From a theoretical oligopolistic model to a generative agent-based simulation

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Outline

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Starting questions

- Do entry, exit and changes in market structure affect the macroeconomy?

- Is there a link between the strategic interactions among oligopolistic firms and the macroeconomic equilibrium?

This questions are certainly not trivial in modern economies, where large oligopolistic firms play a relevant role and so many meetings among statesmen have the explicit scope of promoting contracts for some large and important firms of their countries.

However, surprisingly enough, the most popular theoretical models in the modern macroeconomic literature hardly see any explicit formalization for the macroeconomic effects of changes in market structure, entry, exit and strategic interactions among oligopolists.

Rethinking Macroeconomics with Endogenous Market Structure

Marco Mazzoli, Matteo Morini and Pietro Terna

Mazzoli, M., Morini, M., and Terna, P. 2019. *Rethinking Macroeconomics with Endogenous Market Structure*. Cambridge University Press.

Interactions among oligopolistic firms

We introduce a new macromodel where entry, exit and strategic interactions among oligopolistic firms are explicitly formalized and may generate macroeconomic fluctuations.

About macroeconomic impact of business formation we refer to Gabaix (2011). His "granular hypothesis" was initially studied by Jaimovich and Rebelo (2009).

Gabaix, X. 2011. The granular origins of aggregate fluctuations. *Econometrica*, **79**(3), 733?72.
Jaimovich, N., and Rebelo, S. 2009. Can news about the future drive the business cycle? *American Economic Review*, **99**(4), 1097–118.

Outputs

The output of each firm is given by the production function, where $\psi_{i,t}$ is a positive or negative shock:

$$\varphi_{i,t} = \Lambda L_{i,t}^{\alpha} + \psi_{i,t} \tag{1}$$

Summing up, with P_t the aggregate price level (as weighted average of firms' prices) and having $\psi_{i,t}$ a zero average:

$$Y_t = P_t \Lambda \sum_{i=1}^{H_t} L_{i,t}^{\alpha} \tag{2}$$

H_t is is the total number of oligopolistic firms operating at time t;
Λ the usual constant parameter capturing technology shocks;
$L_{i,t}$ the total amount of labour employed at time t by firm i;
$\psi_{i,t}$ an idiosyncratic stochastic shock reflecting (i) the Cournot-Nash equilibrium among the oligopolisti firms is in mixed strategies, herefore stochastic, plus unpredictable events (e.g., conflicts with the workers).

Aggregate demand

The microfounded optimization problem of the heterogeneous consumers with the same preferences but different budget constraint (depending on wether they are workers, new entrants or incumbent entrepreneurs) yields the following aggregate demand:

$$
D(\cdot)_t = \frac{\Omega(R_t)}{P_t}\{A_t + ((1+r_t)(1+\iota)^{-1}\sum_{i=0}^{\infty}[(1 + E\left(r_{t+i}\right)(1+\iota))]^{-i} \cdot
$$
$$
\cdot E(n_{t+i}(W_{t+i} + h_{t+i}^e\Pi_{t+i}^e + h_{t+i}^{in}\Pi_{t+i}^{in}))\} \tag{3}
$$

Π_{t+i}^{in} and Π_{t+i}^e are the nominal profits of the incumbent and new entrants entrepreneurs;
r_t is the real interest rate at time t; R_t is the nominal interest rate on the financial asset at time t (controlled by the central bank); ι is the «core» inflation rate, assumed to be constant under a given monetary policy regime;
W_{t+i} the nominal wage at time $t+i$; n_{t+i} the total number of employed individuals at time $t+i$, h_{t+i}^{in} and h_{t+i}^e the portion of incumbent entrepreneurs and new entrant over the total labor force;
Pi_{t+i} the price level emerging in the oligopolistic industrial sector (which is also the aggregate price level since we have an indifferentiated good);
Ω a monotonically increasing function in the nominal interest rate.

Equilibrium (1/2)

◇ Since the labor contracts establish the amount of hours to be worked by each worker, the labor is a sunk cost and, as a consequence, there is no need to distinguish between capacity and output decision.

◇ A few assumptions guarantee the existence of the aggregate equilibrium in the goods market.

◇ These assumptions correspond to those contained in Madden (1998), showing that, with a uniformly elastic demand function, the K-S-M two-stage quantity-price game reduces to the Cournot model.

Madden, P. 1998. Elastic demand, sunk costs and the Kreps-Scheinkman extension of the Cournot model. *Economic Theory*, **12**(1), 199–212.

Equilibrium (2/2)

- ◇ With Osborne and Rubinstein (1994, p.39):
  - mixed-strategy equilibria are stochastic steady states, as
  - each occurrence of the game takes place after *n* players are randomly chosen from different populations.
- ◇ The interpretation is consistent with the assumptions of our model, as
  - the firms interacting in the market, due to entry and exit, are not the same in each occurrence of the game
  - and since entry and exit are affected by stochastic shocks, the firms existing in each time *t* are chosen stochastically.
- ◇ The amount of work hired by each firm constitutes a capacity constraint, on the basis of the labor contracts set for time *t*, until time $t + 1$.
- ◇ The quantities decided by the firms, instead of being commodities, are "contracts", i.e., "commitments" to sell commodities to the customers. Commitments are subject to stochastic shocks.

Osborne, M., and Rubinstein, A. 1994. A Course in Game Theory. *MIT press*.

New entrants

If the new entrant at time t is successful, her expected stream of future real income, from time $t + 1$ onwards is

$$
J_{t+1} = \frac{1}{1 + \rho} \{ [\Pr(\Pi^{inR} \geq 0)][E_t(\Pi^{inR}_{t+1}) + E_t(w_{t+1}) - \tau_R + J_{t+2}] + \\
+ [1 - \Pr(\Pi^{inR} \geq 0)][\tau_R \cdot n_{t+1} (l - n_{t+1})^{-1} + \Upsilon_{t+2}] \}
\tag{4}
$$

Υ_{t+1} positively depends on the probability of being hired as a worker by a firm the next period, and negatively on the number of unemployed individuals; therefore the term $\{1 - \Pr(\Pi^{eR}_t \geq 0)]\}(1 + \rho)^{-1} \cdot E_{t-1}[(\tau \cdot n_t (l - n_t)^{-1} + \Upsilon_{t+2}]$ is the expected future stream of income for the unsuccessful entrant from time t onwards, weighted with the probability of going bankrupt in the first period.

ρ is the subjective rate of intertemporal preference;
w_{t+1} the real wage at time $t + 1$;
Υ_{t+2} is the expected net present value of the future income for the unemployed individuals at time $t + 2$;
τ_R the lump sum tax in real terms;
l the total labor force;
$\Pr(\Pi^{inR} \geq 0$ the probability of survival of the incumbents, assumed to be a constant reflecting the probability of the incumbents to collude.

Prediction mistakes

The entrepreneurs can be incumbent, earning at time $t + i$ the incumbent nominal profits Π_{t+i}^{in} (Π_{t+i}^{inR} in real terms) or new entrants, earning the new entrant nominal profits Π_{t+i}^{e} (Π_{t+i}^{eR} in real terms). Π_{t+i}^{e}, in general, diverges from Π_{t+i}^{in} because the new entrants bear some entry costs.

For given average market expectations $E_t(\Pi_{t+1}^{eR})$ and $E_t(\Pi_{t+1}^{inR})$, when the variance of the distributions of these two variables increases, we have a higher frequency of prediction mistakes. We can summarize and simplify that by the following equation

$$\text{Pr(entry)}_t = \beta(var(\Psi_t)) \tag{5}$$

Where Ψ_t) is a shift parameter in the aggregate demand curve, depending on exogenous variables and parameters. The more frequent are the shifts in the aggregate demand, the more common the stochastic prediction mistakes by potential new entrants, although, on average, the expectations are correct and Rational Expectations apply.

Wages

The nominal wage (set by the oligopolistic firms) depends on whether the economy is in full employment ($n = l$) or not ($n < l$), in which case, the firms set the nominal wage to the level w_t^{fu}, discouraging, on average, entry. The new entrants are the workers with a positive (and optimistic) information shock.

$$
W_t = \begin{cases} w_t^* & n < l \\ w_t^{fu} & n = l \end{cases} \tag{6}
$$

Premise

In our model, the agent-based technique allows us to emphasize the role of strategic interaction among oligopolistic firms, as the consequence of subjective decision making, formalizing in the most appropriate way the implication and results of these decisions.

◇ We produce all the actions and reactions designed by the model equations via the behavior of heterogeneous agents actually acting in the simulated time.

◇ We remark that between (a) the formal presentation of the model in the equation based way, strictly necessary to be consistent with the literature upon which our work is grounded, and (b) the agent-based implementation, the consistency is deeply satisfied, but with a few inevitable distinctions.

◇ The same kind of differences that we run up against when we compare (a) the formalization of a phenomenon and (b) the related observation of the reality (here: an artificial one, simulated).
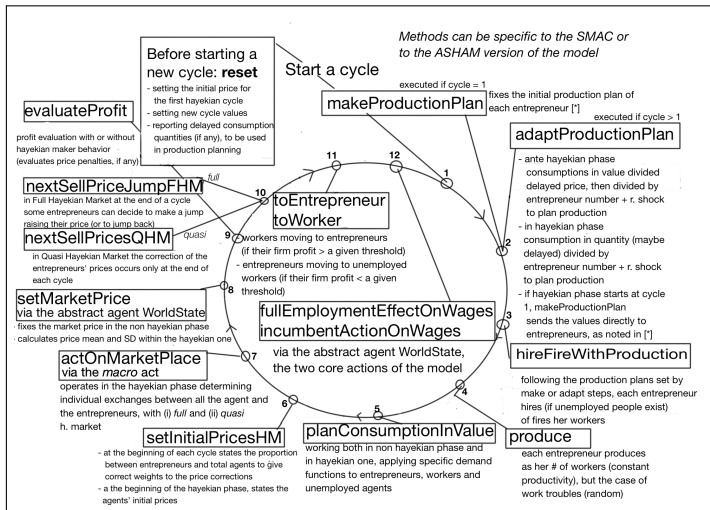
Swarm-Like Agent Protocol in Python (SLAPP)

Outline

Figure 1: Simulation outline