Regression Splines

The basis of regression splines is **piecewise polynomial regression**

- Standard polynomial regression

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \cdots\cdots + \beta_n X^n + \epsilon$$

- Piecewise polynomial regression:

$$Y = \begin{cases} \beta_{01} + \beta_{11}X + \beta_{21}X^2 + \beta_{31}X^3 + \cdots + \beta_{d1}X^d + \epsilon & \text{if } X < c \\ \beta_{02} + \beta_{12}X + \beta_{22}X^2 + \beta_{32}X^3 + \cdots + \beta_{d2}X^d + \epsilon & \text{if } X \geq c \end{cases}$$

- The c is called a **knot**
- When there is no knot we have standard polynomial regression.
- When we include only the intercepts terms, we have step function regression.
- If we have K knots we are fitting $K + 1$ polynomial models

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Example: Wage (ISLR2)

Piecewise cubic polynomial with a single knot placed a age = 50:

$$\text{wage} = \begin{cases} f_1(\text{age}) = \beta_{01} + \beta_{11}X + \beta_{21}X^2 + \beta_{31}X^3 & \text{if age} < 50 \\ f_2(\text{age}) = \ \beta_{02} + \beta_{12}X + \beta_{22}X^2 + \beta_{32}X^3 & \text{if age} \geq 50 \end{cases}$$

Piecewise Cubic

Continuous Piecewise Cubic

Linear Spline