linear independence

example

Let $\vec{v_1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -2 \\ 1 \\ -4 \end{bmatrix}$. Are they linearly dependent?

Let's express each vector as the linear combination of the other two $\vec{v_3} = a\vec{v_2} + b\vec{v_1}$

$$\implies \begin{bmatrix} -2 \\ 1 \\ -4 \end{bmatrix} = a \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

. This gives us three equations, one for each entry:

$$-2 = 0a + 1b \rightarrow b = -2$$

$$1 = 1a + 0b \rightarrow a = 1$$

$$-4 = 1a + 2b \rightarrow -3 = 1 - 4$$

Is this enough to say that they are linearly independent? No, because we have to show that any vector in this set can be expressed as a linear combination of the rest of the vectors.

linear independence

- Given 3 vectors $\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}$, they are called linearly independent if and only if none of them is a linear combination of the others:

$$\overrightarrow{v_1} \neq a\overrightarrow{v_2} + b\overrightarrow{v_3} \text{ for any } a, b \in \mathbb{R}$$

$$\overrightarrow{v_2} \neq a\overrightarrow{v_1} + b\overrightarrow{v_3} \text{ for any } a, b \in \mathbb{R}$$

$$\overrightarrow{v_3} \neq a\overrightarrow{v_1} + b\overrightarrow{v_2} \text{ for any } a, b \in \mathbb{R}$$

- This is equivalent to saying that:

$$a\overrightarrow{v_1} + b\overrightarrow{v_2} + c\overrightarrow{v_3} = 0 \quad \textit{iff} \quad a = b = c = 0$$