Generalities about transformation

I) In the plane referred to an orthonormal system $(O, \vec{i}, \vec{j})$, consider the mapping T of the plane, that associates to every point $M(x,y)$ of the plane a point $M'(x',y')$, such that : $M\begin{cases} x \\ y \end{cases} \longrightarrow M'\begin{cases} x' = x+y+1 \\ y' = -x+y-1 \end{cases}$

1) Find the invariant point I under T.
2) is T involutive ?
3) Determine the transformation T^{-1} inverse of T
4) Let (d) be the straight line of equation $y = 2x - 1$. Determine the image of (d) by T
5) Compare IM and IM' and calculate $(\vec{IM}, \vec{IM'})$

II) consider in the plane of an orthonormal system, the transformatio T which associates to a point $M(x,y)$, a point $M'(x',y')$ such that
$\begin{cases} x' = Kx \\ y' = Ky \end{cases}$ where $K \in \mathbb{R} - \{0,1\}$.

1) what is the inverse transformation of T ? is there an invariant point ? is T reciprocal ?
2) A and B are two points of the plane and $A' = T(A)$ and $B' = T(B)$. Compare $\vec{A'B'}$ and $\vec{AB}$.
3) (C) is the circle of center $I(0,2)$ and radius R. Write the equation of (C) and show that the image of (C) under T is a circle of radius $|K|R$.

III) In the plane refered to an orthonormal system $(O, \vec{i}, \vec{j})$, consider the mapping of the plane T, that to every point $M(x,y)$ distinct of O associates the point $M'(x',y')$, such that :

$$M\begin{cases} x \\ y \end{cases} \longrightarrow M'\begin{cases} x' = \dfrac{x}{x^2+y^2} \\ y' = \dfrac{y}{x^2+y^2} \end{cases}$$

1) Determine the set of invariant points by T
2) show that T is involutive and determine its inverse transformation T
3) Let (C) be the circle of equation $x^2+y^2-2x-4y=0$. Find the image of (C) by T
4) Denote by (Γ) the circle of equation $x^2+y^2-2x-4y+1=0$. Find the image of (Γ) by T.
5) Let (d) be the straight line of equation $y = x-1$, find the image of (d) by T.

IV) the points $A(2,0)$ and $B(-2,0)$ are given in the plane of an orthonormal system. Consider the transformation T which associates to a point $M(x,y)$, the point $M'(x',y')$ which is the intersection of the straight lines (d) and (d') which are perpendicular respectively to MA at A and to MB at B.

1) what is the set in which T is defined ?
2) Calculate x and y in terms of x' and y'.
3) what is the inverse of T ? is T reciprocal ?

V) Designate by (P_1) the plane (P) without the line (d) of equation $x=1$ and by T the transformation that associates to every point $M(x;y) \in (P_1)$ the point $M'(x';y') \in (P_1)$ such that $\begin{cases} x' = \frac{x}{x-1} \\ y' = \frac{y}{x-1} \end{cases}$.

1) is T reciprocal ?
2) Determine the set of invariant points under T
3) show that the points O, M and M' are collinear, Deduce the image of the line (δ) of equation $ax+by=0$ (a and b are two real numbers such that $a \neq 0$ or $b \neq 0$)
4) Find the image under T of a line (L) does not passing through O, without its point of intersection with (d).

VI) Let (P) be the plane in a direct orthonormal system $(O,\vec{i},\vec{j})$ of axes $x'Ox$ and $y'Oy$ and f the transformation defined by

$f: M(x,y) \longrightarrow M'(x';y')$ such that $\begin{cases} x' = \frac{\sqrt{2}}{2}(x+y) \\ y' = \frac{\sqrt{2}}{2}(x-y). \end{cases}$

1) determine the set (d) of invariant points by f.
2) prove that for all point M of (P) its image is M' by f :
   a) the midpoint of $[MM']$ belongs to (d)
   b) the vector $\overrightarrow{MM'}$ has a fixed direction.
   c) compare the direction of (MM') with the direction of (d)
3) Let $M''(x'',y'')$ be the image of $M(x,y)$ by $f \circ f$. Express x'' and y'' in terms of x and y. what can you deduce ?