diagonalization

Matrix diagonalization

diagonalizable

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- Diagonal matrices are the easiest kind of matrices to understand: they just scale the coordinate directions by their diagonal entries.
- Matrix diagonalization is powerful: it transforms a given square matrix into a diagonal matrix,
  - which is much easier to analyze and compute because their non-diagonal elements are zero
  - e.g. calculations like powers and determinants easy to perform

An $n \times n$ matrix A is diagonalizable if it is similar to a diagonal matrix: that is, if there exists an invertible$n \times n$ matrix Q and a diagonal matrix D such that $A = QDQ^{-1}$.

diagonalization

example

$$\begin{bmatrix} -12 & 15 \\ -10 & 13 \end{bmatrix} \text{ is diagonalizable because } \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}^{-1}$$

Note: any diagonal matrix D is diagonalizable because it is similar to itself. For instance,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = I \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} I^{-1}$$