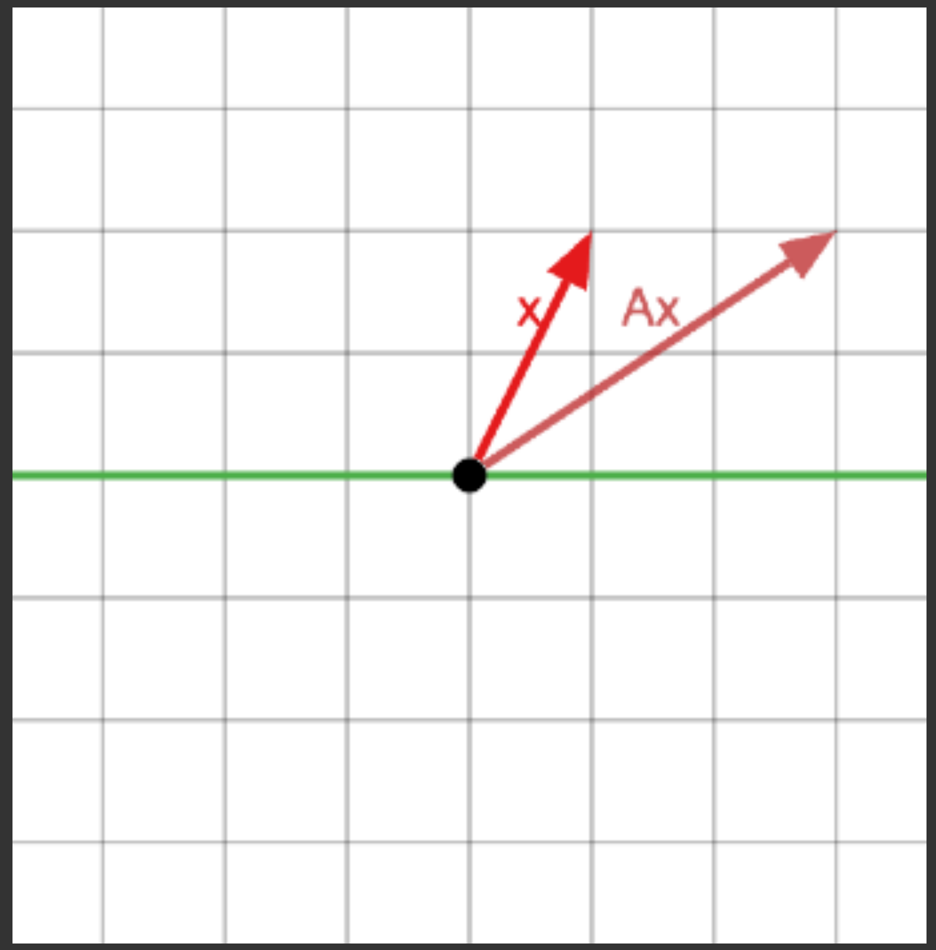
diagonalization

example

Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. The characteristic polynomial of A is $f(\lambda) = (\lambda - 1)^2$ so the eigenvalue of A is 1.

For $\lambda = 1$, solve $A\vec{v} = \lambda\vec{v}$ or $(A - \lambda I)\vec{v} = 0$:

$A - \lambda I = \begin{bmatrix} 1-1 & 1 \\ 0 & 1-1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ which gives equation $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



From the first row: $0x + 1y = 0 \quad \Longrightarrow \quad y = 0$

and there is no restriction on x so let $x = t$ (a free variable).

The eigenvector is a 1-eigenspace:

$$\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ 0 \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \end{bmatrix} \Longrightarrow \text{Basis: } \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

The 1-eigenspace is exactly the x-axis, so all of the eigenvectors of A lie on the x-axis. It follows that A does not admit two linearly independent eigenvectors, so by the diagonalization theorem, it is not diagonalizable.

diagonalization

exercise 2

Diagonalize the matrix $A = \begin{bmatrix} 1/2 & 3/2 \\ 3/2 & 1/2 \end{bmatrix}$.