



what is a vector space?









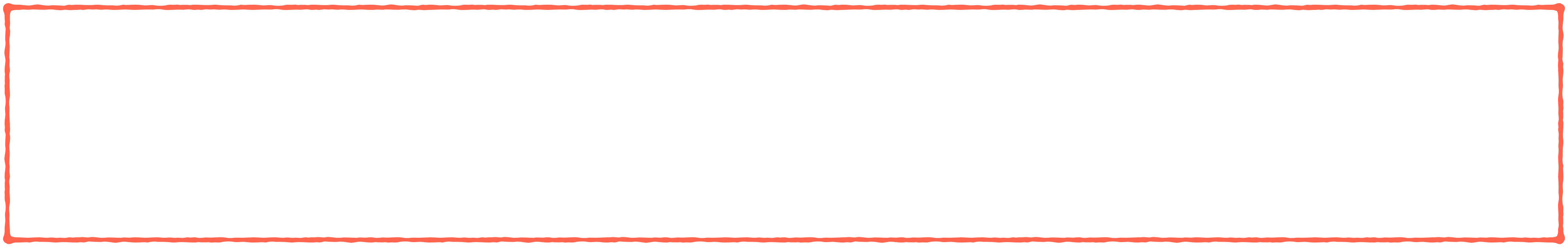












# what is a vector space?

A set  $V$  is called a vector space, if it is equipped with the operations of addition and scalar multiplication in such a way that the 'usual rules' of arithmetic hold.  
The elements of  $V$  are generally regarded as vectors.

The 'rules' (axioms) to hold true for all vectors  $\vec{u}, \vec{v}, \vec{w} \in V$  and scalars  $a, b$  are the following

- Closure:  $\vec{u} + \vec{v} \in V, a\vec{v} \in V$
- Commutative:  $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- Associativity:  $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
- Identity for Addition: There is a zero vector  $\vec{0} \in V$  such that  $\vec{u} + \vec{0} = \vec{u}$
- Distributive Property:  $a(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v}$  and  $(a + b)\vec{u} = a\vec{u} + b\vec{u}$
- Associativity of Scalars:  $(ab)\vec{u} = a(b\vec{u})$
- Multiplicative Identity:  $1\vec{u} = \vec{u}$

# what is a vector space?

Vector spaces are the foundation for:

- Linear transformations and matrices
- Eigenvalues, eigenvectors, and diagonalization
- Applications in
  - physics (e.g., quantum mechanics)
  - engineering (signal processing)
  - computer science (machine learning, graphics)