التاريخ

$\rightarrow \int (x^2+1)\cos x \, dx$

x^2+1		$\cos x$
	$+$	
$2x$		$\sin x$
	$-$	
2		$-\cos x$
	$+$	
0		$-\sin x$

$$= +(x^2+1)\sin x + (2x)(\cos x) + (2)(-\sin x) + C$$

Q.1

Tabular method

$$\int (x^2+2x+3)\,e^{2x}\,dx$$

der. $\downarrow$		Anti der $\downarrow$
x^2+2x+3	$(+)$	e^{2x}
$2x+2$	$(-)$	$\frac{1}{2}e^{2x}$
2	$(+)$	$\frac{1}{4}e^{2x}$
0		$\frac{1}{8}e^{2x}$

$$= +(x^2+2x+3)\left(\frac{1}{2}e^{2x}\right) - (2x+2)\left(\frac{1}{4}e^{2x}\right) + 2\left(\frac{1}{8}e^{2x}\right) + C.$$

$$= e^{2x}\left[\frac{1}{2}x^2 + x + \frac{3}{2} - \frac{1}{2}x - \frac{1}{2} + \frac{1}{4}\right] + C$$

- - -

Integration

§1- Antiderivative (primitive)

→ **Def:**

$$\int f(x)\,dx = g(x) \iff g'(x) = f(x)$$

ex:

- $\int 3\,dx = 3x + C$
- $x \neq 0$, $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$

→ Antiderivative of some particular fts

- $\int k\,dx = kx + C$
- $\int k f(x)\,dx = k \int f(x)\,dx$
- $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $n \neq -1$, $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
- $n \neq -1$, $\int u' u^n\,dx = \frac{u^{n+1}}{n+1} + C$

$$\int (2x+1)^3 dx = \frac{1}{2}\int 2(2x+1)^3 dx$$

$$= \frac{1}{2}\frac{(2x+1)^4}{4} + C$$

$$= \frac{1}{8}(2x+1)^4 + C$$

- $\int \frac{k}{x^2} dx = -\frac{k}{x} + C$
- $\int k\frac{u'}{u^2} dx = -\frac{k}{u} + C$

$$\int \frac{1}{(3x-1)^2} dx = \frac{1}{3}\int \frac{3}{(3x-1)^2} dx$$

$$= \frac{1}{3}\left(\frac{-1}{3x-1}\right) + C$$

$$= \frac{-1}{3(3x-1)} + C$$

- $\int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + C$
- $\int \frac{u'}{2\sqrt{u}} dx = \sqrt{u} + C$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int \frac{u'}{u} dx = \ln|u| + C$

$$\int \frac{\overbrace{\cos x}^{u'}}{\underbrace{\sin x}_{u}} dx = \ln|\sin x| + C$$

$$\int \frac{\sin x}{\underbrace{\cos x}_{u}} = -\int \frac{\overbrace{-\sin x}^{u'}}{\underbrace{\cos x}_{u}} dx$$

Date

$= -\ln|\cos x| + C$.

- $\int \cos x \, dx = \sin x + C$.

$\int \sin x \, dx = -\cos x + C$.

$\int \frac{1}{\cos^2 x} dx$ or $\int (1+\tan^2 x) dx = \tan x + C$

$\int \frac{-1}{\sin^2 x} dx$ or $\int -(1+\cot^2 x) dx = \cot x + C$

- $\int u' \cos u \, dx = \sin u + C$

$\int u' \sin u \, dx = -\cos u + C$.

$\int \frac{u'}{\cos^2 u} dx$ or $\int u'(1+\tan^2 u) dx = \tan u + C$

$\int \frac{-u'}{\sin^2 u} dx$ or $\int -u'(1+\cot^2 u) dx = \cot u + C$

- $\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$

$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$

$\frac{1}{a} \int a \cos(\underbrace{ax+b}_{u}) dx$

$= \frac{1}{a} \sin(ax+b)$.

$$\int \underbrace{x^2}_{v} \cdot \underbrace{e^x}_{u'} \, dx = x^2 e^x - \int 2x e^x \, dx$$

$$= x^2 e^x - 2 \underbrace{\int x e^x \, dx}_{g(x)}$$

let $u' = e^x \rightarrow u = e^x$

$v = x^2 \rightarrow v' = 2x$

$$g(x) = \int x e^x \, dx = x e^x - e^x$$

$$= x^2 e^x - 2[x e^x - e^x] + C.$$

$$= x^2 e^x - 2x e^x + 2e^x + C.$$

Note:

$$\int \underbrace{\text{poly}}_{u'} \times \underbrace{\ln}_{v} \, dx$$

$$\int \underbrace{\text{poly}}_{v} \times \underbrace{\exp}_{u'} \, dx$$

$$\int \underbrace{\text{poly}}_{v} \times \underbrace{\text{trig}}_{u'} \, dx$$

Scanned with CamScanner

- $\int e^x dx = e^x + C$

$\int u' e^u dx = e^u + C$

$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$

→ V.I.P (Integration by parts)

$$\int u' v \, dx = u.v - \int v' u \, dx$$

ex:

- $\int \underset{v}{x} . \underset{u'}{e^x} dx = uv - \int v' u \, dx$

$= xe^x - \int e^x dx$

let $u' = e^x \rightarrow u = e^x$

$v = x \rightarrow v' = 1$

$= xe^x - e^x + C.$

- $\int \underset{u'}{x} . \underset{v}{\ln x} = \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$

$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$

let $u' = x \rightarrow u = \frac{x^2}{2}$

$v = \ln x \rightarrow v' = \frac{1}{x}$

- $\int \underset{v}{(x+1)} \underset{u'}{\cos x} \, dx = (x+1) \sin x - \int \sin x \, dx$

$= (x+1) \sin x + \cos x + C.$

let $u' = \cos x \rightarrow u = \sin x$

$v = x+1 \rightarrow v' = 1$

Scanned with CamScanner