Everest

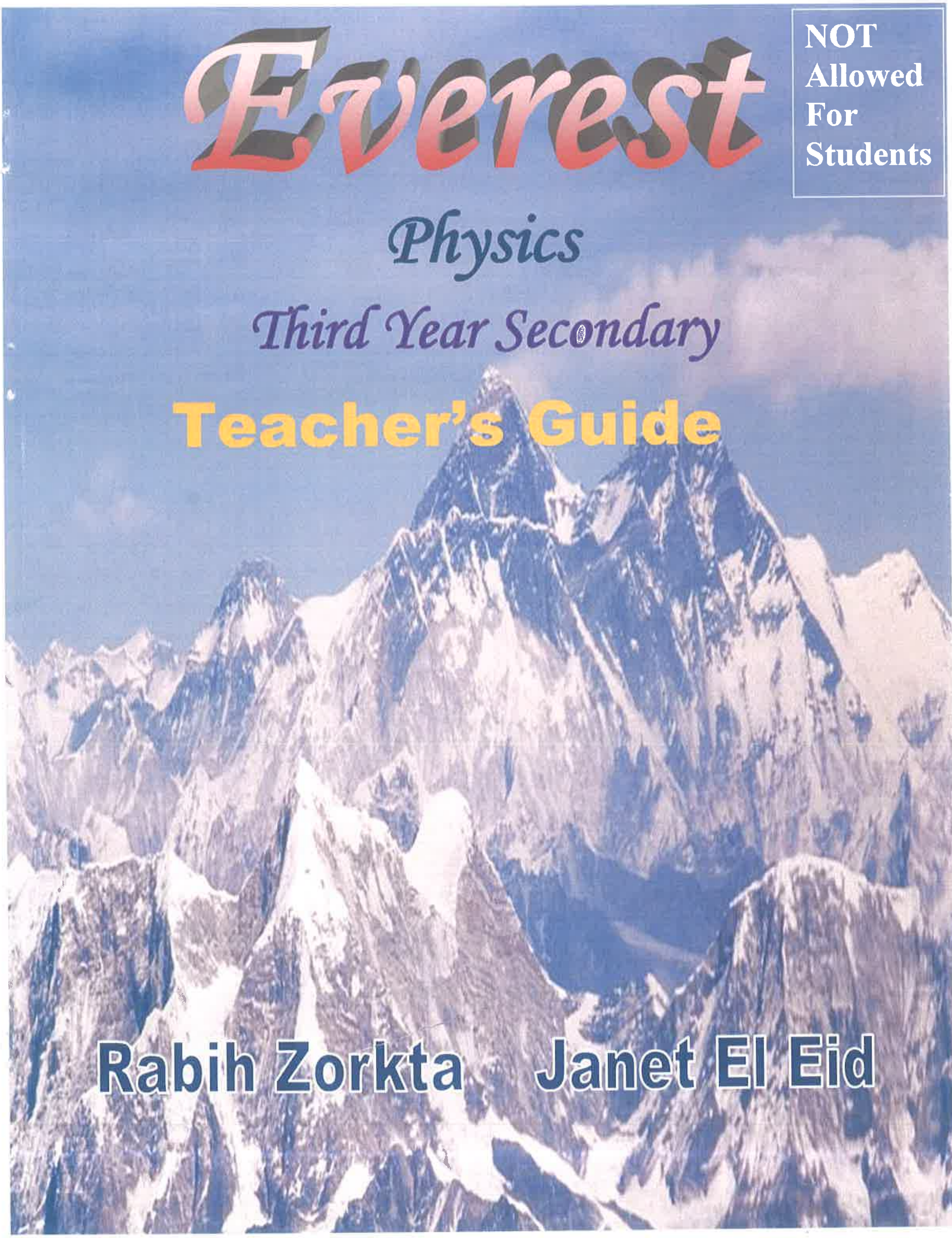
NOT Allowed For Students

Physics

Third Year Secondary

Teacher's Guide

Rabih Zorkta Janet El Eid



Everest

Physics

Third Year Secondary

General Sciences – Life Sciences sections

By

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1st Edition

Preface

This Physics guide book for "Everest" Grade 12 Scientific is written in conformity with the student text book.

In each chapter the teacher finds the detailed solutions of the exercises of the student's book with clear figures and diagrams.
We hope that this guide would meet the needs of teachers and make their job easier.

Give me a fish and I eat for a day. Teach me to fish and I eat for a lifetime.

"Chinese proverb"

Contents

Chapter

Chapter 1: Energy

1.

a) 1st method: No non-conservative forces $\Rightarrow$ The mechanical energy is conserved.

2nd method: The system is an energy-isolated system. Since there is no variation in the internal energy of the system $\Rightarrow$ The mechanical energy is conserved.

b) The internal forces $\vec{F}_{A/B}$ and $\vec{F}_{B/A}$ are non conservative and $\sum W_{\vec{F}_{non\ conservative}} \neq 0 \Rightarrow$ mechanical energy is not conserved. Actually mechanical energy increases.

c) No since the internal forces that cause the explosion are non-conservatives and $\sum W_{\vec{F}_{non\ conservative\ forces}} \neq 0$.

2.

a) Ice cubes gain heat energy to undergo fusion. $\Rightarrow$ U increases.

b) The internal energy of the system increases due to air resistance (non conservative force). Actually mechanical energy is converted into internal energy.

c) ME = constant then chemical energy is converted finally into heat. The produced heat is gained by the car, road and atmosphere $\Rightarrow$

i. U decreases since part of the internal energy is lost to surrounding.

ii. The system does not exchange energy with the surroundings then it is an energy isolated system $\Rightarrow \Delta E = \Delta U + \Delta ME = 0$ but $\Delta ME = 0 \Rightarrow \Delta U = 0 \Rightarrow$ U is constant.

3.

a) Both are vertically upward . $T_1 = k_1\Delta\ell = (80)(0.1) = 8$ N and $mg = (2)(10) = 20$ N.

but $\sum\vec{F}_{ext} = \vec{0}$ (equilibrium) $\Rightarrow m\vec{g} + \vec{T}_1 + \vec{T}_2 = \vec{0} \Rightarrow mg - T_1 - T_2 = 0 \Rightarrow T_2 = 20 - 8 = 12$ N.

b) $T_2 = k_2\Delta\ell \Rightarrow k_2 = \frac{12}{0.1} = 120$ N/m.

c) $EPE = \frac{1}{2}k_1(\Delta\ell)^2 + \frac{1}{2}k_2(\Delta\ell)^2 = \frac{1}{2}(80 + 120)(0.1)^2 = 1$ J.

d) At equilibrium: $mg = k_1(\Delta\ell') \Rightarrow \Delta\ell' = \frac{20}{80} = 0.25$ cm. $EPE_1 = \frac{1}{2}k_1(\Delta\ell')^2 = \frac{1}{2}(80)(0.25)^2 = 2.5$ J.

4.

a)

i. $ME_A = ME_B \Rightarrow m\,g\,Z_A + 0 = \frac{1}{2}m\,v_B^2 + 0 \Rightarrow V_B = \sqrt{2g\,Z} = 4$ m/s.

ii. $\frac{1}{2}m\,v_B^2 - 0 = W_{m\vec{g}} + W_{\vec{N}} = m\,g\,(AB)\sin\alpha + 0 \Rightarrow v_B = 4$ m/s .

b)

i. $\Delta ME = ME_C - ME_B = (\frac{1}{2}m\,v_C^2 + 0) - (\frac{1}{2}m\,v_B^2 + 0) = -12$ J.

ii. $ME_C < M.E_B \Rightarrow$ friction exists.

iii. $\Delta ME = \Sigma\, W_{non\text{-}conservative\ forces} = W_{\vec{f}} = -f(BC) \Rightarrow -12 = -f(10) \Rightarrow f = 1.2$ N.

c)

i. A→B: ME = constant $\Rightarrow \Delta M.E = 0 \Rightarrow \Delta U = 0 \Rightarrow$ U is constant.

ii. B→C: $\Delta ME < 0$. The system does not exchange energy with the surroundings then it is an energy isolated system $\Rightarrow \Delta E = \Delta ME + \Delta U = 0 \Rightarrow \Delta U = -\Delta ME > 0 \Rightarrow$ U increases.

5.

A.

a) $ME_o = GPE_o + K.E_o = 0 + \frac{1}{2}m\,V_o^2 = 100$ J. Air resistance is neglected (no non-conservative forces) $\Rightarrow$ ME is conserved $\Rightarrow$ $M.E_{max\ height} = M.E_o = 100$ J.

b) $M.E_{max\ height} = m\,g\,Z_{max} + \frac{1}{2}mV_{max}^2 \Rightarrow 100 = 2(10)Z_{max} + 0 \Rightarrow Z_{max} = 5$ m.

c) **i.** $M.E = 100\text{ J} = \frac{1}{2}m\,v^2 + m\,g\,Z = \frac{1}{2}(2)v^2 + (2)(10)Z \Rightarrow 100 = v^2 + 20Z$.

ii. $\frac{dM.E}{dt} = 0 = 2vv' + 20Z'$ (but $Z' = v$ and $v' = a$ $\Rightarrow v[2a + 20] = 0$ but $v \neq 0 \Rightarrow a = -10$ m/s².

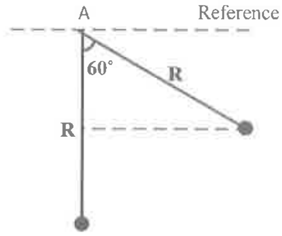
B.

a) $\Delta ME = W_{\vec{f}} = f\,Z'_{max}\cos 180 \Rightarrow m\,g\,Z'_{max} - M.E_o = -f\,Z'_{max} \Rightarrow 2(10)Z'_{max} - 100 = -5\,Z'_{max} \Rightarrow Z'_{max} = 4$ m .

b) **i.** $\frac{1}{2}mv^2 - \frac{1}{2}m\,V_o^2 = W_{m\vec{g}} + W_{\vec{f}} \Rightarrow \frac{1}{2}(2)v^2 - 100 = mg(-Z) - f(Z) \Rightarrow 100 = v^2 + 25Z$.

ii. Derive the obtained relation w.r.t. time $\Rightarrow 0 = 2\,v\,v' + 25\,Z' \rightarrow 0 = v\,[2a + 25]$ but $v \neq 0 \Rightarrow a = -12.5$ m/s².

6.



a) $GPE = m\,g\,Z = m\,g(-R\cos60) = 0.8(10)(-0.4)(0.5) = \underline{-1.6\ J}$.

b) No non-conservative forces ⇒ ME is conserved $ME_f = ME_i$ ⇒

$GPE_f + KE_f = -1.6 \Rightarrow mg(-R) + \frac{1}{2}I(\theta')^2 = -1.6$, but $I = \frac{3}{2}(0.8)(0.4)^2 = 0.192\ kg.m^2$

$\Rightarrow (0.8)(10)(-0.4) + \frac{1}{2}(0.192)(\theta')^2 = -1.6 \Longrightarrow \underline{\theta' = 4.082\ rad/s}$.

7.

a) $ME = KE + GPE = 0 + 0 = \underline{0}$.

b) No, since $\sum W_{non\ conservative\ \ forces} = W_{tensions} \neq 0$.

c)

i. $ME_B = \frac{1}{2}mV_B^2 + m\,g\,Z_B = \frac{1}{2}(10)(1)^2 + 10(10)(1) = \underline{105\ J}$.

ii. $\Delta ME = W_{non\ conservative\ \ forces} \Rightarrow ME_B - ME_o = W_{\vec{T}} \Rightarrow 105 - 0 = T\,(OB) \Rightarrow \underline{T = 105\ N}$.

d)

i. $ME_B = ME_{ground} \Rightarrow 105 = \frac{1}{2}mV_{ground}^2 \Rightarrow V = \sqrt{21}\ m/s$.

ii. $K.E = G.P.E = \frac{M.E}{2} \Rightarrow m\,g\,Z_C = \frac{M.E}{2} \Rightarrow 10(10)Z_C = \frac{105}{2} \Rightarrow \underline{Z_C = 0.525\ m}$.

8.

a)

i. $M.E_o = K.E_o + m_1gZ_1 + m_2\,g\,Z_2 + m\,g\,Z = 0 + 5(10)(20)\sin\alpha + 2(10)(25) + 2(10)(100) = \underline{3300\ J}$.

ii. $K.E_{pulley} = \frac{1}{2}I\,\theta'^2 = \frac{1}{2}(\frac{1}{2}m\,r^2)(\frac{v^2}{r^2}) = \frac{1}{4}mv^2$. $KE_{system} = \frac{1}{2}[m_1 + m_2 + \frac{1}{2}m]v^2 = \frac{1}{2}[5 + 2 + 1](7.75)^2 = 240.25\ J$.

$G.P.E_{system} = 5(10)(0) + 2(10)[20 + 25] + 2(10)(100) = 2900\ J. \Rightarrow M.E_{system} = 2900 + 240.25 = \underline{3140.25\ J}$.

iii. $\Delta M.E = W_{\vec{f}} = -f\,(OC) + 0 \Rightarrow 3140.25 - 3300 = -f\,(20) \Rightarrow \underline{f \cong 7.98\ N}$.

b) $K.E_{system} = \frac{1}{2}[m_1 + m_2]v^2 = 3.5\,v^2$. $GPE_o = GPE_1 + G.P.E_2 + GPE_{pulley} = 0 + 2(10)(20{+}25) + 0 = 900\ J$.

$M.E_{initial} = 5(10)(20)\sin\alpha + 2(10)(25) = 1300\ J \Rightarrow 900 + 3.5v^2 - 1300 = -7.98(20) \Rightarrow \underline{v = 8.28\ m/s}$.

c) The mass of pulley decreases the speed of the particles since 7.75 m/s < 8.28 m/s.

9.

a) No non-conservative forces ⇒ ME is conserved $= M\,E_C = \frac{1}{2}m\,V_C^2 + m\,g\,h_C = \frac{1}{2}(0.4)(3.5)^2 + (0.4)(10)(2) = \underline{10.45\ J}$.

b) $10.45 = ME_A = \frac{1}{2}m\,V_A^2 + m\,g\,l(1-\cos60) \Rightarrow 10.45 = \frac{1}{2}(0.4)(V_A)^2 + (0.4)(10)(1)(1 - 0.5) \Rightarrow \underline{V_A = 6.5\ m/s}$.

c)

i. $10.45 = \frac{1}{2}mV^2 + m\,g\,\ell(1-\cos\theta) = \frac{1}{2}(0.4)(52.25) + (0.4)(10)(1)(1 - \cos\theta)$

$\Rightarrow 1 - \cos\theta = 0 \Rightarrow \cos\theta = 1 \Rightarrow \underline{\theta = 0} \Rightarrow G.P.E = 0 \Rightarrow$ K.E is max. ⇒ V is max.

ii. $K.E = PE \Rightarrow ME = 2PE = 2mg\ell(1 - \cos\theta) \Rightarrow 1 - \cos\theta = \frac{10.45}{2 \times 0.4 \times 10} = 1.306 \Rightarrow \cos\theta = -0.306 \Rightarrow \theta = \pm 107.8°$.

10.

a) The system does not exchange energy with the surroundings then it is an energy isolated ⇒ total energy is conserved.

b) 1st method: speed increases ⇒ KE increases and height increases ⇒ GPE increases ⇒ ME increases ⇒ not conserved.

2nd method: $\Delta E = \Delta U + \Delta ME = 0 \Rightarrow \Delta U = -\Delta ME$ but $\Delta U < 0$ since internal energy decreases $\Rightarrow \Delta ME > 0$.

c)

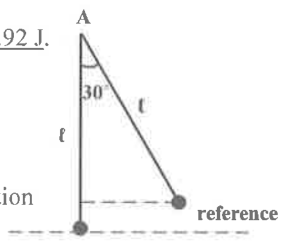
i. $KE_Z = \frac{1}{2}mv^2 = \frac{1}{2}(50)(3.1\times10^3)^2 = 2.4025\times10^8\ J$. $\Delta ME = ME_Z - ME_o = (GPE_Z + KE_Z) - (GPE_o + KE_o)$

$\Rightarrow \Delta ME = \Delta GPE + KE_Z = 2.665 \times 10^9 + 2.4025 \times 10^8 = \underline{2.90525 \times 10^9\ J}$.

ii. The gained M.E represents 10% of the energy given by fuel: $0.1\,E_{fuel} = 2.090525\times10^9 \Rightarrow E_{fuel} = 2.90525 \times 10^{10}\ J$

Energy produced by 1 kg $= \frac{2.90525 \times 10^{10}}{667.874} = \underline{4.35 \times 10^7\ J}$.

11.



a)

i. $ME_A = \frac{1}{2}m\,V_o^2 + m\,g\,h_A = \frac{1}{2}mv^2 + m\,g\ell(1 - \cos30) = \frac{1}{2}(0.5)(1)^2 + 0.5(10)(10(1 - \cos30) = \underline{0.92\ J}$.

ii. $ME_B = \frac{1}{2}mv^2 + 0 = \frac{1}{2}(0.5)(1.5)^2 = 0.5625\ J$. $M.E_B < M.E_A$ ⇒ Air resistance exists.

iii. $\Delta ME = W_{\vec{f}} \Rightarrow 0.5625 - 0.92 = W_{\vec{f}} \Rightarrow W_{\vec{f}} = \underline{-0.3575\ J}$.

b)

i. $KE_f - KE_i = W_{m\vec{g}} + W_{\vec{N}} = 0 + 0$ since these forces are perpendicular to the direction of motion

$\Rightarrow KE_f = KE_i \Rightarrow V_f = V_i$.

ii.

1. The system ((S), spring, Earth) is an energy-isolated system. Since there is no variation in the internal energy then the mechanical energy of the system is conserved $\Rightarrow ME_{Xm} = ME_{\text{before compression}}$

   $\Rightarrow$ KE + GPE + EPE = $KE_{\text{before}} + GPE_{\text{before}} \Rightarrow 0 + 0 + \frac{1}{2}k\,X_m^2 = \frac{1}{2}mv^2 + 0 \Rightarrow \frac{1}{2}(100)\,X_m^2 = 0.5625$

   $\Rightarrow X_m = 0.106$ m = $\underline{10.6 \text{ cm}}$.
2. $ME = \frac{1}{2}k\,x^2 + \frac{1}{2}m\,v^2 + m\,g\,z \Rightarrow 0.5625 = \frac{1}{2}(100)(\frac{0.106}{2})^2 + \frac{1}{2}(0.5)v^2 + 0 \Rightarrow \underline{v = 1.3 \text{ m/s}}$.

12.

a) $\Delta KE = \Sigma W_{\vec{F}_{ext}} \Rightarrow KE_B - KE_A = W_{m\vec{g}} + W_{\vec{N}} + W_{\vec{f}} + W_{\vec{F}} \Rightarrow \frac{1}{2}mv_B^2 - 0 = m\,g(-(AB)\sin\alpha) + 0 - f\,(AB) + F(AB)$

$\Rightarrow \frac{1}{2}(1000)(100) = AB[-1000(10)(0.5) - 200 + 6000] \Rightarrow \underline{AB = 62.5 \text{ m}}$.

b) $KE_C - KE_B = W_{m\vec{g}} + W_{\vec{N}} + W_{\vec{f}} + W_{\vec{F}} \Rightarrow 0 = mg[-(BC)\sin\alpha] + 0 - f\,(BC) + F'(BC) \Rightarrow 0 = BC[-m\,g\sin\alpha - f + F']$

but $BC \neq 0 \Rightarrow -m\,g\sin\alpha - f + F' = 0 \Rightarrow F' = (1000)(10)(0.5) + 200 = \underline{5200 \text{ N}}$.

c) $ME_D - ME_C = W_{\vec{f}} + W_{\vec{f}_{br}} \Rightarrow KE_D + m\,g\,(AD\sin\alpha) - (\frac{1}{2}m\,v_C^2 + m\,g(AC\sin\alpha) = -f\,(CD) - f_{br}\,(CD)$

$\Rightarrow (1000)(10)(0.5)(3.75) - \frac{1}{2}(1000)(100) = -200(3.75) - f_{br}\,(3.75) \Rightarrow \underline{f_{br} = 8133.33 \text{ N}}$.

13.

A.

a) $ME = KE + GPE = \frac{1}{2}mv^2 - mgy$. Resistance is neglected (no non-conservative forces) $\Rightarrow \frac{dME}{dt} = 0$

$\Rightarrow 0 = m\,v\,v' - mgy'$ $\{y' = v$ and $v' = a\} \Rightarrow 0 = m\,v[a - g]$ but $m\,v \neq 0 \Rightarrow a - g = 0 \Rightarrow a = +g = \underline{10 \text{ m/s}^2}$ = constant and $a.v > 0 \Rightarrow$ U.A.R.M.

b) $v^2 - V_0^2 = 2\,a\,(\Delta y) \Rightarrow v^2 - 0 = 2(10)(30) \Rightarrow v = 10\sqrt{6} = \underline{24.49 \text{ m/s}}$.

c) $ME_{final} = ME_{initial} \Rightarrow \frac{1}{2}m\,v^2 - m\,g\,y = 0 + 0 \Rightarrow \frac{1}{2}(75)v^2 - 75(10)(30) = 0 \Rightarrow \underline{v = 10\sqrt{6} \text{ m/s}}$.

B.

a) Since there are no non -conservative forces (tension and weight are conservative forces).

b) Work done by weight results in transformation of GPE into KE.

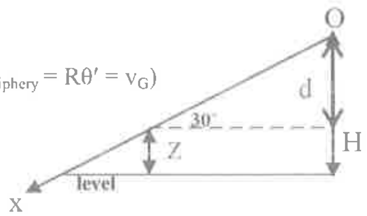
Work done by tension results in transformation of KE into EPE.

c) $ME_{initial} = ME_{\text{max elongation}} \Rightarrow 0 = KE + GPE + EPE = 0 - m\,g\,y + \frac{1}{2}K\,x_m^2 = -75(10)(62) + \frac{1}{2}k(32)^2 \Rightarrow \underline{k \cong 90.82 \text{ N/m}}$.

d) $0 = GPE + KE + EPE = GPE + 2EPE = -m\,g\,y + 2(\frac{1}{2})[k(y - 30)^2] \Rightarrow 0 = (-75)(10)y + 2(\frac{1}{2})(90.82)(y^2 - 60y + 900)$

$90.82y^2 - 6199.2\,y + 81738 = 0 \Rightarrow \underline{y = 50.4 \text{ m}} \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}k(y - \ell_o)^2 \Rightarrow \underline{v = 22.45 \text{ m/s}}$.

14.



a) $ME_o = KE + GPE = 0 + m\,g\,H = 20(10)(10) = \underline{2000 \text{ J}}$.

b) $ME_o = ME_1 = kE_1 + GPE_1 = \frac{1}{2}mv_G^2 + \frac{1}{2}I\,\theta'^2 + 0$. (No sliding $\Rightarrow s = R\theta = x_G$ and $v_{periphery} = R\theta' = v_G$)

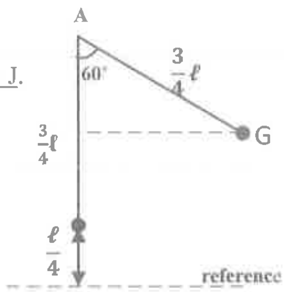
$\Rightarrow 2000 = \frac{1}{2}m\,v_G^2 + \frac{1}{2}(\frac{1}{2}m\,R^2)\frac{v_G^2}{R^2} \Rightarrow 2000 = \frac{3}{4}mv_G^2 \Rightarrow v_G = \underline{11.55 \text{ m/s}}$.

c) $GPE = m\,g\,Z = mg(H - d) = mg(H - x\sin 30)$.

d) $ME = \frac{3}{4}mV_G^2 + m\,g(H - x\sin 30)$, but $\frac{dM.E}{dt} = 0 \Rightarrow 0 = \frac{3}{4}m\,(2\,v_G\,v_G') + m\,g(-x'\sin 30)$

$\{x' = v_G$ and $v_G' = a\} \Rightarrow 0 = x'\,m\,[\frac{3}{2}a - g\sin 30]$, but $x' \neq 0 \Rightarrow \frac{3}{2}a - g\sin 30 = 0 \Rightarrow a = \frac{2\,g\sin 30}{3} = \frac{10}{3}$ m/s^2.

15.



a)

i. $ME_1 = KE + GPE = 0 + M\,g\,Z = Mg(\ell - \frac{3}{4}\ell\cos 60) = (1.2)(10)[1 - 0.75(0.5)] = \underline{7.5 \text{ J}}$.

ii. $ME_1 = ME_2 = M\,g\,Z + \frac{1}{2}I(\theta')^2 \Rightarrow (1.2)(10)\frac{\ell}{4} + \frac{1}{2}(0.8)(\theta')^2 = 7.5 \Rightarrow \underline{\theta' = 3.35 \text{ rad/s}}$.

iii. $V = \ell\theta' = 1 \times (3.35) = \underline{3.35 \text{ m/s}}$.

b)

i. $ME_o = GPE_o + KE_o = 0 + \frac{1}{2}mV_o^2 = \frac{1}{2}(0.6)(\sqrt{11.22})^2 = \underline{3.366 \text{ J}}$.

$V_B = \sqrt{11.22 - 3(2)} = 2.285$ m/s $\Rightarrow M.E_B = 0 + \frac{1}{2}(0.6)(2.285)^2 = \underline{1.566 \text{ J}}$.

ii. $\Delta ME = \Sigma W_{\text{non-conservative forces}} = W_{\vec{f}} \Rightarrow 1.566 - 3.366 = -f\,x \Rightarrow \underline{f = 0.9 \text{ N}}$.

iii. Work done by the friction results in the transformation of ME into heat $\Rightarrow heat_{energy} = -\Delta ME = -[1.566 - 3.3675] = \underline{1.8 \text{ J}}$.

16.

a) $ME_B = KE_B + GPE_B = 6\ J \Rightarrow 6 = 0 + mg(OB)\sin\alpha \Rightarrow OB = \frac{6}{2(10)(0.5)} = 0.6\ m = \underline{60\ cm}$.

$ME_o = ME_B \Rightarrow \frac{1}{2}mv_o^2 + 0 = 6 \Rightarrow \underline{v_o = \sqrt{6}\ m/s}$.

b)

i. $ME = KE + EPE + GPE = \frac{1}{2}mv^2 + \frac{1}{2}k\,x^2 + mg(-x\sin30) = \frac{1}{2}(2)\,v^2 + \frac{1}{2}(200)\,x^2 + 2\,(10)[-x\,(0.5)] = v^2 + 100\,x^2 - 10\,x$.

ii. At C: $v = 0$ and $x = OC \Rightarrow 6 = 100\,x^2 - 10\,x \Rightarrow x = 0.3\ m = \underline{30\ cm = OC}$.

iii. $T_{max} = k\,X_m = (200)(0.3) = \underline{60\ N}$.

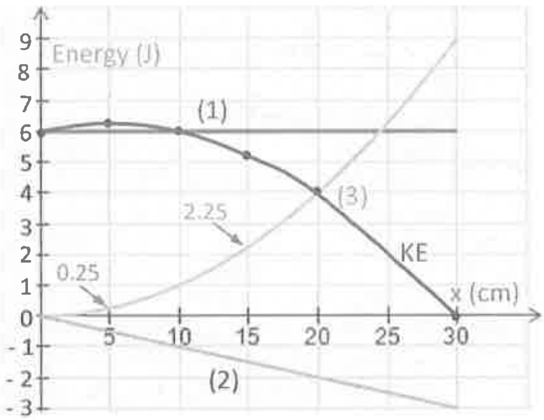
c)

i. Graph 1 represents the ME since it is constant.
Graph 2 represents the GPE since GPE < 0 between O and C.
Graph 3 represents the EPE since as S moves towards C; x increases and so EPE increases.

ii. KE = ME - GPE - EPE

for x = 0 : KE = 6 – 0 – 0 = 6 J;
for x = 5 : KE = 6 - 0.25 + 0.5 = 6.25 J;
for x = 10 : KE = 6 – 1 + 1 = 6 J;
for x = 15 : KE – 6 - (-1.5) - 2.25 – 5.25 J;
for x = 20 : KE = 6 - (-2) – 4 = 4 J;
for x = 30 : KE = 6 - (-3) – 9 = 0.

iii. Figure



17.

A.

a) system: block M. $\sum\vec{F}_{ext} = \vec{0} \Rightarrow m\vec{g} + \vec{N} + \vec{T} = \vec{0} \Rightarrow -m\,g\sin30 + 0 + T = 0 \Rightarrow T = m\,g\sin30 = (0.5)(10)(0.5) = 2.5\ N$

but $T = k\,d \Rightarrow k = \frac{2.5}{0.1} = \underline{25\ N/m}$.

b) $GPE_B = m_1\,g\,z = m_1\,g(-d\sin30) = (0.5)(10)(-0.1(0.5)) = \underline{-0.25\ J}$. $EPE_B = \frac{1}{2}k\,d^2 = \frac{1}{2}(25)(0.1)^2 = \underline{0.125\ J}$.

c) $KE_B - KE_C = W_{m\vec{g}} + W_{\vec{N}} \Rightarrow \frac{1}{2}m_2v_B^2 - \frac{1}{2}m_2\,v_C^2 = m_2\,g(-CB\sin30) + 0 \Rightarrow \frac{1}{2}[4.6 - 3^2] = -CB\,(5) \Rightarrow \underline{CB = 0.44\ m}$.

B.

a) i. $ME_B = KE_B + EPE_B + GPE_B = \frac{1}{2}(0.5)(1.5) + 0.125 - 0.25 = \underline{0.25\ J}$.

ii. No non-conservative force $\Rightarrow$ ME is conserved $\Rightarrow ME_B = ME_D \Rightarrow 0.25 = \frac{1}{2}K(OD)^2 + KE_D + m_1g(OD\sin30)$

$\Rightarrow 0.25 = \frac{1}{2}(25)(OD)^2 + 2.5(OD) \Rightarrow 12.5(OD)^2 + 2.5(OD) - 0.25 = 0 \Rightarrow OD = 0.0732\ m = \underline{7.32\ cm}$.

b) i. $ME = \frac{1}{2}kx^2 + \frac{1}{2}m_1v^2 + m_1\,g\,x\sin30$.

ii. $\frac{dM.E}{dt} = 0 = k\,x\,x' + m_1\,v\,v' + m_1\,g\sin30\,x'$ $\{v = x'$ and $v' = a\} \Rightarrow a = \frac{-k\,x - m_1 g\sin30}{m_1} \Rightarrow a_{(x=0)} = -g\sin30 = \underline{-5\ m/s^2}$.

c) i. $\Delta ME = ME_o - ME_B = (kE_o + GPE_o + EPE_o) - (kE_B + GPE_B + EPE_B) \Rightarrow \Delta ME = (0 + 0 + 0) - (0.25) = \underline{-0.25\ J}$.

ii. $\Delta ME = \Sigma\,W_{\text{non-conservative forces}} = W_{\vec{f}} = -f\,(OB) \Rightarrow f = \frac{-0.25}{-0.1} = \underline{2.5\ N}$.

iii. The system (block, spring, incline, atmosphere, Earth) is energy isolated $\Rightarrow \Delta U = -\Delta M.E = \underline{0.25\ J}$.

18.

a)

i. GPE is transformed into KE.

ii. KE is transformed into electric energy.

b) $M = (22000)(24)(3600) = \underline{1.9008 \times 10^9\ kg}$.

c)

i. $\eta = \frac{P_{useful}}{P_{total}} \Rightarrow 0.34 = \frac{34\times10^6}{P_t} \Rightarrow \underline{P_t = 10^8\ W}$. **ii.** $E_t = P_t.t \Rightarrow P_t.t = m\,g\,h \Rightarrow \frac{m}{t} = \frac{P_t}{g\,h}$.

iii. $22000 = \frac{10^8}{10h} \Rightarrow \underline{h = 454.55\ m}$.

19.

A.

a) **i.** GPE < 0 for Z < 0 and the graph (b_1) is negative for Z < 0.

ii. Z_A is the height elevation ⇒ Graphically $Z_{max} = Z_A = \underline{2\ m}$.

b) KE increases as M moves from A (Z_{max}) to C (Z_{min}).

Graphically the graph (a_1) increases as Z decreases from Z_{max} to Z_{min}.

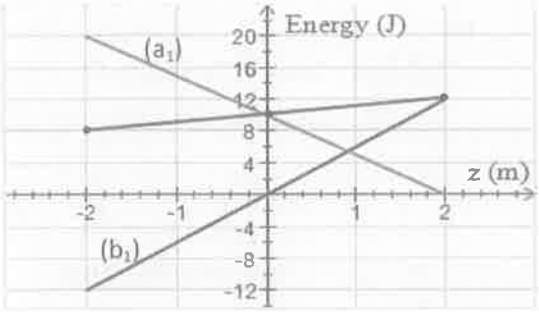
c) **i.** At A: Z = 2 m: $ME_A = KE_A + GPE_A = 0 + 12 = \underline{12\ J}$;

At B: Z = 0 : $ME_B = k.E_B + GPE_B = 10 + 0 = \underline{10\ J}$;

At C: Z = -2 m: $ME_C = 20 + (-12) = \underline{8\ J}$.

ii. $ME_C < ME_B < ME_A$ ⇒ friction exists.

d) Figure.



B.

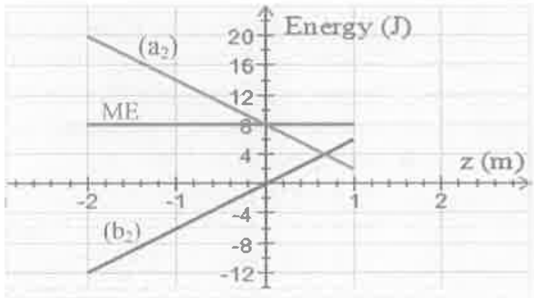
a) From C to D KE decreases till it becomes minimum at D.

Graphically KE is minimum for $Z_D = \underline{1\ m}$.

b) **i.** $ME_C = 8$ J; $ME_{(Z=0)} = 0 + 8 = 8$ J and $M.E_D = 2 + 6 = 8$ J

⇒ $ME_C = ME_{Z=0} = ME_D$ ⇒ ME is conserved

ii. Figure.



c) $\Delta kE = W_{\vec{f}} + W_{m\vec{g}} \Rightarrow KE_D - KE_A = 2 - 0 = W_{\vec{f}} + m\,g\,(h_A - h_D)$

$\Rightarrow 2 = W_{\vec{f}} + 0.6\,(10)\,(2-1) \Rightarrow W_{\vec{f}} = \underline{-4\ J}$.

20.

a)

i. During the motion of D; the GPE of the system (D, Earth) remains constant.

ii. No since the graph is not constant.

iii. The disk starts from rest ⇒ $KE_o = 0$ ⇒ curve (a) represents the variation of the KE of D

OR: ME = GPE + KE at any instant, but KE > 0 and GPE < 0 ⇒ ME < KE at any instant.

b) GPE = - 2 J = m g(-R) ⇒ -2 = 2(10)(-R) ⇒ R = 0.1 m = 10 cm, but $I = \frac{2(0.01)}{2} = \underline{0.01\ kg.m^2}$.

$KE = \frac{1}{2}I(\theta')^2$ at t = 10 s, KE = 32 J ⇒ $32 = \frac{1}{2}(0.01)(\theta')^2 \Rightarrow \underline{\theta' = 80\ rad/s}$.

c)

i. $\sum\mathcal{M} = I\,\theta'' \Rightarrow \mathcal{M}_{m\vec{g}} + \mathcal{M}_{\vec{R}} + \mathcal{M}_{\vec{f}} = I\theta'' \Rightarrow 0 + 0 + \mathcal{M}_{\vec{f}} = I\,\theta'' \Rightarrow \theta''$ = constant and $\theta'\,.\,\theta'' < 0$ ⇒ U. D. Rotation.

ii. $\theta' = \theta''t + \theta'_0 \Rightarrow 0 = \theta''(40) + 80 \Rightarrow \underline{\theta'' = -2\ rad/s^2}$. **iii.** $\mathcal{M}_{\vec{f}} = (0.01)(-2) = \underline{-0.02\ N.m}$.

d) $KE_f - KE_i = W_{m\vec{g}} + W_{\vec{R}} + W_{\vec{f}} + W_{couple} \Rightarrow KE_f - 0 = 0 + 0 + \mathcal{M}_{\vec{f}}\,\Delta\theta + [F\,(2R)\sin\alpha]\,\Delta\theta$

$32 = \Delta\theta[-0.02 + (1.25)(2)(0.1)(0.4)] \Rightarrow \underline{\Delta\theta = 400\ rad}$.

e) $ME_f - ME_i = -2 - (-2) = 0$. $W_{couple} = \mathcal{M}_{couple}.\,\Delta\theta = (0.1)(400) = 40$ J.

Angle covered during the second phase: $(\theta')^2 - (\theta'_0)^2 = 2\theta''(\Delta\theta) \Rightarrow 0 - 80^2 = 2(-2)\Delta\theta \Rightarrow \Delta\theta = 1600$ rad

$W_{\vec{f}} = \mathcal{M}_{\vec{f}}.\,\Delta\theta = -0.02[400 + 1600] = -40$ J ⇒ $\sum W_{non\ conservative\ forces} = W_{couple} + W_{\vec{f}} = 40 - 40 = 0$ ⇒ verified.

21.

a)

i. $GPE_{short} = m_1\,g\,Z_{G(short\ part)}$ where $Z_{G(short\ part)} = -0.2$ m. and $\left.\begin{array}{l}1\ m \to 5\ kg\\ 0.4\ m \to m_1\end{array}\right\} \Rightarrow m_1 = 2$ Kg.

⇒ $GPE_{short} = (2)(10)(-0.2) = \underline{-4\ J}$. Similarly $GPE_{long} = m_2\,g\,Z_{G(long\ part)} = (3)(10)(-0.3) = \underline{-9\ J}$.

ii. $ME_o = GPE_o + KE_o = -4 - 9 = \underline{-13\ J}$.

b)

i. $GPE_{long} = m_2'\,g[-(\frac{0.6+x}{2})]$. {1m has mass 5 kg ⇒ (0.6 + x) has mass $m_2' = 5(0.6 + x) = 3 + 5x$}

$GPE_{long} = (3 + 5x)(10)(-0.3 - \frac{x}{2}) = -25x^2 - 30x - 9$. $GPE_{short} = m_1'\,g[-(\frac{0.4-x}{2})]$

Similarly $m_1' = 2 - 5x \Rightarrow GPE_{short} = (2 - 5x)(10)(-0.2 + \frac{x}{2}) = -25x^2 + 20x - 4$

⇒ $GPE_{Total} = -25x^2 - 30x - 9 - 25x^2 + 20x - 4 = \underline{-50x^2 - 10x - 13}$.

ii. $KE = \frac{1}{2}MV^2 \Rightarrow ME = \underline{2.5v^2 - 50x^2 - 10x - 13}$.

iii. ME is conserved (no friction) ⇒ $-13 = 2.5v^2 - 50x^2 - 10x - 13 \Rightarrow \underline{v^2 = 20x^2 + 4x}$

iv. Derive w.r.t. time : $2v\,v' = 20\,(2\,x\,x') + 4x'$ {$x' = v$ and $v' = a$}⇒ $2a = 40x + 4 \Rightarrow \underline{a = 20x + 2}$.

For x = 0: a is minimum: $\underline{a_{min} = 2\ m/s^2}$. For x = ℓ – 60 = 40 cm = 0.4 m : $\underline{a = 10\ m/s^2}$.

c)

i. t = 0.5 s.

ii. At t = 0.3 s ; a = 4 m/s² ⇒ a = 20x + 2 ⇒ 4 = 20x + 2 ⇒ $\underline{x = 0.1\ m} \Rightarrow v^2 = 20x^2 + 4x = 20(0.01) + 4(0.1)$

⇒ $\underline{v = 0.775 m/s}$.

Chapter 2: linear Momentum

1.

a) $KE = \frac{1}{2}mv^2 = 0 \Rightarrow v = 0$ then $P = m\,v = \underline{0}$.

b)

i. $P = m\,v = 60\,(\frac{72}{3.6}) = \underline{1200\ kg.m/s}$. **ii.** $P_{system} = 1200 + 1200 = \underline{2400\ kg.m/s}$.

iii. The students are at rest relative to the bus $\Rightarrow P = \underline{0}$.

c)

i. $\vec{V}_o = 10\vec{i} - 10\vec{j} \Rightarrow \vec{P}_o = m\vec{V}_o = \underline{20\vec{i} - 20\vec{j}}$ (kg.m/s) . **ii.** $\vec{V}_5 = -5\vec{j} \Rightarrow \vec{P}_5 = m\vec{V}_5 = \underline{-10\vec{j}}$ (kg.m/s)

d) $\vec{P}_1 = m_1\vec{V}_1$ and $\vec{P}_2 = m_2\vec{V}_2$ but $\vec{V}_1 = -\vec{V}_2 \Rightarrow \vec{P}_1 = -\vec{P}_2$ (<u>not equal</u>). $\vec{P}_{Total} = \vec{P}_1 + \vec{P}_2 = \underline{\vec{0}}$.

e) System : (two students) $\Rightarrow \sum\vec{F}_{ext} = m_1\vec{g} + m_2\vec{g} + \vec{N}_1 + \vec{N}_2 = \vec{0} \Rightarrow$ Isolated system $\Rightarrow \vec{P}$ is conserved $\Rightarrow \vec{P}_i = \vec{P}_f$

$\vec{P}_A + \vec{P}_B = \vec{P} \Rightarrow m_A\vec{V}_A + m_B\vec{V}_B = (m_A + m_B)\vec{V} \Rightarrow 60(2\vec{i}) + 50\,\vec{V}_B = (60+50)(-\vec{i}) \Rightarrow \underline{\vec{V}_B = -4.6\,\vec{i}}$ (m/s).

f) $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} \Rightarrow 2\vec{i} = \frac{d\vec{P}}{dt} \Rightarrow \vec{P} = \int 2\vec{i}\,dt \Rightarrow \vec{P} = 2t\,\vec{i} + \vec{P}_o$ but $\vec{P}_o = \vec{0}$ (starts from rest)

$\Rightarrow \vec{P} = 2t\,\vec{i} \Rightarrow \vec{P}_{10} = 2(10)\,\vec{i} = \underline{20\,\vec{i}}$ (kg.m/s).

2.

a) Before collision: $\vec{P} = \vec{P}_1 + \vec{P}_2 = m_t\vec{V}_t + m_b\vec{V}_b = 1(3\vec{i}) + \vec{0} = 3\,\vec{i}$. After collision: $\vec{P}' = (m_t + m_b)\vec{V} = (1+0.5)(2\vec{i}) = 3\,\vec{i}$

$\Rightarrow \vec{P} = \vec{P}' \Rightarrow$ linear momentum is conserved.

b) $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt}$, but $\vec{P}$ is constant $\Rightarrow \frac{d\vec{P}}{dt} = \vec{0} \Rightarrow \sum\vec{F}_{ext} = 0$.

3. $V_1' = (\frac{m_1 - m_2}{m_1 + m_2})V_1 + (\frac{2\,m_2}{m_1 + m_2})V_2 = (\frac{1-2}{1+2})(4) + (\frac{2(2)}{1+2})(1) = \underline{0}$. $V_2' = (\frac{m_2 - m_1}{m_1 + m_2})V_2 + (\frac{2\,m_1}{m_1 + m_2})V_1 = (\frac{2-1}{2+1})(1) + (\frac{2(1)}{1+2})(4) = \underline{3\ m/s}$.

Note: The above expressions of V_1' and V_2' needs a proof.

4.

a) System (Fadi, box, boat). External Forces: The total weight ($m\vec{g}$) of the system and the normal reaction by water (up thrust)

$\sum\vec{F}_{ext} = m\vec{g} + \vec{N} = \vec{0} \Rightarrow \frac{d\vec{P}}{dt} = \vec{0} \Rightarrow \vec{P} = \overline{constant} \Rightarrow \vec{P}_{before} = \vec{P}_{after} \Rightarrow \vec{0} = m_1\vec{V}_1 + m_2\vec{V}_2$

$\vec{V}_2 = -\frac{m_1}{m_2}\vec{V}_1 = -\frac{2}{50}(5\vec{i}) = -0.2\,\vec{i} \Rightarrow \underline{V_2 = 0.2\ m/s}$ and has a <u>direction opposite to $\vec{V}_1$</u>.

b)

i. <u>Zero</u> since h remains 0. **ii.** $\Delta ME = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}(2)\,(5)^2 + \frac{1}{2}(50)\,(0.2)^2 = \underline{26\ J}$.

iii. The work done by the forces of Fadi while throwing the box, results in the transformation of internal energy into ME $\Rightarrow \Delta U = -\Delta ME = \underline{-26\ J}$.

5.

a) $\sum\vec{F}_{ext} = m\vec{a}_G \Rightarrow M\vec{g} = m\vec{a}_G \Rightarrow \vec{a}_G = \vec{g} = \underline{10\,\vec{j}} \Rightarrow a_G = 10\ m/s^2$.

b) $\vec{V}_G = \int \vec{a}_G dt = 10t\vec{j} + \vec{V}_{Go} \Rightarrow \vec{V}_G = 10t\,\vec{j} \Rightarrow \vec{V}_{G1} = 10(1)\vec{j} = \underline{10\,\vec{j}}$ (m/s).

c)

i. Since the magnitude of the internal forces between the parts of the system are much greater than those of the external forces.

ii. $\vec{P}_{G_1} = M\vec{V}_{G_1} = (1 + 2.5 + 1.5)(10)\vec{j} = 50\,\vec{j}$. $\vec{P}_{G_1} = m_A\vec{V}_A + m_B\vec{V}_B + m_C\vec{V}_C \Rightarrow 50\,\vec{j} = 1(3\vec{i} - \vec{j}) + 2.5(-1.5\vec{i} + 2\vec{j}) + 1.5\vec{V}_C$

$\Rightarrow \vec{V}_C = 0.5\vec{i} + 30.67\vec{j}$ (m/s).

6.

a) System: (Man, boat) . $\sum\vec{F}_{ext} = (M+m)\,\vec{g} + \vec{N} = \vec{0} = \frac{d\vec{P}}{dt} \Rightarrow \vec{P}$ is constant $\Rightarrow \vec{P}_{before\ walking} = \vec{P}_{after\ walking} \Rightarrow \vec{0} = m\vec{V}_m + M\vec{V}_b$

collinear velocities $\Rightarrow 0 = mV_m + MV_b \Rightarrow 0 = 80(2) + 400V_b \Rightarrow V_b = -0.4$ m/s

$\Rightarrow$ Magnitude: $\underline{V_b = 0.4\ m/s}$; direction of $\vec{V}_b$ is <u>opposite</u> to that of $\vec{V}_m$.

b) $\vec{P} = \vec{P}_G = \vec{0} \Rightarrow (m+M)\vec{V}_G = \vec{0} \Rightarrow \underline{\vec{V}_G = \vec{0}}$.

7. Since the helicopter is at rest $\Rightarrow F_{up} = mg \Rightarrow F_{up} = u\,\frac{dm}{dt} \Rightarrow (5000)(10) = u(800) \Rightarrow \underline{u = 62.5\ m/s}$.

8.

a) During this collision; linear momentum is conserved: $\vec{P}_{before} = \vec{P}_{after} \Rightarrow m_{bullet}\vec{V}_{bullet} + \vec{0} = (m_{bullet} + m_{block})\vec{V}$

$V = \frac{0.05}{(0.05+2)}(600) = \frac{600}{41}$ m/s = <u>14.63m/s</u>.

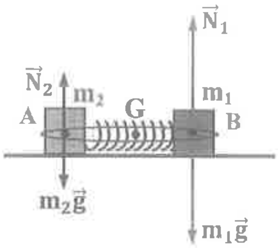
b) System: Block. $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} \Rightarrow m\vec{g} + \vec{N} + \vec{F}_{bullet\,/block} = \frac{d\vec{P}}{dt} \Rightarrow \vec{F}_{bullet\,/block} = \frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t}$

$\Rightarrow F_{bullet/block} = \frac{m_{block}\,V_{block} - 0}{\Delta t} = \frac{2(\frac{600}{41})}{10^{-3}} = \underline{29268.29\ N}$.

c) According to Newton's 3rd law : $\vec{F}_{bullet\,/block} = -\vec{F}_{block\,/bullet} \Rightarrow F_{Block/bullet} = \underline{29268.29\ N}$.

d) $KE_{before} = \frac{1}{2}(0.05)(600)^2 = 9000$ J. $KE_{after} = \frac{1}{2}[\,0.05 + 2](14.63)^2 = 219.39$ J. $KE_{after} < KE_{before} \Rightarrow$ <u>In-elastic collision</u>.

9.



a) The weight of each block and the normal reaction exerted by the table on each block.

b)

i. $\sum\vec{F}_{ext} = m_1\vec{g} + m_2\vec{g} + \vec{N}_1 + \vec{N}_2 = \vec{0}$ but $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} \Rightarrow \frac{d\vec{P}}{dt} = \vec{0} \Rightarrow \vec{P}$ is conserved

$\vec{P}_{before} = \vec{P}_{after} \Rightarrow \vec{0} = \vec{P}_1 + \vec{P}_2 \Rightarrow \vec{0} = m_1\vec{V}_1 + m_2\vec{V}_2 \Rightarrow \vec{0} = 2(-0.2\vec{i}) + 1(\vec{V}_2) \Rightarrow \vec{V}_2 = 0.4\vec{i}$

$\Rightarrow$ $\underline{V_2 = 0.4 m/s}$ and $\vec{V}_2$ is directed <u>horizontally to the right</u>.

ii. $\vec{P}_{system} = \vec{P}_{afterG} = \vec{0} = (m_1 + m_2)\vec{V}_G \Rightarrow \vec{V}_G = \underline{\vec{0}}$.

iii. Since there are no non-conservative forces (the table is frictionless and the tensions exerted by the spring are conservative forces) then $\Rightarrow$ ME is conserved. $ME = GPE_o + EPE_o + KE_o = GPE + EPE + KE$

$\Rightarrow 0 + \frac{1}{2}k(\Delta x)^2 + 0 = 0 + \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 \Rightarrow K(0.05)^2 = 2(0.2)^2 + 1(0.4)^2 \Rightarrow \underline{k = 96\ N/m}$.

10.

a) $\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} = m\vec{a} \Rightarrow \vec{a} = 10\vec{j} \Rightarrow a = 10\ m/s^2 \Rightarrow v^2 - V_0^2 = 2a(\Delta y) \Rightarrow v^2 - 0 = 2(10)(0.4) \Rightarrow v = \sqrt{8}\ m/s = \underline{2.83\ m/s}$.

b) $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow m\vec{g} + \vec{F}_{impact} = \frac{d\vec{P}}{dt}$. Algebraic form: $(0.06)(10) - F_{impact} = \frac{0-(0.06)(\sqrt{8})}{6\times10^{-3}} \Rightarrow F_{impact} = \underline{28.88\ N}$.

c) $(0.06)(10) - F'_{impact} = \frac{0-(0.06)(\sqrt{8})}{60\times10^{-3}} \Rightarrow F'_{impct} = \underline{3.43\ N}$.

d) The egg will break if the stopping time is 6 ms (<u>when it falls on the ground</u>) since in this case $F_{impact} = 28.88\ N > 25\ N$.

11.

a) After derivations we get: $V'_B = (\frac{m_2 - m_1}{m_1 + m_2})V_B + (\frac{2m_1}{m_1 + m_2})V_1$, but $m_1 = m_2 \Rightarrow V'_B = V_A$.

b) $V'_B = V_A \Rightarrow$ the balls will exchange their velocities.

All the balls are identical $\Rightarrow$The same process is repeated when B collides with C, than when C collides with E $\Rightarrow V'_E = V_A$.

12.

a) System: (S_1, Earth). No non-conservative forces $\Rightarrow$ ME is conserved.

$mg\ell(1 - \cos60) = \frac{1}{2}mV^2 \Rightarrow (10)\,l(1 - 0.5) = \frac{1}{2}(25) \Rightarrow \underline{\ell = 2.5\ m}$.

b) After derivations: $V'_1 = (\frac{m_1 - m_2}{m_1 + m_2})V_1 = (\frac{6-6}{12})(5) = \underline{0}$. $V'_2 = (\frac{2m_1}{m_1 + m_2})V_1 = (\frac{2(6)}{6+6})(5) = \underline{5\ m/s}$.

c)

i. During this collision, linear momentum of the system (S_2, wagon) is conserved: $\vec{P}_{before} = \vec{P}_{after}$

Algebraic form: $m_2V'_2 = (m_2 + M)V \Rightarrow 6(5) = (6+9)V \Rightarrow \underline{V = 2\ m/s}$.

ii. $KE_{before} = \frac{1}{2}m_2V_2^2 = \frac{1}{2}(6)(5)^2 = 75\ J$. $KE_{after} = \frac{1}{2}(m_2 + M)V^2 = \frac{1}{2}(6+9)(2)^2 = 30\ J \Rightarrow KE_{after} < KE_{before} \Rightarrow$ <u>In- elastic</u> .

13.

a) System: (Bullet, Earth). No friction (no non-conservative forces) $\Rightarrow$ ME is conserved

$\Rightarrow ME_{Top} = ME_{bottom} \Rightarrow GPE_{Top} + KE_{Top} = GPE_o + kE_0 \Rightarrow (M + m)\,g[\,\ell - \ell\cos30\,] + 0 = 0 + \frac{1}{2}(m + M)V_0^2$

$10\,[1 - \cos30°] = V_0^2 \Rightarrow \underline{V_o = 1.64\ m/s}$.

b) During this collision, linear momentum is conserved $\Rightarrow \vec{P}_{before} = \vec{P}_{after} \Rightarrow m\vec{u} = (m + M)\vec{V}_o$

Algebraic form: $(0.15)\,u = (0.15 + 50)(1.64) \Rightarrow \underline{u = 548.3\ m/s}$.

14.

A.

a) $3t = -2t + 5 \Rightarrow \underline{t = 1\ s}$.

b) $\vec{v}_M = \overrightarrow{OM'} = 2\vec{i} + 3\vec{j} \Rightarrow \vec{P}_1 = 0.2[2\vec{i} + 3\vec{j}] = \underline{0.4\vec{i} + 0.6\vec{j}}$ (kg.m/s). $\vec{v}_N = \overrightarrow{ON'} = 2\vec{i} - 2\vec{j} \Rightarrow \vec{P}_2 = 0.3[2\vec{i} - 2\vec{j}] = \underline{0.6\vec{i} - 0.6\vec{j}}$ (kg.m/s).

c) $\vec{P} = \vec{P}_1 + \vec{P}_2 = \underline{\vec{i}}$ (kg.m/s).

d) $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(\vec{i}) = \vec{0}$ (<u>isolated system</u>).

e) $\vec{P} = \vec{P}_G = (m_1 + m_2)\vec{V}_G \Rightarrow \vec{i} = (0.2 + 0.3)\vec{V}_G \Rightarrow \underline{\vec{V}_G = 2\vec{i}}$ (m/s).

B.

a) $\vec{P}$ is constant $\Rightarrow \vec{P}_{before} = \vec{P}_{after} = (m_1 + m_2)\vec{V} \Rightarrow \vec{i} = (0.2 + 0.3)\vec{V} \Rightarrow \underline{\vec{V} = 2\vec{i}} = \vec{V}_G$.

b) $KE_{before} = \frac{1}{2}m_1\vec{V}_1^2 + \frac{1}{2}m_2\vec{V}_2^2$ { but $V_1 = V_M = \sqrt{13}$ m/s and $V_2 = V_N = \sqrt{8} = 2\sqrt{2}$ m/s}. $KE_{before} = \frac{1}{2}(0.2)(13) + \frac{1}{2}(0.3)(8) = 2.5\ J$.

$KE_{after} = \frac{1}{2}(m_1 + m_2)V^2 = \frac{1}{2}(0.5)(4) = 1\ J$. $K.E_{before} \neq K.E_{after} \Rightarrow$ <u>in elastic collision.</u>

c) System: N. $\sum\vec{F}_{ext\,(av)} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow m_2\vec{g} + \vec{N}_2 + \vec{F}_{M/N} = \frac{m_2[\vec{V} - \vec{V}_N]}{\Delta t}$. $30\,\vec{j} = \frac{0.3[2\vec{i} - (2\vec{i} - 2\vec{j})]}{\Delta t} \Rightarrow \underline{\Delta t = 0.02\ s}$.

15.

a)

i. $\Delta\vec{P} = \vec{P}_{Final} - \vec{P}_{Initial}$. Algebraic form: $\Delta P = 0 - mV_1 = 0 - (60)(8) = \underline{-480\ kg.m/s}$.

ii. System: Driver. $\sum\vec{F}_{ext\,(av)} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow m\vec{g} + \vec{N} + \vec{F}_1 = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow F_1 = \frac{\Delta P}{\Delta t} = \frac{-480}{0.02} = -24000\ N \Rightarrow$ magnitude : $\underline{F_1 = 24000\ N}$.

b)

i. $F_2 = \frac{-480}{0.1} = -4800\ N \Rightarrow$ magnitude : $\underline{F_2 = 4800\ N}$. **ii.** $\underline{F_2 < F_1}$.

iii. the airbag increases the time interval during which the driver is brought to rest, thereby reducing the forces exerted by the steering wheel on the driver

16.

a) During collision the linear momentum is conserved $\Rightarrow m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2' \Rightarrow m_1V_1 = m_1V_1' + m_2V_2'$
for $m_1 = 3$ kg; $V_1' = 0.5$ m/s and $V_2' = 1.5$ m/s $\Rightarrow 3(1) = 3(0.5) + m_2(1.5) \Rightarrow m_2 = 1$ kg.

b) For $m_1 = 3$ kg: $KE_{before} = \frac{1}{2}m_1V_1^2 = \frac{1}{2}(3)(1)^2 = 1.5$ J. $KE_{after} = \frac{1}{2}m_1(V_1')^2 + \frac{1}{2}m_2(V_2')^2 = \frac{1}{2}(3)(0.5)^2 + \frac{1}{2}(1)(1.5)^2 = 1.5$ J

For $m_1 = 1$ kg; $V_1' = 0$ and $V_2' = 1$ m/s. $KE_{before} = \frac{1}{2}(1)(1)^2 = 0.5$ J. $KE_{after} = \frac{1}{2}(1)(0)^2 + \frac{1}{2}(1)(1)^2 = 0.5$ J. ($\Rightarrow$Elastic collision).

c) $0 < m_1 < 1$ kg.

d) $V = 0.5$ m/s and $m_1 = \frac{1}{3}$ kg.

e) $V'_{1\,max} = 0.875$ m/s and $V'_{2max} = 1.875$ m/s.

17.

A.

a) System (A, B, spring, string). $\sum\vec{F}_{ext} = M\vec{g} + \vec{N} = \vec{0} \Rightarrow \vec{P}_{sys}$ is conserved $\Rightarrow (m_A + m_B)\vec{V}_G = m_A\vec{V}_A + m_B\vec{V}_B$
$\Rightarrow (0.2 + 0.1)(\vec{V}_G) = 0.2(3\vec{i}) + \vec{0} \Rightarrow \underline{\vec{V}_G = 2\vec{i}}$ (m/s).

b) $\sum\vec{F}_{ext} = \vec{0}$. G was initially in U.R.M, than when the string is burnt, G remains in U.R.M.

B.

a) $\vec{P} = \vec{P}_A + \vec{P}_C = m_A\vec{V}_A + m_C\vec{V}_C = (0.2)(3\vec{i}) + (0.15)(-2\vec{i}) \Rightarrow \underline{\vec{P} = 0.3\vec{i}}$ (kgm/s).

b) During the collision, $\vec{P}$ is conserved $\Rightarrow \vec{P} = \vec{P}' = m_A\vec{V}_A' + m_C\vec{V}_C' \Rightarrow 0.3\vec{i} = 0.2\vec{V}_A' + (0.15)(3\vec{i}) \Rightarrow \vec{V}_A' = \underline{-0.75\vec{i}}$ (m/s)

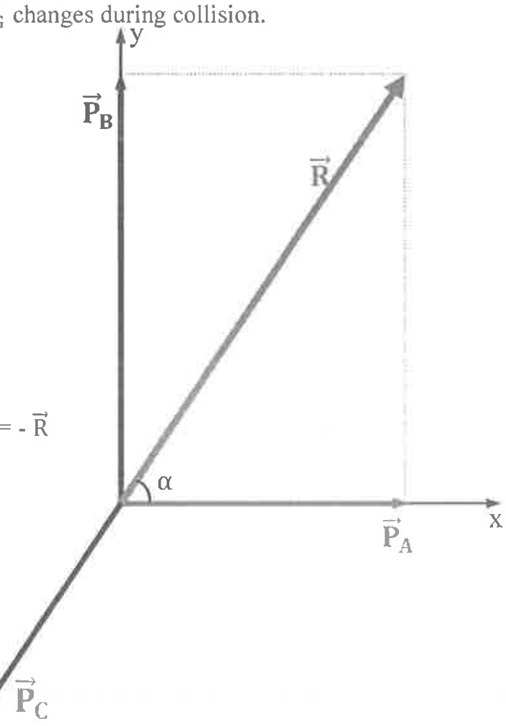
c) $KE_{before} = \frac{1}{2}m_AV_A^2 + \frac{1}{2}m_CV_C^2 = \frac{1}{2}(0.2)(9) + \frac{1}{2}(0.15)(4) = 1.2$ J .

$KE_{After} = \frac{1}{2}m_AV_A'^2 + \frac{1}{2}m_CV_C'^2 = \frac{1}{2}(0.2)(0.75)^2 + \frac{1}{2}(0.15)(3)^2 = 0.73125$ J $\Rightarrow K.E_{before} \neq K.E_{After} \Rightarrow$ Non elastic collision.

C.

a) After collision between A and C: $\vec{P}_{system} = m_A\vec{V}_A' + m_B\vec{V}_B = 0.2\,(-0.75)\vec{i} + \vec{0} = -0.15\,\vec{i} = (m_A + m_B)\vec{V}_G$
$\Rightarrow \vec{V}_G = \underline{-0.5\vec{i}}$ (m/s).

b) During the collision between A and C, the ball C exerts a force $\vec{F}_{C/A}$ on A which is an external force relative to the system (A, B, spring, string), but $\sum\vec{F}_{ext} = (m_A + m_B)\vec{a}_G \Rightarrow \vec{a}_G$ exists $\Rightarrow \vec{V}_G$ changes during collision.



18.

A. **a)** $\vec{P}_o = m\vec{V}_o = (5)(-20)\vec{K} = \underline{-100\vec{K}}$ (kg.m/s).
$\vec{P}_1 = m\vec{V}_1 = (5)(20)\vec{K} = \underline{100\vec{K}}$ (kg.m/s) $\Rightarrow \Delta\vec{P} = \vec{P}_1 - \vec{P}_o = \underline{200\,\vec{K}}$ (kg.m/s).

b) $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow m\vec{g} = \frac{\Delta\vec{P}}{\Delta t} = \frac{200}{4}\vec{K} \Rightarrow \underline{\vec{g} = 10\,\vec{K}}$ (m/s²).

B.

a) $\vec{P} = \vec{0}$.

b) $m\vec{g}$ (weight) and normal reaction ($\vec{N}$). $\sum\vec{F}_{ext} = m\vec{g} + \vec{N} = \vec{0} \Rightarrow$
system (bomb) is isolated.

c) $\vec{P}_A = m\vec{V}_A = (2)(50)\vec{i} = 100\vec{i} \Rightarrow \vec{P}_B = m\vec{V}_B = (2)(75)\vec{j} = 150\,\vec{j}$
Length of $\vec{P}_A = \frac{100}{25} = 4$cm; length of $\vec{P}_B = \frac{150}{25} = 6$ cm.

d) Isolated system: $\vec{P}$ is conserved. $\vec{0} = \vec{P}_A + \vec{P}_B + \vec{P}_C \Rightarrow \vec{P}_C = -(\vec{P}_A + \vec{P}_B) = -\vec{R}$
$\Rightarrow \|\vec{P}_C\| = \|\vec{R}\| = 7.2 \times 25 = 180$ kg.m/s $\Rightarrow \tan\alpha = \frac{P_B}{P_A} = \frac{150}{100} = 1.5$
$\Rightarrow \alpha = 56.3° \Rightarrow \vec{P}_C$ makes $(56.3 + 180)°$ with the x-axis.

B. **a)** linear momentum and kinetic energy.

b) **i)** A rebounds $\Rightarrow V_A'$ and V_A have opposite signs
$\Rightarrow m_A < m_D \Rightarrow m_D > 2$ kg.

ii) $V_A' = 0 \Rightarrow m_A = m_D = 2$ kg.

iii) $V_A' > 0 \Rightarrow m_A > m_D \Rightarrow 0 < m_D < 2$ kg.

19.

A.

a) $\vec{P} = 72\,\vec{i} = m_1\,V_B\,\vec{i}$ then $V_B = \frac{72}{2} = \underline{36\text{ m/s}}$.

b) $\Sigma\vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0}$.

c) $\Sigma\vec{F}_{ext} = m_1\vec{g} + \vec{N} = \vec{0}$ $\Rightarrow$There is no friction acting on M on track AB .

B.

a) $m_1\,\vec{V}_C + 0 = m_1\,\vec{V}_M + m_2\,\vec{V}_S \Rightarrow 2 \times 27 = 2\,V_M + 2 \times 8\sqrt{10}$ (V_M is algebraic) then $\underline{V_M = 1.7\text{ m/s}}$ direction as that of $\vec{V}_C$

b) $KE_i = ½\,m_1\,V_C^2 = (0.5)(2)(27)^2 = 729$ J. $KE_f = ½\,m_1\,V_M^2 + ½\,m_2\,V_S^2 = (0.5)(2)(1.7)^2 + (0.5)(2)(8\sqrt{10})^2 = 642.89$ J
$KE_i > KE_f$ then the collision is not elastic.

c) During collision GPE remain the same but KE decreases then ME decreases .

C.

a) $\vec{P}_o = m_2\vec{V}_D = 2(V_D\cos\alpha\,\vec{i} + V_D\sin\alpha\,\vec{j}) = 2(20\times0.92\vec{i} + 20\times0.4\vec{j}) = \underline{36.8\,\vec{i} + 16\,\vec{j}}$ (kg.m/s).

b) $\Sigma\vec{F}_{ext} = \frac{d\vec{P}}{dt}$ then $m_2\vec{g} = \frac{d\vec{P}}{dt}$ then $2\times(-10)\vec{j} = \frac{d\vec{P}}{dt}$ then $\vec{P} = -20t\,\vec{j} + \vec{P}_o$

$\vec{P} = -20t\,\vec{j} + 36.8\,\vec{i} + 16\,\vec{j} \Rightarrow \underline{\vec{P} = 36.8\,\vec{i} + (16-20t)\,\vec{j}}$ (kg.m/s).

c) $V_y = 0$ then $P_y = 0$ then $16 - 20t = 0$ then $\underline{t = 0.8\text{ s}}$.

20.

A.

a) i. $GPE_{(water\text{-}Earth)} = Mgh = (10)(10)(0.16) = \underline{16\text{ J}}$. **ii.** $GPE_{(car\text{-}Earth)} = \underline{0}$.

b) $ME_{0\,(car-Earth)} = GPE_{0\,(car-Earth)} + KE_{0\,(car)} = 0 + 16 = \underline{16\text{ J}}$.

c) $\sum\vec{F}_{ext} = M_{total}\vec{g} + \vec{N} = \vec{0} \Rightarrow$ isolated system. **d)** $\vec{P}_o = \underline{\vec{0}}$.

B.

a) Isolated system $\Rightarrow \vec{P}$ is conserved $\Rightarrow \vec{P}_o = \vec{P} \Rightarrow \vec{0} = m\vec{V} + M\vec{u} \Rightarrow \vec{V} = -\frac{M}{m}\vec{u} \Rightarrow \vec{V}$ and $\vec{u}$ have opposite directions.

b) i. $ME_{(car,\,Earth)} = KE_{car} + GPE_{(car\text{-}Earth)} = \frac{1}{2}MV^2 = \frac{1}{2}(2)(2.5)^2 + 0 = 6.25$ J.

ii. $\Delta ME = ME_t - ME_o = 6.25 - 0 = \underline{6.25\text{ J}}$.

iii. System (car, water, Earth). ME is conserved since no non-conservative forces $\Rightarrow ME_o = ME_t$

$\Rightarrow ME_{0\,(car-Earth)} + ME_{0\,(water-Earth)} = ME_{t\,(car-Earth)} + ME_{t\,(water-Earth)} \Rightarrow \Delta ME_{(water\text{-}Earh)} = -\Delta ME_{(car\text{-}Earth)} = \underline{-6.25\text{ J}}$.

C.

a) $\Delta\vec{P} = \vec{0} - 2(2.5)\,\vec{i} = \underline{-5\,\vec{i}}$ (kg.m/s).

b) $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow m\vec{g} + \vec{N} + \vec{f} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow -2.5\vec{i} = \frac{-5\vec{i}}{\Delta t} \Rightarrow \underline{\Delta t = 2\text{ s}}$.

21.

a)

i. No non-conservative forces $\Rightarrow$ ME is conserved.

ii. $ME = KE_A + KE_B + GPE = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 + 0 = \frac{1}{2}(1.6)(16) + \frac{1}{2}(2.1)(2.5)^2 \Rightarrow ME = 19.3625$ J.

iii. $\sum\vec{F}_{ext} = m_1\vec{g} + m_2\vec{g} + \vec{N}_1 + \vec{N}_2 = \vec{0} \Rightarrow$ isolated system $\Rightarrow$ the linear momentum is conserved.

$\vec{P} = \vec{P}_1 + \vec{P}_2 = m_1\vec{V}_1 + m_2\vec{V}_2 = (1.6)(4)\vec{i} + (2.1)(-2.5)\vec{i} = 1.15\,\vec{i}$, but $\vec{P} = \vec{P}_G = (m_1+m_2)\vec{V}_G \Rightarrow \vec{V}_G = \frac{1.15\vec{i}}{(1.6+2.1)} = \underline{0.311\,\vec{i}}$ (m/s).

b)

i. Elastic potential energy.

ii.

1. $\vec{P} = \vec{P}_1 + \vec{P}_2 \Rightarrow \vec{P} = m_1\vec{V}_A + m_2\vec{V}_B \Rightarrow 1.15\vec{i} = 1.6(3\vec{i}) + 2.1\,\vec{V}_B \Rightarrow \vec{V}_B = \frac{-73}{42}\vec{i} = \underline{-1.738\,\vec{i}\text{ m/s}}$.
2. $ME = \frac{1}{2}m_1V_A^2 + \frac{1}{2}m_2V_B^2 + \frac{1}{2}Kx^2 + GPE \Rightarrow 19.3625 = \frac{1}{2}(1.6)(9) + \frac{1}{2}(2.1)(1.738)^2 + \frac{1}{2}(600)(x_1^2) + 0 \Rightarrow x_1 = \underline{17.31\text{ cm}}$.

iii.

1. $\vec{P}_G = m_1\vec{V}_A + m_2\vec{V}_B \Rightarrow (m_1+m_2)\vec{V}_G = m_1\vec{V} + m_2\vec{V} = (m_1+m_2)\vec{V} \Rightarrow \vec{V}_G = \vec{V}$.
2. $ME = \frac{1}{2}kx_m^2 + \frac{1}{2}(m_1+m_2)V^2 \Rightarrow 19.3625 = \frac{1}{2}(600)(x_m^2) + \frac{1}{2}(1.6+2.1)(0.311)^2 \Rightarrow X_m = 0.2528\text{ m} = \underline{25.28\text{ cm}}$.

c)

i. The linear momentum is conserved $\Rightarrow m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2' \Rightarrow m_1(V_1 - V_1') = m_2(V_2' - V_2)$...........(1)

KE is conserved: $\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1(V_1')^2 + \frac{1}{2}m_2(V_2')^2 \Rightarrow m_1(V_1^2 - V_1'^2) = m_2(V_2'^2 - V_2^2) \Rightarrow V_1 + V_1' = V_2 + V_2'$.........(2)

solve the two equations: $V_1' = (\frac{m_1-m_2}{m_1+m_2})V_1 + (\frac{2m_2}{m_1+m_2})V_2$ and $V_2' = (\frac{m_2-m_1}{m_1+m_2})V_2 + (\frac{2m_1}{m_1+m_2})V_1$

ii. $m_1 < m_2$ and $V_1 > 0 \Rightarrow (\frac{m_1-m_2}{m_1+m_2})V_1 < 0$ also $V_2 < 0 \Rightarrow (\frac{2m_2}{m_1+m_2})V_2 < 0 \Rightarrow V_1' < 0$ and $V_2' = \frac{3.2}{3.7}(4) + \frac{0.5}{3.7}(-2.5) = 3.121$ m/s > 0.

iii. $|V_2'| > |V_1'| \Rightarrow \frac{32}{37}V_1 + \frac{5}{37}V_2 > -[-\frac{5}{37}V_1 + \frac{42}{37}V_2]$, but $V_2 = -2.5$ m/s $\Rightarrow V_1 > 4.35$m/s $\Rightarrow V_{1(min)} = \underline{4.35^+\text{ m/s}}$.

22.

a) System: (A, Earth). No non-conservative forces $\Rightarrow$ ME is conserved $\Rightarrow ME_{Top} = ME_{bottom}$

$\Rightarrow mgh_A + 0 = 0 + \frac{1}{2}mV_1^2 \Rightarrow h_A = \frac{V_1^2}{2g}$.

b) $V_1' = (\frac{m_1-m_2}{m_1+m_2})V_1$ and $V_2' = (\frac{2m_1}{m_1+m_2})V_1$. *Note these expressions need a proof.*

c) $\vec{V}_1'$ and $\vec{V}_1$ have opposite directions $\Rightarrow \vec{V}_1'$ is directed horizontally to the left.

$\vec{V}_2'$ and $\vec{V}_1$ have same directions $\Rightarrow \vec{V}_2'$ is directed horizontally to the right.

d)

i. $|V_1'| = |V_2'| \Rightarrow -(\frac{m_1-m_2}{m_1+m_2})V_1 = (\frac{2m_1}{m_1+m_2})V_1 \Rightarrow m_2 - m_1 = 2m_1 \Rightarrow m_2 = 3m_1 \Rightarrow \frac{m_1}{m_2} = \frac{1}{3}$.

ii. $h_B = \frac{(V_2')^2}{2g}$ and $h_A = \frac{(V_1')^2}{2g} \Rightarrow \frac{h_B}{h_A} = \frac{(V_2')^2}{(V_1')^2} = 2 \Rightarrow \frac{V_2'}{V_1'} = \sqrt{2} \Rightarrow (\frac{2m_1}{m_1+m_2})V_1 = \sqrt{2}\,[-(\frac{m_1-m_2}{m_1+m_2})V_1] \Rightarrow \frac{m_1}{m_2} = \frac{1}{1+\sqrt{2}}$.

23. Before explosion: $\vec{P} = m\vec{V} \Rightarrow P = (30000)(5000) = 150 \times 10^6$ kg.m/s

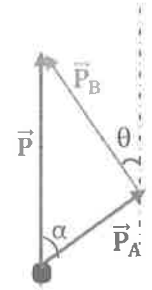
$\vec{P}_A = m\vec{V}_A \Rightarrow P_A = (10000)(9000) = 90 \times 10^6$ kg.m/s.

During explosion linear momentum is conserved $\Rightarrow \vec{P}_{before} = \vec{P}_{after} \Rightarrow \vec{P} = \vec{P}_A + \vec{P}_B$

length of $\vec{P} = \frac{150 \times 10^6}{50 \times 10^6} = 3$ cm. length of $\vec{P}_A = \frac{9 \times 10^7}{50 \times 10^6} = 1.8$ cm.

Graphically: length of $\vec{P}_B = 2.4$ cm $\Rightarrow P_B = 2.4 \times 50 \times 10^6 = 12 \times 10^7$ kg.m/s

$V_B = \frac{P_B}{m_B} = \frac{12 \times 10^7}{2 \times 10^4} =$ <u>6 km/s</u>. We measure θ: <u>$\theta = 40°$.</u>



24.

a) $A_oA_1 = A_1A_2 = A_2A_3 = \ldots\ldots A_{11}A_o$.

b) $V = \frac{A_oA_1}{\tau} = \frac{1.3 \times 10^{-2}}{40 \times 10^{-3}} =$ <u>0.325 m/s.</u>

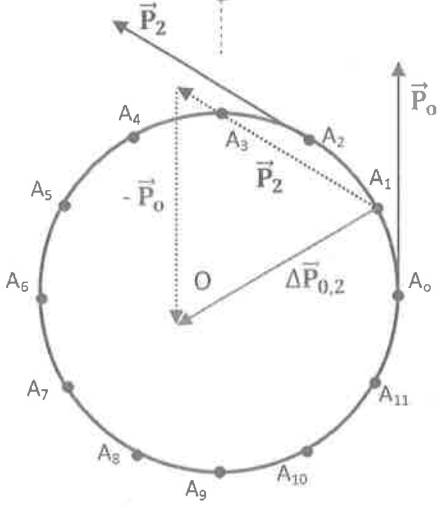
c) $P_o = mV_o = (0.5)(0.325) = 0.1625$ kg.m/s.

$P_2 = mV_2 = 0.1625$ kg.m/s, lengths of $\vec{P}_o$ and $\vec{P}_2$ is $\frac{0.1625 \times 1}{0.05} = 3.25$ cm.

d) Figure.

e) The length of $\overrightarrow{\Delta P}_{0,2}$ is 3.4 cm $\Rightarrow \Delta P_{0,2} = 3.4 \times 0.05 =$ <u>0.17 kg.m/s.</u>

f) $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{\Delta \vec{P}}{\Delta t} \Rightarrow \sum F_{ext} = \frac{0.17}{2 \times 40 \times 10^{-3}} =$ <u>2.125 N.</u>



25.

a) $V_A = \frac{A_1A_2}{\tau} = \frac{1.5 \times 10^{-2}}{50 \times 10^{-3}} =$ <u>0.3 m/s.</u>

$V_B = \frac{B_1B_2}{\tau} = \frac{2 \times 10^{-2}}{50 \times 10^{-3}} =$ <u>0.4 m/s.</u>

b) $P_B = m_BV_B = 0.6 \times 0.4 = 0.24$ kg.m/s

length of $\vec{P}_B$ is $\frac{0.24}{0.05} = 4.8$ cm.

c) $\sum \vec{F}_{ext} = \vec{W}_{system} + \vec{N}_{system} = \vec{0} \Rightarrow$ the system is isolated.

d) Isolated system $\Rightarrow \vec{P}_{before} = \vec{P}_{after} \Rightarrow \vec{P}_o = \vec{P}_A + \vec{P}_B$

$\Rightarrow$ the direction of $\vec{P}_A + \vec{P}_B$ is as that of $\vec{P}_o$ (along BM)

where $\vec{P}_A$ is vertically upward.

e) From the figure the length of $\vec{P}_A$ is 3.6 cm

$\Rightarrow \vec{P}_A = (3.6)(0.05) = 0.18$ kg.m/s

$P_A = m_AV_A \Rightarrow m_A = \frac{0.18}{0.3} =$ <u>0.6 kg.</u>

f) $P_o = m_AV_o$ from the figure the length of $\vec{P}_o$ is 6 cm

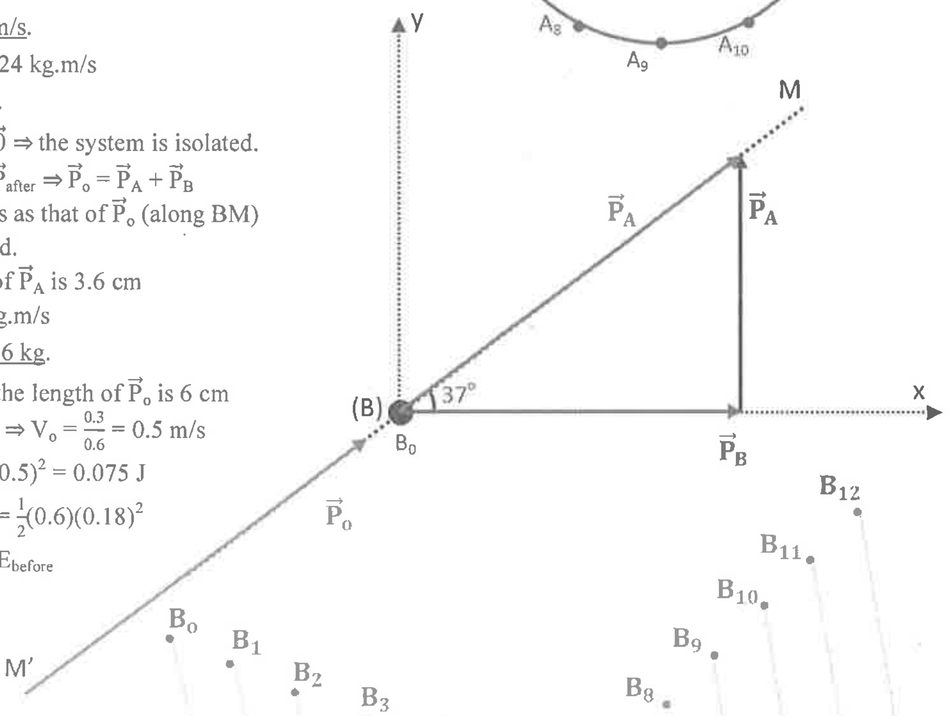
$\Rightarrow P_o = 6 \times 0.05 = 0.3$ kg.m/s $\Rightarrow V_o = \frac{0.3}{0.6} = 0.5$ m/s

$KE_{before} = \frac{1}{2}m_AV_A^2 = \frac{1}{2}(0.6)(0.5)^2 = 0.075$ J

$KE_{after} = \frac{1}{2}m_AV_A^2 + \frac{1}{2}m_BV_B^2 = \frac{1}{2}(0.6)(0.18)^2$

$+ \frac{1}{2}(0.6)(0.4)^2 = 0.075$ J $= KE_{before}$

$\Rightarrow$ <u>the collision is elastic.</u>



26.

a) $A_oG_o = \frac{m_A(A_oA_o) + m_B(A_oB_0)}{m_A + m_B} = \frac{0 + 0.4(7)}{0.4 + 0.6} =$ <u>2.8 cm.</u>

b) $A_1G_1 = \frac{m_A(A_1A_1) + m_B(A_1B_1)}{m_A + m_B} = \frac{0 + 0.4(6.2)}{1} =$

2.48 cm. similarly for A_2G_2, A_3G_3, … $A_{12}G_{12}$.

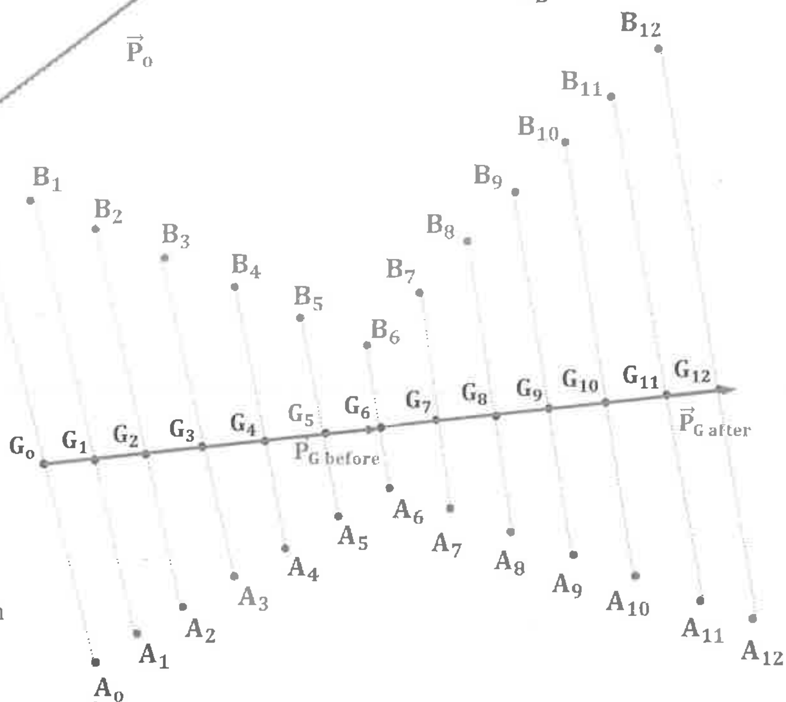
c) Since $G_oG_1 = G_1G_2 = \ldots$and the trajectory of G is straight line $\Rightarrow$ the motion of G is <u>uniform rectilinear.</u>

$V_{G\,before} = V_{G\,after} = \frac{G_oG_2}{2\tau} = \frac{1.5 \times 10^{-2}}{2 \times 20 \times 10^{-3}} =$ <u>0.38 m/s.</u>

d) $P_G = (M_A + M_B)V_G = (0.4 + 0.6)(0.38)$

$= 0.38$ kg.m/s, the length of $\vec{P}_G$ is $\frac{0.38}{0.08} = 4.75$ cm

before and after collision.



Chapter 3: Angular Momentum

1.

A.

a) $\sigma = I\,\theta'$ but $I = mR^2 = (0.05)(0.8)^2 = 0.032$ kg.m^2 $\Rightarrow$ $\sigma = 0.032[-2t + 20] = -0.064t + 0.64$ (S.I).

b) $\sum\mathcal{M} = \frac{d\sigma}{dt} \Rightarrow \sum\mathcal{M} = -0.064$ N.m.

B.

a) $I = I_{disk} + I_A + I_B = \frac{1}{2}Mr^2 + m_1d^2 + m_2D^2 = \frac{1}{2}(1.6)(0.4)^2 + 0.5(0.2)^2 + 1(0.3)^2 = 0.238$ kg.m^2.

b) $\sigma = I\theta' = (0.238)(0.2) = 0.0476$ kg.m^2/s.

2.

a) $I = mR^2 = 1.2(0.3)^2 = 0.108$ kg.m^2 . $\theta' = 2\pi f = 2\pi(160) = 320\pi$ rad/s $\Rightarrow \sigma_o = I\theta' = (0.108)(320\pi) = 108.52$ kg.m^2/s.

b) $\sum\mathcal{M} = \frac{d\sigma}{dt} \Rightarrow \mathcal{M}_{m\vec{g}} + \mathcal{M}_{\vec{R}} + \mathcal{M}_{\vec{F}} = I\theta''$. But $\mathcal{M}_{\vec{R}} = 0$ since $\vec{R}$ passes through (Δ) and $\mathcal{M}_{m\vec{g}} = 0$ since $m\vec{g}$ is parallel to (Δ).

$\Rightarrow$ $0 + 0 - RF = I\,\theta'' \Rightarrow -RF = I\,\theta''$…eq (*). $\theta'' = \frac{-R\,F}{I}$ = const $\Rightarrow \theta' = \theta''t + \theta'_o \Rightarrow 0 = \theta'(8) + 320\pi \Rightarrow \theta'' = -125.6$ rad/s^2.

Substitute in equation (*) $\Rightarrow$ $-(0.3)(F) = 0.108(-125.6)$ $\Rightarrow$ $F = 45.216$ N .

OR: $\sum\mathcal{M} = \frac{d\sigma}{dt} \Rightarrow 0 + 0 - RF = \frac{d\sigma}{dt} \Rightarrow d\sigma = -R\,F\,dt \Rightarrow \sigma = -RF\,t + \sigma_o$. Stops $\Rightarrow 0 = -(0.3)(F)(8) + 108.5$ $\Rightarrow$ $F = 45.216$ N.

c) U.D.R.M $\Rightarrow$ $(\theta')^2 - (\theta'_o)^2 = 2\theta''(\Delta\theta) \Rightarrow 0 - (320\pi)^2 = 2(-125.6)\Delta\theta \Rightarrow \Delta\theta = 4019.2$ rad $\Rightarrow N = \frac{\Delta\theta}{2\pi} = \frac{4019.2}{2\pi} = 640$ revolutions.

3.

a) $V = \sqrt{V_x^2 + V_y^2} = \sqrt{(144\sin^2(2t) + (144\cos^2(2t))} = 12$ m/s. $\theta' = \frac{V}{R} = \frac{12}{6} = 2$ rad/s.

$I = m\,R^2 = (0.2)(36) = 7.2$ kg.m^2 $\Rightarrow \sigma = I\,\theta'' = (7.2)(2) = 14.4$ kg.m^2/s.

b) $\sum\mathcal{M} = \frac{d\sigma}{dt} = 0$ since σ = constant.

c) $\vec{P} = m\vec{V} = 0.2[-12\sin(2t)\vec{i} + 12\cos(2t)\vec{j}] = -2.4\sin(2t)\vec{i} + 2.4\cos(2t)\vec{j}$ (S.I).

d) $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} = -2.4(2)\cos(2t)\vec{i} - 2.4(2)\sin(2t)\vec{j} = -4.8\cos(2t)\vec{i} - 4.8\sin(2t)\vec{j}$ (S.I).

4.

a) $m\vec{g}$: weight of the person, $M\vec{g}$: weight of the turntable, $\vec{R}$: reaction at the axis of rotation (Δ).

b) $\mathcal{M}_{M\vec{g}} = \mathcal{M}_{m\vec{g}} = \mathcal{M}_{\vec{R}} = 0$ ($\vec{R}$ passes through (Δ). $m\vec{g}$ and $M\vec{g}$ are parallel to (Δ)).

$\sum\mathcal{M} = 0 = \frac{d\sigma}{dt} \Rightarrow$ the angular momentum is conserved; σ = constant . $\sigma_{before} = \sigma_{after} \Rightarrow 0 = I_{person}\theta'_{person} + I_{table}\theta'_{table}$

$\Rightarrow \theta'_{table} = -\frac{I_{person}\ \theta'_{person}}{I_{table}} < 0 \Rightarrow$ turntable rotates in the clockwise sense . $\theta'_{table} = \frac{-60(2)^2(\frac{1.5}{2})}{500} = -0.36$ rad/s.

c) Uniform rotation $\Rightarrow$ $\theta = \theta'\,t = \theta_o \Rightarrow \Delta\theta = \theta' t \Rightarrow \frac{\pi}{2} = \frac{1.5}{2}t \Rightarrow t = 2.093$ s.

for turntable : $\Delta\theta = \theta'_{table}\ t = (-0.36)(2.093) = -0.75$ rad $\Rightarrow$ Described angle = 0.75 rad.

d) The kinetic energy increases and the gravitational potential energy of the system remains the same then the ME of the system increases (is not conserved). $\Delta ME = \Delta KE = \frac{1}{2}I_{table}(\theta')^2 + \frac{1}{2}I_{person}(\theta'_{person})^2 = \frac{1}{2}(500)(0.36)^2 + \frac{1}{2}(60)(4)(0.75)^2 = 100$ J.

Work done by the non-conservative forces results in increase in ME $\Rightarrow \Delta U = -\Delta ME = -100$ J.

5.

a) System (two disks). Before collision: $\sigma_{before} = I_1\theta'_i + 0$ {second disk is initially at rest} $\Rightarrow \sigma_{before} = (0.5)(2) = 1$ kg.m^2/s.

$I_2 = \frac{1}{2}mR_2^2 = \frac{1}{2}m(\frac{R_1}{2})^2 = \frac{I_1}{4} = \frac{0.5}{4} = 0.125$ kg.m^2. After collision: $\sigma_{after} = (I_1 + I_2)\theta'_f = (0.5 + 0.125)\theta'_f = 0.625\theta'_f$.

External Forces: Weight of the two disks and the reaction of the axis of rotation. All external forces are confounded with (Δ) $\Rightarrow$ their moments relative to (Δ) are zero $\Rightarrow \sum\mathcal{M} = 0$ but $\sum\mathcal{M} = \frac{d\sigma}{dt} = 0 \Rightarrow$ Angular momentum of the system is conserved.

$\Rightarrow \sigma_{before} = \sigma_{after} \Rightarrow 1 = 0.625\theta'_f$ $\Rightarrow \theta'_f = 1.6$ rad/s.

b) $KE_i = \frac{1}{2}I_1(\theta'_i)^2 = \frac{1}{2}(0.5)(2)^2 = 1$ J. $KE_f = \frac{1}{2}(I_1 + I_2)(\theta'_f)^2 = \frac{1}{2}(0.5 + 0.125)(1.6)^2 = 0.8$ J .

c) Heat energy = $|KE_f - KE_i| = |0.8 - 1| = 0.2$ J.

6.

a) $\sigma_o = I\theta' = (0.45)(10) = 4.5$ kg.m^2/s.

b) The external forces acting on the system are: $M\vec{g}$: weight of the system (karim, stool), $\vec{R}$: the reaction at the ground $m\vec{g}$: weight of the spinning bicycle. $\mathcal{M}_{M\vec{g}} = \mathcal{M}_{m\vec{g}} = 0$ these force are parallel to (Δ), $\mathcal{M}_{\vec{R}} = 0$; $\vec{R}$ passes through (Δ),

$\Rightarrow \sum\mathcal{M} = 0$ but $\sum\mathcal{M} = \frac{d\sigma}{dt} = 0 \Rightarrow \sigma$ is conserved; σ = constant.

c) $\sigma_{before} = \sigma_{after} \Rightarrow \sigma_o + \sigma_{o\,wheel} = \sigma_{karim\,stool} + \sigma_{wheel} = I\,\theta'_f + I_w\theta'_w \Rightarrow 0 + 4.5 = 3\theta'_f + 0.45(-10)$.

$\Rightarrow \theta'_f = 3$ rad/s $> 0 \Rightarrow$ The system (Karim, stool) rotates in the positive sense.

d) System: (Karim, stool, wheel). $\Delta KE\text{ (system)} = \sum W_{\vec{F}_{ext}} + \sum W_{\vec{F}_{int}}$.

But $\sum W_{\vec{F}_{ext}} = 0$ since the moment of each external force is zero $\Rightarrow \Delta KE\text{ (system)} = \sum W_{\vec{F}_{int}} = W_{\vec{F}}$.

$= \frac{1}{2}(0.45)(10)^2 + \frac{1}{2}(3)(0) = 22.5$ J and $= \frac{1}{2}(0.45)(10)^2 + \frac{1}{2}(3)(3)^2 = 36$ J

$\Rightarrow W_{\vec{F}} = KE_f - KE_i = 36 - 22.5 = \underline{13.5\ J}$.

7.

a) The external force is the gravitational force (weight) exerted by the Sun.
The line of action of the gravitational force meets the axis of rotation which passes through the center of the Sun $\Rightarrow$ the moment of this force is zero $\Rightarrow \sum\mathcal{M} = 0$, but $\sum\mathcal{M} = \frac{d\sigma}{dt} = 0 \Rightarrow \sigma$ is conserved; σ = constant.

b) σ = constant $\Rightarrow \sigma_i = \sigma_f \Rightarrow I_1\theta_1' = I_2\theta_2' \Rightarrow mR_1^2(\frac{V_1}{R_1}) = mR_2^2(\frac{V_2}{R_2}) \Rightarrow R_1V_1 = R_2V_2$ but $R_1 \neq R_2 \Rightarrow V_1 \neq V_2$.

c)

i. $\sigma_1 = I_1\,\theta' = \frac{mR_1^2V_1}{R_1} = m\,R_1\,V_1 \Rightarrow \sigma_1 = (6\times 10^{24})(147\times 10^9)(\frac{30.29}{3.6}) = \underline{7.42\times 10^{36}\ kg.m^2/s.}$

ii. $\sigma_1 = \sigma_2 \Rightarrow 7.42\times 10^{36} = m\,R_2V_2 \Rightarrow 7.42\times 10^{36} = (6\times 10^{24})(152\times 10^9)V_2 \Rightarrow V_2 = 8.136\ m/s \cong \underline{29.29\ km/h}$.

8.

a) $P = mV = (30(20) = \underline{6\ kg.m/s.}$

b) $\sigma_o = (6)(1)\sin 90^\circ = \underline{6\ kg.m^2/s.}$

c) System: (particle , rod). During collision, the rod can be considered vertical.
External forces: The weight of the system and the reaction at the axis of rotation. All these forces are passing through (Δ) $\Rightarrow$ they have zero moments $\Rightarrow \sum\mathcal{M} = 0$ but $\sum\mathcal{M} = \frac{d\sigma}{dt} = 0 \Rightarrow \sigma$ = constant $\Rightarrow \sigma_{before} = \sigma_{after} \Rightarrow \sigma_o = \sigma_{particle} + \sigma_{rod} = (I_{particle} + I_{rod})\theta'$.
$\{I_{particle} = m(OA)^2 = 3(1)^2 = 3\ kg.m^2$ and $I_{rod} = \frac{Ml^2}{12} = \frac{1(4)}{12} = \frac{1}{3}\ kg.m^2\} \Rightarrow \sigma_o = 6 = (3 + \frac{1}{3})\theta' \Rightarrow \theta' = \underline{1.8\ rad/s.}$

d) $KE_{before} = \frac{1}{2}mV^2 = \frac{1}{2}(3)(2)^2 = 6\ J$. $KE_{after} = \frac{1}{2}(I_{particle} + I_{rod})(\theta')^2 = \frac{1}{2}(3 + \frac{1}{3})(1.8)^2 = 5.4\ J$.
$kE_{after} < KE_{before} \Rightarrow$ Non-elastic collision.

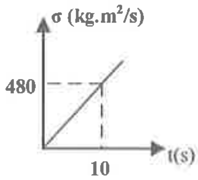
9.

First:

a) Uniformly accelerated rotational $\Rightarrow \theta''$ = constant $\Rightarrow \theta_2' = \theta_2''t + \theta_o' \Rightarrow 0.5 = \theta_2''(10) + 0 \Rightarrow \theta_2'' = 0.05\ rad/s^2 \Rightarrow \theta_2' = \underline{0.05t}$ S.I

b) System (D, Fadi). External forces: $m_1\vec{g}, \vec{R}, m_2\vec{g}$. $\sum\mathcal{M} = \mathcal{M}_{\vec{R}} + \mathcal{M}_{m_1\vec{g}} + \mathcal{M}_{m_2\vec{g}} = \frac{d\sigma}{dt} = \mathcal{M}_{\vec{R}} = \mathcal{M}_{m_1\vec{g}} = \mathcal{M}_{m_2\vec{g}} = 0$
(weight of fadi and disk are parallel to (Δ), $\vec{R}$ is passing through (Δ) $\Rightarrow \sum\mathcal{M} = 0 \Rightarrow \sigma$ = constant

c) $I_1 = \frac{m_1R^2}{2} = \frac{200\times 16}{2} = 1600\ kg.m^2$. $I_2 = m_2R^2 = 60\times 16 = 960\ kg.m^2$
$\sigma_{before} = \sigma_{after} \Rightarrow 0 = \sigma_1 + \sigma_2 \Rightarrow \sigma_1 = -\sigma_2 = -I_2\theta_2' = -[(960)(0.05t)] = \underline{-48t\ S.I}$



d) For disk: $\theta_1' = \frac{\sigma_1}{I_1} = \frac{-48t}{1600} = -0.03t$ S.I $\Rightarrow \theta_1 = \int\theta_1'\,dt = \frac{-0.03}{2}t^2 + \theta_o \Rightarrow \Delta\theta_1 = -0.015t^2$
one rotation: then $\Delta\theta = -2\pi$ (rad) $\Rightarrow -2\pi = -0.015t \Rightarrow t = 20.46$ s , then during this time:
$\theta_2 = \frac{0.05}{2}t^2 + \theta_o \Rightarrow \Delta\theta_2 = 0.025t^2 = 0.025(20.46)^2 = 10.465$ rad $\Rightarrow N_2 = \frac{\Delta\theta_2}{2\pi} = \underline{1.67\ revolutions.}$

e) $\sigma_2 = 48t$.

Second: $I_3 = m_3R^2 = 10\times 4^2 = 160$ and $\theta_3' = \frac{V}{R} = \frac{2}{4} = 0.5$ rad/s.
$\sigma_{before} = \sigma_{after} \Rightarrow 0 = (I_1 + I_2)\,\theta' + I_3\theta_3' = (1600 + 960)\,\theta' + (160\times 0.5) \Rightarrow \underline{\theta' = -0.03125\ rad/s.}$

10.

A.

d. $\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{N} + \vec{T}_o = m\vec{a}_o$. Uniform motion $\Rightarrow a_o = a_{n_0} = \frac{V_0^2}{\ell_o} = \frac{\ell_0^2(\theta_o')^2}{\ell_o} = \ell_o(\theta_o')^2 = 0.5\times 0.4^2 = 0.08\ m/s^2$.
the radial direction oriented positively towards the center: $0 + 0 + T_o = m\,a_o = 2\times 0.08 = \underline{0.16\ N}$.
$= 0$ ($m\vec{g}$ and $\vec{N}$ are parallel to (Δ). $\mathcal{M}_{\vec{T}} = 0$ ($\vec{T}$ passing through Δ) $\Rightarrow \sum\mathcal{M} = 0 \Rightarrow \frac{d\sigma}{dt} = 0 \Rightarrow \sigma$ is conserved.
$_0 = I_f\theta_f' \Rightarrow m\,\ell_o^2\,\theta_o' = m\,\ell_f^2\,\theta_f' \Rightarrow (0.5)^2(0.4) = (0.3)^2(\theta_f') \Rightarrow \theta_f' = \frac{10}{9}$ rad/s.
$)^2 + GPE = \frac{1}{2}(2)(0.5)^2(0.4)^2 + 0 = 0.04\ J$. $ME_f = \frac{1}{2}I_f(\theta_f')^2 + GPE = \frac{1}{2}(2)(0.3)^2(\frac{10}{9})^2 + 0 = 0.111\ J$. $\underline{ME_f > ME_i}$
-conservative forces $= W_{\vec{F}} \Rightarrow W_{\vec{F}} = 0.111 - 0.04 = \underline{0.071\ J.}$

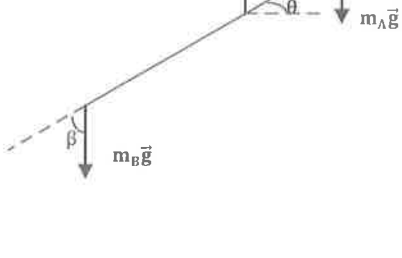
d, string, Earth). No non-conservative forces $\Rightarrow$ ME of the system is conserved.

b) $ME_i = ME_f \Rightarrow \frac{1}{2}I_o(\theta_o')^2 = \frac{1}{2}I_f(\theta_f')^2 \Rightarrow \frac{1}{2}m\ell_o^2(\theta_o')^2 = \frac{1}{2}m\ell^2(\theta')^2 \Rightarrow (0.5)^2(0.4)^2 = (0.3)^2(\theta')^2 \Rightarrow \theta' = 0.666 = \frac{2}{3}$ rad/s.

c) $\sigma_o = I_o\theta_o' = (2)(0.5)^2(0.4) = 0.2\ kg.m^2/s$ and $\sigma_1 = I_1\theta_1' = (2)(0.3)^2(\frac{2}{3}) = 0.12\ kg.m^2/s \Rightarrow \sigma_0 > \sigma_1$.

11.

a) $\mathcal{M}_{m_A\vec{g}} = m_Ag\,(2)\sin\beta = m_Ag(2)\cos\theta = (40)(10)(2)\cos\theta = \underline{800\cos\theta}$ (S.I).
$\mathcal{M}_{m_B\vec{g}} = -\,m_Bg(2.5)\sin\beta = -\,m_Bg\,(2.5)\cos\theta = -\,(30)(10)(2.5)\cos\theta = \underline{-750\cos\theta}$ (S.I).



b) $\sum\mathcal{M} = \frac{d\sigma}{dt}$ since I is constant $\Rightarrow \frac{d\sigma}{dt} = I\,\theta'' \Rightarrow \mathcal{M}_{\vec{R}} + \mathcal{M}_{m_A\vec{g}} + \mathcal{M}_{m_B\vec{g}} = I\,\theta''$
$\Rightarrow 0 + 800\cos\theta - 750\cos\theta = (I_A + I_B)\theta''$.
$\{I_A = m_A(2)^2 = 40(4) = 160\ kg.m^2$ and $I_B = m_B(2.5)^2 = 30(2.5)^2 = 187.5\ kg.m^2\}$
$\Rightarrow 0 + 800\cos\theta - 750\cos\theta = (I_A + I_B)\theta'' \Rightarrow \theta'' = \underline{0.144\cos\theta}$ (S.I).

c) For $\theta = 0 \Rightarrow \theta'' = 0.144\cos(0) = \underline{0.144\ rad/s^2}$

12.

a) $I = \frac{mR^2}{2} = \frac{4\times 0.5^2}{2} = \underline{0.5\ kg.m^2}$. $\sigma = I\,\theta'$. At $t_o = 0$, $\sigma_o = \underline{0}$ since $\theta'_0 = 0$. At t = 5 s, $\sigma = 0.5\,(20) = \underline{10\ kg.m^2/s}$.

b) $\sum\mathcal{M}_{ext} = \frac{d\sigma}{dt} \Rightarrow \sigma = \mathcal{M}_{ext}\,t + \sigma_o = \mathcal{M}_{ext}\,t$. At t = 5 s , $\sigma = 10 \Rightarrow \mathcal{M}_{ext} = \frac{10}{5} = \underline{2\ N.m}$.

c) $\mathcal{M}_{ext} = \mathcal{M}_{couple} + \mathcal{M}_{\vec{fr}} + \mathcal{M}_{\vec{W}} + \mathcal{M}_{\vec{N}} \Rightarrow 2 = 7 + \mathcal{M}_{\vec{fr}} + 0 + 0 \Rightarrow \mathcal{M}_{\vec{fr}} = -5$ N.m.

d) Disk is rigid $\Rightarrow \Delta KE = \sum W_{\vec{F}_{ext}} \Rightarrow \frac{1}{2} I\,\theta_f'^2 - \frac{1}{2} I\,\theta_i'^2 = \mathcal{M}_{couple}(\Delta\theta) + \mathcal{M}_{fr}(\Delta\theta) \Rightarrow \frac{1}{2}(0.5)(20)^2 - 0 = (7-5)(\Delta\theta) \Rightarrow \underline{\Delta\theta = 50\ rad}$.

$\Delta\theta = 2\pi\,n \Rightarrow n = \frac{50}{2\times 3.14} \cong \underline{8\ cycles}$.

e) $E_{loss} = |W_{fr}| = |\,\mathcal{M}_{fr}(\Delta\theta)| = 5\times 50 = \underline{250\ J}$.

f) $\sum\mathcal{M}_{ext} = \mathcal{M}_{\vec{W}_{sys}} + \mathcal{M}_{\vec{N}} = 0$ (parallel to the axis of rotation), then the angular momentum is conserved $\Rightarrow \sigma_i = \sigma_f$

$\Rightarrow I\,\theta_1' = I_2\,\theta_2' \Rightarrow (0.5)(20) = I_2(15) \Rightarrow I_2 = 0.666$ but $I_2 = I + m_1 d^2 \Rightarrow d = \sqrt{\frac{0.666-0.5}{1}} = 0.4\ m = \underline{40\ cm}$.

13.

a)

i. System: rod. External forces: the weight $m\vec{g}$, the reaction $\vec{R}$ of the axis (Δ), the force $\vec{F}$, and the friction $\vec{f}$.
Moments relative to (Δ): $\mathcal{M}_{m\vec{g}} = 0$ since this force is parallel to (Δ), $\mathcal{M}_{\vec{R}} = 0$ since $\vec{R}$ meets (Δ),
$\mathcal{M}_{\vec{F}} = (OA)(F)\sin 90 = (0.5)(2.5)(1) = 1.25$ N.m . $\mathcal{M}_{\vec{f}} = -1$ N.m. $\sum\mathcal{M}_{ext} = \frac{d\sigma}{dt} \Rightarrow 0 + 0 + 1.25 - 1 = \frac{d\sigma}{dt}$
$\Rightarrow \frac{d\sigma}{dt} = 0.25 \Rightarrow \sigma = \int 0.25dt = 0.25t + \sigma_o$ but $\sigma_o = 0$ since the rod starts from rest $\Rightarrow \underline{\sigma = 0.25t}$ S.I

ii. $\sigma_{10} = 0.25\times 10 = 2.5$ kg.m²/s, but $\sigma = I\,\theta' \Rightarrow 2.5 = I\,(2.5) \Rightarrow I = 1$ kg.m².

b) $\sum\mathcal{M}_{ext} = \frac{d\sigma}{dt} \Rightarrow -1 = \frac{d\sigma}{dt} \Rightarrow \sigma = \int -dt = -t + \sigma_o \Rightarrow \sigma = -t + 2.5$ S.I. The rod stops: $\theta' = 0 \Rightarrow \sigma = 0 = -t + 2.5 \Rightarrow \underline{\Delta t = 2.5\ s}$.

c)

i. System (Rod-ring). The weight of the ring is added to the external forces acting. $\sum\mathcal{M} = \frac{d\sigma}{dt} = I\,\theta''$
$\Rightarrow \mathcal{M}_{m\vec{g}} + \mathcal{M}_{\vec{R}} + \mathcal{M}_{\vec{fr}} + \mathcal{M}_{\vec{F}} + \mathcal{M}_{m_s\vec{g}} = I\,\theta'' \Rightarrow 0 + 0 - 1 + (2.5)(OB) + 0 = [m(OB)^2 + 1]\,\theta'' \Rightarrow \theta'' = \frac{2.5(OB)-1}{1+(OB)^2}$.

ii. The rod rotates if $\theta'' > 0 \Rightarrow \frac{2.5(OB)-1}{1+(OB)^2} > 0 \Rightarrow 2.5(OB) - 1 > 0 \Rightarrow OB > 0.4m \Rightarrow x > 0.4\ m \Rightarrow x_{min} = \underline{0.4\ m+}$.

iii.

1. $\theta'' = \frac{2.5(0.5)-1}{1-0.5^2} = \frac{1}{3}$ rad/s². Since $\theta'' = const \Rightarrow \theta' = \theta''t + \theta'_o = (\frac{1}{3})(10) + 0 = \frac{10}{3}$ rad/s.

2. $\theta'^2 - \theta_o'^2 = 2\theta''(\Delta\theta) \Rightarrow (\frac{10}{3})^2 - 0 = 2(\frac{1}{3})(\Delta\theta) \Rightarrow (\Delta\theta) = \frac{50}{3}$ rad. $W_{\vec{F}} = \mathcal{M}_{\vec{F}}\,\Delta\theta = (F\,x)(\Delta\theta) = 2.5(0.5)(\frac{50}{3}) = \underline{20.83\ J}$.

14.

a) System: water bucket. External forces: $m\vec{g}$ weight; $\vec{T}$ tension. $\sum\vec{F}_{ext} = m\,\vec{a}_G \Rightarrow m\vec{g} + \vec{T} = m\vec{a}_G$
project along direction of motion: $mg - T = ma_G \Rightarrow T = mg - ma_G = \underline{30 - 3a_G}$ (S.I).

b) $\mathcal{M}_{m\vec{g}} + \mathcal{M}_{\vec{R}} + \mathcal{M}_{\vec{f}} + \mathcal{M}_{\vec{T}} = \frac{d\sigma}{dt} \Rightarrow 0 + 0 + \mathcal{M}_{\vec{f}} + RT = \frac{d\sigma}{dt} \Rightarrow \sigma = \int(\mathcal{M}_{\vec{f}} + 0.6T)\,dt = (\mathcal{M}_{\vec{f}} + 0.6T)t + \sigma_o$
$\Rightarrow \underline{\sigma = (\mathcal{M}_{\vec{f}} + 0.6T)t}$ S.I.

c) $\sigma = I_1\,\theta' \Rightarrow \theta' = \frac{\sigma}{I_1} = \frac{(\mathcal{M}_{\vec{f}} + 0.6T)t}{I_1}$ $\{ I_1 = \frac{1}{2}MR^2 = \frac{1}{2}(5)(0.6)^2 = \frac{9}{10}$ kg.m²$\}$
$\Rightarrow \theta' = \frac{10}{9}(\mathcal{M}_{\vec{f}} + 0.6T)t \Rightarrow \theta'' = \frac{10}{9}(\mathcal{M}_{\vec{f}} + 0.6T) = \frac{10}{9}(\mathcal{M}_{\vec{f}} + 0.6(30 - 3(0.6\,\theta''))) \Rightarrow \theta'' = \frac{55}{99}\mathcal{M}_{\vec{f}} + \frac{100}{11}$ (S.I).

d) The string is inextensible $\Rightarrow (\Delta S)_{periphery} = (\Delta x)_{bucket} \Rightarrow V_{periphary} = V_{bucket}$. $R\theta' = V_{bucket} \Rightarrow \theta' = \frac{4.8}{0.6} = 8$ rad/s
$\theta'' = const$ (U.V.R.M) $\Rightarrow \theta' = \theta''t + \theta'_o \Rightarrow 8 = \theta''(2) + 0 \Rightarrow \theta'' = 4$ rad/s² substitute in the expression of θ'':
$4 = \frac{55}{99}\mathcal{M}_{\vec{f}} + \frac{100}{11} \Rightarrow \mathcal{M}_{\vec{f}} = \underline{-9.16\ N.m}$.

Chapter 4 : Mechanical Oscillations (1)

1.

a) $d_{total} = (5)(4) = \underline{20\ cm}$.

b)

i. $v = x' = \omega_o X_m \cos(\omega_o t) \Rightarrow V_m = \omega_o X_m$

ii. v is maximum when $\cos(\omega_o t) = \pm 1 \Rightarrow \sin(\omega_o t) = 0$. From the expression of x: $x = X_m \sin(0) = \underline{0}$.

c) $a = v' = -\omega_o^2 X_m \sin(\omega_o t)$, $a_{max} = \omega_o^2 X_m$ { But $\omega_o = 2\pi f = 2\pi(3) = 6\pi$ rad/s} $\Rightarrow a_{max} = (6\pi)^2(0.05) = \underline{17.75\ m/s^2}$.

d) a is maximum when $\sin(\omega_o t) = \pm 1 \Rightarrow \underline{x = \pm X_m}$.

2.

a) $X_m = 8$ cm

b) Since the amplitude X_m of oscillations is constant with time.

c) $T_o = 4\ s \Rightarrow \omega_o = \frac{2\pi}{T_o} = \frac{2\pi}{4} = \frac{\pi}{2}$ rad/s. $\omega_o = \sqrt{\frac{k}{m}} \Rightarrow (\frac{\pi}{2})^2 = \frac{k}{0.4} \Rightarrow \underline{k \approx 1\ N/m.}$

d) At $t_o = 0.5$; $v < 0$ since algebraic value of abscissa is decreasing. At $t_1 = 1.5$; $v < 0$ since algebraic value of abscissa is decreasing. At $t_o = 2.5$; $v > 0$ since algebraic value of abscissa is increasing.

e) $x_o = X_m = X_m \sin(\varphi) \Rightarrow \sin(\varphi) = 1 \Rightarrow \varphi = \frac{\pi}{2}$ rad.

3. $x_o = -0.05\ m = A\cos\varphi \Rightarrow \cos\varphi = \frac{-0.05}{A}$...eq(1) $v = x' = -\omega_o A \sin(\omega_o t + \varphi)$; $v_o = -0.26 = -3(A)\sin\varphi \Rightarrow \sin\varphi = \frac{13}{150\ A}$...eq(2)
$A < 0 \Rightarrow \cos\varphi > 0$ and $\sin\varphi < 0 \Rightarrow \varphi$ is in the 4th quadrant. Divide eq(2) by eq(1) $\Rightarrow \tan\varphi = -1.7333 \Rightarrow \varphi = -60° = -\frac{\pi}{3}$ rad
$\Rightarrow A = -\frac{0.05}{\cos(-\frac{\pi}{3})} = -0.1m = \underline{-10\ cm}$. $A = -X_m \Rightarrow X_m = \underline{10\ cm}$.

4.

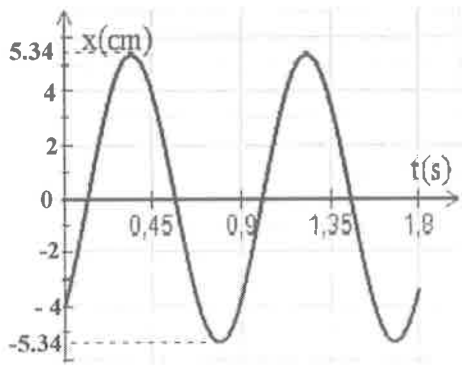
a) $\omega_o = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{0.48}} = 5\sqrt{2}$ rad/s. $T_o = \frac{2\pi}{\omega_o} = \frac{2\pi}{5\sqrt{2}} = \underline{0.89\ s}$.

b) $x_o = -0.04 = X_m\cos\varphi \Rightarrow \cos\varphi = \frac{-0.04}{X_m}$ ... eq(1). $v = x' = -\omega_o X_m \sin(\omega_o t + \varphi)$. $v_o = 0.25 = -5\sqrt{2} X_m \sin\varphi$
$\Rightarrow \sin\varphi = -\frac{\sqrt{2}}{40\ X_m}$ ...eq(2). Divide eq(2) by eq(1) $\Rightarrow \tan\varphi = 0.88388$, but $\cos\varphi < 0$ and $\sin\varphi < 0 \Rightarrow \varphi$ is in the 3rd quadrant
$\Rightarrow \varphi = -138.53° = \underline{-2.42\ rad}$. From eq(2): $\sin(-2.42) = -\frac{\sqrt{2}}{40X_m} \Rightarrow X_m = 0.0534\ m = \underline{5.34\ cm}$.

c) $ME_o = ME_{\text{extreme position}} \Rightarrow \frac{1}{2}mv_o^2 + \frac{1}{2}kx_o^2 = \frac{1}{2}k X_m^2 \Rightarrow (0.48)(0.25)^2 + 24((0.04)^2 = 24X_m^2 \Rightarrow X_m = 0.0534\ m = \underline{5.34\ cm}$.

d) $x = 0.0534\cos(5\sqrt{2}t - 2.42) \Rightarrow 0 = 0.0534\cos(5\sqrt{2}t - 2.42)$
$\Rightarrow 5\sqrt{2}t - 2.42 = \frac{\pi}{2}$ OR $5\sqrt{2}t - 2.42 = -\frac{\pi}{2}$
$\Rightarrow t_2 = 0.5643$ s OR $t_1 = 0.1202$ s
$t_1 = \underline{0.1202\ s}$ is the time needed by the block to pass in O for the 1st time
$t_2 = \underline{0.5643\ s} = t_1 + \frac{T_0}{2}$ is the time needed by the block to pass through O for the 2nd time. $t_3 = t_1 + T_o = 0.1202 + 0.89 = \underline{1.01\ s}$ G passes in O for the third time.

e) Figure



5.

a) $ME = \frac{1}{2}k\,x^2 + \frac{1}{2}m\,v^2$. No non-conservative forces $\Rightarrow$ ME is conserved $\Rightarrow$
$\frac{dM.E}{dt} = 0 \Rightarrow k\,x\,x' + m\,v\,v' = 0$ { $v = x'$ and $v' = a$} $\Rightarrow v\,[k\,x + m\,x''] = 0$
$v = 0$ is rejected $\Rightarrow k\,x + m\,x'' = 0 \Rightarrow x'' + \frac{k}{m}x = 0$.

b) $\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{N} + \vec{T} = m\vec{a} \Rightarrow 0 + 0 - k\,x = m\,x'' \Rightarrow x'' + \frac{k}{m}x = 0$. Differential equation has the form $x'' + \omega_o^2\,x = 0$
$\Rightarrow$ simple harmonic motion

c) $v = x' = -\omega_o X_m \sin(\omega_o t + \varphi)$. $a = v' = -\omega_o^2 X_m \cos(\omega_o t + \varphi) = -\omega_o^2 x$
Substitute in the differential equation: $-\omega_o^2 x + \frac{k}{m}x = 0$ but $\omega_o = \sqrt{\frac{k}{m}} \Rightarrow -\omega_o^2\,x + \omega_o^2\,x = 0$. So it is a solution.

d) At $t_o = 0$; $x_o = 0 = X_m\cos\varphi \Rightarrow \cos\varphi = 0 \Rightarrow \varphi = \frac{\pi}{2}$ rad OR $\varphi = -\frac{\pi}{2}$ rad.
$v = x' = -\omega_o X_m \sin(\omega_o t + \varphi)$. $v_o = -\omega_o X_m \sin\varphi$, but $v_o < 0 \Rightarrow \sin\varphi > 0 \Rightarrow \varphi = \frac{\pi}{2}$ rad.

e)

i. Figure (b), since at $t_o = 0$; $x_o = 0$ but the block is moving in the negative direction

ii. $X_m = \underline{3\ cm}$; $T_o = 2\ s \Rightarrow \omega_o = \frac{2\pi}{T_o} = \pi$ rad/s. **iii.** $V_m = \omega_o X_m = \pi(0.03) = 0.0942\ m/s = \underline{9.42\ cm/s}$.

iii. iv . $v_{av} = \frac{d}{\Delta t} = \frac{3+3}{2.5-1.5} = \underline{6\ cm/s.}$

f) $ME_o = \frac{1}{2} mV_m^2 = \frac{1}{2}(0.5)(0.0942)^2 = \underline{2.22 \times 10^{-3} J}$. $ME_{\substack{extreme\\position}} = \frac{1}{2} kX_m^2 \Rightarrow 2.22 \times 10^{-3} = \frac{1}{2}k\,(0.03)^2 \Rightarrow \underline{k = 4.9 \text{ N/m}}$.

6.

A.

a) $ME = \frac{1}{2}mv^2 + \frac{1}{2}k\,x^2 + \frac{1}{2}k\,x^2 = \frac{1}{2}mv^2 + k\,x^2$.

b) No non-conservative forces $\Rightarrow$ ME is conserved $\Rightarrow \frac{dM.E}{dt} = 0 = m\,v\,v' + 2k\,x\,x' \Rightarrow v[m\,x'' + 2\,k\,x] = 0$

v = 0 is rejected $\Rightarrow x'' + \frac{2k}{m}x = 0$, the equation has the form $x'' + \omega_o^2\ x = 0 \Rightarrow$ simple harmonic motion.

c) $\omega_o^2 = \frac{2k}{m} \Rightarrow \omega_o = \sqrt{\frac{2k}{m}}$.

B.

a) $ME = \frac{1}{2}m\ v^2 + \frac{1}{2}k(\Delta l - x)^2 + \frac{1}{2}k(\Delta l + x)^2$.

b) No non-conservative forces (no friction) $\Rightarrow$ ME is conserved $\Rightarrow \frac{dME}{dt} = 0$

$0 = m\,v\,v' + k(\Delta l - x)(-x') + k(\Delta \ell + x)(x') = v[m\,v' - k\,\Delta\ell + k\,x + k\,\Delta\ell + k\,x]$, v = 0 is rejected $\Rightarrow x'' + \frac{2k}{m}x = 0$

c) $\omega_o^2 = \frac{2k}{m} \Rightarrow \omega_o = \sqrt{\frac{2\,k}{m}}$.

C.

They are equal. ω_o is a character of the horizontal elastic pendulum in the case of free un-damped oscillations, and it depends only on the stiffness of the spring and the mass of the block $\Rightarrow$ it is independent of the initial conditions.

7.

a) $ME = \frac{1}{2}kX_m^2 + \frac{1}{2}kX_m^2 = kX_m^2 = 25(0.08)^2 = \underline{0.16 J}$.

b) ME is conserved $\Rightarrow \frac{1}{2}mV_m^2 = 0.16 J \Rightarrow \frac{1}{2}(2)V_m^2 = 0.16 \Rightarrow V_m = \underline{0.4 \text{ m/s}}$.

c)

i. $ME_{\substack{equilibrium\\position}} = KE + GPE = \frac{1}{2}mV_m^2 + 0 = \underline{0.16 J}$.

ii. $0.16 = \frac{1}{2}kX_m'^2 \Rightarrow 0.16 = \frac{1}{2}(25)\,X_m'^2 \Rightarrow X_m' = 0.113 \text{ m} = \underline{11.3 \text{ cm}}$.

iii. $(\omega_o' = \sqrt{\frac{k}{m}}) < (\omega_o = \sqrt{\frac{2k}{m}}\ )$ so it decreases.

8.

A.

a) $v = x' = \omega_o\,X_m \cos(\omega_o t + \varphi)$. $\quad a = v' = -\ \omega_o^2\ X_m \sin(\omega_o t + \varphi)$.

b) $\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{N} + \vec{T} = m\vec{a} \Rightarrow \vec{T} = m\vec{a} = m(-\omega_o^2\ X_m \sin(\omega_o t + \varphi))\vec{i} = -\,k\,X_m \sin(\omega_o t + \varphi)\,\vec{i} = \frac{-k\ V_m}{\omega_o}\sin(\omega_o t + \varphi)\,\vec{i}$

But $\omega_o = \frac{2\pi}{T_o} \Rightarrow \vec{T} = \frac{-K\,V_m\,T_o}{2\,\pi}\sin(\omega_o t + \varphi)\,\vec{i}$.

B.

a) i. $x_o = 0$ cm; $v_o = 0.2\sqrt{10}$ m/s. **ii.** $T_o = 2$ s.

iii. Speed of S is maximum at the equilibrium position; when x = 0 then $V_m = 0.2\sqrt{10}$ m/s. $v_o = V_m = V_m\cos\varphi \Rightarrow \cos\varphi = 1 \Rightarrow \underline{\varphi = 0}$.

iv. When v = 0, $x = X_m \Rightarrow X_m = \underline{20 \text{ cm}}$. $\omega_o = \frac{V_m}{X_m} = \frac{0.2\sqrt{10}}{0.2} = \sqrt{10}$ rad/s; $w_o^2 = \frac{k}{m} \Rightarrow k = w_o^2\ m = \underline{8 \text{ N/m}}$.

b) $\vec{T} = \frac{-8(0.2\sqrt{10})(2)}{2\pi}\sin(\frac{2\pi}{2}t + 0)\vec{i} = -1.6\sin(\pi t)\vec{i}$ (S.I).

9.

a)

i. $v = x' = -\ \omega_o X_m \sin(\omega_o t + \varphi)$.

ii. $x = X_m\cos(\omega_o t + \varphi) \Rightarrow \cos(\omega_o t + \varphi) = \frac{x}{X_m}$…eq(1). From the expression of v: $\sin(\omega_o t + \varphi) = \frac{v}{-\omega_o\,Xm}$…eq(2).

Square and add$\Rightarrow 1 = \frac{x^2}{X_m^2} + \frac{v^2}{\omega_o^2\ X_m^2} \Rightarrow X_m^2 = x^2 + \frac{v^2}{\omega_o^2} \Rightarrow \omega_o^2\ X_m^2 = \omega_o^2\ x^2 + v^2 \Rightarrow v^2 = \omega_o^2\ [X_m^2 - x^2]$.

b) $ME_t = ME_{\substack{extreme\\position}} \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}k\,X_m^2 \Rightarrow v^2 + \frac{k}{m}x^2 = \frac{k}{m}X_m^2$ but $\omega_o^2 = \frac{k}{m} \Rightarrow v^2 + \omega_o^2\ x^2 = \omega_o^2\ X_m^2$

$\Rightarrow v^2 = \omega_o^2\ [X_m^2 - x^2]$.

c)

i. First evidence: when $x = \pm X_m$; v = 0. second evidence: when x = 0; v = max.

d) $V_m = \omega_o X_m \Rightarrow \omega_o = \frac{V_m}{X_m} = \frac{1.6}{0.16} = \underline{10 \text{ rad/s}}$.

e) $\omega_o = \sqrt{\frac{k}{m}} \Rightarrow 10 = \sqrt{\frac{k}{1}} \Rightarrow \underline{k = 100 \text{ N/m}}$.

10.

A.

a) $ME = \frac{1}{2}m\,v^2 + \frac{1}{2}k\,x^2$.

b) Neglect friction ⇒ ME is conserved. $\frac{dME}{dt} = 0 = m\,v\,v' + 2k\,x\,x$ but $x' = v$ ⇒ $v[m\,v' + k\,x] = 0$, $v = 0$ is rejected

⇒ $m\,v' + k\,x = 0$. Derive w.r.t. time $m\,v'' + k\,v = 0 \Rightarrow v'' + \frac{k}{m}v = 0$.

c) $v' = -V_m\omega_o \sin(\omega_o t + \varphi)$; $v'' = -V_m\,\omega_o^2 \cos(\omega_o t + \varphi) = -\omega_o^2\,v$. Substitute in the differential equation ⇒ $-\omega_o^2\,v + \frac{k}{m}v = 0$

⇒ $v\,[\frac{k}{m} - \omega_o^2\,] = 0 \Rightarrow \frac{k}{m} - \omega_o^2 = 0 \Rightarrow \omega_o^2 = \frac{k}{m} \Rightarrow \omega_o = \sqrt{\frac{k}{m}}$.

d) $x' = v = \omega_o X_m \cos(\omega_o t + \varphi)$ but $v = V_m \cos(\omega_o t + \varphi) \Rightarrow \omega_o X_m = V_m$.....(*)

e) $\frac{1}{2}k\,X_m^2 = \frac{1}{2}k\frac{V_m^2}{\omega_o^2}$, but $\omega_o^2 = \frac{k}{m} \Rightarrow \frac{1}{2}k\,X_m^2 = \frac{1}{2}k\frac{V_m^2}{k/m} = \frac{1}{2}mV_m^2$.

B.

a) free un-damped mechanical oscillations.

b) $v_o = -0.32$ m/s; $V_m = 0.64$ m/s (maximum speed). $T_o = 1.25$ s ⇒ $\omega_o = \frac{2\pi}{T_o} = 5.024$ rad/s.

c) $X_m = \frac{V_m}{\omega_o}$ (from *): $X_m = \frac{0.64}{5.024} = 0.1274$ m = <u>12.74 cm</u>. $v_o = -0.32 = 0.64\cos\varphi \Rightarrow \cos\varphi = -0.5 \Rightarrow \varphi = \pm\frac{\pi}{3}$ rad.

$\cos\varphi < 0$ and $x_0 = X_m \sin\varphi \Rightarrow \sin\varphi > 0 \Rightarrow \varphi$ in the 2nd quadrant ⇒ $\varphi = \frac{\pi}{3}$ rad ⇒ $x_o = (0.1274)\sin(\frac{\pi}{3}) = 0.11033$ m = <u>11.033 cm</u>

d) $\omega_o^2 = \frac{k}{m} \Rightarrow m = \frac{10}{5.024^2} = 0.396$ kg.

11.

A.

a) i. $P_o = -0.4$ kg.m/s. During collision, linear momentum is conserved: $\vec{P}_{before} = \vec{P}_{after} = \vec{P}_o$

$\vec{P}_o = m_1\vec{v}_1 \Rightarrow P_o = m_1 v_1 \Rightarrow m_1 = \frac{-0.4}{-4} =$ <u>0.1 kg</u>. $\vec{P}_o = (m_1 + m_2)\,\vec{V}_m \Rightarrow |P_o| = (m_1 + m_2)V_m \Rightarrow V_m = \frac{|P_o|}{(m_1 + m_2)} =$ <u>0.8 m/s</u>.

ii. At t = 0.2 s, (S) moves in the positive direction (horizontal to the right) ⇒ v > 0.

iii. According to the graph the amplitude of the linear momentum is constant but $\vec{P} = m\vec{v}$, so the amplitude of the velocity is constant ⇒ ME is conserved.

b) $ME_o = \frac{1}{2}(m_1 + m_2)V_m^2 = \frac{1}{2}(0.5)(0.8)^2 \Rightarrow ME_o =$ <u>0.16 J</u>. $M.E_o = ME_{x=X_m} \Rightarrow 0.16 = \frac{1}{2}kX_m^2 \Rightarrow X_m = \sqrt{\frac{2(0.16)}{78}} = 0.064$ m = <u>6.4 cm</u>.

c) ME = KE + EPE ⇒ KE = ME – EPE = ME - $\frac{1}{2}k\,x^2 = 0.16 - [\frac{1}{2}(78)(0.03)^2] = 0.125$ J. $KE = \frac{1}{2}m\,v^2 \Rightarrow v = \sqrt{\frac{2KE}{m}} =$ <u>0.71 m/s</u>.

B.

a) i. $P_{max} = P_o = 0.4$ kgm/s. $T_o = 0.5$ s ⇒ $\omega_o = \frac{2\pi}{T_o} = 4\pi$ rad/s.

$P = 0.4\cos[(4\pi)t + \varphi]$ at $t_o = 0$, $P_o = -0.4$ kg.m/s ⇒ $-0.4 = 0.4\cos(0 + \varphi) \Rightarrow \cos\varphi = -1 \Rightarrow$ <u>$\varphi = \pi$ rad</u>.

ii. $P = 0.4\cos[(4\pi)t + \pi] \Rightarrow \frac{dP}{dt} = -(0.4)(4\pi)\sin(4\pi t + \pi) = -5.03\sin(4\pi t + \pi)$ (S.I).

b) i. $v = \frac{dx}{dt} = (0.064)(4\pi)\cos(4\pi t + \pi) = 0.256\pi\cos(4\pi t + \pi)$. $a = \frac{dv}{dt} = -4\pi^2(0.256)\sin(4\pi t + \pi) = -10.11\sin(4\pi t + \pi)$ (S.I).

ii. $\sum\vec{F}_{ext} = m\,\vec{a} = (0.5)(-10.11)\sin(4\pi t + \pi)\,\vec{i} = -5.05\sin(4\pi t + \pi)\,\vec{i}$ (S.I)

C. The answers are equal $\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt}$. Newton's 2nd law is verified.

12.

A.

a) When x = OA = 10 cm. M is released from rest ⇒ X_m = <u>10 cm</u>. $T_o = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{10}} =$ <u>2 s</u>.

b) $ME = \frac{1}{2}k\,X_m^2 = \frac{1}{2}(10)(0.1)^2 =$ <u>0.05 J</u>.

c) No non-conservative forces ⇒ ME is conserved ⇒ $ME_o = \frac{1}{2}mV_o^2 \Rightarrow 0.05 = \frac{1}{2}(1)\,V_o^2 \Rightarrow V_o =$ <u>0.316 m/s</u>. When $x = X_m$; $v_B = 0$.

B.

a) M reaches B at $t_M = \frac{T_o}{2} =$ <u>1 s</u>. N: $t_N = \frac{CB}{v_N} = \frac{0.75}{0.75} =$ <u>1 s</u> ⇒ $t_M = t_N = 1$ s ⇒ M and N meet at B ⇒ collision takes place at B.

b) During collision linear momentum is conserved ⇒ $\vec{P}_{before} = \vec{P}_{after} \Rightarrow m_2\vec{V}_N + \vec{0} = m_1\vec{V}_M' + m_2\vec{V}_N'$ { At B V_M is zero}

$0.5(0.75\,\vec{i}) = 1(0.5)\vec{i} + 0.5\vec{V}_N' \Rightarrow \vec{V}_N' =$ <u>$-0.25\,\vec{i}$</u> (m/s).

c) $ME_B = \frac{1}{2}kx_B^2 + \frac{1}{2}mV_M'^2 = \frac{1}{2}(10)(0.1)^2 + \frac{1}{2}(1)(0.5)^2 = 0.175$ J. At the new extreme position: $ME = \frac{1}{2}k\,X_M'^2$

⇒ $X_M' = \sqrt{\frac{2(0.175)}{10}} =$ <u>0.187 m</u>.

d) T_o <u>remains constant</u> since it depends only on m and k and not on the initial conditions.

13.

a) Slope = $\frac{20}{-0.1}$ = -200 ⇒ x'' = - 200 x.

b) x'' = -200 x has the form $x'' = -\omega_o^2 x \Rightarrow x'' + \omega_o^2 x = 0$.

The form of the differential equation is $x'' + \omega_o^2 x = 0$ ⇒ Simple harmonic motion.

c)

i. Simple harmonic motion ⇒ block is in free un-damped mechanical oscillations ⇒ No friction.

ii. No friction ⇒ ME is conserved.

d) $x'' = -200x \Rightarrow \omega_o^2 = 200 \Rightarrow \omega_o = 10\sqrt{2}$ rad/s. $\omega_o = \sqrt{\frac{k}{m}} \Rightarrow k = 100$ N/m.

e) $X_m = 0.1$ m. $ME_{\text{extreme position}} = ME_o \Rightarrow \frac{1}{2}kX_m^2 = \frac{1}{2}mV_m^2 \Rightarrow \frac{1}{2}(100)(0.1)^2 = \frac{1}{2}(0.5)V_m^2 \Rightarrow V_m = \sqrt{2}$ m/s.

f)

i. $\Delta ME = \frac{1}{2}kX'^2_m - \frac{1}{2}k X_m^2 = \frac{1}{2}(100)[0.8(0.1)]^2 - \frac{1}{2}(100)(0.1)^2 = -0.18$ J.

ii. This system is energy isolated ⇒ $\Delta U = -\Delta ME = 0.18$ J.

14.

A.

a) Applying the principle of the conservation of ME between the equilibrium position and the maximum position we get: $\frac{1}{2}\mu v_1^2 = \mu g h_{max}$ { GPE at equilibrium position is zero and KE at maximum position is zero} ⇒ $v_1^2 = 2gh_{max} = 2g(\ell - \ell\cos\alpha) = 2g\ell(1-\cos\alpha) \Rightarrow v_1 = \sqrt{2g\ell(1-\cos\alpha)} = \sqrt{2(10)(2)(2.25\times 10^{-3})} = 0.3$ m/s.

b) i. System is initially at rest ⇒ $\sum\vec{F}_{ext} = \vec{0}$. Firing is due to internal forces ($\sum\vec{F}_{ext}$ remains zero during firing) ⇒ the system is an energy isolated system.

ii. During this collision linear momentum is conserved. $\vec{P}_{before} = \vec{P}_{after} \Rightarrow \vec{0} = m\vec{v} + \mu\vec{v}_1$ $\Rightarrow m\vec{v} = -\mu\vec{v}_1 \Rightarrow \vec{v} = \frac{-\mu}{m}\vec{v}_1$ ⇒ the magnitude of $\vec{v}$ is $v = \frac{\mu}{m}v_1 = 500$ m/s.

B.

a) During collision: linear momentum is conserved: $\vec{P}_{before} = \vec{P}_{after} \Rightarrow m\vec{v} + \vec{0} = (m + m_{block})\vec{v}_o$

But $(m + m_{block} = M) \Rightarrow m\vec{v} = M\vec{v}_o \Rightarrow \vec{v}_o = \frac{m}{M}\vec{v} \Rightarrow v_o = \frac{m}{M}v = \frac{(0.03)(500)}{M} = \frac{15}{M}$.

b) $ME = \frac{1}{2}k X_m^2 = \frac{1}{2}M v_o^2 \Rightarrow K(0.15)^2 = M(\frac{15}{M})^2 = \frac{225}{M} \Rightarrow k M = 10^4$ kg.N/m.

c) i. $T_o = \frac{6.3}{10} = 0.63$ s.

ii. $T_o = 2\pi\sqrt{\frac{M}{k}} = 0.63$ and $M k = 10^4 \Rightarrow M = \frac{10^4}{k} \Rightarrow 0.63 = 2\pi\sqrt{\frac{10^4}{k^2}} \Rightarrow k = 996.83$ N/m. **iii.** $M = \frac{10^4}{996.83} = 10.03$ kg.

15.

A.

a) i. $T_1 = K_1\Delta\ell_1 = 2(0.1) = 0.2$ N ($\vec{T}_1$ horizontally to the left). $T_2 = k_2\Delta\ell_2 = 6(0.1) = 0.6$ N ($\vec{T}_2$ horizontally to the right)

ii. $\sum\vec{F}_{ext} = m\vec{g} + \vec{N} + \vec{T}_1 + \vec{T}_2 = \vec{T}_1 + \vec{T}_2 = -0.2\vec{i} + 0.6\vec{i} = 0.4\vec{i} \neq 0$. But G is initially at rest ⇒ G starts its oscillating at $t_0 = 0$. Direction: horizontally to the right since resultant force is directed horizontally to the right.

iii. ME = KE + GPE + EPE = $0 + 0 + \frac{1}{2}k_1\Delta\ell_1^2 + \frac{1}{2}k_2\Delta\ell_2^2 = \frac{1}{2}(2)(0.01) + \frac{1}{2}(6)(0.01) = 0.04$ J.

b) ME = GPE + KE + EPE = $0 + \frac{1}{2}mV^2 + \frac{1}{2}k_1(\Delta\ell + x)^2 + \frac{1}{2}k_2(\Delta\ell - x)^2$.

c) No non-conservative forces ⇒ ME is conserved ⇒ $0 = m v v' + k_1(\Delta\ell + x)(x') + k_2(\Delta\ell - x)(-x') = v[m x'' + k_1\Delta\ell + k_1x - k_2\Delta\ell + k_2x]$, v = 0 is rejected ⇒ $m x'' + (k_1 + k_2)x + \Delta\ell[k_1 - k_2] = 0 \Rightarrow x'' + \frac{k_1 + k_2}{m}x = \frac{\Delta\ell}{m}[k_2 - k_1]$.

d) $\omega_o = \sqrt{\frac{2+6}{0.5}} = 4$ rad/s $\Rightarrow T_o = \frac{2\pi}{\omega_o} = \frac{\pi}{2}$ s.

B.

a) $x_1 = 5$ cm and $X_m = 5$ cm. **b)** $T_o = 0.5\pi$ s . Equal

c) $V_m = 0.2$ m/s.

d) $ME = \frac{1}{2}mV^2 + \frac{1}{2}k_1(\Delta\ell + x)^2 + \frac{1}{2}k_2(\Delta\ell - x)^2$. $ME_o = 0 + \frac{1}{2}(2)[0.1 + 0]^2 + \frac{1}{2}(6)[0.1 - 0]^2 = 0.04$ J, at $t_1 = 0.125\pi$ s; x = 5 cm; v = 0.2 m/s ⇒ $ME = \frac{1}{2}(0.5)(0.2)^2 + \frac{1}{2}(2)[0.1 + 0.05]^2 + \frac{1}{2}(6)[0.1 - 0.05]^2 = 0.04$ J, at $t_2 = 0.25\pi$ s ; x = 10 cm and v = 0 ⇒ $ME = 0 + \frac{1}{2}(2)[0.1 + 0.1]^2 + \frac{1}{2}(6)[0.1-0.1]^2 = 0.04$ J.

e) $ME_o = ME_{(0.125\pi)} = ME_{(0.25\pi)}$ = constant ⇒ ME is conserved.

16.

A.

a) $\sum\vec{F}_{ext} = \vec{0} \Rightarrow m\vec{g} + \vec{T} = \vec{0} \Rightarrow mg = k\Delta\ell \Rightarrow \Delta\ell = \frac{mg}{k} = \frac{0.5(10)}{100} = 0.05$ m.

b) i. ME = GPE + EPE + K.E = $mg(-x) + \frac{1}{2}k[\Delta\ell + x]^2 + \frac{1}{2}mv^2$.

ii. No non-conservative forces (no friction) ⇒ ME is conserved ⇒

$\frac{dM.E}{dt} = 0 = -mgx' + k[\Delta\ell + x](x') + mvv' = x'[-mg + k\Delta\ell + kx + mx''] = x'[kx + mx''] = 0$

$x' = v = 0$ is rejected $\Rightarrow x'' + \frac{k}{m}x = 0$ {the differential equation has the form $x'' + \omega_o^2 x = 0$.

$\Rightarrow$ Motion of G is simple harmonic $\Rightarrow \omega_o^2 = \frac{k}{m} \Rightarrow \omega_o = \sqrt{\frac{k}{m}}$ (ω_o is independent of g).

B.

a) $ME = -mgx + \frac{1}{2}kx^2 + \frac{1}{2}mV^2$.

b) $\frac{dME}{dt} = 0 = -mgx' + kxx' + mvv' = x'[-mg + kx + mx'']$ but $x' = 0$ is rejected $\Rightarrow x'' + \frac{k}{m}x = g$.

c) $v = x' = \omega_o X_m \cos(\omega_o t + \varphi)$ the equation has the form $v = V_m\cos(\omega_o t + \varphi) \Rightarrow$ simple harmonic motion

d) At $t_o = 0$; $x_o = 0$; $0 = X_m \sin\varphi + \frac{mg}{k}$ $\Rightarrow \sin\varphi = \frac{-mg}{kX_m} < 0$. Also $v_o = 0$; $0 = \omega_o X_m\cos\varphi \Rightarrow \cos\varphi = 0 \Rightarrow \varphi = \pm\frac{\pi}{2}$ rad.

But $\sin\varphi < 0 \Rightarrow \varphi = \frac{-\pi}{2}$ rad. $0 = X_m\sin\varphi + \frac{mg}{k} \Rightarrow X_m = \frac{mg}{k}$.

17.

a) $\sum\vec{F}_{ext} = \vec{0} \Rightarrow m\vec{g} + \vec{T} = \vec{0} \Rightarrow mg = k\Delta\ell \Rightarrow \Delta\ell = \frac{3(10)}{300} = 0.1$ m.

b) During collision, linear momentum is conserved $\Rightarrow \vec{P}_{before} = \vec{P}_{after}$.

$m_1\vec{V}_1 = (m + m_1)\vec{V}_o \Rightarrow 0.02\vec{V}_1 = (3 + 0.02)(4\vec{i}) \Rightarrow \vec{V}_1 = 604\,\vec{i}$ (m/s).

c) $ME_o = KE_o + GPE_o + EPE_o = \frac{1}{2}mV_o^2 + 0 + \frac{1}{2}k(\Delta\ell)^2 = \frac{1}{2}(3)(16) + \frac{1}{2}(300)(0.1)^2 = 25.5$ J.

d) $ME = GPE + K.E + EPE = mgx + \frac{1}{2}mV^2 + \frac{1}{2}k(\Delta\ell - x)^2$

No non-conservative forces $\Rightarrow$ ME is conserved $\Rightarrow \frac{dME}{dt} = 0 = mgx' + mvv' + k(\Delta\ell - x)(-x')$

$x'[mg + mx'' - k\Delta\ell + kx] = 0$, but $x' = v = 0$ is rejected and $mg = k\Delta\ell$.

$\Rightarrow x'' + \frac{k}{m}x = 0$ the differential equation has the form $x'' + \omega_o^2 x = 0 \Rightarrow$ simple harmonic motion.

e) $v = -4\cos(10t + \pi) \Rightarrow -2 = -4\cos(10t + \pi) \Rightarrow \cos(10t + \pi) = 0.5 \Rightarrow 10t + \pi = \frac{\pi}{3}$ (rejected)

OR $10t + \pi = \frac{5\pi}{3}$(accepted) $\Rightarrow t = 0.2093$ s.

f)

i. $x = -0.4\sin(10t + \pi)$ while $v = x' = -4\cos(10t + \pi) \Rightarrow GPE = mgx = 3(10)(-0.4)\sin(10t + \pi) = -12\sin(10t + \pi)$ (S.I).

$KE = \frac{1}{2}mV^2 = \frac{1}{2}(3)(16)\cos^2(10t + \pi) = 24\cos^2(10t + \pi)$ (S.I).

ii. $EPE = ME - KE - GPE = 25.5 - 24\cos^2(10t + \pi) + 12\sin(10t + \pi)$ (S.I).

iii. Graph (a): Represents variation of ME since it is constant. Graph (c): represents KE since it starts at $KE_o = 24$ J and $KE_o = 24\cos^2(0 + \pi) \Rightarrow KE_o = 24$ J. Graph (d): Represents variation of GPE since it starts at $GPE_o = 0$ and block is at the reference level at $t_o = 0$. Graph (b): Represents variation of EPE since it starts at $EPE_o = 1.5$ J.

and $EPE_o = \frac{1}{2}k(\Delta\ell)^2 = \frac{1}{2}(300)(0.1)^2 = 1.5$ J.

iv. $T_{KE} = 0.314$ s. $T_o = 0.628$ s $\Rightarrow T_{KE} = \frac{T_o}{2}$.

18.

A.

a) Free damped mechanical oscillations.

b) $T = 1$ s. $T > T_o$. $T_o = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.2}{9.725}} = 0.9$ s.

c) $\omega = \frac{2\pi}{T} = 2\pi$ rad/s. $\omega_o = \frac{2\pi}{T_o} = \frac{2\pi}{0.9} = \frac{20\pi}{9}$ rad/s $\Rightarrow 4\pi^2 = \frac{400}{81}\pi^2 - \frac{h^2}{4(0.04)}$ $\Rightarrow h = 1.22$ kg/s.

d) $v = \frac{dx}{dt}$, when $v = x' = 0 \Rightarrow x = \pm X_m$.

e) At $t = 2$ s: x is maximum $\Rightarrow v = 0 \Rightarrow f_r = -hv = 0$.

f) $W_{\vec{f}_r} = \Delta ME = \frac{1}{2}k[X^2_{m(6T)} - X^2_{m(0)}] = \frac{1}{2}(9.725)[(0.09)^2 - (0.12)^2] = -0.0306$ J.

B.

a) x decreases to zero. **b)** $\frac{h}{2(0.2)} = \frac{20\pi}{9} \Rightarrow h' = 2.791$ kg/s. $h' > h$.

19.

a) $T_o = \frac{42.3}{20} = 2.115$ s.

b)

i. Free damped mechanical oscillations.

ii. $T = \frac{21.3}{9} = 2.367$ s $\Rightarrow T > T_o$.

iii.

1. $|\Delta ME| = \frac{1}{2}k[X^2_{m0} - X^2_{m(T)}]$ = loss of ME $\Rightarrow 1.952 \times 10^{-4} = \frac{1}{2}k[(0.042)^2 - (0.038)^2] \Rightarrow k = 1.22$ N/m.
2. Loss of energy $= \frac{1}{2}k[X^2_{m(8T)} - X^2_{m(9T)}] = \frac{1}{2}(1.22)[(0.019)^2 - (0.017)^2] = 4.392 \times 10^{-5}$ J.

iv.

1. To drive oscillations we supply the system by energy to compensate for the loss in order to oscillate with constant amplitude.
2. $P = \frac{\text{loss during the 1st period}}{T} = \frac{1.952 \times 10^{-4}}{2.367} = \underline{8.247 \times 10^{-5}\ W}$.

20.

a) System: Block. External forces: $m\vec{g}$: weight; $\vec{N}$: normal reaction; $\vec{T}$: tension; $\vec{f}$: friction; and $\vec{F}_{drive}$.

$\sum\vec{F}_{ext} = m\vec{a} \Rightarrow \cancel{m\vec{g} + \vec{N}} + \vec{T} + \cancel{\vec{f} + \vec{F}_{drive}} = m\vec{a} \Rightarrow \vec{T} = m\vec{a} \Rightarrow -k\,x = m\,x' \Rightarrow x'' + \frac{k}{m}x = 0 \Rightarrow$ simple harmonic motion

$\Rightarrow$ oscillator oscillates with its proper angular frequency $\omega_o = \sqrt{\frac{k}{m}}$.

b) $p_{\vec{f}} = \frac{dW_{\vec{f}}}{dt} \Rightarrow W_{\vec{f}} = \int_0^T p_{\vec{f}}\ dt$ {But $p_{\vec{f}} = \vec{f}.\vec{v} = -h\,\vec{v}.\vec{v} = -h\,v^2$} $\Rightarrow W_{\vec{f}} = \int_0^T -h\,v^2 dt$ {But $v = x' = \omega_o X_m \cos(\omega_o t)$}

$W_{\vec{f}} = -h\int_0^T \omega_o^2\ X_m^2 \cos^2(\omega_o t)dt = -h\,\omega_o^2 X_m^2 \int_0^T [\frac{1}{2} + \frac{1}{2}\cos(2\omega_o t)\ dt = \frac{-h\ \omega_o^2\ X_m^2}{2}[t + \frac{\sin(2\omega_o t)}{2\omega_o}]_0^T = \frac{-h\ \omega_o^2\ X_m^2}{2}[T + \frac{\sin(2\omega_o T)}{2\omega_o} - 0]$

{But $T = \frac{2\pi}{\omega_o}$} $\Rightarrow W_{\vec{f}} = \frac{-h\ \omega_o^2\ X_m^2}{2}(\frac{2\pi}{\omega_o}) = -h\omega_o X_m^2\ \pi$.

c) $\vec{F}_{drive} = -\vec{f} \Rightarrow W_{\vec{F}_{drive}} = -W_{\vec{f}} = h\ \omega_o X_m^2\ \pi$.

21.

a) Free damped mechanical oscillations, since the amplitude decreases with time.

b) T = 1s pseudo-period.

c) $ME = \frac{1}{2}k\,x^2 + \frac{1}{2}m\,v^2 \Rightarrow \frac{dME}{dt} = -hv^2 = k\,x\,x' + m\,v\,v' \Rightarrow 0 = hv^2 + k\,x\,x' + mvv' = v[\,h\,x' + k\,x + m\,x'']$

$v = 0$ is rejected $\Rightarrow x'' + \frac{h}{m}x' + \frac{k}{m}x = 0$.

d)

i. $x_o = X_{m_o} = A\,e^0 \sin\varphi \Rightarrow A = \frac{X_{m_o}}{\sin\varphi}$. $v = x' = Ae^{\frac{-ht}{2m}}[\omega\cos(\omega t + \varphi) - \frac{h}{2m}\sin(\omega t + \varphi)]$

$v_o = 0 = A[\omega\cos\varphi - \frac{h}{2m}\sin\varphi]$, but $A \neq 0 \Rightarrow \omega\cos\varphi = \frac{h}{2m}\sin\varphi \Rightarrow \tan\varphi = \frac{2\,m\,\omega}{h}$.

ii. $x_{(2T)} = Ae^{\frac{-h(2T)}{2m}}\sin(\omega(2T) + \varphi) = Ae^{\frac{-hT}{m}}\sin(4\pi + \varphi) = Ae^{\frac{-hT}{m}}\sin(\varphi)$. $x_{(1T)} = Ae^{\frac{-h(T)}{2m}}\sin(\omega(T) + \varphi) = Ae^{\frac{-hT}{2m}}\sin(2\pi + \varphi)$

$= Ae^{\frac{-hT}{2m}}\sin(\varphi) \Rightarrow \frac{x_{(2T)}}{x_{(1T)}} = \frac{Ae^{\frac{-hT}{m}}\sin(\varphi)}{Ae^{\frac{-hT}{2m}}\sin(\varphi)} = e^{\frac{-hT}{2m}} =$ constant.

iii. $C = \frac{1.238}{7.609} = \underline{0.951} \Rightarrow 0.951 = e^{\frac{-h(1)}{2(0.5)}} \Rightarrow -0.05 = -h \Rightarrow \underline{h = 0.05\ kg/s}$.

iv. $\omega^2 = \frac{k}{m} - (\frac{h}{2m})^2 \Rightarrow (6.28)^2 = \frac{k}{0.5} - (\frac{0.05}{2(0.5)})^2$ { $w = \frac{2\pi}{T} = 6.28$ rad/s} $\Rightarrow \underline{k = 90.5\ N/m}$.

$\tan\varphi = \frac{2(0.5)(6.28)}{0.05} = 125.6 \Rightarrow \varphi = 89.527° = \underline{1.56\ rad}$ {$A > 0 \Rightarrow \sin\varphi > 0 \Rightarrow \varphi$ in the 1st quadrant}

$A = \frac{X_{m_o}}{\sin\varphi} = \frac{0.08}{\sin(1.56)} = \underline{0.080004\ m} \approx X_{m_o}$. $x = 0.08e^{-0.05t}\sin(6.28t + 1.56)$ (S.I).

22.

a) Amplitude of oscillations decreases with time $\Rightarrow$ friction exists.

b)

i. At $t_o = 0$; $x_o = 14$ cm $\Rightarrow ME_o = 98$ mJ. $ME_o = \frac{1}{2}k\,x_o^2 \Rightarrow (0.098) = \frac{1}{2}k(0.14)^2 \Rightarrow k = 10$ N/m.

ii. For $x = 0$; $ME = 84$ mJ = KE $\Rightarrow 0.084 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2(0.084)}{0.65}} \approx \underline{0.51\ m/s}$.

iii. $\Delta ME = W_{\text{non-conservative forces}} = W_{\vec{f}} \Rightarrow ME_{(x=-12)} - ME_{(x=14)} = W_{\vec{f}} \Rightarrow 0.07 - 0.098 = -f\,[0.14 + 0.12] \Rightarrow \underline{f = 0.108\ N}$.

c)

i. $W_{\vec{f}} = ME_{(0.5\pi)} - ME_o = \frac{1}{2}(10)[(0.1)^2 - (0.14)^2] = \underline{-0.048\ J}$.

ii. $W_{\vec{f}} = ME_{(1T)} - ME_o \Rightarrow -0.048 = M.E_{(1T)} - 0.098 \Rightarrow M.E_{(1T)} = \underline{0.05\ J}$.

23.

A.

a) $v = \frac{dx}{dt}$ When $x = X_{max} \Rightarrow \frac{dx}{dt} = 0 \Rightarrow v = 0$. **b)** $m\vec{g}$: its weight, $\vec{N}$: normal reaction, $\vec{T}$: tension of the spring, and $\vec{f}$: friction force

c) $\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{N} + \vec{T} + \vec{f} = m\vec{a}$. Project along the x-axis: $0 + 0 - k\,x - h\,v = m\,a \Rightarrow -k\,x - h\,v = m\,v'$

* When $v = \pm V_{max} \Rightarrow \frac{dv}{dt} = v' = 0 \Rightarrow -k\,x - hV_{max} = 0 \Rightarrow x = \pm\frac{hV_{max}}{k}$.

B.

a) i. T = 0.446 s. **ii.** At t_1: $V_{max} = (+4.6\ \text{div})(0.5\frac{m/s}{div}) = \underline{2.3\ m/s}$. $x = (+1\text{div})(5) = 5$ cm $\Rightarrow 0.05 = \frac{-h(-2.3)}{100} \Rightarrow h = 2.17$ kg/s.

b) $ME_1 = \frac{1}{2}mV_m^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.5)(2.3)^2 + \frac{1}{2}(100)(0.05)^2 = \underline{1.4475\ J}$.

c) $ME_2 = \frac{1}{2}kX_m^2 + 0 = \frac{1}{2}(100)(1.7\times 0.05)^2 = \underline{0.36125\ J}$.

Chapter 5: Mechanical Oscillations (2)

1. $T_o = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T_o^2 = 4\pi^2\frac{\ell}{g} \Rightarrow (12)^2 = \frac{4\pi^2\ell}{9.8} \Rightarrow \underline{\ell = 35.78\text{ m}}$.

2.

a) $T_{(compound)} = 2\pi\sqrt{\frac{I}{mg\,a}} = 2\pi\sqrt{\frac{\frac{1}{3}m\ell^2}{mg\frac{\ell}{2}}} = 2\pi\sqrt{\frac{2\,\ell}{3\,g}}$. **b)** $T_{(simple)} = 2\pi\sqrt{\frac{\ell}{g}} > 2\pi\sqrt{\frac{2\,\ell}{3\,g}} \Rightarrow T_{(simple)} > T_{(compound)}$.

c) $T_{(simple)} = 2\pi\sqrt{\frac{\ell'}{g}} = 2\pi\sqrt{\frac{2\,\ell}{3\,g}} \Rightarrow \ell' = \frac{2}{3}\,\ell$.

3.

a) $ME = \frac{1}{2}I\theta'^2 + m\,g\,\ell[1-\cos\theta]$. No non-conservative forces $\Rightarrow$ ME is conserved $\Rightarrow \frac{dME}{dt} = 0 = I\theta'\theta'' + m\,g\,\ell\,\theta'\sin\theta$

$\Rightarrow \theta'[I\,\theta'' + m\,g\,\ell\sin\theta] = 0$ but $\theta' = 0$ is rejected $\Rightarrow \theta'' + \frac{m\,g\,\ell}{I}\sin\theta = 0$ {But $I = m\ell^2$} $\Rightarrow \theta'' + \frac{g}{\ell}\sin\theta = 0$.

b) No since the differential equation does not have the form $\theta'' + \omega_o^2\,\theta = 0$.

c) Free un-damped mechanical oscillations.

d) $ME_{\substack{extreme\\position}} = m\,g\,\ell[1-\cos\theta_m] = (0.2)(10)(0.72)[1-\cos30^\circ] = 0.193$ J. $ME_{\substack{equilibrium\\position}} = \frac{1}{2}I\theta_m'^2 = 0.193$ J $\Rightarrow$

$\frac{1}{2}(0.2)(0.72)^2(\theta_m')^2 = 0.193 \Rightarrow \theta_m' = 1.93$ rad/s $\Rightarrow v = \ell\,\theta' = (0.72)(1.93) = \underline{1.39\text{ m/s}}$.

e) At $t_o = 0$; $\theta_m = 10^\circ$. At $t = 1T$; $\theta_m = \frac{10^\circ}{1.025}$. At $t = 2T$; $\theta_m = \frac{10^\circ}{(1.025)^2} \Rightarrow$ At $t = nT$; $\theta_m = \frac{10^\circ}{(1.025)^n} \Rightarrow 6.9 = \frac{10^\circ}{(1.025)^n} \Rightarrow (1.025)^n = \frac{10^\circ}{6.9} \Rightarrow$ $\underline{n \approx 15\text{ oscillations}}$.

4.

A.

a) $ME = GPE + KE + PE_{torsion} = 0 + \frac{1}{2}I\,\theta'^2 + \frac{1}{2}C\,\theta^2$. No non conservative forces (no friction) $\Rightarrow$ ME is conserved

$\Rightarrow \frac{dME}{dt} = 0 = I\,\theta'\,\theta'' + C\,\theta\,\theta' = \theta'[I\,\theta'' + C\,\theta]$, but $\theta' = 0$ is rejected $\Rightarrow \theta'' + \frac{C}{I}\theta = 0$ The differential equation has the form

$\theta'' + \omega_o^2\,\theta = 0 \Rightarrow$ S.H.M. By comparison: $\omega_o = \sqrt{\frac{C}{I}} \Rightarrow T_o = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{I}{C}}$.

b) $T_o = \frac{30}{10} = 3 = 2\pi\sqrt{\frac{I}{C}} \Rightarrow 9 = 4\pi^2\frac{I}{C} \Rightarrow C = \frac{40}{9}I$ (S.I) ...eq (1)

B.

a) $I_{total} = I + mR^2 = I + (0.14)(0.5)^2 = I + 0.035$ (S.I).

b) $T_o' = \frac{40}{10} = 4\text{ s} = 2\pi\sqrt{\frac{I_{total}}{C}} \Rightarrow 16 = 4\pi^2\frac{I_{total}}{C} = 4(10)\frac{I+0.035}{C} \Rightarrow C = 2.5(I + 0.035)$ .... eq(2)

equ (1) = equ (2): $\frac{40}{9}I = 2.5(I + 0.035) \Rightarrow \underline{I = 0.045\text{ kg.m}^2} \Rightarrow C = \frac{40}{9}(0.045) = \underline{0.2\text{ N.m}}$.

5.

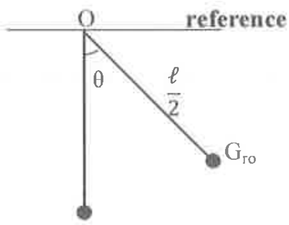
a) System chair: External forces $m\vec{g}$ (vertically downward) and $\vec{R}$ (vertically upward).
Rotational equilibrium: $\sum M_{ext} = M_{\vec{R}} + M_{m\vec{g}} = 0$, but $M_{\vec{R}} = 0$ since $\vec{R}$ is passing through O $\Rightarrow M_{m\vec{g}} = 0 \Rightarrow m\vec{g}$ is passing through O $\Rightarrow G_o$ belongs to the vertical axis passing through O.

b) $ME = \frac{1}{2}I\theta'^2 + m\,g\,a(1-\cos\theta)$, no non conservative forces $\Rightarrow \frac{dME}{dt} = 0 \Rightarrow I\,\theta'\,\theta' + m\,g\,a\,\theta'\sin\theta = 0$
$\{\sin\theta \approx \theta$ and $\theta' = 0$ is rejected$\} \Rightarrow \theta' + \frac{m\,ga}{I}\,\theta = 0$.

c) Since it has the form of $\theta'' + \omega_o^2\,\theta = 0$ (ω_o is positive constant) $\Rightarrow T_o = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{I}{m\,g\,a}}$. **d)** $T_o = \frac{18}{10} = 1.8$ s.

e) When $\theta' = \max \Rightarrow \theta = 0 \Rightarrow ME = \frac{1}{2}I\theta_m'^2 \Rightarrow I = \frac{2\times0.121}{(0.61)^2} = \underline{0.65\text{ kg.m}^2}$.

f) $T_o = 2\pi\sqrt{\frac{I}{m\,g\,a}} \Rightarrow (1.8)^2 = 4(3.14)^2(\frac{0.65}{2\times10\times a}) \Rightarrow a = 0.4\text{ m} = \underline{40\text{ cm}}$.



6.

a) $M.E = \frac{1}{2}I(\theta'^2) + \frac{1}{2}C\,\theta^2 + mg(\frac{\ell}{2} - \frac{\ell}{2}\cos\theta)$.

b) No non-conservative forces (no friction) $\Rightarrow$ ME is conserved

$\Rightarrow \frac{dME}{dt} = 0 = I\,\theta'\,\theta'' + C\,\theta\,\theta' + m\,g\frac{\ell}{2}\sin\theta(\theta') = \theta'[I\,\theta'' + C\,\theta + m\,g\frac{\ell}{2}\theta] = 0$

$\theta' = 0$ is rejected $\Rightarrow \theta'' + \frac{C}{I}\theta + \frac{m\,g\,\ell}{2\,I}\theta = 0 \Rightarrow \theta'' + [\frac{2\,C + m\,g\,\ell}{2\,I}]\theta = 0$.

The differential equation has the form $\theta'' + \omega_o^2\,\theta = 0 \Rightarrow$ S.H.M

c) $\omega_o^2 = \frac{2C + m\,g\,\ell}{2\,I} \Rightarrow \omega_o = \sqrt{\frac{2C + m\,g\,\ell}{2\,I}}$, but $T_o = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{2I}{2C + m\,g\,\ell}} = 2\pi\sqrt{\frac{2\,m\,\ell^2}{3(2C + m\,g\,\ell)}} = 2\pi\sqrt{\frac{2\,m\,\ell^2}{6C + 3\,m\,g\,\ell}}$.

7.

a) $I = \frac{m\ell^2}{3}$ and $a = \frac{\ell}{2} \Rightarrow T_o = 2\pi\sqrt{\frac{I}{m\,g\,a}} = 2\pi\sqrt{\frac{2\,m\,\ell^2}{3\,mg\,\ell}} = 2\pi\sqrt{\frac{2\,\ell}{3\,g}}$.

b) $I'_{system} = I_{rod} + I_m' = \frac{m\,\ell^2}{3} + \frac{m}{4}d^2 = \frac{m}{4}(\frac{\ell}{3} + d^2)$. The position of the center of mass of the system (Rod, particle) is:

$x'_G = a' = \frac{m\frac{\ell}{2} + \frac{m}{4}d}{m + \frac{m}{4}} = \frac{m(\frac{\ell}{2} + \frac{d}{4})}{m(1.25)} = \frac{\ell + 0.5d}{2.5} = \frac{0.5 + 0.5d}{2.5} = \frac{1 + d}{5}$. $T_o = 2\pi\sqrt{\frac{I'}{m'g\,a'}} = 2\pi\sqrt{\frac{2\,\ell}{3g}} \Rightarrow \frac{I'}{m'g\,a'} = \frac{2\,\ell}{3g}$

$\Rightarrow \frac{\frac{m}{4}(\frac{\ell}{3} + d^2)}{(1.25m)(g)[\frac{1+d}{5}]} = \frac{2\,\ell}{3g} \Rightarrow 2(0.5)(1.25)(\frac{1+d}{5}) = \frac{3}{4}(\frac{1}{3} + d^2) \Rightarrow 1 + d = 3(\frac{1}{3} + d^2) \Rightarrow 3d^2 = d \Rightarrow d = \frac{1}{3}\,m = \underline{33.33\text{ cm}}$.

8.

a) $T_o = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.9}{10}} = \underline{1.884\text{ s}}$. $t_1 = \frac{T_o}{4} = \frac{1.884}{4} = \underline{0.471\text{ s}}$.

b)

i. $ME_o = m\,g\,\ell[1 - \cos\theta_{m_1}] = mg\frac{\ell}{2}[1 - \cos\theta_{m_2}] \Rightarrow \theta_{m_2} = \underline{7.07°}$. **ii.** $T'_o = 2\pi\sqrt{\frac{\ell/2}{g}} = 2\pi\sqrt{\frac{0.45}{10}} = \underline{1.332\text{ s}}$.

iii. $t_2 = t_1 + \frac{T'_o}{4} = 0.471 + \frac{1.332}{4} = \underline{0.804\text{ s}}$.

c) $T_o = 2\pi\sqrt{\frac{0.9}{1.63}} = 4.666\text{ s} \Rightarrow t_1 = \underline{1.167\text{ s}}$. $T'_o = 2\pi\sqrt{\frac{0.45}{1.63}} = 3.3\text{ s} \Rightarrow t_2 = 1.167 + \frac{3.3}{4} = \underline{2\text{ s}}$.

9.

A.

a) Since the rod oscillates with almost same amplitude during the first six oscillations.

b) $I_s = I + m(\frac{\ell}{2})^2 + m(\frac{\ell}{2})^2 = I + 2(0.04)(0.1)^2 = I + 8 \times 10^{-4}$. $ME = \frac{1}{2}I_s(\theta')^2 + \frac{1}{2}C\,\theta^2$, but ME is conserved

$\Rightarrow \frac{dME}{dt} = 0 = I_s\theta'\theta'' + C\,\theta\,\theta' = \theta'[I_s\theta'' + C\,\theta]$ $\theta' = 0$ is rejected $\Rightarrow \theta'' + \frac{C}{I + 8\times10^{-4}}\theta = 0$. Differential equation has the

form of $\theta'' + \omega_o^2\,\theta = 0 \Rightarrow \omega_o^2 = \frac{C}{I + (8\times10^{-4})} \Rightarrow T_o = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{I + 8\times10^{-4}}{0.0115}}$ (S.I)

c) $T_o = \frac{18}{6} = 3\text{ s} = 2\pi\sqrt{\frac{I + (8\times10^{-4})}{0.0115}} \Rightarrow I = 1.7875 \times 10^{-3}\text{ kg.m}^2$.

B.

a) $\theta' = -\,\omega_o\,\theta_m\sin(\omega_o t + \varphi) \Rightarrow \theta'' = -\,\omega_o^2\,\theta_m\cos(\omega_o t + \varphi) = -\,\omega_o^2\,\theta$. Substitute in differential equation: $-\,\omega_o^2\,\theta + \frac{C}{I_s}\theta = 0$

But $\frac{C}{I_s} = \omega_o^2 \Rightarrow -\,\omega_o^2\,\theta + \omega_o^2\,\theta = 0$ so it is a solution.

b) $ME = \frac{1}{2}C\,\theta_m^2 = \frac{1}{2}C\,\theta^2 + \frac{1}{2}I_s(\theta')^2 \Rightarrow C[\theta_m^2 - \theta^2] = \frac{1}{2}I_s(\theta')^2 \Rightarrow (\theta')^2 = \frac{C}{I_s}[\theta_m^2 - \theta^2] = \omega_o^2(\theta_m^2 - \theta^2)$.

c) At $t_0 = 0$; $\theta = 0.075$ rad and $\theta' = -\,0.26$ rad/s. Substitute in the obtained relation of part (b): $(-0.26)^2 = (\frac{2\pi}{3})^2[\theta_m^2 - (0.075)^2]$

$\Rightarrow \theta_m = 0.144$ rad. Using $\theta = \theta_m\cos(\omega_o t + \varphi) \Rightarrow 0.075 = 0.144\cos\varphi \Rightarrow \cos\varphi = 0.5357 \Rightarrow \varphi = 57.61°$ or $\varphi = -57.61°$

But $\theta' = -\omega_o\theta_m\sin(\omega_o t + \varphi)$ and $\theta'_0 = -\,0.26 = -\,\omega_o\theta_m\sin\varphi \Rightarrow \sin\varphi > 0 \Rightarrow \varphi = 57.61° \approx 1$ rad.

10.

A.

a) System: Disk. External forces: weight $m\vec{g}$; reaction $\vec{R}$ at axis; the force $\vec{F}$; and the braking couple.

$\mathcal{M}_{m\vec{g}} = \mathcal{M}_{\vec{R}} = 0$ (meet Δ). $\sum\mathcal{M} = \frac{d\sigma}{dt} \Rightarrow \mathcal{M}_{m\vec{g}} + \mathcal{M}_{\vec{N}} + \mathcal{M}_{\vec{F}} + \mathcal{M}_{braking} = \frac{d\,\sigma}{dt} \Rightarrow 0 + 0 + M - b\,\theta' = \sigma' \Rightarrow M = b\theta' + \sigma'$

{ but $\theta' = \frac{\sigma}{I}$ } $\Rightarrow \sigma' + \frac{b}{I}\sigma = M$.

b) $\sigma = \frac{MI}{b}(1 - e^{\frac{-bt}{I}}) \Rightarrow \sigma' = \frac{MI}{b}(0 + \frac{b}{I}e^{\frac{-bt}{I}}) = Me^{\frac{-bt}{I}}$ substitute in the differential equation $\Rightarrow$

$Me^{\frac{-bt}{I}} + \frac{b}{I}\frac{MI}{b}(1 - e^{\frac{-bt}{I}}) = Me^{\frac{-bt}{I}} + M - Me^{\frac{-bt}{I}} = M$ {so it is verified}.

c) $\sigma = \frac{MI}{b}(1 - e^{\frac{-bt}{I}})$ As $t \to \infty$; $e^{\frac{-bt}{I}} \to 0 \Rightarrow \sigma = \frac{MI}{b}(1 - 0) = \frac{MI}{b} \Rightarrow I = \frac{\sigma\,b}{M} = \frac{(4.56\times10^{-3})(3.04\times10^{-5})}{9.12\times10^{-3}} \Rightarrow \underline{I = 1.52 \times 10^{-5}\text{ kg.m}^2}$.

B.

a) $ME = KE + GPE = \frac{1}{2}I_A\theta'^2 + m\,g(-\,R\cos\theta)$.

b) No friction (no non-conservative forces) $\Rightarrow$ ME is conserved $\Rightarrow \frac{dME}{dt} = 0 = I_A\theta'\theta'' + m\,g\,R\sin\theta\,(\theta')$

$\Rightarrow \theta'[I_A\theta'' + m\,g\,R\sin\theta] = 0$, but $\theta' = 0$ is rejected and $\sin\theta \approx \theta \Rightarrow \theta'' + \frac{m\,g\,R}{I_A}\theta = 0$, differential equation has the form

$\theta'' + \omega_o^2\,\theta = 0 \Rightarrow \omega_o^2 = \frac{m\,g\,R}{I_A} \Rightarrow \omega_o = \sqrt{\frac{m\,g\,R}{I_A}}$; $T_o = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{I_A}{m\,g\,R}}$.

c) $T_o = \frac{3.47}{10} = 0.347$ s; $T_o^2 = \frac{4\pi^2 I_A}{m\,g\,R} \Rightarrow (0.347)^2 = \frac{4\pi^2 I_A}{(0.075)(10)(0.02)} \Rightarrow I_A = \underline{4.52 \times 10^{-5}\text{ kg.m}^2}$.

d) $I_A = I + mR^2 \Rightarrow 4.52 \times 10^{-5} = I + (0.075)(0.02)^2 \Rightarrow \underline{I = 1.52 \times 10^{-5}\text{ kg.m}^2}$.

C.

a) ME = KE + GPE + $PE_{torsion}$ = $\frac{1}{2}$ $I\theta'^2 + 0 + \frac{1}{2}C\theta^2 + \frac{1}{2}C\,\theta^2 = \frac{1}{2}I\,\theta'^2 + C\,\theta^2$.

b) No non-conservative forces ⇒ ME is conserved ⇒ $\frac{dME}{dt} = 0 = I\,\theta'\,\theta'' + 2\,C\theta\,\theta' = \theta'[I\,\theta'' + 2C\,\theta]$, but $\theta' = 0$ is rejected

$\Rightarrow \theta'' + \frac{2C}{I}\theta = 0$. The equation has the form $\theta'' + \omega_o^2\,\theta = 0 \Rightarrow \omega_o^2 = \frac{2C}{I} \Rightarrow \omega_o = \sqrt{\frac{2C}{I}}$.

c) $\theta = A\cos(\omega_o t + \varphi)$; $\theta_m = A\cos\varphi \Rightarrow \cos\varphi = \frac{\theta_m}{A} < 0$. $\theta' = -\,\omega_o A\sin(\omega_o t + \varphi)$; $\theta'_o = 0 = -\,\omega_o A\sin\varphi$

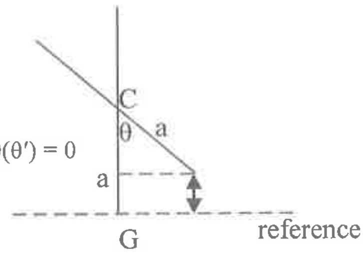
$\Rightarrow \varphi = \pi$ rad Or $\varphi = 0$, but $\cos\varphi < 0 \Rightarrow \underline{\varphi = \pi \text{ rad}} \Rightarrow A = \frac{\theta_m}{\cos\pi} = -\,\theta_m = \underline{-\,0.1 \text{ rad}}$.

d) $\theta' = +\,\omega_o\theta_m\sin(\omega_o t + \pi)$ when disk reaches equilibrium position $t = \frac{T_0}{4} \Rightarrow -10.26 = \omega_o(0.1)\sin(\omega_o\frac{T_0}{4} + \pi)$

$\Rightarrow -10.26 = \omega_o\sin(\frac{\pi}{2} + \pi) \Rightarrow \omega_o = \underline{102.6 \text{ rad/s}}$, but $\omega_o = \sqrt{\frac{2C}{I}} \Rightarrow \omega_o^2 = \frac{2C}{I} \Rightarrow I = \frac{2(0.08)}{102.6^2} = \underline{1.52 \times 10^{-5}\text{ kg.m}^2}$.

11.

A.



a) $X_G = a = CG = \frac{M(0) + mR}{M + m} = \frac{(1)(0) + (0.5)(0.6)}{1.5} = \frac{0.3}{1.5} = \underline{0.2\text{ m}}$.

b) $I_{system} = I_{disk} + I_s \Rightarrow I = I_o + mR^2 = I_o + 0.18$.

c) ME = KE + GPE = $\frac{1}{2}I\theta'^2 + (m + M)g\,a(1 - \cos\theta)$

d) No non-conservative forces ⇒ ME is conserved ⇒ $\frac{dME}{dt} = 0 \Rightarrow I\,\theta'\,\theta'' + (m + M)g\,a\sin\theta(\theta') = 0$

$\theta'[I\theta'' + (m + M)\,g\,a\sin\theta] = 0$ but $\theta' = 0$ is rejected and $\sin\theta \cong \theta \Rightarrow \theta'' + \frac{(m+M)\,g\,a}{I}\theta = 0$

the equation has the form $\theta'' + \omega_o^2\,\theta = 0 \Rightarrow \omega_o^2 = \frac{(m+M)ga}{I} \Rightarrow \omega_o = \sqrt{\frac{(m+M)ga}{I}}$.

e) **i.** $T_o = \frac{13}{6} = 2.167$ s but $\omega_o = \frac{2\pi}{T_0} = 2.898$ rad/s. $I = \frac{(m+M)ga}{\omega_o^2} = \frac{(1.5)(10)(0.2)}{(2.898)^2} = 0.357$ kg.m², but $I_o = I - 0.18 = \underline{0.177\text{ kg.m}^2}$.

ii. $\frac{1}{2}MR^2 = \frac{1}{2}(1)(0.6)^2 = 0.18\text{ kg.m}^2 \approx I_o$.

B.

a) $a' = BG = BC + CG = 0.3 + 0.2 = \underline{0.5\text{ m}}$. **b)** $I' = I_1 + I_s = I_1 + m(BA)^2 = I_1 + 0.405$ (S.I)

c) $\omega'_o = \sqrt{\frac{(m+M)ga'}{I'}}$.

d) **i.** $T'_o = \frac{15}{8} = 1.875$ s but $\omega'_o = \frac{2\pi}{T'_o} = 3.349$ rad/s. $I' = \frac{(m+M)ga'}{\omega_o'^2} = 0.668$, but $I_1 = I - 0.405 = \underline{0.263\text{ kg.m}^2}$.

ii. $I_o + Md^2 = 0.177 + (1)(0.3)^2 = 0.267 \approx I_1$ (S.I).

12.

A.

a) **i.** $a = OG = \frac{m\frac{OA}{2} + m\,x}{m + m'} = \frac{m(0.5) + m(x)}{2m} = \frac{0.5 + x}{2} = \underline{0.25 + 0.5\,x}$. **ii.** $I = I_{rod} + I_S = \frac{1}{3}ml^2 + mx^2 = m(\frac{1}{3} + x^2)$.

b) **i.** ME = KE + GPE = $\frac{1}{2}I\theta'^2 + 2mgz_G = \frac{1}{2}I\theta'^2 + 2mga(1 - \cos\theta)$.

ii. No non-conservative forces ⇒ M.E is conserved ⇒ $\frac{dM.E}{dt} = 0 \Rightarrow I\theta'\,\theta'' + 2\,m\,g\,a\sin\theta(\theta') = 0 \Rightarrow \theta'[I\theta'' + 2\,m\,g\,a\sin\theta] = 0$,

but $\theta' = 0$ is rejected and $\sin\theta \approx \theta \Rightarrow \theta'' + \frac{2\,m\,g\,a}{I}\theta = 0$ the equation has the form $\theta'' + \omega_o^2\,\theta = 0$

$\Rightarrow \omega_o = \sqrt{\frac{2m\,g\,a}{I}} \Rightarrow T_o = \frac{2\pi}{\omega_o} = 2\pi\sqrt{\frac{I}{2\,m\,g\,a}} = 2\pi\sqrt{\frac{m(\frac{1}{3} + x^2)}{2\,m\,g(0.25 + 0.5x)}} = 2\pi\sqrt{\frac{1 + 3x^2}{30\,x + 15}}$.

c) **i.** $\gamma = \frac{1 + 3x^2}{30x + 15} \Rightarrow \frac{d\gamma}{dx} = \frac{(15 + 30x)(6x) - (1 + 3x^2)(30)}{(15 + 30x)^2} =$

$\frac{90x^2 + 90x - 30}{(15 + 30x)^2}$. $\frac{d\gamma}{dx} = 0 \Rightarrow 90x^2 + 90x - 30 = 0$

$x_1 = 0.264$ m and $x_2 = -1.264$ m

γ is minimum for x = 0.264 m and for x: [0.264, 1], γ is increasing ⇒ T_o is increasing

X		0.264	
γ'	-	0	+
γ	↘		↗

ii. For x = 0, $T_o = 2\pi\sqrt{\frac{1}{15}} = \underline{1.622\text{ s}}$,

for x = 0.264, $T_o = 2\pi\sqrt{\frac{1 + 3(0.264)}{15 + 30(0.264)}} = \underline{1.442\text{ s}}$.

B.

a) $\frac{dM.E}{dt} = -\,h(\theta')^2 = I\,\theta'\,\theta'' + 2\,m\,g\,a\sin\theta\,(\theta') \Rightarrow \theta'[I\theta'' + 2\,m\,g\,a\sin\theta + h\,\theta'] = 0$ {$\theta' = 0$ is rejected}$\Rightarrow \theta'' + \frac{h}{I}\theta' + \frac{2\,m\,g\,a}{I}\theta = 0$

b) **i.** T is the pseudo period of oscillations.

ii. $T_o = 1.442$ s. $I = m(\frac{1}{3} + x^2) = 0.403$ kg.m². $\frac{1}{T^2} = \frac{1}{(1.442)^2} - (\frac{0.8}{4\pi(0.403)})^2 \Rightarrow \underline{T = 1.481\text{ s}}$.

iii. $\theta = Ae^{\frac{-ht}{2I}}\sin(\frac{2\pi}{T}t + \varphi)$; At $t = 0$; $\theta_0 = 0.1 = A\sin\varphi \Rightarrow A = \frac{0.1}{\sin\varphi}$. $\theta' = A(\frac{-h}{2I})e^{\frac{-ht}{2I}}\sin(\frac{2\pi}{T}t + \varphi) + Ae^{\frac{-ht}{2I}}\frac{2\pi}{T}\cos(\frac{2\pi}{T}t + \varphi)$;

At $t = 0$; $\theta' = 0 = A(\frac{-h}{2I})\sin\varphi + A\frac{2\pi}{T}\cos\varphi \Rightarrow \frac{A\,h}{2I}\sin\varphi = \frac{2\pi A}{T}\cos\varphi \Rightarrow \tan\varphi = \frac{4\pi I}{T\,h}$.

13.

A.

a) $I = I_{wheel} + I_{particles} = m R^2 + 2m R^2 = 3 m R^2$. **b)** $ME = KE + GPE + EPE = \frac{1}{2}I\theta'^2 + 0 + \frac{1}{2}k x^2 = \frac{3}{2}m R^2\theta'^2 + \frac{1}{2}k r^2\theta^2$.

c) No non-conservative forces ⇒ ME is conserved ⇒ $\frac{dM.E}{dt} = 0 \Rightarrow 3mR^2\theta'\theta'' + k r^2\theta\theta' = 0 \Rightarrow \theta'[3 m R^2\theta'' + k r^2\theta] = 0$

but $\theta' = 0$ is rejected $\Rightarrow \theta'' + \frac{k r^2}{3 m R^2}\theta = 0$

d) i. The differential equation has the form: $\theta'' + \omega_o^2\,\theta = 0 \Rightarrow$ simple harmonic motion.

$\Rightarrow \omega_o = \sqrt{\frac{k r^2}{3 m R^2}} \Rightarrow T_o = 2\pi\sqrt{\frac{3 m R^2}{k r^2}} = \frac{2\pi R}{r}\sqrt{\frac{3 m}{k}}$.

ii. As $r \to 0$, $T_o \to \infty$ (no oscillations).

iii. $T_o = \frac{5.23}{5} = 1.046$ s, but $T_o = \frac{2\pi R}{r}\sqrt{\frac{3m}{k}} \Rightarrow T_o^2 = (\frac{2\pi R}{r})^2(\frac{3m}{k}) \Rightarrow m = \frac{T_o^2 r^2 k}{12\pi^2 R^2} = \frac{(1.046)^2(0.06)^2(30)}{12\pi^2(0.1)^2} = 0.0997$ kg ≈ 100 g.

iv. T_o is minimum when r is maximum ⇒ r = R = 10 cm. $T_{o(min)} = 2\pi\sqrt{\frac{3(0.1)}{30}} = \underline{0.628\text{ s}}$.

e) $ME = \frac{3}{2}m R^2\theta_m'^2 = \frac{1}{2}k r^2\theta_m^2 \Rightarrow \theta_m^2 = \frac{3 m \theta_m'^2}{k} \Rightarrow \theta_m = \sqrt{\frac{3(0.1)(1)^2}{30}} = \underline{0.1\text{ rad}}$.

B.

a) system: (Wheel , S , S'). External forces; weighs $m_w\vec{g}$; $m_{S'}\vec{g}$; $m_S\vec{g}$, Reaction $\vec{R}$. $\mathcal{M}_{m\vec{g}} = \mathcal{M}_{\vec{R}} = 0$ (these forces meet (Δ)).

$\mathcal{M}_{m_{S'}\vec{g}} = -\mathcal{M}_{m_S\vec{g}}$ at any instant $\Rightarrow \sum\mathcal{M} = I\theta'' = 0 \Rightarrow \theta'' = 0$. But $\theta_0' = 1$ so the wheel continues in a uniform rotational motion

b) $\Delta\theta = 2\pi n = 10\pi = 31.4$ but $\Delta\theta = \theta' t \Rightarrow t = \frac{31.42}{1} = \underline{31.4\text{ s}}$.

C.

a) $\sum\mathcal{M} = \frac{d\sigma}{dt} = I\theta'' = \mathcal{M}_{m\vec{g}} + \mathcal{M}_{\vec{R}} + \mathcal{M}_{m_{S'}\vec{g}} + \mathcal{M}_{m_S\vec{g}} + \mathcal{M} \Rightarrow -h\,\theta' = I\theta'' \Rightarrow \theta'' + \frac{h}{I}\theta' = 0$.

b) i. $\theta' = e^{\frac{-t}{\tau}} \Rightarrow \theta'' = \frac{-1}{\tau}e^{\frac{-t}{\tau}} = \frac{-1}{\tau}\theta'$ substitute in the differential equation $\Rightarrow \frac{-1}{\tau}\theta' + \frac{h}{I}\theta' = 0$, but $\theta' \neq 0 \Rightarrow \frac{1}{\tau} = \frac{h}{I} \Rightarrow \tau = \frac{I}{h}$.

ii. $5\tau = 157 \Rightarrow \tau = 31.4$ s. $I = 3 m R^2 = 3(0.1)(0.1)^2 = 0.003$ kg.m² $\Rightarrow h = \frac{I}{\tau} = \underline{9.554 \times 10^{-5}\text{ kg/s}}$.

iii. $\Delta\theta = \int_0^{157}\theta'\,dt = \int_0^{157}e^{\frac{-ht}{I}}dt = -\frac{I}{h}e^{\frac{-ht}{I}}]_0^{157} = \frac{-3\times10^{-3}}{9.554\times10^{-5}}[e^{\frac{-(9.554\times10^{-5})(157)}{3\times10^{-3}}} - 1] = 31.4$ rad. $N = \frac{\Delta\theta}{2\pi} = \frac{31.4}{2\pi} = \underline{5\text{ cycles}}$.

14.

A.

a) $I = \frac{1}{2}mR^2 = \frac{1}{2}(2.7)(0.1)^2 \Rightarrow \underline{I = 0.0135\text{ kg.m}^2}$.

b) System: Disk . External forces: $m\vec{g}$: weight; $\vec{R}$: Reaction at axis; $\vec{T}_1$ and $\vec{T}_2$ tensions of spring. $\sum\vec{F}_{ext} = \vec{0}$

$\Rightarrow m\vec{g} + \vec{R} + \vec{T}_1 + \vec{T}_2 = \vec{0} \Rightarrow -mg + R - 2T = 0 \Rightarrow -mg + R = 2k(\Delta\ell) \Rightarrow -(2.7)(10) + 43 = 2(200)\Delta\ell \Rightarrow \Delta\ell = 0.04\text{ m} = \underline{4\text{ cm}}$.

B.

a) $M.E = K.E + G.P.E + E.P.E = \frac{1}{2}I\theta'^2 + 0 + \frac{1}{2}k(\Delta\ell + x)^2 + \frac{1}{2}k(\Delta\ell - x)^2$

$M.E = \frac{1}{2}I\theta'^2 + \frac{1}{2}k[(\Delta\ell^2 + x^2 + 2\Delta\ell\,x) + (\Delta\ell^2 + x^2 - 2\Delta\ell\,x)] = \frac{1}{2}I\theta'^2 + k(\Delta\ell)^2 + k\,x^2$.

b) No no-conservative forces ⟹ M.E is conserved $\Rightarrow \frac{dM.E}{dt} = 0 = I\theta'\theta'' + 0 + k R^2(2\theta\theta') = \theta'[I\theta'' + 2 k R^2\theta]$

but $\theta' = 0$ is rejected $\Rightarrow \frac{1}{2}m R^2\theta'' + 2 k R^2\theta = 0 \Rightarrow \theta'' + \frac{4k}{m}\theta = 0$.

c) i. $\theta' = -\omega_o\theta_m\sin(\omega_o t + \varphi) \Rightarrow \theta'' = -\omega_o^2\theta_m\cos(\omega_o t + \varphi) = -\omega_o^2\theta$ substitute in the differential equation:

$-\omega_o^2\theta + \frac{4k}{m}\theta = 0 \Rightarrow \theta[-\omega_o^2 + \frac{4k}{m}] = 0$, but $\theta = 0$ is rejected $\Rightarrow \omega_o^2 + \frac{4k}{m} = 0 \Rightarrow \omega_o = 2\sqrt{\frac{k}{m}}$, but $T_o = \frac{2\pi}{\omega_o} = \pi\sqrt{\frac{m}{k}} = \underline{0.36\text{ s}}$.

ii. $\theta_o = 0.05 = 0.1\cos\varphi \Rightarrow \cos\varphi = 0.5 \Rightarrow \varphi = 60°$ Or $\varphi = -60°$. $\theta' = -\omega_o\theta_m\sin(\omega_o t + \varphi)$. But $\theta_o' > 0 \Rightarrow \sin\varphi < 0 \Rightarrow \varphi = -\frac{\pi}{3}$ rad.

C.

a) $\sum\mathcal{M} = \frac{d\sigma}{dt}$ since I = constant $\Rightarrow \sum\mathcal{M} = I\theta'' \Rightarrow -2 R k x - h\theta' = I\theta'' \Rightarrow I\theta'' + 2 R k(R\theta) + h\theta' = 0 \Rightarrow \theta'' + \frac{h}{I}\theta' + \frac{2 k R^2}{I}\theta = 0$.

b) $p_{couple} = \frac{dM.E}{dt} = I\theta'\theta'' + 2 k R^2\theta\theta' = \theta'[I\theta'' + 2 k R^2\theta]$, from differential equation: $I\theta'' + 2 k R^2\theta = -h\theta'$

$\Rightarrow p_{couple} = \theta'[-h\theta'] = -h(\theta')^2$.

c) i. large damping since amplitude decreases to zero during about two periods. **ii.** T = 0.38 s . $\omega_o = \frac{2\pi}{0.38} = 16.53$ rad/s.

iii. $\theta_o = A\sin\varphi \Rightarrow A = \frac{0.1}{\sin\varphi}$. $\theta' = A[e^{\frac{-ht}{2I}}\omega\cos(\omega t + \varphi) + (\frac{-h}{2I}e^{\frac{-ht}{2I}}\sin(\omega t + \varphi))]$. $\theta_o' = 0 = A[\omega\cos\varphi - \frac{h}{2I}\sin\varphi] \Rightarrow \tan\varphi = \frac{2\omega I}{h} =$

$\frac{2(16.530)(0.0135)}{0.2} = 2.23 \Rightarrow \varphi = 65.8° = 1.145$ rad. $A = \frac{0.1}{0.91} = 0.11$ rad.

iv. $0.07 = 0.11e^{\frac{-h(0.05)}{2(0.0135)}}\sin(16.53(0.05) + 1.148] \Rightarrow 0.69 = e^{-h(1.85)} \Rightarrow \underline{h = 0.2\text{ kg/s}}$.

Chapter 6: Forced Oscillations- Resonance

1.

a) Forced mechanical oscillations.

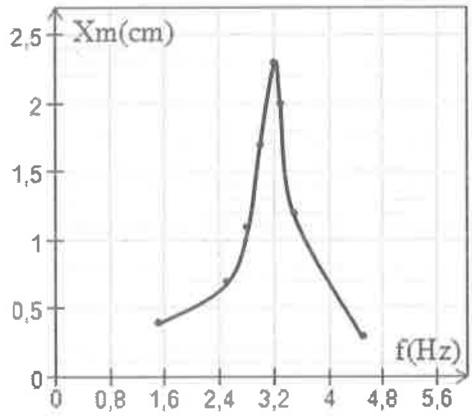
b) Amplitude resonance.

c) Largest damping: curve (c) since: the amplitude at resonance is the smallest . Resonance frequency : $f_c < f_a < f_b$
Curve (c) is more flattened than (a) and (b)

d) When damping is very slight, resonance is sharp and $f_E \approx f_o \Rightarrow \underline{f_o = 1.5\text{ Hz}}$.

e) The pendulum oscillates with a frequency $\underline{f = 1.75\text{ Hz}} = f_E$. Since in forced oscillations, the exciter imposes its frequency on the resonator.

2.



a) Exciter: motor ; Resonator: the mechanical oscillator (C)

b)

i. Figure

ii. Graphically X_m is maximum for f = 3.2 Hz ⇒ Resonance takes place and in this case the frequency of the exciter is approximately equal to the proper frequency of the resonator $\Rightarrow \underline{f_o \approx 3.2\text{ Hz}}$.
$\omega_o = 2\pi f_o = 6.4\pi$ rad/s.
$\omega_o = \sqrt{\frac{k}{m}} \Rightarrow (6.4\pi)^2 = \frac{80}{m} \Rightarrow m = 0.198\text{ kg} \approx \underline{200\text{ g}}$.

iii. No since resonance will not take place.

c) Type: free un-damped mechanical oscillations.
frequency: $f_o = 3.2$ Hz and $X_m = 1.2$ cm.

3.

a) The exciter imposes its frequency on the resonator (resonator oscillates with a frequency equals to that of the exciter). A oscillates with a frequency equals to $f_B \Rightarrow$ B is the exciter and A is the resonator.

b)

i. $f_B = 2$ Hz.

ii. Amplitude resonance.

iii. $f_{B1} = 1.8$ is closer to f_A than $f_{B2} \Rightarrow X_{m1} > X_{m2}$.

4.

a) $T_o = 2\pi\sqrt{\frac{m}{4k}} \Rightarrow T_o^2 = 4\pi^2\frac{m}{4k} \Rightarrow k = \frac{4\pi^2(1000)}{(0.8)^2} = \underline{15405.625\text{ N/m}}$.

b)

i. Exciter: humps (road) . Resonator: car.

ii. $V = \frac{d}{\Delta t} \Rightarrow \Delta t = \frac{d}{5}$ (S.I)

iii. $\Delta t = T_E$ since car receives energy from the road each time when it passes over a hump $T_E \approx T_o = \frac{d}{5} \Rightarrow d = 5(0.8) = \underline{4\text{ m}}$.

c)

i. $V' = \frac{d}{T_o'} = \frac{4}{1.2} = 3.33\text{ m/s} = \underline{12\text{ km/h}}$.

ii. $X_{m_2} << X_{m_1} \Rightarrow$ Shock absorber decreases the amplitude of oscillations ⇒ less damage.

5.

a) The frequency of the resonator is always equal to that of the exciter.

b)

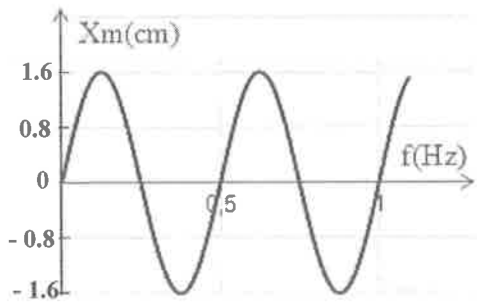
i. $X_{m_1} = 8.8$ cm.

ii. Amplitude resonance.

iii. When $f_E \approx f_o$, resonance takes place and the amplitude of oscillations is maximum ⇒ $f_o = 2.5$ Hz.

iv. $\frac{X_{m1}}{\sqrt{2}} = \frac{8.8}{\sqrt{2}} = 6.223 \Rightarrow 2.4\text{ Hz} < f < 2.6\text{ Hz}$.

c) Figure: for f = 2 Hz; $X_{m\,2} = 1.6$ cm.



d)

i. $m\vec{g}$ weight of the block; $\vec{N}$ normal reaction; $\vec{T}$ tension of the spring; $\vec{F}$ exciting force; $\vec{f}$ friction force.

ii. $\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{N} + \vec{f} + \vec{T} + \vec{F} = m\vec{a}$. project along x-axis:
$0 + 0 + f - k\,x + F = mx'' \Rightarrow f + F = mx'' + k\,x$
Amplitude resonance $\Rightarrow \omega = \omega_o \Rightarrow x = X_{m_1}\sin(\omega_o t)$
$\Rightarrow x' = \omega_o X_{m_1}\cos(\omega_o t) \Rightarrow x'' = -\omega_o^2 X_{m_1}\sin(\omega_o t) = -\omega_o^2 x$
$\Rightarrow f + F = m(-\omega_o^2 x) + kx$ { but $m\,\omega_o^2 = k$}
$\Rightarrow f + F = 0 \Rightarrow f = -F \Rightarrow \vec{f}$ and $\vec{F}$ have same magnitude.

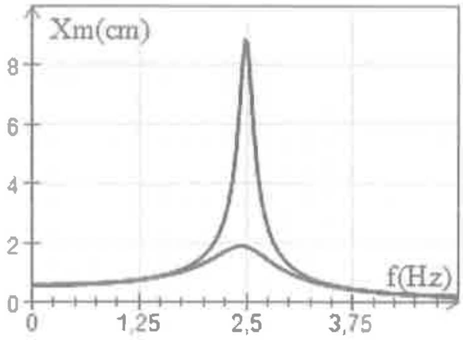
iii. $F = 1.4\cos(\omega t)$. $f = -h\,v = -h\,x' = -h\,X_{m_1}\omega\cos(\omega t)$

$f = F \Rightarrow hX_{m_1}(2\pi f) = 1.4 \Rightarrow h(8.8\times10^{-2})(2\pi\times 2.5) = 1.4 \Rightarrow \underline{h = 1.013\ kg/s}.$

e)

i. Figure (1) does not change since whatever the value of damping forces; the exciter imposes its frequency on the resonator.

ii. Figure.



6.

A.

a) $V_o = V_{max} = \omega_o X_m = 10(0.06) = \underline{0.6\ m/s}$. $\{\omega_o = \frac{2\pi}{T_o} = \frac{2\pi}{0.628} = 10\ rad/s\}$

OR: $ME = \frac{1}{2}kX_m^2 = \frac{1}{2}mV_o^2 \Rightarrow \frac{1}{2}(80)(0.06)^2 = \frac{1}{2}(0.8)V_o^2 \Rightarrow \underline{V_o = 0.6\ m/s}$.

b) $T_o = 2\pi\sqrt{\frac{m}{k}} \Rightarrow$ Factors: mass m and stiffness k.

c) The period of oscillations is the same in both figures, even though we change the initial conditions $\Rightarrow T_o$ depends on m and k and not on the initial conditions

d) $X_{m_a} = 0.1$ m and $X_{m_b} = 0.06$ m $\Rightarrow$ Factors: Initial conditions.

B.

a) $T_a = 0.63$ s and $T_b = 0.642$ s.

b) The coefficient of damping (h).

c) h, mass of S, time and X_{m_o}.

C.

a) $T_a = 0.4188$ s and $T_b = 0.628$ s. Factor: The period T_E of the exciter.

This type is called forced oscillations since the exciter imposes its period on the resonator.

b) i. F_m, h, k, m, T_E (or ω).

ii. Amplitude resonance: $T_E = T_o \Rightarrow \omega = \omega_o \Rightarrow X_m = \frac{F_m}{\omega_o h}$.

iii. In figure (5-b): $T = 0.628\ s = T_o$ and $0.0625 = \frac{F_m}{10\ h} \Rightarrow F_m = 0.625\ h$ (S.I).

iv. $\omega = \frac{2\pi}{T_a} = \frac{2\pi}{0.4188} = 15$ rad/s. (substitute in X_m) $\Rightarrow 0.018 = \frac{F_m}{\sqrt{15^2h^2 + (0.8)^2[10^2-15^2]^2}} \Rightarrow F_m^2 = 0.0729h^2 + 3.24$ (S.I)

Solve the two equations: $\underline{h = 2.975\ kg/s}$ and $F_m = \underline{1.86\ N}$.

7.

A.

a) Exciter: motor Resonator: oscillator (spring-block)

b) $p_{\vec{f}} = \vec{f}.\vec{v} = -h\,v^2$. $p_{\vec{F}} = \vec{F}.\vec{v} = F_m\cos(\omega t)\vec{i}.\,v\,\vec{i} = v\,F_m\cos(\omega t)$.

$ME = \frac{1}{2}k\,x^2 + \frac{1}{2}m\,v^2$; $\frac{dME}{dt} = k\,x\,x' + m\,v\,v' = p_{\vec{F}} + p_{\vec{f}} \Rightarrow k\,x\,x' + m\,v\,v' = -h\,v^2 + v\,F_m\cos(\omega t)$.

$\Rightarrow x'[k\,x + m\,x'' + h\,x'] = x'F_m\cos(\omega t) \Rightarrow k\,x + m\,x'' + h\,x' = F_m\cos(\omega t) \Rightarrow x'' + \frac{h}{m}x' + \frac{k}{m}x = \frac{F_m\cos(\omega t)}{m}$.

B.

a) since $\omega \approx \omega_o$.

b) $x' = \omega_o X_m\cos(\omega_o t)$ and $x'' = -\omega_o^2 X_m\sin(\omega_o t) = -\omega_o^2 x$ substitute in differential equation

$-\omega_o^2 x + \frac{h}{m}\omega_o X_m\cos(\omega_o t) + \frac{k}{m}x = \frac{F_m\cos(\omega_o t)}{m}$ but $\omega_o^2 = \frac{k}{m} \Rightarrow -\omega_o^2 x + \frac{k}{m}x = 0 \Rightarrow$

$\frac{h}{m}\omega_o X_m\cos(\omega_o t) = \frac{F_m\cos(\omega_o t)}{m} \Rightarrow h = \frac{F_m}{\omega_o X_m} = \frac{F_m\sqrt{m}}{\sqrt{k}\,X_m} = \frac{F_m}{X_m}\sqrt{\frac{m}{k}}$.

c) $h = \frac{0.175}{0.12}\sqrt{\frac{0.25}{90}} = 0.077$ kg/s.

d) $\frac{1}{2}k\,X_m^2 = \frac{1}{2}(m\,\omega_o^2)X_m^2$, but $\omega_o X_m = V_m \Rightarrow \frac{1}{2}k\,X_m^2 = \frac{1}{2}mV_m^2$.

e) $ME = \frac{1}{2}m\,v^2 + \frac{1}{2}k\,x^2 = \frac{1}{2}m\,\omega_o^2X_m^2\cos^2(\omega_o t) + \frac{1}{2}k\,X_m^2\sin^2(\omega_o t)$, but $\omega = \omega_0 = \sqrt{\frac{k}{m}} \Rightarrow m\omega_o^2 = k$

$\Rightarrow ME = \frac{1}{2}k\,X_m^2\cos^2(\omega_o t) + \frac{1}{2}k\,X_m^2\sin^2(\omega_o t) = \frac{1}{2}kX_m^2[\cos^2(\omega_o t) + \sin^2(\omega_o t)] = \frac{1}{2}k\,X_m^2 =$ constant

C.

a) $\frac{1}{2}mV_m^2 = \frac{1}{2}m\,\omega^2X_m^2$ while $\frac{1}{2}kX_m^2 = \frac{1}{2}m\omega_o^2X_m^2$, but $\omega \neq \omega_o \Rightarrow \frac{1}{2}m\,\omega^2X_m^2 \neq \frac{1}{2}m\,\omega_o^2X_m^2 \Rightarrow \frac{1}{2}m\,V_m^2 \neq \frac{1}{2}kX_m^2$

b) $ME = \frac{1}{2}kX_m^2\sin^2(\omega t) + \frac{1}{2}m\,\omega^2X_m^2\cos^2(\omega t) = \frac{1}{2}kX_m^2\sin^2(\omega t) + \frac{1}{2}m\,\omega^2X_m^2[1 - \sin^2(\omega t)]$

$ME = \sin^2(\omega t)[\frac{1}{2}k\,X_m^2 - \frac{1}{2}m\,\omega^2\,X_m^2] + \frac{1}{2}m\,\omega^2\,X_m^2 = \sin^2(\omega t)[\frac{1}{2}m\,\omega_o^2X_m^2 - \frac{1}{2}m\,\omega^2X_m^2] + \frac{1}{2}m\,\omega^2\,X_m^2$

$ME = \frac{1}{2}mX_m^2\sin^2(\omega t)[\omega_o^2 - \omega^2] + \frac{1}{2}m\,\omega^2X_m^2 \neq$ constant since $\omega_o \neq \omega$.

Chapter 7: Special Relativity

1.

a)

i. $V_{signal} = c - 0.9\,c = 0.1\,c$.

ii. $V_{signal} = c + 0.9\,c = 1.9\,c$.

b) $V_{signal} = c$ in all inertial frames, whatever the speed of the spaceship and the direction of its motion.

2. The observer on the ground measures $\Delta t_o = \underline{4.5\ s}$.

The astronaut measures $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{4.5}{\sqrt{1-(\frac{0.84\,c}{c})^2}} = \underline{8.3\ s}$.

3. If Fadi observes AB = 8 m = L, he will see a squared plate. Then the plate moves parallel to AB

$\Rightarrow$ Line of action of $\vec{v}$ is parallel to AB.

$L = L_o\sqrt{1-\frac{v^2}{c^2}} \Rightarrow 8 = 10\sqrt{1-\frac{v^2}{c^2}} \Rightarrow (0.8)^2 = 1-(\frac{v}{c})^2 \Rightarrow (\frac{v}{c})^2 = 0.36 \Rightarrow \underline{v = 0.6\,c}$.

4. $d' = d\sqrt{1-\frac{v^2}{c^2}} = 6400\sqrt{1-\frac{(0.8\,c)^2}{c^2}} = \underline{3840\ Km}$.

5. The astronaut measures the proper length of the stick since he is at rest relative to the stick.

The horizontal and the vertical components of the stick measured by the astronaut are x_o = OA cos 60 = (0.5) (0.5) = 0.25 m; and y_o = OA sin 60 = $0.5\frac{\sqrt{3}}{2} = 0.25\sqrt{3}$ m respectively.

The observer, at the Earth, measures the same horizontal component (x_o = 0.25 m) of the stick since no motion along the horizontal axis and only the vertical component of the stick is contracted relative to the observer.

It becomes $y = y_o\sqrt{1-\frac{v^2}{c^2}} = 0.25\sqrt{3}\sqrt{1-\frac{(0.943\,c)^2}{c^2}} \cong 0.1441$ m.

The angle between the stick and the horizontal axis as seen by the observer is $\tan\alpha = \frac{y}{x_o}$

$\tan\alpha = \frac{0.1441}{0.25} = 0.5764 \Rightarrow \underline{\alpha \cong 30^o}$.

6.

a) Relativistic: $KE = m\,c^2[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1] \Rightarrow 10^{-13} = 9.1\times10^{-31}\,(9\times10^{16})\,[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1] \Rightarrow$

$\frac{1000}{819} + 1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 1 - \frac{v^2}{c^2} = 0.20272271 \Rightarrow \underline{v = 0.8929\,c}$.

b) Classical: $KE = \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2\,K.E}{m} = \frac{2\,(10^{-13})}{9.1\times10^{-31}} \Rightarrow v = 4.688\times10^8$ m/s $\cong \underline{1.5627\,c}$.

7.

a) $KE_{relativistic} = E_{rest} \Rightarrow m\,c^2\,[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1] = m\,c^2 \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 = 1 \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = 0.5 \Rightarrow \underline{v = 0.87\,c}$.

b) $m\,c^2\,[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1] = k\,m\,c^2 \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 = k \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{k+1} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{(k+1)^2}$

$v = c\sqrt{1-\frac{1}{(k+1)^2}} \Rightarrow \lim_{k\to\infty} v = c\sqrt{1-0} = \underline{c}$.

8.

a) **i.** System: Lama

External forces: $m\vec{g}$: weight; $\vec{N}$: normal reaction.

$\sum\vec{F}_{ext} = \vec{0} \Rightarrow m\vec{g} + \vec{N} = \vec{0} \Rightarrow m\,\vec{g} = -\vec{N} \Rightarrow mg = N = 600\ N \Rightarrow m = \frac{w}{g} = \frac{600}{10} = \underline{60\ kg}$.

ii. $\sum\vec{F}_{ext} = m\vec{g} + \vec{N} = m\vec{a}$ project along the y-axis oriented positively upward $\Rightarrow$ $-mg + N = ma$

1st phase: $-600 + 660 = 60(a) \Rightarrow \underline{a = 1\ m/s^2}$.
2nd phase: $-600 + 600 = 60(a) \Rightarrow \underline{a = 0}$.
3rd phase: $-600 + 555 = 60(a) \Rightarrow \underline{a = -0.75\ m/s^2}$.

iii. The elevator is an inertial frame in the <u>second phase</u> since a = 0 in this phase.

b) $\sum\vec{F}_{ext} = m\vec{g} + \vec{N} = m\vec{a}$ project along direction of motion $\Rightarrow$ $mg - N = ma \Rightarrow N = mg - ma$ = apparent weight
$N = 600 - 60(1.5) = \underline{510\ N}$.

9.

a) $V = \frac{d}{t} = \frac{d}{\Delta t_o} \Rightarrow d = (0.9994\ c)(2.2 \times 10^{-6}) = \underline{659.604\ m}$.

b) An observer on the ground measures Δt since he is not in the frame of the event, and measures the proper length L_o since he is at rest relative to the covered distance.

$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.2\ \mu s}{\sqrt{1-(\frac{0.9994\ c}{c})^2}} = \underline{63.52\ \mu s}$.

$d = d_o\sqrt{1-\frac{v^2}{c^2}} \Rightarrow 659.604 = d_o(0.034636) \Rightarrow \underline{d_o = 19043.98\ m}$.

OR: $v = \frac{d_o}{\Delta t} \Rightarrow d_o = (0.9994\ c)(63.52 \times 10^{-6}) = 19044.57$ m. $d_o > 15000$ m $\Rightarrow$ The muon <u>reaches</u> the sea-level.

10.

a) The scientist is at rest relative to the distance so he measures $L_o = 45$ Km. But he measures Δt. $v = \frac{L_o}{\Delta t}$

$\Rightarrow \Delta t = \frac{L_o}{v} = \frac{45 \times 10^3}{(0.9954)(3 \times 10^8)} = \underline{1.507 \times 10^{-4}\ s}$.

b) The particle is in motion relative to the distance, then it measures L

$L = L_o\sqrt{1-\frac{v^2}{c^2}} = (45\ Km)\sqrt{1-\frac{(0.9954\ c)^2}{c^2}} = \underline{4.311\ Km}$.

c) 1st method: $\Delta t = \frac{\Delta t_o}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \Delta t_o = \Delta t\sqrt{1-\frac{v^2}{c^2}} = (1.507 \times 10^{-4})(0.0958) = \underline{1.444 \times 10^{-5}\ s}$

2nd method: The particle measures L and Δt_o $\Rightarrow$ $v = \frac{L}{\Delta t_0} \Rightarrow \Delta t_o = \frac{4.311 \times 10^3}{(0.9954)(3 \times 10^8)} = \underline{1.444 \times 10^{-5}\ s}$.

11. Relative to the car (non-inertial frame), each person is at rest $\Rightarrow \Sigma\vec{F}_{ext} + \vec{f}_{fictitious} = \vec{0}$
$\Rightarrow m\vec{g} + \vec{R} + \vec{f}_{fictitious} = \vec{0}$. (m is the mass of a person, $\vec{R}$ is the reaction force acting on the person at the top).
The fictitious force $\vec{f}_{fictitious} = -m\vec{a} = -m\vec{a}_n \Rightarrow$ at the top, the direction of $\vec{f}_{fictitious}$ is vertically upward which is opposite to that of the weight of the person. Project along the vertical axis oriented positively upward: $-mg - R + f_{fictitious} = 0$
The person does not fall if $R > 0 \Rightarrow f_{fictitious} > mg$.

Chapter 8: Electromagnetic Induction

1.

a) Induced current flows in the coil in a direction such that the face D becomes a north face to attract the south pole of the magnet $\Rightarrow$ i_{ind} crosses the coil from A to D.

b) The created couple of electromagnetic forces on the loop tend to rotate the loop in the anti-clockwise sense. According to right hand rule, the induced current flows in the loop in the clock wise sense.

c) The induced current flows in the loop in a direction such that the electromagnetic forces created due to this current tend to rotate the loop opposite to the real sense $\Rightarrow$ i_{ind} flows in the loop in the clockwise sense.

d) The created electromagnetic force at the midpoint of the rod has a direction opposite to that of the motion of the rod $\Rightarrow$ This force is directed horizontally to the right. According to right hand rule, the induced current flows in clock wise sense.

2.

a) Electromagnetic induction. While approaching the magnet, the magnitude of the magnetic field crossing the loop increases at all points $\Rightarrow$ The loop is crossed by a variable magnetic flux $\Rightarrow$ It is the seat of induced electromotive force.

b) **i.** In figure **(a)**, induced current flows in the loop since it forms a closed circuit.

ii. Induced current flows in the loop in a direction such that its upper face acts as a north face to repel the approaching north pole. According to right hand rule i_{ind} flows in the loop in the anti-clockwise sense.

c) The magnet is at rest relative to the loop $\Rightarrow$ No variation in magnetic flux $\Rightarrow$ $e = 0 \Rightarrow i = 0$.

3.

a) $\varphi = \vec{B}.\vec{n}\,S = BS\cos(\vec{n},\vec{B}) \Rightarrow -4.8 \times 10^{-3} = 0.2(a^2)\cos(180) \Rightarrow a = 0.1549\text{ m} \cong$ 15.5 cm.

b) No, since no variation in the magnetic flux crossing the loop during its rotation.

4.

a) **i.** Horizontally to the right.

ii. According to Lenz's law $\vec{B}_{ind}$ has a direction opposite to that of $\vec{B}$ since the bar magnet is approaching the coil $\Rightarrow$ $\vec{B}$ is directed horizontally to the left $\Rightarrow$ (p) is a north pole since the field lines go out from the north pole of a magnet.

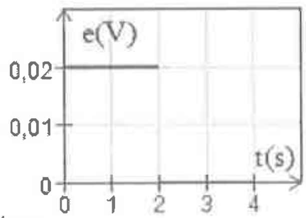
b) Induced current flows in the coil in a direction such that its right face acts as a south face to attract the outgoing north pole $\Rightarrow$ i_{ind} crosses the coil from right to left $\Rightarrow$ LED (1) only glows.

5.

a) **i.** $\varphi = \vec{B}.\vec{n}\,S = B\,S\cos(180) = -B\,S$.

For $0 \le t \le 2$ s: $\varphi = -(2t+1)(0.01) = -0.02\,t - 0.01$ (S.I). For $2\text{ s} \le t \le 4$ s:
$\varphi = -(5)(0.01) = -0.05$ Wb.

ii. $0 \le t \le 2\text{ s}: e = -\frac{d\varphi}{dt} \Rightarrow e = 0.02\text{ V} \Rightarrow i > 0 \Rightarrow$ clockwise;
$2 \le t \le 4\text{ (s)}: e = 0 \Rightarrow i_{ind} = 0$.



b) Figure.

c) **i.** $P_{total} = e.i = (0.02)(0.02) = 4 \times 10^{-4}$ W; $P_{lost} = r\,i^2 = (0.2)(0.02)^2$
$\Rightarrow P_{lost} = 8 \times 10^{-5}$ W. $P_{useful} = P_{total} - P_{lost} = (4 \times 10^{-4}) - (8 \times 10^{-5}) = 3.2 \times 10^{-4}$ W.

ii. Across the generator : $u_{AB} = r\,i - e \Rightarrow u_{BA} = -r\,i + e = -(0.2)(0.02) + 0.02 = 0.016$ V.
Also : $P_{useful} = i\,u_{BA} \Rightarrow 3.2 \times 10^{-4} = (0.02)\,u_{BA} \Rightarrow u_{BA} = 0.016$ V.

iii) $u_{BA} = R\,i \Rightarrow 0.016 = R(0.02) \Rightarrow R = 0.8\ \Omega$.

6.

A.

a) The magnitude of the magnetic field varies, so the magnetic flux that crosses the loop varies causing an establishment of an induced e.m.f. The loop is closed so an induced current flows in it. B increases $\Rightarrow$ induced current Flows in the loop such that the induced magnetic field created by it is opposite to $\vec{B}$, so (i) is clockwise.

b) $\varphi = \vec{B}.\vec{n}.S = B\,S\cos(0) = B\,S$ but $S = (0.3^2) = 0.09\text{ m}^2 \Rightarrow \varphi = 0.09\,B$ (B in T and φ in Wb).

c) $e = -\frac{d\varphi}{dt} = -0.09\frac{dB}{dt}$; $(0 \to 2\text{ ms}): e = -0.09\left(\frac{0.003}{2 \times 10^{-3}}\right) = -0.135\text{ V} < 0 \Rightarrow i < 0 \Rightarrow$ clockwise.

$(2 \to 4\text{ ms}): e = 0$; $(4 \to 7\text{ ms}): e = -0.09\left(\frac{0 - 0.003}{(7-4) \times 10^{-3}}\right) = 0.09\text{ V} > 0 \Rightarrow i > 0 \Rightarrow$ anti-clockwise.

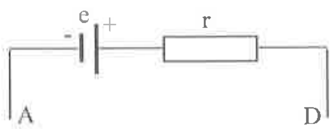
d) $i = \frac{e}{r} \Rightarrow (0 \to 2\text{ ms}): i = \frac{-0.135}{3} = -0.045$ A; $(2 \to 4\text{ ms}): i = 0$; $(4 \to 7\text{ ms}): i = 0.03$ A.

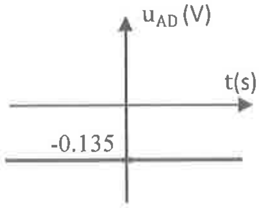
e) $P_{total} = e.\,i = (-0.135)(-4.5 \times 10^{-2}) = 6.075 \times 10^{-3}$ W; $P_{lost} = r\,i^2 = 3 \times (-4.5 \times 10^{-2}) = 6.075 \times 10^{-3}$ W.

B.

a) Figure

b) Oscilloscope measures $u_{AD} = (-r\,i) + e$
$u_{AD} = 0 + e = -0.135$ V.





7.

a) Left face : South; Right face : North

b) L glows during [0, 2s] since (i) is variable $\Rightarrow$ B is variable $\Rightarrow$ C is crossed by a variable magnetic flux $\Rightarrow$ C is the seat of induced e.m.f $\Rightarrow$ current flows in the lamp since the circuit is closed.

c) $\varphi = N\,B\,S\cos(\vec{n},\vec{B}) = (100)(B)(10 \times 10^{-4})\cos(0) = 0.1\,B$;
$[0, 2s]$: $\varphi = 0.1\,(0.02\,i) \Rightarrow \varphi = 2 \times 10^{-3}\,i$ But during $[0,\ 2s]$: $i = 1.5\,t \Rightarrow$ $\varphi = 0.003\,t$ (S.I).

d) $e = -\frac{d\varphi}{dt}$

i. $[0,\ 2s]$: $e = -0.003$ V $\Rightarrow$ $i = \frac{e}{R} = \frac{-3 \times 10^{-3}}{2} = -1.5 \times 10^{-3}$ A < 0 $\Rightarrow$ i flows in the lamp from left to right.

ii. $[2s,\ 5s]$: $e = 0$ (since φ = constant) $\Rightarrow$ $i = 0$.

e) B decreases, then (C) is crossed by a varying magnetic flux, so it becomes the seat of e.m.f. According to Lenz's law, induced current flows in (C) in a direction such that the created induced magnetic field has a direction same as the original one to oppose its decrease . According to R.H.R $\Rightarrow$ i_{ind} flows in the lamp from right to left.

8.

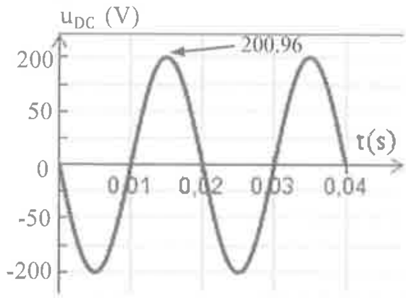
a) ω = constant $\Rightarrow$ $\theta = \omega\,t + \theta_0$ but $\theta_0 = 0 \Rightarrow$ $\theta = 100\,\pi\,t$ (S.I).

b) $\varphi = \vec{B}.\vec{n}\,S = B\,S\cos(\vec{n},\vec{B}) = 100\,(0.2)\,(32 \times 10^{-4})\cos\theta$
$\Rightarrow \varphi = 0.64\cos(100\,\pi\,t)$ (S.I).

c) $e = -\frac{d\varphi}{dt} = -0.64\,[-100\,\pi\sin(100\,\pi t)] = 64\,\pi\sin(100\,\pi\,t)$ (S.I).

d) i. $u_{DC} = r\,i - e$ But $i = 0 \Rightarrow u_{DC} = -e = -64\,\pi\sin(100\,\pi\,t)$; (S.I)
u_{DC} is an alternating sinusoidal voltage.

ii. Figure



9.

a) i. $T = 6.28$ s $\Rightarrow$ $\omega = \frac{2\pi}{T} = \frac{2\pi}{6.28} = 1$ rad/s.

ii. $e_{max} = 20$ V $\Rightarrow$ $e = 20\cos(t)$ S.I.

b) $e = -\frac{d\varphi}{dt} \Rightarrow \varphi = -\int e\,dt$; $\varphi = -\int 20\cos(t)\,dt = -20\sin(t) + \text{constant}$
{ At $t = 0$; $\varphi = N\,B\,S\cos\left(\frac{\pi}{2}\right) = 0$; $0 = -20\sin 0 + \text{constant}$ $\Rightarrow$ constant = 0 } $\Rightarrow$ $\varphi = -20\sin(t)$ S.I

c) $\varphi = N\,B\,S\cos\theta$ { But $\theta = \omega\,t + \theta_0 = 1\,t + \frac{\pi}{2}$ }; $\varphi = N\,B\,S\cos(t + \frac{\pi}{2})$; $\varphi = -N\,B\,S\sin(t)$
By comparison $20 = N\,B\,S \Rightarrow 20 = 1000\,(B)\,(\pi\,(0.12)^2)$; $B = 0.4423$ T.

10.

A.

a) B is variable $\Rightarrow$ coil is crossed by a variable magnetic flux $\Rightarrow$ It is the seat of induced e.m.f.

b) Electromagnetic induction.

c) $\varphi = N\,B\,S\cos(0) = N\,B\,S$; $e = -\frac{d\varphi}{dt} = -\left[N\,S\,\frac{dB}{dt}\right] \Rightarrow e = -N\,S\,\alpha$.

d) i. $u_{DC} = R\,i$ But $u_{DC} > 0 \Rightarrow i > 0 \Rightarrow$ clockwise.

ii. According to R.H.R. the induced magnetic field $\vec{B}_{ind}$ is directed horizontally to the right, in the direction of $\vec{B}$.
According to Lenz's law, $\vec{B}_{ind}$ has the same direction as $\vec{B}$ if $\|\vec{B}\|$ decreases (to oppose its decrease).

iii. 1. Graph (a) : $u_{DC} = -(r\,i) + e = 8$ V $\Rightarrow e = 8$ V.

2. Graph (b) : $u_{DC} = -(r\,i) + e \Rightarrow 6 = -2\,i + 8 \Rightarrow i = 1$ A; $u_{DC} = R\,i \Rightarrow 6 = R\,(1) \Rightarrow R = 6\,\Omega$.

iv. $e = -N\,S\,\alpha \Rightarrow 8 = -(1275)(314 \times 10^{-4})\,\alpha \Rightarrow \alpha = -0.2$ T/s.

B.

a) $\varphi = N\,B\,S\cos(\theta) \Rightarrow \varphi = N\,B\,S\cos(\omega\,t)$; $e = -\frac{d\varphi}{dt} = N\,B\,S\,\omega\sin(\omega\,t)$.

b) i. $\varphi = 0$ for the first time when $\omega\,t = \frac{\pi}{2}$ rad $\Rightarrow t_1 = \frac{\pi}{2\pi} = 0.5$ s .

ii. $e_{(0.5)} = N\,B\,S\,\theta'\sin(\pi\,(0.5)) \Rightarrow e_{(0.5)} = N\,B\,S\,\theta' \neq 0$; $e_{(0.5)} = (1275)(0.4)(314 \times 10^{-4})(\pi) \cong 50.28$ V.

c) $t_2 = 1$ s $\Rightarrow e_{(1)} = N\,B\,S\,\omega\sin(\pi\,(1)) \Rightarrow e = 0$.
$\varphi_1 = N\,B\,S\cos(\omega\,t) = (1275)(0.4)(314 \times 10^{-4})\cos(\pi\,(1)) \Rightarrow \varphi_1 = -160136$ Wb.

d) $\varphi_{0.5} = N\,B\,S\cos\left(\frac{\pi}{2}\right) = 0$ and $e_{(0.5)} = 50.28$ V; $\varphi_1 = -160136$ Wb and $e_1 = 0 \Rightarrow$ e and φ are not directly proportional.

11.

a) $\varphi = B\,S\cos(0) \Rightarrow \varphi = B\,\ell\,x$.

b) x variable $\Rightarrow$ The circuit is crossed by a variable magnetic flux $\Rightarrow$ MN is the seat of induced e.m.f. The circuit MNca is closed $\Rightarrow$ induced current flows in it. According to Lenz's law the direction of the electromagnetic force created due to i_{ind} is horizontally to the left to oppose the motion of MN $\Rightarrow$ i_{ind} crosses MN from M to N (clockwise).

c) Current flows out from N $\Rightarrow$ N : positive and M : negative

d) $e = -\frac{d\varphi}{dt} = -\frac{d(B\ell x)}{dt} = -B\ell\frac{dx}{dt} \Rightarrow e = \underline{-B\ell v}$.

e) **i.** $v_0 = 8e^0 \Rightarrow v_0 = \underline{8\ m/s}$. Decelerated motion.

ii. $i = \frac{e}{r+R} = \frac{-B\ell v}{r+R} = \frac{-(0.8)(0.79)}{(0.8+9.2)}[8e^{-0.33t}] \Rightarrow i = \underline{-0.51e^{-0.33t}}$ (S.I.)

iii. 1) $v_{(3)} = 8e^{-0.33(3)} \Rightarrow V_{(3)} = \underline{2.97\ m/s}$; $e_{(3)} = -B\ell v_{(3)} = (0.8)(0.79)(2.97) = \underline{-1.88\ V}$.

2) $i_{(3)} = \frac{e}{r+R} = \frac{-1.88}{10} = -0.188\ A \Rightarrow u_{NM} = (ri) - e = 0.8(-0.188) + 1.88 = \underline{1.73\ V}$.

$F_{e.m(3)} = i_{(3)}\ell B\sin(90) = -(0.188)(0.79)(0.8) = \underline{0.12\ N}$.

12.

a) $\varphi = \vec{B}.\vec{n}S = BS\cos(0) = \underline{B\ell x}$.

b) $e = -\frac{d\varphi}{dt} \Rightarrow e = \underline{-B\ell v}$; $e = -0.8(0.78)(0.8) \Rightarrow e = \underline{-0.5\ V}$.

c) $R_{eq} = \frac{R_1+R_2}{R_1R_2} = \frac{4\times\frac{20}{3}}{4+\frac{20}{3}} = \underline{2.5\ \Omega}$.

d) $i = \frac{e}{R_{eq}} = \frac{-0.5}{2.5} \Rightarrow i = \underline{-0.2\ A} < 0 \Rightarrow$ i crosses the rod from M to N.

e) $u_{NM} = (ri) - e = 0 - (-0.5) = 0.5\ V$; $u_{NM} = -R_2 i_2 \Rightarrow 0.5 = -\left(\frac{20}{3}\right)i_2 \Rightarrow i_2 = \underline{-0.075\ A}$.

$u_{NM} = -R_1 i_1 \Rightarrow 0.5 = -(4)i_1 \Rightarrow i_1 = \underline{-0.125\ A}$.

13.

A.

a) During the motion of MN, the area of the circuit ACNM varies then it is crossed by a variable magnetic flux. The circuit is open , then no current flowing in it.

b) $\varphi = BS\cos(180) = -BS = B\ell x$; $e = -\frac{d\varphi}{dt} = -\left[-B\ell\frac{dx}{dt}\right] \Rightarrow e = \underline{B\ell v}$.

c) $u_{AC} = -(ri) + e = 0 + B\ell v \Rightarrow u_{AC} = B\ell v$.

d) system : MN; External forces : $m\vec{g}$: weight

$\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} = m\vec{a} \Rightarrow \vec{a} = \vec{g} = g\vec{i} \Rightarrow a = g \Rightarrow v = gt + v_0$ but $v_0 = 0$ (starts from rest) $\Rightarrow v = gt$

substitute in $u_{AC} \Rightarrow u_{AC} = B\ell g$.

e) Graphically : $u_{AC} = \alpha t$ where α is the slope : $\alpha = \frac{0.1}{0.2} = 0.5\ V/s \Rightarrow \alpha = 0.5 = B\ell g \Rightarrow 0.5 = (0.5)(0.1)g$

$\Rightarrow g = \underline{10\ m/s^2}$.

B.

a) **i.** $u_{AC} + u_{CN} + u_{NM} + u_{MA} = 0$; $(Ri) + 0 + (ri) - e + 0 = 0 \Rightarrow i = \frac{e}{r+R} = \frac{B\ell v}{r+R}$.

ii. $v > 0 \Rightarrow i > 0 \Rightarrow$ clockwise.

According to Lenz's law, i_{ind} flows in the circuit in a direction such that the direction of the created electromagnetic force is opposite to $\vec{v}$ (vertically upwards) $\Rightarrow$ (i) cross MN from N to M (clockwise).

b) $u_{AC} = Ri = R\left(\frac{B\ell v}{r+R}\right) \Rightarrow u_{AC} = \frac{B\ell vR}{r+R}$.

c) **i.** $u_{AC\ max} = \underline{0.3\ V}$. **ii.** $u_{AC\ max} = \frac{B\ell v_{max}R}{r+R} \Rightarrow 0.3 = \frac{0.5(0.1)v_{max}(1.5)}{0.5+1.5} \Rightarrow v_{max} = \underline{8\ m/s}$.

d) $F_{em} = i\ell B\sin(90) = i\ell B = \frac{B\ell v}{r+R}(\ell B) \Rightarrow F_{em} = \frac{B^2\ell^2 v}{r+R}$.

e) **i.** $\Sigma\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{F}_{e.m} = m\vec{a}$ project along x-axis $(mg) - F_{e.m} = ma \Rightarrow a = g - \frac{F_{e.m}}{m} \Rightarrow a = g - \frac{B^2\ell^2 v}{m(r+R)}$.

ii. when $v = v_{max}$ = constant ; $\underline{a = 0}$.

iii. $0 = g - \frac{B^2\ell^2 v}{m(r+R)} \Rightarrow g = \frac{B^2\ell^2 v_{max}}{m(r+R)} \Rightarrow v_{max} = \frac{mg(r+R)}{B^2\ell^2} \Rightarrow 8 = \frac{(10^{-3})g(2)}{(0.5)^2(0.1)^2} \Rightarrow g = \underline{10\ m/s^2}$.

14.

A.

a) During the motion of the rod, the area of the circuit increases, then the circuit is crossed by a variable magnetic flux and so it becomes the seat of induced e.m.f. The circuit is closed. According to Lenz's law, the induced current i_{ind} flows in the circuit in a direction such that the created electromagnetic force has a direction opposite to $\vec{v}$ (along the negative direction) $\Rightarrow$ According to R.H.R , i_{ind} crosses the rod from M to N (clockwise).

b) $\varphi = BS\cos(0) = +BS = +B\ell x$; $e = -\frac{d\varphi}{dt} = -\left[-B\ell\frac{dx}{dt}\right] \Rightarrow e = \underline{-B\ell v}$; $v > 0 \Rightarrow e < 0 \Rightarrow i_{ind} < 0 \Rightarrow$ clockwise.

c) $F_{e.m} = i\vec{\ell}\times\vec{B} = i\ell B\sin(90) = i\ell B = \frac{e\ell B}{R} = \frac{B\ell v(B\ell)}{R} \Rightarrow F_{em} = \frac{B^2\ell^2 v}{R}$.

d) $\vec{F}_{e.m}$ is opposite to $\vec{v}$ at any instant $\Rightarrow \vec{F}_{e.m}$ tends to resist the motion of MN $\Rightarrow$ Free damped mechanical oscillations.

B.

a) $F_{e.m} = \underline{0}$ since v = 0 and i = $\underline{0}$.

b) $\vec{T} = -k X_m \vec{i}$. $\vec{T}$ is directed <u>horizontally to the left</u> (towards equilibrium position of MN).

c) $W_{lamp} = \frac{1}{2} k (\Delta \ell)^2 = \frac{1}{2}(100)(0.1)^2 = \underline{0.5\ J}$.

C. i_{ind} crosses MN from N to M (<u>anti-clockwise</u>); $\vec{F}_{em}$ is directed <u>horizontally to the right</u>.

D.

a) $u_{NM} + u_{MP} + u_{PQ} + u_{QN} = 0 \Rightarrow (i r) - e + 0 + (i R) + 0 = 0$; $(B \ell v) + (i R) = 0$.

b) $\Sigma \vec{F}_{ext} = m \vec{a} \Rightarrow m \vec{g} + \vec{N}_N + \vec{N}_M + \vec{F}_{e.m} + \vec{T} = m \vec{a} \Rightarrow (i \ell B) - (k x) = m a = m x'' \Rightarrow m x'' + k x + i \ell B = 0$.

c) $i = \frac{e}{R} = \frac{-B \ell v}{R} \Rightarrow x'' + \frac{B^2 \ell^2}{m R} x' + \frac{k}{m} x = 0$.

d) $M E = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 \Rightarrow k x x' + m v v' = i \ell B v$

$v(m x'' + k x = i \ell B = 0)$, but $v = 0$ is rejected $\Rightarrow x'' + \frac{B^2 \ell^2}{m R} x' + \frac{k}{m} x = 0$

e) At $t = 0$ $x = 0.1 \Rightarrow 0.1 = A \sin\varphi$; $v = x' = A\left[-\frac{B^2 \ell^2}{2 m R} \sin(\omega t + \varphi) + \omega e^{-\left(\frac{B^2 \ell^2}{2 m R}\right) t} \cos(\omega t + \varphi)\right]$

At $t = 0$ $v = 0 \Rightarrow 0 = A\left[-\frac{B^2 \ell^2}{2 m R} \sin(\varphi) + \omega \cos(\varphi)\right] \Rightarrow \tan\varphi = \frac{2 m R \omega}{B^2 \ell}$ and $A = \frac{0.1}{\sin\varphi}$.

15.

a) $\varphi = \vec{B} . \vec{n} S = B S \cos(0) = B \ell x$; $e = -\frac{d\varphi}{dt} \Rightarrow e = \underline{-B \ell v}$.

b) $u_{DA} + u_{AM} + u_{MN} + u_{ND} = 0 \Rightarrow (R i) + 0 - e + 0 = 0 \Rightarrow e = R i \Rightarrow i = \frac{e}{R} = \frac{-B \ell v}{R}$.

c) $|F_{em}| = |i| \ell B \sin(90) = \left|\frac{B \ell v}{R}\right| (\ell B) \Rightarrow |F_{em}| = \frac{B^2 \ell^2 v}{R}$.

d) $\Sigma \vec{F}_{ext} = m \vec{a} \Rightarrow m \vec{g} + \vec{N}_N + \vec{N}_M + \vec{F}_{e.m} = m \vec{a}$ project along x-axis;

$m g \sin(\theta) + 0 + 0 - |F_{e.m}| = m \frac{dv}{dt} \Rightarrow g \sin\theta = \frac{B^2 \ell^2}{m R} v + \frac{dv}{dt}$

e) $v_o = \frac{g m R \sin\theta}{B^2 \ell^2}(1 - 1) = 0 \Rightarrow v_o = \underline{0}$; $v_\tau = 0.63\left(\frac{m g R \sin\theta}{B^2 \ell^2}\right) = 0.63\, v_{max}$;

$v_{5\tau} = 0.99\, v_{max}$. As $t \to \infty$; $v = v_{max}$. $[0, 5\tau]$: <u>Accelerated</u> . $[5\tau, \infty[$: <u>Uniform</u> .

16.

A)

a) Electromagnetic induction.

b) $\varphi = N B S \cos(\vec{n}, \vec{B}) = (1)(B)(\ell x)\cos(0) \Rightarrow \varphi = \underline{B \ell x}$.

c) $e = -\frac{d\varphi}{dt} = -[B \ell v] \Rightarrow e = \underline{-B \ell v}$.

d) $v < 0$ and $i > 0 \Rightarrow \underline{e.i < 0} \Rightarrow$ the rod acts as a receive.

e) system : MN; External forces : weight $m\vec{g}$, Normal reactions $\vec{N}_N$, $\vec{N}_M$, electromotive force $\vec{F}_{e.m}$

$\Sigma \vec{F}_{ext} = m \vec{a} \Rightarrow (m \vec{g} + \vec{N}_N + \vec{N}_M) + \vec{F}_{em} = m \vec{a} \Rightarrow + i \ell B v = m v' \Rightarrow i = \frac{m}{\ell B} \frac{dv}{dt}$

f) $u_{ac} + u_{cN} + u_{NM} + u_{Ma} = -E + 0 + (r i) - e + 0 = 0$; $E = (r i) - (-B \ell v) \Rightarrow E = \frac{r m}{\ell B} \frac{dv}{dt} + B \ell v \Rightarrow \frac{E \ell B}{m r} = \frac{dv}{dt} + \frac{B^2 \ell^2}{m r} v$.

B)

a) $v = A\left(1 - e^{-\frac{t}{\tau}}\right)$; $\frac{dv}{dt} = \frac{A}{\tau} e^{-\frac{t}{\tau}}$ substitute in D.E.

$\frac{E \ell B}{m r} = \frac{A}{\tau} e^{-\frac{t}{\tau}} + \frac{B^2 \ell^2}{m r} A\left(1 - e^{-\frac{t}{\tau}}\right) \Rightarrow \frac{E \ell B}{m r} = A e^{-\frac{t}{\tau}}\left[\frac{1}{\tau} - \frac{B^2 \ell^2}{m r}\right] + \frac{B^2 \ell^2 A}{m r}$

The constant terms are $\frac{E \ell B}{m r} = \frac{B^2 \ell^2 A}{m r} \Rightarrow A = \frac{E}{\ell B}$. The variable term is zero. $A e^{-\frac{t}{\tau}} = 0$ is rejected $\Rightarrow \left[\frac{1}{\tau} - \frac{B^2 \ell^2}{m r}\right] = 0$

$\Rightarrow \tau = \frac{m r}{B^2 \ell^2}$.

b) $i = \frac{m}{\ell B} \frac{dv}{dt} = \frac{m}{\ell B}\left[\frac{E(B^2 \ell^2)}{\ell B(m r)} e^{-t/\frac{m r}{B^2 \ell^2}}\right] \Rightarrow i = \frac{E}{r} e^{-\frac{B^2 \ell^2}{m r} t}$.

C)

a) $v = \frac{E}{\ell B}\left(1 - e^{-\frac{t}{\tau}}\right)$. As $t \to \infty$: $v = \frac{E}{\ell B} = 25 = \frac{E}{(0.4)(0.8)} \Rightarrow E = \underline{8\ V}$.

b) At $t = 0$: $i = \frac{E}{r} = 4 \Rightarrow r = \frac{8}{4} = \underline{2\ \Omega}$.

c) At $t = \tau$: $v = 0.63\, v_{max} = (0.63)(25) = 15.75$ m/s

Graphically $v = 15.75$ m/s when $t = 0.39$ s $= \tau$; but $\tau = \frac{m r}{B^2 \ell^2} \Rightarrow 0.39 = \frac{m(2)}{(0.8)^2 (0.4)^2} \Rightarrow m = 0.02$ kg $= \underline{20\ g}$.

Chapter 9: Self-Induction

1.

a) $u_{PN} = u_{PA} + u_{AD} + u_{DN} \Rightarrow E = 0 + ri + L\frac{di}{dt} + Ri$. In the steady state $\frac{di}{dt} = 0$ and $i = I_m$
$E = rI_m + RI_m \Rightarrow 4 = r(0.1) + 20(0.1) \Rightarrow \underline{r = 20\ \Omega}$.

b) $E = (r + R)i + L\frac{di}{dt}$ At $t_0 = 0,\ i = 0 \Rightarrow 4 = L(61.5) \Rightarrow \underline{L = 0.0065\ H}$.

2.

A. a) i. - When the switch is closed, the current increases, then the coil is crossed by a variable self (proper) flux and so it is the seat of self induced e.m.f which tends to delay the growth of the current ⇒The lamp glows with a certain delay (glows gradually).
- When the steady state is attained, the current is constant and the coil acts as a pure resistor ⇒The brightness of the lamp does not change.

ii. "i" increases, then the self-induced e.m.f. tends to oppose its increase (coil acts as a reeiver ⇒ $e.i < 0$, but $i > 0$ then $e < 0$.

b) The current decreases, then the coil is crossed by a variable (decreasing) self flux, so it is the seat of self-induced e.m.f. which tends to delay the decay of the current ⇒ the brightness of the lamp decreases gradually (Lamp goes off with a certain delay).

B.

a) [2ms, 5ms].

b) $e = -L\frac{di}{dt}$, $L > 0$. During [0, 2 ms]: $\frac{di}{dt} > 0 \Rightarrow e < 0$. But $i > 0 \Rightarrow e.i < 0 \Rightarrow$ the coil acts as <u>a receiver</u>.
During [2ms, 5ms]: $\frac{di}{dt} = 0 \Rightarrow e = 0 \Rightarrow$ coil acts as <u>a pure resistor</u>.
During [5ms, 6ms]: $\frac{di}{dt} < 0 \Rightarrow e > 0$ But $i > 0 \Rightarrow e.i > 0 \Rightarrow$ the coil acts as <u>a generator.</u>

c) [0, 2ms] : the coil is storing energy. [2ms, 5ms] : the coil is neither storing nor supplying energy.
[5ms, 6ms] : the coil is supplying energy to the circuit.

C. a) Self induction

b) Self induction in a circuit (coil) is due to the variation of the magnetic flux created by the same circuit.
Electromagnetic induction in a circuit is due to the variation of an external magnetic flux crossing the circuit.

3.

a) $\varphi = L\,i \Rightarrow L = \frac{0.05}{10} = 0.005\ H = \underline{5\ mH}$.

b) $\vec{B} = \frac{4\pi \times 10^{-7} N\,i}{\ell}\vec{n}$. But $\varphi = N\,S\,\vec{B}\cdot\vec{n} \Rightarrow \varphi = NS\left(\frac{4\pi \times 10^{-7} N\,i}{\ell}\right) = \frac{4\pi \times 10^{-7} N^2\,S\,i}{\ell} = L\,i \Rightarrow L = \frac{4\pi \times 10^{-7} N^2\,S}{\ell}$.

c) $0.005 = \frac{4\pi \times 10^{-7}(500)^2(\pi)}{\ell} \Rightarrow \ell = \underline{0.5\ m}$.

4.

a) The coil acts as a <u>resistor</u>.

b) R and r are connected in parallel $\Rightarrow R_{eq} = \frac{R\,r}{r+R} = \frac{(10)(15)}{15+10} = 6\ \Omega$.
$u_{AC} = u_{AB} + u_{BC} \Rightarrow -r'I + E = R_{eq}I \Rightarrow I = \frac{E}{r + R_{eq}} = \underline{1\ A}$.

c) $u_{AB} = rI_L \Rightarrow IR_{eq} = r\,I_L \Rightarrow 1(6) = 15I_L \Rightarrow I_L = \underline{0.4\ A}$.

5.

a) The voltage across the coil is: $u_{AB} = ri + L\frac{di}{dt}$

b) i. In the steady state $\frac{di}{dt} = 0 \Rightarrow u_{AB} = r\,I \Rightarrow 3 = r(0.6) \Rightarrow \underline{r = 5\ \Omega}$.

ii. $u_{AB} = ri + L\frac{di}{dt}$. At $t = 0.25$ms: $i = 0.38$A, $u_{AB} = 6.3$V and $\frac{di}{dt} = \frac{0.6 - 0.38}{(0.5-0.25)\times 10^{-3}} = 880$ A/s.
$\Rightarrow 6.3 = 5(0.38) + L(880) \Rightarrow L = \underline{5 \times 10^{-3} H}$.

6.

A. a) i. $u_{AD} = u_{AB} + u_{BC} + u_{CD} \Rightarrow E = R_1 i + ri + L\frac{di}{dt} = (R_1 + r)i + L\frac{di}{dt}$. At $t_0 = 0,\ i = 0$
$\Rightarrow E = 0 + L\frac{di}{dt} \Rightarrow 10 = L(5) \Rightarrow L = 0.2H = \underline{200\ mH}$.

ii. At $t = 2$ms: $\frac{di}{dt} = 30$ A/s $\Rightarrow 10 = (40 + 10)\,i + 0.2(30) \Rightarrow i = \underline{0.08\ A}$.

iii. $i = I_{max}$ when $\frac{di}{dt} = 0 \Rightarrow t = \underline{20\ ms}$. **iv)** $e = -L\frac{di}{dt}$, graphically $\frac{di}{dt} > 0 \Rightarrow \underline{e < 0}$.

b) $I_m = \frac{E}{R_1 + r} = \frac{10}{40+10} = \underline{0.2\ A}$. **c)** $W_L = \frac{1}{2}L\,i^2 = \frac{1}{2}(0.2)(0.2)^2 = \underline{4 \times 10^{-3}\ J}$.

B. a) According to Lenz's law, the self induces e.m.f. tends to oppose the decrease in the current flowing in the coil ⇒ e helps the current to flow in the coil in the same original direction ⇒ i flows in the circuit MCDF wise in anti-clock wise.

b) i. At $t_0 = 0$, $I_m = 0.2$ A $\Rightarrow u_{FM} = R_2 I_m = 100(0.2) = \underline{20\ V}$.

ii. $u_{CD} = -u_{FM} = -20$ V.

iii. $u_{CD\,(0)} = ri - e = 10(0.2) - e \Rightarrow -20 = 2 - e \Rightarrow \underline{e = 22\text{ V} > 0} \Rightarrow e.i > 0 \Rightarrow$ coil acts as a generator.

c) $W_L = \frac{1}{2}L\left(\frac{I_m}{2}\right)^2 = \frac{1}{2}(0.2)(0.1)^2 = 10^{-3}$ J $\Rightarrow$ Energy consumed by the resistor: $4.10^{-3} - 10^{-3} = \underline{3 \times 10^{-3}\text{ J}}$.

7.

a) The LED is reversed biased, it acts as an open switch.

b) **i.** $i_0 = 1.2(1 - e^0) = \underline{0}$. As $t \to \infty$; $i = I_m = 1.2(1-0) = \underline{1.2\text{ A}}$.

ii. In the steady state $E = I_m R + I_m r \Rightarrow I_m = \frac{E}{R+r} \Rightarrow R = \underline{8\ \Omega}$.

c) $e = -L\frac{di}{dt} = -L[1.2\,(0 + 500e^{-500t})] = -600\,L\,e^{-500t} < 0$ and $i > 0 \Rightarrow e.\,i < 0 \Rightarrow$ the coil acts as a receiver.

d) **i.** $i = 1.2\left(1 - e^{-500(3\times10^{-3})}\right) = 0.9322$ A. $\quad u_{AB} = R\,i = 8(0.9322) = \underline{7.46\text{ V}}$.

$E = u_{AB} + u_{BC} \Rightarrow u_{BC} = \text{`}12 - 7.46 = \underline{4.54\text{ V}}$.

ii. $u_{BC} = ri - e \Rightarrow 4.54 = 2(0.9322) + 600\,L\,e^{-(500\times0.003)} \Rightarrow L = \underline{0.02\text{ H}}$.

iii. $W_L = \frac{1}{2}LI_m^{\,2} = \frac{1}{2}(0.02)\,(1.2)^2 = \underline{0.0144\text{ J}}$.

e) When we open the switch, the current decreases then the coil is crossed by a decreasing self (proper) flux $\Rightarrow$The coil is the seat of self-induced e.m.f. which tends to delay the decay of the current $\Rightarrow$ an electric current flows in the circuit (LED-R-coil) in the anti-clock wise sense (LED is forward biased).

8.

A. a) i. φ is variable $\Rightarrow$ (M) is crossed by a variable magnetic flux, then it is the seat of induced e.m.f . The circuit is closed, so it curries an electric current.

ii. $e = -\frac{d\varphi}{dt} = -(-0.02) = \underline{0.02\text{ V}} > 0 \Rightarrow i > 0$ (since (M) acts as a generator) then i crosses the coil from D to C.

iii. $i_{ind} = \frac{e}{R+r} \Rightarrow 2\times10^{-3} = \frac{0.02}{8+r} \Rightarrow r = \underline{2\ \Omega}$.

b) i. $\varphi_{self} = N\,B_{ind}\,S\cos(0) = (2000)(6\times10^{-6})(10\times10^{-4}) = \underline{1.2\times10^{-5}\text{ Wb}}$. **ii.** $\varphi_{self} = L\,i \Rightarrow L = \frac{1.2\times10^{-5}}{2\times10^{-3}} = \underline{0.006\text{ H}}$.

B. a) In the steady state $E = I_m R_1 + I_m r \Rightarrow I_m = \frac{E}{R_1+r} \Rightarrow 1 = \frac{16}{r+14} \Rightarrow r = \underline{2\ \Omega}$.

b) i. $i = a\,t$ where $a = \text{slope} = \frac{4\text{ mA}}{2\text{ ms}} = 2\text{ A/s} \Rightarrow i = \underline{2\,t}$ (S.I).

ii) $u_{DC} = ri + L\frac{di}{dt} \Rightarrow u_{DC} = 2(2t) + L\frac{d(2t)}{dt} = \underline{4\,t + 2\,L}$. **iii)** By comparison: $0.012 = 2L \Rightarrow L = 0.006\text{H} = \underline{6\text{ mH}}$.

9.

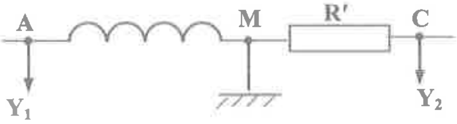
A. a) $\varphi = N\,B\,S\cos(\vec{B}\cdot\vec{n}) = N\,S\left(\frac{4\pi\times10^{-7}Ni}{\ell}\right)\cos(0) = \frac{4\pi\times10^{-7}N^2S\,i}{\ell}$.

b) $L = \frac{\varphi}{i} = \frac{4\pi\times10^{-7}N^2\,S}{\ell}$. $\qquad$ **c)** $L = \frac{4\pi\times10^{-7}(500)^2\,(60\times10^{-4})}{0.628} \cong \underline{0.003\text{ H}}$.

B. a) In the steady state: $I = \frac{E}{R_1+r} = \underline{0.2\text{ A}}$.

b) i) $4.10^{-4} = W_L + 7W_L = 8\,W_L \Rightarrow W_L = \underline{6\times10^{-5}\text{ J}}$

ii) $W_L = \frac{1}{2}LI^2 \Rightarrow L = \frac{6\times10^{-5}\times2}{0.2^2} = \underline{0.003\text{ H}}$.



C. a) figure

b) $u_{AM} = ri + L\frac{di}{dt}$. $\quad u_{CM} = -R'\,i$

c) $\frac{du_{CM}}{dt} = -R'\frac{di}{dt}$, substitute i and $\frac{di}{dt}$ in u_{AM}, then $u_{AM} = r\frac{-u_{CM}}{R'} + L(\frac{-1}{R'})\frac{du_{CM}}{dt}$

d) i. $u_{CM} = 0$. $\qquad$ **ii.** $u_{AM} = -2(0.3) = -0.6$ V. $\qquad$ **iii.** $\frac{du_{CM}}{dt} = \text{slope} = \frac{3\times0.3}{1.5\times1.5\times10^{-6}} = 400000$ V/s.

e) $-0.6 = 0 - \frac{L}{2000}(400000) \Rightarrow L = \underline{0.003\text{ H}}$.

f) The coil acts as a generator when $|i|$ decreases $\Rightarrow$ $[t_1, t_2]$ and $[t_3, t_4]$.

10.

a) $T = 4\text{ div}\times\left(\frac{1\text{ms}}{\text{div}}\right) = 4\text{ ms} = \underline{4\times10^{-3}\text{ s}}$. $\quad U_{max} = 3\text{div}\times\left(\frac{0.1\text{V}}{\text{div}}\right) = \underline{0.3\text{ V}}$.

b) $u_{BC} = ri + L\frac{di}{dt} = 0 + L\frac{di}{dt} = (0.1)\frac{di}{dt}$ (S.I).

c) $di = \frac{u_{BC}}{0.1}dt \Rightarrow i = 10\int u_{BC}\,dt$

i. $\left[0, \frac{T}{2}\right]$: $i = 10\int 0.3\,dt = 3\,t + i_0$. At $t = 0$, $i = 0 \Rightarrow 0 = i_0 \Rightarrow i = \underline{3\,t}$ (S.I).

ii. $\left[\frac{T}{2}, T\right]$: $i = 10\int -0.3\,dt \Rightarrow i = -3\,t + i_0$. At $t = \frac{T}{2}$, $i = 3\,\frac{T}{2} = \frac{3\times4\times10^{-3}}{2} = 6\times10^{-3}$A

$\Rightarrow 6\times10^{-3} = -\frac{3\times4\times10^{-3}}{2} + i_0 = -6\times10^{-3} + i_0 \Rightarrow i_0 = 0.012\text{ A} \Rightarrow i = -3\,t + 0.012$ (S.I).

d) $u_{AB} = R\,i = 10\,i$. $\left[0, \frac{T}{2}\right]$: $u_{AB} = 10(3t) = \underline{30\,t}$ (S.I).

$\left[\frac{T}{2}, T\right]$: $u_{AB} = 10(-3t + 0.012) = \underline{-30\,t + 0.12}$ (S.I).

$u_G = u_R + u_L \Rightarrow$ during $\left[0, \frac{T}{2}\right]$: $u_R = 30\,t$ and $u_L = 0.1(3) = 0.3\text{ V} \Rightarrow u_G = \underline{30\,t + 0.3}$ (S.I).

During $\left[\frac{T}{2}, T\right]$: $u_R = -30\,t + 0.12$ and $u_L = 0.1(-3) = -0.3$ V $\Rightarrow u_G = -30\,t + 0.12 - 0.3 = \underline{-30\,t - 0.18}$ (S.I)

11.

A. a) $I_m = \frac{E}{R_1 + r}$.

b) $u_{CH} = u_{CD} + u_{DF} + u_{FH} \Rightarrow E = R_1 i + 0 + ri + L\frac{di}{dt} \Rightarrow \frac{E}{L} = \frac{(R_1+r)}{L}i + \frac{di}{dt}$. At $t_0 = 0$, $i = 0 \Rightarrow \left.\frac{di}{dt}\right)_{t_0=0} = \frac{E}{L}$.

c) i. $i = A(1 - e^{-\frac{t}{\tau}})$, $\frac{di}{dt} = \frac{A}{\tau}e^{-\frac{t}{\tau}}$ sub. in eq $\Rightarrow \frac{E}{L} = \frac{(R_1+r)}{L}A - \frac{(R_1+r)}{L}Ae^{-\frac{t}{\tau}} + \frac{A}{\tau}e^{-\frac{t}{\tau}} = \frac{(R_1+r)}{L}A + Ae^{-\frac{t}{\tau}}\left[\frac{1}{\tau} - \frac{(R_1+r)}{L}\right]$.

Compare identical terms: $\frac{E}{L} = \frac{(R_1+r)}{L}A \Rightarrow A = \frac{E}{(R_1+r)}$

and $Ae^{-\frac{t}{\tau}}\left[\frac{1}{\tau} - \frac{(R_1+r)}{L}\right] = 0$ but $Ae^{-\frac{t}{\tau}} = 0$ is rejected $\Rightarrow \frac{1}{\tau} - \frac{(R_1+r)}{L} = 0 \Rightarrow \frac{(R_1+r)}{L} = \frac{1}{\tau} \Rightarrow \tau = \frac{L}{(R_1+r)}$.

ii. $A = I_m \Rightarrow i = I_m\left(1 - e^{-\frac{t}{\tau}}\right)$.

d) i. At $t = \tau$; $i = 0.63\,I_m = 0.63\,(0.8) = 0.504$ A ≈ 0.5 A. Graphically, when $i = 0.5$A, $t = \tau = \underline{3\text{ ms}}$.

$\tau = \frac{L}{(R_1+r)} \Rightarrow R_1 = \underline{13\,\Omega}$.

ii) $I_m = \frac{E}{R_1+r} \Rightarrow E = (0.8)\,(13+2) = \underline{12\text{ V}}$.

e) i. 1st method: At $t_0 = 0$, $\left.\frac{di}{dt}\right)_{t_0=0} = \frac{E}{L}$ but $L = \tau(R_1 + r) \Rightarrow \left.\frac{di}{dt}\right)_{t_0=0} = \frac{E}{(R_1+r)} = \frac{I_m}{\tau}$

2nd method: $i = I_m(1 - e^{-\frac{t}{\tau}}) \Rightarrow \frac{di}{dt} = \frac{I_m}{\tau}e^{-\frac{t}{\tau}} \Rightarrow$ at $t_0 = 0$, $\frac{di}{dt} = \frac{I_m}{\tau}$.

ii. $i = \text{slope} \times t = \frac{I_m}{\tau}t$. **iii.** For $i = I_m \Rightarrow I_m = \frac{I_m}{\tau}t \Rightarrow t = \tau = 3$ ms.

B. a) $u_R = u_{MN} = R_2 i$. $R_2 = \text{const} \Rightarrow u_R$ and i are proportional $\Rightarrow$The graph of i is similar to that of u_R to a multiplying factor.

b) $u_{FH} + u_{HM} + u_{MN} + u_{NF} = 0 \Rightarrow ri + L\frac{di}{dt} + 0 + u_R + 0 = 0$, but $u_R = R_2 i$ and $\frac{di}{dt} = \frac{1}{R_2}\frac{du_R}{dt}$

$\Rightarrow \frac{r\,u_R}{R_2} + \frac{L}{R_2}\frac{du_R}{dt} + u_R = 0 \Rightarrow u_R\left(1 + \frac{1}{R_2}\right) + \frac{L}{R_2}\frac{du_R}{dt} = 0 \Rightarrow \frac{(R_2+r)}{L}u_R + \frac{du_R}{dt} = 0.$

c) $\frac{du_R}{dt} = -\frac{U_{Rm}}{\tau'}e^{-\frac{t}{\tau'}}$ sub. in eq $\Rightarrow \frac{(R_2+r)}{L}\left(U_{Rm}e^{-\frac{t}{\tau'}}\right) - \frac{U_{Rm}}{\tau'}e^{-\frac{t}{\tau'}} = 0$

$U_{Rm}e^{-\frac{t}{\tau'}}\left[\frac{(R_2+r)}{L} - \frac{1}{\tau'}\right] = 0$ but $\tau' = \frac{L}{(R_2+r)} \Rightarrow \left[\frac{(R_2+r)}{L} - \frac{(R_2+r)}{L}\right] = 0 \Rightarrow U_{Rm}e^{-\frac{t}{\tau'}}\left[\frac{(R_2+r)}{L} - \frac{1}{\tau'}\right] = 0$ so it is verified.

d) At $t_0 = 0$, $u_R = U_{Rm} = R_2 I_m \Rightarrow U_{Rm} = R_2\frac{E}{R_1+r} = \underline{18.4\text{ V}}$.

e) i. $\Delta t = 5\tau'$. **ii.** At $t = \tau'$, $u_R = 0.37\,U_{Rm} < \frac{U_{Rm}}{2} \Rightarrow$ at $t = 2.5\,\tau'$, $u_R < \frac{U_{Rm}}{2} \Rightarrow t < 2.5\,\tau'$.

iii. $\frac{18.4}{2} = 18.4\,e^{-\frac{t}{\tau'}} \Rightarrow 0.5 = e^{-\frac{t}{\tau'}} \Rightarrow 2 = e^{\frac{t}{\tau'}} \Rightarrow \ell n2 = \frac{t}{\tau'} \Rightarrow t = [\ell n2]\tau' = 0.693\,\tau' < \frac{5}{2}\tau'$.

12.

A. **a)** $u_{AD} = u_{AF} + u_{FC} + u_{CD} \Rightarrow E = 0 + R_1 i + u_L + 0$ $\left\{u_L = L\frac{di}{dt}\right\}$.

Derive w.r.t time: $R_1\frac{di}{dt} + \frac{du_L}{dt} = 0 \Rightarrow \frac{R_1}{L}\frac{di}{dt} + \frac{du_L}{dt} = 0.$

b) $\frac{du_L}{dt} = -\frac{U_{Lmax}}{\tau}e^{-\frac{t}{\tau}} = -\frac{u_L}{\tau}$ sub. in diff.eq: $\frac{R_1}{L}u_L - \frac{u_L}{\tau} = 0 \Rightarrow u_L\left[\frac{R_1}{L} - \frac{1}{\tau}\right] = 0$. But $u_L = 0$ is rejected

$\Rightarrow \frac{R_1}{L} - \frac{1}{\tau} = 0 \Rightarrow \tau = \frac{L}{R_1}$.

c) $R_1 i + u_L - E = 0$; at $t_0 = 0$; $i_0 = 0$: $0 + u_{L_0} - E = 0 \Rightarrow u_{L_0} = E$.

d) i) $E = u_{L_0} = 10$ V. **ii)** At $t = \tau$; $u_L = 0.37\,U_{L_{max}} = 0.37(10) = 3.7$ V.

Graphically for $u_L = 3.7$ V; $t = \tau = \underline{2\text{ ms}} \Rightarrow L = \tau R_1 = \underline{0.01\text{ H}}$.

e) $I_m = \frac{E}{R_1} = \frac{10}{5} = 2$ A.

B. a) When K is turned to position (2), the current decreases, the self induced e.m.f. tends to delay the deacy in "i" $\Rightarrow$ coil acts as a generator and helps the current to flow in the same original direction.

At $t_0 = 0$; W_L is equal to its value at the end of growth process $\Rightarrow i_0 = I_m = 2$ A.

b) $u_{HF} + u_{FC} + u_{CD} + u_{DB} + u_{BH} = 0 \Rightarrow 0 + R_1 i + u_L + 0 + R_2 i = 0 \Rightarrow u_L = -(R_1 + R_2)\,i$ but $i > 0 \Rightarrow u_L < 0$.

c) $u_L = -(R_1 + R_2)i$. At $t_0 = 0$; $i_0 = \frac{E}{R_1} \Rightarrow u_{L_0} = u_2 = -\frac{E}{R_1}(R_1 + R_2)$.

d) At $t_0 = 0$; $u_{L_0} = -12\text{ V} = -\frac{10}{5}(5 + R_2) \Rightarrow R_2 = \underline{1\,\Omega}$.

e) $\frac{R_1+R_2}{R_1} > 1 \Rightarrow |u_2| > |u_1|$.

13.

A.

a) $i = I_m = \frac{E}{r + R_1}$. **b)** $E = (r + R_1)i + L\frac{di}{dt} \Rightarrow \frac{di}{dt} + \left(\frac{r+R_1}{L}\right)i = \frac{E}{L}$.

c) i. $i = ke^{-\frac{t}{\tau}} + M$. As $t \to \infty$; $i \to I_m = k(0) + M \Rightarrow M = I_m = \frac{E}{r + R_1}$.

ii. At $t_0 = 0$; $i = 0 = k(1) + M \Rightarrow k = -M$.

iii. $\frac{di}{dt} = -\frac{k}{\tau}\, e^{-\frac{t}{\tau}}$ sub. in diff . eq: $-\frac{k}{\tau}\, e^{-\frac{t}{\tau}} + \frac{(r+R_1)}{L}\left[ke^{-\frac{t}{\tau}}\right] + \frac{(r+R_1)}{L}M = \frac{E}{L}$

$\Rightarrow ke^{-\frac{t}{\tau}}\left[-\frac{1}{\tau} + \frac{(r+R_1)}{L}\right] + \frac{(r+R_1)}{L}M = \frac{E}{L} \Rightarrow$ the variable term is zero $\Rightarrow \tau = \frac{L}{(r+R_1)}$.

d) **i.** At $t_0 = 0$; $i = 0 \Rightarrow u_R = 0 \Rightarrow$ curve (2) represents the variation of u_R .

ii. $u_G = u_L + u_R$. At $t_0 = 0$; $u_R = 0 \Rightarrow u_L = u_G = 20$ V or $\{u_L = 20 - u_{R_1}\}$.

iii. $i = \frac{E}{r+R_1}\left[1 - e^{-\frac{t}{\tau}}\right] \Rightarrow u_{R_1} = \frac{E\,R_1}{r+R_1}\left[1 - e^{-\frac{t}{\tau}}\right]$. As $t_0 \to \infty$; $16 = \frac{E\,R_1}{r+R_1} \Rightarrow R_1 = 4\,r$.

Or $\left\{I_m = \frac{E}{r+R_1} \Rightarrow \frac{u_{R_{max}}}{R} = \frac{E}{r+R_1} \Rightarrow R_1 = 4r\right\}$. $8 = 4\,r \Rightarrow$ $\underline{r = 2\,\Omega}$.

e) $0.63(16) = 10.08$ V. Graphically when $u_{R_1} = 10.08$ V; $t = \tau = \underline{1\text{ ms}}$.

f) $\tau = \frac{L}{(r+R_1)} \Rightarrow L = \underline{0.01\text{H}}$.

B.

a) $I_m = \frac{E}{r+R_1}$. **b)** $u_{HF} + u_{FD} + u_{DM} = 0 \Rightarrow R_2 i + ri + L\frac{di}{dt} = 0 \Rightarrow \frac{di}{dt} + \frac{(r+R_2)}{L}i = 0$.

c) $\frac{di}{dt} = -\frac{(r+R_2)}{L}e^{-\frac{(r+R_2)}{L}t}\left[\frac{E}{r+R_1}\right] \Rightarrow \frac{di}{dt} = -\frac{(r+R_2)}{L}i$ sub. in diff .eq : $-\frac{(r+R_2)}{L}i + \frac{(r+R_2)}{L}i = 0 \Rightarrow$ It is verified.

d) **i.** $u_{HF} = R\,i$. But $i > 0 \Rightarrow u_{HF} > 0 \Rightarrow$ curve (1) represents the variation of u_R .

ii. $u_{FD} = u_{FH} + u_{HD} \Rightarrow u_{FD} = -u_{HF} = -u_{R_2} \Rightarrow$ curve(2) represents the variation of u_L .

iii. At $t = \tau$; $u_R = 0.37(36) = 13.32$ V. Graphically $u_R = 13.32$ V for $t = \tau = 0.5$ ms. But $\tau = \frac{L}{(r+R_2)} \Rightarrow R_2 = \underline{18\,\Omega}$.

C.

a) $W_{L_0} = \frac{1}{2}L\,I_0^2 = \frac{1}{2}(0.01)\,(2)^2 = \underline{0.02\text{ J}}$.

b) **i.** $P = \frac{dW}{dt} \Rightarrow W = \int_0^\infty P\,dt = \int_0^\infty i^2 R_2 dt = \frac{R_2E^2}{(r+R_1)^2}\int_0^\infty e^{-\frac{2(r+R_2)}{L}t}dt = \frac{R_2E^2}{(r+R_1)^2}\left[\frac{e^{-\frac{2(r+R_2)}{L}t}}{-\frac{2(r+R_2)}{L}}\right]_0^\infty$

$\Rightarrow W = \frac{L\,R_2\,E^2}{2(r+R_1)^2(r+R_2)}\left[-e^{-\frac{2(r+R_2)}{L}t}\right]_0^\infty = \frac{L\,R_2\,E^2}{2(r+R_1)^2(r+R_2)}[-0-(-1)] \Rightarrow W = \frac{L\,R_2\,E^2}{2(r+R_1)^2(r+R_2)}$.

ii. $W_{R_2} = \frac{(0.01)(18)(400)}{2(20)(10)^2} \Rightarrow W_{R_2} = 0.018\text{ J} \Rightarrow W_r = 0.02 - 0.018 \Rightarrow W_r = \underline{2\times10^{-3}\text{ J}}$.

14.

a) $u_{AB} + u_{BC} = u_{AC} \Rightarrow Ri + ri + L\frac{di}{dt} = E$, at the steady state: $\frac{di}{dt} = 0$ and $i = I_m \Rightarrow (R+r)I_m = E \Rightarrow I_m = \frac{E}{r+R} = \underline{0.8\text{ A}}$.

b) **i.** $W_L = \frac{1}{2}Li^2 = \frac{1}{2}L\,I_m^2 e^{-\frac{2t}{\tau}}$. **ii.** At $t_0 = 0$; $W_L = \frac{1}{2}L\,I_m^2 e^{-0} \Rightarrow W_L = \frac{1}{2}L\,I_m^2$.

c) $P_A = i^2\,R = \frac{dW_R}{dt} \Rightarrow W_R = \int_0^t i^2 R\,dt = \int_0^t I_m^2 e^{-\frac{2t}{\tau}} R\,dt = I_m^2\,R\left[\frac{e^{-\frac{2t}{\tau}}}{-\frac{2}{\tau}}\right]_0^t$

$W_R = -\frac{I_m^2\,R\,\tau}{2}\left[e^{-\frac{2t}{\tau}} - 1\right] = \frac{I_m^2 R}{2}\left(\frac{L}{R+r}\right)\left[1 - e^{-\frac{2t}{\tau}}\right] = W_0\left(\frac{R}{R+r}\right)\left[1 - e^{-\frac{2t}{\tau}}\right]$.

d) **i.** As $t \to \infty$; $W_R = W_0\left(\frac{R}{R+r}\right)$ (steady state). Graphically ; $3.2\times10^{-3} = W_0\left(\frac{20}{10+20}\right) \Rightarrow W_0 = \underline{4.8\times10^{-3}\text{ J}}$.

$W_0 = \frac{1}{2}L\,I_m^2 = \frac{1}{2}L\,(0.8)^2 \Rightarrow L = 0.015\text{H} = 15$ mH. $\tau = \frac{L}{(r+R)} = \underline{0.5\text{ ms}}$.

ii. $W_R = 3.2\times10^{-3}\left[1 - e^{-\frac{2t}{\tau}}\right] \cong \underline{2.8\times10^{-3}\text{ J}}$. Or graphically at $t = 0.5$ ms, $W_R \cong \underline{2.8\times10^{-3}\text{ J}}$.

iii. At $t = \tau$; $i = 0.37\,I_m = 0.2368\text{ A} \Rightarrow W_L = \frac{1}{2}L\,i^2 = 4.21\times10^{-4}$ J.

$W_{(R+r)} = W_0 - W_L = 4.8\times10^{-3} - 4.21\times10^{-4} = 4.397\times10^{-3}$ J.

$W_r = W_{(R+r)} - W_R = 4.397\times10^{-3} - 2.8\times10^{-3} = \underline{1.6\times10^{-3}\text{ J}}$.

e) $W_{R_{max}} = 3.2\times10^{-3}$ J . Graphically, At $t = \frac{\tau}{2}$, $W_R = 2\times10^{-3}\text{ J} = 0.63(3.2\times10^{-3}) \Rightarrow$ at $t = \frac{\tau}{2}$, $W_R = 0.63\,W_{R_{max}}$.

15.

A.

a) **i.** $I_m = \frac{E}{r+R} = \frac{20}{2+8} = 2$ A . $W_L = \frac{1}{2}L\,I_m^2 = \frac{1}{2}(1)(2)^2 = \underline{2\text{ J}}$. **ii)** $u_k = 0$.

b) **i.** The current decreases. According to Lenz's law the self-induced e.m.f tents to delay its decay (coil acts as a generator) $\Rightarrow \underline{e > 0}$.

ii. $e = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t} = -1\frac{(0-2)}{2\times10^{-3}} \Rightarrow e = \underline{1000\text{ V}}$.

iii. $u_{AB} + u_{BC} + u_{CM} + u_{MA} = 0 \Rightarrow u_k + ri - e + Ri - E = 0 \Rightarrow u_k = e + E - (r+R)i = 1020 - 10i \Rightarrow u_{k_{max}} = \underline{1020\text{ V}}$.

iv. u_k is very large $\Rightarrow$ electric discharge takes place between the switch terminals $\Rightarrow$ appearance of sparks.

B.

a) $u_{BC} + u_{CF} + u_{FH} + u_{HP} + u_{PB} = 0 \Rightarrow r\,i + L\frac{di}{dt} + 0 + 0 + R'\,i + 0 = 0 \Rightarrow L\frac{di}{dt} + (r+R')i = 0$.

b) **i.** $e = -L\frac{di}{dt} = -L\left(-\frac{I_m}{\tau}\ e^{-\frac{t}{\tau}}\ \right) = \frac{LI_m}{\left[{}^{L}/_{(r+R')}\right]}e^{-\left(\frac{t}{{}^{L}/_{(r+R')}}\right)} \Rightarrow e = I_m(r+R')e^{-\frac{(r+R')t}{L}}$.

ii. $e > 0$ for any value of t, but $i > 0 \Rightarrow e.i > 0 \Rightarrow$ the coil acts as a generator.

iii. $e_{max} = I_m(r+R') = \underline{6\ V}$.

c) $u_k = E + R'i = E + R'I_m e^{-\frac{t}{\tau}}$. u_k is max when $e^{-\frac{t}{\tau}}$ is max $= 1 \Rightarrow u_{k_{max}} = E + R'I_m$

(Or: u_k is max when $i = I_m \Rightarrow u_{k_{max}} = E + R'I_m \Rightarrow u_{k_{max}} = 20 + 1(2) = 22\ V$.

But $\underline{22\ V \ll 1020\ V}$, low voltage across the switch $\Rightarrow$ no sparks.

16.

A.

a) Electromagnetic Induction: due to the variation of the magnetic flux crossing the swept area. MN is the seat of induced e.m.f. and an induced current flows in the circuit.

Self induction in the coil: The coil carries a variable current $\Rightarrow$ it is crossed by a variable proper flux $\Rightarrow$ the coil is the seat of self-induced e.m.f.

b) $\phi = BS\cos(180°) = -B\,S = \underline{-B\ell x}$. $e = -\frac{d\phi}{dt} = \underline{B\ell v}$.

c) $\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{a} + \vec{N} + \vec{F}_{em} = m\vec{a}$ project along x-axis $\Rightarrow 0 + 0 - |F_{em}| = mv' \Rightarrow -i\ell B = mv' \Rightarrow i = -\frac{m}{\ell B}\frac{dv}{dt}$.

d) $u_{MN} + u_{NC} + u_{Ca} + u_{aM} = 0 \Rightarrow -e + 0 + ri + L\frac{di}{dt} + 0 = 0 \Rightarrow -B\ell v + L\left(-\frac{m}{\ell B}\right)\frac{d^2v}{dt^2} = 0 \Rightarrow \frac{B^2\ell^2}{mL}v + v'' = 0$

e) $W = \frac{1}{2}mv^2 + \frac{1}{2}L\,i^2$; $\frac{dW}{dt} = 0 = mvv' + L\,i\,i' = mvv' + L\left(-\frac{m}{\ell B}\frac{dv}{dt}\right)\left(-\frac{m}{\ell B}\right)\frac{d^2v}{dt^2} = v'\left[mv + \frac{m^2L}{B^2\ell^2}v''\right]$. But $v' = 0$ is rejected

$\Rightarrow \frac{B^2\ell^2}{mL}v + v'' = 0$.

B.

a) The rod oscillates since the differential equation in v has the form of $v'' + \omega_0^2 v = 0$.

Type: free-undamped oscillations. $\omega_o = \sqrt{\frac{B^2\ell^2}{mL}} \Rightarrow \omega_o = \frac{B\ell}{\sqrt{mL}}$.

b) **i.** $i = -\frac{m}{B\ell}\left[-\frac{AB\ell}{\sqrt{mL}}\sin\left(\frac{B\ell}{\sqrt{mL}}t + \varphi\right)\right] = A\sqrt{\frac{m}{L}}\sin\left(\frac{B\ell}{\sqrt{mL}}t + \varphi\right)$.

ii. At $t_0 = 0$; $i_0 = 0 = A\sqrt{\frac{m}{L}}\sin(\varphi) \Rightarrow \sin(\varphi) = 0 \Rightarrow \varphi = 0$ or $\varphi = \pi$ rad

At $t_0 = 0$; $v_0 = A\cos(\varphi)$ but $A > 0 \Rightarrow \cos(\varphi) > 0 \Rightarrow \varphi = 0$ and $v_0 = A\cos(\varphi) \Rightarrow A = v_0$.

17.

a) $u_{MN} + u_{NC} + u_{Ca} + u_{aM} = 0 \Rightarrow ri - e + L\frac{di}{dt} + 0 = 0 \Rightarrow r\left(-\frac{m}{\ell B}\right)v' - B\ell v - \frac{Lm}{\ell B}v'' = 0$

$v'' + \left(\frac{r}{L}\right)v' + \left(\frac{B^2\ell^2}{mL}\right)v = 0$

b) **i.** Free damped mechanical oscillations.

ii. $i = -\frac{m}{\ell B}\frac{dv}{dt} = -\frac{mA}{\ell B}\left[e^{-\frac{r}{2L}t}\left(-\omega\sin(\omega t + \varphi) + \cos(\omega t + \varphi)\left(-\frac{r}{2L}\right)e^{-\frac{r}{2L}t}\right)\right] = \frac{mA}{\ell B}e^{-\frac{r}{2L}t}\left[\omega\sin(\omega t + \varphi) + \frac{r}{2L}\cos(\omega t + \varphi)\right]$.

iii. At $t_0 = 0$; $i_0 = 0 = \frac{mA}{\ell B}\left[\omega\sin\varphi + \frac{r}{2L}\cos\varphi\right]$ but $\frac{mA}{\ell B} \neq 0 \Rightarrow \omega\sin\varphi + \frac{r}{2L}\cos\varphi = 0$

$\Rightarrow \omega\sin\varphi = -\frac{r}{2L}\cos\varphi \Rightarrow \tan\varphi = -\frac{r}{2L\omega}$. At $t_0 = 0$; $v_0 = A\cos\varphi \Rightarrow A = \frac{v_0}{\cos\varphi}$.

18.

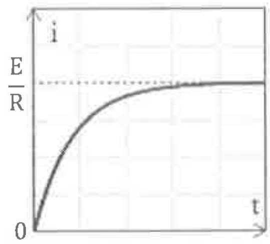
A.

a) $u_{DM} = u_{DA} + u_{AB} + u_{BM} \Rightarrow E = 0 + L\frac{di}{dt} + Ri \Rightarrow E = Ri + L\frac{di}{dt}$

b) **i.** As $t \to \infty$, $i = I_m = \frac{E}{R}(1 - 0) \Rightarrow I_m = \frac{E}{R}$.

ii. $\Delta t = 5\tau = \frac{5L}{R}$.

iii. figure



B.

a) System (D) External forces: weight $m\vec{g}$, and the air resistance $\vec{f}$.

$\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{f} = m\vec{a}$. Project along the direction of motion:

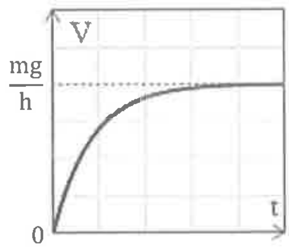
$\Rightarrow mg - f = ma \Rightarrow m\frac{dv}{dt} + hv = mg$.

b) $L \equiv m$; $R \equiv h$; $i \equiv v$, and $E \equiv mg$.

c) **i.** $v = \frac{mg}{h}\left[1 - e^{-\frac{h}{m}t}\right]$. **ii.** As $t \to \infty$, $v = V_m = \frac{mg}{h}[1 - 0] \Rightarrow V_m = \frac{mg}{h}$.

iii. $\Delta t = 5\tau = \frac{5m}{h}$.

iv. Figure



Chapter 10: R-C Series Circuit under Constant and Square Voltage

1.

a) Charging of the capacitor.
b) The free electrons of the lower plate are attracted and transferred to the positive pole of the battery, leaving the lower plate with a positive charge. Similarly the same number of electrons leave the negative pole towards the upper plate which becomes negatively charged .
c) Zero . **d)** The charge remains stored in the capacitor.

2.

a)

Time	u_{AB}	u_{MN}	u_{DM}	i(mA)
0	10	0	10	1
τ	10	6.3	3.7	0.37
2τ	10	8.65	1.35	0.135
5τ	10	9.93	0.07	0.007
∞	10	10	0	0

b)

Time	u_{AB}	u_{MN}	u_{DM}	i(mA)
0	10	0	−10	−1
τ	10	3.7	−3.7	−0.37
5τ	10	0.07	−0.07	−0.007
∞	10	0	0	0

3.

A.

a) $u_R = RI = (16000)(0.25 \times 10^{-3}) = \underline{4\text{ V}}$.
b) $I = \frac{\Delta Q}{\Delta t} \Rightarrow \Delta Q = Q = (0.25 \times 10^{-3})(0.16) = \underline{4 \times 10^{-5}\text{C}}$. $u_{MN} = \frac{Q}{C} = \frac{4 \times 10^{-5}}{4 \times 10^{-6}} = \underline{10\text{ V}}$.
c) $|u_C| = \frac{q}{C} = \frac{It}{C} \Rightarrow |u_C| = \left(\frac{I}{C}\right)t$ But $\frac{I}{C}$ = const $\Rightarrow u_C$ increases linearly with time .
d) $u_{AB} = u_G = u_C + u_R = \frac{I}{C}t + u_R = \frac{0.25 \times 10^{-3}}{4 \times 10^{-6}}t + 4 \Rightarrow u_G = \underline{62.5t + 4}$ (S.I)

B. $\Delta t' = 5\tau = 5RC = 5(16000)(4 \times 10^{-6}) = \underline{0.32\text{ s}}$, $\Delta t' > \Delta t$.

4.

a) $P = i\,u_C = \frac{dW}{dt} \Rightarrow dW = i\,u_C\,dt$ $\left\{\text{But } i = \frac{dq}{dt} = C\frac{du_C}{dt}\right\} \Rightarrow dW = C\frac{du_C}{dt}.u_C\,dt \Rightarrow W = C\int u_C\,du_C = \frac{C\,u_C^2}{2} + \text{const}$.
But when $u_C = 0$; $W = 0 \Rightarrow 0 = 0 + \text{const} \Rightarrow \text{const} = 0 \Rightarrow W = \frac{1}{2}C\,u_C^2$.
b) When the capacitor is fully charged $u_C = E \Rightarrow W = \frac{1}{2}CE^2$. Graphically $W = 0.05 = \frac{1}{2}C(100) \Rightarrow C = \underline{10^{-3}\text{ F}}$.

5.

A.

a) i flows in the anti-clockwise sense .
b) $u_{MN} = u_{MA} + u_{AB} + u_{BD} + u_{DN} \Rightarrow E = 0 + u_C + Ri + 0 \Rightarrow i = \frac{E - u_C}{R}$.
c) At $t_0 = 0$, $u_C = 0 \Rightarrow 0.08 = \frac{80 - 0}{R} \Rightarrow R = \underline{1000\ \Omega}$.
d) The generator supplies energy to the resistor and the capacitor . Then R and C receive energy from the generator.

B.

a) Clockwise in the closed circuit.
b) At t = 2 s = τ, the voltage across the capacitor becomes 0.37 E. $\tau = RC \Rightarrow 2 = 1000\,C \Rightarrow C = 2 \times 10^{-3} = \underline{2\text{ mF}}$.
c) The capacitor supplies energy to the circuit and the resistor consumes energy .
d) **i.** $W_C = \frac{1}{2}CE^2 = \frac{1}{2}(2 \times 10^{-3})(80)^2 = \underline{6.4\text{ J}}$.
ii. At $t = \tau$: $W_C' = \frac{1}{2}(2 \times 10^{-3})(0.37 \times 80)^2 = 0.87616$ J . $P_{av} = \frac{\Delta w}{\Delta t} = \frac{|6.4 - 0.87616|}{2} \cong \underline{2.76\text{ W}}$.

6.

a) At electric equilibrium, the current flowing in the branch of the capacitor is zero. $E = I(R_1 + R_2) = 0.08[100 + 200] = \underline{24\text{ V}}$.
b) $u_{AB} = u_C = i\,R_2 = (0.08)(200) = \underline{16\text{ V}}$.
c) $q = N.e = (2 \times 10^{14})(1.6 \times 10^{-19}) = 3.2 \times 10^{-5}$ C, but $q_A = C\,u_{AB} \Rightarrow C = \frac{3.2 \times 10^{-5}}{16} \Rightarrow C = \underline{2\ \mu\text{F}}$.

7.

a)

i. T = 24 ms and E = 16 V.

ii. Graph (1): voltage across LFG during the charging process. Graph (4): voltage across LFG during the discharging process.
Graph (2): variation of voltage across capacitor during charging.
Graph (3): variation of voltage across capacitor during discharging.

b)

i. $u_{FM} = u_{FD} + u_{DM} \Rightarrow E = Ri + u_C$ $\left\{ i = + \frac{dq}{dt} = C\frac{du_C}{dt} \right\} \Rightarrow E = RC\frac{du_C}{dt} + u_C \Rightarrow \frac{du_C}{dt} + \frac{u_C}{RC} = \frac{E}{RC}$.

ii. $u_C = E - Ee^{-t/\tau} \Rightarrow \frac{du_C}{dt} = \frac{E}{\tau}e^{-t/\tau}$ sub in diff equ: $\frac{E}{\tau}e^{-t/\tau} + \frac{E}{RC} - \frac{E}{RC}e^{-t/\tau} = \frac{E}{RC} \Rightarrow Ee^{-t/\tau}\left(\frac{1}{\tau} - \frac{1}{RC}\right) + \frac{E}{RC} = \frac{E}{RC}$

But $\tau = RC \Rightarrow \frac{E}{RC} = \frac{E}{RC}$ verified.

iii. At $t = \tau$; $u_C = 0.63\ E = 0.63\ [16] \cong 10$ V. Graphically for $u_C = 10$ V ; $t = \tau = \underline{2\text{ ms}}$.

But $\tau = RC \Rightarrow C = \frac{2 \times 10^{-3}}{10000} \Rightarrow C = 2 \times 10^{-7}\ F = \underline{0.2\ \mu F}$.

c)

i. $\frac{du_C}{dt} = \frac{E}{\tau}e^{0} = \frac{E}{\tau}$, But at $t_0 = 0$, $\frac{du_C}{dt}$ = slope of the tangent to u_C at this instant $\Rightarrow$ Eq of tangent : $u = \frac{E}{\tau}$ t.

ii. $u = \frac{E}{\tau}$ t and $u_1 = E \Rightarrow$ the graphs intersect when $u = u_1 \Rightarrow \frac{E}{\tau}\ t = E \Rightarrow t = \tau$. But the coordinates of K are: $(\tau, E) \Rightarrow$ proved.

d)

i. $u_{FM} = u_{FD} + u_{DM} \Rightarrow 0 = Ri + u_C$ $\left\{ i = \frac{dq}{dt} = C\frac{du_C}{dt} \right\} \Rightarrow 0 = \frac{u_C}{RC} + \frac{du_C}{dt}$

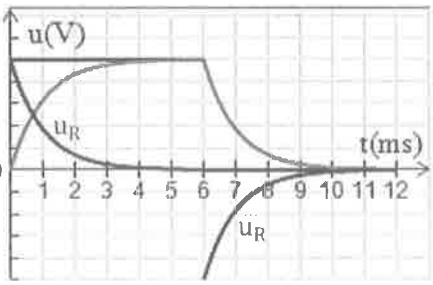
ii. $\frac{du_C}{dt} = \frac{-P}{\tau'}e^{-t/\tau'}$ substitute in the diff equation: $0 = \frac{P}{RC}e^{-t/\tau'} - \frac{P}{\tau'}e^{-t/\tau'}$

$0 = Pe^{-t/\tau'}\left[\frac{1}{RC} - \frac{1}{\tau'}\right]$ But $Pe^{-t/\tau'} = 0$ is rejected $\Rightarrow \frac{1}{RC} - \frac{1}{\tau'} = 0 \Rightarrow \underline{\tau' = RC}$. At $t_0 = 0$: $u_C = E = Pe^{0} \Rightarrow \underline{P = E}$.

iii. $u_C = Ee^{-t/\tau'}$. E and u_C have the same unit $\Rightarrow$ The power of exponential is unit less $\Rightarrow$ t and τ' have the same unit $\Rightarrow$ S.I unit of τ' is s.

8.

A.



a) i. The tangent to u_C at $t_0 = 0$, cuts the asymptote $u_G = E$ at a point of abscissa $\tau \Rightarrow \tau = \underline{1\text{ ms.}}$ $\Delta t_{charging} = \Delta t_{discharging} = 5\tau = 5 \times 1 = \underline{5\text{ ms}}$.

ii. $\tau = RC \Rightarrow C = \frac{10^{-3}}{1000} = \underline{1\ \mu F}$.

b) Graphically, at $t = 5\tau$; $u_C = E$ and $e^{-t/\tau} \approx 0 \Rightarrow E = D$. At $t_0 = 0$, $u_C = 0$
$\Rightarrow 0 = D - 10 \Rightarrow D = 10 = E$.

c) Figure.

d) i. At t = 7 ms ; since the duration of discharging till this instant is $\tau = 1$ ms.
$\Rightarrow u_C = 0.37\ E = 0.37\ (10) = \underline{3.7\ V}$.

ii. During [0 , 6] : $W_R = W_C = \frac{1}{2}C\,E^2 = \frac{1}{2}(1 \times 10^{-6})(10)^2 = 5 \times 10^{-5}$ J.

During [6 , 7] : At t = 7 s: $W_C = \frac{1}{2}C\,u_C^2 = \frac{1}{2}(10^{-6})(3.7)^2 = 6.845 \times 10^{-6}$ J.

$\Rightarrow W_{lost} = 5 \times 10^{-5} - 6.845 \times 10^{-6} = 4.3155 \times 10^{-5}$ J.

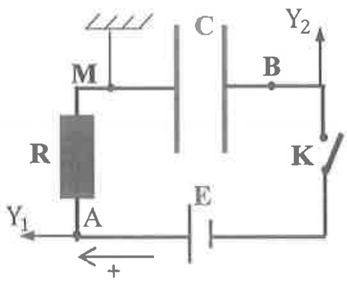
$\Rightarrow$ Total energy lost by R during [0 , 7] is : $W_t = 5 \times 10^{-5} + 4.3155 \times 10^{-5} = \underline{9.3155 \times 10^{-5}\ J}$.

B.

a) In figure (3 – b) since steady state is attained at t = 2 ms < 5 ms (time to reach steady state in figure 2).

b) R and C. We can increase τ by : replacing the resistor by another one of higher resistance, or / and replacing the capacitor by another one of larger C.

9.



a) Figure ("INVERT" button is pressed for Y_2).

b) i. During charging, u_C increases (exponentially) with time $\Rightarrow$ curve (A) represents the variation of u_C as a function of time. $u_R = E - u_C \Rightarrow u_R$ decreases with time
$\Rightarrow$ curve (B) represents the variation of u_R as a function of time.
OR : $[u_{R_0} = E - 0 = E \neq 0 \Rightarrow$ curve B is for u_R].

ii. $u_{R_0} = E - 0 = E = 12\ V = 2.S_{v_1} \Rightarrow S_{v_1} = \frac{12}{2} = \underline{6\ v/div}$.
When steady state is attained $u_C = E = 12$ V. $12 = 4\ S_{v_2} \Rightarrow S_{v_2} = \underline{3\ V/div}$.

c) i. $u_{AB} = u_{AM} + u_{MB} \Rightarrow E = R\,i + \frac{q}{C} \Rightarrow E = R\frac{dq}{dt} + \frac{q}{C} \Rightarrow \frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}$.

ii. $q = Q_m - Q_m e^{-t/\tau} \Rightarrow \frac{dq}{dt} = \frac{Q_m}{\tau}e^{-t/\tau}$ substitute in diff eq: $\frac{Q_m}{\tau}e^{-t/\tau} + \frac{Q_m}{RC} - \frac{Q_m}{RC}e^{-t/\tau} = \frac{E}{R} \Rightarrow Q_m e^{-t/\tau}\left[\frac{1}{\tau} - \frac{1}{RC}\right] + \frac{Q_m}{RC} = \frac{E}{R}$.

The constant terms are equal : $\frac{Q_m}{RC} = \frac{E}{R} \Rightarrow Q_m = \underline{CE}$.

The variable term is zero : $Q_m e^{-t/\tau}\left[\frac{1}{\tau} - \frac{1}{RC}\right] = 0$. But $Q_m e^{-t/\tau} = 0$ is rejected $\Rightarrow \frac{1}{\tau} - \frac{1}{RC} = 0 \Rightarrow \tau = \underline{RC}$.

iii. $0.2\,Q_m = Q_m\left(1 - e^{-t/\tau}\right) \Rightarrow 0.8 = e^{-t/\tau}$...eq (1). $0.8\,Q_m = Q_m\left(1 - e^{-t'/\tau}\right) \Rightarrow 0.2 = e^{-t'/\tau}$...eq (2).

Divide eq(1) by eq(2): $4 = \frac{e^{-t/\tau}}{e^{-t'/\tau}} = e^{-\frac{1}{\tau}[t - t']} = e^{+\frac{1}{\tau}(2.77 \times 10^{-3})}$ $\{t' > t\} \Rightarrow \ln 4 = \frac{(2.77 \times 10^{-3})}{\tau} \Rightarrow \tau = 2 \times 10^{-3}$ s $= \underline{2\text{ ms}}$.

d) **i.** At $t = \tau$: $q = Q_m\,(1 - e^{-1}) = 0.63\,Q_m$. But $u_C = \frac{q}{C} = 0.63\,\frac{Q_m}{C} = 0.63$ E .

ii. $0.63 \times 12 = 7.56$ V $= S_{v2}.y \Rightarrow y = \frac{7.56}{3} = 2.52$ div. Graphically $u_C = 0.63$ E for $t = \tau = 2$ ms $= V_b.\,2 \Rightarrow V_b = \underline{1\text{ ms/div}}$.

10.

A.

a) $u_{AB} = u_{AM} + u_{MB} \Rightarrow E = \frac{q}{C} + Ri$ $\left\{i = +\frac{dq}{dt}\right\}$ Derive w.r.t time $\Rightarrow 0 = \frac{i}{C} + R\,i' \Rightarrow \frac{di}{dt} + \frac{i}{RC} = 0$.

b) $\frac{di}{dt} = \frac{-I_m}{\tau}\,e^{-t/\tau} = \frac{-i}{\tau}$ sub in the differential eqn: $\frac{-i}{\tau} + \frac{i}{RC} = 0 \Rightarrow i\left[-\frac{1}{\tau} + \frac{1}{RC}\right] = 0$ but $\tau = RC \Rightarrow i\left[\frac{-1}{\tau} + \frac{1}{RC}\right] = 0$ verified .

c) **i.** $u_R = R\,i$ but R = const $\Rightarrow u_R$ and i are directly proportional $\Rightarrow$ curve of u_R represents the image of i .

ii. $i = I_m e^{-t/\tau} \Rightarrow$ i decreases exponentially with time $\Rightarrow u_R$ also decreases exponentially with time $\Rightarrow$ curve (a) represents the variation of u_R .

iii. Graphically $u_R > 0 \Rightarrow i > 0 \Rightarrow$ <u>clockwise</u>. $I_m = \frac{u_{Rm}}{R} = \frac{12}{80} = \underline{0.15\text{ A}}$.

d) Curve (b) represents the variation of $u_C = u_{AM}$ during the charging process. $q = q_A = C\,u_{AM}$.
But graphically $u_{AM} > 0 \Rightarrow q > 0$. And i is clockwise $\Rightarrow$ i enters the positive plate of the capacitor $\Rightarrow$ capacitor acts as a receiver .

B.

a) $u_{AM} = \frac{q}{C} > 0 \Rightarrow$ curve (d) represents the variation of $u_C \Rightarrow$ curve (c) represents the variation of u_R.

b) $u_R = Ri$. But $u_R < 0 \Rightarrow i < 0 \Rightarrow$ <u>Anti-clockwise</u>. i flows out from the positive plate of the capacitor $\Rightarrow$ capacitor acts as a generator .

c) **i.** $0 = \frac{q}{C} + u_R$ $\left\{i = +\frac{dq}{dt} \text{ and } i = \frac{u_R}{R}\right\}$ Derive w.r.t time : $0 = \frac{i}{C} + \frac{du_R}{dt} \Rightarrow 0 = \frac{u_R}{RC} + \frac{du_R}{dt}$.

ii. $\frac{du_R}{dt} = -\frac{u_R}{RC}$ At the beginning of discharging ; $u_R = -12$ V $\Rightarrow +2.2 = \frac{-(-12)}{80\,C} \Rightarrow C = 0.06818$ F $= \underline{68.18\text{ mF}}$.

d) On the figure: $u_G = 12$ V during [0, 30 s] and $u_G = 0$ during [30 s, 60 s].

11.

A.

a) $u_C = E\left(1 - e^{-t/\tau}\right)$. $u_{AB} = u_{AD} + u_{DM} + u_{MN} + u_{NB} \Rightarrow E = 0 + u_R + u_C \Rightarrow u_R = E - \left(E - Ee^{-t/\tau}\right) = Ee^{-t/\tau}$.

b) $\ell n\,u_R = \ell n\,E + \ell n\,e^{-t/\tau} \Rightarrow \ell n\,u_R = \ell n\,E - \frac{t}{\tau}$.

c) **i.** Slope $= -\frac{1}{\tau}$. But slope $= \frac{0-3}{(3 \times 10^{-3}) - 0} = -1000 \Rightarrow -\frac{1}{\tau} = -1000 \Rightarrow \tau = 10^{-3}$ s . $\tau = RC \Rightarrow C = \frac{10^{-3}}{500} = \underline{2\ \mu\text{F}}$.

ii. The point (0, 3) satisfies the equation of the line $\Rightarrow 3 = \ell n\,E - 0 \Rightarrow E = e^3 \cong 20$ V.

B.

a) $W_C = \frac{1}{2}C\,E^2 = \frac{1}{2}(2 \times 10^{-6})(20)^2 \Rightarrow W_C = \underline{4 \times 10^{-4}}$ J. **b)** $W_G = 8 \times 10^{-4}$ J.

b) $W_R = W_G - W_C = 8 \times 10^{-4} - 4 \times 10^{-4} = \underline{4 \times 10^{-4}\text{ J}}$. $W_C = W_R$.

C.

a) $u_C = 20\,(1 - e^{-2.5}) = 18.36$ V $\Rightarrow W_C = \frac{1}{2}C\,u_C^2 = \frac{1}{2}(2 \times 10^{-6})(18.36)^2 = \underline{3.37 \times 10^{-4}\text{ J}}$.

b) Graphically when $u_C = 7.4$ V ; $W_C = 7.344 \times 10^{-4}$ J $\Rightarrow W_{R_1} = 7.344 \times 10^{-4} - 3.37 \times 10^{-4} = \underline{3.974 \times 10^{-4}\text{ J}}$.
$W_{R_2} = 4 \times 10^{-4} - 3.974 \times 10^{-4} = \underline{2.6 \times 10^{-6}\text{ J}}$.

c) $W_{R_2} << W_{R_1}$. The average value of i during $[0, 2.5\tau]$ is much greater than its average value during $[2.5\tau,\ 5\tau]$, and $W_{R_1} = i_{av}^2\,R\,(2.5\tau - 0)$, $W_{R_2} = i_{av}^2\,R\,(5\tau - 2.5\tau) \Rightarrow W_{R_2} << W_{R_1}$.

12.

<u>1st experiment :</u>

a) **i.** $E = u_{DA} + u_{AM}$ at $t_0 = 0$, $E = u_{DA} = u_R = I_0 R = 0.12 \times 100 \Rightarrow E = 12$ V.

ii. The tangent to u_C at $t_0 = 0$, cuts the asymptote of equation u = E at a point of abscissa τ so $\tau = \underline{0.2\text{ s}}$.

iii. $\tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{0.2}{100} = 0.002$ F $\Rightarrow C = 2$ mF.

b) **i.** $E = u_{DA} + u_{AM} = u_R + u_C = i\,R + u_C \Rightarrow RC\,\frac{du_C}{dt} + u_C = E$.

ii. $\frac{du_C}{dt} = \alpha\,\beta\,e^{\alpha t} \Rightarrow$ $C\,R\alpha\,\beta e^{\alpha t} + A + \beta\,e^{\alpha t} = E$, then $\beta\,e^{\alpha t}(C\,R\alpha + 1) + A = E$ then :

$\underline{A = E = 12\text{ V}}$ and $\beta\,e^{\alpha t}(CR\alpha + 1) = 0$ but $\beta\,e^{\alpha t} \neq 0$ then $C\,R\,\alpha + 1 = 0$ $\therefore \alpha = -\frac{1}{RC} = -5\ s^{-1}$.

At $t_0 = 0$; $u_C = 0$; $0 = A + B \Rightarrow B = -A = \underline{-12\text{ V}}$.

iii. $u_C = 12 - 12\,e^{-t/RC} \Rightarrow i = C\frac{du_C}{dt} = \frac{12}{RC}\,e^{-\frac{t}{RC}} = 0.12e^{-5t}$. Yes, since $I_0 > 0$ and it is a decreasing function .

iv. $i = 50$ mA $\Rightarrow u_R = i\,R = 0.05 \times 100 = 5$ V $\Rightarrow u_C = E - u_R = 12 - 5 = 7$ V then $W_C = \frac{1}{2}Cu_C^2 = 0.5(2 \times 10^{-3})(49) = \underline{0.049\text{ J}}$.

<u>2nd experiment:</u>

a) At $t_0 = 0$, $u_{FM} = E = \underline{12\text{ V}}$. **b)** $i_0 = 0.06\,e^0 = 0.06$ A ; $u_{FN\,(0)} = 12$ V $\Rightarrow 12 = 0.06\,r \therefore r = 200\,\Omega$. $\Rightarrow i = 0.06\,e^{-2.5\,t}$.

c) $u_{AM} = u_{FN} = i\,r = 12\,e^{-2.5\,t}$.

d) $p = \frac{u^2}{r} \Rightarrow u = \sqrt{0.1 \times 200} = 4.47\text{ V} = 12\,e^{-2.5\,t} \Rightarrow e^{-2.5\,t} = 0.37$ then $\ell n\,0.37 = -2.5\,t$ then $t = \underline{0.4\text{ s}}$.

OR : $W_R = W_C = \frac{1}{2}C\,u_C^2 = \frac{1}{2}(2 \times 10^{-3})(144\,e^{-5t}) \Rightarrow W_C = 0.144\,e^{-5t}$. $p = \frac{dW_C}{dt} = -5\,(0.144)e^{-5t} \Rightarrow p = 0.72\,e^{-5t}$

$0.1 = 0.72\,e^{-5t} \Rightarrow t = \underline{0.395\text{ s}}$.

OR : $p = r\,i^2 \Rightarrow 0.1 = (200)\,i^2 \Rightarrow i = 0.02236 = 0.06\,e^{-2.5\,t} \Rightarrow t = \underline{0.395\text{ s}}$.

13.

A.

a) $u_{AD} + u_{DF} + u_{FM} + u_{MA} = 0 \Rightarrow 0 + u_C + R_1 i - E = 0 \Rightarrow u_C = E - R_1 i$.

b) i. $u_C = E - Ri \Rightarrow$ when $i = 0$; $u_C = E = 24$ V $\Rightarrow E = \underline{24\text{ V}}$.

ii. When $u_C = 0$; $i = 24$ mA $\Rightarrow 0 = 24 - (R_1)(24 \times 10^{-3}) \Rightarrow R_1 = \underline{1000\,\Omega}$.

c) i. $u_C = \frac{q}{C}$. **ii.** $u_C = \frac{q}{C} \Rightarrow C = \frac{q}{u_C} = \frac{48 \times 10^{-6}}{24} = 2 \times 10^{-6}$ F $= \underline{2\,\mu\text{F}}$.

B.

a) $u_{AF} + u_{FD} + u_{DA} = 0 \Rightarrow u_{R_2} - \frac{q}{C} + 0 = 0$ $\left\{i = -\frac{dq}{dt}\right\}$. Derive w.r.t time : $\frac{du_{R2}}{dt} - \frac{1}{C}\frac{dq}{dt} = 0 \Rightarrow \frac{du_{R2}}{dt} + \frac{i}{C} = 0$

$\{u_{R_2} = iR_2\} \Rightarrow \frac{du_{R2}}{dt} + \frac{u_{R2}}{R_2\,C} = 0.$

b) i. $u_{R_2} = u_{AH} > 0 \Rightarrow$ current flows from A to H $\Rightarrow$ g is the + ve plate .

ii. $u_{R_2} = p\,e^{-\frac{t}{\tau'}} + H$. As $t \to \infty$; $u_{R_2} = 0 = p(0) + H \Rightarrow \underline{H = 0}$. At $t_0 = 0$; $u_{R_2} = 24 = pe^0 \Rightarrow \underline{p = 24\text{ V}}$.

0.37 (24) = 8.88 V ; Graphically when $u_{R_2} = 8.88$ V ; $t = \tau' = \underline{1\text{ ms}}$.

iii. $u_{R2(max)} = R_2 I_{max} \Rightarrow 24 = R_2(0.048) \Rightarrow R_2 = 500\,\Omega$. But $\tau' = R_2\,C \Rightarrow 10^{-3} = 500\,C \Rightarrow C = 2 \times 10^{-6}$ F $= \underline{2\,\mu\text{F}}$.

C.

a) <u>No</u> , since in the steady state q = 32 μC and not zero .

b) i. As $t \to \infty$; $q = 32\,\mu C \Rightarrow 32 \times 10^{-6} = \frac{48 \times 10^{-6}}{3}(0) + Z$. $Z = \underline{32 \times 10^{-6}\text{ C}}$.

OR : At $t_0 = 0$: $48 \times 10^{-6} = \frac{Q_m}{3} + Z \Rightarrow Z = 48 \times 10^{-6} - \frac{48 \times 10^{-6}}{3} \Rightarrow Z = \underline{32 \times 10^{-6}\text{ C}}$.

ii. $\frac{dq}{dt} = \frac{Q_m}{3}\left(\frac{-3}{C\,R_3}\right)e^{\frac{-3\,t}{C\,R_3}} \Rightarrow \frac{dq}{dt}\Big|_{t_0=0} = \frac{-Q_m}{C\,R_3}\,e^0 \Rightarrow \frac{dq}{dt}\Big|_{t_0=0} = \frac{-Q_m}{C\,R_3}$.

iii. $\frac{dq}{dt}\Big|_{t_0=0}$ = slope of tangent to q at $t_0 = 0 \Rightarrow \frac{-Q_m}{C\,R_3} = \frac{(0 - 48 \times 10^{-6})}{(1.2 \times 10^{-3})} \Rightarrow \frac{-48 \times 10^{-6}}{C\,R_3} = \frac{-48 \times 10^{-6}}{1.2 \times 10^{-3}} \Rightarrow C\,R_3 = \underline{1.2 \times 10^{-3}\text{ F}\,\Omega}$.

iv. $1.2 \times 10^{-3} = 600\,C \Rightarrow C = \underline{2 \times 10^{-6}\text{ F}}$.

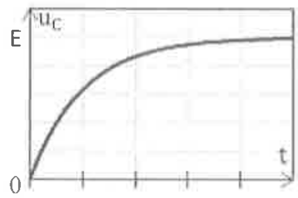
14.

A.

a) $u_{BA} = u_{BM} + u_{MD} + u_{DA} \Rightarrow E = Ri + u_C + 0$ $\left\{\text{But } i = \frac{dq}{dt} = C\frac{du_C}{dt}\right\} \Rightarrow E = RC\frac{du_C}{dt} + u_C$.

b) i. $u_{C\,max} = U_m = \underline{E}$ and $\Delta t = 5\tau = \underline{5\,R\,C}$.

ii. Figure



B.

a) System : spring . External forces : $\vec{T}$, $\vec{f}$, $\vec{F}$.

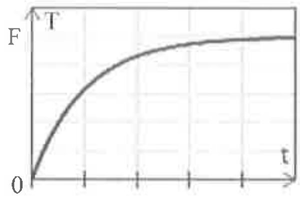
$\sum \vec{F}_{ext} = m\vec{a} \Rightarrow : \vec{T} + \vec{f} + \vec{F} = \vec{0}$ (massless spring) $\Rightarrow -T\vec{i} - h\,v\,\vec{i} + F\vec{i} = \vec{0} \Rightarrow F = T + h\,v$

But $T = k\,x$ and $\frac{dT}{dt} = k\frac{dx}{dt} = k\,v \Rightarrow F = T + h(\frac{1}{k}\frac{dT}{dt}) \Rightarrow F = T + \frac{h}{k}\frac{dT}{dt}$

b) $C \equiv \frac{1}{k}$; $R \equiv h$; $u_C \equiv T$ and $E \equiv F$.

c) i. Using analogy $\Rightarrow T = F\,(1 - e^{-kt/h})$. **ii.** $T_m = F$ and $\Delta t = 5\tau = \frac{5\,h}{k}$.

iii. Figure .



15.

A.

a) Vertically upwards .

b) Electromagnetic induction and charging of the capacitor.

c) $\varphi = NBS\cos(\vec{n},\vec{B}) = BS\cos(0) = B\ell(x)$. $e = -\frac{d\varphi}{dt} \Rightarrow e = \underline{-B\,\ell\,v}$.

d) $i > 0$ and $e < 0 \Rightarrow e.i < 0 \Rightarrow$ MN acts as a receiver.

e) System : MN. External forces : weight $m\vec{g}$, Normal reactions $\vec{N}_N$, $\vec{N}_M$, electromagnetic force $\vec{F}_{e.m}$

$$\sum \vec{F}_{ext} = m\vec{a} \Rightarrow (m\vec{g} + \vec{N}_N + \vec{N}_M) + \vec{F}_{e.m} = m\vec{a} \Rightarrow +i\,\ell\,B = m\,v' \Rightarrow v' = +\frac{\ell B}{m} i.$$

f) $u_{KN} + u_{NM} + u_{Ma} + u_{aD} + u_{DK} = 0$, but $q = cu_{DK}$ and $i = \frac{dq}{dt} \Rightarrow 0 + r\,i - e + 0 - E + \frac{q}{C} = 0$ ... eq(*)

$r\,i + B\,\ell\,v - E + \frac{q}{C} = 0$ Derive w.r.t time $\Rightarrow r\frac{di}{dt} + B\ell\frac{dv}{dt} + \frac{1}{C}\frac{dq}{dt} = 0$

$$r\,i' + B\ell\left(\frac{B\ell}{m} i\right) + \frac{i}{C} = 0 \Rightarrow r\,i' + \left(\frac{i}{C} + \frac{B^2\ell^2}{m}\right) i = 0 \Rightarrow \frac{di}{dt} + \left(\frac{m + C B^2\ell^2}{m r C}\right) i = 0.$$

B.

a) From eq (*) : $ri_0 - 0 - E + 0 = 0 \Rightarrow i_0 = \frac{E}{r}$. From the solution: $i_0 = A(1) = \frac{E}{r} \Rightarrow A = \frac{E}{r}$.

$\frac{di}{dt} = -\frac{A}{\tau} e^{-\frac{t}{\tau}}$ sub in diff eq: $0 = -\frac{A}{\tau} e^{-\frac{t}{\tau}} + \left(\frac{m + C B^2\ell^2}{m r C}\right)\left(Ae^{-\frac{t}{\tau}}\right)$

$Ae^{-\frac{t}{\tau}}\left[-\frac{1}{\tau} + \left(\frac{m + C B^2\ell^2}{m r C}\right)\right] = 0$; but $Ae^{-\frac{t}{\tau}} = 0$ is rejected $\Rightarrow -\frac{1}{\tau} + \frac{m + C B^2\ell^2}{m r C} = 0 \Rightarrow \tau = \frac{m r C}{m + C B^2\ell^2}$.

b) $\frac{dv}{dt} = \frac{\ell B}{m} i \Rightarrow dv = \frac{\ell B}{m} i\,dt \Rightarrow v = \frac{\ell B}{m}\int i\,dt = \frac{\ell B}{m}\int \frac{E}{r} e^{-\left(\frac{m + C B^2\ell^2}{m r C}\right)t} dt = \frac{\ell B}{m}\left(\frac{E}{r}\right)\left[\left(-\frac{m r C}{m + C B^2\ell^2}\right) e^{-\frac{t}{\tau}}\right] + \text{const}$.

$v = \frac{-E\,\ell\,B\,C}{m + C B^2\ell^2} e^{-\frac{t}{\tau}}$ + const. But $v_0 = 0 = \frac{-E\,\ell\,B\,C}{m + C B^2\ell^2}(1)$ + const .

$$\Rightarrow \text{const} = \frac{E\,\ell\,B\,C}{m + C B^2\ell^2} \Rightarrow v = \frac{-E\,\ell\,B\,C}{m + C B^2\ell^2} e^{-\left(\frac{m + C B^2\ell^2}{m r C}\right)t} + \frac{E\,\ell\,B\,C}{m + C B^2\ell^2} = \frac{E\,\ell\,B\,C}{m + C B^2\ell^2}\left[1 - e^{-\left(\frac{m + C B^2\ell^2}{m r C}\right)t}\right].$$

C.

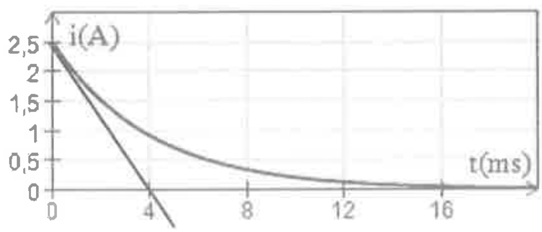
a) $i_0 = \frac{E}{r} \Rightarrow 2.5 = \frac{E}{4} \Rightarrow E = 10$ V .

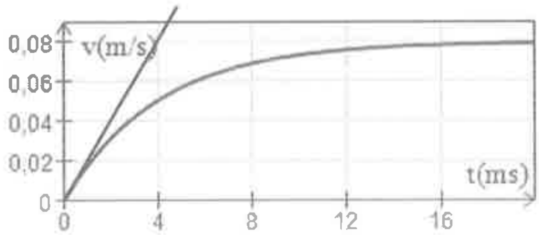
b) From figure (2) : $v_{max} = 0.08$ m/s $= \frac{E\,\ell\,B\,C}{m + C B^2\ell^2} \Rightarrow 0.08 = \frac{(10)(0.2)(0.4)\,C}{0.01 + (C)(0.16)(0.04)} \Rightarrow C = 10^{-3}$F = <u>1 mF</u>.

OR : At $t = \tau$; $v = 0.63\,v_{max} = 0.63\,(0.08) \approx 0.05$ m/s. Graphically $v \cong 0.05$ m/s when $t = \tau = 0.004$ s.

But $\tau = \frac{m r C}{m + C B^2\ell^2} \Rightarrow 0.004 = \frac{(0.01)(4)\,C}{0.01 + C\,(0.16)(0.04)} \Rightarrow C = \underline{10^{-3}\text{ F}}$.

c) Figures.





Chapter 11: Alternating Sinusoidal Current

1. **a)** **i.** u_1 and u_2: $|\Delta\varphi_{1,2}| = |\varphi_2 - \varphi_1| = \left| -\frac{\pi}{6} - \left(+\frac{\pi}{6}\right)\right| = \frac{\pi}{3}$ rad.

u_1 and u_3 : $u_3 = 3\cos\left(\omega t + \frac{\pi}{6}\right) = 3\sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right) = 3\sin\left(\omega t + \frac{2\pi}{3}\right) \Rightarrow |\Delta\varphi_{1,3}| = \varphi_3 - \varphi_1 = \frac{2\pi}{3} - \frac{\pi}{6} = \frac{\pi}{2}$ rad.

ii. $\Delta\varphi_{1,2} = -\frac{\pi}{3} < 0 \Rightarrow u_1$ leads u_2. $\quad \Delta\varphi_{1,3} = \frac{\pi}{2} > 0 \Rightarrow u_3$ leads u_1.

b) u_1 has the higher amplitude and it leads $u_2 \Rightarrow$ figure (2) represents the variations of u_1 and u_2.

2. **a)** $p_{av} = I_{eff}^2\,(R + r) = (1.24)^2\,[50 + 10] = \underline{92.256\text{ W}}$.

b) $p_{av} = I_{eff}\ U_{G\,eff}\cos\varphi \Rightarrow \cos\varphi = \frac{92.256}{(1.24)(120)} = \underline{0.62}$.

3. **a)** $u_{BA} = u_{BM} + u_{MA}$, but $u_{MA} = L\frac{di}{dt} = L\,\omega\,I_m\cos(\omega t) \Rightarrow U\sin(\omega t + \varphi) = RI_m\sin(\omega t) + L\,\omega I_m\cos(\omega t)$

For $\omega t = 0$: $U\sin\varphi = L\omega I_m \Rightarrow \sin\varphi = \frac{L\,\omega\,I_m}{U}$ …eq (1). For $\omega t = \frac{\pi}{2}$: $U\sin\left(\varphi + \frac{\pi}{2}\right) = RI_m \Rightarrow \cos\varphi = \frac{RI_m}{U}$ …eq (2)

eq(1) ÷ eq(2) $\Rightarrow \tan\varphi = \frac{L\,\omega\,I_m}{R\,I_m} = \frac{L\,\omega}{R}$. Square and add: $1 = \frac{L^2\omega^2 I_m{}^2}{U^2} + \frac{R^2 I_m{}^2}{U^2} \Rightarrow U^2 = I_m^2\,[L^2\omega^2 + R^2] \Rightarrow I_m = \frac{U}{\sqrt{L^2\omega^2 + R^2}}$

b) As ω increases, I_m decreases.

4. **a)** $i = \frac{dq}{dt} = c\,\frac{du_c}{dt} \Rightarrow u_c = \frac{1}{C}\int i\,dt = \frac{1}{C}\int I_m\sin(\omega t)\,dt = \frac{I_m}{C}\left[-\frac{\cos(\omega t)}{\omega}\right]$.

$u_{AD} = u_{AB} + u_{BD} \Rightarrow U\sin(\omega t + \varphi) = RI_m\sin(\omega t) - \frac{I_m}{C\,\omega}\cos\omega t$.

For $\omega t = 0$: $U\sin\varphi = -\frac{I_m}{C\,\omega} \rightarrow \sin\varphi = -\frac{I_m}{C\,\omega\,u}$ …eq (1). For $\omega t = \frac{\pi}{2}$: $U\sin\left(\frac{\pi}{2} + \varphi\right) = RI_m \Rightarrow U\cos\varphi = RI_m$

$\Rightarrow \cos\varphi = \frac{RI_m}{U}$ …eq (2). eq (1) ÷ eq (2) $\Rightarrow \tan\varphi = -\frac{I_m}{C\,\omega\,U}\cdot\frac{U}{RI_m} = \frac{-1}{R\,C\,\omega}$.

Square and add $\Rightarrow 1 = \frac{I_m^2}{C^2\omega^2 U^2} + \frac{R^2 I_m^2}{U^2} \Rightarrow U^2 = I_m^2\left[\frac{1}{C^2\omega^2} + R^2\right] \Rightarrow I_m = \frac{U}{\sqrt{\frac{1}{C^2\omega^2} + R^2}}$.

b) As ω increase, I_m increases.

5. **a)** When $\omega = \omega_0$, the amplitude I_m of the current is maximum. Graphically $i = I_m$ when $\omega = 1000$ rad/s.

$\Rightarrow$ Resonance angular frequency $\underline{\omega_o = 1000\text{ rad/s}}$.

b) $u_{G_{max}} = u_{R_{max}}$. For R_3: $u_{G_{max}} = I_3.R_3 = 0.476 \times 210 = \underline{100\text{ V}}$.

c) $u_{G_{max}} = I_2.R_2 \Rightarrow 100 = I_2(500) \Rightarrow I_2 = \underline{0.2\text{ A}}$. $\quad u_{G_{max}} = I_1.R_1 \Rightarrow 100 = 0.05(R_1) \Rightarrow R_1 = \mathbf{200\ \Omega}$.

6. **a)** $R = \frac{U_{ab\,(eff)}}{I_{eff}} = \frac{1}{2\times10^{-3}} = \underline{500\ \Omega}$.

b) $p_{av} = I_{eff}^2\,R = (2\times10^{-3})^2\,(500) = \underline{2\times10^{-3}\text{ W}}$.

$p_{av} = I_{eff}\ U_{ad\,(eff)}\cos\varphi \Rightarrow 2\times10^{-3} = (2\times10^{-3})\,(1)\,\cos\varphi \Rightarrow \cos\varphi = 1 \Rightarrow \underline{\varphi = 0}$.

c) $\varphi = 0 \Rightarrow$ <u>i and u_{ad} are in phase</u> $\Rightarrow$ current resonance takes place.

d) $\omega_o^2\,L\,C = 1 \Rightarrow L = \frac{1}{(5\times10^6)^2\times100\times10^{-12}} \Rightarrow L = 4\times10^{-4}\text{ H} = \underline{0.4\text{ mH}}$.

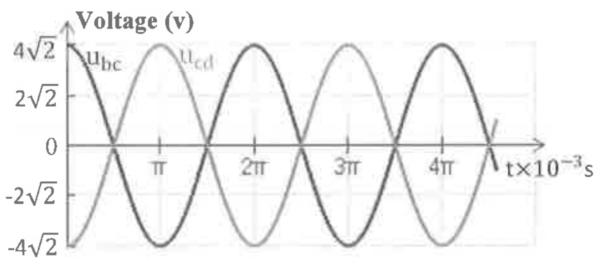
e) $u_{bc} = L\frac{di}{dt} = L\omega I_m\cos\omega t$. $\quad i = C\frac{du_{cd}}{dt} \Rightarrow u_{cd} = \frac{-I_m}{C\,\omega}\cos\omega t$.

But $\omega = \frac{1}{\sqrt{L\,C}} \Rightarrow u_{bc} = I_m\sqrt{\frac{L}{C}}\cos\omega t$, and

$u_{cd} = -I_m\sqrt{\frac{L}{C}}\cos\omega t \Rightarrow u_{bc} = -\,u_{cd}$.

f) $u_{bc} = I_m\sqrt{\frac{L}{C}}\cos\omega t = 2\times10^{-3}\sqrt{2}\sqrt{\frac{4\times10^{-4}}{10^{-10}}}\cos(1000\,t)$

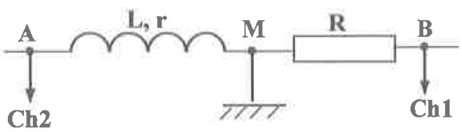
$\Rightarrow u_{bc} = 4\sqrt{2}\cos(1000t)$ (S.I).



7. **a)** Adjacent figure, the "INV" button of Ch2 should be pressed.

b) In the R-L series circuit u_L leads the current i.

$\Rightarrow u_L$ leads u_R and graphically curve (b) leads curve (a).



c) $u_{R\,(max)} = 4 \times 2.5 = \underline{10\ V}$. $T = v_b \,.\, x = (4 \times 10^{-3})\,(5) = \underline{0.02\ s}$.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.02} = \underline{100\pi\ rad/s}$.

$\left.\begin{array}{lll} 2\pi\ rad & \rightarrow & 5\ div \\ |\Delta\varphi| & \rightarrow & 0.8\ div \end{array}\right\} \Rightarrow |\Delta\varphi| = \underline{0.32\,\pi\ rad}$.

d) $u_R = 10\sin(100\pi t)$ (S.I). $u_{L\,max} = 3.72 \times 2.5 = 9.3\ V \Rightarrow u_L = 9.3\sin(100\pi t + 0.32\pi)$ (S.I)

e) i. $u_L = r\,i + L\frac{di}{dt}$ where $i = \frac{u_R}{R} = 0.1\sin(100\pi t)$.

$u_L = r(0.1)\sin(100\pi t) + L\,(0.1)\,(100\pi)\cos(100\pi t) \Rightarrow u_L = 0.1\,r\sin(100\pi t) + 10\pi\,L\cos(100\pi t)$ (S.I).

ii. $9.3\sin(100\pi t + 0.32\pi) = 0.1\,r\sin(100\pi t) + 10\,\pi\,L\cos(100\pi t)$

For $t = 0$: $9.3\sin(100\pi t + 0.32\pi) = 10\pi L \Rightarrow \underline{L = 0.25\ H}$. For $100\,\pi\,t = \frac{\pi}{2}$: $9.3\cos(0.32\,\pi) = 0.1r \Rightarrow r \approx \underline{50\ \Omega}$.

f) $u_{BA} = u_{BM} + u_{MA} \Rightarrow U\sin\left(100\pi t + 0.32\pi - \frac{\pi}{6}\right) = 10\sin(100\pi t) + 9.3\sin(100\pi t + 0.32\pi)$

For $t = 0$: $U\sin\left(\frac{23}{150}\pi\right) = 9.3\sin(0.32\pi) \Rightarrow \underline{U = 17\ V}$.

8. **a)** Ch1 displays u_{AD} and Ch2 displays u_{BD}.

b) i. Graphically, the amplitude of curve (b) is greater than that of (a) (S_v is the same)$\Rightarrow$(b) represents the variation of u_G.
But curve (b) leads (a) $\Rightarrow$ u_G leads u_C.

ii. $u_{G\,max} = 30\ V = S_v\,(3) \Rightarrow S_v = \underline{10\ V/div.}$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{500\pi} = 4 \times 10^{-3} s = V_b.(8) \Rightarrow V_b = \underline{0.5\ ms/div}$.

iii. $u_C = u_{BD} = u_{C\,max}\cos(500\pi t - \varphi)$ where $u_{C\,max} = 1.8(10) = 18\ V$

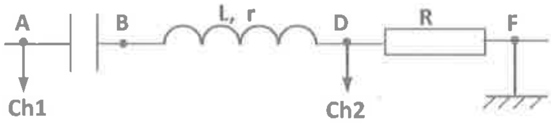
$\left.\begin{array}{l} 8\ div \rightarrow 2\pi\ rad \\ 1.18\ div \rightarrow |\Delta\varphi| \end{array}\right\} \Rightarrow |\Delta\varphi| = 0.295\,\pi\ rad \Rightarrow u_C = 18\cos(500\pi t - 0.295\pi)$ (S.I)

c) i. $i = \frac{dq}{dt} = C\frac{du_C}{dt}$. $i = 3.54 \times 10^{-5}(18)(-500\pi)\sin(500\pi t - 0.295\,\pi) = -\sin(500\pi t - 0.295\,\pi)$ (S.I)

ii. $u_R = R.i = R\sin(500\pi t - 0.295\,\pi)$ (S.I).

d) $(30)^2 = U_{R max}^2 + 18^2 \Rightarrow U_{R max} = 24\ V$. But $U_{R max} = I_{max}\,R \Rightarrow R = \underline{24\ \Omega}$.

9. **A)**



a) Adjacent figure

b) i. Since it has larger amplitude (same vertical sensitivity).

ii. It takes place since i and u_G are in-phase.

iii. $U_{max} = 4 \times 2.5 = 10\ V \Rightarrow U = \frac{U_{max}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}\ V$.

$T_0 = V_b.\ x = 2\ ms/div$. $4\ div = 8\ ms \Rightarrow f_0 = \frac{1}{8 \times 10^{-3}} = 125\ Hz$.

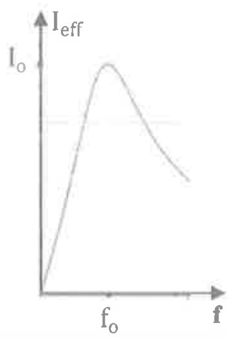
$U_{R max} = 3.6 \times 2.5 = 9\ V \Rightarrow I_m = \frac{9}{100} = 0.09\ A \Rightarrow I_0 = \frac{0.09}{\sqrt{2}}\ A$.

c) $f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow (125)^2 = \frac{1}{4\pi^2 L(3.25 \times 10^{-6})} \Rightarrow L = \underline{0.5\ H}$.

d) $P_{av} = I_{eff}\,U_{G\,eff}\cos\varphi = \frac{(0.09)}{\sqrt{2}}(5\sqrt{2})(1) = 0.45\ W$. $P_{av} = I_{eff}^2\ (r + R)$

$\Rightarrow 0.45 = \left(\frac{0.09}{\sqrt{2}}\right)^2 [r + 100] \Rightarrow r = 11.11\ \Omega$.

e) Figure .



B)

a) $f < f_0 \Rightarrow$ i leads u_G.

b) $u_G = 10\sin(\omega t - 1.4363)$ (S.I).

c) i) $u_L = r\,i + L\frac{di}{dt} = r(0.012)\sin(\omega t) + L\,(0.012)\,\omega\cos(\omega t)$ (S.I).

ii) $u_C = \frac{1}{C}\int i\,dt = -\frac{0.012}{C\omega}\cos(\omega t) \Rightarrow u_C = \frac{-48000}{13\,\omega}\cos(\omega t)$ (S.I).

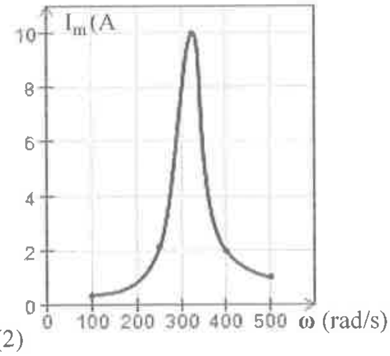
d) $10\sin(\omega t - 1.4363) = \frac{-48000}{13\,\omega}\cos(\omega t) + (0.012)\,(r + R)\sin(\omega t) + (0.012)L\,\omega\cos(\omega t)$

For $\omega t = 0$: $10\sin(-1.4363) = \frac{-48000}{13(100\,\pi)} + (0.012)L(100\pi) \Rightarrow \underline{L = 0.5\ H}$.

For $\omega t = \frac{\pi}{2}$: $10\sin\left(\frac{\pi}{2} - 1.4363\right) = (0.012)(r + 100) \Rightarrow \underline{r = 11.14\ \Omega}$.

10. A)

a) i. $\omega_0 = 314$ rad/s . **ii)** $\varphi = 0$. It is current resonance.



b) $(\omega_0^2) = (314)^2 = 98596.$ $\frac{1}{LC} = \frac{1}{0.318 \times 31.9 \times 10^{-6}} \approx 98596$. So $\omega_0 = \sqrt{\frac{1}{LC}}$.

c) i. i leads u_G since $\varphi < 0$. **ii.** i lags behind u_G since $\varphi > 0$

d) Figure.

B)

a) $u_{NM} = i\,R = I_m\,R \sin\omega t.$ $u_{AB} = L\frac{di}{dt} = L\,I_m\omega\cos\omega t$. $u_{BN} = \frac{1}{C}\int i\,dt = -\frac{I_m}{C\omega}\cos\omega t$

$\Rightarrow U_m \sin(\omega t + \varphi) = I_m R \sin\omega t + L\,I_m\omega\cos\omega t - \frac{I_m}{C\omega}\cos\omega t$

For $\omega t = 0 : U_m \sin\varphi = L I_m \omega - \frac{I_m}{C\omega}$ ... eq (1). For $\omega t = \frac{\pi}{2}$: $U_m \cos\varphi = I_m R$... eq (2)

eq (1) ÷ eq (2) $\Rightarrow \tan\varphi = \frac{L\,I_m\,\omega - \frac{I_m}{C\omega}}{I_m R} \Rightarrow \tan\varphi = \frac{L\omega - \frac{1}{C\omega}}{R}$.

$\sin^2\varphi + \cos^2\varphi = 1 \Rightarrow \frac{I_m^2}{U_m^2}\left(L\omega - \frac{1}{C\omega}\right)^2 + \left(\frac{I_m R}{U_m}\right)^2 = 1 \Rightarrow I_m^2\left[\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2\right] = U_m^2 \Rightarrow I_m = \frac{U_m}{\sqrt{\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2}}$.

b) i. Current resonance $\Rightarrow \tan\varphi = 0 \Rightarrow L\omega = \frac{1}{C\omega} \Rightarrow$ <u>$LC\,\omega^2 = 1$</u>.

ii. From the expression of I_m : $I_m = \frac{U_m}{\sqrt{0 + R^2}} = \frac{U_m}{R} = \frac{100}{10} \Rightarrow I_m =$ <u>10 A</u>.

iii. $U_m = I_m\,R = U_{R\,max}$, also u_R and u_G are in phase $\Rightarrow u_G = u_R \Rightarrow u_{AM} = u_{AB} + u_{BN} + u_{NM} \Rightarrow u_{AB} + u_{BN} = 0$

$\Rightarrow u_{BN} = u_C = -\,u_{AB}$.

11.

A) <u>X is a resistor</u> since the resistor does not oppose the variation of the current then <u>the current reaches its maximum instantly</u>.

<u>Y is a coil</u> since in a coil an induced emf is created which tends to oppose the increase of the current , then <u>the current in coil increases gradually to its maximum value</u>.

<u>Z is a capacitor</u> since <u>the current starts max and decreases gradually to zero</u> .

B)

a) i. $U_m = S_v \times Y_{max} = 5 \times 3 = 15$ V ; U = 5 × 3 = <u>15 V</u>.

ii) 6 div → 2 π
1.5 div → |Δφ| $\Rightarrow$ $|\Delta\varphi| = \frac{\pi}{2}$ rad. u_C lags u_g .

b) u_C lags u_g by $\frac{\pi}{2} \Rightarrow u_C = U\cos\left(100\pi t - \frac{\pi}{2}\right)$; where U = 15 V.

c) i. $i = C\frac{du_C}{dt} = -\,C\,(15 \times 100\pi)\sin\left(100\pi t - \frac{\pi}{2}\right) = -\,4710\,C\sin\left(100\pi t - \frac{\pi}{2}\right) \Rightarrow i = 4710\,C\cos(100\pi t)$ (S.I).

ii. Since i and u_g are in phase ($\Delta\varphi = 0$) current resonance takes place.

iii. $I_m = I\sqrt{2} = 0.1 \times 1.4 = 0.14$ A . But $I_m = (4710\,C) \Rightarrow C = 2.97 \times 10^{-5}$ F ≅ <u>30 μF</u>.

iv. In current resonance $LC\omega^2 = 1 \Rightarrow$ <u>L = 0.34 H</u>.

d) $u_g = u_R + u_L + u_C \Rightarrow 15\cos(100\pi t) = R(0.14)\cos(100\pi t) + (0.33)(0.14)(-100\pi)\sin(100\pi t) + 15\cos\left(100\pi t - \frac{\pi}{2}\right)$

For $\omega t = 100\,\pi t = 0 \Rightarrow 15 = R(0.14) + 15\cos\left(\frac{-\pi}{2}\right) \Rightarrow$ <u>R = 107.14 Ω</u>.

12. a) To measure u_{MB} and not u_{BM} .

b) i. curve (b) represents u_R. Curve (a) represents the variation of u_L since u_L must lead i .

ii. $u_{MB} = R.\,i$, but R = const $\Rightarrow u_{MB}$ and i are directly proportional $\Rightarrow$ The curve of u_{MB} represents the image of i.

iii. Graphically $u_{R\,max} = 4 \times 1 = 4$ V $\Rightarrow R = \frac{u_{R\,max}}{I_{max}} = \frac{4}{0.04} =$ <u>100 Ω</u>.

iv) $T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = 0.02$ s. Graphically: $T = V_b \,.\, x \Rightarrow 0.02 = V_b \,.\, 4 \Rightarrow V_b =$ <u>5 ms/div</u>.

v) 2π → 4 div
|Δφ| → 1 div $\Rightarrow |\Delta\varphi| = \frac{\pi}{2}$ rad . Since u_L leads i by $\frac{\pi}{2}$ rad then the coil is purely inductive; r = 0.

c) $P_{av} = I_{eff}U_{eff}\cos\varphi = \frac{0.04}{\sqrt{2}} \cdot \frac{8}{\sqrt{2}}\cos\left(\frac{-\pi}{3}\right) =$ <u>0.08 W</u>. $P_{av} = I_{eff}^2\,R \Rightarrow 0.08 = \left(\frac{0.04}{\sqrt{2}}\right)^2 R \Rightarrow R = 100\ \Omega$.

d) $u_L = L\frac{di}{dt} = L\,(0.04)(100\pi)\cos(100\pi t) = 4\pi\,L\cos(100\pi t).$ $U_{L\,max} = 4\text{ V} = 4\pi\,L \Rightarrow$ <u>L = 0.318 H</u>.

e) i. $u_C = \frac{1}{C}\int i\,dt = -\frac{0.04}{C\,(100\,\pi)}\cos(100\pi t) \Rightarrow u_C = \frac{-4 \times 10^{-4}}{C\,\pi}\cos(100\pi t)$ (S.I).

ii) $u_{DB} = u_{DA} + u_{AM} + u_{MB} \Rightarrow 8\sin\left(100\pi t - \frac{\pi}{3}\right) = \frac{-4\times10^{-4}}{C\pi}\cos(100\pi t) + 4\cos 100\pi t + 4\sin(100\pi t)$

For $100\pi t = 0$: $8\sin\left(\frac{-\pi}{3}\right) = \frac{-4\times10^{-4}}{C\pi} + 4 \Rightarrow C = 11.7\times10^{-6}$ F $= \underline{11.7\ \mu F}$.

13. A)

a) $W = P.t = (1100)(30)(24)(3600) = \underline{28512\times10^5\ J}$.

b) $P = I_{eff}U_{AB\,eff}\cos\varphi \Rightarrow 1100 = I_{eff}(220)(1) \Rightarrow I_{eff} = \underline{5\ A}$.

c) $W_{lost} = I_{eff}^2 R t = (5)^2(2)(30)(24)(3600) = \underline{1296\times10^5\ J}$.

d) $W_{total} = W_{lost} + W = 1296\times10^5 + 28512\times10^5 = \underline{2.9808\times10^9\ J}$.

B)

a) $W = P.t$: same P and same time t $\Rightarrow$ equal consumption.

b) $P = 1100 = I_{eff}(220)(0.7) \Rightarrow I_{eff} = \frac{50}{7} = 7.14$ A. $W'_{lost} = I_{eff}^2 R t = \left(\frac{50}{7}\right)^2(2)(30)(24)(3600) = 264489795.9$ J.

$W'_{total} = 28512\times10^5 + 264489795.9 = \underline{3.1157\times10^9\ J}$.

C)

a) Cost $= \frac{28512\times10^5}{(1000)(3600)}\times50 = 792\times50 = 39600$ L.L. Equal costs (same consumption of electric energy).

b) $W'_{total} > W_{total}$.

c) As the power factor decreases, the loss of electric energy in the transmission lines increases.

14. A)

a) $T = 30$ ms $\Rightarrow \omega_1 = \frac{2\pi}{T} = \frac{2\pi}{0.03} = \frac{200\pi}{3}$ rad/s $= \underline{209.33\ rad/s}$.

b) i. $\left.\begin{array}{l} 4\text{ div} \longrightarrow 2\pi \\ 2\text{ div} \longrightarrow |\Delta\varphi| \end{array}\right\} \Rightarrow |\Delta\varphi| = \pi$ rad (anti-phase).

ii. If u is u_R then $|\Delta\varphi| = \frac{\pi}{2}$ rad and this is not the case .

c) i. $u_C = \frac{q}{C} \Rightarrow u_C = \frac{4.77\times10^{-3}}{C}\sin\left(\omega_1 t - \frac{\pi}{2}\right)$...eq(*). $i = \frac{dq}{dt} = C\frac{du_C}{dt} = 4.77\times10^{-3}\omega_1\cos\left(\omega_1 t - \frac{\pi}{2}\right)$

$u_L = L\frac{di}{dt} = -L\omega_1^2(4.77\times10^{-3})\sin\left(\omega_1 t - \frac{\pi}{2}\right)$.

ii. $u_L = L\omega_1^2(4.77\times10^{-3})\sin\left(\omega_1 t - \frac{\pi}{2} + \pi\right) = L\omega_1^2(4.77\times10^{-3})\sin\left(\omega_1 t + \frac{\pi}{2}\right)$...eq(**).

From eq(*) and eq(**) : $\Delta\varphi = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$ rad $\Rightarrow u = u_L$.

iii. $u_{L\,max} = L\omega_1^2(4.77\times10^{-3}) = 5 \Rightarrow \underline{L = 0.024\ H}$. $u_{C\,max} = \frac{4.77\times10^{-3}}{C} = 20 \Rightarrow \underline{C = 2.385\times10^{-4}\ F}$.

iv) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.024)(2.385\times10^{-4})}} = 417.97$ rad/s $\neq \omega_1 \Rightarrow$ No current resonance.

B)

a) $u_L + u_C = 0$.

b) $u_{AM} = u_{AB} + u_{BD} + u_{DM} \Rightarrow u_G = u_C + u_L + u_R \Rightarrow u_G = u_R \Rightarrow$ i and u_G are in-phase $\Rightarrow$ current resonance.

c) Current resonance $\Rightarrow \omega_2 = \omega_o = 417.97$ rad/s. $t_1 = T_o = \frac{2\pi}{\omega_0} = \frac{2\pi}{417.97} = 0.015$ s.

d) i. $i = \frac{dq}{dt} \Rightarrow u_C = \frac{1}{C}\int i\,dt = \frac{-I_m}{C\omega_2}\cos(\omega_2 t + \varphi) \Rightarrow u_{C\,max} = \frac{I_m}{C\omega_2} \Rightarrow 20 = \frac{I_m}{(2.385\times10^{-4})(417.97)} \Rightarrow I_m \approx 2$ A.

ii. $u_G = u_R \Rightarrow u_m = u_{R\,max} = R.I_m = (5\sqrt{3})(2) = \underline{10\sqrt{3}\ V}$.

iii. $P_{av} = I_{eff}\,u_{G\,eff}\cos\varphi = \left(\frac{2}{\sqrt{2}}\right)\left(\frac{10\sqrt{3}}{\sqrt{2}}\right)\cos(0) = \underline{10\sqrt{3}\ W}$.

15. A)

a) $u_{AM} = u_{AB} + u_{BD} + u_{DM}$ but $i = \frac{dq}{dt} \Rightarrow U_m\sin(\omega t) = \frac{q}{C} + L\frac{di}{dt} + Ri \Rightarrow \omega U_m\cos(\omega t) = \frac{1}{C}\frac{dq}{dt} + Li'' + Ri'$

$\Rightarrow \frac{\omega U_m\cos(\omega t)}{L} = i'' + \frac{R}{L}i' + \frac{1}{LC}i$.

b) $i = I_m\sin(\omega t + \varphi)$. $u_{DM} = u_R = R\,i = RI_m\sin(\omega t + \varphi)$. $u_{BD} = u_L = L\frac{di}{dt} = L\omega I_m\cos(\omega t + \varphi)$

$u_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int I_m\sin(\omega t + \varphi)\,dt = -\frac{I_m}{C\omega}\cos(\omega t + \varphi)$. $u_{AM} = u_{AB} + u_{BD} + u_{DM} = u_R + u_L + u_C$

$\Rightarrow U_m\sin(\omega t) = RI_m\sin(\omega t + \varphi) + L\omega I_m\cos(\omega t + \varphi) - \frac{I_m}{C\omega}\cos(\omega t + \varphi)$

$\Rightarrow U_m\sin(\omega t) = RI_m\sin(\omega t + \varphi) + I_m\cos(\omega t + \varphi)\left[L\omega - \frac{1}{C\omega}\right]$.

c) For $\omega t = 0$: $0 = RI_m \sin\varphi + I_m \cos\varphi\left[L\omega - \frac{1}{C\omega}\right] \Rightarrow 0 = RI_m \sin\varphi - I_m\cos\varphi\left[\frac{1}{C\omega} - L\omega\right]$...eq (1)

For $\omega t = \frac{\pi}{2}$: $U_{max} = RI_m \sin\left(\frac{\pi}{2} + \varphi\right) + \left(L\omega - \frac{1}{C\omega}\right) I_m \cos\left(\frac{\pi}{2} + \varphi\right) \Rightarrow U_{max} = RI_m\cos\varphi - \left(L\omega - \frac{1}{C\omega}\right) I_m \sin\varphi$

$\Rightarrow U_{max} = RI_m \cos\varphi + \left(\frac{1}{C\omega} - L\omega\right) I_m \sin\varphi$ ...eq(2) square and add

$\Rightarrow U^2_{max} = R^2I^2_m \sin^2\varphi + I^2_m\cos^2\varphi\left[\frac{1}{C\omega} - L\omega\right]^2 - 2RI^2_m \sin\varphi\cos\varphi\left[\frac{1}{C\omega} - L\omega\right] + R^2I^2_m\cos^2\varphi + I^2_m \sin^2\varphi\left[\frac{1}{C\omega} - L\omega\right]^2$

$+ 2RI^2_m \sin\varphi\cos\varphi\left[\frac{1}{C\omega} - L\omega\right] \Rightarrow U^2_{max} = R^2I^2_m\left[\sin^2\varphi + \cos^2\varphi\right] + I^2_m\left[\frac{1}{C\omega} - L\omega\right]^2\left[\sin^2\varphi + \cos^2\varphi\right]$

$\Rightarrow U^2_{max} = I^2_m\left[R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2\right] \Rightarrow I_m = \frac{U_m}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}}$.

d) For $\omega t = \frac{\pi}{2} - \varphi \Rightarrow U_m \sin\left(\frac{\pi}{2} - \varphi\right) = RI_m \sin\left(\frac{\pi}{2} - \varphi + \varphi\right) + I_m\left[L\omega - \frac{1}{C\omega}\right]\cos\left(\frac{\pi}{2} - \varphi + \varphi\right)$

$\Rightarrow U_m\cos\varphi = RI_m \Rightarrow \cos\varphi = \frac{RI_m}{U_m} = \frac{R}{U_m}\left(\frac{U_m}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}}\right) \Rightarrow \cos\varphi = \frac{R}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}}$

B)

a) $P_{av} = I_{eff}\, u_{G\,eff} \cos\varphi$. φ is the phase difference between i and u_G.

b) $P_{av} = \frac{U_m}{(\sqrt{2})\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} \cdot \frac{U_m}{\sqrt{2}} \cdot \frac{R}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}} = \frac{U^2_m R}{2\left(R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2\right)} \Rightarrow P_{av} = \frac{R\,U^2_{eff\,G}}{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}$

c) Current resonance takes place $\Rightarrow LC\omega^2 = 1 \Rightarrow \frac{1}{C\omega} - L\omega = 0 \Rightarrow P_{av} = \frac{(U_{eff})^2}{R}$ = maximum.

d) i. $\omega_o = 10\,000$ rad/s.

ii. $P_{av} = \frac{R\,U^2_{eff}}{R^2 + 0}$ (current resonance) $\Rightarrow P_{av} = \frac{U^2_{eff}}{R} \Rightarrow 40 = \frac{(20)^2}{R} \Rightarrow R = 10\ \Omega$.

iii. $13 = \frac{10\ (400)}{100 + \left(\frac{1}{5000\,C} - 5000\,L\right)^2} \Rightarrow \frac{30\sqrt{39}}{13} = \frac{1}{5000\,c} - 5000\,L$...eq (1) and $LC\,\omega_0^2 = 1 \Rightarrow \frac{1}{C} = 10^8 L$...eq(2)

Solve $\Rightarrow L = 0.96 \times 10^{-3}$ H $\approx$ 1 mH and $C \approx 10^{-5}$ F.

16. A)

a) $u_{AB} + u_{BF} + u_{FD} + u_{DA} = 0 \Rightarrow 0 + u_C + RI - E = 0 \Rightarrow I = \frac{E}{R}$.

b) The resistance R.

B)

a) $u_{AB} + u_{BF} + u_{FD} + u_{DA} = 0 \Rightarrow 0 + L\frac{di}{dt} + RI - E = 0 \Rightarrow I = \frac{E}{R}$.

b) R (the resistance).

C)

a) $Z = \frac{U_m}{I_m} \Rightarrow$ unit of "z" : $[z] = \frac{V}{A} = \Omega \Rightarrow$ unit of Z is Ω.

b) i. $Z = \frac{U_m}{I_m} = U_m \div \frac{U_m}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$.

ii. Resistance , inductance , capacitance and ω.

c) i. $u_C = \frac{1}{C}\int i\,dt = -\frac{I_m}{C\omega}\cos(\omega t) \Rightarrow z_C = \frac{U_{C\,max}}{I_{max}} = \frac{\frac{I_m}{C\omega}}{I_m} \Rightarrow z_C = \frac{1}{C\omega}$.

ii. $U_{C\,max} = \frac{I_m}{C\omega} \Rightarrow \frac{1}{C\omega} = \text{slope} = \frac{10\text{ V}}{0.05\text{ A}} \Rightarrow z_{(c)} = 200\ \Omega$.

d) i. $u_L = L\frac{di}{dt} = LI_m\omega\cos(\omega t) \Rightarrow z_L = \frac{L\,I_m\,\omega}{I_m} \Rightarrow z_L = L\,\omega$.

ii. $U_{L\,max} = LI_m\omega \Rightarrow 10 = L\omega(0.04) \Rightarrow L\omega = z_L = 250\ \Omega$.

e) $\frac{1}{C\omega} = 200 \Rightarrow C = \frac{1}{(200)(5000)} \Rightarrow C = 10^{-6}$ F. $L\,\omega = 250 \Rightarrow L = \frac{250}{5000} \Rightarrow L = 0.05$ H.

Chapter 12: Electromagnetic Oscillations

1.

a) Curve (a) corresponds to electromagnetic energy since it is constant

Curve (b) corresponds to W_C since when q is maximum , $W_c = \frac{q_{max}^2}{2C}$ is maximum.

Curve (c) corresponds to W_L since W_L is maximum when $q = 0$ (capacitor is empty) $\{$or $W_L = W_{em} - W_C\}$

b) $W_{e.m} =$ const, resistance of the circuit is zero.

c) $W_{e.m} = \frac{q_{max}^2}{2C} \Rightarrow 1.6{\times}10^{-5} = \frac{(4{\times}10^{-6})^2}{2C} \Rightarrow C = 0.5 \times 10^{-6}F = \underline{0.5\ mF}$.

d) $W_{e.m} = \frac{1}{2}LI_{max}^2 \Rightarrow L \cong \underline{0.2H}$. **e)** $W_{e.m} = \frac{1}{2}Li^2 + \frac{1}{2}Li^2 = Li^2 \Rightarrow i = \underline{8.94 \times 10^{-3}A}$.

2.

a) **i.** $i = \frac{dq}{dt} = -\omega_o Q_m \sin\omega_o t$.

ii. $q^2 = Q_m^2 \cos^2\omega_o t$ and $i^2 = \omega_o^2 Q_m^2 \sin^2\omega_o t \Rightarrow \frac{i^2}{\omega_o^2} = Q_m^2 \sin^2\omega_o t$

square and add: $q^2 + \frac{i^2}{\omega_o^2} = Q_m^2\cos^2\omega_o t + Q_m^2 \sin^2\omega_o t = Q_m^2 \Rightarrow i^2 = [Q_m^2 - q^2]\omega_o^2$

b) $W_{e.m} = \text{const} = \frac{Q_m^2}{2C} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{2C} \Rightarrow Li^2 = \frac{Q_m^2 - q^2}{C} \Rightarrow i^2 = \frac{1}{LC}[Q_m^2 - q^2] = \omega_o^2[Q_m^2 - q^2]$.

c) **i.** For $q = 0$; $i^2 = Q_m^2\ \omega_o^2$. From the expression of i in part (a- i): $I_m = \omega_o Q_m \Rightarrow Q_m^2\ \omega_o^2 = I_m^2 \Rightarrow$ for $q = 0,\ i = I_m$

ii. For $q = Q_m$; $i^2 = [Q_m^2 - Q_m^2]\omega_o^2 = 0$.

d) **i.** $Q_m = 8\ \mu C$ and $I_m = 40$ mA . **ii.** $I_m = Q_m\omega_o \Rightarrow \omega_o = \underline{5000\ rad/s}$. **iii.** $\omega_o^2 = \frac{1}{LC} \Rightarrow L = \underline{4 \times 10^{-3}H}$.

3.

a) In R-L-C under a constant voltage E: u_C oscillates around E and in the steady state u_C becomes E, but $W_C = \frac{1}{2}Cu_C{}^2 \Rightarrow$ W_C oscillates around a constant value and in the steady state it becomes $\frac{1}{2}CE^2 =$ const.

b) $W_{C(steady)} = \frac{1}{2}CE^2 = 0.9 \times 10^{-3} \Rightarrow C = \underline{4.5\ \mu F.}$

c) $T_{energy} = \frac{1}{2}T_o \Rightarrow T_o = 24 \times 10^{-3}$s. But $T_o = 2\pi\sqrt{LC} \Rightarrow T_o^2 = 4\pi^2 LC \Rightarrow L = \underline{3.25\ H}$.

4.

a) $W_C = \frac{q^2}{2C} = \frac{Q_m^2 \sin^2(\omega_o t + \varphi)}{2C}$.

b) $i = \frac{dq}{dt} = \omega_o Q_m \cos(\omega_o t + \varphi) \Rightarrow W_L = \frac{1}{2}Li^2 = \frac{L\ \omega_o^2\ Q_m^2}{2}\cos^2(\omega_o t + \varphi)$.

c) $W_{em} = W_C + W_L = \frac{Q_m^2}{2C}\sin^2(\omega_o t + \varphi) + \frac{L\ \omega_o^2\ Q_m^2}{2}\cos^2(\omega_o t + \varphi)$. But $\omega_o^2 = \frac{1}{LC}$

$\Rightarrow W_{e.m} = \frac{Q_m^2}{2C}\sin^2(\omega_o t + \varphi) + \frac{LQ_m^2}{2}\left(\frac{1}{LC}\right)\cos^2(\omega_o t + \varphi) = \frac{Q_m^2}{2C}[\sin^2(\omega_o t + \varphi) + \cos^2(\omega_o t + \varphi)] = \frac{Q_m^2}{2C} =$ const.

5.

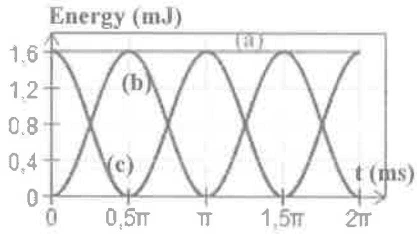
a) **i.** At $t_0 = 0$; $i_0 = 0 \Rightarrow W_L = \frac{1}{2}L\,i^2 = 0 \Rightarrow$ curve (b) represents the variation of W_L.

ii. Curve (b) is for $W_L \Rightarrow$ curve (a) is for $W_{e.m}$ and $W_{e.m} =$ cst

$\Rightarrow$ No resistance in the circuit $\Rightarrow$ purely inductive coil.

iii. Period of energy: $T_{energy} = \pi$ ms . But $T_o = 2T_{energy} \Rightarrow T_o = \underline{2\pi\ ms}$.

b) Figure.



c) $W_{C(max)} = \frac{1}{2}CE^2 \Rightarrow C = \underline{5 \times 10^{-5}F}$. $T_o = 2\pi\sqrt{LC} \Rightarrow L = 2 \times 10^{-2}$H.

d) $W_{L(max)} = \frac{1}{2}LI_m^2 \Rightarrow I_m = \underline{0.4\ A.}$

6.

a) **i.** $W_{C_o} = \frac{Q_m^2}{2C} = 1.5 \times 10^{-5}$J. **ii.** $W_{L_o} = \frac{1}{2}Li^2 = 0$.

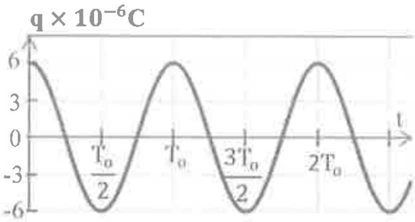
iii. No resistors in the circuit then $W_{e.m}$ is conserved.

$W_{em} = W_{C_o} + W_{L_o} = 1.5 \times 10^{-5}$J.

b) $W_{e.m} = \frac{q^2}{2C} + \frac{1}{2}Li^2 = 1.5 \times 10^{-5} \Rightarrow i = 0.04714A = \underline{47.14\ mA}$.

c) i. $T_o = 2\pi\sqrt{LC} = 7.536 \times 10^{-4}$ s.

ii. Figure



iii. $t = \frac{T_o}{4}$; $q = 0$; $t = \frac{T_o}{2}$; $q = -6 \times 10^{-6}$C, $t = \frac{3T_o}{4}$; $q = 0$; $t = T_o$; $q = +6 \times 10^{-6}$C.

d) **i.** at $t = \frac{T_o}{2}$, the charge of the plate A is $q_A = -Q_{max}$

The capacitor is fully charged so it will discharge through the coil during $[\frac{T_o}{2} ; \frac{3T_o}{4}]$ ⇒ The capacitor acts as a generator.

ii. During discharging, i flows from the positive plate (B) ⇒ current flows in the clockwise.

e) **i.** First method: $u_{AB} = \frac{q}{C} = -L\frac{di}{dt}$ ⇒ $\frac{q}{C} + L\frac{di}{dt} = 0$, but $i = +\frac{dq}{dt}$ ⇒ $i' = q''$: $\frac{q}{C} + Lq'' = 0$ ⇒ $q'' + \frac{1}{LC}q = 0$

Second method: $W_{e.m} = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \text{const}$ ⇒ $\frac{dW_{e.m}}{dt} = L\,i\,i' + \frac{qq'}{C} = 0 \to i\left[L\,i' + \frac{q}{C}\right] = 0$, but $i = 0$ is rejected

⇒ $i' + \frac{q}{LC} = 0$ ⇒ $q'' + \frac{1}{LC}q = 0$.

ii. * $q' = \omega_o A\cos(\omega_o t + \varphi)$ ⇒ $q'' = -\omega_o^2 A\sin(\omega_o t + \varphi) = -\omega_o^2 q$. Substitute in the differential equation:

$-\omega_o^2 q + \frac{1}{LC}q = 0$ ⇒ $q\left[-\omega_o^2 + \frac{1}{LC}\right] = 0$, but $q = 0$ is rejected ⇒ $-\omega_o^2 + \frac{1}{LC} = 0$ ⇒ $\omega_o = \frac{1}{\sqrt{LC}} = \frac{25000}{3}$ rad/s.

* At $t_0 = 0$; $q_0 = 6 \times 10^{-6}\text{C} = A\sin\varphi$ ⇒ $\sin\varphi = \frac{A}{6 \times 10^{-6}}$. $i = \frac{dq}{dt} = \omega_o A\cos(\omega_o t + \varphi)$ and $i_o = 0$;

⇒ $0 = \omega_o A\cos\varphi$ ⇒ $\cos\varphi = 0$ ⇒ $\varphi = \pm\frac{\pi}{2}$rad. But $A > 0$ ⇒ $\sin\varphi > 0$

⇒ $\varphi = \frac{\pi}{2}$rad and $A = 6 \times 10^{-6} \times \sin\frac{\pi}{2}$ ⇒ $A = \underline{6 \times 10^{-6}\text{C}}$.

iii) $u_C = \frac{Q}{C} = \frac{6 \times 10^{-6}}{1.2 \times 10^{-6}}\sin\left(\frac{25000}{3}t + \frac{\pi}{2}\right) = 5\sin\left(\frac{25000}{3}t + \frac{\pi}{2}\right)$.

7.

a) The circuit is under free electromagnetic oscillations.

b) $u_{BD} + u_{DA} + u_{AB} = 0$ ⇒ $L\frac{di}{dt} - E + u_C = 0$ ⇒ $LC\frac{du_C}{dt} + u_C = E$ ⇒ $\frac{du_C}{dt} + \frac{u_C}{LC} = \frac{E}{LC}$ $\left\{i = \frac{dq}{dt} = C\frac{du_C}{dt}\right\}$.

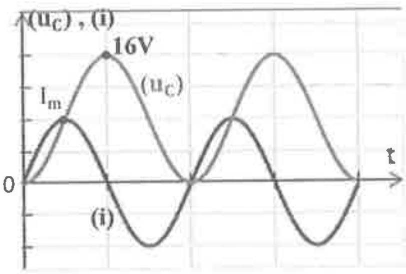
c) **i.** At $t_0 = 0$; $u_C = 0 = F\cos\varphi + E$ ⇒ $\cos\varphi = -\frac{E}{F}$.

ii. $u_C' = -\omega_o F\sin(\omega_o t + \varphi)$ ⇒ $i = -\omega_o F\,C\sin(\omega_o t + \varphi)$.

$i_0 = 0 = -\omega_o F\,C\sin\varphi$ ⇒ $\sin\varphi = 0$ ⇒ $\varphi = 0$ or $\varphi = \pi$ rad.

But $\cos\varphi < 0$ ⇒ $\varphi = \underline{\pi\text{ rad}}$.

iii. $\cos\varphi = -\frac{8}{F}$ ⇒ $F = \frac{-8}{-1} = \underline{8\text{ V}}$.



d) $i = -\omega_o F\,C\sin(\omega_o t + \pi) = \omega_o F\,C\sin(\omega_o t) = I_m\sin(\omega_o t)$.

$u_C = F\cos(\omega_o t + \pi) + E = -8\cos(\omega_o t) + 8$.

8. A. a) zero

b) $q = C\,E = \underline{0.16\text{ C}}$. , $W_C = \frac{1}{2}q\,E = \underline{1.28\text{ J}}$.

B. a) $I = \frac{E}{R_1 + r} = \underline{1.6\text{ A}}$. **b)** $W_L = \frac{1}{2}LI_m^2 = \underline{0.128\text{ J}}$. **c)** $u_{DF} = u_L = ri + L\cancel{\frac{di}{dt}} = r\,I_m = (2)(1.6)$ ⇒ $u_L = \underline{3.2\text{ V}}$.

C. a) $W_{e.m_o} = W_C + W_L = 1.28 + 0.128 = \underline{1.408\text{ J}}$.

b) i. Graphically $i > 0$ ⇒ anti-clockwise. i flows out from + ve plate ⇒ capacitor acts as a generator ⇒ capacitor is discharging.

ii. $|i|$ increases ⇒ W_L increases ⇒ coil is storing energy ⇒ capacitor is discharging through the coil.

c) $T = 247.5 - 27.5 = 220\text{ms} = \underline{0.22\text{ s}}$. $T_o = 2\pi\sqrt{LC} = 0.2$ s ⇒ $T \gtrsim T_o$.

d) At $t = t_1$; $i = 0$, $W_L = 0$ energy is stored in the circuit in the form of electric energy.

e) i. $W_{Lost} = W_{e.m_o} = 1.408$ J. **ii.** $P_{av} = \frac{\Delta W}{\Delta t} = \frac{1.408}{440 \times 10^{-3}} = \underline{3.2\text{ W}}$.

9.

First experiment:

A. $u_{DF} = u_C = E = \underline{12\text{ V}}$. and $i = 0$.

B. **a)** $u_{DF} + u_{FN} + u_{ND} = 0$ ⇒ $\frac{q}{C} + L\frac{di}{dt} + 0 = 0$ $\left\{i = \frac{dq}{dt}\right\}$ ⇒ $\frac{q}{LC} + q'' = 0$

b) $q' = -\omega_o Q\sin(\omega_o t)$ ⇒ $q'' = -\omega_o^2 Q\cos(\omega_o t) = -\omega_o^2 q$ ⇒ $\frac{q}{LC} - \omega_o^2 q = 0$ ⇒ $q\left[\frac{1}{LC} - \omega_o^2\right] = 0$

$q\left[\frac{1}{LC} - \frac{1}{LC}\right] = 0$ So it is verified.

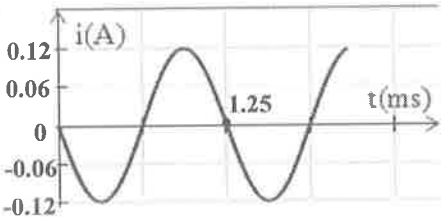
c) i. $Q = 24\ \mu$C. In the steady state: $u_C = E = \frac{Q}{C}$ ⇒ $C = \frac{Q}{E} = \underline{2\ \mu\text{F}}$.

ii. $T_o = 1.25$ ms ⇒ $\omega_o = \frac{2\pi}{T_o} = \underline{5024\,\frac{\text{rad}}{\text{s}}}$. $\omega_o^2 = \frac{1}{LC}$

⇒ $L \cong \underline{0.02\text{ H}}$.

d) i. $i = \frac{dq}{dt} = -\omega_o Q\sin(\omega_o t) = -0.12\sin(5024\,t)$ (S.I)

ii. Figure



Second Experiment

a) $u_{DF} + u_{FH} + u_{HN} + u_{ND} = 0 \Rightarrow \frac{q}{C} + Ri + L\frac{di}{dt} + 0 = 0$

$\frac{q}{C} + \frac{R}{L}q' + \frac{q''}{LC} = 0$

b) i. $q_o = A\cos(-0.204) = 24 \times 10^{-6}C \Rightarrow A = \underline{24.45 \times 10^{-6}\ C}$.

ii. $T = 1.28\ ms \Rightarrow \omega = \frac{2\pi}{T} = 4906.25\ rad/s$. **iii.** $\omega^2 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2 \Rightarrow R = \underline{38.54\ \Omega}$.

10.

A. **a)** $\Delta t = 5 R_1 C$. **b)** $Q_m = CE$.

B. **a)** Free damped electromagnetic oscillation.

b) i. The coil supplies energy to the circuit when $|u_C|$ increases $\Rightarrow$ [0.5 ms, 1 ms].

ii. $T = 2\ ms \Rightarrow \omega = \frac{2\pi}{T} = \underline{1000\pi\ rad/s}$.

c) i. $u_n = 10.005\, e^{-\frac{RnT}{2L}} \cos(\omega nT - 0.0316)$. $u_{n+1} = 10.005\, e^{-\frac{R(n+1)T}{2L}} \cos(\omega(n+1)T - 0.0316)$

$\left\{\omega T = \frac{\omega(2\pi)}{\omega} = 2\pi\right\} \Rightarrow \frac{u_{n+1}}{u_n} = e^{-\frac{RT}{2L}} \frac{\cos[(n+1)2\pi - 0.0316]}{\cos(2\pi n - 0.0316)} \Rightarrow \frac{u_{n+1}}{u_n} = e^{-\frac{RT}{2L}}$.

ii. Graphically $u_{3T} = 5.5\ V$ and $u_{4T} = 4.5\ V \Rightarrow \frac{4.5}{5.5} = e^{-\frac{R(2\times10^{-3})}{2(0.1)}} \Rightarrow R \cong \underline{20\ \Omega}$.

iii. $\omega^2 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2 \Rightarrow C \cong 1 \times 10^{-6}\ F = \underline{1\ \mu F}$.

d) $W_n = \frac{1}{2}Cu_n^2$ and $W_{n+1} = \frac{1}{2}C(u_{n+1})^2 \Rightarrow \frac{W_{n+1}}{W_n} = \frac{(u_{n+1})^2}{u_n^2} = \left(\frac{u_{n+1}}{u_n}\right)^2 = \left[e^{-\frac{RT}{2L}}\right]^2 = e^{-\frac{RT}{L}}$

e) i. $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$, u_C is maximum when $\frac{du_C}{dt} = 0 \Rightarrow i = 0$.

ii. At $t_0 = 0$, u_C is maximum $\Rightarrow i = 0 \Rightarrow W_{e.m_o} = W_C + W_L = \frac{1}{2}Cu_{C_o}^2 = \underline{5 \times 10^{-5} J}$.

iii. $W_{e.m} = 5\times10^{-5} - 4\times10^{-6} = 4.6\times10^{-5} = W_L$ (since $W_C = 0$) $\Rightarrow W_L = \frac{1}{2}Li^2 \Rightarrow i = \underline{0.03\ A}$.

iv. $W_{lost} = W_{e.m(o)} - W_{e.m(f)} = \underline{4.2\times10^{-5}\ J}$.

11.

a) $\omega_o = \frac{1}{\sqrt{LC}} = \underline{500\ rad/s}$.

b) $u_{DF} = u_{DA} + u_{AB} + u_{BF} \Rightarrow E = Ri + L\frac{di}{dt} + u_C$ $\left\{i = \frac{dq}{dt} = C\frac{du_C}{dt}\right\}$

$E = RC\frac{du_C}{dt} + LC\frac{d^2u_C}{dt^2} + u_C \Rightarrow u_C'' + \frac{R}{L}u_C' + \frac{1}{LC}u_C = \frac{E}{LC}$.

c) i. $T_G = 78.6\ ms \Rightarrow \omega_G = \frac{2\pi}{T_G} \cong \underline{80\ rad/s}$.

ii. $T = 13.1\ ms \Rightarrow \omega = \frac{2\pi}{T} = \underline{479.39\ rad/s}$. $\omega < \omega_o$ (ω is not slightly less than ω_o) $\Rightarrow$ large damping.

iii. For $0 \le t \le \frac{T_G}{2}$; $u_G = E = 20\ V$. $\frac{T_G}{2} \le t \le T_G$; $u_G = 0$.

iv. At $t_0 = 0$, $u_C = 0 \Rightarrow 0 = -\frac{125}{6}e^0[\cos\varphi + 20] \Rightarrow \cos\varphi = 0.96$. But $\varphi < 0 \Rightarrow \varphi = \underline{-16.26^\circ = -0.2838\ rad}$.

d) i. For $\left[0, \frac{T_G}{2}\right]$; u_C only increases and does not decrease. **ii.** Damping increases $\Rightarrow R' > R \Rightarrow R' = \underline{900\ \Omega}$.

12.

A.

a) Free damped electromagnetic oscillations. $T = 12.56\ ms \Rightarrow T = \frac{2\pi}{\omega} \Rightarrow \omega = \underline{500\ rad/s}$.

b) $u_{DA} + u_{AB} + u_{BM} + u_{MD} = 0 \Rightarrow Ri + 0 + ri + L\frac{di}{dt} + \frac{q}{C} = 0 \Rightarrow (R+r)\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \Rightarrow q'' + \frac{(R+r)}{L}q' + \frac{q}{LC} = 0$

c) **i.** At $t_0 = 0$, $q = Q_o = F e^0 \cos\phi \Rightarrow Q_o = F\cos\phi$.

ii. $i = \frac{dq}{dt} = F\left[-\frac{(R+r)}{2L}e^{-\frac{R+r}{2L}}\cos(\omega t + \varphi) + e^{-\frac{R+r}{2L}}(-\omega)\sin(\omega t + \varphi)\right]$

$i = -Fe^{-\frac{R+r}{2L}}\left[+\frac{(R+r)}{2L}\cos(\omega t + \varphi) + \omega\sin(\omega t + \varphi)\right]$

iii. At $t_0 = 0$, $i_o = 0$; $0 = -F\left[\frac{(R+r)}{2L}\cos\varphi + \omega\sin\varphi\right]$, but $-F \neq 0 \Rightarrow \frac{(R+r)}{2L}\cos\varphi = -\omega\sin\varphi \Rightarrow \tan\varphi = -\frac{(R+r)}{2L\omega}$

iv. $-0.095 = -\frac{(70+r)}{2(0.8)(500)} \Rightarrow r = \underline{6\ \Omega}$. $2.5\times10^{-5} = F\cos(-5.4268) \Rightarrow F = \underline{2.511\times10^{-5}\ C}$.

d) $W_C = \frac{q^2}{2C} \Rightarrow W_o = \frac{(2.5\times10^{-5})^2}{2(5\times10^{-6})} = 6.25\times10^{-5}J$. At $t = 6.28\ ms$; $W_C = \frac{(-1.8\times10^{-5})^2}{2(5\times10^{-6})} = 3.24\times10^{-5}J$

loss $= 6.25\times10^{-5} - 3.24\times10^{-5} = \underline{3.01\times10^{-5} J}$.

B. a) $P_{av} = \frac{loss}{\Delta t} = \frac{3.01\times10^{-5}}{6.28\times10^{-3}} = 4.793\times10^{-3}\ W$.

b) **i.** $u_{AB} + u_{BM} + u_{MD} + u_{DA} = 0 \Rightarrow \; -(R+r)i + ri + L\frac{di}{dt} + \frac{q}{C} + Ri = 0 \Rightarrow \; Lq'' = 0 \Rightarrow q'' + \frac{q}{LC} = 0$

ii. The differential equation has the form $q'' + \omega_o^2 q = 0 \; \Rightarrow$ q is alternating sinusoidal.

13.

a) $i = \frac{dq}{dt} = \omega Q_m \cos(\omega t) \Rightarrow I_m = \omega\, Q_m \;\Rightarrow I_m = \frac{U_m}{\sqrt{R^2 + \left[L\omega - \frac{1}{\omega C}\right]^2}}$.

b) **i.** When the amplitude of the current is maximum, current resonance takes place and then $\omega = \omega_o = \underline{10000\ rad/s}$.

ii. Current resonance $\omega^2 LC = 1 \Rightarrow L\omega = \frac{1}{\omega C} \Rightarrow I_m = \frac{U_m}{R} \Rightarrow R = \frac{20}{0.2} = \underline{100\ \Omega}$.

c) **i.** Since Q_m is maximum for a certain value of ω.

ii. For $\omega = \omega_o = 10000$ rad/s , Q_m is not maximum $\Rightarrow$ no current resonance for $\omega = \omega_o$.

iii. $\omega_r^2 = (9354.14)^2 = (10000)^2 \times \left(\frac{100}{2L}\right)^2 \Rightarrow L = \underline{0.014\ H}$.

d) $\omega_o^2 = \frac{1}{LC} \Rightarrow C = \underline{7.14 \times 10^{-7} F}$.

14.

A.

a) **i.** $T = 1.57 \times 10^{-3} s \Rightarrow \omega = \frac{2\pi}{T} = \underline{4000\ rad/s}$. $u_R = 6.25 \sin(4000t)$.

ii. $u_L = L\frac{di}{dt}$ but $i = \frac{u_R}{R} = 0.125 \sin(4000t) \Rightarrow u_L = L(0.125)(4000)\cos(4000t) = 500L\cos(4000t)$ (S.I)

iii. $u_C = \frac{1}{C}\int i\, dt = -\frac{3.125 \times 10^{-5}}{C}\cos(4000t)$. Graphically $U_{C_{max}} = 6.25 = \frac{3.125 \times 10^{-5}}{C} \Rightarrow C = \underline{5 \times 10^{-6} F}$.

iv. $\left.\begin{array}{l} 4 \text{ div} \to 2\pi \\ 0.39 \text{ div} \to |\Delta\phi| \end{array}\right\} \Rightarrow \Delta\phi = 0.195\pi$ rad $\Rightarrow u_G = 7.64 \sin(4000t + 0.195\pi)$.

b) $u_G = u_L + u_C + u_R \Rightarrow 7.64\sin(4000t + 0.195\pi) = 500L\cos(4000t) - \frac{3.125 \times 10^{-5}}{C}\cos(4000t) + 6.25\sin(4000t)$

For $\omega t = 0$; $7.64 \sin(0.2\pi) = 500L - \frac{3.125 \times 10^{-5}}{5 \times 10^{-6}} \Rightarrow L \cong \underline{0.0213\ H}$.

c) **i.** $W_L = \frac{1}{2}Li^2 \Rightarrow$ as $|i|$ increases, W_L increases from the graph during [0, 0.3925ms] u_R increases $\Rightarrow$ i increases.

ii. During [0, 0.3925ms], $|u_C|$ decreases $\Rightarrow$ the capacitor is discharging.

iii. u_G and $u_R = i\,R$ have the same sign (positive) $\Rightarrow$ the L.F.G supplies energy to the circuit.

d) u_R and u_G have opposite signs $\Rightarrow$ generator consumes energy. Current decreases $\Rightarrow$ coil supplies energy. u_C increases $\Rightarrow$ capacitor consumes energy. While the resistor always consumes and never supplies energy.

e) $P_{av} = I_{eff} U_{C\,eff} \cos\phi \cong \underline{0.39\ W}$.

B.

a) $T_o = 2\pi\sqrt{LC} \cong 2.05 \times 10^{-3}$ s .

b) $T = 2.05 \times 10^{-3}$ s $\cong T_o$. u_R and u_G are in phase.

c) since u_R and u_G have always same sign.

d) $P_{av} = \frac{7.64}{\sqrt{2}} \frac{7.64}{(50)\sqrt{2}} \cos(0) \cong \underline{0.584\ W}$.

C. In case $\omega = \omega_o$ (current resonance) the generator supplies more energy to the circuit since 0.584 W > 0.39 W .

15.

A.

a) exciter: motor Resonator: oscillator

b) $ME = KE + EPE + GPE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + 0$. $\frac{dME}{dt} = m\,v\,v' + k\,x\,x' = P_{\vec{f}} + P_{\vec{F}} = -h\,v^2 + F\,v$

$\Rightarrow F\,v = v(m\,v' + k\,x + hv)$, $v = x'$, $v' = x''$ and $v = 0$ is rejected $\Rightarrow F_m \sin(\omega t + \varphi) = m\,x'' + h\,x' + kx$

c) $x = x_m \sin(\omega t) \Rightarrow x' = \omega x_m \cos(\omega t) \Rightarrow x'' = -\omega^2 x_m \sin(\omega t)$

$\Rightarrow F_m \sin(\omega t + \varphi) = -m\omega^2 x_m \sin(\omega t) + h\,\omega\,x_m \cos(\omega t) + k\,x_m \sin(\omega t)$

$\Rightarrow F_m \sin(\omega t + \varphi) = [kx_m - m\omega^2 x_m]\sin(\omega t) + h\omega x_m \cos(\omega t)$

For $\omega t = 0$; $F_m \sin(\varphi) = h\omega x_m \Rightarrow \sin(\varphi) = \frac{h\,\omega\,x_m}{F_m}$ ...eq(1)

For $\omega t = \frac{\pi}{2}$; $F_m \cos(\varphi) = kx_m - m\omega^2 x_m \Rightarrow \cos(\varphi) = \frac{k\,x_m - m\,\omega^2 x_m}{F_m}$ ...eq(2)

$\frac{eq(1)}{eq(2)}$: $\tan\varphi = \frac{h\,\omega\,x_m}{F_m} \cdot \frac{F_m}{k\,x_m - m\,\omega^2 x_m} = \frac{h\,\omega}{k - m\,\omega^2}$. But $\omega_o^2 = \frac{k}{m} \Rightarrow k = m\omega_o^2 \Rightarrow \tan\varphi = \frac{h\,\omega}{m(\omega_o^2 - \omega^2)}$

square and add: $1 = \frac{h^2\omega^2 x_m^2}{F_m^2} + \frac{x_m^2(k - m\omega^2)^2}{F_m^2} \Rightarrow F_m^2 = x_m^2[h^2\omega^2 - [m\omega_o^2 - m\omega^2]^2] \Rightarrow x_m = \frac{F_m}{\sqrt{h^2\omega^2 - [m(\omega_o^2 - \omega^2)]^2}}$

B.

a) Exciter: the low frequency generator. Resonator: The L-C circuit.

b) **i.** $W_{e.m} = \frac{1}{2}Li^2 + \frac{q^2}{2C}$

ii. $\frac{dW_{em}}{dt} = L\,i\,i' + \frac{q\,q'}{C} = i\,u_G - Ri^2 \Rightarrow i\,u_G = i\left[Ri + L\,i' + \frac{q}{C}\right]$. But i = 0 is rejected $\Rightarrow u_G = L\,i' + Ri + \frac{q}{C}$

$q' = i$ and $q'' = i' \Rightarrow U\sin(\omega t + \varphi) = L\,q'' + Rq' + \frac{q}{C}$

c) **i.** $q \equiv x$; $i = q' \equiv v = x'$; $L \equiv m$; $R \equiv h$; $C \equiv \frac{1}{k}$ and $U = F_m$.

ii. $q = Q_m \sin(\omega t + \varphi)$. **iii.** $\tan\varphi = \frac{R\omega}{L(\omega_0^2 - \omega^2)}$ and $Q_m = \frac{U}{\sqrt{R\,\omega^2 + [L(\omega_0^2 - \omega^2)]^2}}$

d) **i.** $q = Q_m \sin(\omega t) \Rightarrow i = \frac{dq}{dt} = \omega\,Q_m \cos(\omega t)$.

ii. $i = \omega\,Q_m \sin\left(\omega t + \frac{\pi}{2}\right)$ and $u_G = U\sin(\omega t + \varphi) \Rightarrow \Delta\varphi = \varphi_1 = \frac{\pi}{2} - \varphi$.

iii Current resonance $\Rightarrow \frac{\pi}{2} - \varphi = 0 \Rightarrow \varphi = \frac{\pi}{2}$ (rad) $\Rightarrow \tan\varphi \to \infty \Rightarrow \frac{R\omega}{L(\omega_0^2 - \omega^2)} \to \infty \Rightarrow \omega_o^2 - \omega^2 = 0 \Rightarrow \omega_o = \omega$.

16.

A.

a) Electromagnetic induction in MN and self induction in the coil and discharging of the capacitor.

b) $\phi = \vec{B}.\vec{n}\,S = B\,S\cos(0) = B\,\ell\,x$. $e = -\frac{d\phi}{dt} = -B\,\ell\,v$.

c) System: rod MN, External forces: $m\vec{g}$, $\vec{N}_M$, $\vec{N}_N$, $\vec{F}_{e.m}$

$\sum\vec{F}_{ext} = m\vec{a} \Rightarrow m\vec{g} + \vec{N}_M + \vec{N}_N + \vec{F}_{e.m} = m\vec{a}$ projecting along x-axis

$0 + 0 + 0 + F_{e.m} = ma$ but $F_{e.m} = i\,\ell\,B \Rightarrow i\ell B = m\frac{dv}{dt} \Rightarrow i = \left(\frac{m}{\ell B}\right)\frac{dv}{dt}$.

d) $u_{MN} + u_{Nh} + u_{hS} + u_{Sa} + u_{aM} = 0 \Rightarrow -e + 0 - u_C + ri + L\frac{di}{dt} = 0 \Rightarrow B\,\ell\,v - u_C + ri + L\frac{di}{dt} = 0$ Derive w.r.t time

$B\,\ell\,v' - u_C' + r\,i' + L\,i' = 0 \Rightarrow \frac{B^2\ell^2}{m}i + \frac{i}{C} + r\,i' + L\,i'' = 0$ $\left\{i = -\frac{dq}{dt} = -Cu_C'\right\}$

$\Rightarrow L\,i'' + r\,i' + \left(\frac{B^2\ell^2}{m} + \frac{1}{C}\right)i = 0 \Rightarrow i'' + \frac{r}{L}\,i' + \left(\frac{B^2\ell^2C + m}{m\,C\,L}\right)i = 0.$

B.

a) At $t_0 = 0$; $i_0 = 0 = A\sin\varphi \Rightarrow \sin\varphi = 0 \Rightarrow \varphi = 0$ or $\varphi = \pi$ rad. At $t_0 = 0^+$; $i > 0 \Rightarrow \sin\varphi > 0 \Rightarrow \varphi = 0$.

b) $\frac{di}{dt} = A\left[e^{-\frac{r}{2L}t}\omega\cos(\omega t) + \sin(\omega t)\left(-\frac{r}{2L}e^{-\frac{r}{2L}}\right)\right] = Ae^{-\frac{r}{2L}t}\left[\omega\cos(\omega t) - \frac{r}{2L}\sin(\omega t)\right]$. At $t_0 = 0$; $\frac{di}{dt} = A\,\omega$.

c) $B\,\ell\,v - u_C + r\,i + L\frac{di}{dt} = 0$. At $t_0 = 0$: $v_0 = 0$, $u_{C_0} = E$, $\frac{di}{dt} = A\omega$ and $i_0 = 0$

$\Rightarrow 0 - E + 0 + L(A\omega) = 0 \Rightarrow A = \frac{E}{L\,\omega}$.

d) Figure

Chapter 13: Transformer

1. **a)** When an alternating sinusoidal voltage of frequency (f) is applied across the primary coil, the alternating sinusoidal current flowing in it, creates a variable magnetic field $\vec{B}$. The magnetic field lines are canalized by the soft iron core and across the secondary coil. The secondary coil is crossed by a variable magnetic flux and then it is the seat of an induced electro motive force of frequency (f). The secondary circuit is closed $\Rightarrow$ a current flows in it.
   b) $f_{secondary} = f_{primay} = 20$ Hz , since the transformer does not change the frequency of the alternating voltage.
   c) i) $m = \frac{N_2}{N_1} = \frac{8}{4} \Rightarrow \underline{m = 2}$. $m > 1 \Rightarrow$ step up.
   ii) $m = \frac{I_1}{I_2} > 1 \Rightarrow I_2 < I_1$. $I_1 = m\, I_2 = (2)(2) = 4\ A > I_2$.
   d) i) Ideal transformer $\Rightarrow P_2 = P_1 = I_1\, U_1 \cos\varphi_1 = (4)(12)(0.8) = \underline{38.4\ W}$.
   ii) $P_{av} = \frac{\Delta W}{\Delta t} \Rightarrow \Delta W = (38.4)(30 \times 60) = \underline{69120\ J}$.

2. **a)** $P_2 = I_2\, U_{CD} \cos\varphi_2 \Rightarrow 25 = 2\, U_{CD}\,(1) \Rightarrow U_{CD} = \underline{12.5\ V}$.
   b) $\eta = 0.95 = \frac{P_2}{P_1} \Rightarrow P_1 = \frac{25}{0.95} = 26.316$ W. $P_1 = I_1\, U_{AB} \cos\varphi_1 \Rightarrow 26.316 = 3\,(9) \cos\varphi_1 \Rightarrow \cos\varphi_1 = \underline{0.975}$.
   c) i) open- secondary mode. **ii)** $m = \frac{U_{CD}}{U_{AB}} \Rightarrow U_{CD} = (1.4)(9) \Rightarrow U_{CD} = \underline{12.6\ V}$.

3. **a)** $U_{HN} = I_3\, R_2 = 1\,(16) = \underline{16\ V}$. $m_2 = \frac{U_{HN}}{U_{EF}} \Rightarrow U_{EF} = \frac{16}{4} = \underline{4\ V}$. $m_2 = \frac{I_2}{I_3} \Rightarrow I_2 = 4\,(1) = 4$ A.
   $U_{DC} = U_{DE} + U_{EF} + U_{FC} = I_2\, R_1 + U_{EF} = 4\,(2) + 4 = \underline{12\ V}$. $m_1 = \frac{U_{DC}}{U_{AB}} \Rightarrow U_{AB} = \frac{12}{0.5} = \underline{24\ V}$.
   b) $m_1 = \frac{I_1}{I_2} \Rightarrow I_1 = (0.5)(4) = 2$ A. $P_{av} = I_1\, U_{AB} \cos\varphi_1 = 2\,(24)(1) = \underline{48\ W}$.

4. **A) a)** The lamp glows normally $\Rightarrow U_2 = 11000$ V. But 11000 V > 220 V $\Rightarrow U_2 > U_1 \Rightarrow$ step up. $m = \frac{U_2}{U_1} = \frac{11000}{220} = \underline{50} > 1$
   b) $P_2 = I_2\, U_2 \cos\varphi_2 \Rightarrow 200 = I_2\,(11000)(1) \Rightarrow I_2 = \underline{0.01818\ A}$.
   c) <u>1st method</u> : $m = \frac{I_1}{I_2} \Rightarrow I_1 = 50\,(0.01818) = \underline{0.91\ A}$.
   <u>2nd method</u> : Ideal transformer $\Rightarrow P_1 = P_2 = 200$ W. $P_1 = I_1\, U_1 \cos\varphi_1 \Rightarrow 200 = I_1\,(220)(1) \Rightarrow I_1 = \underline{0.91\ A}$.
   B) a) i) The lamp glows normally $\Rightarrow P_2 = 200\ W \Rightarrow I_2 = \underline{0.01818}$ A. $\eta = \frac{P_2}{P_1} \Rightarrow P_1 = \frac{200}{0.97} = 206.185$ W.
   $P_1 = I_1\, U_1 \cos\varphi_1 \Rightarrow 206.185 = I_1\,(220)(0.98) \Rightarrow I_1 = \underline{0.956\ A}$.
   ii) $\frac{U_2}{U_1} = \frac{11000}{220} = 50$ and $\frac{I_1}{I_2} = \frac{0.956}{0.01818} = 52.585 \Rightarrow \frac{I_1}{I_2} > \frac{U_2}{U_1}$.
   b) $m' = \frac{U'_2}{U_1} = \frac{11220}{220} = \underline{51}$.
   c) The magnetic field is constant at each point of the secondary coil $\Rightarrow$ This coil is crossed by a resultant constant magnetic flux $\Rightarrow$ no e.m.f $\Rightarrow$ no current.

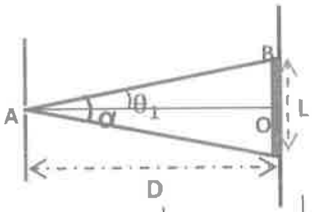
5. **A) a) i)** $r = 8 \times 0.5 = \underline{4\ \Omega}$. **ii)** $P_{lost} = 10\,\%\,P = 0.1 \times (10^7) \Rightarrow P_{lost} = \underline{10^6\ W}$.
   iii) $P_{lost} = r\, I_2^{\,2} \Rightarrow 10^6 = 4\, I_2^{\,2} \Rightarrow I_2 = \underline{500\ A}$.
   b) $P_1 = I_1\, U_1 \cos\varphi_1 \Rightarrow 10^7 = I_1\,(6400)(1) \Rightarrow I_1 = 1562.5$ A. $m = \frac{I_1}{I_2} = \frac{1562.5}{500} \Rightarrow m = 3.125$
   $m = \frac{U_2}{U_1} \Rightarrow U_2 = (3.125)(6400) \Rightarrow U_2 = 20000$ V (Or use directly $\frac{I_1}{I_2} = \frac{U_2}{U_1}$). $U_2 > U_1 \Rightarrow$ <u>step up transformer</u>.
   c) $m = \frac{N_2}{N_1} \Rightarrow N_2 = (3.125)(1000) = \underline{3125\ loops}$.
   d) $P_{Installation} = P_{total} - P_{lost} = 10^7 - 10^6 = \underline{9 \times 10^6\ W}$.

   B) a) $P_{lost} = r\, I^2_1 = (4)(1562.5)^2 = \underline{9765625\ W}$.
   b) Percentage $= \frac{P_{lost}}{P_{total}} = \frac{9765625}{10^7} = 97.66$ %. **c)** To reduce the energy losses in the transmission lines.

6. **A) a)** $m = \frac{U_2}{U_1} = \frac{U_{DN\,(eff)}}{U_{AB\,(eff)}} = \frac{22}{220} = 0.1$. $m = \frac{N_2}{N_1} \Rightarrow N_1 = \frac{50}{0.1} \Rightarrow N_1 = \underline{500}$.
   b) $m = \frac{I_{1(max)}}{I_{2(max)}} \Rightarrow I_{2(max)} = \frac{0.056}{0.1} = 0.56$ A $\Rightarrow i_2 = 0.56 \cos(100\,\pi\, t + 1.2)$ (S.I)
   B) a) $P_{av} = I_{2(eff)}\, U_{2(eff)} \cos\varphi = \left(\frac{0.56}{\sqrt{2}}\right)(22) \cos(1.2\ rad) = \underline{3.18\ W}$.
   b) $P_R = R\, I^2_{2(eff)} = 20\left(\frac{0.56}{\sqrt{2}}\right)^2 = 3.2\ W \cong P_{av} \Rightarrow$ power lost by r is $P_r = r\, I^2_{2(eff)} \cong 0 \Rightarrow r = 0$.
   c) $u_{DN} = u_{DE} + u_{EH} + u_{HN} = R\, i_2 + L\frac{d\,i_2}{dt} + u_C$. $u_C = \frac{1}{C}\int i_2\, dt = \frac{0.56 \sin(100\,\pi\, t + 1.2)}{C\,(100\,\pi)} = 178.34 \sin(100\,\pi\, t + 1.2)$
   $\Rightarrow 22\sqrt{2} \cos(100\,\pi\, t) = 20\,(0.56) \cos(100\,\pi\, t + 1.2) - L\,(0.56)(100\,\pi) \sin(100\,\pi\, t + 1.2) + 178.34 \sin(100\,\pi\, t + 1.2)$. For $100\,\pi\, t = 0$: $22\sqrt{2} = 4.05 - 163.93\, L + 166.26 \Rightarrow \underline{L = 0.85\ H}$.

Chapter 14: Diffraction

1.



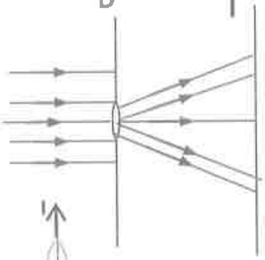
a- Diffraction of light

b- Amplitude of intensity of light varies alternately between maxima and minima during the motion of the detector away from O.

c- $\tan\frac{\alpha}{2} = \frac{\frac{L}{2}}{D} \Rightarrow L = \alpha D \Rightarrow \alpha = \frac{L}{D} = \frac{2.37 \times 10^{-2}\text{m}}{2\text{ m}} = \underline{0.01185 \text{ rad}}.$

d- $\alpha = \frac{2\lambda}{a} \Rightarrow a = \frac{2\,(600 \times 10^{-9})}{0.01185} \cong 1.01 \times 10^{-4}\text{ m} \cong \underline{0.101 \text{ mm}}.$

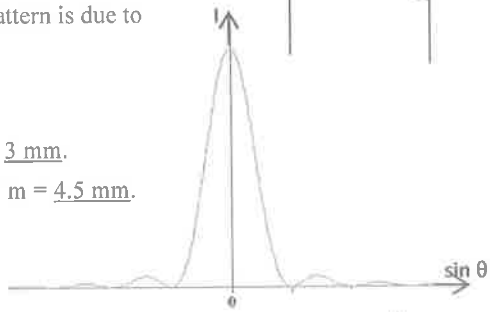
2.



a) **i.** Figure

ii. Alternate bright and dark fringes on both sides of a central spot which is brighter and twice as broad as others. Fringes are aligned along a direction perpendicular to that of the slit.

iii. Each point in the slit acts as a secondary source of wavelets and the pattern is due to superposition of the secondary wave.



b) Figure.

c) $\alpha = \frac{2\lambda}{a} = \frac{2\,(750 \times 10^{-9})}{0.5 \times 10^{-3}} \Rightarrow \alpha = 3 \times 10^{-3}$ rad.

$L = \alpha D$ (figure is necessary) $\Rightarrow L = (3 \times 10^{-3})(1) \Rightarrow L = 3 \times 10^{-3}\text{ m} = \underline{3\text{ mm}}.$

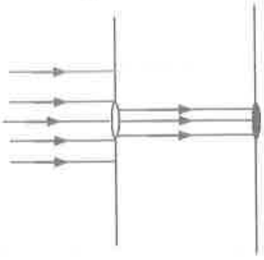
d) 3rd DF: $x_3 = \frac{3\lambda D}{a}$ (with figure) $\Rightarrow x_3 = \frac{3\,(750 \times 10^{-9}) \times 1}{0.5 \times 10^{-3}} \Rightarrow x_3 = 4.5 \times 10^{-3}\text{ m} = \underline{4.5\text{ mm}}.$

Center of 3 B.F is midway between 3DF and 4DF

$\Rightarrow x_{3(\text{bright})} = x_{3(\text{dark})} + \frac{L}{4} = 4.5 + \frac{3}{4} \Rightarrow x_{3(\text{bright})} = \underline{5.25\text{ mm}}.$

e) **i.** $\alpha = \frac{2\lambda}{a}$. As a decreases, α increases $\Rightarrow$ the diffraction pattern <u>becomes broader</u> (diffraction is <u>more clear</u>).

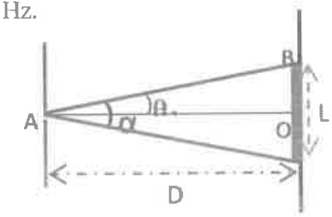
ii. No, since light will undergo diffraction when passing through the hole.



f) 2 cm is very wide relative to this wavelength (a is not of the order of a fraction of 1 mm) $\Rightarrow$ no diffraction, we observe a <u>spot of light</u> of diameter equal to that of the laser beam.

3.

a) **i.** Since it is a light composed of <u>one wavelength</u>. $\nu = \frac{C}{\lambda} = \frac{3 \times 10^{8}}{633 \times 10^{-9}} = 4.74 \times 10^{14}$ Hz.



ii. The phenomenon is diffraction of light.

iii. Wave aspect of light.

b) $L = \alpha D = 2\,\theta_1\,D = \frac{2\lambda D}{a} = \frac{2\,(633 \times 10^{-9})(1.5)}{0.2 \times 10^{-3}} = 9.495 \times 10^{-3}$ (width of C.B.F)

width of other bright fringes is half that of C.B.F : width $= \frac{L}{2} = \underline{4.75 \times 10^{-3}\text{ m}}.$

c) Nearest point of zero intensity corresponds to the 1st dark fringe:

$\theta_1 = \pm\frac{\lambda}{a} = \pm\frac{633 \times 10^{-9}}{0.2 \times 10^{-3}} = \pm\,3.165 \times 10^{-3}$ rad.

$\tan\theta_1 = \frac{x_1}{D}$ but θ_1 is small $\Rightarrow \tan\theta_1 = \theta_1 \Rightarrow x_1 = \theta_1 D = \pm\,(3.165 \times 10^{-3})\,(1.5)$

$\Rightarrow x_1 = \underline{\pm\,4.75 \times 10^{-3}\text{ m}}$. Or: $x_i = \pm\frac{L}{2} = \underline{\pm\,4.75 \times 10^{-3}\text{ m}}.$

d) $L = \frac{2\lambda D}{a} \Rightarrow$ Diffraction pattern is wider (wider fringes) when: λ increases (or/and) D increases (or /and) a decreases.

e) **i.** Same diffraction pattern $\Rightarrow \frac{\lambda' D}{a'} = \frac{\lambda D}{a} \Rightarrow a' = \frac{a\,\lambda'}{\lambda} = \frac{(0.2 \times 10^{-3})\,(400)}{(633)} = \underline{0.126\text{ mm}}.$

ii. Same diffraction pattern $\Rightarrow \frac{\lambda' D'}{a} = \frac{\lambda D}{a} \Rightarrow D' = \frac{\lambda D}{\lambda'} = \frac{(633)\,(1.5)}{(400)} \Rightarrow \underline{D' = 2.37\text{ m}}.$

f) Two perpendicular diffraction patterns are observed on the screen.

4.

a) Ultraviolet. No, since frequency of the radiation is independent of the medium of propagation.

b) $c = f\,.\,\lambda = (5.187 \times 10^{14})\,(578 \times 10^{-9}) \cong 3 \times 10^{8}$ m/s.

c) $\alpha = \frac{2\lambda}{a} \Rightarrow$ broadest central fringe corresponds to the longest wavelength

$\Rightarrow \alpha_{broadest} = \frac{2\,\lambda_{yellow}}{d} = \frac{2\,(578 \times 10^{-9})}{0.4 \times 10^{-3}} = \underline{2.89 \times 10^{-3}\text{ rad}}.$

d) Narrowest pattern corresponds to smallest $\lambda \Rightarrow$ ultraviolet.

$x_3 = \pm\frac{3\,\lambda_D}{d} = \pm\frac{3\,(365 \times 10^{-9}) \times 0.5}{0.4 \times 10^{-3}} \cong \pm\,1.37 \times 10^{-3}\text{ m} \cong 1.37\text{ mm}.$

e) *By drawing a figure* $\Rightarrow d = 2L_{blue} = 2\,\alpha\,D = 2\,(\frac{2\lambda}{d})D = \frac{4\lambda D}{d} = \frac{4\,(436\times10^{-9})\times0.5}{0.4\times10^{-3}} = 2.18\times10^{-3}m = \underline{2.18\ mm}.$

Or: $x_{2DF} = \frac{2\lambda D}{d}$ and $x_{6DF} = \frac{6\lambda D}{d} \Rightarrow d = x_6 - x_2 = \frac{6\lambda D}{d} - \frac{2\lambda D}{d} \Rightarrow d = \frac{4\lambda D}{d} = \underline{2.18\ mm}.$

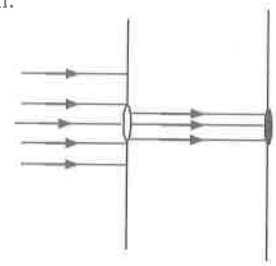
5.

A- a) $\frac{a}{\lambda} = \frac{0.2\times10^{-5}}{800\times10^{-9}} = 250.$

b) $\theta_1 = \frac{\lambda}{a} = 4\times10^{-3}$ rad. $\alpha = 2\,\theta_1 = 2\times4\times10^{-3} = 8\times10^{-3}$ rad. $L = \alpha D = (8\times10^{-3})(1) = 8\times10^{-3}$ m = 8 mm.

c) This distance is equal to the linear width of the central bright fringe $\Rightarrow x = L = 8$ mm.

B- a) The radius of the pin hole is too large for this wavelength. ($\frac{d}{\lambda} = \frac{4\times10^{-2}}{800\times10^{-9}} = 50000$) or d is not of the order of fraction of mm. Diffraction phenomenon doesn't take place and light travels in straight lines. A circular illuminated spot of diameter equal to that of the incident beam of light (see figure).



b) i. $c = \nu\lambda' \Rightarrow \lambda' = \frac{c}{\nu} = \frac{3\times10^8}{187.5\times10^{10}} = \underline{1.6\times10^{-4}\ m}.$ $\frac{d}{\lambda'} = \frac{4\times10^{-2}}{1.6\times10^{-4}} = 250$

ii. Yes, since $\frac{d}{\lambda'} = \frac{a}{\lambda}$, this aperture is small enough relative to this wavelength.

6.

a) $\theta_4 = \frac{-4\lambda}{a} = \frac{-4\,(600\times10^{-9})}{0.4\times10^{-3}} \Rightarrow \theta_4 = \underline{-6\times10^{-3}\ rad}.$ $\tan\theta_4 \approx \theta_4 = \frac{x_4}{D}$

$\Rightarrow x_4 = -\theta_4 D = -6\times10^{-3}\,(2) \Rightarrow x_4 = \underline{-12\times10^{-3}\ m.}$

b) i. Center of B.F is midway between the centers of two consecutive DF $\Rightarrow x_{n(bright)} = \pm\,[\,(n+\frac{1}{2})\frac{\lambda D}{a}\,].$

ii. $7.5 = (n+\frac{1}{2})\frac{\lambda D}{a} \Rightarrow (n+\frac{1}{2}) = \frac{7.5\times10^{-3}\times0.4\times10^{-3}}{600\times10^{-9}\times2} = 2.5 \Rightarrow n = 2$ so B is the center of the $\underline{2^{nd}\ bright\ fringe}$ on the positive side of O.

c) i. $\theta_n = \frac{n\lambda}{a}$; $n = \pm1, \pm2,\ldots$ (D.F) : $x_n = \theta_n D \Rightarrow x_n = \frac{n\lambda D}{a} \Rightarrow$ As D decreases; x_n decreases $\Rightarrow$ pattern of fringes becomes <u>narrower</u>.

ii. 4^{th} DF : $n = 4 \Rightarrow x_4 = \frac{4\lambda D'}{a} \Rightarrow 7.5\times10^{-3} = \frac{4\,(600\times10^{-9})(D')}{0.4\times10^{-3}} \Rightarrow D' = 1.25$ m $\Rightarrow$ displaced distance $\underline{d = 0.75\ m}$

d) $x_n = \frac{n\lambda D}{a} \Rightarrow$
- Replace the monochromatic light by another one of longer wavelength;
- Displace the screen away from the plane of the slit;
- Replace the slit by another one of smaller width.

7.

a) i. Horizontal

ii. Width of the C.B.F is double that of the other bright fringes.

iii. The points are the centers of dark fringes $\Rightarrow$ Intensity of light at these points is zero.

iv. A is the center of the 1 BF to the left side of O while B is the center of the 2 BF to the right side of O $\Rightarrow$ Intensity at B is less than that at A.

v. From the figure : $AB = \frac{L}{4} + L + \frac{L}{2} + \frac{L}{4} = 2L \Rightarrow L = \frac{44}{2} = \underline{22\ mm}.$

b) i. $\theta_1 = \pm\frac{\lambda}{d} \Rightarrow \alpha = 2\,\theta_1 = \frac{2\lambda}{d}$. (Drawing figure): $\tan\theta \cong \theta_1 = \frac{\frac{L}{2}}{D} \Rightarrow L = 2\,\theta_1\,D = L = \alpha D \Rightarrow D = \frac{22\times10^{-3}}{5.5\times10^{-3}}$

$\Rightarrow \underline{D = 4\ m}.$

ii. $\alpha = \frac{2\lambda}{d} \Rightarrow 5.5\times10^{-3} = \frac{2\,(550\times10^{-9})}{d} \Rightarrow d = 0.2\times10^{-3}m = \underline{0.2\ mm}.$

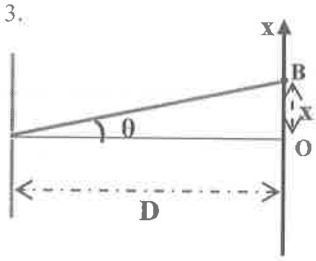
8. a) i. $\sin\theta = \frac{n\lambda}{a}$ ($n = \pm1, \pm2,\ldots$)

ii. $\sin\theta = n\frac{(0.2\times10^{-3})}{0.75\times10^{-3}} \Rightarrow -1 \leq n\,(0.267) \leq 1 \Rightarrow -3.75 \leq n \leq 3.75 \Rightarrow n = \pm1, \pm2, \pm3.$

$\Rightarrow$ the maximum number of centers of DF is (6) (3 D.F on each side of the C.B.F).

b) i. $\tan\theta = \frac{x}{D}$. For small angles $\tan\theta \approx \theta \Rightarrow x = \theta D = \frac{n\lambda D}{a} \Rightarrow n = \frac{x\,a}{\lambda D}$;

$\Rightarrow n = \frac{(2.6\times10^{-3})(0.75\times10^{-3})}{(0.65\times10^{-6})(1)} = 3 \Rightarrow$ B is the center of the 3^{rd} dark fringe on the positive side of O



ii. 1) $\left.\begin{array}{l} c = \nu\,\lambda_{air} \\ v_w = \nu\,\lambda_w \end{array}\right\}$ Divide : $\frac{c}{v_w} = \frac{\lambda_{air}}{\lambda_w} = n \Rightarrow \lambda_w = \frac{\lambda_{air}}{n}.$

2) $L = \alpha D = \frac{2\lambda D}{a} \Rightarrow L' = \frac{2\lambda_w D}{a} = \frac{2\lambda_{air} D}{n a} \Rightarrow L' = \frac{L}{n}$ $(n > 1) \Rightarrow L' < L$. The width becomes smaller.

Or $L = \frac{2\lambda D}{a} \Rightarrow$ L is proportional to λ. $\lambda_w < \lambda_{air} \Rightarrow L_{water} < L_{air} \Rightarrow$ width becomes smaller.

3) $x_B = \frac{n\lambda_w D}{a} \Rightarrow 2.6 \times 10^{-3} = \frac{n\,(0.65 \times 10^{-6})(1)}{(4/3)(0.75 \times 10^{-3})} \Rightarrow n = 4 \Rightarrow$ B is the center of the 4^{th} DF on the positive side of O.

9. a) Diffraction takes place for each monochromatic radiation $\Rightarrow$ all the central bright fringes superpose at O $\Rightarrow$ the color at the center O is white.

b) i) Width of 2^{nd} BF $= \frac{L_{red}}{2} = \frac{6.4}{2} = 3.2$ mm.

ii) (With figure): $L = \alpha D = \frac{2\lambda D}{a} \Rightarrow \frac{D}{a} = \frac{L}{2\lambda} \Rightarrow \frac{L_{red}}{2\lambda_{red}} = \frac{L_{violet}}{2\lambda_{violet}} \Rightarrow L_{violet} = \frac{L_{red}\,\lambda_{violet}}{\lambda_{red}} = \frac{(6.4)(400 \times 10^{-9})}{(800 \times 10^{-9})} = 3.2$ mm.

iii) $L = \alpha D \Rightarrow D = \frac{6.4 \times 10^{-3}}{1.6 \times 10^{-3}} \Rightarrow D = 4$ m; $\alpha = \frac{2\lambda}{a} \Rightarrow 1.6 \times 10^{-3} = \frac{2(800 \times 10^{-9})}{a} \Rightarrow a = 1 \times 10^{-3}$ m $= 1$ mm.

c) $\theta_n = \frac{n\lambda}{a} \Rightarrow x_n = D\,\theta_n = \frac{n\lambda D}{a} \Rightarrow x_n(\text{red})\ n\,\frac{(800 \times 10^{-9})(4)}{10^{-3}} \Rightarrow x_n(\text{red}) = 3.2 \times 10^{-3}\,n\ldots$eq (1)

$x_{n+1}(\text{of }\lambda') = \frac{(n+1)(640 \times 10^{-9})(4)}{10^{-3}} = 2.56 \times 10^{-3}\,(n+1)\ldots$eq (2). $x_n(\text{red}) = x_{n+1}(\text{of }\lambda') \Rightarrow 3.2 \times 10^{-3}\,n = 2.56 \times 10^{-3}\,(n+1) \Rightarrow n = 4$. Substitute in eq (1): $x = 3.2 \times 10^{-3}\,(4) \Rightarrow x = 0.0128$ m $= 12.8$ mm.

d) $x_m = \frac{n\lambda D}{a}$; $x_m > 0 \Rightarrow$ n is a positive whole number $n = \frac{a\,x_m}{\lambda D} = \frac{10^{-3} \times 10 \times 10^{-3}}{\lambda \times 4} = \frac{2.5 \times 10^{-6}}{\lambda}$

but $400 \times 10^{-9} \leq \lambda < 800 \times 10^{-9} \Rightarrow 3.125 \leq n \leq 6.25 \Rightarrow n = 4, 5, 6$. We have $\lambda = \frac{2.5 \times 10^{-6}}{n}$:

For $n = 4 \Rightarrow \lambda = 6.25 \times 10^{-7}$ m $= 625$ nm (4^{th} dark). For $n = 5 \Rightarrow \lambda = 5 \times 10^{-7}$ m $= 500$ nm (5^{th} dark)

For $n = 6 \Rightarrow \lambda = 4.166 \times 10^{-7}$ m $= 416.6$ nm (6^{th} dark)

Chapter 15: Interference of Light

1. **a)** Since the lights falling on the two slits originates from the same laser source.
   b) **i.** Alternate bright and dark fringes, parallel to each other and to the slits, the fringes are rectilinear bands, and their centers are equidistant.
   ii. From the figure the distance between the centers of 7 bright fringes is d = 12 mm ⇒ 6i = d ⇒ $i = \frac{12\text{ mm}}{6} = \underline{2\text{ mm}}$.
   c) $i = \frac{\lambda D}{a} \Rightarrow \lambda = \frac{i\,a}{D} = \frac{(2\times10^{-3})(0.75\times10^{-3})}{2} = 750\times10^{-9}\text{ m} = \underline{750\text{ nm}}$.

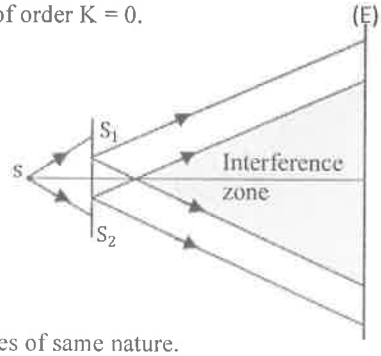
2. **a)** **i.** $x = 0$.
   ii. $x_{2\,D\,F} = 1.5$ mm.
   iii. $i = 1.5 - 0.5 = 1$ mm.
   iv. $d = 2\text{ mm} + 0.5\text{ mm} = 2.5$ mm.
   b) $i = \frac{\lambda D}{a} \Rightarrow 10^{-3} = \frac{(500\times10^{-9})(1.6)}{a} \Rightarrow a = 0.8\times10^{-3}\text{ m} = \underline{0.8\text{ mm}}$.

3. **a)** **i.** The pattern of figure (b) corresponds to diffraction since the C.B.F. is twice as broad as others. The pattern of figure (a) corresponds to interference since the centers of all bright fringes are equidistant.
   ii. L = 1.2 cm.
   b) Diffraction : $L = \frac{2\lambda D}{a_1}$. Interference : $i = \frac{\lambda D}{a}$ and $L = 12\,i \Rightarrow \frac{2\lambda D}{a_1} = 12\frac{\lambda D}{a}$; $a_1 = \frac{2a}{12} = \frac{a}{6} = \frac{1}{6}$ mm.

4. $\tan\theta \cong \theta = \frac{OM}{D} = \frac{x_m}{D} \Rightarrow x_m = \theta D = (4\times10^{-3})(2) = 8\times10^{-3}$ m.
   If $M \in BF$: $\delta = \frac{a\,x}{D} = K\lambda$; $K \in Z$; $K = \frac{a\,x}{\lambda D} = \frac{(0.5\times10^{-3})(8\times10^{-3})}{(500\times10^{-9})(2)} = 4 \Rightarrow M \in \underline{4^{th}\text{ B F}}$ on the positive side of O.

5. **a)** The light must be synchronous and coherent.
   b) The lights are originated:
   1) from the same Laser source, or
   2) from the same point source.
   c) Graphically : a = 2 (OP) = 2 (0.4) = <u>0.8 mm</u>; $i = \frac{\lambda D}{a} = \frac{(0.64\times10^{-6})(1)}{(0.8\times10^{-3})} = 8\times10^{-4}\text{ m} = \underline{0.8\text{ mm}}$.
   d) **i)** $\delta_P = \frac{a\,x}{D} = \frac{(0.8\times10^{-3})(0.4\times10^{-3})}{1} \Rightarrow \delta_P = 32\times10^{-8}$ m.
   ii) $\delta_P = S_2P - S_1P \Rightarrow 32\times10^{-8} = S_2P - 1 \Rightarrow S_2P = 1.00000032$ m.
   iii) $\delta_P = K\lambda$; $K \in Z \Rightarrow K = \frac{\delta_P}{\lambda} = \frac{32\times10^{-8}}{0.64\times10^{-6}} = 0.5 \notin Z \Rightarrow$ P is not the center of a bright fringe.
   $\delta_P = (2K+1)\frac{\lambda}{2}$; $K \in Z$ with $(2K+1) = 1 \Rightarrow K = 0 \Rightarrow P \in \underline{1^{st}\text{ dark}}$ fringe of order K = 0.

6. **a)** Figure.



   b) **i.** The lights which fall on S_1 and S_2 must be issued from the same point source or from the same laser source of light.
   ii. Since S is a point source of light.
   c) **i.** $\delta = K\lambda$; $K \in Z$. $\Rightarrow K\lambda = \frac{a\,x}{D} \Rightarrow x = \frac{K\lambda D}{a}$; $K \in Z$.
   ii. $\delta = (2K+1)\frac{\lambda}{2} = \frac{a\,x}{D} \Rightarrow x = \frac{(2K+1)\lambda D}{2a}$.
   d) $\delta = S_2O - S_1O = 0 = K\lambda$; $K = 0 \Rightarrow$ Bright fringe at O. This fringe is called central bright fringe.
   e) **i.** Interfringe distance is the distance separating the centers of two consecutive fringes of same nature.
   $i = x_{K+1} - x_K = \frac{(K+1)\lambda D}{a} - \frac{K\lambda D}{a} = \frac{K\lambda D}{a} + \frac{\lambda D}{a} - \frac{K\lambda D}{a} \Rightarrow i = \frac{\lambda D}{a}$; $i = \frac{(0.6\times10^{-6})(2)}{(0.5\times10^{-3})} \Rightarrow i = 2.4\times10^{-3}\text{ m} = \underline{2.4\text{ mm}}$.
   ii. 2.B.F on positive side : $x_{2.B.F} = \frac{2\lambda D}{a} = 2\,i$; 3.B.F on negative side : $x_{3.B.F} = \frac{-3\lambda D}{a} = -3\,i$
   $\Rightarrow$ distance $= x_{2.B.F} + |x_{3.B.F}| = 2\,i + 3\,i = 5\,i \Rightarrow$ distance = 5 (2.4) = <u>12 mm</u>.
   iii. 2.D.F on positive side: K = 1 : $x_{2\,D\,F} = \frac{3\lambda D}{2a} = 1.5\,i$; 9 B F on negative side : $x_{9.B.F} = -9\,i$
   $\Rightarrow$ distance $= 1.5\,i + |-9\,i| = 10.5\,i = 10.5\,(2.4) = \underline{25.2\text{ mm}}$.
   f) **i.** $\delta = \frac{a\,x}{D} = \frac{(0.5\times10^{-3})(-7.2\times10^{-3})}{2} = -1.8\times10^{-6}$ m; $\delta = K\lambda$; $K \in Z \Rightarrow K = \frac{-1.8\times10^{-6}}{0.6\times10^{-6}} = -3 \in Z$
   $\Rightarrow P \in \underline{3.B.F}$ to the negative side of O.
   ii. $\delta'_P = \frac{a\,x}{D'} = \frac{(0.5\times10^{-3})(-7.2\times10^{-3})}{4} = -0.9\times10^{-6} = (2K+1)(0.3\times10^{-6}) \Rightarrow 2K+1 = -3 \Rightarrow \underline{K = -2}$
   $\Rightarrow$ P is the center of the 2nd D.F. to the negative side of O $\Rightarrow$ intensity at P is <u>zero</u>.
   g) The wave aspect of light.

7. a) $\tan\frac{\beta}{2} = \frac{\frac{a}{2}}{D}$ Since small angle $\Rightarrow \tan\frac{\beta}{2} = \frac{\beta}{2} = \frac{a}{2D} \Rightarrow D = \frac{a}{\beta}$; $D = \frac{(0.5\times10^{-3})}{(0.0286)\frac{(\pi)}{180}} = 1.002$ m $\cong$ 1 m.

b) i. $\frac{\lambda D}{a} = \frac{(0.5\times10^{-6})(1)}{0.5\times10^{-3}} = 10^{-3} = 1$ mm.

ii. 2.B.F : $x_{2.B.F} = \frac{2\lambda D}{a} = 2$ i and 5.D.F : $x_{5.D.F} = \frac{(2(4)+1)\lambda D}{2a} = 4.5\frac{\lambda D}{a} = 4.5$ i.

$\Rightarrow$ distance = $x_{5.D.F} - x_{2.B.F}$ = 4.5 i – 2 i = 2.5 i = 2.5 (1) = 2.5 mm.

iii. OB = 12 i = 12 (1) = 12 mm.

c) i. $\delta = MS_2 - MS_1$ = 1000.008 – 1000.009 = - 0.001 mm; $\delta = K\lambda$; $K \in Z \Rightarrow K = \frac{-0.001\times10^{-3}}{0.5\times10^{-6}} = -2 \in Z$

$\Rightarrow$ M is the center of the 2nd bright fringe on the negative side of O.

ii. $\delta = \frac{ax}{D} \Rightarrow x = \frac{\delta D}{a} = \frac{(-0.001)\text{mm}\,(1\text{ m})}{(0.5\text{ mm})} = -2\times10^{-3}$ m = -2 mm.

Or : x = - 2 i = - 2 (1 mm) = - 2 mm.

8. A) a) i. i is the distance separating the centers of two consecutive fringes of same nature $\Rightarrow$ AB = 10 i $\Rightarrow$ $i = \frac{12}{10}$ = 1.2 mm.

ii. $i = \frac{\lambda_1 D}{a} \Rightarrow \lambda_1 = \frac{(1.2\times10^{-3})(0.5\times10^{-3})}{1} \Rightarrow \lambda_1 = 600\times10^{-9}$ m = 600 nm.

B) a) No, since these radiations are not synchronous and coherent.

b) The center of the C.B.F of each monochromatic radiation is at O $\Rightarrow$ The color at O is dichromatic.

c) 1 B.F: $x_{1.B.F} = \frac{(1)\lambda_1 D}{a}$. 2.D.F on positive side: K = 1$\Rightarrow$ $x_{2.D.F} = \frac{[2(1)+1]\lambda_2 D}{2a} = \frac{1.5\lambda_2 D}{a}$; $\frac{\lambda_1 D}{a} = \frac{1.5\lambda_2 D}{a}$

$\Rightarrow \frac{\lambda_1 D}{a} = \frac{1.5\lambda_2 D}{a} \Rightarrow \lambda_2 = 400\times10^{-9}$ m = 400 nm.

d) $i_2 = \frac{\lambda_2 D}{a} = \frac{(400\times10^{-9})(1)}{0.5\times10^{-3}} = 0.8\times10^{-3}$ m = 0.8 mm; $\frac{12.2}{0.8} = 15.25$ $\Rightarrow$ number of B.F. = 15 .

C) a) $x_O = 0 \Rightarrow \delta_n = 0$, so O remains the center of the central bright fringe.

b) B.F : $\delta_n = \frac{nax}{D} = K\lambda$; $K \in Z \Rightarrow x_K = \frac{k\lambda D}{na}$ and $x_{K+1} = \frac{(K+1)\lambda D}{na}$ but $i_n = x_{K+1} - x_K$

$\Rightarrow i_n = \frac{(K+1)\lambda D}{na} - \frac{K\lambda D}{na} = \frac{K\lambda D}{na} + \frac{\lambda D}{na} - \frac{K\lambda D}{na} = \frac{\lambda D}{na}$; But $i_{air} = \frac{\lambda D}{a} \Rightarrow i_n = \frac{i_{air}}{n}$.

c) n > 1 $\Rightarrow i_n < i_{air}$ $\Rightarrow$ Fringes become closer to each other.

d) 3.B.F : $x_3 = \frac{3\lambda D}{a} = 3\,i_{air}$; 4.B.F : $x_4 = \frac{4\lambda D}{na} = \frac{4\,i_{air}}{n}$ $\Rightarrow 3\,i_{air} = \frac{4\,i_{air}}{n} \Rightarrow n = \frac{4}{3}$.

9. a) i. $v = \frac{e}{\tau} \Rightarrow \tau = \frac{e}{v}$ but $n = \frac{c}{v} \Rightarrow \tau = \frac{ne}{c}$.

ii. $c = \frac{d_{opt}}{\tau} \Rightarrow d_{opt} = c\,\tau = c\,(\frac{ne}{c}) \Rightarrow d_{opt} = n\,e$.

iii. Increase in optical path = n e – e = e (n – 1).

b) i. $\delta_N = SS_2N - SS_1N = (SS_2 - SS_1)_{optical} + (S_2N - S_1N)$; But: $(SS_2 - SS_1)_{optical} = -e(n-1)$ and $S_2N - S_1N = \frac{ax}{D}$

$\Rightarrow \delta = -e(n-1) + \frac{ax}{D}$.

ii. $\delta_O = -e(n-1) + \frac{a(0)}{D} = e(n-1) \neq 0$ $\Rightarrow$ O is no more center of C.B.F.

iii. Centers of the bright fringes: : $\delta = K\lambda$; $K \in Z$; $K\lambda = -e(n-1) + \frac{ax}{D} \Rightarrow x = \frac{K\lambda D}{a} + \frac{eD}{a}(n-1)$;

Centers of the dark fringes: $\delta = (2K+1)\frac{\lambda}{2}$; $K \in Z$; $(2K+1)\frac{\lambda}{2} = -e(n-1) + \frac{ax}{D} \Rightarrow x = (2K+1)\frac{\lambda D}{2a} + \frac{eD}{a}(n-1)$.

iv. $i = x_{K+1} - x_K = \frac{D}{a}[(K+1)\lambda + e(n-1)] - \frac{D}{a}[K\lambda + e(n-1)]$

$\Rightarrow i = \frac{D}{a}[(K+1)\lambda - K\lambda + e(n-1) - e(n-1)] \Rightarrow i = \frac{\lambda D}{a} = \frac{(600\times10^{-9})(2)}{10^{-3}}$ = 1.2 mm.

c) i. $\delta_O = -e(n-1) = -8\lambda \Rightarrow 0.01\times10^{-3}(n-1) = 8(600\times10^{-9}) \Rightarrow$ n = 1.48.

ii. The 8th B.F. is at d = 8 i from the center of C.B.F $\Rightarrow$ d = 8 (1.2 mm) = 9.6 mm.

iii. $\delta = \frac{ax}{D} - e(n-1) \Rightarrow 0 = \frac{ax_O'}{D} - e(n-1) \Rightarrow x_O' = \frac{eD(n-1)}{a} = \frac{(0.48)(2)(0.01\times10^{-3})}{10^{-3}} = 9.6\times10^{-3}$ m = 9.6 mm.

10. A) a) $\delta = K\lambda$; $K \in Z \Rightarrow K = \frac{3\times10^{-6}\text{ m}}{600\times10^{-9}\text{ m}} = 5 \in Z$ $\Rightarrow$ M $\in$ to the center of the 5th B.F.

b) $i = \frac{\lambda D}{a} = \frac{(600\times10^{-9})(2)}{0.5\times10^{-3}} = 2.4\times10^{-3}$ m = 2.4 mm; x_M = 5 i = 5 (2.4) = 12 mm.

B) a) The optical path of the distance e is n.e $\Rightarrow$ Increase of optical path = n.e – e = e (n – 1)

$\Rightarrow$ New optical path at a point M is $\delta' = (S_2M - S_1M)_{optical} = \frac{ax}{D} + e(n-1)$.

b) D.F : K = 7 ; $\delta' = \frac{[2(7)+1]\lambda}{2} = 7.5(600\times10^{-9}) = 4.5\times10^{-6}$ m.

$\delta' = 4.5\times10^{-6} = 3\times10^{-6} + e(1.5-1) \Rightarrow e = 3\times10^{-6}$ m = 3×10^{-3} mm.

11. a) Vertical since they are parallel to the slit.

b) i = 3.2 mm since all fringes of interference have the same width.

c) The center of the central bright fringe of each monochromatic radiation is at O $\Rightarrow$ color at O is <u>white</u>.

d) **i.** $\delta_M = \frac{a\,x}{D} = \frac{(10^{-3})(12.8\times10^{-3})}{4} = 3.2\times10^{-6}$ m; $\lambda = 800$ nm ; $\delta = K\lambda$; $K \in Z$ $\Rightarrow$ $K = \frac{3.2\times10^{-6}\ m}{800\times10^{-9}\ m} = 4 \in N$

$\Rightarrow$ M is the center of the <u>4th B.F</u> in the positive side of O.

ii. $\lambda = 711$ nm ; $\delta = (2K+1)\frac{\lambda}{2}$; $K \in Z$ $\Rightarrow$ $(2K+1) = \frac{3.2\times10^{-6}}{\frac{711\times10^{-9}}{2}}$ $\Rightarrow$ $(2K+1) \cong 9$ (odd number)

$\Rightarrow$ K = 4 $\Rightarrow$ M is the center of the <u>5th D.F</u> .

e) $\delta_O = \frac{a\,x}{D} = \frac{(10^{-3})(8\times10^{-3})}{4} = 2\times10^{-6}$m. D.F: $\delta = (2K+1)\frac{\lambda}{2}$; $K\in Z \Rightarrow 2\times10^{-6} = \frac{(2K+1)\lambda}{2} \Rightarrow 4\times10^{-6} = (2K+1)\lambda$

For $\lambda = 0.4$ µm $\Rightarrow$ $4\times10^{-6} = (2K+1)(0.4\times10^{-6})$ $\Rightarrow$ $2K+1 = 10$ $\Rightarrow K = 4.5$

For $\lambda = 0.8$ µm $\Rightarrow$ $4\times10^{-6} = (2K+1)(0.8\times10^{-6})$ $\Rightarrow$ $2K+1 = 5$ $\Rightarrow K = 2$

$\Rightarrow 2 \leq K < 4.5 \Rightarrow$ K 2, 3, 4

We have $\lambda = \frac{4\times10^{-6}}{2k+1}$:

For K = 2: <u>3.D.F</u> $\Rightarrow \lambda = \frac{4\times10^{-6}}{5} = 0.8\times10^{-6}$ m = <u>0.8 µm;</u>

For K = 3: <u>4.D.F</u> $\Rightarrow \lambda = \frac{4\times10^{-6}}{2(3)+1} = 0.571\times10^{-6}$ m = <u>0.571µm;</u>

For K = 4 : <u>5.D.F</u> $\Rightarrow \lambda = 0.444\times10^{-6}$ m = <u>0.444 µm</u> .

f) $\lambda_1 = 750$ nm: $x_P = \frac{(2K+1)\lambda_1 D}{2a} = \frac{[2(1)+1]\lambda_1 D}{2a} = \frac{3\lambda_1 D}{2a}$. λ_2: $x_P = \frac{K\lambda D}{a} = \frac{2\lambda_2 D}{a}$

$\Rightarrow \frac{3\lambda_1 D}{2a} = \frac{2\lambda_2 D}{a} \Rightarrow \lambda_2 = \frac{3\lambda_1}{4} = \frac{3(750)}{4} =$ <u>562.5 nm.</u>

12. **A)** Since the two sources are synchronous but not coherent.

B) **a)** the lights falling on S_1 and S_2 are synchronous and coherent since they originate from the same point source.

b) $\delta_O = SS_2O - SS_1O = (SS_2 - SS_1) + (\overset{0}{\cancel{S_2O - S_1O}})$ for any position of S along (IO) ; $SS_1 = SS_2$ $\Rightarrow \delta_O = 0$

$\Rightarrow$ center of C.B.F is at O.

c) $\delta_O = 0 \Rightarrow \delta_O = K\lambda$; $K = 0 \Rightarrow$ In-phase.

C) **a)** **i.** $\delta_O = SS_2O - SS_1O = (SS_2 - SS_1) + (\overset{0}{\cancel{S_2O - S_1O}})$ But $SS_2 \neq SS_1 \Rightarrow \delta_O \neq 0 \Rightarrow$ O is no more the C.B.F.

ii. Let center of C.B.F to be at point O' : $\delta_{O'} = 0 = (SS_2 - SS_1) + (S_2O' - S_1O')$; $SS_2 - SS_1 = S_1O' - S_2O'$

But $SS_2 > SS_1 \Rightarrow S_1O' > S_2O' \Rightarrow$ Center of C.B.F is displaced to the side of S_2.

b) **i.** $\delta_M = SS_2M - SS_1M = (SS_2 - SS_1) + (S_2M - S_1M)$ But $SS_2 - SS_1 = \frac{aZ}{d}$ and $S_2M - S_1M = \frac{a\,x}{D} \Rightarrow \delta_M = \frac{aZ}{d} + \frac{a\,x}{D}$

ii. B.F : $\delta = K\lambda$; $K \in Z$ $K\lambda = \frac{aZ}{d} + \frac{a\,x}{D} \Rightarrow x = \frac{K\lambda D}{a} - \frac{D}{d}Z$.

D.F: $\delta = (2K+1)\frac{\lambda}{2}$; $K \in Z \Rightarrow (2K+1)\frac{\lambda}{2} = \frac{aZ}{d} + \frac{a\,x}{D} \Rightarrow x = \frac{(2K+1)\lambda D}{2a} - \frac{D}{d}Z$.

iii. $i = x_{K+1} - x_K$; $i = [\frac{(K+1)\lambda D}{a} - \frac{D}{d}Z] - [\frac{K\lambda D}{a} - \frac{D}{d}Z] = \frac{K\lambda D}{a} + \frac{\lambda D}{a} - \frac{D}{d}Z - \frac{K\lambda D}{a} + \frac{D}{d}Z = \frac{\lambda D}{a}$.

c) **i.** 2D.F: K = 1 $\Rightarrow$ $\delta_O = \frac{aZ}{d} + 0 = (2K+1)\frac{\lambda}{2} \Rightarrow \frac{aZ}{d} = 1.5\lambda \Rightarrow \frac{(10^{-3})Z}{0.2} = 1.5(0.6\times10^{-6})$; $Z = 1.8\times10^{-4}$m = 0.18 mm.

ii. $\delta_{O'} = 0 = \frac{aZ}{d} + \frac{a\,x_{O'}}{D} \Rightarrow x_{O'} = -\frac{DZ}{d} = \frac{-(2)(1.8\times10^{-4})}{0.2} = -1.8\times10^{-3}$ m = <u>- 1.88 mm.</u>

13.

a) **i.** The optical path through the sheet is d = n . e $\Rightarrow$ The increase of the optical path = ne – e = e (n -1) = 0.00972 (1.48 – 1) = <u>4.6656×10^{-3} mm.</u>

ii. $\delta_P = SS_2P - SS_1P = (SS_2 + S_2P) - (SS_1 + S_1P) = (SS_2 - SS_1) + (S_2P - S_1P)_{optical} \Rightarrow \delta_P = \frac{aZ}{d} + (S_2P - S_1P)_{optical}$

But $(S_2P - S_1P)_{optical} = (S_2P - S_1P)_{geometrical} + e(n-1) = \frac{a\,x}{D} + e(n-1)$.

$\Rightarrow \delta_P = \frac{aZ}{d} + \frac{a\,x}{D} + e(n-1)$.

b) $\delta_P = \frac{0.2\times10^{-3}(0.002)}{0.12} + \frac{0.2\times10^{-3}(0.05)}{4} + [0.00972\times10^{-3}(1.48-1)] \approx 1.05\times10^{-5} = k\lambda \Rightarrow k = \frac{1.05\times10^{-5}}{750\times10^{-9}} \cong 14 \in Z$

$\Rightarrow$ P is the center of the 14th bright fringe.

c) At O, x = 0 $\Rightarrow$ $\delta_O = [\frac{0.2\times10^{-3}(0.002)}{0.12}] + [(0.00972\times10^{-3}(1.48-1)] \cong 8\times10^{-6}$ m.

Bright fringe: $\delta_O = k\lambda \Rightarrow k = 10.66$ (not whole) $\Rightarrow$ O is not the center of a bright fringe.

$\delta_O = 8\times10^{-6}$ m $= (2k+1)\frac{\lambda}{2} \Rightarrow (2k+1) = 21.33$ (not odd) $\Rightarrow$ O is not the center of a dark fringe.

<u>O is neither the center of a bright fringe and nor the center of a dark fringe.</u>

d) Let O' be the center of the new central bright fringe $\Rightarrow \delta_O' = 8\times10^{-6} + \frac{a\,x_O'}{D} = 0 \Rightarrow x_O' = \frac{-8\times10^{-6}\times D}{a} = \frac{-8\times10^{-6}\times4}{0.2\times10^{-3}} = -0.16$ m

$\Rightarrow x_O' =$ <u>- 16 cm.</u>

e) $\delta_o = \frac{aZ'}{d} + \frac{a\,x}{D} + e(n-1) = 0$. At O x = 0 $\Rightarrow \delta_O = \frac{0.2\times10^{-3}Z'}{0.12} + 4.665\times10^{-6} = 0$

$\Rightarrow Z' = -2.8\times10^{-3}$ m = - 2.8 mm so S is displaced <u>towards the side of S_2 by a displacement of 2.8 mm</u> .

Chapter 16: Corpuscular aspect of light- Photoelectric effect

1. Since the liberated electrons (if exist) are attracted again by the zinc plate $\Rightarrow$ the charge of the leaf is not changed $\Rightarrow$ Leaf remains in its initial position.

2. **a)** $E_{ph} = h\nu$; h = constant and ν is independent of the medium of propagation $\Rightarrow$ E_{ph} is not changed.
   b) In vacuum : $E_{ph} = \frac{hc}{\lambda} \Rightarrow E_{ph}\lambda = hc$ = constant.

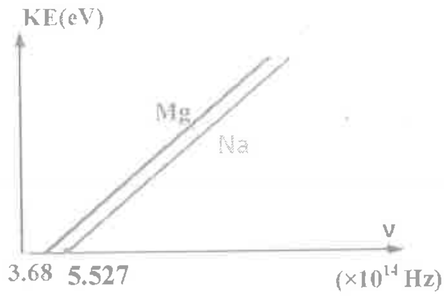
3. **a)** - I do not agree with student A since in photoelectric effect one incident photon may interact only with one electron. 3 eV < 3.4 eV $\Rightarrow$ No photoelectric emission.
   - I do not agree with student B since if the energy of the incident photon is greater or equal to W_o, then we will have photoelectric emission whatever the power of the lamp.

   b) i. $P = \frac{E_{total}}{t} = \frac{N_{receive}}{t} E_{ph} \Rightarrow N_{receive} = \frac{Pt}{E_{ph}} = \frac{Pt}{h\nu}$. But $N_{liberated} = r\, N_{receive} = \frac{rPt}{h\nu} \Rightarrow$ to increases $N_{liberated}$:
   1- increasing the power (P), 2- increasing the exposure time (t), 3- decreasing (ν) such that $\nu > \nu_o$ and keeping the same power.
   ii. $h\nu = W_o + \frac{1}{2} m V^2$ but W_o = constant $\Rightarrow$ speed decreases if (ν) decreases.

4. **a) i.** The maximum wavelength of the incident radiation that is capable to extract electrons from the surface of the metal.
   ii. $W_o = h\nu_o = \frac{hc}{\lambda_o} \Rightarrow \lambda_o = \frac{(6.6\times10^{-34})(3\times10^{8})}{2.25\times1.6\times10^{-19}} = 5.5\times10^{-7}$ m = $\underline{550 \text{ nm}}$.
   b) $\lambda_A = 700$ nm > 550 nm $\Rightarrow$ no liberation of electrons.
   c) $E_{photon} = W_o + KE_e \Rightarrow \frac{hc}{\lambda_B} = W_o + KE_e$
   $\Rightarrow \frac{(6.6\times10^{-34})(3\times10^{8})}{\lambda_B} = (2.25\times1.6\times10^{-19}) + \frac{1}{2}(9.1\times10^{-31})(2.96\times10^{5})^2 \Rightarrow \lambda_B \cong 4.95\times10^{-7}$ m = $\underline{495 \text{ nm}}$.
   d) $E_{ph} = W_o + KE_c \Rightarrow 3.1\times1.6\times10^{-19} = (2.25\times10^{-19}) + \frac{1}{2}(9.1\times10^{-31})V_{max}^{\ 2} \Rightarrow V_{max} \cong \underline{546718.5 \text{ m/s}}$.

5. **a)** $E_{photon} = h\nu = \frac{(6.6\times10^{-34})(3.2\times10^{14})}{1.6\times10^{-19}} = 1.32$ eV. Since (E_{ph} = 1.32 eV) < (W_o = 5.1eV) $\Rightarrow$ No liberation of electrons.
   b) i. $P = \frac{E}{t} \Rightarrow t = \frac{5.1\times1.6\times10^{-19}}{3.2\times10^{-20}} = 25.5$ s.
   ii. According to the classical wavelength, after 25.5 s, the electron must be extracted from the atom. But in part (a), this radiation is not capable of liberating electrons.
   iii. No, the corpuscular aspect.

6. **a)** $\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{200\times10^{-19}} = 1.5\times10^{15}$ Hz > 1.09×10^{15} Hz $\Rightarrow \nu > \nu_o \Rightarrow$ The radiation is capable of liberating electrons.
   b) $\left.\begin{array}{l} 4\pi R^2 \xrightarrow{\text{receives}} 12\text{ W} \\ 4\times10^{-4}\text{ m}^2 \longrightarrow P_r \end{array}\right\} \Rightarrow P_r = \frac{12\times4\times10^{-4}}{4\pi(0.8)^2} = \underline{5.97\times10^{-4}\text{ W}}$.
   c) $E_{total} = P_r t \Rightarrow N E_{photon} = P_r t \Rightarrow N = \frac{P_r\times t}{h\nu} = \frac{(5.97\times10^{-4})(2\times60)}{6.6\times10^{-34}\times1.5\times10^{15}} = \underline{7.238\times10^{16}}$.
   d) $r = \frac{N_{eff}}{N} \Rightarrow N_{eff} = rN = (0.03)(7.238\times10^{16}) = 2.17\times10^{15} \Rightarrow$ number of liberated electrons = $N_{eff} = \underline{2.17\times10^{15} e^-}$



7. **a)** For $\nu = 5.527\times10^{14}$ Hz, the kinetic energy of the liberated electron is zero $\Rightarrow$ this value is the threshold frequency of sodium.
   b) i. $E_{photon} = W_o + KE \Rightarrow h\nu = h\nu_o + KE \Rightarrow \underline{KE = h\nu - h\nu_o}$.
   ii. $(1.845\times1.6\times10^{-19}) = h(10^{15} - 5.527\times10^{14})$
   $\Rightarrow \underline{h = 6.6\times10^{-34}\text{ J.s}}$.
   c) i. h is the slope of the line.
   ii. Figure

8. **a)** $E_{photon} = W_o + KE_e \Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda_o} + KE \Rightarrow KE = \frac{hc}{\lambda} - \frac{hc}{\lambda_o}$.
   b) When KE = 0; $\lambda = \lambda_o \Rightarrow \frac{1}{\lambda_o} = \frac{1}{\lambda} = 1.7\times10^{6}\text{ m}^{-1} \Rightarrow \lambda_o \cong 5.88\times10^{-7}$ m = $\underline{588 \text{ nm}}$.
   c) KE has the form of KE = a $(\frac{1}{\lambda})$ + b $\Rightarrow$ a = slope = $\underline{hc}$.
   slope $= \frac{(4.95\times10^{-19}) - 0}{(4.2-1.7)\times10^{6}} = 1.98\times10^{-25} \Rightarrow h = \frac{\text{slope}}{c} = \frac{1.98\times10^{-25}}{3\times10^{8}} = \underline{6.6\times10^{-34}\text{ J.s}}$.
   d) $W_o = \frac{hc}{\lambda_o} = \frac{(6.6\times10^{-34})(3\times10^{8})}{5.88\times10^{-7}} \cong 3.367\times10^{-19}$ J $\cong \underline{2.1 \text{ eV}}$.

9. a) Non- charged, has zero mass, moves in vacuum at $c = 3 \times 10^8$ m/s.

b) The minimum frequency of the incident radiation that is capable to extract electrons from the surface of the metal.

c) $\lambda_o = \frac{c}{\nu_o} = \frac{3 \times 10^8}{8.96 \times 10^{14}} \Rightarrow \lambda_{o\,(magnesium)} = 3.348 \times 10^{-7}$ m.

d) $E_{ph} = h\nu \Rightarrow \nu = \frac{2.5 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 6.06 \times 10^{14}$ Hz; 6.06×10^{14} Hz $> \nu_{o\,(cesium)} = 4.6 \times 10^{14}$ Hz

$\Rightarrow$ electrons are liberated from the surface of cesium, while 6.06×10^{14} Hz is less than the threshold frequencies of magnesium and iron so no emission of electrons from the surfaces of magnesium and iron.

e) i. $E_{photon} = W_o + \frac{1}{2} m V_{max}{}^2 = h\nu_o + \frac{1}{2} m V_{max}{}^2 \Rightarrow 2.5 \times 1.6 \times 10^{-19} = (6.6 \times 10^{-34})(5.1 \times 10^{14}) + \frac{1}{2}(9.1 \times 10^{-31}) V_{max}{}^2 \Rightarrow V_{max} \cong 373283.61$ m/s $\cong 0.37328 \times 10^6$ m/s .

ii. $E_{received} = N\,E_{ph} = P \times t \Rightarrow P = \frac{N\,E_{ph}}{t}$ but $\frac{N}{t} = 25 \times 10^{15}$ photon/s $\Rightarrow P = (25 \times 10^{15})(2.5 \times 1.6 \times 10^{-19})$ $\Rightarrow P = 0.01$ W.

iii. $r = \frac{N_{eff}}{N_{total}} \Rightarrow N_{eff} = (0.02)(25 \times 10^{15}) = 5 \times 10^{14}$ electrons/s $\Rightarrow I = (5 \times 10^{14})(1.6 \times 10^{-19}) \Rightarrow \underline{I = 8 \times 10^{-5}\text{ A}}$.

f) The corpuscular aspect of light.

10. a) i. $E_{ph} = W_o + \frac{1}{2} m V_{max}{}^2 = (3.5 \times 1.6 \times 10^{-19}) + \frac{1}{2}(9.1 \times 10^{-31})(506659)^2 = \underline{6.768 \times 10^{-19}\text{ J}}$.

$E_{ph} = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_{ph}} = \frac{1.98 \times 10^{-25}}{6.768 \times 10^{-19}} \cong 2.93 \times 10^{-7}\text{m} = \underline{293\text{ nm}}$.

ii. λ_o: maximum wavelength of the radiation capable to liberate electrons from the surface of the metal. $\lambda_o > \lambda$.

iii. $W_o = \frac{hc}{\lambda_o} \Rightarrow \lambda_o = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{3.5 \times 1.6 \times 10^{-19}} \cong 3.54 \times 10^{-7}\text{m} = \underline{354\text{ nm}}$.

iv) $4 \times 10^{-7}\text{m} > 3.54 \times 10^{-7}\text{m} = \lambda_o \Rightarrow$ No liberation of electrons $\Rightarrow N_e \underline{= 0}$.

v) The corpuscular aspect of light.

b) i. $P = \frac{E_{total}}{t} = \frac{N_{total}\ E_{ph}}{t} \Rightarrow N_{total} = \frac{P.t}{E_{ph}} = \frac{(10)(12)}{6.768 \times 10^{-19}} = \underline{1.77 \times 10^{20}\text{ photons}}$.

ii. $r = \frac{N_{eff}}{N_{total}} \Rightarrow N_{eff} = N_{liberated} = (0.02)(1.77 \times 10^{20}) = \underline{3.54 \times 10^{18}\text{ electrons}}$.

c) i. $P = \frac{N_{total}\ E_{ph}}{t} = \frac{N_{total}\ hc}{\lambda\, t} = \frac{N_{lib}\ hc}{\lambda\, t\, r}$.

ii. P = constant $\Rightarrow$ As λ or /and t increases (such that $\lambda < \lambda_o$), N_{lib} $\underline{\text{increases}}$.

11. a) i. $W_o = h\nu_o = \frac{hc}{\lambda_o} \Rightarrow \lambda_o = \frac{hc}{W_o} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{3.68 \times 10^{-19}} = 5.38 \times 10^{-7}\text{m} = \underline{538\text{ nm}}$.

To liberate electrons from lithium : $\lambda \le 540$ nm but $\lambda_1 = 800$ nm $> \lambda_o \Rightarrow$ no liberation of electrons.

ii. $E_{ph\,2} = \frac{hc}{\lambda_2} = \frac{1.98 \times 10^{-25}}{0.5 \times 10^{-6}} = \underline{3.96 \times 10^{-19}\text{ J}}$.

$E_{ph\,2} = W_o + KE_2 \Rightarrow 3.96 \times 10^{-19} = 3.68 \times 10^{-19} + KE_2 \Rightarrow KE_2 = \underline{2.8 \times 10^{-20}\text{ J}}$.

$E_{ph\,3} = \frac{hc}{\lambda_3} = \frac{1.98 \times 10^{-25}}{0.4 \times 10^{-6}} = \underline{4.95 \times 10^{-19}\text{ J}} \Rightarrow KE_3 = (4.95 \times 10^{-19}) - (3.68 \times 10^{-19}) = \underline{1.27 \times 10^{-19}\text{ J}}$.

iii. $KE_3 > KE_2$

iv. As λ decreases, energy of liberated electrons increases.

v) Since $P_2 < P_3$ and $KE_2 > KE_3$.

b) i. $P_r = (\frac{2 \times 10^{-3}}{100})(20) \Rightarrow P_r = \underline{4 \times 10^{-4}\text{ W}}$ But $P_r = \frac{E_{total}}{t} = \frac{N_{incident}\ E_{ph\,2}}{t}$

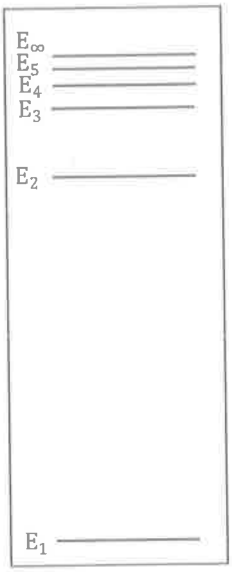
$\Rightarrow N_{incident} = \frac{P_r\,t}{E_{ph\,2}} = \frac{4 \times 10^{-4}\,(1)}{3.96 \times 10^{-19}} \Rightarrow N_{incident} \cong 1.0101 \times 10^{15}$ photon per second.

ii. $r = \frac{\text{number of emitted electrons}}{\text{number of received photons}} \Rightarrow$ number of emitted electrons $= (0.02)(1.0101 \times 10^{15}) = \underline{2.02 \times 10^{13}}$

iii. - $N_2 > N_1$

- As the power of the radiation increases, the number of the liberated electrons $\underline{\text{increases}}$.

Chapter 17: The Atom



1.

a)

i. $E_1 = \frac{-13.6}{(1)^2} = -13.6$ eV. **ii.** $E_2 = \frac{-13.6}{(2)^2} = -3.4$ eV. **iii.** $n \longrightarrow \infty \Rightarrow E_\infty = \frac{-13.6}{\infty} = 0$.

b) Figure (the energy levels are not equally spaced).

c) $W_{min} = E_\infty - E_1 = 0 - (-13.6) = 13.6$ eV. **d)** $n = 3 \longrightarrow n = 2 \longrightarrow n = 1$ and $n = 3 \longrightarrow n = 1$.

e)

i. $E_{ph} = E_3 - E_2 = \frac{-13.6}{9} - (-3.4) = 1.89$ eV . $E_{ph} = h\nu \Rightarrow \nu = \frac{1.89 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = \underline{4.58 \times 10^{14}\text{ Hz}}$.

ii. $\frac{1}{\lambda} = R[\frac{1}{2^2} - \frac{1}{3^2}] = (1.097 \times 10^7)[0.25 - \frac{1}{9}] = 656 \times 10^{-7}$ m $\Rightarrow \nu = \frac{c}{\lambda} \cong \underline{4.58 \times 10^{14}\text{ Hz}}$.

f)

i. $E_{ph} = \frac{hc}{\lambda} \Rightarrow E_{ph}$ and λ are inversely proportional $\Rightarrow$ shortest wavelength correspond to greatest $E_{ph} \Rightarrow E_{ph} = E_\infty - E_1$; $x = \rightarrow \infty$ and $\underline{y = 1}$.

ii. $E_{ph} = E_\infty - E_1 = 13.6$ eV $= \frac{hc}{\lambda} \Rightarrow 13.6 \times 1.6 \times 10^{-19} = \frac{(6.6 \times 10^{34})(3 \times 10^8)}{\lambda}$

$\Rightarrow \lambda = 9.099 \times 10^{-8}$ m $= \underline{91\text{ nm}}$.

iii. Lyman series.

g)

i. $E_{ph} = E_h - E_\ell \Rightarrow E_h = E_{ph} + E_\ell = 10.2 + (-13.6) = -3.4$ eV.
$E_h = -3.4$ eV $= E_2 \Rightarrow$ This photon is <u>absorbed</u> $\Rightarrow$ the atom becomes in the <u>1st excited state</u>.

ii. $E_h = E_{ph} + E_\ell = 11 + (-13.6) = -2.6 = \frac{-13.6}{n^2} \Rightarrow n = 2.29$.
$n \notin N^* \Rightarrow$ the photon is <u>not absorbed</u> $\Rightarrow$ the atom <u>remains in the ground state</u>.

2.

a) For hydrogen $E_1 = -13.6$ eV, $E_2 = -3.4$ eV. The energy levels of this atom are different from those of hydrogen.

b) Third excited state: $n = 4$. Then: $n = 4 \longrightarrow n = 3$, $n = 4 \longrightarrow n = 3$, $n = 4 \longrightarrow n = 1$, $n = 3 \longrightarrow n = 2$, $n = 3 \longrightarrow n = 1$ and $n = 2 \longrightarrow n = 1 \Rightarrow$ <u>6 probable downward transitions</u>.

c) First excited state: $E_2 = -30.6$ eV. $E_{ph} = E_h - E_\ell \Rightarrow E_h = E_{ph} + E_\ell = 9 + (-30.6) = -21.6$ eV.
But -30.6 eV < -21.6 eV < -13.6 eV $\Rightarrow$ the atom does not have an energy level of value -21.6 eV $\Rightarrow$ <u>not absorbed</u>.

d) $E_{ph} = W_{Ion} + KE \Rightarrow KE = E_{ph} - W_{Ion} = [140 - 122.4] \times 1.6 \times 10^{-19} = \underline{2.816 \times 10^{-18}\text{ J}}$.

e) $E_{ph} = E_h - E_\ell = E_5 - E_3 \Rightarrow h\nu = E_5 - E_3 \Rightarrow 6.6 \times 10^{-34}\nu = [-4.9 - (-13.6)][1.6 \times 10^{-19}] \Rightarrow \underline{\nu = 2.109 \times 10^{15}\text{ Hz}}$.

3.

a) (1) and (4).

b)

i. Lyman series.

ii. $E_{ph} = E_5 - E_1 = \frac{hc}{\lambda} \Rightarrow [\frac{-13.6}{25} - \frac{(-13.6)}{1}] \times 1.6 \times 10^{-19} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{\lambda} \Rightarrow \underline{\lambda = 94.78\text{ nm}}$.

iii. <u>Invisible</u> since it is not included in the range [400nm $\longrightarrow$ 800nm].

c) Paschen series. $E_{ph} = E_7 - E_3 = h\nu \Rightarrow 6.6 \times 10^{-34}\nu = [\frac{-13.6}{49} - (\frac{-13.6}{9})]1.6 \times 10^{-19} \Rightarrow \underline{\nu = 2.976 \times 10^{14}\text{ Hz}}$.
this photon is <u>invisible</u>.

d)

i. $E_{ph} = E_h - E_{Ion}$. E_{ph} is minimum $\Rightarrow E_h$ is minimum $\Rightarrow E_{h(min)} = E_2 \Rightarrow E_{ph(min)} = E_2 - E_1 = \frac{-13.6}{4} - (-13.6) = \underline{10.2\text{ eV}}$.

ii. $E_{ph} = W_I + KE$. E_{ph} is minimum if $KE = 0 \Rightarrow E_{ph(min)} = W_{Ion} = 0 - (-13.6) = \underline{13.6\text{ eV}}$.

4.

a) From the figure; the energy has specific values $\Longrightarrow$ the energy of the mercury atom it is quantized.

b) $E_{ph} = E_h - E_\ell = E_h - E_4 \Rightarrow E_h = 3 + (-4.95) = -1.95$ eV. But -2.48 eV < -1.95 eV < -1.57 eV
$\Rightarrow$ the mercury atom does not have this energy level of -1.95 eV $\Rightarrow$ photon is not absorbed.

c) $E_{ph} = E_h - E_\ell = E_5 - E_3 = -3.71 - (-5.52) = 1.81$ eV. But $E_{ph} = h\nu \Rightarrow \nu = \frac{1.81 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = \underline{4.378 \times 10^{14}\text{ Hz}}$.

d)

i. $E_{ph} = W_{Ion} + KE \Rightarrow kE = E_{ph} - W_{Ion} = 13 - 10.38 = \underline{2.62\text{ eV}}$.

ii. $KE_e \geq E_h - E_\ell \Rightarrow E_h \leq KE_e + E_\ell \Rightarrow E_h \leq 5.5 + (-10.38) = -4.88$ eV $\Rightarrow E_h < E_5$.
$\Rightarrow$ the atom might be excited to E_2, to E_3 and to E_4.

5.

A.

a) No. Quantized energy means that this energy has discontinuous (discrete) values (it can not take any value)

b) $E_{ph} = E_3 - E_1 = \frac{hc}{\lambda} \Rightarrow E_1 = E_3 - \frac{hc}{\lambda} = -3.75 - \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{(1.855 \times 10^{-7})(1.6 \times 10^{-19})} = -10.42$ eV.

c) $E_{ph} = E_4 - E_2 = -2.72 - (-4.99) = \underline{2.27\ eV} = 2.27 \times 1.6 \times 10^{-19} J = \underline{3.632 \times 10^{-19} J}$.

d) $\lambda_1 = 0.1855\ \mu m < 0.4\ \mu m$: invisible. $E_{Ph_{4\to2}} = \frac{h\,c}{\lambda_2} \Rightarrow \lambda_2 = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{3.632 \times 10^{-19}} = 5.45 \times 10^{-7}\ m = 0.545\ \mu m$

$0.4\ \mu m < 0.545\ \mu m < 0.8\ \mu m \Rightarrow$ visible.

B.

a) $E_{ph} = E_h - E_\ell = E_4 - E_x \Rightarrow 2.27 = -2.72 - E_x \Rightarrow E_x = \underline{-4.99\ eV} = E_2$.

b) $E_{ph} = W_{Ion} + KE_e = \frac{h\,c}{\lambda} \Rightarrow \lambda_{max}$ corresponds to $kE_e = 0 \Rightarrow \lambda_{max} = \frac{h\,c}{W_{Ion}} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{0-(-4.99 \times 1.6 \times 10^{-19})} = 2.48 \times 10^{-7}\ m = \underline{248\ nm}$.

C.

a) Each bright line appears in the emission spectrum correspond to a wavelength of a certain photon emitted by the atom when de-excited from a higher to a lower energy level.
Similarly each dark line in the absorption spectrum corresponds to the wavelength of a photon absorbed by the atom when excited from a lower to a higher energy level.
But energy of the atom is quantized $\Rightarrow$ the spectrums are line spectra.

b) The corpuscular aspect of light since every line in the spectrum is due to a certain transition of the atom and corresponds to a photon (particle) (emitted or absorbed).

6.

a) Doublet-line

b)

i. $E_{ph} = \frac{h\,c}{\lambda_1} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{(589 \times 10^{-9})(1.6 \times 10^{-19})} = \underline{2.109\ eV}$.

ii. $E_2 - E_1 = -3.03 - (-5.14) = 2.11\ eV \Rightarrow E_{ph} \cong E_2 - E_1$.

iii. $E_{ph} = \frac{h\,c}{\lambda_2} = E_n - E_1 \Rightarrow E_n = E_1 + \frac{h\,c}{\lambda_2} = -5.14 + \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{(589.6 \times 10^{-9})(1.6 \times 10^{-19})} \Rightarrow E_n = \underline{-3.032\ eV} = E_2$.

iv. The 1st excited state is constituted of two energy levels very close to each other (-3.03 and – 3.032) $\Rightarrow$ it is double.

c) Energy given by the electron (received by the atom) is $E = E_3 - E_1 = -1.94 - (-5.14) = 3.2\ eV$.
$KE_1 = KE_2 + E = 1 + 3.2 = \underline{4.2\ eV}$. But $KE_1 = \frac{1}{2} m V^2 \Rightarrow (4.2 \times 1.6 \times 10^{-19}) = \frac{1}{2}(9.1 \times 10^{-31}) V^2$
$\Rightarrow$ The electron hits the atom with a speed of $\underline{V = 1215287.24\ m/s}$.

7.

A.

a) It has specific values (discrete values).

b) **i.** n = 1 : Atom is in the ground state. **ii.** $n \to \infty$: Atom is in the ionized state (reference state).

B.

a) $E_{ph} = E_h - E_\ell = E_h - E_1$. E_{ph} is minimum if $E_h = E_2 \Rightarrow E_{ph(min)} = E_2 - E_1 = \underline{10.2\ eV}$.

b) **i.** $E_{ph} = E_h - E_\ell = E_h - E_1 \Rightarrow E_h = 11.5 + (-13.6) = -2.1 = \frac{-13.6}{n^2} \Rightarrow n = 2.54 \notin N^*$. The atom remains in the ground state.

ii. $KE_e \geq E_h - E_\ell \Rightarrow E_h \leq KE_e + E_1 = 11.5 + (-13.6) = -2.1\ eV \Rightarrow E_h < E_3 \Rightarrow$ the atom is excited to E_2.
$KE_{leave} = KE_{before} - (E_2 - E_1) = 11.5 - (-3.4 - (-13.6)) = \underline{1.3\ eV}$.

c) $E_{ph} = h\,\nu = W_{Ion} + KE = (E_\infty - E_2) + \frac{1}{2} m V^2 \Rightarrow (6.6 \times 10^{-34})\,\nu = (3.4)(1.6 \times 10^{-19}) + \frac{1}{2}(9.1 \times 10^{-31})(956780.3)^2$
$\Rightarrow \underline{\nu = 1.46 \times 10^{15}\ Hz}$.

C.

a) Brackett series : $E_n \longrightarrow E_4$ where n > 4. $E_{Ph} = \frac{h\,c}{\lambda} = E_n - E_4$

$\Rightarrow E_{ph\,max} = E_\infty - E_4 = \frac{h\,c}{\lambda_{min}} \Rightarrow \lambda_{min} = \frac{h\,c}{E_\infty - E_4} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{0-(-0.85)(1.6 \times 10^{-19})} = 1.456 \times 10^{-6}\ m = \underline{1.456\ \mu m}$.

Similarly : $\lambda_{max} = \frac{h\,c}{E_5 - E_4} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{[-0.544 + 0.85](1.6 \times 10^{-19})} = 4.044 \times 10^{-6}\ m = \underline{4.044\ \mu m}$.

b) Transition from $n = 3 \longrightarrow n = 2$.

8.

a) m and n are unit less, then the unit of R is the inverse of the unit of $\lambda \Rightarrow m^{-1}$.

b) $E_{ph} = E_n - E_m = 1.6 \times 10^{-19} [\frac{-13.6}{n^2} - (\frac{-13.6}{m^2})] = \frac{h\,c}{\lambda}$

$2.176 \times 10^{-18} [\frac{1}{m^2} - \frac{1}{n^2}] = h\,c\,[R(\frac{1}{m^2} - \frac{1}{n^2})] = \frac{h\,c\,R}{m^2} - \frac{h\,c\,R}{n^2}$

By comparison: $h\,c\,R = 2.176 \times 10^{-18} \Rightarrow (6.6 \times 10^{-34})(3 \times 10^8) R = 2.176 \times 10^{-18} \Rightarrow \underline{R = 1.096 \times 10^7\ m^{-1}}$.

9.

a) During collision linear momentum of the system two atoms is conserved:
$\vec{P}_{before} = \vec{P}_{after} \Rightarrow m\,\vec{V}_1 + m\,\vec{V}_2 = \vec{0} \Rightarrow \vec{V}_1 = -\vec{V}_2 \Rightarrow \vec{V}_1$ and $\vec{V}_2$ have opposite directions.

b) System: Two atoms. Before collision: $KE_{before} = (\frac{1}{2} m V^2) \times 2 = m V^2 \neq 0$

After collision: $KE_{after} = 0 \Rightarrow K.E_{before} \neq K.E_{after} \Rightarrow$ Inelastic collision.

c) The KE before collision is transformed into energy for the two atoms.

$KE_{before} = 2 (E_{ph}) = m V^2 \Rightarrow E_{ph} = \frac{1}{2}(1.7 \times 10^{-27})(48989.79)^2 = 2.04 \times 10^{-18}$ J

$\frac{hc}{\lambda} = 2.04 \times 10^{-18}$ J $\Rightarrow \lambda = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{2.04 \times 10^{-18}} = \underline{97.06 \times 10^{-9}\ m}$.

d) $E_{ph} = E_h - E_\ell = E_n - E_1 \Rightarrow E_n = E_{ph} + E_1 = \frac{2.04 \times 10^{-18}}{1.6 \times 10^{-19}} + (-13.6) = -0.85$ eV $\Rightarrow E_n = \frac{-13.6}{n^2} \Rightarrow n = 4 \Rightarrow k = 4 - 1 = \underline{3}$.

10.

a) The second energy level of n = 2.

b) $n = 3 \longrightarrow n = 2$, $n = 4 \longrightarrow n = 2$, $n = 5 \longrightarrow n = 2$ and $n = 6 \longrightarrow n = 2$.

c) $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2}) = R(\frac{n^2 - m^2}{m^2 n^2}) \Rightarrow \lambda = (\frac{m^2 n^2}{n^2 - m^2})\frac{1}{R}$.

For $m = 2 : \lambda = (\frac{4 n^2}{n^2 - 4})\frac{1}{R} = \frac{4}{R}(\frac{n^2}{n^2 - 4}) \Rightarrow B = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 3.646 \times 10^{-7}$ m .

d) $E_{ph} = \frac{hc}{\lambda} \Rightarrow$ Greatest energy correspond to shortest $\lambda \Rightarrow$ shortest λ corresponds to $n \longrightarrow \infty$:

$\lambda_{min} = B \lim_{n \to \infty}(\frac{n^2}{n^2 - 4}) = B \lim_{n \to \infty}(\frac{n^2}{n^2(1 - \frac{4}{n^2})}) = B(1) = 3.646 \times 10^{-7}$ m $= \underline{364.6\ nm}$.

longest λ corresponds to $n = 3 : \lambda_{max} = 3.646 \times 10^{-7}(\frac{9}{9-4}) = 6.5628 \times 10^{-7}$ m $= \underline{656.28\ nm}$.

11.

A. $E_3 - E_2 = \frac{-2.176 \times 10^{-18}}{9} - \frac{-2.176 \times 10^{-18}}{4} = \underline{3.02 \times 10^{-19}\ J}$. Similarly $E_2 - E_1 = \underline{1.632 \times 10^{-18}\ J}$.

$E_3 - E_2 \neq E_2 - E_1 \Rightarrow$ the energy levels are not equally spaced.

B.

a) $E_n = \frac{-2.176 \times 10^{-18}}{n^2 (1.6 \times 10^{-19})} = \frac{-13.6}{n^2}$ (eV).

b) i. electron α: $KE_\alpha = E_h - E_\ell \Rightarrow E_h = E_\alpha + E_1 = 3 + (-13.6) = -10.6$ eV

$-10.6 = \frac{-13.6}{n^2} \Rightarrow n = 1.132 < 2 \Rightarrow$ this electron is not capable to excite the atom then it does not interact with it

$\Rightarrow$ The <u>atom remains in the ground state</u>

ii. electron β : $E_h = 12.5 + (-13.6) = -1.1$ eV $\Rightarrow$ $-1.1 = \frac{-13.6}{n^2} \Rightarrow n = 3.51$. $3 < n < 4 \Rightarrow$ the atom <u>might be excited to E_2</u> or to $E_3 \Rightarrow$ this electron interact with the atom.

c) $KE_{leave} = KE_\beta - (E_h - E_\ell)$. From n = 1 to n = 2 $\Rightarrow KE_{leave} = 12.5 - [-3.4 - (-13.6)] = \underline{2.3\ eV}$.

From n = 1 to n = 3: From n = 1 to n = 2 $\Rightarrow KE_{leave} = KE_\beta - (E_3 - E_1) = 12.5 - [-1.51 - (-13.6)] = \underline{0.41\ eV}$.

C.

a) $E_{ph} = E_n - E_m \Rightarrow \frac{hc}{\lambda} = \frac{-2.176 \times 10^{-18}}{n^2} - \frac{-2.176 \times 10^{-18}}{m^2} = 2.176 \times 10^{-18}(\frac{1}{m^2} - \frac{1}{n^2}) \Rightarrow \frac{1}{\lambda} = \frac{2.176 \times 10^{-18}}{hc}(\frac{1}{m^2} - \frac{1}{n^2})$

By comparison: $R = \frac{2.176 \times 10^{-18}}{hc} \Rightarrow h = \frac{2.176 \times 10^{-18}}{1.097 \times 10^7 \times 3 \times 10^8} = 6.6 \times 10^{-34}$ J.s.

b) i. from $n = 4 \longrightarrow n = 1$; $n = 4 \longrightarrow n = 2$; $n = 4 \longrightarrow n = 3$; $n = 3 \longrightarrow n = 1$; $n = 3 \longrightarrow n = 2$ and $n = 2 \longrightarrow n = 1$.

ii. Lyman series: $n = 4 \longrightarrow n = 1$; $n = 3 \longrightarrow n = 1$ and $n = 2 \longrightarrow n = 1$. Balmer series: $n = 4 \longrightarrow n = 2$ and $n = 3 \longrightarrow n = 2$.

Paschen series: $n = 4 \longrightarrow n = 3$.

c) $\lambda = \frac{c}{\nu} \Rightarrow \frac{1}{\lambda} = \frac{\nu}{c} \Rightarrow R(\frac{1}{1^2} - \frac{1}{n^2}) = \frac{\nu}{c} \Rightarrow \frac{\nu}{cR} = 1 - \frac{1}{n^2} \Rightarrow \frac{1}{n^2} = 1 - \frac{\nu}{cR} = 1 - \frac{2.9253 \times 10^{15}}{3 \times 10^8 \times 1.097 \times 10^7} = 0.1112 \Rightarrow \underline{n = 3}$.

12.

a) System: (electron, photon, atom)

Before capturing the electron $E_{before} = KE_e + E_{atom}$

After capturing the electron $E_{after} = \frac{hc}{\lambda} + E_n = \frac{hc}{\lambda} - \frac{13.6(1.6 \times 10^{-19})}{n^2}$

$E_{before} = E_{after} \Rightarrow KE_e + 0 = \frac{hc}{\lambda} - \frac{2.176 \times 10^{-18}}{n^2} \Rightarrow \frac{hc}{\lambda} = kE_e + \frac{2.176 \times 10^{-18}}{n^2}$

b)

i. λ_{min} corresponds to $KE_{max} = 2.256$ eV $\Rightarrow \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{\lambda_{min}} = (2.256)(1.6 \times 10^{-19}) + \frac{2.176 \times 10^{-18}}{16} \Rightarrow$

$\lambda_{min} = \underline{3.98 \times 10^{-7}\ m}$.

λ_{max} corresponds to $KE_{min} = 1.553$ eV substitute in the above equation $\Rightarrow \lambda_{max} = \underline{5.15 \times 10^{-7}\ m}$.

ii. For the downward transition to n = 4, the wavelength λ takes any value within the range [398 nm $\leq \lambda \leq$ 515 nm] $\Rightarrow$ continuous spectrum.

Chapter 18: Atomic Nucleus

1.

a) ${}^{62}_{28}Ni$ Since it has the greatest binding energy per nucleon.

b) ${}^{9}_{4}Be$ and ${}^{238}_{92}U$

2.

a) The figure shows that only specific values of energy are allowed ⇒ energy of the nucleus is quantized.

b) Transition (1) : $E_{ph1} = E_3 - E_2 = 1.02 - 0.83 = 0.19$ MeV

Transition (2) : $E_{ph2} = E_2 - E_1 = 0.83 - 0 = 0.83$ MeV

c) $E_{ph3} = 1.02 - 0 = 1.02$ MeV, but $E_{ph1} + E_{ph2} = 0.19 + 0.83 = 1.02$ MeV

$\Rightarrow E_{ph3} = E_{ph1} + E_{ph2}$.

3.

a) $A = N + P = 7 + 7 = 14$. Symbol : ${}^{14}_{7}N$

b) $r = r_o A^{\frac{1}{3}} = (1.2 \times 10^{-15})(14)^{\frac{1}{3}} = 2.89 \times 10^{-15}$ m.

c) $\frac{r_{atom}}{r_{radius}} = \frac{0.7 \times 10^{-10}}{2.89 \times 10^{-15}} = 24221.45$.

d)

i. $m = A\, m_{nucleon} = 14\,(1.66 \times 10^{-27}) = \underline{2.324 \times 10^{-26}\ kg}$

ii. $\rho = \frac{m}{V} = \frac{2.324 \times 10^{-26}}{\frac{4}{3}\pi (2.89 \times 10^{-15})^3} \approx \underline{2.3 \times 10^{17}\ kg/m^3}$.

iii. $V' = \frac{m}{\rho_{mercury}} = \frac{2.324 \times 10^{-26}}{13600} = \underline{1.71 \times 10^{-30}\ m^3}$.

4.

a) Since they have the same charge number Z, but different mass number

b) Binding energy : minimum energy needed to break a nucleus into its individual nucleons.

c) ${}^{56}_{27}$**Co** : $\Delta m = [27\, m_p + 29\, m_n] - m_x = [27\,(1.00727) + 29\,(1.00866)] - 55.925018$

$\Delta m = 0.522682\ (u) = 0.522682 \times 931.5\ \frac{MeV}{c^2} = 486.878\ \frac{MeV}{c^2}$

$E_B = \Delta m.c^2 = 486.878\ \frac{MeV}{c^2} c^2 = 486.878\ MeV \Rightarrow \frac{E_B}{A} = \frac{486.878}{56} = \underline{8.69\ MeV}$.

${}^{57}_{27}$**Co** : $\Delta m = [27\, m_p + 30\, m_n] - m_x = [27\,(1.00727) + 30\,(1.00866)] - 56.921479$

$\Delta m = 0.534881\ (u) = 498.242\ \frac{MeV}{c^2}$

$\Rightarrow E_B = 498.242\ MeV \Rightarrow \frac{E_B}{A} = \underline{8.74\ MeV}$.

${}^{59}_{27}$**Co** : $\Delta m = 0.555297\ (u) = 517.259\ \frac{MeV}{c^2} \Rightarrow E_B = 517.259\ MeV \Rightarrow \frac{E_B}{A} = 8.767\ MeV$

${}^{60}_{27}$**Co** : $\Delta m = 0.563335\ (u) = 524.7466\ \frac{MeV}{c^2} \Rightarrow \frac{E_B}{A} = \underline{8.747\ MeV}$.

d) ${}^{59}_{27}Co$ is the most stable nucleus since it has the greatest binding energy per nucleon.

5.

a) For ${}^{4}_{2}He$: $A < 20$; For ${}^{238}_{92}U$: $A > 190$

While for ${}^{56}_{26}Fe$: $20 < A < 190$ ⇒ It is the most stable nucleus.

b) ${}^{4}_{2}$**He** : $E_B = 28.2945\ MeV \Rightarrow \frac{E_B}{A} = \underline{7.074\ MeV}$.

$E_B = \Delta m.c^2 \Rightarrow \Delta m = 28.2945\ \frac{MeV}{c^2} = \underline{0.030375\ u}$.

$\Delta m = m_{nucleons} - m_{nucleus} \Rightarrow m_{nucleus} = 4.03188 - 0.030375 = \underline{4.001505\ u}$.

${}^{56}_{Z}$**Fe** : $E_B = 8.789434 \times 56 = 492.208304$ MeV

$E_B = \Delta m.c^2 \Rightarrow \Delta m = 492.208304\ \frac{MeV}{c^2} = \underline{0.528404\ u}$

$\Delta m = m_{nucleons} - m_{nucleus} \Rightarrow m_{nucleons} = 0.528404 + 55.920676 = \underline{56.44908\ u}$.

${}^{238}_{92}$**U** : $\Delta m = 239.93412 - 238.000312 = \underline{1.933808\ u}$.

$E_B = \Delta m.c^2 = 1.933808 \times 931.5 = \underline{1801.342\ MeV} \Rightarrow \frac{E_B}{A} = \underline{7.5687\ MeV}$.

6.

a) $\Delta m = (Z\,m_p + N\,m_n) - m\,^{14}_{6}C \Rightarrow 0.1130096 = [(6 \times 1.00728) + (8 \times 1.00866)] - m\,^{14}_{6}C$

$\Rightarrow m\,^{14}_{6}C = \underline{13.9999504\text{ u}}.$

b) $N = \frac{m}{M} N_A \Rightarrow m = \frac{14 \times 10^{15}}{6.022 \times 10^{23}} = \underline{2.32 \times 10^{-8}\text{g}}.$

c)

i. The binding energy per nucleon is the minimum average energy needed to extract one nucleon from the nucleus.

ii. $E_B = \Delta m.c^2 = 0.1130096 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 1.688 \times 10^{-11}$ J.

$\Rightarrow \frac{E_B}{A} = \frac{E_B}{14} = \underline{1.206 \times 10^{-12}\text{J}}. \qquad \frac{E_B}{A} = \frac{1.206 \times 10^{-12}}{1.6 \times 10^{-13}} = \underline{7.537\text{ MeV}}.$

iii. $A < 20 \Rightarrow \frac{E_B}{A} < 8$ MeV agrees with Aston curve.

7.

A.

a) 92 proton; 235 – 92 = 143 neutrons

b) $r = r_o A^{\frac{1}{3}} = (1.2 \times 10^{-15})(235)^{\frac{1}{3}} \cong \underline{7.4052 \times 10^{-15}\text{m}}.$

c) $\Delta m = [143\,m_n + 92\,m_p] - m\,^{235}_{92}U = [143(1.00866) + 92(1.00727)] - 234.9935 = \underline{1.91464\text{ u}}.$

d) $\Delta m = 1.91464 \times 931.5 = 1783.49 \frac{\text{MeV}}{c^2}. \quad E_B = \Delta m.c^2 = 1783.49\text{ MeV} \Rightarrow \frac{E_B}{A} = \frac{1783.49}{235} = \underline{7.589\text{ MeV}}.$

B.

a) Binding energy: $E_B = 7.67957 \times 12 = 92.15484$ MeV.

$E_B = \Delta m.c^2 \Rightarrow \Delta m = \frac{E_B}{c^2} = 92.15484 \frac{\text{MeV}}{c^2} = \frac{92.15484}{931.5}\text{ u} = 0.098932\text{ u}.$

$\Delta m = (6\,m_P + 6\,m_P) - m_{precise} = [6(1.00727) + 6(1.00866)] - m_{precise} \Rightarrow m_{precise} = 12.09558 - 0.098932 = \underline{11.996648\text{ u}}.$

$m_{precise} = 11.996648 \times 1.666 \times 10^{-27} = \underline{1.99 \times 10^{-26}\text{ kg}}.$

b) $m_{approximate} = A\text{ (u)} = 12$ u.

c) $\frac{E_B}{A} = 7.67957$ MeV.

d) $^{12}_{6}C$ is more stable since it has a greater binding energy per nucleon.

Chapter 19: Radioactivity

1.

a) Reaction (3) is provoked.

b) ${}^{87}_{37}Rb \longrightarrow {}^{87}_{38}Sr + {}^{\ 0}_{-1}e + {}^{0}_{0}\bar{\nu}$. Type : β^- decay

c) ${}^{11}_{6}C \longrightarrow {}^{11}_{5}B + {}^{\ 0}_{+1}e + {}^{0}_{0}\nu$.

$Z({}^{11}_{6}C) > Z({}^{11}_{5}B) \Rightarrow$ A proton decays into a neutron and a positron and a neutrino $\Rightarrow {}^{1}_{1}H \longrightarrow {}^{1}_{0}n + {}^{0}_{1}e + {}^{0}_{0}\nu$

2.

a) B^+ decay.

b) M is a neutrino. Neutrino is uncharged and does not interact with matter.

c) ${}^{11}_{6}C \longrightarrow {}^{11}_{x}B + {}^{\ 0}_{+1}e + {}^{0}_{0}\nu$. Law of conservation of charge number: $6 = x + 1 + 0 \Rightarrow \underline{x = 5}$.

d) $A = \frac{A_0}{2^n} \Rightarrow 6.25 \times 10^{-3} = \frac{0.8}{2^n} \Rightarrow 2^n = 128 \Rightarrow n \ln 2 = \ell n\, 128 \Rightarrow n = 7$ but $t = n\,T \Rightarrow T = \frac{142.1}{7} = \underline{20.3 \text{ min}}$.

3.

a)

i. ${}^{218}_{84}X \longrightarrow {}^{214}_{82}M + {}^{4}_{2}He$. ${}^{210}_{83}N \longrightarrow {}^{210}_{84}Y + {}^{\ 0}_{-1}e + {}^{0}_{0}\bar{\nu}$.

ii. Two α decays and four β decay.

b) ${}^{218}_{84}X \longrightarrow {}^{210}_{84}Y + K\,{}^{4}_{2}He + n\,{}^{\ 0}_{-1}e + n\,{}^{0}_{0}\nu^-$. Soddy's laws: $218 = 210 + K(4) + n(0) \Rightarrow \underline{K = 2}$

$84 = 84 + 2(2) + n(-1) + n(0) \Rightarrow \underline{n = 4}$.

c) ${}^{218}_{84}X \longrightarrow {}^{210}_{84}Y + 2\,{}^{4}_{2}He + 4\,{}^{\ 0}_{-1}e + 4\,{}^{0}_{0}\bar{\nu}$.

4. $A = A_o\, e^{-\lambda t} \Rightarrow 250 = 474\, e^{-\lambda t} \Rightarrow e^{-\lambda t} = \frac{125}{237} \Rightarrow -\lambda t = -0.6397 \Rightarrow 0.6397 = \frac{\ell n\, 2}{5730}\, t \Rightarrow \underline{t = 5288.55 \text{ years}}$.

5.

a) $\left.\begin{array}{l} 238 \text{ g} \longrightarrow 6.022 \times 10^{23} \text{ nuclei} \\ 8 \text{ g} \longrightarrow N = ? \end{array}\right\} \Rightarrow N = \underline{2.0242 \times 10^{22} \text{ nuclei}}$.

b) $A = \lambda N \Rightarrow \lambda = \frac{A}{N} = \frac{3.558 \times 10^{8} \text{ decay /h}}{2.0242 \times 10^{22}} = 1.7577 \times 10^{-14} \text{ h}^{-1} = (1.7577 \times 10^{-14})(365)(24) = 1.54 \times 10^{-10} \text{ y}^{-1}$.

$T = \frac{\ell n\, 2}{\lambda} = \underline{4.5 \times 10^9 \text{ years}}$.

c) $m = \frac{m_0}{2^n} \Rightarrow 8 = \frac{m_0}{2^n}$. $n = \frac{t}{T} = \frac{11.25 \times 10^9}{4.5 \times 10^9} = 2.5 \Rightarrow m_o = 8\,(2^{2.5}) = \underline{45.255 \text{ g}}$.

$A = \frac{A_0}{2^n} \Rightarrow A_o = \frac{3.558 \times 10^8}{3600} \times 2^{2.5} = \underline{559085.8 \text{ Bq}}$.

6.

a) ${}^{27}_{12}Mg \longrightarrow {}^{27}_{13}A\ell + {}^{A}_{Z}X$

law of conservation of mass number : $27 = 27 + A \Rightarrow A = 0$

law of conservation of charge number: $12 = 13 + Z \Rightarrow Z = -1 \Rightarrow {}^{A}_{Z}X$ is an electron. ${}^{27}_{12}Mg \longrightarrow {}^{27}_{13}Al + {}^{\ 0}_{-1}e + {}^{0}_{0}\nu^-$

b) β^- decay

c) $\Delta m = m_{before} - m_{after} = [\,26.977758 - (26.974406 + 5.486 \times 10^{-4})] = 2.8034 \times 10^{-3}$ u

$\Delta m = 2.8034 \times 10^{-3} \times 931.5 \frac{MeV}{c^2} = 2.611 \frac{MeV}{c^2} \Rightarrow E = \Delta m.c^2 = 2.611 \frac{MeV}{c^2} c^2 = \underline{2.611 \text{ MeV}}$.

d)

i. $E = E_\gamma + KE_{e^-} \Rightarrow 2.611 = [\,1.02 - 0\,] + KE_{e^-} \Rightarrow KE_{e^-} = \underline{1.591 \text{ MeV}}$.

ii. $2.611 = [\,0.83 - 0\,] + KE_{e^-} \Rightarrow KE_{e^-} = \underline{1.781 \text{ MeV}}$.

7.

a)

i. β^- decay.

ii. ${}^{A}_{Z}Th \longrightarrow {}^{234}_{91}Pa + {}^{\ 0}_{-1}e + {}^{0}_{0}\bar{\nu} + \gamma$

Soddy's laws : $A = 234 + 0 + 0 = 234$. $Z = 91 - 1 + 0 = 90$.

b) γ-ray is emitted due to the de excitation of the daughter nucleus. ${}^{234}_{91}Pa^* \longrightarrow {}^{234}_{91}Pa + \gamma$

c) In radioactivity, an unstable nucleus decays to a more stable nucleus $\Rightarrow$ the daughter nucleus (protactinium) is more stable than the parent nucleus.

d)

i. $\lambda = \frac{\ell n\,2}{T} = \frac{\ell n\,2}{(24.1)(24)(3600)} = \underline{3.3289 \times 10^{-7} s^{-1}}$. $N_o = \frac{80 \times 10^{-3}}{(233.994219)(1.66 \times 10^{-27})} = 2.0596 \times 10^{23}$ nuclei.

$A_o = \lambda N_o = (3.3289 \times 10^{-27})(2.0596 \times 10^{23}) = \underline{6.8561 \times 10^{16}\ Bq}$.

ii. $A = \frac{A_o}{2^n} = \frac{6.8561 \times 10^{16}}{2^{(\frac{64}{24.1})}} = 1.0881 \times 10^{16}\,Bq$. $A = \lambda N \Rightarrow N = \frac{1.0881 \times 10^{16}}{3.3289 \times 10^{-7}} = 3.2686 \times 10^{22}$ nuclei

$\Rightarrow N_{decay} = N_o - N = 2.0596 \times 10^{23} - 3.2686 \times 10^{22} = \underline{1.733 \times 10^{23}\ nuclei}$.

iii. $A_{av} = \frac{N_{decay}}{t} = \frac{1.733 \times 10^{23}}{64 \times 24 \times 3600} = \underline{3.134 \times 10^{16}\ Bq}$.

iv. $P_{av} = \frac{\Delta W}{\Delta t} = \frac{N_{decay} \times E_{liberated}}{\Delta t} = A_{av} \times E_{liberated} = (3.134 \times 10^{16})(270 \times 10^3 \times 1.6 \times 10^{-19}) = \underline{1353.89\ W}$.

8.

a) $N = N_o\, e^{-\lambda t}$. $A = -\frac{dN}{dt} \Rightarrow (\frac{dN}{dt})_{t_0=0} = -A_o = -x \Rightarrow x = A_o \Rightarrow$ This quantity (x) is the initial activity of the sample

b) Slope $= \frac{(0-10^{20})}{25-0} = -4 \times 10^{18}$ Bq. $A_o = \lambda N_o \Rightarrow \lambda = \frac{4 \times 10^{18}}{10^{20}} = \underline{0.04\ s^{-1}}$. $T = \frac{\ell n\,2}{\lambda} = \underline{17.33\ s}$.

c) For M: $N = \frac{N_0}{2} \Rightarrow t = 1T = \underline{17.33\ s}$.

d) $N_{positrons} = N_{decays} = N_o - N_{remaining} = 10^{20} - (0.6 \times 10^{20}) = \underline{0.4 \times 10^{20}\ positrons}$.

9.

a) At t = 1 T; $N_{remaining} = \frac{N_o}{2} \Rightarrow N_{decays} = N_o - \frac{N_o}{2} = \frac{N_o}{2} = N_{remaining} \Rightarrow$ P: $(1, \frac{N_o}{2}) \Rightarrow$ P: $(1,\ 0.5 \times 10^{12})$

b) $\lambda = \frac{\ell n\,2}{T} = \frac{\ell n\,2}{30 \times 365 \times 24 \times 3600} = 7.3265 \times 10^{-10} s^{-1}$. $N = \frac{A}{\lambda} = \frac{183.1625}{7.3265 \times 10^{-10}} = 0.25 \times 10^{12}$ nuclei.

Graphically for $N = 0.25 \times 10^{12}$, $\underline{n = 2}$.

c) $N_{remaining} + N_{daughter} = N_o$ (figure).

d) $N_{decay} = N_o - N = N_o - \frac{N_o}{2^n} = N_o(1 - \frac{1}{2^n})$, but $N_{dt} = N_{decay} = N_o(1 - \frac{1}{2^n}) = (10^{12})(1 - 2^{-2.2}) = 0.7824 \times 10^{12}$ nuclei.

e)

i. ${}^{137}_{55}Cs \longrightarrow {}^{137}_{56}Ba + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$.

ii. The decayed mass: $m_{decay} = m_o - m$ { m: remaining}

$m_{decay} = m_{dt} + m_{{}^{0}_{-1}e} + (\frac{E_{liberated}}{c^2}) \Rightarrow m_{dt} < m_{decay} \Rightarrow m_{dt} < m_o - m$.

10.

a) At t = 1 T; $m = \frac{m_o}{2} = 0.5$ kg $\Rightarrow$ P: (138.38 days, 0.5 kg).

b) $m = m_o\, e^{-\lambda t} \Rightarrow \frac{dm}{dt} = m_o(-\lambda)\, e^{-\lambda t} = -\lambda\, m_o\, e^{-\lambda t}$.

c) at t = 200 days: $\frac{dm}{dt} = \frac{0 - 0.37}{400 - 200} = -1.85 \times 10^{-3}$ kg/day. $\lambda = \frac{\ell n\,2}{T} = \frac{\ell n\,2}{138.38} = 5.009 \times 10^{-3} day^{-1}$

$\Rightarrow -\lambda\, m_o\, e^{-\lambda t} = -(5.009 \times 10^{-3})(1)\, e^{-(5.009 \times 10^{-3})(200)} = -1.84 \times 10^{-3}$ kg/day $\Rightarrow \frac{dm}{dt} = -\lambda\, m_o\, e^{-\lambda t}$.

11.

a) At t = 1 T, $N = \frac{N_o}{2}$. Graphically for $N = \frac{N_o}{2}$, T_3 is the longest $\Rightarrow$ curve (3).

b)

i. $N = N_o\, e^{-\lambda t}$.

ii. $0.63\, N_o = N_o\, e^{-\lambda \tau} \Rightarrow 0.63 = e^{-\lambda \tau} \Rightarrow \ell n\, 0.63 = -\lambda \tau \Rightarrow \tau = \frac{-\ell n\, 0.63}{\lambda}$.

iii. $1\ day = \frac{-\ell n\, 0.63}{\lambda_2} \Rightarrow \lambda_2 = \underline{0.462\ day^{-1}}$. $T_2 = \frac{\ell n\,2}{\lambda_2} = \underline{1.5\ days}$.

iv. $\lambda_1 = \frac{\ell n\,2}{T_1} = \frac{\ell n\,2}{0.6} = \underline{1.155\ day^{-1}}$. $\tau_1 = \frac{-\ell n\, 0.63}{\lambda_1} = \underline{0.4\ day} = t_1$.

12.

a) $A = \frac{-dN}{dt}$

b) $A = \lambda N$

c) $\frac{-dN}{dt} = \lambda N \Rightarrow \frac{dN}{dt} + \lambda N = 0$.

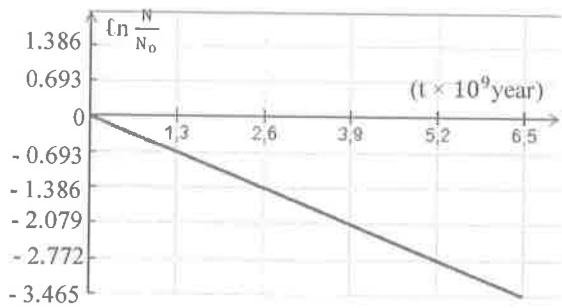
d) $N = k\, e^{\alpha t} \Rightarrow \frac{dN}{dt} = k\, \alpha\, e^{\alpha t} = \alpha N$ Substitute in the differential equation: $\alpha N + \lambda N = 0 \Rightarrow N(\alpha + \lambda) = 0$, but $N \neq 0$

$\Rightarrow \alpha + \lambda = 0 \Rightarrow \underline{\alpha = -\lambda}$. At $t_o = 0$: $N = N_o = k\, e^{\alpha(0)} \Rightarrow \underline{k = N_o} \Rightarrow N = N_o\, e^{-\lambda t}$.

13.

a) $\frac{N}{N_o} = \frac{1}{2} \Rightarrow N = \frac{N_o}{2} \Rightarrow t = 1\,T = \underline{1.3 \times 10^9 \text{ years}}.$

b) Adjacent figure.



c)

i. $N = N_o\, e^{-\lambda t} \Rightarrow \frac{N}{N_o} = e^{-\lambda t} \Rightarrow \ln\frac{N}{N_o} = -\lambda t.$

ii. Slope represents $-\lambda$. λ: decay constant

iii. Slope $= -\lambda = \frac{-(-1.386-0)}{2.6 \times 10^9} = 5.33 \times 10^{-10}\ y^{-1}$

$T = \frac{\ell n 2}{\lambda} = 1.3 \times 109$ years.

14.

A.

a) Activity: number of disintegrations per unit time. $A_o = 3.7 \times 10^{14}$ Bq.

b) $A = \frac{A_0}{2^n} \Rightarrow 0.23125 \times 10^{14} = \frac{3.7 \times 10^{14}}{2^n} \Rightarrow 2^n = 16 \Rightarrow n = 4 \Rightarrow T = \frac{t}{n} = \frac{16}{4} = \underline{4\ s}$. $\lambda = \frac{\ell n 2}{T} = \underline{0.1733\ s^{-1}}$.

c) $A_o = \lambda N_o \Rightarrow N_o = \frac{A_0}{\lambda} = \frac{3.7 \times 10^{14}}{0.1733} = \underline{2.135 \times 10^{15} \text{ nuclei}}.$

B.

a) ${}^{219}_{86}Rn \longrightarrow {}^{A}_{Z}Po + {}^{4}_{2}He$

Soddy's laws: $219 = A + 4 \Rightarrow A = 215$. $86 = Z + 2 \Rightarrow Z = 84$

b) i. Polonium emits radiation when it is in the excited state. The type of this radiation is γ-ray emission. The nature of this radiation is electromagnetic radiation.

ii. $\Delta m = m_{before} - m_{after} = [\,218.962297 - 214.953336 + 4.0015)] = 7.461 \times 10^{-3}\ u = 7.461 \times 10^{-3} \times 931.5\ \frac{MeV}{c^2} = 6.9499\ \frac{MeV}{c^2}$

$\Rightarrow E = \Delta m.c^2 = \underline{6.9499 \text{ MeV}}.$

iii. The decay is accompanied with γ-rays if $KE_\alpha = 6.43$ MeV. $E = KE_\alpha + E_\gamma \Rightarrow E\gamma = 6.9499 - 6.43 = \underline{0.52 \text{ MeV}}.$

15.

A.

a) ${}^{60}_{27}Co \longrightarrow {}^{A}_{Z}X + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$

$60 = A + 0 + 0 \Rightarrow A = 60$. $27 = Z - 1 + 0 \Rightarrow Z = 28 \Rightarrow$ the daughter nucleus is $\underline{\text{nickel } {}^{60}_{28}Ni}$.

$\Rightarrow {}^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$.

b) $Z_{Ni} > Z_{Co} \Rightarrow$ A neutron decays into a proton and an electron and anti-neutrino $\Rightarrow {}^{1}_{0}n \longrightarrow {}^{1}_{1}H + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$

c) i. Law of conservation of total energy.

ii. Common: Both travels in vacuum at the speed c .

Both are uncharged.

Distinct: The photon has a zero mass; the anti-neutrino has an extremely small mass.

The anti-neutrino does not interact with matter while a photon may interact.

d) i. $\lambda = \frac{\ell n 2}{T} = \frac{\ell n 2}{5.3 \times 365 \times 24 \times 3600} = 4.147 \times 10^{-9} s^{-1}$. $N_o = \frac{0.02}{(59.919817)(1.66 \times 10^{-27})} = \underline{2.0107 \times 10^{23} \text{ nuclei}}.$

ii. $N = \frac{N_0}{2^n} = \frac{2.0107 \times 10^{23}}{2^{(\frac{10}{5.3})}} = \underline{5.4371 \times 10^{22} \text{ nucleis}}.$

B.

a) $\Delta m = m_{before} - m_{after} = [\,59.919 - (\,59.91544 + 5.486 \times 10^{-4})] = 3.0114 \times 10^{-3}\ u$

$\Rightarrow \Delta m = 3.0114 \times 10^{-3} \times 931.5\ \frac{MeV}{c^2} = 2.8051\ \frac{MeV}{c^2}$. $E = \Delta m.c^2 = \underline{2.8051 \text{ MeV}} = 2.8051 \times 1.6 \times 10^{-13}\ J = \underline{4.48816 \times 10^{-13} J}.$

b) ${}^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$. $m_{Co}\, c^2 = m_{Ni}\, c^2 + KE_{Ni} + m_e\, c^2 + KE_e + E_{\bar{\nu}} + E\gamma \Rightarrow \Delta m\, c^2 = KE_e + E_{\bar{\nu}} + E\gamma$

$2.8051 = 1.2 + E_{\bar{\nu}} + 1.332 \Rightarrow E_{\bar{\nu}} = \underline{0.2731 \text{ MeV}}.$

c) $KE = m\, c^2 [\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1] \Rightarrow 1.2 \times 1.6 \times 10^{-13} = (9.1 \times 10^{-31})(9 \times 10^{16})\, [\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1]$

$\Rightarrow 1 - \frac{v^2}{c^2} = 0.0894095 \Rightarrow v \approx 0.95425\ c \approx \underline{2.863 \times 10^8\ m/s}$.

d) i. At t = 1 T, $N = \frac{N_o}{2} \Rightarrow N_{decay} = N_o - \frac{N_o}{2} = \frac{N_o}{2} = 1.00535 \times 10^{23}$ nuclei. $E_{total} = N_{decay}\ E$

$\Rightarrow E_{total} = (1.00535 \times 10^{23})(2.8051) = \underline{2.82 \times 10^{23}\ MeV}$.

ii. At t = 2 T, $N = \frac{N_o}{4} \Rightarrow N_{decay} = \frac{N_o}{2} - \frac{N_o}{4} = \frac{N_o}{4} = 5.02675 \times 10^{22}$ nuclei.

$\Rightarrow E_{total} = (5.02675 \times 10^{22})(2.8051) = \underline{1.41 \times 10^{23}\ MeV}$.

C. One nucleus of cobalt: $m = (59.919 \times 1.66 \times 10^{-27}) = 9.9466 \times 10^{-26}$ kg

$$\left.\begin{array}{lcl} 9.9466 \times 10^{-26} & \longrightarrow & 4.48816 \times 10^{-13}\ J \\ m & \longrightarrow & 3 \times 10^7\ J \end{array}\right\} \Rightarrow m = 6.6485 \times 10^{-6} kg \approx \underline{6.65\ mg}.$$

16.

A.

a) Since the emission of a positron is accompanied by the emission of a neutrino.

b) Law of conservation of mass number: $40 = M + 0 + 0 \Rightarrow M = 40$.

Law of conservation of charge number: $19 = N + 1 + 0 \Rightarrow N = 18$

$\Rightarrow {}^{40}_{19}K \longrightarrow {}^{40}_{18}Ar + {}^{0}_{+1}e + {}^{0}_{0}\nu$

c) $E = \Delta m.c^2 \Rightarrow \Delta m = \frac{E}{c^2} = \frac{1.5\ MeV}{c^2} = \frac{1.5}{931.5}(u) = \underline{1.610306 \times 10^{-3}}u$.

d) $\Delta m = m_K - (m_{Ar} + m_e) \Rightarrow 1.610306 \times 10^{-3} = 39.953577 - (m_{Ar} + 5.486 \times 10^{-4}) \Rightarrow m_{Ar} = 39.951418$ u

$\frac{m_K}{m_{Ar}} = \frac{39.953577}{39.951418} = 1.000054 \Rightarrow m_K = 1.000054\ m_{Ar}$.

e) $E_\nu + E_e = 0.44 + 0.48 = 0.92\ MeV < 1.5\ MeV$

$\Rightarrow E_\nu + E_e \neq E_{librated} \Rightarrow \gamma$-ray(s) is emitted.

$E_{librated} = E_\gamma + E_e + E_\nu \Rightarrow E_\gamma = 1.5 - 0.92 = \underline{0.58\ MeV}$.

B.

a) $m_K = 1.000054\ m_{Ar} \Rightarrow m = 1.676007 + (1.000054 \times 1.396673) = \underline{3.07275542}$ g.

b) $m = \frac{m_o}{2^n} \Rightarrow 1.676007 = \frac{3.07275542}{2^n} \Rightarrow n \approx 0.8745 \Rightarrow t = (0.8745)(1.3 \times 10^9) = \underline{1.137 \times} 10^9$ y.

Chapter 20: Stimulated Nuclear Reaction

1.

a) Reaction (1) is fission. Reaction (2) is fusion. Reaction (3) is radioactivity (α-decay).

b) Yes, since 3 neutrons are produced in this reaction.

2.

a) Since in this reaction the reaction takes place under external intervention (the impact of the neutron).

b) This neutron must be thermal; its speed is about 2200 km/s.

c) Law of conservation of mass number: $235 + 1 = 88 + 136 + 12(A) \Rightarrow a = 1$

Law of conservation of charge number: $92 + 0 = 38 + 54 + 12(Z) \Rightarrow Z = 0$

${}^{A}_{Z}X$ is a neutron ${}^{1}_{0}n$. 12 neutrons are produced in this reaction, then it leads to a chain reaction.

d) $\Delta m = 0.1357649 \times 931.5 \frac{MeV}{c^2} = 126.4650044 \frac{MeV}{c^2}$

$E = \Delta m.c^2 = 126.465 \frac{MeV}{c^2} c^2 = 126.465 \text{ MeV} = \frac{126.465}{1.6 \times 10^{-13}} = \underline{2.02344 \times 10^{-11} \text{ J}}.$

e)

i. $\left.\begin{array}{l} 235 \text{ g} \longrightarrow 6.022 \times 10^{23} \\ 1 \text{ g} \quad \longrightarrow \quad N \end{array}\right\} \Rightarrow N = 2.5626 \times 10^{21}$. $E_{total} = E.N = (2.02344 \times 10^{-11})(2.5626 \times 10^{21}) = \underline{5.1852 \times 10^{10} \text{ J}}$.

ii. $\left.\begin{array}{l} \text{One reaction} \longrightarrow 2.02344 \times 10^{-11} \text{ J} \\ N' \qquad\qquad \longrightarrow 10^{10} \text{ J} \end{array}\right\} \Rightarrow N' = \underline{4.9421 \times 10^{20} \text{ reactions}}.$

3.

a) ${}^{12}_{6}C + {}^{12}_{6}C \longrightarrow {}^{24}_{Z}Mg$

Law of conservation of charge number $6 + 6 = Z \Rightarrow Z = 12$.

b) $\Delta m = (11.996708)(2) - 23.978459 = 0.014957 \text{ u} = 0.014957 \times 1.66 \times 10^{-27} \text{ Kg} = \underline{2.483 \times 10^{-29} \text{ Kg}}.$

$E = \Delta m.c^2 = (2.483 \times 10^{-29})(9 \times 10^{16}) = \underline{2.559 \times 10^{-12} \text{ J}}.$

c) $E = KE_{Mg} = \frac{1}{2} m V^2 \Rightarrow 2.559 \times 10^{-12} = \frac{1}{2}(23.978459)(1.66 \times 10^{-27}) V^2 \Rightarrow \underline{V = 1.13 \times 10^{7} \text{ m/s}}.$

4.

a) $\Delta m = m_{before} - m_{after} = [1.00866 + 10.010193] - [7.014357 + 4.0015] = 2.996 \times 10^{-3}$ u. $\quad \underline{\Delta m > 0} \Rightarrow$ Exo-ergic.

b) $E = \Delta m.c^2 = [2.996 \times 10^{-3} \times 931.5] \frac{MeV}{c^2} c^2 = 2.790774$ MeV. $\quad E = KE_{Li} + KE_{\alpha}$

$\Rightarrow (2.790774 \times 1.6 \times 10^{-13}) = KE_{Li} + \frac{1}{2}(4.0015)(1.66 \times 10^{-27})(9.3 \times 10^{6})^2 \Rightarrow KE_{Li} = \underline{1.5927 \times 10^{-13} \text{ J} \cong 0.9954 \text{ MeV}}.$

5.

a) The neutron has no charge (neutral) while the proton is positively charged. They have the same mass number.

b) ${}^{9}_{4}Be + {}^{4}_{2}He \longrightarrow {}^{A}_{6}C + {}^{1}_{0}n$. Law of conservation of mass number: $9 + 4 = A + 1 \Rightarrow \underline{A = 12}$.

c) $E = \Delta m.c^2 \Rightarrow 5.70078 = \Delta m\, c^2 \Rightarrow \Delta m = 5.70078 \frac{MeV}{c^2} = \frac{5.70078}{931.5} \text{ u} = \underline{6.12 \times 10^{-3} \text{ u}}.$

$\Delta m = m_{before} - m_{after} \Rightarrow 6.12 \times 10^{-3} = [9.009988 + 4.0015] - [11.996708 + m_n] \Rightarrow \underline{m_n = 1.00866 \text{ u}}.$

d) $KE_{\alpha} + KE_{Be} + m_{before}\, c^2 = KE_{after} + m_{bafter}\, c^2 \Rightarrow KE_{\alpha} + 0 + \Delta m\, c^2 = KE_{after} \Rightarrow KE_{after} = E_{liberated} + KE_{\alpha}$.

6.

a) ${}^{1}_{1}H + {}^{2}_{1}H \longrightarrow {}^{3}_{2}He + \gamma$.

b)

i. $\Delta m = m_{before} - m_{after} = [1.6726 \times 10^{-27} + 3.3436 \times 10^{-27}] - [5.0064 \times 10^{-27}] = \underline{9.8 \times 10^{-30} \text{ kg}}.$

ii. $E_{liberated} = \Delta m.c^2 = (9.8 \times 10^{-30})(9 \times 10^{16}) = \underline{8.82 \times 10^{-13} \text{ J}}.$

c) The photon is emitted due to the de-excitation of the produced helium nucleus.

d)

i. $E = E_{lib}.N = (8.82 \times 10^{-13})(7.174 \times 10^{24}) = \underline{6.3275 \times 10^{12} \text{ J}}.$

ii. $\left.\begin{array}{l} 1 \text{ kg petroleum} \longrightarrow 4.4 \times 10^{7} \text{ J} \\ m \qquad\qquad\quad \longrightarrow 6.3275 \times 10^{12} \text{ J} \end{array}\right\} \Rightarrow \underline{m \cong 143806 \text{ kg}}.$

7.

a) Soddy's laws: $94 + 0 = 42 + 52 + kZ \Rightarrow K\,Z = 0$. The particle is neutron $\Rightarrow Z = 0$ and $A = 1 \Rightarrow K\,Z = 0$

$239 + 1 = 100 + 134 + k(1) \Rightarrow \underline{k = 6}$. So 6 neutrons are produced.

b) $\Delta m = m_{before} - m_{after} = [239.000589 + 1.00866] - [133.882842 + 99.884435 + 6(1.00866)] = \underline{0.190012 \text{ u}}.$

c) $\frac{\Delta m}{m_{Pu}} = \frac{0.190012}{239.000589} = 0.0795\,\%$.

d) $E = \Delta m.c^2 = (0.190012 \times 1.66 \times 10^{-27})(9 \times 10^{16}) = \underline{2.8388 \times 10^{-11} \text{ J}} = \frac{2.8388 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV} = \underline{177.43 \text{ MeV}}.$

e) $\eta = \frac{P_{useful}}{P_{nuclear}} \Rightarrow P_{nuclear} = \frac{500}{0.4} = 1250$ MW. $E = P\,t = (1250 \times 10^{6})(24 \times 3600) = 1.08 \times 10^{14}$ J.

The mass of one plutonium nucleus is $m_{nucleus} = (239.000589)(1.66 \times 10^{-27})\text{ Kg} \cong 3.96741 \times 10^{-25}$ Kg.

$\left.\begin{array}{l} 3.96742 \times 10^{-25} \text{ Kg} \longrightarrow 2.8388 \times 10^{-11} \text{ J} \\ m \qquad\qquad\qquad \longrightarrow 1.08 \times 10^{14} \text{ J} \end{array}\right\} \Rightarrow \underline{m = 1.5094 \text{ Kg}}.$

8.

a) $\Delta m = m_{before} - m_{after} = (1.00728 + 3.0155) - (2 \times 2.0136) = -4.42 \times 10^{-3}$ u. $\underline{\Delta m < 0} \Rightarrow$ endoergic reaction.

b) $m\,{}_1^1H\,.\,c^2 + m\,{}_1^3H.\,c^2 + KE\,({}_1^1H) = 2\,m_{deutron}\,.\,c^2 + KE_{deuterons}$

$\Rightarrow [(m\,{}_1^1H + m\,{}_1^3H) - (2\,m_{deutron})]\,c^2 + KE\,({}_1^1H) = KE_{deuterons} \Rightarrow \Delta m.c^2 + KE_{proton} = KE_{deuterons}$.

c) $-4.42 \times 10^{-3} \times 931.5\,\frac{MeV}{c^2}\,c^2 + 5.7 = 1.43 + KE_{deuteron} \Rightarrow KE_{deuteron} = \underline{0.15277\text{ MeV}}$.

9.

a) $\Delta m = m_{before} - m_{after} = [\,13.999234 + 4.0015\,] - [\,16.994743 + 1.00728\,] = \underline{-1.289 \times 10^{-3}\text{ u}}$. $\Delta m < 0 \Rightarrow$ endoergic.

b) $KE_\alpha + KE_N + m_{before}\,.c^2 = m_{after}\,.c^2 + KE_{after} + E\gamma \Rightarrow KE_\alpha + \Delta m.c^2 = KE_{after} + E\gamma$

KE_α is minimum if $KE_{after} + E\gamma = 0 \Rightarrow KE_{\alpha(min)} = -\Delta m.c^2 = -[-1.289 \times 10^{-3} \times 931.5\,\frac{MeV}{c^2}\,]\,c^2 \Rightarrow KE_{\alpha(min)} \cong 1.2$ MeV.

10.

a) $KE = \frac{1}{2}\,m\,V^2 = \frac{1}{2}\,(1.00866)\,(1.66 \times 10^{-27})\,(12.661743 \times 10^6)^2 = \underline{1.342 \times 10^{-13}\text{ J}} = \underline{838750\text{ eV}}$.

b) $n = 1$: $KE_1 = \frac{KE_o}{2}$, $n = 2$: $KE_2 = \frac{KE_o}{2}\,(\frac{1}{2}) = \frac{KE_o}{2^2} \Rightarrow$ n: $KE_n = \frac{KE_o}{2^n}$ thus $0.025 = \frac{838750}{2^n} \Rightarrow \underline{n = 25\text{ collisions}}$.

11.

a) ${}_7^{15}N$: $E_N = \Delta m\,c^2 = [\,7m_p + 8m_n - m_N\,]\,.\,c^2 \Rightarrow \frac{E_N}{c^2} = 7m_p + 8m_n - m_N \Rightarrow m_N = 7m_p + 8m_n - \frac{E_N}{c^2}$

Similarly: $m_C = 6m_p + 6m_n - \frac{E_C}{c^2}$ and $m_\alpha = 2m_p + 2m_n - \frac{E_\alpha}{c^2}$.

b) Mass defect: $\Delta m = [\,m_{{}_7^{15}N} + m_{{}_1^1H}\,] - [\,m_\alpha + m_{{}_6^{12}C}\,] = [7m_p + 8m_n - \frac{E_N}{c^2} + m_p\,] - [2m_p + 2m_n - \frac{E_\alpha}{c^2} + 6m_p + 6m_n - \frac{E_C}{c^2}\,]$

$E_{liberated} = \Delta m.c^2 = [-\frac{E_N}{c^2} + \frac{E_\alpha}{c^2} + \frac{E_C}{c^2}\,].\,c^2 = E_C + E\alpha - E_N$.

12.

a) $\Delta m = [\,56.921479 + 5.486 \times 10^{-4}\,] - [\,56.921132\,] = 8.956 \times 10^{-4}$ u.

$E = \Delta m.c^2 = (8.956 \times 10^{-4})\,(1.66 \times 10^{-27})\,(9 \times 10^{16}) = 1.33 \times 10^{-13}$ J. $\Rightarrow E_\gamma = \underline{1.33 \times 10^{-13}\text{ J}}$.

b)

i. $A = \frac{A_o}{2^n} = \frac{74000}{2^{(\frac{20}{271.8})}} = 70320.32574$ Bq. $N_{decay} = N_o - N = \frac{A_o}{\lambda} - \frac{A}{\lambda} = \frac{1}{\lambda}\,(A_o - A) = \frac{T}{\ell n\,2}\,(A_o - A)$

$\Rightarrow N_{decay} = \frac{(271.8)\,(24)\,(3600)}{\ell n\,2}\,[\,74000 - 70320.32574\,] = \underline{1.2467 \times 10^{11}\text{ nuclei}}$.

ii. $E = (1.246 \times 10^{11})(1.33 \times 10^{-13}) = 0.01658$ J.

iii. $D = \frac{E}{m} = \frac{0.01658}{70 \times 2} = 1.1843 \times 10^{-4}$ Gy.

13.

a) $ED = D \times QF = (50\text{ Gy})\,(1) = \underline{50\text{ Sv}}$.

b)

i. $D = \frac{E}{m} \Rightarrow E = D.m = (50)(0.021) = \underline{1.05\text{ J}}$. ii. $N_{decay} = \frac{1.05}{(0.34)\,(1.709)\,(1.6 \times 10^{-13})} = 1.1294 \times 10^{13}$ nuclei.

c) $A_o = \lambda\,N_o \Rightarrow N_o = \frac{A_o}{\lambda} = \frac{A_o\,T}{\ell n\,2} = \frac{(10^{-3} \times 3.7 \times 10^{10})\,(14.3)\,(24)\,(3600)}{\ell n\,2} = 6.5952 \times 10^{13}$ nuclei.

d) $N_{remaining} = N_o - N_{decay} = 6.5952 \times 10^{13} - 1.1294 \times 10^{13} = 5.4658 \times 10^{13}$ nuclei.

$N = \frac{N_o}{2^n} \Rightarrow 5.4658 \times 10^{13} = \frac{6.5952 \times 10^{13}}{2^n} \Rightarrow n = 0.270984 \Rightarrow t = 0.270984 \times 14.3 = \underline{3.8751\text{ days}}$.

14.

a) ${}_5^{11}B + {}_1^1H \longrightarrow 3\,{}_2^4He$.

b) $\Delta m = [\,11.006562 + 1.00728\,] - [\,3(4.0015)\,] = 9.342 \times 10^{-3}$ u.

$E = \Delta m.c^2 = [9.342 \times 10^{-3} \times 931.5\,\frac{MeV}{c^2}\,]\,.\,c^2 = \underline{8.7021\text{ MeV}}$.

c) ${}_2^4He + {}_5^{11}B \longrightarrow {}_Z^AN + {}_0^1n$. Soddy's laws: $4 + 11 = A + 1 \Rightarrow \underline{A = 14}$. $2 + 5 = Z + 0 \Rightarrow \underline{Z = 7}$.

d) $\eta = \frac{P_{output}}{P_{input}} \Rightarrow P_{input} = \frac{200\text{ MW}}{0.85} = 235.2941$ MW. $E_{yearly} = P_{input}\,.\,t = \frac{(235.2941)\,(365)\,(24)\,(3600)}{1.6 \times 10^{-13}} = 4.6376 \times 10^{22}$ MeV

The mass of one boron nucleus is $m_{boron} = (11.006562)\,(1.66 \times 10^{-27}) \cong 1.8271 \times 10^{-26}$ Kg.

$$\left.\begin{array}{lcl} 1.8271 \times 10^{-26}\text{ Kg} & \longrightarrow & 8.7021\text{ MeV} \\ m & \longrightarrow & 4.6376 \times 10^{22}\text{ MeV} \end{array}\right\} \Rightarrow m = 9.7372 \times 10^{-5}\text{ kg} = \underline{97.372\text{ mg}}$$

e) Deuterium-tritium fusion requires a much smaller temperature to take place, so it is more practical.

15.

a) ${}_1^2H + {}_1^2H \longrightarrow {}_2^4He$.

b)

i. To overcome the repulsive force between the two positive nuclei.

ii. $KE = \frac{3}{2}\,K\,T \Rightarrow (0.05175)\,(1.6 \times 10^{-13}) = \frac{3}{2}\,(1.38 \times 10^{-23})\,T \Rightarrow \underline{T = 4 \times 10^8\text{ K}}$.

c)

i. $$\left.\begin{array}{lcl} \text{Two nuclei of mass } (2 \times 2.0136)\,(1.66 \times 10^{-27})\text{ kg} & \longrightarrow & 3.8408 \times 10^{-12}\text{ J} \\ 2\text{ kg} & \longrightarrow & E \end{array}\right\} \Rightarrow \underline{E = 1.15 \times 10^{15}\text{ J}}$$

ii.

$\eta = \frac{P_{output}}{P_{input}} \Rightarrow P_{input} = P_{fusion} = \frac{60 \times 10^6}{0.3} = 2 \times 10^8$ W .

$E = P.t \Rightarrow t = \frac{E}{P} = \frac{1.15 \times 10^{15}}{2 \times 10^8} = 5.75 \times 10^6$ s $\Rightarrow t = \frac{5.75 \times 10^6}{(3600)(24)(30)} = \underline{2.218 \text{ months}}$.

iii. 10^3 kg $\longrightarrow 0.033$ kg

m $\longrightarrow 2$ kg $\Rightarrow m = 60606.06$ kg $= \underline{60.6 \text{ ton}}$.

iv. $m = \frac{(1\text{ kg})(1.15 \times 10^{15})}{3 \times 10^7} = 38.333 \times 10^6$ kg.

d) $\eta = \frac{P_{useful}}{P_{total}} \Rightarrow P_{total} = \frac{4800}{0.3} = 16000$W. $E = P.t = (16000)(24)(3600) = 1.3824 \times 10^9$ J.

$$\left.\begin{array}{ll} 2\,(2.0136)\,(1.66 \times 10^{-27}) & \longrightarrow 3.8408 \times 10^{-12} \text{ J} \\ m' & \longrightarrow 1.3824 \times 10^9 \text{ J} \end{array}\right\} \Rightarrow m' = 2.4 \times 10^{-6} \text{ kg}.$$

e) See the course.

16.

a) Surface area of Earth $A = 4\pi R^2$

$$\left.\begin{array}{ll} 200 \text{ W} & \longrightarrow 1 \text{ m}^2 \\ P & \longrightarrow 4\pi (6400 \times 10^3)^2 \end{array}\right\} \Rightarrow P = 1.0289 \times 10^{17} \text{ W}. \quad E = Pt = (1.0289 \times 10^{17})(24)(3600) \cong \underline{8.8897 \times 10^{21} \text{ J}}.$$

b)

i. $E = P.t = (4 \times 10^{26})(60) = 2.4 \times 10^{28}$ J. $E = \Delta m.c^2 \Rightarrow \Delta m = \frac{E}{c^2} = \frac{2.4 \times 10^{28}}{9 \times 10^{16}} = 2.67 \times 10^{11}$ kg.

mass decrease rate $= \frac{\Delta m}{\Delta t} = \frac{2.67 \times 10^{11}}{60} = 44.44 \times 10^8$ kg/s.

ii. Δm of the reaction: $\Delta m = \frac{E}{c^2} = \frac{25 \text{ MeV}}{c^2} \; \frac{25}{931.5}$ u $= 0.02683843264$ u

$$\left.\begin{array}{ll} 4 \text{ nuclei} & \longrightarrow \Delta m = 0.02683843264\,(u) \times 1.66 \times 10^{-27} \text{ Kg} \\ n & \longrightarrow 44.44 \times 10^8 \text{ Kg} \end{array}\right\} \Rightarrow \underline{n = 3.99 \times 10^{38} \text{ nuclei}}.$$

iii. $m_{lost} = \frac{\Delta m}{\Delta t} \times t = 44.44 \times 10^8 \times 10^{10} \times 365 \times 24 \times 3600 = \underline{1.4 \times 10^{27} \text{ kg}}$.

17.

a) ${}^{235}_{92}U$: 92 protons and 235 – 92 = 143 neutrons. ${}^{238}_{92}U$: 92 protons and 238 – 92 = 146 neutrons.
We call them isotopes

b) It is a nucleus that might undergo fission.

c)

i. Soddy's laws: $1 + 325 = x + 142 + 3(1) + y(0) + y(0) \Rightarrow \underline{x = 91}$. $0 + 92 = 40 + 58 + 3(0) + y(-1) + y(0) \Rightarrow \underline{y = 6}$.

${}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{91}_{40}Z + {}^{142}_{58}Ce + 3\,{}^{1}_{0}n + 6\,{}^{0}_{-1}e + 6\,{}^{0}_{0}\bar{\nu}$

ii. $KE = \frac{1}{2} m V^2 \Rightarrow (0.025 \times 1.6 \times 10^{-19}) = \frac{1}{2}(1.00866)(1.66 \times 10^{-27}) V^2 \Rightarrow \underline{V = 2186 \text{ m/s}}$.

iii.

1. $\underline{\Delta m > 0} \Rightarrow$ Exo-ergic reaction.
2. $E = \Delta m.c^2 = 197 \frac{\text{MeV}}{c^2}.c^2 = \underline{197 \text{ MeV}} = 197 \times 1.6 \times 10^{-13}$ J $= \underline{3.152 \times 10^{-11}}$ J.

iv.

1. $KE_{{}^{1}_{0}n} = \frac{(0.03)(197)}{3} = 1.97$ MeV.
2. 1.97 MeV = 1970000 eV >> 0.025 eV. The produced neutrons <u>are not thermal</u>.

v. The moderator.

d)

i. The chain reaction becomes out of control and nuclear <u>explosion takes place.</u>

ii. Control rods <u>absorb neutrons</u>. The rod can be raised or lowered into the reactor core according to the required reaction rate.

e)

i. $\eta = \frac{P_{electric}}{P_{nuclear}} \Rightarrow P_{nuclear} = \frac{900 \text{ MW}}{0.3} = 3000$ MW. $E = P.t = (3000 \times 10^6)(1) = 3 \times 10^9$ J.

One thermal neutron stimulates one fission $\Rightarrow$

$$\left.\begin{array}{ll} 1 \text{ neutron} & \longrightarrow 3.152 \times 10^{-11} \text{ J} \\ N & \longrightarrow 3 \times 10^9 \text{ J} \end{array}\right\} \Rightarrow \underline{N = 9.52 \times 10^{19} \text{ neutrons}}.$$

ii. Energy in one year: $E_{yearly} = (3000 \times 10^6)(365)(24)(3600) = 9.4608 \times 10^{16}$ J

The mass of the molecule of UO_2 is $[\,234.993586 + 2\,(15.990526)\,]\,(1.66 \times 10^{-27}) \cong 4.4318 \times 10^{-25}$ Kg.

$$\left.\begin{array}{ll} 4.4318 \times 10^{-25} \text{ Kg} & \longrightarrow 3.152 \times 10^{-11} \text{ J} \\ m_o & \longrightarrow 9.4608 \times 10^{16} \text{ J} \end{array}\right\} \Rightarrow m_o = \underline{1330.2 \text{ kg}}.$$