indirect proofs: proof by contrapositive


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Proof.

indirect proofs: proof by contrapositive

Theorem

For any $n \in \mathbb{Z}$, if n^2 is even, then n is even.

Proof.

- By contrapositive; we prove that if n is odd, then n^2 is odd
- Let n be an arbitrary odd integer.
- Since n is odd, there is some integer k such that $n = 2k + 1$.
- Squaring both sides of this equality and simplifying yields the following:

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \end{aligned}$$

- From this we see that there is an integer m (namely $2k^2 + 2k$) such that $n^2 = 2m + 1$.
- Therefore n^2 is odd. ■

indirect proofs: proof by contradiction

Theorem

For any $n \in \mathbb{Z}$ n, if n^2 is even, then n is even.

Proof.