the symmetry of second partial derivatives

example

$$f(x, y) = x^2y + 3xy^3$$

$$\frac{\partial f}{\partial x} = 2xy + 3y^3$$

$$\frac{\partial f}{\partial y} = x^2 + 9xy^2$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = 2y$$

$$\frac{\partial^2 f}{\partial xy} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = 2x + 9y^2$$

$$\frac{\partial^2 f}{\partial yx} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = 2x + 9y^2$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = 18xy$$

the symmetry of second partial derivatives

Schwarz's theorem

If the second partial derivatives are continuous, the order of differentiation is not important and we therefore have:

$$\frac{\partial^2 f}{\partial xy} = \frac{\partial^2 f}{\partial yx}$$