AMJAD

MATHEMATICS DEPARTMENT
Final Exam

Class: GS

Date: 11-5- 2023

Duration: 3 hours

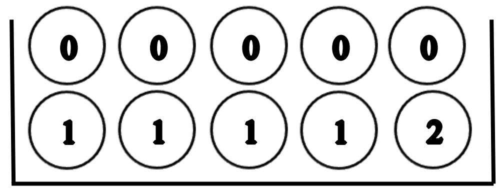
Name of the Student: ______________________________________

Instructions:

1. **Scientific calculators are allowed.**
2. **The exam consists of 6 pages (including this cover page) and 4 exercises.**
3. **If the figures in this exam are used for construction or other additional information then submit the question sheet with your answer sheet as well.**
4. **Full mark is 40.**
5. **Answer Problems I and II on a separate answer sheet for Mr Nabil and problems III and IV for Mr Fadi.**

I- (8 points)

A bag U contains ten balls:



- 5 balls numbered 0
- 4 balls numbered 1
- One ball numbered 2.

Part A:

Three balls are randomly and simultaneously selected from this bag.

Consider the following events:

A: « The sum of numbers on the three selected balls is equal to zero »

B: « The sum of numbers on the three selected balls is equal to one »

C: « The sum of numbers on the three selected balls is equal to two »

Prove that the probability P(C) is equal to $\frac{1}{3}$ and calculate P(A) and P(B).

Part B

A second bag V contains six envelops of which **two** of them contain each one ticket to travel to Canada to watch the 2026 world cup.

A player selects randomly and simultaneously three balls from the bag U.

- If the sum of numbers on the three selected balls is equal to 0, the player leaves the game.
- If the sum of numbers on the three selected balls is equal to 1 or 2, the player draws randomly and simultaneously two envelops from the bag V.
- If the sum of numbers on the three selected balls is equal to 3 or 4, the player draws an envelope from the bag V. If the player wins the ticket then the game stops, if not then the player puts it in the bag V and draws a second envelope and the game stops.
  (The player wins the travel if he draws at least one envelope containing the ticket)

Consider the following events:

N : « The player leaves the game »

E : « The sum of numbers on the three selected balls is equal to 1 or 2 »

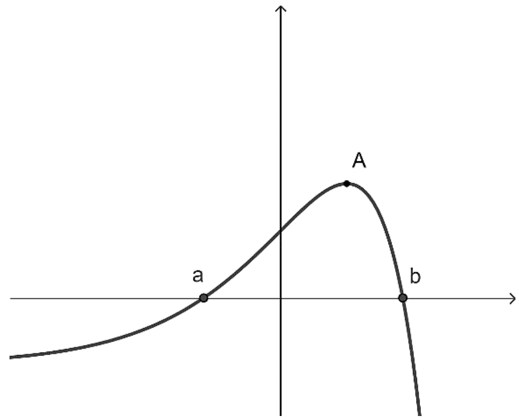
F : « The sum of numbers on the three selected balls is equal to 3 or 4 »

W : « The player wins the travel »

1. Verify that $P(E) = \frac{2}{3}$ and $P(F) = \frac{1}{4}$
2. Calculate the probability $P(W / E)$ and deduce that $P(E \cap W) = \frac{2}{5}$
3. Prove that $P(W) = \frac{97}{180}$
4. The player doesn't win the travel, what is the probability that the sum of numbers on the three selected balls is 1 or 2?

II- (12 points)

Part A



The adjacent curve is the representative curve of the function h defined over $]-\infty,+\infty[$ by $h(x)=(m-x)e^{x}+n$.
The tangent to the curve of h at its point A of abscissa $x=1$ has an equation $y=e-1$.

1) Show that $m=2$ and $n=-1$.
2) Verify that $-1.2<a<-0.8$ and $1.7<b<1.9$
3) Study, in terms of a and b, the sign of h(x).

In what follows take $a=-1$ *and* $b=1.8$

Part B

Consider the function f defined over $]-\infty,+\infty[$ by $f(x)=\dfrac{e^{x}-1}{e^{x}-x}$ and let (C) its representative curve in an orthonormal system.

1) Calculate $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$. Deduce two horizontal asymptotes.
2) Show that $f'(x)=\dfrac{h(x)}{\left(e^{x}-x\right)^{2}}$ and set up the table of variations of f.
3) Show that $y=x$ is an equation of the tangent (T) to (C) at $x=0$.
4) Let g be the function defined over $]-\infty,+\infty[$ by $g(x)=e^{x}-x-1$
   a. Study the variations of g.
   b. Deduce the sign of g(x).
   c. Show that $f(x)-x=\dfrac{(1-x)g(x)}{e^{x}-x}$.
   d. Deduce the relative position of (C) and (T).
5) Draw (T) and (C).

Part C

Let k be the function defined by $k(x)=\ln[f(x)]$.

1) Determine the domain of definition of k.
2) Solve $k(x)\geq\ln(x)$.

III-(10 points)

In the complex plane referred to an orthonormal system $(O,\vec{u},\vec{v})$, consider the points A , B , M and M' of respective affixes $3i$,-i, z and z' ,such that $z'=\dfrac{2iz+6}{z+i}$ with $z \neq - i$.

1) Write z' in exponential form when $z=4+3i$ then deduce that $(z')^{2020}$ is real number.

2) a) Verify that $z'=\dfrac{2i(z-3i)}{z+i}$.

b) Show that $|z'|=\dfrac{2AM}{BM}$, and $(\vec{u};\overrightarrow{OM'})=(\overrightarrow{BM};\overrightarrow{AM})+\dfrac{\pi}{2}+2k\pi$.

c) Show that if (BM) and (AM) are perpendicular, then z' is real.

d) Find the locus of M' as M moves on the straight line with equation y = 1.

3) Let $z=x+iy$ and $z'=x'+iy'$.

Given that $x'=\dfrac{8x}{x^2+(y+1)^2}$ and $y'=\dfrac{2(x^2+y^2-2y-3)}{x^2+(y+1)^2}$.

Show that if M moves on a circle of center I (0;1) and radius R=2 then M' moves on axis of abscissa.

4) Let $z_C=3e^{i\frac{2\pi}{3}}\times z_B$.

a) Point C is the image of B by a transformation T.
Determine the nature and the elements of T.

b) Write the algebraic form of z_C .

c) Write $\dfrac{z_A-z_C}{z_A}$ in exponential form. Deduce the nature of triangle OAC and calculate its area.

IV-(10 points)

Triangle ABC is right at B with AB=1 and $\left(\overrightarrow{CA};\overrightarrow{CB}\right)=\dfrac{\pi}{6}$

Triangle ACE is right at C and $\left(\overrightarrow{AC};\overrightarrow{AE}\right)=\dfrac{\pi}{3}$.

Designate by **S** the direct plane similitude that maps <u>**B onto C**</u> and <u>**C onto E**</u>.

1) a- Verify that the ratio of S is equal to 2 and that an angle of S is $\dfrac{\pi}{3}$.

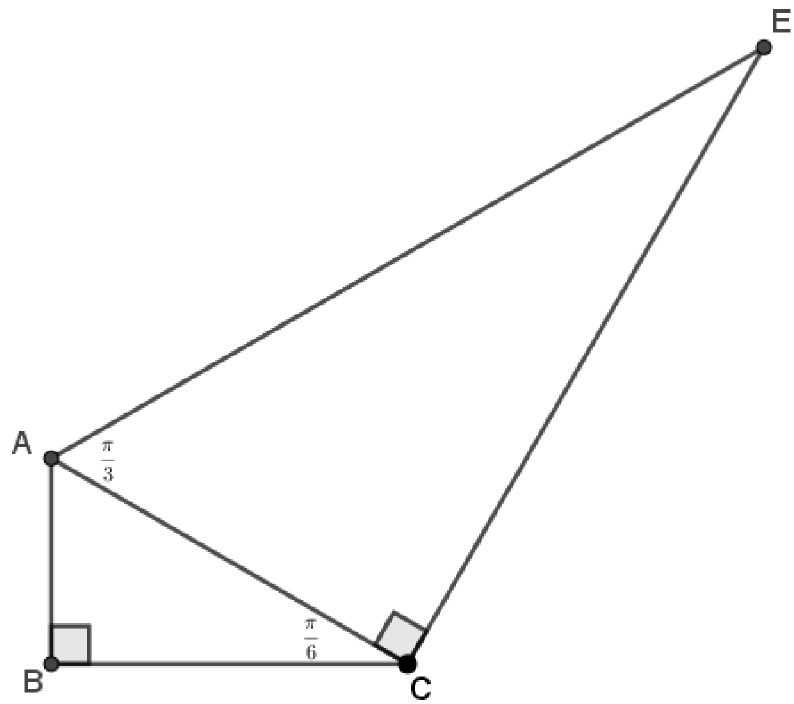
   b- Prove that A is the center of S.

2) M is the midpoint of [BC] and N is that of [CE].
   Prove that S(M)=N then deduce that AMN is semi-equilateral triangle.

3) F is the point of intersection of (MN) and (AC).
   The parallel through F to (CE) intersects (AE) at L.
   a- Show that S(F)=L.
   b- Deduce that triangle ANL is right at N.

4) S' is the similitude with center C that maps A onto B.
   a) Determine the scale factor of S'.
   b) Determine the angle and the scale factor of $S'\circ S$.
   c) Let I be the orthogonal projection of B on (AC).
      i) Determine $S'\circ S$ (A).
      ii) Show that I is the center of $S'\circ S$.

5) The complex plane is referred to the orthonormal system $\left(B;\vec{u},\vec{v}\right)$ such that $\vec{v}=\overrightarrow{BA}$.
   a) Verify that the complex form of $S'\circ S$ is : $z'=\sqrt{3}\,i\,z+\sqrt{3}$.
   b) Determine z_I



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