

Smoothing Splines

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- Unlike regression splines and natural splines, there are no knots!
- The discrete problem of selecting a number of knots into a continuous penalization problem
- We seek a function g among all possible functions (linear + non-linear) which minimizes

$$\underbrace{\text{model fit} + \text{penalty term}}_{\text{not the usual RSS}} = \sum_{i=1}^n (y_i - g(x_i))^2 + \lambda \int \underbrace{(g''(t))^2 dt}_{\text{catches wiggles or non-linearities}}$$

- The function g that minimizes the above quantity is called a **smoothing spline**
- $\lambda \geq 0$ is the tuning penalty parameter, also called **roughness penalty**
 - when $\lambda = 0$ we get an extremely wiggly non-linear function g (completely useless)
 - as λ increases, the function becomes smoother
 - theoretically: when $\lambda \rightarrow \infty$, g'' is zero everywhere $\implies g(X) = \beta_0 + \beta_2 X$ i.e. linear model
- the solution for any finite and non-zero λ is that the function g is a natural cubic spline but with knots placed on each individual sample point $x_1, x_2, x_3, \dots, x_n$