

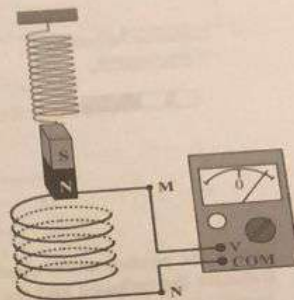
2 Exercises and Problems

N° 1

The phenomenon of electromagnetic induction

In the adjacent figure, we consider a spring which lies vertically and to which we attach a magnet. Just below the magnet, we fix a coil branched to an analogue voltmeter.

- 1) What is the position of the magnet when the needle is fixed?
- 2) The magnet oscillates above the coil.
- a) Indicate the induced and the inductor in this experiment.
- b) Prove the existence of an induced electromotive force « e » in the coil.
- c) Is there a current in the coil? Justify.
- d) Does the induced electromotive force « e » have a constant sign? Justify.
- e) Determine the direction of the deviation of the needle of the voltmeter along with the direction of the displacement of the magnet.

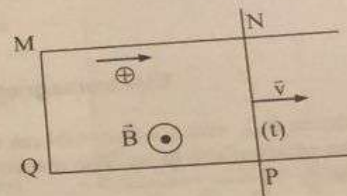


N° 2

Electromagnetic induction by varying the area

We consider the circuit in the adjacent figure of constant resistance $R = 2 \Omega$. (t) is a conducting rod displaced with a speed v . Given : $v = 50 \text{ cm/s}$, $B = 0.4 \text{ T}$, $MQ = d = 20 \text{ cm}$ and $MN = x$.

- 1) Give the direction and the line of action of the normal to the surface of the circuit \vec{n} .
- 2) Find, as a function of x , the expression of the magnetic flux crossing the surface MNPQ.
- 3) Deduce the induced electromotive force.
- 4) Indicate the direction of the induced current in the circuit and calculate its value.

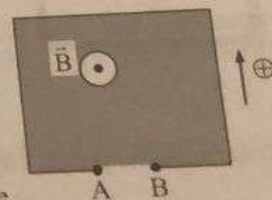


N° 3

Electromagnetic induction by varying the magnetic field (1)

A square coil, of side $a = 12 \text{ cm}$ and carries $N = 100$ turns, is placed normal to a uniform magnetic field of magnitude, at an instant t , is : $B = 0.08 \text{ t}$ (B in T and t in s).

- 1) Give the direction and the line of action of the normal to the surface of the circuit \vec{n} .
- 2) Express, as a function of t , the magnetic flux in the square. Deduce the induced electromotive force « e » in the coil.
- 3) Calculate the voltage u_{AB} .
- 4) We connect A and B by a resistor of resistance $R = 10 \Omega$. Calculate the value of the induced current in the coil and indicate its direction. We neglect the resistance of the coil.

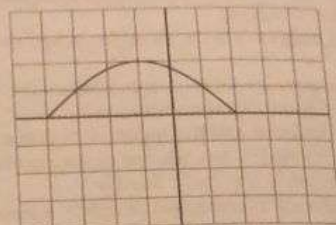
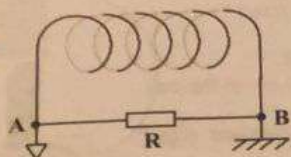


N° 4
Verification of Lenz's law

A coil is branched to a resistor of resistance R . The terminals A and B of the resistor are connected to an oscilloscope.

We approach a magnet from the coil, the oscilloscope displays the curve shown in the figure below.

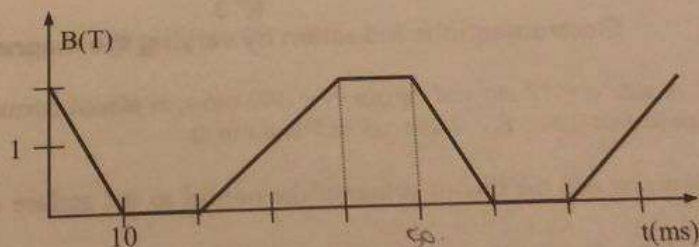
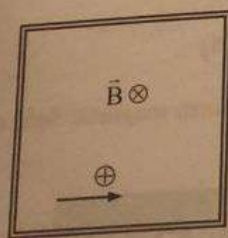
Direction of motion
of the magnet



- 1) Name the phenomenon observed in this experiment.
- 2) Indicate the induced and the inductor.
- 3) Justify the existence of this phenomenon.
- 4) Which voltage is displayed on the oscilloscope: u_{AB} or u_{BA} ?
- 5) Specify the direction of the induced current in R .
- 6) This experiment verifies Lenz's law. Justify.

N° 5
Electromagnetic induction by varying the magnetic field (2)

A square coil, with a side $a = 20$ cm and carrying $N = 100$ turns, is placed in a uniform magnetic field normal to the plane of the turns. The magnetic field varies periodically as a function of time as shown in the figure below.

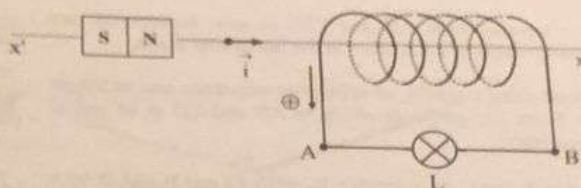


- 1) Determine the period T of the magnetic field \vec{B} .
- 2) Calculate, as a function of $\frac{dB}{dt}$, the induced electromotive force in the coil.
- 3) Deduce the values of the electromotive force induced in the interval $[0, T]$.
- 4) Indicate the direction of the induced current in the coil in the interval $[0, T]$.

N° 6 Theoretical approach

A solenoid, carrying $N = 200$ turns, each of area $S = 12 \text{ cm}^2$, is branched to a lamp L , equivalent to a resistor, sensitive to a voltage larger than 1 mV .

A magnetic bar is made to displace along the axis $x'x$ of the solenoid.



The resistance of the coil and that of the lamp are respectively : $r = 0.125 \Omega$ and $R = 0.025 \Omega$.

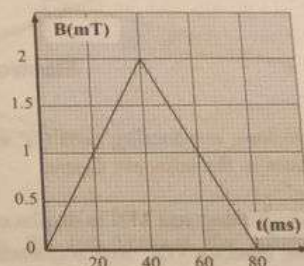
I - Qualitative study

- 1) During the displacement of the magnet, the lamp lights up. Justify.
- 2) State Lenz's law.
- 3) Specify, applying Lenz's law, the direction of the current when the magnet :
 - a) Approaches the solenoid.
 - b) Made to move away from the solenoid.

II - Quantitative study

We suppose that the magnet during its motion creates in the solenoid a uniform magnetic field and parallel to $x'x$: $\vec{B} = B \cdot \vec{i}$.

The variation of B as a function of time is represented in the adjacent figure.



- 1) Verify that the induced electromotive force is given by the expression : $e = k \frac{dB}{dt}$ where k is a constant depending on N and S .
- 2) Determine the values of « e » in the intervals $[0 ; 40 \text{ ms}]$ and $[40 \text{ ms} ; 80 \text{ ms}]$. Deduce the intensities of the current in these intervals.
- 3) Calculate, in the preceding intervals, the values of the voltage u_{AB} .
- 4) Does the lamp light up ? Justify.
- 5) Determine the value of u_{AB} become if the lamp is to light up.

N° 7 Induced current in a solenoid

Consider two coaxial and independent solenoids, the exterior solenoid carries 20 turns per centimeter of length and the interior one is formed of $N = 500$ spires of a cross section of $S = 100 \text{ cm}^2$. The resistance of the interior solenoid is $r = 10 \Omega$.

The exterior solenoid is traversed by an electric current i which varies between 0 and 10 amperes between $t_0 = 0$ and $t_1 = 5 \text{ s}$.

- 1) Find i as a function of time.
- 2) Determine the characteristics of the magnetic induction created by the external solenoid on the internal solenoid.
- 3) Specify the phenomenon which takes place in the interior solenoid.
- 4) Calculate the value of the current which appears in the interior solenoid when its terminals are joined.

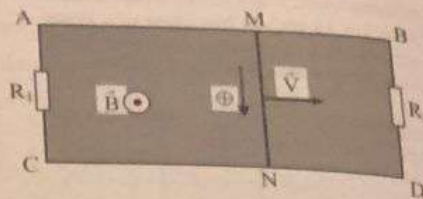
N° 8 Induced current in many branches

Two metallic rods AB and CD, of zero resistances, are placed parallel to each other in the horizontal plane.

A conducting rod MN, of negligible resistance and of length $\ell = 5 \text{ cm}$, lies perpendicularly to AB and CD at M and N respectively.

We join A and C by a resistor $R_1 = 0.1 \Omega$ and B and D by a resistor $R_2 = 0.5 \Omega$.

We move MN, parallel to itself, with a constant speed $V = 8 \text{ cm/s}$ in a vertical magnetic field of magnitude $B = 0.4 \text{ T}$ (see the figure).



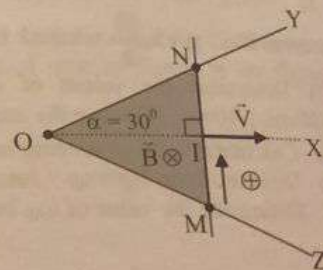
- 1) Choosing the surface AMNC, determine, as a function of B , ℓ and V , the expression of the induced electromotive force « e » which appears in the rod MN.
- 2) The rod MN is equivalent to a generator of electromotive force e and of internal resistance $r = 0$. Calculate i_{MN} .
- 3) Determine the intensity and the direction of the induced current in R_1 , R_2 and MN.

N° 9 Electromagnetic induction through area variation (1)

Two long conducting rails OZ and OY form an angle $\widehat{YOZ} = 2\alpha = 60^\circ$ as shown in the adjacent figure.

A conducting rod MN is displaced on the rails but remaining perpendicular to the bisector OX, with a constant velocity \vec{V} of magnitude $V = 2 \text{ m/s}$. At $t_0 = 0$, the rod MN is found at O.

The plane of the rails is perpendicular to the lines of the uniform magnetic field of vector \vec{B} , and of magnitude $B = 1 \text{ T}$.



- 1) Prove the existence of an induced current in the circuit.
- 2) Applying Lenz's law, indicate the direction of the induced current in the circuit.
- 3) Verify that the area of the triangle OMN at an instant t , is $S = \frac{4}{\sqrt{3}} t^2$ (in m^2).
- 4) Calculate, as function of time, the value of the induced electromotive force in the circuit.
- 5) Deduce the intensity of the induced current knowing the resistance per unit length of the circuit $\lambda = 0.5 \Omega/\text{m}$.
- 6) Determine the characteristics of the electromagnetic force exerted on MN.
 - a) Calculate as function of time :
 - i – The total electric power which appears in the circuit,
 - ii – The mechanical power supplied by the operator which displaces the rod.
 - b) Compare the preceding powers and give a conclusion.

N° 10
A rod sliding, in a magnetic field, on inclined rails

A copper homogeneous rod MN, of mass $m = 20 \text{ g}$ and of length $\ell = 10 \text{ cm}$, can slide without friction, along two metallic rails AC and A'C' forming a plane inclined by an angle $\alpha = 30^\circ$ with respect to the horizontal plane [figure (1)].

During its motion, the rod MN remains perpendicular to the rails AC and A'C' and joins them at M and N.

Given $g = 10 \text{ m/s}^2$.

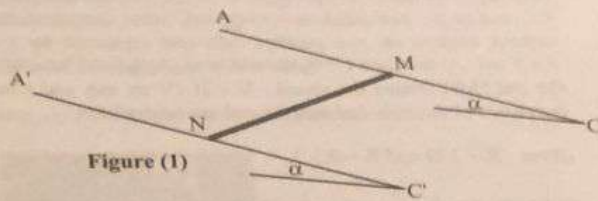


Figure (1)

A – The rod is launched, along the rails, without initial speed. After covering a distance L , the measurement of the speed is $v = 3 \text{ m/s}$. Calculate L .

B – The points A and A' are joined by a conducting wire, of resistance $R = 0.2 \Omega$, the resistance of the rails and of the rod are negligible.

When the rod covers a distance L , it passes, at $t_0 = 0$, with a speed $v = 3 \text{ m/s}$, in a region of space where the magnetic field is uniform, vertically upwards and of a magnitude $B = 0.6 \text{ T}$ [figure (2)].

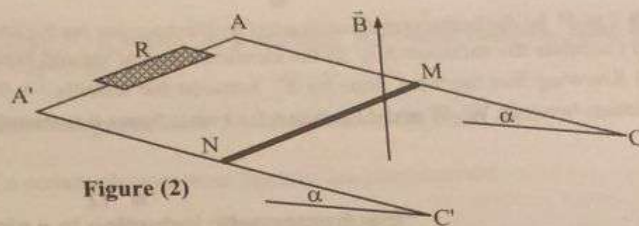


Figure (2)

1) a) Show that the induced electromotive force « e » which appears in the rod verifies the relation : $|e| = B\ell v \cos \alpha$ where v is the speed of the rod at time t .

b) Deduce the direction of the induced current.

2) a) Show that the rod is subjected to, at the instant, an electromagnetic force \vec{F} .

b) Precise the line of action and the direction of \vec{F} .

c) Does the force \vec{F} verify Lenz's law ? Justify.

d) Show that : $F = \frac{B^2 \ell^2 v}{R} \cos \alpha$.

3) a) Applying Newton's second law, show that the acceleration of the rod : $a = g \sin \alpha - \frac{B^2 \ell^2 v}{mR} \cos^2 \alpha$.

b) The graph of v as a function of time, given in the figure (3), shows that the motion passes through two phases. Specify these phases.

c) Calculate the limit speed acquired by the rod when its motion becomes uniform.

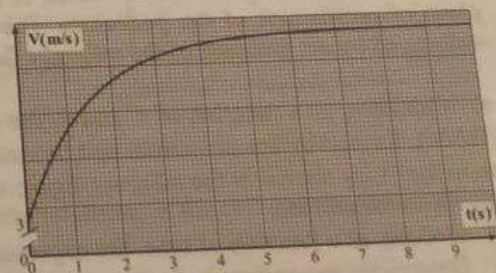
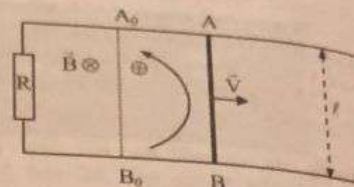


Figure (3)

N° 11
Energetic plan of a system

The circuit of the adjacent figure is situated in the horizontal plane. The rod (AB), of mass $m = 2 \text{ g}$, can move perpendicularly and without friction on two parallel rails and separated by a distance $\ell = 5 \text{ cm}$. At the time $t = 0$, the rod is at (A_0B_0) and later at a time t , the rod (AB) attains the speed: $V = 2t$ (V en m/s and t in s). The resistance of the rails and of the rod are negligible.

Given: $R = 1 \Omega$ and $B = 0.3 \text{ T}$.



- 1) Verify that the i.e.m.f. in the circuit is: $\mathcal{E} = 3 \times 10^{-2} t$ (SI).
- 2) a) Determine Laplace's force \vec{F} applied on the rod.
b) Is this force in agreement with Lenz's law? Justify.
- 3) a) Calculate, at the time t , the total electric power received by the circuit.
b) Find the total electric energy W collected in the circuit between the times $t = 0$ and $t = 10 \text{ s}$.

Given the total electric power: $P = \frac{dW}{dt}$.

- 4) Let \vec{F} be the horizontal force exerted by the operator on the rod.
a) Calculate the variation ΔE_K of the kinetic energy of the rod between times $t = 0$ and $t = 10 \text{ s}$.
b) Knowing that the work done by \vec{F} , between the instants $t = 0$ and $t = 10 \text{ s}$, is $W' = 0.7 \text{ J}$. Find a relation between W' , W and ΔE_K then tell to what forms it is transformed W' .

N° 12
*Electromagnetic induction in a simple pendulum

A simple pendulum is formed of a copper wire, of length $\ell = 80 \text{ cm}$ and of negligible mass, carrying at its extremity a mass m .

The pendulum undergoes harmonic oscillations of small amplitudes. We neglect friction.

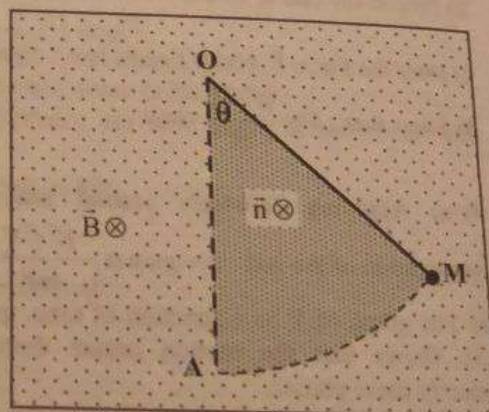
- 1) Calculate the period T_0 of the oscillations of the pendulum. Given $g = 9.8 \text{ m/s}^2$.
- 2) Determine the expression of the angular elongation of the pendulum, knowing that at $t_0 = 0$, the pendulum passes through the position of maximum elongation $\theta = \theta_m = 0.1 \text{ rad}$.
- 3) The pendulum is placed in a region of a magnetic field of constant magnitude $B = 0.15 \text{ T}$ which is perpendicular to the plane of oscillations of the pendulum. The oscillations are not influenced by the presence of the magnetic field.

a) Calculate, as a function of ℓ and θ , the area S of the sector (OAM), formed by the pendulum and its equilibrium position at an instant t , as shown in the adjacent figure where \vec{n} is a vector normal to the surface.

b) Deduce, as a function of time, the expression of S .

c) Determine the induced electromotive force which appears across the pendulum.

d) The oscillations of the pendulum are not influenced by the presence of the magnetic field. Justify.

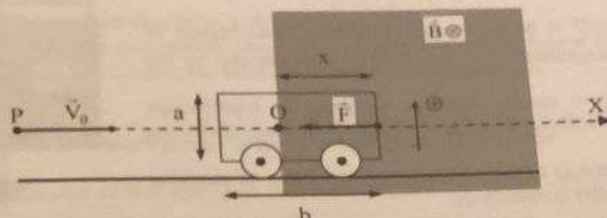


N° 13
Friction on a chariot in a magnetic field

A chariot (C) is formed of rectangular coils (of dimensions: $a = 40 \text{ cm}$ and $b = 1 \text{ m}$ and of resistance $R = 0.3 \Omega$). The chariot is placed on rectilinear and horizontal rails. The total mass of (C) is $M = 1 \text{ kg}$.

We launch (C) with a velocity \vec{V}_0 oriented along OX and of magnitude $V_0 = 6 \text{ m/s}$.

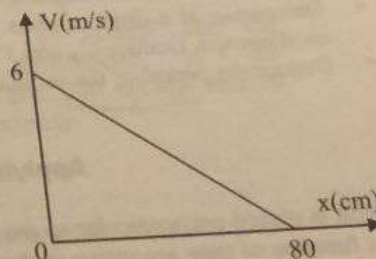
The center of inertia G of (C) moves, without friction, on POX as shown in the figure below.



At instant $t_0 = 0$, the coil enters, from point O, a region of a uniform, horizontal magnetic field \vec{B} perpendicular to the plane of the coils and of magnitude B .

The chariot is then under the action of an electromagnetic force \vec{F} as indicated in the figure.

- 1) The presence of \vec{F} gives an evidence of a certain phenomenon. Specify this phenomenon.
- 2) Justify the existence and the direction of the force \vec{F} .
- 3) a) Calculate, as a function of B , a and x , the magnetic flux in the coil.
b) Deduce the value of the electromagnetic force induced as a function of B , a and the speed V of the chariot at a time t .
c) Calculate as a function of B , a , V and R , the induced current in the coil, deduce the expression of the magnitude F of the force \vec{F} .
- 4) The variation of the speed of the chariot as a function of x is given in the adjacent figure.



- a) Extract the distance covered by the chariot in the magnetic field region, and justify if the chariot is totally in the magnetic region or not.
- b) Express V as a function of x and then the linear momentum P of the chariot as a function of x .
- c) Applying Newton's second law, $\sum \vec{F}_{ex} = \frac{d\vec{P}}{dt}$, on the chariot, Calculate B .

5) Energy Study:

- a) Express, as a function of B , a , V and R , the total electric power produced in the coil and the instantaneous power of the force \vec{F} . Give a conclusion.
- b) The new reference of gravitational potential energy is the horizontal plane passing through PX. Calculate the initial mechanical energy of the system [Earth; chariot]. Deduce the electric energy produced the coil.

N° 14

Studying the oscillations of a pendulum using an oscilloscope

The circuit of figure 1 is formed of a :
spring of unjoint turns, of negligible mass and constant k ; rectangular coil formed of $N = 250$ conducting and joint turns, of width $b = 5$ cm and length $a = 8$ cm ; a marked mass, attached to the coil at its lower extremity.

The total mass (coil - marked mass) : $m = 250$ g.

The lower part of the coil is in a constant magnetic field region of induction vector \vec{B} , of magnitude $B = 0.4$ T and perpendicular to the plane of the coil.

We move the mass m by a certain distance x_m from its equilibrium position and we release it without speed at $t = 0$, the pendulum oscillates with an amplitude x_m .

At a given instant t , the center G of the coil is found at a point of abscissa x , along an axis $x'Ox$ vertically downwards.

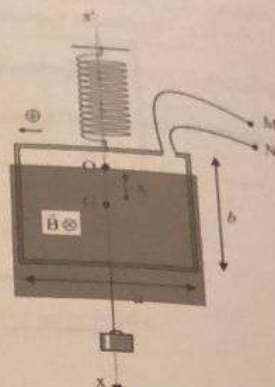


Figure 1

- 1) Show that the coil is a seat of an induced e.m.f.
- 2) When the extremities of the wire forming the coil are attached to each other, the oscillations of the pendulum are damped. When these two extremities are free, the oscillations are no more damped. Interpret these two observations.
- 3) The two extremities being connected to an oscilloscope. The oscilloscope visualizes the oscillogram in figure 2. The vertical and the horizontal sensitivities of the oscilloscope are $S_v = 500$ mV/div and $S_h = 200$ ms/div respectively.

- a) Verify that the induced electromagnetic force in the coil is :

$$e = N.B.a \frac{dx}{dt}$$

- b) Knowing that the oscillations are simple harmonic with an equation :

$$x = x_m \cos\left(\sqrt{\frac{k}{m}} t\right)$$

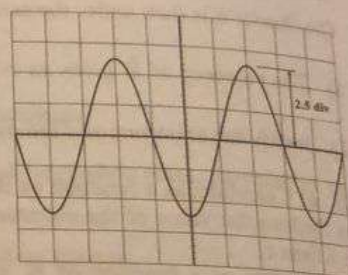


Figure 2

- Extract, from the oscillogram, the maximum value U_m and the period T of the voltage u_{MN} .
- Determine, at a time t and as a function of N , B , a , x_m , k and m , the voltage u_{MN} displayed on the oscilloscope. Deduce U_m and T as a function of N , B , a , x_m , k and m .
- Deduce the period of the oscillations of the pendulum, k and x_m .

N° 15

Applying Lenz's and Faraday's laws - Energy

A coil is placed perpendicular to the lines of a uniform magnetic field $B(t)$ as shown in figure (a). $B(t)$ varies as a function of time according to the graph in figure (b).

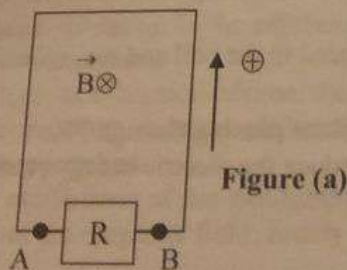


Figure (a)

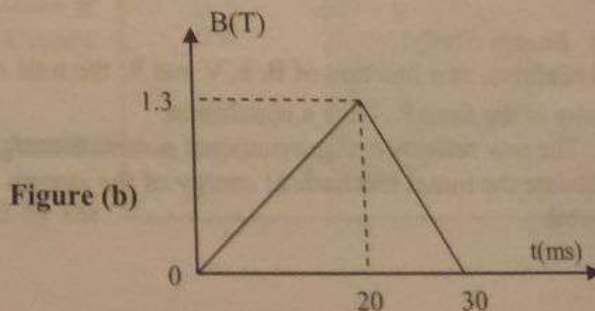


Figure (b)

The coil carries $N = 40$ rectangular turns (length : $a = 10$ cm and width : $b = 5$ cm). The total resistance of the coil is $r = 2 \Omega$ and its extremities A and B are connected to a resistor of resistance $R = 8 \Omega$.

- 1) Prove that there is an induced current in the coil during the time interval $[0 ; 30 \text{ ms}]$.
- 2) Applying Lenz's law, specify the direction of the induced current in each of the intervals $[0, 20 \text{ ms}]$ and $[20 \text{ ms}, 30 \text{ ms}]$.
- 3) Indicate, in each of the intervals $[0, 20 \text{ ms}]$ and $[20 \text{ ms}, 30 \text{ ms}]$, the sign of the induced electromotive force \mathcal{E} .
- 4) Show that the intensity of the induced current in the circuit can be written in the form: $i = k \frac{\Delta B}{\Delta t}$ where k is constant to be determined as a function of N, r, R, a and b .
- 5) Determine u_{AB} and represent in a system, the graph of u_{AB} as a function of time in the interval $[0, 30 \text{ ms}]$.
- 6) Calculate in the interval $[0 ; 30 \text{ ms}]$, the total electric energy in the circuit.

N° 16

Verifying Faraday's law in an alternator

We consider a rectangular coil of $N = 100$ turns and cross-section $S = 40 \text{ cm}^2$. This coil is fixed on a support.

A magnet bar, placed at the center of the coil, can turn, around an axis (Δ) , with a constant angular speed ω , in a plane perpendicular to the turns as shown in **figure (1)**. The system [coil ; magnet bar] forms an alternator.

The terminals M and N of the coil are connected to an oscilloscope displaying the curve in **figure (2)**.

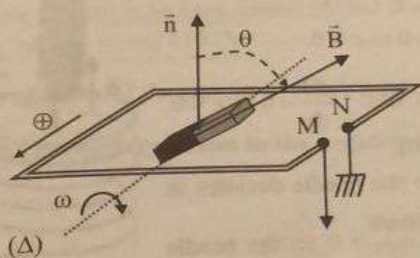


Figure (1): \vec{n} is normal to the plane of the turns and \vec{B} is the magnetic field of the magnet bar.

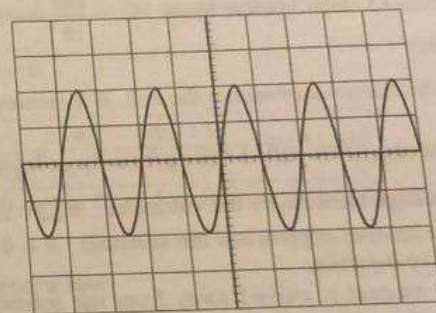


Figure (2): The vertical and the horizontal sensitivities of the oscilloscope are 200 mV/div and 10 ms/div respectively.

- 1) Extract the nature of the voltage produced by the alternator.
- 2) The voltage displayed on the oscilloscope is due to a physical phenomenon.
 - a) Name this phenomenon ?
 - b) Interpret the existence of this phenomenon.
- 3) Knowing that at $t_0 = 0$, the magnetic field \vec{B} , created by the magnet, confounds with the normal \vec{n} coil. At an instant t , \vec{B} forms with \vec{n} an angle θ .
 - a) Using Faraday's law, show that: $u_{MN} = NBS\omega \sin(\omega t)$.
 - b) Tell whether Faraday's law verifies the voltage displayed on the oscilloscope or not.
 - c) Extract the period and the maximum voltage delivered by the alternator.
 - d) Deduce the values of ω and B .