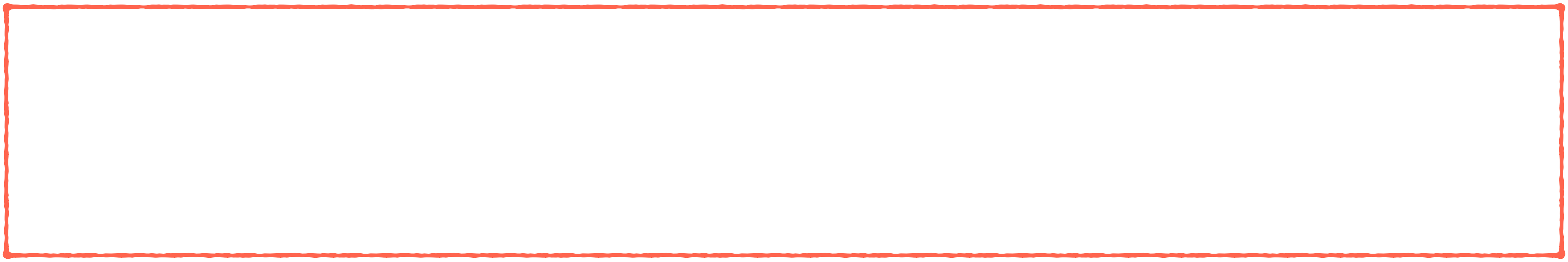




matrix multiplication

inner dimensions

the outer dimensions





# matrix arithmetic: matrix multiplication

Let  $A$  be an  $m \times r$  matrix, and let  $B$  be an  $r \times n$  matrix.

The matrix product of  $A$  and  $B$ , denoted  $A \cdot B$  or  $AB$ , is the  $m \times n$  matrix  $M$  whose entry in the  $i^{th}$  row and  $j^{th}$  column is the product of the  $i^{th}$  row of  $A$  and the  $j^{th}$  column of  $B$ .

- In order to multiply two matrices  $A$  and  $B$ , the number of columns of  $A$  must be the same as the number of rows of  $B$  (the **inner dimensions** must be the same)
- The resulting matrix has same number of rows as  $A$  and same number of columns as  $B$  (i.e. **the outer dimensions**)

final dimensions are outer dimensions

$$(m \times r) \times (r \times n)$$

inner dimensions must match

# matrix arithmetic: matrix multiplication

Let matrix  $A$  have rows  $\vec{a}_1, \vec{a}_2, \dots, \vec{a}_m \implies A = \begin{bmatrix} - & \vec{a}_1 & - \\ - & \vec{a}_2 & - \\ & \vdots & \\ - & \vec{a}_m & - \end{bmatrix}$

and let matrix  $B$  have columns  $\vec{b}_1, \vec{b}_2, \dots, \vec{b}_n \implies B = \begin{bmatrix} | & | & & | \\ \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_n \\ | & | & & | \end{bmatrix}$

Then  $AB = \begin{bmatrix} \vec{a}_1 \vec{b}_1 & \vec{a}_1 \vec{b}_2 & \cdots & \vec{a}_1 \vec{b}_n \\ \vec{a}_2 \vec{b}_1 & \vec{a}_2 \vec{b}_2 & \cdots & \vec{a}_2 \vec{b}_n \\ \vdots & \vdots & \ddots & \vdots \\ \vec{a}_m \vec{b}_1 & \vec{a}_m \vec{b}_2 & \cdots & \vec{a}_m \vec{b}_n \end{bmatrix}$