Where the Hessian, gradient and Jacobian meet

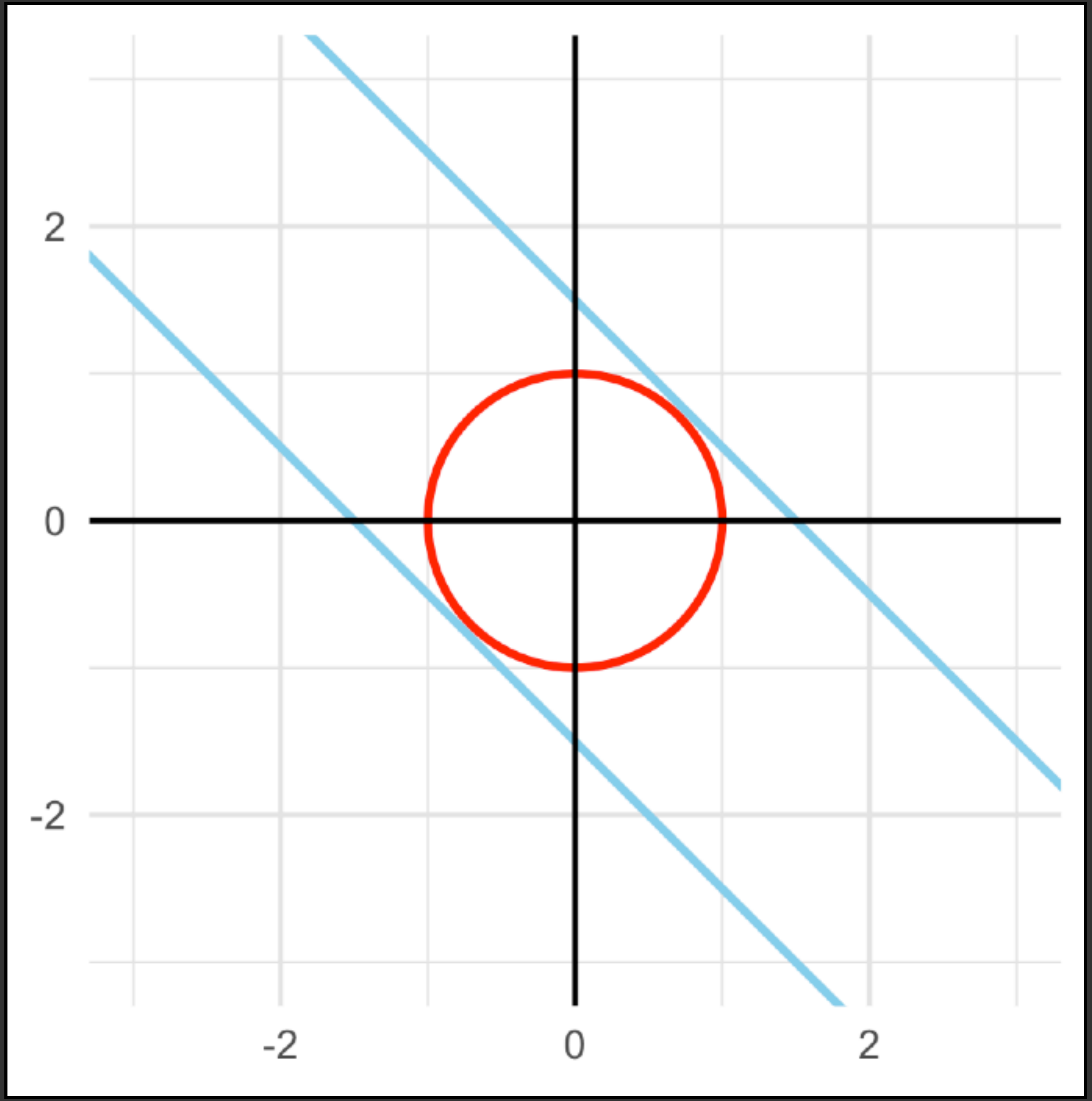
- The gradient points in the direction of steepest ascent.
- The Jacobian describes how the components of a vector function change with respect to changes in input variables
- The Hessian describes the local curvature of a scalar function

Matrix	**Purpose**	**Function Type**	**Size**
Gradient ∇f	First-order derivatives	$f : \mathbb{R}^n \to \mathbb{R}$	$n \times 1$
Jacobian J	First-order derivatives of vector functions	$f : \mathbb{R}^n \to \mathbb{R}^m$	$m \times n$
Hessian H	Second-order derivatives	$f : \mathbb{R}^n \to \mathbb{R}$	$n \times n$

- Gradient is the Jacobian of a scalar function $f : \mathbb{R}^n \to \mathbb{R}$: $\quad \nabla f = J$
- Hessian is the Jacobian of the Gradient ∇f : $\quad H = J_{\nabla f}$

for your awareness: constrained optimization

optimize $f(x, y)$ subject to $g(x, y) = k$



$$f(x, y) = 2x + y$$

$$g(x, y) = x^2 + y^2 = 1$$