Taylor approximation


example

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Find the Taylor polynomial of $f(x) = \dfrac{1}{1+x}$ of degree 3 at $x = 0$.

We have that $TP_3(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2}x^2 + \dfrac{f'''(0)}{3!}x^3$

Computing the successive derivatives we get

$$f(x) = \frac{1}{1+x}, \quad f'(x) = \frac{-1}{(1+x)^2}, \quad f''(x) = \frac{2}{(1+x)^3}, \quad f'''(x) = \frac{-6}{(1+x)^4}$$

Substituting $x = 0$ we get

$$f(0) = 1, \quad f'(0) = -1, \quad f''(x) = 2, \quad f'''(x) = -6$$

Therefore, the Taylor polynomial of f of degree 3 at $x = 0$ is equal to

$$TP_3(x) = 1 - x + x^2 - x^3$$

Taylor approximation

exercise 2

Approximate the function $f(x) = \sqrt{1+x}$ using a Taylor polynomial of degree 3 centered at $x = 0$.
Compare it to the true value.