substitution

- Choose the easiest variable to solve for and plug this expression for the variable into the other two equations you did not yet use
- Three things can happen:

1. There is the same number of equations as unknowns $\Longrightarrow$ uniquely determined
2. There are more unknowns than equations i.e. infinite number of solutions $\Longrightarrow$ underdetermined
3. There are more equations than unknowns (equations are contradictory) $\Longrightarrow$ overdetermined

substitution

$$2x - y + 3z = 9 \quad (1)$$
$$x + 4y - 5z = -6 \quad (2)$$
$$x - y + z = 2 \quad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- We solve equation (3) and substitute it into equation (2) and (1): $x = y - z + 2$

  $\implies 2(y - z + 2) - y + 3z = 9 \implies 2y - 2z + 4 - y + 3z = 9 \implies y + z = 5 \quad (4)$

  $\implies (y - z + 2) + 4y - 5z = -6 \implies y - z + 2 + 4y - 5z = -6 \implies 5y - 6z = -8 \quad (5)$

- Now solve equations (4) and (5):

  From (4) we have $y = 5 - z$ which is substituted into equation (5)

  $5(5 - z) - 6z = -8 \implies 25 - 5z - 6z = -8 \implies -11z = -33 \implies z = 3$

  Substitute back into y: $y = 5 - 3 = 2$

  and finally from $x = y - z + 2$ we get $x = 2 - 3 + 2 = 1$