



Entrance Exam 2002-2003

Physics

Duration: 2 hours

First Exercise: [7 pts].

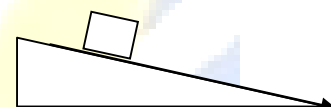
Mechanical energy:

We suggest to determine the variation of the mechanical energy of a system between two given instants. Let us consider an air table, inclined 15° with respect to the horizontal plane, with its accessories. During its motion, the puck, of mass 0.22 kg , undergoes resistive forces, whose resultant $\vec{f} = -f \vec{i}$ is constant and in opposite direction with the velocity $\vec{V} = V \vec{i}$ ($V > 0$).

A computer, having a certain recording system, records, at equal intervals of time $\tau = 40 \text{ ms}$, the abscissa x and the velocity V of the center of inertia G of the puck along an axis $(O; \vec{i})$ parallel to the line of greatest slope. The results of measurements are put in the following table. ($g = 9.80 \text{ m/s}^2$)

Date	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}	t_{12}
Position	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}
Abscissa $x(\text{m})$	0.0000	0.0116	0.0271	0.0465	0.0697	0.0968	0.1278	0.1626	0.2013	0.2439	0.2904	0.3407	0.3949
speed $V(\text{m/s})$	0.2420	0.3388	0.4356	0.5324	0.6292	0.7260	0.8228	0.9196	1.0164	1.1132	1.2100	1.3068	1.4036

1. Calculate the algebraic value of the linear momentum of G at the instants t_0 , t_2 , t_5 , t_7 , t_{10} , and t_{12} .
2. Calculate the algebraic value of the instantaneous variation ΔP of the linear momentum at the instants t_1 , t_6 and t_{11} . Compare the different results.
3. What are the forces acting on the puck?
4. a) Find the algebraic value F of the sum \vec{F} of these forces.
b) Determine, applying Newton's second law, the value of f .
5. Calculate the work $W(\vec{f})$ done by \vec{f} between the points M_1 and M_{11} .
6. Calculate the height separating the horizontal planes passing through M_1 and M_{11} .
7. a) Calculate the mechanical energy of the system (puck- Earth) at the instants t_1 and t_{11} knowing that the horizontal level passing by M_{11} is chosen as the reference for the gravitational potential energy.
b) Deduce the variation ΔE_m of the mechanical energy between the instants t_1 and t_{11} . Due to what is this variation ΔE_m ?
c) Compare ΔE_m to $W(\vec{f})$.





Second Exercise: [14 pts]

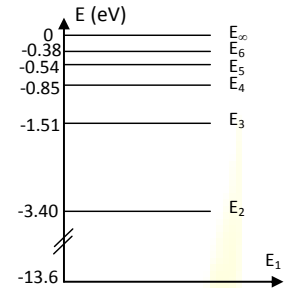
Hydrogen atom

A. Energy level

The adjacent energy diagram shows some energy levels E_n of a hydrogen atom.

The expression giving the respective value of these energies is $E_n = -\frac{13.6}{n^2}$,

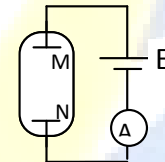
where E_n is expressed in eV and n is an integer number.



1. a) In which state does the atom exist when its energy is zero?
b) Is the electron of this atom free or linked?
2. a) Determine the ionization energy of the hydrogen atom taken in the fundamental state.
b) Show that the absorption of a radiation of wavelength $\lambda = 91.2$ nm allows the passage of the atom from the fundamental state to the ionized state.
3. a) Show that the wavelength λ' of the emitted radiation resulting from the transition from the second excited state towards the fundamental level has a value $\lambda' = 102.6$ nm.
b) The passage of an atom (disexcitation) from the second excited level towards the fundamental level can be done through different transitions. Calculate the value of the energies of the radiations associated to these transitions.

B. Absorption of radiations:

We have two sources of radiation S_1 and S_2 emitting respectively two monochromatic radiations of wavelengths $\lambda_1 = 80$ nm and $\lambda_2 = 102.6$ nm, an ammeter (A) very sensitive to very weak currents, a generator of e.m. f E and a glass bulb, transparent to the considered radiations, and having two electrodes M and N and containing hydrogen under low pressure.



The bulb is successively irradiated with the two radiations of wavelengths λ_1 and λ_2 . Show that one of these two radiations allows the ammeter to detect the passage of a current, specifying the phenomenon shown in evidence.

Take: $e = 1.602 \times 10^{-19}$ C; $c = 2.998 \times 10^8$ m/s; $h = 6.626 \times 10^{-34}$ J.s;
visible spectrum: $400 \text{ nm} \leq \lambda \leq 750 \text{ nm}$.

Third Exercise: [22 pts].

Natural frequency of oscillations

We want to determine the natural frequency f_a of the oscillations in an (R,L,C) circuit, by using two methods. To do this, we use a resistor (R) of resistance $R = 120 \Omega$, a capacitor (C) of capacitance $C = 1 \mu\text{F}$, a coil (B) of inductance $L = 0.06$ H and of negligible resistance, two generators G_1 and G_2 which can deliver, respectively across their terminals, a constant voltage



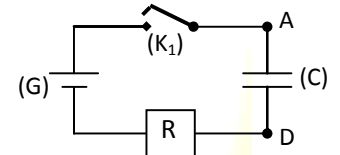
$U_1 = 6 \text{ V}$ and an alternating sinusoidal voltage u of adjustable frequency f , two switches (K_1) and (K_2), and connecting wires. Take $0.32\pi = 1$.

A. Undamped free oscillations:

I- Charge of the capacitor (C):

We set up the circuit shown in the adjacent figure.

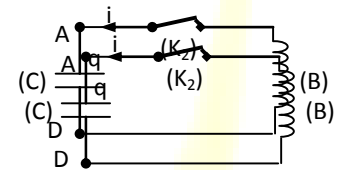
We close the switch (K_1). Calculate, in steady state, the charge of the armature A and the energy stored in (C).



II- Oscillating circuit:

The capacitor, initially charged under the voltage U_1 , is connected to the coil (B) as shown in the adjacent figure.

We close the switch (K_2) at the instant $t = 0$. At the instant t , the armature A has the charge q and the circuit carries a current i .



1. Write down, at the instant t :

- The expression of the electrical energy E_e stored in (C).
- The expression of the magnetic energy E_m stored in (B).

2. Taking into account the conservation of the expression ($E_e + E_m$) :

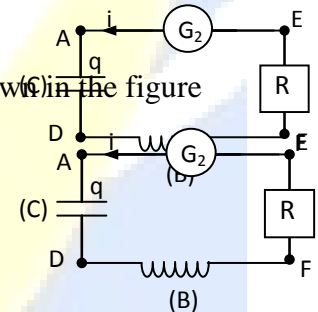
- Establish, by using the derivative of the expression ($E_e + E_m$) with respect to time, the differential equation which describes the variation of the charge q as a function of time.
- Deduce the natural frequency f_0 of the oscillations in the (L,C) circuit.

B. Forced oscillations:

(C) is initially uncharged. (G_2), (R), (C) and (B) are connected in series as shown in the figure below.

In steady state G_2 delivering the voltage $u = V_A - V_E = U_m \sin(2\pi ft)$, the circuit carries an alternating sinusoidal current i whose expression is;

$$i = I_m \sin(2\pi ft - \phi), \quad (u \text{ in V; } I \text{ in A; } f \text{ in Hz; } t \text{ in s})$$



- Establish, in terms of I_m , f , t and ϕ , the expressions of the alternating sinusoidal voltages ($V_A - V_D$), ($V_D - V_E$) and ($V_F - V_E$).
 - Give the instantaneous expression resulting from the addition of voltages.
 - Replacing the time t by the particular values: i) $t = 0$ and ii) $t = 1/4f$, establish, in terms of f and U_m the expression giving I_m^2 .
 - Determine, from the expression of I_m^2 , the numerical value f'_0 of f for which I_m^2 takes a maximum value.
 - What is the phenomenon obtained in this case?
- C. Comparison of f_0 and f'_0 : Are these two methods valid?



Entrance Exam 2002-2003

Solution of Physics

Duration: 2 hours

First Exercise

1. $P = mV$; $P_0 = 0.05324 \text{ kg m/s}$; $P_2 = 0.09583 \text{ kg m/s}$; $P_5 = 0.1597 \text{ kg m/s}$;
 $P_7 = 0.2023 \text{ kg m/s}$; $P_{10} = 0.2662 \text{ kg m/s}$; $P_{12} = 0.3088 \text{ kg m/s}$.
2. $\Delta P_1 = m (V_2 - V_0) = 0.04259 \text{ kg m/s}$; $\Delta P_6 = m (V_7 - V_5) = 0.04260 \text{ kg m/s}$;
 $\Delta P_{11} = m (V_{12} - V_{11}) = 0.04260 \text{ kg m/s}$.
3. Forces: weight $m\vec{g}$; \vec{N} normal reaction of the support; \vec{f} force due to friction.
4. a) $\vec{F} = m\vec{g} + \vec{N} + \vec{f}$.
Projection: $F = mg \sin \alpha - f = 0.22 \times 9.8 \times 0.2538 - f$.
b) $F = \frac{\Delta P}{\Delta t} = \frac{\Delta P}{2\tau}$;
 $0.5580 - f = 0.04260 / 0.08 \Rightarrow f = 0.02550 \text{ N}$
5. $W(\vec{f}) = \vec{f} \cdot \vec{d} = -f d = -0.02550 (0.3407 - 0.0116) = -0.00839 \text{ J} \approx -0.0084 \text{ J}$.
6. $h = d \sin \alpha = (0.3407 - 0.0116) \sin 15^\circ = 0.08517 \text{ m}$
7. a) $ME = KE + PE = \frac{1}{2}mV^2 + m g z$; $ME_1 = \frac{1}{2}mV_1^2 + m g h = 0.1963 \text{ J}$; $ME_2 = \frac{1}{2}mV_{11}^2 + 0 = 0.1878 \text{ J}$;
b) $\Delta ME = ME_2 - ME_1 = -0.0085 \text{ J}$. Variation due to friction.
c) $\Delta ME = W(\vec{f})$ with the errors of the experiment.

Second exercise

- A. 1. a) Ionized State
b) Free
2. a) $E_i = E_\infty - E_1 = 13.6 \text{ eV}$.
b) $\Delta E = hc/\lambda = 6.626 \times 10^{-34} \times 2.998 \times 10^8 / (91.2 \times 10^{-9} \times 1.602 \times 10^{-19}) = 13.596 \approx 13.6 \text{ eV}$
 $= E_\infty - E_1$ ou $\lambda = hc/\Delta E$
3. a) $\lambda' = hc/\Delta E = hc/(E_3 - E_1) = 102.56 \approx 102.6 \text{ nm}$.
b) $\Delta E_{31} = -1.51 + 13.6 = 12.09 \text{ eV}$; $\Delta E_{32} = -1.51 + 3.4 = 1.89 \text{ eV}$;
 $\Delta E_{21} = -3.4 + 13.6 = 10.2 \text{ eV}$.
- B- Since $\lambda_1 = 80 \text{ nm} < \lambda = 91.2 \text{ nm} \Rightarrow (E_1) > E_i \Rightarrow$ ionization of the atom and emission of an electron. The electron in presence of a p.d. $E \Rightarrow$ passage of a current. Phenomenon of ionization.



Third exercise

A- I) $Q = C U_1 = 6 \times 10^{-6} \text{ C}$; $W = \frac{1}{2} C U_1^2 = 1.8 \times 10^{-5} \text{ J}$

II) 1. a) $E_e = \frac{1}{2} C u^2 = \frac{1}{2} q^2 / C$.

b) $E_m = \frac{1}{2} L i^2$.

2. a) $\frac{1}{2} q^2 / C + \frac{1}{2} L i^2 = \text{constant}$ and $i = \frac{dq}{dt} \neq 0$: $\frac{1}{C} q \frac{dq}{dt} + L i \frac{di}{dt} = 0$

$$\Rightarrow \ddot{q} + \frac{1}{LC} q = 0 \text{ of the form } \ddot{q} + \omega_0^2 q = 0 \Rightarrow \omega_0^2 = \frac{1}{LC}.$$

$$\Rightarrow f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \approx 635 \text{ Hz}.$$

B- 1. $V_A - V_D = q/C = \frac{\int i dt}{C} = -\frac{I_m}{2\pi f C} \cos(2\pi f t - \phi)$; $V_F - V_E = R i = R I_m \sin(2\pi f t - \phi)$;

$$V_D - V_F = L \frac{di}{dt} = L 2\pi f I_m \cos(2\pi f t - \phi).$$

2. a) $U_m \sin(2\pi f t) = R I_m \sin(2\pi f t - \phi) + L 2\pi f I_m \cos(2\pi f t - \phi) - \frac{I_m}{2\pi f C} \cos(2\pi f t - \phi).$

b) For $t = 0$, $\Rightarrow 0 = R I_m \sin(\phi) + [L 2\pi f - \frac{1}{2\pi f C}] I_m \cos(\phi)$;

For $t = 1/4f$, $\Rightarrow U_m = R I_m \cos(\phi) + [L 2\pi f - \frac{1}{2\pi f C}] I_m \sin(\phi)$;

$$\Rightarrow \text{Calculation: } \frac{I_m^2}{R^2 + [L 2\pi f - 1/(2\pi f C)]^2}$$

3. I_m^2 is max... if.. $[L 2\pi f - \frac{1}{2\pi f C}] = 0 \Rightarrow f'_0 = \frac{1}{2\pi\sqrt{LC}} \approx 653 \text{ Hz}.$

4. Phenomenon of resonance of current.

C- Yes. Since $f_0 = f'_0 \approx 653 \text{ Hz}.$