matrix arithmetic: matrix multiplication

example

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$. What is AB?

Compute each element of resulting matrix $C = A \times B$ by summing products of rows of A and columns of B:

$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$ where

$$c_{11} = 1 \cdot 5 + 2 \cdot 7 = 5 + 14 = 19$$
$$c_{12} = 1 \cdot 6 + 2 \cdot 8 = 6 + 16 = 22$$
$$c_{21} = 3 \cdot 5 + 4 \cdot 7 = 15 + 28 = 43$$
$$c_{22} = 3 \cdot 6 + 4 \cdot 8 = 18 + 32 = 50$$

$$\implies C = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

trace

Let A be an $n \times n$ matrix. The trace of A, denoted $tr(A)$, is the sum of the diagonal elements of A. That is,

$$tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$$

Properties of trace:

- $tr(A + B) = tr(A) + tr(B)$
- $tr(A - B) = tr(A) - tr(B)$
- $tr(kA) = k \cdot tr(A)$
- $tr(AB) = tr(BA)$
- $tr(A^T) = tr(A)$

The trace will come up again in reference to eigenvalues.