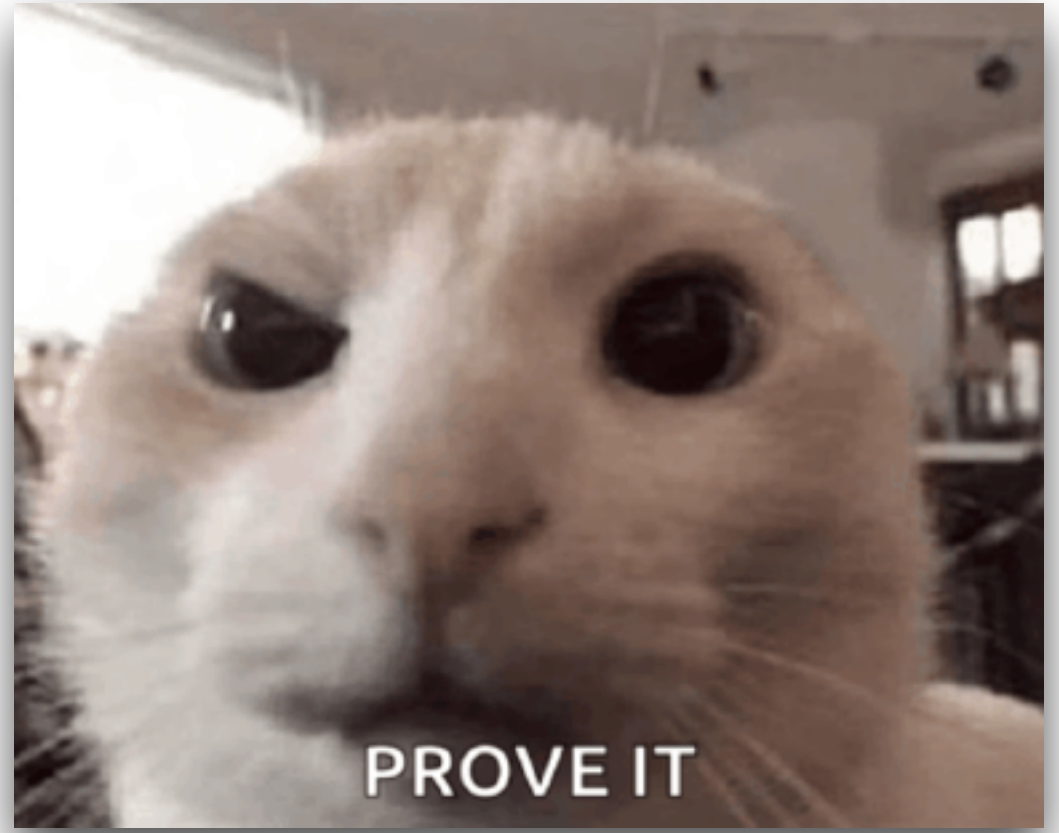

PROVE IT

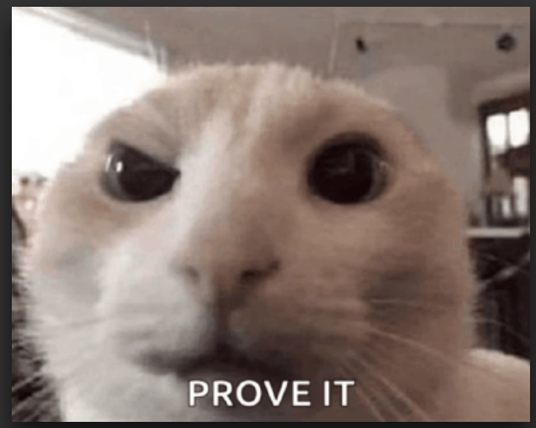
proof by induction

Proof cont'd.

proof by induction

Theorem

The sum of the first n powers of two is $2^n - 1$.



Proof cont'd.

- The inductive step:
  - the goal here is to prove "if $P(k)$ then $P(k+1)$ is true"
  - to do this we choose an arbitrary k, assume $P(k)$ is true, then try to prove $P(k+1)$

    $\Longrightarrow$ assume that for some arbitrary $k \in \mathbb{N}$ that $P(k)$ holds, meaning that

    $$2^0 + 2^1 + \cdots + 2^{k-1} = 2^k - 1$$

  - we need to show that $P(k+1)$ holds, meaning the sum of the first $k+1$ powers of two is $2^{k+1} - 1$

    $$2^0 + 2^1 + \cdots + 2^{k-1} + 2^k = 2^k - 1 + 2^k$$
    $$= 2(2^k) - 1$$
    $$= 2^{k+1} - 1 \checkmark$$

- Therefore, $P(k+1)$ is true, completing the induction. ■

indirect proofs