the Jacobian

If we have a function $f : \mathbb{R}^n \to \mathbb{R}^m$ mapping an $n-$dimensional input to an $m-$dimensional output,

$$f(x_1, x_2, \ldots, x_n) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ f_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_m(x_1, x_2, \ldots, x_n) \end{bmatrix}$$

, then the Jacobian matrix contains all first-order partial derivatives of f :

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

the Jacobian

- The Jacobian matrix is a matrix of all first-order partial derivatives of a vector-valued function. It generalizes the concept of a derivative to multiple variables and dimensions.
- Measures how a function transforms space:
  It describes the local scaling, rotation, or shearing of a function.
- Useful in nonlinear transformations
- The Jacobian determinant represents the factor by which the transformation stretches or squishes the $n-$dimensional volumes around a certain input.