

linear independence

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linear independence

- Two equivalent (but informal) definitions of linear dependency for a set of vectors:
 1. One of more of the vectors can be expressed as a linear combination of the remaining vectors
 2. One or more of the vectors is inside the span of the remaining vectors
- Note: Linear dependence is a property of the set! (i.e. an entire set of vectors can be linearly dependent simply because two of its vectors are dependent on each other)

A set K of n vectors is called linearly independent iff:

$$\sum_{i=1}^n a_i \vec{v}_i = 0 \quad \text{iff} \quad a_1 = a_2 = \cdots = a_n = 0$$

- We call a set of linearly independent vectors that span a vector space a **basis** of that vector space
- The **dimension** of a vector space is equal to the number of vectors in its basis

linear independence

example

Let $\vec{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -2 \\ 1 \\ -4 \end{bmatrix}$. Are they linearly dependent?

Let's express each vector as the linear combination of the other two $\vec{v}_3 = a\vec{v}_2 + b\vec{v}_1$

$$\Rightarrow \begin{bmatrix} -2 \\ 1 \\ -4 \end{bmatrix} = a \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}. \text{ This gives us three equations, one for each entry:}$$

$$-2 = 0a + 1b \rightarrow b = -2$$

$$1 = 1a + 0b \rightarrow a = 1$$

$$-4 = 1a + 2b \rightarrow -3 = 1 - 4$$

Is this enough to say that they are linearly independent? No, because we have to show that any vector in this set can be expressed as a linear combination of the rest of the vectors.