





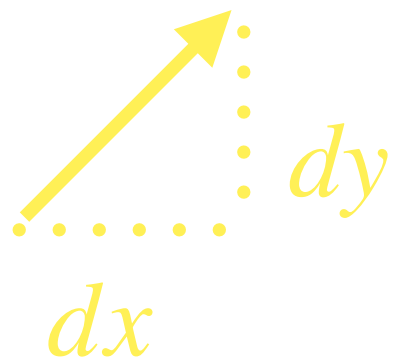
chain rule: multivariable functions



*dt*

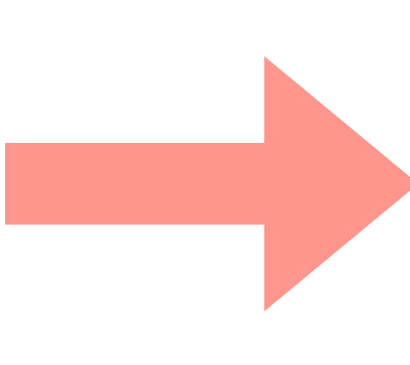











$$df = \frac{\partial f}{\partial x} \frac{dx}{dt} dt + \frac{\partial f}{\partial y} \frac{dy}{dt} dt$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

df



$$dx = \frac{dx}{dt} dt$$

$$dy = \frac{dy}{dt} dt$$

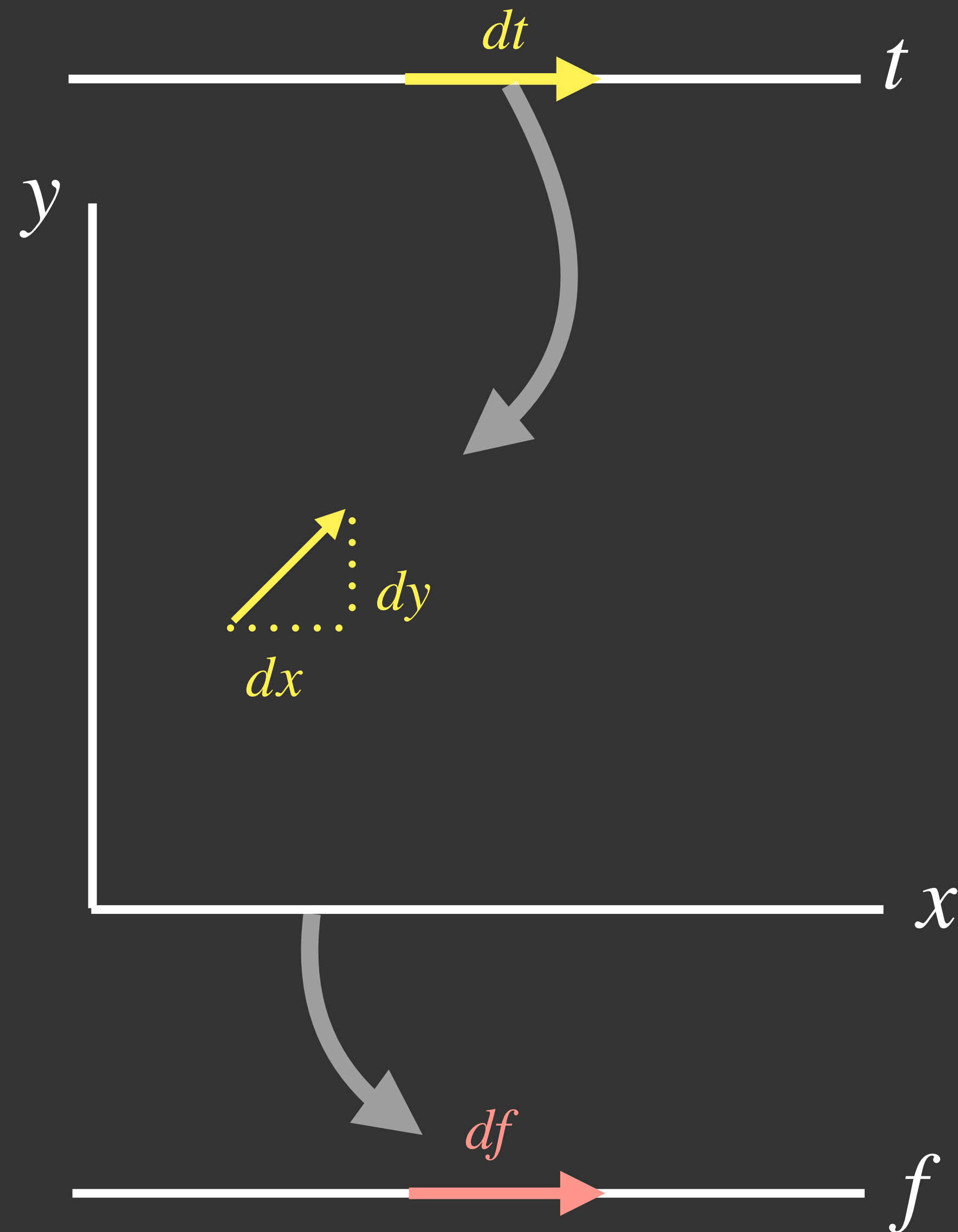
2

$$df_{dx} = \frac{\partial f}{\partial x} dx$$

$$df_{dy} = \frac{\partial f}{\partial y} dy$$

# chain rule: multivariable functions

$$\frac{df(x(t), y(t))}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$



**1**  $dx = \frac{dx}{dt} dt$        $dy = \frac{dy}{dt} dt$

**2**  $df_{dx} = \frac{\partial f}{\partial x} dx$        $df_{dy} = \frac{\partial f}{\partial y} dy$

$\Rightarrow df = \frac{\partial f}{\partial x} \frac{dx}{dt} dt + \frac{\partial f}{\partial y} \frac{dy}{dt} dt$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

# the Jacobian

If we have a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$  mapping an  $n$ —dimensional input to an  $m$ —dimensional output,

$$f(x_1, x_2, \dots, x_n) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) \\ f_2(x_1, x_2, \dots, x_n) \\ \vdots \\ f_m(x_1, x_2, \dots, x_n) \end{bmatrix}, \text{ then the Jacobian matrix contains all first-order partial derivatives of } f :$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$