ExaMath groups	**Mathematics exam** **Section:** L.S.	**Prepared by:** Dr. Ali Moussawi **Edited by:** H. Ahmad
Number of questions: 3	**Sample 01 - 2022** **Duration:** 90 min	**Name:** **N°:**

- *This exam consists of three problems. It is inscribed on three pages, numbered from 1 to 3.*
- *The use of a non-programmable calculator is allowed.*

I - (3 points)

In the table below, only one of the answers given to each question is correct.
Choose, **with justification**, the correct answer.

N°	Question	Proposed answers A	B	C
1.	The domain of definition of the function f defined by: $f(x) = \ln(x^2 - 4)$ is:	$]2\ ;\ +\infty[$	$]-\infty\ ;\ 2[$	$]-\infty\ ;\ 2[\ \cup\ ]2\ ;\ +\infty[$
2.	$e^{\frac{1}{2}\ln 9} \times e^{-\ln\frac{1}{3}} =$	e^3	6	9
3.	Let A and B be two independent events such that: $p(A\cap B) = 0.32$ and $p(B) = 2p(A)$. The probability of the event B is equal to:	0.04	0.08	0.8
4.	In a sports club, 75% of members are women. One out of five women plays tennis, while seven out of ten men do. A person, chosen at random, plays Tennis. The probability that this person is a woman has the value rounded to the nearest thousandth:	0.750	0.462	0.150

II - (5 points)

Consider two urns U_1 and U_2 containing balls that are indistinguishable to the touch.

- The urn U_1 contains one white ball and three red balls.
- The urn U_2 contains two white balls and two red balls.

1) In this part we draw one ball from U_1 and one ball from U_2.
Consider the following events:
R: "The two drawn balls are red";

C: "The two drawn balls are of different colors".
Calculate $p(R)$ and verify that $p(C) = \frac{1}{2}$.

2) In this part, we consider a well balanced six-sided cubic die, two of which carry the number 1 and the others carry the number 2.
We proceed to the following random experiment:
We roll the die once:

- If the upper face carries the number 1, we draw at random one ball from the urn U_1.
- If the upper face carries the number 2, we draw at random one ball from the urn U_2.

Consider the following events:
D: "The upper face of the die carries the number 1";
B: "Draw a white ball".

a) Show that $p(D) = \frac{1}{3}$ and calculate $p(B/D)$ and $p(B \cap D)$.

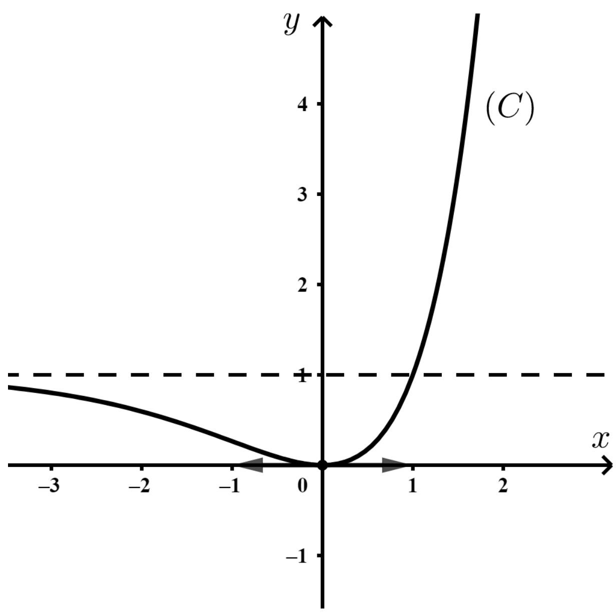
b) Calculate $p\left(B \cap \overline{D}\right)$ and deduce that $p(B) = \frac{5}{12}$.

c) Knowing that we have drawn a white ball, what is the probability that it comes from the urn U_2?

III - (12 points)

Part A:
The adjacent curve (C) is the representative curve, in an orthonormal system, of the function g defined over $\mathbb{R}$ by: $g(x) = (ax + b)e^x + c$, where a, b and c are real numbers.

- The x-axis is tangent to the curve (C) at the point of coordinates $(0\ ;\ 0)$.
- The line of equation $y = 1$ is an asymptote to (C) at $-\infty$.



1) Prove that $a = 1$, $b = -1$ and $c = 1$.

2) Use the curve (C) to determine the sign of $g(x)$ according to the values of x in $\mathbb{R}$.

Part B:
Let f be the function defined over $\mathbb{R}$ by $f(x) = (x-2)e^x + x$.
Designate by (C_f) the representative curve of f in an orthonormal system $\left(O\ ;\ \vec{i}\ ;\ \vec{j}\right)$.

1) a) Calculate the limits of f at $+\infty$ and at $-\infty$.

b) Show that the line (Δ) of equation $y = x$ is an asymptote to (C_f) at $-\infty$.

c) Study the relative position of (C_f) with respect to (Δ).

2) Show that $f'(x) = g(x)$ and draw the table of variations of the function f.

3) Show that the equation $f(x) = 0$ admits a unique solution α in $\mathbb{R}$ and verify that: $1.6 < \alpha < 1.7$.

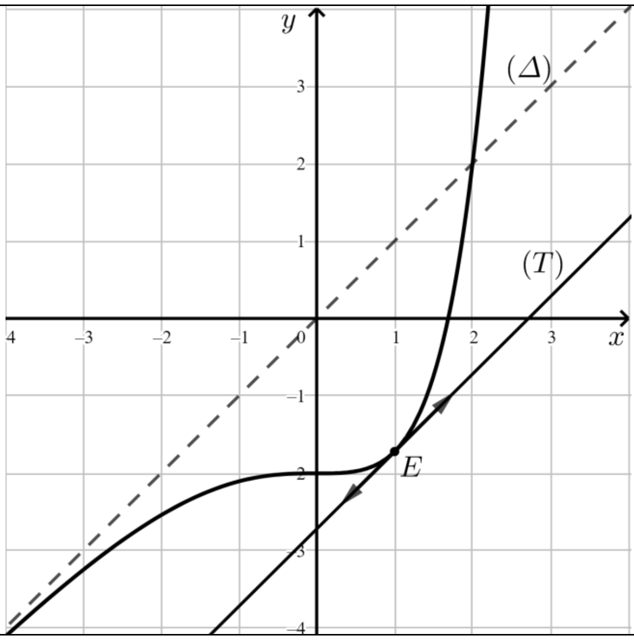
4) Show that there exist a point E of the curve (C_f) where the tangent (T) to (C_f) is parallel to the line (Δ). Calculate the coordinates of E and give an equation of the tangent (T).

5) Draw (Δ), (T) and (C_f).

QI	Answers	Note
1.	The function is f is defined if $x^2-4>0$ therefore for $x\in\left]-\infty\,;\,-2\right[\cup\left]2\,;\,+\infty\right[$; The correct answer is then C.	¾
2.	$e^{\frac{1}{2}\ln 9}\times e^{-\ln\frac{1}{3}}=e^{\ln 9^{\frac{1}{2}}}\times e^{\ln\left(\frac{1}{3}\right)^{-1}}=\sqrt{9}\times 3=9$; The correct answer is then C.	¾
3.	A and B are two independent events then $p(A\cap B)=p(A)\times p(B)$ and as $p(A)=\frac{p(B)}{2}$ we obtain the equation $0.32=\frac{p(B)}{2}\times p(B)$ therefore $\left[p(B)\right]^2=0.64$ $p(A)=0.8$; The correct answer is then C.	¾
4.	Consider the events: F : "The chosen person is a woman" and T : "The chosen person practices Tennis"; It is a question of calculating $p(F/T)=\frac{p(F\cap T)}{p(T)}$; $p(T\cap F)=p(T/F)\times p(F)=\frac{1}{5}\times\frac{75}{100}=\frac{3}{20}$; $p(T\cap\overline{F})=p(T/\overline{F})\times p(\overline{F})=\frac{7}{10}\times\frac{25}{100}=\frac{7}{40}$ $p(T)=p(T\cap F)+p(T\cap\overline{F})=\frac{3}{20}+\frac{7}{40}=\frac{13}{40}$, Finally $p(F/T)=\frac{P(F\cap T)}{P(T)}=\frac{3/20}{13/40}=\frac{6}{13}\approx 0.462$ rounded to the nearest thousandth.	¾

QII	Answers	Note
1.	$p(R)=\frac{C_3^1\times C_2^1}{C_4^1\times C_4^1}=\frac{3}{8}$; $p(C)=\frac{C_1^1\times C_2^1+C_3^1\times C_2^1}{C_4^1\times C_4^1}=\frac{1}{2}$.	1½
2.a.	$p(D)=\frac{2}{6}=\frac{1}{3}$; $p(B/D)=\frac{1}{4}$; $p(B\cap D)=p(B/D)\times p(D)=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$	1½
2.b	$p(B\cap\overline{D})=p(B/\overline{D})\times P(\overline{D})=\frac{2}{4}\times\frac{4}{6}=\frac{1}{3}$; Using on the total probability formula: $p(B)=p(B\cap D)+p(B\cap\overline{D})=\frac{1}{12}+\frac{1}{3}=\frac{5}{12}$	1
2.c.	$p(\overline{D}/B)=\frac{p(\overline{D}\cap B)}{p(B)}=\frac{1/3}{5/12}=\frac{4}{5}$.	1

QIII	Answers	Note
A.1.	• $\lim\limits_{x\to-\infty} g(x)=1$; $\lim\limits_{x\to\infty}\left(ae^x+be^x+c\right)=1$; $0+0+c=1$; $c=1$; • $g(0)=0$; $b+c=0$; $b=-c=-1$; • $g'(0)=0$; $g'(x)=(ax+a+b)e^x$; $a+b=0$; $a=-b=1$;	1½
A.2.	Using the curve (C) : • $g(x)=0$ when $x=0$; • $g(x)>0$ when $x\in\left]-\infty\,;\,0\right[\cup\left]0\,;\,+\infty\right[$	1
B.1.a.	$\lim\limits_{x\to+\infty} f(x)=+\infty\times(+\infty)+\infty=+\infty$; $\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\left(xe^x-2e^x+x\right)=0-0-\infty=-\infty$.	1
B.1.b.	$\lim\limits_{x\to-\infty}\left[f(x)-x\right]=\lim\limits_{x\to-\infty}\left(xe^x-2e^x\right)=0-0=0$; the line (Δ) of equation $y=x$ is an oblique asymptote to (C_f) at $-\infty$.	1
B.1.c.	$f(x)-x=(x-2)e^x$ has the same sign as $x-2$ since $e^x>0$ for every $x\in\mathbb{R}$; • $f(x)-x>0$ if $x>2$; (C_f) is above (Δ) if $x\in\left]2\,;\,+\infty\right[$; • $f(x)-x>0$ if $x<2$; (C_f) is below (Δ) if $x\in\left]-\infty\,;\,2\right[$; • $f(x)-x=0$ if $x=2$; (C_f) cut (Δ) at the point of coordinates $(2\,;\,2)$.	1
B.2.	$f'(x)=e^x+(x-2)e^x+1=(x-1)e^x+1=g(x)$; $f'(x)$ and $g(x)$ have the same sign over $\mathbb{R}$; using part A.2. we obtain the following table of variation of f : $\begin{array}{c\|ccccc} x & -\infty & & 0 & & +\infty \\ \hline f'(x) & & + & 0 & + & \\ \hline f(x) & -\infty & \nearrow & -2 & \nearrow & +\infty \end{array}$	2
B.3.	The function f is continuous and strictly increasing over $\left]0\,;\,+\infty\right[$ and changes the sign from negative $(-\infty)$ to positive $(+\infty)$, then the equation $f(x)=0$ admits on $\left]0\,;\,+\infty\right[$ a unique solution α ; In addition, $f(1.6)\approx-0.38<0$ and $f(1.7)\approx0.06>0$ therefore $1.6<\alpha<1.7$.	1
B.4.	(T) is parallel to (Δ) then $f'(x_E)=1$; $(x_E-1)e^{x_E}+1=1$; $(x_E-1)e^{x_E}=0$; So $x_E=1$; $y_E=f(1)=-e+1$; so the coordinates of E are $E(1\,;\,-e+1)$; An equation of (T) : $y=f'(1)(x-1)+f(1)$; $(T): y=x-e$.	1½

B.5.		2