diagonalization

example

$$\begin{bmatrix} -12 & 15 \\ -10 & 13 \end{bmatrix} \text{ is diagonalizable because } \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}^{-1}$$

Note: any diagonal matrix D is diagonalizable because it is similar to itself. For instance,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = I \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} I^{-1}$$

powers of diagonalizable matrices

Multiplying diagonal matrices together just multiplies their diagonal entries:

$$\begin{bmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix} \begin{bmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{bmatrix} = \begin{bmatrix} x_1y_1 & 0 & 0 \\ 0 & x_2y_2 & 0 \\ 0 & 0 & x_3y_3 \end{bmatrix}$$

so it is easy to take powers of a diagonal matrix:

$$\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}^n = \begin{bmatrix} x^n & 0 & 0 \\ 0 & y^n & 0 \\ 0 & 0 & z^n \end{bmatrix}$$

$\Longrightarrow$ if $A = QDQ^{-1}$ where D is the diagonal matrix, then $A^n = QD^nQ^{-1}$