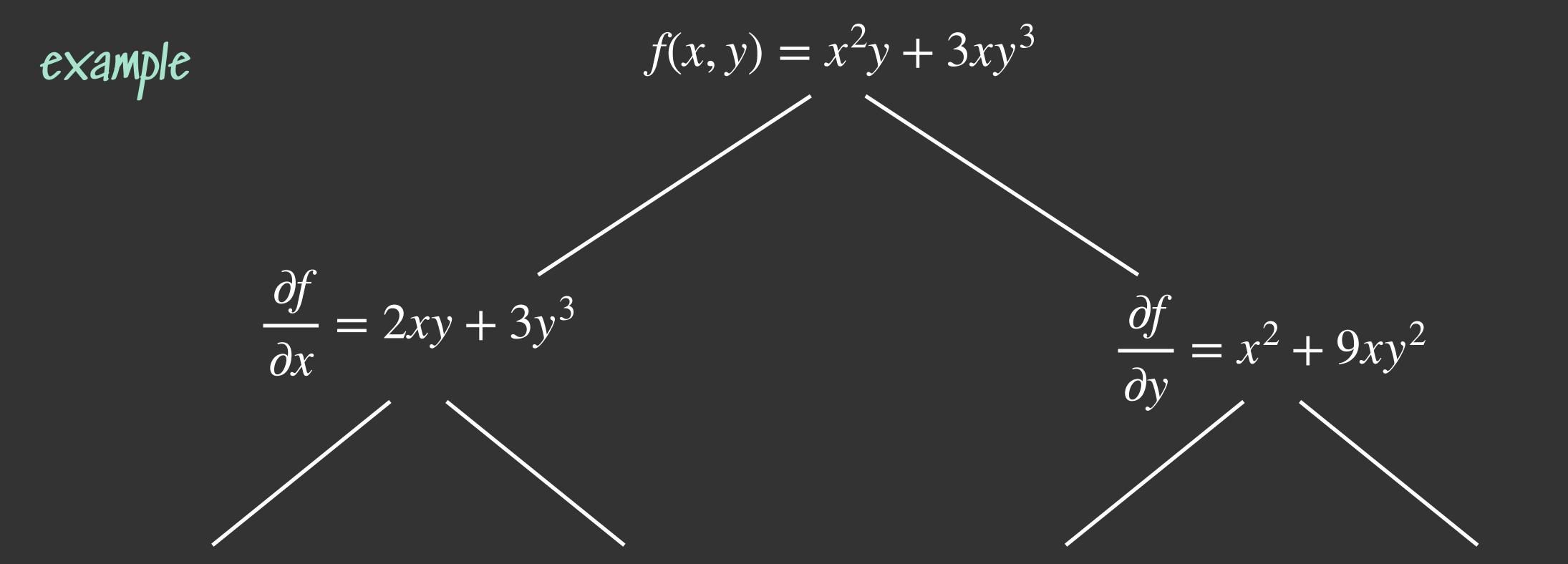
the symmetry of second partial derivatives



$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right) \qquad \frac{\partial^2 f}{\partial xy} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right)$$
$$= 2y \qquad = 2x + 9y^2$$

$$\frac{\partial^2 f}{\partial yx} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) \qquad \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right)$$
$$= 2x + 9y^2 \qquad = 18xy$$

the symmetry of second partial derivatives

example

$$f(x,y) = x^2y + 3xy^3$$

$$\frac{\partial f}{\partial x} = 2xy + 3y^3$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial x} \right)$$
$$= 2y$$

$$\frac{\partial^2 f}{\partial xy} = \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right)$$
$$= 2x + 9y^2$$

$$\frac{\partial^2 f}{\partial yx} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right)$$
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$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial y} \right)$$
$$= 18xy$$