Solved Problems

N°1.

Calculate. Justify your work:

1) $\arccos\left[\cos\left(\frac{11\pi}{3}\right)\right]$
2) $\arccos\left[\sin\left(\frac{11\pi}{3}\right)\right]$
3) $\sin\left[\arccos\left(-\frac{1}{2}\right)\right]$
4) $\cos\left[\arcsin\left(-\frac{\sqrt{2}}{2}\right)\right]$
5) $\arccos\left[\cos\left(\frac{7\pi}{6}\right)\right]$
6) $\arctan\left[\tan\left(\frac{7\pi}{6}\right)\right]$

N°2.

Find a relation between :

1) $\arccos(-x)$ and $\arccos(x)$
2) $\arcsin(-x)$ and $\arcsin(x)$
3) $\arctan(-x)$ and $\arctan(x)$

N°3.

1) Given $\alpha = \arccos\frac{3}{5}$ and $\beta = \arcsin\frac{12}{13}$.

   Calculate $\cos(\alpha+\beta)$.
2) Calculate $\arctan 2 + \arctan 3$.

N°4.

Simplify each of the following expressions :

1) $\tan(2\arctan x)$
2) $\cos(2\arccos x)$
3) $\cos(4\arctan x)$
4) $\sin(2\arcsin x)$
5) $\cos(2\arctan x)$
6) $\cos^2(2\arcsin x)$

N°5.

Given the equation (E) : $\arcsin x + \arcsin \frac{x}{2} = \frac{\pi}{4}$

Does this equation admit solutions for $-1 < x < 0$?

N°6.

Solve each of the following equations :

1) $\arctan 2x + \arctan 3x = \frac{\pi}{4}$.

2) $\arcsin 2x + \arcsin \frac{1}{2} = \frac{\pi}{2}$.

3) $\arctan x + \arctan 3 = \frac{3\pi}{4}$.

4) $\arctan 2x + \arccos x = \frac{\pi}{2}$.

5) $\arcsin(2x-1) + 2\arctan\sqrt{\frac{1-x}{x}} = \frac{\pi}{2}$.

6) $\arcsin x + \arccos \frac{1}{3} = \arcsin \frac{1}{2}$.

7) $\arctan 4x + \arctan \frac{12}{13} = \arctan 1$.

8) $\arcsin\sqrt{\frac{2x}{1+x}} = \frac{\pi}{2} - \arcsin\sqrt{x}$.

N°7.

Prove each of the following equalities :

1) $\arctan\frac{1}{3} + \arctan\frac{1}{4} = \arctan\frac{7}{11}$

2) $2\arctan\frac{2}{3} = \arctan\frac{12}{5}$

3) $\arctan\frac{1}{2} + \arccos\frac{\sqrt{5}}{5} = \frac{\pi}{2}$

4) $2\arccos\frac{2}{3} = \pi - \arccos\frac{1}{9}$

N°8.

Calculate each of the following integrals :

1) $\displaystyle\int_0^{\frac{1}{4}} \frac{1}{\sqrt{1-4x^2}}\,dx$

2) $\displaystyle\int_0^{1} \frac{1}{\sqrt{9-x^2}}\,dx$

3) $\displaystyle\int_{\frac{3}{2}}^{2} \frac{1}{\sqrt{-x^2+4x-3}}\,dx$

4) $\displaystyle\int_0^{\frac{1}{2}} \frac{\arcsin t}{\sqrt{1-t^2}}\,dt$

5) $\displaystyle\int_0^{1} \frac{\arctan t}{1+t^2}\,dt$

6) $\displaystyle\int_1^{2} \frac{1}{4x^2+4x+2}\,dx$

7) $\displaystyle\int_{-1}^{+1} \frac{dx}{x^2+2x+5}$

8) $\displaystyle\int_0^{1} \frac{dx}{2x^2-2x+1}$

9) $\displaystyle\int_{-1}^{0} \frac{x}{x^2+2x+2}\,dx$.

N°9.

Consider the function f defined over IR by $f(x)=\sqrt{x^2+1}-x$.

1) Show that f is strictly decreasing over IR.
2) Determine $\lim\limits_{x\to+\infty} f(x)$ and deduce the sign of $f(x)$ over IR.
3) Let g be the function defined over IR by $g(x)=\arctan f(x)$.
   a- Show that the equation $g(x)=\frac{\pi}{2}+\arccos x$ has no solutions in IR.
   b- Prove that $g(x)=\frac{\pi}{4}-\frac{1}{2}\arctan x$ and deduce $\tan\frac{\pi}{8}$.

N°10.

f is the function defined over $\left[-\frac{1}{2};+\frac{1}{2}\right]$ by :

$f(x)=\arccos 2x+\arcsin 2x$.

1) Show that $f(x)=\frac{\pi}{2}$.
2) Solve the equation $\arcsin x+\arcsin 2x=\arccos x+\arccos 2x$.

N° 11

Consider the function f defined over $]-1;+1[$ by $f(x)=\arctan\frac{2x}{1-x^2}$.

1) Calculate $f'(x)$ and deduce that $f(x)=2\arctan x$.
2) Solve the equation $f(x)=\frac{\pi}{2}-2\arctan\frac{1}{2}$.

N° 12

Consider the function f defined over IR by $f(x)=\frac{1-x^2}{1+x^2}$.

1) Study the variations of f and deduce that $-1<f(x)\leq 1$.
2) Determine the domain of definition of the function g defined by $g(x)=\arccos\left(\frac{1-x^2}{1+x^2}\right)$.
3) Calculate $g'(x)$ and deduce that $g(x)=2\arctan x$ for $x\in]0;+\infty[$.

N° 13

Consider the function f defined over $[0;2]$ by $f(x)=2+\arcsin(x-1)$ and designate by (C) its representative curve in a direct orthonormal system $(O;\vec{i},\vec{j})$.

1) Study the variations of f and trace (C).
2) Let (C') be the curve representing a function g defined over $[-1;1]$ by $g(x)=\arcsin x$.
   Show that (C') can be deduced from (C) by the translation of vectror $\vec{v}(-1;-2)$ then trace (C').
3) Show that f admits an inverse function f^{-1}.
   Determine the domain of definition of f^{-1} and find $f^{-1}(x)$
   Trace the curve representative of f^{-1}.

N° 14

Let f be the function defined over $]0;+\infty[$ by $f(x)=\arctan\sqrt{x}$.

Designate by (C) its representative curve in a direct orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $f(1)$ and $f(3)$.
2) Show that $\arctan\sqrt{x}+\arctan\dfrac{1}{\sqrt{x}}=\dfrac{\pi}{2}$.
3) Calculate $f'(x)$ and draw the table of variations of f.
4) Trace (C).

N°15.

Let f be the function defined over $IR-\{0\}$ by $f(x)=\arctan\left(1+\dfrac{2}{x}\right)$, and designate by (C) its representative curve in a direct orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$, $\lim\limits_{x\to+\infty} f(x)$, $\lim\limits_{\substack{x\to0\\x<0}} f(x)$ and $\lim\limits_{\substack{x\to0\\x>0}} f(x)$.
2) Calculate $f'(x)$ and draw the table of variations of f.
3) Find the coordinates of A, the point of intersection of (C) with the axis $x'x$ and give an equation of the tangent (T) at A to (C).
4) Trace (C).

N°16.

1) a- Prove that $arctan\,u+arc\,cot\,u=\dfrac{\pi}{2}$ for $u>0$.

 b- Deduce the derivative of $h(u)=arc\,cot\,u$.

2) Let f be the function defined over IR by :
 $f(x)=arc\cot(2x-1)-arc\cot(2x+1)$.

 a- Calculate $f'(x)$.

 b- Show that $f(x)=arc\cot(2x^2)$.

3) Deduce a simple expression of the sum:
 $S_n=arc\cot(2\times1^2)+arc\cot(2\times2^2)+\dots\dots\dots\dots+arc\cot(2\times n^2)$.
 where n is a non-zero natural integer.
 Calculate $\lim\limits_{n\to+\infty} S_n$.

N°17.
Prove that if $\arccos\alpha + \arccos\beta + \arccos\gamma = \pi$ then $\alpha^2 + \beta^2 + \gamma^2 = 1 - 2\alpha\beta\gamma$.

N°18.
Solve the equation:
$2\arctan x + \arctan 3x = \operatorname{arccot} x + 2\operatorname{arccot} 3x$.

N°19.
Consider the function f defined over $\left[\frac{\sqrt{2}}{2};1\right]$ by :

$f(x) = \arcsin\left(2x\sqrt{1-x^2}\right)$ and designate by (C) its representative curve in a direct orthonormal system $(O;\vec{i},\vec{j})$.

1) Show that f is differentiable over $\left]\frac{\sqrt{2}}{2};1\right[$ and calculate $f'(x)$.
2) Show that $f(x) = \pi - 2\arcsin x$.
3) Trace (C).

N°20.
x is real number greater than or equal to 1.
1) Prove that $\arctan\frac{1}{2x-1} - \arctan\frac{1}{2x+1} = \arctan\frac{1}{2x^2}$.
2) Deduce a simple expression of the sum :
$S_n = \arctan\frac{1}{2} + \arctan\frac{1}{8} + \arctan\frac{1}{18} + \cdots\cdots + \arctan\frac{1}{2n^2}$.
Calculate $\lim\limits_{n\to+\infty} S_n$.

N°21.
Let f be the function defined by $f(x) = \arctan\sqrt{\frac{1-x}{1+x}} + \frac{1}{2}\arcsin x$.
1) Determine the domain of definition of f.
2) Show that $f(x)$ is a constant to be determined.