

matrix arithmetic: matrix multiplication

the outer dimensions















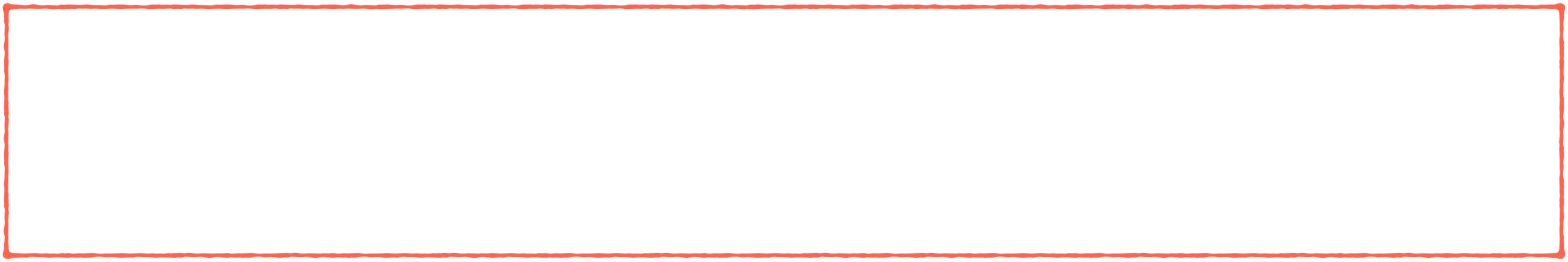


inner dimensions









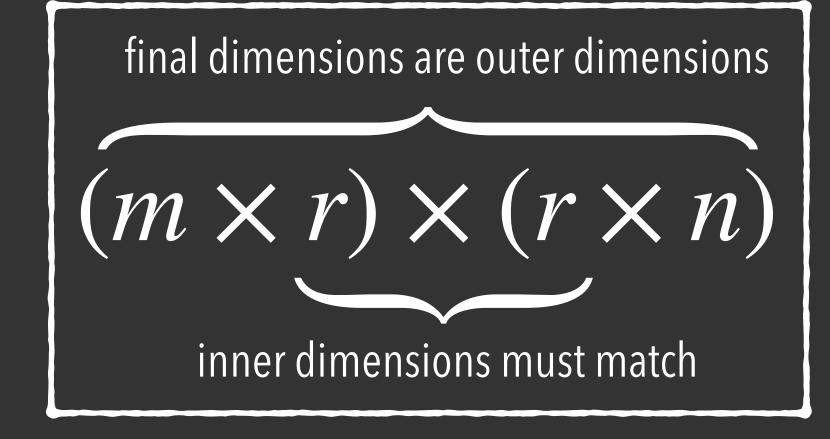
matrix arithmetic: matrix multiplication

Let A be an $m \times r$ matrix, and let B be an $r \times n$ matrix.

The matrix product of A and B, denoted $A \cdot B$ or AB, is the $m \times n$ matrix M whose entry in the ith row and jth column is the product of the ith row of A and the jth column of B.

- In order to multiply two matrices A and B, the number of columns of A must be the same as the number of rows of B (the inner dimensions must be the same)
- ullet The resulting matrix has same number of rows as A and same number of columns as B

(i.e. the outer dimensions)



matrix arithmetic: matrix multiplication

Let matrix
$$A$$
 have rows $\vec{a}_1, \vec{a}_2, ..., \vec{a}_m \implies A = \begin{bmatrix} - & \vec{a}_1 & - \\ - & \vec{a}_2 & - \\ & \vdots & \\ - & \vec{a}_m & - \end{bmatrix}$

and let matrix
$$B$$
 have columns $\vec{b}_1, \vec{b}_2, ..., \vec{b}_m \implies B = \begin{bmatrix} | & | & | & | \\ \vec{b}_1 & \vec{b}_2 & ... & \vec{b}_n \\ | & | & | & | \end{bmatrix}$

Then
$$AB = \begin{bmatrix} \vec{a}_1 \vec{b}_1 & \vec{a}_1 \vec{b}_2 & \cdots & \vec{a}_1 \vec{b}_n \\ \vec{a}_2 \vec{b}_1 & \vec{a}_2 \vec{b}_2 & \cdots & \vec{a}_2 \vec{b}_n \\ \vdots & \vdots & \ddots & \vdots \\ \vec{a}_m \vec{b}_1 & \vec{a}_m \vec{b}_2 & \cdots & \vec{a}_m \vec{b}_n \end{bmatrix}$$