univariate entropy

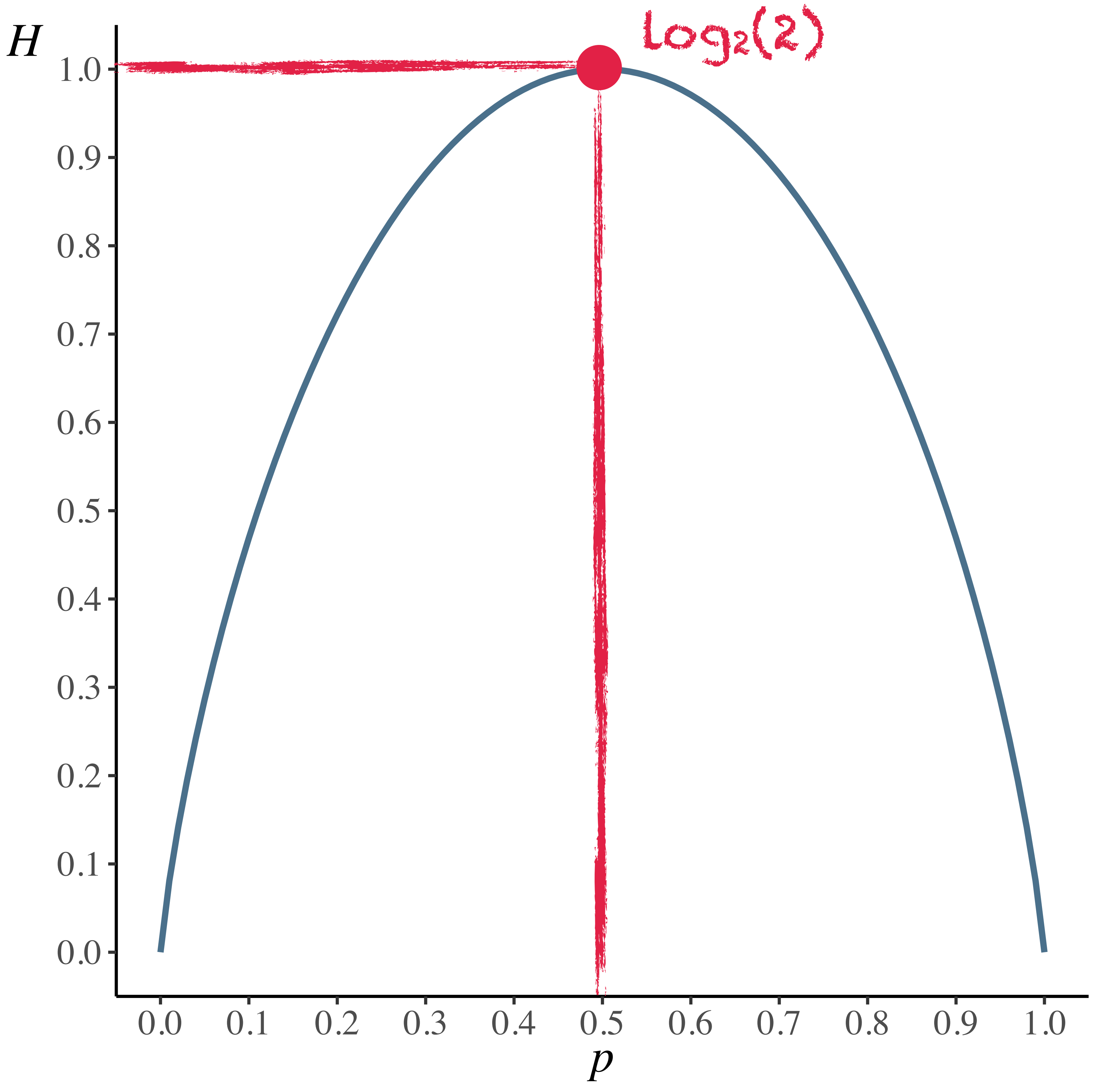
statistical entropy is a measure of uncertainty of random variables

for a discrete random variable X with a finite range space of size r_X

$$H(X) = \sum_x p(x) \log_2 \frac{1}{p(x)} \qquad p(x) > 0, \ \sum_x p(x) = 1$$

- ☑ minimal zero entropy has no uncertainty
- ☑ maximum entropy $\log_2(r_X) \implies$ uniform distribution

example:
entropy of a binary variable



bivariate and joint entropies

for two discrete random variables X and Y the bivariate entropy is given by

$$H(X,Y) = \sum_x \sum_y p(x,y) \log_2 \frac{1}{p(x,y)}$$

and bounded according to

$$H(X) \leq H(X,Y) \leq H(X) + H(Y)$$

variable selection

equality to the left iff $X \to Y$ equality to the right iff $X \perp Y$

the two increments of the inequalities around $H(X,Y)$:

- joint entropy

$$J(X,Y) = H(X) + H(Y) - H(X,Y)$$

non-negative and equal to 0 iff $X \perp Y$

association graphs

divergence statistic

- expected conditional entropy

$$EH(Y|X) = H(X,Y) - H(X)$$

non-negative and equal to 0 iff $X \to Y$

prediction power