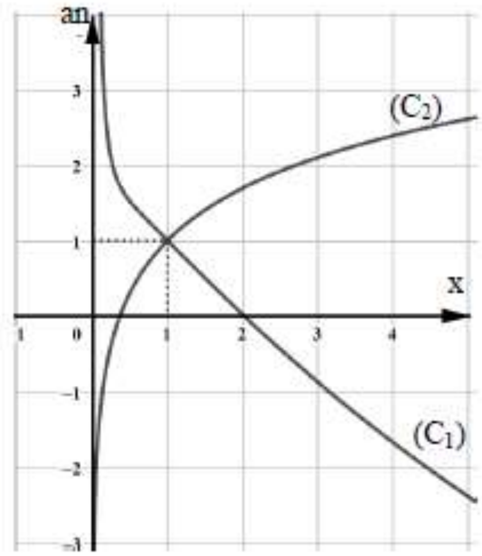
## Problem VI

### Part A

In the adjacent figure where the coordinate system is orthonormal:

- $(C_1)$ is the representative curve of the function defined over $]0\,;+\infty[$ by: $x \mapsto \frac{1}{x} - (\ln x)^2$;
- $(C_2)$ is the representative curve of the function defined over $]0\,;+\infty[$ by: $x \mapsto \ln x + 1$;
- $(C_1)$ and $(C_2)$ intersect at the point of abscissa 1.




1) Study graphically according to the values of x in $]0\,;+\infty[$ the relative position of $(C_1)$ and $(C_2)$.
2) Deduce according to the values of x in $]0\,;+\infty[$ the sign of $g(x) = \frac{1}{x} - (\ln x)^2 - \ln x - 1$.

### Part B

Let f be the function defined over $]0\,;+\infty[$ by: $f(x) = \frac{1+\ln x}{1+x\ln x}$.

Denotes by $(C_f)$ the representative curve of f in an orthonormal system $(O\,;\vec{i}\,;\vec{j})$.

1) Calculate $\lim\limits_{x\to 0^+} f(x)$ and show that $\lim\limits_{x\to+\infty} f(x) = 0$. Interpret graphically both results.
2) Show that for every x in $]0\,;+\infty[$, $f'(x) = \frac{g(x)}{(1+x\ln x)^2}$.
3) Deduce the sense of variation of the function f and then set up its table of variations.
4) Let (T) be the tangent to the curve $(C_f)$ at its point of intersection with the x-axis. Show that $y = \left(\frac{e^2}{e-1}\right)x - \frac{e}{e-1}$ is an equation of (T).
5) Draw (T) and $(C_f)$.
6) Determine graphically the values of the real parameter m for which the equation $(e-1)f(x) = e^2x - me$ admits two distinct roots.
7) Determine an antiderivative F of f over $]0\,;+\infty[$ such that $F(e) = 0$.