the Jaconian

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If we have a function
$$f:\mathbb{R}^n \to \mathbb{R}^m$$
 mapping an $n-$ dimensional input to an $m-$ dimensional output,
$$f(x_1,x_2,\ldots,x_n) = \begin{bmatrix} f_1(x_1,x_2,\ldots,x_n) \\ f_2(x_1,x_2,\ldots,x_n) \\ \vdots \\ f_m(x_1,x_2,\ldots,x_n) \end{bmatrix}, \text{ then the Jacobian matrix contains all first-order partial derivatives of } f:$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

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the Jacobian

- The Jacobian matrix is a matrix of all first-order partial derivatives of a vector-valued function. It generalizes the concept of a derivative to multiple variables and dimensions.
- Measures how a function transforms space:
 It describes the local scaling, rotation, or shearing of a function.
- Useful in nonlinear transformations
- The Jacobian determinant represents the factor by which the transformation stretches or squishes the n- dimensional volumes around a certain input.