linear independence

basis

dimension

linear independence

- Two equivalent (but informal) definitions of linear dependency for a set of vectors:
    1. One of more of the vectors can be expressed as a linear combination of the remaining vectors
    2. One or more of the vectors is inside the span of the remaining vectors
- Note: Linear dependence is a property of the set! (i.e. an entire set of vectors can be linearly dependent simply because two of its vectors are dependent on each other)

A set K of n vectors is called linearly independent iff:

$$\sum_{i=1}^{n} a_i \vec{v_i} = 0 \quad iff \quad a_1 = a_2 = \cdots = a_n = 0$$

- We call a set of linearly independent vectors that span a vector space a basis of that vector space
- The dimension of a vector space is equal to the number of vectors in its basis

linear independence

example

Let $\vec{v_1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -2 \\ 1 \\ -4 \end{bmatrix}$. Are they linearly dependent?

Let's express each vector as the linear combination of the other two $\vec{v_3} = a\vec{v_2} + b\vec{v_1}$

$\implies \begin{bmatrix} -2 \\ 1 \\ -4 \end{bmatrix} = a \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. This gives us three equations, one for each entry:

$$-2 = 0a + 1b \rightarrow b = -2$$
$$1 = 1a + 0b \rightarrow a = 1$$
$$-4 = 1a + 2b \rightarrow -3 = 1 - 4$$

Is this enough to say that they are linearly independent? No, because we have to show that any vector in this set cannot be expressed as a linear combination of the rest of the vectors.