

# linear and non-linear transformation

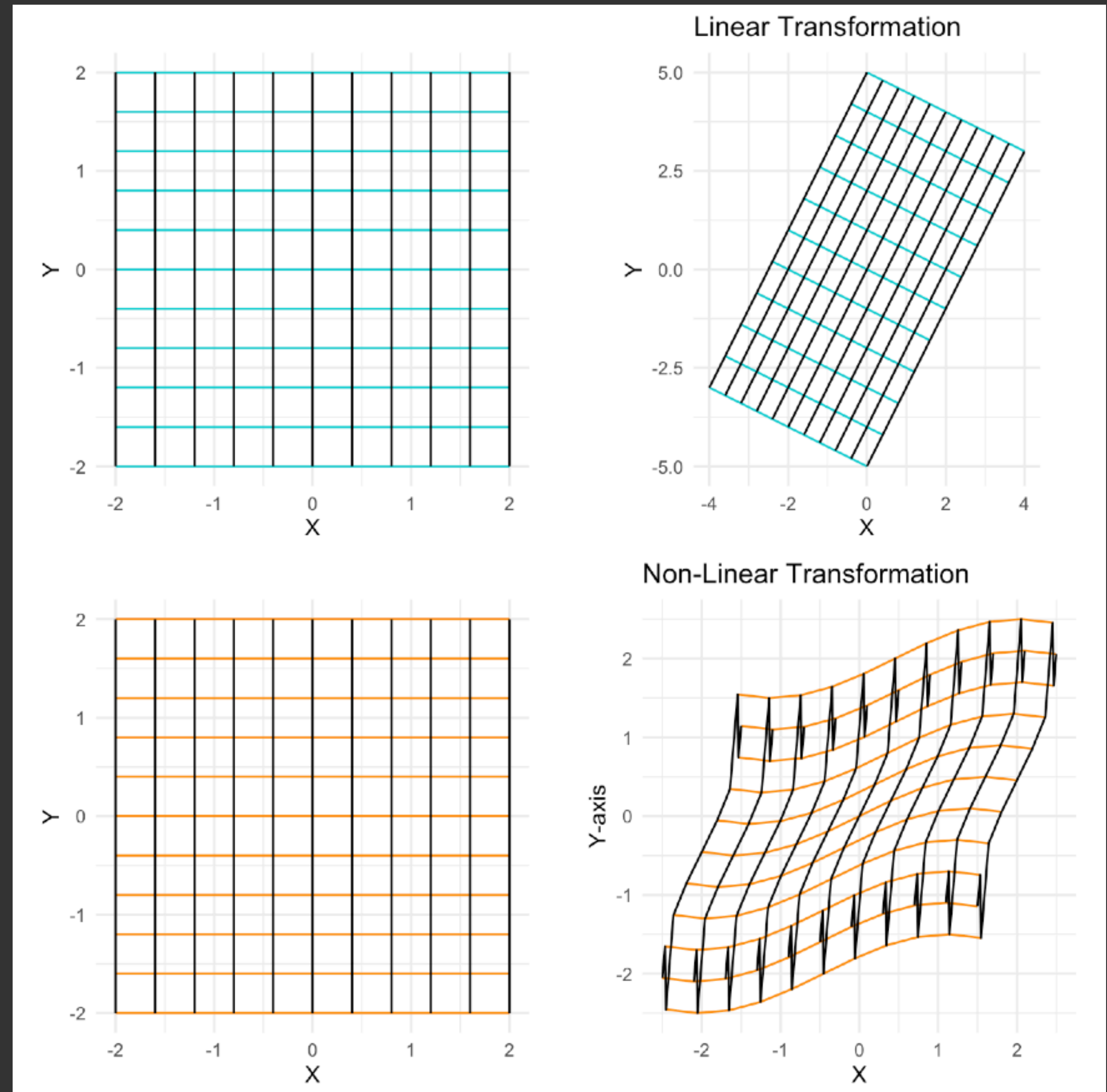
Linear:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A \cdot \begin{bmatrix} x \\ y \end{bmatrix} \text{ where } A = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$$

Non-Linear:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} f(x, y) \\ g(x, y) \end{bmatrix} = \begin{bmatrix} x + 0.5 \sin(y) \\ y + 0.5 \sin(x) \end{bmatrix}$$

*but locally linear when we zoom in!*



# the Jacobian

## example

Let  $f$  be a transformation from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  with the following Jacobian matrix:

$$J = \begin{bmatrix} 3x^2 - 4 & 0 \\ 0 & 3y^2 - 4 \end{bmatrix}$$

What is the determinant of  $f$ ? How will  $f$  stretch or squish the space around the point  $(1, -1)$ ?