Gaussian elimination

- We eliminate one variable by combining equations

$$2x - y + 3z = 9 \quad (1)$$
$$x + 4y - 5z = -6 \quad (2)$$
$$x - y + z = 2 \quad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- Eliminate x:
  - subtracting equation (3) from (2): $(x + 4y - 5z) - (x - y + z) = -6 - 2 \implies 5y - 6z = -8 \quad (4)$
  - subtract $2\times$ equation (3) from equation (1): $(2x - y + 3z) - 2(x - y + z) = 9 - 2(2) \implies y + z = 5 \quad (5)$
- Solve for y and z:
  - from equation (5) we get a new equation: $y = 5 - z \quad (6)$ which is substituted into (4):
    $5(5 - z) - 6z = -8 \implies 25 - 5z - 6z = -8 \implies -11z = -33 \implies z = 3$
  - from (6) we get $y = 5 - 3 = 2$ and from (3) we get $x = y - z + 2 \implies x = 2 - 3 + 2 = 1$

matrix inversion