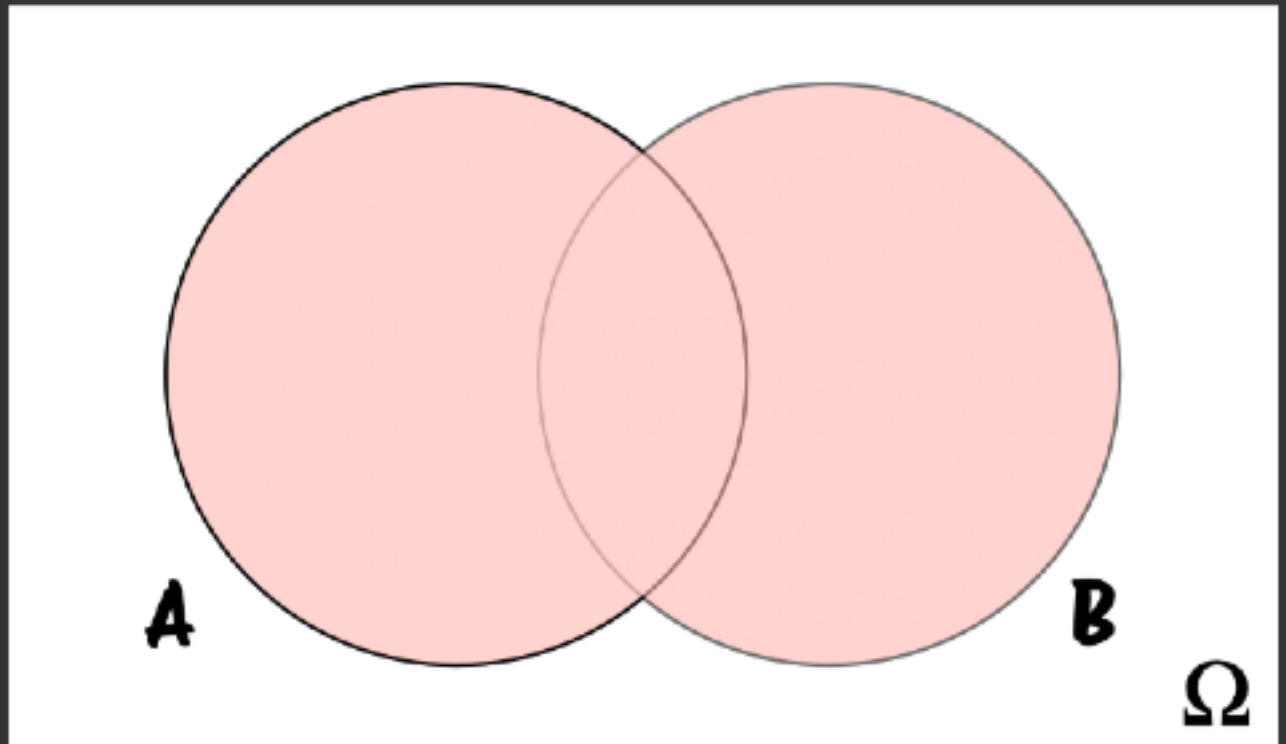
probability rules

The Addition Rule

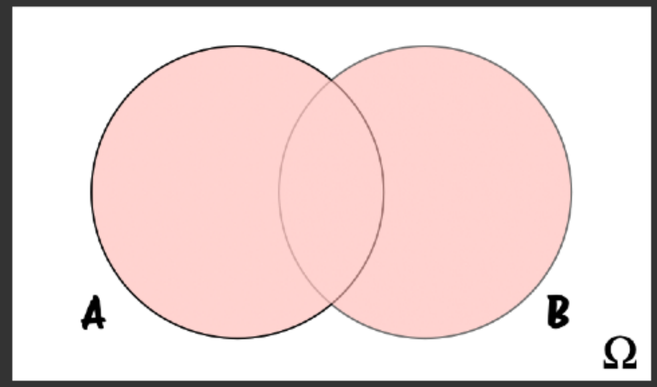

A
B
Ω

probability rules

The Addition Rule

If A and B are events in the sample space Ω, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Proof:

Let $a = P(A \cap \overline{B}), b = P(A \cap B), c = P(B \cap \overline{A})$

$$P(A \cup B) = a + b + c$$

$$P(A) + P(B) - P(A \cap B) = [a + b] + [b + c] - b = a + b + c$$

probability rules

Inclusion-Exclusion Rule

For three events $A,\ B,\ C$

$$P(A \cup B \cup C) = P(A) + P(\overline{A} \cap B) + P(\overline{A} \cap \overline{B} \cap C)$$

$$\begin{aligned} P(A \cup B \cup C) = P(A) &+ P(B) + P(C) \\ &- P(A \cap B) - P(A \cap C) - P(B \cup C) \\ &+ P(A \cap B \cap C) \end{aligned}$$