Maximal Margin Classifier: The Math

The maximal margin classifier solves a constrained optimization problem:

$$\arg\max_{\beta_0,\beta} \left\{ \frac{1}{\|\beta\|} \min_i \left[y_i(\beta_0 + \beta^T x_i) \right] \right\}$$

easier to minimize
$$\arg\min_{\beta_0,\beta} \frac{1}{2}\|\beta\|^2$$

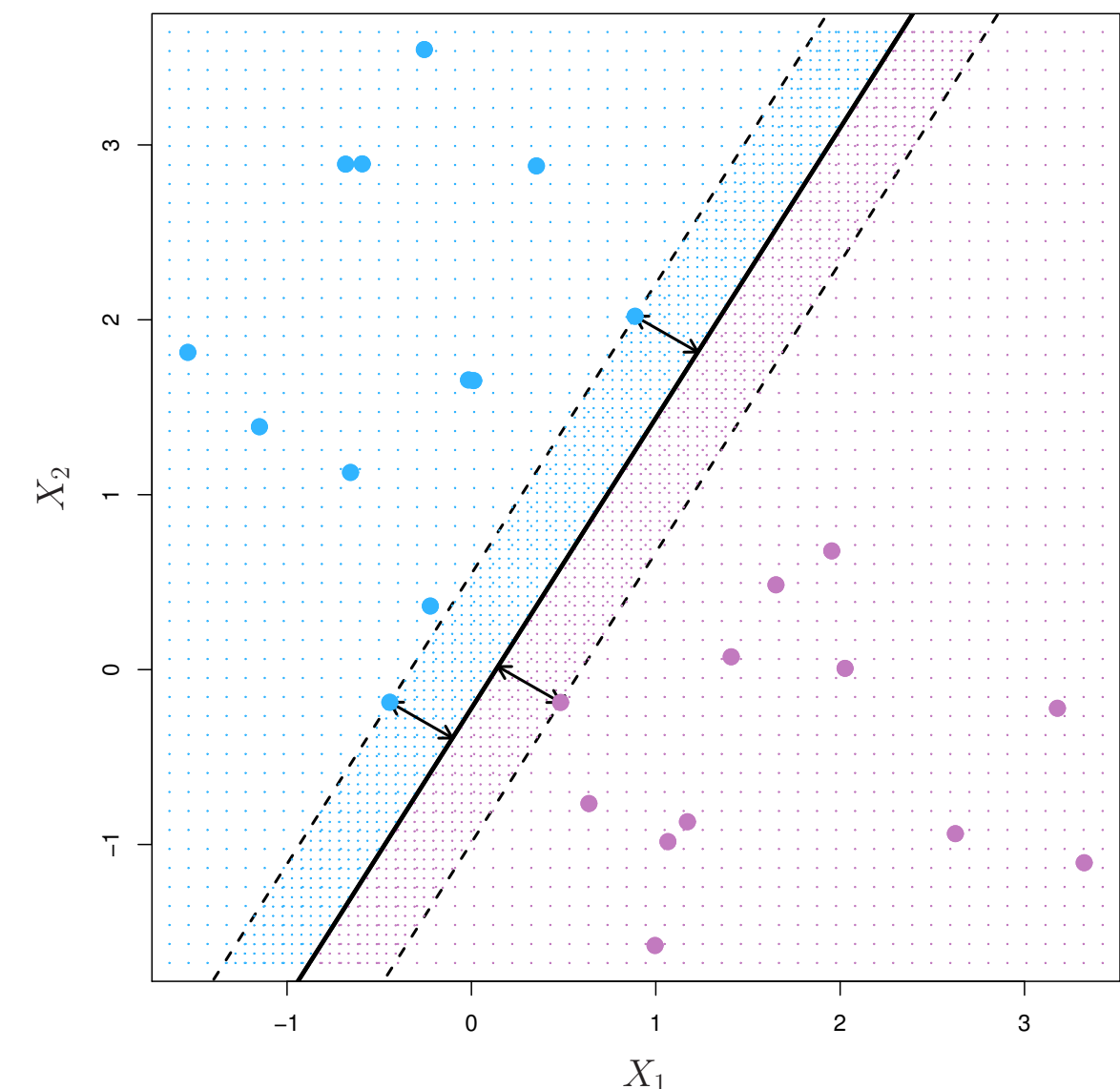maximize the minimum distance between hyperplane and point

$$\text{subject to: } y_i(\beta_0 + \beta^T x_i) \geq 1, \quad \forall i = 1,\ldots,n$$

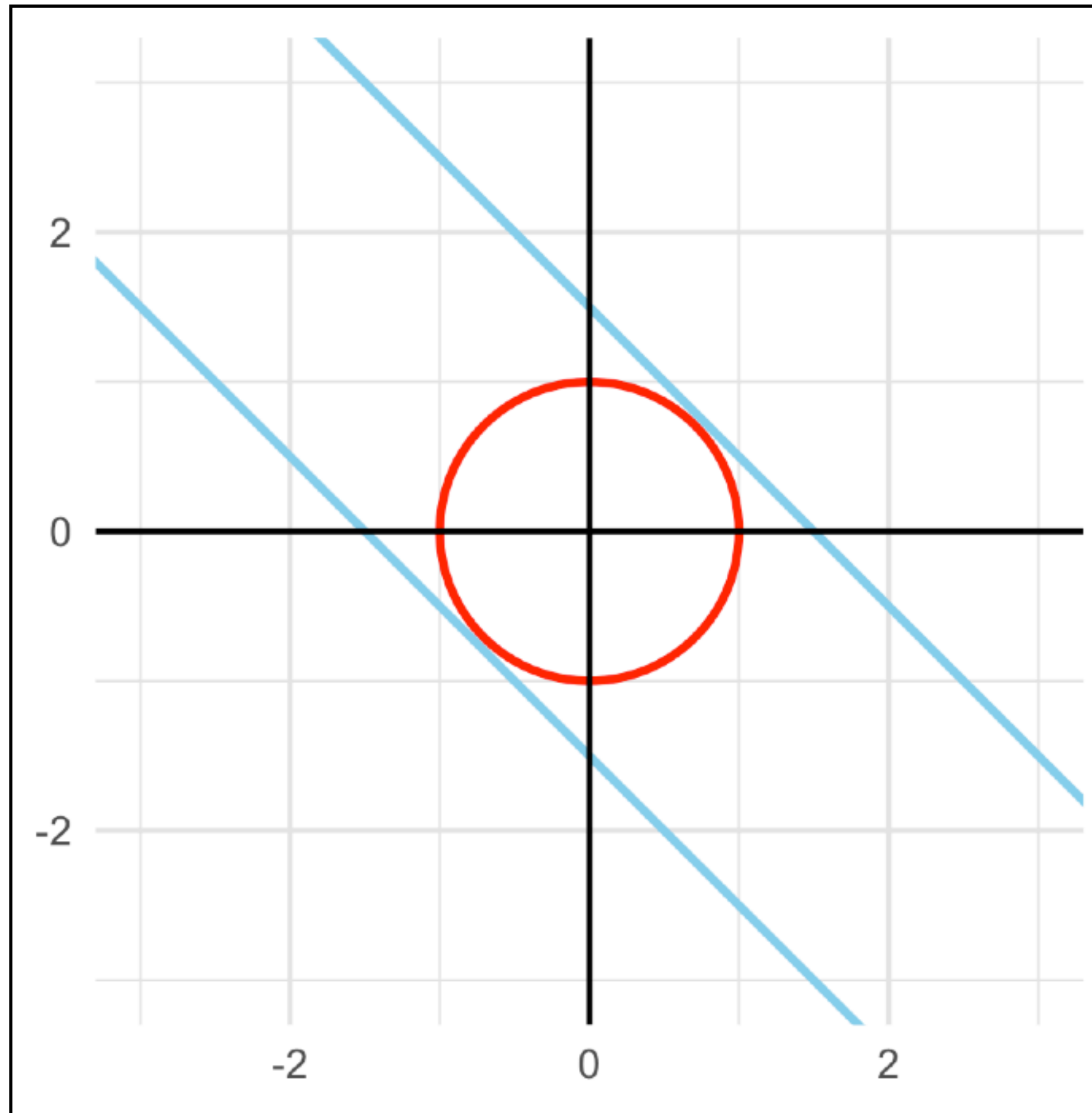distance between x_i and line where

$\|\beta\| = \sqrt{\sum_{j=1}^{p} \beta_j^2}$ is the Euclidean norm of β

$$\left\{ \frac{y_i(\beta_0 + \beta^T x_i)}{\|\beta\|} \right.$$

What is a Constrained Optimization Problem?

Optimize $f(x, y)$ subject to $g(x, y) = k$

$$f(x, y) = 2x + y$$
$$g(x, y) = x^2 + y^2 = 1$$