| ExaMath Groups | Mathematics Exam<br>Class: LS | Prepared by: Khodor Hannoush<br>Edited by: Randa Chehade |
|---|---|---|
| **Number of questions: 3** | **Sample 10 – year 2023**<br>**Duration: 1½ hours** | **Name:**<br>**N°:** |



- *This exam consists of three problems inscribed on two pages.*
- *The use of a non-programmable calculator is allowed.*

**I – (4 points)**

In the table below, only one of the proposed answers to each question, is correct.
Write down the number of each question and give, **with justification**, the answer corresponding to it.

| N° | Questions | Answers a | Answers b | Answers c |
|---|---|---|---|---|
| 1) | Consider the function $g$ defined over $]0;+\infty[$ as $g(x)=\frac{1}{x}-2\ln x$. The image of the interval $[e;+\infty[$ by $g$ is | $\left]-\infty;\frac{1}{e}-2\right[$ | $\left]-\infty;\frac{1}{e}-2\right]$ | $\left[\frac{1}{e}-2\,;0\right[$ |
| 2) | Let $h$ be the function defined over $]0;+\infty[$ by $h(x)=\ln^2(x)-2$, then the point of inflection of the representative curve of function $h$ is of coordinates | $(1\,;-2)$ | $(e;0)$ | $(e;-1)$ |
| 3) | The representative curve of the function $f$ defined over $\mathbb{R}$ by $f(x)=\ln\left(e^x+1\right)+2$, admits at $-\infty$ a horizontal asymptote of equation | $y=2$ | $y=\frac{5}{2}$ | $y=3$ |
| 4) | An urn contains 10 identical balls indistinguishable by touch, such as: 6 balls are red and 4 balls are green. We draw at random and **successively without replacement** two balls form the urn. The probability of the following event $D$: "The two drawn balls have different colors" is | $\frac{4}{15}$ | $\frac{8}{15}$ | $\frac{11}{15}$ |

**II – (6 points)**

In a high school 40% of the students of the 3rd secondary year are enrolled in the LS section and the others are enrolled in the GS section. Among the students in the LS section, 30% decided to continue their studies at the Faculty of Engineering while 40% of the students in the GS section decided not to continue their studies at this faculty.
A student from the 3rd secondary year is randomly chosen.
Consider the following events:
*G*: "The chosen student is in the GS section";
*E*: "The chosen student decided to continue his studies at the Faculty of Engineering".

**1) a)** Determine $P(E/G)$ and deduce that $P(E\cap G)=0.36$.

**b)** Show that $P(E)=0.48$.

2) The chosen student decides not to continue his studies in the Faculty of Engineering. Calculate the probability that he is in the LS section (give the answer to the nearest $10^{-3}$).

3) In this part, suppose that the total number of students in the 3rd secondary year is 200. We choose 4 students simultaneously. Calculate the probability that at least one student will continue his studies in the Faculty of Engineering (give the answer to the nearest $10^{-3}$).

**III – (10 points)**

**Part A**

Consider the function $g$ defined over $\mathbb{R}$ by: $g(x)=1-x-4e^{x}$.

**1)** Calculate $\lim\limits_{x\to-\infty} g(x)$ and $\lim\limits_{x\to+\infty} g(x)$.

**2)** Set up the table of variations of function $g$.

**3)** Show that the equation $g(x)=0$ has a unique solution $\alpha$ and verify that $-0.9<\alpha<-0.7$.

**4)** Study, according to the values of $x$, the sign of $g(x)$.

**Part B**

Consider the function $f$ defined over $\mathbb{R}$ by $f(x)=(2-x-2e^{x})e^{x}$.

Denote by $(C)$ the representative curve of the function $f$ in an orthonormal system $(O\,;\vec{i}\,;\vec{j})$.

**1)** Calculate $\lim\limits_{x\to-\infty} f(x)$. Interpret graphically.

**2)** Determine $\lim\limits_{x\to+\infty} f(x)$ and calculate $f\left(\dfrac{1}{2}\right)$ to the nearest $10^{-2}$.

**3)** Prove that $f'(x)=g(x)e^{x}$, then set up the table of variations of $f$.

**4) a)** Verify that $f(\alpha)=\dfrac{(3-\alpha)(1-\alpha)}{8}$.

**b)** Deduce that $f(\alpha)<1$.

**5)** Write an equation of the tangent $(T)$ to the curve $(C)$ at the point of abscissa 0.

**6)** Draw $(T)$ and $(C)$

**Part C**

Consider the function $h$ defined by $h(x)=\ln[1-f(x)]$.

1) Determine the domain of definition of $h$.

2) Set up the table of variations of the function $h$.

| QI | Answers | 4 pts. |
|---|---|---|
| 1) | $g'(x)=\frac{-1}{x^2}-\frac{2}{x}=\frac{-1-2x}{x^2}<0$, for every $x$ belongs to $]0\,;+\infty[$.<br>$g$ is continuous and strictly decreasing over its domain and in particular over $[e;+\infty[$.<br>$g(e)=\frac{1}{e}-2\ln e=\frac{1}{e}-2$ ; and $\lim\limits_{x\to+\infty} g(x)=\lim\limits_{x\to+\infty}\left(\frac{1}{x}-2\ln x\right)=-\infty$.<br>Hence $g\left([e;+\infty[\right)=\left]\lim\limits_{x\to+\infty} g(x);\ g(e)\right]=\left]-\infty;\frac{1}{e}-2\right]$.<br>Thus, the correct answer is **b.** | 1 |
| 2) | $h(x)=\ln^2(x)-2$ ;<br>$h'(x)=\frac{2\ln(x)}{x}$ ; $h''(x)=\frac{-2\ln(x)+2}{x^2}$, same sign as $-2\ln(x)+2$ since $x^2>0$ over $]0\,;\,+\infty[$.<br>If $h''(x)=0\,;-2\ln(x)+2=0$ ; $\ln(x)=1$ ; $x=e$.<br>$h''$ vanishes at $x=e$ and changes its sign, hence the representative curve of $h$ admits an inflection point of coordinates $(e;\ f(e))$, that is $(e;-1)$.<br>Thus, the correct answer is **c.** | 1 |
| 3) | $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\left[\ln\left(e^x+1\right)+2\right]=\ln(0+1)+2=2$.<br>Thus, the correct answer is **a.** | 1 |
| 4) | $P(D)=\frac{A_6^1\times A_4^1}{A_{10}^2}\times\frac{2!}{1!\times 1!}=\frac{8}{15}$.<br>Thus, the correct answer is **b.** | 1 |

| QII | Answers | 6 pts. |
|---|---|---|
| 1) a) | $P(E/G)=1-0.4=0.6$;<br>$P(E\cap G)=P(E/G)\times P(G)=0.6\times0.6=0.36$. | 1½ |
| 1) b) | $P(E)=P(E\cap G)+P(E\cap\overline{G})=0.36+P(E/\overline{G})\times P(\overline{G})=0.36+0.3\times0.4=0.48$. | 1½ |
| 2) | $P(\overline{G}/\overline{E})=\frac{P(\overline{G}\cap\overline{E})}{P(\overline{E})}=\frac{P(\overline{E}/\overline{G})\times P(\overline{G})}{1-P(E)}=\frac{0.7\times0.4}{1-0.48}\approx0.538$ (by default). | 1 |
| 3) | The number of students who will continue their studies in the Faculty of Engineering is: $200\times P(E)=96$; Let $F$ be the event: "at least one student will continue his studies in the Faculty of Engineering";<br>$P(F)=1-\frac{C_{104}^4}{C_{200}^4}\approx0.928$ (by default) | 2 |

<table>
<tr><th>QIII</th><th>Answers</th><th>10 pts.</th></tr>
<tr><td>A1)</td><td>$\lim\limits_{x\to-\infty} g\left(x\right)=1+\infty-0=+\infty$.<br>$\lim\limits_{x\to+\infty} g\left(x\right)=1-\infty-\infty=-\infty$.</td><td>½</td></tr>
<tr><td>A2)</td><td>$g'\left(x\right)=-1-4e^{x}<0$ for every $x\in\mathbb{R}$ (since $e^{x}>0$ for every $x\in\mathbb{R}$),<br>thus, $g$ is strictly decreasing over $\mathbb{R}$.<br>Table of variations of $g$ :<br>$$\begin{array}{c|ccc} x & -\infty & \alpha & +\infty \\ \hline g'(x) & & - & \\ \hline g(x) & +\infty & \searrow 0 \searrow & -\infty \end{array}$$</td><td>1</td></tr>
<tr><td>A3)</td><td>$g$ is continuous, strictly decreasing and changes its sign from positive ($+\infty$) to negative ($-\infty$) then the equation $g\left(x\right)=0$ admits a unique solution $\alpha$ over $\mathbb{R}$.<br>Moreover $g\left(-0.9\right)\approx0.27>0$ and $g\left(-0.7\right)\approx-0.28>0$ , therefore $-0.9<\alpha<-0.7$ .</td><td>1</td></tr>
<tr><td>A4)</td><td>According to the table of variations of $g$ :<br>• $g\left(x\right)>0$ if $x<\alpha$.<br>• $g\left(x\right)=0$ if $x=\alpha$.<br>• $g\left(x\right)<0$ for $x>\alpha$.</td><td>½</td></tr>
<tr><td>B1)</td><td>$\lim\limits_{x\to-\infty} f\left(x\right)=\lim\limits_{x\to-\infty}\left(2e^{x}-xe^{x}-2e^{2x}\right)=2\left(0\right)-0-2\left(0\right)=0.$<br>Thus, the x – axis is a horizontal asymptote to $(C)$ at $\left(-\infty\right)$.</td><td>¾</td></tr>
<tr><td>B2)</td><td>$\lim\limits_{x\to+\infty} f\left(x\right)=\lim\limits_{x\to+\infty}\left(2-x-2e^{x}\right)e^{x}=\left(2-\infty-\infty\right)\left(+\infty\right)=-\infty$.</td><td>¼</td></tr>
<tr><td>B3)</td><td>$f'\left(x\right)=\left(-1-2e^{x}\right)e^{x}+e^{x}\left(2-x-2e^{x}\right)=\left(1-x-4e^{x}\right)e^{x}=g\left(x\right)e^{x}$, verified.<br>$f'\left(x\right)$ and $g\left(x\right)$ have same sign over $\mathbb{R}$ $\left(e^{x}>0 \text{ for every } x\in\mathbb{R}\right)$.<br>Table of variations of $f$ :<br>$$\begin{array}{|c|ccccc|} \hline x & -\infty & & \alpha & & +\infty \\ \hline f'\left(x\right) & & + & 0 & - & \\ \hline f\left(x\right) & 0 & \nearrow & f\left(\alpha\right) & \searrow & -\infty \\ \hline \end{array}$$</td><td>1 ½</td></tr>
<tr><td>B4)a)</td><td>$g\left(\alpha\right)=1-\alpha-4e^{\alpha}=0;\ e^{\alpha}=\dfrac{1-\alpha}{4}.$<br>$f\left(\alpha\right)=\left(2-\alpha-2e^{\alpha}\right)e^{\alpha}=\left(2-\alpha-\dfrac{1-\alpha}{2}\right)\left(\dfrac{1-\alpha}{4}\right)=\left(\dfrac{4-2\alpha-1+\alpha}{2}\right)\left(\dfrac{1-\alpha}{4}\right)=$<br>$\left(\dfrac{3-\alpha}{2}\right)\left(\dfrac{1-\alpha}{4}\right)=\dfrac{\left(3-\alpha\right)\left(1-\alpha\right)}{8}$, verified.</td><td>1</td></tr>
<tr><td>B4)b)</td><td>$-0.9<\alpha<-0.7;\ 0.7<-\alpha<0.9$ ; $1.7<1-\alpha<1.9$, and $3.7<3-\alpha<3.9$ , thus<br>$\dfrac{1.7\times3.7}{8}<\dfrac{\left(3-\alpha\right)\left(1-\alpha\right)}{8}<\dfrac{1.9\times3.9}{8}<1$, hence $f\left(\alpha\right)<1$, verified.</td><td>½</td></tr>
<tr><td>B5)</td><td>$\left(T\right): y-f\left(0\right)=f'\left(0\right)\left(x-0\right)$ ; $y-0=-3x$ ; $y=-3x$. Thus $\left(T\right): y=-3x$.</td><td>½</td></tr>
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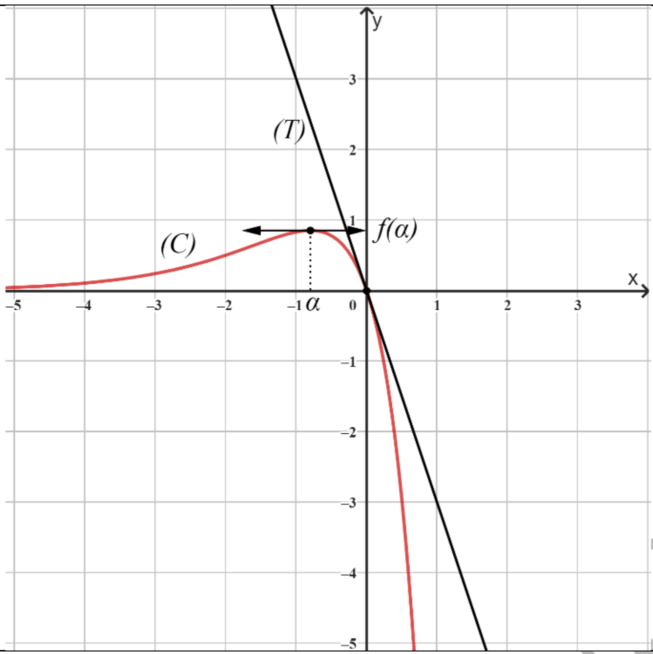
<tr>
<td>B6)</td>
<td>
</td>
<td>1</td>
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<tr>
<td>C1)</td>
<td>$h$ is defined for $1-f(x)>0$; $f\left(x\right)<1$ .<br>
Graphically: $\left(C\right)$ is below the line of equation $y=1$ for every $x\in\mathbb{R}$ , so $D_h=\mathbb{R}$ .<br>
<u>OR:</u><br>
For all values of $x$ : $f\left(x\right)\leq f\left(\alpha\right)<1$ ; thus $1-f\left(x\right)>0$ for every $x\in\mathbb{R}$ , so $D_h=\mathbb{R}$</td>
<td>½</td>
</tr>
<tr>
<td>C2)</td>
<td>$\lim\limits_{x\to-\infty} h(x)=\ln\left[1-\lim\limits_{x\to-\infty} f(x)\right]=\ln\left[1-0\right]=\ln\left(1\right)=0.$<br>
$\lim\limits_{x\to+\infty} h(x)=\ln\left[1-\lim\limits_{x\to+\infty} f(x)\right]=\ln\left[1+\infty\right]=\ln\left(+\infty\right)=+\infty.$<br>
$h'(x)=\dfrac{-f'(x)}{1-f\left(x\right)}$ , thus $h'(x)$ and $f'(x)$ have opposite signs over $\mathbb{R}$ ,<br>
since $1-f\left(x\right)>0$ for every $x\in\mathbb{R}$ .<br>
Table of variations of $h$ :
<table>
<tr><td>$x$</td><td>$-\infty$</td><td></td><td>$\alpha$</td><td></td><td>$+\infty$</td></tr>
<tr><td>$h'\left(x\right)$</td><td></td><td>$-$</td><td>$0$</td><td>$+$</td><td></td></tr>
<tr><td>$h\left(x\right)$</td><td>$0$</td><td>↘</td><td>$\ln\left[1-f(\alpha)\right]$</td><td>↗</td><td>$+\infty$</td></tr>
</table></td>
<td>1</td>
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