the Jacobian

example

Let f be a transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ with the following Jacobian matrix:

$$J = \begin{bmatrix} 3x^2 - 4 & 0 \\ 0 & 3y^2 - 4 \end{bmatrix}$$

What is the determinant of f? How will f stretch or squish the space around the point $(1, -1)$?

where the Hessian, gradient and Jacobian meet

- The gradient points in the direction of steepest ascent.
- The Jacobian describes how the components of a vector function change with respect to changes in input variables
- The Hessian describes the local curvature of a scalar function

Matrix	Purpose	Function Type	Size
Gradient ∇f	First-order derivatives	$f : \mathbb{R}^n \to \mathbb{R}$	$n \times 1$
Jacobian J	First-order derivatives of vector functions	$f : \mathbb{R}^n \to \mathbb{R}^m$	$m \times n$
Hessian H	Second-order derivatives	$f : \mathbb{R}^n \to \mathbb{R}$	$n \times n$

- Gradient is the Jacobian of a scalar function $f : \mathbb{R}^n \to \mathbb{R}$: $\nabla f = J$
- Hessian is the Jacobian of the Gradient ∇f : $H = J_{\nabla f}$