chain rule: multivariable functions

$$\xrightarrow{dt}$$

dy
dx

$$\Rightarrow \quad df = \frac{\partial f}{\partial x}\frac{dx}{dt}dt + \frac{\partial f}{\partial y}\frac{dy}{dt}dt$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

df

1

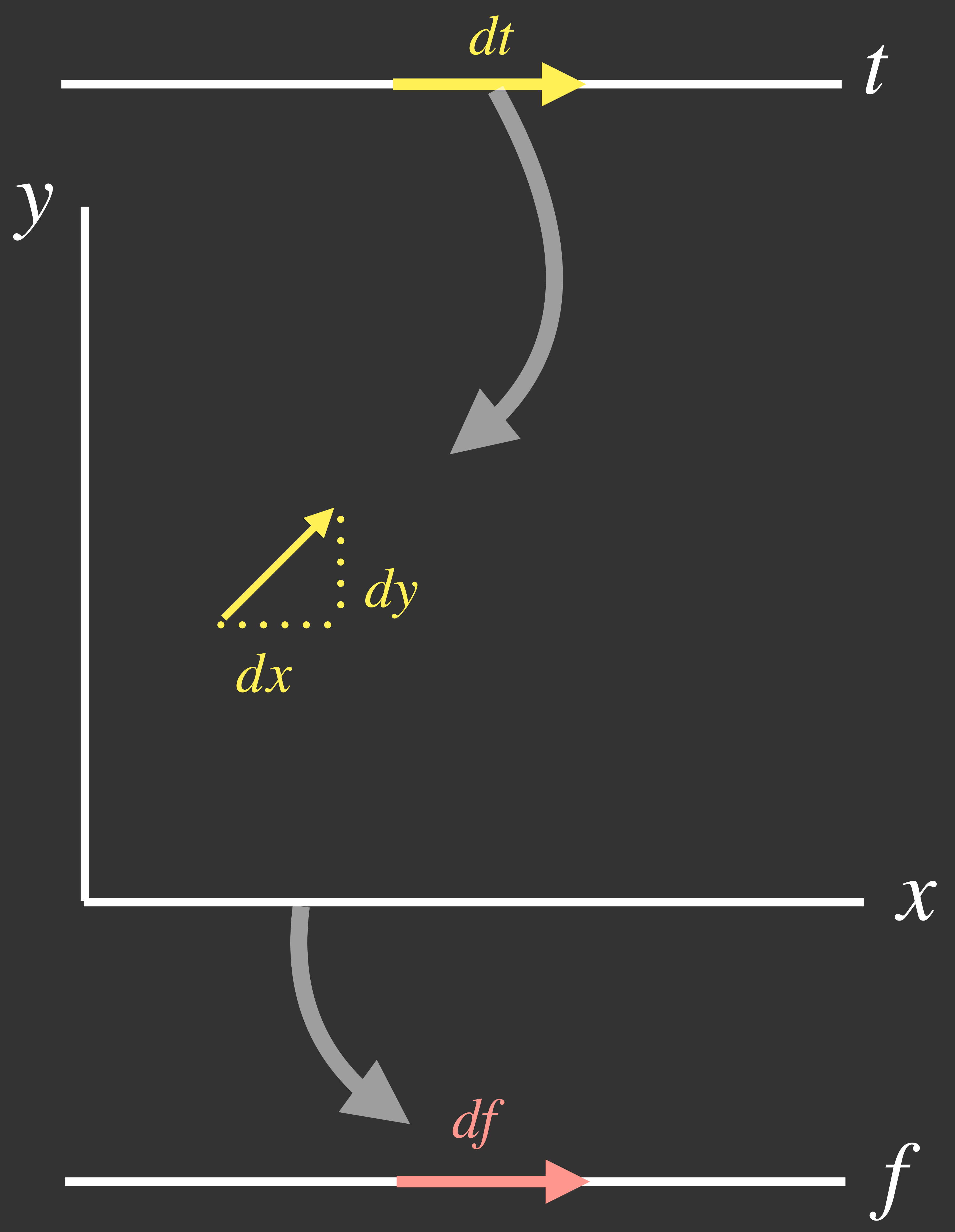
$$dx = \frac{dx}{dt}dt \qquad dy = \frac{dy}{dt}dt$$

2

$$df_{dx} = \frac{\partial f}{\partial x} dx \qquad df_{dy} = \frac{\partial f}{\partial y} dy$$

chain rule: multivariable functions

$$\frac{df(x(t), y(t))}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$



1 $dx = \dfrac{dx}{dt}dt \qquad dy = \dfrac{dy}{dt}dt$

2 $df_{dx} = \dfrac{\partial f}{\partial x}dx \qquad df_{dy} = \dfrac{\partial f}{\partial y}dy$

$$df = \frac{\partial f}{\partial x}\frac{dx}{dt}dt + \frac{\partial f}{\partial y}\frac{dy}{dt}dt$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

the Jacobian

If we have a function $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ mapping an $n-$dimensional input to an $m-$dimensional output,

$$f(x_1, x_2, \ldots, x_n) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_m(x_1, x_2, \ldots, x_n) \end{bmatrix}$$

, then the Jacobian matrix contains all first-order partial derivatives of f :

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$