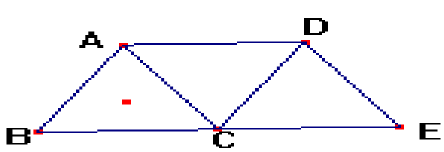
Amal Educational Institutions	**Final Exam** Grade: 12 GS	Year: 2020 / 2021 Date: 16/ 6 / 2021
Name: ____________________	Subject: Math	Time: 240 minutes

I- (3 points)

In the given table, for every question there is only one correct answer. Write the number of each question then choose **with justification** its corresponding answer.

Nº	Questions	Answers		
		A	**B**	**C**
1)	$z = \sqrt{2} - 2e^{i\frac{\pi}{4}}$. An argument of z is :	0	π	$-\frac{\pi}{2}$
2)	The equation: $e^x - 1 - 2e^{-x} = 0$ has	No roots	1 root	2 roots
3)	Let f be a function defined by: $f(x) = e^{2x}$. $f^{(n)}$ is the n^{th} derivative of the function f. (n is a non-zero natural number) $f^{(n)}(0) =$	2^{n-1}/.	2^n	2^{2n-1}
4)	Let $F(x) = \int_0^{2x} \frac{t}{e^t + e^{-t}} dt$ where $x \succ 0$ then $F'(x) =$	$\frac{4xe^{2x}}{e^{4x}+1}$	$\frac{4x}{e^{4x}+1}$	$\frac{2xe^{2x}}{e^{4x}+1}$
5)	Let (C) be a circle of center O(0 ;0) and of radius R = 2 and (C') be the circle of center A(5 ; 0) and R ' = 3 then the affix of the center I of the positive dilation that transforms (C) onto (C ') is m =	2	-10	$2 + 10i$
6)	 ABC ,ACD and CDE are equilaterals, let R_A, R_C and R_D be rotations of centers A ,C and D and of same angle $\frac{\pi}{3}$, then the composite $R_D \circ R_C \circ R_A$ is a	Translation of vector $\overrightarrow{BE}$	Central symmetry of center C	Identity mapping

II- (3.5 points)

In the complex plane referred to a direct orthonormal system $\left(O;\vec{u},\vec{v}\right)$, consider the points A and B of respective affixes $-2i$ and $1-2i$. Let M and M' be the points of affixes z and z' such that $z'=\dfrac{-2iz+4+2i}{z+2i}$ $(z\neq -2i)$..

1) Determine the coordinates of the point M when $z' = -2\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$.

2)
 a- Show that $z'=-2i\dfrac{(z+2i-1)}{z+2i}$.
 b- Show that $OM'=2\dfrac{BM}{AM}$.
 c- Find the set of points M′ when the point M moves on a line (d) of equation $x=\frac{1}{2}$.

3)
 a- Show that $(z'+2i)(z+2i)=2i$.
 b- Show that AM′ × AM = 2 and $\left(\vec{u},\overrightarrow{AM'}\right)=\frac{\pi}{2}-\left(\vec{u},\overrightarrow{AM}\right)+2k\pi,\ k\in\mathbb{Z}$.
 c- Deduce that, if M describes the circle of center A and radius 1, M′ moves on a circle whose radius and center are to be determined.

4) Let $z = x + iy$ and $z' = x' + iy'$, where x, y, x', and y' are real numbers.
 a- Express x′ and y′ in terms of x and y.
 b- Deduce that if M belongs to the line of equation $y = x - 2$, deprived from A, then $y'=x'-2$.

III- (3 points)

Consider an urn U containing 7 coins:

- **Five** red coins of two faces numbered 0 and 1.
- **Two** black coins of two faces numbered 1.

A player selects randomly and simultaneously three coins from the urn, then he throws them only once.
Consider the following events:

T: « The chosen coins are three red »
O: « Only one of the chosen coins is red »
W: « Two of the chosen coins are red »
Z: « The product of the numbers obtained is zero »

1) Show that P (T) = $\frac{2}{7}$.

2) Calculate P(W) and P(O).

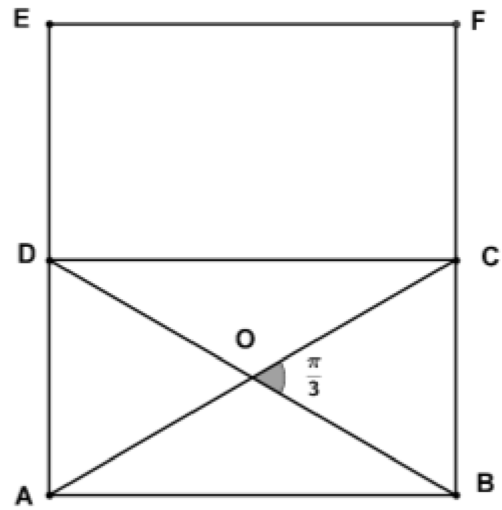
3)
 a- Show that P ($\overline{Z}$/W) = 0.25.
 b- Deduce P(Z/W) and show that P (Z ∩W) = $\frac{3}{7}$. .

4) Calculate P (Z ∩T) and show that P (Z)= 0.75.

5) The product of the three numbers obtained is zero. Calculate the probability that at least one of the thrown coins is black.

IV- (3.5 points)

In the adjacent figure:



- ABCD is a direct rectangle of center O such that AB = 3.
- $\left(\overrightarrow{OB}, \overrightarrow{OC}\right) = \frac{\pi}{3}\ [2\pi]$
- E is the symmetric of A with respect to D.
- DCFE is a direct rectangle.

1) Show that ACE is an equilateral triangle.

2) Show that there exists a rotation r that transform C onto E and A onto C, and determine its angle.

3) Consider the similitude S that transform A onto E, and D into F.

a- Determine the angle of S and show that the ratio of S is $\sqrt{3}$.

b- Show that the image by S of (AC) is (OE), and that of (DB) is (OF).

c- Deduce that O is the center of S.

d- Determine the nature of the triangle OFE.

4) Let E' = S(E)

a- Determine the nature and the characteristics of h = SoS.

b- Show that $\overrightarrow{OE'} = 3\overrightarrow{AO}$.

5) The plane is referred to a direct orthonormal system of center A such that $Z_C = 3 + i\sqrt{3}$.

a- Find the complex form of r.

b- Determine the affix of G the center of r.

V- (3.5 points)

f and g are two functions defined, on]0 ; +∞[, by: $f(x) = \frac{x^2 - 1 + \ln x}{x}$ and $g(x) = x^2 + 2 - \ln x$.

Designate by (C) the representative curve of f in an orthonormal system $(O;\ \vec{i}, \vec{j})$.

Part A

1) Set up the table of variations of g.

2) Deduce the sign of g(x).

Part B

1) Calculate f(1), and solve the equation f(x) = x.

2)

a- Calculate $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to +\infty} f(x)$.

b- Show that the line (L) of equation y = x is an asymptote to (C).

c- Study the relative positions of (C) and (L).

3)

a- Show that $f'(x) = \frac{g(x)}{x^2}$.

b- Set up the table of variations of f.

4) Prove that the equation f(x) = 3 has a unique solution α such that 2 < α < 3.

5) Draw (C).

6) Calculate, in terms α, of the area of the region bounded by (C), (x′Ox), (y′Oy) and the line of equation y = 3.

VI- (3.5 points)

Consider the function f defined on $\mathbb{R}$ by $f(x) = (3x + 3)e^{-x}$, and designate by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$.
2) Calculate $\lim\limits_{x\to+\infty} f(x)$. Deduce the equation of an asymptote to (C).
3) Calculate $f'(x)$ and set up the table of variations of f.
4) Show that (C) admits an inflection point I to be determined.
5) Calculate $f(-1)$ and $f(-1.5)$ then Draw (C).
6)
   a- Verify that $f(x) = 3e^{-x} - f'(x)$.
   b- Deduce the area of the domain limited by (C), $(x'Ox)$ and the line $x = 1$.
7) Let F be the function that is defined over $\mathbb{R}$ by $F(x) = \int_0^x f(t)dt$. Determine the sense of variations of F.
8) Consider the function g defined by $g(x) = \ln[f(x)]$.
   a- Determine the domain of definition of g.
   b- Study the variations of g and draw its table of variations.