the symmetry of second partial derivatives

Schwarz's theorem

If the second partial derivatives are continuous, the order of differentiation is not important and we therefore have:

$$\frac{\partial^2 f}{\partial xy} = \frac{\partial^2 f}{\partial yx}$$

gradient

The gradient of a scalar function $f(x_1, x_2, \ldots, x_n)$ is a vector field that points in the direction of the greatest rate of increase of f.

For a function $f : \mathbb{R}^n \to \mathbb{R}$, the gradient is denoted as:

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right)$$

where each component is a partial derivative of f with respect to one of the variables.

Direction: The gradient points in the direction of the steepest ascent of f

Magnitude: The magnitude $||\nabla f||$ represents the rate of the steepest increase.

Zero Gradient: If $\nabla f = 0$, the point is a critical point (possible max, min, or saddle point).