the characteristic polynomial

example

Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

We have

$$f(\lambda) = \det(A - \lambda I) = \det\left(\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)$$

$$= \det\begin{bmatrix} 5-\lambda & 2 \\ 2 & 1-\lambda \end{bmatrix} = (5-\lambda)(1-\lambda) - 2 \cdot 2$$

$$= \lambda^2 - 6\lambda + 1$$

the characteristic polynomial

Eigenvalues are roots of the characteristic polynomial

Let A be a $n \times n$ matrix and let $f(\lambda) = \det(A - \lambda I)$ be its characteristic polynomial.

Then a number λ_0 is an eigenvalue of A if and only if $f(\lambda_0) = 0$.

example cont'd

$$f(\lambda) = \lambda^2 - 6\lambda + 1 = 0$$

$$\Longrightarrow \lambda = 3 - 2\sqrt{2} \quad \text{and} \quad \lambda = 3 + 2\sqrt{2}\,.$$

To compute the eigenvectors, we solve the homogeneous system of equations $(A - \lambda I)\vec{v} = 0$ for each eigenvalue λ.