

# the matrix

- A **diagonal matrix** is a square matrix with non-zero elements only on the main diagonal
- An **identity matrix** is a diagonal matrix in which all elements on the main diagonal are 1:

$$D_{n \times n} = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \quad I_{n \times n} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The identity matrix is special because, when multiplied by another matrix, it produces the original matrix back again (we'll return to this later after covering matrix multiplication)
- A **lower triangular matrix** has non-zero elements only on or below the main diagonal
- An **upper triangular matrix** has non-zero elements only on or above the main diagonal
- A **symmetric matrix** is a square matrix with elements symmetric such that  $a_{ij} = a_{ji}$

# the transpose of a matrix

Let  $A$  be an  $m \times n$  matrix. The transpose of  $A$ , denoted  $A^T$  or  $A'$ , is the  $n \times m$  matrix whose columns are the respective rows of  $A$ .

- A matrix is symmetric if it doesn't change when you take its transpose

*example*

If you take the transpose of matrices  $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

we get  $A^T = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$  and  $B^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ .

Note: matrix  $B$  is thus symmetric.