the recipe

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Let A be an $n \times n$ matrix. To diagonalize A:

1. Find the eigenvalues of A using the characteristic polynomial.
2. For each eigenvalue λ of A, compute the basis B_λ for the $\lambda-$eigenspace.
3. If there are fewer than n total vectors in all of the eigenspace bases B_λ, then the matrix is not diagonalizable.
4. Otherwise, the n vectors $\overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_n}$ in the eigenspace bases are linearly independent, and

$$A = QDQ^{-1} \text{ for } Q = \begin{bmatrix} | & | & | & | \\ \overrightarrow{v_1} & \overrightarrow{v_2} & \ldots & \overrightarrow{v_1} \\ | & | & | & | \end{bmatrix}, \quad D = \begin{bmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_n \end{bmatrix}$$

where λ_i is the eigenvalue for $\overrightarrow{v_i}$.

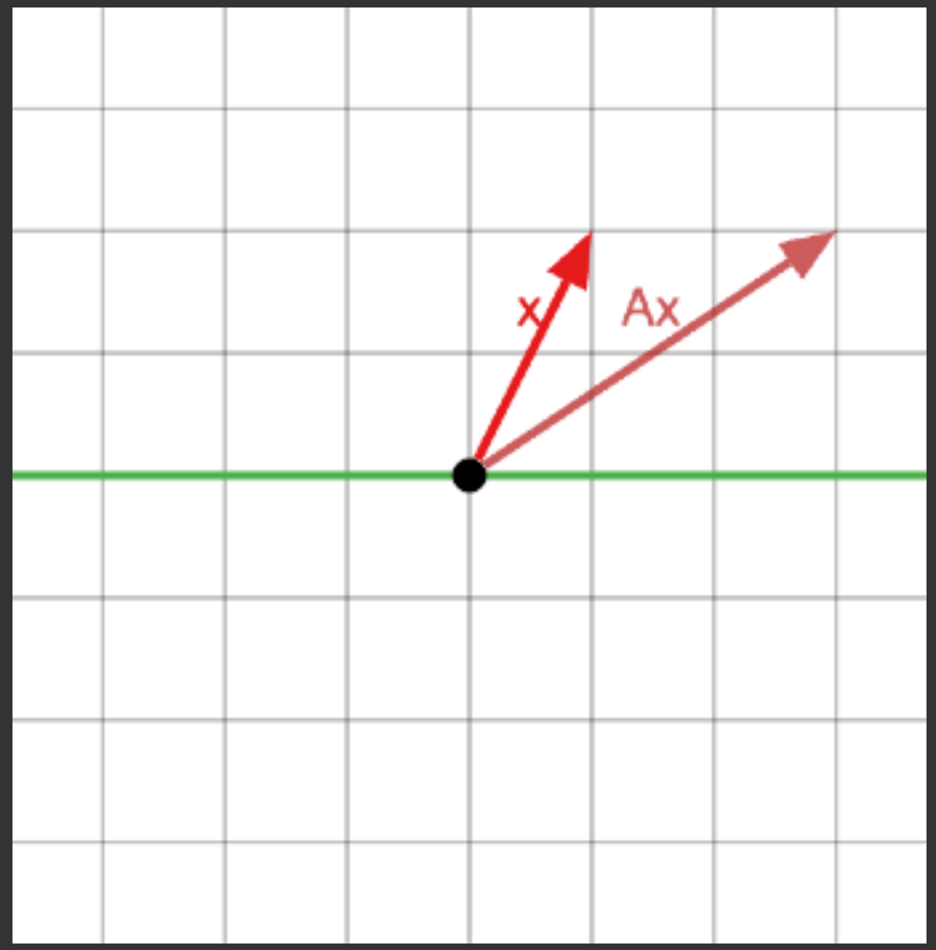
diagonalization

example

Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. The characteristic polynomial of A is $f(\lambda) = (\lambda - 1)^2$ so the eigenvalue of A is 1.

For $\lambda = 1$, solve $A\vec{v} = \lambda\vec{v}$ or $(A - \lambda I)\vec{v} = 0$:

$$A - \lambda I = \begin{bmatrix} 1-1 & 1 \\ 0 & 1-1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$ which gives equation $$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



From the first row: $0x + 1y = 0 \implies y = 0$
and there is no restriction on x so let $x = t$ (a free variable).
The eigenvector is a 1-eigenspace:

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \end{bmatrix} \implies \text{Basis: } \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

The 1-eigenspace is exactly the x-axis, so all of the eigenvectors of A lie on the x-axis. It follows that A does not admit two linearly independent eigenvectors, so by the diagonalization theorem, it is not diagonalizable.