matrix inversion

augmented matrix

row echelon form

matrix inversion

- $A\vec{x} = \vec{b}$ can be used to create an augmented matrix by taking our A and adding the column $\vec{b}$ to it as a new column on the right
- We can then attempt to get the identity matrix in all but the last column
- If we can do this, then we get what is called reduced row echelon form
- We can do this, then we can read off the answers to the system by returning to equation form, since each row will provide the value of one of the variables
- If the rank of A is equal to its number of rows, then the system has at least one solution (i.e. each equation is linearly independent, and so cannot produce a contradiction)
- If the rank of A is equal to its number of columns, then the system has at most one solution (i.e. the number of independent rows, corresponding to equations, is at least as great as the number of variables)
- When the matrix A is square and nonsingular, we can invert the matrix to figure out what the unique solution is to the equation: $\vec{x} = A^{-1}\vec{b}$

matrix inversion

$$2x - y + 3z = 9 \quad (1)$$
$$x + 4y - 5z = -6 \quad (2)$$
$$x - y + z = 2 \quad (3)$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -5 \\ 1 & -1 & 1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} \implies A\vec{x} = \vec{b}$$

- Find the determinant of A to make sure it's invertible (via cofactor expansion along first row): $\det(A) = -11$
- This is not zero, the matrix is nonsingular, and we can invert it
- To invert the matrix we compute 9 minors:

$$M_{11} = -1,\ M_{12} = 6,\ M_{13} = -5,\ M_{21} = 2,\ M_{22} = -1,\ M_{23} = -1,\ M_{31} = -7,\ M_{32} = -13,\ M_{33} = 9$$

- Now use the formula for the inverse from the previous lecture $A^{-1} = \dfrac{1}{|A|}C^T$ where $C_{ij} = (-1)^{i+j}M_{i,j}$
- This gives: $\mathbf{A}^{-1} = \dfrac{1}{-11} \cdot \begin{bmatrix} -1 & -2 & -7 \\ -6 & -1 & 13 \\ -5 & 1 & 9 \end{bmatrix}$
- And finally we multiply with $\vec{b}$ to get the solution: $\vec{x} = A^{-1}\vec{b} = \dfrac{1}{-11} \cdot \begin{bmatrix} -1 & -2 & -7 \\ -6 & -1 & 13 \\ -5 & 1 & 9 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -6 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$