

I-(6.5 points)

Velocity Diagram in a Collision

We consider:

- ⊗ a horizontal track NP found in a vertical plane;
- ⊗ an object (S_1) considered as a particle of mass $m_1 = 0.4\text{kg}$ and placed at the origin O of a horizontal x -axis of unit vector \vec{i} .
- ⊗ an object (S_2) considered as a particle of mass $m_2 = 0.1\text{kg}$.

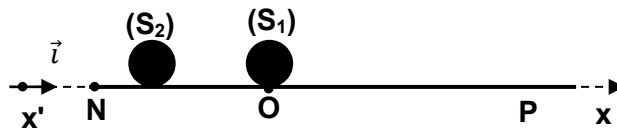


Figure 1

Neglect all forces of friction. (S_2) is launched towards (S_1) and reaches O with a velocity $\vec{v} = 2\vec{i} \text{ (m/s)}$, and enters into a head-on elastic collision with (S_1) initially at rest.

Part A

Elastic collision

Right after collision (S_2) rebounds with a velocity \vec{v}_2 , while (S_1) moves to the right with a velocity \vec{v}_1 .

1. What are the physical quantities that remain conserved upon collision?
2. Show that $v_1 = 0.8 \text{ m/s}$ and then determine v_2 .

Part B

Graphical study

The diagram of figure 2 represents the change versus time of the velocities of (S_1) and (S_2) upon collision.

1. State the forces acting on (S_2) upon collision and represent them.
2. Show that that of the average force $F_{1/2}$ exerted by (S_1) on (S_2) is given by:

$$F_{1/2} = \frac{m_2(v_2 - v)}{\Delta t}$$

where Δt is the duration of the collision.

3. Among the segments labelled (a), (b), (c) and (d), specify those which are associated to (S_1) and those to (S_2).
4. Deduce:
 - a) the collision duration;
 - b) the force of interaction exerted by (S_1) on (S_2).

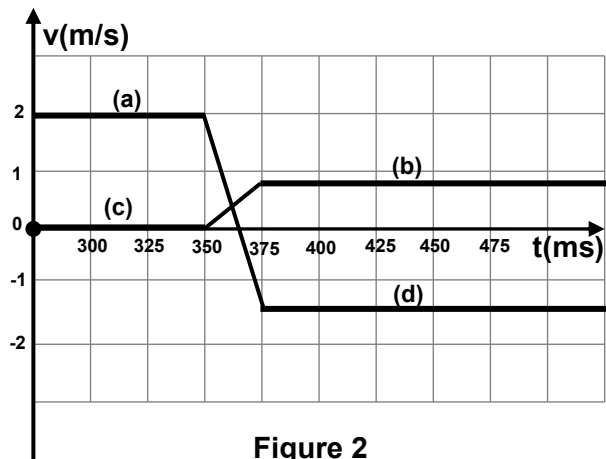


Figure 2

II-(07 points)

Laser to Measure the Diameter of a Spherical Grain

A laser source illuminates by a monochromatic light a wavelength $\lambda = 635\text{nm}$ in air, under normal incidence, a slit of width $a = 0.1\text{mm}$. (E) is a screen placed parallel to the plane of the slit at a distance $D = 2\text{m}$.

The position of a point P on the screen is defined by its abscissa $x = \overline{OP}$ and O is the center of the central bright fringe and θ_1 is the angular position of the first dark point.

P is a point on the screen of abscissa 30mm .

The speed of light in vacuum $c = 3 \times 10^8\text{m/s}$.

- Calculate the frequency of the laser beam used.
- Show that, for small angles where $\sin \theta \approx \tan \theta \approx \theta(\text{rad})$, the expression of the linear width ℓ of the central bright fringe is given:

$$\ell = \frac{2 \lambda D}{a}.$$

- Deduce:

- the value of ℓ .
- in degrees, the angular width of the central bright fringe.

- The laser source is now replaced by a white light source whose wavelengths spectrum is continuous $[375\text{nm}; 750\text{nm}]$. Specify the radiations that would give at P the center of a dark fringe.

- The success of a chocolate recipe depends on taste characteristics and particles granulometry, which is the size of powder particles. Particles that are too finely ground will make chocolate sticky while too large particles will give it a grainy appearance to the eye and mouth.

It is possible to determine the size of a powder grain by light diffraction, to identify the best use of chocolate either as cover chocolates used for topping, milk chocolates or agglomerated chocolates used for instant recipes.

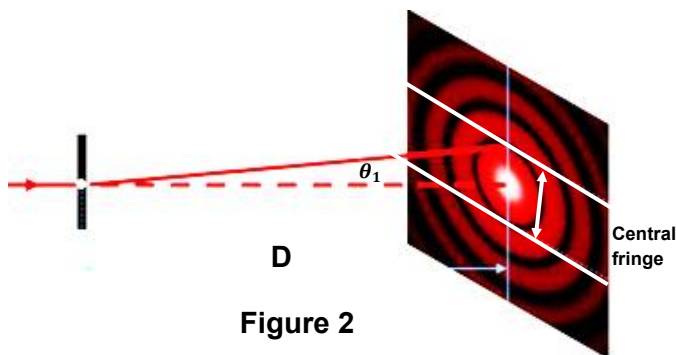


Figure 2

Chocolate use	cover	milk	agglomerated
Recommended size of grain chocolate $a(\mu\text{m})$	[5; 15[[15; 75[[75; 300]

The diffraction pattern (Figure 2) obtained by a thin circular hole or a small spherical grain of diameter a consists of concentric circles alternately bright and dark. θ_1 the half-angle of the central bright fringe is such that:

$$\sin \theta_1 = 1.22 \frac{\lambda}{a}$$

The curve (figure 3), shows the change of light relative intensity versus the angular position θ . Specify the best use of this chocolate powder (use ruler to get accurate values).

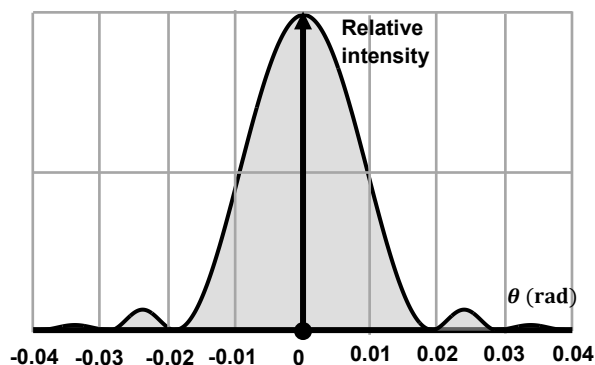


Figure 3

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III-(6.5 points)

Capacitor to Deploy Airbag

An airbag is a vehicle occupant-restraint system using a bag designed to inflate (figure 1). Typically, the decision to deploy an airbag in a frontal crash is commanded by an electronic system that uses a set of capacitors. When a current through the circuit of these capacitors, is detected, a signal is sent to deploy the airbag. The set that triggers the airbag could be modeled by a single capacitor conveniently connected having a fixed armature while the other one is mobile. During accidents, the mobile armature approaches from the fixed one.

A simplified series circuit of the setup used could be represented by an ideal generator of *e.m.f.* E , a resistor of resistance R and a capacitor of capacitance $C = 100\text{pF}$ taken neutral (figure 2).

The switch K is closed at an instant taken as origin of time $t_0 = 0$.

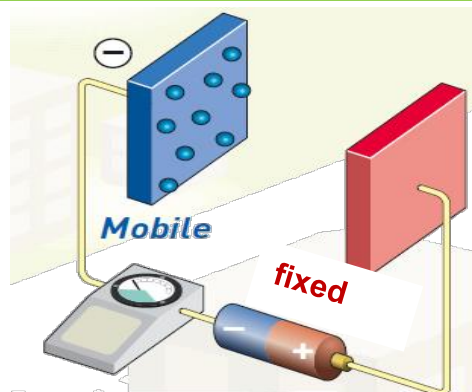


Figure 1

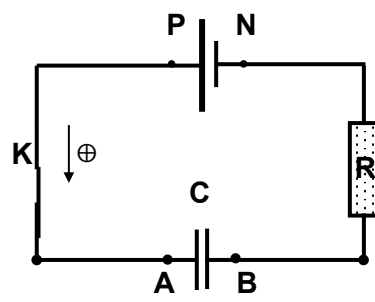


Figure 2

1. Indicate the sign of the charges carried by each armature of the capacitor during charging.
2. Show that the differential equation that governs the change of the voltage across the capacitor, $u_C = u_{AB}$, is given by:

$$u_C + RC \frac{du_C}{dt} = E$$

3. The solution of the differential equation may be written $u_C = a + b e^{-\frac{t}{\tau}}$.

Determine the expressions of the constants a , b & τ in terms of E , R & C .

4. Give, in steady state:

- a) the expression of u_C .
- b) the current carried by the circuit.

5. Knowing that the capacitance of the capacitor C is inversely proportional to the distance d between armatures, explain how the airbag deployment is triggered.

6. An appropriate device traces the voltage across the capacitor $u_C = u_{AB}$ (Figure 3) versus time.

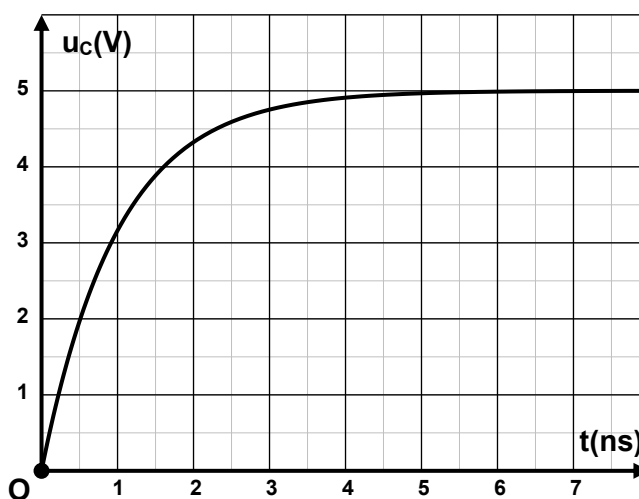
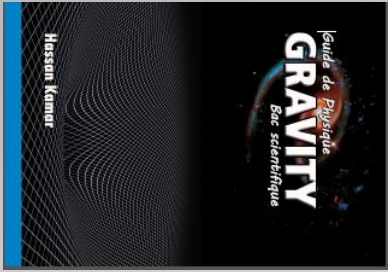
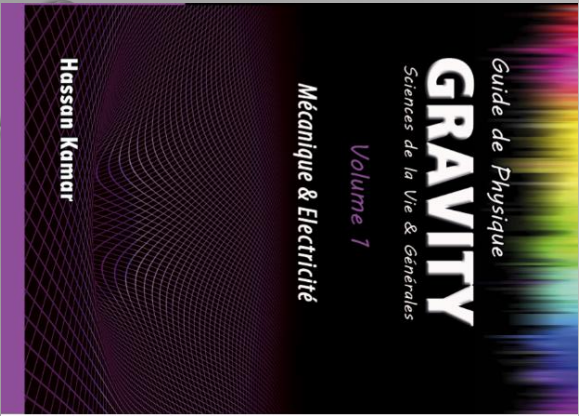
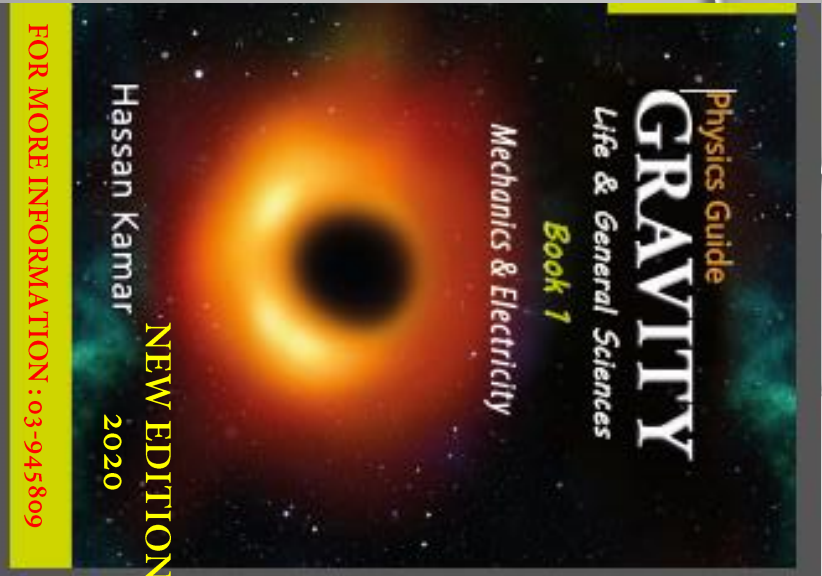
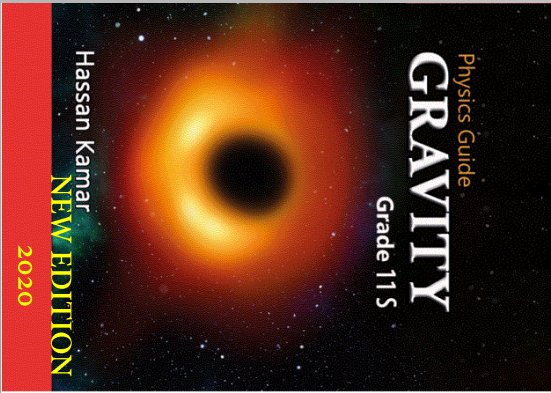
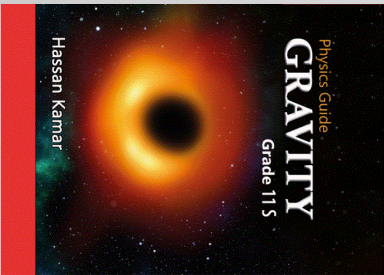


Figure 3

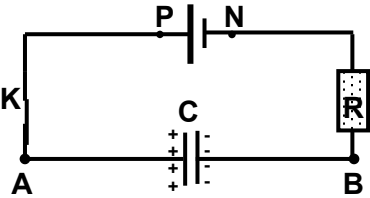
- a) Give the value of E .
- b) Determine the time constant τ , and compare it to the collision duration that is of the order of 250ms .
- c) Explain why the airbag is deployed very fast.
- d) Deduce R .



Mark Scheme – Sample 2022/2023

Exercise I (6.5 points)			
Part A (03 points)	1.	The linear momentum and kinetic energy.	½
	2.	Conservation of linear momentum: $m_1 \vec{0} + m_2 \vec{v} = m_1 \vec{v}_1 + m_2 \vec{v}_2$; The velocities are collinear, so: $m_2(v - v_2) = m_1 v_1 \dots \dots \textcircled{1}$	½
		Conservation of kinetic energy: $0 + \frac{1}{2} m_2 v^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$; $m_2(v - v_2) \times (v + v_2) = m_1 v_1^2 \dots \dots \textcircled{2}$	½
		$\frac{\textcircled{2}}{\textcircled{1}}$: $v + v_2 = v_1$;	½
		& $v - v_2 = 4 v_1$; $v_1 = 0.8m/s$ & $v_2 = v_1 - v = -1.2m/s$.	½
Part B (3.5 points)	1.	Forces acting: weight, normal reaction and the force of interaction Representation	½ ½
		Newton's second law applied on (S_2) upon collision: So, $\vec{w}_2 + \vec{N}_2 + \vec{F}_{1/2} = \frac{d\vec{P}_2}{dt}$;	½
		However $\vec{w}_2 + \vec{N}_2 = \vec{0}$,	¼
		& $\frac{d\vec{P}_2}{dt} = \frac{\Delta \vec{P}_2}{\Delta t}$ because the change is linear.	½
		We get: $\vec{F}_{1/2} = \frac{\Delta \vec{P}_2}{\Delta t} = \frac{m_2(\vec{v}_2 - \vec{v})}{\Delta t}$.	½
	2.	(a) & (d) for (S_2); (c) & (b) for (S_1).	½
	3.a)	The duration of the collision: $\Delta t = (375 - 350)ms = 25ms$.	¼
	3.b)	$F_{1/2} = \frac{0.1(-1.2-2)}{25 \times 10^{-3}} = -12.8N$	½

Exercise II (07 points)		
1.	$\nu = \frac{c}{\lambda}$ $= 4.72 \times 10^{14} \text{ Hz.}$	$\frac{1}{4}$ $\frac{1}{2}$
2.	<p>Diagram</p> <p>The angular position of the first dark point is given by:</p> $\tan \theta_1 = \frac{\ell/2}{D};$ <p>For small angles: $\tan \theta_1 \approx \theta_1 (\text{rad});$</p> $\theta_1 = \frac{\ell/2}{D};$ <p>The positions of the dark points are given by:</p> $\sin \theta_n = n \frac{\lambda}{a}; \text{ with } n \text{ is nonzero integer.}$ $\theta_1 = \frac{\lambda}{a};$ <p>Thus, $\ell = \frac{2\lambda D}{a}.$</p>	$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
3.	$\ell = \frac{2\lambda D}{a} = 2.54 \text{ cm.}$ <p>The angular width of the central bright fringe:</p> $\alpha = 2\theta_1 = \frac{\ell}{D} = 1.27 \times 10^{-2} \text{ rad};$ $\alpha = 1.27 \times 10^{-2} \times \frac{180}{\pi} \approx 0.7^\circ.$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
4.	$\tan \theta_n = \frac{x}{D} \text{ \& } \sin \theta_n = n \frac{\lambda}{a}$ <p>For small angles: $\tan \theta \approx \sin \theta \approx \theta_1 (\text{rad}): x = \frac{n \lambda D}{a};$</p> <p>Then, $\lambda = \frac{a x}{n D} = \frac{1.5 \times 10^{-6}}{n}$ where n is a nonzero integer & λ in m</p> <p>For visible radiations:</p> $375 \times 10^{-9} \leq \frac{1.5 \times 10^{-6}}{n} \leq 750 \times 10^{-9};$ <p>Then, $2 \leq n \leq 4$ and n is an integer; So, $n = 2, 3 \text{ \& } 4.$</p> <p>For $n = 2, \lambda = 750 \text{ nm};$ For $n = 3, \lambda = 500 \text{ nm};$ For $n = 4, \lambda = 375 \text{ nm}.$</p>	$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{2}$
5.	<p>Then, $\theta_0 = 0.0188 \text{ rad};$</p> <p>However, $\sin \theta_0 = 1.22 \frac{\lambda}{a} \approx \theta_0;$</p> $a = \frac{1.22 \times 635 \times 10^{-3}}{0.0188} \mu\text{m} \approx 41 \mu\text{m} \in [15 \mu\text{m}, 75 \mu\text{m}];$ <p>Then, the chocolate is suitable for milk.</p>	$\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{4}$

Exercise III (6.5 points)		
1.		$\frac{1}{2}$
2.	Law of addition of voltages: $u_{PN} = u_{PA} + u_{AB} + u_{BN}$;	$\frac{1}{2}$
	$u_{BN} = Ri$ where $i = +\frac{dq}{dt}$ & $q = Cu_C$, then $u_{BN} = RC \frac{du_C}{dt}$; $E = u_C + RC \frac{du_C}{dt}$	$\frac{1}{2}$
3.	So, $\frac{du_C}{dt} = -\frac{b}{\tau} e^{-\frac{t}{\tau}}$;	$\frac{1}{4}$
	Substitution in the differential equation: $E = a + b e^{-\frac{t}{\tau}} \left(1 - \frac{RC}{\tau}\right)$	$\frac{1}{2}$
	However: $b e^{-\frac{t}{\tau}} \neq 0$, then $a = E$ &	$\frac{1}{2}$
	$1 - \frac{RC}{\tau} = 0$, thus $\tau = RC$	$\frac{1}{2}$
	At $t_0 = 0$, the capacitor was taken neutral $u_{C0} = 0$; But $u_{C0} = a + b e^0 = a + b = 0$; Then, $b = -a = -E$	$\frac{1}{4}$
4.a)	In steady state: $u_C = E$.	$\frac{1}{4}$
4.b)	In steady state: $i = 0$.	$\frac{1}{4}$
5.	<p>During accidents, the mobile armature approaches from the fixed armature, then the capacitance of the capacitor increases (C is inversely proportional to the distance).</p> <p>The electric charge carried by the armature positively charged, in the steady, is:</p> $Q = C \times E$ <p>As C increases, the electric charge should increase; a current is detected in the circuit, thus the airbag will be deployed.</p>	$\frac{1}{2}$
6.a)	$E = 5V$ (steady state)	$\frac{1}{4}$
6.b)	At $t = \tau$, $u_C = 0.63 \times 5V = 3.15V$	$\frac{1}{4}$
	Graphically $\tau = 1ns$	$\frac{1}{2}$
6.c)	$\tau = 1ns$ is very short compared to $250ms$;	$\frac{1}{2}$
6.d)	$\tau = RC = 1 \times 10^{-9}s$; $R = \frac{\tau}{C} = \frac{1 \times 10^{-9}}{100 \times 10^{-12}} = 10\Omega$	$\frac{1}{2}$