Course Summary

1. Definition:

The exponential function $f(x) = e^x$ is defined, continuous and strictly increasing over $\mathbb{R}$, such that for every real number x, $e^x > 0$.

Remarks

- $e^0 = 1$ and $e^1 = e$
- The two equations $y = \ln x$ and $x = e^y$ are equivalent
- For every real number x, we have, $\ln e^x = x$
- For every strictly positive real number x, we have, $e^{\ln x} = x$

2. Rules of Calculations:

Let x and y be two real numbers:

- Exponential of a sum: $e^{x+y} = e^x . e^y$
- Exponential of a difference: $e^{x-y} = \frac{e^x}{e^y}$
- Exponential of a product of a real number by a rational number: $e^{rx} = (e^x)^r$

3. Equations and Inequations:

Let a and b be two real numbers, then, the following hold:

- The two equations: $a = b$ and $e^a = e^b$ are equivalent.
- The two inequations: $a < b$ and $e^a < e^b$ are equivalent.
- The two inequations: $a > b$ and $e^a > e^b$ are equivalent.
- The two equations: $e^x = a$ and $x = \ln a$ are equivalent.
- The two inequations: $e^x < a$ and $x < \ln a$ are equivalent.
- The two inequations: $e^x > a$ and $x > \ln a$ are equivalent.

4. Limits :

- $\lim\limits_{x \to -\infty} e^x = 0$
- $\lim\limits_{x \to +\infty} e^x = +\infty$

5. Derivative and Primitive:

Derivative

- $(e^x)' = e^x$
- If u is a differentiable function of x then, $(e^u)' = u'e^u$

Primitive

- $\int e^x dx = e^x + c$
- $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$
- $\int u'e^u dx = e^u + c$

6. **Study of the Exponential Function:**

Consider the function $f(x)=e^x$ and designate by (C) be its representative curve in an orthnormal system $(O;\vec{i};\vec{j})$

- $f'(x)=e^x>0$, so f is defined, continuous, differentiable and strictly increasing for every real number x.
- $\lim\limits_{x\to-\infty} f(x)=0$ and $\lim\limits_{x\to+\infty} f(x)=+\infty$, the axis of abscissas is a horizontal asymptote to the curve (C) at $-\infty$.
- The curve (C) passes through the two point $A(0;1)$ and $B(1;e)$.
- The adjacent table is the table of variations of the function $f(x)=e^x$

x	$-\infty$	$+\infty$
$f'(x)$	+	
$f(x)$	0	$+\infty$

- The curve below is (C) the representative curve of f