eigenvectors and eigenvalues

Let A be a $n \times n$ matrix.

1. An eigenvector of A is a nonzero vector $\vec{v}$ in $\mathbb{R}^n$ such that $A\vec{v} = \lambda\vec{v}$, for some scalar λ
2. An eigenvalue of A is a scalar λ such that the equation $A\vec{v} = \lambda\vec{v}$ has a *non-trivial** solution

If $A\vec{v} = \lambda\vec{v}$ for $\vec{v} \neq 0$, we say that λ is the eigenvalue for $\vec{v}$, and that $\vec{v}$ is an eigenvector for λ.

*means that the solution vector $\vec{v}$ is not the zero vector ($\vec{v} \neq \vec{0}$), and ensures that it represents a meaningful direction in the vector space.

verifying eigenvectors

How to check if a given $\vec{v}$ is the eigenvector of a given matrix A

- multiply $\vec{v}$ by A and see if $A\vec{v}$ is a scalar multiple of $\vec{v}$, i.e. $A\vec{v} = \lambda\vec{v}$
- what happens when a matrix hits a vector?

example

Consider matrix $A = \begin{bmatrix} 2 & 2 \\ -4 & 8 \end{bmatrix}$ and vector $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Which are eigenvectors? What are their eigenvalues?

$$A\vec{v} = \begin{bmatrix} 2 & 2 \\ -4 & 8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4\vec{v}$$

$\implies$ $\vec{v}$ is an eigenvector of A

$$A\vec{w} = \begin{bmatrix} 2 & 2 \\ -4 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$\implies$ $\vec{w}$ is not an eigenvector of A