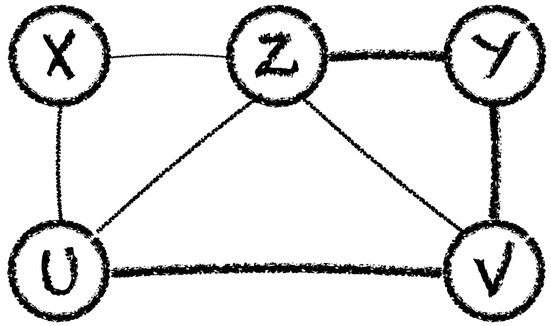
divergence tests of goodness of fit

testing conditional independence

of random variable X, Y and Z with r_X, r_Y and r_Z outcomes

$p =$ model based on empirical distribution $p(x, y, z)$ with $d(p) = r_X r_Y r_Z - 1$

$q = X \perp Y | Z$ such that $p(x, z)p(y, z)/p(z)$ with $d(q) = r_Z - 1 + r_Z(r_X - 1 + r_Y - 1)$



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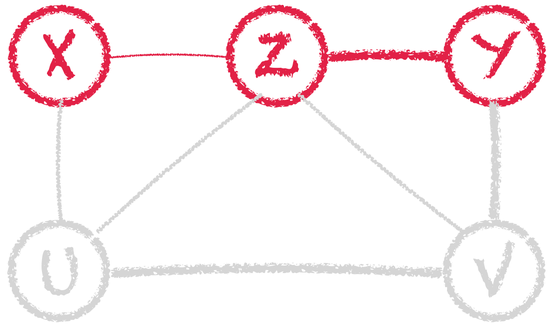

☑ log likelihood ratio test statistic

$$\begin{aligned}\chi^2((r_X - 1)(r_Y - 1)r_Z) &= 2nD(p, q) \\ &= 2n[H(X, Z) + H(Y, Z) - H(Z) - H(X, Y)] \\ &= 2nEJ(X, Y | Z)\end{aligned}$$

☑ independence is rejected if

$$\chi^2((r_X - 1)(r_Y - 1)r_Z) \geq (r_X - 1)(r_Y - 1)r_Z + \sqrt{8(r_X - 1)(r_Y - 1)r_Z}$$

or if the empirical expected joint entropy $J(X, Y)$ is larger than

$$[(r_X - 1)(r_Y - 1)r_Z + \sqrt{8(r_X - 1)(r_Y - 1)r_Z}]/2n$$