

# Gaussian elimination

- We eliminate one variable by combining equations

$$2x - y + 3z = 9 \quad (1)$$

$$x + 4y - 5z = -6 \quad (2)$$

$$x - y + z = 2 \quad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- Eliminate  $x$ :

- subtracting equation (3) from (2):  $(x + 4y - 5z) - (x - y + z) = -6 - 2 \implies 5y - 6z = -8 \quad (4)$

- subtract  $2 \times$  equation (3) from equation (1):  $(2x - y + 3z) - 2(x - y + z) = 9 - 2(2) \implies y + z = 5 \quad (5)$

- Solve for  $y$  and  $z$ :

- from equation (5) we get a new equation:  $y = 5 - z \quad (6)$  which is substituted into (4):

$$5(5 - z) - 6z = -8 \implies 25 - 5z - 6z = -8 \implies -11z = -33 \implies z = 3$$

- from (6) we get  $y = 5 - 3 = 2$  and from (3) we get  $x = y - z + 2 \implies x = 2 - 3 + 2 = 1$

matrix inversion