

micro Hayekian Market

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July 25, 2018

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Introduction to a micro Hayekian Market

The purpose of the note is that of introducing a very simple agent-based model of a market, with emergent (quite interesting) price dynamics.

A counter example is also introduced, showing how with tiny modification we generate implausible price dynamics.

The code uses the IPython¹ language (interaction with Python²) and can be downloaded from <https://github.com/terna/microHayekianMarket> using the *Clone or download* button; it is also possible to run it directly on line at <https://mybinder.org/v2/gh/terna/microHayekianMarket/master?filepath=microHayekianMarket.ipynb>.

A suggested reading about Hayek is a quite recent paper of Bowles *et al.* (2017).

1 The technical setup

The IPython (or Python 3.x) code requires the following starting setup:

Listing 1: Setup of the program

```
%pylab inline
import statistics as s
import numpy as np
import pylab as plt
from IPython.display import clear_output
import time
import math
```

`%pylab inline` is a *magic* command of Jupyter.³

¹<https://ipython.org>.

²<https://www.python.org>.

³<http://jupyter.org>.

2 The structure of the model and the *warming up* phase

Our agents are simply prices, to be interpreted as reservation prices.⁴

We have two price vectors: pL^b with item pL_i^b for the buyers, and pL^s with item pL_j^s for the sellers. The i^{th} or the j^{th} elements of the vectors are prices, but we can use them also as agents.

Both in the hayekian perspective (Section 3) and in the unstructured one (Section 4) we have to pre-run the *warming up* action.

In this phase, we define:

- d_0 - the lower bound of the random uniform numbers, both for the buyers and the sellers, in the warming up phase;
in the running phase, the lower bound is 0;
- d_1 - the upper bound of the random uniform numbers for the buyers;
- d_2 - the upper bound of the random uniform numbers for the sellers;
- $nCycles$ - number of simulation cycles;
- $nBuyers$ - number of the buyers;
- $nSellers$ - number of the sellers;
- $seed$ - the seed of the random numbers;
- the initial buyer i reservation price, different for each buyer: $p_{b,i} = \frac{1}{1+u_i}$ with $u_i \sim \mathcal{U}(d_0, d_1)$;
- the initial seller j reservation price, different for each seller: $p_{s,j} = 1 + u_j$ with $u_j \sim \mathcal{U}(d_0, d_2)$.

With $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, sorting in decreasing order the vector pL^b and in increasing order the vector pL^s we obtain two not overlapping price sequences that we can interpret as a demand curve and an offer one (Fig. 1).

This is the *warming up*, or starting situation, of the model. To generate new examples related to Section 3 and to Section 4, it is necessary to repeat this phase.

The IPython (or Python 3.x) code is:

⁴The *max* price a buyer could pay and the *min* one a seller could accept.

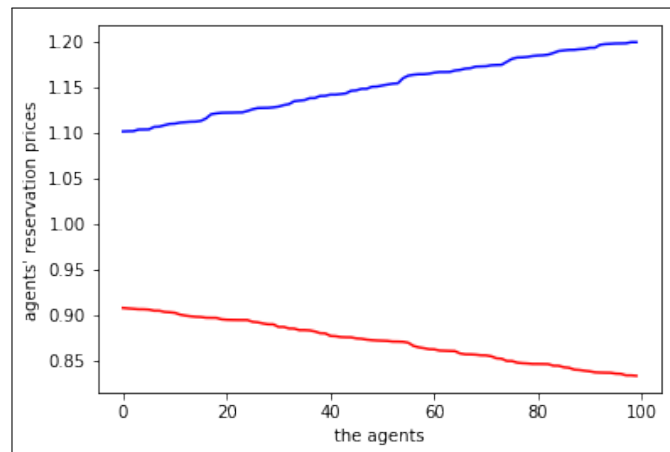


Figure 1: An example of initial not overlapping demand curve and offer curve

Listing 2: Warming up of the model

```
# warming up

# run it before executing both - the hayekian perspective or
#                               - the unstructured case

nCycles=10000
nBuyers= 100
nSellers=100

seed=111
np.random.seed(seed)

d0=0.1
d1=0.2
d2=0.2

buyerPriceList=[]
sellerPriceList=[]

for i in range(nBuyers):
    buyerPriceList.append(1/(1+np.random.uniform(d0,d1)))
for j in range(nSellers):
    sellerPriceList.append(1+np.random.uniform(d0,d2))

plt.plot(sorted(buyerPriceList,reverse=True),"r");
plt.plot(sorted(sellerPriceList),"b");
xlabel("the_agents");
ylabel("agents' reservation_prices");
```

3 The hayekian version

The buyers and the sellers meet randomly. Buyer i and seller j exchange if $pL_i^b \geq pL_j^s$; the deal is recorded at the price of the seller pL_j^s .⁵

In this version, representing the key point in this note, the running prices are changing being multiplied in each cycle by following the correction coefficients:

- for the buyer: (i) $c_b = \frac{1}{1+u_b}$ if the deal succeeds (trying to pay less next time) or (ii) $c_b = 1 + u_b$ if the deal fails (preparing to pay more next time); in (i) and (ii) we have $u_b \sim \mathcal{U}(0, d_1)$
- for the seller: (iii) $c_s = 1 + u_s$ if the deal succeeds (preparing to obtain a higher revenue next time) or (iv) $c_s = \frac{1}{1+u_s}$ if the deal fails (preparing to obtain a lower revenue next time); in (iii) and (iv) we have $u_s \sim \mathcal{U}(0, d_2)$.

With $seed = 111$, $d_1 = 0.2$, $d_2 = 0.2$ and $nCycles$ set to 10,000 we obtain sequences of mean prices (mean within each cycle) quite realistic, with a very low variance within each cycle (see Fig. 2 and 3).

The *coefficient of variation* at time t is calculated as:

$$\frac{standard\ deviation_t}{mean_t}$$

A comment: we have a plausible series of mean prices, with a complicated behavior, and with a high stability of the dispersion of the values within each cycle.

The right side of the buyer and seller curves shows another plausible situation: that of the presence of agents not exchanging. **A note for Matteo and Pietro: this is a very important effect for the *Oligopoly* model.**

Have a look to the Appendix 1 for the cases of not balancing number of buyers and sellers.

The IPython (or Python 3.x) code is:

Listing 3: The model in the hayekian perspective

```
# hayekian perspective
meanPrice_ts=[]
meanPriceStDev_ts=[]
meanPriceVar_ts=[]
```

⁵In the *mall*, sell prices are public.

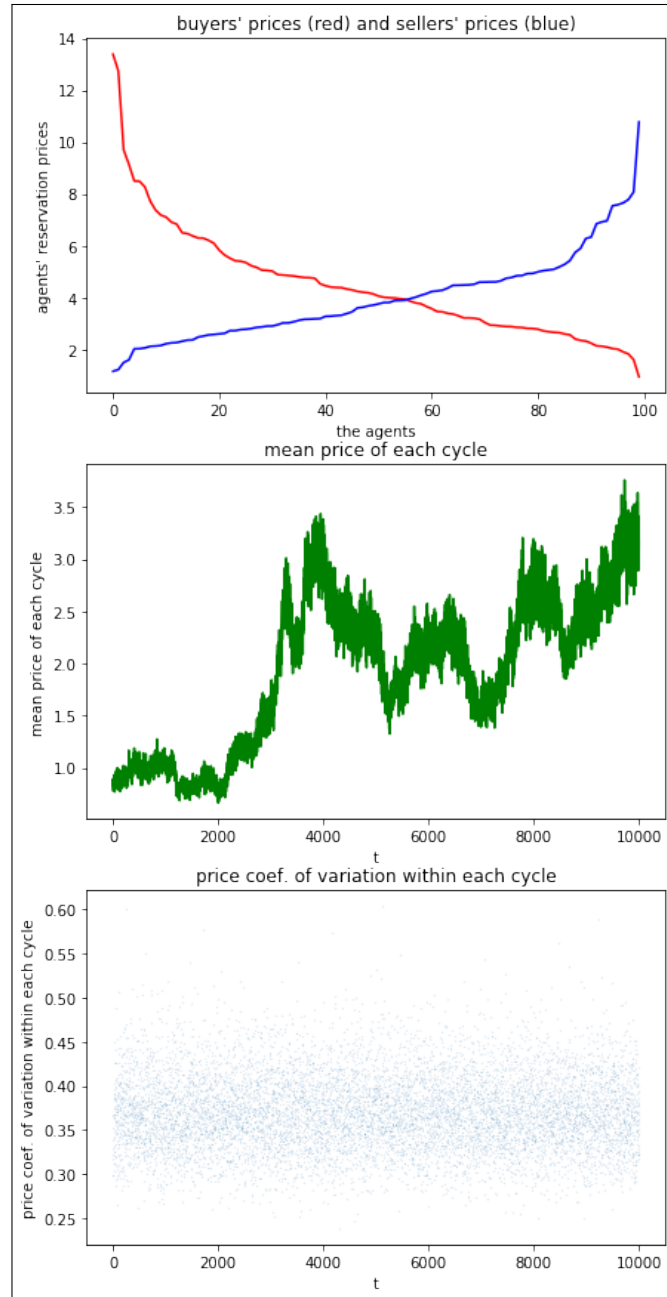


Figure 2: Hayekian case: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

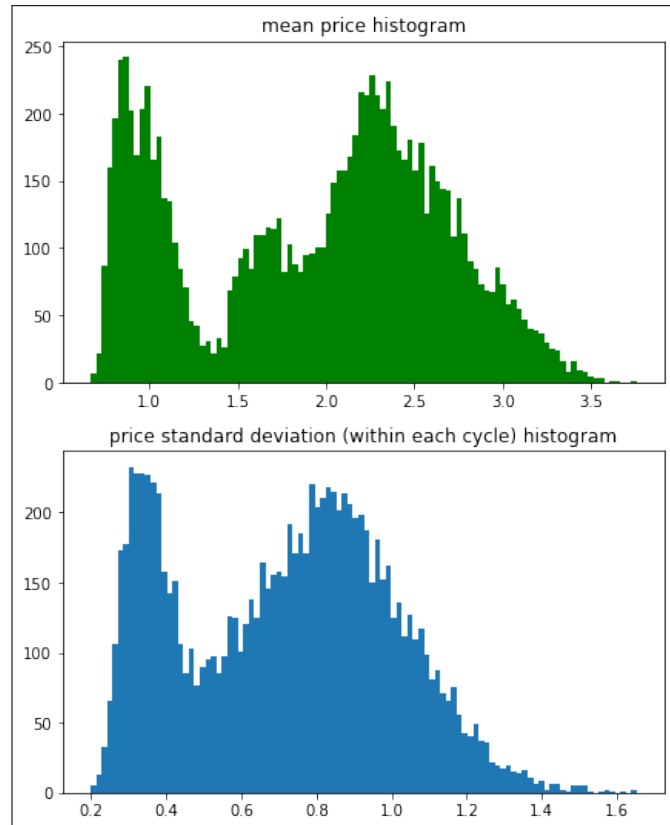


Figure 3: Hayekian case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

```

for t in range(1,nCycles+1):
    dealPrices=[]
    agNum=max(nBuyers,nSellers)

    for n in range(agNum):

        i = np.random.randint(0,nBuyers)
        j = np.random.randint(0,nSellers)

        if buyerPriceList[i]>=sellerPriceList[j]:
            dealPrices.append(sellerPriceList[j])
            buyerPriceList[i] *=1/(1+np.random.uniform(d1))
            sellerPriceList[j]*=1+np.random.uniform(d2)
        else:
            buyerPriceList[i] *=1+np.random.uniform(d1)
            sellerPriceList[j]*=1/(1+np.random.uniform(d2))

    if len(dealPrices) > 2:
        meanPrice_ts.append(s.mean(dealPrices))
        meanPriceVar_ts.append(s.variance(dealPrices))
        meanPriceStDev_ts.append(s.stdev(dealPrices))
    else:
        meanPrice_ts.append(np.nan)
        meanPriceStDev_ts.append(np.nan)

    if t % 1000==0:
        clear_output()
        print('time', t, 'and n. of exchanges in the last cycle', \
              len(dealPrices))
        print(\
        'mean and var of exchange prices in the last cycle: %.3f, %.3f' %\
        (meanPrice_ts[-1],meanPriceVar_ts[-1]))

    plt.figure(1,figsize=(7,15),clear=True)

    plt.subplot(311)
    plt.plot(sorted(buyerPriceList,reverse=True),"r")
    plt.plot(sorted(sellerPriceList),"b")
    plt.title(\
        "buyers' prices (red) and sellers' prices (blue)")
    xlabel("the agents")
    ylabel("agents' reservation prices")

    plt.subplot(312)
    plt.title("mean price of each cycle")
    xlabel("t")
    ylabel("mean price of each cycle")
    plt.plot(meanPrice_ts,"g")

```

```

plt.subplot(313)
plt.title("price_coef_of_variation_within_each_cycle")
coefOfVariation=[]
for m in range(len(meanPriceStDev_ts)):
    coefOfVariation.append(meanPriceStDev_ts[m]/
                           meanPrice_ts[m])
plt.plot(coefOfVariation, ".", markersize=0.1)
xlabel("t")
ylabel("price_coef_of_variation_within_each_cycle")
show()
#time.sleep(0.1)

# hist crashes with NaN
meanPrice_ts_hist=[]
for k in range(len(meanPrice_ts)):
    if not math.isnan(meanPrice_ts[k]):
        meanPrice_ts_hist.append(meanPrice_ts[k])
meanPriceStDev_ts_hist=[]
for k in range(len(meanPriceStDev_ts)):
    if not math.isnan(meanPriceStDev_ts[k]):
        meanPriceStDev_ts_hist.append(meanPriceStDev_ts[k])
plt.figure(2, figsize=(7, 9))
plt.subplot(211)
if meanPrice_ts_hist != []:
    plt.title("mean_price_histogram")
    plt.hist(meanPrice_ts_hist, 100, color="g");
plt.subplot(212)
if meanPriceStDev_ts_hist != []:
    plt.title("price_standard_deviation_(within_each_cycle)_histogram")
    plt.hist(meanPriceStDev_ts_hist, 100);

```

4 The unstructured version

The buyers and the sellers meet randomly as in Section 3. Buyer i and seller j exchange in any case; the deal is recorded at the mean of the price of the seller pL_j^s and of the price pL_i^b of the buyer.

In this version the running prices are changing being multiplied in each cycle by following the correction coefficients:

- with the same probability for the buyer: (i) $c_b = \frac{1}{1+u_b}$ or (ii) $c_b = 1 + u_b$; in (i) and (ii) we have $u_b \sim \mathcal{U}(0, d_1)$
- with the same probability for the seller: (iii) $c_s = 1 + u_s$ or (iv) $c_s = \frac{1}{1+u_s}$; in (iii) and (iv) we have $u_s \sim \mathcal{U}(0, d_2)$.

With $seed = 111$, $d_1 = 0.2$, $d_2 = 0.2$ and $nCycles$ set to 10,000 we obtain exploding sequences of mean prices (mean in each cycle), and exploding standard deviation within each cycle (see Fig. 4 and 5).

The *coefficient of variation* at time t is calculated as:

$$\frac{standard\ deviation_t}{mean_t}$$

A comment: this counter-example shows that, missing the *intelligence* in the correction of the prices (implicitly propagating among all the agents), a system of pure random price settings is absolutely far from being plausible.

The IPython (or Python 3.x) code is:

Listing 4: The unstructured version

```
# unstructured case (remember the warming up step)
meanPrice_ts=[]
meanPriceStDev_ts=[]
meanPriceVar_ts=[]

for t in range(1,nCycles+1):
    dealPrices=[]
    agNum=max(nBuyers,nSellers)
    for n in range(agNum):
        i = np.random.randint(0,nBuyers)
        j = np.random.randint(0,nSellers)

        dealPrices.append((sellerPriceList[j]+buyerPriceList[i]/0.5))

    if np.random.uniform(0,1)>=0.5:
        buyerPriceList[i] *=1/(1+np.random.uniform(d1))
        sellerPriceList[j]*=1+np.random.uniform(d2)
    else:
        buyerPriceList[i] *=1+np.random.uniform(d1)
        sellerPriceList[j]*=1/(1+np.random.uniform(d2))

    if len(dealPrices) > 2:
        meanPrice_ts.append(s.mean(dealPrices))
        meanPriceVar_ts.append(s.variance(dealPrices))
        meanPriceStDev_ts.append(s.stdev(dealPrices))
    else:
```

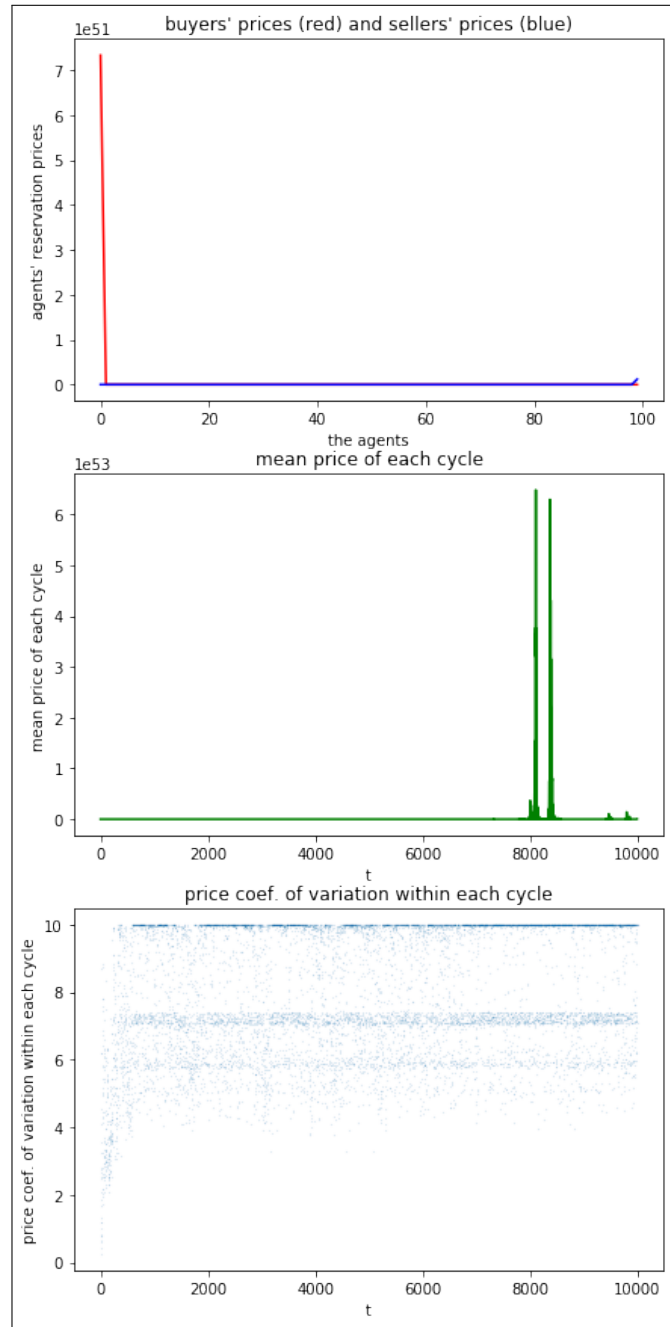


Figure 4: Unstructured case: ((i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

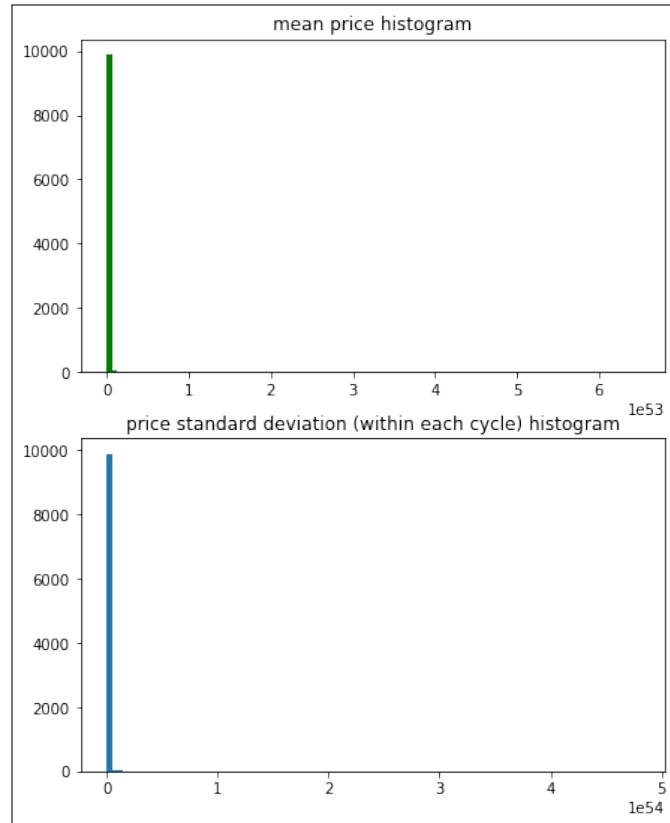


Figure 5: Unstructured case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

```

meanPrice_ts.append(np.nan)
meanPriceStDev_ts.append(np.nan)

if t % 1000==0:
    clear_output()
    print('time', t, 'and n. of exchanges in the last cycle', \
          len(dealPrices))
    print(\
    'mean and var of exchange prices in the last cycle: %.3f, %.3f' %\
    (meanPrice_ts[-1], meanPriceVar_ts[-1]))

plt.figure(1, figsize=(7, 15), clear=True)

plt.subplot(311)
plt.plot(sorted(buyerPriceList, reverse=True), "r")
plt.plot(sorted(sellerPriceList), "b")
plt.title(\
    "buyers' prices (red) and sellers' prices (blue)")
xlabel("the agents")
ylabel("agents' reservation prices")

plt.subplot(312)
plt.title("mean price of each cycle")
xlabel("t")
ylabel("mean price of each cycle")
plt.plot(meanPrice_ts, "g")

plt.subplot(313)
plt.title("price coef. of variation within each cycle")
coefOfVariation=[]
for m in range(len(meanPriceStDev_ts)):
    coefOfVariation.append(meanPriceStDev_ts[m]/
                             meanPrice_ts[m])
plt.plot(coefOfVariation, ".", markersize=0.1)
xlabel("t")
ylabel("price coef. of variation within each cycle")
show()
#time.sleep(0.1)

# hist crashes with NaN
meanPrice_ts_hist=[]
for k in range(len(meanPrice_ts)):
    if not math.isnan(meanPrice_ts[k]):
        meanPrice_ts_hist.append(meanPrice_ts[k])
meanPriceStDev_ts_hist=[]
for k in range(len(meanPriceStDev_ts)):
    if not math.isnan(meanPriceStDev_ts[k]):
        meanPriceStDev_ts_hist.append(meanPriceStDev_ts[k])
plt.figure(2, figsize=(7, 9))

```

```
plt.subplot(211)
if meanPrice_ts_hist != []:
    plt.title("mean_price_histogram")
    plt.hist(meanPrice_ts_hist,100,color="g");
plt.subplot(212)
if meanPriceStDev_ts_hist != []:
    plt.title("price_standard_deviation_(within_each_cycle)_histogram")
    plt.hist(meanPriceStDev_ts_hist,100);
```


Appendices

1 Two cases of not balancing numbers of buyers and sellers

1.1 Case $nBuyers \gg nSellers$

If $nBuyers \gg nSellers$ (e.g., $nBuyers = 100$ and $nSellers = 30$, as in Fig. 6), we observe that in Figs. 7 and 8 the prices are higher than in Figs. 2 and 3. We have always $seed = 100$.

The result is highly consistent with the numbers of the agents in the two sides of the market.

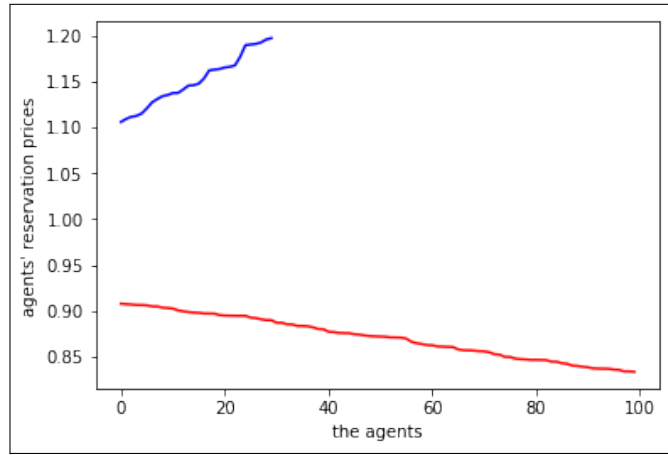


Figure 6: An example of initial not overlapping demand curve and offer curve, case $nBuyers \gg nSellers$

1.2 Case $nBuyers \ll nSellers$

If $nBuyers \ll nSellers$ (e.g., $nBuyers = 30$ and $nSellers = 100$, as in Fig. 9), we observe that in Figs. 10 and 11 the prices are lower than in Figs. 2 and 3. We have always $seed = 100$.

The result is highly consistent with the numbers of the agents in the two sides of the market.

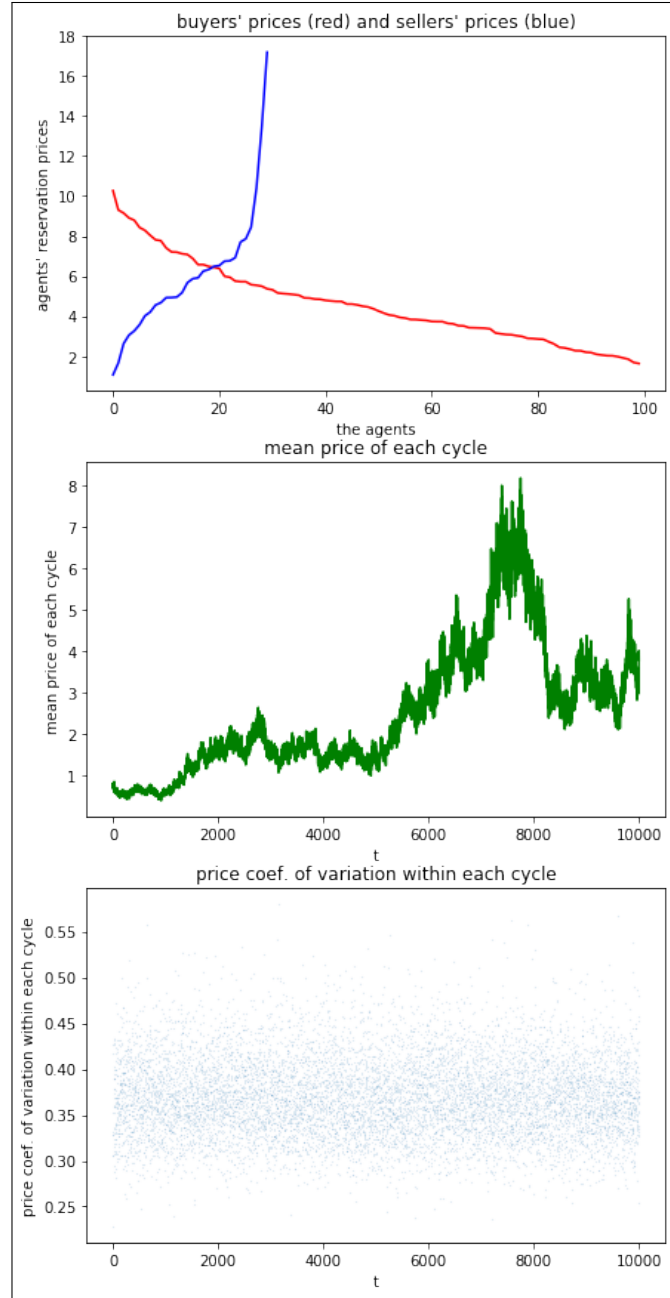


Figure 7: Hayekian case, with $nBuyers \gg nSellers$: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

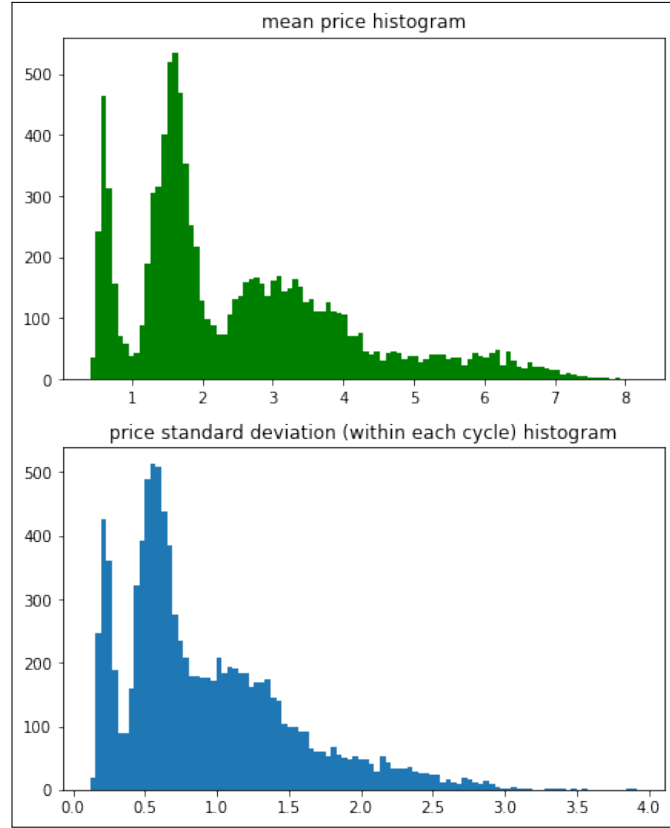


Figure 8: Hayekian case, with $nBuyers \gg nSellers$: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

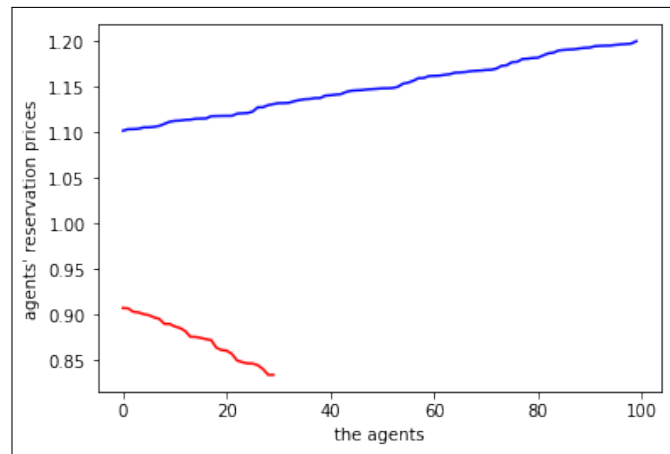


Figure 9: An example of initial not overlapping demand curve and offer curve, case $nBuyers \ll nSellers$

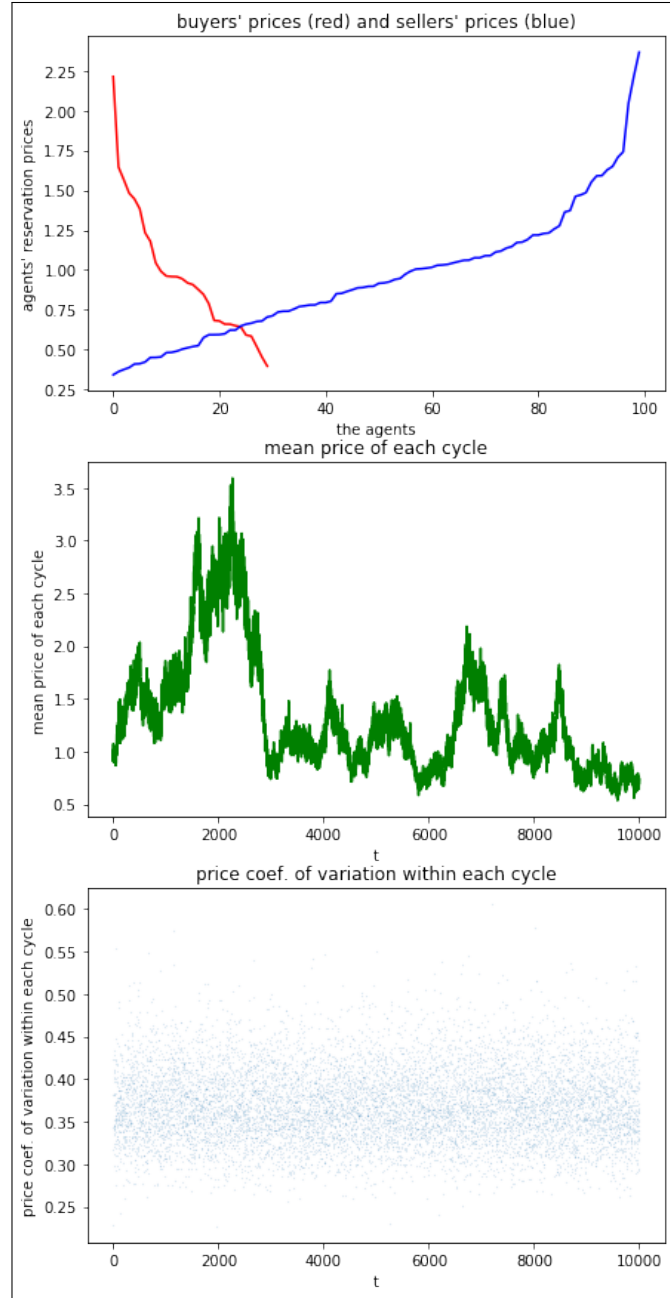


Figure 10: Hayekian case, with $nBuyers \ll nSellers$: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

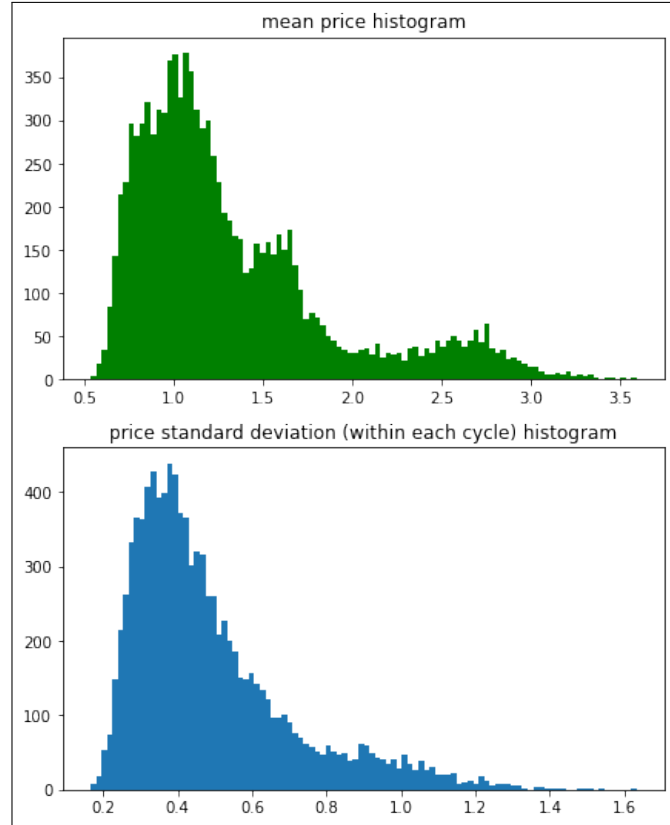


Figure 11: Hayekian case, with $nBuyers \ll nSellers$: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

Bibliography

Bowles, S., Kirman, A. and Sethi, R. (2017). *Retrospectives: Friedrich Hayek and the Market Algorithm*. In «Journal of Economic Perspectives», vol. 31(3), pp. 215-30.

URL <http://www.aeaweb.org/articles?id=10.1257/jep.31.3.215>

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