

eigenvalue decomposition summarized

- Let $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ be the eigenvectors of matrix A and let $\lambda_1, \lambda_2, \dots, \lambda_n$ be corresponding eigenvalues
- Consider now a matrix Q whose columns are $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$
- We have now

$$\begin{aligned} AQ &= A \begin{bmatrix} | & | & | & | \\ \vec{v}_1 & \vec{v}_2 & \dots & \vec{v}_n \\ | & | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ A\vec{v}_1 & A\vec{v}_2 & \dots & A\vec{v}_n \\ | & | & | & | \end{bmatrix} \\ &= \begin{bmatrix} | & | & | & | \\ \lambda_1 \vec{v}_1 & \lambda_2 \vec{v}_2 & \dots & \lambda_n \vec{v}_n \\ | & | & | & | \end{bmatrix} \\ &= \begin{bmatrix} | & | & | & | \\ \vec{v}_1 & \vec{v}_2 & \dots & \vec{v}_n \\ | & | & | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} = QD \end{aligned}$$



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- If Q^{-1} exists, then we can write

$$A = QDQ^{-1} \quad \text{eigenvalue decomposition}$$

$$Q^{-1}AQ = D \quad \text{diagonalization of } A$$

