

Exercise 1:

In the following table, only one of the proposed answers to each question is correct. Write the number of each question and give, with justification, the answer that corresponds to it.

N	Question	Answers		
		A	B	C
1)	$\lim_{x \rightarrow 0} \frac{1}{x^3 \ln x}$	$+\infty$	$-\infty$	1
2)	x is a real number. The number of solutions of the equation: $\ln^2(x) + 3 \ln(x) + 2 = 0$ is:	0	1	2
3)	Let $f(x) = \ln(1 - \ln x)$ then f ...	is strictly decreasing	is strictly increasing	is not monotonic
4)	$\lim_{x \rightarrow +\infty} x \ln \left(1 + \frac{1}{x} \right)$	$+\infty$	0	1
5)	Let f be a function defined on IR by: $f(x) = x^4 - 4x^3 + 6x^2$ then f ...	Has a single inflection point	Has a double inflection point	Don't has inflection point
6)	The domain of definition of the function f, such that $f(x) = \ln(4 - x^2)$, is:	$] - \infty; -2[\cup]2; +\infty[$	$] -2; 2[$	$] - \infty; 2] \cup [2; +\infty[$
7)	$\lim_{x \rightarrow +\infty} x^3 - x \ln x + 2$	$+\infty$	+2	$-\infty$

Exercise2:

Part (A):

Let g be a function defined over $]0; +\infty[$ by: $g(x) = 1+x - x \ln x$.

- 1) Calculate the limits of $g(x)$ at 0 and $+\infty$.
- 2) Calculate $g'(x)$ then construct the table of variation of g .
- 3) Show that the equation $g(x) = 0$ admits a unique solution α and verify that $3.59 \leq \alpha \leq 3.6$.
- 4) Study the sign of $g(x)$.

Part (B):

Consider the function f defined over $]0; +\infty[$ by $f(x) = 1 + \frac{2 \ln x}{x+1}$, where (C) is its representative curve in an orthonormal system .

- 1) Verify that $f(\alpha) = 1 + \frac{2}{\alpha}$
- 2) Calculate $\lim_{x \rightarrow 0} f(x)$. Deduce an asymptote to (C).
- 3) a. Show that the line (d) of equation $y = 1$ is an asymptote to (C).
b. Study the relative position of (C) and (d).
- 4) Verify that $f'(x) = \frac{2g(x)}{x(x+1)^2}$ and set up the table of variation of f .
- 5) Determine the equation of (T), the tangent to (C) at a point A of abscissa 1.
- 6) Draw (d), (T) and (C). (take $\alpha = 3.6$)