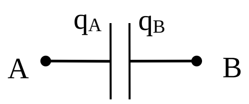
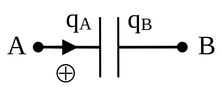
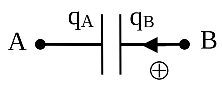
Chapter 9
Capacitors

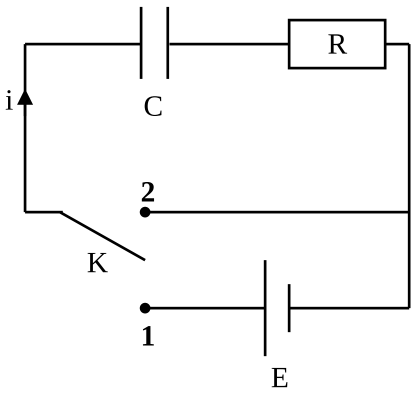
1 Summing up the lesson

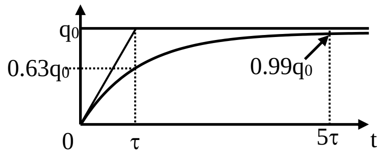
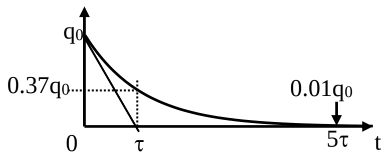
- **Recall**

Symbol of the capacitor	
Capacitance : C	Expressed in farad « F » in SI
Relation between the charges of the capacitors	$q_A = -q_B$ (at each instant)
Relation tension-charge	$q_A = Cu_{AB}$ et $q_B = Cu_{BA}$
Stored electric energy	$W = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}Cu^2 = \frac{1}{2}qu$ (if $q = q_A$ then $u = u_{AB}$ and vice versa)
Intensity of the current	$i = \frac{dq_A}{dt}$ (positive direction entering armature A)  $i = \frac{dq_B}{dt}$ (positive direction entering armature B) 

- **Differential equations, time equations and graphs of the charge q of a capacitor during charging and during discharging.**

We connect the circuit in the figure below.



	Initial conditions	**Differential equation**	**Solution of the differential equation**	**Graph of q as a function of time**
Charging of a capacitor (K on 1)	t = 0, q = 0	$\frac{dq}{dt}+\frac{q}{RC}=\frac{E}{RC}$	$q=q_0\left(1-e^{-\frac{t}{\tau}}\right)$	
Discharging of a capacitor (K on 2)	t = 0, q = q_0	$\frac{dq}{dt}+\frac{q}{RC}=0$	$q=q_0e^{-\frac{t}{\tau}}$	

Where : $q_0 = E\times C$; $\tau = R\times C$

Definition of the time constant τ : This is the time in which the capacitor charges up to 63 %.

Remarks :

— If $i=\frac{dq}{dt}\Rightarrow i=\begin{cases} I_0e^{-\frac{t}{\tau}} & \text{(During charging)} \\ -I_0e^{-\frac{t}{\tau}} & \text{(During discharging)} \end{cases}$ Where : $I_0=\frac{E}{R}$.

— The intensities of the currents in the cases of charging and discharging have opposite signs, which signifies that the currents are of opposite directions with respect to the capacitor.

— The intensity of the current, in absolute value, is always decreasing (during charging or during discharging)

— The tangent at the origin of time cuts the asymptote to the curve of the function q = f(t) at a point of abscissa τ (see the figures in the table).
This rule is valuable for the functions i = f(t) and u_C = f(t).

— At the end of charging or discharging the current in the capacitor is zero (i = 0).

2 Exercises and problems

N⁰ 1
Charging of a capacitor « 1 »

A capacitor of capacitance $C = 470\ \mu F$ is connected across a battery ($E = 6$ V).

1) Calculate the final charge of the capacitor and the electric energy stored in capacitor at end of charging.
2) Determine the intensity of the current in the circuit at the end of charging.

N⁰ 2
Charging of a capacitor « 2 »

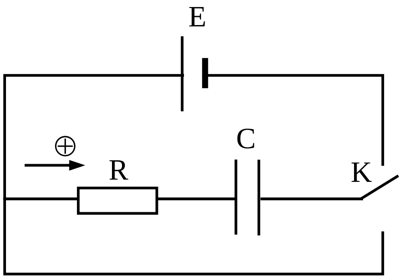
A capacitor of capacitance $C = 100\ \mu F$ is placed in series with a resistor of resistance $R = 1\ k\Omega$ across the terminals of the battery $E = 12$ V. At $t_0 = 0$ we close the switch.

1) Define the time constant τ of the circuit and calculate its value.
2) Find, in the steady state, the charge Q of the capacitor.
3) Calculate the value of the charge of the capacitor at $t = \tau$. Deduce at this same instant the voltage across the capacitor and the resistor and the intensity of current in the circuit.

N⁰ 3
Charging and discharging of a capacitor (1)

In the electric circuit of the adjacent figure the capacitor passes through two phases : Phase of charging and phase of discharging.

We represent, in figures (1) and (2) below, the curves, as a function time, of the voltage u_C across the terminals C and of the intensity i of current corresponding to phases of charging and discharging.



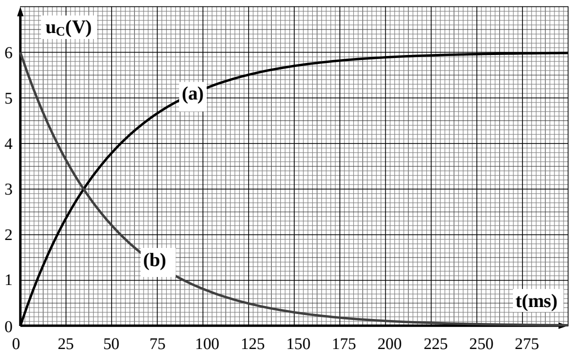


Figure (1)

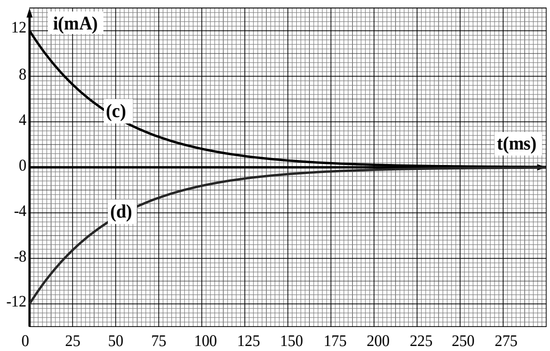
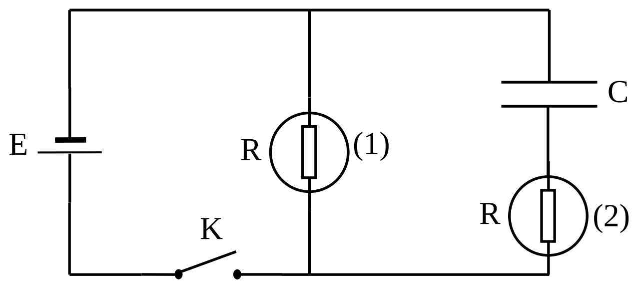


Figure (2)

1) On which position must we put the switch K in order to charge the capacitor?
2) a) Specify the curves which correspond to the phase of charging of the capacitor.
b) What are, the values of u_C and i at the beginning and at the end of charging?
c) Deduce E.
3) What are, the values of u_C and i at the beginning and at the end of discharging?
4) What is the signification of the negative sign of i in curve (d) ?
5) Verify that the time constant τ of the (R-C) circuit has the same values in the two phases and on any of the curves in figures (1) and (2).

N⁰ 4
Aspect of lighting of a lamp

We consider two identical lamps each carrying a resistor of resistance R = 50 Ω, a capacitor of capacitance C = 500 mF initially neutral, a battery of electromotive force E = 12 V and of negligible internal resistance and a switch K.
Using the preceding dipoles we connect the circuit represented in the adjacent figure.



1) At t = 0, we close K.
a) Write, with justification, the aspect of brightness of each lamp when the switch is closed.
b) Calculate, in the steady state, the intensities of the current in the different branches of the circuit and the electric energy stored in the capacitor.
c) Represent, in the same system of axes, the curves of the intensities of the currents i_1 and i_2 circulating in the lamps (1) and (2) respectively and precise on each curve two particular points.
2) The steady phase is attained, at a new instant taken as an origin of time, we open the switch.
a) Describe, with justification, the aspect of each lamp.
b) Represent the curve of the intensity of the current i circulating in the lamps. Precise on the curve two particular points.

N⁰ 5
Differential equation of charging

A capacitor of capacitance C = 100 μF is placed in series with a resistor of resistance R = 1 kΩ across the terminals of a battery E = 12 V. At $t_0 = 0$ we close the circuit.

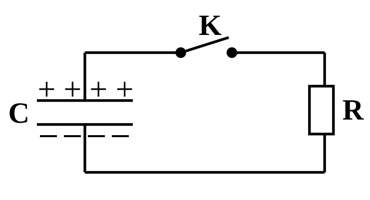
1) Give, as a function of C, R and E, the expression of the differential equation in terms of the charge q of the capacitor.

2) The solution of the differential equation is under the form : $q = \alpha e^{-\frac{t}{\tau}} + \beta$. Determine the expressions of α, τ and β as a function of C, R and E.
3) Deduce the expression of the current i in the circuit.
4) At what instant does the intensity of the current become 10 mA ?

N⁰ 6
Differential equation of discharging

We consider the set up in the adjacent figure.

The switch K is open. The capacitor is initially charged under a constant voltage E = 12 V. We close K_2 at the instant $t_0 = 0$.



Given : C = 500 μF ; R = 2 kΩ and $u_{AB} = u_C$.

1) Specify on a figure the direction of the current in the circuit.
2) Determine the following differential equation : $\frac{du_C}{dt} + \frac{u_C}{RC} = 0$ (The positive direction is that of current).
3) Show that the solution of the preceding differential equation is under the form : $u_C = Ae^{-Bt}$ where A and B are constants to be determined.
4) At what instant does the capacitor store half its initial energy ?

N⁰ 7
Characteristics of an electric dipole

An electric dipole (D) is formed in series with a capacitor of capacitance C and a resistor of resistance R.

We branch the dipole (D) across the terminals of a battery delivering a constant voltage E as shown in figure (a).

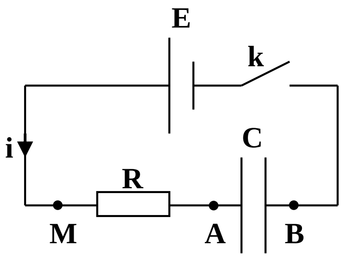
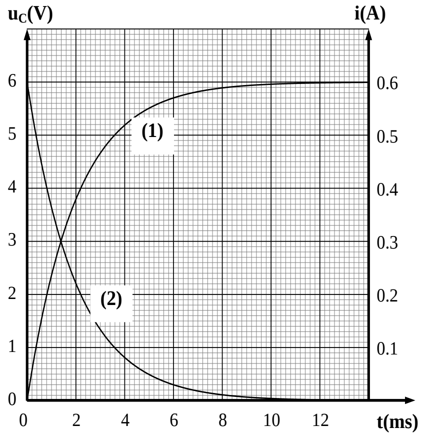


Figure (a)

Figure (b)

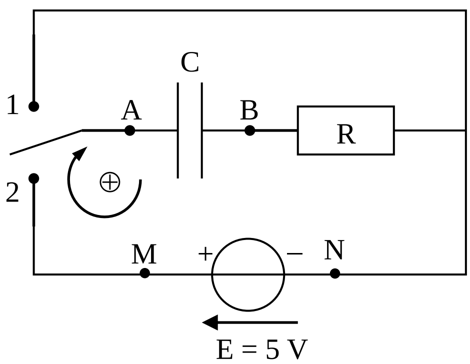


At $t_0 = 0$ we close the switch.

1) Name the phenomenon which takes place in the circuit.
2) Determine the differential equation that governs the variation of the voltage $u_C = u_{AB}$ as a function of time.
3) Verify that the solution of the preceding differential equation is : $u_C = E(1 - e^{-\frac{t}{\tau}})$ with $\tau = RC$.
4) Determine the time equation of the current i.
5) Deduce the expressions of u_C at the instants $t = \tau$ and $t \to +\infty$ and that of i at the instant $t_0 = 0$.
6) In the figure (b) we represent the graphs of u_C and i as a function of time.
a) Associate, with justification, to u_C and i the corresponding curve.
b) Deduce the values of E, R and C.

N⁰ 8
Charging and discharging of a capacitor (2)

We want to study the discharging of a capacitor, initially neutral, of capacitance C = 60 μF, across a resistor of resistance R = 10 kΩ. We use, for this aim, the setup of the adjacent figure.



I – We close the switch on position 2.
1) Name the phase of the capacitor.
2) Determine, applying law of addition of voltage and respecting the chosen positive direction, the differential equation which governs the evolution of u_{AB} as a function of time.
3) Verify that in the steady state we have : $u_{AB} = E$.
4) What minimal duration must we wait, measured from closing the switch, in order to attain the steady state ?
5) Calculate, in the steady state, the electric energy stored in the capacitor.

II – After several dozens of seconds, of closing the switch on position 2, we close the sitch on position 1 at the time t = 0.
1) The discharging of the capacitor starts at the time t = 0. What can we say, if at this time, the charging of the capacitor is completed under the voltage E?

2) Verify that :

a) The differential equation of the discharging of the capacitor is : $\alpha \frac{du_{AB}}{dt} + u_{AB} = 0$ where α is a positive constant that we must identify.

b) The solution of the differential equation is : $u_{AB}(t) = Ee^{-\frac{t}{\alpha}}$.

3) We define the duration $t_{1/2}$ such that : $u_{AB}(t_{1/2}) = \frac{E}{2}$. Calculate $t_{1/2}$ as a function of α.

4) a) Trace, in a reference, the graph of u_{AB} as a function of time and its tangent at $t = 0$.
Scale : 1 cm ↔ 0.5 s (abscissa) and 1 cm ↔ 1 V (ordinate).
b) Trace, in the preceding reference, the graph of $u_{AB}(t)$ as a function of time when we replace the resistor by another one of resistance R' = 20 kΩ and when the generator delivers a voltage E' = 2.5 V.

N⁰ 9
Evaluating physical quantities of a dipole in a circuit

We consider the circuit of figure (1) :

• C is a capacitor initially neutral;

• D_1 and D_2 are two resistors of respective resistances R_1 and R_2 ;

• The battery delivers a constant voltage U_0.

We suppose : $R = R_1 + R_2$.

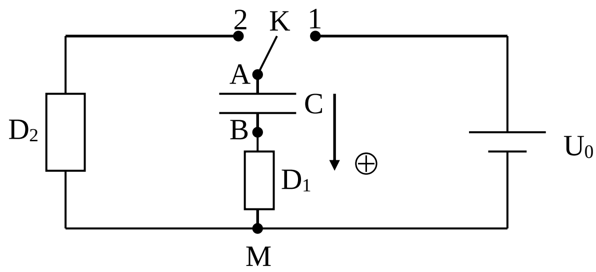


Figure (1)

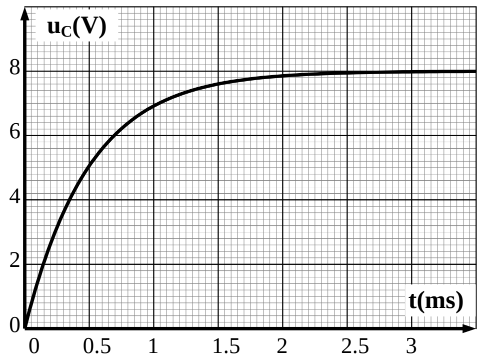


Figure (2)

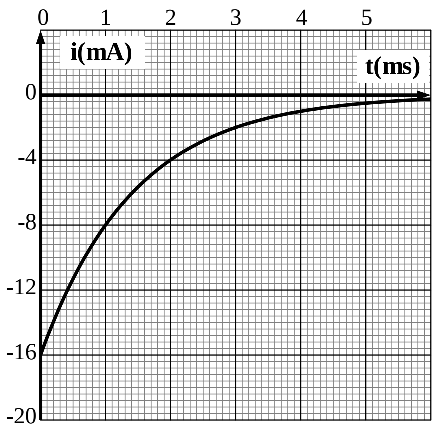


Figure (3)

I – Charging of the capacitor

At $t_0 = 0$, we close the switch K on 1.
An appropriate device permits us to record the voltage $u_{AB} = u_C$ of the capacitor as a function of time as indicated in figure (2).

1) Verify that : $i = C\frac{du_C}{dt}$

2) Determine the expression of the differential equation (E_1) of u_C as a function of time.

3) The solution of the differential equation (E_1) is under the form: $u_C = Ae^{-\frac{t}{\tau_1}} + B$. Calculate A, τ_1 and B as a function of R_1, C and U_0.

4) Verify that τ_1 is time.
5) Using the graph in figure 2, calculate U_0 and τ_1.

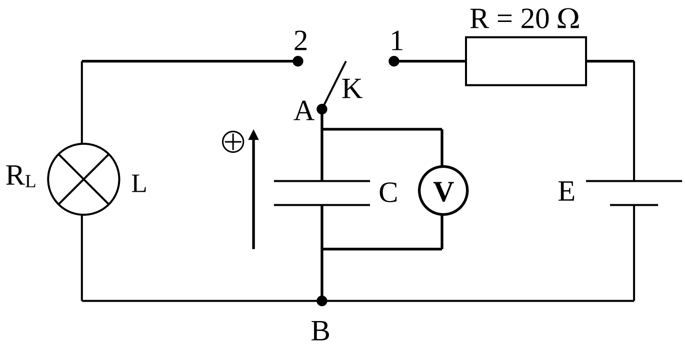
II – Discharging of the capacitor
We open K and we close it to 2 at an instant taken as a new origin of time.
An appropriate device permits us to record the intensity of the current, in the circuit, as a function of time as indicated in figure (3).

1) **a)** Show that : $u_C = -Ri$
b) Deduce the differential equation (E_2) which governs i as a function of time.
c) What is, as a function of R and U_0, the expression of the intensity of the current at $t_0 = 0$?

d) The solution of the differential equation (E_2) is under the form : $i = \alpha e^{-\frac{t}{\tau}}$. Calculate α and τ as a function of R, C and U_0.
2) Using the graph in figure 3, calculate R and τ.
3) Deduce the values of C, R_1 and R_2.

N⁰ 10
Flash of a lamp

The principle of functioning of the flash of a photographic device is represented in the circuit of the adjacent figure.



At the start of charging, the electronic switch K is in position 1. The switch K leaves position 1 and is fixed in position 2 during about **10 ms**, then automatically K retains its precious position.

A – Study of charging
We suppose that at $t_0 = 0$, K is on position 1 and the capacitor is without any charge. In the table below we indicate the effective value by the voltmeter during each five seconds.

t (s)	0	5	10	15	20	25	30	35	40	45	50	55
u_{AB} (V)	0	1.85	2.95	3.58	3.96	4.2	4.31	4.40	4.44	4.46	4.48	4.49

1) Represent, on a graph paper, the voltage u as a function of time.
Scale : 1 cm ↔ 5 s for abscissa ; 1 cm ↔ 1 V for ordinate.
2) Calculate graphically, by two methods, the charging constant τ_C of the capacitor.
3) Deduce the capacitance C of the capacitor.
4) Calculate the electric energy stored in the capacitor, at the end of charging.

B – Study of discharging
The capacitor is totally charged under the voltage of the battery. At a new origin of time, K is on position 2.

We suppose that : $u_{AB} = u$ and the lamp is assimilated to a resistor of resistance R_L.

1) **a)** Determine, as a function of R_L, C, u and $\frac{du}{dt}$, the expression of the differential equation which governs the evolution of the voltage of the capacitor as a function of time.

b) The solution of the preceding equation is under the form $u = \alpha e^{-\beta t}$. Calculate, as a function of E, R_L and C, the constants of α and β.

2) After **10 ms** from the start of discharging K is opened.

a) Calculate, at the moment of opening K, the stored energy ?

b) Deduce the average electric power consumed by the lamp.

c) Why does the lamp emit an intense flash of light ?

d) Calculate the resistance of the lamp.

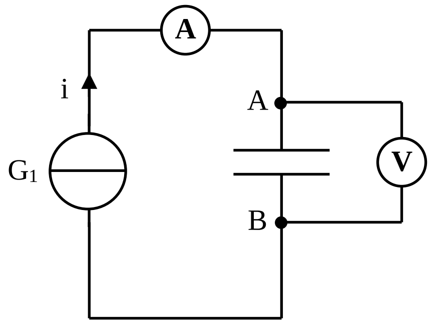
N^0 11
Calculating the capacitance and verifying some expressions

The aim of this exercise is to evaluate the capacitance C of a capacitor to verify the expression $\tau = RC$ and $i = C\frac{du_{AB}}{dt}$ when the (R-C) circuit is under a constant voltage.

To achieve this objective we have :
A capacitor of capacitance C.
A resistor of resistance R = 100 Ω.
An analogue voltmeter.
An analogue ammeter.
A generator G_1 delivering a constant current of intensity I_0.
A generator G_2 delivering a constant voltage U_0.

A – Calculating the capacitance C

We connect the circuit in the adjacent figure and each 10 seconds we give the values recorded by the multimeters in the table below.



t(s)	0	10	20	30	40	50
i (mA)	50	50	50	50	50	50
u_{AB} (V)	0	0.5	1	1.5	2	2.5

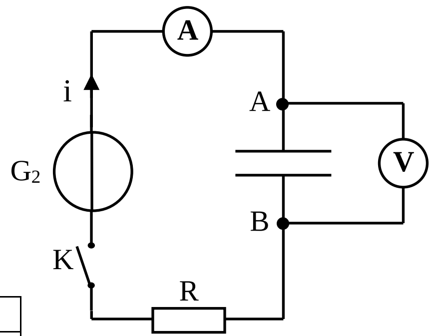
1) What is the value of I_0 ?

2) Verify that the charge of the capacitor at an instant t : $q_A = I_0 t$.

3) Show that : $C = \frac{I_0 t}{u_{AB}}$. Calculate C.

B – Verification of the expressions $\tau = RC$ and $i = C\frac{du_{AB}}{dt}$

We connect the adjacent figure and every 10 seconds we record the values indicated by the multimeters in the table below :



t (s)	0	10	20	30	40	50	60	70	80
u_{AB}(V)	0	1.14	2.17	3.11	3.95	4.72	5.41	6.0	6.6
i(mA)	120	108.6	98.25	89	80.43	72.8	65.8	59.6	54

t (s)	90	100	150	200	300	400	500	600	700
u_{AB}(V)	7.12	7.6	9.3	10.4	11.4	11.8	11.9	11.97	11.99
i(mA)	48.8	44.14	26.8	16.24	6	2.2	0.8	0.3	0.11

1) Represent u_{AB} as a function of time.

2) What is the value U_0 of the voltage of the generator ?

3) Let τ be the time constant of the circuit. We define τ to be the time at which the voltage across the capacitor becomes 63% of U_0.

a) Determine graphically the value of τ.

b) Using another graphical method find the value of τ.

c) Verify the value of τ using its expression.

4) For small intervals of time with respect to τ, we can write : $\dfrac{du_{AB}}{dt} = \dfrac{\Delta u_{AB}}{\Delta t}$.

a) Verify that the unit, in SI, of the expression $C\dfrac{\Delta u_{AB}}{\Delta t}$ is the ampere.

b) Fill in the table below.

t (s)	0	10	20	30	40	50	60
u_{AB}(V)	0	1.14	2.17	3.11	3.95	4.72	5.41
$C\dfrac{du_{AB}}{dt}$ (A)		108.5×10^{-3}		89×10^{-3}		73×10^{-3}	
i(mA)	120	108.6	98.25	89	80,43	72.8	65.8

Give a conclusion.

N^0 12
Electric energy transformed in mechanical energy

In the circuit of figure (a) we consider a capacitor (C) of capacitance C = 1 F, a generator delivering a constant voltage u_G = E = 12 V, a resistor (D) of resistance $R = 10\,\Omega$, an electric motor (M) and a switch k.

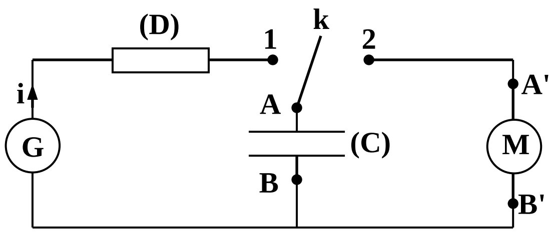


Figure (a)

A – First phase

The capacitor is initially neutral, we close at t_0 = 0, the switch to plot 1.

1) Name the phase of the capacitor.

2) Show that the intensity of the current at an instant t is given by : $i = C\dfrac{du_{AB}}{dt}$.

3) **a)** Determine the differential equation which governs the evolution of u_{AB} as a function of time.

b) The solution of the differential equation is under the form : $u_{AB} = a.e^{-b.t} + c$. Find the expressions of the constants a,b, and c as a function of E, R and C.

c) Deduce the value of u_{AB} at the end of this phase.

4) Calculate the final electric energy stored in the capacitor.

B – Second phase

The switch is set for several minutes on plot 1 then it passes automatically to plot 2, at an instant taken as a new origin of time t_0 = 0.

The variation of the voltage $u_{A'B'}$ as a function of the intensity i of current which traverses the motor is given in figure (b) in the following page.

1) Justify the direction of the electric current in the circuit as shown in figure (c) in the following page.

2) Using the graph calculate the intensity of the current in the circuit at t_0 = 0.

3) **a)** Determine the equation giving $u_{A'B'}$ as a function of i.

b) Verify that the differential equation which governs the evolution of u_{AB} as a function of time is : $2\frac{du_{AB}}{dt}+u_{AB}=6$.

c) Verify that the solution of the preceding differential equation is : $u_{AB}=6(e^{-0.5t}+1)$.

d) Calculate when $t\rightarrow+\infty$ the value of u_{AB}. Is the capacitor completely discharges? Why ?
e) Calculate the electric energy W supplied by the capacitor to the motor.

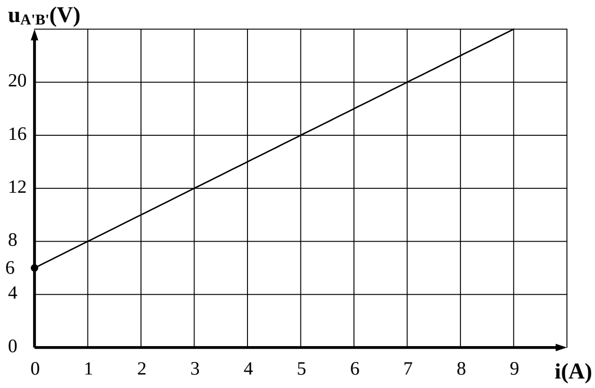


Figure (b)

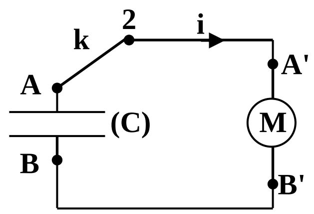
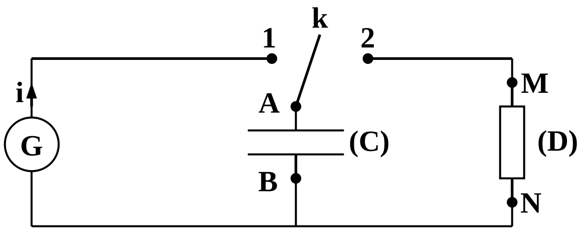


Figure (c)

4) The motor is used to lift, from rest to a height « h » and then bring back to rest, a body of mass $m=1\,kg$. Knowing that 10 % of W is transformed by the motor into mechanical energy. Calculate h.
Given g = 10 m/s^2.

N^0 13
The capacitor is a temporary storing device of energy

In the circuit of figure (a) we consider a capacitor (C) of capacitance C = 1 F, a generator (G), a resistor (D) of resistance R and a switch k.



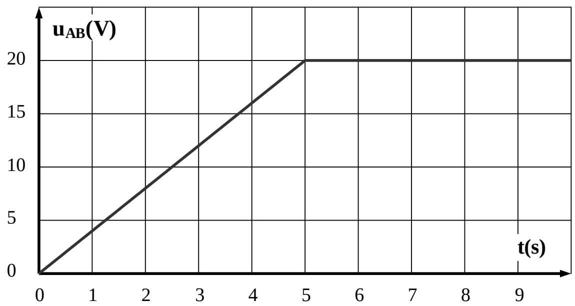


Figure (b)

A – First phase
The capacitor is initially neutral, we close at t_0 = 0, the switch to plot 1. The variation of the voltage u_{AB} of the capacitor, as a function of time, is represented in figure (b).

1) Name the phase of the capacitor.

2) Show that the intensity of the current i at an instant t is given by : $i=C\frac{du_{AB}}{dt}$.

3) Deduce the values of i during this phase.
4) At what instant is this phase over? Why ?
5) Calculate the electric energy stored in the capacitor.

B – Second phase

The switch passes, at an instant taken as an origin of time, automatically from 1 to 2.

1) Specify the direction of the electric current i in the resistor (D).

2) To what form of energy is the stored electric energy in the capacitor transformed to ? Why ?

3) Show that : $u_{MN} = -RC\dfrac{du_{AB}}{dt}$.

4) Write the differential equation which describes the variation of u_{AB} as a function of time.

5) The solution of the differential equation is under the form : $u_{AB} = a.e^{-b.t}$. Determine the expressions of the constants a and b as a function of U_0, R and C.

6) Calculate at $t = \dfrac{1}{b}$, the value of u_{AB}.

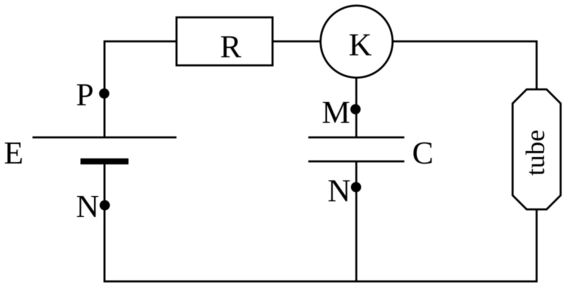
7) Represent as a function of time and in the same system, the curve of u_{AB} corresponding to $R = R_1 = 10\ \Omega$ and $R = R_2 = 50\ \Omega$.

8) The resistor (D) represents the filament of a lamp, of temporary lighting in a building. Why should we choose a lamp of resistance R_2 and not R_1 ?

N⁰ 14
Lighting of a neon tube

The neon discharge tube is an electric dipole which lights if the voltage across its terminals attains a value V_a, the tube remains lit for voltages smaller than V_a and goes off when the voltage across it becomes equal to a value V_e called voltage of extinction. ($V_e < V_a$).

This dipole acts as an open switch when turned off and as a resistor of resistance R' when it is lit.



In order to study the functioning of this dipole we consider the following circuit where K is an automatic switch used to connect and disconnect the generator in well-determined intervals of time.

A – The tube is off. (R = 100 kΩ ; C = 10 μF).

1) What is the phenomenon observed in the capacitor ?

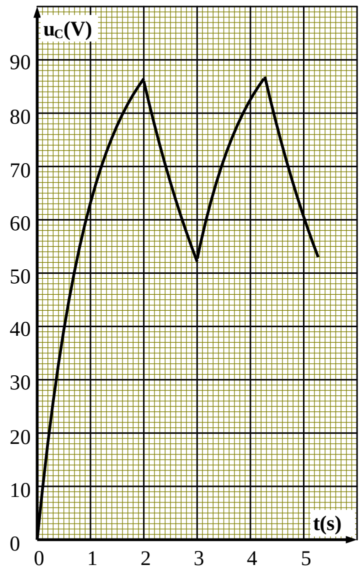
2) Establish the differential equation which describes the variation of the voltage $u_C = u_{MN}$ across the terminals of the capacitor.

3) The solution of this equation is $u_C = A(1 - e^{-\frac{t}{\tau}})$. Determine the expressions of constants A and τ. Calculate the value of τ.

4) Verify that, if the tube is not connected to the circuit, the maximum voltage attained by u_C is E.

5) After what interval of time is this maximum voltage attained ?

6) The adjacent figure represents u_C as a function of time during the charging and the discharging of the capacitor. Determine, by the aid of the graph, the value of E.



B – Tube Lit.

1) Determine, graphically, the value V_a where the tube lights up.

2) Find the extinction voltage V_e.

3) Deduce the duration of lighting of the tube.
4) Establish the differential equation in u_C during the discharging.

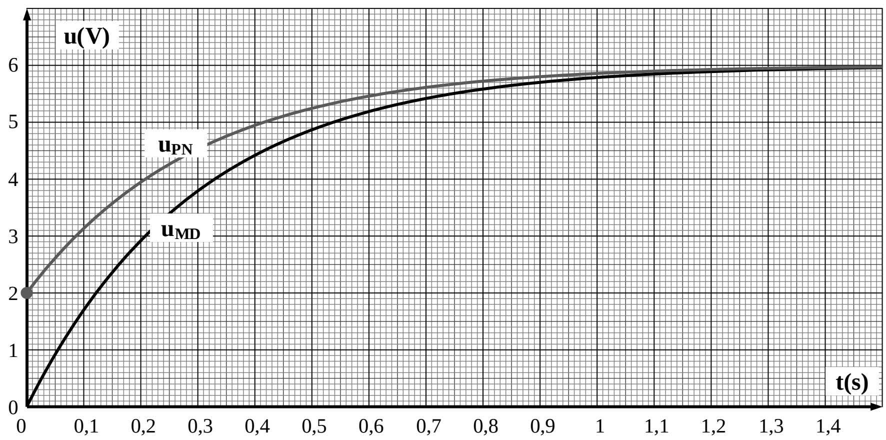
5) Let $u_C = V_a e^{-\frac{t}{\tau'}}$. We choose the instant where the tube lights as an origin of time. Calculate the value of τ'. Deduce the value of R'.

N⁰ 15
Calorific energy dissipated by a battery

A neutral capacitor of capacitance C = 1 F, is charged by a battery of e.m.f. E and of internal resistance r. An ammeter, of resistance r' is branched in series, with the capacitor and a voltmeter, of very high resistance, is branched in parallel across the capacitor.

At the time $t_0 = 0$, we close the switch, the capacitor starts charging.

A convenient device, branched in the circuit, represents the variation of the voltages u_{PN} and u_{MD} across the battery and the capacitor respectively as shown in the figure below.



1) Make a figure showing the circuit.

2) How could the ammeter be considered as an indicator to indicate that the charging phase of the capacitor has terminated ?

3) a) Find, a relation between E, r, r', C, the intensity i of current and u_{MD}.

b) Determine a relation between i, C and $\frac{du_{MD}}{dt}$.

c) Deduce the differential equation which governs the evolution of u_{MD} as a function of time.

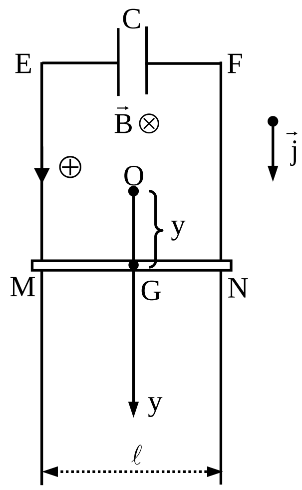
d) Show that the solution of this differential equation is : $u_{MD} = A\left(1 - e^{-\frac{t}{B}}\right)$ where A and B are constants to be determined.

4) Find u_{PN} as a function of E, r, r', C and t.

5) Choosing particular times and by the aid of the graph, calculate E ; r and r' .

6) The calorific energy dissipated by joule's effect by the battery is given by : $W = \int_0^{+\infty} ri^2 dt$. Calculate W.

N^0 16
A rod is a generator of current of constant intensity



A rectilinear rod, of length ℓ, of mass m, is launched from point O without speed at time $t_0 = 0$, is in downward translational motion along two vertical rails remaining parallel to a horizontal direction and closing the rectangular circuit (EMNF) which contains a capacitor of capacitance C initially neutral. The whole setup is placed in a uniform magnetic field and perpendicular to the plane of the rails.
We neglect friction an the resistance in the circuit.

We designate by $y = \overline{OG}$ the ordinate of the center of inertia G of the rod and by $\vec{V} = V.\vec{j}$ its speed at the time t and by g the gravitational field strength.

1) a) Verify that the rod is the seat of an i.e.m.f. such that : $u_{NM} = B\ell V$.

b) Specify the direction of current, of intensity i, which appears in the circuit.

2) a) the rod is under the action of two forces. Name these forces and give their literal expressions.

b) Applying Newton's 2nd law, show that the acceleration of the rod is : $a = g - \frac{B\ell}{m} i$.

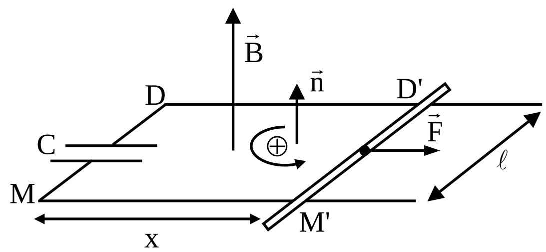
3) a) Applying on the circuit MNFE the law of addition of voltage, find the charge q_E of the armature E of the capacitor as a function of C, B, ℓ and V.

b) Deduce i as a function of *a*, C, B and ℓ.

4) a) Show that the motion of G is uniformly accelerated rectilinear translational motion.

b) Deduce that the rod is a generator of current of constant intensity : $I = \frac{m}{m + B^2\ell^2 C} B\ell C g$.

N^0 17
The capacitor moves a rod



A capacitor, of capacitance C = 1 F, is charged by a voltage of 6 V then branched across two horizontal conducting rails, separated by a distance $\ell = 10$ cm and situated in the same horizontal plane. The whole setup is placed in a uniform and vertical magnetic field $\vec{B}$ and of constant magnitude B. At the time t = 0, we place, perpendicular to the rails, a rectilinear and conducting rod, of mass m = 10 g and of resistance R, the rod starts to move remaining parallel to itself under the action of an electromagnetic force $\vec{F}$ as shown in the adjacent figure.

We neglect the resistance of the rails and friction.

1) a) Specify the direction of current i in the circuit.

b) Give, as a function of i, B and ℓ, the magnitude F of $\vec{F}$.

2) Two physical phenomena appear in the circuit. Name these phenomena.

3) a) Calculate, as a function of B, ℓ and x, the magnetic flux traversing the surface MM'D'D.

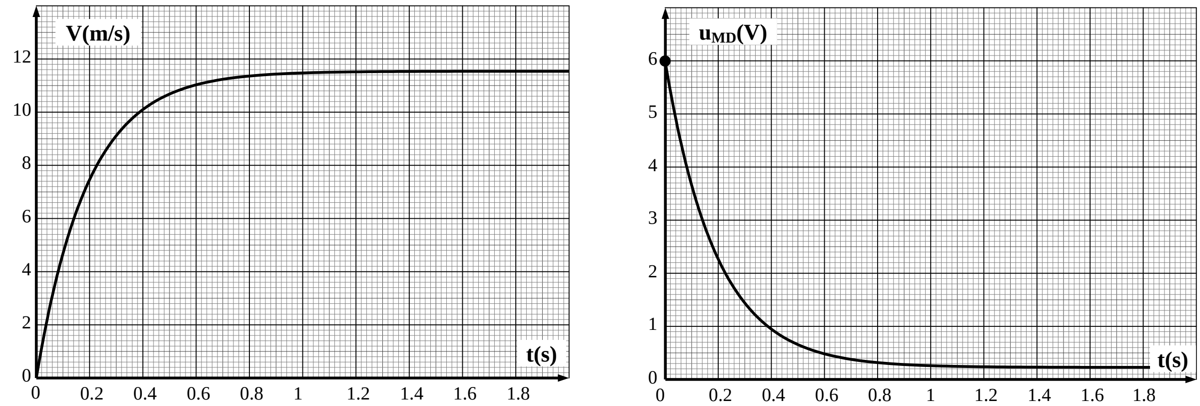
b) Deduce that the i.e.m.f. which appears in the circuit : $e = -B\ell V$ where V is the speed at time t.

4) a) Applying law of addition of voltage, show that : $u_{MD} + RC\frac{du_{MD}}{dt} - B\ell V = 0$.

b) Show, applying Newton's 2^{nd} law on the rod, that : $\frac{dV}{dt} = -\frac{CB\ell}{m}\frac{du_{MD}}{dt}$. Deduce V as a function of C, B, ℓ , m and u_{MD}.

c) Determine the differential equation which governs the evolution of u_{MD} as a function of time. Deduce its solution.

5) An advanced study permits to represent the graphs of V and of u_{MD} as a function of time.



a) The graphs show that the rod attains a limit speed and the capacitor attains a limiting voltage. Find these limiting values.

b) The capacitor does not discharge totally. Justify.

c) By the aid of the preceding equations , find the values of B, m and R.

d) Calculate the electric energy liberated by the capacitor and the variation of the kinetic energy of the rod. Making the energetic diagram of the circuit, calculate the energy dissipated by joule's effect.

3 Solutions

N^0 1
Charging of a capacitor « 1 »

1) At the end of charging the voltage across the capacitor is that across the generator : $U_C = E = 6$ V.
The charge of the capacitor : $Q = CU_C = 470\ \mu F \times 6\ V = 2820\ \mu C$. $\boxed{Q = 2820\ \mu C}$.

The energy stored in the capacitor :
$W_e = \frac{1}{2}CU_C^2 = \frac{1}{2}470\times10^{-6}\times6^2 = 8.46\times10^{-3}\,J$. $\boxed{W_e = 8.46\times10^{-3}\,J}$.

2) At the end of charging, the charge of the capacitor is constant : q = Q = constant, hence :
$i = \frac{dq}{dt} = 0\,A$. $\boxed{i = 0\,A}$.

N^0 2
Charging of a capacitor « 2 »

1) The time constant τ is the duration at which the capacitor has 63% of its maximum charge.
The time constant is given by : $\tau = RC = 1\times10^3\times100\times10^{-6} = 0.1\,s = 0.1\,s$. $\boxed{\tau = 0.1\,s}$.

2) In the steady state, which means at the end of charging : $U_C = E = 6$ V.
$Q = CU_C = 100\times10^{-6}\ F \times 12\ V = 12\times10^{-4}\ C$. $\boxed{Q = 12\times10^{-4}\,C}$.

3) $q(t=\tau) = \frac{63}{100}Q = 0.63\times12\times10^{-4} = 7.56\times10^{-4}\,C$. $\boxed{q(t=\tau) = 7.56\times10^{-4}\,C}$

$u(t=\tau) = \frac{q(t=\tau)}{C} = \frac{7.56\times10^{-4}}{100\times10^{-6}} = 7.56\ V$ $\boxed{u_C(t=\tau) = 7.56\ V}$.

From the law of addition of voltages : $E = u_C + u_R \Rightarrow u_R = E - u_C$
At $t = \tau$: $u_R = 12 - 7.56 = 4.44\ V$ $\boxed{u_R = 4.44\ V}$.
From Ohm's law : $i = \frac{u_R}{R} = \frac{4.44}{1000} = 4.44\times10^{-3}\,A$ $\boxed{i(t=\tau) = 4.44\times10^{-3}\,A}$.

N^0 3
Charging and discharging of a capacitor (1)

1) We must put K on plot 1.
2) a) During charging the voltage across the capacitor increases with time and the current circulates in the positive direction and this is verified by the curves (a) and (c).

b) Charging phase : From the curves (a) and (b).

	u_C(V)	i(mA)
Beginning : t = 0	0	12
End : t = ∞	6	0

c) At the end of charging the voltage across the capacitor is that of the generator : E = 12 V.

3) Discharging phase : From the curves (b) and (d).

	u_C(V)	i(mA)
Beginning : t = 0	6	–12
End : t = ∞	0	0

4) The negative sign of i, in discharging phase, indicates that the current circulates in the negative direction.

5)

	Charging phase	**Discharging phase**
$u_C(t = \tau)$	$0.63u_{Cmax} = 0.63\times6 = 3.78$ V	$0.37u_{C0} = 0.37\times6 = 2.22$ V
$i(t = \tau)$	$0.37i_0 = 0.37\times12 = 4.44$ mA	$0.37i_0 = 0.37\times(–12) = –4.44$ mA

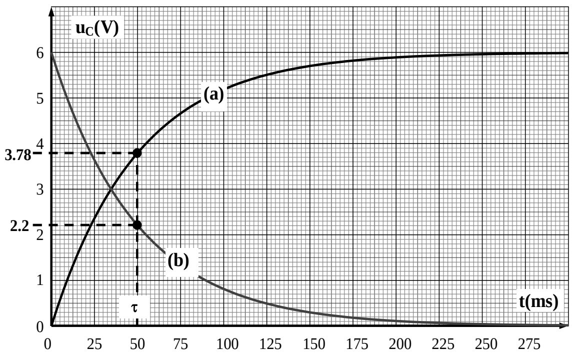


Figure (1)

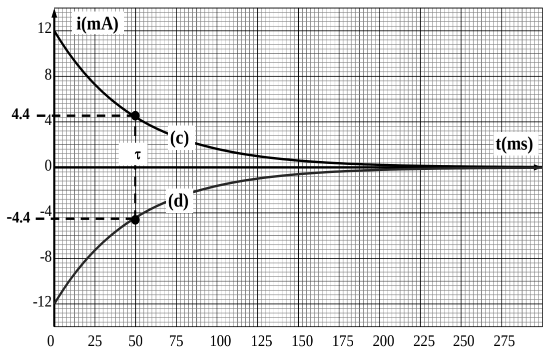


Figure (2)

N⁰ 4
Aspect of lighting of a lamp

1) a) The lamp (1) is in parallel with the battery : $u_{L1} = E = 12$ V = constant independent of time, thus the lamp (1) glows immediately and conserves the same glowing.

The lamp (2) and the capacitor are branched across the battery : $u_{L2} + u_C = E$ or $u_{L2} = 12 – u_C$
When we close K, the capacitor starts charging and its voltage increases from 0 V to 12 V and thus (from the expression : $u_{L2} = 12 – u_C$) the voltage across the lamp (2) decreases from 12 V to 0 V. Thus lamp (2) glows then turns off.
Or we say : during charging i decreases then lamp (2) glows then turns off.

b) We always have : $u_{L1} = E = 12$ V, then the intensity of current in the lamp (1) is : $i_1 = I_1 = \frac{u_{L1}}{R} = \frac{12}{50} = 0.24\,A$.

In the steady state the capacitor is totally charged, and the intensity of current is zero. Thus the intensity of current, at the end of charging, in the branch [C ; Lamp(2)] is : $i_2 = 0$ A.

The intensity i of current delivered by the generator is determined by the law of addition of current:
$i = i_1 + i_2$
In the steady state : i = 0.24 + 0 = 0.24 A.
The energy stored in the capacitor at a given time : $W = \frac{1}{2}Cu_C^2$
In the steady state : $u_C = 12$ V, then : $W = \frac{1}{2}\times0.5\times12^2 = 36\,J$

c) $i_1 = 0.24$ A = 240 mA = constant, then its graph is a horizontal straight line.

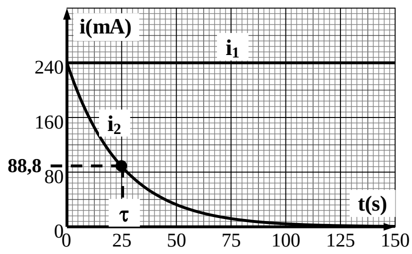
i_2 is an exponential decreasing function.

At beginning of the closure of K, the voltage of the lamp (2) is : $u_{02} = E = 12$ V (see part **1) a)**), thus the initial intensity of current : $i_{02} = \frac{u_{02}}{R} = \frac{12}{50} = 0.24\,\text{A} = 240\,\text{mA}$.

Let τ be the time constant of the circuit [C ; lamp (2)] : $\tau = RC = 50\times0.5 = 25$ s.

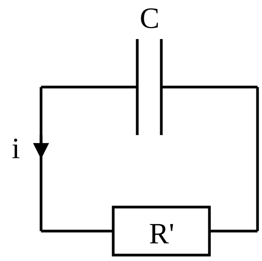
At the time $t = \tau$, the intensity of current represents 37% of its initial value (i_2 is a decreasing exponential function):
$i_2(\tau) = 0.37 i_{02} = 0.37\times240 = 88.8$ mA.

The particular points of i_2 : (0 ; 240 mA) and (25 s ; 88.8 mA).



2) a) When we open the switch, the closed circuit [C ; two lamps] is isolated, since C is initially charged, it will discharge in the lamps delivering to the circuit a current of intensity which decreases exponentially. Thus the lamps glow then turn off together.

b) The circuit [C ; two lamps] is equivalent to the circuit of the following figure (R' is a resistor equivalent to the two lamps : R' = 100 Ω):

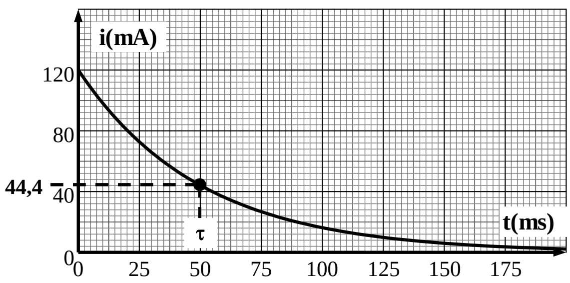


The capacitor is branched in parallel with R', thus always we have :
$u_{R'} = u_C$

At the moment we open K: $u_{0R'} = u_{0C} = 12$ V, the intensity of current at this time :
$i_0 = \frac{u_{0R'}}{R'} = \frac{12}{100} = 0.12\,\text{A} = 120\,\text{mA}$.

The time constant of the circuit: $\tau' = R'C = 100\times0.5 = 50$ s.

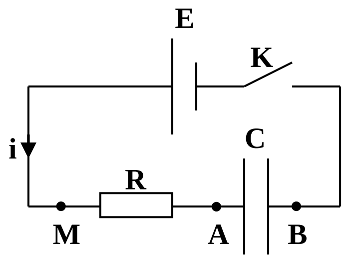
At the time $t = \tau'$: $i = 0.37 i_0 = 0.37\times120 = 44.4$ mA.



The particular points of i : (0 ; 120 mA) and (50 s ; 44.4 mA).

N⁰ 5
Differential equation of charging

1) The adjacent figure represents the electric circuit in question.



From the law of addition of voltages : $u_{MB} = u_{MA} + u_{AB}$.

With : $i = \frac{dq_A}{dt} = \frac{dq}{dt}$, $u_{MB} = E$, $u_{MA} = Ri = R\frac{dq}{dt}$ et $u_{AB} = \frac{q_A}{C} = \frac{q}{C}$.

The differential equation in q : $\boxed{E = R\frac{dq}{dt} + \frac{q}{C}}$.

2) We can write : $\frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}$

This differential equation is under the form : $\frac{dq}{dt} + aq = b$ with : $a = \frac{1}{RC}$ and $b = \frac{E}{R}$.

The solution is under the form : $q = ke^{-at} + \frac{b}{a}$ with k is a constant.

Calculating k :

At $t_0 = 0$, the capacitor is uncharged, q = 0, then : $0 = ke^0 + \frac{b}{a} \Rightarrow k = -\frac{b}{a}$

Hence : $q = -\frac{b}{a}e^{-at} + \frac{b}{a} = -ECe^{-\frac{t}{RC}} + EC$ hence : $\boxed{q = -ECe^{-\frac{t}{RC}} + EC}$

Therefore : $\alpha = -EC$, $\beta = EC$ and $\tau = RC$.

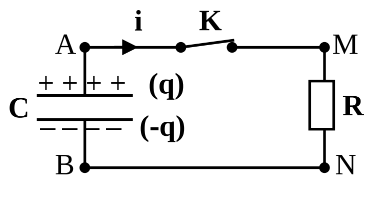
3) $i = \frac{dq}{dt} = (-ECe^{-\frac{t}{RC}} + EC)' = \frac{-EC}{-RC}e^{-\frac{t}{RC}} = \frac{E}{R}e^{-\frac{t}{RC}}$. $\boxed{i = \frac{E}{R}e^{-\frac{t}{RC}}}$.

4) We have : $i = \frac{E}{R}e^{-\frac{t}{RC}} \Rightarrow e^{-\frac{t}{RC}} = \frac{R}{E}i \Rightarrow -\frac{t}{RC} = \ln\left(\frac{R}{E}i\right) \Rightarrow t = -RC\ln\left(\frac{R}{E}i\right)$.

$t = -10^3 \times 10^{-4}\ln\left(\frac{10^3}{12}10^{-2}\right) = 0.01823s = 18.23\,ms$. $\boxed{t = 18.23\,ms}$

N⁰ 6
Differential equation of discharging

1) The electric current circulates from the positive to the negative armature through R.



2) From the law of uniqueness of voltages : $u_{AB} = u_{MN}$

with : $u_{AB} = u_C$;

The current entering armature B : $i = \frac{dq_B}{dt} = \frac{d(Cu_{BA})}{dt} = C\frac{du_{BA}}{dt} = -C\frac{du_{AB}}{dt} = -C\frac{du_C}{dt}$;

$u_{MN} = Ri = -RC\frac{du_C}{dt}$.

Then : $u_C = -RC\frac{du_C}{dt}$ The required differential equation : $\boxed{\frac{du_C}{dt} + \frac{u_C}{RC} = 0}$.

3) The preceding differential equation is under the form : $\frac{du_C}{dt} + a u_C = 0$

with $a = \frac{1}{RC} = \frac{1}{2\times10^3 \times 500\times10^{-6}} = 1\,s^{-1}$ and the solution is under the form : $u_C = ke^{-at}$.

Calculating k : At $t_0 = 0$, $u_C = E = 12$ V then $12 = ke^0 \Rightarrow k = 12$ V.

Hence : $\boxed{u_C = 12e^{-t}}$ with $\boxed{A = 12 \text{ V and } B = 1\,s^{-1}}$.

4) The electric energy stored in the capacitor at an instant t :

$$W = \tfrac{1}{2}Cu_C^2 = \tfrac{1}{2}500\times10^{-6}(12e^{-t})^2 = 0.036e^{-2t} \text{ (Expressed in J)}$$

The initial energy is at $t_0 = 0$: $W_0 = 0.036$ J

Let t_1 be the instant at which the stored energy is half : $0.036e^{-2t_1} = \frac{0.036}{2}$

$\Rightarrow e^{-2t_1} = 0.5 \Rightarrow -2t_1 = \ln(0.5) \Rightarrow t_1 = -\frac{\ln(0.5)}{2} = 0.346\,s$ or $t_1 = 346$ ms . $\boxed{t_1 = 346 \text{ ms}}$

N[0] 7
Characteristics of an electric dipole

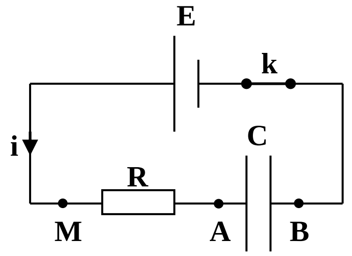
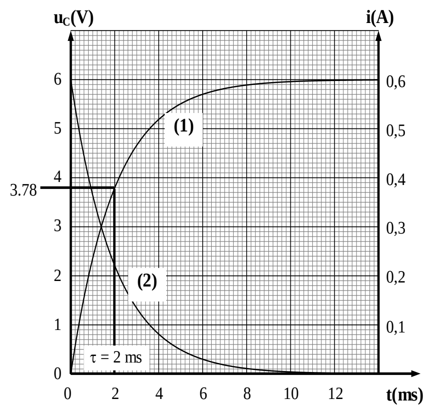


Figure (a)

Figure (b)



1) The phenomenon is called charging the capacitor.

2) From the law of addition of voltages : $u_{MB} = u_{MA} + u_{AB}$.

The current entering the armature A : $i = \frac{dq_A}{dt} = \frac{d(Cu_{AB})}{dt} = C\frac{du_{AB}}{dt}$;

with : $u_{MB} = E$, $u_{MA} = Ri = RC\frac{du_{AB}}{dt}$.

The differential equation in u_{AB} is : $\boxed{E = RC\frac{du_{AB}}{dt} + u_{AB}}$.

3) The preceding differential equation is : $\frac{du_{AB}}{dt} + \frac{u_{AB}}{RC} = \frac{E}{RC}$, hence its general form is : $\frac{du_{AB}}{dt} + a u_{AB} = b$

with : $a = \frac{1}{RC}$ and $b = \frac{E}{RC}$.

The solution of the differential equation is under the form : $u_{AB} = ke^{-at} + \frac{b}{a}$ with k being a constant.

Calculating k :

At $t_0 = 0$, the capacitor is neutral, $u_{AB} = 0$, hence : $0 = ke^0 + \frac{b}{a} \Rightarrow k = -\frac{b}{a}$

Hence : $u_{AB} = -\frac{b}{a}e^{-at} + \frac{b}{a} = \frac{b}{a}(1-e^{-at}) = E(1-e^{-\frac{t}{RC}})$ with $\tau = RC$.

4) Using : $i = C\frac{du_{AB}}{dt} = C\left[E(1-e^{-\frac{t}{RC}})\right]' = CE\left(0+\frac{1}{RC}e^{-\frac{t}{RC}}\right) = \frac{E}{R}e^{-\frac{t}{RC}}$. $\boxed{i = \frac{E}{R}e^{-\frac{t}{RC}}}$.

5) $u_{AB}(t=\tau=RC) = E(1-e^{-\frac{RC}{RC}}) = E(1-e^{-1}) = 0.63E \Rightarrow \boxed{u_{AB}(t=\tau) = 0.63E}$

$u_{AB}(t\to+\infty) = E(1-e^{-\infty}) = E(1-0) = E \Rightarrow \boxed{u_{AB}(t\to+\infty) = E}$

$i(t=0) = \frac{E}{R}e^0 = \frac{E}{R}$ or $\boxed{i_0 = \frac{E}{R}}$

6) a) We have : $u_{AB} = E(1-e^{-\frac{t}{RC}})$ is an increasing function of time, then u_{AB} corresponds to curve (1), hence i corresponds to curve (2).

b) From curve (1) : $u_{AB}(t\to+\infty) = 6\text{ V}$ hence : $\boxed{E = 6\text{ V}}$.

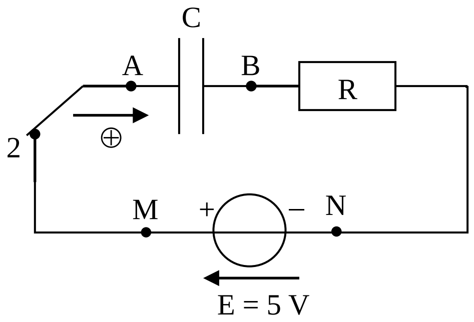
From curve (2) : i_0 = 0,6 A, but $i_0 = \frac{E}{R} \Rightarrow R = \frac{E}{i_0} = \frac{6}{0.6} = 10\,\Omega$. $\boxed{R = 10\,\Omega}$.

We can show that : $u_{AB}(t=\tau) = 0.63E = 0.63\times 6 = 3.78\text{ V}$. From curve (1) : τ = 2 ms.

But $\tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{2\times10^{-3}}{10} = 2\times10^{-4}\text{ F}$ or $\boxed{C = 200\ \mu F}$.

N⁰ 8
Charging and discharging of a capacitor (2)

I – We close switch K on position 2.



1) This is the charging phase of the capacitor.

2) According to the law of addition of voltage :

$$u_{MN} = u_{MA} + u_{AB} + u_{BN}$$

$\Leftrightarrow E = 0 + u_{AB} + Ri$; but $i = \frac{dq_A}{dt} = \frac{d(Cu_{AB})}{dt} = C\frac{du_{AB}}{dt}$

Thus the differential equation is : $\boxed{E = u_{AB} + RC\frac{du_{AB}}{dt}}$.

3) In the steady state, u_{AB} = cst $\Rightarrow \frac{du_{AB}}{dt} = 0$, from the differential equation we obtain: u_{AB} = E.

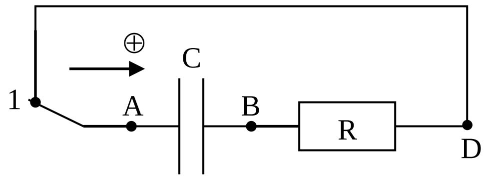
4) The minimal duration that that the capacitor needs to reach the steady state is :

$5\tau = 5RC = 5\times10\times10^3\times60\times10^{-6} = 3\text{ s}$. $\boxed{\text{This duration is 3 s}}$.

5) The electric energy stored in the capacitor :

$W = \frac{1}{2}CE^2 = \frac{1}{2}\times60\times10^{-6}\times5^2 = 7.5\times10^{-4}\text{ J}$. $\boxed{W = 7.5\times10^{-4}\text{ J}}$.

II – After several dozens of seconds, of closing the switch on position 2, we close the switch on position 1 at time t = 0.



1) We measure by a voltmeter the voltage across the terminals of a capacitor which is equal to E = 5 V at the end of charging.

2) a) $u_{AB} + u_{BD} = u_{AD} = 0 \Leftrightarrow u_{AB} + Ri = 0$

But : $i = C\dfrac{du_{AB}}{dt}$

Then : $u_{AB} + RC\dfrac{du_{AB}}{dt} = 0$, we deduce : $\boxed{\alpha = RC \text{ this is the time constant of the circuit}}$.

b) The preceding differential equation can be written : $\dfrac{du_{AB}}{dt} + \dfrac{1}{RC}u_{AB} = 0$

The general form of the differential equation : $\dfrac{du_{AB}}{dt} + au_{AB} = b$, with $a = \dfrac{1}{RC}$ and b = 0.

The solution of this equation is : $u_{AB} = ke^{-at} + \dfrac{b}{a} = ke^{-at}$

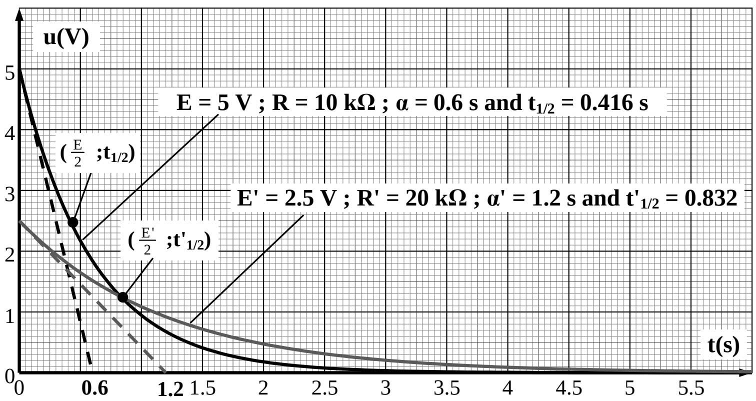
But at : t = 0, u_{AB} = E, then : $E = ke^{-a\times 0} \Rightarrow k = E$

Finally : $u_{AB}(t) = Ee^{-\frac{t}{RC}} = Ee^{-\frac{t}{\alpha}}$.

3) We have : $u_{AB}(t_{1/2}) = \dfrac{E}{2}$, then : $\dfrac{E}{2} = Ee^{-\frac{t_{1/2}}{\alpha}} \Rightarrow e^{-\frac{t_{1/2}}{\alpha}} = \dfrac{1}{2} \Rightarrow e^{\frac{t_{1/2}}{\alpha}} = 2 \Rightarrow \dfrac{t_{1/2}}{\alpha} = \ln 2 \Rightarrow t_{1/2} = \alpha \ln 2$.

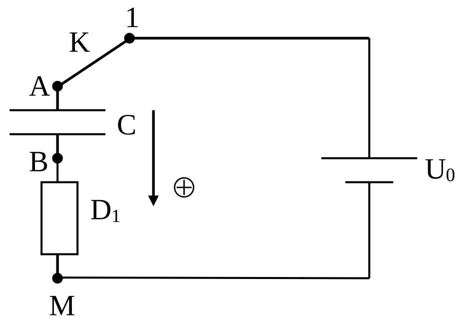
Finally : $\boxed{t_{1/2} = \alpha \ln 2}$.

4) a) b) The tangent at the origin of time cuts the asymptote (which is the axis of abscissas) at point of abscissa α = RC.



N^0 9
Evaluating physical quantities of a dipole in a circuit

I – Charging of the capacitor



1) The positive direction entering the armature A : $i = \dfrac{dq_A}{dt}$

On the other hand : $q_A = Cu_{AB} \Rightarrow i = \dfrac{d(Cu_{AB})}{dt} = C\dfrac{du_{AB}}{dt} = C\dfrac{du_C}{dt}$

2) Applying, the law of addition of voltage in the circuit in the adjacent figure:$U_0 = u_{AB} + u_{BM} = u_C + R_1 i = u_C + R_1 C\dfrac{du_C}{dt}$

The differential equation which describes the voltage across the capacitor u_C : $\boxed{(E_1): \frac{du_C}{dt} + \frac{1}{R_1C} u_C = \frac{U_0}{R_1C}}$

3) The general form of the preceding differential equation: $\frac{du_C}{dt} + au_C = b$ with $a = \frac{1}{R_1C}$ and $b = \frac{U_0}{R_1C}$ and the solution is under the form: $u_C = ke^{-at} + \frac{b}{a}$

Calculating k : At t = 0, the capacitor starts charging, $u_C = 0$: $0 = ke^0 + \frac{b}{a} \Rightarrow k = -\frac{b}{a}$

Hence : $u_C = -\frac{b}{a}e^{-at} + \frac{b}{a} = -U_0 e^{-\frac{t}{R_1C}} + U_0$.

In comparison with : $u_C = Ae^{-\frac{t}{\tau}} + B$, we find : $\boxed{A = -U_0,\ B = U_0 \text{ and } \tau_1 = R_1C}$.

And we write : $\boxed{u_C = U_0\left(1 - e^{-\frac{t}{R_1C}}\right)}$.

4) We have : $[R_1C] = \Omega \times F$; but : $C = \frac{q}{u} \Rightarrow F = \frac{C}{V}$,

$$\text{With } \left.\begin{array}{l} U = RI \Rightarrow V = \Omega \times A \\ \text{and} \\ i = \frac{dq}{dt} \Rightarrow A = \frac{C}{s} \end{array}\right\} \Rightarrow V = \Omega \times \frac{C}{s}$$

where : $F = \frac{C}{\Omega \times \frac{C}{s}} = \frac{s}{\Omega}$.

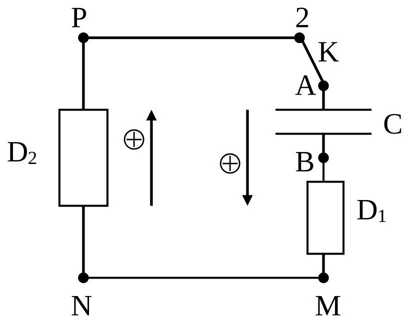
Therefore : $[R_1C] = \Omega \times \frac{s}{\Omega} = s$, hence τ_1 is expressed in s in SI and therefore it stands for time.

5) We have : $u_C = U_0\left(1 - e^{-\frac{t}{\tau_1}}\right)$, hence, when $t \to +\infty \Rightarrow e^{-\frac{t}{\tau_1}} \to 0 \Rightarrow u_C \to U_0$

From the graph, we have : $u_C \to 8\text{ V}$ when $t \to +\infty$. Hence : $U_0 = 8$ V .
The point of coordinates (0,7 ms ; 6 V) is a point on the curve u_C, hence it verifies he equation $u_C = U_0\left(1 - e^{-\frac{t}{\tau_1}}\right)$, hence : $6 = 8\left(1 - e^{-\frac{0.7}{\tau_1}}\right) \Rightarrow 1 - e^{-\frac{0.7}{\tau_1}} = \frac{6}{8} \Rightarrow e^{-\frac{0.7}{\tau_1}} = 0.25$

$\Rightarrow -\frac{0.7}{\tau_1} = \ln(0.25) \Rightarrow \tau_1 = -\frac{0.7}{\ln(0.25)} = 0.5\text{ ms}$. $\boxed{\tau_1 = 0.50\text{ ms}}$

II – Discharging of a capacitor



1) a) Applying the law of addition of voltage on the circuit in the adjacent figure : $u_{PN} = u_{AB} + u_{BM} \Rightarrow -R_2 i = u_C + R_1 i$
$\Rightarrow u_C = -(R_1 + R_2)i = -Ri$ (since $R = R_1 + R_2$).

b) But : $i = C\frac{du_C}{dt} = C\frac{d(-Ri)}{dt} = -RC\frac{di}{dt}$

$$\Rightarrow i + RC\frac{di}{dt} = 0 \text{ or } \boxed{(E_2): \frac{di}{dt} + \frac{1}{RC}i = 0}$$

c) The voltage across the terminals of C at the instant K is set on 2 is :

$u_{AB0} = u_{C0} = U_0 = 8$ V.

We have shown in the preceding part that : $u_C = -Ri \Rightarrow u_{C0} = -Ri_0 \Rightarrow i_0 = -\frac{U_0}{R}$

d) The general form of the differential equation (E_2) : $\frac{di}{dt} + a'i = 0$ with $a' = \frac{1}{RC}$, and whose solution is under the form: $i = k'e^{-a't}$.

Calculating k' : At t = 0, we have : $i = i_0 = -\frac{U_0}{R}$, hence : $-\frac{U_0}{R} = k'e^0 \Rightarrow k' = -\frac{U_0}{R}$.

$\boxed{i = -\frac{U_0}{R}e^{-\frac{t}{RC}}}$, we deduce that : $\boxed{\alpha = -\frac{U_0}{R} \text{ and } \tau = RC}$.

2) From the graph of figure 3 : $i_0 = -16\text{mA}$;

with $i_0 = -\frac{U_0}{R} \Rightarrow R = -\frac{U_0}{i_0} = -\frac{8}{16\times10^{-3}} = 500\ \Omega$

From the graph of figure 3 : the point of coordinates (3 ms ; – 2 mA) is a point on the graph.

With : $i = -\frac{U_0}{R}e^{-\frac{t}{RC}} = i_0 e^{-\frac{t}{\tau}} \Rightarrow -2 = -16e^{-\frac{3}{\tau}} \Rightarrow e^{-\frac{3}{\tau}} = \frac{2}{16} \Rightarrow -\frac{3}{\tau} = \ln(\frac{2}{16}) \Rightarrow \tau = -\frac{3}{\ln(\frac{2}{16})} \approx 1.44\text{ms}$.

$$\boxed{R = 500\ \Omega \text{ et } \tau = 1.44\text{ ms}}.$$

3) We have : $\tau = RC \Rightarrow C = \frac{\tau}{R} = \frac{1.44\times10^{-3}}{500} = 2.88\times10^{-6}\text{F} = 2.88\ \mu\text{F}$. $\boxed{C = 2.88\ \mu\text{F}}$

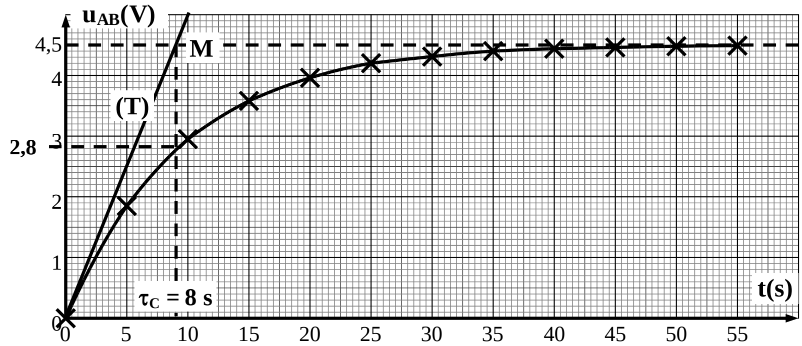
We have : $\tau_1 = R_1C \Rightarrow R_1 = \frac{\tau_1}{C} = \frac{0.5\times10^{-3}}{2.88\times10^{-6}} \approx 173.6\Omega$ $\boxed{R_1 \approx 173.6\ \Omega}$.

We have : $R = R_1 + R_2$, hence : $R_2 = R - R_1 = 500 - 173.6 = 326.4\ \Omega$. $\boxed{R_2 = 326.4\ \Omega}$.

N^0 10
Flash of a lamp

A – Study of charging

1)



2) When the capacitor is completely charge its voltage remains constant and equal to E. From the graph it is: $U_0 = E = 4.5$ V.

First method : At t = τ_C, $u_{AB} = 0.63U_0 = 2.835$ V, using the graph we get : $\boxed{\tau_C = 8\text{ s}}$.

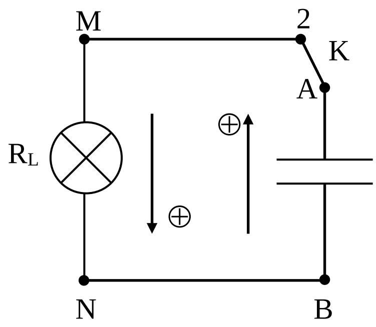
Second method : The tangent (T) at the origin of time cuts the asymptote of equation $u_{AB} = 4.5$ V at a point M of abscissa τ_C. We find : $\boxed{\tau_C = 8\text{ s}}$.

3) We have : $\tau_C = RC \Rightarrow C = \frac{\tau_C}{R} = \frac{8}{20} = 0.4\text{F}$. $\boxed{C = 0.4\text{ F}}$.

4) The electric energy stored in the capacitor at the end of charging:
$W = \frac{1}{2}CU_0^2 = \frac{1}{2}0.4 \times 4.5^2 = 4.05\text{J}$. $\boxed{W = 4.05\text{ J}}$.

B – Study of discharging

1) a) Taking the positive direction represented in the adjacent figure :



$$i = \frac{dq_B}{dt} = -\frac{dq_A}{dt} = -\frac{d(Cu_{AB})}{dt} = -C\frac{du_{AB}}{dt} = -C\frac{du}{dt}$$

On the other hand : $u_{AB} = u_{MN} \Rightarrow u = R_L i \Rightarrow u = -R_L C\frac{du}{dt}$

Where: $\boxed{\frac{du}{dt} + \frac{1}{R_L C}u = 0}$.

b) The general form of the preceding differential equation : $\frac{du}{dt} + au = 0$

With $a = \frac{1}{R_L C}$ and with a solution under the form : $u = ke^{-at}$

Calculating k : At t = 0, the capacitor starts discharging, u = E = 4.5 V : $E = ke^0 \Rightarrow k = E \Rightarrow u = Ee^{-\frac{t}{R_L C}}$

Comparing with : $u = \alpha e^{-\beta t}$, we find : $\boxed{\alpha = E}$ and $\boxed{\beta = \frac{1}{R_L C}}$.

2) a) The electric energy stored in the capacitor at the instant t = 10 ms :
$W' = \frac{1}{2}Cu^2 = \frac{1}{2}0.4 \times 2^2 = 0.8\text{ J}$. $\boxed{W' = 0.8\text{ J}}$.

b) The electric energy initially stored in the capacitor: W = 4.05 J.
The electric energy stored in the capacitor after 10 ms of discharging : W' = 0.8 J.

The transfer of energy to the lamp during 10 ms = the energy lost by the capacitor during the same time = $\Delta W = W - W' = 4.05 - 0.8 = 3.25$ J.

The average power of the lamp : $\boxed{P_m = \frac{\Delta W}{\Delta t} = \frac{3.25}{10 \times 10^{-3}} = 325\text{ W}}$

c) The lamp emits a flash since it lights up in a very short duration (10 ms) consuming a relatively large energy during this duration.

d) We have shown that the voltage across the lamp is : $u = Ee^{-\frac{t}{R_L C}} = 4.5e^{-\frac{t}{0.4R_L}}$, the lamp turns off at t = 10 ms and when u = 2 V, then :

$$2 = 4.5e^{-\frac{10\times10^{-3}}{0.4R_L}} \Rightarrow e^{-\frac{1}{40R_L}} = \frac{2}{4.5} \Rightarrow -\frac{1}{40R_L} = \ln\left(\frac{2}{4.5}\right) \Rightarrow R_L = -\frac{1}{40\ln\left(\frac{2}{4.5}\right)} = 0.03\,\Omega \text{ . } \boxed{R_L = 0.03\,\Omega}.$$

N⁰ 11
Calculating the capacitance and verifying some expressions

A – Calculating the value of C

1) From the table : $\boxed{I_0 = 50\text{ mA}}$.

2) We have : $i = I_0 = \frac{dq_A}{dt} \Rightarrow q_A = \int I_0 dt = I_0 t + q_{0A}$

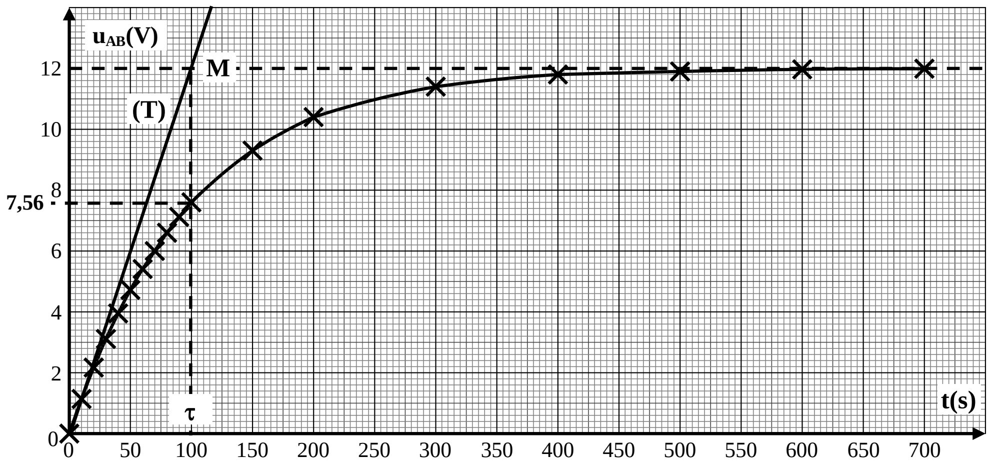
At t = 0, $u_{AB} = 0$ hence $q_A = 0$ or $q_{0A} = 0$, hence : $\boxed{q_A = I_0 t}$.

3) On the other hand : $q_A = Cu_{AB} \Rightarrow C = \frac{q_A}{u_{AB}} = \frac{I_0 t}{u_{AB}}$.

At t = 30 s, u_{AB} = 1.5 V, hence : $\boxed{C = \frac{I_0 t}{u_{AB}} = \frac{50\times10^{-3}\times30}{1.5} = 1\,F}$.

B – Verification of the expressions $\tau = RC$ and $i = C\frac{du_{AB}}{dt}$

1)



2) In the steady state the voltage of the capacitor becomes constant and equal to that of the generator: U_0 = 12 V.

3) a) At t = τ, $u_C = 0.63U_0 = 0.63\times12 = 7.56$ V, from the graph : τ = 100 s.

b) The tangent (T) at the origin of time cuts the asymptote of an equation u = 12 V at a point M of abscissa τ. We find : $\boxed{\tau = 100\text{ s}}$.

c) $\tau = RC = 100\times1 = 100$ s, hence the expression is verified.

4) a) $$\left[C\frac{\Delta u_{AB}}{\Delta t}\right] = \underbrace{F}_{C=\frac{q}{U}} \times \frac{V}{s} = \frac{C}{V}\times\frac{V}{s} = \frac{C}{\underbrace{s}_{i=\frac{q}{t}}} = A$$

b) At t_2 = 20 s : $C\frac{\Delta u_{AB}}{\Delta t} = C\frac{u_{AB}(t_3 = 30s) - u_{AB}(t_1 = 10s)}{t_3 - t_1} = 1\frac{3.11 - 1.14}{30 - 10} = 98.5\times10^{-3}A$

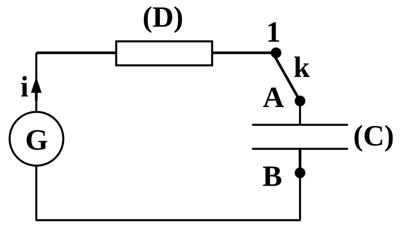
At t_4 = 40 s : $C\frac{\Delta u_{AB}}{\Delta t} = C\frac{u_{AB}(t_5 = 50s) - u_{AB}(t_3 = 30s)}{t_3 - t_1} = 1\frac{4.72 - 3.11}{30 - 10} = 80.5\times10^{-3}A$.

t (s)	0	10	20	30	40	50	60
u_{AB}(V)	0	1.14	2.17	3.11	3.95	4.72	5.41
$C\frac{du_{AB}}{dt}$ (A)		108.5×10^{-3}	$\mathbf{98.5\times10^{-3}}$	89×10^{-3}	$\mathbf{80.5\times10^{-3}}$	73×10^{-3}	
i(mA)	120	108.6	98.25	89	80.43	72.8	65.8

Conclusion: The table verifies the expression: $i = C\frac{du_{AB}}{dt}$ **.**

N⁰ 12
Electric energy transformed into mechanical energy

A – First phase



1) This is the charging phase of the capacitor.

2) Referring to the adjacent figure, the electric current entering the armature A of the capacitor, hence : $i = \frac{dq_A}{dt} = \frac{d(Cu_{AB})}{dt} = C\frac{du_{AB}}{dt}$.

3) a) From the law of addition of voltages : $u_G = u_D + u_{AB} \Rightarrow E = Ri + u_{AB} \Rightarrow \boxed{E = RC\frac{du_{AB}}{dt} + u_{AB}}$

b) The differential equation can be written under the form : $\frac{du_{AB}}{dt} + \frac{u_{AB}}{RC} = \frac{E}{RC}$ whose general form is : $\frac{du_{AB}}{dt} + \alpha u_{AB} = \beta$ with $\alpha = \frac{1}{RC}$ and $\beta = \frac{E}{RC}$ and whose general solution : $u_{AB} = k.e^{-\alpha.t} + \frac{\beta}{\alpha}$.

At t = 0, we have : u_{AB} = 0 hence : $0 = ke^0 + \frac{\beta}{\alpha} \Rightarrow k = -\frac{\beta}{\alpha} = -E$.

Where : $u_{AB} = -E.e^{-\frac{t}{RC}} + E$.

By comparison with : $u_{AB} = a.e^{-b.t} + c$ we get : $\boxed{a = -E\ ,\ b = \frac{1}{RC}\ \text{and}\ c = E}$.

c) At the end of charging : $t \to +\infty$, then : $u_{AB} \to -E.e^{-\infty} + E \Rightarrow u_{AB} \to E$ or $\boxed{u_{AB} = E = 12\text{ V}}$.

4) The electric energy stored in the capacitor at the end of charging : $W_e = \frac{1}{2}CU_0^2 = \frac{1}{2}\times1\times12^2 = 72\,J$.

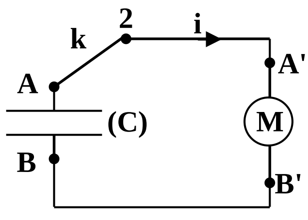
B – Second phase

1) We have : $u_{AB} > 0$, hence $V_A > V_B$ and the current circulates towards the less potential, which means from A to B in the circuit.

2) At each instant we have : $u_{A'B'} = u_{AB}$.
At $t_0 = 0$, we have : u_{AB} = 12 V, then $u_{A'B'}$ = 12 V, from the graph we find : $\boxed{i = 3\text{ A}}$.

3) a) $u_{A'B'}$ is a linear function of i, hence : $u_{A'B'} = m.i + n$
With : $m = \text{slope} = \frac{\Delta u_{A'B'}}{\Delta i} = \frac{12-6}{3-0} = 2\text{ V/A}$ and n = 6 V, hence : $\boxed{u_{A'B'} = 2.i + 6}$.

b) Referring to the adjacent figure :



$$i = \frac{dq_B}{dt} = -\frac{dq_A}{dt} = -\frac{d(Cu_{AB})}{dt} = -C\frac{du_{AB}}{dt} = -\frac{du_{AB}}{dt}$$ ….. (C = 1 F).

Using the law of uniqueness of voltages : $u_{AB} = u_{A'B'} \Rightarrow u_{AB} = 2.i + 6$

Hence : $u_{AB} = -2\frac{du_{AB}}{dt} + 6 \Rightarrow \boxed{2\frac{du_{AB}}{dt} + u_{AB} = 6}$.

c) The differential equation can be written under the form : $\frac{du_{AB}}{dt} + \frac{u_{AB}}{2} = 3$ of general form : $\frac{du_{AB}}{dt} + \alpha' u_{AB} = \beta'$ with $\alpha' = \frac{1}{2}$ and $\beta' = 3$ and whose general solution is : $u_{AB} = k'.e^{-\alpha.t} + \frac{\beta'}{\alpha'} = k'.e^{-\frac{t}{2}} + 6$.

At t = 0, we have : u_{AB} = 12 V hence : $12 = k'.e^0 + 6 \Rightarrow k' = 6$ .Where : $\boxed{u_{AB} = 6.e^{-\frac{t}{2}} + 6 = 6(e^{-0.5t} + 1)}$.

d) When $t \to +\infty$: $u_{AB} \to 6(e^{-\infty} + 1) = 6V \Rightarrow \boxed{u_{AB} = 6V}$.

e) The capacitor is not discharges totally since $t \to +\infty$, u_{AB} = 6 V $\neq$ 0 V.

The electric energy stored in the capacitor when $t \to +\infty$: $W'_e = \frac{1}{2}Cu_{AB}^2 = \frac{1}{2}\times 1\times 6^2 = 18\,J$.

The electric energy transferred to the motor: $\boxed{W = W_e - W'_e = 72 - 18 = 54\ J}$.

4) The 10 % of the electric energy consumed by the motor is transformed into gravitational potential energy : $\frac{10}{100}W = mgh \Rightarrow h = \frac{W}{10mg} = \frac{54}{10\times 1\times 10} = 0.54$ m .$\boxed{h = 0.54\ m}$

N⁰ 13
The capacitor as a temporary storing device of energy

A – First phase

1) This is the charging phase of the capacitor.

2) The electric current entering the armature A of the capacitor : $i = \frac{dq_A}{dt} = \frac{d(Cu_{AB})}{dt} = C\frac{du_{AB}}{dt}$.

3) Figure (b) shows that the graph of u_{AB} as a function of time is formed of segments, then : $\frac{du_{AB}}{dt} = \frac{\Delta u_{AB}}{\Delta t}$.

In the interval of time [0 ; 5 s] : $\boxed{i = C\frac{\Delta u_{AB}}{\Delta t} = 1\frac{20-0}{5-0} = 4\,A}$.

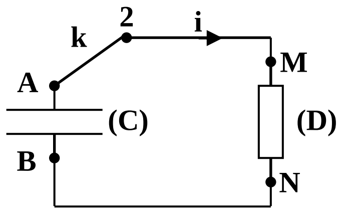
In the interval of time [5 s ; +∞] : $\boxed{i = 0\ A}$ since u_{AB} is constant.

4) The charging of the capacitor ends at t_1 = 5 s, since at this instant i = 0 A or u_{AB} becomes constant.

5) Using the graph: $\boxed{U_0 = 20\ V}$ and the electric energy stored in the capacitor :

$\boxed{W_e = \frac{1}{2}CU_0^2 = 200\ J}$.

B – Second phase



1) We have : $u_{AB} > 0$, hence $V_A > V_B$ and the current circulates towards the lower potential, which means from A to B in the circuit or from M to N in (D).

2) The capacitor discharges in (D), hence the electric energy stored in the capacitor is transformed totally into thermal energy by (D).

3) Referring to the adjacent figure : $u_{MN} = Ri = R\dfrac{dq_B}{dt} = -R\dfrac{dq_A}{dt} = -R\dfrac{d(Cu_{AB})}{dt} = -RC\dfrac{du_{AB}}{dt}$.

4) Using the law of addition of voltages : $u_{AB} = u_{MN} \Rightarrow u_{AB} = -RC\dfrac{du_{AB}}{dt} \Rightarrow \boxed{RC\dfrac{du_{AB}}{dt} + u_{AB} = 0}$.

5) The differential equation : $\dfrac{du_{AB}}{dt} + \dfrac{u_{AB}}{RC} = 0$ whose general form is : $\dfrac{du_{AB}}{dt} + \alpha u_{AB} = \beta$ with $\alpha = \dfrac{1}{RC}$ and $\beta = 0$ and whose general solution : $u_{AB} = k.e^{-\alpha.t} + \dfrac{\beta}{\alpha} = k.e^{-\alpha.t}$.

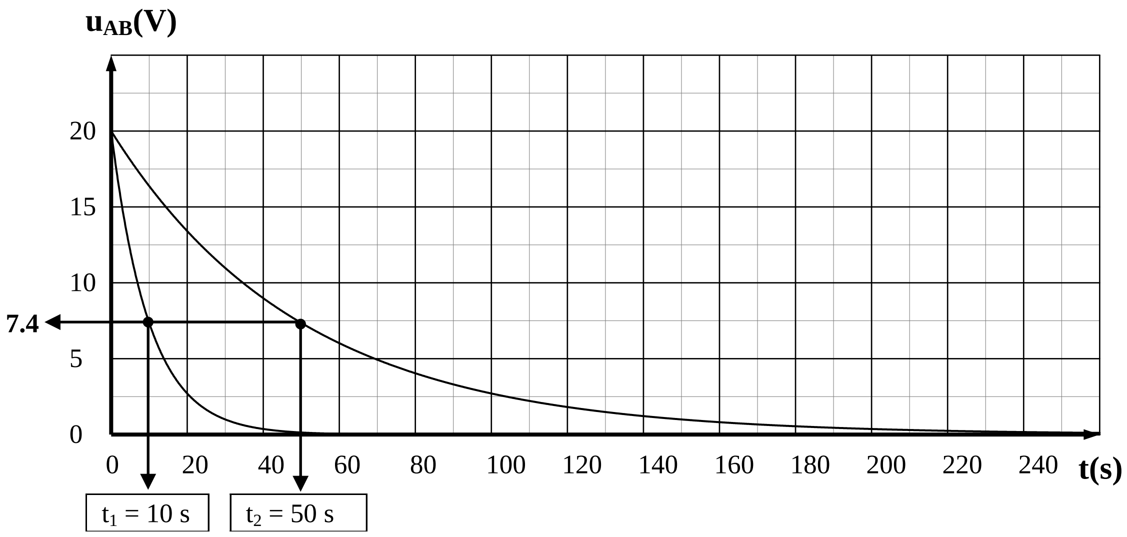
At t = 0, we have : $u_{AB} = U_0$ hence : $U_0 = k.e^0 \Rightarrow k = U_0$.

Where : $u_{AB} = U_0.e^{-\frac{t}{RC}}$.

Comparing with : $u_{AB} = a.e^{-b.t}$ we have : $\boxed{a = U_0 \text{ and } b = \dfrac{1}{RC}}$.

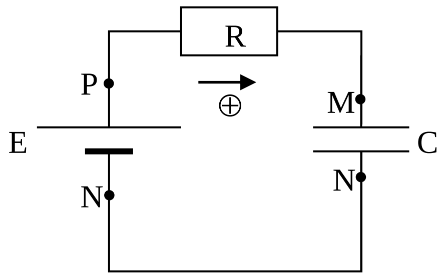
6) At $t = \dfrac{1}{b} = RC$: $u_{AB} = U_0.e^{-\frac{RC}{RC}} = U_0e^{-1} = 0.37U_0 = 0.37 \times 20 = 7.4$ V . $\boxed{u_{AB} = 7.4 \text{ V}}$

7) For $t_1 = R_1C = 10$ s or $t_2 = R_2C = 50$ s we have : $u_{AB} = 7.4$ V.



8) We choose a lamp of resistance R_2 since the capacitor discharges slowly and the lamp lights up for a longer time.

N^0 14
Lighting of a neon tube



A – The tube is off. (R = 100 kΩ ; C = 10 μF).

1) This is the phenomenon of charging of the capacitor.

2) Applying the law of addition of voltage :

$u_{PN} = u_{PM} + u_{MN} \Leftrightarrow E = Ri + u_C$;

on the other hand : $i = \frac{dq_M}{dt} = \frac{d(Cu_{MN})}{dt} = C\frac{du_{MN}}{dt} = C\frac{du_C}{dt}$;

and therefore : $E = RC\frac{du_C}{dt} + u_C \Leftrightarrow \frac{du_C}{dt} + \frac{1}{RC}u_C = \frac{E}{RC}$.

3) The preceding differential equation is under the form : $\frac{du_C}{dt} + au_C = b$ where $a = \frac{1}{RC}$ and $b = \frac{E}{RC}$

And whose solution is : $u_C = ke^{-at} + \frac{b}{a}$.

But at t = 0, first when closing the switch the capacitor is uncharged, $u_C = 0$, then :

$0 = ke^{-a\times 0} + \frac{b}{a} \Rightarrow k = -\frac{b}{a}$

Finally : $u_C = \frac{b}{a}(1 - e^{-at}) = E(1 - e^{-\frac{t}{RC}})$ and identification of the expression with $u_C = A(1 - e^{-\frac{t}{\tau}})$ we obtain :

$\boxed{A = E \text{ and } \tau = RC}$. $\tau = 100 \times 10^5 \times 10 \times 10^{-6} = 1\,s$.$\boxed{\tau = 1\,s}$.

4) In this case the capacitor continues charging as time passes, at the end and when t is large enough ($t = +\infty$) : $u_{C\max} = u_C(t = +\infty) = E(1 - e^{-\infty}) = E$.

5) The capacitor is said to be charged or attains a maximal voltage after a duration equal to : $\boxed{5\tau = 5\ s}$.

6) At the time : t = τ = 1 s we have, from the graph, that : $u_C(\tau) \approx 63$ V.

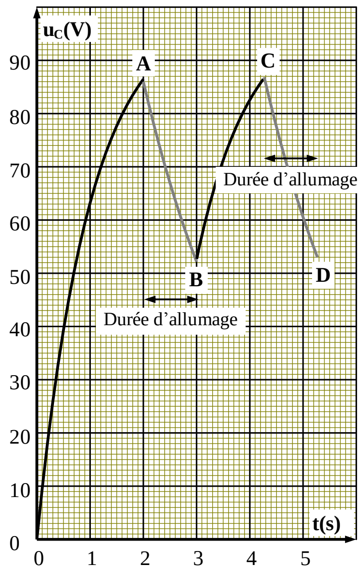
But we know that : $u_C(\tau) = 0.63E \Leftrightarrow E = \frac{u_C}{0.63} = \frac{63}{0.63} = 100\,V$.

Another method (we choose a point from the graph during charging phase) :

From the graph we have : $u_C(t = 0.5\,s) = 40$ V .

But : $u_C = E(1 - e^{-\frac{t}{RC}}) \Leftrightarrow E = \frac{u_C}{1 - e^{-\frac{t}{RC}}} = \frac{40}{1 - e^{-\frac{0.5}{1}}} \Leftrightarrow E = 101.6\,V$

or $\boxed{E \cong 100\ V}$.



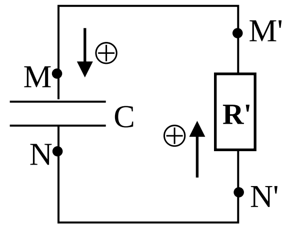
B – Tube lit.

1) When the tube is lit it is equivalent to a resistor and the capacitor starts to discharge this is the end of charging and the discharging started (point A or C or …), then from the graph : $\boxed{V_a = 86\ V}$.

2) When the capacitor starts charging the tube is off, thus this is the end of discharging and the start of the charging (point B or D or ….), then from the graph : $\boxed{V_e = 52\ V}$.

3) The duration of lighting of the tube this is the duration of discharging (between A and B or between C and D), from the graph : $\boxed{1\,s}$.

4) From the adjacent figure :



$u_{MN} = u_{M'N'} \Leftrightarrow u_C = -Ri$

$\Leftrightarrow u_C = -RC\dfrac{du_C}{dt}$

$\Leftrightarrow \dfrac{du_C}{dt} + \dfrac{1}{RC}u_C = 0$.

(i remains always $RC\dfrac{du_C}{dt}$ since we didn't change the positive sense with respect to the capacitor)

5) The instant where the tube lights (t_1 = 2 s) considered as origin of time.
At the end of the first discharge (u_C = 52 V and t_2 = 3 s) according to the new origin of time :

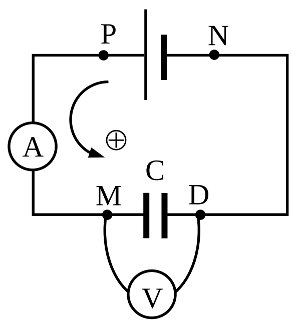
t = 1 s and u_C = 52 V and from : $u_C = V_a e^{-\frac{t}{\tau'}}$ we have :

$$52 = 86e^{-\frac{1}{\tau'}} \Leftrightarrow e^{-\frac{1}{\tau'}} = \frac{52}{86} \Leftrightarrow -\frac{1}{\tau'} = \ln\left(\frac{52}{86}\right) \Leftrightarrow \tau' = -\frac{1}{\ln\left(\frac{52}{86}\right)} \approx 2s\,.$$

But : $\boxed{R' = \dfrac{\tau'}{C} = 200\,k\Omega}$.

N⁰ 15
Calorific energy dissipated by a battery

1) See the adjacent figure.



2) At the end of charging, the intensity of current becomes zero.

3) a) From the law of addition of voltage : $u_{PN} = u_{PM} + u_{MD} + u_{DN}$

Then : $E - ri = r'i + u_{MD} + 0 \Leftrightarrow \boxed{E = (r + r')i + u_{MD}}$.

b) $i = \dfrac{dq_M}{dt} = \dfrac{d(Cu_{MD})}{dt} = C\dfrac{du_{MD}}{dt}$. $\boxed{i = C\dfrac{du_{MD}}{dt}}$.

c) From the preceding : $E = (r + r')i + u_{MD}$ and $i = C\dfrac{du_{MD}}{dt}$

We obtain the following differential equation : $\boxed{E = (r + r')C\dfrac{du_{MD}}{dt} + u_{MD}}$.

d) We have : $u_{MD} = A\left(1 - e^{-\frac{t}{B}}\right)$ then : $\dfrac{du_{MD}}{dt} = A\left(0 + \dfrac{1}{B}e^{-\frac{t}{B}}\right) = \dfrac{A}{B}e^{-\frac{t}{B}}$

Replacing these expressions in the differential equation :

$$E = (r + r')C \times \frac{A}{B}e^{-\frac{t}{B}} + A(1 - e^{-\frac{t}{B}}) \Leftrightarrow E = A + A\left(1 - \frac{(r + r')C}{B}\right)e^{-\frac{t}{B}}$$

$$E + 0.e^{-\frac{t}{B}} = A + A\left(1 - \frac{(r + r')C}{B}\right)e^{-\frac{t}{B}}$$

Identifying the two members of the equation : $\boxed{A = E}$ and $A\left(1 - \dfrac{(r + r')C}{B}\right) = 0 \Leftrightarrow \boxed{B = (r + r')C}$.

4) We have : $u_{MD} = E\left(1 - e^{-\frac{t}{(r+r')C}}\right)$.

Then : $i = C\dfrac{du_{MD}}{dt} = CE\left(0 + \dfrac{1}{(r + r')C}e^{-\frac{t}{(r+r')C}}\right) = \dfrac{E}{r + r'}e^{-\frac{t}{(r+r')C}}$.

But : $u_{PN} = E - ri = E - r\dfrac{E}{r + r'}e^{-\frac{t}{(r+r')C}} \Leftrightarrow \boxed{u_{PN} = E\left(1 - \dfrac{r}{r + r'}e^{-\frac{t}{(r+r')C}}\right)}$.

5)

	From the graph		Expression of	
	u_{MD}	u_{PN}	u_{MD}	u_{PN}
$t_1 = +\infty$	6 V	6 V	$E(1 - \underbrace{e^{-\infty}}_{0}) = E$	$E\left(1 - \dfrac{r}{r + r'}\underbrace{e^{-\infty}}_{0}\right) = E$
$t_2 = 0$	0 V	2 V	$E(1 - e^{0}) = 0$	$E\left(1 - \dfrac{r}{r + r'}e^{0}\right) = \dfrac{Er'}{r + r'}$
$t_3 = (r + r')C$	0.3 s		$E(1 - e^{-1}) = 0.63E$	

From the table we find :

E = 6 V ;

$(r + r')C = 0.3\text{ s} \Rightarrow (r + r') = \dfrac{0.3}{C} = \dfrac{0.3}{1} = 0.3\,\Omega$;

$\dfrac{Er'}{r + r'} = 2\text{ V} \Rightarrow \dfrac{6r'}{0.3} = 2 \Rightarrow r' = 0.1\,\Omega$;

$r + r' = 0.3\,\Omega \Rightarrow r = 0.3 - r' = 0.2\,\Omega$. $\boxed{E = 6\text{ V} \;;\; r = 0.2\,\Omega \text{ and } r' = 0.1\,\Omega}$.

6) We have : $i = \dfrac{E}{r + r'}e^{-\frac{t}{(r+r')C}} = 20e^{-\frac{t}{0.3}}$.

$$W = \int_0^{+\infty} 0.2\left(20e^{-\frac{t}{0.3}}\right)^2 dt = \int_0^{+\infty} 80e^{-\frac{2t}{0.3}}dt = 80 \times \left(-\frac{0.3}{2}\right)e^{-\frac{2t}{0.3}}\Big]_0^{+\infty} = -12e^{-\frac{2t}{0.3}}\Big]_0^{+\infty} = -12(e^{-\infty} - e^{0}) = 12\text{ J}.$$

The dissipated energy by joule's effect in the battery is : $\boxed{W = 12\text{ J}}$.

N⁰ 16
A rod is a generator of current of constant intensity

1) a) • The magnetic flux traversing the circuit EFNM : $\Phi = BS\cos(\vec{B}, \vec{n})$

S is the surface of rectangle EFNM : S = MN×EM = $\ell \times (L + y)$ with L the fixed distance between O and (EF).

$\vec{n}$ is the normal to the plane of the figure, which is, based on the positive sense and the right hand rule, is parallel and opposite direction of $\vec{B}$ ($\vec{n} \triangleq$), then : $(\vec{B}, \vec{n}) = 180^{0}$.

Thus : $\Phi = -B\ell(L + y)$

• According to Faraday's law the e.m.f. in the rod : $e = -\frac{d\Phi}{dt} = B\ell\frac{dy}{dt} = B\ell V$.

• Respecting the positive direction : $u_{NM} = -(\underbrace{ri}_{0} - e) = B\ell V$.

b) The rod plays the role of a generator delivering a voltage : $u_{NM} = B\ell V > 0$ thus the current of the rod is from N to enter at M thus following the positive direction.

2) a) The rod is under the action of its weight : $m\vec{g} = mg.\vec{j}$ and electromagnetic force vertically upward : $\vec{F} = -iB\ell.\vec{j}$.

b) According to Newton's 2nd law : $\sum \vec{F}_{ext} = m\vec{a}$

The motion of the rod is translational rectilinear along $\vec{j}$ then : $\vec{a} = a.\vec{j}$.

Thus : $(mg - iB\ell)\vec{j} = ma.\vec{j} \Leftrightarrow mg - iB\ell = ma \Leftrightarrow a = g - \frac{B\ell}{m}i$.

3) a) In the circuit MNFE : $u_{NM} = u_{NE} + u_{EF} + u_{FM} \Leftrightarrow B\ell V = 0 + \frac{q_E}{C} + 0 \Leftrightarrow \boxed{q_E = B\ell CV}$.

b) We know that : $i = \frac{dq_E}{dt} = \frac{d}{dt}(B\ell CV) = B\ell C\frac{dV}{dt} = B\ell Ca$. $\boxed{i = B\ell Ca}$.

4) a) Replacing i by $B\ell Ca$ in : $a = g - \frac{B\ell}{m}i$, we have :

$$a = g - \frac{B\ell}{m}B\ell Ca \Leftrightarrow a = g - \frac{B^2\ell^2 C}{m}a \Leftrightarrow \boxed{a = \frac{mg}{m + B^2\ell^2 C}}$$

The preceding expression shows that a is a positive constant, the motion of G is thus uniformly accelerated rectilinear.

b) We have : $i = B\ell Ca$ and from : $a = \frac{mg}{m + B^2\ell^2 C}$ we obtain : $\boxed{i = \frac{m}{m + B^2\ell^2 C}B\ell Cg = cte}$.

The rod is a generator of current of intensity : $I = \frac{m}{m + B^2\ell^2 C}B\ell Cg$.

N⁰ 17
The capacitor moves a rod

1) a) According to the right hand rule, the current flows from M' to D' in the rod or in the positive sense in the circuit.

b) $F = iB\ell\sin(\underbrace{\vec{B}, M'D'}_{90^0})$ or $\boxed{F = iB\ell}$.

2) • The capacitor is in a closed circuit without a generator : First phenomenon is **discharging of the capacitor**.

• The magnetic flux in the circuit MND'M' is variable because of the variation of its surface : Second phenomenon is the **electromagnetic induction**.

3) a) The magnetic flux traversing the surface MM'D'D : $\Phi = BS\cos(\vec{B}, \vec{n})$.

S is the surface of rectangle MM'D'D : S = $M'D' \times MM' = \ell \times x$ and $(\vec{B}, \vec{n}) = 0^0$

Therefore : $\boxed{\Phi = B\ell x}$

b) The i.e.m.f. is the opposite of the derivative, with respect to time, of the magnetic flux : $e = -\frac{d\Phi}{dt}$

$e = -\frac{d(B\ell x)}{dt} = -B\ell\frac{dx}{dt} = -B\ell V$. $\boxed{e = -B\ell V}$

4) a) According to the law of addition of voltage : $u_{MD} = u_{MM'} + u_{M'D'} + u_{D'D}$

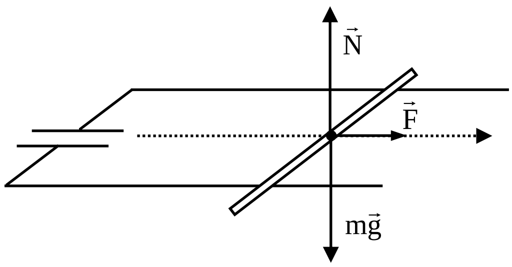
With : $i = \frac{dq_D}{dt} = \frac{d(Cu_{DM})}{dt} = -C\frac{du_{MD}}{dt}$

Then : $u_{MD} = 0 + (\underbrace{r}_{R}i - e) + 0 \Leftrightarrow u_{MD} = -RC\frac{du_{MD}}{dt} + B\ell V$ or $u_{MD} + RC\frac{du_{MD}}{dt} - B\ell V = 0$.

b) • The forces acting on the rod M'D' are : the weight $m\vec{g}$, the reaction of the rails $\vec{N}$ and the force $\vec{F}$.

Applying Newton's 2nd law : $m\vec{g} + \vec{N} + \vec{F} = m\vec{a}$

Projecting this relation on an oriented axis in the direction of $\vec{F}$:



$F = ma \Leftrightarrow iB\ell = m\frac{dV}{dt} \Leftrightarrow -C\frac{du_{MD}}{dt}B\ell = m\frac{dV}{dt} \Leftrightarrow$

$\frac{dV}{dt} = -\frac{CB\ell}{m}\frac{du_{MD}}{dt}$.

• We have : $\frac{dV}{dt} = -\frac{CB\ell}{m}\frac{du_{MD}}{dt} \Leftrightarrow dV = -\frac{CB\ell}{m}du_{MD} \Leftrightarrow V = \int -\frac{CB\ell}{m}du_{MD} = -\frac{CB\ell}{m}u_{MD} + cte$.

At the time t = 0, V = 0 and $u_{MD} = U_0$, then : $0 = -\frac{CB\ell}{m}U_0 + cte \Leftrightarrow cte = \frac{CB\ell}{m}U_0$.

Therefore : $V = -\frac{CB\ell}{m}u_{MD} + \frac{CB\ell}{m}U_0$ or $\boxed{V = \frac{CB\ell}{m}(U_0 - u_{MD})}$.

c) • Replacing the expression of V in the equation : $u_{MD} + RC\frac{du_{MD}}{dt} - B\ell V = 0$

$\Leftrightarrow u_{MD} + RC\frac{du_{MD}}{dt} - B\ell\frac{CB\ell}{m}(U_0 - u_{MD}) = 0 \Leftrightarrow u_{MD} + RC\frac{du_{MD}}{dt} + \frac{CB^2\ell^2}{m}u_{MD} = \frac{CB^2\ell^2}{m}U_0$

$\Leftrightarrow \boxed{\frac{du_{MD}}{dt} + \frac{CB^2\ell^2 + m}{mRC}u_{MD} = \frac{B^2\ell^2}{mR}U_0}$.

• The preceding differential equation is of the form : $\frac{du_{MD}}{dt} + \alpha u_{MD} = \beta$

With $\alpha = \frac{CB^2\ell^2 + m}{mRC}$ and $\beta = \frac{B^2\ell^2}{mR}U_0$

The solution is of the form : $u_{MD} = ke^{-\alpha t} + \frac{\beta}{\alpha} = ke^{-\frac{CB^2\ell^2+m}{mRC}t} + \frac{B^2\ell^2C}{B^2\ell^2C + m}$

At the time t = 0, we have : $u_{MD} = U_0$, then : $U_0 = ke^0 + \frac{B^2\ell^2C}{B^2\ell^2C + m} \Leftrightarrow k = U_0 - \frac{B^2\ell^2C}{B^2\ell^2C + m}$

Hence the solution : $\boxed{u_{MD} = \left(U_0 - \frac{B^2\ell^2C}{B^2\ell^2C + m}\right)e^{-\frac{CB^2\ell^2+m}{mRC}t} + \frac{B^2\ell^2C}{B^2\ell^2C + m}}$.

5) a) The limiting speed attained by the rod is : $\boxed{V_\ell = 11.6\ m/s}$

The limiting voltage of the capacitor : $\boxed{u_\ell = 0.2\ V}$.

b) In the circuit the phenomenon of electromagnetic induction appears at each instant where the rod moves then the rod plays th role of a generator and therefore the capacitor cannot discharge totally.

c) • From the differential equation : $u_{MD} + RC\frac{du_{MD}}{dt} - B\ell V = 0$.

At the limit : $u_{MD} = u_\ell$ = cst = 0.2 V $\Rightarrow \frac{du_{MD}}{dt} = 0$ and V = V_ℓ = cst = 11.6 m/s.

Then : $u_\ell + RC\times 0 - B\ell V_\ell = 0 \Rightarrow B = \frac{u_\ell}{\ell V_\ell} = \frac{0.2}{0.1\times 11.6} = 0.172$ T. $\boxed{B = 0.172\ T}$.

• From the equation : $V = \frac{CB\ell}{m}(U_0 - u_{MD})$ we can write : $V_\ell = \frac{CB\ell}{m}(U_0 - u_\ell) \Leftrightarrow m = \frac{CB\ell(U_0 - u_\ell)}{V_\ell}$

Numerically : $m = \frac{1\times 0.172\times 0.1(6-0.2)}{11.6} = 8.6\times 10^{-3}$kg. $\boxed{m = 8.6\,g}$.

• $u_{MD} = \left(U_0 - \frac{B^2\ell^2C}{B^2\ell^2C+m}\right)e^{-\frac{CB^2\ell^2+m}{mRC}t} + \frac{B^2\ell^2C}{B^2\ell^2C+m}$

By the aid of the graph of u_{MD}, we find for t = 0.2 s that u_{MD} = 2.25 V, replacing in the solution

$u_{MD} = \left(U_0 - \frac{B^2\ell^2C}{B^2\ell^2C+m}\right)e^{-\frac{CB^2\ell^2+m}{mRC}t} + \frac{B^2\ell^2C}{B^2\ell^2C+m}$, we get :

$$2.25 = (6 - 0.0332)e^{-\frac{0.20688}{R}} + 0.0332 \Leftrightarrow e^{-\frac{0.20688}{R}} = 0.371 \Leftrightarrow -\frac{0.20688}{R} = -0.99 \Leftrightarrow \boxed{R \approx 2.1\Omega}$$

d) The electric energy liberated by the capacitor :

$W_e = \frac{1}{2}C\left(u^2_{MD}(t=0) - u^2_{MD}(t=+\infty)\right) = \frac{1}{2}\times 1\times(6^2 - 0.2^2) = 17.98$ J.

The variation of the kinetic energy of the rod :.

$\Delta E_K = \frac{1}{2}m\left(V^2(t=+\infty) - V^2(t=0)\right) = \frac{1}{2}\times 10^{-2}(11.6^2 - 0^2) = 0.6728$ J.

Energetic diagram:

The initial electric energy stored in the capacitor : $W_e(0) = \frac{1}{2}CU_0^2 = \frac{1}{2}\times 1\times 6^2 = 18$ J.

A part of $W_e(0)$ is transformed into energy first part into kinetic energy to move the rod (ΔE_K = 0.6728 J), second part remains stored in the capacitor ($W'_e = \frac{1}{2}Cu_\ell^2 = \frac{1}{2}\times 1\times 0.2^2 = 0.02$ J) and the third part is transformed into thermal energy due to joule's effect (presence of an electric resistance).

The energy dissipated by joule's effect by the circuit : $18\ J - (0.02 + 0.6728)\ J = 17.3072\ J$.