Solved Problems

N°1

Calculate each of the following integrals:

1) $\int_{1}^{4}\left(\sqrt{x}+\frac{3}{\sqrt{x}}\right)dx$ 2) $\int_{0}^{2}\frac{x^2}{\sqrt{x^3+1}}dx$ 3) $\int_{0}^{4}\sqrt{2x+1}\,dx$

4) $\int_{-1}^{0}(3x+2)^4dx$ 5) $\int_{2}^{6}\frac{dx}{\sqrt{2x-3}}$ 6) $\int_{0}^{2}x(x^2-1)^4dx$

7) $\int_{-2}^{-1}\left(x^2+\frac{3}{x^2}\right)dx$ 8) $\int_{1}^{2}\frac{3x^4-2x^3+5}{x^2}dx$ 9) $\int_{-3}^{-2}\frac{dx}{(x+1)^3}$

10) $\int_{0}^{1}x\sqrt{x^2+1}\,dx$ 11) $\int_{0}^{2}\frac{xdx}{\sqrt{2x^2+1}}$ 12) $\int_{0}^{3}|2x-4|\,dx$

13) $\int_{-2}^{0}|x^2-x-2|\,dx$ 14) $\int_{-3}^{1}(3x^2-|2x+4|)dx$

N°2

Calculate each of the following integrals:

1) $\int_{\pi/4}^{\pi/2}\cos^3x\sin x\,dx$ 2) $\int_{0}^{\frac{\pi}{4}}\cos 2x\sin^3 2x\,dx$ 3) $\int_{0}^{\frac{\pi}{4}}\frac{\sin x}{\cos^3 x}dx$

4) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cot x}{\sin^2 x}dx$ 5) $\int_{0}^{\frac{\pi}{4}}(\tan^2 x+3)dx$ 6) $\int_{0}^{\frac{\pi}{8}}\frac{dx}{\cos^2 2x}$

7) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{\sin^2 2x}$ 8) $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}}\cot^2(2x)dx$ 9) $\int_{0}^{\frac{\pi}{2}}\frac{\sin 2x}{\sqrt{1+\cos^2 x}}dx$

10) $\int_{0}^{\frac{\pi}{4}}\frac{\sin(2t)}{(1+\cos 2t)^3}dt$ 11) $\int_{0}^{\frac{\pi}{2}}\sin 2x.\sqrt{1+3\cos^2 x}\,dx$

12) $\int_{0}^{\frac{\pi}{4}}\frac{1}{\cos^4 x}dx$

N°3.

After linearization, calculate each of the following integrals:

1) $\int_{0}^{\frac{\pi}{4}} \cos 3x \cos x dx$ 2) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos 2x \sin 4x dx$

3) $\int_{0}^{\frac{\pi}{4}} \sin 3x \sin x dx$ 4) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2 2x . dx$

5) $\int_{0}^{\pi} \sin^2 3x . dx$ 6) $\int_{0}^{\frac{\pi}{2}} \cos^3 x dx$

7) $\int_{0}^{\frac{\pi}{2}} \sin^3 x \cos^2 x dx$ 8) $\int_{\frac{\pi}{2}}^{3\frac{\pi}{4}} \cos^4 x dx$

9) $\int_{0}^{\frac{\pi}{2}} \sin^3 x dx$

N°4.

Calculate each of the following integrals:

1) $\int_{-3}^{3} x\sqrt{x^2+1}\, dx$ 2) $\int_{-2}^{2} \frac{5x}{x^2+1} dx$ 3) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\sin^3 x + \tan x + x^5\right) dx$

N°5.

1) Calculate the following integrals using integration by parts:

a- $\int_{0}^{\pi} x \cos x\, dx$ b- $\int_{0}^{\pi} x^2 \cos x\, dx$ c- $\int_{0}^{\pi} x \cos^2 x dx$

2) Deduce the integral $I = \int_{0}^{\pi} (x^2 - 2x) \cos x\, dx$.

3) Calculate $\int_{1}^{2} x\sqrt{-x+3}\, dx$.

N°6

Given the two integrals: $I=\int_0^{\pi} x\cos^2 x\,dx$ and $J=\int_0^{\pi} x\sin^2 x\,dx$.

1) a- Calculate $I+J$.
   b- Calculate $I-J$.
2) Deduce the values of I and J.

N°7

Consider the two integrals: $I=\int_0^{\pi}\cos^4(x)\,dx$ and $J=\int_0^{\pi}\sin^4(x)\,dx$.

1) a- Show that $I=\int_0^{\pi}\cos x\,(\cos x-\cos x\sin^2 x)dx$.

   b- Show that:
   $I=\frac{-1}{3}J+\int_0^{\pi}\sin^2 x\,dx$. (You may use integration by parts).

   c- Similarly prove that $J=\frac{-1}{3}I+\int_0^{\pi}\cos^2 x\,dx$.

2) a- Show that $I+J=\frac{3\pi}{4}$ and $I-J=0$.

   b- Deduce I and J.

N°8

Given: $I_n=\int_0^1\left(1-x^n\right)\sqrt{1-x^2}\,dx$, $J_n=\int_0^1 x^n\sqrt{1-x^2}\,dx$.

Let $J_0=\int_0^1\sqrt{1-x^2}\,dx$ where $n\in IN^*$.

1) Justify that $J_0=\frac{\pi}{4}$.

2) Calculate J_1 and deduce the value of I_1.

3) Show that, for any $n\in IN^*$ we have: $J_n\le\int_0^1 x^n\,dx$.

   Deduce $\lim\limits_{n\to+\infty} J_n$.

N°9

Consider the function f defined over $[-1;+\infty[$ by $f(x)=\sqrt{x+1}$.
Designate by (C), its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.(1 unit = 2 cm).

1) Calculate, in cm^2, the area of the region (D), limited by (C), $x'Ox$ and the two straight lines of equations $x=0$ and $x=3$.
2) Calculate, in cm^3, the volume of the solid generated by rotating (D) about $x'Ox$.

N°10

Consider the function f defined over IR by $f(x)=\dfrac{x}{\sqrt{x^2+1}}$.

(C) is its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to+\infty} f(x)$, and deduce that (C) has an asymptote (d).
2) Study the relative positions of (C) and (d).
3) Show that f is odd and deduce the asymptote of (C) at $-\infty$.
4) a- Calculate the area of the region limited by (C), (d) and the straight lines of equations $x=0$ and $x=3$.
   b- Deduce the area of the domain limited by (C), and the straight lines of equations $y=-1$, $x=0$ and $x=-3$.

N°11

Consider the function f defined over $IR-\{0\}$ by $f(x)=x+\dfrac{4}{x^2}$.

(C) is the curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Show that the straight line (d) of equation $y=x$ is an asymptote to (C), and study the relative positions of (C) and (d).
2) Calculate the area of the region bounded by (C), (d) and the straight lines of equations $x=1$ and $x=2$.

N°12

Consider the integrals $I_n=\displaystyle\int_0^{\frac{\pi}{2}} x^n\cos x\,dx$ and $J_n=\displaystyle\int_0^{\frac{\pi}{2}} x^n\sin x\,dx$,

where n is a non zero natural number.

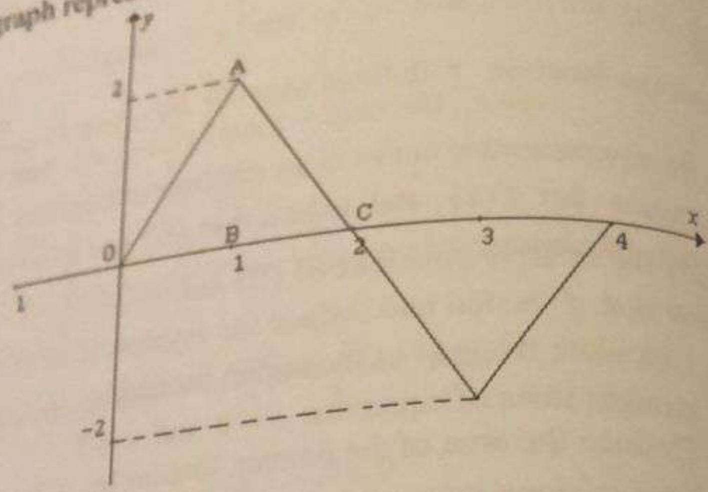
1) a- Using integration by parts, find a relation between I_n and J_{n-1}.

b- Using integration by parts, find a relation between J_n and I_{n-1}.

2) Deduce I_n in terms of I_{n-2}.

N°13

The given graph represents a function f defined over $[\,0\,;4\,]$.



Let $F(x)=\int_0^x f(t)\,dt$.

1) Calculate $F(0)$, $F(1)$, $F(2)$, $F(3)$ and $F(4)$.
2) Calculate $F'(x)$, draw the table of variations of F on $[\,0\,;4\,]$.
3) Show that F is defined over $[\,0\,;4\,]$ by

$$F(x)=\begin{cases} x^2 & 0\le x\le 1 \\ -x^2+4x-2 & 1\le x\le 3 \\ x^2-8x+16 & 3\le x\le 4 \end{cases}$$

N°14

Calculate the derivative function f' of each of the functions f defined as:

1) $f(x)=\int_x^1 \sqrt{2t+6}\,dt$

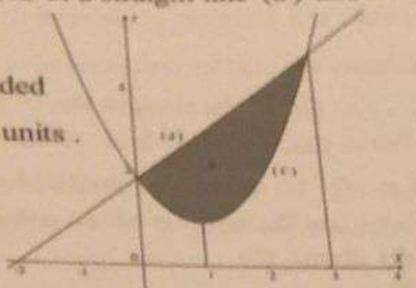
2) $f(x)=\int_1^{x^2} \frac{t^3+2}{t+1}\,dt$

N°15

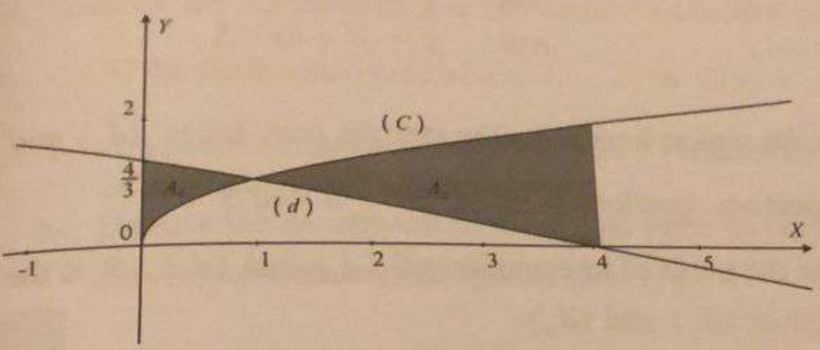
The figures below represent the graphs of a straight line (d) and a curve (C) of a function f.

1) In this figure the area of the shaded region A is equal to $\frac{9}{2}$ square units.

Calculate $\int_0^3 f(x)\,dx$.



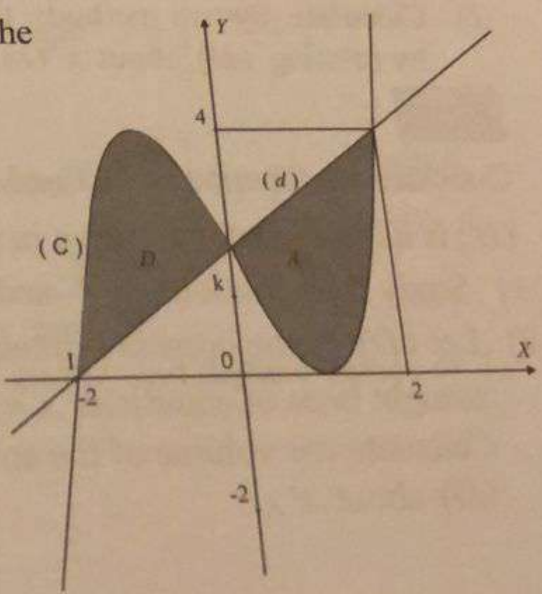
2) In the figure below the area of the shaded region A_1 is $\frac{1}{2}$ and the area of the shaded region A_2 is $\frac{19}{6}$, calculate $\int_0^4 f(x)\,dx$.



3) In the adjacent figure, the area of the shaded region A is 4 , and $w(0;2)$ is a center of symmetry of (C).

Calculate $\int_0^2 f(x)\,dx$

and deduce $\int_{-2}^2 f(x)\,dx$.



N°16

Consider the function f defined over $[1;3]$ by $f(x)=\frac{3}{x}$.

Designate by (H) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$, (d) is the straight line of equation $y=-x+4$.

1) Prove that (H) is below (d) in the interval $[1;3]$.
2) Let (D) be the region limited by (H) and (d).
   Calculate the volume of the solid generated by revolving (D) about $x'x$.

N°17

Calculate the area of each figure limited by the following lines:

1) $y=x(x-1)(x-2)$ and $x'ox$.
2) $y=x^2(x-1)$ and $x'ox$.
3) $y=-x^2+2x$ and $y=-x$.
4) $y=(x-1)^2$ and $y=-x^2+6x-5$.

N°18

Let (D) be the region limited by the straight lines (d_1), (d_2) and (d_3) of respective equations $y=2x$, $y=\frac{1}{2}x$ and $x=2$.

A designates the point of intersection of (d_1) and (d_3), B is the common point of (d_2) and (d_3).

1) Calculate the area of the domain (D).
2) Calculate ,by two methods, the volume of the solid generated by rotating (D) about $x'Ox$.

N°19

Consider the function f defined over $[2;+\infty[$ by $f(x)=\sqrt{3x^2-4}$.

(H) is its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Study the variations of f and draw its table of variations.
2) Let (D) be the domain limited by (H), the axis $x'x$ and the two straight lines of equations $x=2$ and $x=4$.
   Calculate the volume of the solid generated by revolving (D) about $x'x$.

Supplementary Problems

N°1.

Given the two integrals:

$I = \int_0^{\frac{\pi}{4}} (2x+1)\cos^2 x \, dx$ and $J = \int_0^{\frac{\pi}{4}} (2x+1)\sin^2 x \, dx$

1) Calculate $I + J$, and $I - J$.
2) Deduce the values of I and J.

N°2.

Consider the function g defined over $[0;2]$ by:

$$g(x) = \begin{cases} x & 0 \le x \le 1 \\ -x+2 & 1 \le x \le 2 \end{cases}.$$

1) Trace the line representing g in an orthonormal system $(O;\vec{i},\vec{j})$.
2) Let G be the function defined over $[0;2]$ by $G(x) = \int_0^x g(t)\,dt$.
   - a- Study the variations of G over $[0;2]$ and set up the table of variations of G.
   - b- Deduce the expression of $G(x)$ and trace its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

N°3.

Given the two integrals $I = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}$ and $J = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 x}$.

1) Calculate I.
2) Let f be the function defined over $\left[0;\frac{\pi}{4}\right]$ by $f(x) = \frac{\sin x}{\cos^3 x}$.
   - a- Show that $f'(x) = \frac{3}{\cos^4 x} - \frac{2}{\cos^2 x}$.
   - b- Find a relation between I and J and deduce values of I and J.

N°4

Let (C) be the curve representing, in an orthonormal system $(O;\vec{i},\vec{j})$, of the function f defined over $[0,\pi]$ by $f(x)=\cos^2 x$.

Let (D) be the domain limited by (C), $x'x$ and the straight lines of equations $x=0$ and $x=\pi$.

1) Calculate the area of (D)

2) Calculate the volume of the solid generated by rotating (D) about $x'x$.

N°5

Consider the function f defined over $[0;3]$ by $f(x)=x\sqrt{3-x}$.

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Study the variations of f and set up its table of variations.

2) Let (D) be the domain limited by (C) and $x'x$.

Calculate the volume of the solid generated by rotating (D) about $x'x$.