

# the symmetry of second partial derivatives

example

$$f(x, y) = x^2y + 3xy^3$$

$$\frac{\partial f}{\partial x} = 2xy + 3y^3$$

$$\frac{\partial f}{\partial y} = x^2 + 9xy^2$$

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial x} \right) \\ &= 2y\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 f}{\partial xy} &= \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) \\ &= 2x + 9y^2\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 f}{\partial yx} &= \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) \\ &= 2x + 9y^2\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 f}{\partial y^2} &= \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial y} \right) \\ &= 18xy\end{aligned}$$

# the symmetry of second partial derivatives

## Schwarz's theorem

If the second partial derivatives are continuous, the order of differentiation is not important and we therefore have:

$$\frac{\partial^2 f}{\partial xy} = \frac{\partial^2 f}{\partial yx}$$