17 PROBABILITY I

- **Exercises: 1, 7** Page 210
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Ω: the sample space

A: is an event subset of Ω

- $P(A) = \dfrac{Card(A)}{Card(\Omega)}$
- $0 \leq P(A) \leq 1$ since $Card(A) \leq Card(\Omega)$

Random Experiment

1 A coin is tossed and its outcomes are observed.

a) Is it possible that heads appears?

b) Is it certain that tails appears?

a) Yes, it is possible since

$\Omega = \{ H , T \}, \text{set of all possible outcomes}$

b) No, it is not certain but possible as head

$\Omega = \{ H , T \}, \text{set of all possible outcomes}$

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Random Experiment

1 A coin is tossed and its outcomes are observed.

c) Is it possible that heads and tails appear?

d) Is it possible that heads or tails appear?

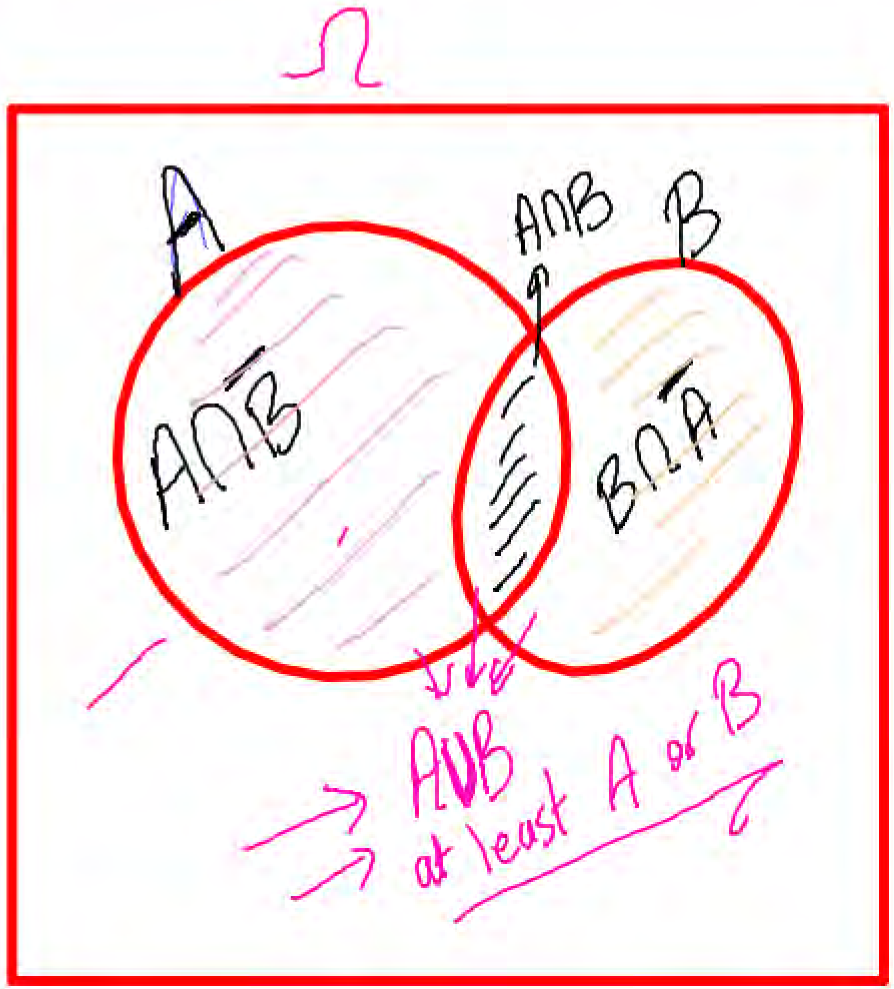
e) Is it impossible that neither heads or tails appear?

c) No, only one of them will appear

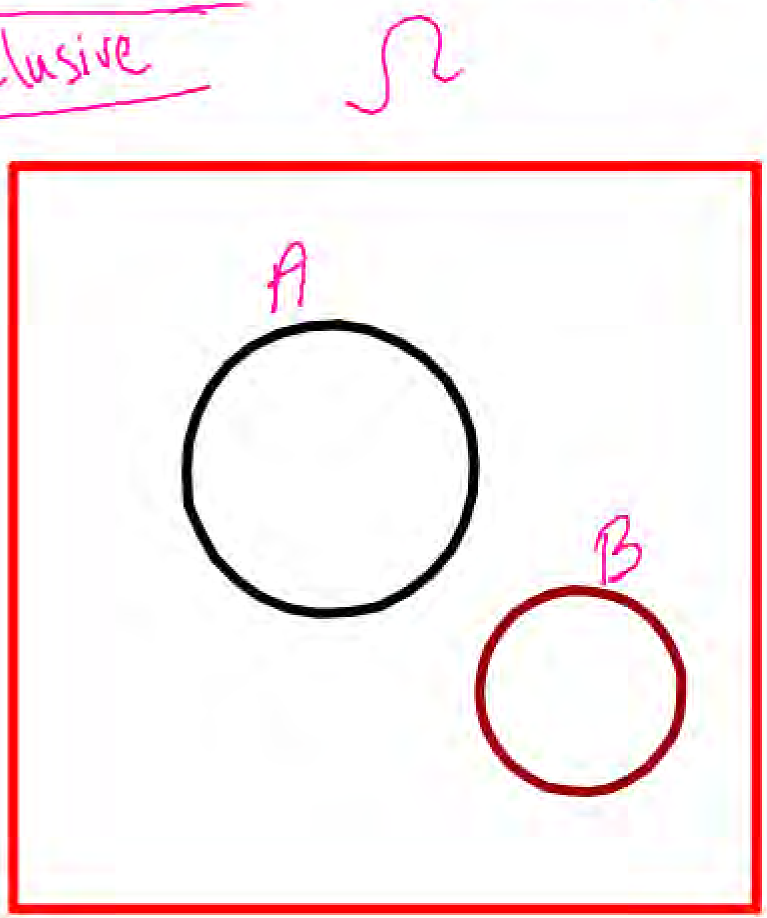
d) Yes it is possible and certain that one of them will appear.

e) Yes , it is impossible that neither head or tail

Two events are disjoint when their intersection is empty.
We say also (mutually exclusive or incompatible)



Disjoint events: $A \cap B = \emptyset$ (Impossible events)
Mutually exclusive



Example

Two dice are rolled. We are interested in the numbers appearing on the top faces.

Let A and B be the two events:

A: “both numbers are odd”

B: “both numbers are equal”

The event (A and B) is: “both numbers are odd and equal”:

$$A \cap B = \{(1, 1), (3, 3), (5, 5)\}.$$

When two dice are rolled:

Sample space Ω : { (1,1) , (1, 2) , (1, 3), (1 ,4) , (1, 5) , (1, 6)
(2,1) , (2, 2) , (2, 3), (2 ,4) , (2, 5) , (2, 6)
(3,1) , (3, 2) , (3, 3), (3 ,4) , (3, 5) , (3, 6)
(4,1) , (4, 2) , (4, 3), (4 ,4) , (4, 5) , (4, 6)
(5,1) , (5, 2) , (5, 3), (5 ,4) , (5, 5) , (5, 6)
(6,1) , (6, 2) , (6, 3), (6 ,4) , (6, 5) , (6, 6)}

Operations on Events: Intersection ∩

- **Sample space Ω :** { (1,1) , (1, 2) , (1, 3), (1 ,4) , (1, 5) , (1, 6)
  (2,1) , (2, 2) , (2, 3), (2 ,4) , (2, 5) , (2, 6)
  (3,1) , (3, 2) , (3, 3), (3 ,4) , (3, 5) , (3, 6)
  (4,1) , (4, 2) , (4, 3), (4 ,4) , (4, 5) , (4, 6)
  (5,1) , (5, 2) , (5, 3), (5 ,4) , (5, 5) , (5, 6)
  (6,1) , (6, 2) , (6, 3), (6 ,4) , (6, 5) , (6, 6)}

A: "both numbers are odd"

B: "both numbers are equal"

A={ (1,1) , (1,3), (1,5), (3,1), (3,3) , (3,5), (5,1), (5,3) , (5,5)}

$\bar{A} = \{(1,2), (1,4), (1,6),$ (2,1) , (2, 2) , (2, 3), (2 ,4) , (2, 5) , (2, 6), .. } = **Ω-A**

B={ (1,1), (2,2) , (3,3) , (4,4) , (5,5) , (6,6) }

$\bar{B} = \{(1,2), (1,3), (1,4), \ldots\ldots\}$ = **Ω−B**

A&B=$\boldsymbol{A} \cap \boldsymbol{B} = \{(1,1)\ , (3,3)\ , (5,5)\ \}$

Operations on Events: Union (0r) ∪

- **Sample space Ω :** { (1,1) , (1, 2) , (1, 3), (1 ,4) , (1, 5) , (1, 6)
(2,1) , (2, 2) , (2, 3), (2 ,4) , (2, 5) , (2, 6)
(3,1) , (3, 2) , (3, 3), (3 ,4) , (3, 5) , (3, 6)
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(5,1) , (5, 2) , (5, 3), (5 ,4) , (5, 5) , (5, 6)
(6,1) , (6, 2) , (6, 3), (6 ,4) , (6, 5) , (6, 6)}

A={ (1,1) , (1,3), (1,5), (3,1), (3,3) , (3,5), (5,1), (5,3) , (5,5)}

$\bar{A} = \{(1,2), (1,4), (1,6),$ (2,1) , (2, 2) , (2, 3), (2 ,4) , (2, 5) , (2, 6), .. } = **Ω-A**

B={ (1,1), (2,2) , (3,3) , (4,4) , (5,5) , (6,6) }

$\bar{B} = \{(1,2), (1,3), (1,4), \ldots\ldots\} =$ **Ω−B**

AorB=$\boldsymbol{A} \cup \boldsymbol{B} = \{$(1,1) , (1,3), (1,5), (3,1), (3,3) , (3,5), (5,1), (5,3) , (5,5),(2,2),(4,4),(6,6)}

7 An urn contains 12 balls numbered from 1 to 12. A ball is drawn at random from this urn. Let:

A: be the event: "the ball's number is a multiple of 2,"

B: the event: "the ball's number is a multiple of 3,"

C: the event: "the ball's number is a multiple of 4."

Determine the events:

$\overline{A}$, $\overline{B}$, $\overline{C}$, (A and B), (A or B), (B and C),(B or C).

- $\Omega = \{1, 2, 3 , 4,5, 6, 7, 8, 9, 10,11 , 12\}$
- $A = \{2, 4, 6,8, 10 , 12\}$ $then$ $\bar{A} = \{1, 3, 5, 7, 9,11\}$ or **Ω-A**
- $B = \{3, 6, 9, 12\}$ $then$ $\overline{B} = \{ 1 , 2 , 4, 5, 7 , 8 , 10 , 11\}$
- $C = \{4,8, 12\}$ $then$ $\overline{C} = \{1, 2, 3, 5, 6, 7, 9, 10 , 11\}$
- $A\ and\ B = multiple\ of\ 2\ and\ 3 = A \cap B = Common\ between\ A\ and\ B$

So $A\ and\ B = \{6, 12\}$

- $A\ or\ B = multiple\ of\ 2\ or\ 3 = A\ U\ B = at\ least\ A\ or\ B$

So $A\ or\ B = \{2, 3, 4,6, 8,9, 10,12\}$

- $C\ and\ B = multiple\ of\ 4\ and\ 3 = C \cap B = Common\ between\ C\ and\ B$ So $C\ and\ B = \{12\}$

$C\ or\ B = multiple\ of\ 4\ or\ 3 = C\ U\ B = at\ least\ C\ or\ B$

So $C\ or\ B = \{3, 4,6, 8,9,12\}$

10 We meet, at random, a person in a population attacked by two illnesses M_1 and M_2. Denote by:

A: the event "the person is attacked by the illness M_1;"

B: the event "the person is attacked by the illness M_2."

In one-sentence phrases, explain each of the following events:

$$\overline{A},\quad \overline{B},\quad A \cap B,\quad A \cup B,\quad \overline{A \cup B}\,.$$

A: the event "the person is attacked by the illness M_1;"

B: the event "the person is attacked by the illness M_2."

$\bar{A}$: " the person that is **not** attacked by the illness M1 "

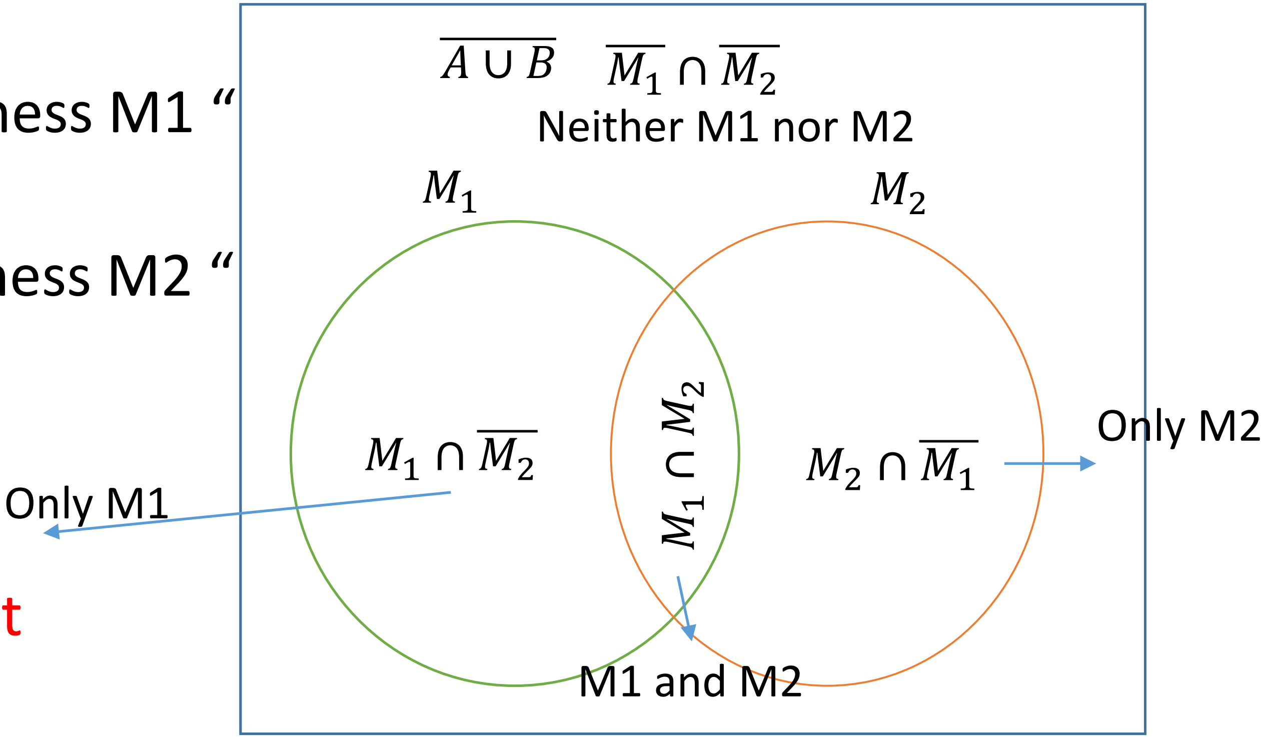
$\bar{B}$: " the person that is **not** attacked by the illness M2 "

$A \cap B$: " the person that is attacked by both the illnesses M1 and M2 "

$A \cup B$: " the person that is attacked by at least the illnesses M1 and M2 "

$\overline{A \cup B}$: " the person that is not attacked by M1 and M2 "
Neither M1 nor M2

Note $\overline{A \cup B} = \bar{A} \cap \bar{B}$



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PROBABILITY II

GRADE LS

1

What is a Random Experiment ? Event ?

- **It is a trial** that, given a well determined conditions and from possible outcomes, leads to an unpredictable outcomes.

Ex: Tossing a fair coin, roll a die , draw a ball at random

- **Sample Space** / **Universe** :
  Set of all possible outcomes denoted by Ω
- **Event:**
  Is a set of outcomes subset of sample space
- **Impossible Event** : ϕ

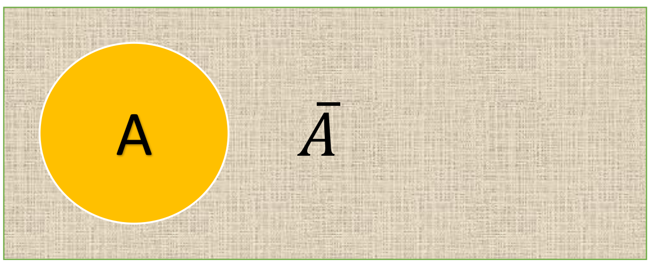
2

Definition:

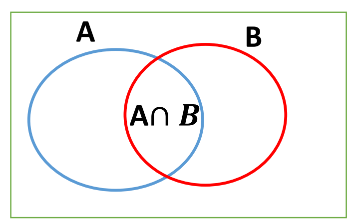
Probability is **Meaningless** if the experiment **was not randomly operated**.
Consider a random experiment: Ω :Sample space that is the set of all possible outcomes.
Suppose $\Omega = \{a_1, a_2, ...a_n\}$. $\quad Card(\Omega) = n. \quad 0 \le P(A) \le 1.$

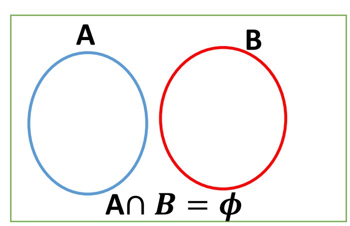
$$P(A) = \frac{Card(A)}{Card(\Omega)} \quad or \quad P(A) = \frac{Number \text{ of Favorable outcomes}}{Total \text{ number of outcomes}}$$

Ω



Sets $A, B, ...$ are subsets	Events $A, B, ...$ are Events	Notation: $A \subset \Omega$, $B \subset \Omega$.	Probability
Set Ω	**Certain** (Sure) event	Ω	$P(\Omega) = 1$
Empty set	Impossible event	ϕ	$P(\phi) = 0$
ω **is an element of** A	The outcome ω occurs if A occurs	$\omega \in A$	$P(\omega) = \frac{nb. \text{ of existance of } \omega}{n}$
ω **is an element of** $\bar{A}$ **(not** A**)**	The outcome ω does not occur if A occurs	$\omega \in \bar{A}$ or $\omega \notin A$	$P(\omega) = \frac{nb. \text{ of } \omega}{n}$
The Union of A **&** B **(we say** A **or** B**)**	The outcome A or B	$A \cup B$	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
The intersection of A & B **(we say** A **&** B**)**	The outcome A & B	$A \cap B$	$P(A \cap B) \le P(A)$ $P(A \cap B) \le P(B)$
The Complement of A **We say Not** A	The Event is not A $(\bar{A})$	$\bar{A}$	$P(\bar{A}) + P(A) = 1 \Leftrightarrow P(\bar{A}) = 1 - P(A)$
A & B **are disjoint**	A & B are mutually exclusive (incompatible)	$A \cap B = \varnothing$ (impossible Events)	$P(A \cap B) = 0$. $P(A \cup B) = P(A) + P(B)$.

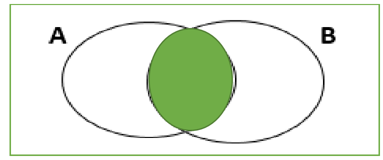




$$Note : P(A \cap B) = P(B \cap A) \;\; \& \;\; P(A \cup B) = P(B \cup A).$$

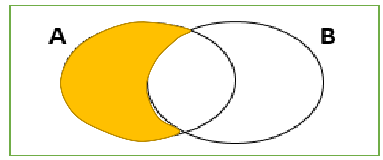
❖ **Activity – 1-(Set Operation Review)**

Translate the following set-theoretic notation into event language. For example, "$A \cup B$" means " A or B occurs". Then shade the corresponding rejoin.



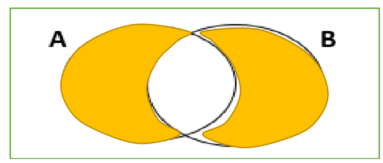
1) $A \cap B$:

A and B occurs at the same time



2) $A \cap \overline{B}$

A occurs and no B occurs at the same time



3) $(A \cup B) \cap (\overline{A \cap B})$

A occurs and no B occurs at the same time

Or B occurs and no A occurs at the same time

❖ **Activity -2- (Set Operation Review)**

Given the probabilities $\boldsymbol{P}(\boldsymbol{A}) = 0.5$ & $\boldsymbol{P}(\boldsymbol{B}) = 0.5$ & $\boldsymbol{P}(\boldsymbol{A} \cup \boldsymbol{B}) = 0.8$

1) Find the probabilities: $P(\bar{A})$, $P(\bar{B})$, $\boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B})$

2) Are A and B mutually exclusive (Disjoint)?

3) Are A and B independent?

1) $P(\bar{A}) = 1 - P(A) = 1 - 0.5 = 0.5$

$P(\bar{B}) = 1 - P(B) = 1 - 0.5 = 0.5$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.5 + 0.5 - P(A \cap B)$

Then : $P(A \cap B)$ =0.2

2) $P(A \cap B)$ =0.2$\neq$ 0

So A and B are not disjoint

3) A and B are independent

If $P(A \cap B) = P(A) \times P(B)$

$0.2 = 0.5 \times 0.5$

0.2=0.25 False

Then A and B are

not Independent (Dependent)

$\bar{\bar{A}} = A$

I) A class is composed of 4 boys and 6 girls.

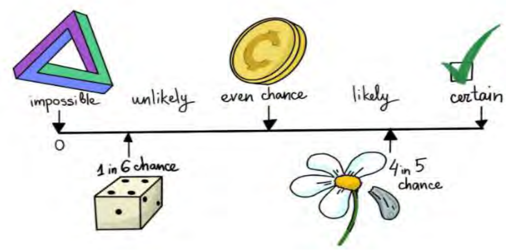
A student is chosen at random

1) What is the probability of that the chosen student is a boy?
2) What is the probability of that the chosen student is not a boy?

$1)P(student\ is\ a\ boy)$

P(Boy)=$\frac{\mathbf{4}}{\mathbf{10}} = 0.4$

2) $P(\overline{Boy}) = 1 - P(Boy) = 0.6$

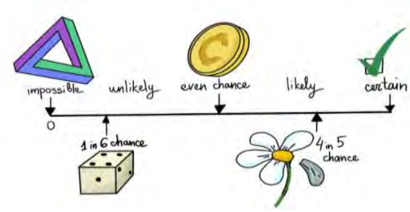


7

II) An urn contains 8 balls as below:

- **4 balls colored red**
- **3 balls colored** white
- **1 ball colored green**

A ball is chosen at random.



1) What is the probability it is of white color?
2) What is the probability that the chosen ball is **not white**?
3) What is the probability that the chosen ball is green?
4) **Knowing that** the chosen ball is **not white**, what is the probability it is of green color?

$1)P(White) = \frac{3}{8} = 0.375$

$2)P(\overline{White}) = 1 - \mathrm{P(white)} = 1 - 0.375 = 0.625$

$3)P(Green) = \frac{1}{8} = 0.125$

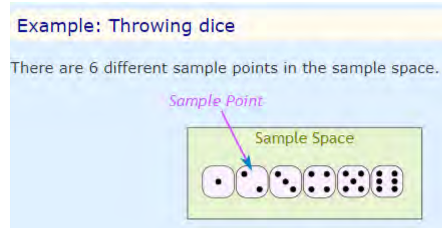
4) $P(\text{ Green}/\overline{white}) = \frac{1}{5}$

Knowing that not red :

P(question / given)

P(question / not red)

8



Ex 2.2 (MCQ)
Choose the correct answer among the proposed answers:

1) An experiment is composed of tossing a fair coin at random:
What is the probability of obtaining: head **or** tail? $\Omega = \{H, T\}$
a) 0 b) 0.5 c) 1

2) An experiment is composed of tossing a fair coin at random:
What is the probability of obtaining: head **and** tail? $Impossible = \{\ \}$
a) 0 b) 0.5 c) 1

3) An experiment is composed of rolling a fair die at random: $\Omega = \{1,2,3,4,5,6\}$
What is the probability of obtaining: a natural number?
a) 0 b) 0.5 c) 1

4) An experiment is composed of rolling a fair die at random:
What is the probability of obtaining: a number zero on the die?
a) 0 b) 0.5 c) 1

5) An experiment is composed of tossing a fair coin at random: $\Omega = \{H, T\}$
What is the probability of obtaining: a **head**?
a) 0 b) 0.5 c) 1

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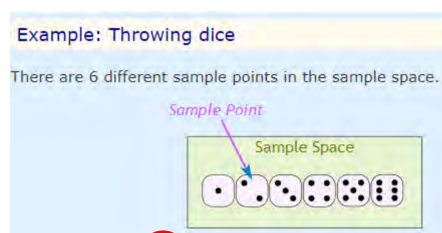

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What is the probability of obtaining: a **head**?
a) 0 b) 0.5 c) 1

10

6) An experiment is composed of rolling a fair die at random:
What is the probability of obtaining: number **one** on the die?
a) 1/3 b) 0.5 c) 1/6

7) An experiment is composed of rolling a fair die at random:
What is the probability of obtaining: an **odd** number on the die?
a) 1/3 b) 0.5 c) 1/6

8) An experiment is composed of selecting a ball at random from an urn containing identical balls holding the colors: red (4 balls), white (3 balls) and green (2 balls) :
What is the probability of obtaining: a red ball?
a) 1/4 b) 4/9 c) 5/9

9) An experiment is composed of rolling a fair die twice at random:
What is the probability of obtaining: same number twice on the die?
a) 1/3 b) 1/6 $\frac{6}{36}$ c) 5/6

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	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)			
3			(3,3)			
4				(4,4)		
5					(5,5)	
6						(6,6)

8) An experiment is composed of selecting a ball at rand
holding the colors: red (4 balls), white (3 balls) and g
What is the probability of obtaining: a red ball?
a) 1/4 b) 4/9

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a) 1/3 b) 1/6 $\frac{6}{36}$ c) 5/6

10) An experiment is composed of selecting a ball at random from an urn containing identical balls holding the colors: red (4 balls), white (3 balls) and green (2 balls) :
What is the probability of obtaining: a non-red ball?
a) 1/4 b) 4/9 c) 5/9

12) An experiment is composed of selecting a ball at random from an urn containing identical balls holding the colors: red (4 balls), white (3 balls) and green (2 balls) :
What is the probability of obtaining: a ball have the colors red and green?
a) 1/4 b) 0 c) 5/9

13) An experiment is composed of rolling a fair die at random:
What is the probability of obtaining: an odd number and even on the die?
a) 1/3 b) 0 c) 1/6

16) An experiment is composed of selecting a ball at random from an urn containing identical balls holding the colors: red (4 balls), white (3 balls) and green (2 balls) :
What is the probability of obtaining: a ball holding the red or green?
P(RorG)=P(R)+P(G)-P(R&G)=6/9
a) 2/3 b) 0 c) 8/9

13

10) An experiment is composed of selecting a ball at random from an urn containing identical balls holding the colors: red (4 balls), white (3 balls) and green (2 balls) : Total =9
What is the probability of obtaining: a non-red ball?
a) 1/4 b) 4/9 P(none red)=1-P(Red)=1-4/9=5/9 c) 5/9

12) An experiment is composed of selecting a ball at random from an urn containing identical balls holding the colors: red (4 balls), white (3 balls) and green (2 balls) :
What is the probability of obtaining: a ball have the colors red and green?
a) 1/4 b) 0 c) 5/9

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What is the probability of obtaining: a ball holding the red or green?
P(R or G)=P(R)+P(G)-P(R&G)=6/9
a) 2/3 b) 0 c) 8/9

14

The table below shows the distribution of students in a certain school:

	1st secondary	2nd secondary	3rd secondary
Boys	**100**	**80**	**70**
Girls	**160**	**120**	**80**

A student is chosen at random

1) What is the probability of that the chosen student is a boy?
2) What is the probability of that the chosen student is not a boy?
3) What is the probability of that the chosen student is a second secondary student?
4) What is the probability of that the chosen student is boy in 2^{nd} secondary?
5) What is the probability of that the chosen student is a girl in 1^{st} secondary?
6) What is the probability of that the chosen student either a girl **or** from 2^{nd} secondary?
7) What is the probability of that the chosen student is either a boy **or** from 1^{st} secondary?
8) Knowing that the chosen student is a boy, what is the probability that he is of 1^{st} secondary?
9) Knowing that the chosen student is of 2^{nd} secondary, what is the probability that he is a boy?

15

The table below shows the distribution of students in a certain school:

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Boys	**100**	**80**	**70**	
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4) What is the probability of that the chosen student is boy in 2^{nd} secondary?
5) What is the probability of that the chosen student is a girl in 1^{st} secondary?
6) What is the probability of that the chosen student either a girl **or** from 2^{nd} secondary?
7) What is the probability of that the chosen student is either a boy **or** from 1^{st} secondary?

8) Knowing that the chosen student is a boy, what is the probability that he is of 1^{st} secondary?
9) Knowing that the chosen student is of 2^{nd} secondary, what is the probability that he is a boy?

16

The table below shows the distribution of students in a certain school:

	1st secondary	2nd secondary	3rd secondary	Total
Boys	100	80	70	250
Girls	160	120	80	360
Total	260	200	150	610

A student is chosen at random

1) What is the probability of that the chosen student is a boy? $P(Boy) = \frac{250}{610} = \frac{25}{61}$
2) What is the probability of that the chosen student is not a boy? $P(\overline{Boy}) = 1 - \frac{250}{610} = \frac{360}{610} = \frac{36}{61}$
3) What is the probability of that the chosen student is a second secondary student? $P(2nd) = \frac{200}{610} = \frac{20}{61}$
4) What is the probability of that the chosen student is boy in 2nd secondary? $P(boy\ in\ 2nd) = \frac{80}{610} = \frac{8}{61}$
5) What is the probability of that the chosen student is a girl in 1st secondary? $P(girl\ in\ 1st) = \frac{160}{610} = \frac{16}{61}$
6) What is the probability of that the chosen student either a girl **or** from 2nd secondary? $P(girl\ or\ 2nd) = P(girl) + P(2nd) - P(girl\ in\ 2nd) = \frac{360}{610} + \frac{200}{610} - \frac{120}{610} = \frac{440}{610} = \frac{44}{61}$
7) What is the probability of that the chosen student is either a boy **or** from 1st secondary? $P(boy\ or\ 1st) = P(boy) + P(1st) - P(boy\ in\ 1st) = \frac{250}{610} + \frac{260}{610} - \frac{100}{610} = \frac{41}{61}$
8) Knowing that the chosen student is a boy, what is the probability that he is of 1st secondary? $P(1st/boy) = \frac{100}{250}$
9) Knowing that the chosen student is of 2nd secondary, what is the probability that he is a boy? $P(boy/2nd) = \frac{80}{200}$

Application:

V) Given : P(A)=0.5 , P(B)=0.6 , P(A∩B)=0.2.

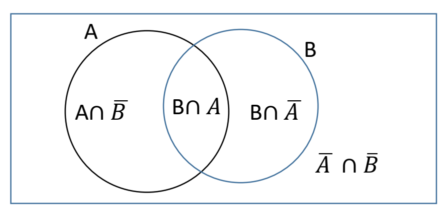
a) Calculate P(AUB).

b) **Are** A and B **independent**? Justify.

Note: A and B are independent if P(A ∩B)=P(A)P(B)???

c) Calculate $P(\bar{A} \cap B)$, $P(\bar{B} \cap A)$, $P(\overline{A \cap B})$ and $P(\overline{AUB})$.

d) Calculate $P(\bar{A} \cap \bar{B})$ and $P(\bar{A}\, U \bar{B})$.



a) P(AUB)=$P(A) + P(B) - P(A \cap B) = 0.5 + 0.6 - 0.2 = 0.9$.

b) $P(A) \times P(B) = 0.5 \times 0.6 = 0.3 \neq P(A \cap B)$.
so , A and B are not independent.

Total Probability : $P(A) = P(A \cap \bar{B}) + P(A \cap B)$

c) $P(\bar{A} \cap B) = P(B) - P(B \cap A) = 0.6 - 0.2 = 0.4$,
$P(\bar{B} \cap A) = P(A) - P(A \cap B) = 0.5 - 0.2 = 0.3$
$P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - 0.2 = 0.8$
$P(\overline{AUB}) = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

d) $P(\bar{A} \cap \bar{B}) = P(\overline{AUB}) = 0.1$
$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 0.8$

18

PROBABILITY III

Selection of more than one Object
Application on Combination

PROBABILITY III
Selection of more than one Object

GRADE 11 S

Solution of the three exercises on Combination-Probability

$$C(n,k) = \frac{n!}{(n-k)!k!}$$

II) Simultaneous Selection

Remember:
Order is not important!

Selection Simultaneously

An urn contains *12 balls* distributed as follows:

- **2 red balls** numbered **1.**
- **5 blue balls** numbered **2.**
- **4 green** balls numbered **3.**
- 1 yellow ball numbered **4.**

A student selects randomly **two balls** **simultaneously** from the urn

Consider the following events:

R: " the two chosen balls are of red color"
S: " The two chosen balls are of same color"
~~T: " The balls are chosen as follow red , and then blue"~~
O: " Only one among the chosen balls is red"
A: " At least one of the chosen balls is blue"
N: " None of the chosen balls has a red color"
~~E: " the 1st ball is the only green ball"~~

1) Calculate the probabilities of all events R, S ,~~T~~ , O , A , N and ~~E~~

2) Knowing that no red ball is chosen , what is the probability of that the two balls are of same color?

Remember:
Order is not important!

An urn contains *12 balls* distributed as follows:

- **2 red balls** numbered **1.**
- **5 blue balls** numbered **2.**
- **4 green** balls numbered **3.**
- **1 yellow** ball numbered **4.**

A student selects randomly **two balls** **simultaneously** from the urn

Consider the following events:

R: " the two chosen balls are of red color"

S: " The two chosen balls are of same color"

O: " Only one among the chosen balls is red"

A: " At least one of the chosen balls is blue"

N: " None of the chosen balls has a red color"

1) Calculate the probabilities of all events R, S , O , A , N

Total Cases $=C_n^r = nCr$

Where n is the total and r is the chosen

n=12 and r=2; Total cases = $C_{12}^2 = 12C2 = 66$

P(R) =P(2 red balls) = $\frac{C_2^2}{C_{12}^2} = \frac{2C2}{12C2} = \frac{1}{66}$

P(S) =P(2 balls of same color) =

P(2red) + P(2 blue) + P(2 green)=

$\frac{1}{66} + \frac{C_5^2}{C_{12}^2} + \frac{C_4^2}{C_{12}^2} = \frac{17}{66}$

P(O) =P(Only one is red) = P(1Red and 1 not red)=P($1R$ & $1\ \bar{R}$)=$\frac{C_2^1 C_{10}^1}{C_{12}^2}$

Note: Inside the parenthesis the number should be equal to the chosen

And when we finish precising we then can say **or (+)** if we have another possibility

An urn contains *12 balls* distributed as follows:

Remember:
Order is not important!

- **2 red balls** numbered **1.**
- **5 blue balls** numbered **2.**
- **4 green** balls numbered **3.**
- 1 yellow ball numbered **4.**

A student selects randomly **two balls** **simultaneously** from the urn

Consider the following events:

R: " the two chosen balls are of red color"

S: " The two chosen balls are of same color"

O: " Only one among the chosen balls is red"

A: " At least one of the chosen balls is blue"

N: " None of the chosen balls has a red color"

1) Calculate the probabilities of all events R, S , O , A , N

Total Cases $= C_n^r = nCr$

Where n is the total and r is the chosen

n=12 and r=2; Total cases = $C_{12}^2 = 12C2 = 66$

P(A) =P(At least one is blue)=

= P(1 is blue) + P(2 are blue)

= P(1Blue&1$\overline{Blue}$) + P(2Blue)

$$=\frac{C_5^1 C_7^1}{C_{12}^2} + \frac{C_5^2}{C_{12}^2}$$

or 1- P(none is blue)=1-$\frac{C_7^2}{C_{12}^2}$

$$P(N)=P(2\overline{Red}) = \frac{C_{10}^2}{C_{12}^2}$$

Note: Inside the parenthesis the **number should be equal to the chosen**

And when we finish precising we then can say **or (+)** if we have another possibility

II) Simultaneous Selection

Remember:
Order is not important!

Selection Simultaneously

An urn contains *12 balls* distributed as follows:

- **2 red balls** numbered **1.**
- **5 blue balls** numbered **2.**
- **4 green** balls numbered **3.**
- 1 yellow ball numbered **4.**

A student selects randomly **Three balls** **simultaneously** from the urn

Consider the following events:

R: " the three chosen balls are of red color"
S: " The three chosen balls are of same color"
D: " The three chosen balls are of three different colors"
O: " Only one among the chosen balls is red"
A: " At least one of the chosen balls is blue"
N: " None of the chosen balls has a red color"

1) Calculate the probabilities of all events R, S ,~~T~~ , O , A , N and ~~E~~

2) Knowing that no red ball is chosen , what is the probability of that the two balls are of same color?

II) Simultaneous Selection

Remember:
Order is not important! And so we use Combination $C_n^r = nCr \ \ where\ r \leq n$

Selection Simultaneously

An urn contains ***12 balls*** distributed as follows:

- **2 red balls** numbered **1.**
- **5 blue balls** numbered **2.**
- **4 green** balls numbered **3.**
- 1 yellow ball numbered **4.**

A student selects randomly **three balls simultaneously** from the urn

Consider the following events:

R: " the three chosen balls are of red color"

S: " The three chosen balls are of same color"

D: " The three chosen balls are of **three** different colors"

1) Calculate the probabilities of all events R, S , O , A , and N

Total Cases $= C_n^r = nCr$

Where n is the total and r is the chosen

n=12 and r=3; Total cases = $C_{12}^3 = 12C3 = 220$

P(R) =P(3 red balls) = 0 ($Impossible\ event$)

$$P(S) = P(3\ blue) + P(3\ green)$$

$$= \frac{C_5^3}{C_{12}^3} + \frac{C_4^3}{C_{12}^3} = \frac{7}{110}$$

$$P(D) = P(1red\&1Blue\&1Green) + P(1red\&1Blue\&1Yellow) + P(1Blue\&1Green\&1Yellow) +$$

$$P(1Red\&1Green\&1Yellow) = \frac{C_2^1\times C_5^1\times C_4^1+C_2^1\times C_5^1\times C_1^1+C_5^1\times C_4^1\times C_1^1+C_2^1\times C_4^1\times C_1^1}{C_{12}^3} = \frac{39}{110}$$

II) Simultaneous Selection

Selection Simultaneously

Remember:

Order is not important! And so we use Combination $C_n^r = nCr \ \ where\ r \leq n$

An urn contains ***12 balls*** distributed as follows:

- 2 red balls numbered **1.**
- 5 blue balls numbered **2.**
- 4 green balls numbered **3.**
- 1 yellow ball numbered **4.**

A student selects randomly **three balls** **simultaneously** from the urn

Consider the following events:

O: " Only (Exactly) one among the chosen balls is red"

A: " At least one of the chosen balls is blue"

N: " None of the chosen balls has a red color"

1) Calculate the probabilities of all events R, S , O , A , and N

Total cases = $C_{12}^3 = 12C3 = 220$

P(O) =P$(1Red\&2\overline{Red})$ =$\frac{C_2^1 C_{10}^2}{C_{12}^3} = \frac{9}{22}$

P(A) =$1 - P(\ none\ is\ blue)$

$= 1 - P(3\ \overline{Blue})$

$= 1 - \frac{C_7^3}{C_{12}^3} = \frac{37}{44}$

We have
5 blue balls and 7
are none blue

P(N) =$P(3\ \overline{Red})$

$= \frac{C_{10}^3}{C_{12}^3} = \frac{6}{11}$

We have
2 red balls and 10
are none red

1

In a shop there are **18 packs** of cigarettes are **Put** on a stand :

- **7** Marlboro packs.
- **5 Gitane** packs.
- **6 Winston** packs.

A customer wants to buy 3 packs of cigarettes. For this he ***selects 3 packs*** from the Stand ***simultaneously & randomly***.

1) How many possible selections are there?
2) Calculate the probability of each event:

A" *The 3 chosen packs are 2Gitane & 1 Winston* " ; **B**" *The 3 chosen packs are of same type*"
C" The 3 chosen packs are one of each type " ; **D**" *at least one Winston pack* "

3) The prices of each kind of cigarettes are given in the table below:

	Marlboro	*Winston*	*Gitane*
Prices	3000LL	2000LL	4000LL

a) Calculate the probability of the event:
E" *the amount paid by the customer is* **9000**LL"

b) Calculate the probability of the event :
" the *amount paid by the customer doesn't exceed 8000LL"* is.

1 In a shop there are **18 packs** of cigarettes are putted on a stand :

- 7 Marlboro packs.
- 5 **Gitane** packs.
- 6 **Winston** packs.

A customer wants to buy 3 packs of cigarettes. For this he ***selects 3 packs*** from the Stand ***simultaneously & randomly***.

1) How many possible selections are there?

- **Simultenously**, then the order is not considered. We use C_n^r
- Total number is 18 = n= Number of total elements
- Chosen : 3 = r = number of chosen elements simultaneously.

$$Number\ of\ total\ cases:\ C_{18}^{3}=816$$

1 In a shop there are **18 packs** of cigarettes are putted on a stand :

- **7** Marlboro packs.
- **5 Gitane** packs.
- **6 Winston** packs.

A customer wants to buy 3 packs of cigarettes. For this he ***selects 3 packs*** from the Stand ***simultaneously & randomly***.

2) Calculate the probability of each event:

A" *The 3 chosen packs are 2Gitane & 1 Winston* " ; **B**" *The 3 chosen packs are of same type*"

C" *The 3 chosen packs are one of each type* " ; **D**" *at least one Winston pack* "

$$P(A) = P(2Gitane\&1Winston) = \frac{C_5^2 \times C_6^1}{C_{18}^3} = 5/68$$

$$P(B) = P(all\ of\ same\ type) = P(3Marlboro) + P(3Gitane) + P(3\ winston)$$

$$= \frac{C_7^3}{C_{18}^3} + \frac{C_5^3}{C_{18}^3} + \frac{C_6^3}{C_{18}^3} = \frac{65}{816}$$

In a shop there are **18 packs** of cigarettes are putted on a stand :

- **7** Marlboro packs.
- **5 Gitane** packs.
- **6 Winston** packs.

A customer wants to buy 3 packs of cigarettes. For this he ***selects 3 packs*** from the Stand ***simultaneously & randomly***.

2) Calculate the probability of each event:

A" *The 3 chosen packs are 2Gitane & 1 Winston* " ; **B**" *The 3 chosen packs are of same type*"

C" *The 3 chosen packs are one of each type* " ; **D**" *at least one Winston pack* "

$$P(C) = P(1Marlboro\&1Gitane\&1Winstton) = \frac{C_7^1 \times C_5^1 \times C_6^1}{C_{18}^3} = \frac{35}{136}$$

$$P(D) = P(at\ least\ one\ winstone) = 1 - P(None\ winstone)$$

$$= 1 - P(3\ none\ winstone) = 1 - \frac{C_{12}^3}{C_{18}^3} = \frac{149}{204}$$

1 In a shop there are **18 packs** of cigarettes are putted on a stand :

- **7** Marlboro packs.
- **5 Gitane** packs.
- **6 Winston** packs.

3) The prices of each kind of cigarettes are given in the table below:

	Marlboro	***Winston***	***Gitane***
Prices	3000LL	2000LL	4000LL

a) Calculate the probability of the event:

E" *the amount paid by the customer is* **9000**LL"

b) Calculate the probability of the event :

" the *amount paid by the customer doesn't exceed 8000LL"* is.

$3)\ a)\ P(E) = P(Amount\ Paid\ is\ 9000LL) = P(3\ marlboro) + P(1Marlboro\&1Win\&1Git)$

$$=\frac{C_7^3}{C_{18}^3} + \frac{C_7^1 \times C_5^1 \times C_6^1}{C_{18}^3} = \frac{245}{816}$$

1 In a shop there are **18 packs** of cigarettes are putted on a stand :

- **7** Marlboro packs.
- **5 Gitane** packs.
- **6 Winston** packs.

3) The prices of each kind of cigarettes are given in the table below:

	Marlboro	*Winston*	*Gitane*
Prices	3000LL	2000LL	4000LL

a) Calculate the probability of the event:

E" *the amount paid by the customer is* **9000**LL"

b) Calculate the probability of the event :

" the *amount paid by the customer doesn't exceed 8000LL"* is.

$3)\ b)\ P(F) = P(Amount\ Paid\ does\ not\ exceed\ 8000LL) =$

$P(3\ Winston) + P(2\ Winston\&1Mar) + P(2\ Winston\&1Git) + P(2Marlboro\&1Win)$

$$=\frac{C_6^3}{C_{18}^3} + \frac{C_6^2 \times C_7^1}{C_{18}^3} + \frac{C_6^2 \times C_5^1}{C_{18}^3} + \frac{C_7^2 \times C_6^1}{C_{18}^3} = \frac{163}{408}$$

2 An urn contains 7 balls :

- 1 red ball carrying the number 0
- 2 white balls carrying the number 1
- 1 green ball carrying the number 2
- 3 black balls carrying the number 3.

3 balls are drawn ***simultaneously & randomly***.

1) How many possible drawings are there?
2) Calculate the probabilities of each event:

   A" *the 3 balls are of same color "* ; **B**" *2 black & 1 white"*

   C" *1red ,1green ,1 white"* ; **D**" *at least one black"* ; **L**" *at least 2 black"*
3) The 3 drawn balls form a 3-digit number(abc)

   Consider the events:

   E" *the product of the digits is 0"* ; **F**"*the sum of the digits is 6"*

   a) Calculate P(E) , P(F).

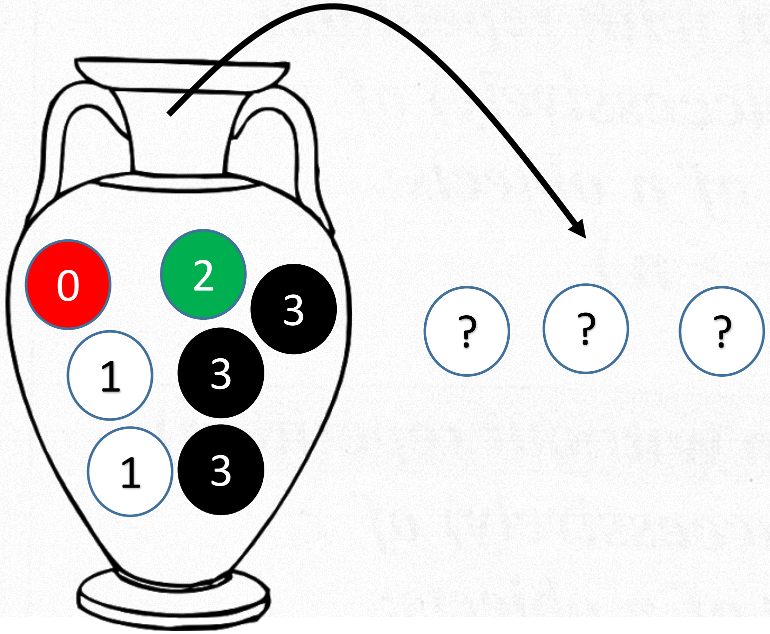
   b) Calculate: $P(E \cap F)$. Deduce $P(E \cup F)$.

2 An urn contains 7 balls :

- 1 red ball carrying the number 0
- 2 white balls carrying the number 1
- 1 green ball carrying the number 2
- 3 black balls carrying the number 3.

3 balls are drawn ***simultaneously & randomly***.

1) How many possible drawings are there?

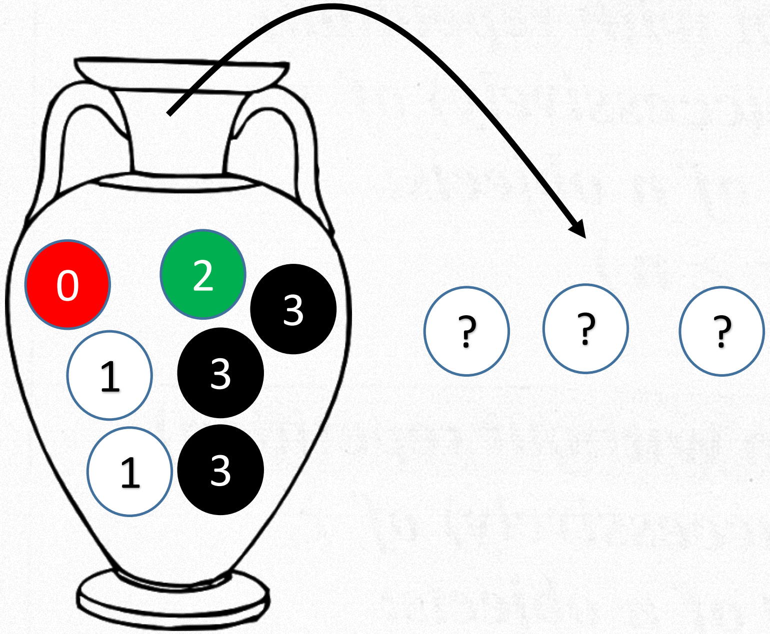


- **Simultenously**, then the order is not considered. We use C_n^r
- Total number is 7 = n= Number of total elements
- Chosen : 3 = r = number of chosen elements simultaneously.

$$Number\ of\ total\ cases:\ C_7^3$$

2 An urn contains 7 balls :

- 1 red ball carrying the number 0
- 2 white balls carrying the number 1
- 1 green ball carrying the number 2
- 3 black balls carrying the number 3.



3 balls are drawn ***simultaneously & randomly***.

1) How many possible drawings are there?
2) Calculate the probabilities of each event:

A" *the 3 balls are of same color "* ; **B**" *2 black & 1 white"*

C" *1red ,1green ,1 white"* ; **D**" *at least one black"* ; **L**" *at least 2 black"*

2) P(A)=P(3 black)=$\frac{3C3}{7C3} = \frac{1}{35}$

P(B)=P(2 black and 1 white)=$\frac{3C2\times2C1}{7C3} = \frac{6}{35}$

P(C)=P(1 red and 1 green and 1 white)=$\frac{1C1\times1C1\times2C1}{7C3} = \frac{2}{35}$

P(D)=P(At least 1 black)=

1- P(no black)=1-$\frac{4C3}{7C3} = \frac{31}{35}$

P(D)=P(At least 2 black)=

P(2black&1$\overline{black}$)+P(3black)=13/35

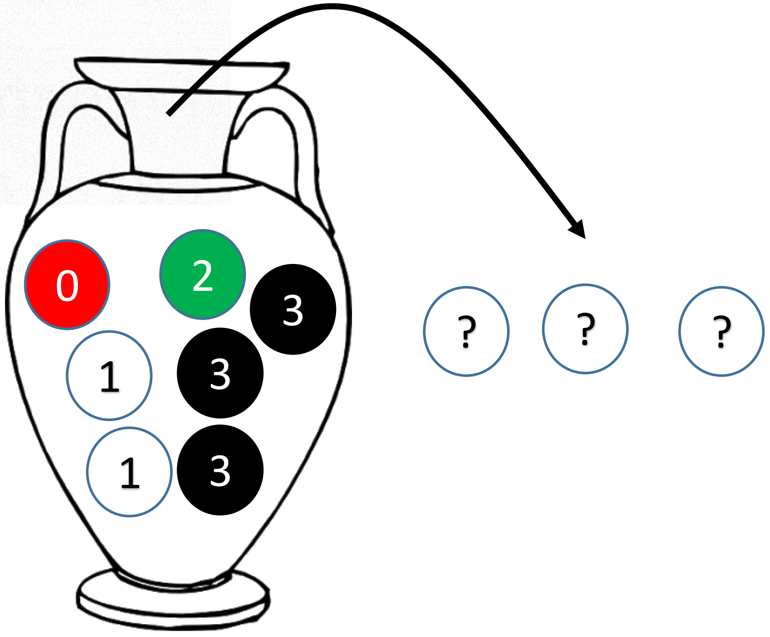
3) The 3 drawn balls form a 3-digit number(abc)

Consider the events:

E" *the product of the digits is 0"* ; *F"the sum of the digits is 6"*

a) Calculate P(E) , P(F).

b) Calculate: $P(E\cap F)$. Deduce $P(E\cup F)$.



3) a) $P(E) = P(at\ least\ one\ zero)$

$$= P(1\ zero\ and\ 2\ digits\ are\ none\ zero) = \frac{1C1 \times 6C2}{7C3}$$

Or 1 – P(no zero)=1-$\frac{6C3}{7C3} = \frac{3}{7}$

$P(F) = P(\ 1\ \&2\&3) + P(2\ digits\ are\ 3\&1\ digit\ is\ 0)$

$$= \frac{2C1 \times 1C1 \times 3C1}{7C3} + \frac{3C2 \times 1C1}{7C3} = \frac{9}{35}$$

3)b) $P(E\ \cap F) = P(product\ is\ zero\ and\ sum\ is\ 6) = P(2\ digits\ are\ 3\&1\ digit\ is\ 0)$=

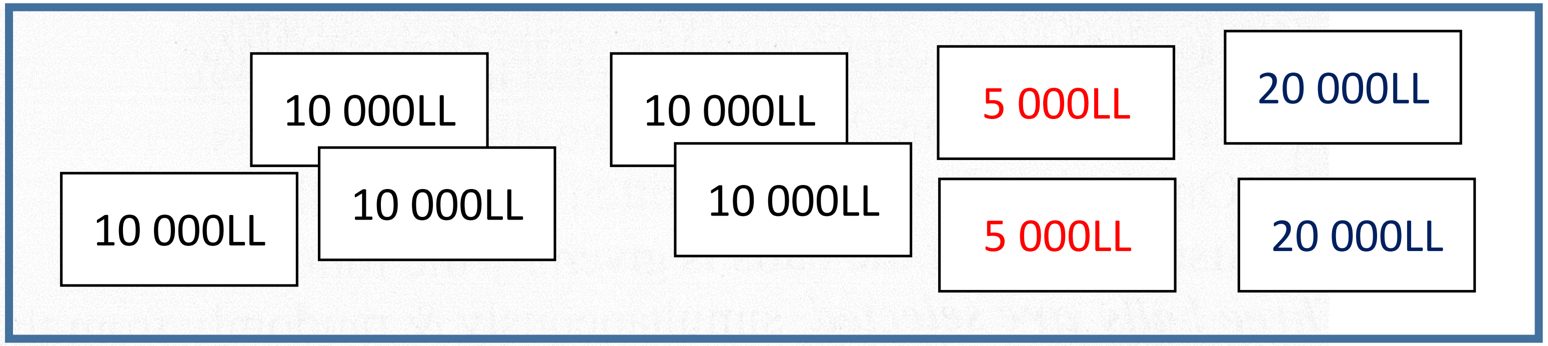
$$\frac{3C2 \times 1C1}{7C3} = \frac{3}{35}$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{3}{7} + \frac{9}{35} - \frac{3}{35} = \frac{3}{5}$$

3

In a drawer there are 9 bills :

- 5 bills of 10 000LL.
- 2 bills of 5000LL.
- 2 bills of 20 000LL.



3 *bills* are drawn from the drawer ***simultaneously & randomly***.

1) How many possible drawings are there?

2) Consider the events :

A" *the 3 bills are of* 10 000LL." ; **B**" *2 bills* of 5000LL *& one bill* of 20 000LL."

C' *at least one bill* of 10 000LL" ; **S**" the ***total amount*** of the 3 bills is 30 000LL."

Calculate the probabilities of A , B ,C & S.

3) The 9 bills are put in use in years 2003; 2004 & 2005 as shown in the table below:

year / ***Type***	***2003***	***2004***	***2005***
Bills of 5000LL	0	1	1
Bills of 10, 000LL	2	1	2
Bills of 20, 000LL	0	1	1

a) A bill of 10 000LL is selected from the drawer.
What is the probability that the selected bill puts in use in 2003?

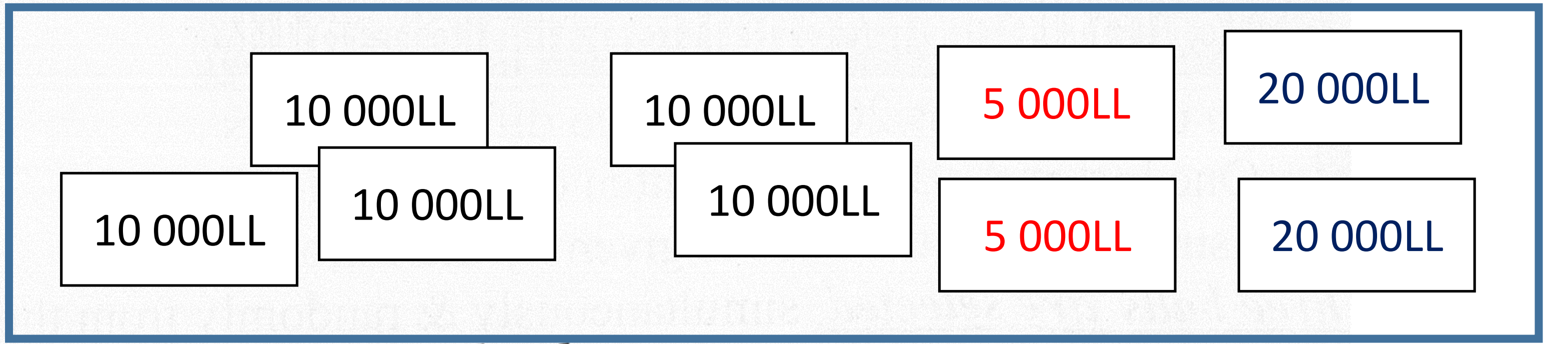
b) A bill puts in use in the year 2005 is selected from the drawer.
What is the probability that this bill is of 5000LL?

3 In a drawer there are 9 bills :

- 5 bills of 10 000LL.
- 2 bills of 5000LL.
- 2 bills of 20 000LL.

3 ***bills*** are drawn from the drawer ***simultaneously & randomly***.

1) How many possible drawings are there?

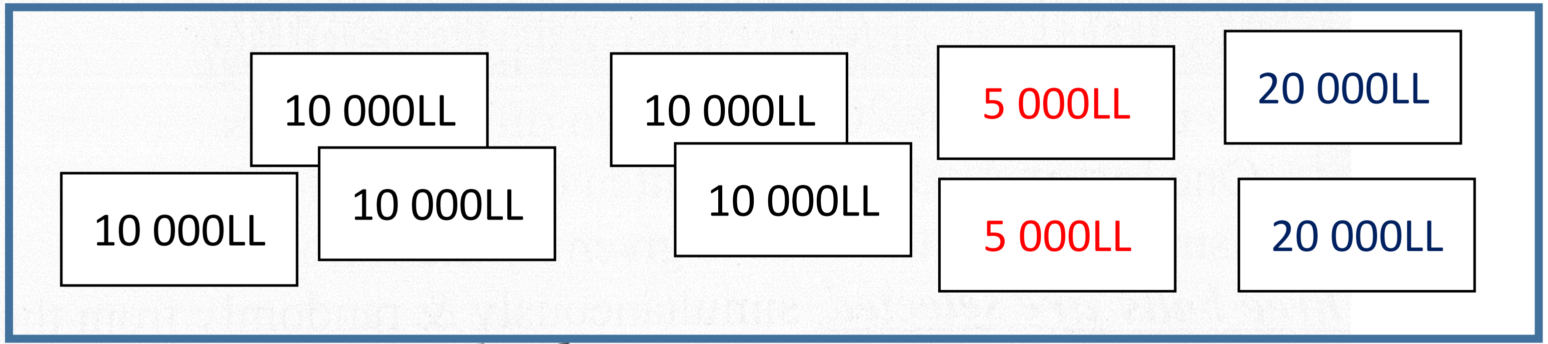


- **Simultenously**, then the order is not considered. We use C_n^r
- Total number **is 9 = n=** Number of total elements
- Chosen **: 3 = r =** number of chosen elements simultaneously.

$$Number\ of\ total\ cases:\ C_9^3 = 84$$

3 In a drawer there are 9 bills :

- 5 bills of 10 000LL.
- 2 bills of 5000LL.
- 2 bills of 20 000LL.



3 *bills* are drawn from the drawer ***simultaneously & randomly***.

1) How many possible drawings are there?

2) Consider the events :

A" *the 3 bills are of* 10 000LL." ; **B**" *2 bills* of 5000LL *& one bill* of 20 000LL."

C' *at least one bill* of 10 000LL" ; **S**" the ***total amount*** of the 3 bills is 30 000LL."

Calculate the probabilities of A , B ,C & S.

$$2)\ P(A) = P(3\ bills\ are\ of\ 10000LL)$$

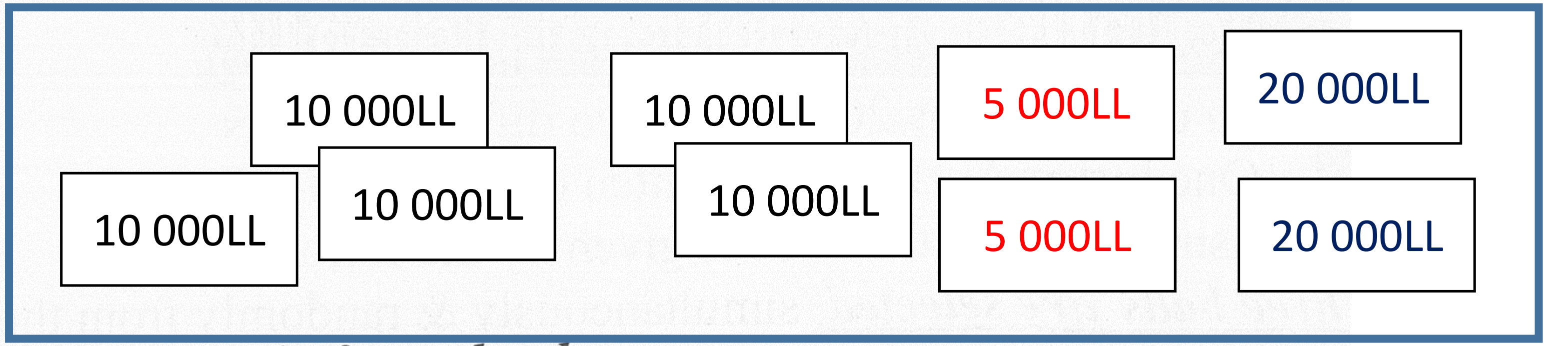
$$= \frac{C_5^3}{C_9^3}$$

$$2)\ P(B) = P(2\ bills\ are\ of\ 5000LL\&1\ of\ 20000LL)$$

$$= \frac{C_2^2 \times C_2^1}{C_9^3}$$

3 In a drawer there are 9 bills :

- 5 bills of 10 000LL.
- 2 bills of 5000LL.
- 2 bills of 20 000LL.



3 *bills* are drawn from the drawer ***simultaneously & randomly***.

1) How many possible drawings are there?

2) Consider the events :

A" *the 3 bills are of* 10 000LL." ; **B**" *2 bills* of 5000LL & *one bill* of 20 000LL."

C' *at least one bill* of 10 000LL" ; **S**" the ***total amount*** of the 3 bills is 30 000LL."

Calculate the probabilities of A , B ,C & S.

$$2)\ P(C) = P(at\ least\ one\ bill\ of\ 10000LL)$$

=1-P(no bill of 10000LL)= 1 – P(3bills are of no 10000LL)

$$= 1 - \frac{C_4^3}{C_9^3} = \frac{20}{21}$$

$$2)\ P(S) = P(Amount\ of\ bills\ is\ 30000LL)$$

=P(3 bills of 10000LL) +P(1 bill of 20000 & 2 bills of 5000LL)

$$= \frac{C_5^3}{C_9^3} + \frac{C_2^2 \times C_2^1}{C_9^3} = \frac{1}{7}$$

3) The 9 bills are put in use in years 2003; 2004 & 2005 as shown in the table below:

year / Type	2003	2004	2005
Bills of 5000LL	0	1	1
Bills of 10, 000LL	2	1	2
Bills of 20, 000LL	0	1	1

10 000LL	5 000LL	20 000LL
10 000LL	5 000LL	20 000LL
10 000LL	10 000LL	10 000LL

a) A bill of 10 000LL is selected from the drawer.
What is the probability that the selected bill puts in use in 2003?

b) A bill puts in use in the year 2005 is selected from the drawer.
What is the probability that this bill is of 5000LL?

3) $a)\ (Given\ informtaion) There\ are\ 5\ bills\ of\ 10\ 000Ll, so\ new\ total\ is\ 5$

P(in use in 2003/(it is 10000LL)=$\frac{2}{5}$

3) $b)(Given\ informtaion)\ A\ bill\ put\ in\ use\ in\ 2005$

P(bill of 5000LL/(it is in use 2005LL)=$\frac{1}{4}$

3) The 9 bills are put in use in years 2003; 2004 & 2005 as shown in the table below:

Type \ year	2003	2004	2005
Bills of 5000LL	0	1	1
Bills of 10, 000LL	2	1	2
Bills of 20, 000LL	0	1	1

10 000LL	5 000LL	20 000LL
10 000LL	5 000LL	20 000LL
10 000LL	10 000LL	10 000LL

c) A bill is chosen at random .
What is the probability it is of 2004?

$$P(\,in\ year\ 2004) = \frac{3}{9}$$

d) A bill of 20,000LL is chosen at random .
What is the probability it is of 2004?

$$P\left({}^{in\ year\ 2004}/_{20{,}000}\right) = \frac{1}{2}$$

e) A bill is chosen at random .
What is the probability it is of 10,000LL and in year 2004?

$$P(10{,}000\ and\ year\ 2004) = \frac{1}{9}$$

3) The 9 bills are put in use in years 2003; 2004 & 2005 as shown in the table below:

year / *Type*	*2003*	*2004*	*2005*
Bills of 5000LL	0	1	1
Bills of 10, 000LL	2	1	2
Bills of 20, 000LL	0	1	1

10 000LL	5 000LL	20 000LL
10 000LL	5 000LL	20 000LL
10 000LL	10 000LL	10 000LL

4) In this case, 3 bills are chosen successively and with replacement at random.

What is the probability of each of the following events?

A: " 3 bills of same kind" B: " 3 bills of sum equal to 25,000LL"

C: " 3 bills of <u>sum</u> more than 20,000 LL"

D:" 3 bills of the same year in use"

E: " 3 bills of sum 30,000LL , knowing that all in the year 2005"

F: " At least one of the bills in the year 2003"

$$P(A) = P(5000{,}5000{,}5000) + P(10000{,}10000{,}10000) + P(20000{,}20000{,}20000) =$$

$$= \frac{2}{9} \times \frac{2}{9} \times \frac{2}{9} + \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} + \frac{2}{9} \times \frac{2}{9} \times \frac{2}{9} = \frac{47}{243}$$

3) The 9 bills are put in use in years 2003; 2004 & 2005 as shown in the table below:

Type \ *year*	*2003*	*2004*	*2005*
Bills of 5000LL	0	1	1
Bills of 10, 000LL	2	1	2
Bills of 20, 000LL	0	1	1

10 000LL	5 000LL	20 000LL
10 000LL	5 000LL	20 000LL
10 000LL	10 000LL	10 000LL

4) In this case, 3 bills are chosen successively and with replacement at random.

What is the probability of each of the following events?

A: " 3 bills of same kind" B: " 3 bills of sum equal to 25,000LL"

C: " 3 bills of sum more than 20,000 LL"

$$P(B) = P(10000{,}10000{,}5000) \times 3 = \frac{5}{9} \times \frac{5}{9} \times \frac{2}{9} \times 3 = \frac{50}{243}$$

$$P(C) = P(sum\ more\ than\ 20{,}000) = 1 - P(20000\ or\ less)$$

$$= 1 - [P(sum = 2\,0000) + P(sum = 15000)] =$$

$$1 - [P(10000{,}5000{,}5000) \times 3 + P(5000{,}5000{,}5000)]$$

$$= 1 - [\frac{5}{9} \times \frac{2}{9} \times \frac{2}{9} \times 3 + \frac{2}{9} \times \frac{2}{9} \times \frac{2}{9}] = \frac{661}{729}$$

3) The 9 bills are put in use in years 2003; 2004 & 2005 as shown in the table below:

year / Type	2003	2004	2005
Bills of 5000LL	0	1	1
Bills of 10, 000LL	2	1	2
Bills of 20, 000LL	0	1	1

10 000LL	5 000LL	20 000LL
10 000LL	5 000LL	20 000LL
10 000LL	10 000LL	10 000LL

4) In this case, 3 bills are chosen successively and with replacement at random.

What is the probability of each of the following events?

D:" 3 bills of the same year in use"

E: " 3 bills of sum 30,000LL , knowing that all in the year 2005"

F: " At least one of the bills in the year 2003"

$$P(D) = P(2003,2003,2003) + P(2004,2004,2004) + P(2005,2005,2005) = \left(\frac{2}{9}\right)^3 + \left(\frac{3}{9}\right)^3 + \left(\frac{4}{9}\right)^3 = \frac{11}{81}$$

$$P(E) = P((10000,10000,10000)/2005) + P((5000,5000,20000)/2005) \times 3 =$$

$$\left(\frac{2}{4}\right)^3 + \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right) \times 3 = \frac{11}{64}$$

2005
1
2
1

$$P(F) = 1 - P(no\ bill\ in\ 2003) = 1 - P(\overline{2003}, \overline{2003}, \overline{2003}\ ) = 1 - \left(\frac{7}{9}\right)^3 = \frac{386}{729}$$