dot product

The dot product is key for calculating vector projections, vector decompositions, and determining orthogonality

The dot product of two vectors $\vec{a}$ and $\vec{b}$ is

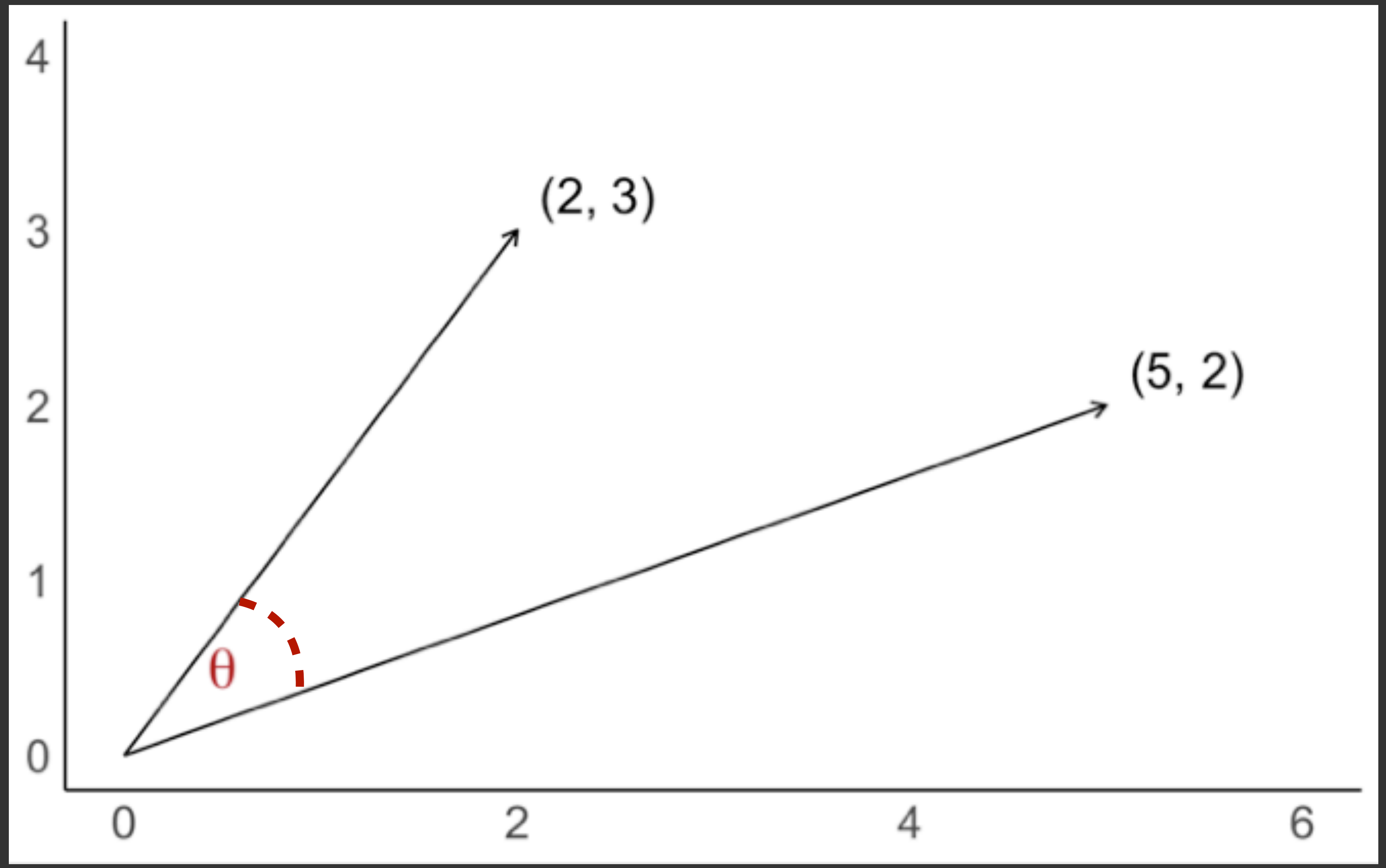
$$\vec{a} \cdot \vec{b} = \sum_{i=1}^{n} a_i b_i + a_2 b_2 + \cdots + a_n b_n$$

The angle θ of between two vectors is determined by the formula

$$\vec{a} \cdot \vec{b} = ||\vec{a}||\,||\vec{b}|| \cos\theta$$

where $||\vec{a}||$ is the length or norm or magnitude of a vector. Thus

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}||\,||\vec{b}||} = \frac{\vec{a}}{||\vec{a}||} \cdot \frac{\vec{b}}{||\vec{b}||}$$



orthogonality

- $\cos 0° = 1$ the vectors point in exactly the same direction (they coincide)
- $\cos 90° = 0$ means the vectors are perpendicular (aka orthogonal) to each other in 2D or 3D

Two vectors are orthogonal to one another if the dot product of those two vectors is equal to zero

- Orthogonal vectors point in completely independent directions, meaning one vector cannot be expressed as a scalar multiple of the other