global min/max

example

For Function: $f(x, y) = 4 - x^2 - y^2$ the Hessian is given by

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

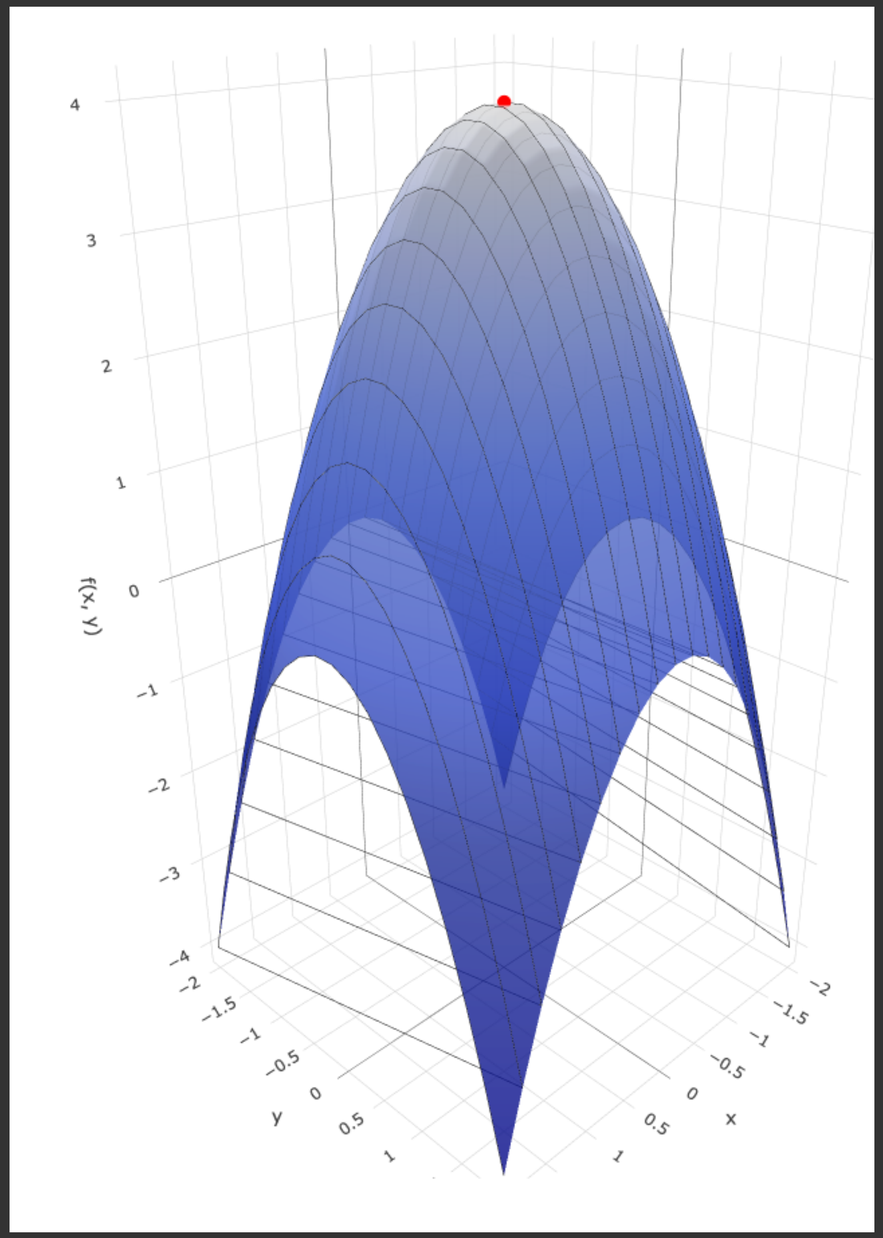
Both eigenvalues are negative (-2, -2), so H is negative definite

$\Longrightarrow$ Local max at (0,0)

Is it global?

Since $f(x, y) \to -\infty$ as $|x|, |y| \to \infty$, the function is unbounded and has only one maximum, which must be global

$\Longrightarrow$ (0,0) is the global maximum, and f(0,0) = 4



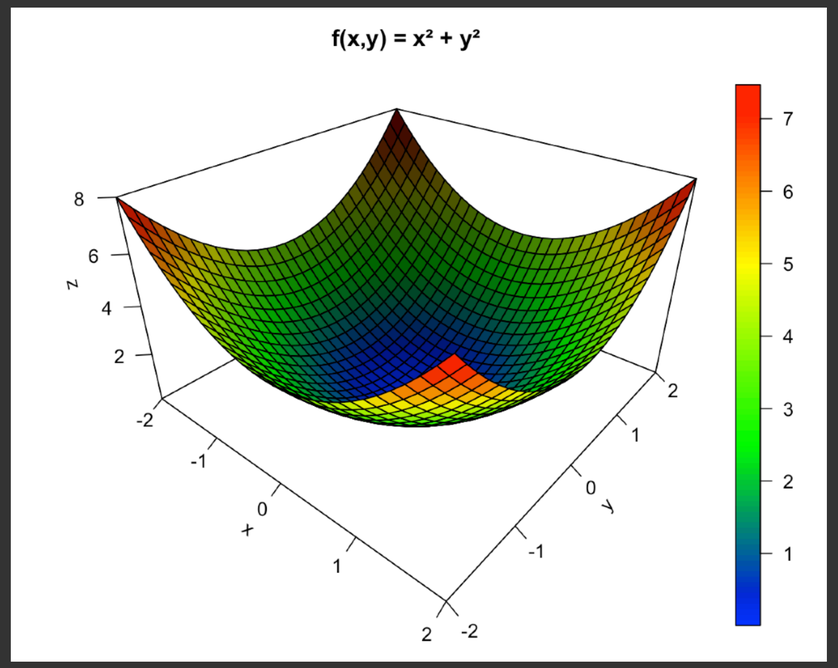
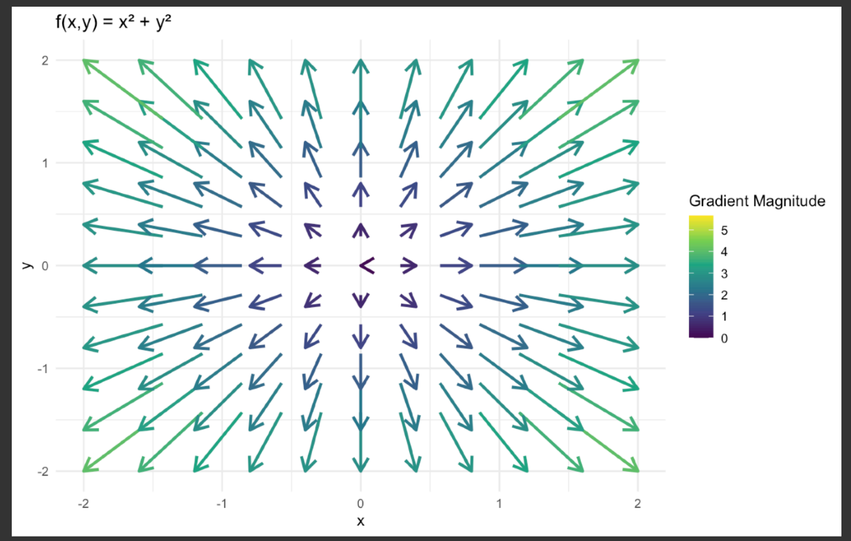
Hessian

The Hessian provides a way to classify critical points (where the gradient is zero).

example

For $f(x, y) = x^2 + y^2$, the Hessian is:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Diagonal matrix with positive values indicating function is convex

Eigenvalues?