matrix inversion

$$2x - y + 3z = 9 \qquad (1)$$
$$x + 4y - 5z = -6 \qquad (2)$$
$$x - y + z = 2 \qquad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- Find the determinant of A to make sure it's invertible (via cofactor expansion along first row): $\det(A) = -11$
- This is not zero, the matrix is nonsingular, and we can invert it
- To invert the matrix we compute 9 minors:

$$M_{11} = -1,\ M_{12} = 6,\ M_{13} = -5,\ M_{21} = 2,\ M_{22} = -1,\ M_{23} = -1,\ M_{31} = -7,\ M_{32} = -13,\ M_{33} = 9$$

- Now use the formula for the inverse from the previous lecture $A^{-1} = \frac{1}{|A|}C^T$ where $C_{ij} = (-1)^{i+j}M_{i,j}$
- This gives: $\mathbf{A}^{-1} = \frac{1}{-11} \cdot \begin{bmatrix} -1 & -2 & -7 \\ -6 & -1 & 13 \\ -5 & 1 & 9 \end{bmatrix}$
- And finally we multiply with $\vec{b}$ to get the solution: $\vec{x} = A^{-1}\vec{b} = \frac{1}{-11} \cdot \begin{bmatrix} -1 & -2 & -7 \\ -6 & -1 & 13 \\ -5 & 1 & 9 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$

Cramer's rule

- only works when there an equal number of equations and unknowns (i.e. a square matrix A) and A is nonsingular
- this rules states that we can solve for $\vec{x}$ using the formula:

$$x_i = \frac{|B_i|}{|A|}$$

where the matrix B_i is formed by replacing the ith column of A (the column corresponding to variable x_i) with $\vec{b}$