substitution

$$2x - y + 3z = 9 \qquad (1)$$
$$x + 4y - 5z = -6 \qquad (2)$$
$$x - y + z = 2 \qquad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- We solve equation (1) and substitute it into equation (2) and (3): $x = y - z + 2$

$$\implies 2(y - z + 2) - y + 3z = 9 \implies 2y - 2z + 4 - y + 3z = 9 \implies y + z = 5 \qquad (4)$$
$$\implies (y - z + 2) + 4y - 5z = -6 \implies y - z + 2 + 4y - 5z = -6 \implies 5y - 6z = -8 \qquad (5)$$

- Now solve equations (4) and (5):

From (4) we have $y = 5 - z$ which is substituted into equation (5)

$$5(5 - z) - 6z = -8 \implies 25 - 5z - 6z = -8 \implies -11z = -33 \implies \boxed{z = 3}$$

Substitute back into y: $\boxed{y = 5 - 3 = 2}$

and finally from $x = y - z + 2$ we get $\boxed{x = 2 - 3 + 2 = 1}$

Gaussian elimination

- We eliminate one variable by combining equations

$$\begin{aligned} 2x - y + 3z &= 9 && (1)\\ x + 4y - 5z &= -6 && (2)\\ x - y + z &= 2 && (3) \end{aligned}$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- Eliminate x:
  - subtracting equation (3) from (2): $(x + 4y - 5z) - (x - y + z) = -6 - 2 \implies 5y - 6z = -8 \quad (4)$
  - subtract $2 \times$ equation (3) from equation (1): $(2x - y + 3z) - 2(x - y + z) = 9 - 2(2) \implies y + z = 5 \quad (5)$

- Solve for y and z:
  - from equation (5) we get a new equation: $y = 5 - z \quad (6)$ which is substituted into (4):
    $5(5 - z) - 6z = -8 \implies 25 - 5z - 6z = -8 \implies -11z = -33 \implies \boxed{z = 3}$
  - from (6) we get $\boxed{y = 5 - 3 = 2}$ and from (3) we get $x = y - z + 2 \implies \boxed{x = 2 - 3 + 2 = 1}$