}_1 + \vec{F}_2 = \frac{d\vec{P}}{dt}$$

$$\vec{F}_1 + \vec{F}_2 = \frac{d\vec{P}}{dt}$$

$$5\vec{i} + 15\vec{j} + 10\vec{i} + 10\vec{j} = \frac{d\vec{P}}{dt} \rightarrow \mathbf{15\vec{i} + 25\vec{j}}$$

Integrate both sides:

$$\vec{P} = 15t\vec{i} + 25t\vec{j} + \vec{P}_0$$

At $t_0 = 0$; $V_0 = 0$ then: $P_0 = mV_0 = 5 \times (0) = 0$

$$\vec{P} = 15t\vec{i} + 25t\vec{j}$$



Newton's second law & Linear Momentum.

Given: $M = 5\text{kg}$; starts from rest; $\vec{F}_1 = 5\vec{i} + 15\vec{j}$ and $\vec{F}_2 = 10\vec{i} + 10\vec{j}$

2. Deduce the velocity of the center of mass of the solid in terms of time.

$$\vec{P} = M\vec{V}_G$$

$$15t\vec{i} + 25t\vec{j} = 5 \times \vec{V}_G$$

$$\vec{V}_G = \frac{15t\vec{i} + 25t\vec{j}}{5}$$

$$\vec{V}_G = 3t\vec{i} + 5t\vec{j}$$

Objectives



1

To apply conservation of Linear momentum.



Newton's second law & Linear Momentum.

The Linear momentum of a system is given by:

$$\vec{P} = \mathbf{M}\vec{V}_G$$

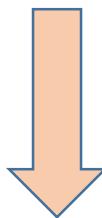
derive w.r.t time: $\frac{d\vec{p}}{dt} = \mathbf{M}\vec{V}' \rightarrow \frac{d\vec{p}}{dt} = M\vec{a}$

According to Newton's second law: $\sum \vec{F}_{ex} = M\vec{a}$

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ex}$$



Can be applied for system or for particle.



The sum of all external forces acting on a system of particles is equal to the time derivative of the linear momentum of the system.

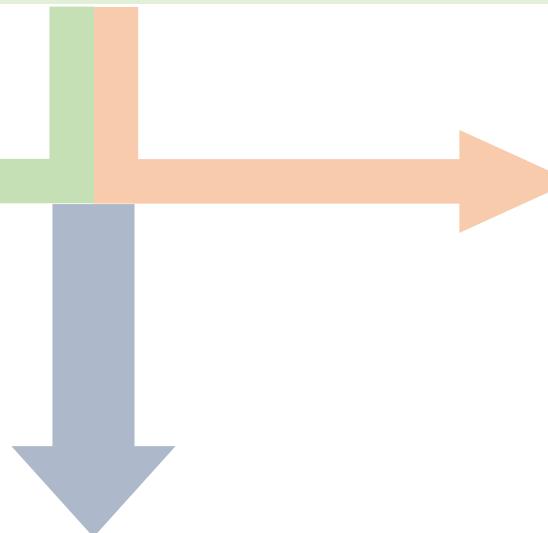
Conservation of Linear Momentum.

Consider a system subjected to external forces $\vec{F}_1, \vec{F}_2, \vec{F}_3\dots$

A system is called **mechanically isolated**, if the sum of external forces applied on the system is zero ($\sum \vec{F}_{\text{ex}} = 0$); then:

$$\frac{d\vec{P}}{dt} = 0 \rightarrow \vec{P} \text{ is constant}$$

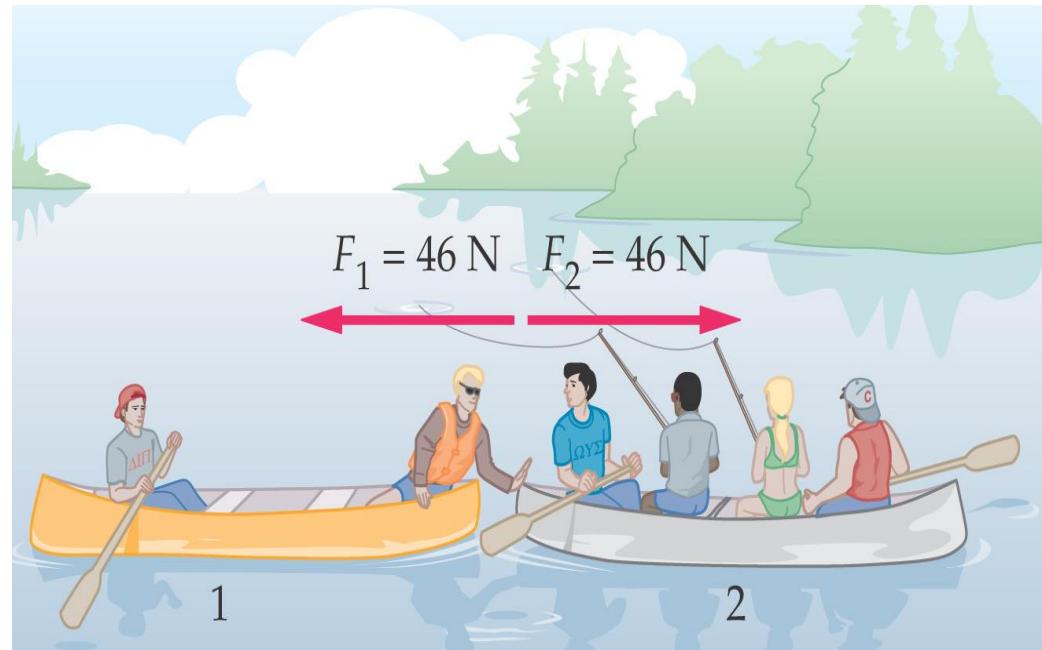
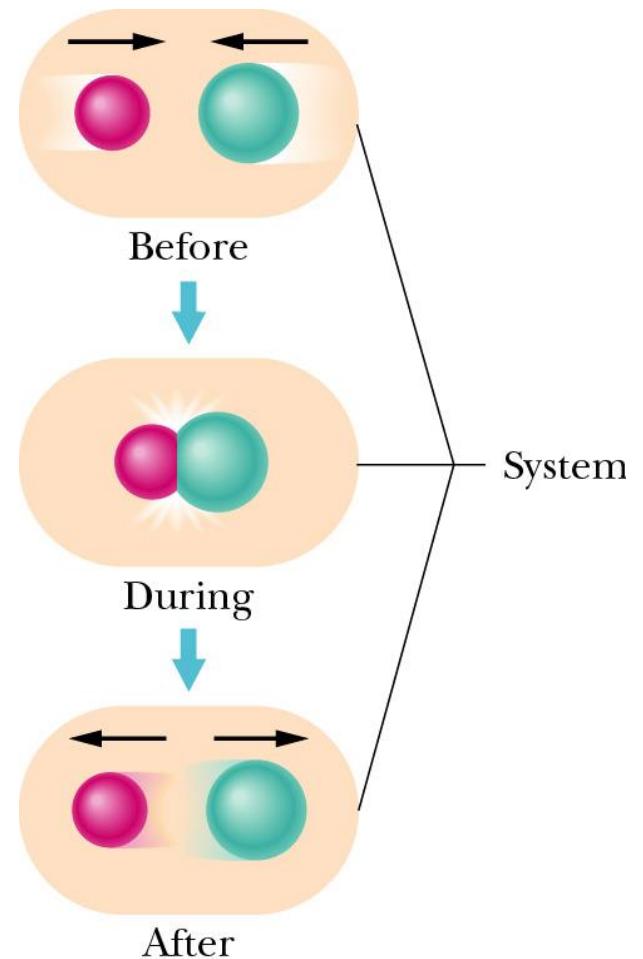
$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{\text{ex}}$$



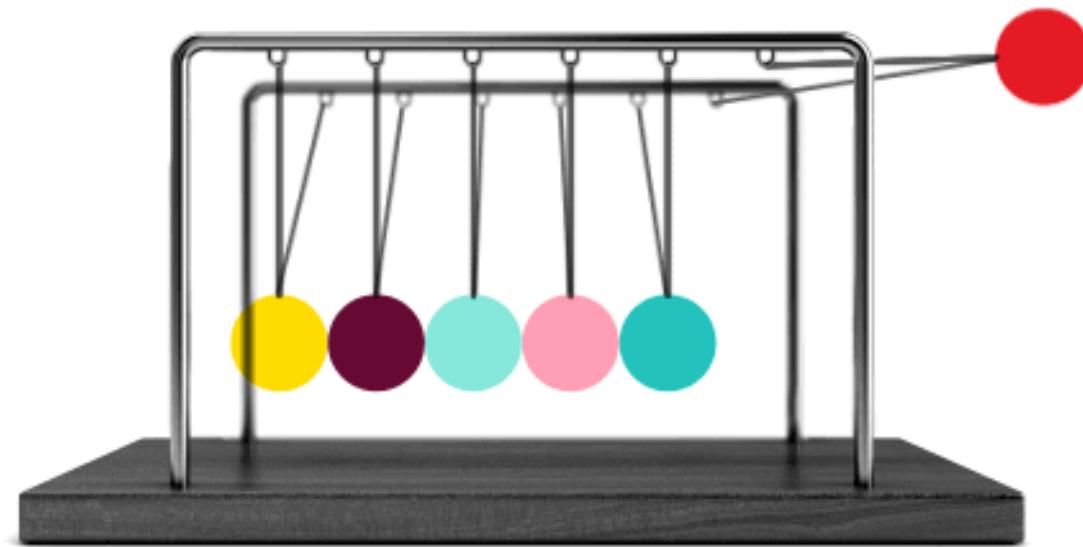
$$\vec{P}_i = \vec{P}_f$$

Linear momentum is conserved

Conservation of Linear Momentum.



Conservation of Linear Momentum.

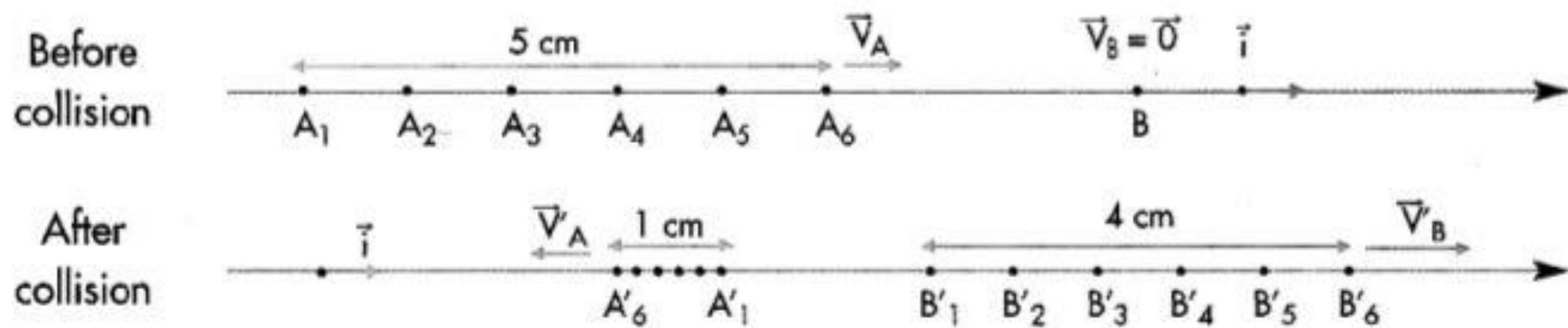


Application



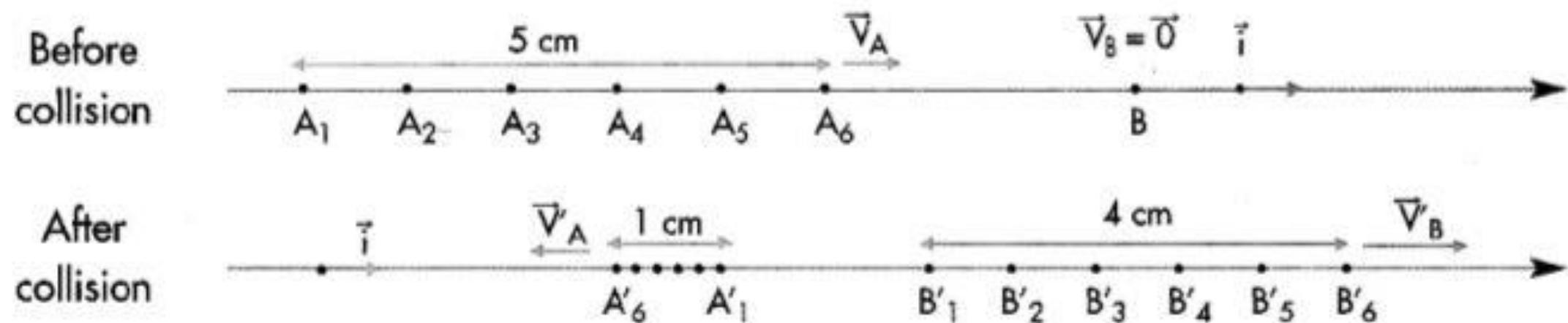
Consider an equipped horizontal air table and two pucks (A) and (B) of respective masses $m_A = 0.2\text{kg}$ and $m_B = 0.3\text{ kg}$.

(A), moves with the velocity $\vec{V}_A = V_A \vec{i}$, enters in a head-on collision with (B), **initially at rest**. (A) **rebounds** with the velocity $\vec{V}'_A = V'_A \vec{i}$ and (B) is moves with the velocity $\vec{V}'_B = V'_B \vec{i}$. The figure below shows the positions of the centers of masses of (A) and (B) obtained. The time interval separating two successive dots is $\tau = 20\text{ms}$.



Application

- Calculate the algebraic values V_A , V'_A and V'_B .
- Determine the linear momentums \vec{P}_A and \vec{P}'_A of the puck (A) before and after collision respectively and \vec{P}'_B of the puck (B) after collision.
- Deduce the linear momentums \vec{P} and \vec{P}' of the center of mass of the system [(A) and (B)] before and after collision respectively.
- Compare \vec{P} and \vec{P}' then conclude.



Application

Particle (A): $m_A = 0.2 \text{ kg}$; $\vec{V}_A = V_A \vec{i}$; $\vec{V}'_A = V'_A \vec{i}$

Particle (B): $m_B = 0.3 \text{ kg}$; $\vec{V}_B = 0$; $\vec{V}'_B = V'_B \vec{i}$

1. Calculate the algebraic values V_A , V'_A and V'_B .

$$V_A = \frac{A_1 A_6}{5\tau} = \frac{5 \times 10^{-2}}{5 \times (20 \times 10^{-3})} = 0.5 \text{ m/s}$$

$$V'_A = \frac{A'_1 A'_6}{5\tau} = \frac{1 \times 10^{-2}}{5 \times (20 \times 10^{-3})} = 0.1 \text{ m/s}$$

$$V'_B = \frac{B'_1 B'_6}{5\tau} = \frac{4 \times 10^{-2}}{5 \times (20 \times 10^{-3})} = 0.4 \text{ m/s}$$

Application



Particle (A): $m_A = 0.2 \text{ kg}$; $\vec{V}_A = 0.5\vec{i}$; $\vec{V}'_A = -0.1\vec{i}$

Particle (B): $m_B = 0.3 \text{ kg}$; $\vec{V}_B = 0$; $\vec{V}'_B = 0.4\vec{i}$

2. Determine the linear momentums \vec{P}_A and \vec{P}'_A of the puck (A) before and after collision respectively and \vec{P}'_B of the puck (B) after collision.

- $\vec{P}_A = m_A \cdot \vec{V}_A = 0.2 \times (0.5\vec{i}) \rightarrow \vec{P}_A = 0.1\vec{i} \text{ kg.m.s}$
- $\vec{P}'_A = m_A \cdot \vec{V}'_A = 0.2 \times (-0.1\vec{i}) \rightarrow \vec{P}'_A = -0.02\vec{i} \text{ kg.m/s}$
- $\vec{P}'_B = m_B \cdot \vec{V}'_B = 0.3 \times (0.4\vec{i}) \rightarrow \vec{P}'_B = 0.12\vec{i} \text{ kg.m/s}$

Application

Particle (A): $m_A = 0.2 \text{ kg}$; $\vec{V}_A = 0.5\vec{i}$; $\vec{V}'_A = -0.1\vec{i}$

Particle (B): $m_B = 0.3 \text{ kg}$; $\vec{V}_B = 0$; $\vec{V}'_B = 0.4\vec{i}$

3. Deduce the linear momentums \vec{P} and \vec{P}' of the center of mass of the system [(A) and (B)] before and after collision respectively.

- $\vec{P} = \vec{P}_A + \vec{P}_B = 0.1\vec{i} + 0 \rightarrow \vec{P} = 0.1\vec{i} \text{ kg.m.s}$
- $\vec{P}' = \vec{P}'_A + \vec{P}'_B = -0.02\vec{i} + 0.12\vec{i} \rightarrow \vec{P}' = 0.1\vec{i} \text{ kg.m/s}$

4. Compare \vec{P} and \vec{P}' then conclude.

$\vec{P} = \vec{P}' = 0.1\vec{i} \text{ kg.m/s}$; then the linear momentum of the system is conserved.



A Very Special
"Thank You!"



Physics/ Grade 12 LS

Unit one – Mechanics

Chapter 2 – Linear momentum

Teacher Mohamad Seif

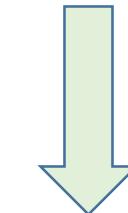
2020 – 2021

Revision about previous period.

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ex}$$



$$\sum \vec{F}_{ex} = 0$$



$$\frac{d\vec{P}}{dt} = 0$$

The sum of all external forces acting on a system of particles is equal to the time derivative of the linear momentum of the system.

- **The system is mechanically isolated.**
- Linear momentum is conserved.

$$\vec{P}_i = \vec{P}_f$$

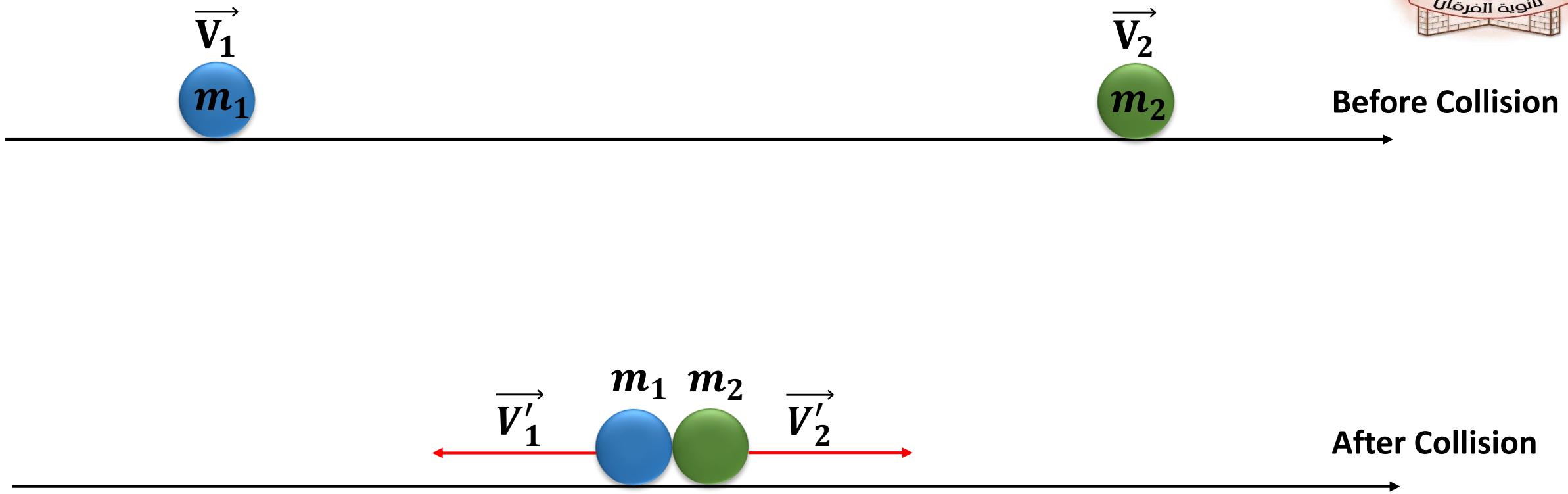
Application

Consider two objects A of mass $m_1 = 500\text{g}$ and B of mass $m_2 = 750\text{g}$. A moves horizontally with a velocity magnitude $V_1 = 4\text{m/s}$, while B moves with a velocity of magnitude $V_2 = 2\text{m/s}$. At a certain time a collision takes place between the two objects as shown in the figure.

After collision A returns with a velocity $V_1^1 = 5\text{m/s}$ and B moves with a velocity $V_2^1 = 4\text{m/s}$.

1. Determine the forces acting on each object.
2. Determine the sum of external forces acting on the system [A, B]. What can you deduce.
3. Calculate the linear momentum \vec{P}_A of object A and \vec{P}_B of object B before collision.
4. Calculate the linear momentum \vec{P}'_A of object A and \vec{P}'_B of object B after collision.
5. Compare \vec{P}_A of object A and \vec{P}_B . Prove that it is confirm with part 2.

Application



Application

Before collision: A: $m_1 = 500\text{g}$; $V_1 = 4\text{m/s}$; B: $m_2 = 750\text{g}$; $V_2 = 2\text{m/s}$.

After collision: A: $V_1^1 = 5\text{m/s}$ and B : $V_2^1 = 4\text{m/s}$.

1. Determine the forces acting on each object, then the sum of external forces of the system.

- For object A: weight ($\vec{W}_1 = m_1 \vec{g}$); normal (\vec{N}_1): $\sum \vec{F}_{ex} = \vec{W}_1 + \vec{N}_1 = 0$
- For object B: weight ($\vec{W}_2 = m_2 \vec{g}$); normal (\vec{N}_2): $\sum \vec{F}_{ex} = \vec{W}_2 + \vec{N}_2 = 0$



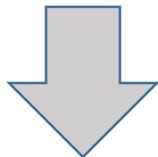
Application

Before collision: A: $m_1 = 500\text{g}$; $V_1 = 4\text{m/s}$; B: $m_2 = 750\text{g}$; $V_2 = 2\text{m/s}$.

After collision: A: $V_1^1 = 5\text{m/s}$ and B : $V_2^1 = 4\text{m/s}$.

2. Determine the sum of external forces acting on the system [A, B]. What can you deduce.

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ex} = 0$$



The linear momentum is conserved

Application

Before collision: A: $m_1 = 500\text{g}$; $V_1 = 4\text{m/s}$; B: $m_2 = 750\text{g}$; $V_2 = 2\text{m/s}$.

After collision: A: $V_1^1 = 5\text{m/s}$ and B : $V_2^1 = 4\text{m/s}$.

3. Calculate the linear momentum \vec{P}_A of object A and \vec{P}_B of object B before collision.

- For object A before collision: $\vec{P}_A = m_1 \times \vec{V}_1 = 0.5 \times (+4)$

$$\vec{P}_A = 2\text{kg.m/s}.$$

- For object B before collision: $\vec{P}_B = m_2 \times \vec{V}_2 = 0.75 \times (-2)$

$$\vec{P}_B = -1.5\text{kg.m/s}.$$



Application

Before collision: A: $m_1 = 500\text{g}$; $V_1 = 4\text{m/s}$; B: $m_2 = 750\text{g}$; $V_2 = 2\text{m/s}$.

After collision: A: $V_1' = 5\text{m/s}$ and B : $V_2' = 4\text{m/s}$.

4. Calculate the linear momentum \vec{P}'_A of object A and \vec{P}'_B of object B after collision

- For object A after collision: $\vec{P}'_A = m_1 \times \vec{V}'_1 = 0.5 \times (-5)$

$$\vec{P}'_A = -2.5 \text{kg.m/s.}$$

- For object B after collision: $\vec{P}'_B = m_2 \times \vec{V}'_2 = 0.75 \times (4)$

$$\vec{P}'_B = 3 \text{kg.m/s.}$$



Application

Before collision: A: $m_1 = 500\text{g}$; $V_1 = 4\text{m/s}$; B: $m_2 = 750\text{g}$; $V_2 = 2\text{m/s}$.

After collision: A: $V_1^1 = 5\text{m/s}$ and B : $V_2^1 = 4\text{m/s}$.

5. Deduce the linear momentums \vec{P} and \vec{P}' of the system [(A) and (B)] before and after collision respectively.

- For the system before collision: $\vec{P}_1 = \vec{P}_A + \vec{P}_B = 2 - 1.5$

$$\vec{P}_1 = 0.5\text{kg. m/s.}$$

- For the system after collision: $\vec{P}_2 = \vec{P}'_A + \vec{P}'_B = -2.5 + 3$

$$\vec{P}_2 = 0.5\text{kg. m/s}$$

Application

4. Compare \vec{P} and \vec{P}' then conclude.



$$\vec{P} = \vec{P}' = 0.5 \text{ kg} \cdot \text{m/s}$$
A large, light-grey cloud-shaped callout box is centered on the page. Inside the box, the equation $\vec{P} = \vec{P}' = 0.5 \text{ kg} \cdot \text{m/s}$ is displayed in a large, black, serif font. A single blue downward-pointing arrow is positioned below the cloud, pointing towards the text "The linear momentum is conserved".

The linear momentum is conserved



A Very Special
"Thank You!"

Objectives

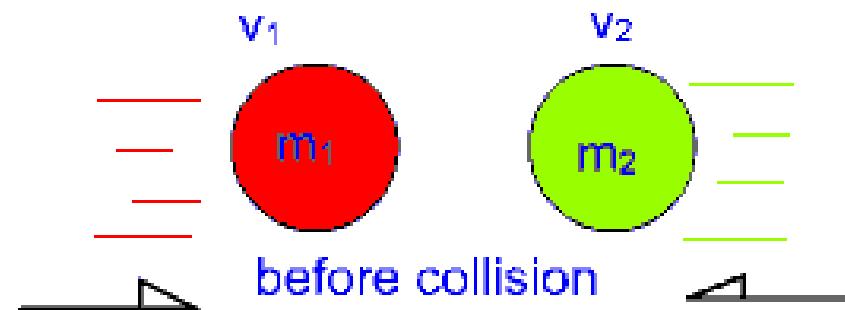


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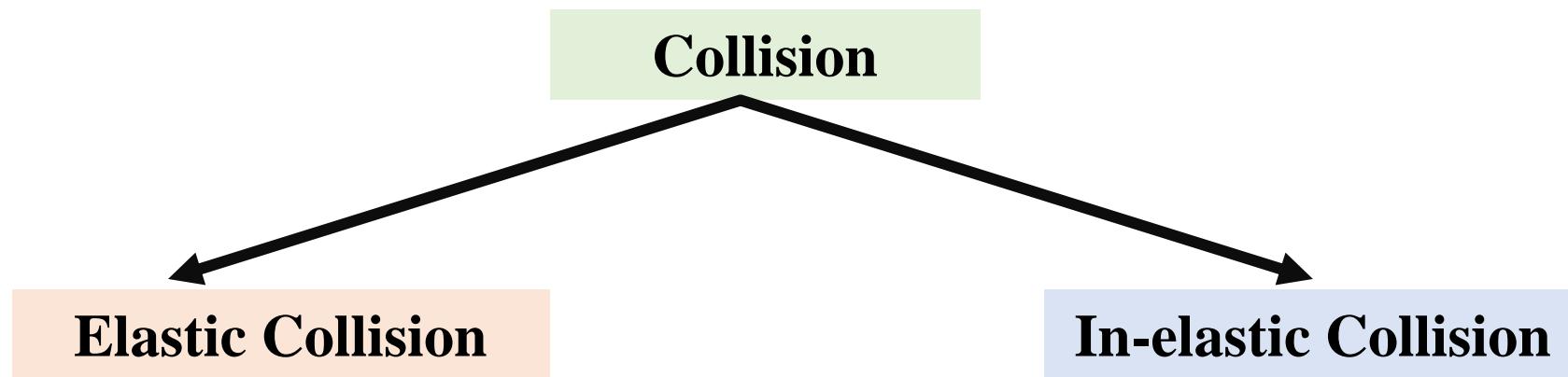
Elastic Collision.

Types of Collision between two particles

- **Collision:** are observed between billiards balls or between two cars...
- Usually collision last for a very short time, so external forces are neglected with respect to internal forces.



Types of Collision between two particles



- Linear momentum is conserved:

$$\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$$

- Kinetic energy is conserved

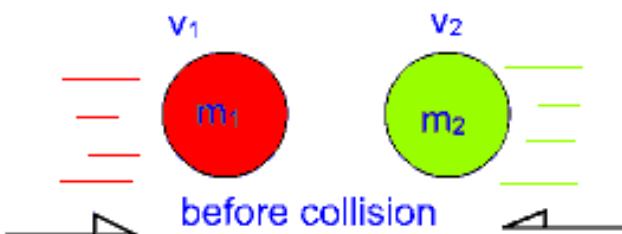
$$KE_{\text{before}} = KE_{\text{after}}$$

- Linear momentum is conserved

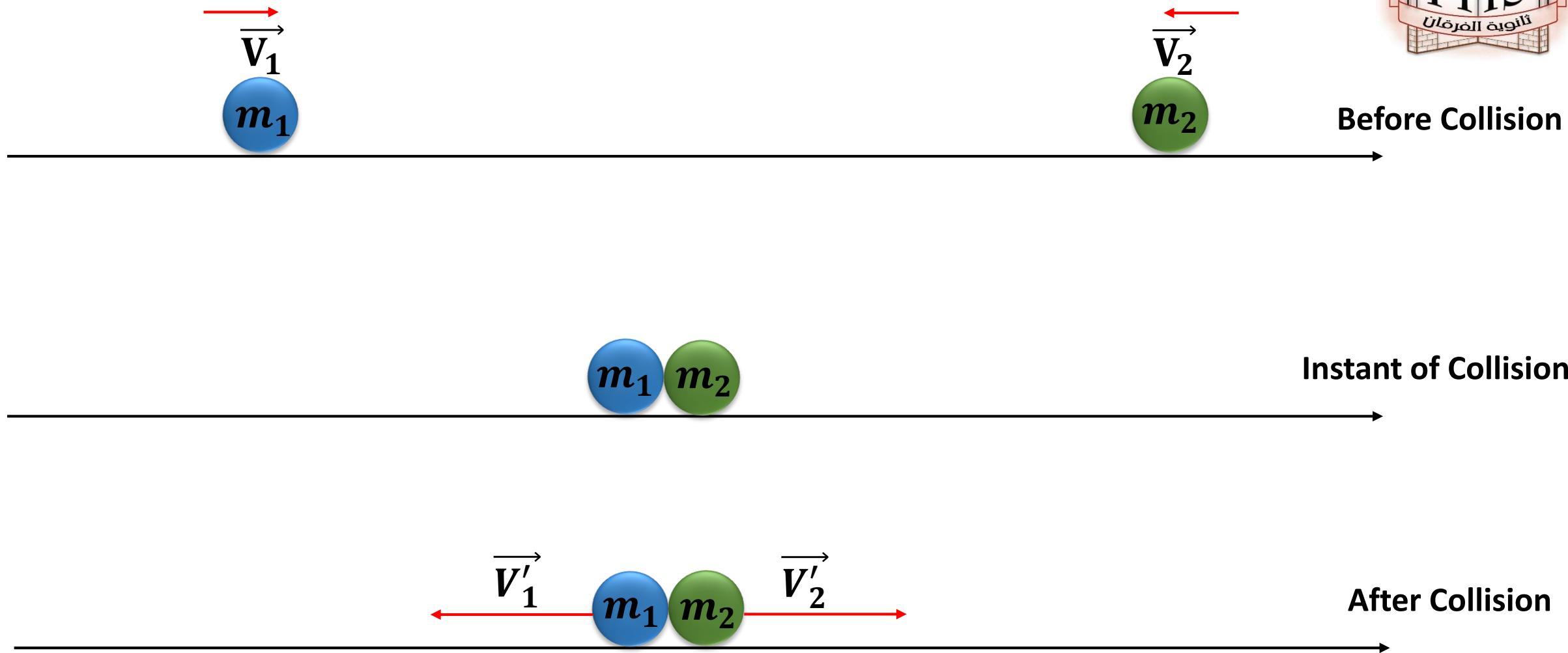
$$\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$$

- Kinetic energy is not conserved

$$KE_{\text{before}} \neq KE_{\text{after}}$$



Elastic Collision of two particles



Elastic Collision of two particles

The linear momentum of the system is conserved:

$$\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$$

$$m_1 \vec{V}_1 + m_2 \vec{V}_2 = m_1 \vec{V}'_1 + m_2 \vec{V}'_2$$

The velocities are collinear then:

$$m_1 V_1 + m_2 V_2 = m_1 V'_1 + m_2 V'_2$$

$m_1(V_1 - V'_1) = m_2(V'_2 - V_2) \dots \dots \dots (1)$

The total kinetic energy of the system is conserved:

$$K.E_{\text{before}} = K.E_{\text{after}}$$

$$\frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m_1 {V'_1}^2 + \frac{1}{2} m_2 {V'_2}^2$$

$$m_1 (V_1^2 - {V'_1}^2) = m_2 ({V'_2}^2 - V_2^2)$$



Elastic Collision of two particles

$$m_1 (v_1^2 - v_1'^2) = m_2 (v_2'^2 - v_2^2)$$

$$m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2' + v_2) \dots \dots \dots (2)$$

Divide equation (2) by equation (1):

$$\frac{m_1(v_1 - v_1')(v_1 + v_1')}{m_1(v_1 - v_1')} = \frac{m_2(v_2' - v_2)(v_2' + v_2)}{m_2(v_2' - v_2)}$$

$$(v_1 + v_1') = (v_2' + v_2) \dots \dots \dots (3)$$



Elastic Collision of two particles

Solve the system of equation (1) and (3):

$$\begin{cases} m_1(V_1 - V'_1) = m_2(V'_2 - V_2) \dots \dots \dots (1) \\ (V_1 + V'_1) = (V'_2 + V_2) \dots \times (m_1) \dots \dots (3) \end{cases}$$

$$\begin{cases} m_1V_1 - m_1V'_1 = m_2V'_2 - m_2V_2 \\ m_1V_1 + m_1V'_1 = m_1V'_2 + m_1V_2 \end{cases}$$

Add the two equations:

$$m_1V_1 + m_1V_1 = m_2V'_2 + m_1V'_2 - m_2V_2 + m_1V_2$$

$$2m_1V_1 = V'_2(m_1 + m_2) + V_2(m_1 - m_2)$$

$$2m_1V_1 - V_2(m_1 - m_2) = V'_2(m_1 + m_2)$$

Elastic Collision of two particles

$$2m_1V_1 - V_2(m_1 - m_2) = V'_2(m_1 + m_2)$$

Then:

$$V'_2 = \frac{2m_1V_1 + V_2(m_2 - m_1)}{(m_1 + m_2)}$$

$$V'_2 = \left[\frac{2m_1}{(m_1 + m_2)} \right] \cdot V_1 + \left[\frac{m_2 - m_1}{m_1 + m_2} \right] \cdot V_2$$

Substitute V'_2 in equation:

$$V'_1 = \left[\frac{2m_2}{(m_1 + m_2)} \right] \cdot V_2 + \left[\frac{m_1 - m_2}{m_1 + m_2} \right] \cdot V_1$$



A Very Special
"Thank You!"

Objectives



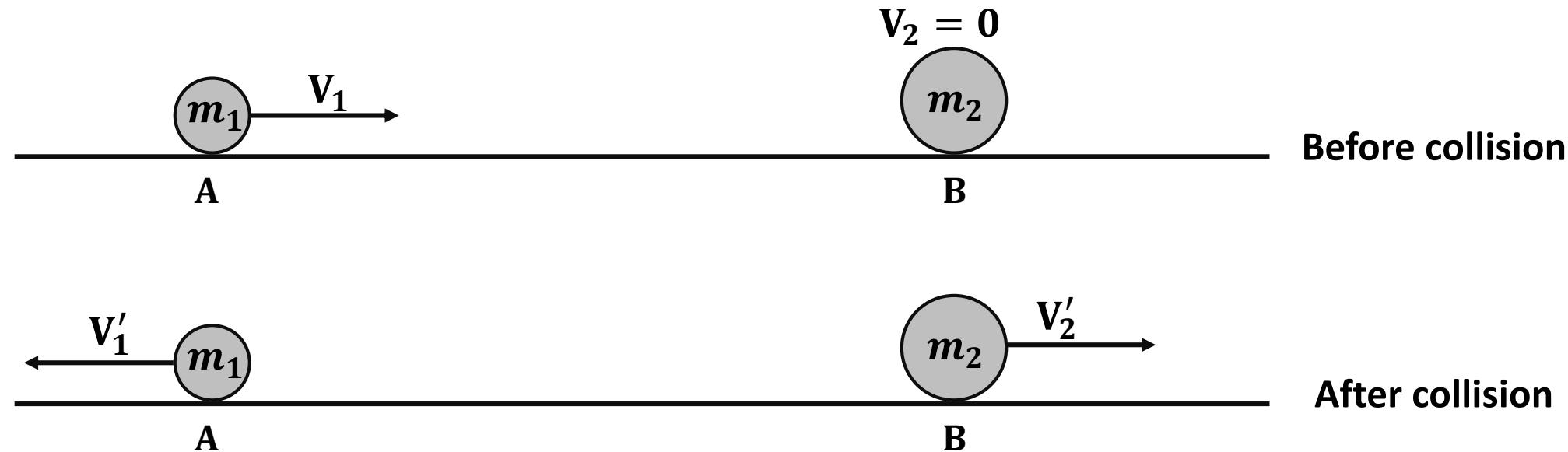
1

Application on Elastic Collision.

Elastic Collision: Application

Consider a body (A), of mass $m_1 = 400 \text{ g}$, moves on a horizontal plane with a speed $V_1 = 3.464 \text{ m/s}$ enters in a head on **perfectly elastic collision** with a body (B) of mass $m_2 = 800 \text{ g}$ **initially at rest**. After collision (A) rebounds with a speed V'_1 and (B) moves forward with moves with a speed V'_2 .

Calculate the speeds V'_1 of (A) and V'_2 of (B) after collision.



Elastic Collision: Application



Before collision: (A): $m_1 = 0.4 \text{ g}$; $V_1 = 3.464 \text{ m/s}$; (B): $m_2 = 0.8 \text{ g}$; $V_2 = 0$

After collision: (A) rebounds with V'_1 ; (B) moves forward with V'_2 .

Solution:

Elastic collision then:

Conservation of linear momentum of the system [(A), (B)]:

$$\vec{P}_{\text{before}} = \vec{P}'_{\text{after}}$$

$$m_1 \vec{V}_1 = m_1 \vec{V}'_1 + m_2 \vec{V}'_2$$

The velocities are collinear then:

$$m_1 V_1 = m_1 V'_1 + m_2 V'_2$$

$$m_1 V_1 - m_1 V'_1 = m_2 V'_2$$

$$m_1(v_1 - v'_1) = m_2 v'_2 \quad \dots \dots \dots \quad (1)$$

Elastic Collision: Application



Before collision: (A): $m_1 = 0.4 \text{ g}$; $V_1 = 3.464 \text{ m/s}$; (B): $m_2 = 0.8 \text{ g}$; $V_2 = 0$

After collision: (A) rebounds with V'_1 ; (B) moves forward with V'_2 .

Conservation of kinetic energy of the system [(A), (B)]:

$$KE_{before} = KE_{after}$$

$$\frac{1}{2}m_1V_1^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2$$

$$m_1 V_1^2 = m_1 {V'_1}^2 + m_2 {V'_2}^2$$

$$m_1 V_1^2 - m_1 {V'_1}^2 = m_2 {V'_2}^2$$

$$m_1(V_1^2 - V_1'^2) = m_2 V_2'^2$$

Elastic Collision: Application



Before collision: (A): $m_1 = 0.4 \text{ g}$; $V_1 = 3.464 \text{ m/s}$; (B): $m_2 = 0.8 \text{ g}$; $V_2 = 0$

After collision: (A) rebounds with V'_1 ; (B) moves forward with V'_2 .

$$m_1(V_1 - V'_1) = m_2 V'_2 \quad \dots \dots \dots \dots \dots \dots \dots \quad (1)$$

Divide equation (1) by equation (2):

$$\frac{m_1(V_1 - V'_1)(V_1 + V'_1)}{m_1(V_1 - V'_1)} = \frac{m_2 {V'_2}^2}{m_2 V'_2}$$

$$V_1 + V'_1 = V'_2 \dots \dots \dots (3)$$

Elastic Collision: Application



Substitute: $V_2' = V_1 + V_1'$ in equation (1)

$$m_1(v_1 - v'_1) = m_2(v_1 + v'_1)$$

$$m_1 V_1 - m_1 V'_1 = m_2 V_1 + m_2 V'_1$$

$$m_1 V_1 - m_2 V_1 = m_2 V'_1 + m_1 V'_1$$

$$V_1(m_1 - m_2) = V'_1(m_2 + m_1)$$

$$V'_1 = \frac{(m_1 - m_2)V_1}{(m_1 + m_2)} = \frac{(0.4 - 0.8)3.464}{(0.4 + 0.8)}$$

$$V_1' = -1.15 \text{ m/s}$$

Elastic Collision: Application



Now using equation (3)

$$\mathbf{V}'_2 = \mathbf{V}_1 + \mathbf{V}'_1$$

$$\mathbf{V}'_2 = 3.464 - 1.15$$

$$\mathbf{V}'_2 = 2.31 \text{ m/s}$$



A Very Special
"Thank You!"

Objectives



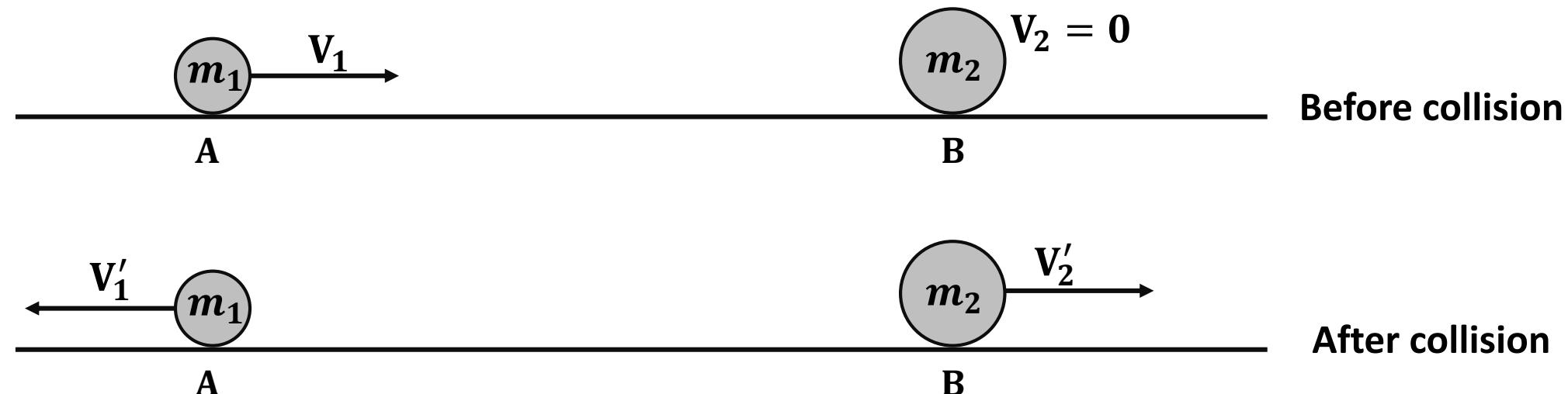
1

Application on Elastic Collision.

Elastic Collision: Application

Consider a body (A), of mass $m_1 = 400 \text{ g}$, moves with a speed $V_1 = 3.464 \text{ m/s}$ enters in a head on collision with a body (B) of mass $m_2 = 800 \text{ g}$ **initially at rest**. After collision (B) moves with a speed $V'_2 = 2.31 \text{ m/s}$ while (A) rebounds with V'_1

1. Which variable is conserved during collision.
2. Apply the conservation of the preceding variable, determine the speed V'_1 of body (A) after collision.
3. Calculate the kinetic energy of the system [(A), (B)] before and after collision.
4. What is the nature of collision.



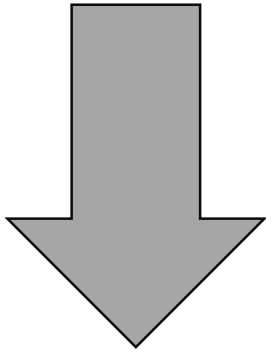


Elastic Collision: Application

$m_A = 0.4 \text{ kg}$; $V_1 = 3.464m/s$; $m_B = 0.8\text{kg}$; $V_2 = 0$ (initially at rest).

After collision: (B): $V'_2 = 2.31\text{m/s}$; (A) rebounds with V'_1

1. Which variable is conserved during collision.



The variable which is conserved during collision
is **linear momentum** of the system [(A), (B)].



Elastic Collision: Application

$m_A = 0.4 \text{ kg}$; $V_1 = 3.464 m/s$; $m_B = 0.8 \text{ kg}$; $V_2 = 0$ (initially at rest).

After collision: (B): $V'_2 = 2.31 \text{ m/s}$; (A) rebounds with V'_1

2. Apply the conservation of the preceding variable, determine the speed V'_1 of (A) after collision.

Using law of conservation of linear momentum of the system [(A), (B)]:

$$\vec{P}_{\text{sys}} = \vec{P}'_{\text{sys}}$$

$$m_1 \vec{V}_1 = m_1 \vec{V}'_1 + m_2 \vec{V}'_2$$

$$0.4 \times (+3.464) = 0.4 \times \vec{V}'_1 + 0.8 \times (+2.31)$$

$$1.385 = 0.4 \times \vec{V}'_1 + 1.848$$

$$1.385 - 1.848 = 0.4 \times \vec{V}'_1$$

$$-0.463 = 0.4 \times \vec{V}'_1$$

$$\vec{V}'_1 = \frac{-0.463}{0.4}$$

$$\vec{V}'_1 = -1.15 \text{ m/s}$$



Elastic Collision: Application

$m_A = 0.4 \text{ kg}$; $V_1 = 3.464 m/s$; $m_B = 0.8 \text{ kg}$; $V_2 = 0$ (initially at rest).

After collision: (B): $V'_2 = 2.31 \text{ m/s}$; (A) rebounds with V'_1

3. Calculate the kinetic energy of the system [(A), (B)] before and after collision.

Before collision:

$$KE_{\text{sys}} = KE_A + KE_B$$

$$KE_{\text{sys}} = \frac{1}{2} m_1 V_1^2 + 0$$

$$KE_{\text{sys}} = 0.5 \times 0.4 \times (3.464)^2$$

$$\text{KE}_{\text{sys}} = 2.39 \text{ J}$$

After collision:

$$KE_{\text{sys}} = KE_A + KE_B$$

$$KE_{\text{sys}} = \frac{1}{2} m_1 V'_1{}^2 + \frac{1}{2} m_2 V'_2{}^2$$

$$KE_{\text{sys}} = 0.5 \times 0.4 \times (-1.15)^2 + 0.5 \times 0.8 \times (2.31)^2$$

$$KE_{\text{sys}} = 0.26 + 2.13$$

$$\text{KE}_{\text{sys}} = 2.39 \text{ J}$$



Elastic Collision: Application

$m_A = 0.4 \text{ kg}$; $V_1 = 3.464 m/s$; $m_B = 0.8 \text{ kg}$; $V_2 = 0$ (initially at rest).

After collision: (B): $V'_2 = 2.31 \text{ m/s}$; (A) rebounds with V'_1

4. What is the nature of collision.

$$\text{KE}_{(\text{sys})\text{before}} = \text{KE}_{(\text{sys})\text{after}} = 2.39 \text{ J}$$

Then the collision is elastic

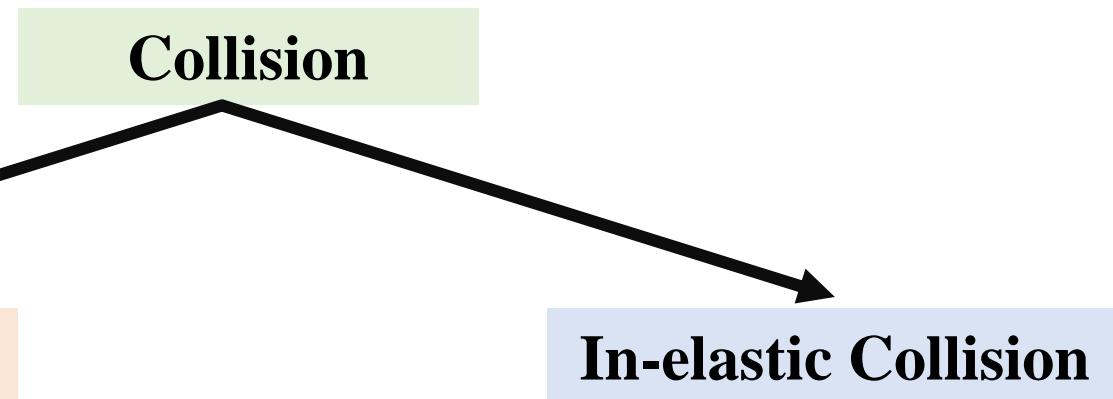


Objectives



- 1 To verify the In – Elastic Collision.

Types of Collision between two particles



- Linear momentum is conserved:

$$\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$$

- Kinetic energy is conserved

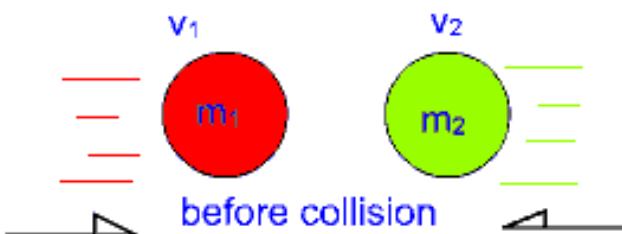
$$KE_{\text{before}} = KE_{\text{after}}$$

- Linear momentum is conserved

$$\vec{P}_{\text{before}} = \vec{P}_{\text{after}}$$

- Kinetic energy is not conserved

$$KE_{\text{before}} \neq KE_{\text{after}}$$

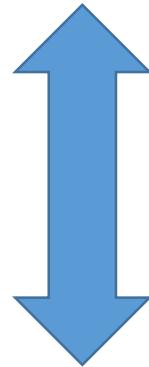


In – Elastic Collision

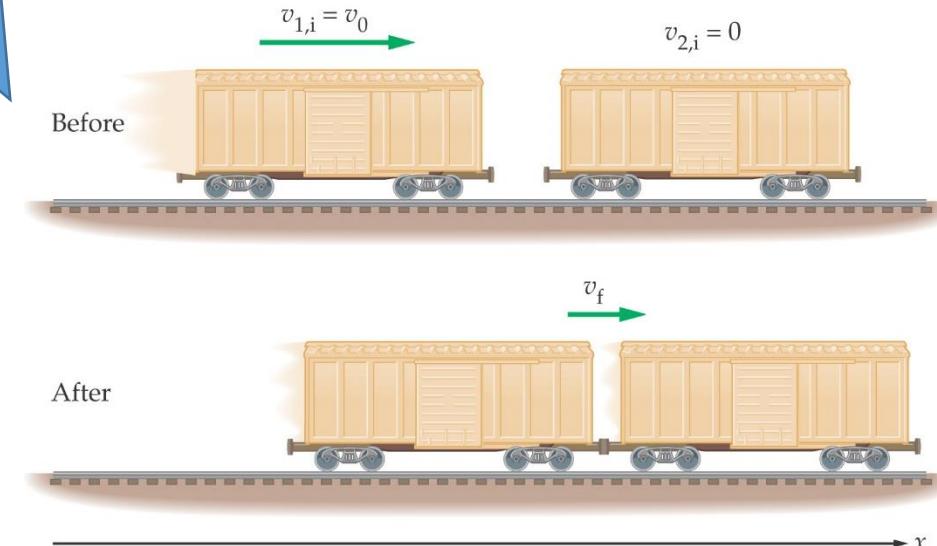


In – Elastic Collision

Normal In-elastic collision



Completely inelastic collision:
objects stick together afterwards



For In-Elastic collision

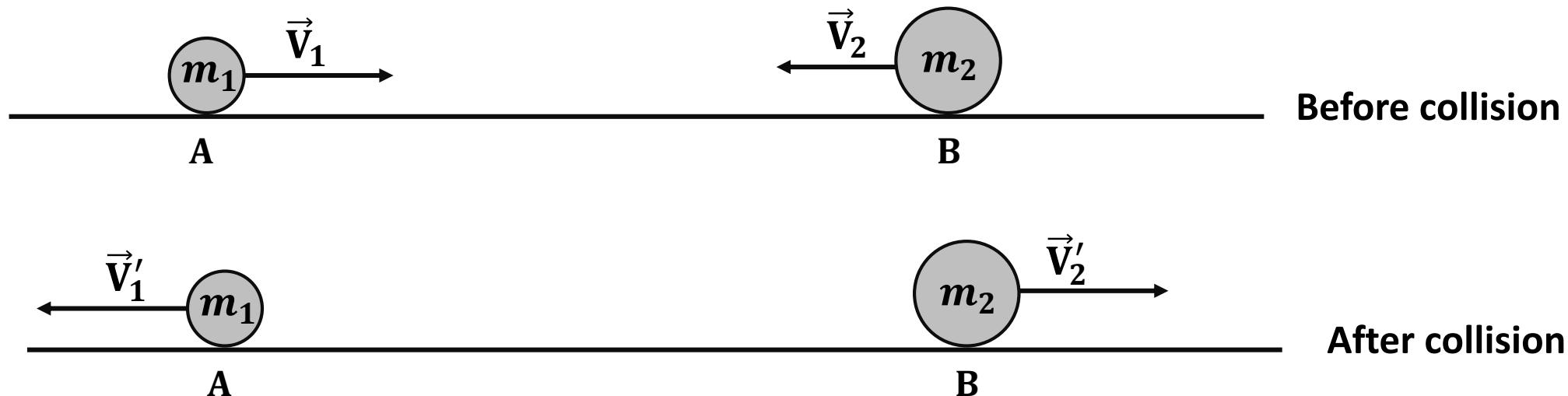
$$\vec{P}_{\text{sys(before)}} = \vec{P}_{\text{sys(after)}}$$

$$KE_{\text{sys(before)}} \neq KE_{\text{sys(after)}}$$

In – Elastic Collision: Application

Consider a solid (A), of mass $m_1 = 0.5\text{kg}$, moves with a speed $V_1 = 1.5\text{m/s}$ enters in a head on collision with a solid (B) of mass $m_2 = 1\text{kg}$ moves with a speed $V_2 = 0.9\text{m/s}$. After collision (A) moves back with a speed $V'_1 = 3\text{m/s}$ and (B) Moves with a speed V'_2 .

1. Which variable is conserved during collision.
2. Apply the conservation of the preceding variable, determine the speed V'_2 of body (B) after collision.
3. Calculate the kinetic energy of the system [(A), (B)] before and after collision.
4. What is the nature of collision.



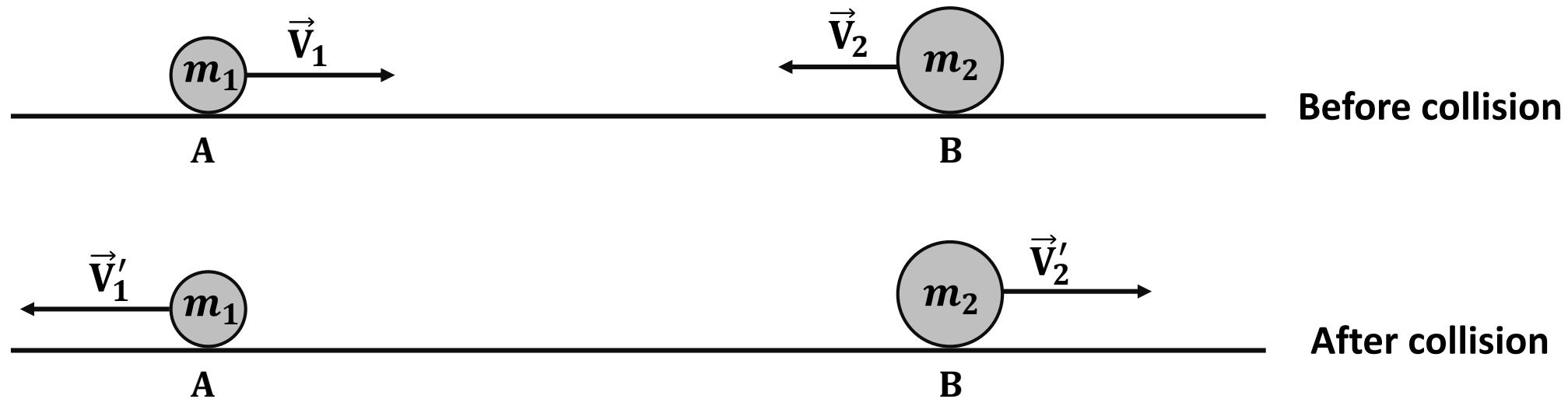
In – Elastic Collision: Application

before collision: (A): $m_1 = 0.5\text{kg}$; $V_1 = 1.5\text{m/s}$; (B): $m_2 = 1\text{kg}$; $V_2 = 0.9\text{m/s}$.

After collision: (A) moves back $V'_1 = 3\text{m/s}$ and (B) Moves: V'_2 .

1. Which variable is conserved during collision.

During collision linear momentum is conserved.





In – Elastic Collision: Application

before collision: (A): $m_1 = 0.5\text{kg}$; $V_1 = 1.5\text{m/s}$; (B): $m_2 = 1\text{kg}$; $V_2 = 0.9\text{m/s}$.

After collision: (A) moves back $V'_1 = 3\text{m/s}$ and (B) Moves: V'_2 .

1. Apply the conservation of the preceding variable, determine the speed V'_2 of body (B) after collision.

$$\vec{P}_{\text{sys(before)}} = \vec{P}_{\text{sys(after)}}$$

$$\vec{P}_1 + \vec{P}_2 = \vec{P}'_1 + \vec{P}'_2$$

$$m_1 \vec{V}_1 + m_2 \vec{V}_2 = m_1 \vec{V}'_1 + m_2 \vec{V}'_2$$

$$0.5 \times (+1.5) + 1 \times (-0.9) = 0.5 \times (-3) + 1 \times \vec{V}'_2$$

$$0.75 - 0.9 = -1.5 + \vec{V}'_2$$

$$-0.15 + 1.5 = \vec{V}'_2$$

$$\vec{V}'_2 = 1.35\text{m/s}$$



In – Elastic Collision: Application

before collision: (A): $m_1 = 0.5\text{kg}$; $V_1 = 1.5\text{m/s}$; (B): $m_2 = 1\text{kg}$; $V_2 = 0.9\text{m/s}$.

After collision: (A) moves back $V'_1 = 3\text{m/s}$ and (B) Moves: $V'_2 = 1.35\text{m/s}$

3. Calculate the kinetic energy of the system [(A), (B)] before and after collision.

$$\text{KE(sys)}_{\text{before}} = \text{KE}_A + \text{KE}_B$$

$$\text{KE(sys)}_{\text{before}} = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$$

$$\text{KE(sys)}_{\text{before}} = 0.5 \times 0.5 \times (1.5)^2 + 0.5 \times 1 \times (0.9)^2$$

$$\text{KE(sys)}_{\text{before}} = 0.56 + 0.405$$

$$\text{KE(sys)}_{\text{before}} = 0.965\text{J}$$

$$\text{KE(sys)}_{\text{after}} = \text{KE}_A + \text{KE}_B$$

$$\text{KE(sys)}_{\text{after}} = \frac{1}{2}m_1V'_1{}^2 + \frac{1}{2}m_2V'_2{}^2$$

$$\text{KE(sys)}_{\text{after}} = 0.5 \times 0.5 \times (3)^2 + 0.5 \times 1 \times (1.35)^2$$

$$\text{KE(sys)}_{\text{after}} = 2.25 + 0.911$$

$$\text{KE(sys)}_{\text{after}} = 3.16\text{J}$$



In – Elastic Collision: Application

before collision: (A): $m_1 = 0.5\text{kg}$; $V_1 = 1.5\text{m/s}$; (B): $m_2 = 1\text{kg}$; $V_2 = 0.9\text{m/s}$.

After collision: (A) moves back $V'_1 = 3\text{m/s}$ and (B) Moves: $V'_2 = 1.35\text{m/s}$

4. What is the nature of collision.

Since

$$\text{KE(sys)}_{\text{before}} \neq \text{KE(sys)}_{\text{after}}$$

Then the collision is not Elastic



A Very Special
"Thank You!"