The Radial Kernel (RBF)

The **Radial Kernel**

$$K(a,b) = e^{-\gamma}(a-b)^2$$

projects to **infinite dimensional** space
works similar to nearest neighbors classifier

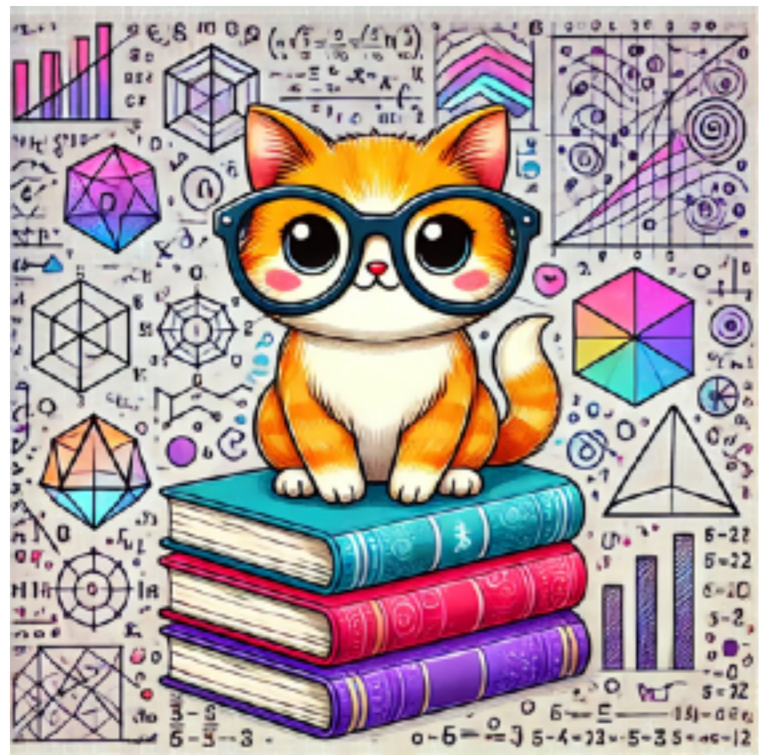
we can use the Polynomial Kernel to get the intuition
behind how Radial Kernel works in infinite dimensions

$$K(a,b) = (a \cdot b)^d$$

$$\boxed{ab + a^2b^2 + a^3b^3 + \cdots + a^\infty b^\infty = (a, a^2, a^3, \ldots, a^\infty)(b, b^2, b^3, \ldots, b^\infty)}$$

take sum for infinite terms gives dot product with infinite dimensions!

The Radial Kernel: Taylor Series Expansion



$$K(a,b) = e^{-\gamma}(a-b)^2 = e^{-\gamma(a^2+b^2-2ab)} = e^{-\gamma(a^2+b^2)}e^{\gamma 2ab}$$

set $\gamma = \dfrac{1}{2} \implies e^{-\frac{1}{2}\gamma(a^2+b^2)}\boxed{e^{ab}}$ Taylor expansion of this term

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(\infty)}(a)}{\infty!}(x-a)^\infty$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^\infty}{\infty!}$$

$$e^{ab} = 1 + (ab) + \frac{(ab)^2}{2!} + \frac{(ab)^3}{3!} + \cdots + \frac{(ab)^\infty}{\infty!}$$

each term contains Polynomial Kernel with $r = 0$ and d from 0 to $d = \infty$