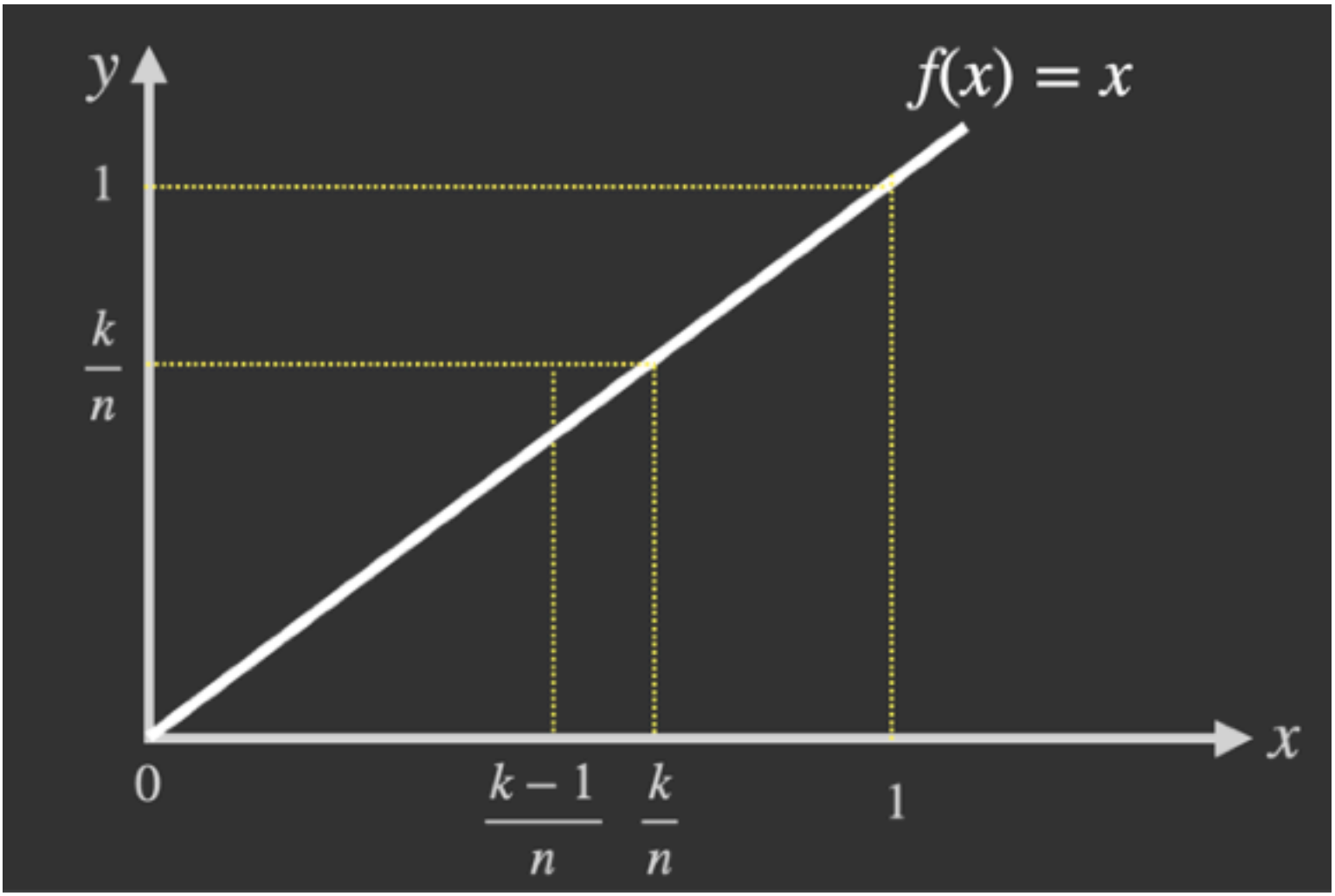
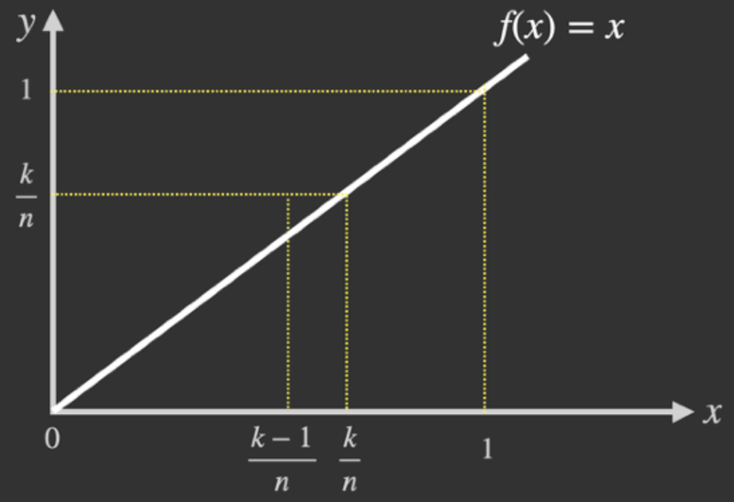

y
f(x) = x
1
k/n
0
(k − 1)/n
k/n
1
x

Riemann sum

Riemann sum



- we divide $[0,1]$ into n equal pieces

  $\Longrightarrow$ the divisions occur at

$$0,\ \frac{1}{n},\ \frac{2}{n},\ \cdots,\ \frac{k-1}{n},\ \frac{k}{n},\ \cdots,\ \frac{n-1}{n},\ \frac{n}{n} = 1$$

- we have $n+1$ points and we put a rectangle on each point
- the rectangle between $\dfrac{k-1}{n}$ and $\dfrac{k}{n}$ has height $f\left(\dfrac{k}{n}\right) = \dfrac{k}{n}$ and area of this rectangle is

$$\underbrace{\frac{k}{n}}_{\text{height}} \cdot \underbrace{\frac{1}{n}}_{\text{width}} = \frac{k}{n^2}$$

Riemann sum