2 Exercises and problems

N°1
Potential energy

An elastic spring (R) of negligible mass and stiffness $k = 50$ N/m and of unjoint turns lies in the vertical plane and carries at its lower extremity a small solid (S) of mass $m = 250$ g. $g = 10\ m/s^2$.

1) Calculate at equilibrium, the elongation of the spring. Deduce its elastic potential energy.
2) Let O be the position of the center of inertia of (S) at equilibrium. Take the zero reference of gravitational potential energy as the horizontal plane passing through O.
We displace (S) from its equilibrium position at O, vertically downward, by a distance ($b = 5$ cm). Calculate then, the potential energy of the system [Earth ; (R) ; (S)].

N° 2
Conservation of mechanical energy

A solid (S), of mass $m = 0.5$ kg, can move freely on a track situated in the vertical plane.
The kinetic energy E_K, potential energy E_{PG} and the mechanical energy E_m of the system [Earth ; (S)] at different instants are given in the table below :

Date	t_1	t_2	t_3	t_4
E_K (J)	0.6	0.8		0.1
E_{PG}(J)	0.7		−1.6	
E_m(J)				

1) Complete the table.
2) Determine the position of (S) with respect to the reference at the instants t_2 and t_3. $g = 10\ m/s^2$.

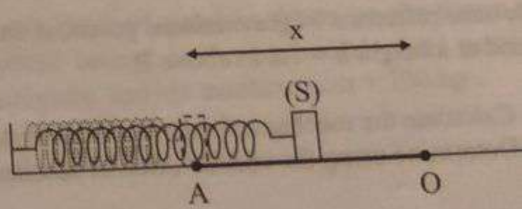
N°3
Mechanical energy of a particle in freefall

In a vertical plane of an orthonormal system $(O ; \vec{i} ; \vec{j})$, a particle M of coordinates (x,y) and mass $m = 500$ g moves with : $x = 30t$ and $y = -5t^2 + 40t$ (t in seconds, x and y in meter)where $\vec{j}$ is a unit vector which is vertically upwards. The horizontal plane passing through O is taken as a reference level for the gravitational potential energy. Take : $g = 10\ m/s^2$.

1) Determine the coordinates and the magnitude of the velocity vector of the particle at the instants 0 ; 2 s ; 10 s.
2) Calculate the mechanical energy of the system S (Earth – particle) at the preceding instants.
3) Show that the mechanical energy of the system S is conserved.

N° 4
Conservation of the mechanical energy « 1 »

A solid (S), of mass $m = 250$ g, is connected to a spring (R) with a horizontal axis and of a negligible mass, unjoint turns and constant $k = 40$ N/m. (S) can move on a horizontal rail X'X.



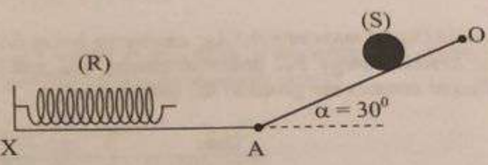
At equilibrium, (S) is at O. We displace (S) to a point A, the spring is then compressed by a distance $x = OA = 10cm$.

At $t = 0$, we release (S) without speed. At the instant t_1 the solid passes by the point O with a speed $\vec{V}_0$ and the spring regains its free length. We neglect friction. The zero level of gravitational potential energy is the horizontal plane passing through the axis of (R).

1) Indicate the form of the energy stored in the system [Earth ; (S) ; (R)] at $t = 0$.
2) Calculate the mechanical energy of the system [Earth ; (S) ; (R)] at the instant $t = 0$. Deduce the value V_0 of the speed $\vec{V}_0$.
3) Find the speed of (S) when it is midway between A and O.
4) Determine the magnitude of the tension in the spring when (S) has a speed $V = 0.8$ m/s.

N° 5
Conservation of mechanical energy « 2 »

A solid (S), of mass $m = 200$ g and of small dimensions, is launched without initial speed from the top O of an inclined plane. (S) moves along the line of greatest slope OA as shown in the adjacent figure. (R) is a spring of constant $k = 40$ N/m.

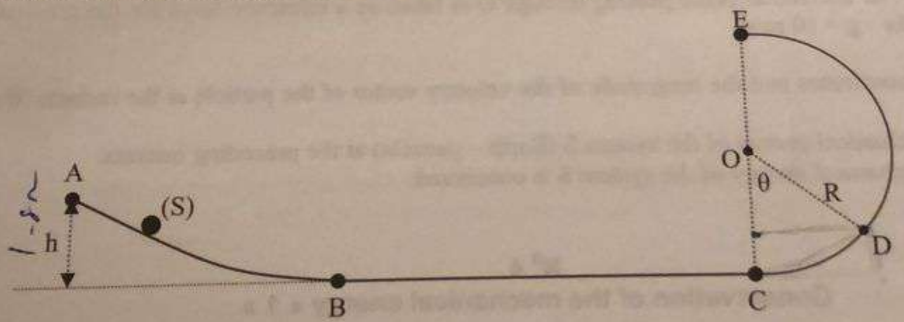


We consider the horizontal plane passing through AX as the reference level of gravitational potential energy. In this exercise we neglect the frictional forces. Take : $g = 10$ m/s^2, OA = 50 cm .

1) Calculate the mechanical energy of the system [(S) ; spring ; Earth].
2) Find the speed of (S) when it passes through the point A.
3) The solid (S) passes by the point A, slides on a horizontal and rectilinear track AX, and compresses the spring (R). Determine the maximum compression of the spring.

N° 6
Conservation of mechanical energy « 3 »

A solid (S), of mass $m = 0{,}1$ kg, is launched without speed from a point A on the track ABCDE situated in the vertical plane as indicated in the figure below. Friction is negligible. Take $g = 10$ m/s^2.



The zero reference of gravitational potential energy is the horizontal plane passing through BC. The point A is found at a height $h = 1.8$ m above B.

1) Calculate the mechanical energy of the system [(S) , earth] when (S) leaves point A.
2) Determine using the conservation of mechanical energy, the speed V_B of (S) when it passes by point B.

3) CDE is a half circular part of radius $R = OC = OD = 2$ m.

a) Applying the conservation of mechanical energy, verify that the speed of (S) when it reaches the point D on the circular track ($\widehat{COD} = \theta$) is given by : $V_D = \sqrt{2g[h - R(1 - \cos\theta)]}$.

b) Deduce the value θ_m of θ corresponding to highest position reached by (S) along CDE.

N°7
Conservation of mechanical energy « 4 »

Two identical springs, of unjoint turns, such that each one is characterized by a free length $L_0 = 20$ cm and a stiffness $k = 25$ N/m.

The two springs have the same horizontal axis x'Ox and intercalate between them a small mass $M = 400$ g as shown in the figure. O is the midpoint of the segment [AB] with $AB = 60$ cm.
We neglect frictional forces and we take the gravitational potential energy zero at O.

1) Calculate at equilibrium, the potential energy of the system S (Earth – springs – M – supports).
2) We place M at a point I of abscissa $x = +5$ cm. Calculate the potential energy of the system S.
3) M being at I, is released without speed. Determine the velocity vector of M when it passes for the first time by O.

N° 8
Variation in the internal energy of a system

A ping-pong, of mass $m = 2.7$ g, and of radius $R = 2$ cm, is left, in air without initial speed, from a point O of height $H = 24$ m from the ground. The ball moves, vertically downward, on a rectilinear path and reaches a limiting speed $V_\ell = 9\,\text{m/s}$ after covering 10 m.

The ball is subjected to air resistance represented by a force $\vec{f}$ opposite to its displacement and of a value f proportional to the speed V such that: $f = 6\pi\eta RV$ (η is a positive constant called the coefficient of viscosity of air). Given $g = 9.8$ m/s^2.

1) Applying Newton's second law « $\sum \vec{F} = m\dfrac{d\vec{V}}{dt}$ » on the ball :

a) Calculate $\dfrac{dV}{dt}$ as a function of m, g, η, R and V.

b) The limiting speed of the ball is reached when its motion becomes uniform. Determine the expression of the limiting speed as a function of m, g, η, R.

c) Deduce the value of η and indicate its SI unit.

2) a) Give the speed of the ball when it reaches the ground.

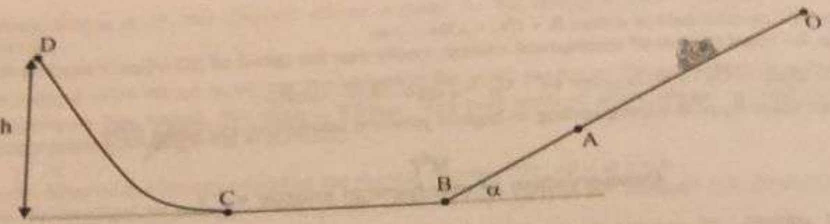
b) Calculate for the system [Earth, ball, air], the variations of the following energies: kinetic, gravitational potential, mechanical, total and internal from when the ball leaves O till it reaches the ground.

c) Give the form of energy that the variation of mechanical energy of the system appears in.

N° 9
Motorcyclist and internal energy

A motorcyclist descends with a turned off engine, from point O and without speed, along the line of greatest slope OB of an inclined plane by an angle $\alpha = 30^0$ with respect to the horizontal as shown on the figure of the following page. The reference level of gravitational potential energy is the horizontal plane passing through B. We neglect all resistive forces. The mass of the motorcyclist and his machine is $m = 100$ kg .

Take : $OA = 50$ m, $AB = 30$ m and $g = 10$ m/s^2.



1) a) Calculate the mechanical energy of the system (Earth ; motorcyclist ; machine) at O.
b) Deduce the speed of the motorcyclist at A.
2) At A, the motorcyclist starts braking and attains the point B with a speed of 63 km/h. We suppose that the magnitude f of the braking force is constant. Find f.
3) On BC the motorcyclist starts his motor and climbs the track CD, situated in a vertical plane, with constant speed of 63 km/h. Given : h = 35 m.
a) Calculate the variation of the mechanical energy of the system (Earth ; motorcyclist ; machine ; air ; track CD) along the track CD.
b) What can we say about the total energy of this system ? Why?
c) Deduce the variation of its internal energy. Interpret this variation.

N⁰ 10
Non conservation of mechanical energy

A solid (S) of mass m = 0.6 kg is launched with a speed $\vec{V}_0$ along the line of greatest slope inclined by an angle α with respect to the horizontal as shown in figure (a).

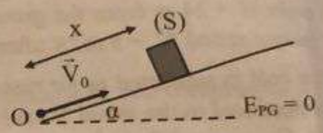


Figure (a)

In figure (b) we represent the graphs of the mechanical energies E_m and of the gravitational potential energy E_{PG} of the system [(S) ; Earth] as a function of the position x of (S).

The zero level of gravitational potential energy is taken to be the horizontal plane passing through O. Given $g = 10\ m/s^2$.

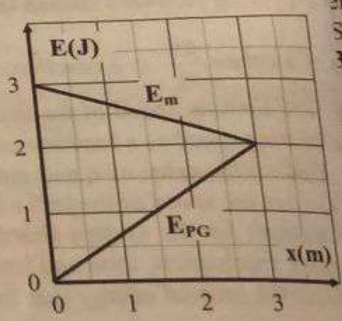


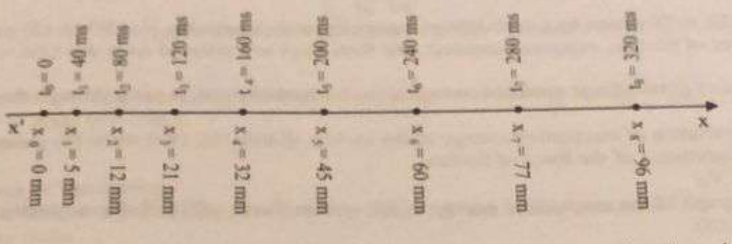
Figure (b)

1) The graph shows that the mechanical energy of the system is not conserved. Justify.
2) Extract the value of the potential energy of the system at x = 3m. Deduce the kinetic energy of the system for x = 3 m.
3) Find α.
4) Calculate the initial value of the kinetic energy. Deduce the value V_0 of $\vec{V}_0$.
5) The non conservation of the mechanical energy is due to a force of friction between (S) and the support. Calculate the magnitude of this force supposed constant.

N⁰ 11
Energetic study of a moving puck

puck (S) of mass M = 600 g is placed on a horizontal table inclined by an angle α with respect to the horizontal. Once (S) is launched, it descends without friction along the line of greatest slope x'x of the table

At an instant $t_0 = 0$, taken as an origin of time, the puck starts recording the successive positions of its center of inertia G for successive and constant intervals of time $\tau = 0.04$ s. The position G_i of G, at the instant t_i, is represented on x'x by an abscissa x_i as shown in the figure below.



The zero reference of gravitational potential energy is the horizontal plane passing through the position of the puck at the instant t_5. Take $g = 10\ m/s^2$.

1) a) The mechanical energy of the system [Earth ; (S)] is conserved. Justify.
b) Calculate the mechanical energy of the system [Earth ; (S)].
2) a) Complete the empty boxes in the following table:

t(ms)	0	40	80	120	160	200	240	280	320
V(m/s)									
E_m(mJ)									
E_K(mJ)									
E_{PG}(mJ)									

b) Represent as a function of time and in the same reference, the curves of the mechanical energy, the kinetic energy, and the gravitational potential energy of the system [Earth ; (S)].
Scale : 1 cm ↔ 0.04 s for abscissa and 1 cm ↔ 0.005 J for ordinate.
3) a) Show that the gravitational potential energy of the system at the instant t_1 is :

$$E_{PG(t_1)} = Mg.(x_5 - x_1)\sin\alpha .$$

b) Calculate the angle α and the speed of G at the instant $t_0 = 0$.

N° 12
Measuring the force of friction

A spring (R) of a horizontal axis with negligible mass, of un-joint turns and stiffness k = 40 N/m, is fixed by one of its extremities to a support.
The spring's other extremity is free and found at point O [figure (a)].

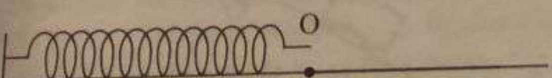


Figure (a)

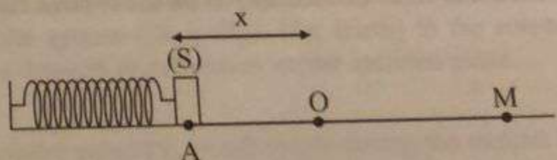


Figure (b)

- **Course reminders (Mind map)**
- **Action Verbs**
- **Method sheet**
- **Problems**

We displace the free extremity of the spring along its axis from the point O to the point A by distance $x = 10$ cm. We place in front of the spring a (S) of mass $m = 250$ g [figure (b)].

At $t_0 = 0$, we release (S) without initial speed. The spring expands and moves (S) without friction from A to O.

At the instant t_1 the solid passes by O and leaves the spring with a velocity $\vec{V}_0$. When (S) passes by O it is subjected to a force of friction, supposed constant, and then stops at a point M such that $OM = 20$ cm.

The zero reference of gravitational potential energy is the horizontal plane passing through the axis of spring.

1) Calculate the variation of mechanical energy of the system [Earth; (S); (R)] when (S) passes from A to M.
2) Deduce the magnitude f of the force of friction.
3) Find the speed V_0.
4) Represent the graph of the mechanical energy of the system [Earth ; (S) ; (R)] as a function of the distance "d" between A and (S).

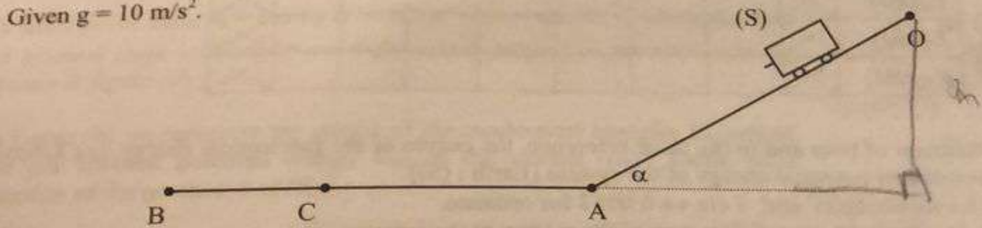
N° 13

Conservation and non conservation of mechanical energy

A chariot (S), of small dimensions and mass $m = 300$ g, is released without speed from the top O of inclined plane OA ($OA = 40$ cm) making with the horizontal an angle $\alpha = 30^0$. The chariot descends, without friction, on the inclined plane and attains the lowest point A of the plane.

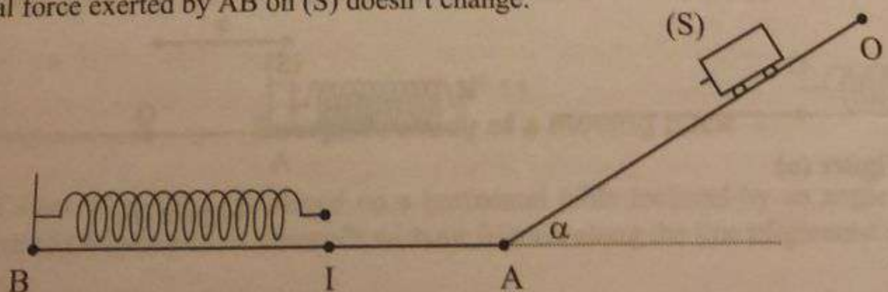
The reference of the gravitational potential energy is the horizontal plane passing through A.

Given $g = 10\ m/s^2$.



After reaching A, (S) surpasses the point A on the horizontal support AB and it stops at the point C under the action of a constant friction force of value $f = 3$ N.

1) Calculate the mechanical energy of the system [(S) ; Earth ; support], when (S) leaves the point O.
2) Deduce the speed of (S) at the point A.
3) Calculate the variation of the mechanical energy of the system [(S) ; Earth ; support] when (S) passes from A to C. Deduce the distance AC.
4) The system [(S) ; Earth ; support ; air] is energetically isolated . Determine the variation of the internal energy of the system [(S) ; Earth ; support ; air] when (S) passes from A to C. Interpret the result.
5) We do the preceding experiment again, releasing the chariot without speed from O, but on AB we place a spring of unjoint turns BI ($IA = 10$ cm) having a spring constant $k = 20$ N/m as shown in the figure below. The frictional force exerted by AB on (S) doesn't change.



a) Find the speed of (S) at I.
b) Deduce the value of the maximum compression x_m of the spring (Establish a second order equation in x_m).

N° 14
Energetic study and measuring the force of friction

We consider:

- Two springs of unjoined turns (R_1) and (R_2) of respective constants k_1 and k_2;
- A solid (S) of mass m = 250 g.

Given: $g = 10\ m/s^2$.

I – Identification of the springs

We attach (S) to the extremity of (R_1), as shown in figure (a), the spring elongates at equilibrium by 5 cm.

We repeat the experiment but with (R_2), the spring elongates at equilibrium by 10 cm. Calculate k_1 and k_2.

Figure (a)

II – Energetic study

We enroll (R_1) on a rectilinear and horizontal rail (EB) where one of its extremities is fixed at E and the other is free and is found at O [figure (b)]. The spring (R_2) is enrolled on a rectilinear rail (BF) inclined by an angle $\alpha = 30^0$ with respect to the horizontal where one of its extremities is fixed at F and the other is free and is found at C ($BC = \ell_2 = 5$ cm).

On the horizontal rail we put a rough paper represented on the figure by AB ($AB = \ell_1 = 10$ cm).

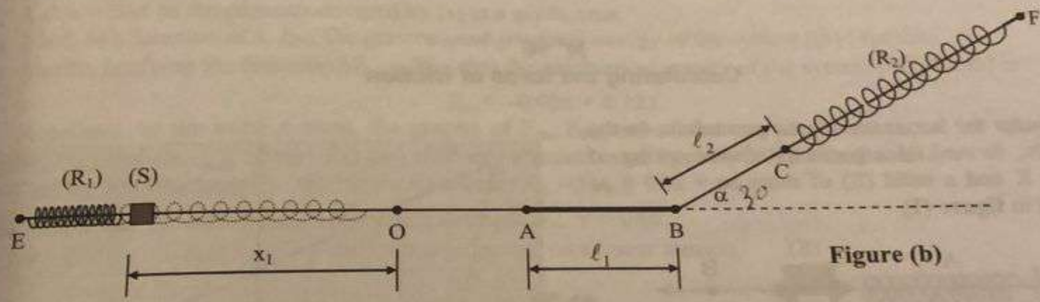


Figure (b)

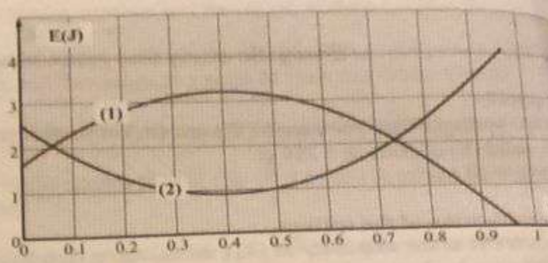
We compress (R_1) by a distance $x_1 = 15$ cm and place just in front of its free extremity a solid (S). We leave the system without initial speed, R_1 dilates and (S) moves on the horizontal rail then climbs, from B, the inclined plane with a velocity $\vec{V}_0$, it then moves towards the spring (R_2) and compresses it by a maximum distance $x_2 = 6$ cm.

In the entire problem we neglect all frictional forces except for those on [AB]. The zero level of gravitational potential energy is the horizontal plane passing through (AB).

) Is the mechanical energy of the system [(R_1); (R_2); (S); Earth] conserved? Why ?
) Determine the mechanical energies E_{m1} and E_{m2} of the system [(R_1); (R_2); (S); Earth] in the respective ases : **a)** (S) is before A ; **b)** after (S) passes through B and moves on the inclined plane.
) Calculate the magnitude V_0 of the velocity $\vec{V}_0$
Determine the magnitude of the force of friction, supposed constant, which exists during the motion of (S (AB).
(S) re-descends the inclined plane and stops at a point M between A and B. Find the length of BM.

N° 15
The energies as a function of time

A ball (S), of mass m = 200 g, is launched at the instant $t_0 = 0$ s in space, from a point O at a height h from the ground with a speed $\vec{V}_0$ making an angle α with the horizontal. The reference of gravitational potential energy is horizontal level of the ground.



The curves (1) and (2) in the figure above represent, as a function of time, the kinetic and the potential energy of the system [Earth ; (S)]. Given g = 10 m/s².

1) Curve (1) corresponds to the gravitational potential energy. Justify.
2) Verify at three instants of your choice, the conservation of the mechanical energy of the system [Earth ; (S)].
3) Represent in the preceding reference, the graph of the mechanical energy of the system [Earth ; (S)].
4) With the aid of the graph, determine :
a) The speed V_0 and the height h.
b) The instant of collision of (S) with the ground and its speed at this instant.
c) The minimal speed V_{min} of (S).
5) a) Give the direction of the minimal velocity $\vec{V}_{min}$ of (S).
b) Knowing that the horizontal component of the velocity of (S) is constant. Calculate α.

N° 16
Calculating the force of friction

We consider the horizontal elastic pendulum, on the axis x'Ox, formed of a perfectly elastic spring of constant K and a solid (S) of mass m = 250 g as indicated in **figure (1)**.

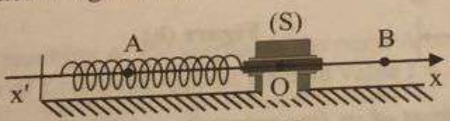


Figure (1)

When (S) is at O, the spring is neither compressed nor elongated. We displace (S) along x'Ox, to a point A, of abscissa $x_A = -5$ cm, and then we release it at the instant $t_0 = 0$, without speed, (S) displaces in the positive direction and reaches the point B of abscissa $x_B = +4$ cm at which the spring elongates to its maximum.

An appropriate device records, as (S) moves from point A to point B, the variation of the mechanical energy E_m and that of the potential energy E_P of the system (Pendulum ; Earth) as a function of the abscissa x of the center of mass (S) as shown in **figure (2)**.

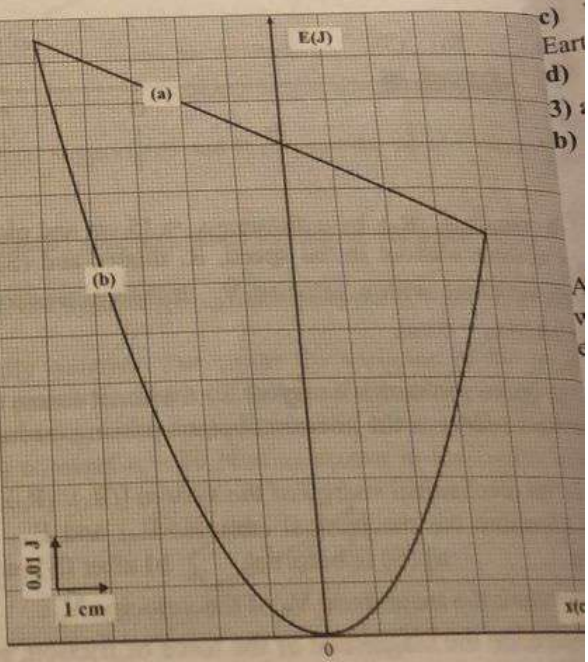


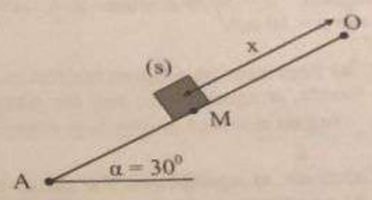
Figure (2)

The zero level of the gravitational potential energy is the horizontal plane passing through x'Ox.

1) Give the expression of the potential energy E_P.
2) Specify, between the graphs (a) and (b) the one corresponding to E_P.
3) The mechanical energy of the system (Pendulum ; Earth) is not conserved. Justify.
4) Using graphs (a) and (b), calculate the speed V of (S) when it passes through the point of abscissa $x = +2$ cm.
5) a) Using the graph shows that : $\frac{dE_m}{dx} = \mu$ where μ is a constant whose value and unit are to be determined.
b) Deduce the magnitude of the force of friction between (S) and the support ?

N° 17
Sliding of a solid on a rough support

We place at the top O of a plane, inclined by an angle $\alpha = 30^0$ with respect to the horizontal, a solid (s) of mass $m = 50\,g$. We release (s) without speed, it slides with friction along the line of greatest slope OA ($OA = d = 50\,cm$) and reaches the lowest point A of the plane with a speed $V_A = 2\,m/s$.
The reference level for gravitational potential energy is the horizontal plane passing through A. Take : $g = 10\ m/s^2$.



1) a) Verify, by calculation, that the mechanical energy of the system [(s) ; Earth] is not conserved.
b) Deduce the magnitude of the force of friction supposed constant.
2) Let $x = OM$ be the distance covered by (s) at a given time.
a) Find, as a function of x, E_{PG} the gravitational potential energy of the system [(s) ; Earth].
b) Verify, applying the formula $\Delta E_m = W_{\vec{f}}$, that the mechanical energy of the system [(s) ; Earth] is :

$$E_m = -0.05x + 0.125.$$

c) Represent, in the same system, the graphs of E_m, E_{PG} and of the kinetic energy E_K of the system [(s) ; Earth] as a function of x. Scale : 1 cm ↔ 0.05 m (abscissa) and 1 cm ↔ 0.025 J (ordinate).
d) Find by **two methods** the value of x such that : $E_K = E_{PG}$.
3) a) Verify that the speed of (S) at a given moment is : $V = \sqrt{8x}$
b) Show that the motion of (S) is uniform accelerated rectilinear motion.

N° 18
Launching a solid

A spring (R), of unjoined turns, of negligible mass and stiffness K = 25 N/m, is placed in a vertical tube whose lower end is fixed. We place at the highest extremity of the spring a solid (S) of mass M = 50 g. At equilibrium, the spring is compressed a distance *a*. [**figure (1)**]. **We neglect the friction**. $g = 10\ m/s^2$.

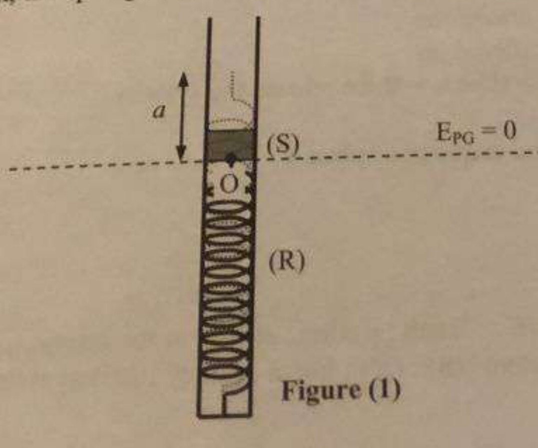


Figure (1)

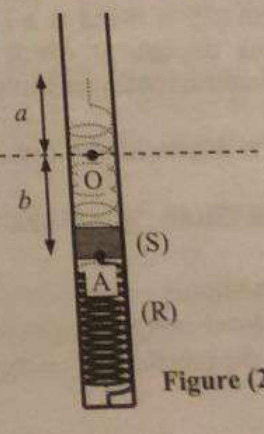
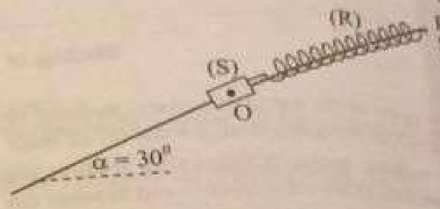


Figure (2)

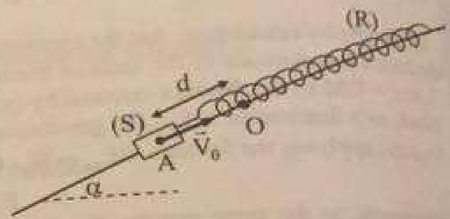
1) Calculate a. Deduce that the potential energy of system [Earth-(S)-(R)], at equilibrium, is : 5×10^{-3} J
2) We press on (S), the spring is compressed by a supplementary distance $b = 3$ cm.
a) Calculate the potential energy of the system [Earth-(S)-(R)].
b) We release (S) without speed. Calculate :
— the speed of (S) when it passes again by its equilibrium position.
— the maximum height attained by (S), after it leaves the spring, measured with respect to the reference level of gravitational potential energy.

N° 19
Oscillation of a solid on an inclined plane



A solid (S), of mass $m = 200$ g, can slide without friction on a rail inclined by an angle $\alpha = 30^0$ with respect to the horizontal, is attached to the free extremity of a spring (R), of stiffness $k = 50$ N/m whose axis is parallel to the rail.
Take $g = 10$ m/s^2.

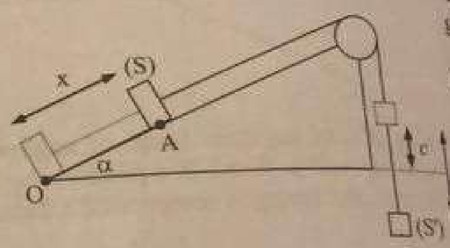
1) a) Represent on a figure the forces acting on (S).
b) Verify, at equilibrium, that the elongation of the spring is:
$$\Delta\ell = \frac{mg\sin\alpha}{k}$$
c) Calculate, at equilibrium, the elastic potential energy of the spring.
2) At equilibrium, (S) is at O. We displace (S), along the rail, to the point A at a distance $d = OA = 8$ cm and we launch it, at the time $t_0 = 0$, with a speed $V_0 = 1$ m/s parallel to rail as shown in the adjacent figure. The reference level for gravitational potential energy is the horizontal plane passing through O. Calculate the :



a) mechanical energy of system [(S)-Earth] at the time $t_0 = 0$.
b) speed of (S) when it passes by O.
c) maximum dilation $\Delta\ell_{max}$ of spring.

N° 20
Acceleration of a solid sliding on an inclined plane

In the setup of the adjacent figure, consider a solid (S), of mass $m = 120$ g, is connected to a solid (S'), of mass $m' = 80$ g, by an inextensible string and of negligible mass which passes on the periphery of a pulley of negligible mass.



At the time $t_0 = 0$, the solid (S) is at O the lowest point of the plane, inclined by an angle $\alpha = 30^0$ with respect to the horizontal, and the solid (S') is found at a height c above the horizontal plane passing through O which is considered as reference level for gravitational potential energy. At the time $t_0 = 0$ we release the system.

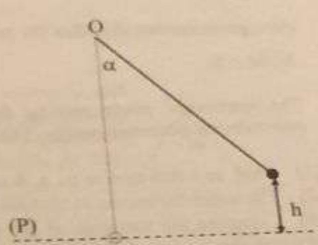
At a time t, the solid (S) is displaced by a distance x and acquires a speed V.

We neglect friction forces. Given : $g = 10$ m/s^2.

1) Calculate, as a function of :
a) m', g and c, the mechanical energy of system [(S) ; (S') ; Earth ; pulley ; string] at the instant $t_0 = 0$.
b) m, m', g, c, α, x and V, the mechanical energy of system [(S) ; (S') ; Earth ; pulley ; string] at the instant

2) Deduce the speed V as a function of m, m', g, α and x.
3) Find the acceleration a of (S).

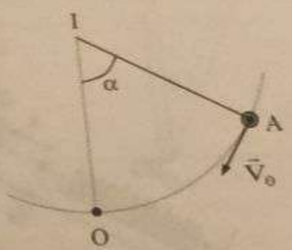
N° 21
Study of graphs



In the adjacent figure, we represent a simple pendulum, formed of an inextensible wire of length $L = 1.6$ m and a particle of mass $m = 20$ g. The reference level for gravitational energy is the horizontal plane (P) passing through the equilibrium position of the particle. We neglect friction. Take: $g = 10$ m/s^2.
We shift the pendulum from its equilibrium position by an angle $\alpha_0 = 60°$ and we release it without speed. At a given instant the pendulum forms with the vertical an angle α and the particle is found at a height h from plane (P) and acquires a speed V.

1) a) Calculate the mechanical energy of the system (pendulum ; Earth).
b) Find, in function of h, the kinetic and potential energy of the system.
2) a) Represent the graphs of mechanical, potential and kinetic energies of the system in function of h.
b) By the aid of the graphs calculate :
i – the speed of the particle when it passes by its equilibrium position.
ii – the value of α where the potential energy is equal to the kinetic energy.

N° 22
Oscillation of a simple pendulum



A simple pendulum (P), of length $L = 80$ cm, and mass $m = 50$ g, is at the position (IA) of angular displacement $\alpha = 60°$ measured with respect to the stable equilibrium position (IO).

We release (P) with a speed $V_0 = 2$ m/s as indicated in the adjacent figure.

The horizontal plane passing through O is taken as the zero reference for the gravitational potential energy.

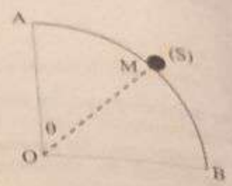
Given $g = 10$ m/s^2.

1) The mechanical energy of the system [Earth ; pendulum] is conserved ; calculate the speed of the mass m when it passes through O.
2) Determine the maximum elongation α_1 of the pendulum.
3) Find the value of α at which the potential energy of the system [Earth, pendulum] is equal to its kinetic energy.
4) The pendulum (P) oscillates. We fix at O a horizontal, plane, and rough piece of carton. Each time the mass m passes over the carton the maximum elongation decreases by 5 % from its previous value.
a) What is this decrease in the amplitude due to ? Is the mechanical energy conserved ?
b) Represent the graph of α as a function of time.
c) The initial maximum elongation of the pendulum is α_1 and that of the second (of the other side) is α_2. Verify that the nth maximum elongation has the expression : $\alpha_n = (0.95)^{n-1}\alpha_1$.
d) Calculate the variation of the mechanical energy of the system [Earth ; pendulum] between the 1st and the 20th maximum elongation.

• Course reminders (Mind map)
• Action Verbs
• Method sheet
• Problems and Exercises
• Typical subj...

N° 23
Application on the variation of mechanical energy

A punctual solid (S), of mass $m = 50$ g, is launched from a point A with negligible speed, on a spherical portion of center O. (S) describes then, in the vertical plane, a circular trajectory of radius $R = 20$ cm as shown in the adjacent figure.



At a given instant (S) takes the position of a point M on the arc AB such that: $\widehat{AOM} = \theta$.

The horizontal plane passing through A is taken as the zero level of gravitational potential energy. Take : $g = 10$ m/s^2.

1) Find, as a function of m, g, R and θ, the gravitational potential energy of the system [Earth ; (S)].
2) The solid (S) leaves its circular trajectory with a speed $V = 1$ m/s when $\theta = 70^\circ$.
a) Calculate the variation of the mechanical energy of the system [Earth ; (S)] between $\theta = 0$ and $\theta = 70°$. What is this variation due to ?
b) Calculate the magnitude of the force of friction, supposed constant, between (S) and its trajectory.

N° 24
Mechanical energy of a system

In the figure (a) we consider a solid (S) of mass $m = 100$ g ; a spring (R) of unjoint turns, negligible mass, and constant k ; a plane inclined by an angle α with respect to the horizontal.

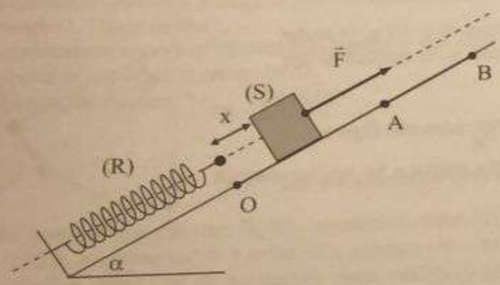


Figure (a)

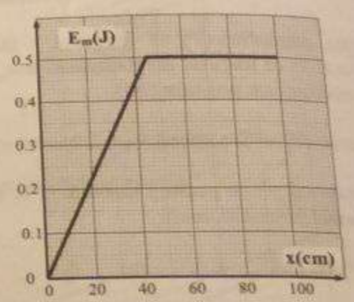


Figure (b)

The forces of friction are negligible in the entire problem. Given $g = 10$ m/s^2.

The axis of the spring is taken along the line of greatest slope of the inclined plane. (S) is at O and the spring is neither compressed nor elongated. We pull (S) with a constant force $\vec{F}$ parallel to the axis of the spring. When (S) reached the point A of the inclined plane, (S) climbs the plane towards the point B, the highest point in the trajectory. In the figure (b) we represent the graph of the mechanical energy of the system [Earth ; (S) ; spring], when (S) climbs from O towards B, as a function of the distance x between O and (S). The zero level of gravitational potential energy is taken to be the horizontal plane passing through O.

1) Using the graph, answer the following questions :
a) Is the mechanical energy of the system [Earth ; (S) ; spring] conserved ? Why ?
b) Extract the distances OA and AB.

c) Verify that : $\alpha = 30^0$.

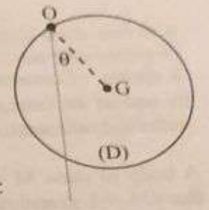
d) Knowing that : $\Delta E_m = W(\vec{F})$. Deduce the magnitude F of $\vec{F}$.

2) At the point B, (S) rebounds back and re-descends compressing the spring to a maximum distance $x_m = 12\,cm$.

a) What is the mechanical energy of the system [Earth ; (S) ; spring] in this phase ?

b) Calculate k.

N⁰ 25
*A Solid in rotation « 1 »

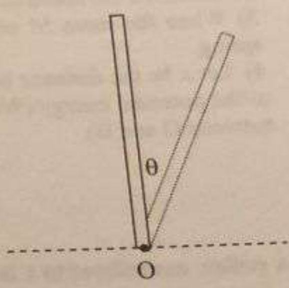


A homogeneous disk (D) of mass m, of radius R = 5 cm and of center of inertia G, can oscillate without friction around a horizontal axis perpendicular to its plane and passing through a point O of its periphery. We displace (D) from its stable position of equilibrium by an angle $\theta_1 = 40^0$ and we release it without speed. The moment of inertia of (D) : $I = \frac{1}{2} mR^2$.

1) Calculate the angular speed ω_0 of (D) when it passes through the equilibrium position.

2) Determine the value of θ_2 corresponding to an angular speed of the disk of : $\omega_2 = \frac{1}{2}\omega_0$. $g = 10\ m/s^2$.

N⁰ 26
*Launching a rod



A rectilinear and homogeneous rod, of mass m = 600 g and of length $L = 30\,cm$, can rotate around a horizontal axis (Δ) horizontal and perpendicular to its extremity O.

The rod rests vertically, as shown in the adjacent figure, leaves from rest and rotates in the vertical plane and forms, at a given instant, an angle θ with the vertical and acquires an angular speed θ'.

The reference level for gravitational potential energy is the horizontal plane passing through O.

Moment of inertia of the rod with respect to (Δ) is: $I = 18 \times 10^{-3}\,kg.m^2$.

Take : $g = 10\ m/s^2$.

1) We neglect friction. Calculate :

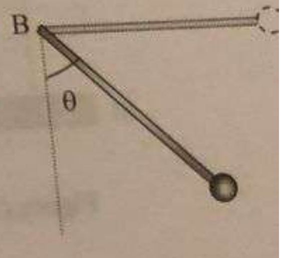
a) The mechanical energy of the system [Earth-rod].

b) The angular speed θ'_1 of the rod when it becomes horizontal.

2) In reality friction is not negligible between the rod and the axis of rotation, and the angular speed of the rod when it becomes horizontal for the first time is : $\theta'_2 = 8\,rad/s$.

We designate by $\mathcal{M}$ the moment of the friction force supposed constant. Calculate $\mathcal{M}$. Given : the work of the friction force $W = \mathcal{M}\theta$ where θ is the angle of rotation expressed in radian.

N⁰ 27
*A Solid in rotation « 2 »



A homogeneous rod AB, rigid, of length L = 60 cm and mass $M = 0.6\,kg$, can rotate, without friction, around an axis horizontal and perpendicular at B to (AB). At the extremity A of the rod we attach a punctual object of mass m = 60 g. The moment of inertia of the rod with respect to the axis of rotation is : $I_{rod} = \frac{1}{3} M\ell^2$.

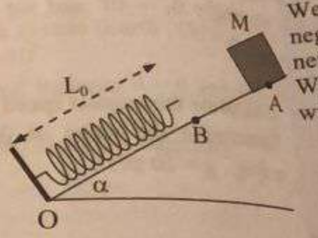
1) Verify that the moment of inertia of the pendulum is $I = 0.0936\ kgm^2$.

2) Knowing that G is the center of mass of pendulum (rod ; mass). Verify that $BG = \frac{18}{55}\text{ m} \approx 0.327\text{ m}$.

3) The pendulum initially in the horizontal position, is released without initial speed at the instant $t_0 = 0$.
The horizontal plane passing through B is taken as the zero level of gravitational potential energy.
a) Calculate the mechanical energy of the system [Earth ; pendulum ; support] at the instant $t_0 = 0$.
b) Write the expression of the mechanical energy of the system [Earth ; pendulum ; support] as a function of the angular speed of the pendulum θ' and the angle θ between pendulum and the vertical at a certain instant.
c) Determine the position of the pendulum where the angular speed is $\theta' = 3$ rad/s.

N⁰ 28
Equality between kinetic and potential energies

A spring, of constant $k = 90$ N/m and free length $L_0 = OB = 40$ cm, is fixed by one of its extremities at a point O of a plane, inclined by angle $\alpha = 30^0$ with the horizontal, as indicated in the adjacent figure.



A body of mass $M = 600$ g is placed at a point A on the inclined plane such that OA = 1 m and released without speed.
We neglect the frictional forces between the mass M and the inclined plane.
The point B is on the zero reference of gravitational potential energy.

1) Determine the mechanical energy of the system S (Earth – spring – M).
2) Calculate the speed of the mass M at the point B.
3) When the mass M reaches point B, the spring is compressed. Find the maximum compression of the spring.
4) Let x be the distance between B and the body. Determine the values of x for which kinetic energy is equal to the potential energy (We can consider two cases : **first case :** M is between A and B and **second case :** M is between O and B).

N⁰ 29
*Rotation and translation

A pulley, assimilated to a homogeneous disk, of mass m = 2 kg and of radius r, can turn without friction about a horizontal axis confounded with its axis of revolution. We enroll on the periphery of the pulley an inextensible and mass less string. Given: $g = 10\text{ m/s}^2$ and the moment of inertia of the disk : $I = \frac{1}{2}mr^2$.

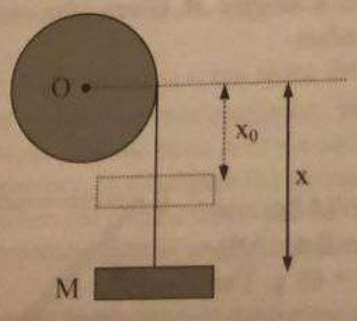


Figure (1)

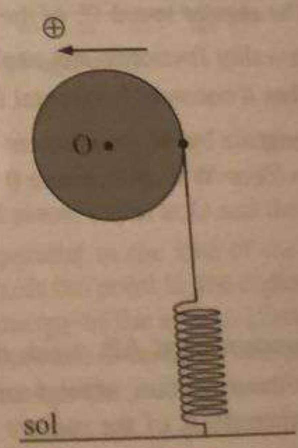


Figure (2)

A – First experiment

At the free extremity of the string we attach a mass $M = 0.5$ kg (M is of small dimensions) as indicated in figure (1).

The string is stretched, we release the mass M, which is found initially at a distance x_0 from O, without initial speed at the instant $t = 0$. At an instant t the mass M is at a distance x from O.

In this experiment the string doesn't slide on the periphery of the pulley.

1) Calculate, as a function of M, m, x, x_0 and g, the speed V of the mass M at the instant t.
2) Deduce the acceleration of M.

B – Second experiment

We fix to a point on the periphery of the pulley one of the extremities of a spring, of unjoined turns and of a negligible mass, of constant $k = 100$ N/m as indicated in the adjacent figure (2). In this case the spring is neither elongated nor compressed.

We rotate the pulley by a quarter of a turn, in the positive direction shown in the figure, and we release it without speed. Calculate the angular speed of the pulley at which it passes by its equilibrium position again.