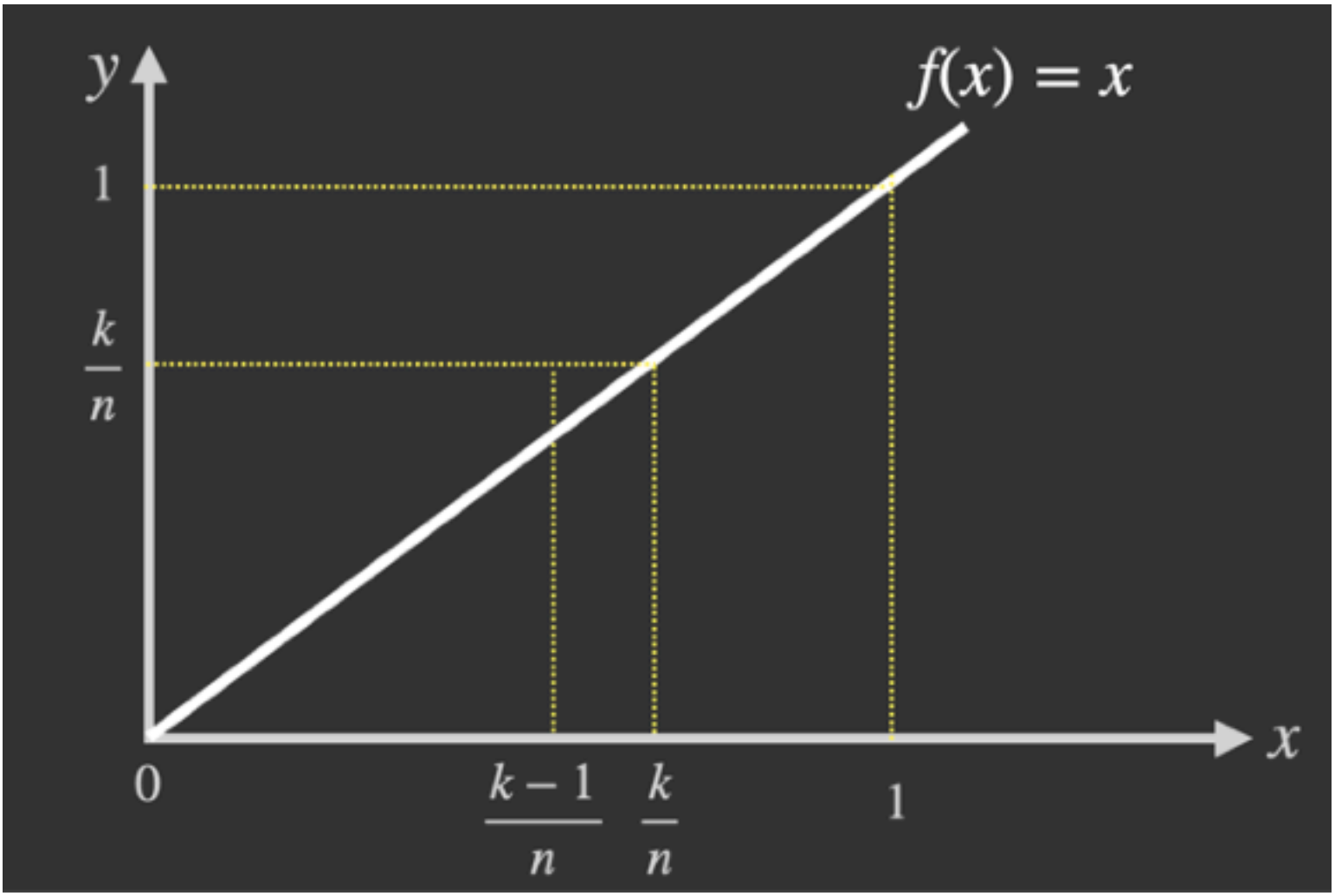
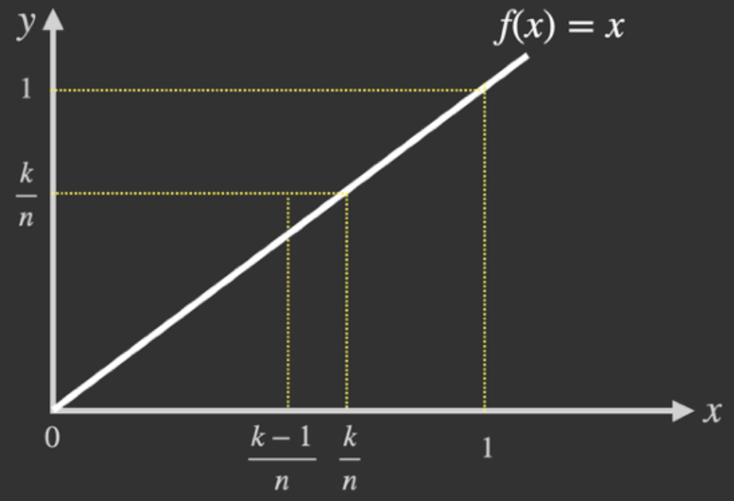

y
f(x) = x
1
k/n
0
(k − 1)/n
k/n
1
x

Riemann sum

Riemann sum



- The sum of the area of all rectangles on the interval is

$$\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{k}{n^2}+\cdots+\frac{n}{n^2}=\frac{1}{n^2}(1+2+\cdots+k+\cdots+n)$$

$$=\frac{1}{n^2}\frac{n(n+1)}{2}$$

$$=\frac{1}{2}\left(\frac{n+1}{n}\right)$$

$$=\frac{1}{2}\left(1+\frac{1}{n}\right)$$

Riemann sum