Integral

Grade: 12 LS

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Formulas

$$\int K\,dx = Kx + c$$

$$Note:\ (Kx)' = k$$

Application

$$\int 5\,dx = 5x + c$$

$$\int -4\,dx = -4x + c$$

Formulas

$$\int x^n dx = \frac{x^n+1}{n+1} + \mathrm{c}$$

Note : $(kx^n)' = knx^{n-1}$

Application

$$\int x^2\, dx = \frac{x3}{3} + c$$

$$\int x^4\, dx = \frac{x^5}{5} + c$$

$$\int 3x^2 dx = 3\,\frac{x^3}{3} + c = x^3 + c$$

Formulas

$$\int U^n U' dx = \frac{U^{n+1}}{n+1} + \mathrm{C}$$

Note : $(\mathrm{U}^{\mathrm{n}})' = \mathrm{n}U^{n-1}\, U'$

Application

$$\int 6(6x+1)^2\, \mathrm{dx}$$

$U = 6x + 1 \quad U' = 6$

$$\int U' U^2 dx = \frac{U^3}{3} + c = \frac{(6x+1)^3}{3} + \mathrm{c}$$

Formulas

$$\int U^n U' dx = \frac{U^{n+1}}{n+1} + C$$

Note : $(\mathrm{U}^{\mathrm{n}})' = \mathrm{n}U^{n-1}\,U'$

Application

$$\int 4x(2x^2+1)^3 dx$$

$$U = 2x^2 + 1 \quad then\; U' = 4x$$

$$\int U' U^3 dx = \frac{U^4}{4} + c = \frac{(2x^2+1)^4}{4} + c$$

Formulas

$\int U^n U' dx = \frac{U^{n+1}}{n+1} + C$

Note : $(U^n)' = nU^{n-1}\, U'$

Application

Calculate $\int 5(5x-4)^4 dx$

$= \int U'U^4 dx = \frac{U^5}{5} + c = \frac{(5x-4)^5}{5} + c$ $\quad U = 5x - 4 \;\; then \;\; U' = 5$

Formulas

$$\int U^n U' dx = \frac{U^{n+1}}{n+1} + \text{C}$$

Note : $(\mathrm{U}^{\mathrm{n}})' = \mathrm{n}U^{n-1}\, U'$

Application

$$\int (-2x+1)^3 dx$$

$U = -2x + 1 \quad U' = -2$

$$= \frac{1}{-2}\int -2(-2x+1)^3 dx$$

$$= \frac{-1}{2}\int U' U^3 dx = \frac{-1}{2}\frac{U^4}{4} + c = \frac{-1}{2}\frac{(-2x+1)^4}{4} + c.$$

Note :

$$\int (x^2 + 1)^3 dx$$

$$U = x^2 + 1 \quad then\, U' = 2x$$

$$\int \frac{2x}{2x} (x^2 + 1)^3 \mathrm{dx}$$

This is wrong because there is a condition $x \neq 0$

So what can we do ?

We can expand $(x^2 + 1)^3$ and ......

Application

Find $\int \frac{1}{x^2}$ dx

$$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{x^{-1}}{-1} = \frac{-1}{x^1} = -\frac{1}{x}$$

Formulas

$\int \frac{1}{x} dx = \ln x + \text{c}$ with $x > 0$

Note : $(\ln x)' = \frac{1}{x}$

Application

$$\int \frac{1}{x} dx = \ln x + c$$

$$\int \frac{dx}{x} = \int \frac{1}{x} dx = \ln x + c$$

Formulas

$\int \frac{U'}{U} dx = \ln U + \mathrm{c}$ with $U > 0$

Note : $(\ln U)' = \frac{U'}{U}$

Application

$$\int \frac{2}{2x+1} dx$$

$U = 2x + 1$ *then* $U' = 2$

$$= \int \frac{U'}{U} dx = \ln U + \mathrm{c} = \ln(2x + 1) + \mathrm{c}$$

Formulas

$\int \frac{U'}{U} dx = \ln U + \text{c}$ with $U > 0$

Note : $(\ln U)' = \frac{U'}{U}$

Application

$$\int \frac{1}{-3x+2} dx$$

$U = -3x + 2$ *then* $U' = -3$

$$-\frac{1}{3} \int \frac{-3}{-3x+2} dx = -\frac{1}{3} \int \frac{U'}{U} dx = -\frac{1}{3} \ln U + \text{c} = -\frac{1}{3} \ln(-3x + 2) + \text{c}$$

Formulas

$\int \frac{U'}{U} dx = \ln U + \text{c}$ with $U > 0$

Note : $(\ln U)' = \frac{U'}{U}$

Application

$$\int \frac{4x-3}{2x^2-3x} dx$$

$U = 2x^2 - 3x$ *then* $U' = 4x - 3$

$$= \int \frac{U'}{U} dx = \ln U + \text{c} = \ln(2x^2 - 3x) + \text{c}$$

Formulas

$\int e^x dx = e^x + \mathrm{c}$

Note :$(e^x)' = e^x$

Application

$$\int e^x dx = e^x + c$$

Formulas

$\int U'e^U dx = e^U + \mathrm{c}$

Note :$(e^U)' = U'e^U$

Application

$\int 2e^{2x} dx = \int U'e^U + \mathrm{c} = e^{2x} + c$

$U = 2x \quad then\ U' = 2$

Formulas

$\int U'e^{U}dx = e^{U} + \text{c}$

Note :$(e^{U})' = U'e^{U}$

Application

$\int -3e^{-3x}dx = \int U'e^{U} + \text{c} = e^{-3x} + c$

$U = -3x$ $then\ U' = -3$

Formulas

$$\int U' e^{U} dx = e^{U} + \mathrm{c}$$

Note : $(e^{U})' = U' e^{U}$

Application

$$\int e^{2x} dx =$$

$U = 2x$ then $U' = 2$

$$\frac{1}{2}\int 2\, e^{2x} dx = \frac{1}{2}\int U' e^{U} + \mathrm{c} = \frac{1}{2} e^{2x} + c$$

Formulas

$\int U' e^{U} dx = e^{U} + \mathrm{c}$

Note :$(e^{U})' = U' e^{U}$

Application

$\int e^{-x} dx =$

$U \ = -x \ \ then\ U' = -1$

$-\int -1\, e^{2x} dx = -\int U' e^{U} + \mathrm{c} \ = -e^{-x} + c$

Formulas

$$\int (kU + mV)dx = k\int U\,dx + m\int V\,dx \;\; where\; k\; and\; m\; are\; real\; numbers$$

Application

$$\int (2e^{3x} + \frac{3}{x})\,dx = 2\int e^{3x}\,dx + 3\int \frac{1}{x}\,dx = 2\frac{e^{3x}}{3} + 3\mathrm{lnx} + \mathrm{c}$$

Formulas

$$\int K\,dx = Kx + c$$

$$\int \frac{1}{x}dx = \ln x + \text{c with } x > 0$$

$$\int e^{x}dx = e^{x} + \text{c}$$

$$\int x^{n}dx = \frac{x^{n}+1}{n+1} + \text{c}$$

$$\int \frac{U\prime}{U}dx = \ln U + \text{c}$$

with $U > 0$

$$\int U'e^{U}dx = e^{U} + \text{c}$$

$$\int U^{n}U'dx = \frac{U^{n+1}}{n+1} + C$$

$$\int (mU + nV)dx = m\int U\,dx + n\int V\,dx$$

Application

$$\int \frac{\ln x}{x} dx = \int \frac{1}{x} \ln x \, dx$$

$U = \ln x$ *then* $U' = \frac{1}{x}$

$$= \int U'U \, dx$$

$$= \frac{U^2}{2} + c = \frac{(\ln x)^2}{2} + c$$

Application

$$\int \frac{\ln^2 x}{x}\, dx = \int \frac{1}{x} \ln^2 x \, dx$$

$$U = ln\, x \quad then\ U' = \frac{1}{\mathrm{x}}$$

$$= \int U' U^2 \, dx$$

$$= \frac{U^3}{2} + c \quad = \frac{(\ln x)^3}{2} + c$$

Definite integral

Let f be a continuous function over an interval L ,F is its antiderivative , a and b are two real numbers belongs to L.

We call integral of f from a to b the real number $F(b) - F(a)$.
This number is denoted by $\int_a^b f(x)dx = F(b) - F(a)$

Example

$$\int_0^1 e^x \, dx = \left. e^x \right]_0^1 = e^1 - e^0 = e - 1$$

Example

$$\int_1^3 2\,dx \quad = 2x]_1^3 \quad = 2(3) - 2(1) = 6 - 2 = 4$$

$$\int_0^2 e^{2x}\,dx \quad = \frac{e^{2x}}{2}]_0^2 = \frac{e^{2(2)}}{2} - \frac{e^{2(0)}}{2} = \frac{\mathrm{e}^4}{2} - \frac{1}{2}$$

Properties

$$\int_a^a f(x)dx = 0$$

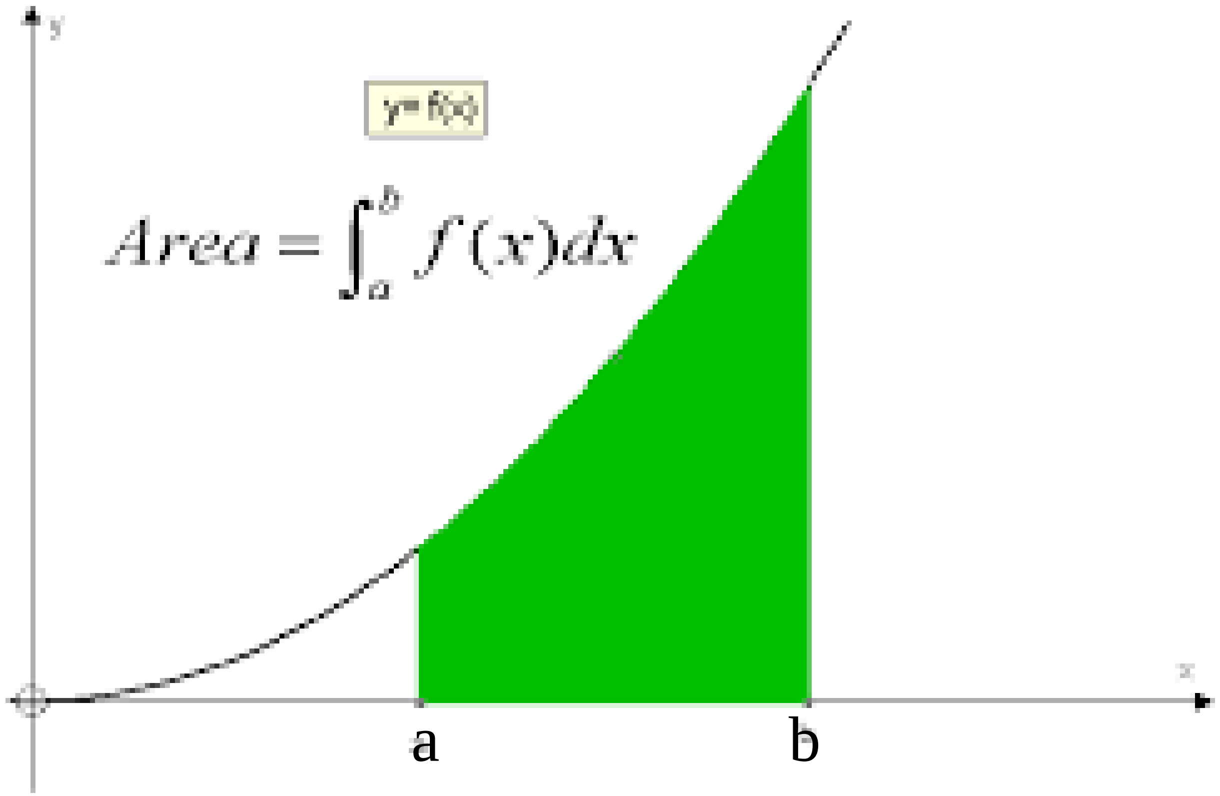
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

Graphical representation

$$\int_a^b f(x)\,dx$$

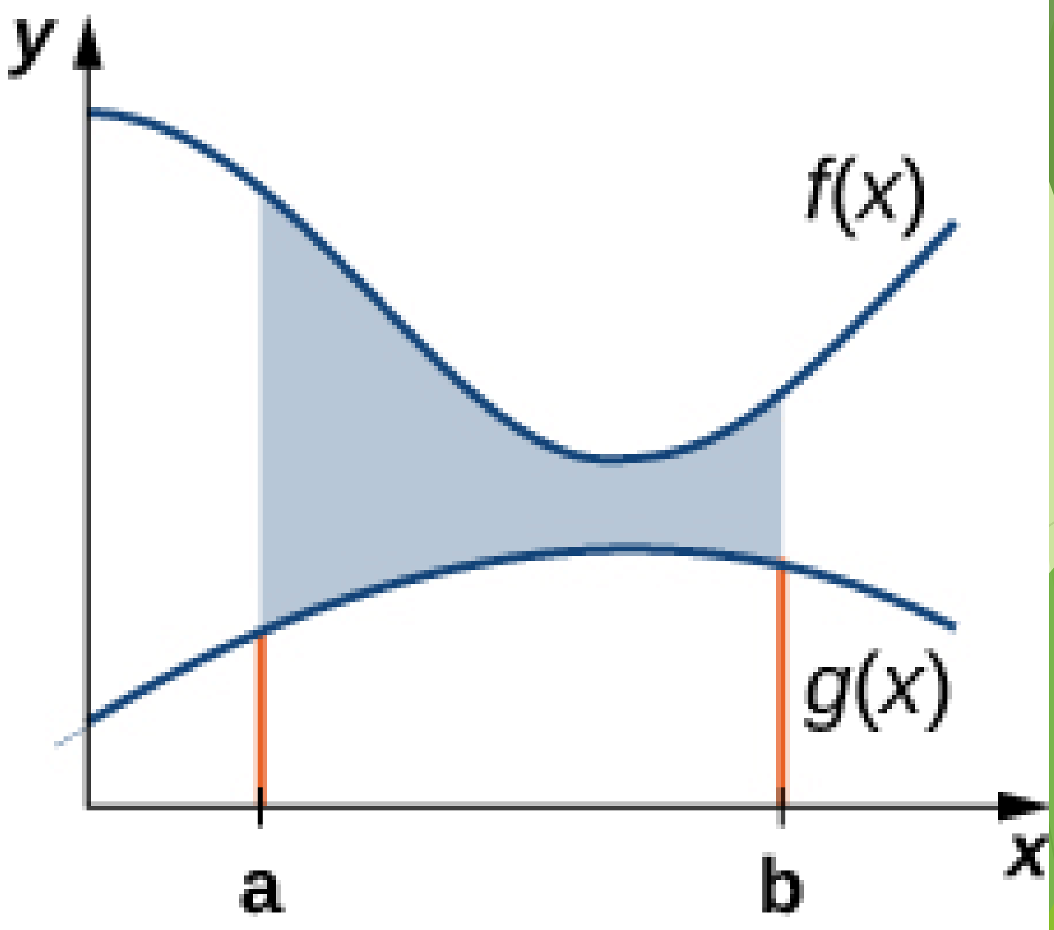
Is the area of the region bounded by the curve of f (f(x) >0) , the x-axis and the two vertical lines x =a and x =b.



Graphical representation

$$\int_a^b (f(x) - g(x))\, dx$$

Is the area of the region bounded by the curve of f,g the x-axis and the two vertical lines x =a and x =b.



Graphical representation

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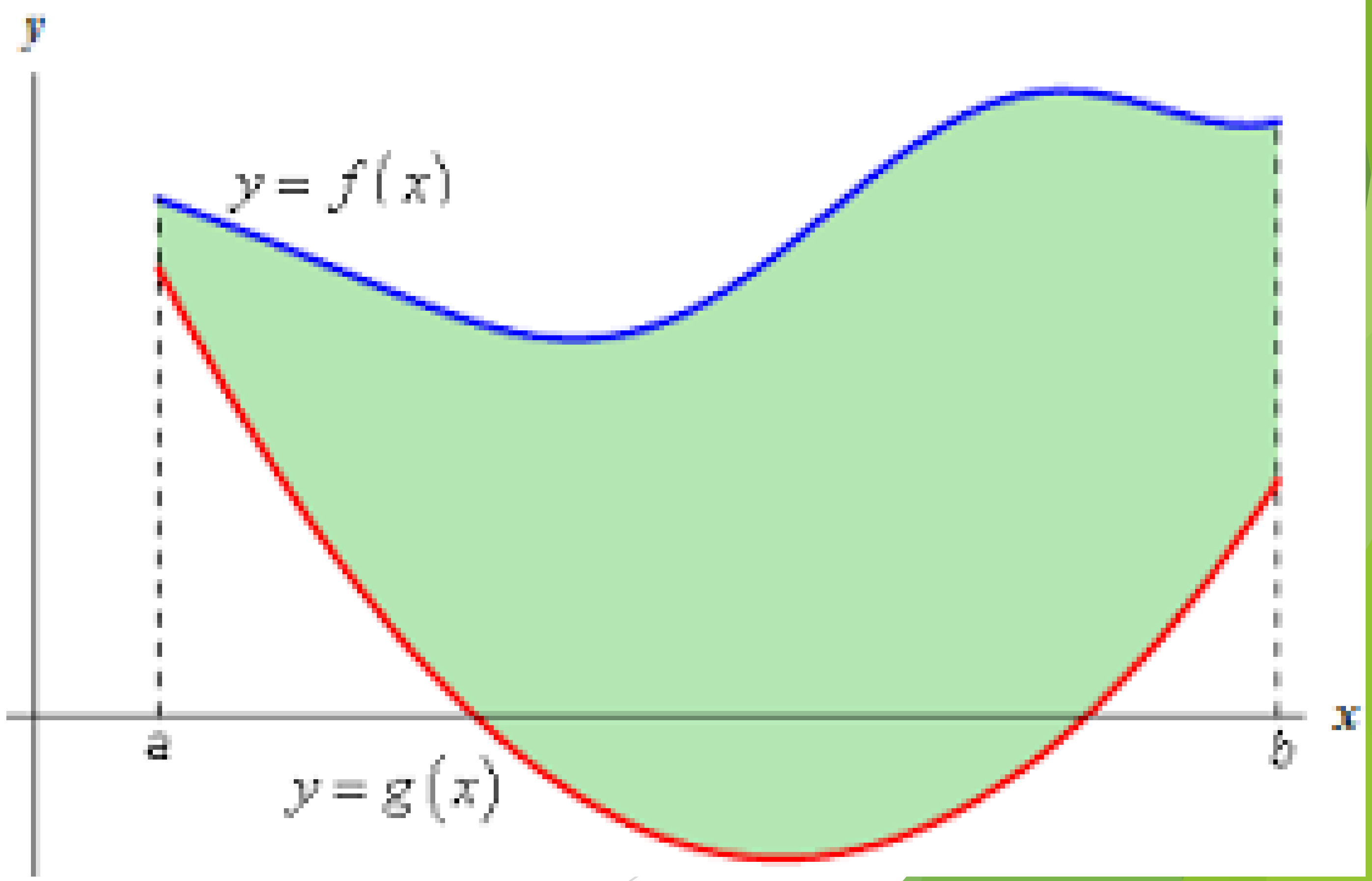
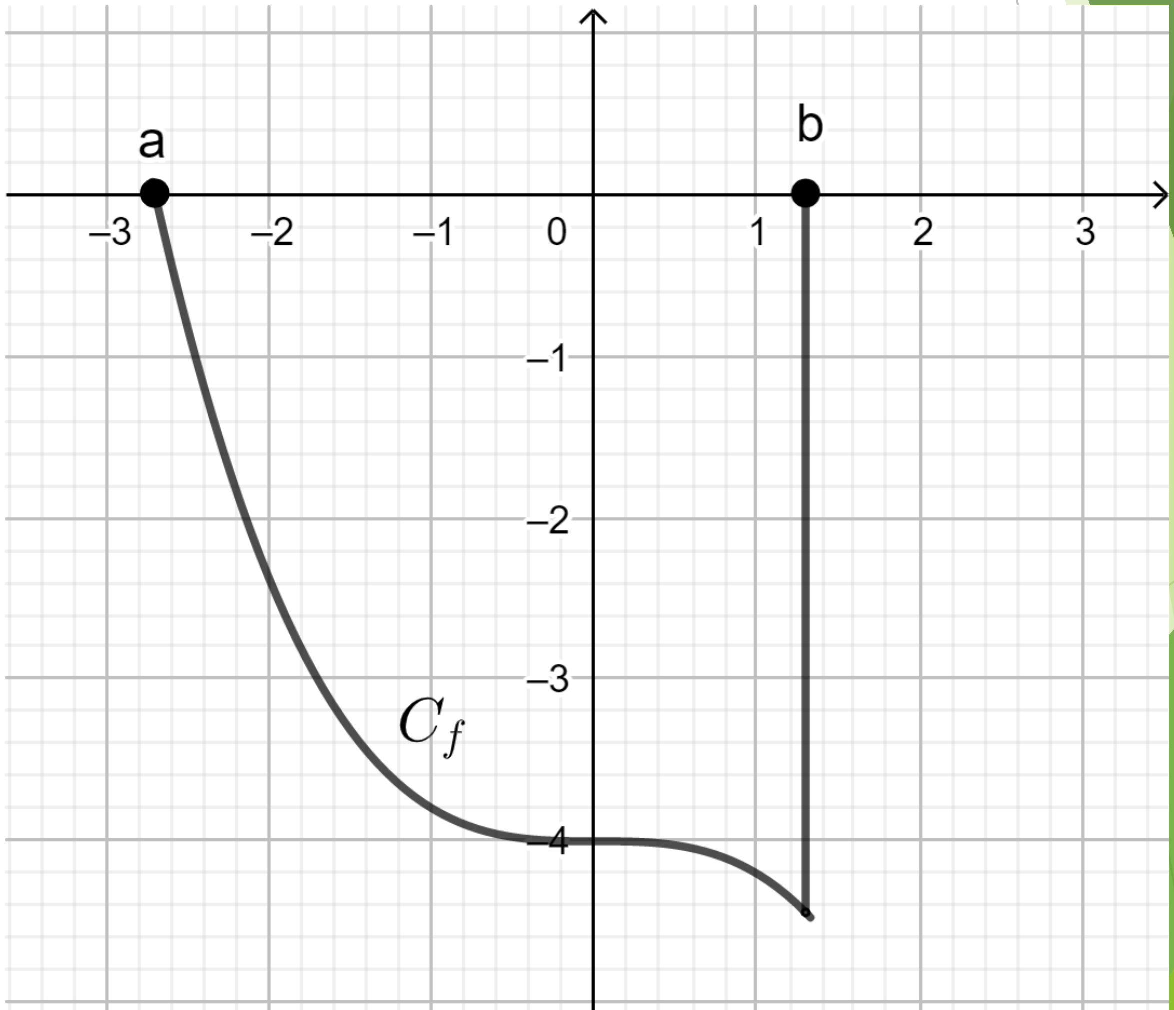
Is the area of the region bounded by the curve of f,g the x-axis and the two vertical lines x =a and x =b.



Graphical representation

$$\int_a^b -f(x)\,dx$$

Is the area of the region bounded by the curve of f (f(x) >0) , the x-axis and the two vertical lines x =a and x =b.



Example

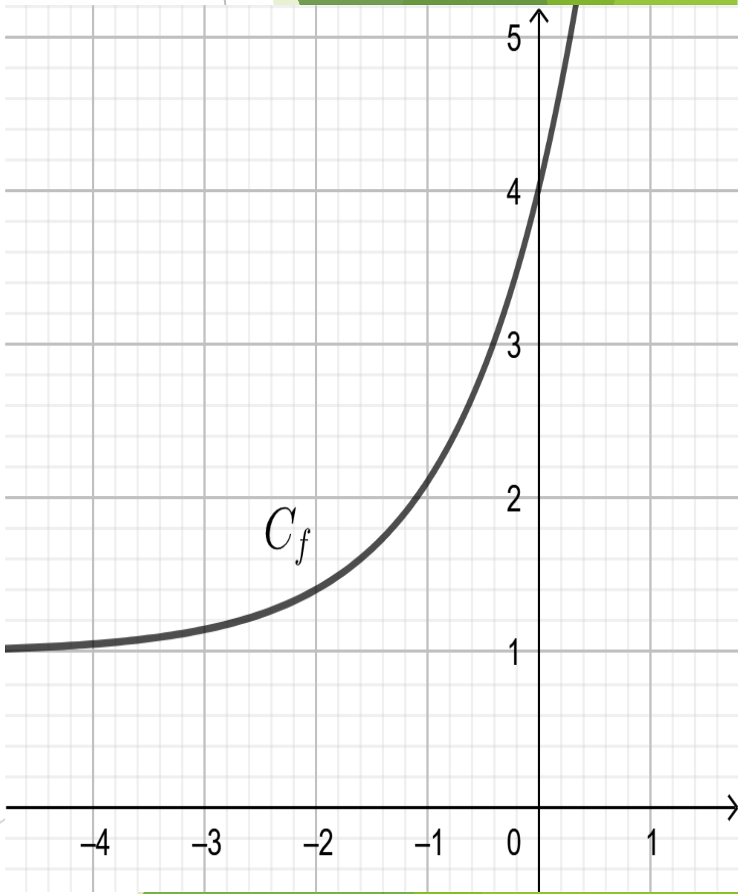
Calculate the area of the region bounded by the curve of f ,($f(x) = 3e^x + 1$) the x-axis ,the y axis and the two vertical lines $x = -2$.

Solution

$$\int_{-2}^{0} f(x)\, dx = \int_{-2}^{0} (3e^x + 1)dx = \int_{-2}^{0} 3e^x dx + \int_{-2}^{0} 1 dx$$

$$= 3e^x + x]_{-2}^{0} = 3\mathrm{e}^0 + 0 - 3\mathrm{e}^{-2} - (-2) = 3 - \frac{3}{\mathrm{e}^2} + 2$$

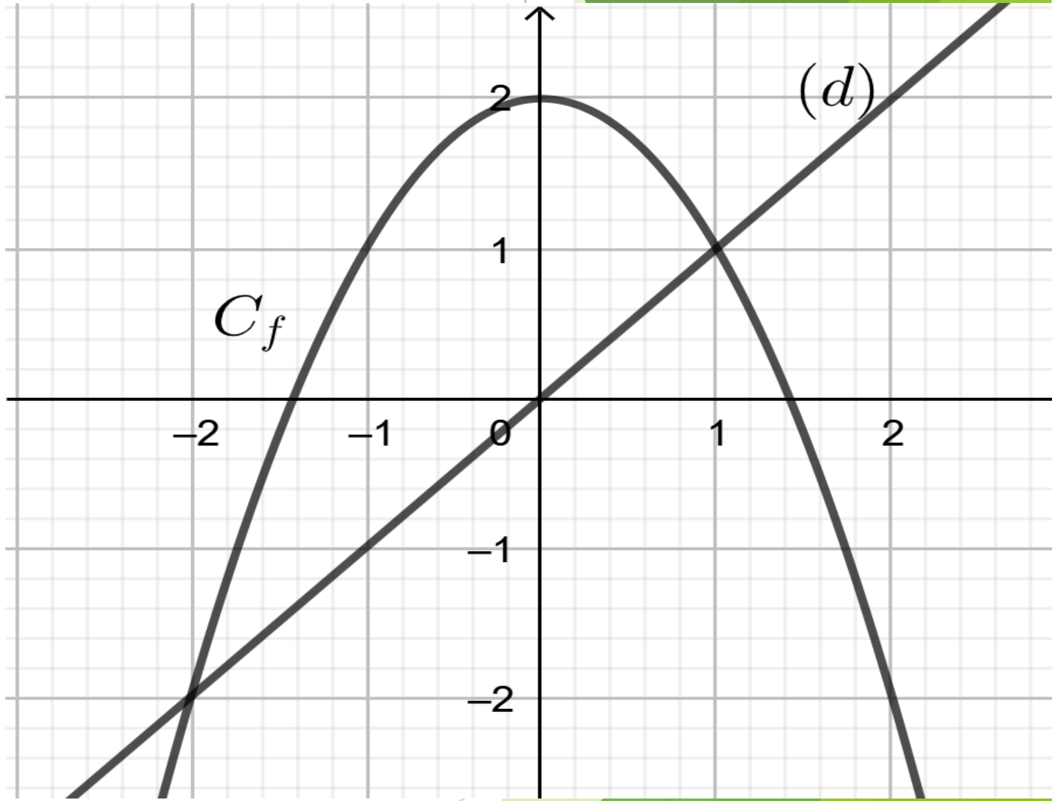
$$= 5 - \frac{3}{\mathrm{e}^2} \ \text{unit}^2$$



Example

Given that $f(x) = 2 - x^2$ $\quad (d): y = x$

Calculate the area of the region bounded by the curve of f and (d).



Solution

$$\int_{-2}^{1} (f(x) - y)\, dx \quad = \int_{-2}^{1} (2 - x^2 - x) dx$$

$$= \int_{-2}^{1} 2dx + \int_{-2}^{1} -x^2 dx + \int_{-2}^{1} -xdx$$

$$= 2x - \frac{x^3}{3} - \frac{x^2}{2}\Big]_{-2}^{1} = 2(1) - \frac{1^3}{3} - \frac{1^2}{2} - 2(-2) + \frac{(-2)^3}{2} + \frac{(-2)^2}{2} = 3.17 \text{ unit}^2$$