Grade 12	GS – LS – ES	Calculus	Functions	Worksheet 14	Logarithmic Functions (4)

I. Determine each of the limits below.

1) $\lim\limits_{x\to 0^+}(x - \ln x + 2)$

2) $\lim\limits_{x\to -\infty}\ln\left(\frac{x+1}{x-1}\right)$

3) $\lim\limits_{x\to +\infty}(x - \ln x + 2)$

4) $\lim\limits_{x\to +\infty}\left(x^2 + 1 - \frac{\ln x}{2x}\right)$

5) $\lim\limits_{x\to 0^+} x(\ln x + x - 2)$

6) $\lim\limits_{x\to +\infty}((\ln x)^2 - \ln x - 2)$

7) $\lim\limits_{x\to 0^+}\left(\frac{x-1}{x}\ln x\right)$

8) $\lim\limits_{x\to 1^+}\frac{\ln x}{x-1}$

9) $\lim\limits_{x\to +\infty}\frac{\ln x}{x-1}$

10) $\lim\limits_{x\to 0^+} 2(\ln x - 1)^2$

11) $\lim\limits_{x\to +\infty}(2x^2 - 2 - \ln x)$

12) $\lim\limits_{x\to -\infty}(5 - x)\ln(5 - x)$

13) $\lim\limits_{x\to 0^+}(x^3 - 1 + 2\ln x)$

14) $\lim\limits_{x\to +\infty}\left(x - \frac{\ln x}{x^2}\right)$

15) $\lim\limits_{x\to 5^-}(5 - x)\ln(5 - x)$

Grade 12	GS – LS – ES	Calculus	Functions	Worksheet 15 (A)	Logarithmic Functions (5)

I. Let f be the function defined, on $]0, +\infty[$, as $f(x) = (x - e)(\ln x - 1)$.
Let (C) be the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Determine the limits of f at 0 and at $+\infty$.
2) Determine the derivative function of f.
3) Let g be the function defined, on $]0, +\infty[$, as $g(x) = \ln x - \frac{e}{x}$.
   a- Determine the limits of g at the boundaries of its domain of definition.
   b- Study the variations of g.
   c- Calculate g(e) and deduce, using part **b)**, the sign of g(x) on $]0, +\infty[$.
4) Using the results of the function g, study the variations of f.
5) Determine the equation of (T), the tangent to (C) at the point of abscissa 1.
6) Let (D) be the line of equation $y = -x + e$. Study the relative positions of (D) and (C).
7) Draw (D), (T), and (C).
8) Find the number of roots of the equation: $f(x) = m$, where m is a real number.

Grade 12	GS – LS – ES	Calculus	Functions	Worksheet 13	Logarithmic Functions (3)

I. Determine the derivative of each of the functions below.

1) $f(x) = x + 1 + 2\ln x$

2) $f(x) = x^2 + 1 - \frac{\ln x}{2x}$

3) $f(x) = x\ln x + x + \frac{1}{2}$

4) $f(x) = (\ln x)^2 + \ln x - 2$

5) $f(x) = \frac{-1}{x+1}\ln x$

6) $f(x) = \ln\left(\frac{x-2}{x+1}\right)$

7) $f(x) = \frac{x-1}{x}\ln x$

8) $f(x) = (x - e)(\ln x - 1)$

9) $f(x) = \frac{x-1}{x} - 3\ln x$

10) $f(x) = 2(\ln x - 1)^2$

11) $f(x) = 2x^2 - 2 + \ln x$

12) $f(x) = 2x + \frac{1 - \ln x}{x}$

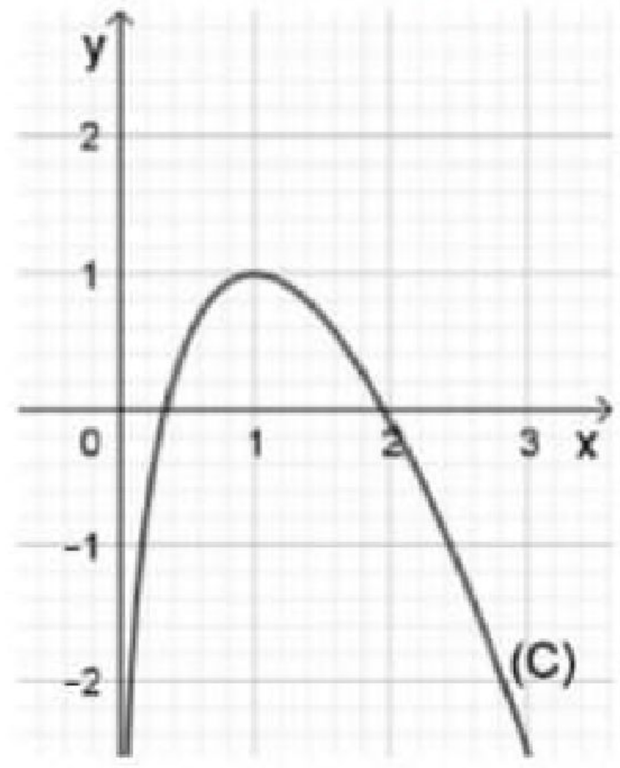
13) $f(x) = (5 - x)\ln(5 - x)$

14) $f(x) = x^3 - 1 - 2\ln x$

15) $f(x) = x - \frac{\ln x}{x^2}$

Grade 12	GS – LS – ES	Calculus	Functions	Worksheet 15 (C)	Logarithmic Functions (5)

III. Part A Let f be the function defined, on $]0\ ,+\infty[$, as $f(x) = ax + (bx + c)\ln x$, where a, b, and c are three real numbers. The curve (C) at right is the representative curve of f in an orthonormal system $(O;\ \vec{\imath},\vec{\jmath})$. Find a, b, and c, knowing that $f(2) = 2 - 3\ln 2$, $A(1\ ,\ 1)$ is a point of (C), and A is an extremum of f.



Part B Let g be the function defined, on $]0\ ,+\infty[$, as $g(x) = x + (1 - 2x)\ln x$.
Without using the given curve:

1) Calculate the limit of g at 0 and at $+\infty$.
2) a- Determine the derivative function of g.
   b- Study the sign of $-2\ln x$ and that of $\frac{1-x}{x}$.
   Deduce the sign of $g'(x)$ and the variations of g.
3) Let (Δ) the line of equation $y = x$.
   a- Solve, in $\mathbb{R}$, the equation $g(x) - x = 0$, then give a graphical interpretation of the roots.
   b- Study the position of the representative curve of g with respect to the line (Δ).

Grade 12	GS	Calculus	Functions	Worksheet 15 (D)	Logarithmic Functions (5)

IV. Consider the function f defined, on $]0\ ,+\infty[$, as $f(x) = \frac{1}{2}x - 1 + \frac{\ln x}{x}$.
Let (C) be its representative curve in an orthonormal system $(O;\ \vec{\imath},\vec{\jmath})$.

1) a- Study the limits of f at 0 and at $+\infty$.
   b- Prove that the line (Δ) of equation $y = \frac{1}{2}x - 1$ is an oblique asymptote to (C).
   c- Specify the relative positions of (C) and (Δ).
2) a- Calculate $f'(x)$ and $f''(x)$. Deduce the **sense of variations** of f' and the sign of $f'(x)$.
   b- Prove that (C) has a point of inflection whose coordinates are to be determined.
   c- Set up the table of variations of f.
3) a- Show that the equation $f(x) = 0$ has a unique root α. Verify that $1.4 < \alpha < 1.5$
   b- Verify that $\ln\alpha = \alpha - \frac{\alpha^2}{2}$.
4) Draw (Δ), then (C).

Grade 12	GS – LS – ES	Calculus	Functions	Worksheet 15 (B)	Logarithmic Functions (5)

II. Part A Consider the function g defined, on $]0\ ,+\infty[$, as $g(x) = -x^2 + 1 - \ln x$.

1) Study the variations of the function g.
2) Calculate g(1), then study the sign of g(x) on $]0\ ,+\infty[$.

Part B Consider the function f defined, on $]0\ ,+\infty[$, as $f(x) = -\frac{1}{2}x + 1 + \frac{\ln x}{2x}$.
Let (C) be the representative curve of f in an orthonormal system $(O;\ \vec{\imath},\vec{\jmath})$.

1) Study the limits of f at 0 and at $+\infty$.
2) Find a relation between $f'(x)$ and $g(x)$, and then set up the table of variations of f.
3) Prove that the equation $f(x) = 0$ has two roots; one on $]0.4\ ,\ 0.5[$ and one on $]2.3\ ,\ 2.4[$.
4) Prove that (C) has a point of inflection I with abscissa $e\sqrt{e}$.
5) Let (Δ) be the line of equation $y = -\frac{1}{2}x + 1$.
   a- Give the sign of $D(x) = f(x) + \frac{1}{2}x - 1$. What do you deduce about (C) and (Δ)?
   b- Prove that the line (Δ) is an asymptote to (C).
6) Draw (C) and (Δ).

Grade 12	GS – LS – ES	Calculus	Functions	Worksheet 12	Logarithmic Functions (2)

I. Find the domain of definition of each of the functions below.

1) $f(x) = \ln x - \ln(2x - 6)$ 2) $f(x) = \ln\frac{x+1}{x-2}$

3) $f(x) = \frac{1}{\ln x}$ 4) $f(x) = \sqrt{\ln x}$

II. Solve each of the equations below.

1) $\ln(3x - 3) = \ln x$ with $x > 1$

2) $\ln(x^2 - 9) = \ln x$ with $x \geq 3$

3) $\ln(x + 4) + \ln(x + 1) = \ln(x + 10)$ with $x > -1$

4) $(\ln x)^2 - \ln x - 30 = 0$

III. Solve each of the inequalities below.

1) $\ln(3x - 3) > \ln x$ with $x > 1$

2) $\ln(x2 - 9) < \ln x$ with $x \geq 3$

3) $\frac{\ln x}{(1+x)^2} \leq 0$ with $x > 0$

4) $\ln(x + 4) + \ln(x + 1) \geq \ln(x + 10)$ with $x > -1$

5) $(\ln x)2 - \ln x - 30 \leq 0$

IV. Find, mentally, the sign of each expression below, with $x > 0$.

1) $f(x) = \frac{1}{x} + 1$ 2) $f(x) = \ln x$ 3) $f(x) = -\frac{1}{x}$

4) $f(x) = \frac{x+1}{x+3}$ 5) $f(x) = (\ln x)^2$ 6) $f(x) = -\ln x$