Work Sheet (logarithms)		Date: 07- 12 – 2023
Class : 12 – LS		
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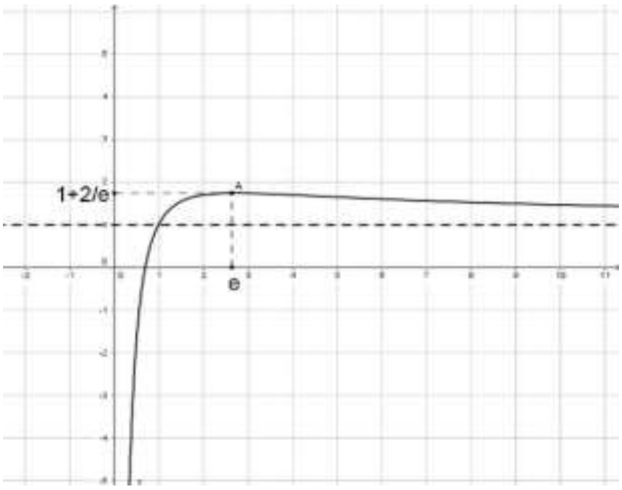
I-

Consider the function f defined over $]0;+\infty[$ by $f(x) = ax + b\ln^2 x$.

In the adjacent figure: (C') represents the **derivative function f ' of f**.

(C') admits a maximum at $A\left(e;1+\dfrac{2}{e}\right)$.



1) Determine $f'(x)$ in terms of a, b and x.
2) Verify that a = b = 1.
3) Show that the equation $f'(x)=0$ has a unique root α such that $0.7<\alpha<0.8$.
4) Study the sign of $f'(x)$ over $]0;+\infty[$.
5) Show that the curve (C) of f, has an inflection point W of coordinates to be determined.

II-

Given the table of variations of a continuous function f defined on $]0;+\infty[$ by : $f(x) = m + n\dfrac{\ln x}{x}$.

x	0		e		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$	$+\infty$	$\searrow$	$1-2e^{-1}$	$\nearrow$	1

1- Determine $f'(x)$ in terms of n and x.
2- Prove that $m=1$ and $n=-2$.
3- Prove that , for all x in $]0;+\infty[$, $\ln x \le \dfrac{x}{e}$.
4- Prove that the representative curve of any antiderivative of f on $]0;+\infty[$ admits a point of inflection I.
5- Determine the antiderivative F of f for which the point I belongs to the line of equation $y=x$.

III-

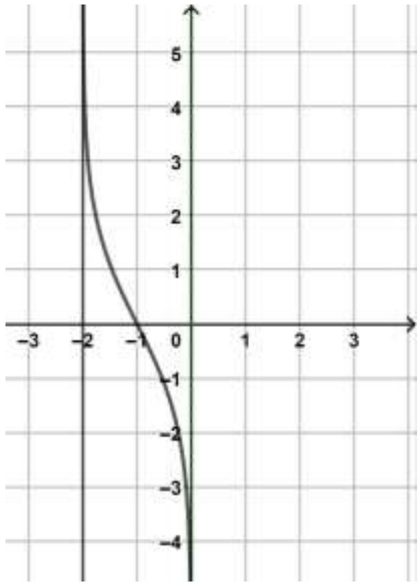
Consider the function : $g(x) = ax^2 + b\ln x + c$, defined over $]0\ ;+\infty[$.

let g' be its derivative, and denote by (G) the curve of g , such that (G) passes through the point $A(1;b+2)$.

Given the table of variation of the function g' the derivative of g.

1- Show that g admits a point of inflection whose coordinates to be determined.
2- Study the sign of g'.
3- Show that $a=-1$, $b=-2$ and $c=1$.
4- Find the equation of tangent (T) to (G) at the point A.
5- Draw table of variation of g.
6- Calculate $g(1)$, deduce the sign of g .

IV- The following curve (C) represents the function g defined on $]-2;0[$ by $g(x)=\ln\left(\dfrac{ax}{x+b}\right)$ where $b\neq 0$.



1) Verify that $a=-1$ and $b=2$.

2) Prove that the point $W\,(-1;0)$ is a center of symmetry.

3) Verify that $g''(x)=\dfrac{-4(x+1)}{x^2(x+2)^2}$.

4) Verify that g admits an inflection point whose coordinates are to be determined.

V- The adjacent table is the table of variations of a function f defined, on $]0;+\infty[$ .Denote by (C) the representative curve of f in an orthonormal system.

x	0		1		$e^{\frac{3}{4}}$		$e^{\frac{3}{2}}$		$+\infty$
$f(x)$	‖ $+\infty$	↘	1	↘	$-\frac{1}{8}$	↗	1	↗	$+\infty$

1) Solve $f(x)=1$.

2) Suppose that the function f is defined by $f(x)=2(\ln x)^2-3\ln x+1$.

a) Solve $f(x)=0$.

b) Show that $f''(x)=\dfrac{-4\ln x+7}{x^2}$.

c) Deduce that the curve (C) admits an inflection point I.

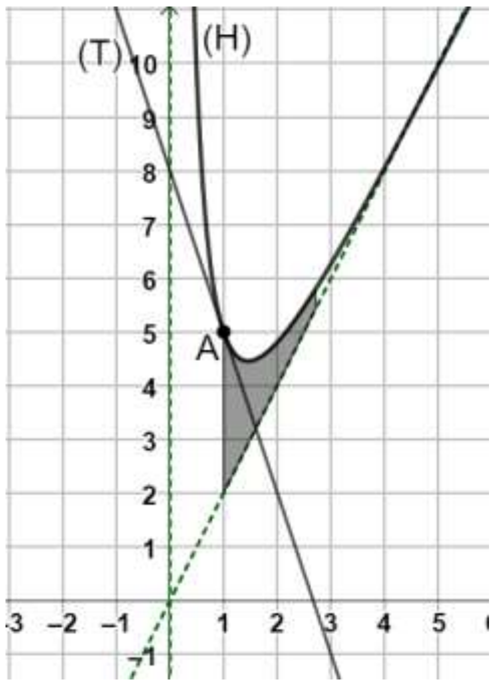
3) Let h be the function defined by $h(x)=\ln[f(x)]$.

a) Determine the domain of definition of h and set up its table of variations.

b) Solve the inequality $h(x)\geq 0$.

VI-

<u>Part A:</u> The adjacent figure represent the representative curve of the function h defined, over $]0;+\infty[$, by $h(x)=ax-\dfrac{2\ln x}{x}+\dfrac{b}{x}$.



- (H) passes through $A(1;5)$.
- (T) : $y=-3x+8$ is the tangent to (H) at A .

1) Prove that $a=2$ and $b=3$.

2) Show that the line $(d): y=2x$ is an asymptote to (H).

3) Calculate the area of the shaded part limited by (H), (d) and the two lines of equations $x=1$ and $x=e$.

<u>Part B:</u> Let g be the function defined over $]0;+\infty[$ by

$g(x)=x^2-1-(\ln x)^2+3\ln x$.

1) Prove that $g'(x)=h(x)$ and set up the table of variations of g.

2) Calculate $g(1)$ Deduce the sign of $g(x)$ according to the values of x.