Stochastic Actor Oriented Models

Coevolution of Networks and Behaviors

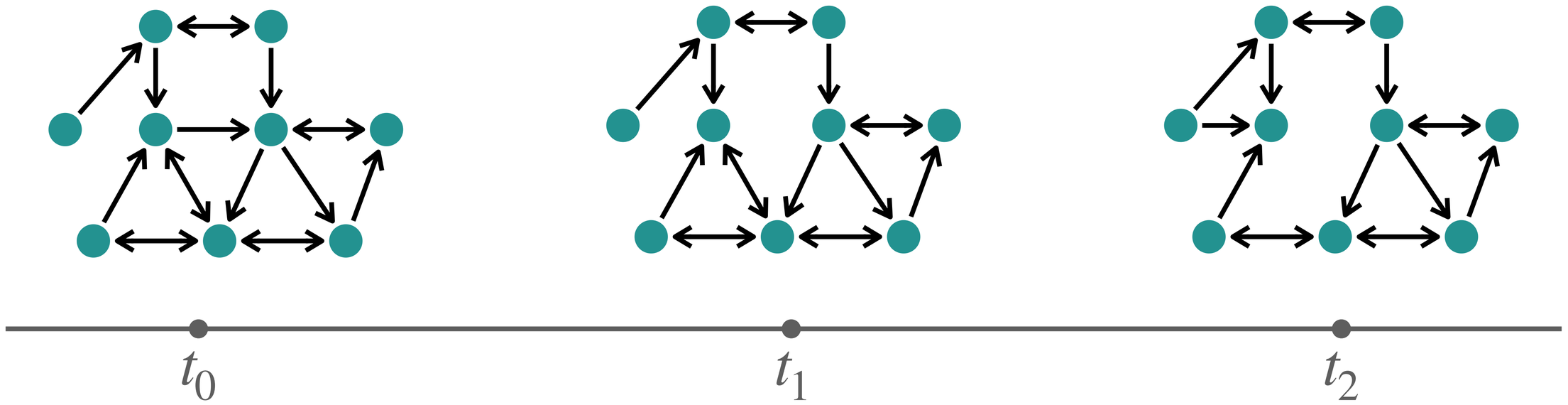
distinguishing between influence and selection

$\Longrightarrow$ **Stochastic Actor Oriented Models (SAOMs)**

after this session we should have sufficient grasp of the basic assumptions of the SAOM to

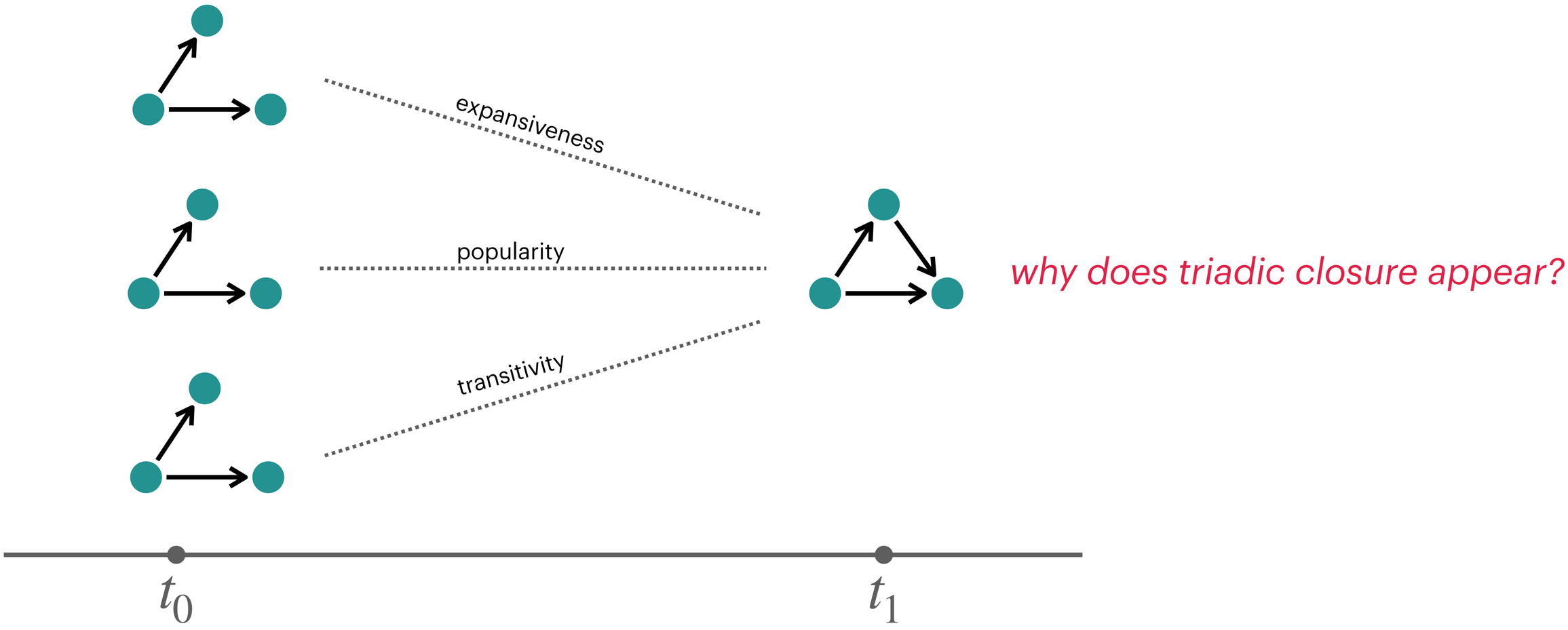
- understand what the model does
- interpret output

network dynamics



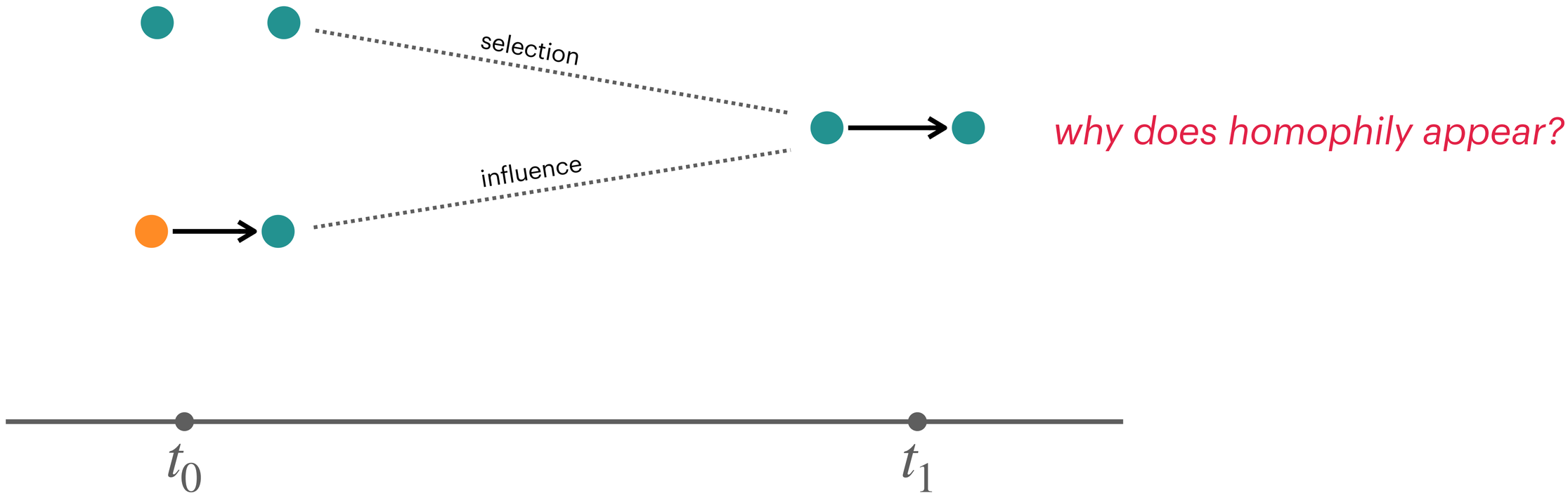
networks are dynamic by nature:
the observed networks are the results of change over time
ties can be created, deleted or maintained

network dynamics



why does triadic closure appear?

network dynamics



why does homophily appear?

structure and individual characteristics change over time

in what order though?

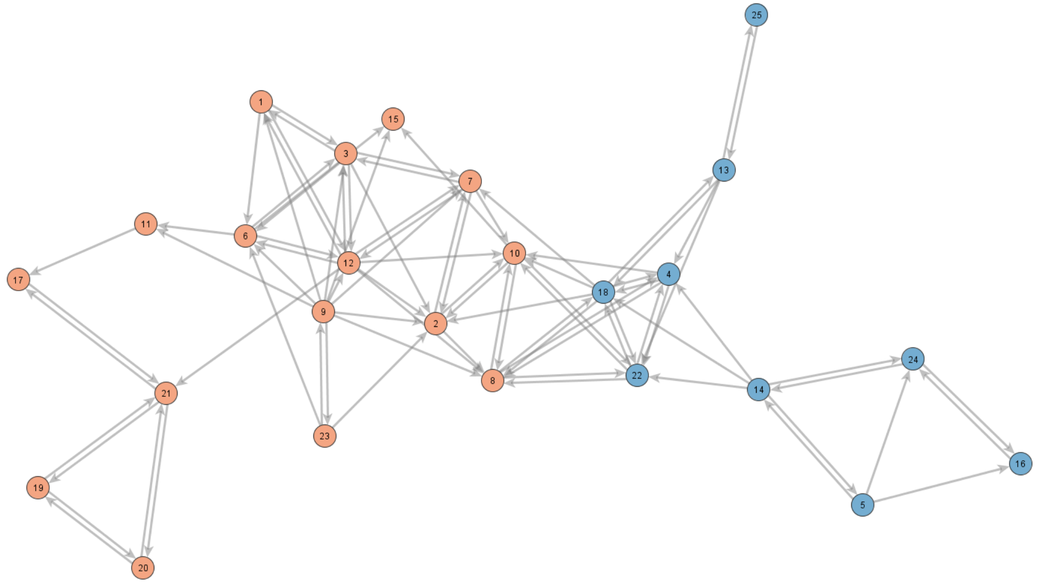
network dynamics

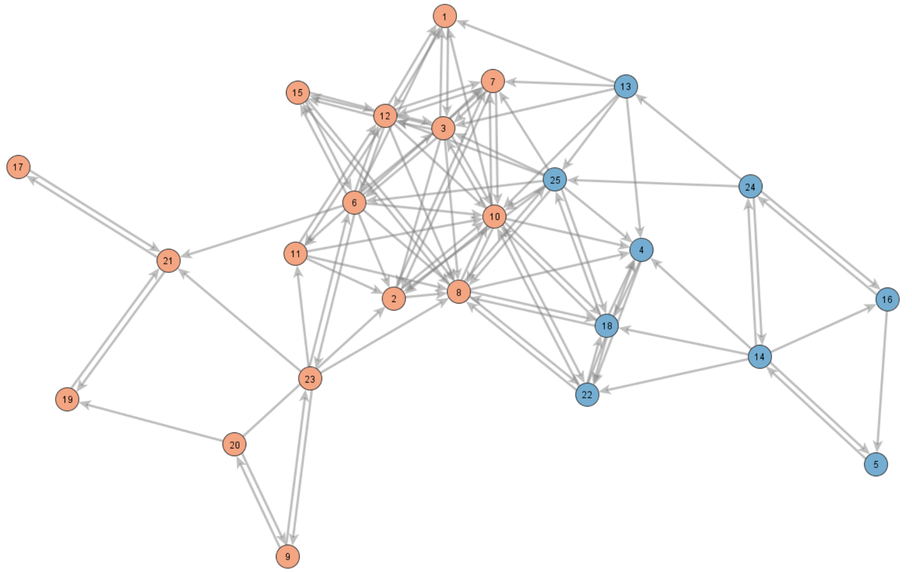
Questions:

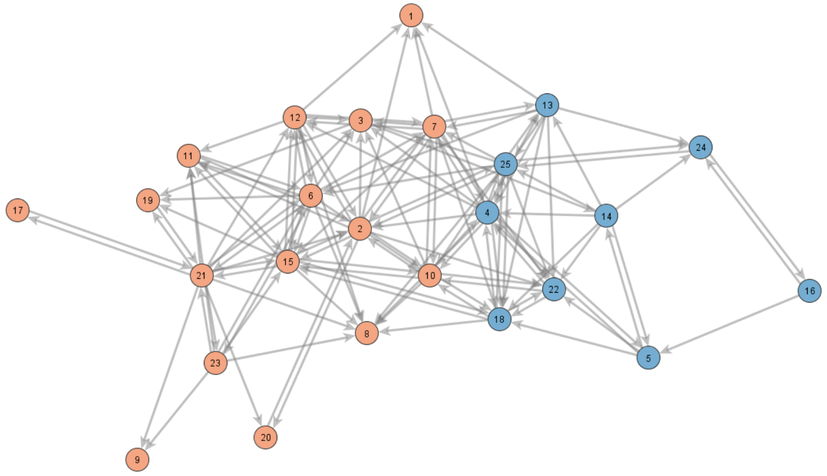
1. how frequently do actors change ties?
2. what are the reasons that lead to a tie change?
3. how might the network appear in the future?

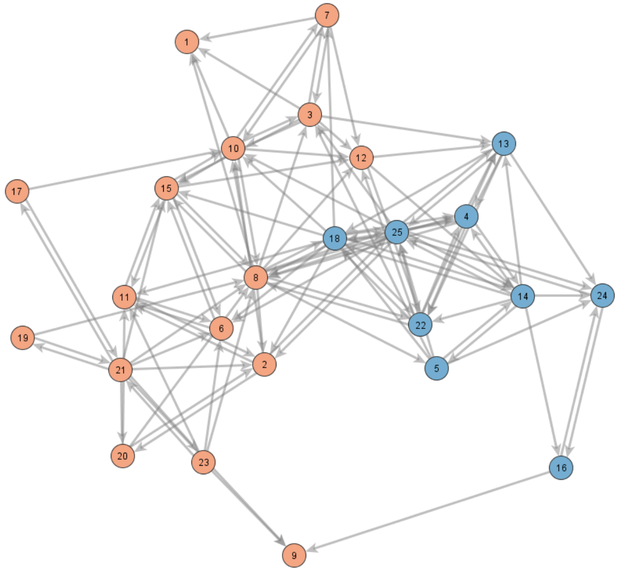
how can we model network evolution over time?

A. Knecht (2008): "Friendship Selection and Friends' Influence"









four time points in the pupils' first year at secondary school (color gender)

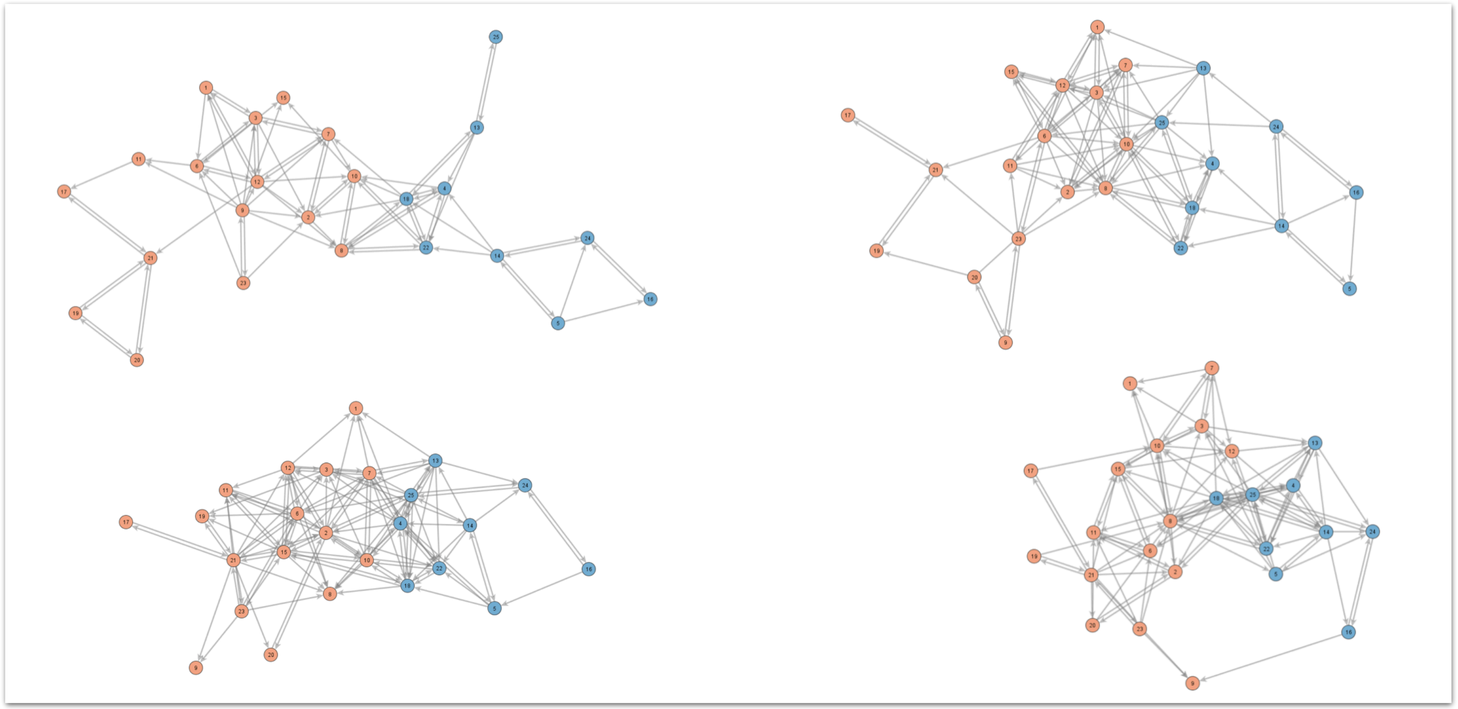
A. Knecht (2008): "Friendship Selection and Friends' Influence"

is there any tendency in friendship formation

- towards **reciprocity**?

- towards **transitivity**?

- towards gender **homophily**?



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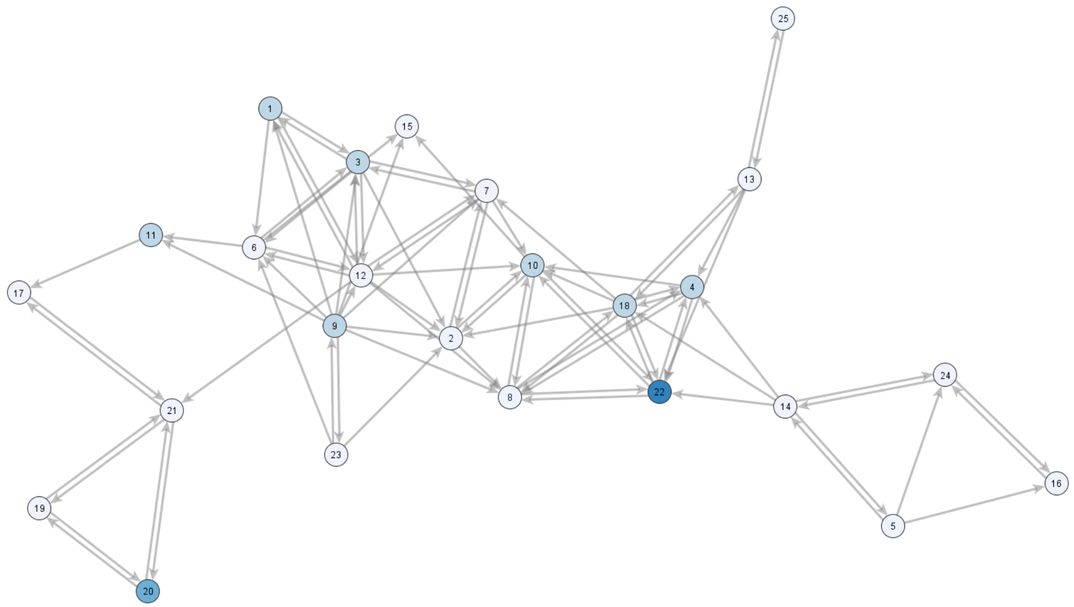
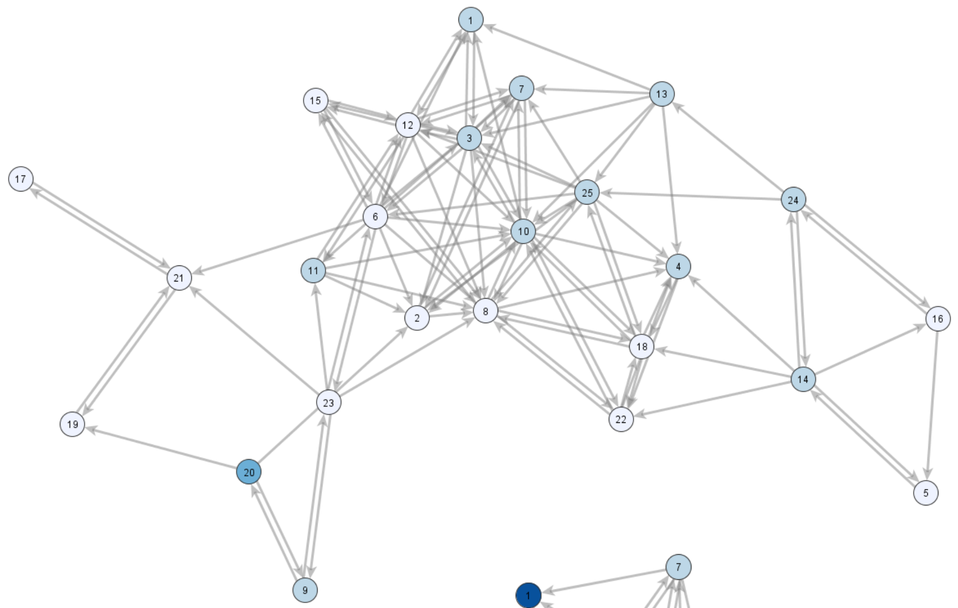
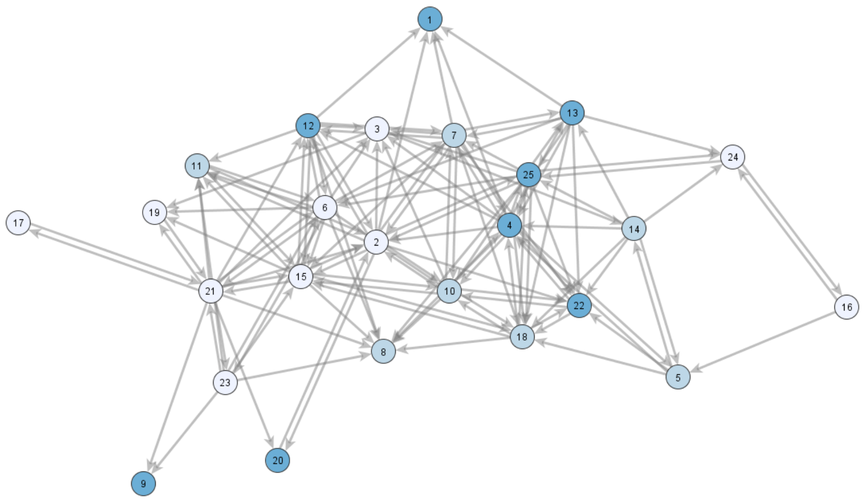
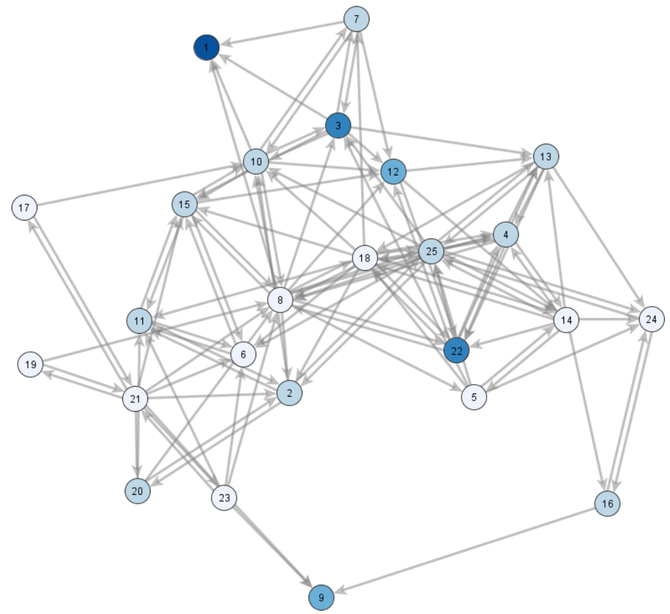






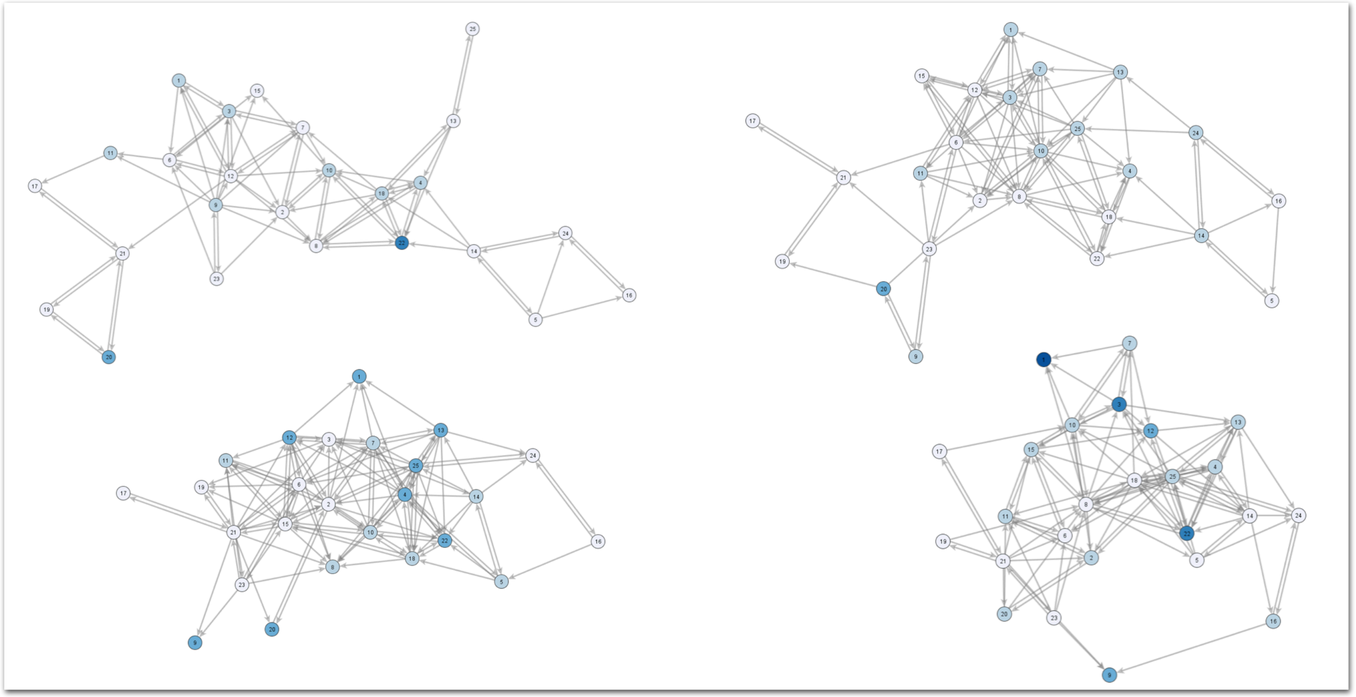


four time points in the pupils' first year at secondary school (color delinquency)

A. Knecht (2008): "Friendship Selection and Friends' Influence"

- is there **social selection**?

- is there **social influence**?



Networks Models for Longitudinal Data

- Stochastic actor-oriented models (SAOMs)
- Temporal exponential random graph models (TERGMS and STERGMs)

Aim is to explain network evolution as a result of

- **endogenous variables:**
  structural effects depending on the network only
  (e.g. reciprocity, transitivity, etc.)
- **exogenous variables:**
  actor-dependent and dyadic-dependent features
  (e.g. effect of an attribute on the existence of a tie)

simultaneously.

```
Network evolution is the outcome of a continuous-time Markov chain,
ties are formed as a reaction to the existence of other ties.
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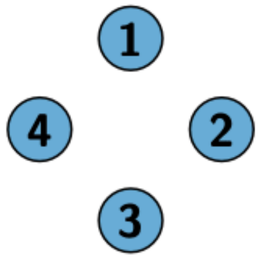
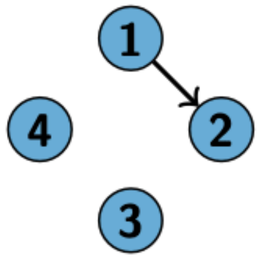
Continuous-Time Markov chain

The **finite set** of all possible adjacency matrices defined on $2^{n(n-1)}$

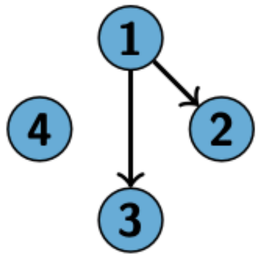
Example



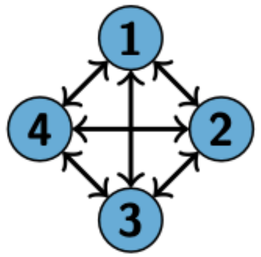
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0



0	1	0	0
0	0	0	0
0	0	0	0
0	0	0	0



0	1	1	0
0	0	0	0
0	0	0	0
0	0	0	0

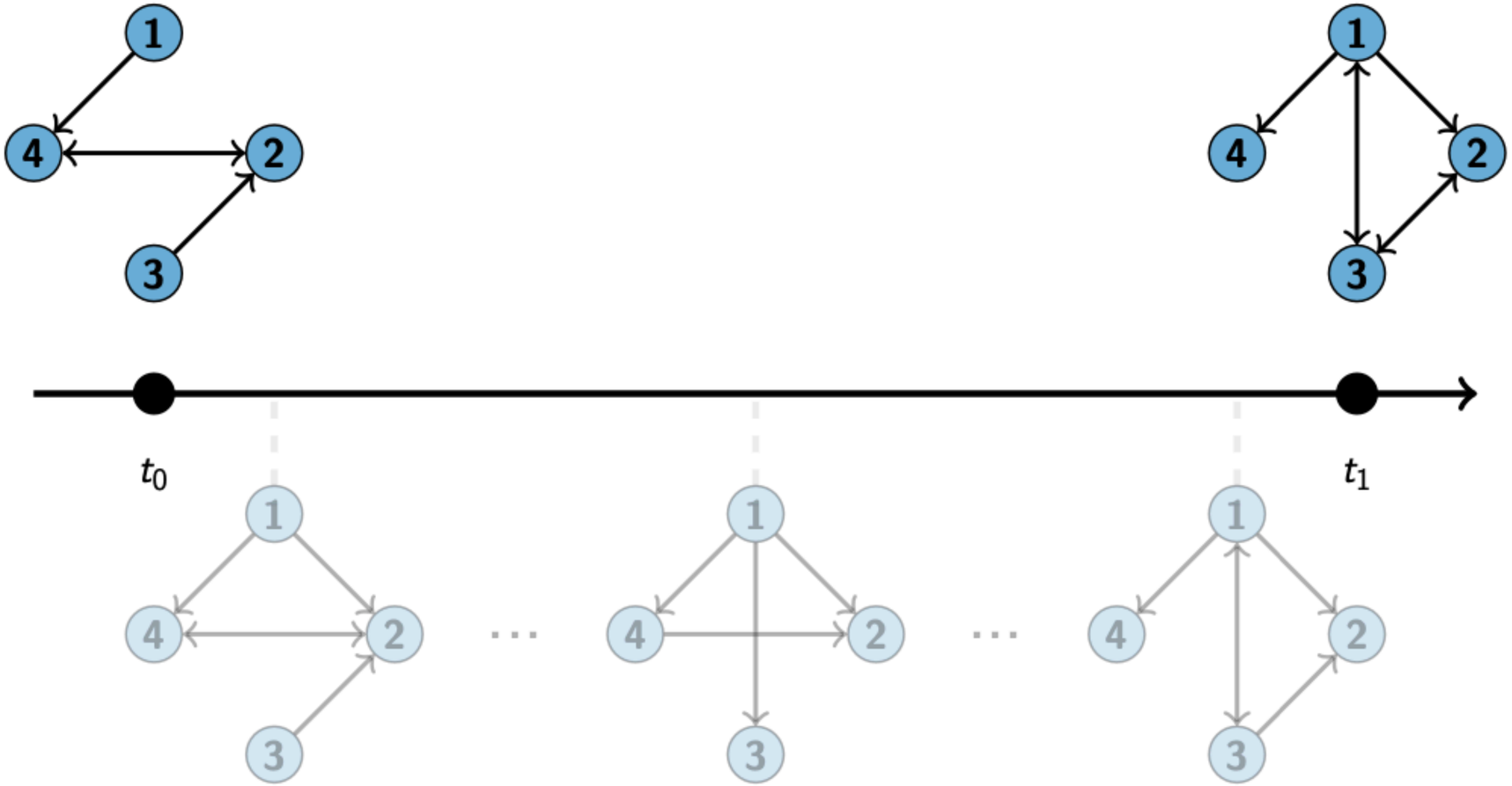


0	1	1	1
1	0	1	1
1	1	0	1
1	1	1	0

Continuous-Time Markov chain

Continuous-time process

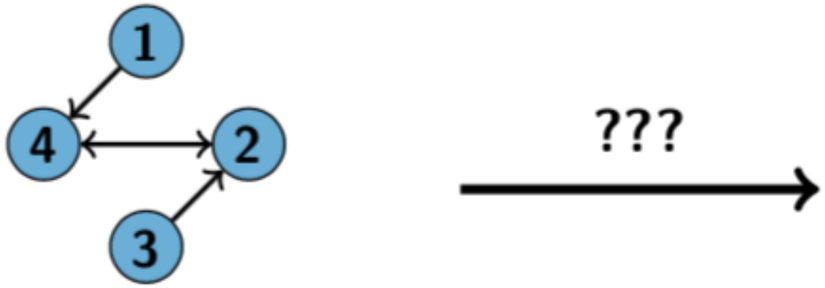
network evolves latently in continuous-time but we observed it only at discrete time points



Continuous-Time Markov chain

Given the current network (x) what is the next network (x')?

Since we had $2^{n(n-1)}$ possible networks, there are $2^{n(n-1)} - 1$ possibilities $\Longrightarrow$ ***too may!***



The **Markov property** states that the future state of a process depends only on the present state, not on the sequence of past states

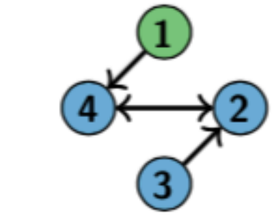
$\Longrightarrow$ the future network structure depends only on current network, not on the full history.

Actor Oriented Model

The model is **actor-oriented**:

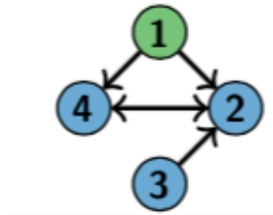
- **Opportunity to change**
  at any given moment t one actor has the opportunity to change
- **Absence of co-occurrence**
  no more than one tie can change at any given moment *t*
- **Actor's decision**
  change in ties are made by the actor who sends the ties

SAOMs



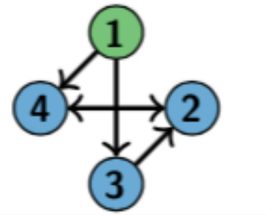
0	0	0	1
0	0	0	1
0	1	0	0
0	1	0	0

x=current state



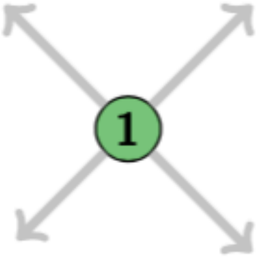
0	1	0	1
0	0	0	1
0	1	0	0
0	1	0	0

$x(i \rightsquigarrow 2)$

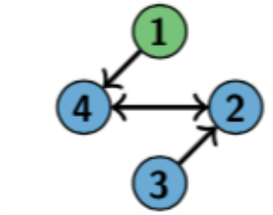


0	0	1	1
0	0	0	1
0	1	0	0
0	1	0	0

$x(i \rightsquigarrow 3)$

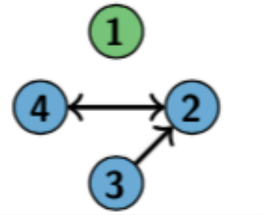


do nothing



0	0	0	1
0	0	0	1
0	1	0	0
0	1	0	0

$x(i \rightsquigarrow 4)$

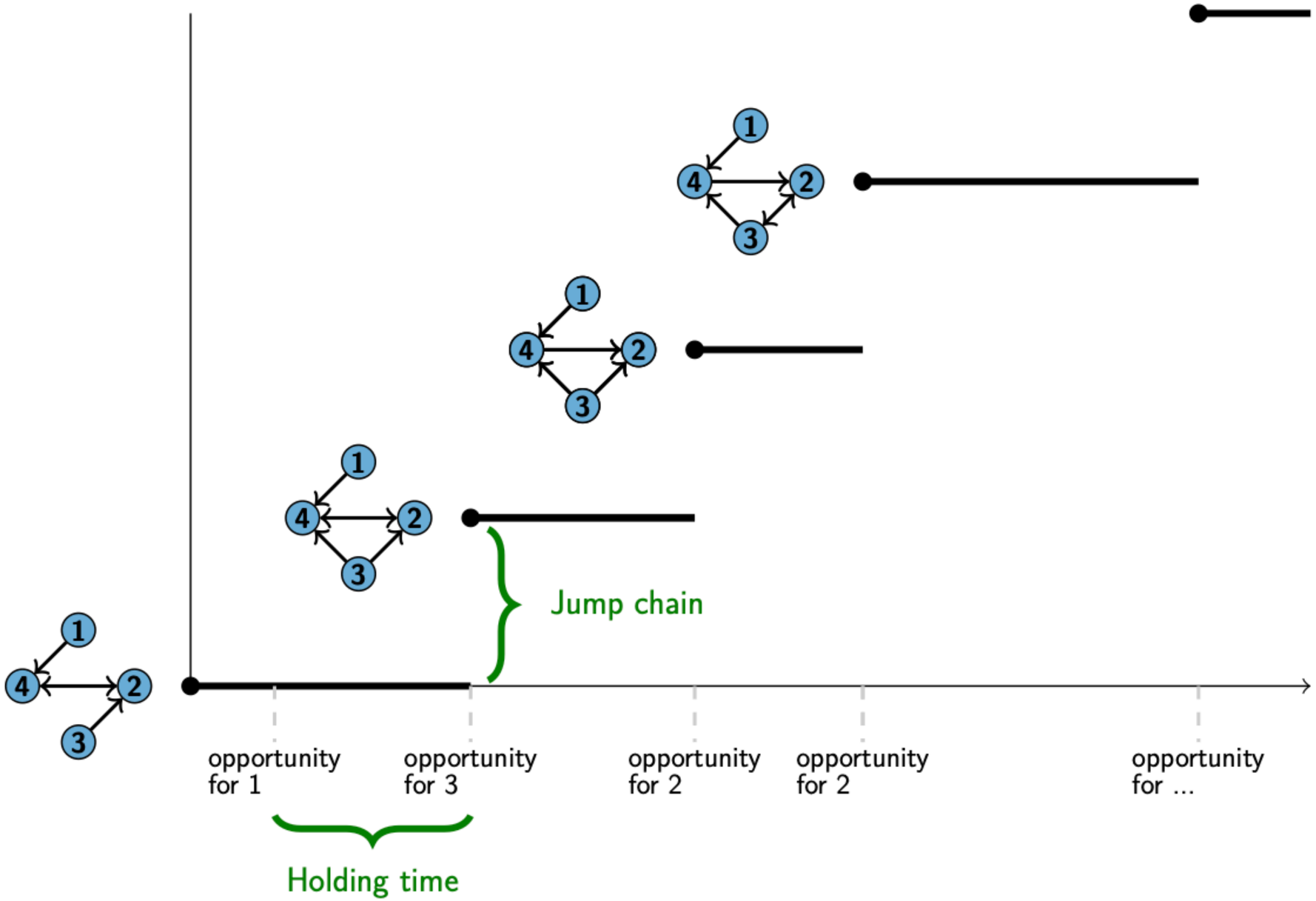


0	0	0	0
0	0	0	1
0	1	0	0
0	1	0	0

$x(i \rightsquigarrow\!\!\!\!\rightarrow j)$ denote the network x where the tie from i to j is turned into its opposite

SAOMs

The evolution process can be decomposed into mini-steps



SAOMs

the general form of the model:

- **network function** for structural change (endogenous effects)
- **behavior function** for attribute change (exogenous effects)
- **rate function** for when the change occurs (holding time)

	decision time **when changes occur**	**decision rules** **how changes occur**
network change	network rate function	network objective function
behaviour change	behaviour rate function	behaviour objective function

Distribution of the holding time: rate function
Transition matrix of the jump chain: objective function

Network Objective Function

What is the direction of change?

$$f_i(\beta, x) = \sum_k \beta_k s_{ki} + \epsilon$$

- β current set of parameters
- x state of the network
- s statistics/effects
- ϵ random component

goal of model is to estimate each β_k

when actor i gets an opportunity for change,
they have the possibility to change one outgoing tie $x(i \rightsquigarrow j)$,
or leave everything unchanged

actors make changes such as to optimize/maximize $f_i(\beta, x)$

higher probabilities of moving toward new states
that have higher values of the objective function

Network Objective Function

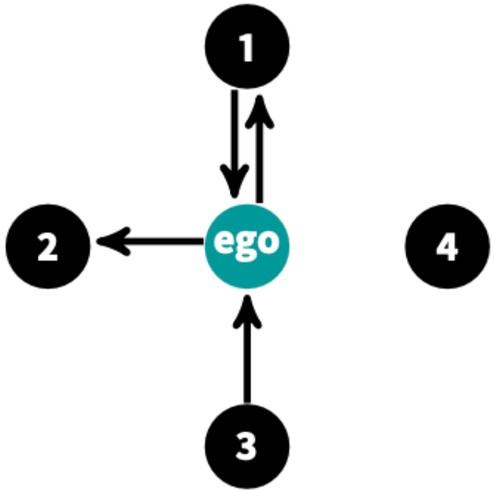
example.

$$f_{ego}(\beta, x) = -2(\text{outdegree}) + 1.8(\text{reciprocity})$$

goal is to maximize objective function f_{ego}

if...	outdegree	reciprocity	sum
no change	-2×2=-4	1.8×1=1.8	-2.2

current state:



Network Objective Function

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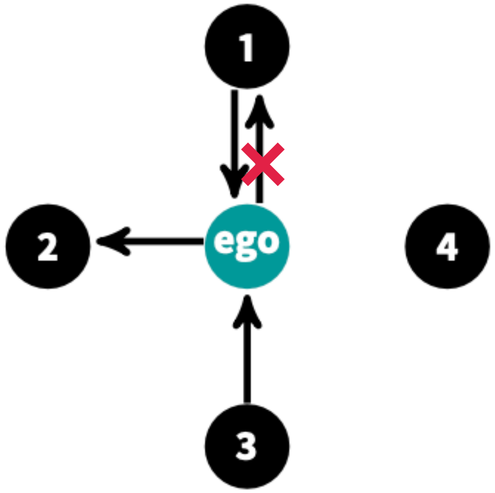
if...	outdegree	reciprocity	sum
no change	-2×2=-4	1.8×1=1.8	-2.2
drop 1	-2×1=-2	1.8×0=0	-2

current state:



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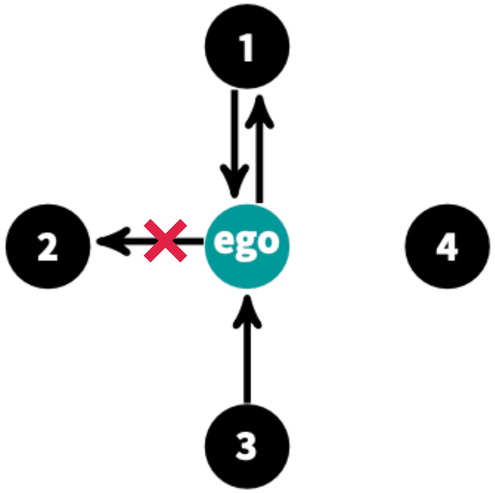
drop 1	-2×1=-2	1.8×0=0	-2
drop 2	-2×1=-2	1.8×1=1.8	-0.2

current state:



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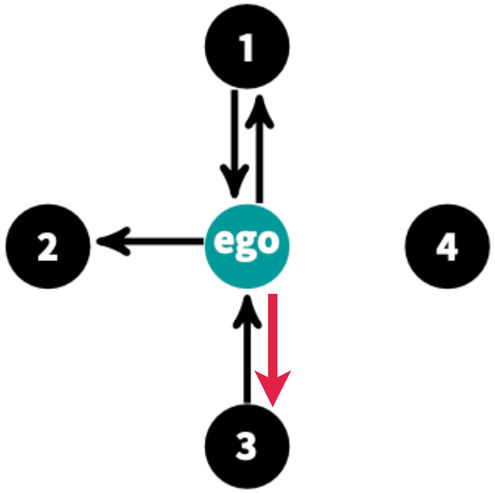
drop 1	-2×1=-2	1.8×0=0	-2
drop 2	-2×1=-2	1.8×1=1.8	-0.2
add 3	-2×3=-6	1.8×2=3.6	-2.4

current state:



Network Objective Function

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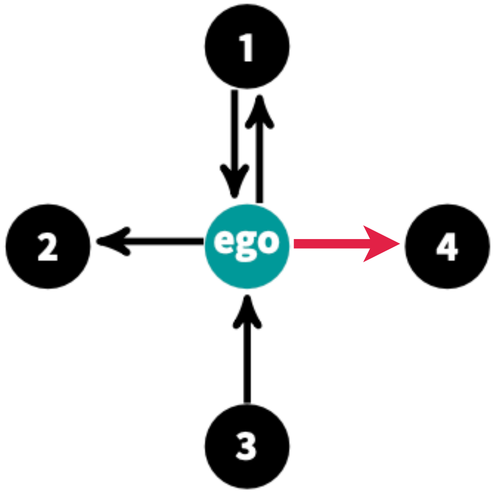
drop 2	-2×1=-2	1.8×1=1.8	-0.2
add 3	-2×3=-6	1.8×2=3.6	-2.4
add 4	-2×3=-6	1.8×1=1.8	-4.2

current state:



Network Objective Function

example.

$$f_{ego}(\beta, x) = -2(\text{outdegree}) + 1.8(\text{reciprocity})$$

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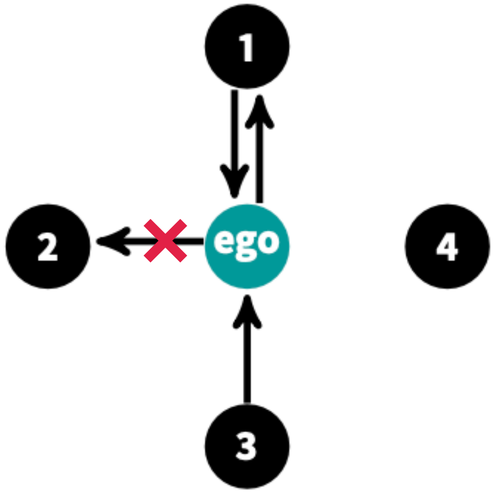
current state:



if...	outdegree	reciprocity	sum
no change	-2×2=-4	1.8×1=1.8	-2.2
drop 1	-2×1=-2	1.8×0=0	-2
drop 2	-2×1=-2	1.8×1=1.8	-0.2
add 3	-2×3=-6	1.8×2=3.6	-2.4
add 4	-2×3=-6	1.8×1=1.8	-4.2

drop the tie to node 2 decision maximizes objective function

Behavior Objective Function

$$f_i^z(\beta, x, z) = \sum_k \beta_k^z s_{ki}^z + \epsilon$$

z	behavior
β	current set of parameters
x	state of the network
s	statistics/effects
ϵ	random component

- used to model changes in behavior (attributes like beliefs, interests, attitude, etc)
- ordinal measurements on the attribute
- it is an optional function

actors evaluate all possible changes: increase/decrease by one unit, or no change

Rate Function

How fast is the opportunity for changing?

- specified for both network and behavior objective function
- the waiting time until actor's chance to make decisions

the waiting time between opportunities of change for an actor i
is exponentially distributed with parameter λ_i

λ_i is called the rate function

simple specification: all actors have the same the same rate of change

complex specification:

- modeled based on actor attributes:
  do some types of actors experience more/less change?
- modeled based on degrees:
  do actors with more/fewer ties have different volume of changes?

SAOMs

In summary, change process is decomposed into two sub-models

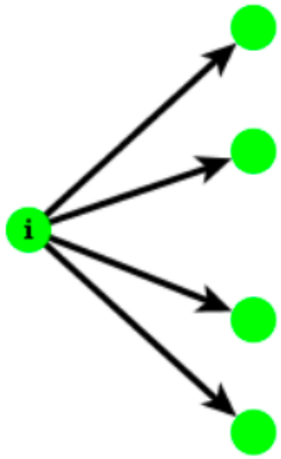
1. **Objective functions:**
   probabilities of changing ties conditional on such an opportunity for change
2. **Rate functions:**
   Waiting times until the next opportunity for a change made by an actor

the distinction between rate function and objective function separates
the model for how many changes are made
from the model for which changes are made

Objective Function Specification

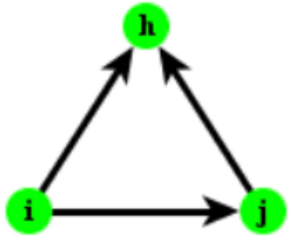
Endogenous effects = *dependent on the network structures*

outdegree

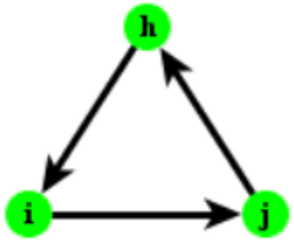


reciprocity

transitivity



three cycles

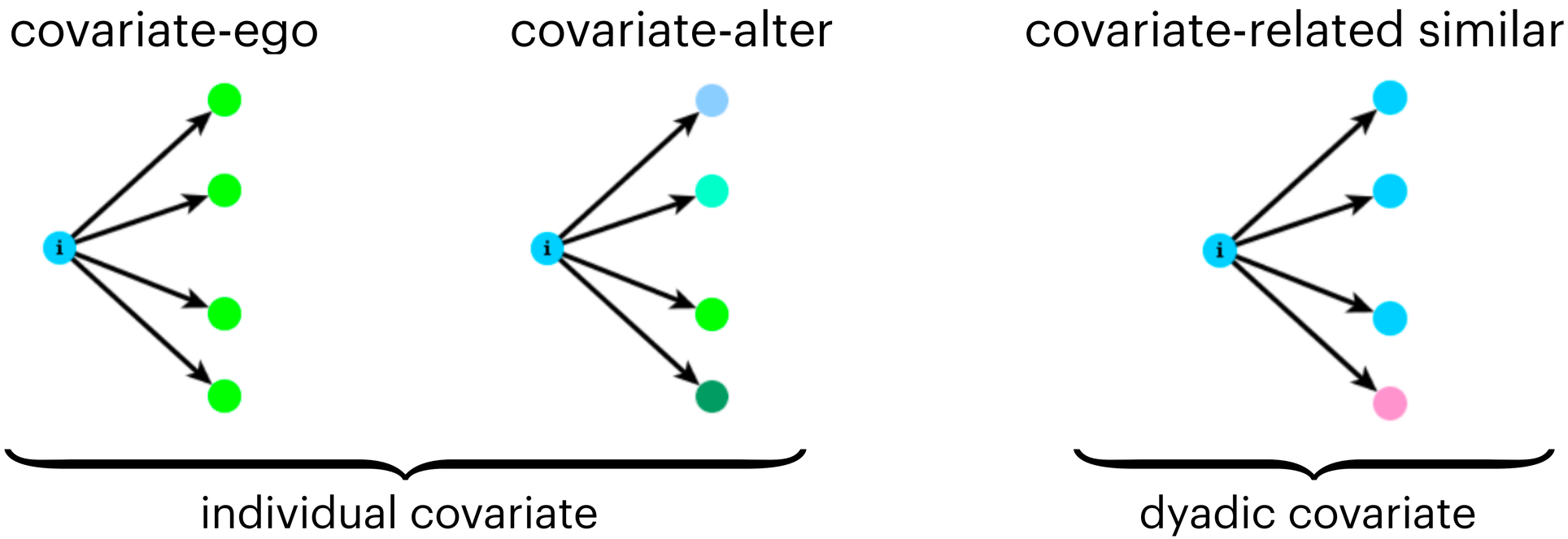


Objective Function Specification

Exogenous effects *= related to actor's attributes*

Examples:

- Friendship among pupils:
  - Smoking: non, occasional, regular
  - Gender: boys, girls
- Trade/Trust (Alliances) among countries:
  - Geographical area: Europe, Asia, North-America,...
  - Worlds: First, Second, Third, Fourth



Objective Function Specification

Which effects must be included in the objective function?

Outdegree and **Reciprocity** must always be included.
The choice of the other effects must be determined according to hypotheses derived from theory

Example. Friendship Network

Theory		Effect
the friend of my friend is also my friend	$\Rightarrow$	transitive effect
girls trust girls boys trust boys	$\Rightarrow$	covariate-related similarity

Parameter Interpretation

The parameter β_k quantifies the role of the effect s_{ik} in the network evolution:

- $\beta_k = 0 \Longrightarrow s_{ik}$ plays no role in the network dynamics
- $\beta_k > 0 \Longrightarrow$ higher probability of moving into networks where s_{ik} is higher
- $\beta_k < 0 \Longrightarrow$ higher probability of moving into networks where s_{ik} is lower

Parameter Interpretation

Which β_k are “significantly” different from 0?

Hypothesis test:

The null hypothesis (H_0):

the observed increase or decrease in the number of network configurations related to a certain effect results purely from chance

$$H_0 : \beta_k = 0$$

The alternative hypothesis (H_1):

the observed increase or decrease in the number of network configurations related to a certain effect is influenced by some non-random cause.

$$H_1 : \beta_k \neq 0$$

Parameter Interpretation

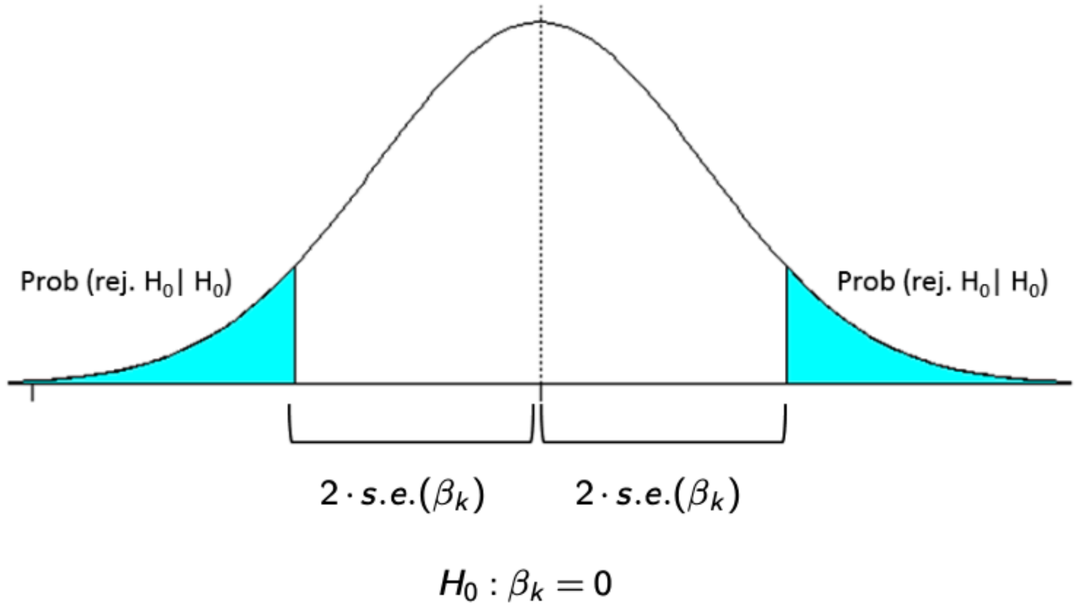
Decision Rule:

| parameter estimate | > twice the standard error

i.e.

$$\left| \frac{\beta_k}{s.e.(\beta_k)} \right| \geq 2 \qquad \text{reject } H_0$$

$$\left| \frac{\beta_k}{s.e.(\beta_k)} \right| < 2 \qquad \text{fail to reject } H_0$$



Example: Study of Smoking Initiation and Friendship

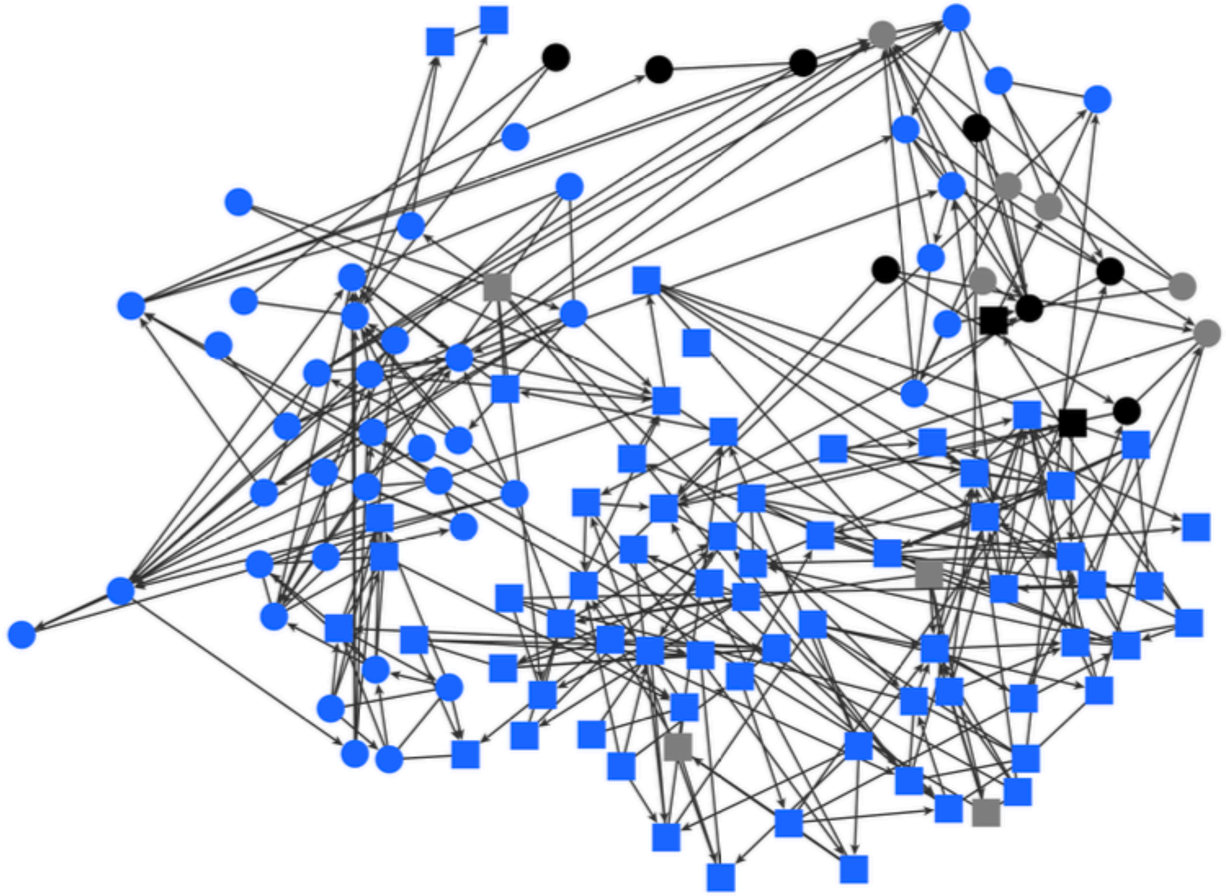
The Teenage Friends and Lifestyle Study analyzes smoking behavior and friendship one school year group from a Scottish secondary school

- starting at age 12-13 years
- monitored over 3 years
- total of 160 pupils, of which 129 pupils present at all 3 observations
- sociometric and behavior questionnaires at three moments ≈1 year intervals
- covariates:
  - gender (1-2)
  - smoking of parents and siblings (1-2)
  - money available (range 0–40 pounds/week)
  - smoking: values 1–3
  - drinking: values 1–5

Example: Study of Smoking Initiation and Friendship

Wave 1

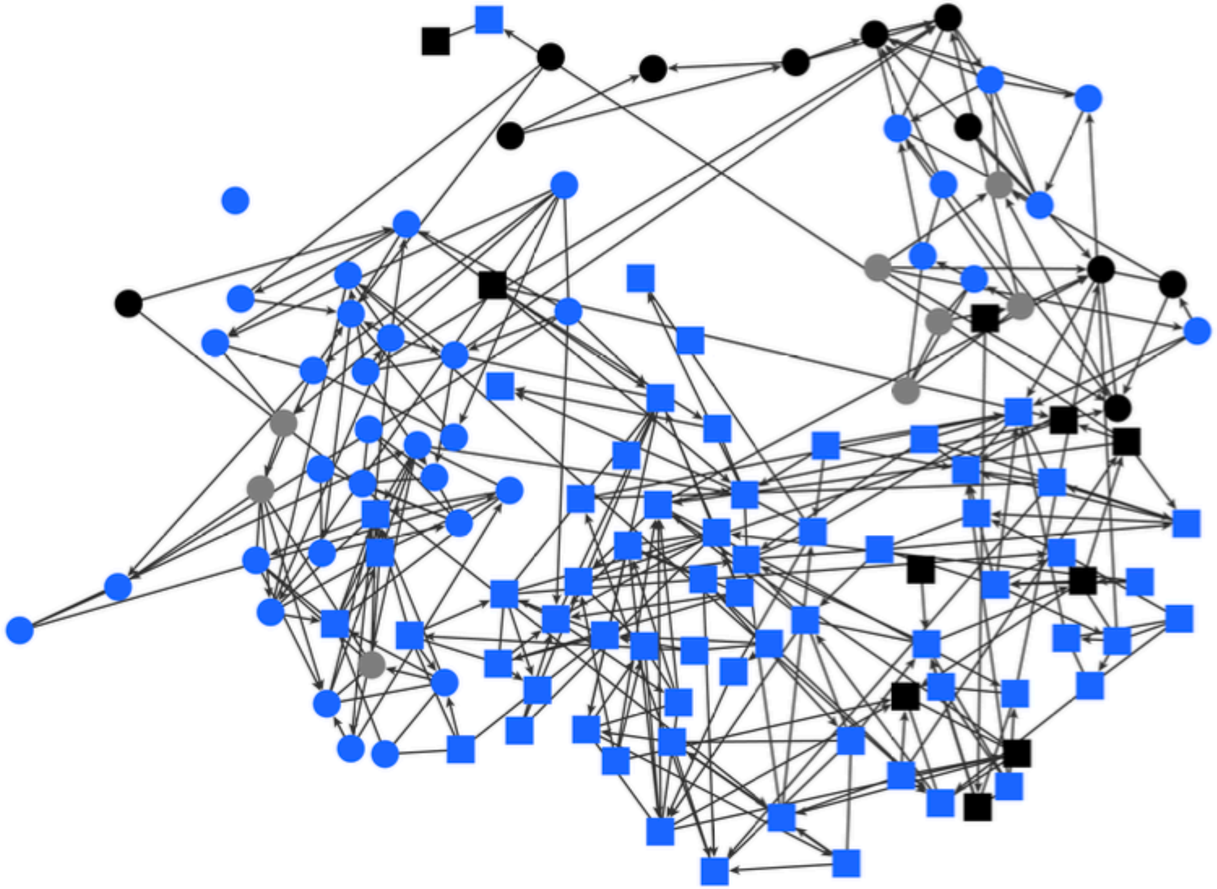
arrows = friendship relation
gender:
  circle = girl,
  square = boy
smoking behavior:
  blue = non,
  gray = occasional,
  black = regular



Example: Study of Smoking Initiation and Friendship

Wave 2

arrows = friendship relation
gender:
  circle = girl,
  square = boy
smoking behavior:
  blue = non,
  gray = occasional,
  black = regular



Example: Study of Smoking Initiation and Friendship

Wave 3

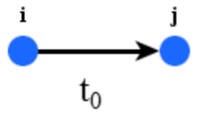
arrows = friendship relation
gender:
  circle = girl,
  square = boy
smoking behavior:
  blue = non,
  gray = occasional,
  black = regular

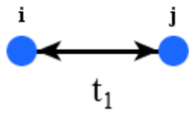
Example: Study of Smoking Initiation and Friendship

Some questions:

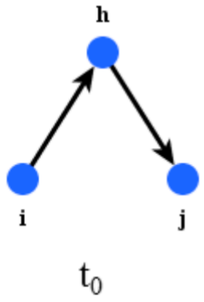
is there any tendency in friendship formation...

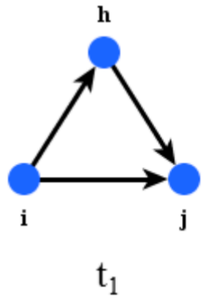
➡ towards reciprocity?

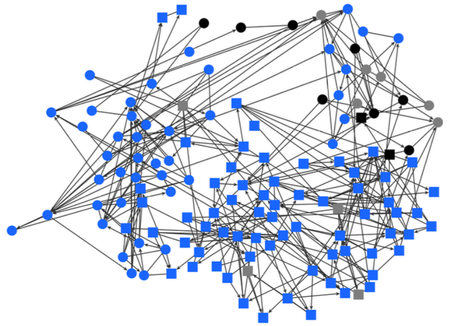


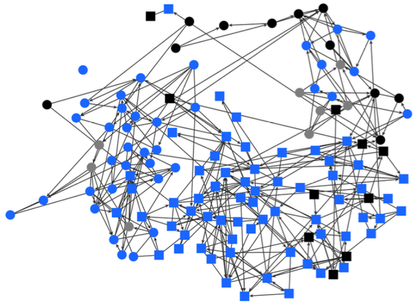


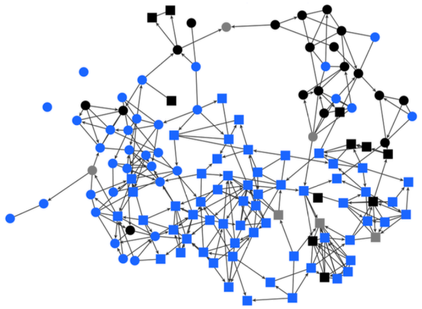
➡ towards transitivity?









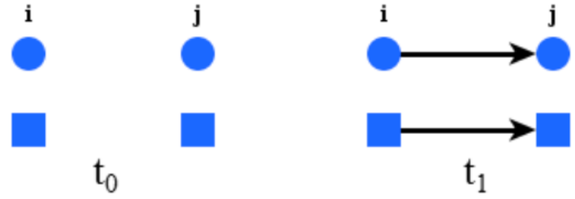


Example: Study of Smoking Initiation and Friendship

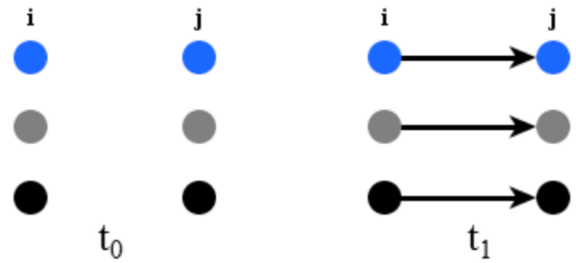
Some questions:

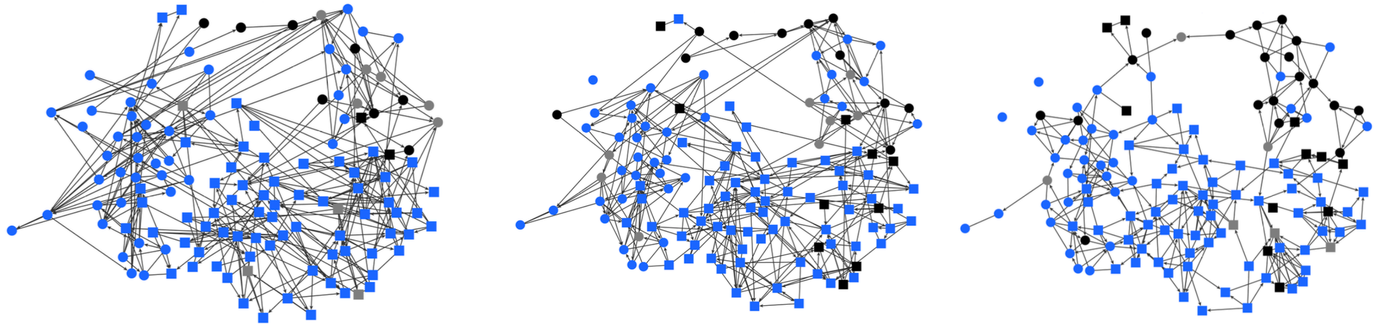
is there any homophily in friendship formation with respect to...

➡ gender?



➡ smoking behavior?





Example: Study of Smoking Initiation and Friendship

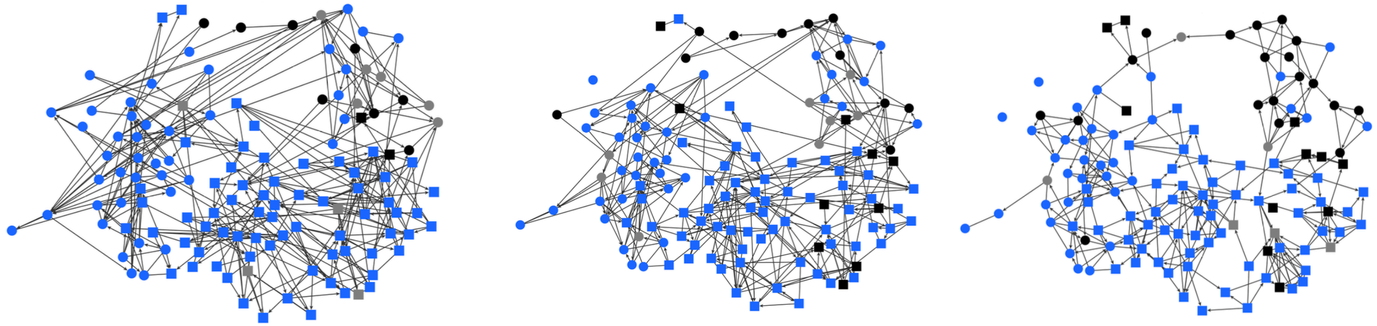
Networks: relation = friendship

actors = 129 pupils present at all three measurement points

Covariates: gender (1 = Male, 2 = Female)

smoking behavior (1 = no, 2= occasional, 3 = regular)

Let's start with a very simple model...



Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Rate parameters:			
Rate parameter period 1	8.5948	(0.7091)	
Rate parameter period 2	7.2115	(0.5751)	
Other parameters:			
outdegree (density)	-2.4147	(0.0387)	-62.3875
reciprocity	2.7106	(0.0811)	33.4061

Example: Study of Smoking Initiation and Friendship

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outdegree (density)	-2.4147	(0.0387)	-62.3875
reciprocity	2.7106	(0.0811)	33.4061

- **Rate parameter:** expected frequency, between two consecutive network observations, with which actors get the opportunity to change a network tie
  - about 9 opportunities for change in the first period
  - about 7 opportunities for change in the second period

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Rate parameters:			
Rate parameter period 1	8.5948	(0.7091)	
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- **outdegree parameter:** the observed networks have low density

Example: Study of Smoking Initiation and Friendship

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Other parameters:			
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reciprocity	2.7106	(0.0811)	33.4061

- **outdegree parameter:** the observed networks have low density

- **reciprocity parameter**: strong tendency towards reciprocated ties

  assume we have that there is a tie between node i and j, that is $x_{ij} = 1$
  adding a reciprocated tie $x_{ji} = 1$ gives −2.4147 + 2.7106 = 0.2959
  while adding a non-reciprocated tie$x_{ji} = 0$ gives −2.4147

  $\Longrightarrow$ reciprocated ties are valued positively and
  non-reciprocated ties are valued negatively by actors

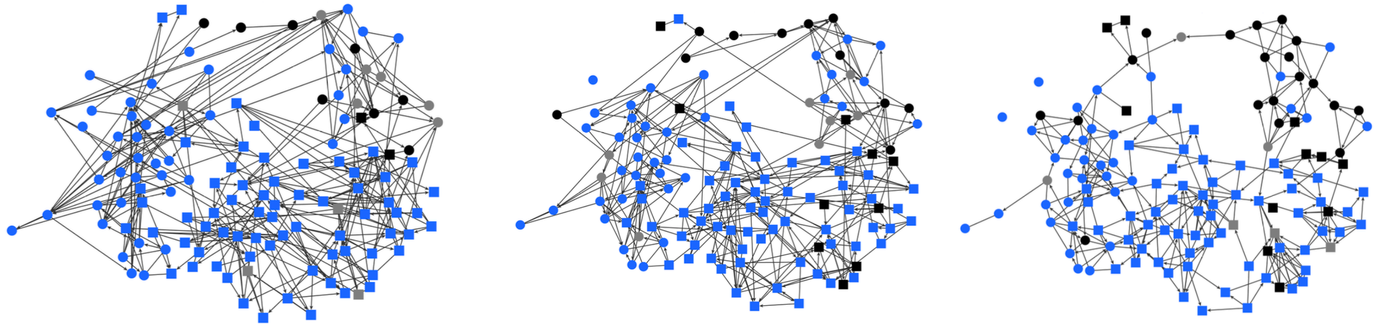
Example: Study of Smoking Initiation and Friendship

Let's continue with a more complex model...

- covariate similarity:

- covariate ego effect

- covariate alter effect:



Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Rate parameters:			
Rate parameter period 1	10.6809	(1.0425)	
Rate parameter period 2	9.0116	(0.8386)	
Other parameters:			
outdegree (density)	-2.8597	(0.0608)	-47.0288
reciprocity	1.9855	(0.0876)	22.6765
transitive triplets	0.4480	(0.0257)	17.4558
sex alter	-0.1513	(0.0980)	-1.5445
sex ego	0.1571	(0.1072)	1.4659
sex similarity	0.9191	(0.1076)	8.5440
smoke alter	0.1055	(0.0577)	1.8272
smoke ego	0.0714	(0.0623)	1.1469
smoke similarity	0.3724	(0.1177)	3.1647

- **outdegree parameter:** the observed networks have low density
- **reciprocity parameter:** strong tendency towards reciprocated ties
- **transitivity parameter:** preference for being friends with friends' friends

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
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smoke similarity	0.3724	(0.1177)	3.1647

- **sex alter:** gender does not affect actor popularity
- **sex ego:** gender does not affect actor activity
- **sex similarity:** tendency to choose friends with the same gender

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Rate parameters:			
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- **smoke alter:** smoking behavior does not affect actor popularity
- **smoke ego:** smoking behavior not affect actor activity
- **smoke similarity:** tendency to choose friends with the same smoking behavior

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Rate parameters:			
Rate parameter period 1	10.6809	(1.0425)	
Rate parameter period 2	9.0116	(0.8386)	

NOTE:

WE HAVE SO FAR ONLY CONSIDERED EFFECTS OF NETWORK DYNAMICS AND WE FOUND EVIDENCE FOR SOCIAL SELECTION PROCESSES BASED ON GENDER AND SMOKING BEHAVIOR. WE COULD HAVE USED TERGM/STERGM FOR THIS PART AS WELL.

WE NOW CONTINUE WITH MODEL SPECIFICATIONS INCLUDING BEHAVIORAL DYNAMICS AND THEIR EFFECT ON NETWORK STRUCTURE.

THIS ALLOWS US TO UNDERSTAND THE CO-EVOLUTION OF NETWORK AND BEHAVIOR.

- **smoke alter:** smoking behavior does not affect actor popularity
- **smoke ego:** smoking behavior not affect actor activity
- **smoke similarity:** tendency to choose friends with the same smoking behavior

Example: Study of Smoking Initiation and Friendship

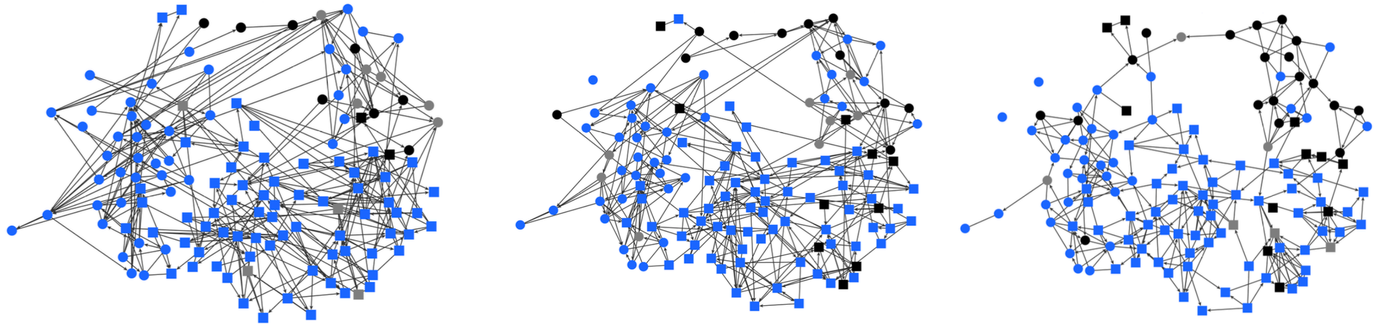
The co-evolution of networks and behavior

We will now use the SAOM for the co-evolution of networks and behaviors to distinguish influence from selection.

1. Do pupils select friends based on similar smoking behavior?
2. Are pupils influenced by friends to adjust to their smoking behavior?

Dependent variables: friendship networks and smoking behavior
Covariate: gender

Let's start with a very simple model...



Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	8.6287	(0.6666)	
constant friendship rate (period 2)	7.2489	(0.5466)	
outdegree (density)	-2.4084	(0.0407)	-59.1268
reciprocity	2.7024	(0.0823)	32.8337
Behavior Dynamics			
rate smokebeh (period 1)	3.8922	(1.9689)	
rate smokebeh (period 2)	4.4813	(2.3679)	
behavior smokebeh linear shap	-3.5464	(0.4394)	-8.0712
behavior smokebeh quadratic shape	2.8464	(0.3628)	7.8447

- **Network rate parameter:**
  - about 9 opportunities for change in the first period
  - about 7 opportunities for change in the second period

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	8.6287	(0.6666)	
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- **Network objective function parameters:**
  - **outdegree parameter:** the observed networks have low density
  - **reciprocity parameter:** strong tendency towards reciprocated ties

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	8.6287	(0.6666)	
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behavior smokebeh quadratic shape	2.8464	(0.3628)	7.8447

- **Behavioral rate parameters:**
  - about 4 opportunities for a behavioral change in the first period
  - about 4 opportunities for a behavioral change in the second period

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	8.6287	(0.6666)	
constant friendship rate (period 2)	7.2489	(0.5466)	
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behavior smokebeh quadratic shape	2.8464	(0.3628)	7.8447

- **Behavioral objective function parameters:**
  - determine the shape of the behavior function: what the smoking distribution looks like
  - attractiveness of different behavioral levels based on the current structure of the network and the behavior of the others

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	8.6287	(0.6666)	
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- **Behavioral objective function parameters:**
  - determine the shape of the behavior function: what the smoking distribution looks like

what does this mean?

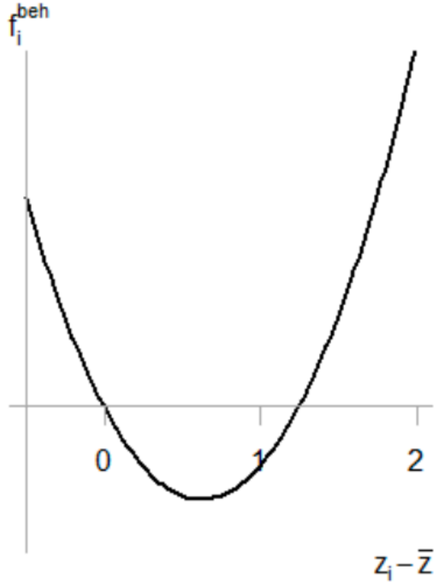
Example: Study of Smoking Initiation and Friendship

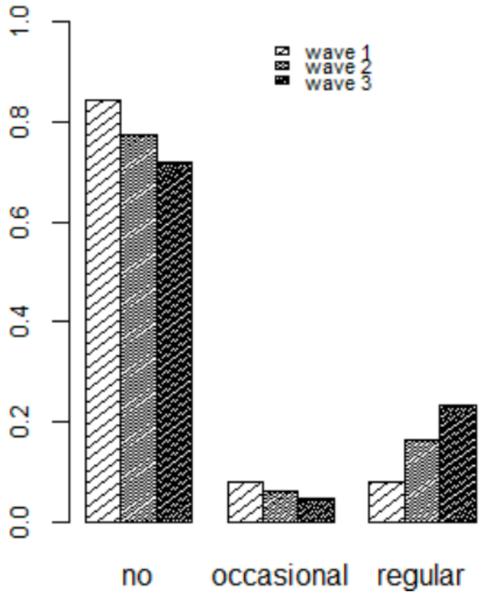
- **Smoking behavior** z: coded with 1 for "no", 2 for "occasional", and 3 for "regular" smokers
- The mean of this covariate in the data is equal to $\bar{z} = 1.377$
- Then

$$z_i - \bar{z} = \begin{cases} -0.377 & \text{for no smokers} \\ 0.623 & \text{for occasional smokers} \\ 1.623 & \text{for regular smokers} \end{cases}$$

- Thus the contribution to the behavioral objective function is

$$\gamma_{\text{linear}}(z_i - \bar{z}) + \gamma_{\text{quadratic}}(z_i - \bar{z})^2$$
$$= -3.5464(z_i - \bar{z}) + 2.8464(z_i - \bar{z})^2$$





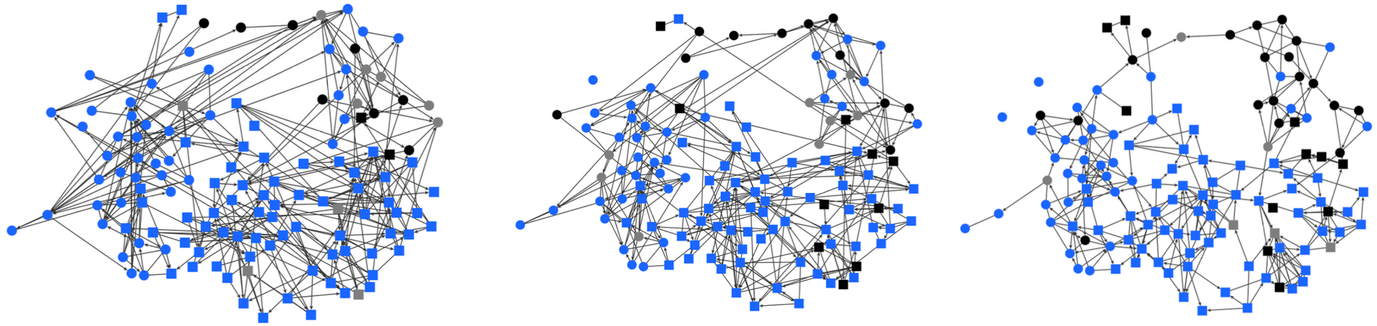
Example: Study of Smoking Initiation and Friendship

Let's continue with a more complex model...

- covariate similarity:

- covariate ego effect

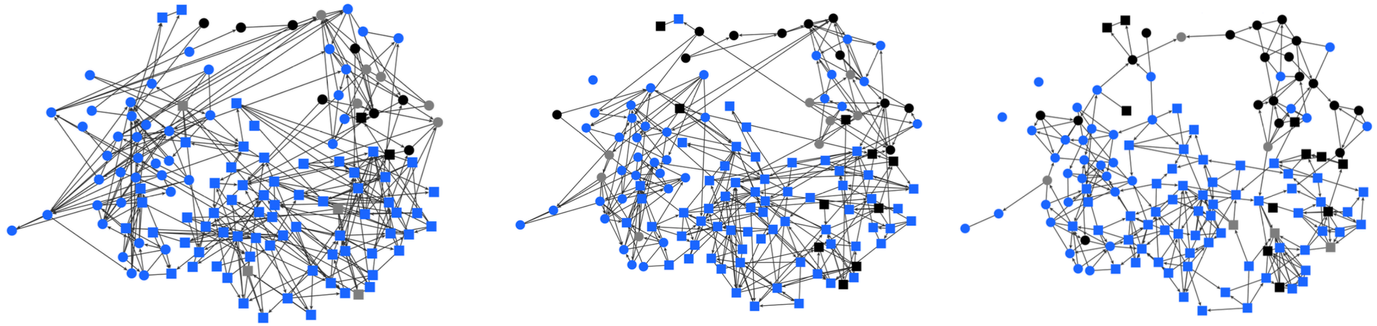
- covariate alter effect:



Example: Study of Smoking Initiation and Friendship

- This baseline model does not provide any information about selection and influence processes:
  - network dynamics are explained by preference towards creating and reciprocating ties
  - behavior dynamics are described only by the distribution of behavior in the population

- If we want to distinguish selection from influence we should include in the objective functions specification:
  - the effects capturing dependence of social network dynamics on actor's characteristic
  - the effects tcapturing the dependence of behavior dynamics on social network



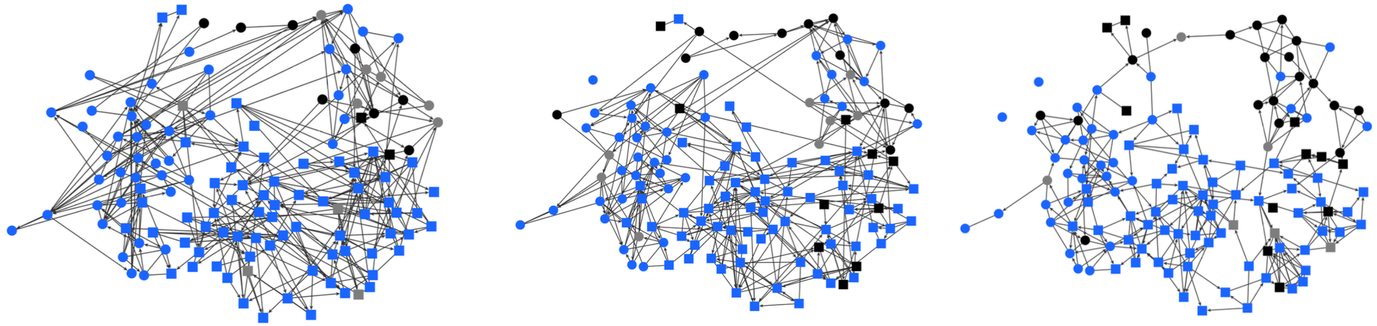
Example: Study of Smoking Initiation and Friendship

Effects for the dependence of network dynamics on actor's characteristic (as before)

- covariate similarity:

- covariate ego effect

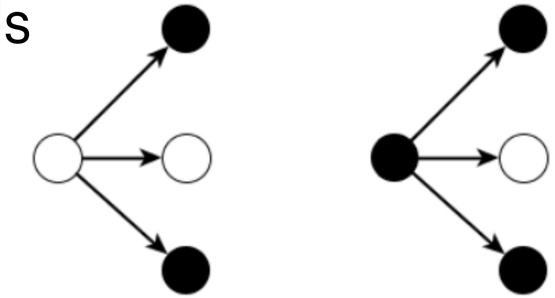
- covariate alter effect:



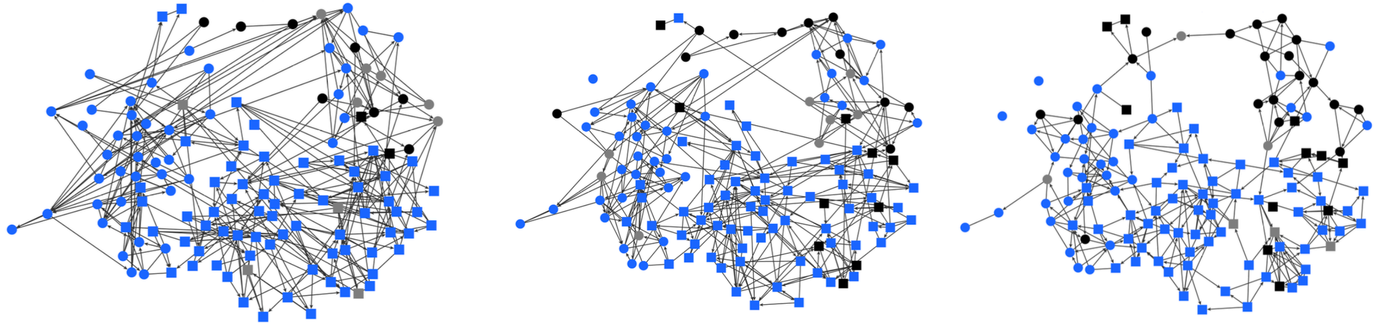
Example: Study of Smoking Initiation and Friendship

Effects for the dependence of behavior dynamics on network

- **average similarity effect**: pupils tend to adjust their smoking behavior according to the behaviors of their friends



- This effect must be controlled for the indegree and the outdegree effects
  - Indegree effect
  - Outdegree effect



Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	10.7166	(1.4036)	
constant friendship rate (period 2)	9.0005	(0.7709)	
outdegree (density)	-2.8435	(0.0572)	-49.6776
reciprocity	1.9683	(0.0933)	21.1077
transitive triplets	0.4447	(0.0322)	13.7964
sex ego	0.1612	(0.1206)	1.3368
sex alter	-0.1476	(0.1064)	-1.3871
sex similarity	0.9104	(0.0882)	10.3244
smoke ego	0.0665	(0.0846)	0.7857
smoke alter	0.1121	(0.0761)	1.4719
smokebeh similarity	0.5114	(0.1735)	2.9479

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	10.7166	(1.4036)	
constant friendship rate (period 2)	9.0005	(0.7709)	
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reciprocity	1.9683	(0.0933)	21.1077
transitive triplets	0.4447	(0.0322)	13.7964
sex ego	0.1612	(0.1206)	1.3368
sex alter	-0.1476	(0.1064)	-1.3871
sex similarity	0.9104	(0.0882)	10.3244
smoke ego	0.0665	(0.0846)	0.7857
smoke alter	0.1121	(0.0761)	1.4719
smokebeh similarity	0.5114	(0.1735)	2.9479

- **Network objective function parameters:**
  tendency towards reciprocity, transitivity and homophily with respect to gender

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Network Dynamics			
constant friendship rate (period 1)	10.7166	(1.4036)	
constant friendship rate (period 2)	9.0005	(0.7709)	
outdegree (density)	-2.8435	(0.0572)	-49.6776
reciprocity	1.9683	(0.0933)	21.1077
transitive triplets	0.4447	(0.0322)	13.7964
sex ego	0.1612	(0.1206)	1.3368
sex alter	-0.1476	(0.1064)	-1.3871
sex similarity	0.9104	(0.0882)	10.3244
smoke ego	0.0665	(0.0846)	0.7857
smoke alter	0.1121	(0.0761)	1.4719
smokebeh similarity	0.5114	(0.1735)	2.9479

- **Network objective function parameters:**
  pupils selected others with similar smoking behavior as friends
  $\Longrightarrow$ evidence for selection process

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Behavior Dynamics			
rate smokebeh (period 1)	3.9041	(1.7402)	
rate smokebeh (period 2)	3.8059	(1.4323)	
behavior smokebeh linear shape	-3.3573	(0.5678)	-5.9129
behavior smokebeh quadratic shape	2.8406	(0.4125)	6.8864
behavior smokebeh indegree	0.1711	(0.1812)	0.9444
behavior smokebeh outdegree	0.0128	(0.1926)	0.0662
behavior smokebeh average similarity	3.4361	(1.4170)	2.4250

- **Behavioral objective function parameters:**
  U-shaped distribution of the smoking behavior

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Behavior Dynamics			
rate smokebeh (period 1)	3.9041	(1.7402)	
rate smokebeh (period 2)	3.8059	(1.4323)	
behavior smokebeh linear shape	-3.3573	(0.5678)	-5.9129
behavior smokebeh quadratic shape	2.8406	(0.4125)	6.8864
behavior smokebeh indegree	0.1711	(0.1812)	0.9444
behavior smokebeh outdegree	0.0128	(0.1926)	0.0662
behavior smokebeh average similarity	3.4361	(1.4170)	2.4250

- **Behavioral objective function parameters:**
  indegree and outdegree effects are not significant

Example: Study of Smoking Initiation and Friendship

	Estimates	s.e.	t-score
Behavior Dynamics			
rate smokebeh (period 1)	3.9041	(1.7402)	
rate smokebeh (period 2)	3.8059	(1.4323)	
behavior smokebeh linear shape	-3.3573	(0.5678)	-5.9129
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behavior smokebeh average similarity	3.4361	(1.4170)	2.4250

- **Behavioral objective function parameters:**
  pupils are influenced by the smoking behavior of the others

  $\Longrightarrow$ evidence for influence process

  i.e. actors adopt smoking levels that bring them closer to the average of their friends

Example: Study of Smoking Initiation and Friendship

There is evidence for

- friendship selection based on gender and smoking behavior
- social influence based on smoking behavior