

Maximum Likelihood Estimation



$$p(x) = \sum_{k=1}^K w_k \mathcal{N}(x | \mu_k, \Sigma_k)$$

$$p(\mathbf{X} | \mathbf{w}, \mu, \Sigma) = p(x_1, x_2, \dots, x_n | \mathbf{w}, \mu, \Sigma) = \prod_{n=1}^N \sum_{k=1}^K w_k \mathcal{N}(x_n | \mu_k, \Sigma_k)$$

$$\log p(\mathbf{X} | \mathbf{w}, \mu, \Sigma) = \sum_{n=1}^N \log \left[\sum_{k=1}^K w_k \mathcal{N}(x_n | \mu_k, \Sigma_k) \right]$$

Goal: choose w, μ, Σ that maximizes the log likelihood

GMM: EM Algorithm

1. Choose **k** random points to be cluster centers (or estimate using k-means...)
2. For each data point, calculate the **probability** of belonging to each cluster
3. Using these probability weights, recalculate the **means + variances** (and weights)
4. Repeat 2 and 3 until **distributions converge**