

Eigendecomposition



For a given square matrix A , there are **special vectors**
which refuse to stray from their path

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

These vectors are called **eigenvectors**

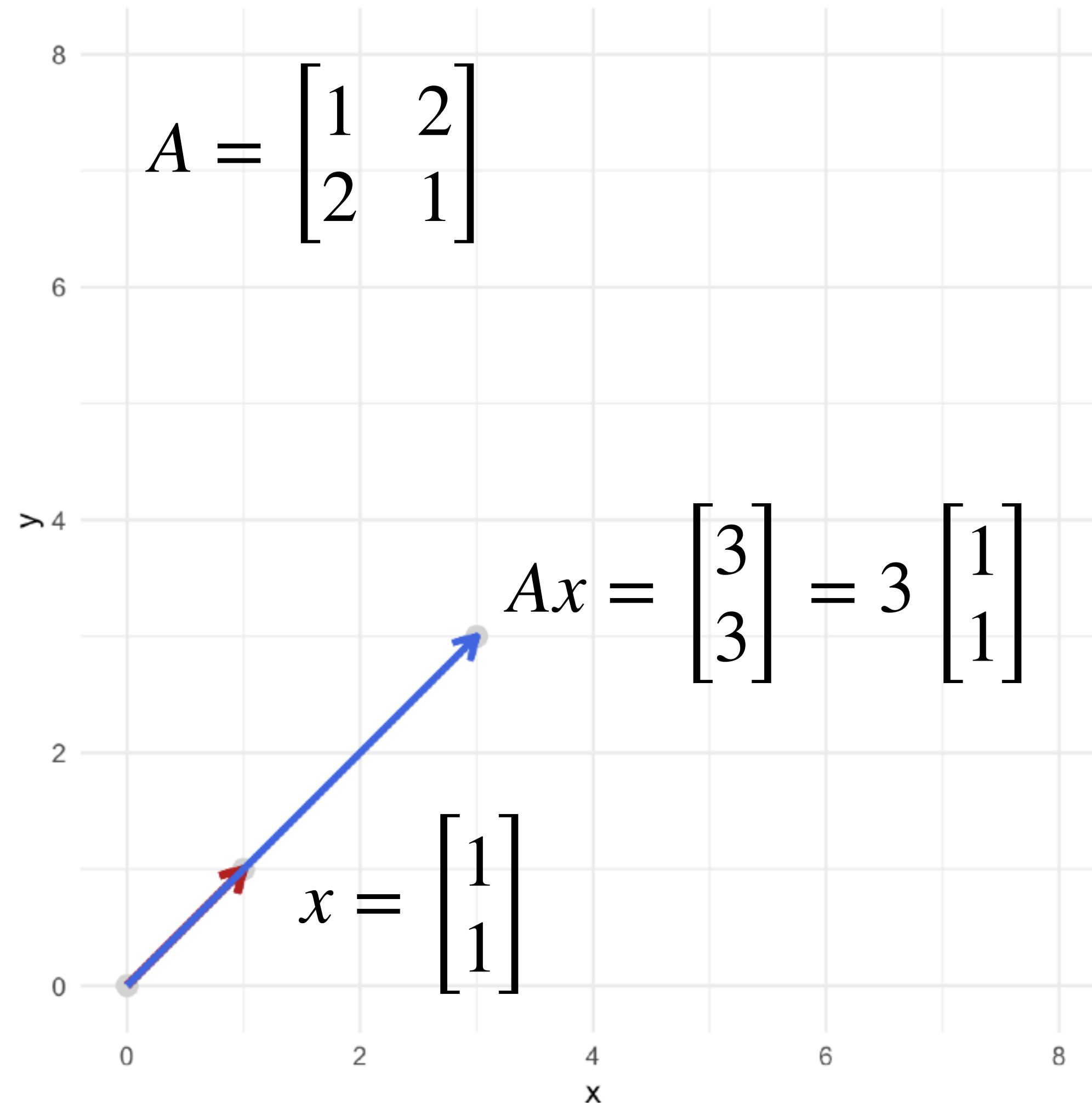
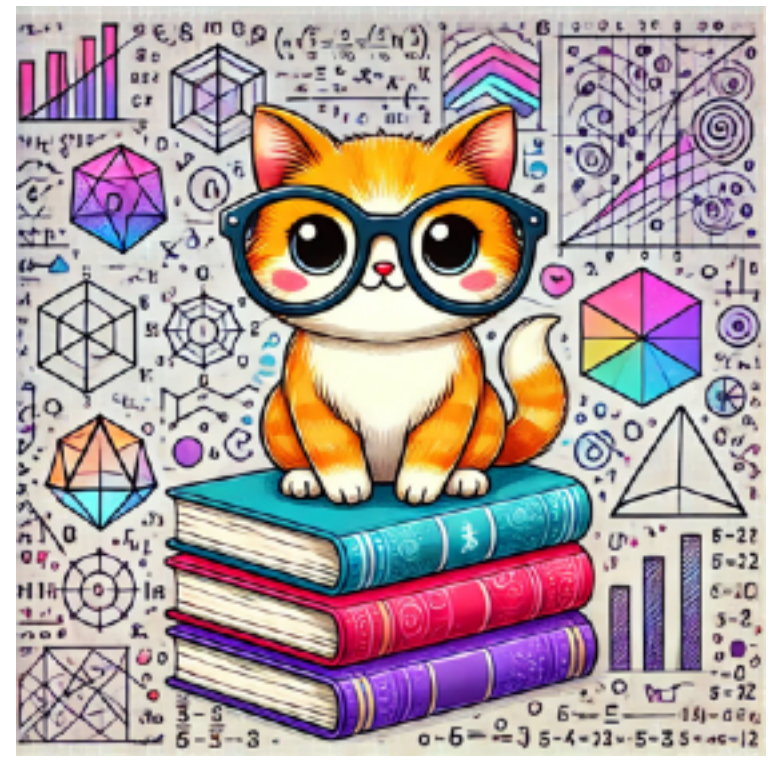
Formally, $Ax = \lambda x$

where λ are the eigenvalues determining the scale,

but directions remains the same (x)

Several properties of matrices can be analyzed based on their eigenvalues.

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The eigenvectors of a square matrix A having distinct eigenvalues are linearly independent.

The eigenvectors of a square symmetric matrix are orthogonal.

The eigenvectors of a square symmetric matrix can thus form a convenient basis.

$$\text{Cov}(\mathbf{x}) = \begin{bmatrix} \text{Var}(x_1) & \text{Cov}(x_1, x_2) & \text{Cov}(x_1, x_3) & \cdots & \text{Cov}(x_1, x_n) \\ \text{Cov}(x_2, x_1) & \text{Var}(x_2) & \text{Cov}(x_2, x_3) & \cdots & \text{Cov}(x_2, x_n) \\ \text{Cov}(x_3, x_1) & \text{Cov}(x_3, x_2) & \text{Var}(x_3) & \cdots & \text{Cov}(x_3, x_n) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \text{Cov}(x_n, x_1) & \text{Cov}(x_n, x_2) & \text{Cov}(x_n, x_3) & \cdots & \text{Var}(x_n) \end{bmatrix}$$