Physics

Notes

LS & GS

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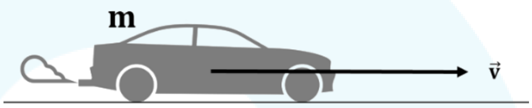
Contents

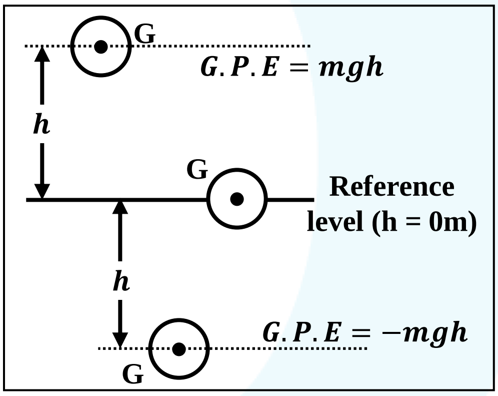
ENERGY: *Energy is the capacity for doing work.*

Kinetic energy ($K.E$ or E_k): energy possessed by an object (or system) due to its motion.

$$\boldsymbol{K.E = \frac{1}{2}mv^2}$$



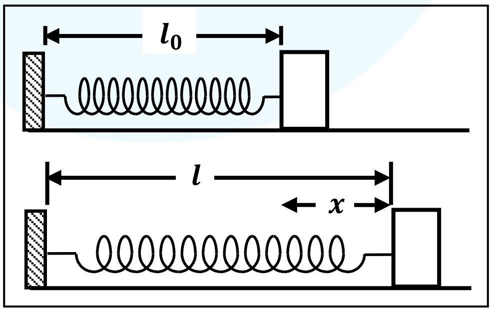
Variation in kinetic energy
(Work-kinetic energy theorem):
$\Delta K.E = \sum W_{ext}$ (For a rigid system)



Gravitational potential energy ($G.P.E$ or $P.E_g$ or E_{pg}): energy possessed by an object (or system) due to its position in the gravitational field.

$$G.P.E = mgh$$

Variation in gravitational potential energy: $\Delta G.P.E = -W_{\vec{W}}$



Elastic potential energy ($E.P.E$ or $P.E_e$ or E_{pe}): is the potential energy stored as a result of deformation of elastic objects.
Magnitude of tension in an ideal spring (obeys Hooke's law):
$T = k|\Delta l| = k|x|$ where $x = \Delta l = l - l_0$

$$E.P.E = \frac{1}{2}kx^2$$

Variation in elastic potential energy: $\Delta E.P.E = -W_{\vec{T}}$

Mechanical energy ($M.E$ or E_m): sum of kinetic and potential energies.

$$M.E = K.E + G.P.E + E.P.E = \frac{1}{2}mV^2 + mgh + \frac{1}{2}kx^2$$

Mechanical energy

Conserved
The non-conservative forces (friction - traction force …) are neglected.

Principle of conservation of mechanical energy:
$\boldsymbol{\Delta M.E = 0}$
$\boldsymbol{M.E_i = M.E_f}$
$\boldsymbol{\Delta K.E = -\Delta P.E}$

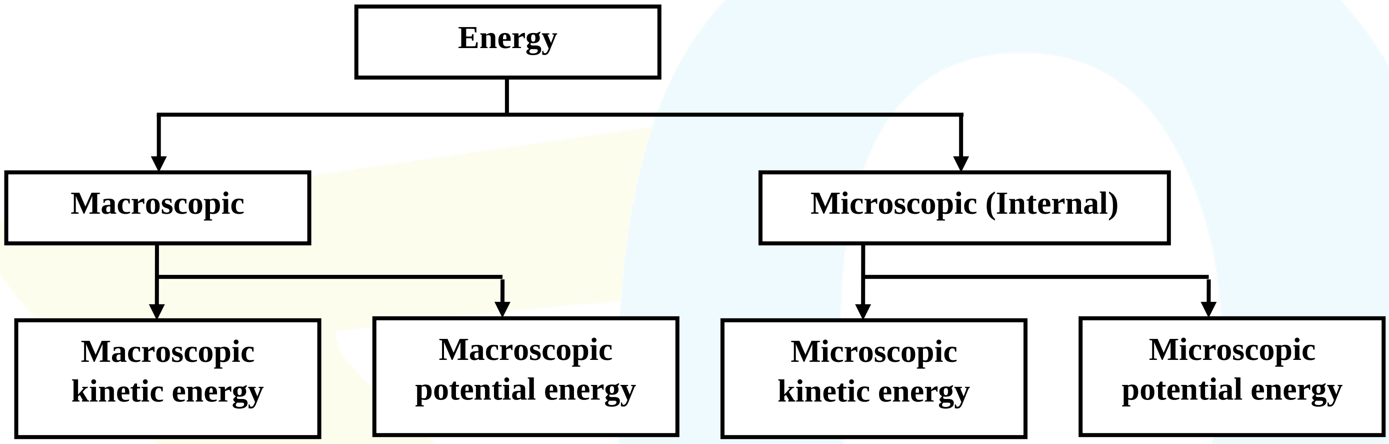
Not conserved
The non-conservative forces (friction - traction force) exist

Principle of non-conservation of mechanical energy:
$\boldsymbol{\Delta M.E = W_{NC}}$
$\boldsymbol{M.E_f - M.E_i = W_{NC}}$

In the case where friction is the only non-conservative force:

$$\Delta M.E = W_{\vec{f}} \Longrightarrow M.E_f - M.E_i = -f \times d$$

The energy lost appears in form of thermal energy: $Q = |\Delta M.E|$



Total energy: $E = M.E + U$.
Energy-isolated system: no exchange of energy with the surrounding ($E = constant \Longrightarrow \Delta E = 0$).

$$\Delta E = \Delta M.E + \Delta U = 0 \Longrightarrow \Delta M.E = -\Delta U.$$

2.1- RECALL

Case of a particle		
	One dimensional motion	**Two-dimensional motion**
Position	$\boldsymbol{x}$ $v = \frac{dx}{dt} \Rightarrow x = \int v dt$	$\vec{r}$ $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{r} = \int \vec{v} dt$
Velocity	$v = \frac{dx}{dt} = x'$ $a = \frac{dv}{dt} \Rightarrow v = \int a dt$	$\vec{v} = \frac{d\vec{r}}{dt} = \vec{r}'$ $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{v} = \int \vec{a} dt$
Acceleration	$a = \frac{dv}{dt} = v'$	$\vec{a} = \frac{d\vec{v}}{dt} = \vec{v}'$
Case of a system of particles		
	One dimensional motion	**Two-dimensional motion**
Position	$X_G = \frac{m_1x_1+m_2x_2+m_3x_3+\cdots}{m_1+m_2+m_3+\cdots}$	$\vec{r}_G = \frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3+\cdots}{m_1+m_2+m_3+\cdots}$
Velocity	$V_G = \frac{dx_G}{dt} = x'_G$	$\vec{V}_G = \frac{d\vec{r}}{dt} = \vec{r}'_G$
Acceleration	$a_G = \frac{dV_G}{dt} = V'_G$	$\vec{a}_G = \frac{d\vec{V}_G}{dt} = \vec{V}'_G$

2.2- LINEAR MOMENTUM AND NEWTON'S 2ND LAW

	Particle	**System of Particles (S) = [(S1); (S2)]**
Linear momentum	$\vec{P} = m\vec{V}$	$\vec{P}_s = \vec{P}_1 + \vec{P}_2 = m_1\vec{V}_1 + m_2\vec{V}_2$ $\vec{P}_S = \vec{P}_G = M\vec{V}_G$ with $M = m_1 + m_2$
Newton's 2nd law	$\sum \vec{F}_{ext} = m\vec{a} = \frac{d\vec{P}}{dt}$	$\sum \vec{F}_{ext} = M\vec{a}_G = \frac{d\vec{P}_s}{dt}$

2.3- CONSERVATION OF LINEAR MOMENTUM

A system is said to be isolated if the sum of the external forces acting on it is zero.

$$\sum \vec{F}_{ext} = \frac{d\vec{P}_s}{dt} = \vec{0} \Rightarrow \vec{P}_s = constant \Rightarrow \vec{P}_i = \vec{P}_f$$

The linear momentum of the system is conserved

Applications of conservation of linear momentum

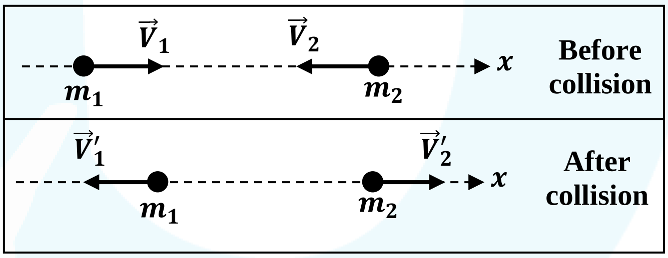
- **Explosions**
- **Recoil of cannons, rifles, …**
- **Collisions**

Collisions

Perfectly elastic: the kinetic energy is conserved	Non-elastic: the kinetic energy is not conserved	Perfectly in-elastic: the colliding particles stick together and move with a common velocity
$K.E_b = K.E_a$	$K.E_b \neq K.E_a$	

During collisions, explosions and recoil, the internal forces are much stronger than the external forces acting on the system; then, the external forces can be considered neglected relative to the internal forces.

2.4- Perfectly Elastic Collision



Two particles m_1 and m_2 enter in a perfectly elastic head-on collision.

Elastic collision:

- Linear momentum of the system [m₁; m₂] is conserved.
- The Kinetic energy of the system [m₁; m₂] is conserved.

Required to find: V_1' and V_2' the respective velocities of m_1 and m_2 just after collision.

To find these two unknowns we need two equations:

The first equation is obtained by applying the principle of conservation of linear momentum.
The second equation is obtained by applying the principle of conservation of kinetic energy.

During collision, the system [m_1; m_2] is isolated.

$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \vec{0} \Longrightarrow \vec{P} = constant$

Principle of conservation of linear momentum: $\vec{P}_b = \vec{P}_a$

$m_1\vec{V}_1 + m_2\vec{V}_2 = m_1\vec{V}_1' + m_2\vec{V}_2'$

The collision is head-on, so the above equation can be written in its algebraic form:

$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$

$m_1V_1 - m_1V_1' = m_2V_2' - m_2V_2$

$$\boxed{m_1(V_1 - V_1') = m_2(V_2' - V_2) \ldots (1)}$$

Note:
Perfectly in-elastic collision

$$\boldsymbol{V_1' = V_2' = V}$$
$$\boldsymbol{m_1V_1 + m_2V_2 = (m_1 + m_2)V}$$
$$\boldsymbol{V = \frac{m_1V_1 + m_2V_2}{m_1 + m_2}}$$

The collision is elastic; then, the kinetic energy of the system [m_1; m_2] is conserved.

$K.E_b = K.E_a$

$\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2$

$m_1V_1^2 + m_2V_2^2 = m_1V_1'^2 + m_2V_2'^2$

$m_1V_1^2 - m_1V_1'^2 = m_2V_2'^2 - m_2V_2^2$

$m_1(V_1^2 - V_1'^2) = m_2(V_2'^2 - V_2^2)$

$$\boxed{m_1(V_1 - V_1')(V_1 + V_1') = m_2(V_2' - V_2)(V_2' + V_2) \ldots (2)}$$

Divide Eq (2) by Eq (1)

$\frac{m_1(V_1 - V_1')(V_1 + V_1')}{m_1(V_1 - V_1')} = \frac{m_2(V_2' - V_2)(V_2' + V_2)}{m_2(V_2' - V_2)}$

$$\boxed{V_1 + V_1' = V_2' + V_2 \ldots (3)}$$

From Eq (3):
$V_2' = V_1 + V_1' - V_2$
Replace in Eq (1)
$m_1(V_1 - V_1') = m_2(V_1 + V_1' - V_2 - V_2)$
$m_1V_1 - m_1V_1' = m_2V_1 + m_2V_1' - 2m_2V_2$
$m_1V_1 - m_2V_1 + 2m_2V_2 = m_2V_1' + m_1V_1'$
$(m_1 - m_2)V_1 + 2m_2V_2 = (m_1 + m_2)V_1'$
$V_1' = \frac{m_1-m_2}{m_1+m_2}V_1 + \frac{2m_2}{m_1+m_2}V_2$
Similarly:
$V_2' = \frac{2m_1}{m_1+m_2}V_1 + \frac{m_2-m_1}{m_2+m_1}V_2$

For $V_2 = 0$

$$V_1' = \frac{m_1 - m_2}{m_1 + m_2}V_1$$

$$V_2' = \frac{2m_1}{m_1 + m_2}V_1$$

For $m_1 = m_2$

$$V_1' = V_2$$

$$V_2' = V_1$$

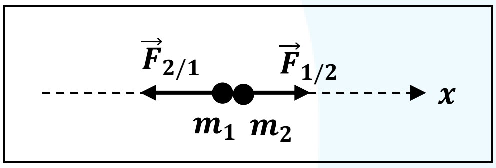
The particles exchange their velocities

2.5- INTERACTION BETWEEN m₁ AND m₂

$\vec{F}_{1/2} = -\vec{F}_{2/1}$ (Newton's 3rd law)

$\sum \vec{F}_{ext/m_1} = \vec{F}_{2/1} = \frac{\Delta\vec{P}_1}{\Delta t}$

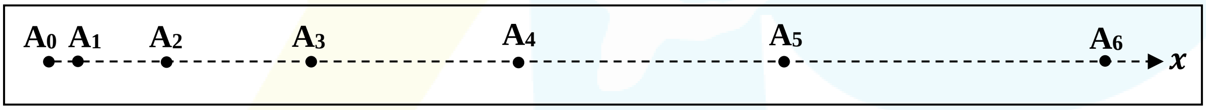
$\sum \vec{F}_{ext/m_2} = \vec{F}_{1/2} = \frac{\Delta\vec{P}_2}{\Delta t}$



Note: The external forces acting on m_1 and m_2 are neglected relative to the internal forces

2.6- MOTION OF A PUCK ALONG AN AIR TABLE

A_0 is taken as an origin of space and time.
Time constant τ: time separating two consecutive dots.



1- Time needed to reach the dot A_i is $t_i = i\tau$
Example: the time needed to reach the dot A_2 is $t_2 = 2\tau$

2- The position (abscissa) of the dot A_i is $x_i = \overline{A_0A_i}$
Example: the position (abscissa) of the dot A_3 is $x_3 = \overline{A_0A_3}$

3- The velocity of the puck at the dot A_i is $V_i = V_{av(i-1,i+1)} = \frac{\Delta x}{\Delta t} = \frac{x_{i+1}-x_{i-1}}{t_{i+1}-t_{i-1}} = \frac{\overline{A_0A_{i+1}}-\overline{A_0A_{i-1}}}{2\tau}$
Example: the velocity of the puck at the dot A_2 is:
$V_2 = V_{av(1,3)} = \frac{\Delta x}{\Delta t} = \frac{x_3-x_1}{t_3-t_1} = \frac{\overline{A_0A_3}-\overline{A_0A_1}}{t_3-t_1} = \frac{\overline{A_0A_3}-\overline{A_0A_1}}{3\tau-\tau} = \frac{\overline{A_1A_3}}{2\tau}$ $(A_1A_3\ in\ [m] and\ \tau\ in\ [s])$

4- The acceleration of the puck at the dot A_i is: $a_i = a_{av(i-1,i+1)} = \frac{\Delta V}{\Delta t} = \frac{V_{i+1}-V_{i-1}}{t_{i+1}-t_{i-1}} = \frac{V_{i+1}-V_{i-1}}{2\tau}$

Example: the acceleration at the dot A_3 is $a_3 = a_{av(2,4)} = \frac{\Delta V}{\Delta t} = \frac{V_4-V_2}{t_4-t_2} = \frac{V_4-V_2}{2\tau}$

8.1- RECALL

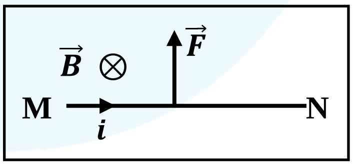
Magnetic field: invisible region of space created by:
- A magnet (Bar, U, needle, …).
- Earth (Terrestrial magnetic field).
- Current carrying-wire (Hans Christian Oersted 1819).

Magnetic field created by a current
The intensity of the magnetic field created by a wire is directly proportional to the intensity of the current traversing it:

$$B = ki$$

Rectilinear wire: $B = \frac{\mu_0}{2\pi d} i$
Flat coil: $B = \frac{\mu_0 N}{2R} i$
Solenoid: $B = \frac{\mu_0 N}{L} i$
The direction of $\vec{B}$ is determined by applying the right-hand rule.

The electromagnetic force (Laplace's force): is a force acting on conductor MN:
- Placed in a magnetic field $\vec{B}$.
- Traversed by an electric current i.

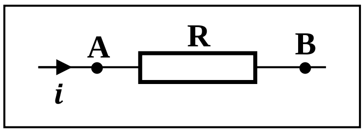


$$\vec{F} = i\vec{l} \times \vec{B} \text{ with } \vec{l} = \overrightarrow{MN}$$
$$F = ilB \sin\alpha \text{ where } \alpha = \left(\overrightarrow{MN}; \vec{B}\right)$$

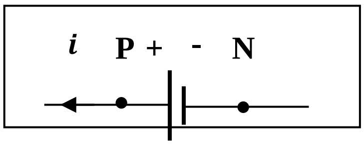
The electromagnetic force is perpendicular to the plane containing $\overrightarrow{MN}$ and $\vec{B}$
The direction of the electromagnetic force is determined by applying the RHR.

Lorentz force: on an electric charge placed in a magnetic field.

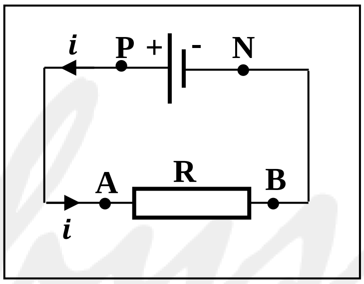
$$\vec{F} = q\vec{V} \times \vec{B}$$



Ohms law for a resistor: $u_{AB} = u_R = Ri$
The electric power received by a resistor is $p = V_{AB} i > 0$



Ohm's law for a generator: $\boldsymbol{u_{PN} = u_G = e - ri}$
The electric power received by a generator is: $p = -u_{PN} i < 0$
e: Electromotive force in [V].
r: Internal resistance in [Ω].



Expression of the intensity of the electric current traversing a circuit that consists of a generator and a resistor.
$u_{PN} = u_{PA} + u_{AB} + u_{BN}$
$e - ri = Ri$
$i = \frac{E}{r+R} = \frac{e}{R_{eq}}$ where $R_{eq} = r + R$

8.2- ELECTROMAGNETIC INDUCTION

Question: Can a magnetic field produce electricity?

Michael Faraday the English scientist was the first person to prove that a magnet can create a current. He thought that the reverse must be true, that a magnetic field could produce an electric current.
Goal: convert magnetism to electricity.

Experimental evidence of the induction phenomenon

To test this, he moved a magnet towards and away from the coil of wire connected to a galvanometer. Deflection in the galvanometer indicates that that a current is induced in the coil.
The current obtained due to the relative motion between the magnet and the coil is called induced current.
The phenomenon by which an electromotive force is induced in a conductor due to change in the magnetic field near the conductor is known as **electromagnetic induction**.

Let's now look into some experiments performed by Michael Faraday

Faraday arrived at a few conclusions by moving a bar magnet in and out of the coil of wire.
Displaced here a circular insulating wire with many turns connected to a galvanometer.

Observe the deflection of the galvanometer needle when:

1- Magnet is moved in and out
2- Different poles are introduced
3- Number of turns is changed.

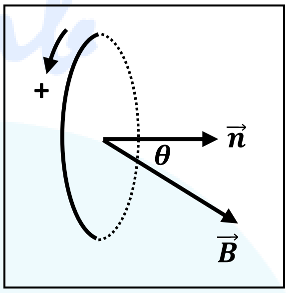
Drag the magnet in and out of the coil. Observe the deflection in the galvanometer.

1- The deflection of the galvanometer indicates the presence of current in the coil.
2- The direction of the deflection gives the direction of flow of current.
3- The speed of deflection gives the rate at which the current is induced.

Conclusions:

1- Deflection in the galvanometer indicates that the current is induced in the coil due to the relative motion between the magnet and the coil. The deflection in the galvanometer lasts as long there is a relative motion between the magnet and the coil.
2- The deflection is more if the magnet is moved faster and less when the magnet is moved slowly. That is, the rate at which the current is induced is more when the magnet is moved faster.
3- The deflection in the galvanometer is reversed when the same pole of the magnet is moved in the opposite direction or when the opposite pole is moved in the same direction. The direction of the deflection indicates the direction of flow of current.
4- The deflection in the galvanometer changes with the change in the number of turns of the coil – more the number of turns in the coil greater the deflection. The magnetic field goes around each loop of wire in the coil, so if we increase the number of turns in the wire the change in magnetic field is more.

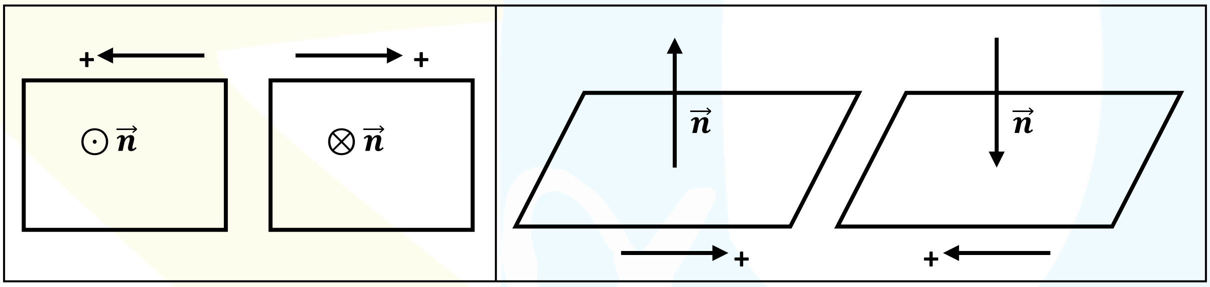
8.3- MAGNETIC FLUX



Magnetic flux: is a measurement of the total magnetic field which passes through a given area.

$$\phi = N\vec{B}.\vec{S} = N\vec{B}.\vec{n}S = NBS\cos\theta$$

ϕ: magnetic flux in [Wb]
N: number of turns
B: intensity of the magnetic field in [T]
S: area of the loop in [m^2]
$\theta = (\vec{B};\ \vec{n})$: angle between the magnetic field and the normal to the plane
Remark: the normal to the plane depends on the positive chosen sense.



Electromagnetic induction: is the generation or establishment of an e.m.f. in a circuit when the magnetic flux crossing it is varied. When such a circuit is closed, it is traversed by an induced electric current.
Variation in magnetic flux:
- Variation in the intensity of the magnetic field B.
- Variation in area S.
- Variation in θ.

8.4- LAWS OF ELECTROMAGNETIC INDUCTION

Faraday's law: the induced electromotive force "e" at any instant is equal to the opposite of the derivative with respect to time of the magnetic flux crossing the circuit.

$$e = -\frac{d\phi}{dt}$$

Lenz's law: the direction of the induced current is such that its electromagnetic effects always oppose the cause that has established this current.

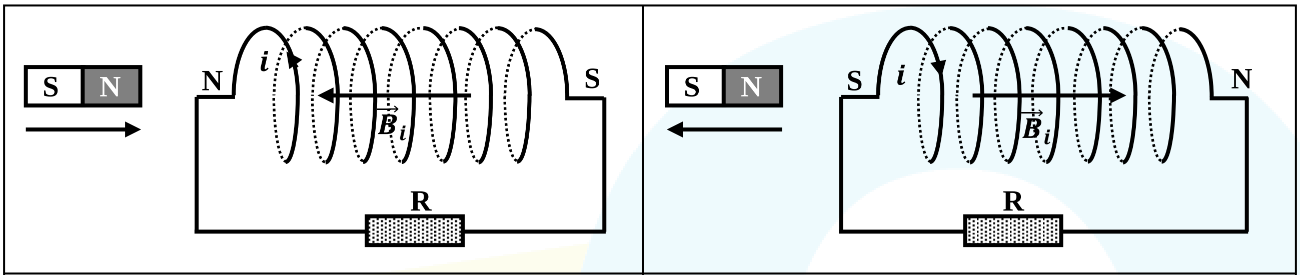
In simple words:
Systems don't like changes and try to minimize it

Attention:
- The induced current i creates an induced magnetic field $\vec{B}_i$ (direction is determined by RHR).
- The induced emf e and the induced current i are algebraic quantities that have the same sign.
- The signs of e and i depends on the positive chosen sense.

$$\phi\ varies \Rightarrow e \Rightarrow i \Rightarrow \vec{B}_i$$

The direction of the induced current *i* can be determined by the:

- **Opposition to pole movement:**



Repulsion	Attraction

Inside the coil, the magnetic field $\vec{B}_i$ is directed from S to N.

- **Opposition to change in the intensity of the magnetic field:**

$B \nearrow \Longrightarrow \vec{B}$ **and** $\vec{B}_i$ **have opposite directions**	$B \searrow \Longrightarrow \vec{B}$ **and** $\vec{B}_i$ **have the same direction**

- **Opposition to the movement of the conductor.**

$\phi = NBS\cos\theta = -Blx$

$e = -\frac{d\phi}{dt} = -\frac{d(-Blx)}{dt} = Bl\frac{dx}{dt} = Blv$

$i = \frac{e}{R_{eq}} = \frac{e}{R+r} = \frac{Blv}{R+r}$

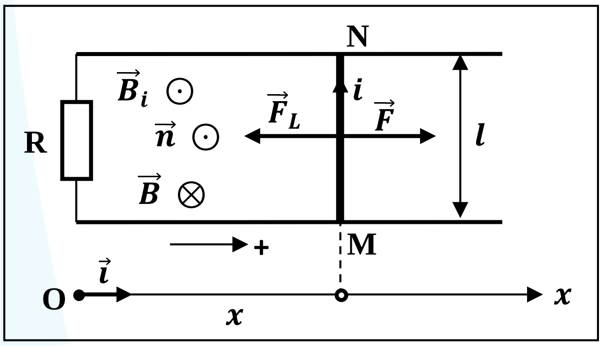
$F = ilB\sin\alpha = ilB = \frac{B^2l^2v}{R+r}$

URM $\Longrightarrow \sum \vec{F}_{ext} = \vec{0}$

$\vec{N} + \vec{W} + \vec{F} + \vec{F}_L = \vec{0}$ with $\vec{N} + \vec{W} = \vec{0}$

$\vec{F} + \vec{F}_L = \vec{0} \Longrightarrow \vec{F} = -\vec{F}_L \Longrightarrow F = F_L$

$u_{MN} = ri - e$



Uniform Rectilinear motion (URM):
$x = Vt + x_0$

- **Variation in θ**

$\phi = NBS\cos\theta = NBS\cos(\omega t + \theta_0)$

$e = -\frac{d\phi}{dt} = NBS\omega\sin(\omega t + \theta_0)$ with $e_m = NBS\omega$.

Equivalent generator:

The positive sense is oriented from A to B:

$u_{AB} = ri - e$

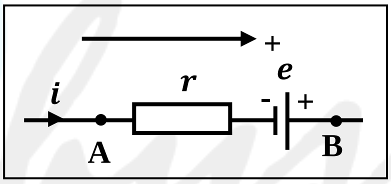
$u_{BA} = -u_{BA} = -ri + e$

$ei = V_{BA}i + ri^2$

$P = ei$ (Total electric power due to the variation of the magnetic flux)

$P = u_{BA}i$ (The electric power transferred to the external circuit)

$P = ri^2$ (Power dissipated due to Joule's effect)



Self-induction: is the generation of an e.m.f. in a circuit when the current in that circuit is varied.
Self-flux:

$$\boldsymbol{\phi = Li}$$

L: inductance of the coil in Henry [H]
Self-induced emf:

$$\boldsymbol{e = -\frac{d\phi}{dt} = -\frac{d(Li)}{dt}}$$

If $L = constant$

$$\boldsymbol{e = -L\frac{di}{dt}}$$

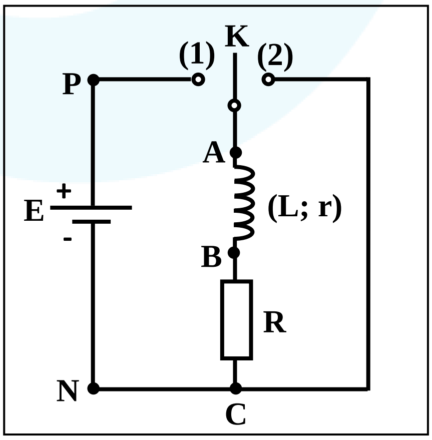
Voltage-current relation for a coil:

$$\boldsymbol{u = ri - e = ri + L\frac{di}{dt}}$$

r: resistance of the coil in ohm [Ω]
Difference between r and L: r opposes current while L opposes the change in current.
Magnetic energy stored in a coil:

$$\boldsymbol{W_{mag} = E_{mag} = \frac{1}{2}Li^2}$$

Roles of a coil:
Generator if: $e.i > 0$
Receiver if: $e.i < 0$



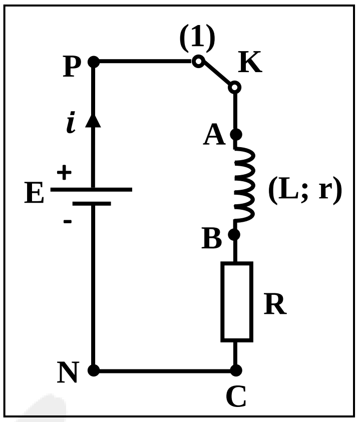
RL circuit under DC source
Growth of current (switch at position 1)

Law of addition of voltages:
$u_{PN} = u_{PA} + u_{AB} + u_{BC} + u_{CN} \Longrightarrow u_G = u_L + u_R$

Differential equation in i:
$E = ri + L\frac{di}{dt} + Ri \Longrightarrow E = L\frac{di}{dt} + (R+r)i$

Solution: $i = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$ with $I_0 = \frac{E}{R+r}$ and $\tau = \frac{L}{R+r}$

Differential equation in u_R:
$E = L\frac{di}{dt} + (R+r)i$ with $i = \frac{u_R}{R}$ and $\frac{di}{dt} = \frac{1}{R}\frac{du_R}{dt}$
$E = \frac{L}{R}\frac{du_R}{dt} + \frac{R+r}{R}u_R \Longrightarrow \frac{ER}{L} = \frac{du_R}{dt} + \frac{R+r}{L}u_R$



Differential equation in u_L (case of a purely inductive coil):
$E = u_L + Ri$ with $u_L = L\frac{di}{dt} \Longrightarrow \frac{di}{dt} = \frac{u_L}{L}$
Derive both sides with respect to time: $0 = \frac{du_L}{dt} + R\frac{di}{dt}$
$\frac{du_L}{dt} + \frac{R}{L}u_L = 0$
Solution: $u_L = U_{L_0}e^{-\frac{t}{\tau}}$ with $U_{L_0} = E$ and $\tau = \frac{L}{R}$
At t = 0s; $i = 0 \Longrightarrow u_R = 0$ and $u_L = E$
At $t = \tau$; $i = 0.63I_0$ and $u_R = 0.63RI_0$
At the steady state $t \geq 5\tau$ or $t \to \infty$: $i = I_0 = constant \Longrightarrow \frac{di}{dt} = 0$
$E = (R+r)I_0 \Longrightarrow I_0 = \frac{E}{R+r}$
$u_R = RI_0 = \frac{RE}{R+r}$ and $u_L = rI_0$ and $u_R = RI_0$.

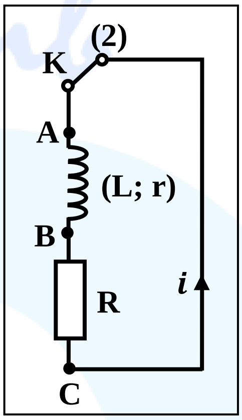
Decay of current (switch at position 2):

Law of addition of voltages: $u_L + u_R = 0$

$ri + L\frac{di}{dt} + Ri = 0$

$L\frac{di}{dt} + (R + r)i = 0$

Solution: $i = I_0 e^{-\frac{t}{\tau}}$ with $I_0 = \frac{E}{R}$ and $\tau = \frac{L}{R}$



Differential equation in u_R:

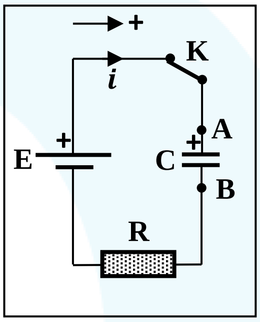
$L\frac{di}{dt} + (R + r)i = 0$ with $i = \frac{u_R}{R}$ and $\frac{di}{dt} = \frac{1}{R}\frac{du_R}{dt}$

$\frac{L}{R}\frac{du_R}{dt} + \frac{R+r}{R}u_R = 0$

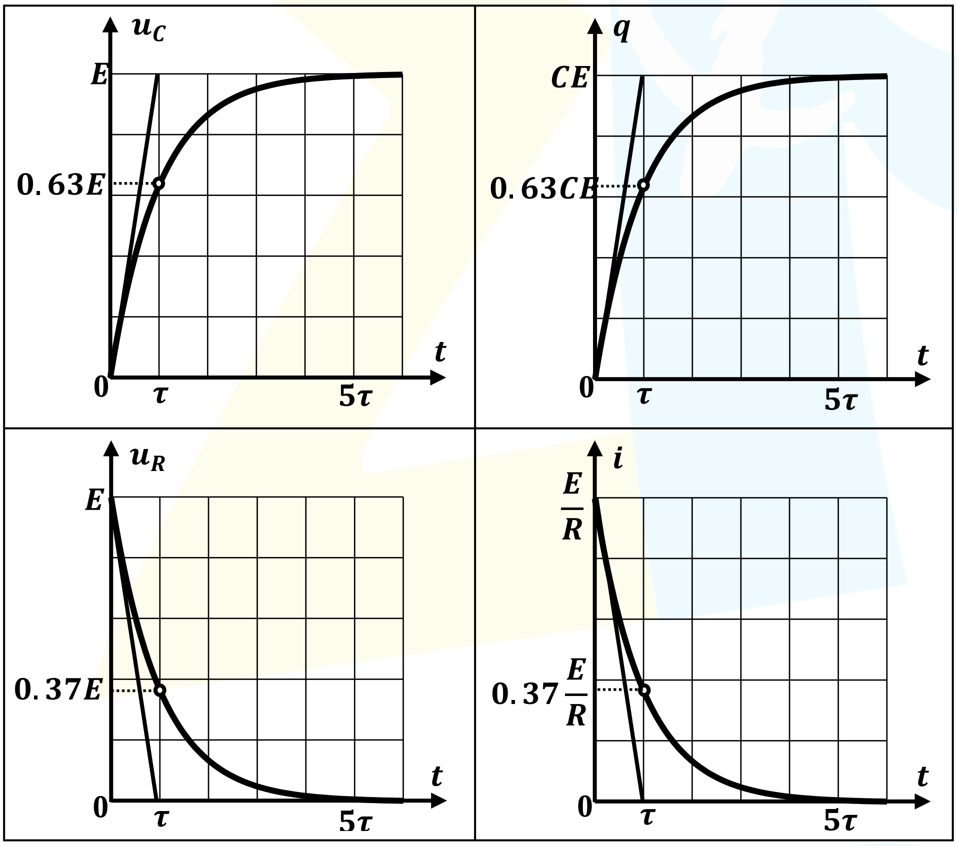
$\frac{du_R}{dt} + \frac{R+r}{L}u_R = 0$

RC charging circuit

Law of addition of voltages: $E = u_C + u_R \Longrightarrow u_R = E - u_C$



t	u_C	$q = Cu_C$	u_R	$i = \frac{u_R}{R}$
$t = 0$	0	0	$U_{Rmax} = E$	$I_{max} = \frac{E}{R}$
$t = \tau = RC$	$0.63E$	$0.63CE$	$0.37E$	$0.37\frac{E}{R}$
$t = 5\tau = 5RC$	$U_{Cmax} = E$	$Q_{max} = CE$	0	0



Forms of the solution of the differential equation in u_C:

$u_C = A\left(1 - e^{-\frac{t}{\tau}}\right)$ with $A = E$ and $\tau = RC$

$u_C = A(1 - e^{\alpha t})$ with $A = E$ and $\alpha = -\frac{1}{\tau}$

$u_C = A + Be^{-\frac{t}{\tau}}$ with $A = E$ and $B = -A = -E$

The electric energy stored in a capacitor is:
$W = \frac{1}{2}Cu_C^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qu_C$
The electric power dissipated by a resistor is:
$P = Ri^2$

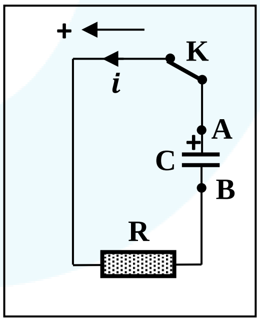
Differential Equations (charging)

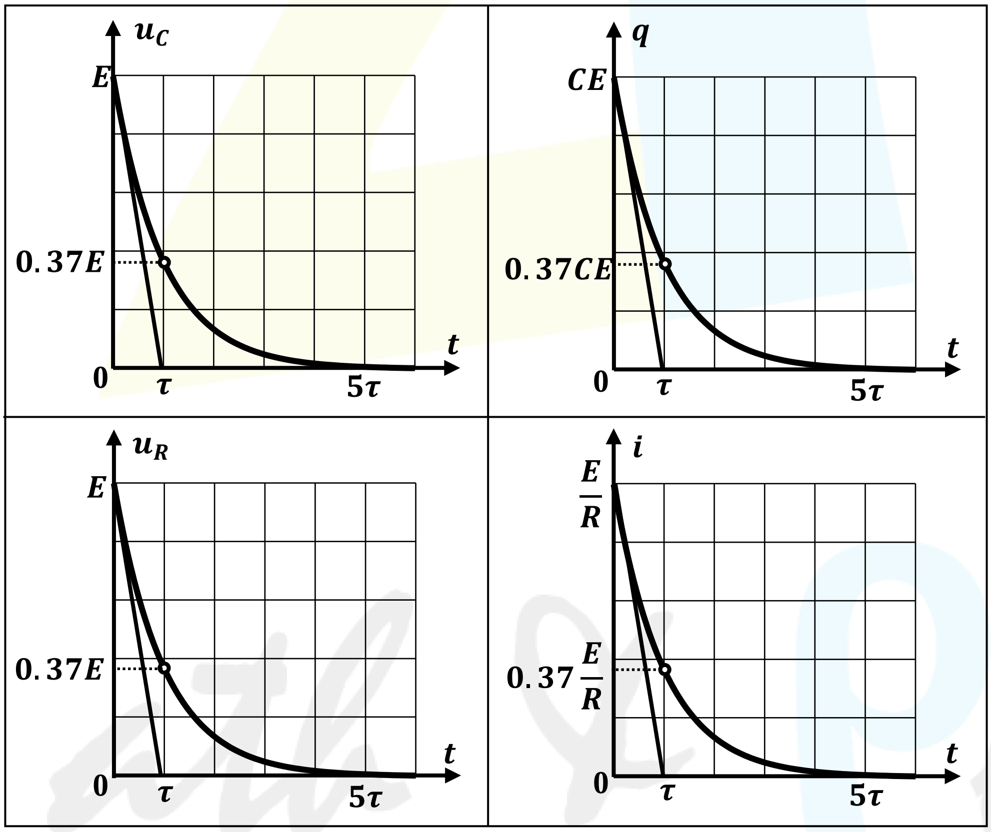
Differential Equation in u_C	**Differential Equation in q**
Law of addition of voltages: $u_G = u_C + u_R$ $E = u_C + Ri$ with $q = Cu_C$ and $i = \frac{dq}{dt} = C\frac{du_C}{dt}$ $E = u_C + RC\frac{du_C}{dt}$	Law of addition of voltages: $u_G = u_C + u_R$ $E = \frac{q}{C} + Ri$ with $u_C = \frac{q}{C}$ and $i = \frac{dq}{dt}$ $E = \frac{q}{C} + R\frac{dq}{dt}$
Differential Equation in u_R Law of addition of voltages: $u_G = u_C + u_R$ $E = u_C + u_R$ with $u_C = \frac{q}{C}$ and $i = \frac{dq}{dt} = \frac{Cdu_C}{dt}$ Derive both sides with respect to time: $0 = \frac{du_C}{dt} + \frac{du_R}{dt}$ $\mathbf{0 = \frac{i}{C} + \frac{du_R}{dt}}$ **and** $\mathbf{0 = \frac{u_R}{RC} + \frac{du_R}{dt}}$	**Differential Equation in i** Law of addition of voltages: $u_G = u_C + u_R$ $E = u_C + Ri$ with $u_C = \frac{q}{C}$ and $i = \frac{dq}{dt} = \frac{Cdu_C}{dt}$ Derive both sides with respect to time: $0 = \frac{du_C}{dt} + R\frac{di}{dt}$ $\mathbf{0 = \frac{i}{C} + R\frac{di}{dt}}$

RC discharging circuit

Law of uniqueness of voltage: $u_C = u_R$

$\boldsymbol{t}$	$\boldsymbol{u_C}$	$\boldsymbol{q = Cu_C}$	$\boldsymbol{u_R}$	$\boldsymbol{i = \frac{u_R}{R}}$
$\boldsymbol{t = 0}$	$U_{Cmax} = E$	$Q_{max} = CE$	$U_{Rmax} = E$	$I_{max} = \frac{E}{R}$
$\boldsymbol{t = \tau = RC}$	$0.37E$	$0.37CE$	$0.37E$	$0.37\frac{E}{R}$
$\boldsymbol{t = 5\tau = 5RC}$	0	0	0	0





Solution of the differential equation:

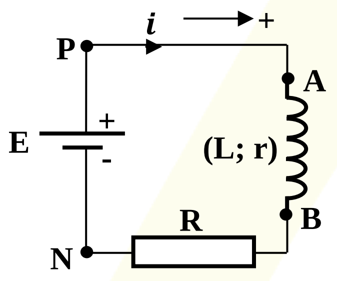
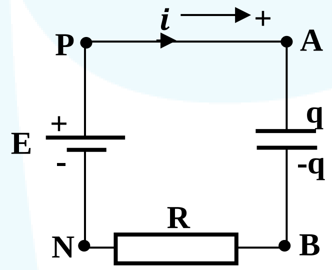
$u_C = Ae^{-\frac{t}{\tau}}$ with $A = E$ and $\tau = RC$

$u_C = Ae^{\alpha t}$ with $A = E$ and $\alpha = -\frac{1}{\tau}$

Differential Equations (discharging)

Differential Equation in u_C Law of uniqueness of voltage: $u_C = u_R$ $u_C = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$ $u_C = -RC\frac{du_C}{dt}$ $u_C + RC\frac{du_C}{dt} = 0$	**Differential Equation in q** Law of uniqueness of voltage: $u_C = u_R$ $\frac{q}{C} = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$ $\frac{q}{C} = -R\frac{dq}{dt}$ $\frac{q}{C} + R\frac{dq}{dt} = 0$
Differential Equation in u_R Law of uniqueness of voltage: $u_C = u_R$ $u_C = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$ Derive both sides with respect to time: $\frac{du_C}{dt} = \frac{du_R}{dt} \Rightarrow -\frac{i}{C} = \frac{du_R}{dt}$ $-\frac{u_R}{RC} = \frac{du_R}{dt} \Rightarrow \frac{du_R}{dt} + \frac{u_R}{RC} = 0$	**Differential Equation in i** Law of uniqueness of voltage: $u_C = u_R$ $u_C = Ri$ with $q = Cu_C$ and $i = -\frac{dq}{dt} = -C\frac{du_C}{dt}$ Derive both sides with respect to time: $\frac{du_C}{dt} = R\frac{di}{dt} \Rightarrow -\frac{i}{C} = R\frac{di}{dt}$ $\mathbf{R\frac{di}{dt} + \frac{i}{C} = 0}$

RC Versus RL Circuits

GS **RL Circuit**	**LS & GS** **RC Circuit**
 **Law of addition of voltages:** $u_{PN} = u_{PA} + u_{AB} + u_{BN}$ $u_G = u_L + u_R$ $E = ri + L\frac{di}{dt} + Ri$ $E = (r+R)i + L\frac{di}{dt}$ At t = 0s; $i = 0$ **At the steady state:** $i = I_0 = constant \Rightarrow \frac{di}{dt} = 0$ $E = (r+R)I_0 \Rightarrow I_0 = \frac{E}{r+R}$	 **Law of addition of voltages:** $u_{PN} = u_{PA} + u_{AB} + u_{BN}$ $u_G = u_C + u_R$ $E = u_C + Ri$ with $q = Cu_C$ and $i = \frac{dq}{dt} = C\frac{du_C}{dt}$ $E = u_C + RC\frac{du_C}{dt}$ At t = 0s; $u_C = 0$ **At the steady state:** $u_C = u_{Cmax} = constant \Rightarrow \frac{du_C}{dt} = 0$ $E = u_C$

Definition: the process by which a beam of light or other system of waves is spread out as a result of passing through a narrow aperture (slit) or across an edge.

Conditions for diffraction to take place:
The width of the slit should be smaller than 1mm (a < 1mm)
The width of the slit must be comparable with the wavelength of light.

Characteristics of the diffraction pattern:
- Alternating bright and dark fringes.
- The size of the central bright fringe is double that of any other bright fringe.
- The direction of the pattern of fringes is perpendicular to that of the slit.

Small angles approximation:

$$\sin\theta \simeq \tan\theta \simeq \theta$$

Angles of diffraction of the centers of dark fringes

$\sin\theta_n = n\frac{\lambda}{a} \Longrightarrow \theta_n = n\frac{\lambda}{a}$

Angular width of the central fringe

For $n = 1;\ \theta_1 = \frac{\lambda}{a}$

$\alpha = 2\theta_1 = 2\frac{\lambda}{a}$

Width of the central bright fringe

For $n = 1;\ \theta_1 = \frac{\lambda}{a}$

$\tan\theta_1 = \frac{L}{2D} \Longrightarrow \theta_1 = \frac{L}{2D}$

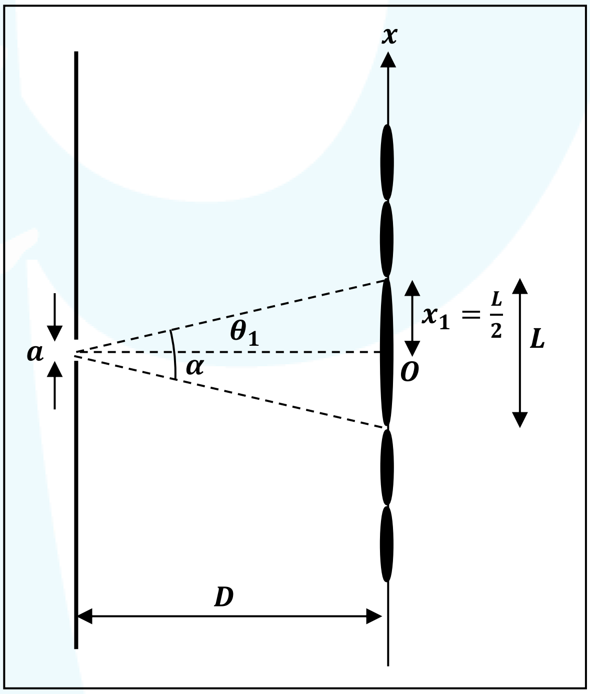
$\frac{L}{2D} = \frac{\lambda}{a} \Longrightarrow L = \frac{2\lambda D}{a}$

Abscissa of the n^th^ dark fringe

$\theta_n = n\frac{\lambda}{a}$

$\tan\theta_n = \frac{x_n}{D} \Longrightarrow \theta_n = \frac{x_n}{D}$

$\frac{x_n}{D} = n\frac{\lambda}{a} \Longrightarrow x_n = n\frac{\lambda D}{a} = n\frac{L}{2}$



Note

The wavelength of an electromagnetic wave of frequency ν and propagating at a speed c in vacuum is:

$$\lambda = \frac{c}{\nu}$$

The wavelength of an electromagnetic wave of frequency ν' and propagating at a speed V in a medium of index of refraction n is:

$$\lambda' = \frac{V}{\nu'} = \frac{c}{n\nu} = \frac{\lambda}{n}$$

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