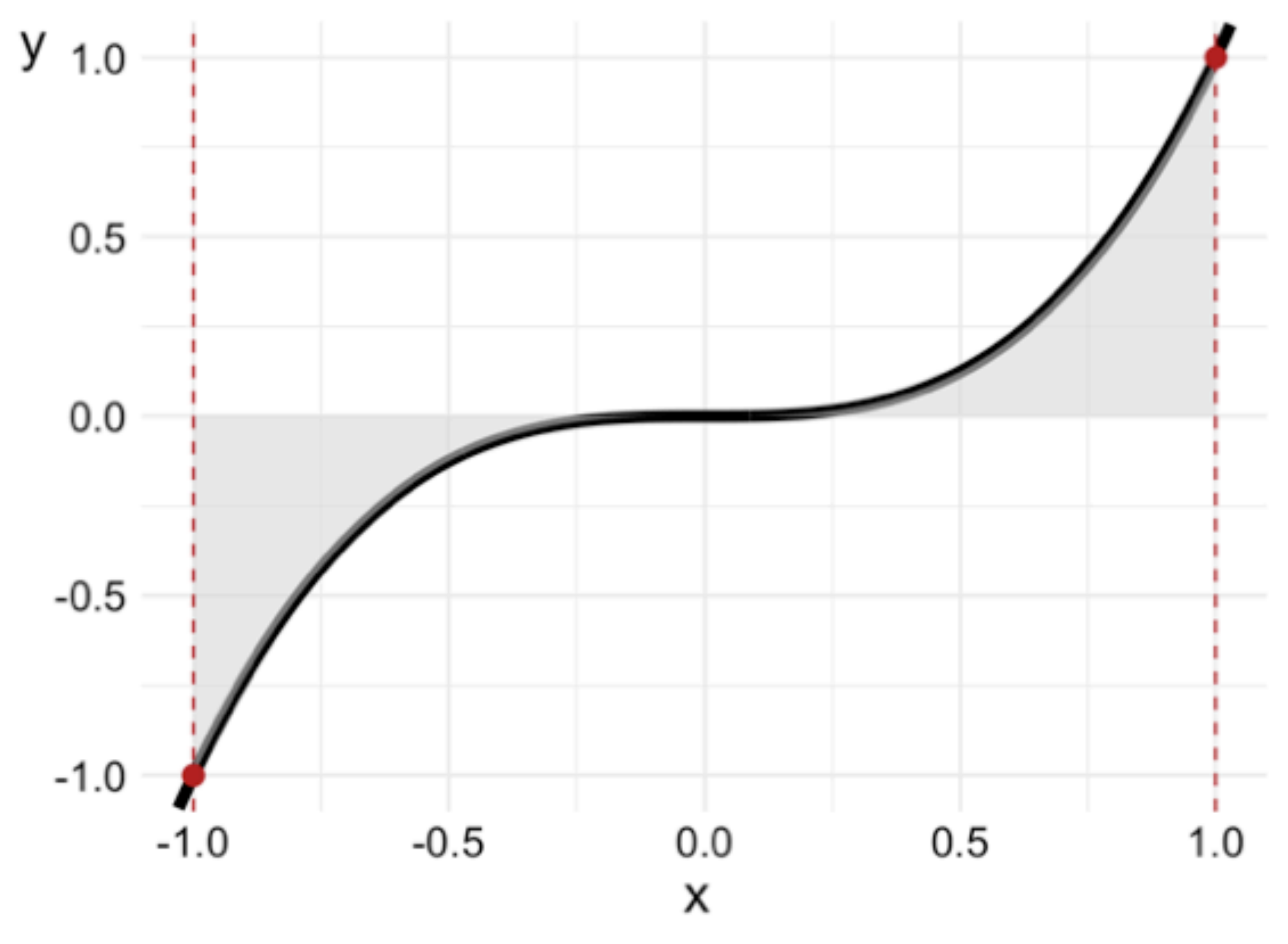
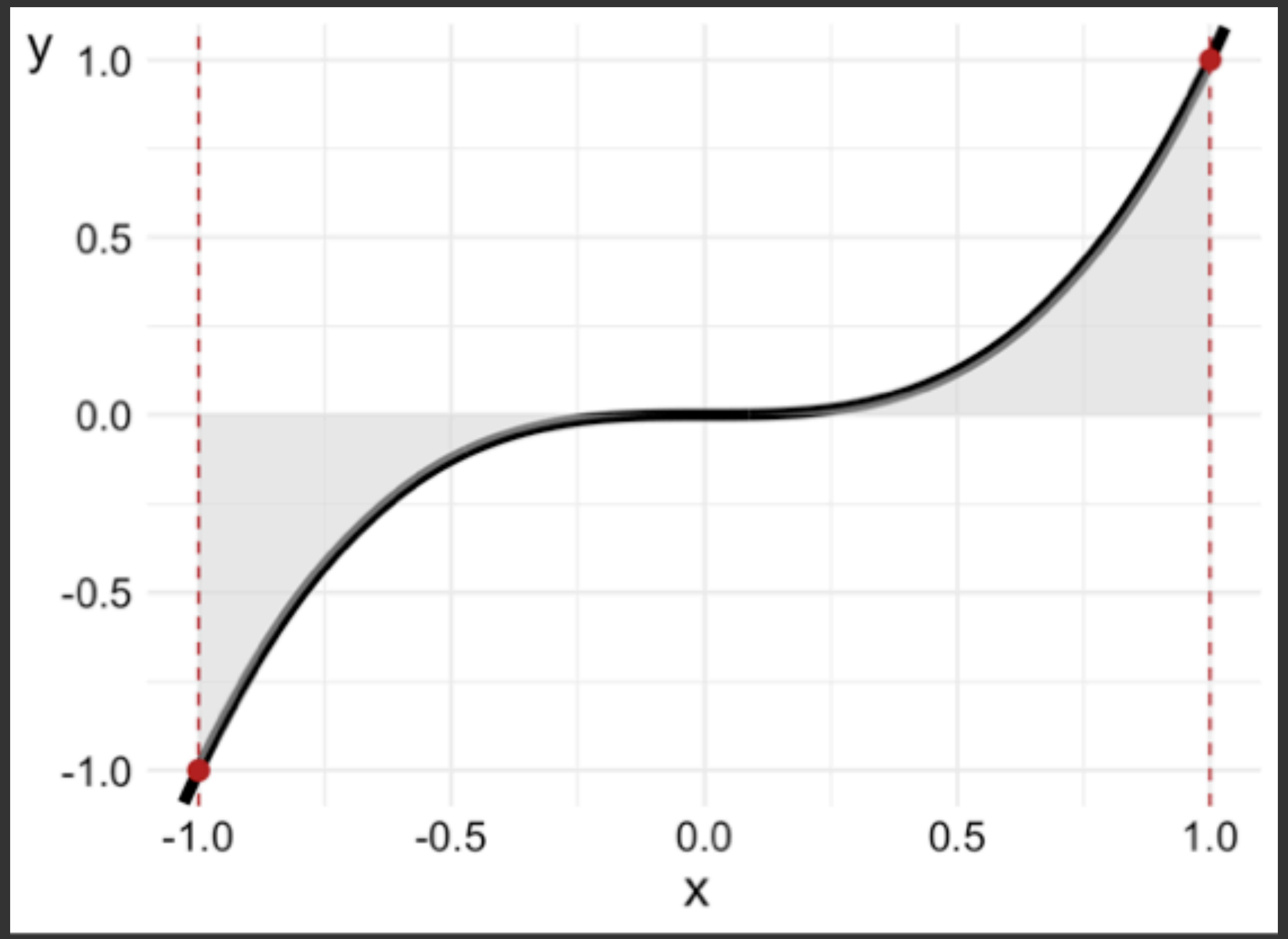
example

area under the curve value of the definite integral


y
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0
x

example

Consider the function $f(x) = x^3$, find the integral over the interval $[-1, 1]$



1. Write the integral $\int_{-1}^{1} x^3\, dx$

2. Find the antiderivative $\int x^3\, dx = \frac{x^4}{4} + C$

3. Apply fundamental theorem of calculus

$$\int_{-1}^{1} x^3\, dx = \left[\frac{x^4}{4}\right]_{-1}^{1} \implies \int_{-1}^{1} x^3\, dx = \frac{(1)^4}{4} - \frac{(-1)^4}{4} \implies \int_{-1}^{1} x^3\, dx = \frac{1}{4} - \frac{1}{4} = 0$$

Why Is the integral zero?

The function x^3 is odd, meaning $f(-x) = -f(x)$. For any odd function integrated over a symmetric interval $[-a, a]$ the integral is always zero because the positive and negative contributions cancel out

area under the curve and the value of the definite integral are not always the same

some antiderivatives

function $f(x)$	antiderivative $\int f(x)\,dx$
$f(x) = a$	$\int f(x)\,\mathrm{d}x = ax + C$
$f(x) = ax^n$	$\int f(x)\,\mathrm{d}x = \frac{ax^{(n+1)}}{n+1} + C$
$f(x) = ax^{-1}$	$\int f(x)\,\mathrm{d}x = a\ln\lvert x\rvert + C$
$f(x) = ae^{kx}$	$\int f(x)\,\mathrm{d}x = \frac{1}{k}ae^{kx} + C$
$f(x) = a\cos(kx)$	$\int f(x)\,\mathrm{d}x = \frac{1}{k}a\sin(kx) + C$
$f(x) = a\sin(kx)$	$\int f(x)\,\mathrm{d}x = -\frac{1}{k}a\cos(kx) + C$