Cramer's rule

$$2x - y + 3z = 9 \qquad (1)$$
$$x + 4y - 5z = -6 \qquad (2)$$
$$x - y + z = 2 \qquad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- we already know the determinant of A: $\det(A) = -11$
- form the B_i by replacing each of the three columns by $\vec{b}$

$$B_1 = \begin{bmatrix} 9 & -1 & 3 \\ -6 & 4 & -5 \\ 2 & -1 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 2 & 9 & 3 \\ 1 & -6 & -5 \\ 1 & 2 & 1 \end{bmatrix},\ B_3 = \begin{bmatrix} 2 & -1 & 9 \\ 1 & 4 & -6 \\ 1 & -1 & 2 \end{bmatrix}$$

- compute determinant for each B_i: $\det(B_1) = -11,\ \det(B_2) = -22,\ \det(B_3) = -33$
- apply Cramer's rule:

$$x = \frac{\det(B_1)}{\det(A)}, \quad y = \frac{\det(B_2)}{\det(A)}, \quad z = \frac{\det(B_3)}{\det(A)} \implies \boxed{x = \frac{-11}{-11} = 1, \quad y = \frac{-22}{-11} = 2, \quad z = \frac{-33}{-11} = 3}$$

Cramer's rule

exercise 2

Use Cramer's Rule to solve the linear system where $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$