Chapter 3 - Capacitors.

25OCT2023

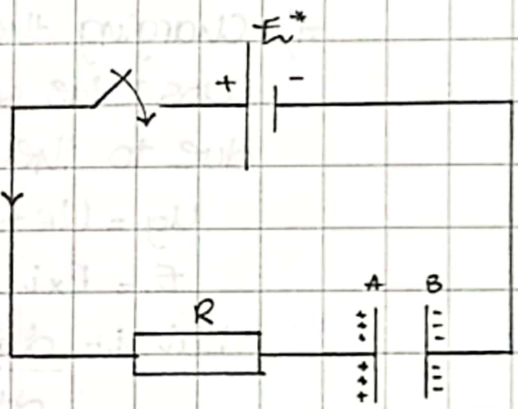
$(e^{u(x)})' = u'(x)\,e^{u(x)}$

↳ exp. function.

ex.

$$(e^{5x^2+2x})' = (10x+2)\,e^{5x^2+2x}$$

I) Introduction:



* E: constant voltage source.
* a voltage is being established across the Capacitor.
* at $t=0$, $U_C=0$, $U_R=E$ & $i=I_0=I_{max}$
* with time, $i \searrow \Rightarrow U_R \searrow$, $U_C \nearrow$
* at $t=t_{final}$, $i=0\,A$, $U_R=0$, $U_C=E$.

↳ Energy stored in a Capacitor:

* $W_C = \frac{1}{2} C U_C^2$

W_C → energy stored "J"; C → capacitance "F"; U_C → voltage "V"

↳ Charge of Capacitor:

* $Q = C \times U$

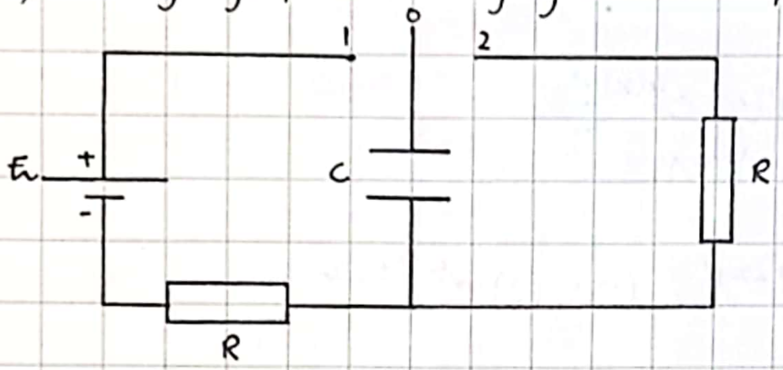
Q → charge "C"

↳ * $i = \dfrac{dq}{dt}$

↳ U_R is the image of i

$U_R = Ri$

II) Charging & Discharging of a Capacitor:



A) Charging the Capacitor:

(close the switch at 1)

due to the law of addition of voltages

$U_g = U_R + U_C$

$E = R \times i + U_C$

but $i = \frac{dq}{dt} = \frac{d}{dt}(C \times U_C) = C\frac{dU_C}{dt}$

$\Rightarrow \left(E = RC\frac{dU_C}{dt} + U_C\right) \times \frac{1}{RC}$

$\frac{E}{RC} = \frac{dU_C}{dt} + \frac{1}{RC}U_C$ → 1st order differential eq. that governs the growth of U_C with time.

↳ the solution of this differential equation:

$U_C = E(1 - e^{-t/RC})$

$U_C = E(1 - e^{-t/\tau})$

proof:

* $U_G = U_C + U_R$

$E = U_C + Ri$

$i = \frac{dq}{dt} = \frac{d}{dt}(C \times U_C) \Rightarrow i = C\frac{dU_C}{dt}$

$$E = U_C + RC\frac{dU_C}{dt}$$

$$\frac{E}{RC} = \frac{1}{RC}U_C + \frac{dU_C}{dt}$$

* $U_C = E(1 - e^{-t/\tau})$ $(\tau = RC)$

$$U_C = E - Ee^{-t/\tau}$$

$$\frac{dU_C}{dt} = 0 - (Ee^{-t/\tau})'$$

$$= -E(e^{-t/\tau})'$$

$$= -E\left(\frac{-1}{\tau}e^{-t/\tau}\right)$$

$$\frac{dU_C}{dt} = \frac{E}{\tau}e^{-t/\tau}$$

$$\Rightarrow \frac{E}{\tau}e^{-t/\tau} + \frac{1}{RC}(E - Ee^{-t/\tau}) \overset{?}{=} \frac{E}{RC}$$

$$\frac{E}{\tau}e^{-t/\tau} + \frac{E}{RC} - \frac{E}{RC}e^{-t/\tau} \overset{?}{=} \frac{E}{RC}$$

$$\frac{E}{RC} = \frac{E}{RC}$$ verified

* at $t = \tau$

$$U_C = E(1 - e^{-t/\tau})$$

$$= E(1 - e^{-1})$$

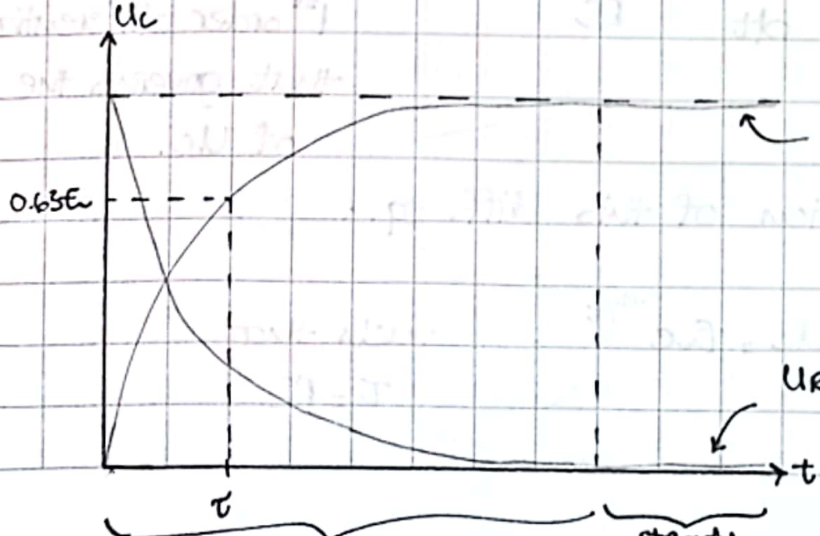
$$= 0.63E$$

at $t = 5\tau$

$$U_C = E(1 - e^{-5\tau/\tau})$$

$$= E(1 - e^{-5})$$

$$= 0.99E \approx E.$$



U_C exponential growth $f(x) = a(1 - e^{b(x)})$

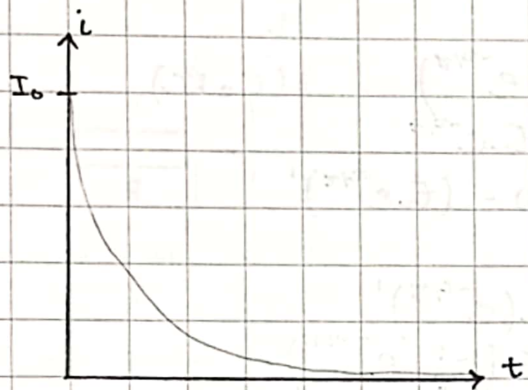
U_R exponential decay $f(x) = a e^{b(x)}$

transient phase ← 5τ steady phase

at $t \rightarrow \infty$

$U_C = E(1 - \cancel{e^{-\infty}}^{\,0})$ $\quad\left(\lim\limits_{x \to -\infty} e^{x} = 0\right)$ } extra

$U_C = E$



$U_g = U_C + U_R$

$E = U_R + E - E e^{-t/\tau}$

$\cancel{E} - \cancel{E} + E e^{-t/\tau} = U_R$

$U_R = E e^{-t/\tau}$

B) Discharging of a capacitor:

(close the switch at 2)

due to the law of addition of voltages

$U_R + U_C = 0$

but $U_R = Ri$

$Ri + U_C = 0$

but $i = \dfrac{dq}{dt} = C\dfrac{dU_C}{dt}$

$RC\dfrac{dU_C}{dt} + U_C = 0$

$\dfrac{dU_C}{dt} + \dfrac{1}{RC} U_C = 0$ → 1st order differential eq. that governs the decay of U_C.

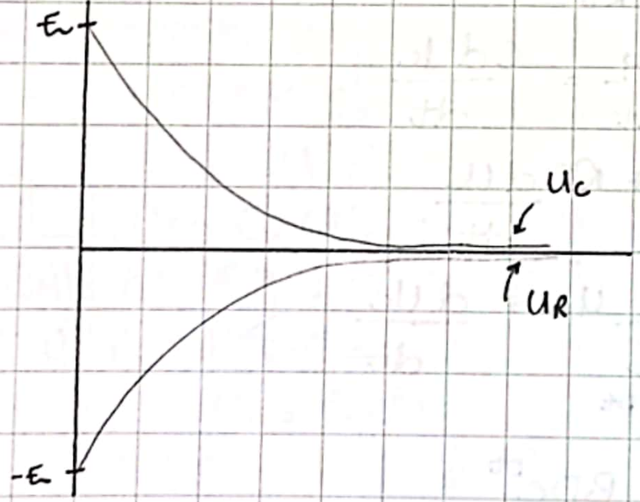
↳ solution of this diff. eq.:

$U_C = E e^{-t/\tau}$ such that $\tau = RC$

proof: at $t=0$, $u_c = E e^{-0}$
$= E(1)$
$u_c = E$

at $t=\tau$, $u_c = E e^{-\tau/\tau}$
$= E(0.37)$
$u_c = 0.37E$

at $t=5\tau$, $u_c = E e^{-5\tau/\tau}$
$= E(0.00\ldots)$
$u_c = 0\,V$



$u_R + u_c = 0$
$u_R = -u_c$

↳ 2018 session 2 - LS :

ex. 2 :

$C = 4\mu F = 4 \times 10^{-6} F$

$U_{AM} = E$

1) theoretical study:

1) due to the law of addition of voltages

$$u_g = u_C + u_R$$

$$E = u_C + u_R$$

but $u_R = Ri$

$$E = u_C + Ri$$

but $i = \frac{dq}{dt} = \frac{C\,du_C}{dt}$

$$E = u_C + RC\frac{du_C}{dt}$$

$$\frac{E}{RC} = \frac{1}{RC}u_C + \frac{du_C}{dt}$$

2) $u_C = A + Be^{Dt}$

$$\frac{du_C}{dt} = 0 + BDe^{Dt}$$

replace u_C & $\frac{du_C}{dt}$ in the diff. eq.

$$BDe^{Dt} + \frac{1}{RC}(A + Be^{Dt}) = \frac{E}{RC}$$

$$BDe^{Dt} + \frac{A}{RC} + \frac{B}{RC}e^{Dt} = \frac{E}{RC}$$

$$\frac{A}{RC} + Be^{Dt}\left(D + \frac{1}{RC}\right) = \frac{E}{RC}$$

$$\frac{A}{RC} = \frac{E}{RC} \Rightarrow A = E$$

$$Be^{Dt}\left(D+\frac{1}{RC}\right)=0$$

$$D+\frac{1}{RC}=0 \quad , \quad D=\frac{-1}{RC}$$

$U_c = E_n + Be^{-t/RC}$

at $t_0=0$, $U_c=0$

$0 = E_n + B$

$B = -E_n$

$U_c = E_n - E_n e^{-t/RC}$

$U_c = E_n(1-e^{-t/RC})$

3) at $t = 5\tau$

$U_c = E_n(1-e^{-5\tau/\tau})$

$= E_n(1-e^{-5})$

$U_c = 0.99E_n \approx E_n$

4) as $R\nearrow$, $\tau\nearrow$

2) 1) 1) at $t = t_{final}$

$U_c = E_n = 8V$

2) (a) $\longrightarrow R_1$

(b) $\longrightarrow R_2$

(a) reaches its maximum value before (b)

$\Rightarrow R_1 < R_2$

3) at $t=\tau$

$U_c = 0.63E_n$

$= 0.63(8)$

$= 5.04\ V$

$\tau_1 = 0.4\ ms$

$= 0.4\times10^{-3}\ s$

$\tau_1 = R_1C$

$$R_1 = \frac{\tau_1}{C} = \frac{0.4\times10^{-3}}{4\times10^{-6}} = 100\ \Omega$$

$\tau_2 = 0.8\ ms$

$= 0.8 \times 10^{-3}\ s$

$\tau_2 = R_2 C$

$R_2 = \frac{\tau_2}{C} = \frac{0.8 \times 10^{-3}}{4 \times 10^{-6}} = 200\ \Omega.$

2) $u_C = E$

1) $W_C = \frac{1}{2} C u_C^2$

$= \frac{1}{2} C E^2$

$\Rightarrow$ W_C is independent of R.

2) $W_C = \frac{1}{2} C E^2$

$= \frac{1}{2} (4 \times 10^{-6})(8)^2$

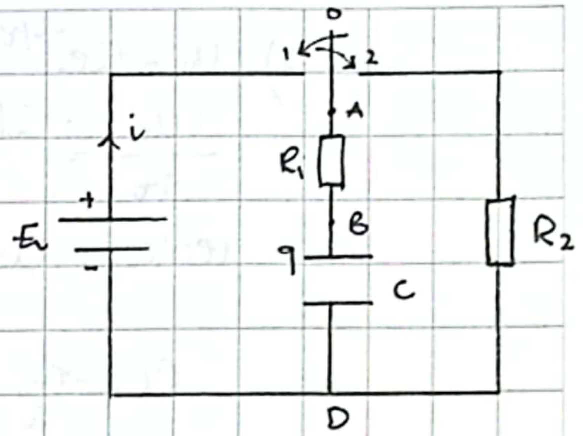
$= 1.28 \times 10^{-4}\ J$

LS ~~2018~~ 2021 session 1:

ex.2:

$E = 10V$

$R_1 = R_2 = 4k\Omega = 4\times10^3\,\Omega$



1) charging the capacitor:

1) during the charging process of the capacitor, u_C increases until $u_C = E$ (at $t = 5\tau$), which corresponds to curve (L).
while u_{R_1} decreases with time, until $u_{R_1} = 0$ at $t = 5\tau$ which corresponds to curve (a).

2) $\tau_1 = R_1C$

1) at $t = \tau$

$u_C = 0.63E$
$= 0.63(10)$
$= 6.3V$ $\Rightarrow$ $\underbrace{t = \tau_1 = 0.4s}$

or, $t = 5\tau_1$, $u_C = E$
$2 = 5\tau_1$
$\underbrace{\tau_1 = 0.4s}$

2) $\tau_1 = R_1C$

$C = \dfrac{\tau_1}{R_1} = \dfrac{0.4}{4\times10^3} = 10^{-4}F$ $\quad \underbrace{C = 10^{-4}F}$

3) $t_1 = 5\tau_1$
$= 5(0.4)$
$\underbrace{t_1 = 2s}$

2) discharging the capacitor:

1) due to the law of addition of voltages

$u_C + u_{R_1} + u_{R_2} = 0$

$u_C + R_1 i + R_2 i = 0$ $\quad$ but $i = \dfrac{dq}{dt} = \dfrac{d(C\times u_C)}{dt} = \dfrac{C\,du_C}{dt}$

$u_C + (R_1+R_2)i = 0$

$u_C + \underbrace{(R_1+R_2)}_{R}\dfrac{C\,du_C}{dt} = 0$

$RC\dfrac{du_C}{dt} + u_C = 0$ verified.

2) $u_C = E e^{-t/\tau_2}$

$$\frac{du_C}{dt} = E\left(\frac{-1}{\tau_2} e^{-t/\tau_2}\right) = \frac{-E}{\tau_2} e^{-t/\tau_2}$$

replace u_C & $\frac{du_C}{dt}$ in the diff. eq.

$$RC\left(-\frac{E}{\tau_2} e^{-t/\tau_2}\right) + E e^{-t/\tau_2} = 0$$

$$\underbrace{E e^{-t/\tau_2}}_{\neq 0}\left(-\frac{RC}{\tau_2} + 1\right) = 0$$

$$-\frac{RC}{\tau_2} + 1 = 0$$

$$\frac{RC}{\tau_2} = 1$$

$$\underline{\tau_2 = RC}$$

3) $t_2 = 5\tau_2$ (R.T.P)

$u_C = E e^{-t/\tau_2}$

$= E e^{-5\tau/\tau_2}$

$= E(6.73 \times 10^{-3})$

$u_C = 0\ V$

3) $t_1 = 5R_1C$

$t_2 = 5(R_1 + R_2)C$

$\Rightarrow (R_1 + R_2) > R_1$

$t_2 > t_1$ verified

LS 2019 I

08NOV2023

ex. 2:

$E = 10\,V$

1) $i = \frac{dq}{dt}$ but $q = C \times u_C$

$i = \frac{d}{dt}(C \times u_C)$

$$\boxed{i = C\frac{du_C}{dt}}$$

2) due to the law of addition of voltages:

$u_G = u_C + u_R$

$E = u_C + Ri$ but $i = C\frac{du_C}{dt}$

$E = u_C + RC\frac{du_C}{dt}$

$$\boxed{RC\frac{du_C}{dt} + u_C = E}$$

$(e^u)' = u'e^u$

3) $u_C = a + be^{-t/\tau}$

$\frac{du_C}{dt} = b\left(\frac{-1}{\tau}e^{-t/\tau}\right)$

$\frac{du_C}{dt} = \frac{-b}{\tau}e^{-t/\tau}$ replace u_C & $\frac{du_C}{dt}$ in diff. eq.

$RC\left(\frac{-b}{\tau}e^{-t/\tau}\right) + a + be^{-t/\tau} = E$

$a + be^{-t/\tau}\left(\frac{-RC}{\tau} + 1\right) = E$ $(a = E)$

$be^{-t/\tau}\left(\frac{-RC}{\tau} + 1\right) = 0$ $(be^{-t/\tau} \neq 0)$

$\frac{-RC}{\tau} = -1$

$$\boxed{\tau = RC}$$

at $t=0$, $u_c=0$

$$E + be^0 = 0$$

$$\boxed{b = -E}$$

4) $i = \frac{E}{R} e^{-t/RC}$??

$\Rightarrow i = \frac{C\,du_c}{dt}$

$$i = \cancel{C}\left(+\frac{E}{R\cancel{C}} e^{-t/RC}\right)$$

$$i = \frac{E}{R} e^{-t/RC}$$ verified

5) $I_{0\,(t=0)} = 5\,mA = 5\times10^{-3}\,A$

$I_0 = \frac{E}{R} e^0$, $I_0 = \frac{E}{R}$ $\Rightarrow R = \frac{E}{I_0} = \frac{10}{5\times10^{-3}}$

$$\underbrace{R = 2\times10^3\ \Omega}$$

6) $u_R = Ri$

$= \cancel{R}\left(\frac{E}{\cancel{R}} e^{-t/RC}\right)$

$u_R = Ee^{-t/RC}$

or// $u_c + u_R = E$

$E(1-e^{-t/RC}) + u_R = E$

$\cancel{E} - Ee^{-t/RC} + u_R = \cancel{E}$

$u_R = Ee^{-t/RC}$

7) 1) R.T.P. $t_1 = RC\ln 2$

$u_c = u_R$

$E - Ee^{-t_1/RC} = Ee^{-t_1/RC}$

$E = Ee^{-t_1/RC} + Ee^{-t_1/RC}$

$E = 2Ee^{-t_1/RC}$

$\frac{E}{2E} = e^{-t_1/RC}$

$\frac{1}{2} = e^{-t_1/RC}$

$\ln(\frac{1}{2}) = \ln(e^{-t_1/RC})$

$\ln 1 - \ln 2 = -\frac{t_1}{RC}$

$\frac{t_1}{RC} = \ln 2$

$t_1 = RC\ln 2$

verified

CS CamScanner

2) $t_1 = 1.4\ ms$

$= 1.4 \times 10^{-3}\ s$

$1.4 \times 10^{-3} = 2000 \times C \times \ln 2$

$C = \dfrac{1.4 \times 10^{-3}}{2000 \times \ln 2}$

$C = 1 \times 10^{-6}\ F$

8) 1) $\tau = CR$ corresponds to ~~the~~ an equation of st. line passing through the origin ($y = ax$)

$\Rightarrow$ verified.

2) $\tau = RC$

at $\tau = 3 \times 10^{-3}\ s$, $R = 3 \times 10^{-3}\ \Omega$ (graphically)

$\Rightarrow C = \dfrac{\tau}{R} = \dfrac{3 \times 10^{-3}}{3 \times 10^{-3}} = 1 \times 10^{-6}\ \Omega$ verified.

* 2020 session (private):

ex. 2:

$R = 1k\Omega = 10^{+3}\ \Omega$.

1) figure

2) due to the law of addition of voltages:

$U_G = U_C + U_R$

$E = U_C + Ri$ but $i = \dfrac{dq}{dt} = \dfrac{\cancel{d(Cu_C)}}{\cancel{dt}}$

$= C\dfrac{du_C}{dt}$

$E = U_C + RC\dfrac{dU_C}{dt}$

~~RCU_C~~ $RC\dfrac{dU_C}{dt} + U_C = E$

3) $U_C = E(1 - e^{-t/RC})$

$U_C = E - Ee^{-t/RC}$

$$\frac{dU_C}{dt} = 0 - E\left(\frac{-1}{RC} e^{-t/RC}\right)$$

$$\frac{dU_C}{dt} = \frac{E}{RC} e^{-t/RC}$$

replace U_C & $\frac{dU_C}{dt}$ in the diff. eq.

$$RC\left(\frac{E}{RC} e^{-t/RC}\right) + E - Ee^{-t/RC} \stackrel{?}{=} E$$

$$\cancel{Ee^{-t/RC}} + E - \cancel{Ee^{-t/RC}} \stackrel{?}{=} E$$

$$E = E$$

$\Rightarrow$ $U_C = E(1-e^{-t/RC})$ is a solution.

4) $U_C + U_R = E$

$E - Ee^{-t/RC} + U_R = E$

$$U_R = \cancel{E} - \cancel{E} + Ee^{-t/RC}$$

$$U_R = Ee^{-t/RC}$$

5) 1) $U_R = Ee^{-t/RC}$ is a function of exponential decay that is in agreement with the shape of the curve.

2) at $t_0 = 0$, $U_C = 0$ & $U_R = E$

$U_R = 8V$.

6) * at $t = \tau$

$U_R = Ee^{-t/\tau}$ $\Rightarrow$ statement 1: true.

$= Ee^{-\tau/\tau}$

$= Ee^{-1}$

$U_R = 0.37E$

* statement ~~2~~ 3: true

as $\tau \nearrow$, $5\tau \nearrow$

$\Rightarrow$ time needed to reach the steady state $\nearrow$

7) $t = \tau$

$U_R = 0.37E$

$= 0.37(8)$

$U_R = 2.96\ V$

graphically: $\tau \approx 5\ ms \approx 5 \times 10^{-3}\ s$

8) $\tau = RC$

$5 \times 10^{-3} = (1 \times 10^{-3})C$

$C = \frac{5 \times 10^{-3}}{1 \times 10^{-3}} = 5 \times 10^{-6}\ F$.

classwork * 2013 session 2:

third exercise:

$E = 10\ V$

$R_1 = R_2 = 10\ k\Omega = 10 \times 10^{-3}\ \Omega = 10^4\ \Omega$

A) 1) a) due to the law of addition of voltages

$U_E = U_C + U_{R_1}$

$E = U_C + R_1 i$ but $i = \frac{dq}{dt} = C\frac{dU_C}{dt}$

$E = U_C + R_1 C\frac{dU_C}{dt}$

$\Rightarrow R_1 C\frac{dU_C}{dt} + U_C = E$

b) $U_C = A(1 - e^{-t/\tau_1})$

$\frac{dU_C}{dt} = 0 + \frac{A}{\tau_1} e^{-t/\tau_1}$

$\frac{dU_C}{dt} = \frac{A}{\tau_1} e^{-t/\tau_1}$ replace in diff. eq.

$\Rightarrow R_1 C\left(\frac{A}{\tau_1} e^{-t/\tau_1}\right) + A - Ae^{-t/\tau_1} = E$

$A + Ae^{-t/\tau_1}\left(\frac{R_1 C}{\tau_1} - 1\right) = E$

$A = E$ and $\underbrace{Ae^{-t/\tau_1}}_{\neq 0}\left(\frac{R_1 C}{\tau_1} - 1\right) = 0$

$\frac{R_1 C}{\tau_1} - 1 = 0 \Rightarrow \frac{R_1 C}{\tau_1} = 1$

$\tau_1 = R_1 C$

c) $U_C = A(1-e^{\frac{-t}{R_1C}}) = E(1-e^{-t/R_1C})$

at $t = 5\tau$

$U_C = E(1-e^{-5\tau/\tau_1})$

$= E(1-e^{-5})$

$= 0.99E$

$\boxed{U_C = E}$ verified

OR at the end of the charging process

$t \longrightarrow \infty$

$U_C = E(1-e^{-\infty})$

$= E(1-0)$

$U_C = E$ verified

d) $U_{R_1} = Ee^{-t/R_1C}$??

$U_C + U_{R_1} = E$

$E(1-e^{-t/R_1C}) + U_{R_1} = E$

$U_{R_1} = E - E(1-e^{-t/R_1C})$

$= E - E + Ee^{-t/R_1C}$

$\boxed{U_R = Ee^{-t/R_1C}}$ verified

e) $\ln(U_{R_1}) = ??$

$U_{R_1} = Ee^{-t/R_1C}$

$\ln(U_{R_1}) = \ln(Ee^{-t/R_1C})$

$\ln(U_{R_1}) = \ln E + \ln(e^{-t/R_1C})$

$\ln(U_{R_1}) = \frac{-1}{R_1C} t + \ln E$ $(y = ax + b)$

2) a) $\ln(U_{R_1}) = \frac{-1}{R_1C} t + \ln E$

is an equation of straight ~~th~~ line that corresponds to $y = ax + b$ where $y = \ln(U_{R_1})$

$b = \ln E$

$a = -1/R_1C$

and $a < 0$, so the straight line is decreasing.

b) $t = 10\ ms = 10 \times 10^{-3} s = 0.01 s$

$\ln(U_{R_1}) = 1.3 V$

$\Rightarrow 1.3 = \frac{-1}{R_1C}(0.01) + \ln 10$

$1.3 - \ln 10 = \frac{-0.01}{10^4 \times C}$

$-1 = -1 \times 10^{-6} \times C$

$C = 1 \times 10^{-6} F$.

$C = 1\ \mu F$

B) 1) during discharging, the plate next to side B is positively charged.

2) due to the law of addition of voltages

$$u_C + u_R = 0$$

$$u_C + Ri = 0 \qquad \text{but } i = \frac{dq}{dt} = \frac{C\,du_C}{dt}$$

$$u_C + RC\frac{du_C}{dt} = 0$$

$$u_C + (R_1+R_2)C\frac{du_C}{dt} = 0$$

3) $u_C = E e^{-t/\tau_2}$

$$\frac{du_C}{dt} = \frac{-E}{\tau_2} e^{-t/\tau_2}$$ replace u_C & $\frac{du_C}{dt}$ in diff. eq.

$$E e^{-t/\tau_2} + (R_1+R_2)C\left(-\frac{E}{\tau_2} e^{-t/\tau_2}\right) = 0$$

$$\underbrace{E e^{-t/\tau_2}}_{\neq 0}\left(1 - \frac{(R_1+R_2)C}{\tau_2}\right) = 0$$

$$1 - \frac{(R_1+R_2)C}{\tau_2} = 0$$

$$\tau_2 = (R_1+R_2)C$$

4) at $t = \tau$ $\qquad \Leftarrow \quad C = \frac{\tau_2}{R_1+R_2} = \frac{0.02}{20000} = 10^{-6}\,F$

$$u_C = 10 e^{-\tau/\tau_2}$$

$$u_C = 10 e^{-1}$$

$$u_C = 3.67V$$

//

III) Differential Equation in terms of q:

due to the law of addition of voltages

$U_C + U_R = E$ but $U_R = Ri$

$U_C + Ri = E$ but $i = \frac{dq}{dt}$ $q = C \times U_C$, $U_C = \frac{q}{C}$

$$\left(\frac{q}{C} + R\frac{dq}{dt} = E\right) \times \frac{1}{R}$$

$$\frac{dq}{dt} + \frac{1}{RC} q = E$$ $\rightarrow$ 1st ordre differential eq. in terms of q.

IV) Differential Equation in terms of i:

$U_C + U_R = E$

$U_C + Ri = E$ but $U_C = \frac{q}{C}$

$$\frac{d}{dt}\left(Ri + \frac{q}{C} = E\right)$$

$$R\frac{di}{dt} + \frac{1}{C}\frac{dq}{dt} = 0$$ but $i = \frac{dq}{dt}$

$$\left(R\frac{di}{dt} + \frac{1}{C} i = 0\right) \times \frac{1}{R}$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

14NOV'23

LS 2013 session 1:

first exercise:

$C = 1\mu F = 10^{-6} F$.

A) analytical study:

1) $q = U_{AB} \times C$

2) due to the law of addition of voltages

$$U_g = U_C + U_R$$

$$E = U_C + Ri$$

$$\left(E = \frac{q}{C} + R\frac{dq}{dt} \right) \times \frac{1}{R}$$

$$\frac{dq}{dt} + \frac{q}{CR} = \frac{E}{R}$$ $\rightarrow$ 1st order differential eq. that governs the growth of q with time.

3) a) at $t=0$, $I_0 \overset{?}{=} \frac{E}{R}$

at $t=0$, $q = 0C$ & $U_{AB} = 0$.

$$\Rightarrow E = R\frac{dq_{max}}{dt}$$

$$E = RI_0 \Rightarrow I_0 = \frac{E}{R}$$ verified

b) $Q_{max} = ??$

at $t = t_{final}$, $\frac{dq}{dt} = 0$ $(q = cst)$

$$0 + \frac{1}{RC} Q_{max} = \frac{E}{R}$$

$$Q_{max} = EC$$

B) exploitation of the curve.

1) a) at $t = t_{final}$, $q = Q_{max} = 10^{-5} C$

b) $Q_{max} = EC$

$10^{-5} = E \times 10^{-6}$

$E = 10 V$

2) a) $I_0 = \frac{dq_{max}}{dt} = \frac{\Delta q}{\Delta t}$ tangent

$= \frac{10^{-6} - 0}{10^{-2} - 0}$

$= 10^{-3}\ A$

$= 1 \times 10^{-3}\ A = 1\ mA$

b) $I_0 = \frac{E}{R}$

$R = \frac{E}{I_0} = \frac{10}{10^{-3}} = 10^4\ \Omega$

3) $t_1 = 10^{-2} s$

at $t_1 \rightarrow q = 0.63 \times 10^{-5}$

$q = C \times U_C$

$U_C = \frac{q}{C} = \frac{0.63 \times 10^{-5}}{10^{-6}} = 6.3\ V$

$U_R + U_C = E$

$U_R = E - U_C$

$U_R = 10 - 6.3$

$U_R = 3.7 V$

$U_R = Ri$

$i = \frac{U_R}{R} = \frac{3.7}{10^4} = 3.7 \times 10^{-4}\ A$

c) energy stored in a capacitor:

$w = \frac{1}{2}\frac{q^2}{C}$

1) $w_0 = \frac{1}{2}\frac{q_0^2}{C} = 0 J$

$w_{t_1} = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(0.63 \times 10^{-5})^2}{10^{-6}} = 0.19845 \times 10^{-4}\ J$

2) $P = \frac{|\Delta w|}{\Delta t} = \frac{0.19845 \times 10^{-4} - 0}{10^{-2}} = 1.9845 \times 10^{-3}$ watt

GS 2023 session 2

ex.3:

1) charging the capacitor:

$$R\frac{di}{dt} + \frac{1}{C} i = 0$$

1) $i = \frac{E}{R} e^{-t/RC}$ $\quad \frac{di}{dt} = \frac{E}{R}\left(-\frac{1}{RC} e^{-t/RC}\right) = \frac{-E}{R^2C} e^{-t/RC}$

replace in diff. eq. i & $\frac{di}{dt}$:

$$\cancel{R}\left(-\frac{E}{R^{\cancel{2}}C} e^{-t/RC}\right) + \frac{1}{C}\left(\frac{E}{R} e^{-t/RC}\right) \stackrel{?}{=} 0$$

$$-\frac{E}{RC} e^{-t/RC} + \frac{E}{RC} e^{-t/RC} \stackrel{?}{=} 0$$

$0 = 0$ verified.

2) at $t_0 = 0$, $i_0 = \frac{E}{R} e^0 = \frac{E}{R}$

3) at $t_0 = 0$, $i_0 = 1.2 \text{ mA} = 1.2 \times 10^{-3}$ A.

$\Rightarrow i_0 = \frac{E}{R}$, $R = \frac{E}{i_0} = \frac{12}{1.2 \times 10^{-3}} = 10^4 \ \Omega$

4) due to the law of addition of voltages:

$U_G = U_R + U_C$

$E = Ri + U_C$ $\quad$ but $q = U_C \times C \Rightarrow U_C = \frac{q}{C}$

$E = R\left(+\frac{E}{R} e^{-t/RC}\right) + \frac{q}{C}$

$\frac{q}{C} = E - E e^{-t/RC}$

$q = EC - EC e^{-t/RC}$ verified

5) $Q = 12 \times 10^{-4}$ C, $C = 100 \mu F = 10^{-4}$ F

$Q = EC - EC e^{-5RC/RC}$

$Q = EC(1 - e^{-5RC/RC}) = EC(1 - e^{-5}) = EC$

$C = \frac{Q}{E} = \frac{12 \times 10^{-4}}{12} = 10^{-4} F = 100 \mu F$.

6) $\tau = RC$

$= 10^4 \times 10^{-4}$

$= 1\ s$

2) discharging the capacitor:

1) $u_c = Ee^{-t/\tau'}$

at $t = \tau'$, $u_c = 0.37E$

$u_c = 0.37(12)$

$u_c = 4.44V$

2) $\tau = \tau'$ since both circuits have the same R & C

3) $u_c \geqslant 1V$

graphically, $t = 2.5\ s$

4) τ & C are directly proportional

$\Rightarrow$ To increase τ, you must ~~include~~ increase C.

LS 2023 session 2:

ex.3:

$R = 100\ k\Omega = 100{,}000\ \Omega$

$U_{PN} = E = 12\ V$

1) charging the capacitor:

1) due to the law of addition of voltages

$$U_G = U_C + U_R$$

$$E = U_C + Ri \qquad \text{but } i = \frac{dq}{dt} = \frac{C\,du_C}{dt}$$

$$E = U_C + RC\frac{du_C}{dt}$$

$$\Rightarrow RC\frac{du_C}{dt} + U_C = E \quad \text{verified.}$$

2) $U_C = E - Ee^{-t/\tau}$

$$\frac{du_C}{dt} = 0 - E\left(-\frac{1}{\tau}e^{-t/\tau}\right), \quad \frac{du_C}{dt} = \frac{E}{\tau}e^{-t/\tau}$$

replace U_C & $\frac{du_C}{dt}$ in diff. eq.

$$\Rightarrow RC\left(\frac{E}{\tau}e^{-t/\tau}\right) + \left(E - Ee^{-t/\tau}\right) \stackrel{?}{=} E$$

$$RC\left(\frac{E}{\tau}e^{-t/\tau}\right) + \cancel{E} - Ee^{-t/\tau} \stackrel{?}{=} \cancel{E}$$

$$\underbrace{Ee^{-t/\tau}}_{\neq 0}\left(\frac{RC}{\tau} - 1\right) \stackrel{?}{=} 0$$

$$\frac{RC}{\tau} - 1 = 0$$

$$\frac{RC}{\tau} = 1$$

$$\boxed{\tau = RC}$$

3) $t_1 = 7s$, $U_C = E_1/2$

$\Rightarrow$ $U_C = E_1 - E_1 e^{-t_1/\tau}$

$$\frac{E_1}{2} = E_1 - E_1 e^{-t_1/\tau}$$

$$\frac{E_1}{2} = E_1 e^{-t_1/\tau}$$

$$\frac{1}{2} = e^{-t_1/\tau}$$

$$\ln 1 - \ln 2 = \frac{-t_1}{\tau}$$

$$-\ln 2 = -\frac{t_1}{\tau}$$

$$\tau = \frac{t_1}{\ln 2}$$

$$\tau = 10.098 \approx 10s .$$

4) $\tau = RC$

$10 = 100{,}000\, C$

$C = 10^{-4} F$

2) $U_C = E_1 e^{-t/\tau}$

1) R.T.P. $\ln\left(\frac{E_1}{U_C}\right) = \frac{t}{\tau}$