

indirect proofs: proof by contradiction



Proof.



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Theorem

For any $n \in \mathbb{Z}$, if n^2 is even, then n is even.



Proof.

- Assume for the sake of contradiction that n is an integer and that n^2 is even, but that n is odd.
- Since n is odd, there is some integer k such that $n = 2k + 1$.
- Squaring both sides of this equality and simplifying yields the following:

$$\begin{aligned}n^2 &= (2k + 1)^2 \\&= 4k^2 + 4k + 1 \\&= 2(2k^2 + 2k) + 1\end{aligned}$$

- This tells us that n^2 is odd, which is impossible, by assumption n^2 is even.
- We have a contradiction so our assumption is incorrect
 \implies if n is an integer and n^2 is even then n is also even. ■

