Bayes Rule

probability rules

{rule of total probability}

the Bayesian interpretation

probability rules

Bayes Rule

If events $A_1, A_2, \ldots, A_k$ constitute a partition of the sample space Ω and $P(A_i) \neq 0 \ \forall i$, then for any event B in Ω such that $P(B) \neq 0$

$$
\begin{aligned}
P(A_i \mid B) &= \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \cdots + P(A_k)P(B \mid A_k)} \\
&= \frac{P(A_i)P(B \mid A_i)}{\sum_{j=1}^{k} P(A_j \mid B)P(A_j)} \quad \text{\{rule of total probability\}} \\
&= \frac{P(A_i)P(B \mid A_i)}{P(B)}
\end{aligned}
$$

This theorem is consistent with the Bayesian interpretation of probability theory

probability rules

exercise 5

In an experiment on human memory, participants have to memorize a set of words (B_1), numbers (B_2), and pictures (B_3). These occur in the experiment with the probabilities $P(B_1) = 0.5,\ P(B_2) = 0.4,\ P(B_3) = 0.1.$

Then participants have to recall the items (where A is the recall event). The results show that $P(A \mid B_1) = 0.4,\ P(A \mid B_2) = 0.2,\ P(A \mid B_3) = 0.1.$

(a) Compute $P(A)$, the probability of recalling an item.

(b) What is the probability that an item that is correctly recalled (A) is a picture (B_3)?