bivariate and joint entropies

for two discrete random variables X and Y the bivariate entropy is given by

$$H(X,Y) = \sum_x \sum_y p(x,y) \log_2 \frac{1}{p(x,y)}$$

and bounded according to

$$H(X) \leq H(X,Y) \leq H(X) + H(Y)$$

variable selection

equality to the left iff $X \rightarrow Y$ equality to the right iff $X \perp Y$

the two increments of the inequalities around $H(X,Y)$:

- ☑ **joint entropy**

  $$J(X,Y) = H(X) + H(Y) - H(X,Y)$$

  non-negative and equal to 0 iff $X \perp Y$

  association graphs
  divergence statistic

- ☑ **expected conditional entropy**

  $$EH(Y|X) = H(X,Y) - H(X)$$

  non-negative and equal to 0 iff $X \rightarrow Y$

  prediction power

trivariate and higher order entropies

for three discrete random variables X, Y and Z the trivariate entropy is bounded

$$H(X, Y) \leq H(X, Y, Z) \leq H(X, Z) + H(Y, Z) - H(Z)$$

the two increments of the inequalities around $H(X, Y, Z)$:

☑ **expected joint entropy**

$$EJ(X, Y | Z) = H(X, Z) + H(Y, Z) - H(Z) - H(X, Y, Z)$$

non-negative and equal to 0 iff $(X, Y) \perp Z$

association graph
divergence statistic

☑ **expected conditional entropy**

$$EH(Z | X, Y) = H(X, Y, Z) - H(X, Y)$$

non-negative and equal to 0 iff $X \to Y$

prediction power

similarily for $H(X, Y, Z, U)$, $H(X, Y, Z, U, V)$, $H(X, Y, Z, U, V, W)$, …