indirect proofs: proof by contradiction


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Proof.

indirect proofs: proof by contradiction

Theorem

For any $n \in \mathbb{Z}\, n$, if n^2 is even, then n is even.

Proof.

- Assume for the sake of contradiction that n is an integer and that n^2 is even, but that n is odd.
- Since n is odd, there is some integer k such that $n = 2k + 1$.
- Squaring both sides of this equality and simplifying yields the following:

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \end{aligned}$$

- This tells us that n^2 is odd, which is impossible, by assumption n^2 is even.
- We have a contradiction so our assumption is incorrect
  $\Longrightarrow$ if n is an integer and n^2 is even. then n is also even. ■