

# divergence tests of goodness of fit

testing nested model specifications

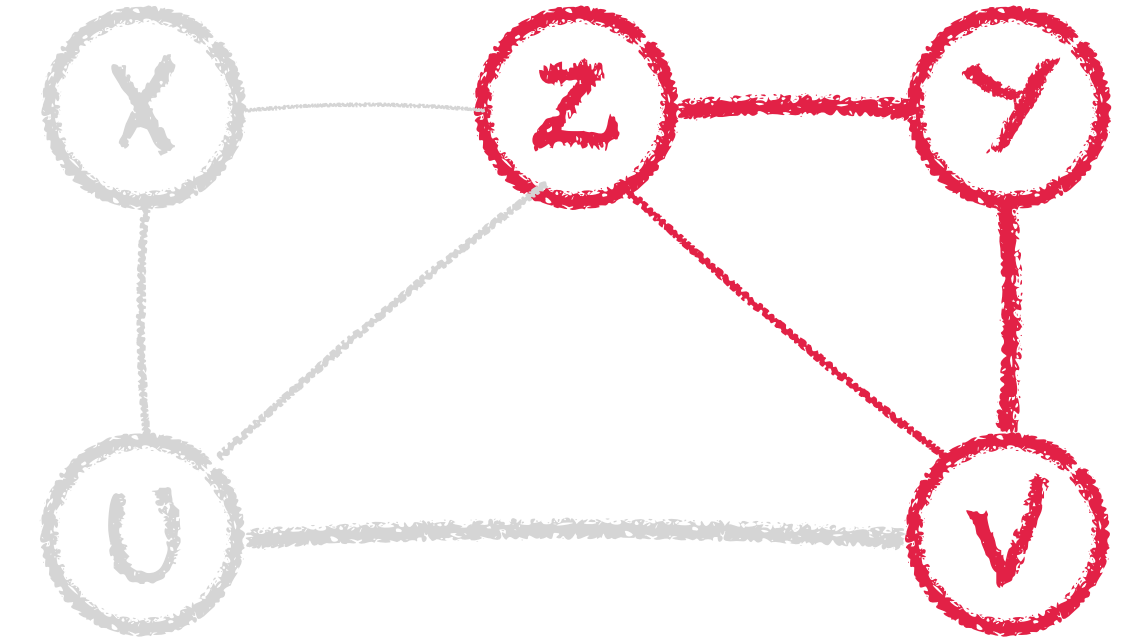
example: five dimensional data  $(X, Y, Z, U, V)$  with  $r_X, r_Y, r_Z, r_U, r_V$  outcomes

$p$  = model based on empirical distribution  $p(x, y, z, u, v)$  with  $d(p) = r_X r_Y r_Z r_U r_V - 1$

$q$  = model with listed imposed independence and conditional independence assumptions

examples:  $q_1 = X \perp (Y, Z, U, V)$  and  $U \perp (Y, Z, V)$

$q_2 = X \perp (Y, Z, U, V)$  and  $U \perp (Y, Z, V)$  and  $Z \perp V | Y$



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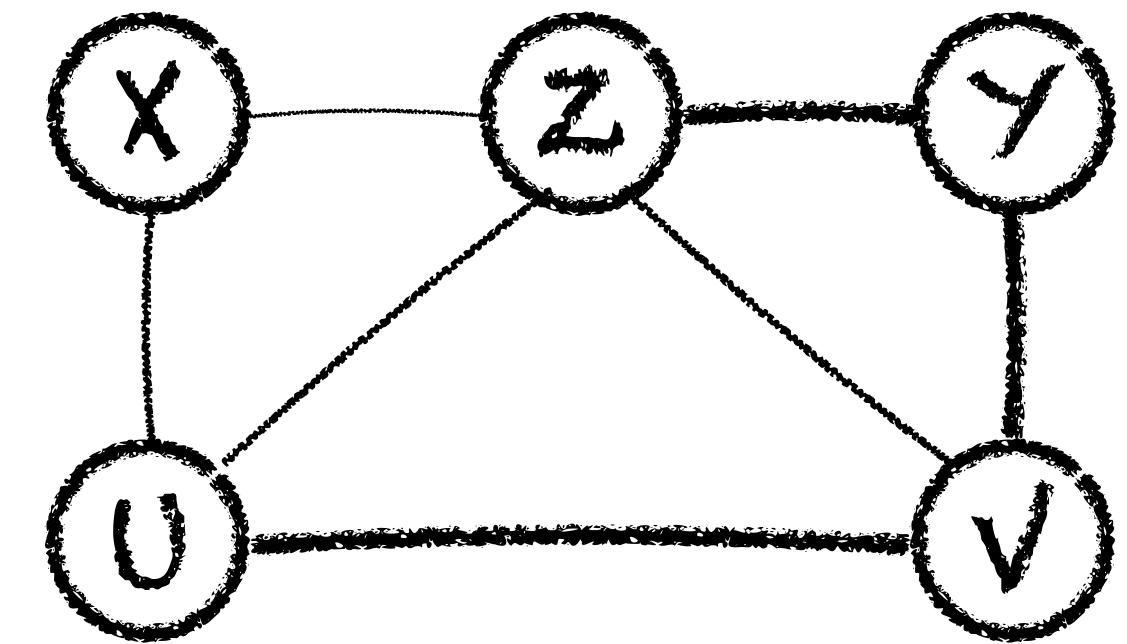
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☑  $d(q)$  is sum of the degrees of freedom of its independent components

☑ divergence  $D(p, q)$  of each model is the sums of the divergences of its nested specifications

$$\chi^2(d) = 2nD(p, q_1) = 2n[D(X \perp (Y, Z, U, V)) + D(U \perp Y, Z, V)]$$

$$\chi^2(d) = 2nD(p, q_2) = 2n[D(X \perp (Y, Z, U, V)) + D(U \perp Y, Z, V) + D(Z \perp V | Y)]$$

where  $d$  can be obtained as either

✓ the sums of degrees of freedom for the divergences of the nested specifications

✓ the difference between degrees of freedom of the general and the specified model  $d(p) - d(q)$