the characteristic polynomial

the characteristic polynomial

characteristic polynomial

the characteristic polynomial

Is there a method for computing all of the eigenvalues of a matrix?

YES!

by finding the roots of the characteristic polynomial (i.e. solving a nonlinear equation in one variable)

> Let A be a $n \times n$ matrix. The characteristic polynomial of A is the function $f(\lambda)$ given by
>
> $$f(\lambda) = \det(A - \lambda I)$$

- The characteristic polynomial is in fact a polynomial
- The point of the characteristic polynomial is that we can use it to compute eigenvalues
- Finding the characteristic polynomial means computing the determinant of the matrix $\det(A - \lambda I)$ whose entries contain the unknown λ

the characteristic polynomial

example

Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

We have

$$f(\lambda) = \det(A - \lambda I) = \det\left(\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)$$

$$= \det\begin{bmatrix} 5-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix} = (5-\lambda)(1-\lambda) - 2\cdot 2$$

$$= \lambda^2 - 6\lambda + 1$$