Teacher Name:			Subject:	Physics
Grade/Section:	LS		Chapter 1	Energy
Semester:	1st Semester		Academic Year	2020 - 2021

First Exercise

Consider an inclined plane that makes an angle $\alpha = 30^0$ with the horizontal plane. An object (S), supposed as a particle, of mass $m = 0.5\ kg$ is launched from the bottom O of the inclined plane, at the instant $t_0 = 0$, with a velocity $\vec{v}_0 = v_0\vec{\imath}$ along the line of the greatest slope (OB).

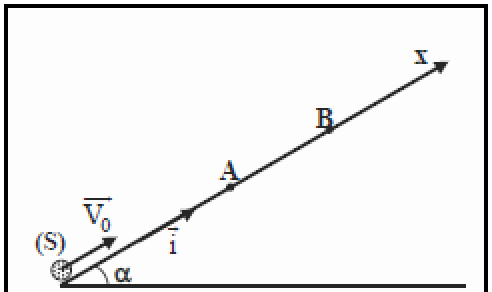


Fig.1

Let A be a point of OB such that $OA = 5\ m$ (fig.1). The position of (S), at the instant t, is given by $\overrightarrow{OM} = x\vec{\imath}$, where $x = f_{(t)}$. The variation of the mechanical energy of the system $[(S)\,, Earth]$ as a function of Ox, is represented by the graph of figure 2.

Take the horizontal plane passing through OH as a gravitational potential energy reference; $g = 10\,m/s^2$.

1) Using the graph of figure 2:
   1-1) Show that (S) is submitted to a force of friction between the points of abscissas $x_0 = 0$ and $x_A = 5m$.
   1-2) Calculate the variation of the mechanical energy of the system $[(S)\,, Earth]$ between the instants of the passage of (S) through the points O and A.
   1-3) Deduce the magnitude of the force of friction, supposed constant, between O and A.
   1-4) Determine, for $0 \leq x \leq 5m$, the expression of the mechanical energy of the system $[(S)\,, Earth]$ as function of x.
   1-5) Determine the speed of (S) at the point of abscissa $x = 6\ m$.

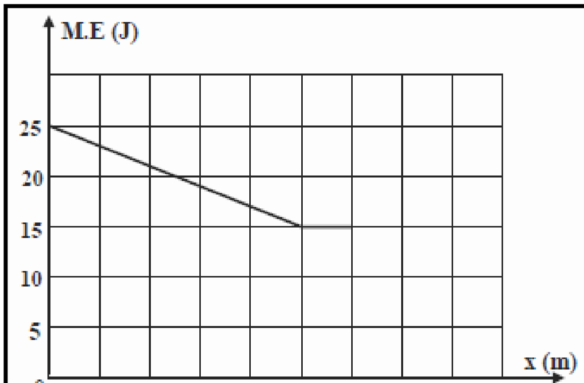


Fig.2

2) Let v be the speed of (S) when it passes through the point M of abscissa x so that $0 \leq x \leq 5m$.
   2-1) Determine the relation between v and x.
   2-2) Deduce that the algebraic value of the acceleration of (S) is $a = -9\ m/s^2$.
   2-3) Determine the values of the speed of (S) at O and at A.
   2-4) Calculate the duration $\Delta t = t_A - t_0 = t$ of the displacement of (S) from O to A, knowing that the algebraic value of the velocity of (S) is given by: $v = at + v_0$

Solution:

1-1) ME of the system decreases from $25\ J$ to $15\ J$ between $x_0 = 0$ and $x_A = 5\ m$. Then friction exists.

1-2) $\Delta ME = ME_A - ME_O = 15 - 25 = -10\ J$.

1-3) Apply the work-energy theorem:

$\Delta ME = W_{\vec{f}}$

Then $-10 = -f.OA$ where $OA = \Delta x = x_A - x_O = 5\ m$

Then $f = \frac{10}{5} = 2\ N$.

1-4) Referring to the graph, the variation of the ME as function of x is a straight line. Then:

$ME = ax + b$

Where (a) is the slope of the line and (b) is the point of intersection of the straight line with the vertical axis.

$a = \frac{\Delta ME}{\Delta x} = \frac{-10}{5} = -2\ J/m$

$b = 25\ J$

Then $ME = -2x + 25$

1-5) At the point of abscissa $x = 6\ m$.

$ME = 15\ J$

$PE_g = mgz$

Where $z = \Delta x . \sin 30 = 6 \times 0.5 = 3\ m$

Then $PE_g = 0.5 \times 10 \times 3 = 15\ J$

Then $KE = 15 - 15 = 0\ J$

Then the speed at this point is zero.

2-1) At any point on the inclined planed where $0 \leq x \leq 5m$, the ME is:

$ME = KE + PE_g = \frac{1}{2}mv^2 + mgz$

Where $z = x \sin 30 = 0.5x$

Then $-2x + 25 = 0.5 \times 0.5 \times v^2 + 0.5 \times 10 \times 0.5x$

Then $0.25v^2 + 4.5x = 25$

2-2) Derive the equation obtained in the previous part w.r.t time.

$0.25 \times 2vv' + 4.5x' = 0$

Where $v = x' \neq 0$ and $v' = a$

Then $0.5va = -4.5v$

Then $a = \frac{-4.5}{0.5} = -9\ m/s^2$.

2-3) $0.25v^2 + 4.5x = 25$

At O, $x = 0$ then:

$v_O^2 = \frac{25}{0.25} = 100$

Then $v_O = 10\ m/s$.

At $(A), x_A = 5\ m$

Then $0.25v_A^2 = 25 - 4.5 \times 5 = 2.5$

Then $v_A^2 = \frac{2.5}{0.25} = 10$

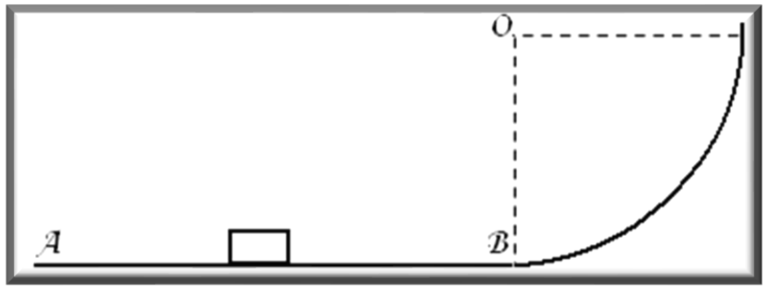
Then $v_A = \sqrt{10} = 3.16\ m/s$.

2-4) $v = at + v_0$

Then $3.16 = -9t + 10$

Then $t = 0.76\ s$

Second Exercise



A rectangular body of mass $m = 400\, g$ considered as a particle is projected with an initial speed $v_A = 5\ m/s$ from point A on a horizontal rough surface of length $l = 2\, m$ and reaches B with a speed $v_B = \sqrt{10}\ m/s$ then continues up a circular frictionless track of radius $R = 1\, m$ and center O. Take the horizontal through A and B as a reference for the gravitational potential energy and $g = 10\, m/s^2$.

1) Calculate the mechanical energy of the system $(body\ , Earth)$ at A and at B.
2) Calculate the value of the friction force assumed constant acting on the body on the horizontal rough surface.
3) Calculate by applying the work energy theorem the acceleration of the body on the horizontal plane. What is the nature of this motion on the horizontal surface?
4) Calculate the maximum height reached by the body on the circular track. Calculate the value of the maximum angle that the position of the body at the maximum height forms with the vertical equilibrium position.
5) Express the gravitational potential energy and the kinetic energy as function of α on the circular track.
6) Calculate the value of α, the angle formed with the vertical equilibrium position when the kinetic and potential energies are equal.
7) Plot the graph of variation of the gravitational potential energy and the kinetic energy as function of α between B and the maximum point reached by the body.

Solution

1) $ME_A = KE_A + PE_{g(A)} = \frac{1}{2}mv_A^2 + 0$ since (A) lies on the reference then $z_A = 0$.
   Then $ME_A = 0.5 \times 0.4 \times 5^2 = 5\, J$
   $ME_B = KE_B + PE_{g(B)} = \frac{1}{2}mv_B^2 + 0$ since (B) lies on the reference then $z_B = 0$.
   Then $ME_B = 0.5 \times 0.4 \times \left(\sqrt{10}\right)^2 = 2\, J$
2) Apply the work-energy theorem:
   $\Delta ME = W_{\vec{f}}$
   Then $ME_B - ME_A = -f \times AB$
   Then $f = \frac{2-5}{-2} = 1.5\, N$
3) Let M be any point between A and B at distance x from A. The speed of the particle at M is v.
   Apply the work-energy theorem:
   $\Delta ME = W_{\vec{f}}$
   $ME_M - ME_A = -f \times AM$
   Then $\frac{1}{2}mv^2 + 0 - 5 = -1.5 \times x$
   Then $0.2v^2 - 5 = -1.5x$
   Derive w.r.t time.
   $0.4vv' = -1.5x'$
   Where $v = x' \neq 0$ and $v' = a$
   Then $0.4va = -1.5v$
   Then $a = -3.75\ m/s^2$.

Since $a = cst < 0$ and the speed decreases along the path between A and B then the motion is uniformly decelerated rectilinear.

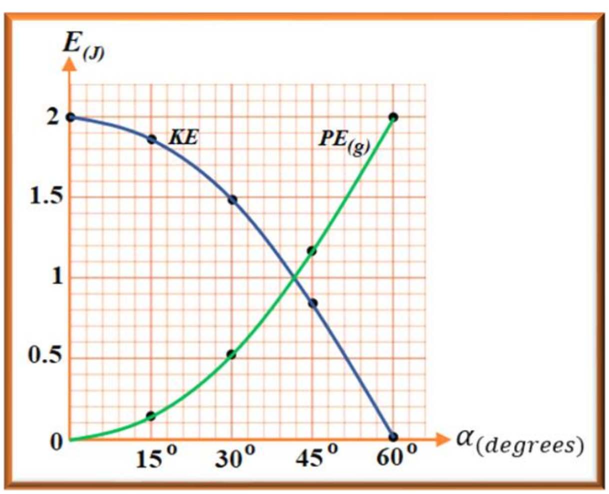
4) At maximum height, the speed of the particle is $v = 0$. Then $KE = 0$.
Apply the law of conservation of mechanical energy since friction is neglected, then:
$ME_{at\ \max\ height} = ME_B$
$KE + PE_{g(\max)} = 2$
$mgz_{max} = 2$
Then $z_{max} = \frac{2}{0.4 \times 10} = 0.5\ m$
To find the maximum angle α_{max}, we use the value of z_{max}, where:
$z_{max} = R - R\cos\alpha_{max}$
Then $\cos\alpha_{max} = 1 - 0.5 = 0.5$
Then $\alpha_{max} = 60^0$.

5) At any point on the circular track the height is:
$z = R - R\cos\alpha$
Then $PE_g = mgz = 0.4 \times 10 \times (1 - \cos\alpha) = 4 - 4\cos\alpha$
The circular track is frictionless, then ME is conserved. Then the mechanical energy at any point is equal to that at B.
$ME = ME_B$
Then $KE = 2 - PE_g = 2 - 4 + 4\cos\alpha$
Then $KE = -2 + 4\cos\alpha$

6) $KE = PE_g$
Then $-2 + 4\cos\alpha = 4 - 4\cos\alpha$
Then $8\cos\alpha = 6$
Then $\cos\alpha = 0.75$
Then $\alpha = 41.4^0$

7) To plot the graph of variation of KE and PE_g as function of α we must find the value of these energies for different values of $0 \le \alpha \le 60^0$.

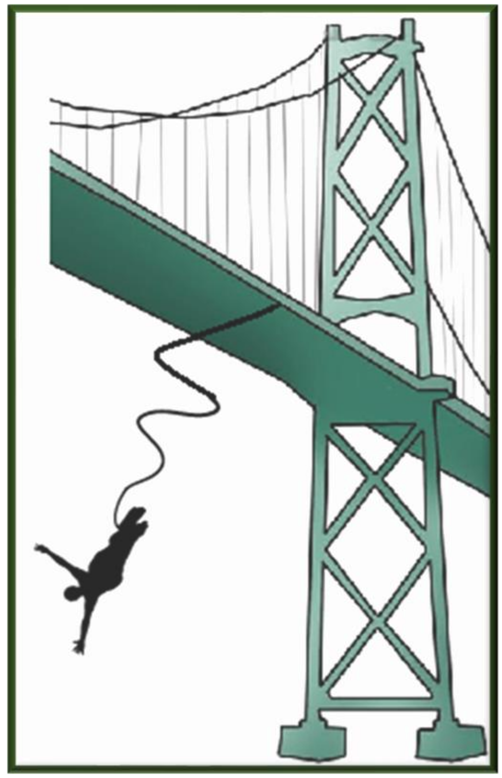
α	0	15^0	30^0	45^0	60^0
$KE_{(J)}$	2	1.86	1.464	0.83	0
$PE_{g(J)}$	0	0.14	0.535	1.17	2



Third Exercise

A bungee jumper of mass $m = 80\,kg$ jumps from rest at $t_0 = 0$ from a bridge $100\,m$ above a river. An elastic rope of natural length $l_0 = 60$ m and constant k is attached to the ankles of the jumper. When reaching its natural length, at (A), the rope starts stretching and the jumper is momentarily at rest at (B) at a height $30\,m$ above the water surface. Neglect air resistance and take $g = 10\,m/s^2$.

Take the surface of water as a reference for the gravitational potential energy.



1) Calculate the kinetic energy, gravitational potential energy, and the mechanical energy of the system (jumper, rope, and Earth) at $t_0 = 0$, the instant of jumping from the bridge.
2) What is the magnitude of the mechanical energy of the system when the rope has its natural length at (A) and at the point of maximum stretching (B)? Why?
3) Calculate the gravitational potential energy, elastic potential energy, and kinetic energy of the jumper when the rope has its natural length $l_0 = 60\,m$ at (A). Deduce the speed of the jumper at that instant.
4) Calculate the gravitational potential energy, kinetic energy of the jumper and the elastic potential energy when the rope is at maximum stretching at (C). Deduce the value of k.

Solution:

1) $KE_0 \frac{1}{2}mv_0^2 = 0\,J$

   $PE_{g(0)} = mgz_0 = 80 \times 10 \times 100 = 80000\,J$

   At $t_0 = 0$, the spring was not stretched, then $PE_{e(0)} = 0\,J$

   $ME_0 = KE_0 + PE_{g(0)} + PE_{e(0)} = 80000\,J$
2) Air resistance is neglected then ME is conserved.

   Then $ME_A = ME_B = ME_O = 80000\,J$
3) At (B):

   $PE_{g(A)} = mgz_{(A)} = 80 \times 10 \times (100 - 60) = 32000\,J$

   $PE_{e(A)} = 0$ since the rope is not stretched yet at (A).

   $KE_{(A)} = ME_{(A)} - PE_{g(A)} - PE_{e(A)} = 80000 - 32000 - 0 = 48000\,J$

   $KE_{(A)} = \frac{1}{2}mv_A^2 = 48000\,J$

   $v_A^2 = \frac{2\times48000}{80} = 1200$

   Then $v_A = 20\sqrt{3}\ m/s.$
4) At (B), the jumper momentarily stops. Then $v_B = 0$

   Then $KE_{(B)} = 0\,J$

   $PE_{g(B)} = mgz_B = 80 \times 10 \times 30 = 24000\,J$

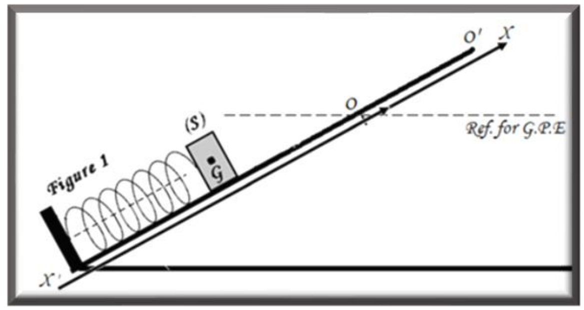
   $PE_{e(B)} = ME_{(B)} - PE_{g(B)} - KE_{(B)} = 80000 - 24000 - 0 = 56000\,J$

   $PE_{e(B)} = \frac{1}{2}kx^2$ where $x = l - l_0$ and $l = 100 - 30 = 70\,m$ then $x = 70 - 60 = 10\,m$

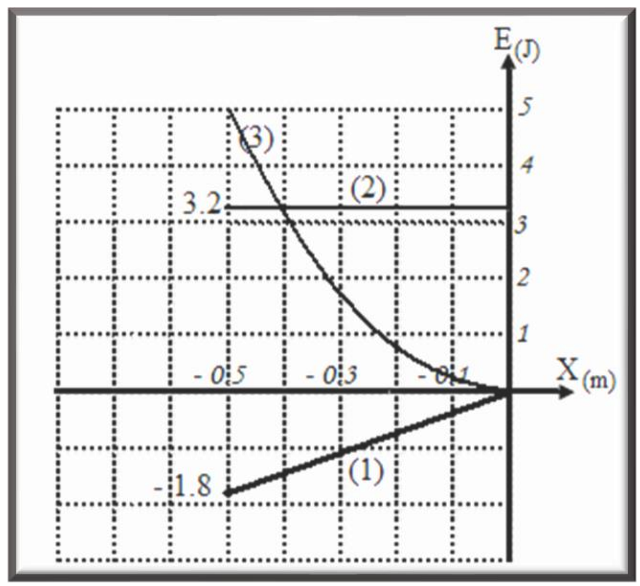
   $\frac{1}{2}kx^2 = 56000$ then $k = \frac{2\times56000}{100} = 1120\ N/m.$

Fourth Exercise

A spring of constant k is placed on a plane inclined by an angle α with respect to the horizontal where $\sin\alpha = 0.9$ and of length $150\,cm$. The lower end of the spring is fixed to a support while its free end is found at point O taken as an origin of abscissas. A solid of mass m is placed in contact with free end of the spring and the formed system is compressed by $50\,cm$ and then released from rest at $t_0 = 0$ along the frictionless path between the releasing point and O. The variation in the elastic potential, gravitational potential, and mechanical energies between the point of maximum compression and the launching point at O are shown on the graph of figure 2. The reference for the gravitational potential energy is taken as the horizontal plane through the point O. Take $g = 10\,m/s^2$ and neglect air resistance.



1) Express as a function of x, the gravitational and elastic potential energies at any instant during launching.
2) Referring to figure 2 indicate which of graphs (1), (2) and (3) represents mechanical energy, gravitational potential energy, and elastic potential energy.
3) Using the graph show that the spring constant is $k = 40\ N/m$ and $m = 0.4\,kg$.
4) Express the kinetic energy of the solid at any instant during launching in terms of x. Calculate the kinetic energy and the launching speed of the solid when passing through O.
5) Plot the graph of variation of the kinetic energy of the solid between the points of maximum compression and O.
6) After leaving O, the solid reaches a maximum point A, $40\,cm$ away from O. Show that friction exists then calculate its value assumed constant.
7) Determine as a function of x the expressions of the potential, mechanical then kinetic energies and plot their graphs between O and the maximum position up the plane.



Solution

1) $PE_e = \frac{1}{2}kx^2$

   $PE_g = mgz$

   Where z is a function of x.

   Note: In this exercise x is not a distance but an abscissa. Then x can be either positive or negative which is important when determining z. For all the points above O where the reference of gravitational potential energy passes the values of z and x are positive and for all the points below O and the reference z and x are negative. Then z and x have the same sign.

   $\sin\alpha = \frac{z}{x}$ then $z = x\sin\alpha = 0.9x$

   Then $PE_g = 0.9mgx$

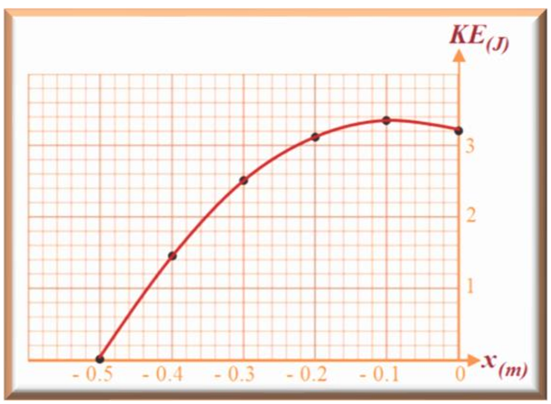
2) Between the launching point and O, friction is neglected then the mechanical energy is conserved (constant). Then curve (2) represents ME.
The elastic potential energy is a quadratic function in x and positive for all values of x. Then curve (3) represents the elastic potential energy.
The gravitational potential energy is a linear function of x and its values are negative for negative values of x. Then curve (1) represents the gravitational potential energy.

3) For $x = -0.5\ m$, $PE_e = 5\ J$ and $PE_g = -1.8\ J$.
$\frac{1}{2}k(-0.5)^2 = 5$ then $k = \frac{2\times5}{0.25} = 40\ \ N/m$.
$0.9m(10)(-0.5) = -1.8$ then $m = \frac{-1.8}{0.9\times10\times(-0.5)} = 0.4\ kg$

4) $ME = KE + PE_e + PE_g$
Then $KE = ME - PE_e - PE_g$
Then $KE = -20x^2 - 3.6x + 3.2$
When passing through O, $x = 0$.
Then $KE = 3.2\ J$
$\frac{1}{2}mv^2 = 3.2$ then $v^2 = \frac{2\times3.2}{0.4} = 16$
Then $v = 4\ \ m/s$.

5) To plot the graph of variation of the kinetic energy as function of x for $-0.5\ m \leq x \leq 0$, we must find the value of the kinetic energy for different values of x.

$x_{(m)}$	-0.5	-0.4	-0.3	-0.2	-0.1	0
$KE_{(J)}$	0	1.44	2.48	3.12	3.36	3.2

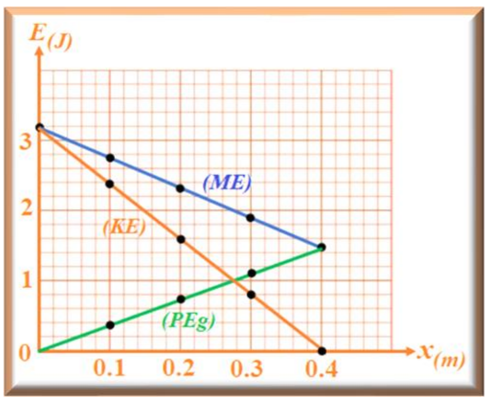


6) At maximum point (A), $v = 0$. Then $KE_A = 0$.
$PE_e = 0$ since after leaving point O the body is detached from the spring since it is not connected to it.
$PE_g = 0.9 \times 0.4 \times 10 \times 0.4 = 1.44\ J$
So, between O and the maximum point, the mechanical energy decreases from $3.2\ J$ to $1.44\ J$. Therefore, friction exists.
Apply the work-energy theorem:
$\Delta ME = W_{\vec{f}}$
Then $1.44 - 3.2 = -f(0.4 - 0)$
Then $f = \frac{-1.76}{-0.4} = 4.4\ N$.

7) At any point between O and the maximum point reached up the plane, the gravitational potential energy is:
$PE_g = 0.9mgx = 3.6x\ \ (J)$
To find the expression of the mechanical energy at any point, we apply the work-energy theorem:

$\Delta ME = W_{\vec{f}}$
$ME - ME_O = -fx$
$ME = -4.4x + 3.2$
$ME = KE + PE_g$
Then $KE = ME - PE_g = -4.4x + 3.2 - 3.6x$
Then $KE = -8x + 3.2$
To plot the graph of variation of the mechanical, kinetic and gravitational potential energies as function of x for $0 \leq x \leq 0.4\ m$ we have to find the value of these energies for different values of the abscissa x.

$x_{(m)}$	0	0.1	0.2	0.3	0.4
$ME_{(J)}$	3.2	2.76	2.32	1.88	1.44
$KE_{(J)}$	3.2	2.4	1.6	0.8	0
$PE_{g(J)}$	0	0.36	0.72	1.08	1.44



Fifth Exercise
A car which is considered as a particle of mass $m = 1000\ kg$, starts from rest at the bottom A of an inclined plane AD which makes an angle $\alpha = 30^0$ with the horizontal. The force of friction acting on the car is supposed constant and of magnitude $f = 200\ N$.

Take the horizontal plane containing A as a reference level for gravitational potential energy.
1) Motion along the path AB.
The car reaches point B with a speed $v_B = 10\ m/s$ under the action of the traction force which is parallel to AD and of magnitude $F = 6000\ N$. Apply the work-energy theorem to determine the distance AB.
2) Motion along the path BC.
As the car reaches B the driver changes the magnitude of the traction force so that the car continues its motion along the path BC at a constant speed $10\ m/s$.
Apply the work-energy theorem to determine the new value F' of the taction force.

3) Motion along the path CD.
   As the car reaches C the driver applies the brakes, so the traction become zero. As a result, the car stops at point D which is 3.75 m away from C.
   Apply the work-energy theorem to determine the value of the constant braking force $\vec{F}_{br}$.

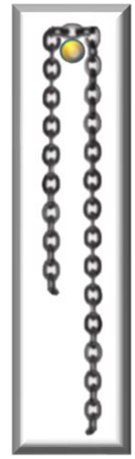
Solution:

1) Consider the system $(Car\,;Earth)$.
   Apply the work-energy theorem: $\Delta ME = W_{\vec{F}} + W_{\vec{f}}$.
   $ME_B - ME_A = F.AB - f.AB$
   At (A):
   $ME_{(A)} = KE_{(A)} + PE_{g(A)} = 0$, since $v_A = 0$ and $z_A = 0$.
   At (B):
   $ME_{(B)} = KE_{(B)} + PE_{g(B)} = \frac{1}{2}mv_B^2 + mgz_B$
   Where $z_B = AB.\sin 30 = 0.5AB$
   Then $ME_{(B)} = 0.5 \times 1000 \times 100 + 1000 \times 10 \times 0.5AB = 50000 + 5000AB$
   Then $50000 + 5000AB - 0 = 6000AB - 200AB$
   Then $800AB = 50000$
   Then $AB = 62.5\ m$
2) $\Delta ME = W_{\vec{F}'} + W_{\vec{f}}$
   $ME_C - ME_B = F'.BC - f.BC$
   $KE_C + PE_{g(C)} - KE_B - PE_{gB} = F'.BC - f.BC$
   Where $KE_C - KE_B = 0$ since $v = cst$.
   Then $mgz_C - mgz_B = F'.BC - f.BC$
   Then $mg(z_C - z_B) = (F' - f)BC$
   Where $z_C - z_B = BC.\sin 30 = 0.5BC$
   Then $0.5 \times 1000 \times 10 \times BC = (F' - 200)BC$
   Then $5000 = F' - 200$
   Then $F' = 5200\ N$.
3) $\Delta ME = W_{\vec{F}} + W_{\vec{f}}$
   $ME_D - ME_C = -F_{br}.CD - f.CD$
   $KE_D + PE_{g(D)} - KE_C - PE_{gC} = -(F_{br} + f).CD$
   $0 + mgz_D - \frac{1}{2}mv_C^2 - mgz_C = -(F_{br} + f).CD$
   Then $mg(z_D - z_C) - \frac{1}{2}mv_C^2 = -(F_{br} + f).CD$
   Where $z_D - z_C = CD.\sin 30 = 3.75 \times 0.5 = 1.875\ m$
   Then $1000 \times 10 \times 1.875 - 0.5 \times 1000 \times 100 = -(F_{br} + 200) \times 3.75$
   Then $-31250 = -(F_{br} + 200) \times 3.75$
   Then $F_{br} + 200 = \frac{31250}{3.75} = 8333.33$
   $F_{br} = 8533.33\ N$

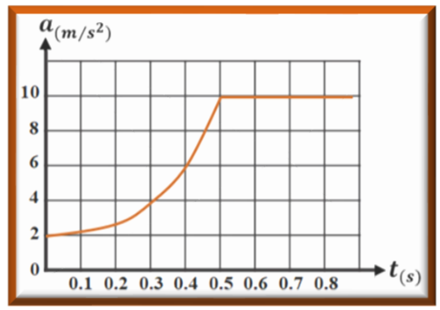
Sixth Exercise

A uniform chain of mass $M = 5\ kg$ of length $l = 1m$ can move in the vertical plane over a horizontal cylindrical tube.
The chain starts from rest at $t_0 = 0$, when the length of one of its parts is 40 cm and that of the other is 60 cm.

Neglect all friction and take the horizontal plane containing the tube as a reference for the gravitational potential energy.



1) Determine at $t_0 = 0$, the mechanical energy of the system (Chain; Earth).
2) At an instant t the long part falls down a distance x.
   - 2-1) For $0 \leq x \leq 0.4\ m$, prove that the expression of the gravitational potential energy of the system (Chain; Earth) as function of x is: $PE_g = -50x^2 - 10x - 13$ (SI).
   - 2-2) Deduce the expression of the mechanical energy of the formed system as a function of x and the speed v of the chain.
   - 2-3) Deduce the expression of v^2 as a function of x.
   - 2-4) Deduce the expression of the acceleration a of the chain as a function of x.
   - 2-5) Determine the minimum and maximum accelerations of the chain.
3) The adjacent figure shows the variation of the value of the acceleration a of the chain as a function of the time t.
   Refer to the graph to:
   - 3-1) Pick out the time needed by the chain to leave the tube.
   - 3-2) Determine the distance covered by the chain and its speed at $t = 0.3\ s$.



Solution:

1) At $t_0 = 0$, the chain was at rest, then $v_0 = 0$. Therefore $KE_0 = 0$.
   $ME_0 = KE_0 + PE_{g1(0)} + PE_{g2(0)}$
   Then $ME_0 = m_1 g z_{1(0)} + m_2 g z_{2(0)}$
   Where m_1 and m_2 are the masses of the hanging parts which is proportional to the length of the suspended parts of the chain.
   Since the mass of the chain is uniformly distributed then the mass per unit length of the chain is constant.
   $\mu = \frac{M}{l} = \frac{m_1}{l_1} = \frac{m_2}{l_2}$.
   Then $m_1 = \frac{M.l_1}{l} = \frac{5\times 0.6}{1} = 3\ kg$ and $m_2 = \frac{M.l_2}{l} = \frac{5\times 0.4}{1} = 2\ kg$
   Then $ME_0 = 3 \times 10 \times (-0.3) + 2 \times 10 \times (-0.2) = -13\ J$
   Note: The negative signs in z is because the center of mass of each part of the chain is below the reference level for gravitational potential energy. In addition, we measure the distances from the center of each part of the chain.

2-1) If the long part falls down a distance, x then the short part rises by x. Then the new length of the suspended parts is $l_1 + x$ and $l_2 - x$ for the long and short parts, respectively. Then their masses are:
   $m'_1 = \frac{M.(l_1+x)}{l} = 5(0.6 + x) = 3 + 5x$
   $m'_2 = \frac{M.(l_2-x)}{l} = 5(0.4 - x) = 2 - 5x$
   $PE_g = PE_{g1} + PE_{g2} = m'_1 g z_1 + m'_2 g z_2$
   Then $PE_g = (3 + 5x)(10)\left(-\frac{(l_1+x)}{2}\right) + (2 - 5x)(10)\left(-\frac{(l_2-x)}{2}\right)$
   Then $PE_g = -(15 + 25x)(0.6 + x) - (10 - 25x)(0.4 - x)$
   Then $PE_g = -9 - 15x - 15x - 25x^2 - 4 + 10x + 10x - 25x^2$
   Then $PE_g = -50x^2 - 10x - 13$

2-2) Since the two parts move with the same speed (they move the same distance during the same time) then the total kinetic energy of the chain is:

$KE = \frac{1}{2}Mv^2$.

$ME = KE + PE_g = \frac{1}{2}(5)v^2 - 50x^2 - 10x - 13$

Then $ME = 2.5v^2 - 50x^2 - 10x - 13$

2-3) Apply the law of conservation of energy since friction is neglected.

$ME = ME_0$

$2.5v^2 - 50x^2 - 10x - 13 = -13$

Then $2.5v^2 = 50x^2 + 10x$

Then $v^2 = 20x^2 + 4x$

2-4) To determine the acceleration of the chain we must derive with respect to time the expression obtained in the part (2-3).

$2vv' = 40xx' + 4x'$

Where $v' = a$ and $x' = v \neq 0$.

Then $2a = 40x + 4$

Then $a = 20x + 2$

2-5) The minimum acceleration is when $x = 0$. Then $a_{min.} = 2\ m/s^2$.

The maximum acceleration is when $x = 0.4\ m$. Then $a_{max.} = 20 \times 0.4 + 2 = 10\ m/s^2$

3-1) The chain leaves the tube at $t = 0.5\ s$.

3-2) At $t = 0.3\ s$, the acceleration is $4\ m/s^2$.

Substituting in the equation obtained in part $(2-4)$:

$4 = 20x + 2$

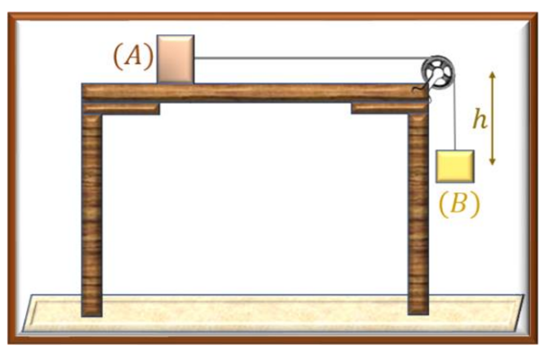
Then $x = 0.1\ m$

To find the speed v, substitute for x in the equation of part $(2-3)$:

$v^2 = 20(0.1)^2 + 4(0.1) = 0.6$

Then $v = 0.77\ m/s$.

Seventh Exercise



Two particles A and B of respective masses m_A and m_B, are connected by means of an inextensible light string. The string passes over the groove of a light pulley as shown in the figure.

At instant $t_0 = 0$, the system S (Pulley, masses, string) is left without initial speed, with particle B being at a distance h from the axis of the pulley. The reference level of the gravitational potential energy is that of the horizontal plane containing the axis of the pulley and the particle A.

Neglect all resistive forces during the motion of the particles.

1) Calculate at $t_0 = 0$, the mechanical energy of the formed system (S; Earth) in terms of m_B, g and h.
2) Calculate the mechanical energy of the system (S ; Earth) after covering a distance x.
3) Calculate by applying the principle of conservation of mechanical energy, the speed v of A or B in terms of x, g, m_A and m_B. Deduce the acceleration of the motion.

Solution:

1) At $t_0 = 0$, both particles were at rest then $v_0 = 0$. In addition, particle (A) was on the reference for the gravitational potential energy, then the initial height of (A) is zero while the height of (B) is $z = -h$ since it is below the reference.

$ME_0 = KE_{0(A)} + PE_{g0(A)} + KE_{0(B)} + PE_{g0(B)} + KE_{0(P)} + PE_{g0(P)}$

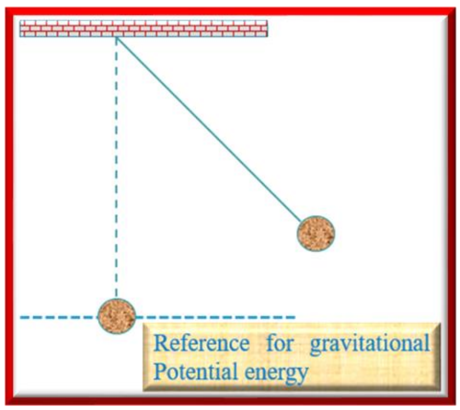
The pulley is massless then $KE_{0(P)} = 0$ and $PE_{g0(P)} = 0$.
Then $ME_0 = m_B g z_{0(B)} = m_B g(-h) = -m_B gh$.

2) After covering a distance x, both particles have the same speed v.
At any time t we have:
$ME = KE_{(A)} + PE_{g(A)} + KE_{(B)} + PE_{g(B)}$
$ME = \frac{1}{2}m_A v^2 + 0 + \frac{1}{2}m_B v^2 + m_B g[-(h+x)]$
Then $ME = \frac{1}{2}(m_A + m_B)v^2 - m_B g(h+x)$

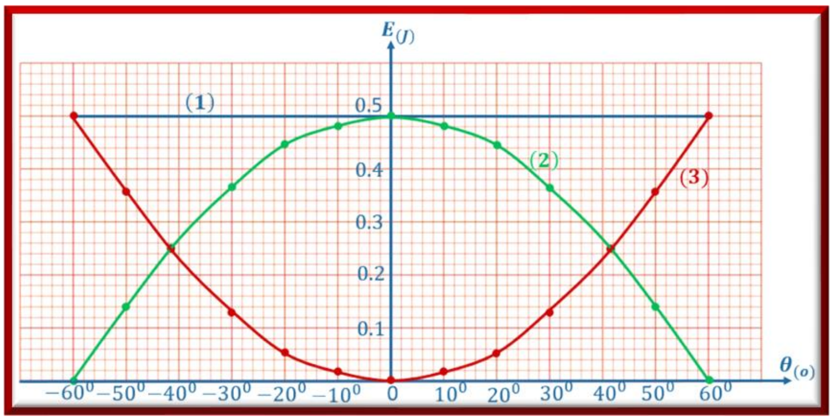
3) Applying the principle of conservation of mechanical energy.
$ME = ME_0$
$\frac{1}{2}(m_A + m_B)v^2 - m_B g(h+x) = -m_B gh$
$\frac{1}{2}(m_A + m_B)v^2 = -m_B gh + m_B g(h+x) = m_B gx$
Then $v^2 = \frac{2m_B g}{m_A+m_B}.x$ is the relation between the speed v and distance x.
To determine the acceleration of the motion we must derive with respect to time the expression of v obtained in the previous part.
$2vv' = \frac{2m_B g}{m_A+m_B}.x'$
Where $v' = a$ and $x' = v \neq 0$.
Then $2va = \frac{2m_B g}{m_A+m_B}.v$
Then $a = \frac{m_B g}{m_A+m_B}$

Eighth Exercise

A pendulum is formed of a particle, of mass $m = 100\ g$, suspended from the lower end of an inextensible light string of length $l = 100\ cm$. The pendulum is shifted, from the vertical equilibrium position by an angle 60^0, about a horizontal axis passing through the other end of the string and released without initial velocity at $t_0 = 0$. The string is always stretched during the motion.
The horizontal plane through the position of the particle at its equilibrium position as a reference for the gravitational potential energy. Neglect all resistive forces during the motion of the pendulum.



1) Calculate the mechanical energy of the system (Pendulum, Earth, Support).
2) Find in terms of θ ($\theta < 60^0$), the expressions for the kinetic and gravitational potential energies of the system.
3) The graphs 1, 2 and 3 in the following figure represent energies of S. What type of energy does each graph represent?
4) Find in two different ways the positions of the pendulum for which the kinetic energy is equal to the gravitational potential energy.



Solution:

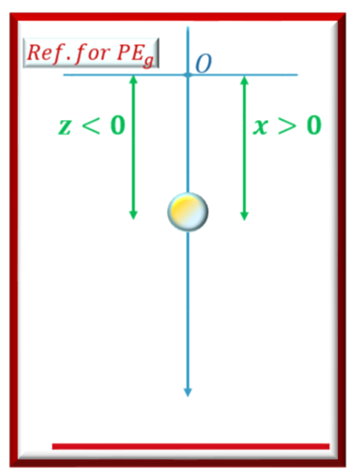
1) At $t_0 = 0$, the mechanical energy of the system is:
   $ME_0 = KE_0 + PE_{g(0)}$
   But the pendulum was released from rest, then $v_0 = 0$.
   $ME_0 = 0 + mgz_0$
   Where $z_0 = l - l\cos\theta_0 = 1 - 0.5 = 0.5\ m$
   Then $ME_0 = 0.1 \times 10 \times 0.5 = 0.5\ J$
   Since friction is neglected then the mechanical energy of the system is conserved.
   Then $ME = ME_0 = 0.5\ J$
2) $PE_g = mgz = mgl(1-\cos\theta) = 0.1 \times 10 \times 1(1-\cos\theta) = 1 - \cos\theta$.
   $KE = ME - PE_g = 0.5 - 1 + \cos\theta = -0.5 + \cos\theta$
3) Graph (1) represents the mechanical energy of the system since it is conserved.
   Graph (2) represents the variation kinetic energy since at extreme points the kinetic energy is zero and has a maximum value when passing through the vertical equilibrium position.
   Graph (3) represents the variation of the gravitational potential energy as function of the angular abscissa θ, since it has a maximum value at the extreme points and a minimum value at the equilibrium position.
4) There are two ways to determine the position of the pendulum for which the kinetic and gravitational potential energies are equal.
   1st way:
   The point of intersection of the curves representing kinetic and gravitational potential energies. Then $\theta \approx 41^0$.
   2nd way:
   Substitute for the equations obtained in the previous part.
   $KE = PE_g$
   $-0.5 + \cos\theta = 1 - \cos\theta$
   $2\cos\theta = 1.5$
   Then $\cos\theta = 0.75$ and $\theta = 41.4^0$

Eleventh Exercise

Using the principle of conservation of mechanical energy, show that the acceleration $\vec{a}$ of a freely falling body has the value g.

Solution:

To show that the acceleration $\vec{a}$ has a value g we are going to solve the exercise taking different references for gravitational potential energy and once by taking x as abscissa and again as distance.

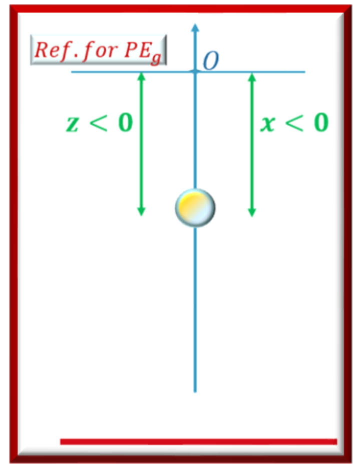


Note: If x is taken as abscissa then it can be either positive or negative but if its taken as distance then it is always positive.

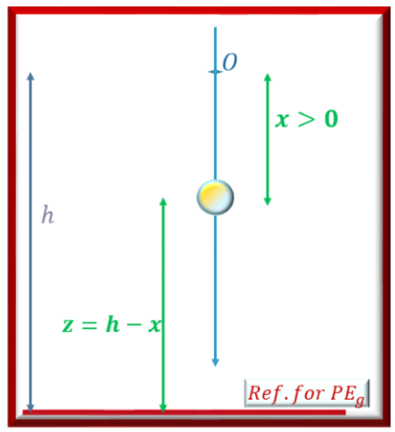
1) Take x as abscissa and the releasing point as origin of abscissas.
   a) Take the horizontal plane through the releasing point as a reference for the gravitational potential energy and the positive direction to be vertically downward.
      At any time we have:
      $ME = KE + PE_g = \frac{1}{2}mv^2 + mgz$
      Where $z = -x$ (x and z have opposite signs).
      Then $ME = \frac{1}{2}mv^2 - mgx = cst.$

Derive with respect to time.
$\frac{dME}{dt} = \frac{1}{2}m2vv' - mgx' = 0$
Where $v = x' \neq 0$ and $v' = a$
Then $mva = mgv$, then $a = g$.

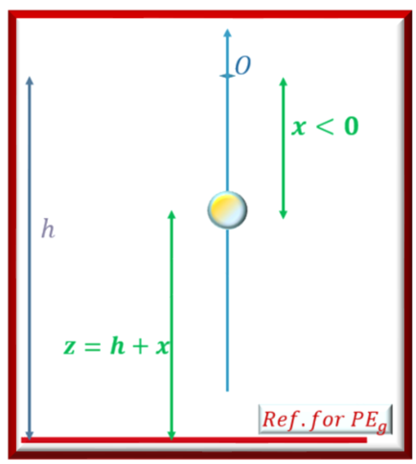
b) Take the horizontal plane through the releasing point as a reference for the gravitational potential energy and the positive direction to be vertically upward.
At any time, we have:
$ME = KE + PE_g = \frac{1}{2}mv^2 + mgz$
Where $z = x$ (x and z have same signs).
Then $ME = \frac{1}{2}mv^2 + mgx = cst.$
Derive with respect to time.
$\frac{dME}{dt} = \frac{1}{2}m2vv' + mgx' = 0$
Where $v = x' \neq 0$ and $v' = a$
Then $mva = -mgv$, then $a = -g$.



c) Take the ground level as a reference for the gravitational potential energy and the positive direction to be vertically downward.
At any time, we have:
$ME = KE + PE_g = \frac{1}{2}mv^2 + mgz$
Where $z = h - x$.
Then $ME = \frac{1}{2}mv^2 + mg(h - x) = cst.$
Derive with respect to time.
$\frac{dME}{dt} = \frac{1}{2}m2vv' - mgx' = 0$
Where $v = x' \neq 0$ and $v' = a$
Then $mva = mgv$, then $a = g$.



d) Take the ground level as a reference for the gravitational potential energy and the positive direction to be vertically upward.
At any time, we have:
$ME = KE + PE_g = \frac{1}{2}mv^2 + mgz$
Where $z = h + x$.
Then $ME = \frac{1}{2}mv^2 + mg(h + x) = cst.$
Derive with respect to time.
$\frac{dME}{dt} = \frac{1}{2}m2vv' + mgx' = 0$
Where $v = x' \neq 0$ and $v' = a$
Then $mva = -mgv$, then $a = -g$.



2) Take x as distance.
a) Take the horizontal plane through the releasing point as a reference for the gravitational potential energy.
At any time, we have:

$ME = KE + PE_g = \frac{1}{2}mv^2 + mgz$

Where $z = -x$ (x and z have opposite signs).

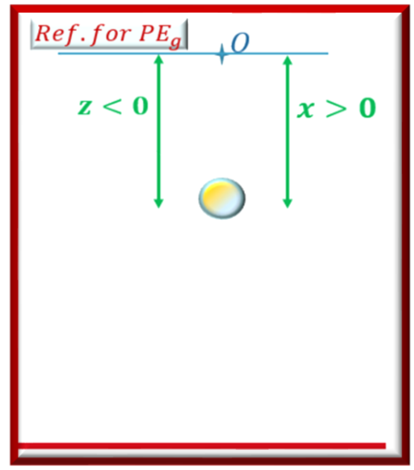
Then $ME = \frac{1}{2}mv^2 - mgx = cst.$

Derive with respect to time.

$\frac{dME}{dt} = \frac{1}{2}m2vv' - mgx' = 0$

Where $v = x' \neq 0$ and $v' = a$

Then $mva = mgv$, then $a = g$.



b) Take the ground level as a reference level for the gravitational potential energy.

At any time, we have:

$ME = KE + PE_g = \frac{1}{2}mv^2 + mgz$

Where $z = h - x$.

Then $ME = \frac{1}{2}mv^2 + mg(h - x) = cst.$

Derive with respect to time.

$\frac{dME}{dt} = \frac{1}{2}m2vv' - mgx' = 0$

Where $v = x' \neq 0$ and $v' = a$

Then $mva = mgv$, then $a = g$.