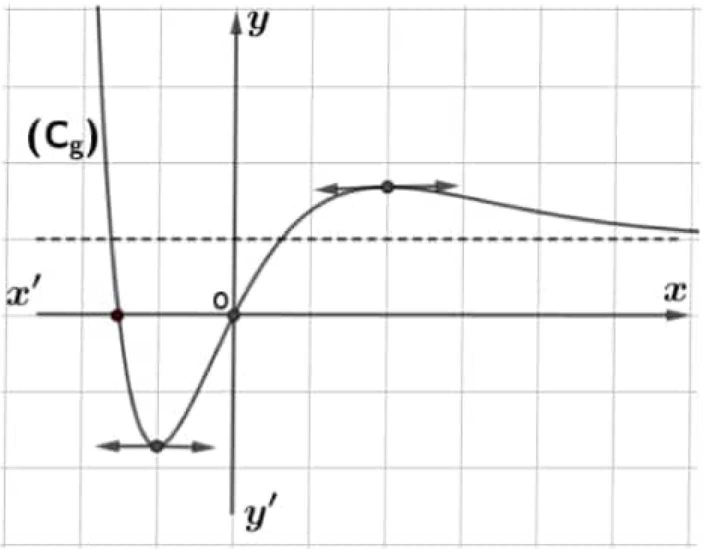
**V-( 6.5 points)**

**Part A**

In the adjacent figure $(C_g)$ is the graph of the function $g$ defined over IR by $g(x) = 1 + (x^2 + x - 1)e^{-x}$

The graph$(C_g)$ intersects the axis of abscissas $x'ox$ at two points of abscissas 0 and $\alpha$

The line $y = 1$ is an asymptote to $(C_g)$ at $+\infty$




1) Knowing that $g(-1) = 1 - e$ and $g(2) = 1 + 5e^{-2}$
   Set up the table of variations of $g$
2) Show that $\alpha$ verifies two conditions
   $-1.52 < \alpha < -1.5$ and $\alpha^2 + \alpha = 1 - e^{\alpha}$
3) Study the sign of $g(x)$, according to the values of $x$
4) Let $h(x) = \sqrt{g(x)} + \ln(-x)$ . Find domain of definition of $h$

**Part B**

Consider a function $f$ defined on IR by $f(x) = -x + (x^2 + 3x + 2)e^{-x}$

Let (C ) be the representative curve of $f$ in an orthonormal system ( O , $\vec{\imath}$ , $\vec{\jmath}$ )

1) Calculate $\lim\limits_{x\to-\infty} . f(x)$ and $\lim\limits_{x\to-\infty} . \frac{f(x)}{x}$
2) a)Calculate $\lim\limits_{x\to+\infty} . f(x)$
   b) Show that the line (D):$y = -x$ is an oblique asymptote to (C ) at $+\infty$
   c) Study the relative positions of (C ) and (D )
3) a) Show that $f'(x) = -g(x)$
   b) Set up the table of variations of $f$
4) Show that (C ) admits two points of inflection of coordinates to be determined
5) Plot (C ) . Choose $\alpha = -1.51$

**III.** **(11 points)**

**Part A**

The below table is the table of variations of the function g defined over ℝ by $g(x) = axe^{2x} + b$, where $a$ and $b$ are two real numbers.

| $x$ | $-\infty$ | | $\frac{-1}{2}$ | | $+\infty$ |
| --- | --- | --- | --- | --- | --- |
| $g'(x)$ | | $+$ | $0$ | $-$ | |
| $g(x)$ | $-1$ | $\nearrow$ | $2e^{-1}-1$ | $\searrow$ | $-\infty$ |

**1)** Prove that $g(x) = -4xe^{2x} - 1$.

**2)** Study, according to the values of $x$, the sign of g.

**Part B**

Consider the function $f$ defined over ℝ by $f(x) = (2x-1)e^{2x} + x$. Denote by *(C)* the representative curve of $f$ in an orthonormal system $(O; \vec{i}, \vec{j})$.

**1)** **a.** Calculate $\lim\limits_{x \to -\infty} f(x)$.

**b.** Prove that the line *(d)* of equation $y = x$ is an asymptote to *(C)* at $-\infty$.

**c.** Study, according to the values of $x$, the relative position of *(C)* and *(d)*.

**2)** Calculate $\lim\limits_{x \to +\infty} f(x)$ and find $f(1)$ to the nearest $10^{-2}$.

**3)** Prove that $f'(x) = -g(x)$, then setup the table of variation of $f$.

**4)** Show that *(C)* admit an inflection point W whose coordinates are to be determined.

**5)** **a.** Prove that the equation $f(x) = 0$ admit a unique solution $\alpha$ and verify that $0.4 < \alpha < 0.5$.

**b.** Show that $e^{2\alpha} = \frac{-\alpha}{2\alpha-1}$.

**6)** **a.** Precise the point of intersection *A* of *(C)* with the ordinate axis.

**b.** Write the equation of tangent *(T)* to *(C)* at *A*.

**7)** Draw *(d)* and *(C)*. (Take $\alpha = 0.45$)

**8)** Let $h$ be the function defined as $h(x) = \ln(-1 - f(x))$.

**a.** Use $(C)$ to determine the domain of definition $D_h$ of $h$.

**b.** Prove that $h$ is strictly decreasing over its domain $D_h$.