







proof by induction

Proof cont'd.



































# proof by induction

## Theorem

The sum of the first  $n$  powers of two is  $2^n - 1$ .



## Proof cont'd.

- The inductive step:
  - the goal here is to prove "if  $P(k)$  then  $P(k + 1)$  is true"
  - to do this we choose an arbitrary  $k$ , assume  $P(k)$  is true, then try to prove  $P(k + 1)$   
 $\implies$  assume that for some arbitrary  $k \in \mathbb{N}$  that  $P(k)$  holds, meaning that
$$2^0 + 2^1 + \dots + 2^{k-1} = 2^k - 1$$
  - we need to show that  $P(k + 1)$  holds, meaning the sum of the first  $k + 1$  powers of two is  $2^{k+1} - 1$ 
$$\begin{aligned} 2^0 + 2^1 + \dots + 2^{k-1} + 2^k &= 2^k - 1 + 2^k \\ &= 2(2^k) - 1 \\ &= 2^{k+1} - 1 \quad \checkmark \end{aligned}$$
- Therefore,  $P(k + 1)$  is true, completing the induction. ■

indirect proofs