

Entrance exam 2018-2019 Physics (Bac. Lebanese) July 2018 Duration 2 h

Exercise I: Damping in different situations (25 points)

A- Free mode

A solid (S), supposed to be a point mass of mass m, is connected to one end of a horizontal spring (R) of negligible mass and stiffness k=300 N/m. Shifted from its equilibrium position by 10 cm, (S) starts from rest at the instant $t_0=0$ and then moves on a horizontal support, with respect to its

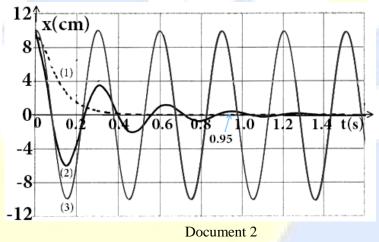


equilibrium position O, origin of the axis $(O, \vec{1})$. During its motion, the center of inertia G of the solid is located, at an instant t, by its abscissa x and its velocity $\vec{v} = v \vec{i}$ where $v = \frac{dx}{dt} = x'$, and (S) is subjected to several forces including the tension \vec{F} of the spring $(\vec{F} = -k \times \vec{i})$ and the force of friction $\vec{f} = -b \vec{v}$, where b is a constant of adjustable value.

1. Applying Newton's second law, $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$, show that the differential equation verified by x is of the

$$form: \frac{\text{d}^2x}{\text{d}t^2} + 2\lambda \frac{\text{d}x}{\text{d}t} + \omega_0^2 \ x = 0.$$

- 2. The document 2 shows the curves (1), (2) and (3). Each curve represents the motion of (S) for a value of b.
- **2.1.** Give the name of the mode associated with each curve.
- **2.2.1.** One of the curves represents a pseudoperiodic motion of pseudoperiod T. Determine the value of T.
- **2.2.2.** Referring to document 2, determine the proper period T_0 and m.
- **2.3.** Knowing that the pseudo-angular frequency ω is given by:



$$\omega = \sqrt{\omega_0^2 - \lambda^2}$$
, determine the corresponding value of b.

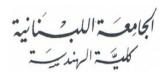
B- Radius of a fog droplet

The fog, consisting of very fine droplets of water, is a form of cloud that touches the ground, each droplet being of radius r and mass m.

I- Motion of a water droplet

At the instant $t_0 = 0$, it is assumed that a droplet starts from rest from a point O situated at an altitude h relative to the ground. O is considered as the origin of a downward vertical axis ($(0, \vec{1})$.





At an instant t, the droplet, of abscissa x and velocity $\vec{v} = v \vec{i}$ where $v = \frac{dx}{dt} = x'$, is subjected to its weight $m\vec{g} = mg\vec{i}$ and to a force of friction, $\vec{f} = -kv\vec{i}$, where k is a constant of expression $k = 6\pi\eta r$. (The Archimedes' upthrust due to air is neglected with respect to the weight of (S)).

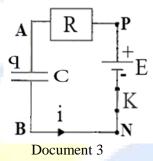
Given: volume of a sphere of radius $r: V = \frac{4}{3}\pi r^3$; the viscosity of air : $\eta = 1.8 \times 10^{-5} \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$; the density of water: $\rho = 1.0 \times 10^3 \text{ kg} \cdot \text{m}^{-3}$; and $g = 9.8 \text{ m} \cdot \text{s}^{-2}$.

- **1.1.** By choosing the ground as the reference level of the gravitational potential energy, write, at an instant t, the expression of the mechanical energy of the system (droplet, Earth) in terms of x, h, m, g and v.
- **1.2.** Knowing that the power of \vec{f} is given by $P = \frac{dME}{dt}$, deduce the differential equation in v that desribes the motion of the droplet.
- 2. The solution of this differential equation is of the form $v = v_{\lambda}$ (1 e^{-t/τ_1}). Show that the expressions of the constants are $v_{\lambda} = \frac{m \cdot g}{k}$ and $\tau_1 = \frac{m}{k}$.

II- Charging of a capacitor

The adjacent circuit (Doc.3) consists of a capacitor (C) of capacitance C connected in series with a resistor (R) of resistance $R=10~M\Omega$ across the terminals P and N of a generator delivering a constant voltage $U_{PN}=E$.

The switch (K) is closed at the instant $t_0 = 0$. At an instant t, (C) carries the charge q and the circuit carries a current i. $u_C = u_{AB}$ is the voltage across the capacitor and $u_R = u_{PA}$ the voltage across the resistor (R).

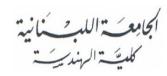


- **1.** Derive the differential equation in u_C .
- 2. The solution of this equation is of the form: $u_C = A(1 e^{-t/\tau_2})$. Determine the expressions of the constants A and τ_2 .
- 3. Determine the equation of the tangent to the curve at the origin of time. Deduce the coordinates of the point of intersection of this tangent with the asymptote to the curve.
- **4.** An appropriate device can register the variation of u_C as a function of time. Referring to the graph in Document 4, determine the values of E and C.

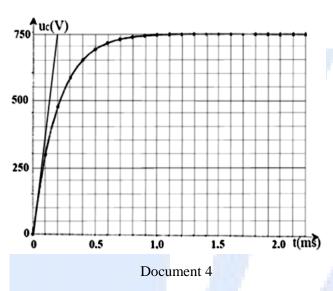
III- Analogy

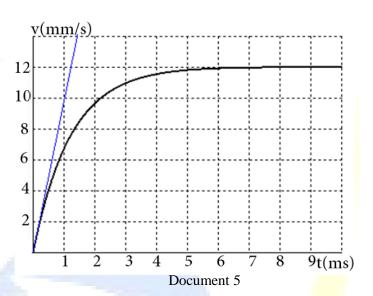
1. By comparing the time variation of v with that of u_C , determine, referring to the appropriate graph, the value of v_{λ} and that of τ_1 .





2. Deduce the value of the radius r of the droplet.





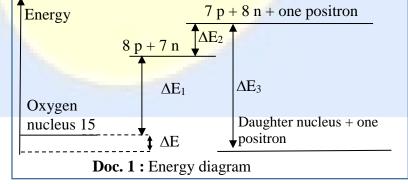
Exercise II: Use of radioactive water in medicine: The PET (17 points)

Radioactive water is obtained by replacing the oxygen nucleus 16 ($^{16}_{8}$ O), with an oxygen nucleus 15 ($^{15}_{8}$ O), a β^+ emitter. Performing an intravenous injection to a patient, with radioactive water, a large amount of these water molecules are found in the brain.

Given: $h = 6.626 \times 10^{-34} \text{ J·s}$; $c = 2.998 \times 10^8 \text{ m.s}^{-1}$; $1 \text{ u} = 931.5 \text{ MeV/c}^2$; $1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}$ Binding energy per nucleon (in MeV/nucleon): $\frac{E_{\lambda}}{A}(^{16}C) = 6.676$; $\frac{E_{\lambda}}{A}(^{15}N) = 7.699$; $\frac{E_{\lambda}}{A}(^{18}O) = 7.463$. Mass of particles (in u): electron and positron: 5.486×10^{-4} ; neutron: 1.00866; proton: 1.00727.

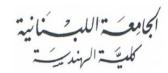
1. The disintegration of oxygen 15

- 1.1. Determine the equation of the decay reaction of the oxygen nucleus 15, the daughter nucleus obtained being in the ground state.
- **1.2.** ΔE is the energy released by the decay of an oxygen nucleus 15; it is indicated on the energy diagram of the document 1, where ΔE_1 and ΔE_3 represent respectively the binding energies of the oxygen nucleus 15 and the daughter nucleus.
- **1.2.1.** ΔE_2 represents the energy required to transform one particle into another.
- **a)** Name these two particles and write the equation giving this transformation.
- **b**) Explain why an energy must be provided for this transformation to take place.
- c) Show that $\Delta E_2 = 1.806$ MeV.
- **1.2.2.** Calculate, in MeV, the binding energy ΔE_3 of the daughter nucleus.



1.2.3. Knowing that $\Delta E_1 = 111.945$ MeV, deduce that the value of ΔE is equal to 1.734 MeV.





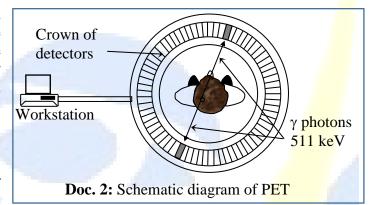
2. The use of oxygen 15 in PET

"The β^+ positron is the antimatter of the electron. Matter and antimatter annihilate (disappear) as soon as they are in contact with each other; a positron and an electron of the surrounding medium annihilate by releasing a pair of γ photons, each of energy 511 keV. The two γ photons are emitted in two diametrically opposite directions.

The purpose of the PET is to locate the photons, which largely pass through the brain and crane, so that they can be detected outside the cranial box.

Finally, the half-life time of these oxygen nuclei 15, positron emitters, is short: 123 seconds. This property allows us to do several studies on the same patient. This short half-life nevertheless requires that two successive injections must be spaced by 8 to 10 minutes apart. "

- **2.1.** The time variation of the number of oxygen nuclei 15 is given by the decay law where N_0 is the number of oxygen nuclei 15 present when the injection is performed at the instant $t_0 = 0$ s. Show that the value of the radioactive constant of oxygen 15 is $\lambda = 5.64 \times 10^{-3} \text{ s}^{-1}$.
- **2.2.** If it is desired to continue the PET examination, one think that it is necessary to carry out a new injection into the patient's body when the number $N(t_1)$ of oxygen nuclei remaining, at the instant t_1 , is close to 5% of the number N_0 .

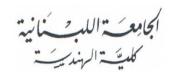


- **2.2.1.** Calculate the value of the instant t₁.
- **2.2.2.** The spacing time of the injections mentioned in the text is then adequate. Explain why.

3. The detection of gamma radiation

- **3.1.** Referring to the text, write the equation of the reaction taking place when a positron, issued from the decay of an oxygen nucleus 15, meets an electron of the surrounding medium, the kinetic energy of the positron being negligible.
- **3.2.** It is supposed that the energy released by this reaction is shared equally between the two photons whose mass is zero. The energy of each emitted gamma photon is in agreement with that given in the text. Why?



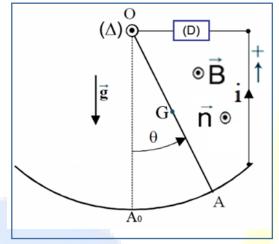


Exercise III: Motion of a pendulum in a magnetic field (18 pts)

A homogeneous metallic rod OA, of mass m and length λ , constitutes a pendulum that can rotate about a

horizontal axis (Δ) perpendicular at O to the plane of the figure (Doc.1). The moving end A slides on a circular conductor, so that, at every instant, the rod-conductor set closes an electric circuit consisting of a component (D), the amplitude of motion of the pendulum being small at any instant. Neglect any force of friction.

The set is immersed in a uniform and constant magnetic field \overrightarrow{B} perpendicular to the plane of the figure. At the instant $t_0=0,\,\theta_0=0^o,$ the rod OA is set into motion. At an instant t, the circuit carries a current i and the angular elongation of the pendulum is θ . The moment of inertia of the rod with respect to (Δ) is given by: $I=\frac{m\lambda^2}{3}$.



Document 1

A- Let S_0 be the surface area of the electric circuit when the rod is in its equilibrium position ($\theta = 0^{\circ}$). At an instant t, the surface area of the circuit is written as: $S = S_0 - \frac{\lambda^2 \theta}{2}$.

- **1.1.** Determine, in terms of B, λ , and $\frac{d\theta}{dt}$, the expression of the induced e.m.f that appears across the terminals O and A of the rod.
- **1.2.** Deduce the expression of the voltage u_{AO} across the rod.
- 2. At the instant t, the rod is subjected, to its weight, the Laplace force \vec{F} and the reaction of (Δ) .
- **2.1.** Determine the expression of the moment of each of these forces with respect to the axis (Δ) .
- **2.2.** Deduce that the algebraic sum of the moments of these forces is written as:

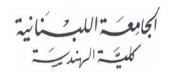
$$\Sigma \mathbf{M} / \Delta (\vec{F}_{ext}) = - mg \frac{\lambda}{2} \theta - i \cdot \frac{B\lambda^2}{2}.$$

- **B-** (D) is assumed to be a resistor of adjustable resistance R.
- 1. Show that the differential equation in θ that describes the motion of the rod is written as:

$$\frac{m\lambda^2}{3}\frac{d^2\theta}{dt^2} + \frac{B^2\lambda^4}{4R}\frac{d\theta}{dt} + mg\frac{\lambda}{2}\theta = 0$$

- **2.** Identify the different modes of motion that the rod can perform according to the values of R (R of very low value, R of finite value, R of very large value).
- **C-** (D) is supposed to be a coil of inductance L and of negligible resistance.
- **1.** Knowing that, when passing through the equilibrium position, the current i is zero (i = 0 for θ = 0), show that the expression of i is given by: $i = \frac{B\lambda^2}{2L}\theta$.
- **2.** Deduce the expression of the moment of the Laplace force with respect to (Δ) .
- **3.** Derive the differential equation in θ that describes the oscillations of the rod.
- **4.** Deduce the proper angular frequency of these oscillations.





Entrance exam 2018-2019 Answer of Physics

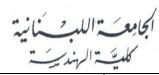
July 2018 Duration 2 h

Exercise I : Damping in different situations (25 points)

Q		Marks
A-	Applying Newton's second law: $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$. $m\vec{g} + \vec{N} + \vec{F} + \vec{f} = \frac{d\vec{P}}{dt} = m\frac{d\vec{v}}{dt} = m \cdot \vec{a}$ (m is constant)	2.5
1.	Projection along (O, \vec{i}), we get: $0 + 0 - k \cdot x - b \cdot v = b \cdot a = m \cdot x''$ and $x'' + \frac{b}{m}x' + \frac{k}{m}x = 0$	
	We thus obtain the differential equation in the form:	
	$\frac{d^2x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega_0^2 x = 0$, with $\lambda = \frac{b}{2m}$ and $\omega_0^2 = \frac{k}{m}$.	
2.1.	(1) Aperiodic mode, (2) pseudoperiodic mode and (3) periodic mode.	1.5
2.2.1	The curve (2) of document 2 gives: $3 T = 0.95 s$ et $T = 0.317 s$.	1
2.2.2.	The curve (3) of document 2 gives: $2T_0 = 0.6$ s and $T_0 = 0.3$ s.	1.5
	$T_0 = 2\pi \sqrt{\frac{m}{k}}$ and $m = \frac{kT_0^2}{4\pi^2} = \frac{300 \times 0.09}{39.5} = 0.684$ kg.	
2.3.	The proper angular frequency: $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{0.3} = 20.9 \text{ rad/s}$ and $\omega_0^2 = 438.65 \text{ rad}^2/\text{s}^2$.	2.5
	The pseudo-angular frequency $\omega = \frac{2\pi}{T} = \frac{0.3}{0.317} = 19.82 \text{ rad/s and } \omega^2 = 392.86 \text{ rad}^2/\text{s}^2$	
	The pseudo-angular frequency w = T = 0.317	
	We have: $\omega = \sqrt{\omega_0^2 - \lambda^2}$, $\omega^2 = \omega_0^2 - \lambda^2$ and $392.86 = 438.65 - \lambda^2$ and $\lambda = \sqrt{45.79} = 6.77$ rad/s.	
	$\lambda = \frac{b}{2m}$ and $b = 2\lambda m = 2 \times 6.77 \times 0.684 = 9.26 \text{ kg} \cdot \text{s}^{-1}$.	
B-I	The mechanical energy of the system (droplet, Earth) is given by:	1
1.1.	$ME = KE + PE_g = \frac{1}{2}mv^2 + mg(h-x)$	
1.2.	From the non-conservation of the mechanical energy : $P = \frac{dE_m}{dt} = \vec{f} \cdot \vec{v} = -kv^2$.	2
	- $kv^2 = mv\frac{dv}{dt}$ - mgv that gives: $m\frac{dv}{dt}$ + $kv = mg$. therefore, the differential equation in v that	7
	governs the motion of the droplet is: $\frac{dv}{dt} + \frac{k}{m}v = g$.	- 46
2.	$\frac{dv}{dt} = \frac{v_{\lambda}}{\tau_{1}} e^{-t/\tau_{1}}. \text{ Thus, } \frac{v_{\lambda}}{\tau_{1}} e^{-t/\tau_{1}} + \frac{k}{m} \frac{v_{\lambda} - \frac{k}{m} v_{\lambda}}{v_{\lambda}} e^{-t/\tau_{1}} = g. \ (\frac{1}{\tau_{1}} - \frac{k}{m}) v_{\lambda} \ e^{-t/\tau_{1}} = g - \frac{k}{m} v_{\lambda},$	2
	identifying, we get $g - \frac{k}{m} v_{\lambda} = 0$ and $v_{\lambda} = \frac{m \cdot g}{k}$ and $(\frac{1}{\tau_1} - \frac{k}{m}) = 0 \Rightarrow \tau_1 = \frac{m}{k}$.	
II-	From the law of addition of voltages: $u_{PN} = u_{PA} + u_{AB}$. With $u_{PN} = E$ and $u_{PA} = R \cdot i$ (Ohm's law). i	2
1.	$=\frac{dq}{dt}$ and $q=C\cdot u_C$, therefore $i=C\frac{du_C}{dt}$.	
	The differential equation: $E = u_C + RC \frac{du_C}{dt} \text{ or } \frac{du_C}{dt} + \frac{1}{RC} u_C = \frac{E}{RC}$.	
2.	We have: $u_C = A(1 - e^{-t/\tau_2})$ and $\frac{du_C}{dt} = \frac{A}{\tau_2} e^{-t/\tau_2}$. Repalcing in the differential equation each term	2
	by its expression, we get: $E = A - A e^{-t/\tau_2} + RC^{\frac{A}{\tau}} e^{-t/\tau_2}$.	
	E-A + $(1-\frac{RC}{\tau_2})$ A $e^{-t/\tau_2} = 0$ $\forall t$. Identifying, we get A = E and τ_2 = RC.	
3.	E-A + $(1-\frac{RC}{\tau_2})$ A $e^{-t/\tau_2} = 0$ $\forall t$. Identifying, we get A = E and τ_2 = RC. We have $\frac{du_C}{dt} = \frac{E}{\tau_2} e^{-t/\tau_2}$, at $t_0 = 0$ the slope of the tangent is $\frac{du_C}{dt}(t_0=0) = \frac{E}{\tau_2} e^0 = \frac{E}{\tau_2}$. As the tangent	2
	passes through the origin, then the equation of the tangent becomes $u_1 = \frac{E}{\tau_0}t$, and the equation of	
	the asymptote is $u_2 = E$. The coordinates of the point of intersection of the tangent to the curve at	
	the origin of time with the asymptote are given by $u_1 = u_2 \Rightarrow t = \tau_2$,. So (τ_2, E) .	

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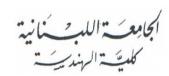
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4.	At the end of charging $(t = \infty)$, $u_{C(\infty)} = A = E$ and from the graph: $E = \frac{1}{750} \text{ V}$.	2
	Graphically, we find $\tau_2 = 0.2$ ms which is the abscissa of the point of	
	intersection of the tangent to the curve at the origin of time with the	
	asymptote.	
	As $\tau_2 = RC$, then $C = \frac{\tau_2}{R} = \frac{0.2 \times 10^{-3}}{10 \times 10^6} = 2 \times 10^{-11} \text{F}.$	
III-	Referring to document 5, for $t = \infty$, v_{λ} equals 1.2×10^{-2} m/s and we find $\tau_1 = 1.2$ ms which is the	1.5
1.	abscissa of the ingtersection point of the tangent to the curve at the origin with the asymptote due	
	to the analogy.	
2.	As $v_{\lambda} = \frac{m \cdot g}{k} = \frac{\rho_3^4 \pi r^3 \cdot g}{6\pi \eta r} = \frac{\rho 2 r^2 \cdot g}{9\eta}$, then $r^2 = \frac{9\eta v_{\lambda}}{2\rho g} = \frac{9 \times 1.8 \times 10^{-5} \times 1.2 \times 10^{-2}}{2 \times 1.0 \times 10^3 \times 9.8} = 9.92 \times 10^{-11}$ and the value of	1.5
	the radius is: $r = 9.96 \times 10^{-6} \text{ m} \approx 10 \mu\text{m}$.	



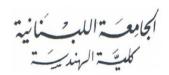




Exercise II: Use of the radioactive water in medicine: The PET (17 points)

Q	11. Ose of the fauloactive water in medicine. The FET (17 points)	Marks
1.	The oxygen 15 being a β^+ emitter, then the equation of disintegration of the oxygen	2.5
1.1.	nucleus 15 is given by: ${}^{15}_{8}O \rightarrow {}^{A}_{z}Y + {}^{0}_{1}e + {}^{0}_{0}v$. Also, there is no emission of γ radiation	
	since the daughter nucleus is obtained in the ground state.	
	From the law of conservation of the mass number : $15 = 0 + A + 0 \Rightarrow A = 15$.	
	From the law of conservation of the charge number: $8 = 1 + Z + 0 \Rightarrow Z = 7$.	
	\Rightarrow the daughter nucleus is the nitrogene. ${}^{15}_{8}O \rightarrow {}^{0}_{1}e + {}^{15}_{7}N + {}^{0}_{0}v$.	
1.2.1.a)	Proton in neutron. ${}_{1}^{1}p \rightarrow {}_{0}^{1}n + {}_{1}^{0}e$.	1.5
1.2.1.b)	For this transformation to take place, an energy must be provided because the mass of	1
	a proton is smaller than that of a neutron.	
1.2.1.c)	The energy $\Delta E_2 = \Delta m \cdot c^2 = (m_P - m_n - m_{positron}) \cdot c^2$	2
	$\Delta E_2 = (1.00727 - 1.00866 - 5.486 \times 10^{-4}) \times 931.5 \text{ MeV/}c^2 \times c^2$	
	$\Delta E_2 = 0.0019386 \times 931.5 = 1.806 \text{ MeV}.$	
1.2.2.	The binding energy ΔE_3 of the daughter nucleus :	1
1.2.2.		
	$\Delta E_3 = E_{\lambda}({}^{15}_{7}N) = A \times \frac{E_{\lambda}}{A} = 15 \times 7.699 = 115.485 \text{ MeV.}$	
1.2.3.	From the energy diagram : $\Delta E_3 = \Delta E + \Delta E_1 + \Delta E_2$, with $\Delta E_3 = E_l(^{15}_{7}N)$	2
	$\Delta E = \Delta E_3 - \Delta E_1 - \Delta E_2 = 115.485 - 111.945 - 1.806 = 1.734 \text{ MeV}.$	- 6
2.	The decay law is as: $N = N_0 \times e^{-\lambda \cdot t_{1/2}}$.	2
2.2.1.	After a period equal to $t_{1/2}$, we get: $N(t_{1/2}) = \frac{N_0}{2}$, so $\frac{N_0}{2} = N_0 \times e^{-\lambda \cdot t_{1/2}}$.	1887
	Then $\frac{1}{2} = e^{-\lambda \cdot t_{1/2}}$; so $\ln(\frac{1}{2}) = -\lambda \times t_{1/2}$, $\ln 2 = \lambda \times t_{1/2}$ and $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{123} = 5.64 \times 10^{-3} \text{ s}^{-1}$.	r ,
2.2.2.	$N(t_1) = 0.05 \times N_0 = N_0 \times e^{-\lambda \cdot t_1}$; so $e^{-\lambda \cdot t_1} = 0.05$; $\ln(e^{-\lambda \cdot t_1}) = \ln(0.05)$	1.5
	$-\lambda \times t_1 = \ln(0.05)$ $t_1 = \frac{-\ln 0.05}{\lambda} = 532 \text{ s}$ So $t_1 \approx 9 \text{ min}$	
222	According to the coloulation of tent. One in the number of quality remaining is 50% of	1
2.2.3.	According to the calculation, after $t_1 = 9$ min, the number of nuclei remaining is 5% of the number of the initially injected nuclei. It is then necessary to carry out a new	1
	injection. Our calculation of t_1 is consistent with the text which indicates that the	
	injections are spaced by 8 to 10 min.	
3. 3.1.	$_{+1}^{0}e+_{-1}^{0}e \rightarrow 2_{0}^{0}\gamma$.	1
3.2.1.	The energy liberated by the reaction becomes:	1.5
	$E = 2 \times m_e \times c^2 = 2 \times 5.486 \times 10^{-4} \times 931.5 \text{ MeV/}c^2 \times c^2 = 1.022 \text{ MeV}$	
	The energy of each emitted photon is then equal to: $E_1 = \frac{E}{2} = 511 \text{ keV}$, which is	
	consistent with the text data.	



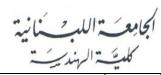


Exercice III: Motion of a pendulum in a magnetic field (18 points)

Q		Marks
A- 1.1	We have $\vec{S} = S \cdot \vec{n}$. he magnetic flux is: $\phi = S \cdot \vec{n} \cdot \vec{B} = S \cdot B \cos(0^\circ) = S \cdot B$, because according to the orientation of the circuit, the unit surface vector \vec{n} of the circuit has the same	3
	direction as that of \vec{B} . As a result : $\phi = B(S_0 - \frac{\lambda^2 \theta}{2})$ and according to Faraday's law, the	
1.0	e.m.f.: $e = -\frac{d\phi}{dt} = \frac{B\lambda^2}{2} \frac{d\theta}{dt}$.	
1.2.	The current i is directed so that it leaves the point A, then: $u_{AO} = e = \frac{B\lambda^2}{2} \frac{d\theta}{dt}$.	1
2.1.	We have: $\mathbf{M}_{\Delta}(\vec{N}) = 0$ since \vec{N} meets (Δ) ;	3
	$\mathbf{M}/\Delta \ (\mathbf{mg}) = -\mathbf{mg}\frac{\lambda}{2}\sin\theta = -\mathbf{mg}\frac{\lambda}{2}\theta \ ; \tag{(1)}$	
	$F = i \cdot \lambda \cdot B \sin(90^{\circ}) = i \cdot \lambda \cdot B \text{ and } \mathbf{M}_{/\Delta} (\vec{F}) = -i \cdot \lambda \cdot B \frac{\lambda}{2} = -i \cdot \frac{B\lambda^{2}}{2}.$ (Because \vec{F} is perpendicular to the rod at G)	
	₩mg A	
2.2.	$\begin{split} \Sigma \mathbf{M}_{\Delta} (\overrightarrow{F}_{ext}) &= 0 - mg_{2}^{\lambda} \theta - i \cdot \frac{B\lambda^{2}}{2} = - mg_{2}^{\lambda} \theta - i \cdot \frac{B\lambda^{2}}{2}. \\ \text{We have : } u_{AO} &= e = Ri = \frac{B\lambda^{2}}{2} \frac{d\theta}{dt}. \text{ With } i = \frac{B\lambda^{2}}{2R} \frac{d\theta}{dt}. \end{split}$	0,5
B-	We have: $u_{AO} = e = Ri = \frac{B\lambda^2}{B} \frac{d\theta}{d\theta}$. With $i = \frac{B\lambda^2}{B} \frac{d\theta}{d\theta}$.	2.5
1.	$\Sigma \mathbf{M}_{/\!\Delta} (\vec{\mathbf{F}}_{\text{ext}}) = \mathbf{I} \cdot \ddot{\boldsymbol{\theta}} \Longrightarrow -\mathbf{i} \cdot \mathbf{B} \frac{\lambda^2}{2} - \mathbf{m} \mathbf{g} \frac{\lambda}{2} \boldsymbol{\theta} = \frac{\mathbf{m} \lambda^2}{3} \ddot{\boldsymbol{\theta}}.$	
	The differential equation in θ is: $\frac{m\lambda^2}{3} \frac{d^2\theta}{dt^2} + \frac{B^2\lambda^4}{4R} \frac{d\theta}{dt} + mg_2^{\lambda}\theta = 0 \ \forall t.$	1000
2.	For a low value of R, aperiodic mode because the coefficient $\frac{B^2\lambda^4}{4R}$ becomes very large.	1.5
	For a finite value of R, pseudoperiodic mode because the coefficient $\frac{B^2\lambda^4}{4R}$ becomes weak.	- 4
	For a very large value of R, un-damped periodic mode because the coefficient $\frac{B^2\lambda^4}{4R}$	
	becomes practically nil.	
	212.10	1.5
C- 1.	We have : $u_{AO} = u_L = e$. Therefore, $u_L = L \frac{di}{dt} = e = \frac{B\lambda^2}{2} \frac{d\theta}{dt} \Rightarrow \frac{di}{dt} = \frac{B\lambda^2}{2L} \frac{d\theta}{dt}$. By integrating:	1.5
	$i = \frac{B\lambda^2}{2L}\theta$, since $i = 0$ for $\theta = 0$.	
2.	The expression of the moment of the Laplace force with respect to (Δ) is:	1.5
	$\mathbf{M}/\Delta(\vec{\mathbf{F}}) = -i \cdot \frac{B\lambda}{2} = -\frac{B^{-}\lambda}{4L} \theta.$	
3.	$\mathbf{M}_{\Delta}(\vec{F}) = -i \cdot \frac{B\lambda^{2}}{2} = -\frac{B^{2}\lambda^{4}}{4L}\theta.$ $\Sigma \mathbf{M}_{\Delta}(\vec{F}_{ext}) = I_{\Delta} \cdot \ddot{\theta} \Rightarrow -\frac{B^{2}\lambda^{4}}{4L}\theta - mg\frac{\lambda}{2}\theta = \frac{m\lambda^{2}}{3}\ddot{\theta}, \text{ therefore: } \frac{m\lambda^{2}}{3}\frac{d^{2}\theta}{dt^{2}} + (mg\frac{\lambda}{2} + \frac{B^{2}\lambda^{4}}{4L})\theta = 0.$	2.5
	The differential equation in θ that describes the oscillations of the rod is $\frac{d^2\theta}{dt^2} + \frac{mg_2^{\lambda} + \frac{B^2\lambda^4}{4L}}{\frac{m\lambda^2}{3}}$	
	$\theta = 0.$	
4.	The proper angular frequency of these oscillations is:	1

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$$\omega_0^2 = \frac{mg_2^{\lambda} + \frac{B^2\lambda^4}{4L}}{\frac{m\lambda^2}{3}} = \frac{6mgL + 3B^2\lambda^3}{4 m \lambda L} \text{ et } \omega_0 = \sqrt{\frac{6mgL + 3B^2\lambda^3}{4 m \lambda L}}.$$

