

matrix inversion

$$2x - y + 3z = 9 \quad (1)$$

$$x + 4y - 5z = -6 \quad (2)$$

$$x - y + z = 2 \quad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- Find the determinant of A to make sure it's invertible (via cofactor expansion along first row): $\det(A) = -11$
- This is not zero, the matrix is nonsingular, and we can invert it
- To invert the matrix we compute 9 minors:

$$M_{11} = -1, M_{12} = 6, M_{13} = -5, M_{21} = 2, M_{22} = -1, M_{23} = -1, M_{31} = -7, M_{32} = -13, M_{33} = 9$$

- Now use the formula for the inverse from the previous lecture $A^{-1} = \frac{1}{|A|} C^T$ where $C_{ij} = (-1)^{i+j} M_{i,j}$

- This gives: $A^{-1} = \frac{1}{-11} \cdot \begin{bmatrix} -1 & -2 & -7 \\ -6 & -1 & 13 \\ -5 & 1 & 9 \end{bmatrix}$

- And finally we multiply with \vec{b} to get the solution: $\vec{x} = A^{-1}\vec{b} = \frac{1}{-11} \cdot \begin{bmatrix} -1 & -2 & -7 \\ -6 & -1 & 13 \\ -5 & 1 & 9 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Cramer's rule

- only works when there an equal number of equations and unknowns (i.e. a square matrix A) and A is nonsingular
- this rules states that we can solve for \vec{x} using the formula:

$$x_i = \frac{|B_i|}{|A|}$$

where the matrix B_i is formed by replacing the i^{th} column of A (the column corresponding to variable x_i) with \vec{b}