

# Micro Simplified Hayekian Market

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### Abstract

We propose a simplified version of the Hayek’s decentralized market hypothesis, considering elementary processes of price adaptation in exchanges.

Sections 1 and 2 report the technical setup and the structure of the model. In Section 3 we introduce an elementary agent-based model of a market, with emergent (quite interesting) price dynamics.

A counter example is also introduced in Section 4, showing that—with tiny modification—we generate implausible price dynamics.

In Appendix A we report some technical analyses of the cases with unmatching numbers of buyers and sellers. These analyses are strongly related to the [Oligopoly](#)<sup>1</sup> simulation project.

In Appendix B we deepen the problem of the agents that seem to be idle, having the level of their reservation price too low to buy. The Appendix is mainly of interest to understand the [Oligopoly](#) project.

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<sup>1</sup>Click to go to <https://terna.github.io/oligopoly/>

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# Introduction to a Micro Simplified Hayekian Market

The purpose of the note is that of introducing a very simple agent-based model of a market, with emergent (quite interesting) price dynamics.

A counter example is also introduced, showing how—with tiny modifications—we generate implausible/impossible price dynamics.

The code uses the IPython<sup>2</sup> language (an interactive layer upon Python<sup>3</sup> using the Jupyter<sup>4</sup> infrastructure) and can be downloaded from <https://github.com/terna/microHayekianMarket> via the *Clone or download* button; it is also possible to run it directly on line thanks to the [MyBinder project](#)<sup>5</sup>.

A suggested reading about Hayek is a quite recent paper of [Bowles \*et al.\* \(2017\)](#).

Quoting from the introduction:

Friedrich A. Hayek (1899-1992) is known for his vision of the market economy as an information processing system characterized by spontaneous order, the emergence of coherence through the independent actions of large numbers of individuals each with limited and local knowledge, coordinated by prices that arise from decentralized processes of competition.

A simplified version—proposed here—is that of considering decentralized elementary processes of price adaptation in exchanges, with surprising results.

## 1 The technical setup

The IPython (or Python 3.x) code requires the following setup to start:

Listing 1: Setup of the program

```
%pylab inline
%pylab inline
import statistics as s
import numpy as np
import pylab as plt
from ipywidgets import Output
```

---

<sup>2</sup><https://ipython.org>

<sup>3</sup><https://www.python.org>

<sup>4</sup><http://jupyter.org>

<sup>5</sup>Click to go to <https://mybinder.org/v2/gh/terna/microHayekianMarket/master?filepath=microHayekianMarket.ipynb>

```
from IPython.display import clear_output
from IPython.display import display
import time
import math
```

`%pylab inline` is a *magic* command of Jupyter.

## 2 The structure of the model and the *warming up* phase

Our agents are simply prices, to be interpreted as reservation prices.<sup>6</sup>

We have two price vectors:  $pL^b$  with item  $pL_i^b$  for the buyers, and  $pL^s$  with item  $pL_j^s$  for the sellers. The  $i^{th}$  or the  $j^{th}$  elements of the vectors are prices, but we can use them also as agents.

Both in the simplified hayekian perspective (Section 3) and in the unstructured one (Section 4) we have to pre-run a *warming up* action. This happens automatically, calling the specific function in the beginning of both the cases.

With the *warming up* phase, we define:

- $d_0$  - the lower bound of the random uniform numbers, both for the buyers and the sellers, in the warming up phase;  
in the running phase, the lower bound is 0;
- $d_1$  - the upper bound of the random uniform numbers for the buyers;
- $d_2$  - the upper bound of the random uniform numbers for the sellers;
- $nCycles$  - number of simulation cycles;
- $nBuyers$  - number of the buyers;
- $nSellers$  - number of the sellers;
- $seed$  - the seed of the random numbers;
- the initial buyer  $i$  reservation price, different for each buyer:  $p_{b,i} = \frac{1}{1+u_i}$  with  $u_i \sim \mathcal{U}(d_0, d_1)$ ;
- the initial seller  $j$  reservation price, different for each seller:  $p_{s,j} = 1 + u_j$  with  $u_j \sim \mathcal{U}(d_0, d_2)$ ;
- $buyersSellersRatio$  - the ratio  $\frac{nBuyers}{nSellers}$ ;
- $sellersBuyersRatio$  - the ratio  $\frac{nSellers}{nBuyers}$ ;
- $usingRatios$  - a logic variable activating limitations to  $d_1$  or  $d_2$
- $squeezeRate$  - always  $< 1$ , as further compression of  $d_1$  or  $d_2$
- $usingSqueezeRate$  - a logic variable to further squeeze  $d_1$  or  $d_2$

---

<sup>6</sup>The *max* price a buyer could pay and the *min* one a seller could accept.

With  $d_0 = 0.1$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$ , sorting in decreasing order the vector  $pL^b$  and in increasing order the vector  $pL^s$ , we obtain in Fig. 1 two not overlapping price sequences that we can interpret as a demand curve (red) and an offer one (blue).

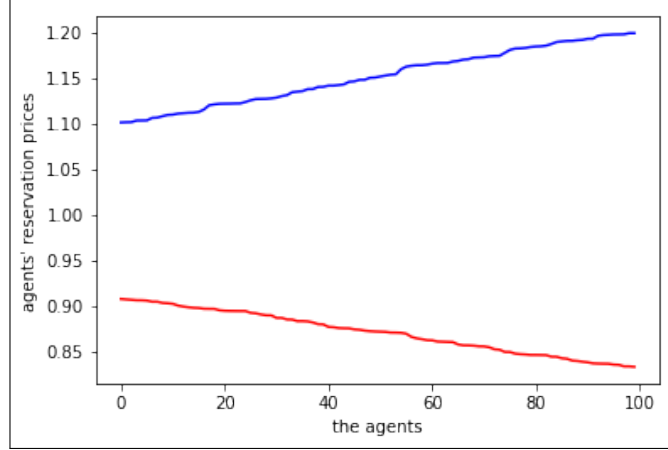


Figure 1: An example of initial not overlapping demand curve (red) and offer curve (blue)

To generate new examples related to Section 3 and to Section 4, it is necessary to repeat this phase. This happens automatically, calling the specific function in the beginning of both the cases.

The IPython (or Python 3.x) code is:

Listing 2: Warming up of the model

```
# warming up

# execute before both:
# - the hayekian perspective or
# - the unstructured case
def warmingUp():
    global nCycles, nBuyers, nSellers,\
           buyersSellersRatio, sellersBuyersRatio,\
           usingRatios, usingSqueezeRate, squeezeRate,\
           d0, d1, d2, buyerPriceList, sellerPriceList

    nCycles=10000
    nBuyers= 50
    nSellers=100

    buyersSellersRatio=nBuyers/nSellers
    sellersBuyersRatio=nSellers/nBuyers
    usingRatios=True
    squeezeRate=0.3 # always < 1
    usingSqueezeRate=False

    seed=111
    np.random.seed(seed)

    d0=0.1
    d1=0.2
    d2=0.2
```



```

buyerPriceList=[]
sellerPriceList=[]

for i in range(nBuyers):
    buyerPriceList.append(1/(1+np.random.uniform(d0,d1)))
for j in range(nSellers):
    sellerPriceList.append(1+np.random.uniform(d0,d2))

plt.figure(0)
plt.plot(sorted(buyerPriceList,reverse=True),"r");
plt.plot(sorted(sellerPriceList),"b");
xlabel("the_agents");
ylabel("agents'_reservation_prices");

```

### 3 The simplified hayekian version

The buyers and the sellers meet randomly. Buyer  $i$  and seller  $j$  exchange if  $pL_i^b \geq pL_j^s$ ; the deal is recorded at the price of the seller  $pL_j^s$ .<sup>7</sup>

In this version, representing the key point in this note, the running prices are multiplied in each cycle by the following correction coefficients:

- for the buyer: (i)  $c_b = \frac{1}{1+u_b}$  if the deal succeeds (trying to pay less next time) or (ii)  $c_b = 1 + u_b$  if the deal fails (preparing to pay more next time); in (i) and (ii) we have  $u_b \sim \mathcal{U}(0, d_1)$
- for the seller: (iii)  $c_s = 1 + u_s$  if the deal succeeds (trying to obtain a higher revenue next time) or (iv)  $c_s = \frac{1}{1+u_s}$  if the deal fails (preparing to obtain a lower revenue next time); in (iii) and (iv) we have  $u_s \sim \mathcal{U}(0, d_2)$ .

With  $seed = 111$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$  and  $nCycles$  set to 10,000 we obtain sequences of mean prices (mean within each cycle) quite realistic, with a very low variance within each cycle (see Fig. 2 and 3).

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<sup>7</sup>In the *mall*, sell prices are public.

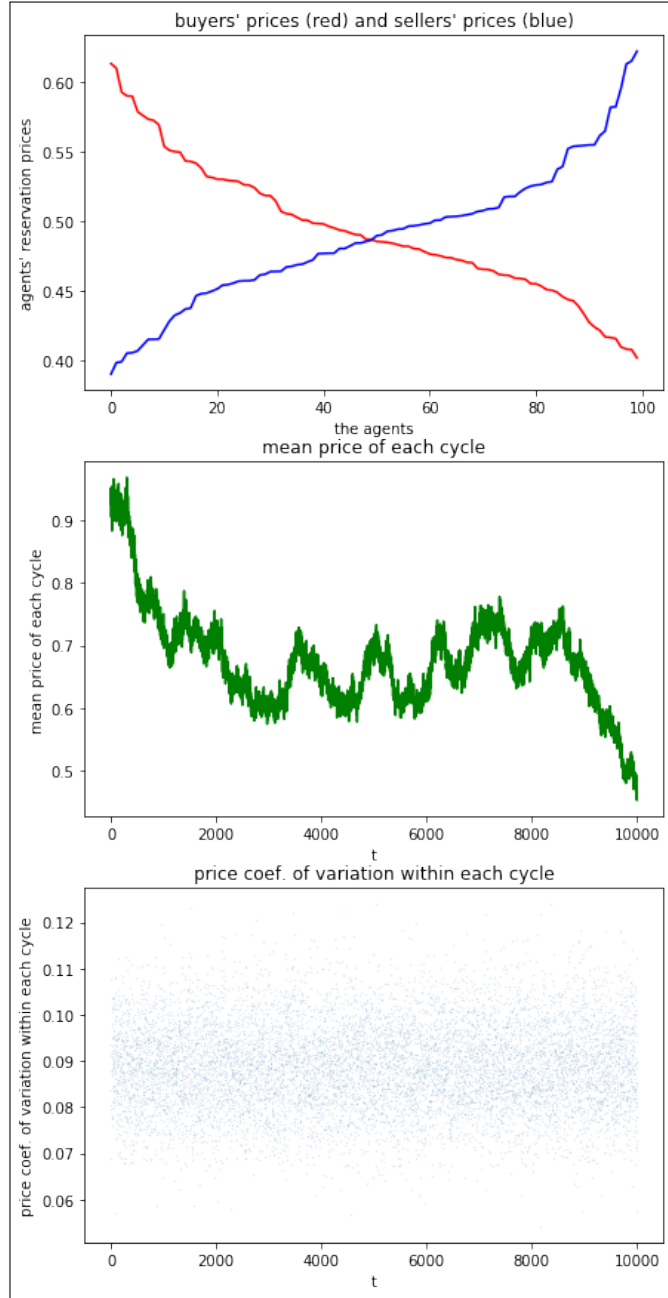


Figure 2: Simplified Hayekian case: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

The *coefficient of variation* at time  $t$  is calculated as:

$$\frac{\text{standard deviation}_t}{\text{mean}_t}$$

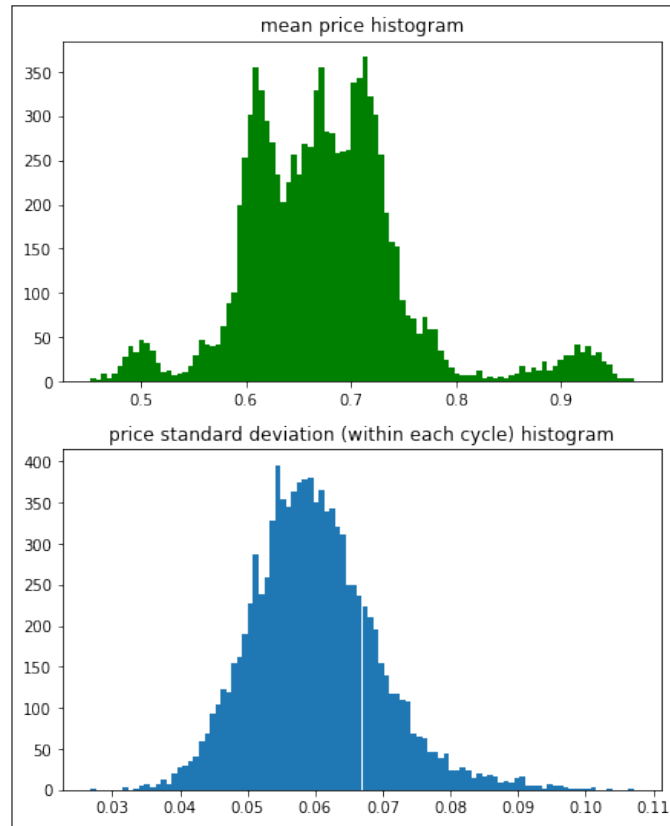


Figure 3: Simplified Hayekian case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A comment: we have a plausible series of mean prices, with a complicated behavior, and with a high stability of the dispersion of the values within each cycle.

The right side of the buyer and seller curves shows another plausible situation: that of the presence of agents not exchanging. A note: this observation is related to a very important emergent effect for the *Oligopoly* model.

Have a look to the Appendix A for the cases of not balancing number of buyers and sellers. The IPython (or Python 3.x) code is:

Listing 3: The model in the simplified hayekian perspective

```
# hayekian perspective
warmingUp()
```

```

out = Output()
display(out)

meanPrice_ts=[]
meanPriceStDev_ts=[]
meanPriceVar_ts=[]

if usingRatios and not usingSqueezeRate:
    if buyersSellersRatio>1: d2*=sellersBuyersRatio
    if sellersBuyersRatio>1: d1*=buyersSellersRatio

if usingRatios and usingSqueezeRate:
    if buyersSellersRatio>1: d2*=sellersBuyersRatio*squeezeRate
    if sellersBuyersRatio>1: d1*=buyersSellersRatio*squeezeRate

for t in range(1,nCycles+1):
    dealPrices=[]
    agNum=max(nBuyers,nSellers)

    for n in range(agNum):

        i = np.random.randint(0,nBuyers)
        j = np.random.randint(0,nSellers)

        if buyerPriceList[i]>=sellerPriceList[j]:
            dealPrices.append(sellerPriceList[j])
            buyerPriceList[i] *=1/(1+np.random.uniform(0,d1))
            sellerPriceList[j]*=1+np.random.uniform(0,d2)
        else:
            buyerPriceList[i] *=1+np.random.uniform(0,d1)
            sellerPriceList[j]*=1/(1+np.random.uniform(0,d2))

    if len(dealPrices) > 2:
        meanPrice_ts.append(s.mean(dealPrices))
        meanPriceVar_ts.append(s.variance(dealPrices))
        meanPriceStDev_ts.append(s.stdev(dealPrices))
    else:
        meanPrice_ts.append(np.nan)
        meanPriceStDev_ts.append(np.nan)

    if t % 1000==0:
        with out:
            clear_output()
        with out:
            print('time', t, 'and_n_of_exchanges_in_the_last_cycle', \
                  len(dealPrices))
            print(\
'mean_and_var_of_exchange_prices_in_the_last_cycle:%1.3e,%1.3e' %\
      (meanPrice_ts[-1],meanPriceVar_ts[-1]))

plt.figure(1,figsize=(7,15),clear=True)

plt.subplot(311)
plt.plot(sorted(buyerPriceList,reverse=True),"r")
plt.plot(sorted(sellerPriceList),"b")
plt.title(\
    "buyers'prices(red)and_sellers'prices(blue)")
xlabel("the_agents")
ylabel("agents'reservation_prices")

plt.subplot(312)

```

```

plt.title("mean_price_of_each_cycle")
xlabel("t")
ylabel("mean_price_of_each_cycle")
plt.plot(meanPrice_ts, "g")

plt.subplot(313)
plt.title("price_coef_of_variation_within_each_cycle")
coefOfVariation=[]
for m in range(len(meanPriceStDev_ts)):
    coefOfVariation.append(meanPriceStDev_ts[m]/
                           meanPrice_ts[m])
plt.plot(coefOfVariation, ".", markersize=0.1)
xlabel("t")
ylabel("price_coef_of_variation_within_each_cycle")
#plt.show() #activate to see intermediate plots
#time.sleep(0.1)

# hist crashes with NaN
meanPrice_ts_hist=[]
for k in range(len(meanPrice_ts)):
    if not math.isnan(meanPrice_ts[k]):
        meanPrice_ts_hist.append(meanPrice_ts[k])
meanPriceStDev_ts_hist=[]
for k in range(len(meanPriceStDev_ts)):
    if not math.isnan(meanPriceStDev_ts[k]):
        meanPriceStDev_ts_hist.append(meanPriceStDev_ts[k])
plt.figure(2, figsize=(7, 9))
plt.subplot(211)
if meanPrice_ts_hist != []:
    plt.title("mean_price_histogram")
    plt.hist(meanPrice_ts_hist, 100, color="g");
plt.subplot(212)
if meanPriceStDev_ts_hist != []:
    plt.title("price_standard_deviation_(within_each_cycle)_histogram")
    plt.hist(meanPriceStDev_ts_hist, 100);
    
```

## 4 The unstructured version

The buyers and the sellers meet randomly as in Section 3. Buyer  $i$  and seller  $j$  exchange in any case; the deal is recorded at the mean of the price of the seller  $pL_j^s$  and of the price  $pL_i^b$  of the buyer.

In this version the running prices are multiplied in each cycle by the following correction coefficients:

- with the same probability for the buyer: (i)  $c_b = \frac{1}{1+u_b}$  or (ii)  $c_b = 1 + u_b$ ; in (i) and (ii) we have  $u_b \sim \mathcal{U}(0, d_1)$
- with the same probability for the seller: (iii)  $c_s = 1 + u_s$  or (iv)  $c_s = \frac{1}{1+u_s}$ ; in (iii) and (iv) we have  $u_s \sim \mathcal{U}(0, d_2)$ .

With  $seed = 111$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$  and  $nCycles$  set to 10,000 we obtain exploding sequences of the means of the prices (mean in each cycle), and exploding standard deviation within each cycle (see Fig. 4 and 5).

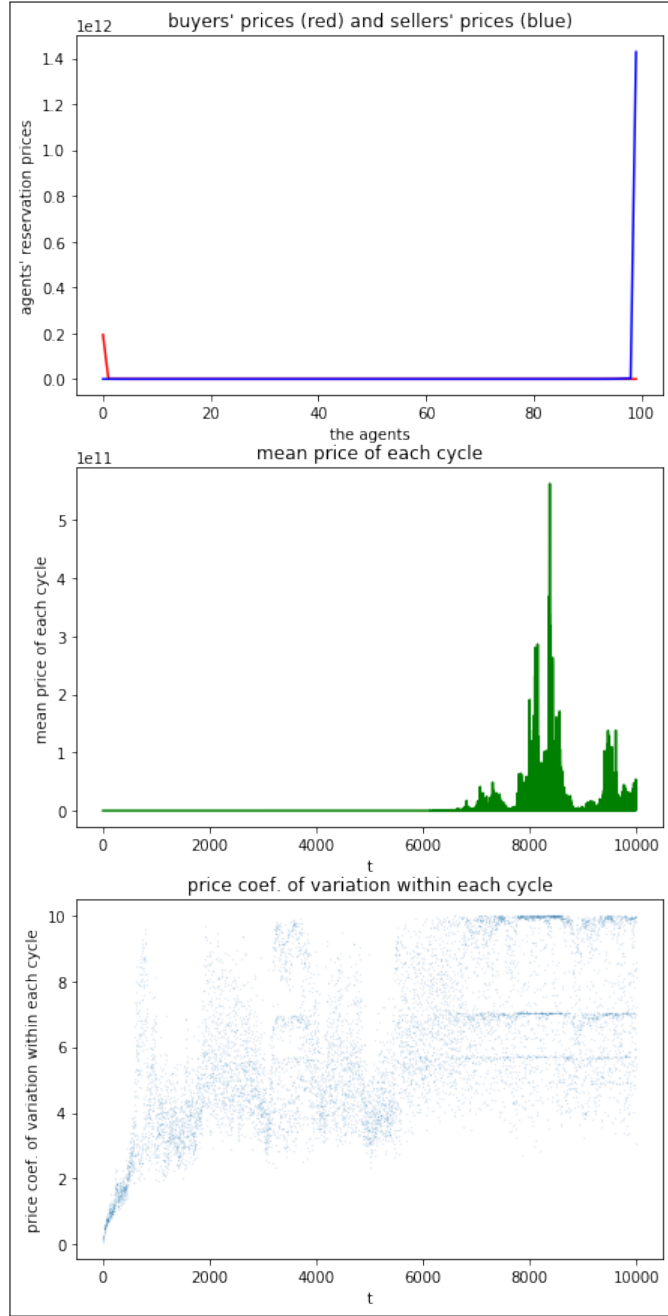


Figure 4: Unstructured case: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

The *coefficient of variation* at time  $t$  is calculated as:

$$\frac{\text{standard deviation}_t}{\text{mean}_t}$$

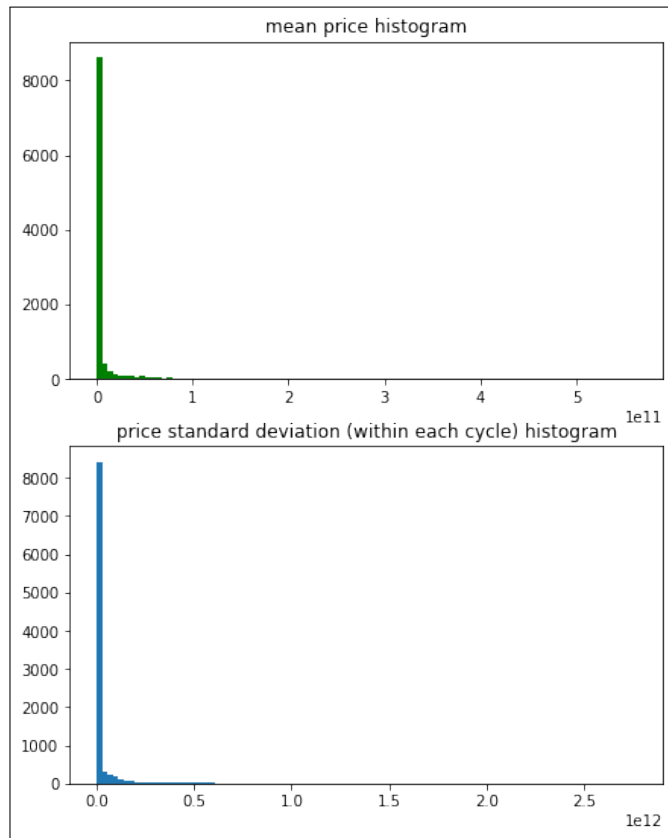


Figure 5: Unstructured case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A comment: this counter-example shows that, missing the *intelligence* in the correction of the prices (implicitly propagating among all the agents), a system of pure random price settings is absolutely far from being plausible.

The IPython (or Python 3.x) code is:

Listing 4: The unstructured version

```
# unstructured case

warmingUp()

out = Output()
display(out)
```

```

meanPrice_ts=[]
meanPriceStDev_ts=[]
meanPriceVar_ts=[]

if usingRatios and not usingSqueezeRate:
    if buyersSellersRatio>1: d2*=sellersBuyersRatio
    if sellersBuyersRatio>1: d1*=buyersSellersRatio

if usingRatios and usingSqueezeRate:
    if buyersSellersRatio>1: d2*=sellersBuyersRatio*squeezeRate
    if sellersBuyersRatio>1: d1*=buyersSellersRatio*squeezeRate

for t in range(1,nCycles+1):
    dealPrices=[]
    agNum=max(nBuyers,nSellers)
    for n in range(agNum):
        i = np.random.randint(0,nBuyers)
        j = np.random.randint(0,nSellers)

        dealPrices.append((sellerPriceList[j]+buyerPriceList[i]/0.5))

        if np.random.uniform(0,1)>=0.5:
            buyerPriceList[i] *=1/(1+np.random.uniform(0,d1))
            sellerPriceList[j]*=1+np.random.uniform(0,d2)
        else:
            buyerPriceList[i] *=1+np.random.uniform(0,d1)
            sellerPriceList[j]*=1/(1+np.random.uniform(0,d2))

    if len(dealPrices) > 2:
        meanPrice_ts.append(s.mean(dealPrices))
        meanPriceVar_ts.append(s.variance(dealPrices))
        meanPriceStDev_ts.append(s.stdev(dealPrices))
    else:
        meanPrice_ts.append(np.nan)
        meanPriceStDev_ts.append(np.nan)

if t % 1000==0:
    with out:
        clear_output()
    with out:
        print('time', t, 'and n. of exchanges in the last cycle', \
              len(dealPrices))
        print(\
              'mean and var of exchange prices in the last cycle: %1.3e, %1.3e' %\
              (meanPrice_ts[-1],meanPriceVar_ts[-1]))

plt.figure(3,figsize=(7,15),clear=True)

plt.subplot(311)
plt.plot(sorted(buyerPriceList,reverse=True),"r")
plt.plot(sorted(sellerPriceList),"b")
plt.title(\
    "buyers' prices (red) and sellers' prices (blue)")
xlabel("the agents")
ylabel("agents' reservation prices")

plt.subplot(312)
plt.title("mean price of each cycle")
xlabel("t")
ylabel("mean price of each cycle")
plt.plot(meanPrice_ts,"g")

```



```

plt.subplot(313)
plt.title("price_coef_of_variation_within_each_cycle")
coefOfVariation=[]
for m in range(len(meanPriceStDev_ts)):
    coefOfVariation.append(meanPriceStDev_ts[m]/
                           meanPrice_ts[m])
plt.plot(coefOfVariation, ".", markersize=0.1)
xlabel("t")
ylabel("price_coef_of_variation_within_each_cycle")
#plt.show() #activate to see intermediate plots
#time.sleep(0.1)

# hist crashes with NaN
meanPrice_ts_hist=[]
for k in range(len(meanPrice_ts)):
    if not math.isnan(meanPrice_ts[k]):
        meanPrice_ts_hist.append(meanPrice_ts[k])
meanPriceStDev_ts_hist=[]
for k in range(len(meanPriceStDev_ts)):
    if not math.isnan(meanPriceStDev_ts[k]):
        meanPriceStDev_ts_hist.append(meanPriceStDev_ts[k])
plt.figure(4,figsize=(7,9))
plt.subplot(211)
if meanPrice_ts_hist != []:
    plt.title("mean_price_histogram")
    plt.hist(meanPrice_ts_hist,100,color="g");
plt.subplot(212)
if meanPriceStDev_ts_hist != []:
    plt.title("price_standard_deviation_(within_each_cycle)_histogram")
    plt.hist(meanPriceStDev_ts_hist,100);

```

# Appendices

## Appendix A

# Two triple cases of not balancing numbers of buyers and sellers

### A.1 Case $nBuyers \gg nSellers$

With  $nBuyers \gg nSellers$  (e.g.,  $nBuyers = 100$  and  $nSellers = 50$ , as in Fig. A.1), we have three possible paths of analysis.

#### A.1.a Case $nBuyers \gg nSellers$ , with different rates of per-capita correction

If  $nBuyers \gg nSellers$ , we have in each cycle one call—in mean—to a *seller* from each *buyer*, the number of per-capita actions of the *sellers* in each cycle is greater of the number of per-capita actions of the *buyers*.

As a consequence, the probability that a *seller* decreases her price to meet that of a *buyer* is greater than the probability that a *buyer* increases her price to meet that of a *seller*.

We can observe that in Figs. A.2 and A.3 the prices are—in the end—lower than in Figs. 2 and 3 and, must of all, the price tendency has a strong negative slope. We always have  $d_0 = 0.1$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$ , and  $seed = 111$ .

This result is inconsistent with the microeconomic theory, where we could expect that an excess of demand will generate the rise of the prices.

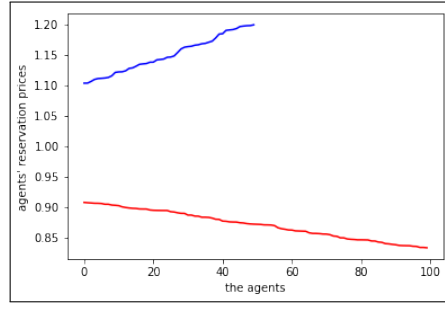


Figure A.1: An example of initial not overlapping demand curve and offer curve, case  $nBuyers \gg nSellers$

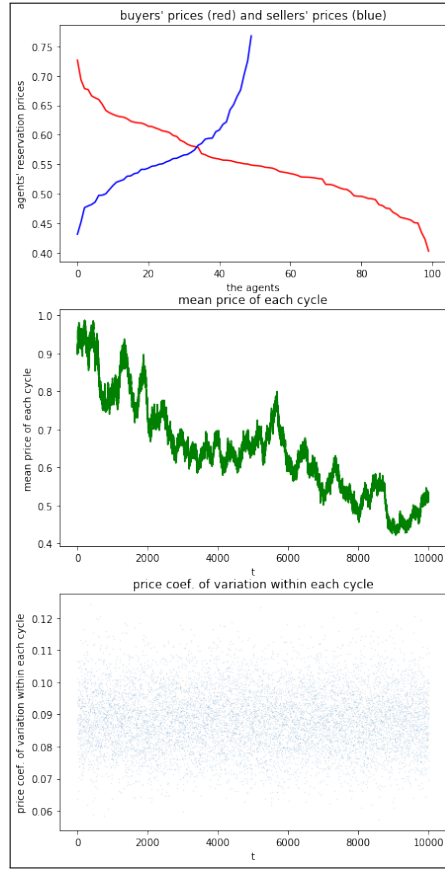


Figure A.2: Simplified Hayekian case, with  $nBuyers \gg nSellers$ : (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

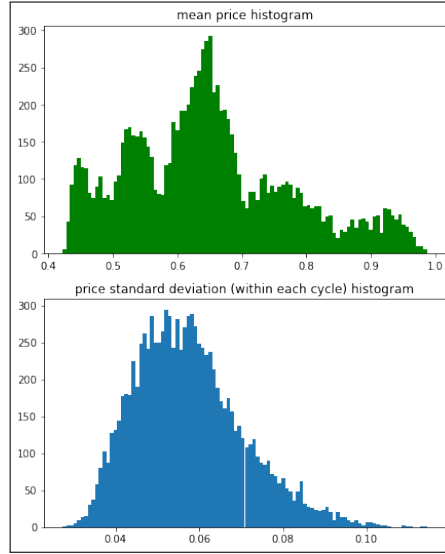


Figure A.3: Simplified Hayekian case, with  $nBuyers \gg nSellers$ : (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

#### A.1.b Case $nBuyers \gg nSellers$ , with unequal rates of per-capita correction, with equivalent effects

Again, with  $nBuyers \gg nSellers$ , and always having in each cycle one call—in mean—to a *seller* from each *buyer*, the number of per-capita actions of the *sellers* in each cycle is greater of the number of per-capita actions of the *buyers*.

In this second version of the case  $nBuyers \gg nSellers$ , we always have  $d_0 = 0.1$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$ , and  $seed = 111$ .

The novelty is that of setting *usingRatios* = *True*, so we are activating limitations to  $d_1$  or  $d_2$ .

The limitations work as follow:

- if the  $\frac{nBuyers}{nSellers} > 1$  (our case in this example),  $d_2$ , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by  $\frac{nSellers}{nBuyers}$ ,<sup>8</sup>
- if the  $\frac{nSellers}{nBuyers} > 1$ ,  $d_1$ , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by  $\frac{nBuyers}{nSellers}$ .

We have now unequal rates of per-capita correction, with equivalent effects. The interpretation is that if the number of sellers is smaller than the number of buyers, the sellers act with a slow pace of price correction (proportional to  $\frac{nSellers}{nBuyers}$ ) because in this way they can cherry-pick the best buyers (those with the higher reservation price). In this way, they avoid to contribute to the fall of the prices.

<sup>8</sup>An example to clarify: in this Section we have  $nBuyers = 100$  and  $nSellers = 50$ , so  $\frac{nBuyers}{nSellers} \equiv 2$  and  $\frac{nSellers}{nBuyers} \equiv 0.5$ ;  $d_2$  is reduced of the 50%.

Always with Fig. A.1 as the starting configuration of the prices, in Figs A.4 and A.5 we see now interesting price oscillations roughly confined between the limits of Fig. A.1.

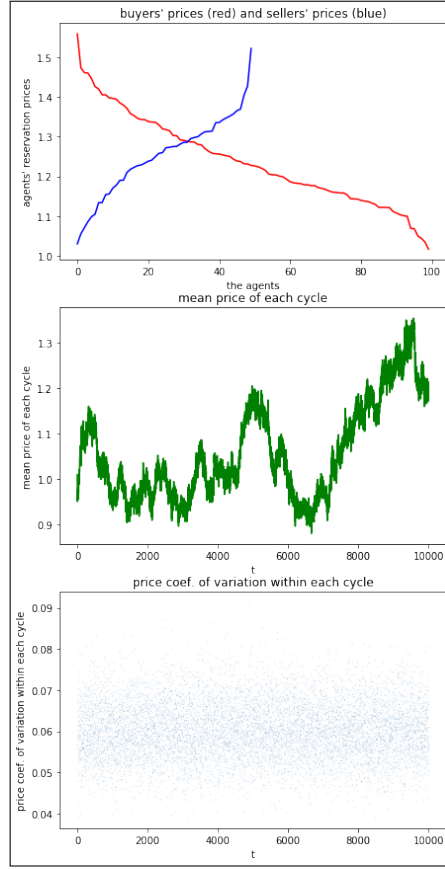


Figure A.4: Simplified Hayekian case, with  $nBuyers \gg nSellers$  but with equivalent effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

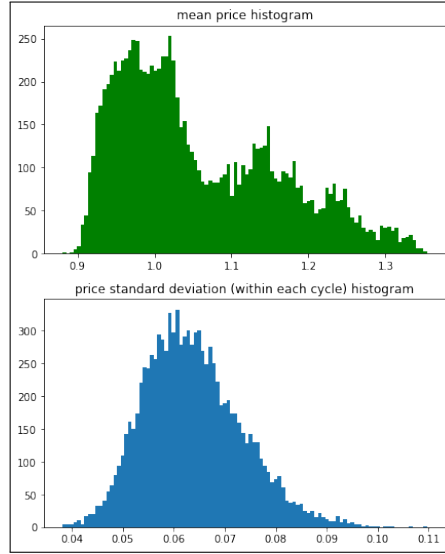


Figure A.5: Simplified Hayekian case, with  $nBuyers \gg nSellers$  but with equivalent effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

### A.1.c Case $nBuyers \gg nSellers$ , with unequal rates of per-capita correction, but squeezing the effects

In this third version of the case  $nBuyers \gg nSellers$ , we always have  $d_0 = 0.1$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$ , and  $seed = 111$ .

The second novelty, after that of Section A.1.b, is that of setting  $usingSqueeze = True$  (and setting  $usingRatios = True$  as in Section A.1.b), so we are activating further limitations to  $d_1$  or  $d_2$ . We also have  $squeezeRate = 0.3$ .

- if the  $\frac{nBuyers}{nSellers} > 1$  (our case in this example),  $d_2$ , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by  $squeezeRate$ ;
- if the  $\frac{nSellers}{nBuyers} > 1$ ,  $d_1$ , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by  $squeezeRate$ ;

Always with Fig. A.1 as the starting configuration of the prices, in Figs A.6 and A.7 we see a limited price dynamics, very close to the top band of Fig. A.1. This result is perfectly consistent with microeconomic theory.

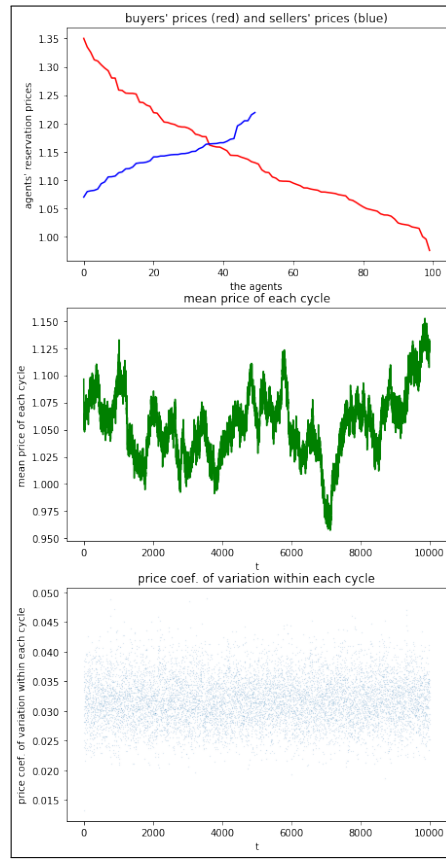


Figure A.6: Simplified Hayekian case, with  $nBuyers \gg nSellers$  but squeezing the effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)



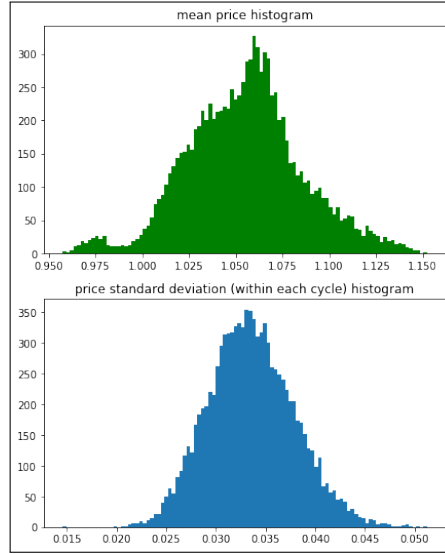


Figure A.7: Simplified Hayekian case, with  $nBuyers \gg nSellers$  but squeezing the effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

## A.2 Case $nBuyers \ll nSellers$

With  $nBuyers \ll nSellers$  (e.g.,  $nBuyers = 50$  and  $nSellers = 100$ , as in Fig. A.8), we again have three possible paths of analysis.

### A.2.a Case $nBuyers \ll nSellers$ , with different rates of per-capita correction

If  $nBuyers \ll nSellers$ , we have in each cycle one call—in mean—to a *buyer* from each *seller*, the number of per-capita actions of the *buyers* in each cycle is greater of the number of per-capita actions of the *sellers*.

As a consequence, the probability that a *buyer* increases her price to meet that of a *seller* is greater than the probability that a *seller* decreases her price to meet that of a *buyer*.

We can observe that in Figs. A.9 and A.10 the prices are—in the end—greater than in Figs. 2 and 3 and, must of all, the price tendency has a strong positive slope. We always have  $d_0 = 0.1$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$ , and  $seed = 111$ .

This result is inconsistent with the microeconomic theory, where we could expect that an excess of offer will generate the fall of the prices.

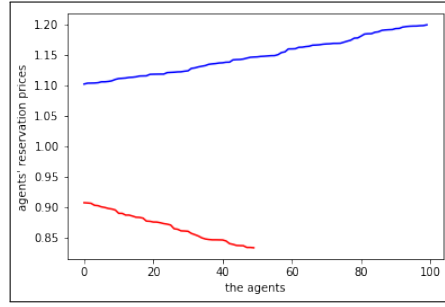


Figure A.8: An example of initial not overlapping demand curve and offer curve, case  $nBuyers \ll nSellers$

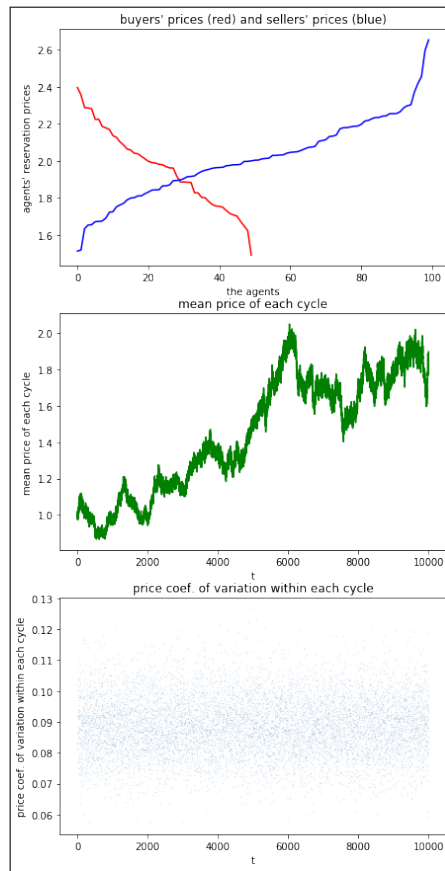


Figure A.9: Simplified Hayekian case, with  $nBuyers \ll nSellers$ : (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

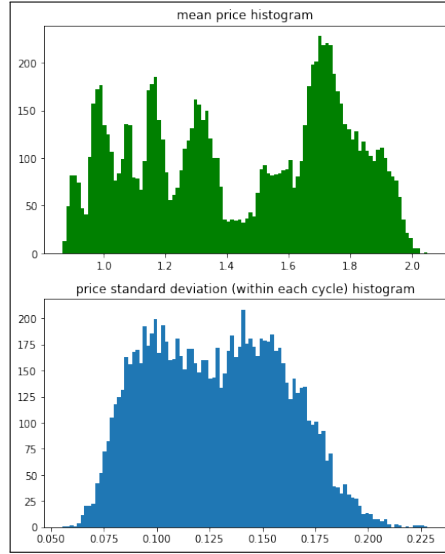


Figure A.10: Simplified Hayekian case, with  $nBuyers \ll nSellers$ : (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

### A.2.b Case $nBuyers \ll nSellers$ , with unequal rates of per-capita correction, with equivalent effects

In this second version of the case  $nBuyers \ll nSellers$ , we always have  $d_0 = 0.1$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$ , and  $seed = 111$ .

As in Section A.1.b, the novelty is that of setting  $usingRatios = True$ , so we are activating limitations to  $d_1$  or  $d_2$ .

The limitations work as follow:

- if the  $\frac{nBuyers}{nSellers} > 1$ ,  $d_2$ , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by  $\frac{nSellers}{nBuyers}$ ;
- if the  $\frac{nSellers}{nBuyers} > 1$  (our case in this example),  $d_1$ , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by  $\frac{nBuyers}{nSellers}$ .<sup>9</sup>

We have now unequal rates of per-capita correction, with equivalent effects. The interpretation is that if the number of buyers is smaller than the number of sellers, the buyers act with a slow pace of price correction (proportional to  $\frac{nBuyers}{nSellers}$ ) because in this way they can cherry-pick the best sellers (those with the lower reservation price). In this way, they avoid to contribute to the rise of the prices.

Always with Fig. A.8 as the starting configuration of the prices, in Figs A.11 and A.12 we see now a compressed price oscillations roughly close to the bottom limits of Fig. A.8.

<sup>9</sup>An example to clarify: in this Section we have  $nBuyers = 50$  and  $nSellers = 100$ , so  $\frac{nSellers}{nBuyers} \equiv 2$  and  $\frac{nBuyers}{nSellers} \equiv 0.5$ ;  $d_1$  is reduced of the 50%.

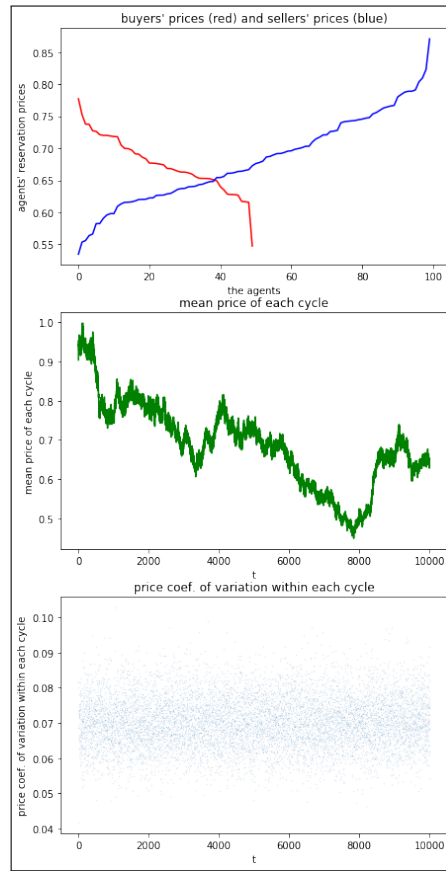


Figure A.11: Simplified Hayekian case, with  $n_{Buyers} \gg n_{Sellers}$  with equivalent effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

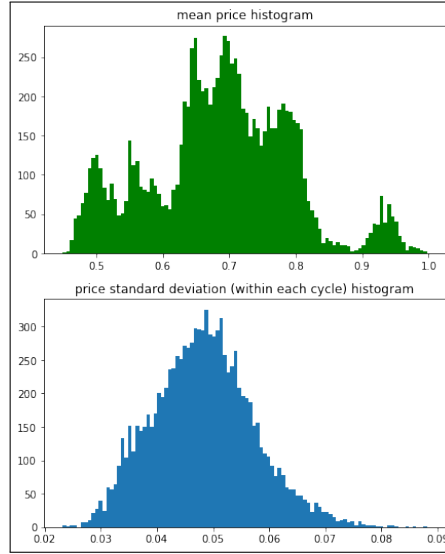


Figure A.12: Simplified Hayekian case, with  $nBuyers \gg nSellers$  but with equivalent effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

### A.2.c Case $nBuyers \ll nSellers$ , with unequal rates of per-capita correction, but squeezing the effects

In this third version of the case  $nBuyers \ll nSellers$ , we always have  $d_0 = 0.1$ ,  $d_1 = 0.2$ ,  $d_2 = 0.2$ , and  $seed = 111$ .

The second novelty, after that of Section A.2.b, is that of setting  $usingSqueeze = True$  (and setting  $usingRatios = True$  as in Section A.2.b), so we are activating further limitations to  $d_1$  or  $d_2$ . We also have  $squeezeRate = 0.3$ .

- if the  $\frac{nBuyers}{nSellers} > 1$ ,  $d_2$ , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by  $squeezeRate$ ;
- if the  $\frac{nSellers}{nBuyers} > 1$  (our case in this example),  $d_1$ , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by  $squeezeRate$ ;

Always with Fig. A.8 as the starting configuration of the prices, in Figs A.13 and A.14 we see a limited price dynamics, very close to the bottom band of Fig. A.8. This result is perfectly consistent with microeconomic theory.

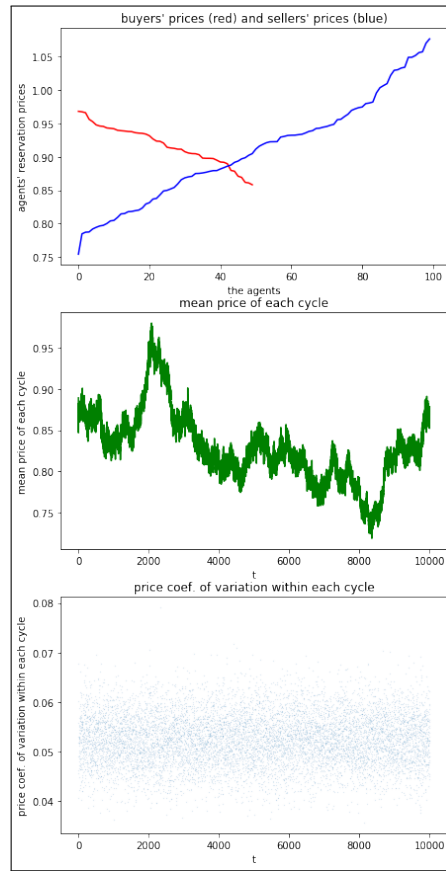


Figure A.13: Simplified Hayekian case, with  $nBuyers \gg nSellers$  but squeezing the effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

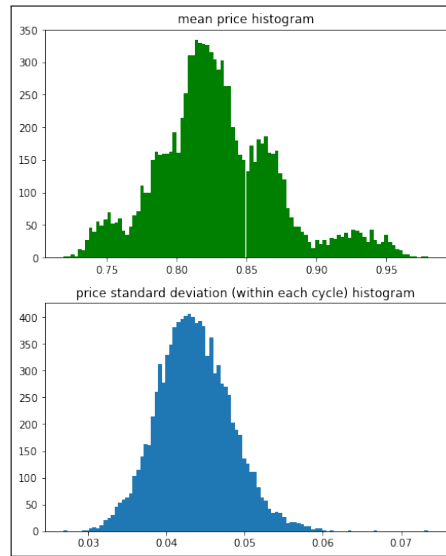


Figure A.14: Simplified Hayekian case, with  $nBuyers \gg nSellers$  but squeezing the effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

## Appendix B

# Trying to activate the *idle* agents, with a fundamental unexpected by-product

### B.1 Corrupting the simplified hayekian market model to activate the idle agents: an ingenuous quest for the solution

As the paper shows up to here, in all the runs of the model, looking at the demand and offer curves we can see that the larger part of the agents are not acting: all the sellers with reservation price greater than that of the crossing point of the curves and all the buyers with reservation price lower than that value.

Trying to activate those idle agents, we add two new parameters to the list of p. 4:

- *buyerThreshold* - over this number of failures, a special price correction occurs, multiplying the buyer price by  $c_b = 1 + u_b$  with  $u_b \sim \mathcal{U}(0, d_{1_{overT}})$ ;
- *sellerThreshold* - over this number of failures, a special price correction occurs, multiplying the seller price by  $c_s = \frac{1}{1+u_s}$  with  $u_s \sim \mathcal{U}(0, d_{2_{overT}})$ .

In this experiment we have *buyerThreshold* = 5, *sellerThreshold* = 50,  $d_{1_{overT}} = 0.4$ ,  $d_{2_{overT}} = 0.4$ , and *seed* = 111. We have *nBuyers* = *nSellers* = 100, so none of the corrections of Appendix A are working here.

As you can see in Listing B.1, Listing 2 is modified in the following way: now each agent holds a two element vector reporting her reservation price and a counter.

Listing B.1: Warming up for the corrupted version of the model

```
# warming up for corrupted version

# execute before the corrupted hayekian version
def warmingUp():
    global nCycles, nBuyers, nSellers, \
        buyersSellersRatio, sellersBuyersRatio, \
        usingRatios, usingSqueezeRate, squeezeRate, \
        d0, d1, d2, buyerPriceList, sellerPriceList, \
```



```

sellerThreshold, buyerThreshold,\
d1overT, d2overT, fig0

nCycles=10000
nBuyers= 100
nSellers=100

buyersSellersRatio=nBuyers/nSellers
sellersBuyersRatio=nSellers/nBuyers
usingRatios=True
squeezeRate=0.3 # always < 1
usingSqueezeRate=False

seed=111
np.random.seed(seed)

d0=0.1
d1=0.2
d2=0.2

d1overT=0.4
d2overT=0.4

sellerThreshold=50
buyerThreshold =5

buyerPriceList=[]
sellerPriceList=[]

for i in range(nBuyers):
    buyerPriceList.append([1/(1+np.random.uniform(d0,d1)),0])
for j in range(nSellers):
    sellerPriceList.append([1+np.random.uniform(d0,d2),0])

#zip is an iterator and * means iterable
fig0=plt.figure(0)
plt.plot(list(zip(*sorted(buyerPriceList,reverse=True)))[0],"r");
plt.plot(list(zip(*sorted(sellerPriceList)))[0],"b");
xlabel("the agents");
ylabel("agents' reservation prices");

```

Also Listing 3 is modified, to build Listing B.2, where the counter of each agent records the number of consecutive failures in buying or in selling.

In case of success the counter is set again to 0.

If the counter is *greater than* one of the thresholds, *buyerThreshold* or *sellerThreshold*, the buyer or the seller make a special correction of their reservation prices multiplying the buyer price by  $c_b = 1 + u_b$  (with  $u_b \sim \mathcal{U}(0, d_{1overT})$ ) or the seller price by  $c_s = \frac{1}{1+u_s}$  (with  $u_s \sim \mathcal{U}(0, d_{2overT})$ ).

The idea behind this construction is that of forcing the *idle* agents to operate. The effect is quite disappointing.

Considering that as a starting point we always have the situation of Fig. 1, in Figs B.1 and B.2 we simply see that in our case, with *buyerThreshold* = 5 and *sellerThreshold* = 50, the price series is higher than in Fig. 2 and that a lot of buyers (and sellers too, but here we focus on buyers) still are not operating. Our attempt of *corrupting* the simplified hayekian market to force the buyers to buy, has been a complete failure. But ... the next paragraph shows that it has been a source of inspiration.

Listing B.2: The model in the corrupted simplified hayekian perspective

*# corrupted simplified hayekian perspective*

```

warmingUp()

out = Output()
display(out)

meanPrice_ts=[]
meanPriceStDev_ts=[]
meanPriceVar_ts=[]

if usingRatios and not usingSqueezeRate:
    if buyersSellersRatio>1: d2*=sellersBuyersRatio
    if sellersBuyersRatio>1: d1*=buyersSellersRatio

if usingRatios and usingSqueezeRate:
    if buyersSellersRatio>1: d2*=sellersBuyersRatio*squeezeRate
    if sellersBuyersRatio>1: d1*=buyersSellersRatio*squeezeRate

for t in range(1,nCycles+1):
    dealPrices=[]
    agNum=max(nBuyers,nSellers)
    for n in range(agNum):

        i = np.random.randint(0,nBuyers)
        j = np.random.randint(0,nSellers)

        if buyerPriceList[i][0]>=sellerPriceList[j][0]:
            dealPrices.append(sellerPriceList[j][0])
            buyerPriceList[i][0] *=1/(1+np.random.uniform(0,d1))
            buyerPriceList[i][1] =0 # success
            sellerPriceList[j][0]*=1+np.random.uniform(0,d2)
            sellerPriceList[j][1] =0 # success
        else:
            buyerPriceList[i][0] *=1+np.random.uniform(0,d1)
            buyerPriceList[i][1] +=1 # failure
            sellerPriceList[j][0]*=1/(1+np.random.uniform(0,d2))
            sellerPriceList[j][1]+=1 # failure

        # corrections

        if buyerPriceList[i][1] > buyerThreshold:
            buyerPriceList[i][0] *=1+np.random.uniform(0,d1overT)
            buyerPriceList[i][1] =0
        if sellerPriceList[j][1] > sellerThreshold:
            sellerPriceList[j][0]*=1/(1+np.random.uniform(0,d2overT))
            sellerPriceList[j][1]=0

    if len(dealPrices) > 2:
        meanPrice_ts.append(s.mean(dealPrices))
        meanPriceVar_ts.append(s.variance(dealPrices))
        meanPriceStDev_ts.append(s.stdev(dealPrices))
    else:
        meanPrice_ts.append(np.nan)
        meanPriceStDev_ts.append(np.nan)

if t % 1000==0:
    with out:
        clear_output()
    with out:
        print('time', t, 'and n. of exchanges in the last cycle', \

```

```

        len(dealPrices))
    print(\\
'mean_and_var_of_exchange_prices_in_the_last_cycle:%1.3e,%1.3e' %\\
        (meanPrice_ts[-1],meanPriceVar_ts[-1]))

fig1=plt.figure(1,figsize=(7,15),clear=True)

plt.subplot(311)
plt.plot(list(zip(*sorted(buyerPriceList, reverse=True))))[0], "r")
plt.plot(list(zip(*sorted(sellerPriceList))))[0], "b")
plt.title(\\
    "buyers'prices(red)and_sellers'prices(blue)")
xlabel("the_agents")
ylabel("agents'reservation_prices")

plt.subplot(312)
plt.title("mean_price_of_each_cycle")
xlabel("t")
ylabel("mean_price_of_each_cycle")
plt.plot(meanPrice_ts, "g")

plt.subplot(313)
plt.title("price_coef_of_variation_within_each_cycle")
coefOfVariation=[]
for m in range(len(meanPriceStDev_ts)):
    coefOfVariation.append(meanPriceStDev_ts[m]/
                            meanPrice_ts[m])
plt.plot(coefOfVariation, ".", markersize=0.1)
xlabel("t")
ylabel("price_coef_of_variation_within_each_cycle")
#plt.show() #activate to see intermediate plots
#time.sleep(1)

# hist crashes with NaN
meanPrice_ts_hist=[]
for k in range(len(meanPrice_ts)):
    if not math.isnan(meanPrice_ts[k]):
        meanPrice_ts_hist.append(meanPrice_ts[k])
meanPriceStDev_ts_hist=[]
for k in range(len(meanPriceStDev_ts)):
    if not math.isnan(meanPriceStDev_ts[k]):
        meanPriceStDev_ts_hist.append(meanPriceStDev_ts[k])
fig2=plt.figure(2,figsize=(7,9))
plt.subplot(211)
if meanPrice_ts_hist != []:
    plt.title("mean_price_histogram")
    plt.hist(meanPrice_ts_hist,100,color="g");
plt.subplot(212)
if meanPriceStDev_ts_hist != []:
    plt.title("price_standard_deviation_(within_each_cycle)_histogram")
    plt.hist(meanPriceStDev_ts_hist,100);

```

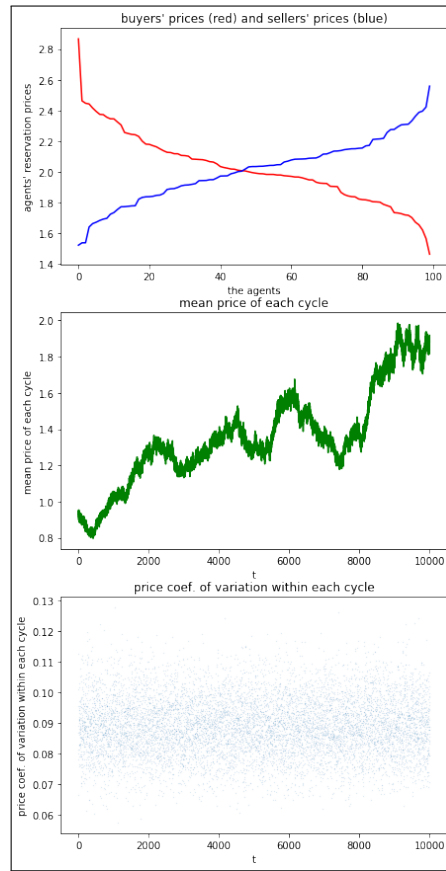


Figure B.1: Corrupting the Simplified Hayekian case: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

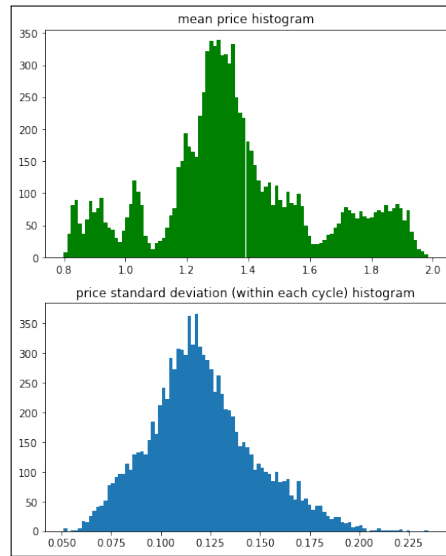


Figure B.2: Corrupting the Simplified Hayekian case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

Observing now the Fig. [B.3](#), that we generate with the code of the Listing

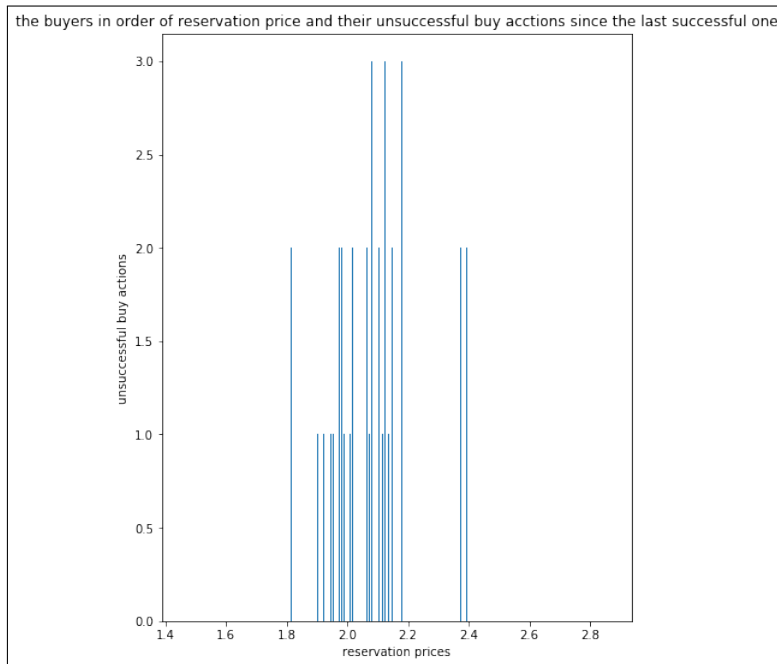


Figure B.3: Extending the Simplified Hayekian case: after 10,000 actions we see here a *flash* image of the buyers ordered following their reservation prices, with the number of recent consecutive unsuccessful buying attempts

## B.2 A fundamental unexpected by-product: the solution is here, under our eyes

Fixed!!!! seconda presentazione con la scoperta che sempre tutti comperano solo circa il 50% dei casi ...

As starting point we always have the situation of Fig. 1. In Figs B.4 and B.5 doppio periodo

Must of all, the bar plot in Fig. B.6 demonstrates that

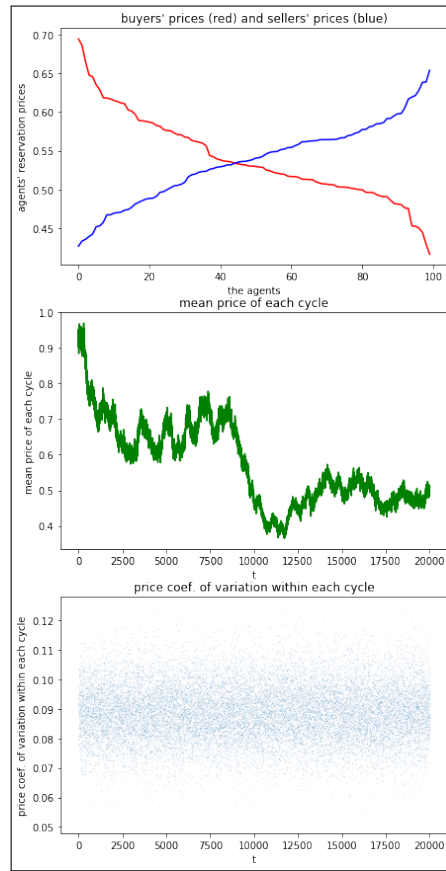


Figure B.4: Extending the Simplified Hayekian case: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

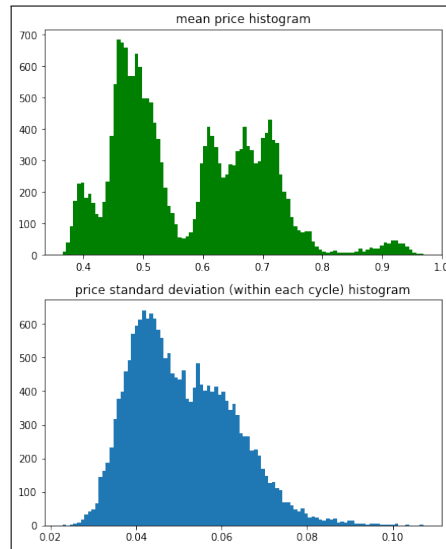


Figure B.5: Extending the Simplified Hayekian case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)



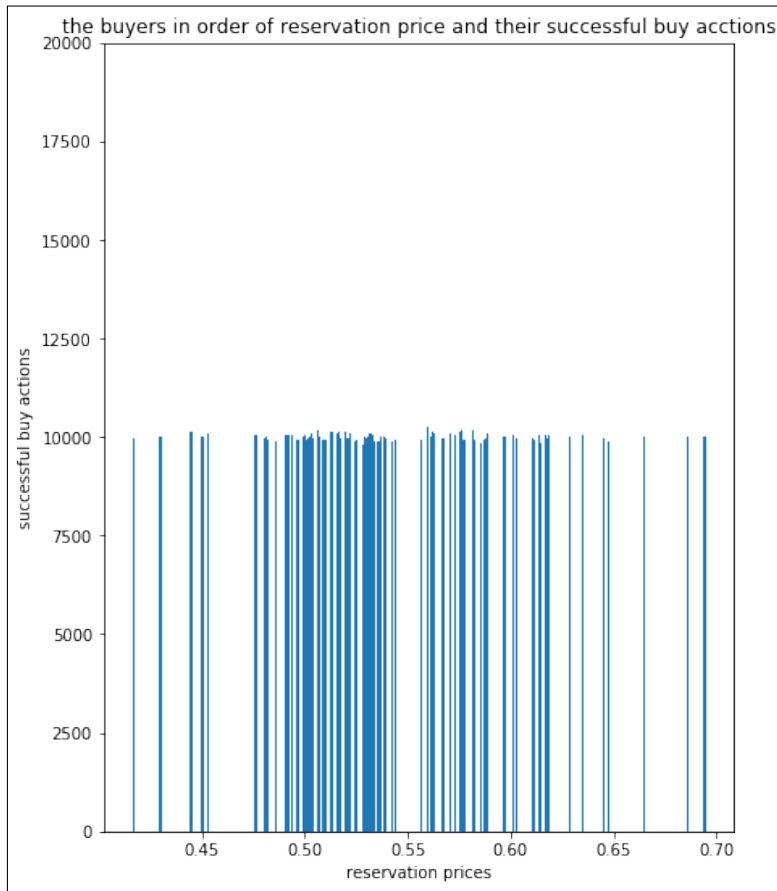


Figure B.6: Extending the Simplified Hayekian case: after 20,000 actions, each buyer (here the buyers are ordered following their reservation prices) has been successful in concluding a deal only in about 10,000 cases

### B.3 Final considerations related to the Oligopoly project

The key point for the developers of the [Oligopoly](https://terna.github.io/oligopoly/)<sup>10</sup> project are:

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<sup>10</sup><https://terna.github.io/oligopoly/>

# Bibliography

Bowles, S., Kirman, A. and Sethi, R. (2017). *Retrospectives: Friedrich Hayek and the Market Algorithm*. In «Journal of Economic Perspectives», vol. 31(3), pp. 215-30.  
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