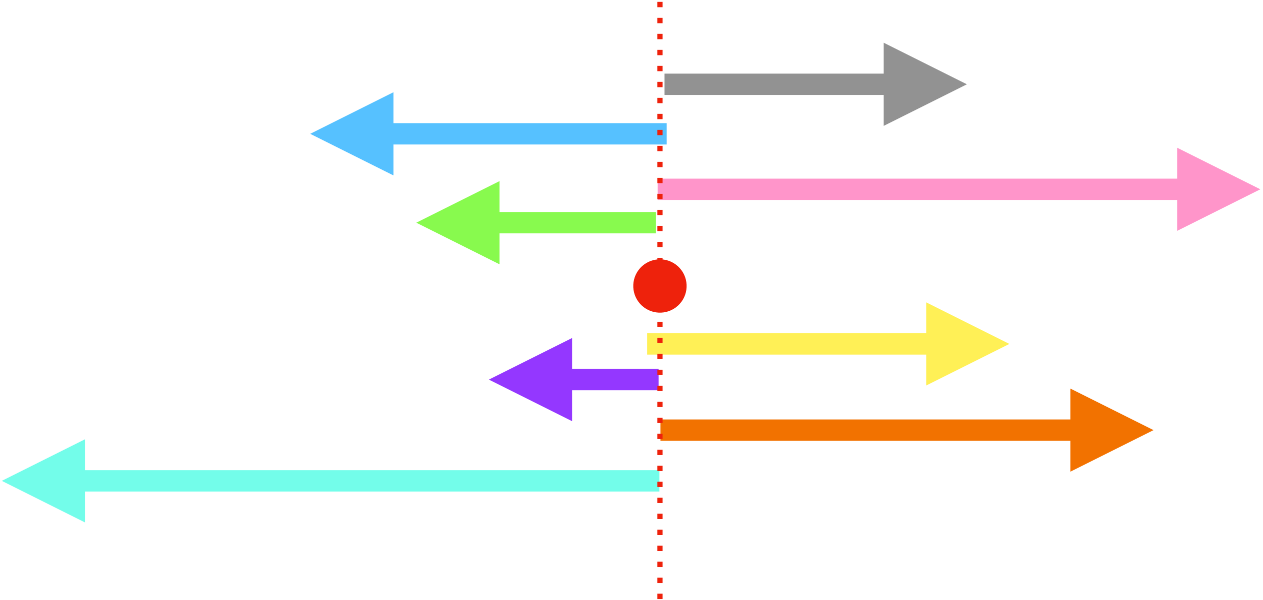
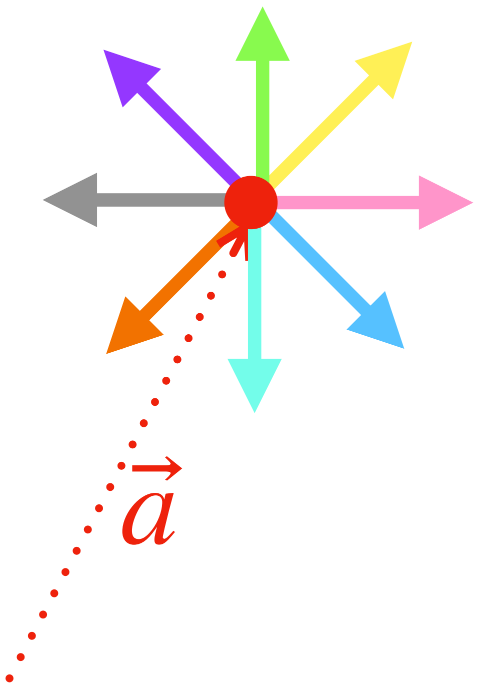
gradient



directional derivatives


$\vec{a}$

$$\boxed{\max_{\|\vec{v}=1\|} \nabla f(a,b)\cdot\vec{v}}$$

gradient

$$f(x,y) = x^2 + y^2 \implies \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

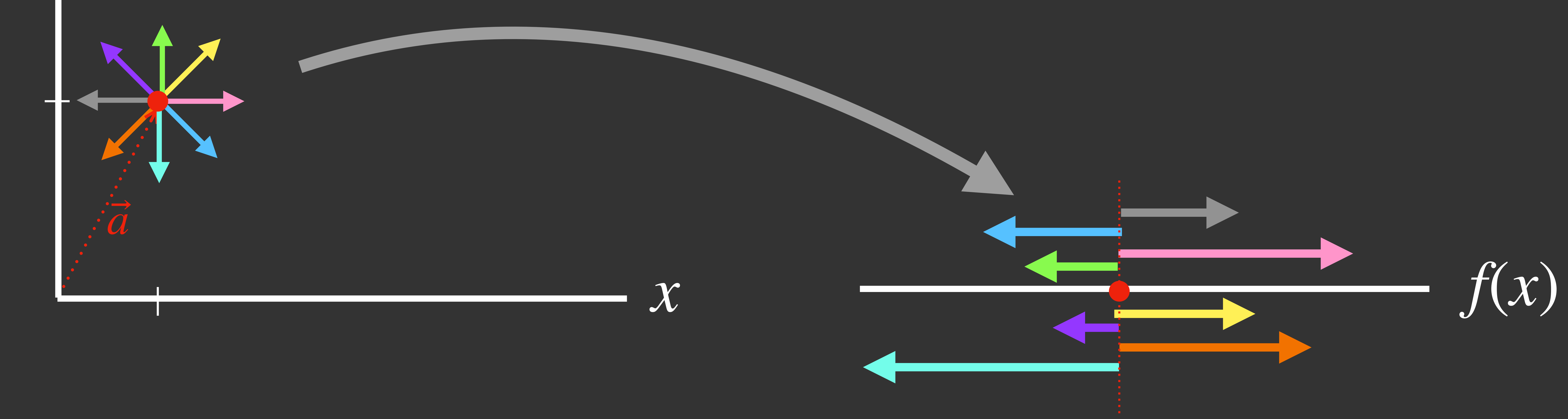
directional derivatives

$$\boxed{\nabla_{\vec{v}} f(\vec{a}) = \lim_{h \to \infty} \frac{f(\vec{a} + h \cdot \vec{v}) - f(\vec{a})}{h}}$$

not a graphical function!

$$\nabla_{\vec{v}} f = v_1 \frac{\partial f}{\partial x} + v_2 \frac{\partial f}{\partial y} \quad \text{where} \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\boxed{\max_{||\vec{v}=1||} \nabla f(a,b) \cdot \vec{v}}$$



gradient

Zero Gradient:

If $\nabla f = 0$, the point is a critical point (max, min, or saddle point)

example

For $f(x, y) = x^2 + y^2$, the gradient is:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (2x, 2y)$$

Find minimum (we see from image):

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \vec{0}$$