gradient

The gradient of a scalar function $f(x_1, x_2, \ldots, x_n)$ is a vector field that points in the direction of the greatest rate of increase of f.

For a function $f : \mathbb{R}^n \to \mathbb{R}$, the gradient is denoted as:

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right)$$

where each component is a partial derivative of f with respect to one of the variables.

Direction: The gradient points in the direction of the steepest ascent of f

Magnitude: The magnitude $||\nabla f||$ represents the rate of the steepest increase.

Zero Gradient: If $\nabla f = 0$, the point is a critical point (possible max, min, or saddle point).

gradient

The gradient captures all the partial derivative information of a multivariable function.

example

For $f(x, y) = x^2 + y^2$, the gradient is:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (2x, 2y)$$