the characteristic polynomial

Eigenvalues are roots of the characteristic polynomial

Let A be a $n \times n$ matrix and let $f(\lambda) = \det(A - \lambda I)$ be its characteristic polynomial.

Then a number λ_0 is an eigenvalue of A if and only if $f(\lambda_0) = 0$.

example cont'd

$$f(\lambda) = \lambda^2 - 6\lambda + 1 = 0$$

$$\implies \lambda = 3 - 2\sqrt{2} \quad \text{and} \quad \lambda = 3 + 2\sqrt{2}.$$

To compute the eigenvectors, we solve the homogeneous system of equations $(A - \lambda I)\vec{v} = 0$ for each eigenvalue λ.

the characteristic polynomial: a shortcut

Recall the trace of the square matrix: Let A be an $n \times n$ matrix. The trace of A, denoted $tr(A)$, is the sum of the diagonal elements of A. That is,

$$tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$$

The characteristic polynomial for a 2×2 matrix

Let A be a 2×2 matrix. All coefficients of the characteristic polynomial can be found via

$$f(\lambda) = \lambda^2 + tr(A)\lambda + \det(A)$$

this is generally the fastest way to compute the characteristic polynomial of a 2×2 matrix.

example cont'd

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \implies f(\lambda) = \lambda^2 - tr(A)\lambda + \det(A) = \lambda^2 - 6\lambda + 1$$