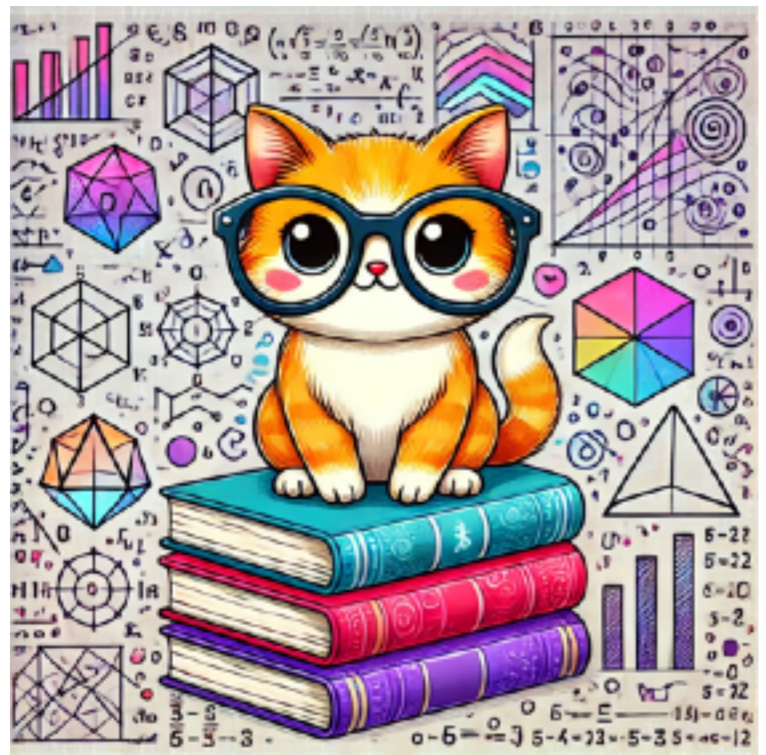
Distortion



metric that assesses the performance of K-means (smaller values better)

actual data point n

$$J = \sum_{n=1}^{N} \sum_{k=1}^{K} r_{nk} \|x_n - \mu_k\|^2$$

center of cluster k

Goal: choose r_{nk} and μ_k that minimizes J

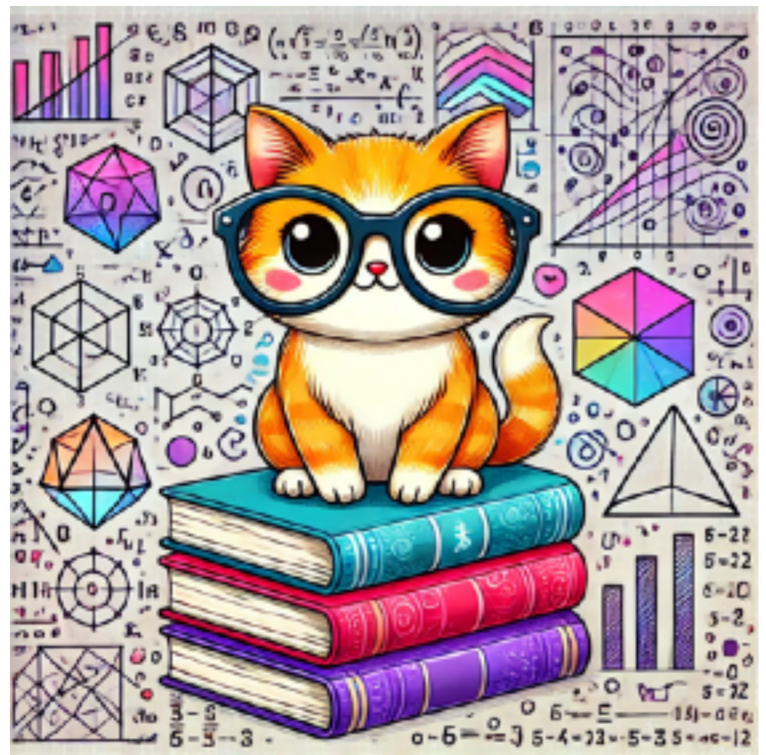
hard assignments!

$$r_{nk} = \begin{cases} 1 & \text{if } k = \arg\min_j \|x_n - \mu_j\|^2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{dJ}{d\mu_k} = 2 \sum_{n=1}^{N} r_{nk}(x_n - \mu_k) = 0 \implies \mu_k = \frac{\sum_n r_{nk} x_n}{\sum_n r_{nk}} = \frac{1}{N_k} \sum_n r_{nk} x_n$$

optimal value for μ_k minimizing our loss is the mean of all data points in that cluster

Distortion



metric that assesses the performance of K-means (smaller values better)

actual data point n

$$J = \sum_{n=1}^{N} \sum^{K} \cdots$$

minimizes J

hard assignmen

1. choose **k** random points as cluster centers
2. for each data point, assign it the cluster whose centroid is the closest
3. using these assignments, recalculate the centers
4. reiterate from step (2) until **convergence**:
   - cluster membership does not change
   - center only changes very very little

$$\frac{dJ}{d\mu_k} = 2 \sum_{n=1} r_{nk}(x_n \;\; \mu_k) = 0 \;\; \mu_k = \frac{\cdots}{\sum_n r_{nk}} = \frac{1}{N_k} \sum_n r_{nk} x_n$$

optimal value for μ_k minimizing our loss is the mean of all data points in that cluster