Grade 12 GS

Probability ex 17

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By Mr.Kassem Hodeib

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Exercise 17:

An urn contains 6 identical balls of which 4 are red and 2 are black.

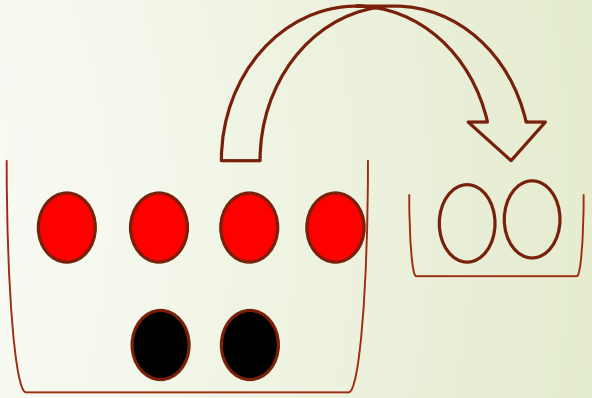
1) We randomly draw two balls from the urn. Consider the three events:

A_0 : the two drawn balls are red

A_1 : the two drawn balls have different colors.

A_2 : the two drawn balls are black.

Calculate the probability of each of A_0, A_1 and A_2.



- $P(A_0) = P(2r) = \frac{C_4^2}{C_6^2} = \frac{2}{5}$
- $P(A_1) = P(rb) = \frac{C_4^1 \times C_2^1}{C_6^2} = \frac{8}{15}$
- $P(A_2) = P(2b) = \frac{C_2^2}{C_6^2} = \frac{1}{15}$

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2) After the first drawing, the urn contains 4 balls. We randomly draw two new balls from the urn.

Consider the three events:

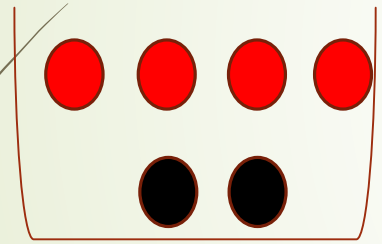
B_0 : the two drawn balls are red

B_1 : the two drawn balls have different colors.

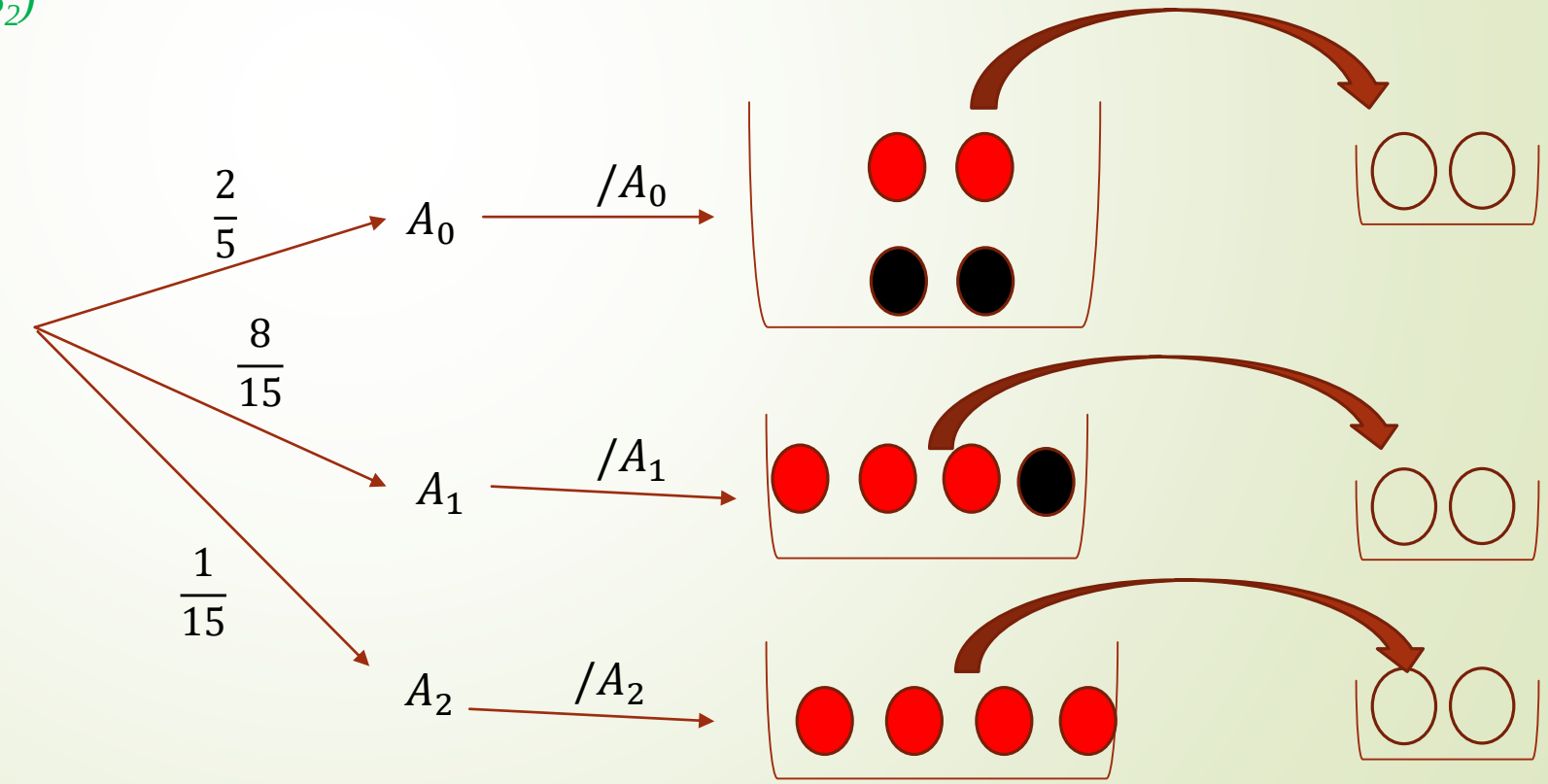
B_2 : the two drawn balls are black.

a) Calculate $p(B_0 / A_0)$, $p(B_0 / A_1)$ and $p(B_0 / A_2)$. Deduce that $p(B_0) = 0.4$

b) Calculate $p(B_1)$ and $p(B_2)$



- $2\ balls\ drawn\ from\ urn$:



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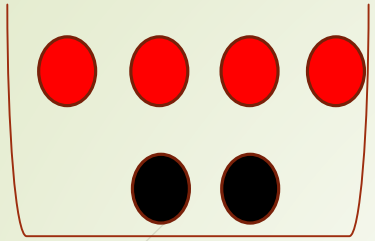
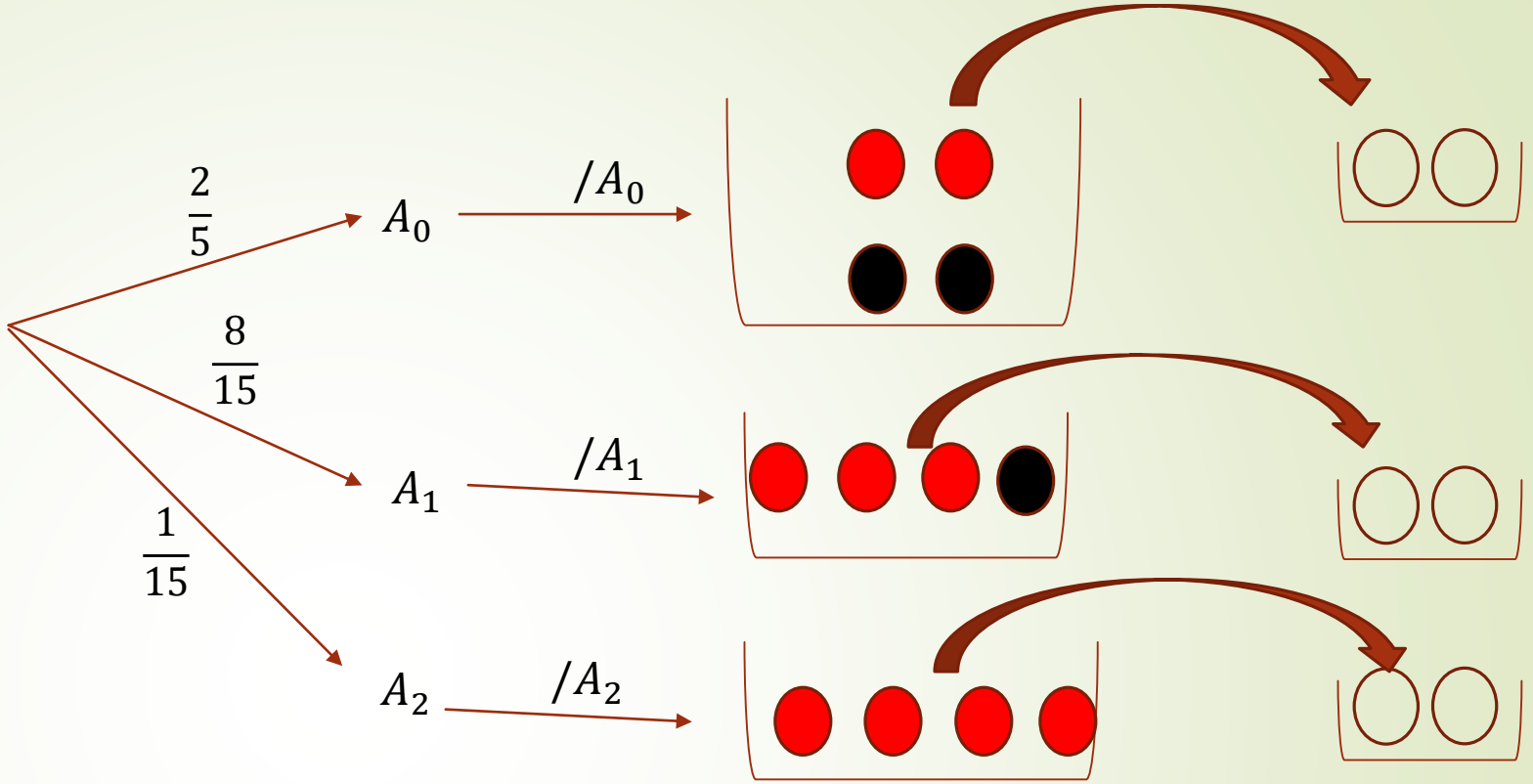


- *2 balls drawn from urn:*



a) Calculate $p(B_0 / A_0)$, $p(B_0 / A_1)$ and $p(B_0 / A_2)$. Deduce that $p(B_0) = 0.4$

- $P(B_0/A_0) = \frac{C_2^2}{C_4^2} = \frac{1}{6}$
- $P(B_0/A_1) = \frac{C_3^2}{C_4^2} = \frac{1}{2}$
- $P(B_0/A_2) = \frac{C_4^2}{C_4^2} = 1$

- $\mathrm{P}(B_0) = P(B_0 \cap A_0) + \mathrm{P}(B_0 \cap A_1) + P(B_0 \cap A_2)$

$$= P(B_0/A_0) \times P(A_0) + P(B_0/A_1) \times P(A_1) + P(B_0/A_2) \times P(A_2) = \frac{1}{15} + \frac{4}{15} + \frac{1}{15} = \frac{2}{5} = 0.4$$

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b) Calculate $p(B_1)$ and $p(B_2)$

- $\mathrm{P}(B_1) = P(B_1 \cap A_0) + \mathrm{P}(B_1 \cap A_1) + P(B_1 \cap A_2)$

$$= P(B_1/A_0) \times P(A_0) + P(B_1/A_1) \times P(A_1) + P(B_1/A_2) \times P(A_2)$$

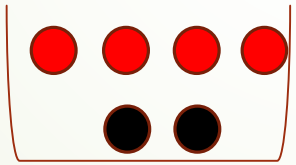
$$= \frac{C_2^1 \times C_2^1}{C_4^2} \times \frac{2}{5} + \frac{C_1^1 \times C_3^1}{C_4^2} \times \frac{8}{15} + 0 = \frac{8}{15}$$

- $\mathrm{P}(B_2) = P(B_2 \cap A_0) + \mathrm{P}(B_2 \cap A_1) + P(B_2 \cap A_2)$

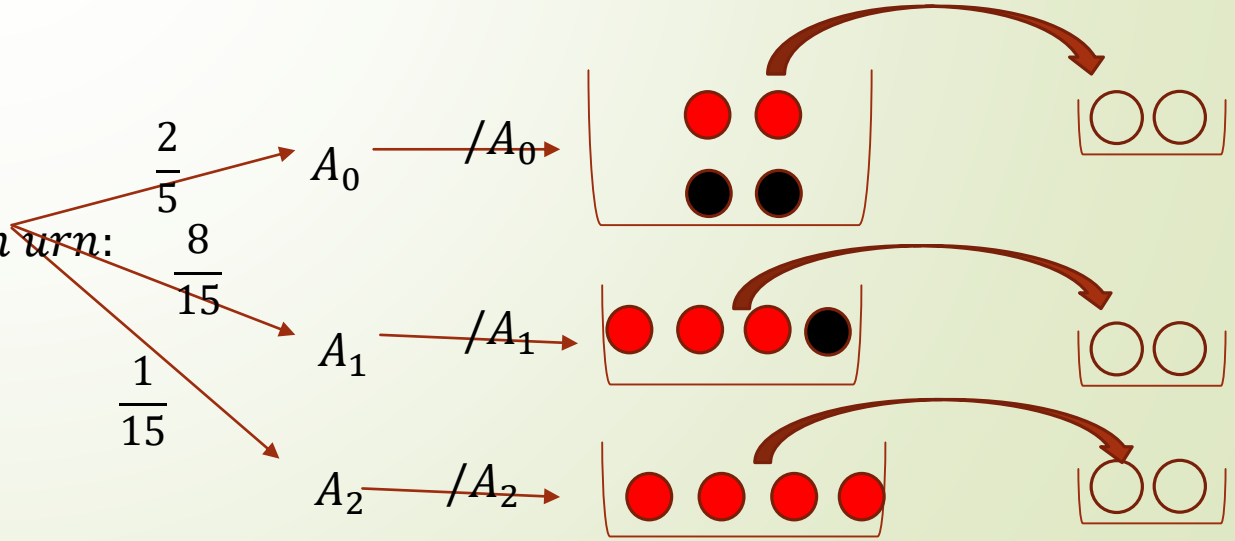
$$= P(B_2/A_0) \times P(A_0) + P(B_2/A_1) \times P(A_1) + P(B_2/A_2) \times P(A_2)$$

$$= \frac{C_2^2}{C_4^2} \times \frac{2}{5} + 0 + 0$$

$$= \frac{1}{15}$$



- *2 balls drawn from urn:*



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c) Knowing that only one black ball is obtained in the second drawing, calculate the probability that only one black ball has been obtained in the first drawing.

- $\mathrm{P}(A_1/B_1)= \dfrac{\mathrm{P}(A_1 \cap B_1)}{P(B_1)} = \dfrac{\frac{4}{15}}{\frac{8}{15}} = \dfrac{1}{2}$

3) Calculate the probability that, after the two drawing, the remaining two balls in the urn are red.

- $P(A_0 \cap B_2) + \mathrm{P}(A_1 \cap B_1) + \mathrm{P}(A_2 \cap B_0) = \dfrac{1}{15} + \dfrac{4}{15} + \dfrac{1}{15} = \dfrac{6}{15}$

- *2 balls drawn from urn*: