Work and Energy

Work :

Work done by a force :

$$\vec{W_F} = \underline{\vec{F} \cdot \vec{AB}}$$

work done by F (J) ← W_F

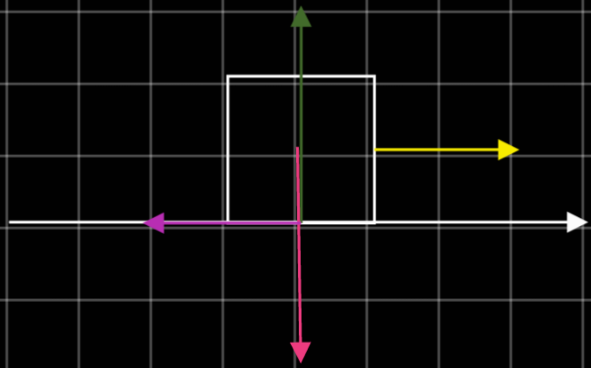
$= F \cdot AB \cdot \cos\alpha$

AB → distance (m)

F → magnitude of $\vec{F}$ (N)

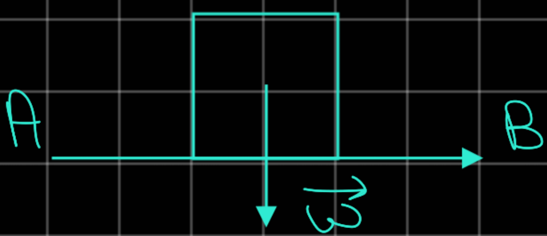
Forces acting on an object :

Applied Force ($\vec{F}$)
Normal Reaction of Support ($\vec{N}$)
Weight ($\vec{W}$)
Friction ($\vec{Fr}$)



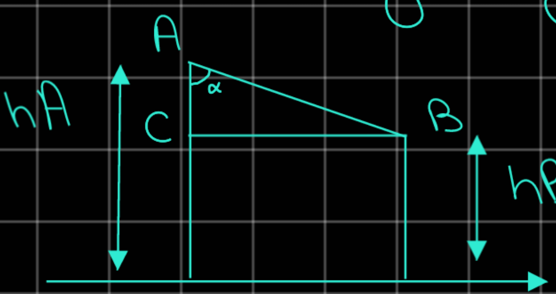
Work done by Weight ($\vec{W}$) : $W = mg$

Motion Along Horizontal Plane :



- $W_{\vec{W}} = 0$ ($\vec{W} \perp \vec{AB}$)

Motion along any trajectory :



$W_{\vec{W}} = W \cdot AB \cdot \cos\alpha$

$\cos\alpha = \dfrac{adj}{hyp} = \dfrac{AC}{AB}$

$\cos\alpha = hA - hB$

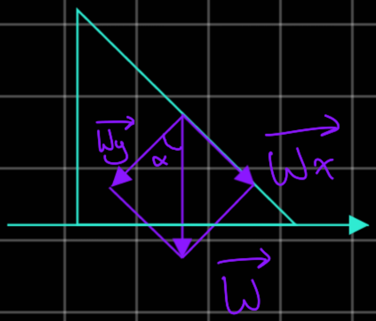
- $W_{\vec{w}, A-B} = mg(h_A - h_B)$

AB

$AB\cos\alpha = h_A - h_B$

$$\boxed{W_{\vec{w}} = mg(h_i - h_f)}$$

Motion along an inclined plane:



$$\alpha(\vec{W}, \vec{W_y})$$

Magnitude: $W_x = mg\sin\alpha$
$W_y = mg\cos\alpha$

Declined plane: $W_{\vec{w}} = mg\sin\alpha d$
Inclined plane: $W_{\vec{w}} = -mg\sin\alpha d$

Energy:

Kinetic Energy (KE): energy due to motion

$KE = \frac{1}{2} mV^2$

Kinetic Energy (J) ← KE; V → Speed (m/s); m → Mass (Kg)

→ If the speed is doubled, KE is quadrupled

$\boxed{V \times 2 \rightarrow KE \times 4}$ - $V\uparrow \Rightarrow KE\uparrow$

→ Work-Energy Theorem: $\boxed{\Delta KE = \Sigma \vec{W_{Fext}}}$

Gravitational Potential Energy (GPE): energy due to position

$GPE = mgh$ → h: Height (m)

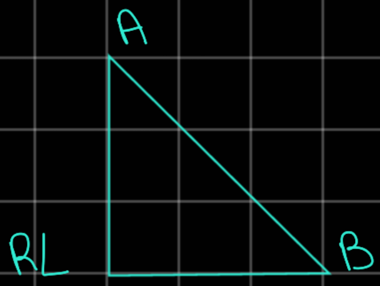
Gravitational Potential ← GPE; g: Gravity (10 m/s²)

Energy (J)   Mass (kg)   g

→ $h_B = 0$; $\sin\alpha = \frac{opp}{hyp}$

$\sin\alpha = \frac{h_A}{AB}$

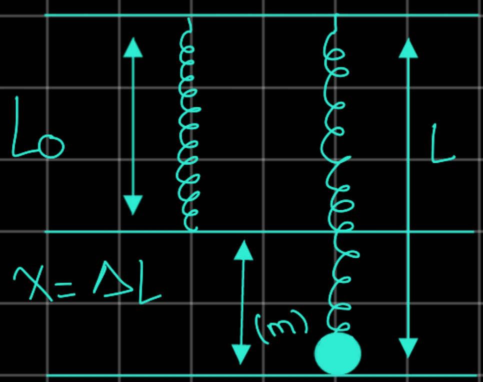
$h_A = AB\sin\alpha$



- $h\uparrow \Rightarrow GPE\uparrow$

→ Height in a circle = $R - R\cos\theta$

Elastic Potential Energy (EPE): energy stored in deformed elastic objects ($T = Kx$)



- Characteristics of a spring:
  L_0 : natural / free length (m)
  K : Stifness

- Forces acting on (m): Weight ($\vec{W}$)
  Tension ($\vec{T}$)

- Condition of Equilibrium: $\sum \vec{F}_{ext} = 0$
  $\vec{T} + \vec{W} = 0$
  $\vec{T} = -\vec{W} \rightarrow \boxed{T = W}$

- Hooke's Law: $T = W \rightarrow \boxed{Kx = mg}$

$EPE = \frac{1}{2}Kx^2$
Elastic Potential Energy (J)
K: stifness
x → Elongation : ΔL (m)

→ Mechanical Energy (ME): sum of energies

$$\boxed{\begin{aligned} ME &= GPE + KE + EPE \\ &= mgh + \frac{1}{2}mV^2 + \frac{1}{2}Kx^2 \end{aligned}}$$

Conservation of ME	Non-Conservation of ME
The non-conservative forces (friction, air resistance .....) are neglected, so ME is conserved	The non-conservative forces (friction, air resistance .....) are applied, so ME varies
$\boxed{ME_i = ME_f}$	$ME_i \neq ME_f$
Variation of ME (ΔME): $\Delta ME = ME_f - ME_i = 0$	Variation of ME (ΔME): $\boxed{\Delta ME = ME_f - ME_i = W_{\vec{fr}}}$
$fr = 0$; $ME = \text{constant}$	Energy lost, appears in the form of thermal energy $\boxed{Q = \lvert \Delta ME \rvert}$

At Max Height : $V = 0 \rightarrow KE = 0$

Linear Momentum ($\vec{p}$)

The time constant (τ) is the time separating two consecutive dot-points. (τ = constant)

If the dot-points are equally separated, same distance is covered in the same time, then motion is said to be :

$\boxed{U.R.M \Longrightarrow a = 0 \text{ , } V = \text{constant}}$

If the distance between dot-points is increasing, then more distance is covered in the same time, the motion is:

$$\boxed{U.A.R.M \Rightarrow a>0\ ,\ V\uparrow}$$

If the distance between dot-points is decreasing, then less distance is covered in the same time, motion is said to be:

$$\boxed{U.D.R.M \Rightarrow a<0\ ,\ V\downarrow}$$

$\vec{r} = x\vec{i} + y\vec{j}$ (position Vector)

$\vec{V} = \vec{r}'$, $V = \sqrt{(V_x)^2 + (V_y)^2}$

$\vec{a} = \vec{V}'$, $a = \sqrt{(a_x)^2 + (a_y)^2}$, $a = \dfrac{\Delta V}{t}$

Linear Momentum / Quantity of motion ($\vec{P}$):

The more momentum an object has the harder it is to be stopped.

$$\vec{P} = m\vec{V}$$

Linear Momentum (Kg·m/s)

mass (Kg)

Speed (m/s)

Linear Momentum of a System ($\vec{P}_{sys}$):

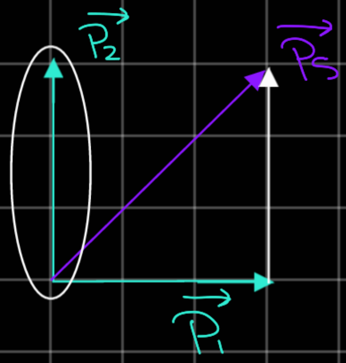
$\vec{P}_{sys}$ = Sum of $\vec{P}$ for all objects in the system

$$\boxed{\vec{P}_{sys} = \vec{P}_A + \vec{P}_B + \dots}$$

Note: Adding Vectors

- Same Line of action, Same direction : $P_S = P_1 + P_2$

- Same Line of action, Opposite direction : $P_S = P_1 - P_2$

- 

By displacing $\vec{P_2}$ and applying pythagoras theorem : $P_S = \sqrt{P_2^2 + P_1^2}$

When vectors are collinear, substitute value of vectors directly.

$$\boxed{\vec{P'} = m\vec{a} = \Sigma \vec{Fext}}$$

- $\vec{P}$ and $\vec{V}$ are always collinear + have same direction

Note : Center of Mass (G)

$$xG = \frac{mAxA + mBxB + mCxC}{mA + mB + mC}$$

The Linear Momentum of a system is equal to the Linear momentum of its center.

$$\boxed{\vec{Psys} = \vec{P_G}}$$

Conservation of Linear Momentum ($\vec{P}$) :

$\vec{P'} = \Sigma \vec{Fext}$, If $\Sigma \vec{Fext} = 0$, the system is said to be isolated.

Then $\vec{P'} = 0$, so linear momentum is conserved.

$$\boxed{\vec{Psys} = \text{Constant}}$$

In an Elastic Collision, KE is conserved