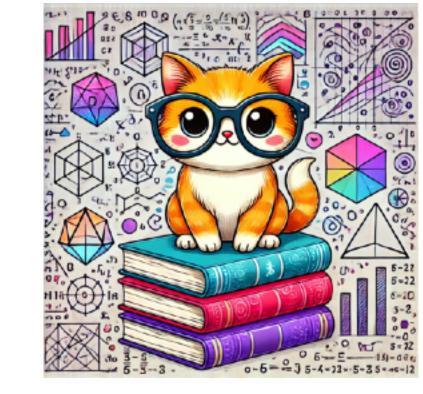
## Maximum Likelihood Estimation



$$p(x) = \sum_{k=1}^{K} w_k \mathcal{N}(x | \mu_k, \Sigma_k)$$

$$p(\mathbf{X} \mid \mathbf{w}, \mu, \mathbf{\Sigma}) = p(x_1, x_2, \dots, x_n \mid \mathbf{w}, \mu, \mathbf{\Sigma}) = \prod_{n=1}^{N} \sum_{k=1}^{N} w_k \mathcal{N}(x_n \mid \mu_k, \mathbf{\Sigma}_k)$$

$$\log p(\mathbf{X} \mid \mathbf{w}, \mu, \mathbf{\Sigma}) = \sum_{n=1}^{N} \log \left[ \sum_{k=1}^{K} w_k \mathcal{N}(x_n \mid \mu_k, \mathbf{\Sigma}_k) \right]$$

Goal: choose  $w, \mu, \Sigma$  that maximizes the log likelihood

## GMM: EM Algorithm

- 1. Choose **k** random points to be cluster centers (or estimate using k-means...)
- 2. For each data point, calculate the probability of belonging to each cluster
- 3. Using these probability weights, recalculate the means + variances (and weights)
- 4. Repeat 2 and 3 until distributions converge