harmonic series

harmonic series

harmonic series

the harmonic series is the infinite series formed by summing all positive unit fractions

The **harmonic series** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (does not have a finite limit).

Proof by contradiction.

- Suppose the series converges to S: $S = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots$
- Then: $\frac{1}{2}S = \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2n} + \cdots$
- Therefore, the sum of the odd-numbered terms: $1 + \frac{1}{3} + \cdots + \frac{1}{2n-1} + \cdots$ must be the other half of S
- However this is impossible since $\frac{1}{2n-1} > \frac{1}{2n}$ for each positive integer n. ■

limit of a function