chapter	quantity	formula	Instruction
Energy And motion	Average velocity: Instantaneous:	$\vec{V}_{av} = \frac{\Delta\vec{r}}{\Delta t}$ $\vec{V}_{inst} = \frac{d\vec{r}}{dt}$	memorize
	Average acceleration: Instantaneous:	$\vec{a}_{av} = \frac{\Delta\vec{V}}{\Delta t}$ $\vec{a}_{inst} = \frac{d\vec{V}}{dt}$	Memorize
	U.R.M.	$x = vt + x_0$ v=constant	Memorize
	U.A.R.M.	$x = \frac{1}{2}at^2 + v_0t + x_0$ $v = at + v_0$ a=constant	Memorize
	Newton's second law	$\sum \vec{F}_{ext.} = m\vec{a}$	Memorize
	Kinetic energy	$K.E. = \frac{1}{2}mv^2$	Memorize
	Gravitational potential energy	$P.G. = mgh$	Memorize
	Elastic potential energy	$P.E. = \frac{1}{2}K(L - L_0)^2$ Or $P.E. = \frac{1}{2}kx^2$	Memorize
	Mechanical energy	M.E.=K.E+P.G.+P.E.	Memorize
	Work done by a force along a distance d and making an angle θ	W=F.d.cos(θ)	Memorize
	Work-Energy theorem:	$W_{fr} = \Delta M.E$ Where $W_{fr} = fr.d.cos(\theta)$	Memorize $(\theta = \pi = 180^0)$
	Power of a force	$P = \frac{W}{t}$	Memorize

chapter	quantity	formula	Instruction
linear momentum	Linear momentum	$\vec{P} = m.\vec{V}$	memorize
	Fundamental relation of dynamics	$\sum \vec{F}_{ext.} = \frac{d\vec{P}}{dt}$	Memorize
	Conservation of linear momentum	$\sum \vec{F}_{ext.} = \frac{d\vec{P}}{dt} = \vec{0}$ So $\vec{P} = constant$ (conserved)	Memorize
	Elastic collision	$\vec{P}$ *and K.E. are conserved*	Memorize
	Before and after collision velocities	$V_1' = \frac{m_1 - m_2}{m_1 + m_2} V_1 + \frac{2m_2}{m_1 + m_2} V_2$ $V_2' = \frac{2m_1}{m_1 + m_2} V_1 + \frac{m_2 - m_1}{m_1 + m_2} V_1$	derive
mechanical oscillations	Differential equation of S.H.M.	$x'' + \frac{k}{m} x = 0$	Derive
	Proper angular frequency	$\omega_0 = \sqrt{\frac{k}{m}} = 2\pi f_0$	Memorize
	Proper period	$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$	Memorize
	Solution	$x = X_m sin(\omega_0 t + \varphi)$ $x = X_m cos(\omega_0 t + \varphi)$	given

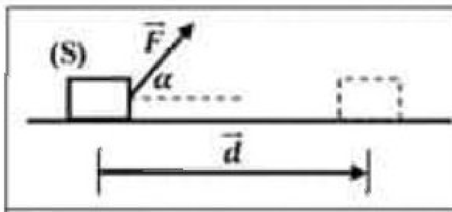
chapter	quantity	formula	Instruction
Electromagnetic induction	Magnetic flux	$\emptyset = N.B.S.cos(\theta)$	memorize
	Faraday's law	$e = -\frac{d\emptyset}{dt}$	Memorize
	Laplace force, electromagnetic force:	$F = i.L.B.sin(\propto)$	Memorize
	Ohm's law across a generator	U_G=e-ri	Memorize
	Electric power	P=U.I	Memorize
Self-induction	Ohm's law across a coil	$u = r.i - e$ Or $u = ri + L\frac{di}{dt}$	memorize

chapter	quantity	formula	Instruction
Alternating current	capacitance	$$C = \frac{Q}{U}$$	Memorize
	Stored electric energy in a capacitor	$$W_{elect.} = \frac{1}{2}CU^2$$	memorize
	Differential equation in charging of a capacitor (U_G=E)	$$u_c + RC\frac{du_c}{dt} = E$$	Derive
	solution	$$u_c = E(1 - e^{-\frac{t}{\tau}})$$ Where $\tau = RC$	Given derive
	Differential equation in discharging of a capacitor (U_G=0)	$$u_c + RC\frac{du_c}{dt} = 0$$	derive
	Solution	$$u_c = Ee^{-\frac{t}{\tau}}$$	given
	Alternating sinusoidal voltage	$$u = U_m cos(\omega_0 t + \varphi)$$ Or $$u = U_m sin(\omega_0 t + \varphi)$$	given
	Alternating current	$$i = I_m cos(\omega_0 t + \varphi)$$ Or $$i = I_m sin(\omega_0 t + \varphi)$$	given

chapter	quantity	formula	Instruction
Alternating current	Maximum voltage Period	$U_m = S_v \times y$ $T = S_h \times X$	memorize
	Maximum current in a series circuit containing a resistor R	Ohm's Law $I_m = \frac{U_R}{R}$	meemorize
	Effective voltage and current	$U_{eff} = \frac{U_{max}}{\sqrt{2}}$ $I_{eff} = \frac{I_{max}}{\sqrt{2}}$	memorize
	R-L-C circuit: Current resonance relation of RLC circuit	$LC\omega^2 = 1$	memorize
	Across a coil:	The voltage leads the current. v *leads* i *or* i *lags behind* v	Memorize
	Across a capacitor:	The current leads the voltage. i *leads* v *or* v *lags behind* i	Memorize
	For a pure inductance coil where the internal resistance of the coil is: r = 0 Ω The phase difference: $\varphi = \frac{\pi}{2}$ $rd.$		Memorize
	Average power	$P_{av} = U.I.\cos(\varphi)$	memorize

CHAPTER 1 – WORK AND ENERGY

1.1- WORK DONE BY A FORCE

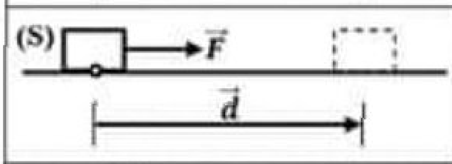


The work done by a constant force $\vec{F}$, acting on a solid (S), along a rectilinear displacement $\vec{d}$ is:

$$W_{\vec{F}} = \vec{F}.\vec{d} = F \times d \times \cos(\vec{F};\vec{d}) = F \times d \times \cos\alpha$$

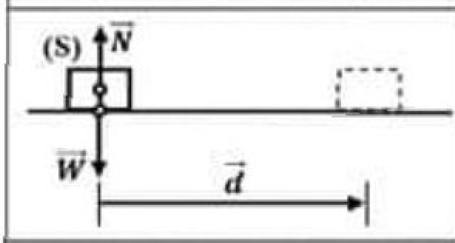
Special cases:

1- **Maximum work:**



$$W_{\vec{F}} = \vec{F}.\vec{d} = F \times d \times \cos(\vec{F};\vec{d}) = F \times d \times \cos 0° = F \times d$$

2- **The work done by forces that are perpendicular to the displacement is zero.**



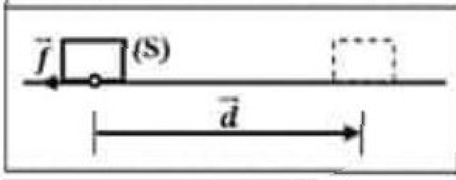
Work done the normal reaction of a support force:

$$W_{\vec{N}} = \vec{N}.\vec{d} = 0J \ (\vec{N} \perp \vec{d})$$

Work done by weight along a horizontal displacement:

$$W_{\vec{W}} = \vec{W}.\vec{d} = 0J \ (\vec{W} \perp \vec{d})$$

3- **Work done by the force of friction:**

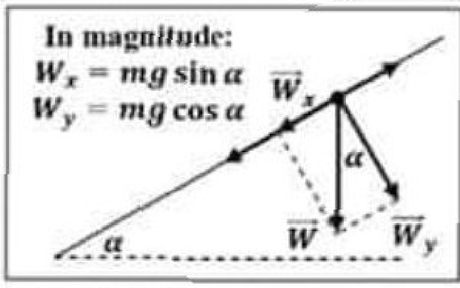


$$W_{\vec{f}} = \vec{f}.\vec{d} = f \times d \times \cos(\vec{f};\ \vec{d}) = f \times d \times \cos 180° = -f \times d$$

4- **Work done by weight along any displacement:**

$$W_{\vec{W}} = mg(h_i - h_f)$$

5- **Work done by weight along an inclined plane:**



$$W_{\vec{W}} = W_{\vec{W}_x} + W_{\vec{W}_y} = \vec{W}_x.\vec{d} + \vec{W}_y.\vec{d} = W_x \times d \times \cos(\vec{W}_x;\ \vec{d}) + 0$$

Downwards motion: $(\vec{W}_x;\ \vec{d}) = 0° \Rightarrow W_{\vec{W}} = +mgd\sin\alpha$

Upwards motion: $(\vec{W}_x;\ \vec{d}) = 180° \Rightarrow W_{\vec{W}} = -mgd\sin\alpha$

1.2- ENERGY

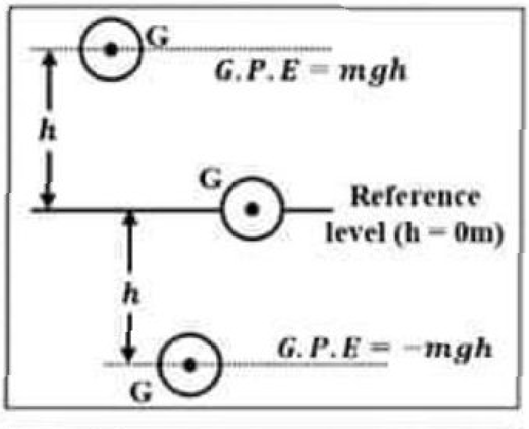
Energy is the capacity for doing work.

Kinetic energy ($K.E$ or E_k): energy possessed by an object (or system) due to its motion.

$$K.E = \frac{1}{2}mV^2$$

Variation in kinetic energy (work-kinetic energy theorem):

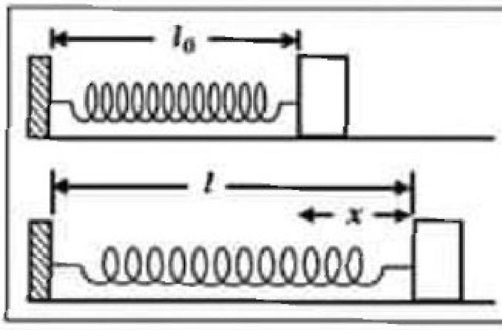
$$\Delta K.E = \sum W_{ext} \text{ (For a rigid system)}$$



Gravitational potential energy ($G.P.E$ or $P.E_g$ or E_{pg}): energy possessed by an object (or system) due to its position in the gravitational field.

$$G.P.E = mgh$$

Variation in gravitational potential energy: $\Delta G.P.E = -W_{\vec{W}}$



Elastic potential energy ($E.P.E$ or $P.E_e$ or E_{pe}): is the potential energy stored as a result of deformation of elastic objects.

Magnitude of tension in an ideal spring (obeys Hooke's law):

$T = k|\Delta l| = k|x|$ where $x = \Delta l = l - l_0$

$$E.P.E = \frac{1}{2}kx^2$$

Variation in elastic potential energy: $\Delta E.P.E = -W_{\vec{T}}$

Mechanical energy ($M.E$ or E_m): sum of kinetic and potential energies.

$$M.E = K.E + G.P.E + E.P.E = \frac{1}{2}mV^2 + mgh + \frac{1}{2}kx^2$$

Mechanical energy

- **Conserved**
  The non-conservative forces (friction - traction force …) are neglected.
  - **Principle of conservation of mechanical energy:**
    $\Delta M.E = 0$
    $M.E_i = M.E_f$
    $\Delta K.E = -\Delta P.E$
- **Not conserved**
  The non-conservative forces (friction - traction force) exist
  - **Principle of non-conservation of mechanical energy:**
    $\Delta M.E = W_{NC}$
    $M.E_f - M.E_i = W_{NC}$

In the case where friction is the only non-conservative force:

$$\Delta M.E = W_{\vec{f}} \Rightarrow M.E_f - M.E_i = -f \times d$$

The energy lost appears in form of thermal energy: $Q = |\Delta M.E|$

Variation in mechanical energy with respect to time: $\frac{\Delta M.E}{\Delta t} = \frac{W_{\vec{f}}}{\Delta t} = P_{\vec{f}(av)} = \vec{f}.\vec{V}_{av}$

$$\frac{dM.E}{dt} = P_{\vec{f}} = \vec{f}.\vec{V}$$

Energy

- **Macroscopic**
  - **Macroscopic kinetic energy**
  - **Macroscopic potential energy**
- **Microscopic (Internal)**
  - **Microscopic kinetic energy**
  - **Microscopic potential energy**

Total energy: $E = M.E + U$.
Energy-isolated system: no exchange of energy with the surrounding ($E = constant \Rightarrow \Delta E = 0$).
$\Delta E = \Delta M.E + \Delta U = 0 \Rightarrow \Delta M.E = -\Delta U$.

CHAPTER 2 – LINEAR MOMENTUM

2.1- RECALL

Case of a particle		
	One dimensional motion	**Two dimensional motion**
Position	x $v = \frac{dx}{dt} \Rightarrow x = \int v dt$	$\vec{r}$ $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{r} = \int \vec{v} dt$
Velocity	$v = \frac{dx}{dt} = x'$ $a = \frac{dv}{dt} \Rightarrow v = \int a dt$	$\vec{v} = \frac{d\vec{r}}{dt} = \vec{r}'$ $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{v} = \int \vec{a} dt$
Acceleration	$a = \frac{dv}{dt} = v'$	$\vec{a} = \frac{d\vec{v}}{dt} = \vec{v}'$
Case of a system of particles		
	One dimensional motion	**Two dimensional motion**
Position	$X_G = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$	$\vec{r}_G = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$
Velocity	$V_G = \frac{dx_G}{dt} = x'_G$	$\vec{V}_G = \frac{d\vec{r}}{dt} = \vec{r}'_G$
Acceleration	$a_G = \frac{dV_G}{dt} = V'_G$	$\vec{a}_G = \frac{d\vec{V}_G}{dt} = \vec{V}'_G$

2.2- LINEAR MOMENTUM AND NEWTON'S 2ND LAW

	Particle	**System of Particles** **(S) = [(S$_1$); (S$_2$)]**
Linear momentum	$\vec{P} = m\vec{V}$	$\vec{P}_s = \vec{P}_1 + \vec{P}_2 = m_1\vec{V}_1 + m_2\vec{V}_2$ $\vec{P}_S = \vec{P}_G = M\vec{V}_G$ with $M = m_1 + m_2$
Newton's 2nd law	$\sum \vec{F}_{ext} = m\vec{a} = \frac{d\vec{P}}{dt}$	$\sum \vec{F}_{ext} = M\vec{a}_G = \frac{d\vec{P}_s}{dt}$

2.3- CONSERVATION OF LINEAR MOMENTUM

A system is said to be isolated if the sum of the external forces acting on it is zero.

$$\sum \vec{F}_{ext} = \frac{d\vec{P}_s}{dt} = \vec{0} \Rightarrow \vec{P}_s = constant \Rightarrow \vec{P}_i = \vec{P}_f$$

The linear momentum of the system is conserved

Applications of conservation of linear momentum

- **Explosions**
- **Recoil of cannons, rifles, …**
- **Collisions**

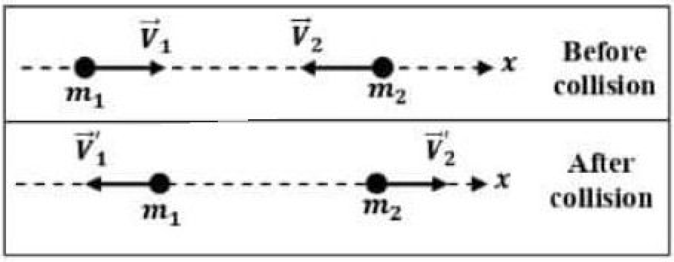
Collisions

- **Perfectly elastic: the kinetic energy is conserved**
  K.E$_b$ = K.E$_a$
- **Non-elastic: the kinetic energy is not conserved**
  K.E$_b$ ≠ K.E$_a$
- **Perfectly in-elastic: the colliding particles stick together and move with a common velocity**

During collisions, explosions and recoil, the internal forces are much stronger than the external forces acting on the system; then, the external forces can be considered neglected relative to the internal forces.

2.4- Perfectly Elastic Collision

Two particles m_1 and m_2 enter in a perfectly elastic head-on collision.



Elastic collision:

- Linear momentum of the system [m_1; m_2] is conserved.
- The Kinetic energy of the system [m_1; m_2] is conserved.

Required to find: V_1' and V_2' the respective velocities of m_1 and m_2 just after collision.

To find these two unknowns we need two equations:

The first equation is obtained by applying the principle of conservation of linear momentum.
The second equation is obtained by applying the principle of conservation of kinetic energy.

During collision, the system [m_1; m_2] is isolated.

$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Rightarrow \vec{P} = constant$

Principle of conservation of linear momentum: $\vec{P}_b = \vec{P}_a$

$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$

The collision is head-on, so the above equation can be written in its algebraic form:

$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$

$m_1V_1 - m_1V_1' = m_2V_2' - m_2V_2$

Note:
Perfectly in-elastic collision

$$\boldsymbol{V_1' = V_2' = V}$$
$$\boldsymbol{m_1V_1 + m_2V_2 = (m_1 + m_2)V}$$
$$\boldsymbol{V = \frac{m_1V_1 + m_2V_2}{m_1 + m_2}}$$

$$\boxed{m_1(V_1 - V_1') = m_2(V_2' - V_2) \ldots (1)}$$

The collision is elastic; then, the kinetic energy of the system [m_1; m_2] is conserved.

$K.E_b = K.E_a$

$\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2$

$m_1V_1^2 + m_2V_2^2 = m_1V_1'^2 + m_2V_2'^2$

$m_1V_1^2 - m_1V_1'^2 = m_2V_2'^2 - m_2V_2^2$

$m_1(V_1^2 - V_1'^2) = m_2(V_2'^2 - V_2^2)$

$$\boxed{m_1(V_1 - V_1')(V_1 + V_1') = m_2(V_2' - V_2)(V_2' + V_2) \ldots (2)}$$

Divide Eq (2) by Eq (1)

$\frac{m_1(V_1 - V_1')(V_1 + V_1')}{m_1(V_1 - V_1')} = \frac{m_2(V_2' - V_2)(V_2' + V_2)}{m_2(V_2' - V_2)}$

$$\boxed{V_1 + V_1' = V_2' + V_2 \ldots (3)}$$

From Eq (3):

$V_2' = V_1 + V_1' - V_2$

Replace in Eq (1)

$m_1(V_1 - V_1') = m_2(V_1 + V_1' - V_2 - V_2)$

$m_1V_1 - m_1V_1' = m_2V_1 + m_2V_1' - 2m_2V_2$

$m_1V_1 - m_2V_1 + 2m_2V_2 = m_2V_1' + m_1V_1'$

$(m_1 - m_2)V_1 + 2m_2V_2 = (m_1 + m_2)V_1'$

$V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1 + \frac{2m_2}{m_1 + m_2}V_2$

Similarly:

$V_2' = \frac{2m_1}{m_1 + m_2}V_1 + \frac{m_2 - m_1}{m_2 + m_1}V_2$

For $V_2 = 0$

$$V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1$$

$$V_2' = \frac{2m_1}{m_1 + m_2}V_1$$

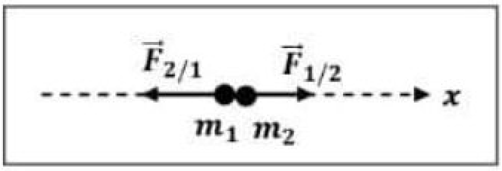
For $m_1 = m_2$

$$V_1' = V_2$$

$$V_2' = V_1$$

The particles exchange their velocities

2.5- INTERACTION BETWEEN m_1 AND m_2



$\vec{F}_{1/2} = -\vec{F}_{2/1}$ (Newton's 3rd law)

$\sum \vec{F}_{ext/m_1} = \vec{F}_{2/1} = \frac{\Delta\vec{P}_1}{\Delta t}$

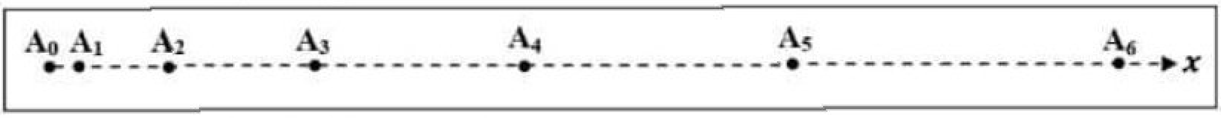
$\sum \vec{F}_{ext/m_2} = \vec{F}_{1/2} = \frac{\Delta\vec{P}_2}{\Delta t}$

Note: The external forces acting on m_1 and m_2 are neglected relative to the internal forces

2.6- MOTION OF A PUCK ALONG AN AIR TABLE

A_0 is taken as an origin of space and time.

Time constant τ: time separating two consecutive dots.



1- Time needed to reach the dot A_i is $t_i = i\tau$
   Example: the time needed to reach the dot A_2 is $t_2 = 2\tau$
2- The position (abscissa) of the dot A_i is $x_i = \overline{A_0A_i}$
   Example: the position (abscissa) of the dot A_3 is $x_3 = \overline{A_0A_3}$
3- The velocity of the puck at the dot A_i is $V_i = V_{av(i-1,i+1)} = \frac{\Delta x}{\Delta t} = \frac{x_{i+1}-x_{i-1}}{t_{i+1}-t_{i-1}} = \frac{\overline{A_0A_{i+1}}-\overline{A_0A_{i-1}}}{2\tau}$
   Example: the velocity of the puck at the dot A_2 is:
   $V_2 = V_{av(1,3)} = \frac{\Delta x}{\Delta t} = \frac{x_3-x_1}{t_3-t_1} = \frac{\overline{A_0A_3}-\overline{A_0A_1}}{t_3-t_1} = \frac{\overline{A_0A_3}-\overline{A_0A_1}}{3\tau-\tau} = \frac{\overline{A_1A_3}}{2\tau}$ $(A_1A_3\ in\ [m] and\ \tau\ in\ [s])$
4- The acceleration of the puck at the dot A_i is: $a_i = a_{av(i-1,i+1)} = \frac{\Delta V}{\Delta t} = \frac{V_{i+1}-V_{i-1}}{t_{i+1}-t_{i-1}} = \frac{V_{i+1}-V_{i-1}}{2\tau}$

Example: the acceleration at the dot A_3 is $a_3 = a_{av(2,4)} = \frac{\Delta V}{\Delta t} = \frac{V_4-V_2}{t_4-t_2} = \frac{V_4-V_2}{2\tau}$

2.7- PROJECTILE MOTION

A projectile is an object that moves through space acted upon only by Earth's gravity.

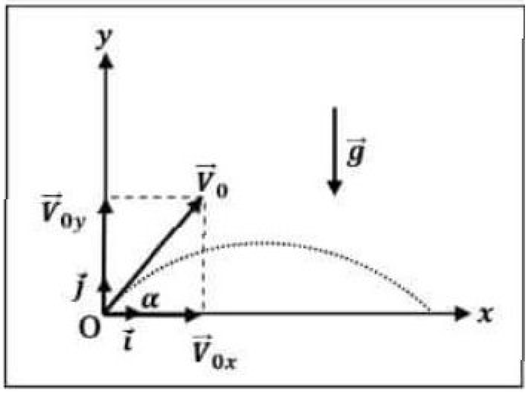
At the instant $t_0 = 0$, a particle is launched from O with an initial velocity vector making an angle α with the horizontal.

Initial velocity vector at $t_0 = 0$.

$$\vec{V_0}\left|\begin{array}{l} V_{0x} = V_0\cos\alpha \\ V_{0y} = V_0\sin\alpha \end{array}\right.$$

Initial linear momentum at $t_0 = 0$.

$$\vec{P_0}\left|\begin{array}{l} P_{0x} = mV_{0x} = mV_0\cos\alpha \\ P_{0y} = mV_{0y} = mV_0\sin\alpha \end{array}\right.$$



Newton's 2nd law:

$$\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

$$m\vec{g} = \frac{d\vec{P_x}}{dt} + \frac{d\vec{P_y}}{dt}$$

$$-mg\vec{\imath} = \frac{dP_x}{dt}\vec{\imath} + \frac{dP_y}{dt}\vec{\imath}$$

$$\frac{dP_x}{dt} = 0 \Rightarrow P_x = constant = P_{0x} = mV_0\cos\alpha$$

$$\frac{dP_y}{dt} = -mg \Rightarrow P_y = -mgt + P_{0y} = -mgt + mV_0\sin\alpha$$

If y is directed vertically downwards:

$$\vec{a} = \vec{g} = g\vec{\jmath}$$

$$\vec{W} = mg\vec{\jmath}$$

Acceleration vector at any time t:

$$\sum\vec{F}_{ext} = m\vec{a}$$

$$m\vec{g} = m\vec{a}$$

$$\vec{a} = \vec{g} = -g\vec{\jmath}$$

$$\vec{a}\left|\begin{array}{l} a_x = 0\ (URM) \\ a_y = -g\ (UVRM) \end{array}\right.$$

In the case of horizontal launching:

$\alpha = 0$

Velocity vector at any instant t:

$$V_x = \frac{P_x}{m} = V_0\cos\alpha$$

$$V_y = \frac{P_y}{m} = -gt + V_0\sin\alpha$$

Position vector at any time t

$x = \int V_x dt = V_0 \cos\alpha\, t + x_0 = V_0 \cos\alpha\, t$

$y = \int V_y dt = -\frac{1}{2} g t^2 + V_0 \sin\alpha\, t + y_0 = -\frac{1}{2} g t^2 + V_0 \sin\alpha\, t$

Time needed to reach maximum height

$V_y = 0 \Longrightarrow -gt + V_0 \sin\alpha = 0 \Longrightarrow t = \frac{V_0 \sin\alpha}{g}$

Maximum height

$t = \frac{V_0 \sin\alpha}{g} \Rightarrow H_{max} = y_{max} = \frac{V_0^2 \sin^2\alpha}{2g}$

Range

$y = 0 \Rightarrow x_R = \frac{V_0^2 \sin 2\alpha}{g}$

Time needed to reach the range

$t = \frac{x}{V_0 \cos\alpha}$ *and* $x_R = \frac{V_0^2 \sin 2\alpha}{g} = \frac{2V_0^2 \sin\alpha \times \cos\alpha}{g}$

$t = \frac{1}{v_0 \cos\alpha} \times \frac{2V_0^2 \sin\alpha \times \cos\alpha}{g} = \frac{2V_0 \sin\alpha}{g}$

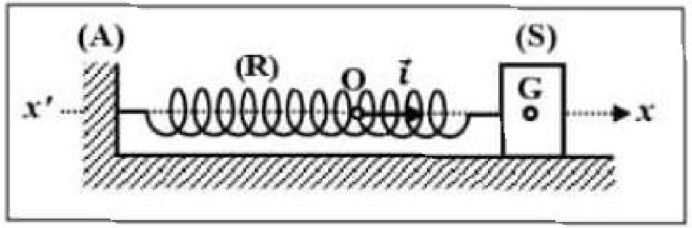
Speed of the projectile as it reaches the range

$$V_x = V_{0x} = V_0 \cos\alpha$$

$$V_y = a_y t + V_{0y} = -g\left(\frac{V_0 \sin\alpha}{g}\right) + V_0 \sin\alpha = -2V_0 \sin\alpha + V_0 \sin\alpha = -V_0 \sin\alpha$$

Mechanical energy at the instant *t*: $M.E = K.E + E.P.E + G.P.E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + 0$

Differential equation that governs the motion of G:



$F_{NC} = 0N \Rightarrow ME = constant \Rightarrow \frac{dM.E}{dt} = 0$

$\frac{1}{2}m(2vv') + \frac{1}{2}k(2xx') = 0$

$mvv' + kxx' = 0$ with $v = x'$ and $v' = x''$

$mx'x'' + kxx' = 0 \Rightarrow x'(mx'' + kx) = 0$ with $x' \neq 0$

$mx'' + kx = 0 \Rightarrow x'' + \frac{k}{m}x = 0$

Expressions of the proper angular frequency, proper frequency and proper period:

The above equation is a 2nd order differential equation that can be written in the form of $x'' + \omega_0^2 x = 0$ where $\omega_0 = \sqrt{\frac{k}{m}}$ is the proper angular frequency.

Type or nature of motion: SHM (simple harmonic motion) or RSM (rectilinear sinusoidal motion).

The proper frequency is $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

A free un-damped oscillator has the minimum possible period of oscillations, called proper period $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$

During on oscillation, G covers a distance $d = 4X_m$

Time equations of motion

Solution of the differential equation: $x = X_m \sin(\omega_0 t + \phi)$ or $x = X_m \cos(\omega_0 t + \phi)$

	Time equations of motion	Time equations of motion
Position	$x = X_m \sin(\omega_0 t + \phi)$	$x = X_m \cos(\omega_0 t + \phi)$
Velocity	$v = x' = X_m \omega_0 \cos(\omega_0 t + \phi)$	$v = x' = -X_m \omega_0 \sin(\omega_0 t + \phi)$
Acceleration	$a = x'' = -X_m \omega_0^2 \sin(\omega_0 t + \phi)$	$a = x'' = -X_m \omega_0^2 \cos(\omega_0 t + \phi)$

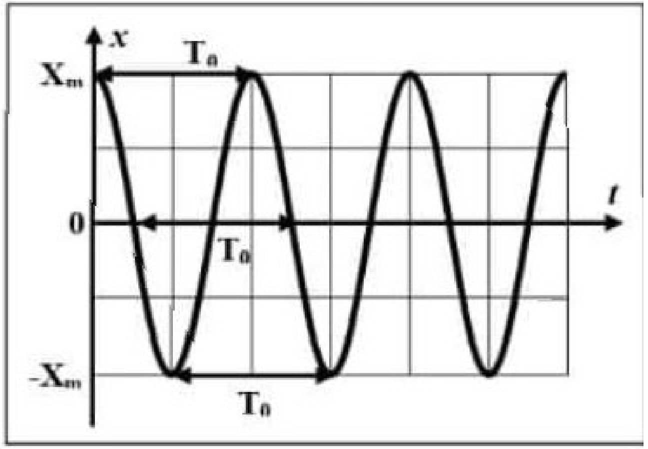
Determination of the expression of ω_0

Replace x and x'' in the differential equation:

$-X_m \omega_0^2 \sin(\omega_0 t + \phi) + \frac{k}{m} X_m \sin(\omega_0 t + \phi) = 0$

$X_m \sin(\omega_0 t + \phi)\left[-\omega_0^2 + \frac{k}{m}\right] = 0$

$-\omega_0^2 + \frac{k}{m} = 0 \Rightarrow \omega_0^2 = \frac{k}{m} \Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$



Electromagnetic induction: is the generation or establishment of an e.m.f. in a circuit when the magnetic flux crossing it is varied. When such a circuit is closed, it is traversed by an induced electric current.
Variation in magnetic flux:
- Variation in the intensity of the magnetic field B.
- Variation in area S.
- Variation in θ.

8.4- LAWS OF ELECTROMAGNETIC INDUCTION

Faraday's law: the induced electromotive force "e" at any instant is equal to the opposite of the derivative with respect to time of the magnetic flux crossing the circuit.

$$e = -\frac{d\phi}{dt}$$

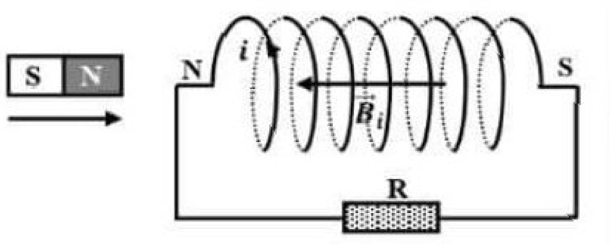
Lenz's law: the direction of the induced current is such that its electromagnetic effects always oppose the cause that has established this current.

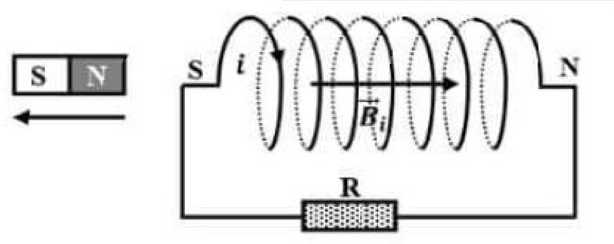
In simple words:
Systems don't like changes and try to minimize it

Attention:
- The induced current i creates an induced magnetic field $\vec{B}_i$ (direction is determined by RHR).
- The induced emf e and the induced current i are algebraic quantities that have the same sign.
- The signs of e and i depends on the positive chosen sense.

$$\phi\ varies \Rightarrow e \Rightarrow i \Rightarrow \vec{B}_i$$

The direction of the induced current i can be determined by the:
- **Opposition to pole movement:**

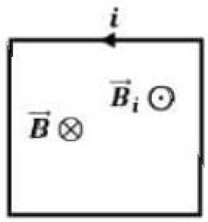


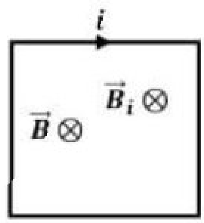


Repulsion	**Attraction**

Inside the coil, the magnetic field $\vec{B}_i$ is directed from S to N.

- **Opposition to change in the intensity of the magnetic field:**

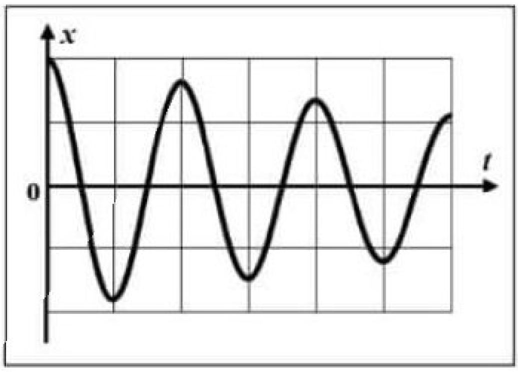




$B \nearrow \Rightarrow \vec{B}$ **and** $\vec{B}_i$ **have opposite directions**	$B \searrow \Rightarrow \vec{B}$ **and** $\vec{B}_i$ **have the same direction**

4.4- FREE DAMPED OSCILLATION

The amplitude of a free damped oscillator is not constant. In the presence of damping (due to friction for example), the period of oscillation T, called the pseudo-period, becomes larger than T_0 ($T > T_0$). In the case of slight damping: $T \cong T_0$.

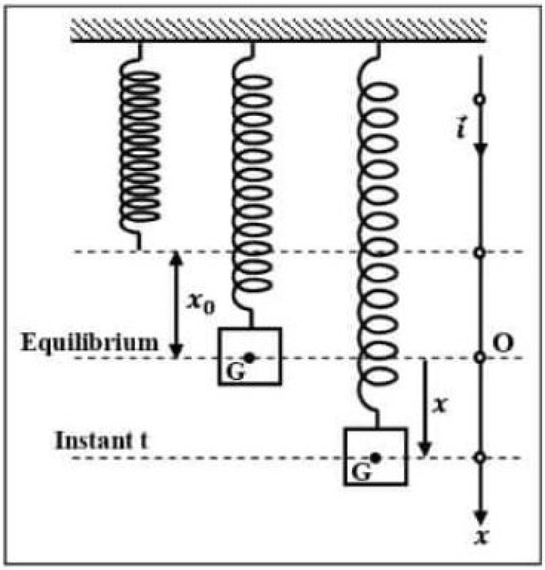


4.5- DRIVEN OSCILLATION

To provide the oscillator with the necessary energy needed to compensate for the losses and maintain its amplitude constant.

Average power: $P_{av} = \frac{|\Delta M.E|}{\Delta t}$

4.6- VERTICAL ELASTIC PENDULUM



Newton's 1st law: $\sum \vec{F}_{ext} = \vec{0} \Longrightarrow \vec{T} + \vec{W} = \vec{0}$

$\vec{T} = -\vec{W} \Longrightarrow T = W \Longrightarrow kx_0 = mg$

Mechanical energy at an instant t:

$M.E = K.E + E.P.E + G.P.E$

$M.E = \frac{1}{2}mv^2 + \frac{1}{2}k(x + x_0)^2 - mgx$

Differential equation

$F_{NC} = 0 \Longrightarrow M.E = constant \Longrightarrow \frac{dM.E}{dt} = 0$

$\frac{1}{2}m(2vv') + \frac{1}{2}k[2(x + x_0)x'] - mgx' = 0$

$mvv' + k(x + x_0)x' - mgx' = 0$ with $v = x'$ and $v' = x''$

$mx'x'' + kxx' + kx_0x' - mgx' = 0$

$x'(mx'' + kx + kx_0 - mg) = 0$ with $kx_0 - mg = 0$

$mx'' + kx = 0 \Longrightarrow x'' + \frac{k}{m}x = 0$

4.7- TWO IDENTICAL SPRINGS

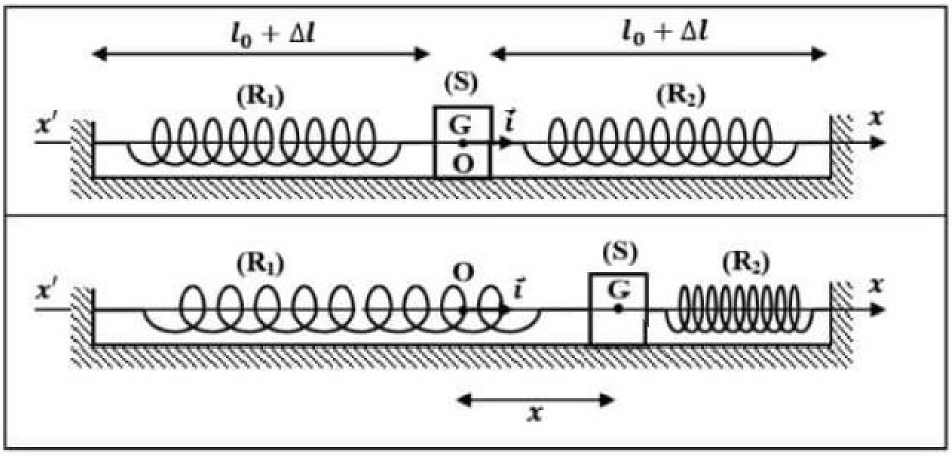
$M.E = K.E + E.P.E + G.P.E = \frac{1}{2}MV^2 + \frac{1}{2}k(\Delta l + x)^2 + \frac{1}{2}k(\Delta l - x)^2 = \frac{1}{2}mV^2 + k(x^2 + \Delta l^2)$

$F_{NC} = 0 \Longrightarrow M.E = constant \Longrightarrow \frac{dM.E}{dt} = 0$

$\frac{1}{2}M(2VV') + k(2xx') = 0 \Longrightarrow Mx'x'' + 2kxx' = 0$ with $V = x'$ and $V' = x''$

$x'(Mx'' + 2kx) = 0$ with $x' \neq 0$

$x'' + \frac{2k}{M}x = 0$



Application 1:

For a horizontal elastic pendulum:

1- Write the expression of the mechanical energy in terms of m, k, x, and v.
2- Establish the 2[nd] order differential equation that governs the motion of x.
3- Deduce the expressions of the proper angular frequency and the proper period.

1- $M.E = K.E + E.P.E + G.P.E = \frac{1}{2}mV^2 + \frac{1}{2}kx^2$

2- $f = 0 \Rightarrow M.E = cst \Rightarrow \frac{dM.E}{dt} = 0$

$\frac{1}{2}m(2VV') + \frac{1}{2}k(2xx') = 0$

$mVV' + kxx' = 0$ with $V = x'$ and $V' = x''$

$mx'x'' + kx = 0$

$x'(mx'' + kx) = 0$ with $x' \neq 0$

$mx'' + kx = 0$

$x'' + \frac{k}{m}x = 0$ (2[nd] order differential equation)

3- The above differential equation can be written in the form: $x'' + \omega_0^2 x = 0$

$\omega_0^2 = \frac{k}{m} \Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$

$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$

Application 2:

For a horizontal elastic pendulum:

1- Write the expression of the mechanical energy in terms of m, k, x, and v.
2- Establish the 2[nd] order differential equation that governs the motion of x.
3- The solution of the differential equation has the form $x = X_m \sin(\omega_0 t + \phi)$. Determine the expressions of the proper angular frequency and the proper period.

1- $M.E = K.E + E.P.E + G.P.E = \frac{1}{2}mV^2 + \frac{1}{2}kx^2$

2- $f = 0 \Rightarrow M.E = cst \Rightarrow \frac{dM.E}{dt} = 0$

$\frac{1}{2}m(2VV') + \frac{1}{2}k(2xx') = 0$

$mVV' + kxx' = 0$ with $V = x'$ and $V' = x''$

$mx'x'' + kx = 0$

$x'(mx'' + kx) = 0$ with $x' \neq 0$

$mx'' + kx = 0$

$x'' + \frac{k}{m}x = 0$ (2[nd] order differential equation)

3- $x = X_m \sin(\omega_0 t + \phi)$

$x' = X_m \omega_0 \cos(\omega_0 t + \phi)$

$x'' = -X_m \omega_0^2 \sin(\omega_0 t + \phi)$

$-X_m \omega_0^2 \sin(\omega_0 t + \phi) + \frac{k}{m} X_m \sin(\omega_0 t + \phi) = 0$

$X_m \sin(\omega_0 t + \phi)\left[-\omega_0^2 + \frac{k}{m}\right] = 0$

$-\omega_0^2 + \frac{k}{m} = 0 \Rightarrow \omega_0^2 = \frac{k}{m} \Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$

$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$

Application 3:

For a horizontal elastic pendulum:

1- Write the expression of the mechanical energy in terms of m, k, x, and v.

2- Establish the 2nd order differential equation that governs the motion of x.

3- The solution of the differential equation has the form $x = X_m \cos\left(\frac{2\pi}{T_0}t + \phi\right)$. Determine the expressions of the proper angular frequency and the proper period.

1- $M.E = K.E + E.P.E + G.P.E = \frac{1}{2}mV^2 + \frac{1}{2}kx^2$

2- $f = 0 \Rightarrow M.E = cst \Rightarrow \frac{dM.E}{dt} = 0$

$\frac{1}{2}m(2VV') + \frac{1}{2}k(2xx') = 0$

$mVV' + kxx' = 0$ with $V = x'$ and $V' = x''$

$mx'x'' + kx = 0$

$x'(mx'' + kx) = 0$ with $x' \neq 0$

$mx'' + kx = 0$

$x'' + \frac{k}{m}x = 0$ (2nd order differential equation)

3- $x = X_m \cos\left(\frac{2\pi}{T_0}t + \phi\right)$

$x' = -X_m\left(\frac{2\pi}{T_0}\right)\sin\left(\frac{2\pi}{T_0}t + \phi\right)$

$x'' = -X_m\left(\frac{2\pi}{T_0}\right)^2\cos\left(\frac{2\pi}{T_0}t + \phi\right)$

$-X_m\left(\frac{2\pi}{T_0}\right)^2\cos\left(\frac{2\pi}{T_0}t + \phi\right) + \frac{k}{m}X_m\cos\left(\frac{2\pi}{T_0}t + \phi\right) = 0$

$X_m\cos\left(\frac{2\pi}{T_0}t + \phi\right)\left[-\left(\frac{2\pi}{T_0}\right)^2 + \frac{k}{m}\right] = 0$

$-\left(\frac{2\pi}{T_0}\right)^2 + \frac{k}{m} = 0 \Rightarrow \left(\frac{2\pi}{T_0}\right)^2 = \frac{k}{m} \Rightarrow \frac{2\pi}{T_0} = \sqrt{\frac{k}{m}}$

$\frac{T_0}{2\pi} = \sqrt{\frac{m}{k}} \Rightarrow T_0 = 2\pi\sqrt{\frac{m}{k}}$

$\omega_0 = \frac{2\pi}{T_0} = \sqrt{\frac{k}{m}}$

CHAPTER 8 - ELECTROMAGNETIC INDUCTION (1831)

8.1- RECALL

Magnetic field: invisible region of space created by:
- A magnet (Bar, U, needle, …).
- Earth (Terrestrial magnetic field).
- Current carrying-wire (Hans Christian Oersted 1819).

Magnetic field created by a current

The intensity of the magnetic field created by a wire is directly proportional to the intensity of the current traversing it:

$$B = ki$$

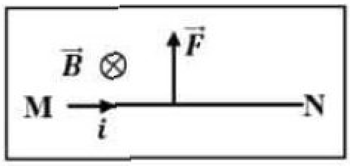
Rectilinear wire: $B = \frac{\mu_0}{2\pi d} i$

Flat coil: $B = \frac{\mu_0 N}{2R} i$

Solenoid: $B = \frac{\mu_0 N}{L} i$

The direction of $\vec{B}$ is determined by applying the right hand rule.

The electromagnetic force (Laplace's force): is a force acting on conductor MN:
- Placed in a magnetic field $\vec{B}$.
- Traversed by an electric current i.



$$\vec{F} = i\vec{l} \times \vec{B} \text{ with } \vec{l} = \overrightarrow{MN}$$

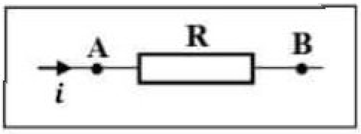
$$F = ilB \sin\alpha \text{ where } \alpha = (\overrightarrow{MN}; \vec{B})$$

The electromagnetic force is perpendicular to the plane containing $\overrightarrow{MN}$ and $\vec{B}$

The direction of the electromagnetic force is determined by applying the RHR.

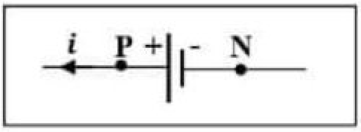
Lorentz force: on an electric charge placed in a magnetic field.

$$\vec{F} = q\vec{V} \times \vec{B}$$



Ohms law for a resistor: $u_{AB} = u_R = Ri$

The electric power received by a resistor is $p = V_{AB} i > 0$

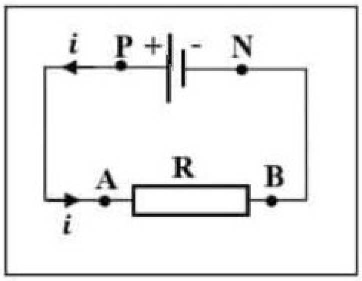


Ohm's law for a generator: $u_{PN} = u_G = e - ri$

The electric power received by a generator is: $p = -u_{PN} i < 0$

e: Electromotive force in [V].

r: Internal resistance in [Ω].



Expression of the intensity of the electric current traversing a circuit that consists of a generator and a resistor.

$u_{PN} = u_{PA} + u_{AB} + u_{BN}$

$e - ri = Ri$

$i = \frac{E}{r+R} = \frac{e}{R_{eq}}$ where $R_{eq} = r + R$

8.2- ELECTROMAGNETIC INDUCTION

Question: Can a magnetic field produce electricity?

Michael Faraday the English scientist was the first person to prove that a magnet can create a current. He thought that the reverse must be true, that a magnetic field could produce an electric current.

Goal: convert magnetism to electricity.

Experimental evidence of the induction phenomenon
To test this he moved a magnet towards and away from the coil of wire connected to a galvanometer.
Deflection in the galvanometer indicates that that a current is induced in the coil.
The current obtained due to the relative motion between the magnet and the coil is called induced current.
The phenomenon by which an electromotive force is induced in a conductor due to change in the magnetic field near the conductor is known as **electromagnetic induction**.

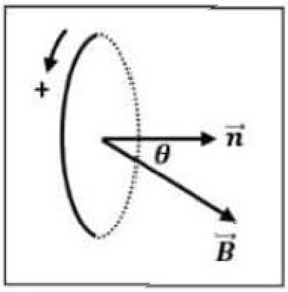
Let's now look into some experiments performed by Michael Faraday
Faraday arrived at a few conclusions by moving a bar magnet in and out of the coil of wire.
Displaced here a circular insulating wire with a many turns connected to a galvanometer.

Observe the deflection of the galvanometer needle when:
1- Magnet is moved in and out
2- Different poles are introduced
3- Number of turns is changed.

Drag the magnet in and out of the coil. Observe the deflection in the galvanometer.
1- The deflection of the galvanometer indicates the presence of current in the coil.
2- The direction of the deflection gives the direction of flow of current.
3- The speed of deflection gives the rate at which the current is induced.

Conclusions:
1- Deflection in the galvanometer indicates that the current is induced in the coil due to the relative motion between the magnet and the coil. The deflection in the galvanometer lasts as long there is a relative motion between the magnet and the coil.
2- The deflection is more if the magnet is moved faster and less when the magnet is moved slowly. That is, the rate at which the current is induced is more when the magnet is moved faster.
3- The deflection in the galvanometer is reversed when the same pole of the magnet is moved in the opposite direction or when the opposite pole is moved in the same direction. The direction of the deflection indicates the direction of flow of current.
4- The deflection in the galvanometer changes with the change in the number of turns of the coil – more the number of turns in the coil greater the deflection. The magnetic field goes around each loop of wire in the coil, so if we increase the number of turns in the wire the change in magnetic field is more.

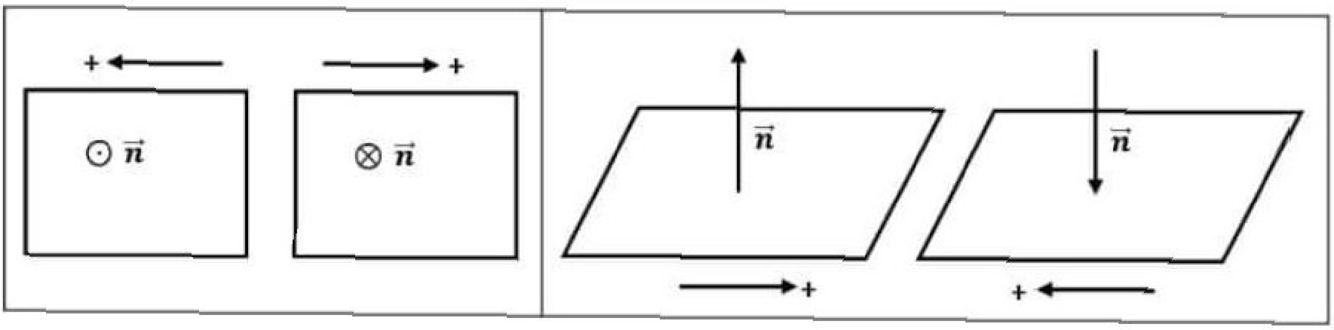
8.3- MAGNETIC FLUX



Magnetic flux: is a measurement of the total magnetic field which passes through a given area.

$$\phi = N\vec{B}.\vec{S} = N\vec{B}.\vec{n}S = NBS\cos\theta$$

ϕ: magnetic flux in [Wb]
N: number of turns
B: intensity of the magnetic field in [T]
S: area of the loop in [m^2]
$\theta = (\vec{B};\ \vec{n})$: angle between the magnetic field and the normal to the plane
Remark: the normal to the plane depends on the positive chosen sense.



Application 1:

A resistor of resistance R and a capacitor of capacitance C are connected in series with an ideal DC generator of emf E.

1- Establish the differential equation that governs the variation of the voltage u_C across the terminals of the capacitor.

2- The solution of the differential equation has the form $u_C = A\left(1 - e^{-\frac{t}{\tau}}\right)$. Determine the expressions of A and τ in terms of E, R and C.

Solution:

1- Law of addition of voltages in series connection: $E = u_R + u_C$

$E = Ri + u_C$ with $i = \frac{dq}{dt} = \frac{d(Cu_C)}{dt} = C\frac{du_C}{dt}$

$\boldsymbol{E = RC\frac{du_C}{dt} + u_C}$

2- $u_C = A\left(1 - e^{-\frac{t}{\tau}}\right) = A - Ae^{-\frac{t}{\tau}}$ and $\frac{du_C}{dt} = 0 - A\left(-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}} = \frac{A}{\tau}e^{-\frac{t}{\tau}}$

Replace u_C and $\frac{du_C}{dt}$ in the differential equation:

$E = RC\frac{A}{\tau}e^{-\frac{t}{\tau}} + A - Ae^{-\frac{t}{\tau}}$

$E = Ae^{-\frac{t}{\tau}}\left(\frac{RC}{\tau} - 1\right) + A$

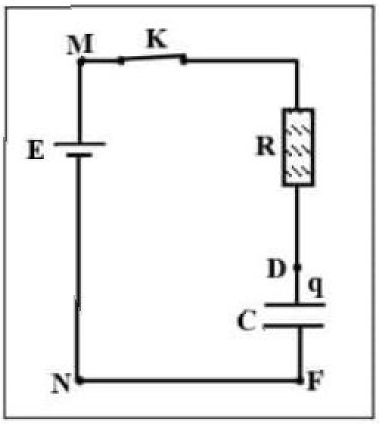
By identification: $E = A$ and $\frac{RC}{\tau} - 1 = 0 \Rightarrow \tau = RC$

$u_C = E\left(1 - e^{-\frac{t}{\tau}}\right)$

Application 2:

Consider the RC circuit shown in the adjacent document.

The capacitor is being neutral, we close the switch K at the instant t = 0s.



1- Establish the differential equation that governs the variation of the voltage u_C across the terminals of the capacitor.

2- The solution of the differential equation has the form $u_C = A + Be^{-\frac{t}{\tau}}$.
Determine the expressions of A, B and τ in terms of E, R and C.

Solution

1- Law of addition of voltages: $E = u_R + u_C$

$E = Ri + u_C$ with $i = \frac{dq}{dt} = \frac{d(Cu_C)}{dt} = C\frac{du_C}{dt}$

$E = RC\frac{du_C}{dt} + u_C$

2- $u_C = A + Be^{-\frac{t}{\tau}}$ and $\frac{du_C}{dt} = 0 + B\left(-\frac{1}{\tau}\right)e^{-\frac{t}{\tau}} = -\frac{B}{\tau}e^{-\frac{t}{\tau}}$

Replace u_C and $\frac{du_C}{dt}$ in the differential equation

$E = -RC\frac{B}{\tau}e^{-\frac{t}{\tau}} + A + Be^{-\frac{t}{\tau}}$

$E = Be^{-\frac{t}{\tau}}\left(-\frac{RC}{\tau} + 1\right) + A$

By identification: $E = A$ and $-\frac{RC}{\tau} + 1 = 0 \Rightarrow \tau = RC$

At t = 0s; $u_C = 0$

$0 = A + Be^0 \Rightarrow B = -A = -E$

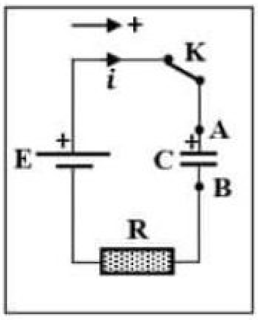
$u_C = E - Ee^{-\frac{t}{\tau}} = E\left(1 - e^{-\frac{t}{\tau}}\right)$

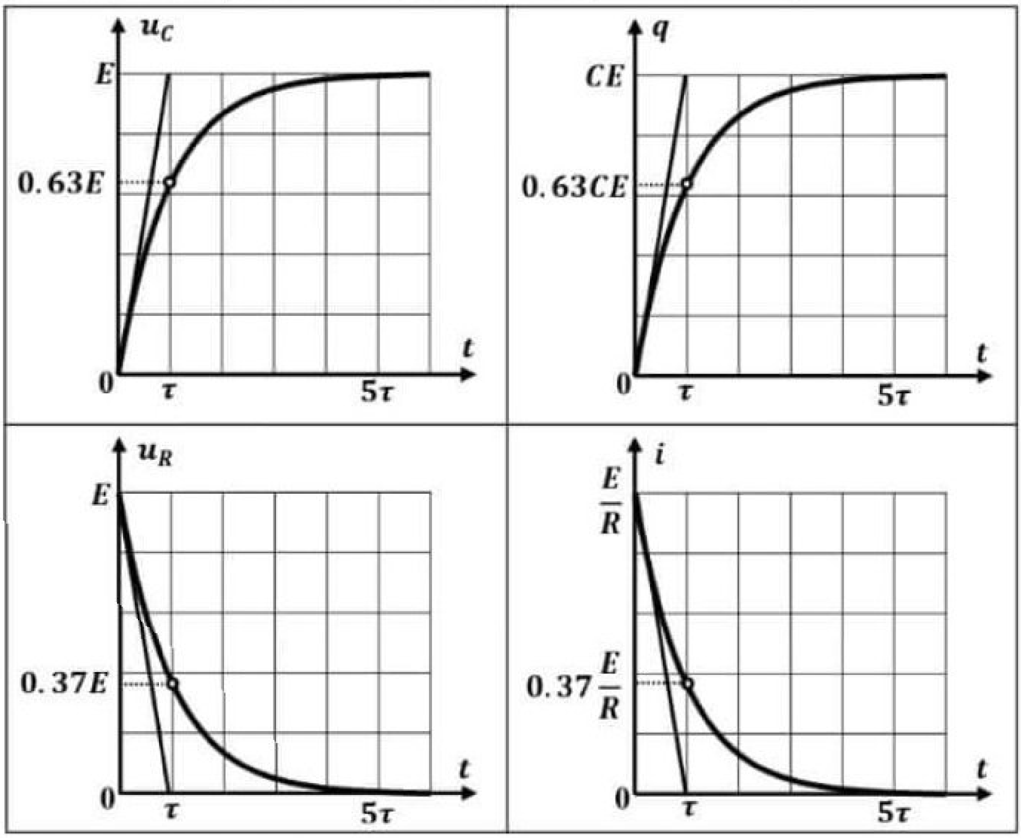
CHAPTER 10.1 – CAPACITORS

RC Charging circuit

Law of addition of voltages: $E = u_C + u_R \Rightarrow u_R = E - u_C$

t	u_C	$q = Cu_C$	u_R	$i = \frac{u_R}{R}$
$t = 0$	0	0	$U_{Rmax} = E$	$I_{max} = \frac{E}{R}$
$t = \tau = RC$	$0.63E$	$0.63CE$	$0.37E$	$0.37\frac{E}{R}$
$t = 5\tau = 5RC$	$U_{Cmax} = E$	$Q_{max} = CE$	0	0





Forms of the solution of the differential equation in u_C:

$u_C = A\left(1 - e^{-\frac{t}{\tau}}\right)$ with $A = E$ and $\tau = RC$

$u_C = A(1 - e^{\alpha t})$ with $A = E$ and $\alpha = -\frac{1}{\tau}$

$u_C = A + Be^{-\frac{t}{\tau}}$ with $A = E$ and $B = -A = -E$

The electric energy stored in a capacitor is:
$W = \frac{1}{2}Cu_C^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qu_C$
The electric power dissipated by a resistor is:
$P = Ri^2$

Differential Equations

Differential Equation in u_C

Law of addition of voltages: $u_G = u_C + u_R$

$E = u_C + Ri$ with $q = Cu_C$ and $i = \frac{dq}{dt} = C\frac{du_C}{dt}$

$E = u_C + RC\frac{du_C}{dt}$

Differential Equation in q

Law of addition of voltages: $u_G = u_C + u_R$

$E = \frac{q}{C} + Ri$ with $u_C = \frac{q}{C}$ and $i = \frac{dq}{dt}$

$E = \frac{q}{C} + R\frac{dq}{dt}$

Differential Equation in u_R

Law of addition of voltages: $u_G = u_C + u_R$

$E = u_C + u_R$ with $u_C = \frac{q}{C}$ and $i = \frac{dq}{dt} = \frac{Cdu_C}{dt}$

Derive both sides with respect to time:

$0 = \frac{du_C}{dt} + \frac{du_R}{dt}$

$0 = \frac{i}{C} + \frac{du_R}{dt}$

$0 = \frac{u_R}{RC} + \frac{du_R}{dt}$

Differential Equation in i

Law of addition of voltages: $u_G = u_C + u_R$

$E = u_C + Ri$ with $u_C = \frac{q}{C}$ and $i = \frac{dq}{dt} = \frac{Cdu_C}{dt}$

Derive both sides with respect to time:

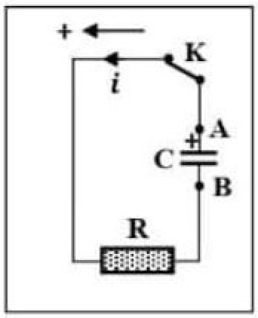
$0 = \frac{du_C}{dt} + R\frac{di}{dt}$

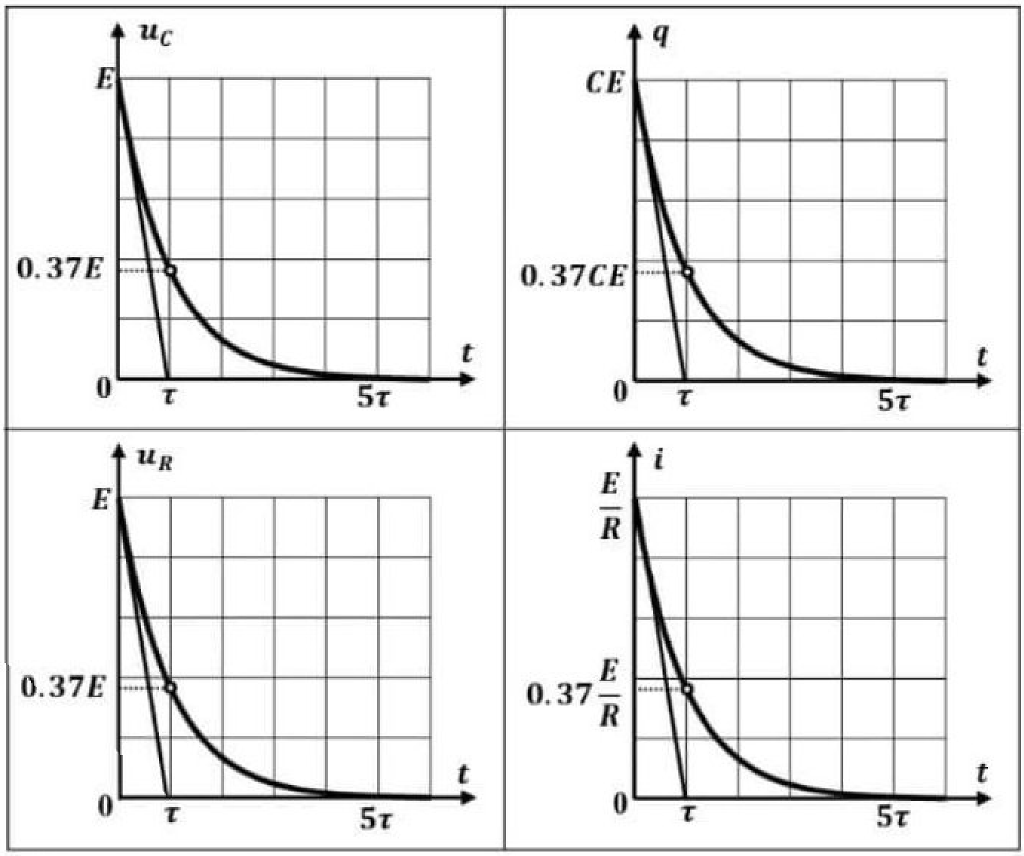
$0 = \frac{i}{C} + R\frac{di}{dt}$

RC discharging circuit

Law of uniqueness of voltage: $u_C = u_R$

t	u_C	$q = Cu_C$	u_R	$i = \frac{u_R}{R}$
$t = 0$	$U_{Cmax} = E$	$Q_{max} = CE$	$U_{Rmax} = E$	$I_{max} = \frac{E}{R}$
$t = \tau = RC$	$0.37E$	$0.37CE$	$0.37E$	$0.37\frac{E}{R}$
$t = 5\tau = 5RC$	0	0	0	0





Solution of the differential equation:

$u_C = Ae^{-\frac{t}{\tau}}$ with $A = E$ and $\tau = RC$

$u_C = Ae^{\alpha t}$ with $A = E$ and $\alpha = -\frac{1}{\tau}$

Differential Equations

Differential Equation in u_C

Law of uniqueness of voltage:

$u_C = u_R$

$u_C = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$

$u_C = -RC\frac{du_C}{dt}$

$u_C + RC\frac{du_C}{dt} = 0$

Differential Equation in q

Law of uniqueness of voltage:

$u_C = u_R$

$\frac{q}{C} = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$

$\frac{q}{C} = -R\frac{dq}{dt}$

$\frac{q}{C} + R\frac{dq}{dt} = 0$

Differential Equation in u_R

Law of uniqueness of voltage:

$u_C = u_R$

$u_C = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$

Derive both sides with respect to time:

$\frac{du_C}{dt} = \frac{du_R}{dt} \Longrightarrow -\frac{i}{C} = \frac{du_R}{dt}$

$-\frac{u_R}{RC} = \frac{du_R}{dt} \Longrightarrow \frac{du_R}{dt} + \frac{u_R}{RC} = 0$

Differential Equation in i

Law of uniqueness of voltage:

$u_C = u_R$

$u_C = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$

Derive both sides with respect to time:

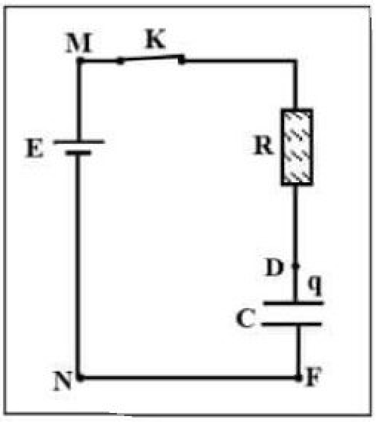
$\frac{du_C}{dt} = R\frac{di}{dt} \Longrightarrow -\frac{i}{C} = R\frac{di}{dt}$

$R\frac{di}{dt} + \frac{i}{C} = 0$

Application 3:

Consider the RC circuit shown in the adjacent document.

The capacitor is being neutral, we close the switch K at the instant t = 0s.



1- Establish the differential equation that governs the variation of the voltage u_C across the terminals of the capacitor.
2- The solution of the differential equation has the form $u_C = A + Be^{-\alpha t}$.
Determine the expressions of A, B and α in terms of E, R and C.

Solution

1- Law of addition of voltages: $E = u_R + u_C$

$E = Ri + u_C$ with $i = \frac{dq}{dt} = \frac{d(Cu_C)}{dt} = C\frac{du_C}{dt}$

$E = RC\frac{du_C}{dt} + u_C$

2- $u_C = A + Be^{-\alpha t}$ and $\frac{du_C}{dt} = 0 + B(-\alpha)e^{-\alpha t} = -B\alpha e^{-\alpha t}$

Replace u_C and $\frac{du_C}{dt}$ in the differential equation

$E = -RCB\alpha e^{-\alpha t} + A + Be^{-\alpha t}$

$E = Be^{-\alpha t}(-RC\alpha + 1) + A$

By identification: $E = A$ and $-RC\alpha + 1 = 0 \Rightarrow \alpha = \frac{1}{RC}$

At t = 0s; $u_C = 0$

$0 = A + Be^{0} \Rightarrow B = -A = -E$

- **Opposition to the movement of the conductor.**

$\phi = NBS\cos\theta = -Blx$

$e = -\frac{d\phi}{dt} = -\frac{d(-Blx)}{dt} = Bl\frac{dx}{dt} = Blv$

$i = \frac{e}{R_{eq}} = \frac{e}{R+r} = \frac{Blv}{R+r}$

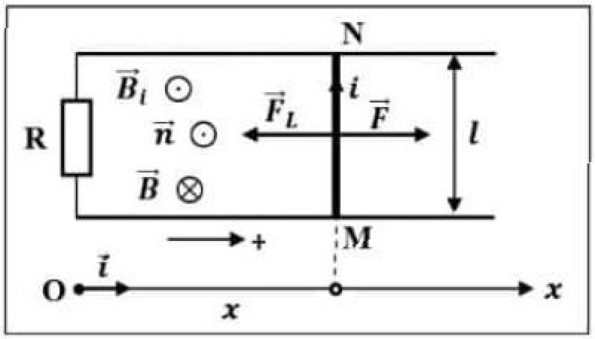
$F = ilB\sin\alpha = ilB = \frac{B^2l^2v}{R+r}$

URM $\Longrightarrow \sum \vec{F}_{ext} = \vec{0}$

$\vec{N} + \vec{W} + \vec{F} + \vec{F}_L = \vec{0}$ with $\vec{N} + \vec{W} = \vec{0}$

$\vec{F} + \vec{F}_L = \vec{0} \Rightarrow \vec{F} = -\vec{F}_L \Rightarrow F = F_L$

$u_{MN} = ri - e$



Uniform Rectilinear motion (URM):
$x = Vt + x_0$

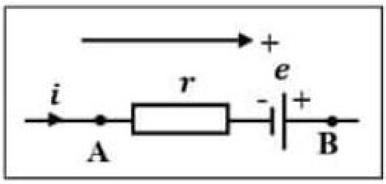
- **Variation in θ**

$\phi = NBS\cos\theta = NBS\cos(\omega t + \theta_0)$

$e = -\frac{d\phi}{dt} = NBS\omega\sin(\omega t + \theta_0)$ with $e_m = NBS\omega$.

Equivalent generator:

The positive sense is oriented from A to B:



$u_{AB} = ri - e$

$u_{BA} = -u_{BA} = -ri + e$

$ei = V_{BA}i + ri^2$

$P = ei$ (Total electric power due to the variation of the magnetic flux)

$P = u_{BA}i$ (The electric power transferred to the external circuit)

$P = ri^2$ (Power dissipated due to Joule's effect)