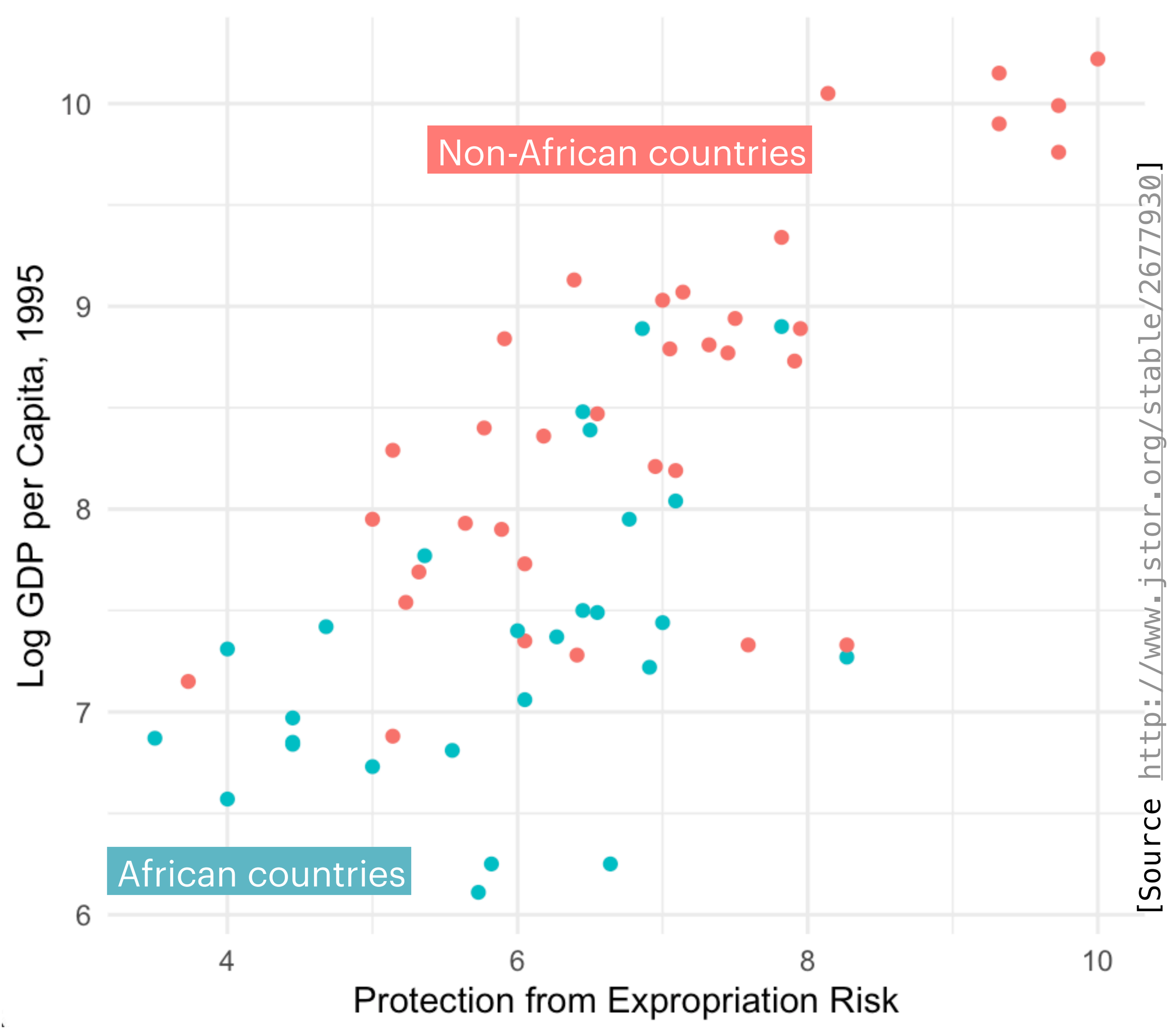
Interactions

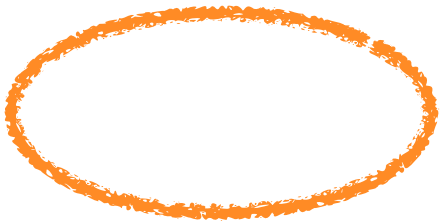

Non-African countries
African countries
Log GDP per Capita, 1995
Protection from Expropriation Risk
10
9
8
7
6
4
6
8
10
[Source http://www.jstor.org/stable/2677930]

Is the relationship between **X** and **Y** different when you consider values of **Z**?

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

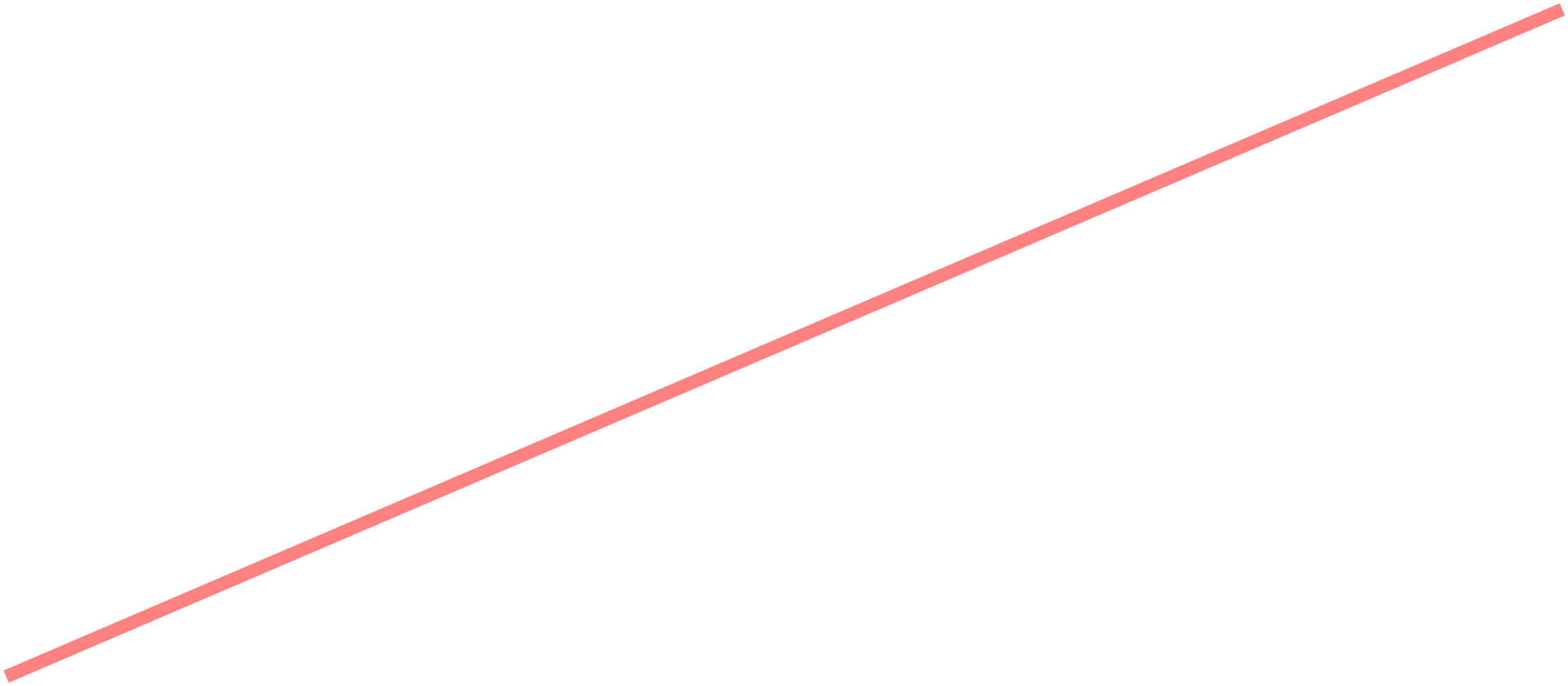
$$
\begin{aligned}
Z = 0 \implies Y &= \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 ZX \\
&= \beta_0 + \beta_1 X + \beta_2 \cdot 0 + \beta_3 \cdot 0 \\
&= \beta_0 + \beta_1 X
\end{aligned}
$$

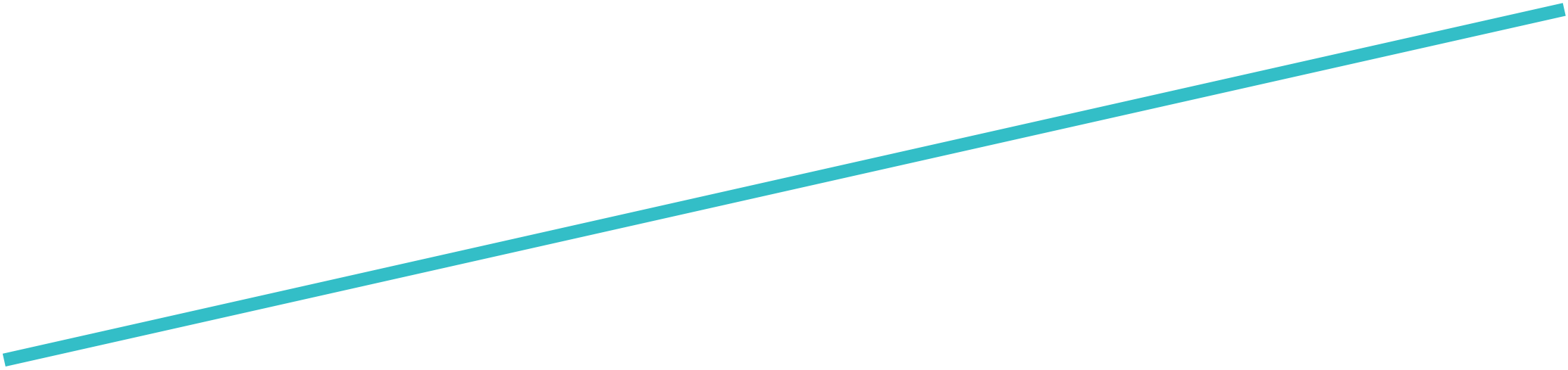
$$
\begin{aligned}
Z = 1 \implies Y &= \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 ZX \\
&= \beta_0 + \beta_1 X + \beta_2 \cdot 1 + \beta_3 \cdot 1 \cdot X \\
&= (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X
\end{aligned}
$$



different intercepts

different slope





Interactions

Is the relationship between **X** and **Y** different when you consider values of **Z**?

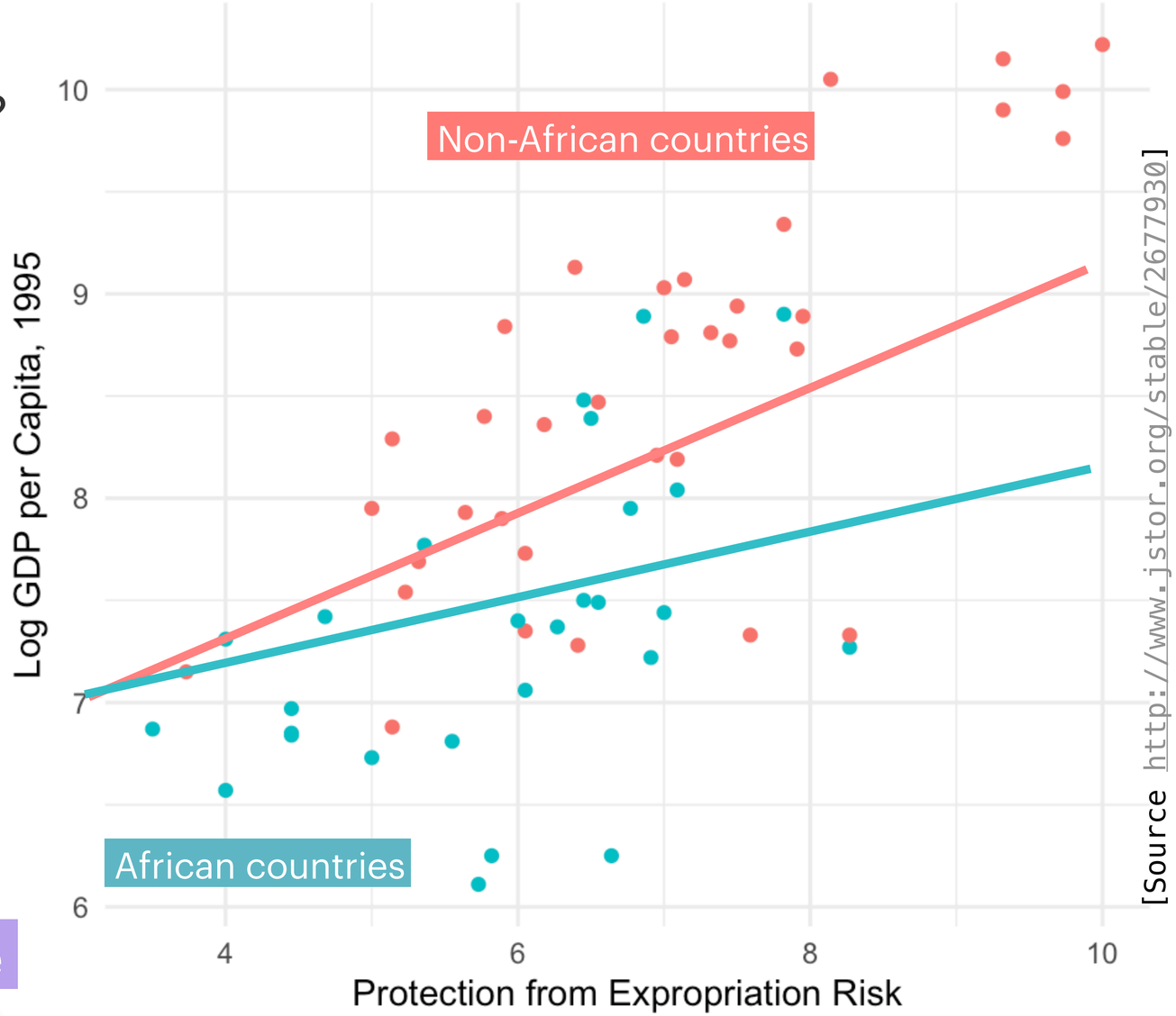
$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 XZ$$

$$\begin{aligned} Z = 0 \implies Y &= \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 ZX \\ &= \beta_0 + \beta_1 X + \beta_2 \cdot 0 + \beta_3 \cdot 0 \\ &= \beta_0 + \beta_1 X \end{aligned}$$

$$\begin{aligned} Z = 1 \implies Y &= \beta_0 + \beta_1 X + \beta_2 Z + \beta_3 ZX \\ &= \beta_0 + \beta_1 X + \beta_2 \cdot 1 + \beta_3 \cdot 1 \cdot X \\ &= (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X \end{aligned}$$

different intercepts

different slope



Interactions

	Income	Limit	Rating	Cards	Age	Education	Own	Student	Married	Region	Balance
	14.891	3606	283	2	34	11	No	No	Yes	South	333
	106.025	6645	483	3	82	15	Yes	Yes	Yes	West	903
	104.593	7075	514	4	71	11	No	No	No	West	580
	148.924	9504	681	3	36	11	Yes	No	No	West	964
	55.882	4897	357	2	68	16	No	No	Yes	South	331
	80.180	8047	569	4	77	10	No	No	No	South	1151

[Source: First six rows of dataset “Credit”, ISLR2]

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 (X_1 \times X_2)$$