binomial random variable

- A r.v. modeling the number of successes in a fixed number of independent Bernoulli trials.
- Discrete outcomes $\{0,1,2,3\ldots,n\}$
- Two parameter
  - p - probability of a success for each trial
  - n - number of trials
- Shorthand notation: $X \sim \text{Binom}(n,p)$
- $E(X) = np,\ V(X) = np(1-p)$

$$P(X = x \,|\, n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

X	$P(X = x)$
0	$\binom{n}{0} p^0 (1-p)^n$
1	$\binom{n}{1} p^1 (1-p)^{n-1}$
2	$\binom{n}{2} p^2 (1-p)^{n-2}$
$\vdots$	$\vdots$
n-1	$\binom{n}{(n-1)} p^{n-1} (1-p)^1$
n	$\binom{n}{n} p^n (1-p)^0$

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example (cont'd...)

Toss a coin 3 times: the sample space is $\Omega : \{H, T\} \times \{H, T\} \times \{H, T\}$

Define the random variable: $X =$ the number of heads

What is the probability distribution of X?

$$X \sim Bin(n = 3, p = 0.5)$$

$$\implies P(X = x) = \binom{x}{n} p^x (1 - p)^{n-x} = \binom{x}{3} 0.5^x (0.5)^{3-x}$$