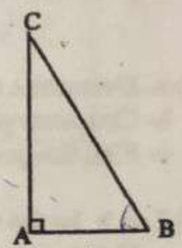
I) In an oriented plane, consider a direct triangle ABC right angled at A, such that $AB = 2cm$ and $(\overrightarrow{BC};\overrightarrow{BA}) = \frac{\pi}{3}\ (2\pi)$.



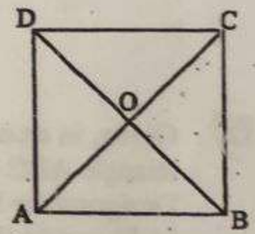
Let S be the direct similitude that transforms A onto B and B onto C.

1) Determine the ratio and the angle of S.

2) a- Construct the point C', the image of C under S. (Give the steps of this construction).
b- Calculate the area of triangle BCC'.

3) Let O be the midpoint of [AB], and consider the direct orthonormal system $(O;\vec{u},\vec{v})$ such that $\vec{u} = \overrightarrow{OB}$.
a- Find the complex form of the similitude S.
b- Determine the affix of point W, the center of S.
c- Let S^{-1} be the inverse transformation of S. Give the complex form of S^{-1}.

II) Consider, in an oriented plane, the direct square ABCD with center O such that $(\overrightarrow{AB},\overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$.



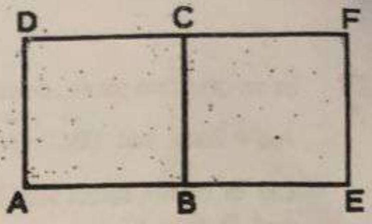
Let r be the rotation with center O and angle $\frac{\pi}{2}$ and h be the dilation (homothecy) with center C and ratio 2.
Designate by S the transformation $r \circ h$.

1) Determine the nature of S and specify its ratio and its angle.

2) Designate by W the center of S.
a- Show that $S(C) = D$ and that $S(O) = B$.
b- Construct the point W, specifying clearly the steps of this construction.

3) The plane is referred to an orthonormal system $(A;\overrightarrow{AB},\overrightarrow{AD})$.
a- Write the complex form of S and deduce the affix of the center W.
b- Determine the image of the square ABCD under S.

III) Consider, in an oriented plane, the two direct squares ABCD and BEFC.
Let S be the direct plane similitude that transforms A onto E and E onto F.

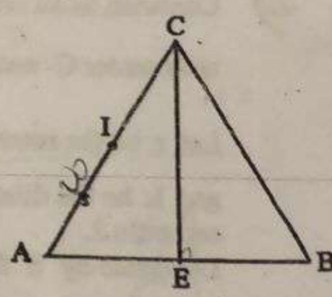


1) a- Determine the ratio k and an angle α of S.
   b- Construct geometrically the center W of S.
   c- Find the point G that is the image of F under S.

2) Let h be the transformation that is defined by $h = S \circ S$.
   a- Determine the nature and the elements of h.
   b- Specify h(A), and express $\overrightarrow{WA}$ in terms of $\overrightarrow{WF}$.

3) The complex plane is referred to an orthonormal system $(A; \overrightarrow{AB}, \overrightarrow{AD})$.
   a- Determine the affixes of the points E, F and W.
   b- Find the complex form of S.
   c- Give the complex form of h and find the affix of h(E).

IV) Given, in an oriented plane, a direct equilateral triangle ABC of side 4 cm.
Designate by E and I the mid points of [AB] and [AC] respectively.
Let S be the direct plane similitude that transforms A onto E and E onto C.

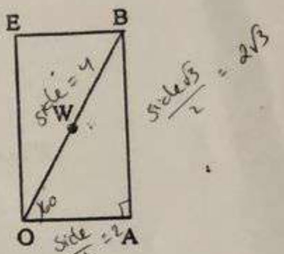


1) a- Determine the ratio and an angle of S.
   b- Construct the image under S of each of the straight lines (AC) and (EI), and deduce the image of I under S.

2) Suppose that the plane is referred to a direct orthonormal system $(A; \vec{u}, \vec{v})$ where $\vec{u} = \frac{1}{4}\overrightarrow{AB}$.
   a- Give the complex form of S.
   b- Find the affix of the point W, the center of S.
   c- Prove that W is a point on [AC].
   d- Let J be the image of the point I under $S \circ S$; Compare WC and WJ.

V) Consider in an oriented plane the rectangle OABE such that $OA = 2$ and $(\overrightarrow{OA}, \overrightarrow{OB}) = \frac{\pi}{3}$ (2π).

Designate by (C) the circle with diameter [OB] and center W.



Let S be the direct plane similitude with center O, ratio $\sqrt{3}$ and angle $\frac{\pi}{3}$.

A-

1) Let A' be a point on the semi-straight line [OB) such that $OA' = 2\sqrt{3}$.

   Prove that A' is the image of A under S.

2) a- Verify that the triangle OAW is equilateral.
   b- Determine the image under S of triangle OAW.
   c- Construct then the circle (C'), the image of (C) under S.

B-

The complex plane is referred to a direct orthonormal system $(O ; \vec{u}, \vec{v})$ such that:

$z_A = 2$ and $z_E = 2\sqrt{3}i$.

1) Write the complex form of S.
2) Find the affix of W and that of W' the image of W under S.
3) Let f be the plane transformation with complex form $z' = iz + 4 + 2i\sqrt{3}$.
   a- Show that f is a rotation whose angle and center H are to be determined.
   b- Verify that $f(W') = W$ and determine $f \circ S(W)$.
   c- Determine the nature and the characteristic elements of $f \circ S$.

VI) In an oriented plane, consider the rectangle ABCD such that:

$(\overrightarrow{AB}; \overrightarrow{AD}) = \frac{\pi}{2} (\text{mod } 2\pi)$, $AB = 4$ and $AD = 3$.

Let H be the orthogonal projection of A on (BD) and h be the dilation, of center H, that transforms D to B.

1) a- Determine the image of the straight line (AD) by h.

   b- Deduce the image E of point A by h. Plot E.

   c- Construct the point F image of B by h and the point G image of C by h, then determine the image of rectangle ABCD by h.

2) Let S be the direct similitude that transforms A onto B and D onto A.

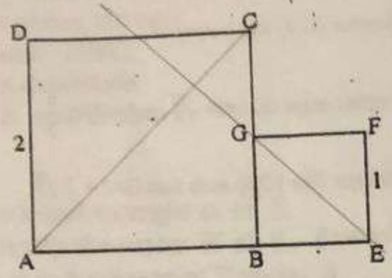
   a- Determine an angle of S.

   b- Determine the image of the straight line (AH) by S and the image of the straight line (BD) by S.

   c- Deduce that H is the center of S.

3) Show that $S(B) = E$ and deduce that $S \circ S(A) = h(A)$.
4) Show that $S \circ S = h$.

VII) In an oriented plane, consider two direct squares ABCD and BEFG of sides 2 and 1 respectively.



Let S be the direct plane similitude, of ratio k and angle α, that transforms E onto A and G onto C.

A- 1) Verify that $k = 2$ and $\alpha = -\frac{\pi}{2}\ (2\pi)$.

2) Prove that the image of point F by S is B then deduce S(B).

3) Construct the center W of S.

B- The complex plane is referred to a direct orthonormal system $(B;\ \overrightarrow{BE};\ \overrightarrow{BG})$.

1) Determine the affixes of the points A and E.

2) Write the complex form of S.

3) Determine the algebraic form of the affix of W.

4) Show that the points C, W and E are collinear.

C- Let R be the rotation with center A and angle $\frac{\pi}{2}$.

Prove that RoS is a dilation whose center and ratio are to be determined.

VIII) ABCD is a square of side 2 and of center O such that $(\overrightarrow{AB}, \overrightarrow{AD}) = \frac{\pi}{2}\ (2\pi)$.

E and F are the midpoints of [AB] and [BC] respectively and G is the midpoint of [BF].

Let S be the direct plane similitude that transforms A onto B and D onto E.

1) Calculate an angle and the ratio of S.

2) Verify that $S(B) = F$, and determine S(E).

3) Let $h = S o S$.

a- Show that h is a dilation and precise its ratio.

b- Prove that the center I of S is the point of intersection of (AF) and (DG).

c- Determine the image by S of the square ABCD and deduce the nature of triangle OIC.

4) Let (A_n) be the sequence of points defined by: $A_0 = A$ and $A_{n+1} = S(A_n)$ for all natural integers n.

a- Let $L_n = A_nA_{n+1}$ for all n.

Prove that (L_n) is a geometric sequence whose common ratio and first term are to be determined.

Calculate $S_n = L_0 + L_1 + \dots\dots\dots + L_n$ and $\lim\limits_{n \to +\infty} S_n$.

b- Calculate $(\overrightarrow{IA}, \overrightarrow{IA_n})$ in terms of n and prove that if n is even then the points I, A and A_n are collinear.

The plane is referred to a direct orthonormal system $(O;\vec{u},\vec{v})$. Let Ω be the point with affix 2.

Denote by S the transformation with complex form $z'=(1+i)z-2i$ that maps each point M in the plane with affix z onto the point M' with affix z'.

A-

1) Show that S is a similitude with center Ω. Specify the ratio and an angle of S.

2) a- Show that, for $z\neq 2$, $\dfrac{z'-z}{z-2}=i$.

b- Deduce that triangle $\Omega MM'$ is right isosceles.

B-

Let M_0 be the point with affix $2+i$.

Consider, for every natural number n, the points (M_n) in the plane such that $M_{n+1}=S(M_n)$.

1) Let (d_n) be the sequence defined, for every n, by $d_n=\Omega M_n$

a- Calculate ΩM_{n+1} in terms of ΩM_n and deduce that (d_n) is a geometric sequence whose first term is equal to 1.

b- Determine the values of n when M_n is interior to the circle with center Ω and radius 5.

2) a- Justify that a measure of the angle $(\overrightarrow{\Omega M_0},\overrightarrow{\Omega M_n})$ is equal to $\dfrac{n\pi}{4}$.

b- Determine n when Ω, M_0 and M_n are collinear.

IV) The complex plane is referred to a direct orthononormal system $(O;\vec{u},\vec{v})$.

Let A be the point of affix 2, and B be the point of affix $2i$.

Designate by E the image of A under the rotation R with center O and angle $\frac{\pi}{3}$, and by F the image of B under the transformation T that is defined by the complex form: $z'=(\frac{-1}{2}-i\frac{\sqrt{3}}{2})z$.

1) a- Determine the nature and the characteristic elements of T.

b- Prove that the four points A, B, E and F belong to the same circle with center O and whose radius is to be determined.

2) a- Prove that $\dfrac{z_B - z_A}{z_F - z_B}$ is real.

b- Verify that $\dfrac{z_F - z_A}{z_E - z_B} = -i$.

c- Deduce that AEBF is an isosceles trapezoid and that $(\overrightarrow{BE},\overrightarrow{AF}) = -\frac{\pi}{2}\ (2\pi)$.

3) Consider: the dilation (homothecy) h that transforms A onto F and E onto B, and the rotation r with angle $\frac{\pi}{2}$, that transforms B onto F.

a- Determine W, the center of h.

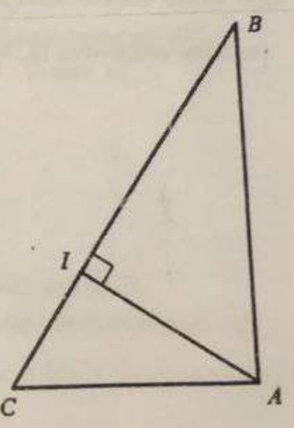
b- Prove that $h \circ r = r \circ h$.

c- Let $S = h \circ r$.
Determine the nature and the characteristic elements of S.

II)

Given a triangle ABC such that $AB = 6$, $AC = 4$ and $(\overrightarrow{AB};\overrightarrow{AC}) = \frac{\pi}{2}\ (2\pi)$

Let I be the orthogonal projection of A on (BC).



1- Let h be the dilation of center I that transforms C onto B.
Construct the image (d) of the line (AC) under h.
Deduce the image D of A under h.

2- Let S be the similitude that transforms A onto B, and C onto A.

a) Determine the ratio and an angle of S.

b) Determine the image by S of each of the two straight lines (AI) and (CB). Deduce that I is the center of S.

c) Determine the image of (AB) by S.
Deduce that $S(B) = D$.

3- a) Determine the nature and the characteristic elements of $S \circ S$.

b) Prove that $S \circ S(A) = h(A)$.

c) Prove that $S \circ S = h$.

4- Let E be the mid point of [AC].

a) Determine the points F and G such that $F = S(E)$ and $G = S(F)$.

b) Show that the points E, I and G are collinear.