In his name
Adloun official high school

Subject: Worksheet **Date: 10 February 2021**

Chapter: 3 **Class: BE12 – Life sciences section**

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INTEGRATION

Exercise 1

1) Verify that F is an antiderivative of f on $\mathbb{R}$ such that:
$F(x) = x^2 + 2x - 3$ and $f(x) = 2(x+1)$

2) Verify that F is an antiderivative of f on $\mathbb{R}^{*+}$ such that:
$F(x) = x.\ln x - x - 1$ and $f(x) = \ln x$

Exercise 2

Determine the antiderivatives F over their domain of definition of each of the following functions:

1) $f(x) = (2x+1)^2$
2) $f(x) = x + \frac{1}{\sqrt{x}} + \frac{1}{x^2}$
3) $f(x) = \frac{\left(x^2-2\right)^2}{x^2}$
4) $f(x) = \frac{3}{\sqrt{x}} + 2$
5) $f(x) = \sqrt{x} + \sqrt[3]{x}$
6) $f(x) = \sqrt[3]{x^2} + x$

Exercise 3

Give an antiderivative F of the function f on their domain of definition and verify the given condition:

1) $f(x) = x^2 - \frac{1}{x^2} + x + 1$ and $F(1) = 0$
2) $f(x) = (x^2 + x^{-2})^2$ and $F(-1) = -2$
3) $f(x) = \frac{1}{x^2} - \sqrt{x} + x\sqrt{x}$ and $F(1) = -1$

Exercise 4

Calculate the following indefine integrals

Part A

1) $\int (2x+3)^5\,dx$
2) $\int x\sqrt{x^2+3}\,dx$
3) $\int \frac{2x}{(x^2+1)^2}\,dx$
4) $\int \frac{x+1}{(x^2+2x)^3}\,dx$
5) $\int \frac{3x}{\sqrt{1+x^2}}\,dx$
6) $\int x\sqrt{2x+1}\,dx$

Part B

1) $\int \frac{x^4+2x^3+x^2-1}{x^2}\,dx$
2) $\int \frac{x^3}{x^2-1}\,dx$
3) $\int \frac{2x-1}{\sqrt{x+1}}\,dx$

Part C

1) $\int \frac{dx}{2x+3}$
2) $\int \frac{3x+5}{x+1}\,dx$
3) $\int \frac{(x^2+1)^2}{x-1}\,dx$

4) $\int \frac{x^4+3x^2+x-1}{x}dx$

5) $\int \frac{2x}{x^2+3}dx$

6) $\int \frac{\ln x}{x}dx$

Exercise 7

Calculate the following define integrals

1) $\int_{-1}^{2} 0\,dx$
2) $\int_{-2}^{2} dx$
3) $\int_{0}^{3} 2\,dx$
4) $\int_{1}^{3} \left(3x^2+\frac{2}{x^2}+4\right)dx$
5) $\int_{0}^{1} \frac{2x}{\sqrt{4-x^2}}dx$
6) $\int_{2}^{5} \frac{2x}{\sqrt{x-1}}dx$
7) $\int_{-1}^{0} \frac{dx}{(x-1)^3}$
8) $\int_{-1}^{1} (2x-3)^3\,dx$
9) $\int_{0}^{1} 2x\,(x^2-1)^{99}dx$
10) $\int_{4}^{9} \frac{\sqrt{x}+1}{\sqrt{x}}dx$
11) $\int_{0}^{\sqrt{28}} x\,\sqrt[3]{1-x^2}dx$
12) $\int_{1}^{4} \frac{x-2}{\sqrt{x}}dx$
13) $\int_{0}^{1} x^2\,\sqrt{2x^3+3}dx$
14) $\int_{-\sqrt{3}}^{\sqrt{2}} \frac{x}{(x^2+2)^2}dx$
15) $\int_{0}^{\sqrt{3}} \frac{2x.2^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}dx$

Exercise 8

Calculate the following integrals

1) $\int_{1}^{2} \frac{dx}{x+1}$
2) $\int_{0}^{1} \frac{x+1}{x^2+2x+3}dx$
3) $\int_{-2}^{-1} \frac{2x+3}{3x+2}dx$
4) $\int_{1}^{e^2} \frac{\ln x}{2x}dx$
5) $\int_{1}^{e} \frac{dx}{x.\ln x}dx$
6) $\int_{e}^{e^2} \frac{2}{x(3+\ln x)}dx$
7) $\int_{1}^{e} \frac{x^2+x+\ln x}{x}dx$
8) $\int_{e}^{e^2} \frac{1+\ln x}{x}dx$

Exercise 9

1) Find a continuous function f defined over $]0,+\infty[$ such that $\int_0^x f(t)dt = \frac{1}{2}ln^2x$ then study its sens of variations.
2) Given the continuous function g defined over $]0,+\infty[$ by $g(x) = \int_1^x f(t)dt$ and $f(1) = 2$. Find the euqation of the tangente (T) to the cuve of g at the point of abscissa 1.

Exercise 10

1) Determine the real numbers a, b and c such that: $\frac{x^2+1}{x^2(x-1)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x-1}$
2) Deduce $I = \int_2^3 \frac{x^2+1}{x^2(x-1)}dx$

Exercise 11

Calculate the following integrals:

1) $\int_{-4}^{4} f(x)dx$ such that $f(x) = \begin{cases} 3x^2 & if\ x \le -1 \\ x-1 & if -1 < x < 1 \\ \sqrt{x} & if\ x \ge 1 \end{cases}$
2) $\int_0^5 |x-3|dx$
3) $\int_{-3}^{2} |x^2+5x-6|\,dx$
