matrix arithmetic: matrix multiplication

inner dimensions

the outer dimensions

matrix arithmetic: matrix multiplication

Let A be an $m \times r$ matrix, and let B be an $r \times n$ matrix.

The matrix product of A and B, denoted $A \cdot B$ or AB, is the $m \times n$ matrix M whose entry in the ith row and jth column is the product of the ith row of A and the jth column of B.

- In order to multiply two matrices A and B, the number of columns of A must be the same as the number of rows of B (the inner dimensions must be the same)
- The resulting matrix has same number of rows as A and same number of columns as B (i.e. the outer dimensions)

$$\overbrace{(m \times \underbrace{r) \times (r}_{\text{inner dimensions must match}} \times n)}^{\text{final dimensions are outer dimensions}}$$

matrix arithmetic: matrix multiplication

Let matrix A have rows $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_m \implies A = \begin{bmatrix} - & \vec{a}_1 & - \\ - & \vec{a}_2 & - \\ & \vdots & \\ - & \vec{a}_m & - \end{bmatrix}$

and let matrix B have columns $\vec{b}_1, \vec{b}_2, \ldots, \vec{b}_m \implies B = \begin{bmatrix} | & | & & | \\ \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_n \\ | & | & & | \end{bmatrix}$

Then $AB = \begin{bmatrix} \vec{a}_1\vec{b}_1 & \vec{a}_1\vec{b}_2 & \cdots & \vec{a}_1\vec{b}_n \\ \vec{a}_2\vec{b}_1 & \vec{a}_2\vec{b}_2 & \cdots & \vec{a}_2\vec{b}_n \\ \vdots & \vdots & \ddots & \vdots \\ \vec{a}_m\vec{b}_1 & \vec{a}_m\vec{b}_2 & \cdots & \vec{a}_m\vec{b}_n \end{bmatrix}$