School:Amjad ***Homework (3)***

Subject: Conditional Probability (Probability tree)

I-A box contains questions of two type "Cinema" & "Music". $\frac{1}{3}$ of the questions are "Cinema" & the others are "Music".

The candidate for this game is called Hani.

Hani chooses from the box a question

- if it's a "Cinema" question, the probability to answer it correctly is 0.5
- if it's a "Music" question, the probability to answer it correctly is 0.75

Consider the events **C** *" The selected question is Cinema"*

M *"The selected question is Music"*

R *" Hani answers correctly the question"*

1) Calculate the following probabilities: $P(R \cap C)$, $P(R \cap M)$

2) Deduce that: $P(R) = \frac{2}{3}$.

3)Knowing that Hani , didn't answer correctly ($\bar{R}$) to the proposed question , what is the probability That the selected question by Hani is "Cinema". → $P(C/\bar{R})$

4) We propose to Hani three questions to answer them.(selection of questions are independent).

"In each selection for a question , we replace in the box a question carry the same theme. "

a) Calculate the probability that Hani answers 2 questions correctly.

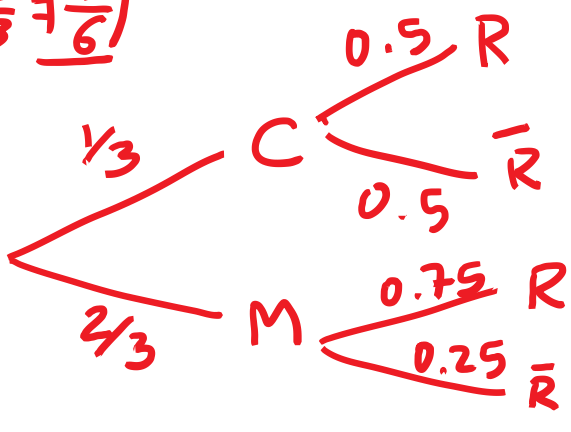
b) Calculate the probability that Hani answers at least one question correct.

1) $P(R \cap C) = P(R/C) \times P(C) = 0.5 \times \frac{1}{3} = \boxed{\frac{1}{6}}$

$P(R \cap M) = P(R/M) \times P(M)$

$= 0.75 \times \frac{2}{3}$

$= \boxed{\frac{1}{2}}$



2) $P(R) = P(\overline{R} \cap C) + P(R \cap M) = \frac{1}{6} + \frac{1}{2} = \frac{4}{6} = \boxed{\frac{2}{3}}$.

3) $P(C/\overline{R}) = \frac{P(C \cap \overline{R})}{P(\overline{R})} = \frac{0.5 \times \frac{1}{3}}{1 - \frac{2}{3}} = \frac{\frac{1}{6}}{\frac{1}{3}} = \boxed{\frac{1}{2}}$

II- (Lebanese BAC)

In order to encourage students to improve reading habits, a teacher uses two urns *A* and *B* such that :

The **urn *A*** contains 6 **white** balls and 5 **red** balls .
The **urn *B*** contains 4 **red** balls and 7 **green** balls .

He proposes the following game :
The student draws at random one ball from the urn *A* .
• If the drawn ball is white , then the student does not get anything .
• If the ball is red, the student draws randomly a ball from urn *B* .
- If it is red, the student gets a gift of 10 books .
- If it is green, he again draws, without replacing the ball in *B* , another ball from *B* .
If this last ball is red, then he gets 5 books; if not, he does not get anything .

Consider the following events :
F : «The student gets 10 books» . *E* : «The student gets 5 books» .
N : «The student does not get anything» .

1° What is the probability of the event : «the student does not get anything for the draw from urn *A*» ?

2° Calculate the probability *P*(*F*) and show that $P(E) = \frac{14}{121}$.

3° Calculate *P*(*N*) .

III- A game is organized between the students of the 3rd secondary class , each student throws a ball.

- $\frac{5}{6}$ of the students are right hand players.
- $\frac{1}{6}$ of the students are left hand players.
- For a right hand player , the probability to put the ball in the basket is 0.25.
- For a left hand player , the probability to put the ball in the basket is 0.5.

A student is chosen at random from the class.

Consider the events :**L**" *the student is a left hand player*"

R" *the student is a right hand player"* ; **I**" *the student puts the ball in the basket"*

1) a) Calculate: $P(I \cap L)$, $P(I \cap R)$.Deduce P(I).

b) Knowing that the student puts the ball in the basket , what is the probability that he is a right hand player?

2) The player *gets* **5 points** if he puts the ball in the basket , if not he *loses* **3 points**.

Fadi is a player , he throws the ball twice one after the other in independent way.

Let X be the random variable , which denotes *the algebraic gain of Fadi.*

a) Find the possible values of X.

b) Determine the probability distribution of X.

c) Calculate the average gain of Fadi.