

Maximal Margin Classifier: The Math



The maximal margin classifier solves a constrained optimization problem:

$$\underbrace{\arg \max_{\beta_0, \beta} \left\{ \frac{1}{\|\beta\|} \min_i [y_i(\beta_0 + \beta^T x_i)] \right\}}_{\text{maximize the minimum distance between hyperplane and point}} \leftarrow \begin{array}{l} \text{easier to minimize} \\ \arg \min_{\beta_0, \beta} \frac{1}{2} \|\beta\|^2 \end{array}$$

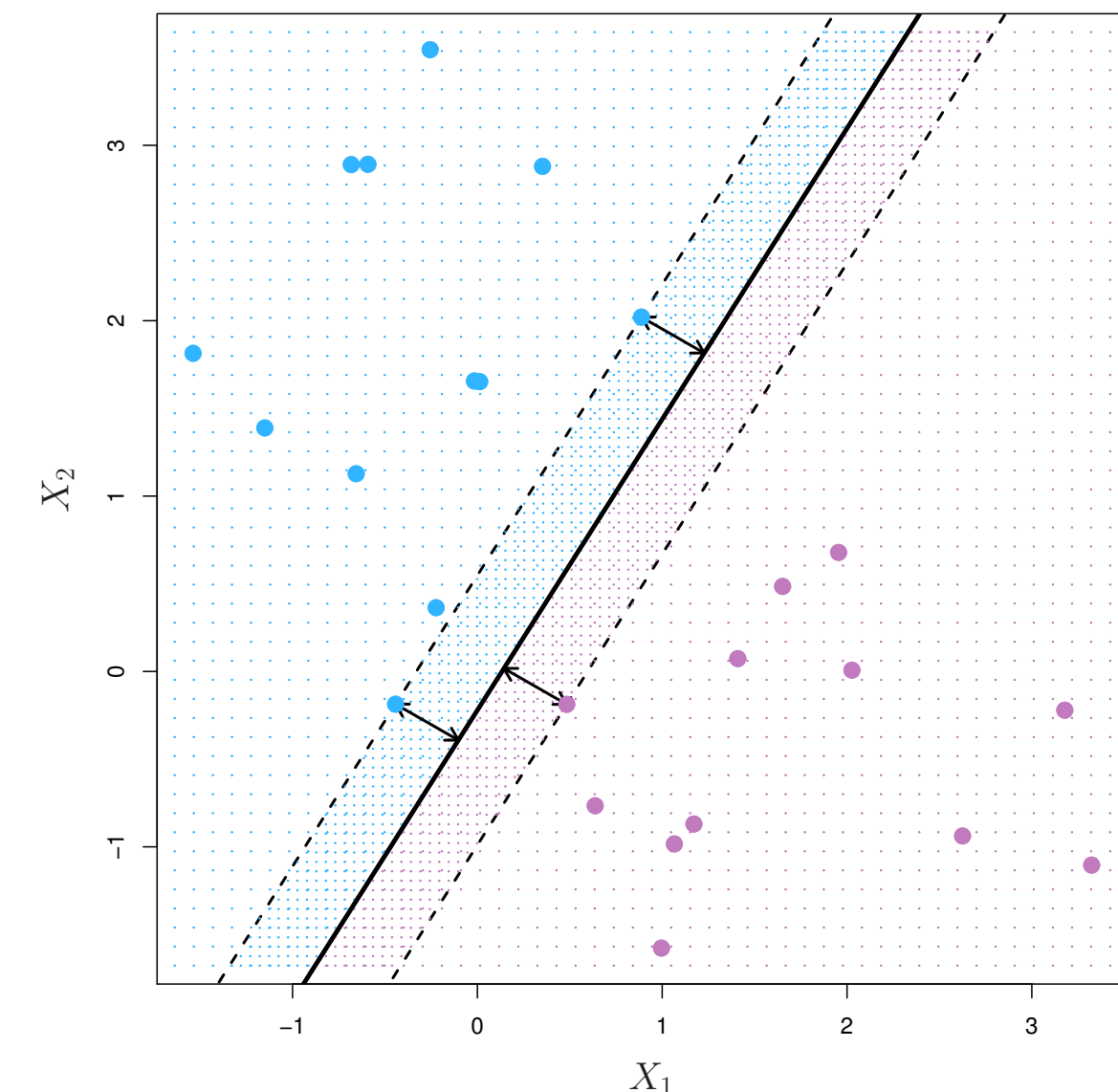
maximize the minimum distance between hyperplane and point

$$\text{subject to: } y_i(\beta_0 + \beta^T x_i) \geq 1, \quad \forall i = 1, \dots, n$$

distance between x_i and line where

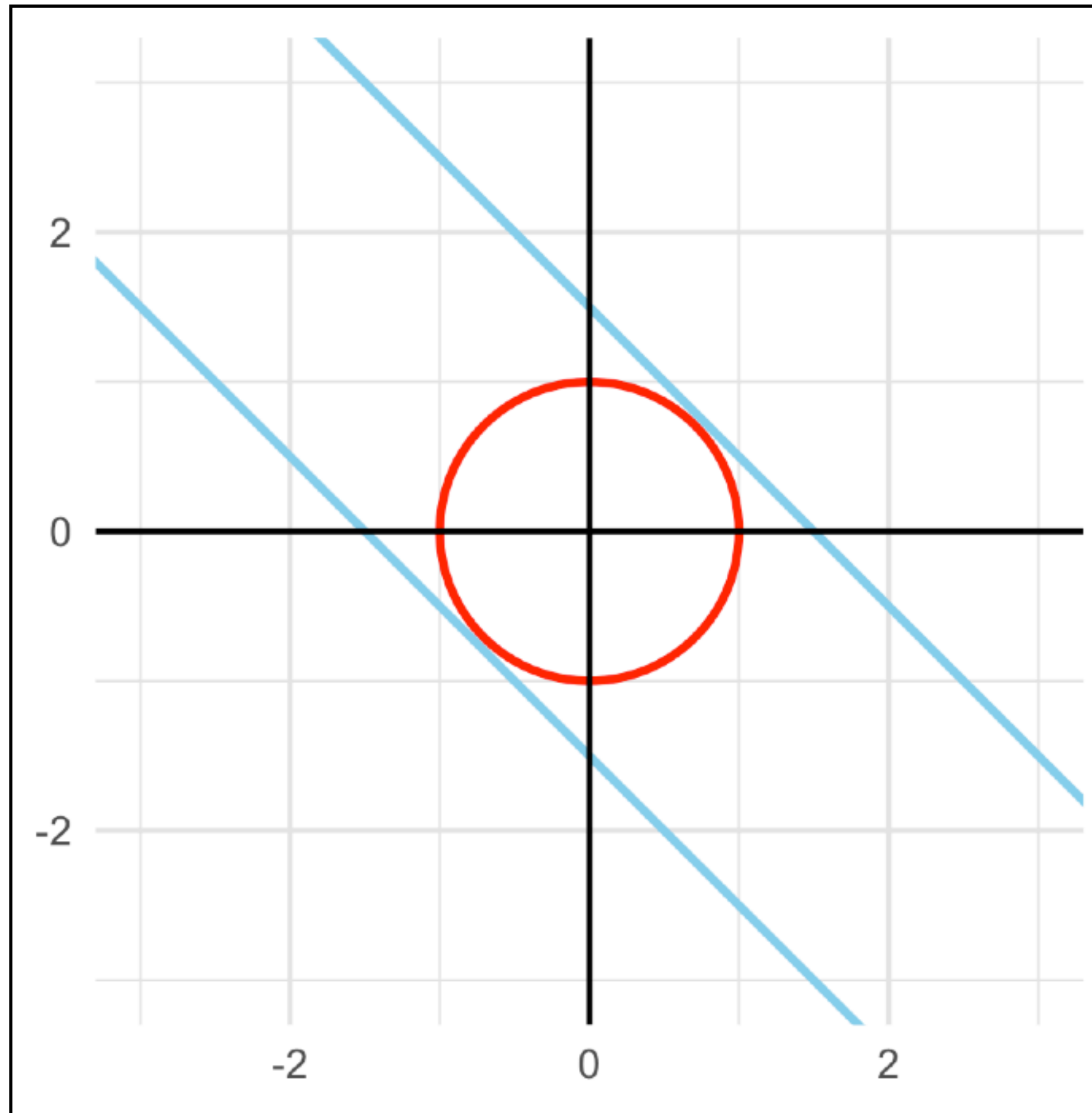
$$\|\beta\| = \sqrt{\sum_{j=1}^p \beta_j^2} \text{ is the Euclidean norm of } \beta$$

$$\left\{ \frac{y_i(\beta_0 + \beta^T x_i)}{\|\beta\|} \right\}$$



What is a Constrained Optimization Problem?

Optimize $f(x, y)$ subject to $g(x, y) = k$



$$f(x, y) = 2x + y$$

$$g(x, y) = x^2 + y^2 = 1$$