Maximum Likelihood Estimation

$$\theta_{MLE} = \underset{\theta \in \Theta}{\arg\max}\, L(\theta)$$

the value we pick for
our parameters

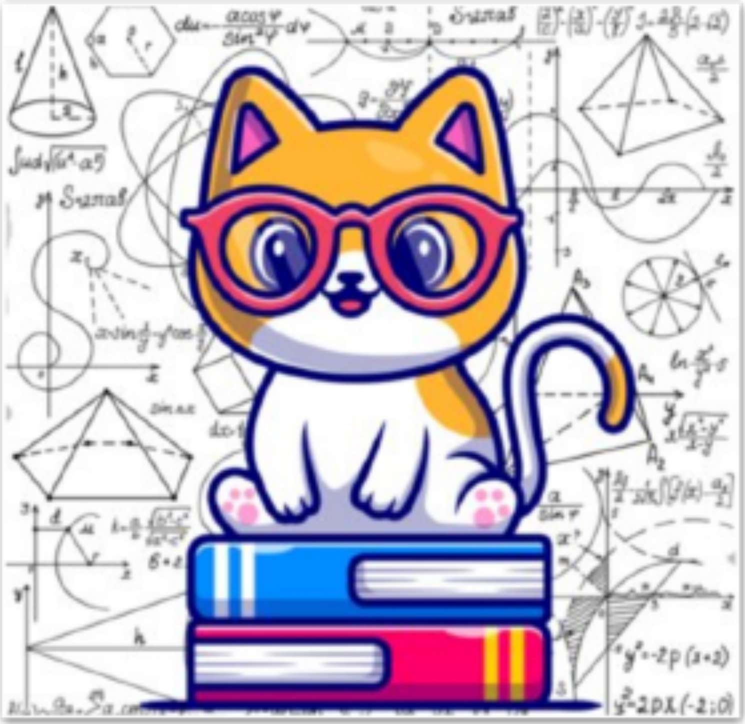
are the parameter values
(out of all possible parameter
values) that maximize

the likelihood of the data using these parameters

likelihood function $$L(y \mid \beta_0, \beta_1, \sigma^2) = \prod_{i=1}^{n} p(y_i \mid \beta_0, \beta_1, \sigma^2)$$

log-likelihood function $LL(\beta_0, \beta_1, \sigma^2) = \log L$

solved by taking partial derivatives and setting equal to 0



$$\frac{\partial LL}{\partial \beta_0} = 0$$

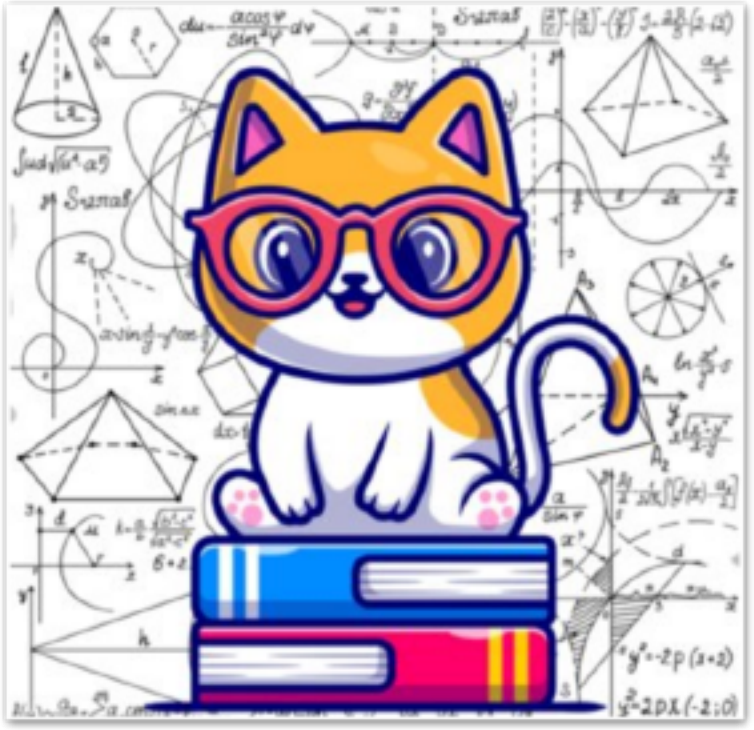
$$\frac{\partial LL}{\partial \beta_1} = 0$$

$$\Longrightarrow \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\Longrightarrow \hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{y}\overline{x}}{\sum_{i=1}^{n} x_i^2 + n\overline{x}} = \frac{Cov(x, y)}{Var(x)}$$

[full proof: https://statproofbook.github.io/P/slr-mle]

Maximum Likelihood Estimation



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