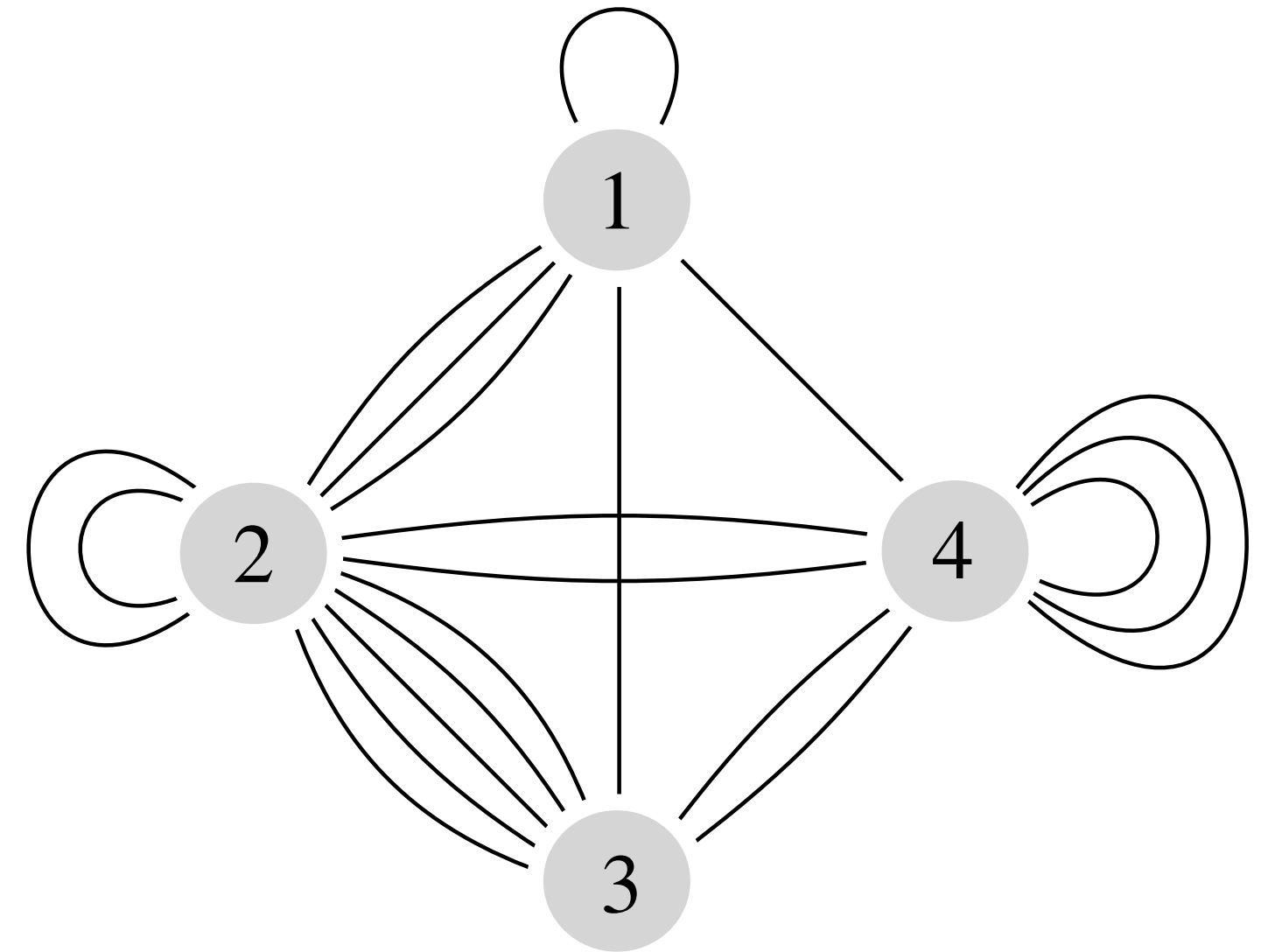




no digital representation of data

**example:**



 the number of vertex pair sites

$$r = \binom{n+1}{2} = \frac{5 \times 4}{2} = 10$$

 edge multiplicity sequence

$$\mathbf{M} = (M_{11}, M_{12}, M_{13}, M_{14}, M_{22}, M_{23}, M_{24}, M_{33}, M_{34}, M_{44})$$

$$= (1, 3, 1, 1, 2, 5, 2, 0, 2, 3)$$

 edge multiplicities as entries in a matrix

$$\mathbf{M} = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\mathbf{M} + \mathbf{M}' = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 1 & 5 & 0 & 2 \\ 1 & 2 & 2 & 6 \end{bmatrix}$$

# multigraph representation of network data

example:

☑ the number of vertex pair sites

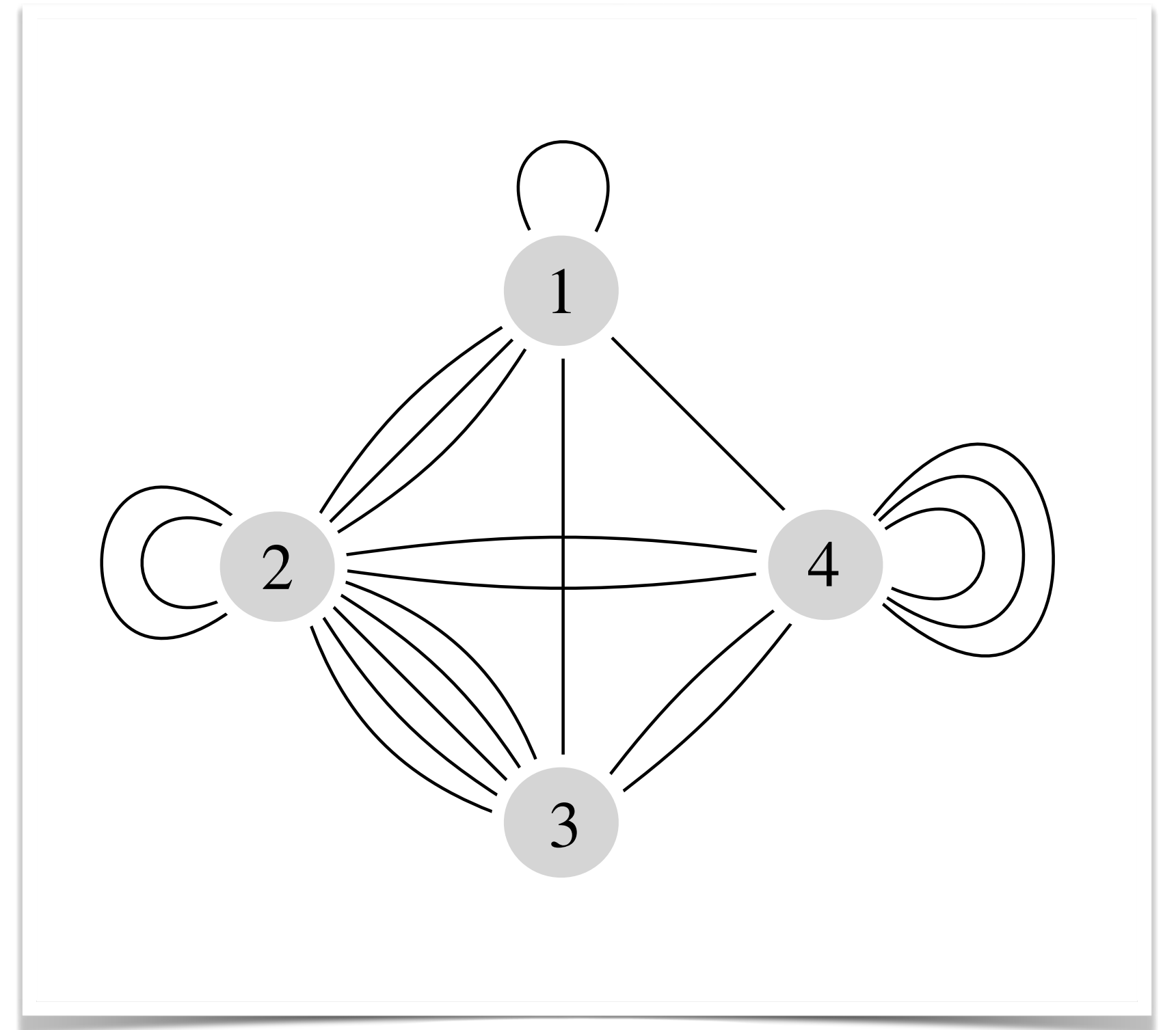
$$r = \binom{n+1}{2} = \frac{5 \times 4}{2} = 10$$

☑ edge multiplicity sequence

$$\begin{aligned} \mathbf{M} &= (M_{11}, M_{12}, M_{13}, M_{14}, M_{22}, M_{23}, M_{24}, M_{33}, M_{34}, M_{44}) \\ &= (1, 3, 1, 1, 2, 5, 2, 0, 2, 3) \end{aligned}$$

☑ edge multiplicities as entries in a matrix

$$\mathbf{M} = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix} \quad \mathbf{M} + \mathbf{M}' = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 1 & 5 & 0 & 2 \\ 1 & 2 & 2 & 6 \end{bmatrix}$$



**statistics under random multigraph models**