Exercises on Exponential Functions for LS(sheet 2): **2020-2021**

Nº1) Calculate the following limits:

1) $\lim\limits_{x \to +\infty}\left(\frac{e^x}{1 + x^2}\right)$

2) $\lim\limits_{x\to +\infty}\left(\frac{e^x + 3}{2\ e^x - 5}\right)$

3) $\lim\limits_{x\to +\infty}(x^3 - e^x)$

4) $\lim\limits_{x\to 0}\left(\frac{e^{2x} + e^x - 2}{e^x - 1}\right)$

5) $\lim\limits_{x \to 0}\left(\frac{e^x - 1}{\sqrt{x}}\right)$

6) $\lim\limits_{x\to 0} x\ e^{\frac{1}{x}}$

7) $\lim\limits_{x \to -\infty}\left(\frac{e^x + 1}{e^x - 1}\right)$

8) $\lim\limits_{x\to -\infty}(x^2 + 1)e^x$

9) $\lim\limits_{x\to +\infty}\left(\frac{x}{e^x + 1}\right)$

10) $\lim\limits_{x\to +\infty}(x + 1)e^{-x}$

11) $\lim\limits_{x\to 0}\left(\frac{e^{-x} + 2\ e^x - 3}{e^x - 1}\right)$

Nº2) Let f be the function defined by $f(x) = e^x + 1 + \frac{3}{e^x - 3}$.

1)Calculate $f(0)$, $f(\ln 2)$, $f\left(\ln\frac{1}{3}\right)$ and $f(-\ln 2)$.

2)What is the domain of definition of f?

3)Determine the intersection points of the curve (C) representing f in an orthonormal system $(O\ ;\ \vec{i},\vec{j})$ with:

a. the abscissa axis.

b. the line of equation $y = -2$.

Nº3) Consider the function $f(x) = e^{2x} - 2\ e^x$ defined over IR , of representative curve (C) in an orthonormal system $(O\ ;\ \vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to -\infty} f(x)$ & $\lim\limits_{x\to +\infty} f(x)$. Deduce the equation of the asymptote.
2) Calculate $f'(x)$.
3) Verify that the curve (C) has asymptotic direction parallel to $y'Oy$.
4) Set up the table of variations of f.
5) Draw the curve (C).

Nº4) Extra Math Consider the function f defined over IR by $f(x) = e^x - x - 2$, of representative curve (C) in an orthonormal system $(O\ ;\ \vec{i},\vec{j})$.

1) Calculate $\lim\limits_{x\to -\infty} f(x)$ & $\lim\limits_{x\to +\infty} f(x)$.
2) Determine $f'(x)$ then set up the table of variations of f.
3) Prove that the straight line (d) with equation $y = -x - 2$ is an oblique asymptote to the curve (C) at $-\infty$ then study the relative position of (C) & (d).
4) a. Show that the equation $f(x) = 0$ admits two distinct roots a and b such that $a < 0$ and $b > 0$.
   b. Prove that $-1.9 < \alpha < -1.8$ and $1.1 < b < 1.2$.
   c. Show that $f'(a) = 1 + a$
   d. Trace (d) and (C).

Nº5) Succeed in Bac

Part A Consider the function g defined over $]0,+\infty[$ by $g(x) = x - \ln 2x$.

1) Determine $g'(x)$ then set up the table of variations of g.
2) Show that $g(x) > 0 \quad for\ x > 0$.

Part B Consider the function f defined over IR by $f(x) = x^2 - e^x$.

1) Determine $f'(x)$.
2) Study the sign of $f'(x)$ for $x < 0$, $x = 0$ & $for\ x > 0$.
3) Set up the table of variations of f.

Nº6) Succeed in Bac f is a function described by $f(x) = x + 1 + \ln(e^{2x} - e^x + 1)$ and let (C) be its representative curve.

1) Verify that the domain of definition of f is IR.
2) Calculate $\lim_{x\to -\infty}(f(x) - x)$ and $\lim_{x\to +\infty}(f(x) - 3x)$.Write four immediate conclusions.
3) Show that f is strictly increasing on IR.

Nº7)Mastering Mathematics

Part A Consider the function g defined over IR by $g(x) = 2e^x + 2x - 7$.

(Γ) is its representative curve in an orthonormal system $(O\,;\,\vec{\imath},\vec{\jmath})$.

1) Calculate $\lim_{x\to +\infty} g(x) \quad and \quad \lim_{x\to -\infty} g(x)$.
2) Show that the straight line (d) of equation $y = 2x - 7$ is an asymptote to (Γ).
3) Study the variations of g over IR and set up its table of variations.
4) Justify the equation $g(x) = 0$ admits a unique root α such that $0.94 < \alpha < 0.941$ and deduce the sign of $g(x)$.
5) Draw (Γ).

Part B Consider the function f defined over IR by $f(x) = (2x - 5)(1 - e^{-x})$ and designate by (C) its representative curve in an orthonormal system $(O\,;\,\vec{\imath},\vec{\jmath})$.

1) Study the sign of f over IR.
2) Calculate $\lim_{x\to -\infty} f(x)$ & $\lim_{x\to +\infty} f(x)$.
3) a. Verify that $f'(x)$ and $g(x)$ have the same sign .
   b. Set up the table of variations of f.
   c. Show that the straight line (D) of equation $y = 2x - 5$ is an asymptote to (C) at $+\infty$.
   d. Study the relative position of (C) with respect to (D).
   e. Taking $\alpha = 0.94$, draw (C).