Cramer's rule

Cramer's rule

- only works when there an equal number of equations and unknowns (i.e. a square matrix A) and A is nonsingular
- this rules states that we can solve for $\vec{x}$ using the formula:

$$x_i = \frac{|B_i|}{|A|}$$

where the matrix B_i is formed by replacing the ith column of A (the column corresponding to variable x_i) with $\vec{b}$

1. take the determinant of A, to check to make sure that we can apply this rule and determine the denominator of each x_i
2. we form the B_i by replacing each of the three columns by $\vec{b}$
3. compute the determinants of the matrices B_i
4. apply Cramer's rule according to formula

Cramer's rule

$$\begin{aligned} 2x - y + 3z &= 9 \qquad (1) \\ x + 4y - 5z &= -6 \qquad (2) \\ x - y + z &= 2 \qquad (3) \end{aligned}$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- we already know the determinant of A: $\det(A) = -11$
- form the B_i by replacing each of the three columns by $\vec{b}$

$$B_1 = \begin{bmatrix} 9 & -1 & 3 \\ -6 & 4 & -5 \\ 2 & -1 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 2 & 9 & 3 \\ 1 & -6 & -5 \\ 1 & 2 & 1 \end{bmatrix}, B_3 = \begin{bmatrix} 2 & -1 & 9 \\ 1 & 4 & -6 \\ 1 & -1 & 2 \end{bmatrix}$$

- compute determinant for each B_i: $\det(B_1) = -11,\ \det(B_2) = -22,\ \det(B_3) = -33$
- apply Cramer's rule:

$$x = \frac{\det(B_1)}{\det(A)}, \quad y = \frac{\det(B_2)}{\det(A)}, \quad z = \frac{\det(B_3)}{\det(A)} \implies \boxed{x = \frac{-11}{-11} = 1, \quad y = \frac{-22}{-11} = 2, \quad z = \frac{-33}{-11} = 3}$$