Micro Simplified Hayekian Market

Matteo Morini 1 and Pietro Terna 2 August 7, 2018

 $^{^1{\}rm University}$ of Torino, Italia $^2{\rm University}$ of Torino, Italia

Abstract

We propose a simplified version of the Hayek's decentralized market hypothesis, considering elementary processes of price adaptation in exchanges.

Sections 1 and 2 report the technical setup and the structure of the model. In Section 3 we introduce an elementary agent-based model of a market, with emergent (quite interesting) price dynamics.

A counter example is also introduced in Section 4, showing that—with tiny modification—we generate implausible price dynamics.

In Appendix A we report some technical analyses of the cases with unmatching numbers of buyers and sellers. These analyses are strongly related the the Oligopoly¹ simulation project.

In Appendix B we deepen the problem of the agents that seem to be idle, having the level of their reservation price too low to buy. The Appendix is mainly of interest for the developers of the Oligopoly project.

¹Clic to go to https://terna.github.io/oligopoly/

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Introduction to a Micro Simplified Hayekian Market

The purpose of the note is that of introducing a very simple agent-based model of a market, with emergent (quite interesting) price dynamics.

A counter example is also introduced, showing how—with tiny modifications—we generate implausible/impossible price dynamics.

The code uses the IPython² language (an interactive layer upon Python³ using the Jupyter⁴ infrastructure) and can be dowloaded from https://github.com/terna/microHayekianMarket via the *Clone or download* button; it is also possible to run it directly on line thanks to the MyBinder project⁵.

A suggested reading about Hayek is a quite recent paper of Bowles *et al.* (2017). Quoting from the introduction:

Friedrich A. Hayek (1899-1992) is known for his vision of the market economy as an information processing system characterized by spontaneous order, the emergence of coherence through the independent actions of large numbers of individuals each with limited and local knowledge, coordinated by prices that arise from decentralized processes of competition.

A simplified version—proposed here—is that of considering decentralized elementary processes of price adaptation in exchanges, with surprising results.

1 The technical setup

The IPython (or Python 3.x) code requires the following setup to start:

Listing 1: Setup of the program

```
import time
import math
```

%pylab inline is a magic command of Jupyter.

2 The structure of the model and the warming up phase

Our agents are simply prices, to be interpreted as reservation prices.⁶

We have two price vectors: pL^b with item pL_i^b for the buyers, and pL^s with item pL_j^s for the sellers. The i^{th} or the j^{th} elements of the vectors are prices, but we can use them also as agents.

Both in the simplified hayekian perspective (Section 3) and in the unstructured one (Section 4) we have to pre-run a *warning up* action. This happens automatically, calling the specific function in the beginning of both the cases.

With the warming up phase, we define:

- d_0 the lower bound of the random uniform numbers, both for the buyers and the sellers, in the warming up phase;
 - in the running phase, the lower bound is 0;
- ullet d₁ the upper bound of the random uniform numbers for the buyers;
- d_2 the upper bound of the random uniform numbers for the sellers;
- nCycles number of simulation cycles;
- *nBuyers* number of the buyers;
- nSellers number of the sellers:
- seed the seed of the random numbers;
- the initial buyer *i* reservation price, different for each buyer: $p_{b,i} = \frac{1}{1+u_i}$ with $u_i \sim \mathcal{U}(d_0, d_1)$;
- the initial seller j reservation price, different for each seller: $p_{s,j} = 1 + u_j$ with $u_j \sim \mathcal{U}(d_0, d_2)$;
- buyersSellersRatio the ratio $\frac{nBuyers}{nSellers}$;
- sellersBuyersRatio the ratio $\frac{nSellers}{nBuyers}$;
- usingRatios a logic variable activating limitations to d_1 or d_2
- squeezeRate always < 1, as further compression of d_1 or d_2
- usingSqueezeRate a logic variable to further squeeze d_1 or d_2

With $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, sorting in decreasing order the vector pL^b and in increasing order the vector pL^s , we obtain in Fig. 1 two not overlapping price sequences that we can interpret as a demand curve (red) and an offer one (blue).

⁶The max price a buyer could pay and the min one a seller could accept.

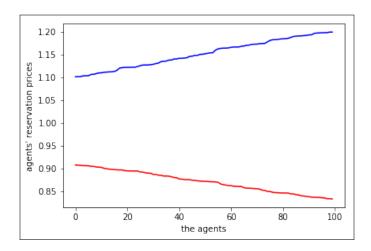


Figure 1: An example of initial not overlapping demand curve (red) and offer curve (blue)

To generate new examples related to Section 3 and to Section 4, it is necessary to repeat this phase. This happens automatically, calling the specific function in the beginning of both the cases.

The IPython (or Python 3.x) code is:

Listing 2: Warming up of the model

```
warming up
  execute before both:
                         the hayekian perspective or
                        the unstructured case
def warmingUp():
    global nCycles, nBuyers, nSellers,\
        buyersSellersRatio, sellersBuyersRatio,\
        usingRatios, usingSqueezeRate, squeezeRate,
        d0, d1, d2, buyerPriceList, sellerPriceList
    nCycles=10000
    nBuyers= 50
    nSellers=100
    buyersSellersRatio=nBuyers/nSellers
    sellersBuyersRatio=nSellers/nBuyers
    usingRatios=True
    squeezeRate=0.3 # always < 1
    usingSqueezeRate=False
    seed=111
    np.random.seed(seed)
    d0 = 0.1
    d1 = 0.2
    d2=0.2
    buyerPriceList=[]
    sellerPriceList=[]
```

```
for i in range(nBuyers):
        buyerPriceList.append(1/(1+np.random.uniform(d0,d1)))
for j in range(nSellers):
        sellerPriceList.append(1+np.random.uniform(d0,d2))

plt.figure(0)
plt.plot(sorted(buyerPriceList,reverse=True),"r");
plt.plot(sorted(sellerPriceList),"b");
xlabel("the_uagents");
ylabel("agents'ureservation_prices");
```

3 The simplified hayekian version

The buyers and the sellers meet randomly. Buyer i and seller j exchange if $pL_i^b \ge pL_j^s$; the deal is recorded at the price of the seller pL_i^s .

In this version, representing the key point in this note, the running prices are multiplied in each cycle by the following correction coefficients:

- for the buyer: (i) $c_b = \frac{1}{1+u_b}$ if the deal succeeds (trying to pay less next time) or (ii) $c_b = 1 + u_b$ if the deal fails (preparing to pay more next time); in (i) and (ii) we have $u_b \sim \mathcal{U}(0, d_1)$
- for the seller: (iii) $c_s = 1 + u_s$ if the deal succeeds (trying to obtain a higher revenue next time) or (iv) $c_s = \frac{1}{1+u_s}$ if the deal fails (preparing to obtain a lower revenue next time); in (iii) and (iv) we have $u_s \sim \mathcal{U}(0, d_2)$.

With seed = 111, $d_1 = 0.2$, $d_2 = 0.2$ and nCycles set to 10,000 we obtain sequences of mean prices (mean within each cycle) quite realistic, with a very low variance within each cycle (see Fig. 2 and 3).

⁷In the *mall*, sell prices are public.

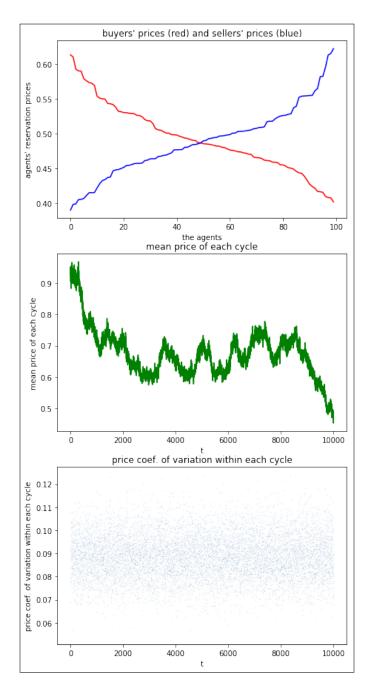


Figure 2: Simplified Hayekian case: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

The coefficient of variation at time t is calculated as:

 $\frac{standard\ deviation_t}{mean_t}$

.

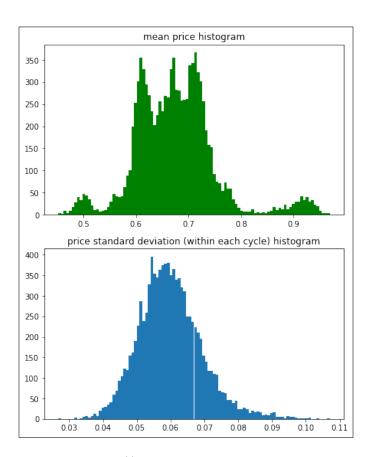


Figure 3: Simplified Hayekian case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A comment: we have a plausible series of mean prices, with a complicated behavior, and with a high stability of the dispersion of the values within each cycle.

The right side of the buyer and seller curves shows another plausible situation: that of the presence of agents not exchanging. A note for Matteo and Pietro: this is a very important emergent effect for the Oligopoly model.

Have a look to the Appendix ?? for the cases of not balancing number of buyers and sellers. The IPython (or Python 3.x) code is:

Listing 3: The model in the simplified hayekian perspective

hayekian perspective

warmingUp()

```
out = Output()
display(out)
meanPrice_ts=[]
meanPriceStDev_ts=[]
meanPriceVar_ts=[]
if usingRatios and not usingSqueezeRate:
    if buyersSellersRatio >1: d2*=sellersBuyersRatio
    if sellersBuyersRatio >1: d1*=buyersSellersRatio
if usingRatios and usingSqueezeRate:
    if buyersSellersRatio >1: d2*=sellersBuyersRatio*squeezeRate
    if sellersBuyersRatio>1: d1*=buyersSellersRatio*squeezeRate
for t in range(1,nCycles+1):
    dealPrices=[]
    agNum=max(nBuyers,nSellers)
    for n in range(agNum):
        i = np.random.randint(0,nBuyers)
        j = np.random.randint(0,nSellers)
         if buyerPriceList[i]>=sellerPriceList[j]:
             dealPrices.append(sellerPriceList[j])
             buyerPriceList[i] *=1/(1+np.random.uniform(0,d1))
             sellerPriceList[j]*=1+np.random.uniform(0,d2)
         else:
             buyerPriceList[i] *=1+np.random.uniform(0,d1)
             sellerPriceList[j]*=1/(1+np.random.uniform(0,d2))
    if len(dealPrices) > 2:
        meanPrice_ts.append(s.mean(dealPrices))
        meanPriceVar_ts.append(s.variance(dealPrices))
        meanPriceStDev_ts.append(s.stdev(dealPrices))
    else:
        meanPrice_ts.append(np.nan)
        meanPriceStDev_ts.append(np.nan)
    if t % 1000==0:
        with out:
             clear_output()
        with out:
             print('time', t, 'and _{\square}n. _{\square}of _{\square}exchanges _{\square}in _{\square}the _{\square}last _{\square}cycle', \setminus
               len(dealPrices))
             print(\
         (meanPrice_ts[-1], meanPriceVar_ts[-1]))
        plt.figure(1,figsize=(7,15),clear=True)
        plt.subplot(311)
        plt.plot(sorted(buyerPriceList,reverse=True),"r")
        plt.plot(sorted(sellerPriceList),"b")
        plt.title(\
             "buyers' _{\sqcup} prices _{\sqcup} (red) _{\sqcup} and _{\sqcup} sellers' _{\sqcup} prices _{\sqcup} (blue)")
        \verb|xlabel("the|| agents")|\\
        {\tt ylabel("agents'_{\sqcup}reservation_{\sqcup}prices")}
        plt.subplot(312)
```

```
plt.title("meanupriceuofueachucycle")
         xlabel("t")
        {\tt ylabel("mean\_price\_of\_each\_cycle")}
         plt.plot(meanPrice_ts,"g")
        plt.subplot(313)
        plt.title("price_{\square}coef._{\square}of_{\square}variation_{\square}within_{\square}each_{\square}cycle")
         coefOfVariation=[]
         for m in range(len(meanPriceStDev_ts)):
             coefOfVariation.append(meanPriceStDev_ts[m]/
                                       meanPrice_ts[m])
         plt.plot(coefOfVariation,".",markersize=0.1)
         xlabel("t")
         \verb|ylabel("price_ucoef._uof_uvariation_within_ueach_ucycle")| \\
         #plt.show() #activate to see intermadiate plots
         #time.sleep(0.1)
# hist crashes with NaN
meanPrice_ts_hist=[]
for k in range(len(meanPrice_ts)):
    if not math.isnan(meanPrice_ts[k]):
        meanPrice_ts_hist.append(meanPrice_ts[k])
meanPriceStDev_ts_hist=[]
for k in range(len(meanPriceStDev_ts)):
    if not math.isnan(meanPriceStDev_ts[k]):
        meanPriceStDev_ts_hist.append(meanPriceStDev_ts[k])
plt.figure(2,figsize=(7,9))
plt.subplot(211)
if meanPrice_ts_hist != []:
    {\tt plt.title("mean_{\sqcup}price_{\sqcup}histogram")}
    plt.hist(meanPrice_ts_hist,100,color="g");
plt.subplot(212)
if meanPriceStDev_ts_hist != []:
    \tt plt.title("price\_standard\_deviation\_(within\_each\_cycle)\_histogram")
    plt.hist(meanPriceStDev_ts_hist,100);
```

4 The unstructured version

The buyers and the sellers meet randomly as in Section 3. Buyer i and seller j exchange in any case; the deal is recorded at the mean of the price of the seller pL_j^s and of the price pL_i^b of the buyer.

In this version the running prices are are multiplied in each cycle by the following correction coefficients:

- with the same probability for the buyer: (i) $c_b = \frac{1}{1+u_b}$ or (ii) $c_b = 1+u_b$); in (i) and (ii) we have $u_b \sim \mathcal{U}(0, d_1)$
- with the same probability for the seller: (iii) $c_s = 1 + u_s$ or (iv) $c_s = \frac{1}{1 + u_s}$; in (iii) and (iv) we have $u_s \sim \mathcal{U}(0, d_2)$.

With seed = 111, $d_1 = 0.2$, $d_2 = 0.2$ and nCycles set to 10,000 we obtain exploding sequences of the means of the prices (mean in each cycle), and exploding standard deviation within each cycle (see Fig. 4 and 5).

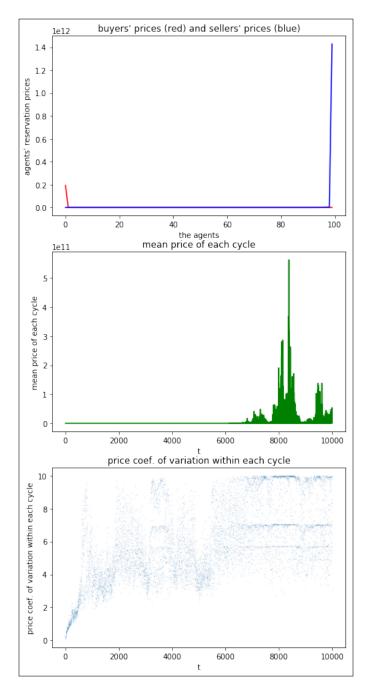


Figure 4: Unstructured case: ((i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

The $coefficient\ of\ variation$ at time t is calculated as:

 $\frac{standard\ deviation_t}{mean_t}$

.

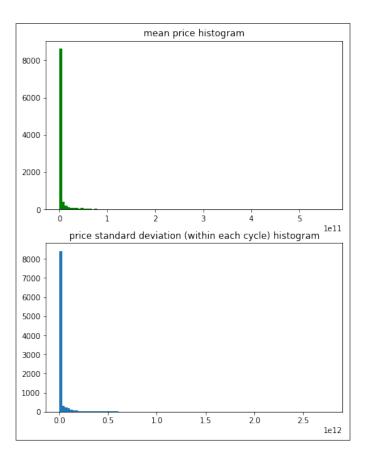


Figure 5: Unstructured case: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A comment: this counter-example shows that, missing the *intelligence* in the correction of the prices (implicitly propagating among all the agents), a system of pure random price settings is absolutely far from being plausible.

The IPython (or Python 3.x) code is:

Listing 4: The unstructured version

```
# unstructured case
warmingUp()
out = Output()
display(out)
```

```
meanPrice_ts=[]
meanPriceStDev_ts=[]
meanPriceVar_ts=[]
if usingRatios and not usingSqueezeRate:
    if buyersSellersRatio >1: d2*=sellersBuyersRatio
    if sellersBuyersRatio >1: d1*=buyersSellersRatio
if usingRatios and usingSqueezeRate:
    if buyersSellersRatio >1: d2*=sellersBuyersRatio*squeezeRate
    if sellersBuyersRatio >1: d1*=buyersSellersRatio*squeezeRate
for t in range(1,nCycles+1):
    dealPrices=[]
    agNum=max(nBuyers,nSellers)
    for n in range(agNum):
        i = np.random.randint(0,nBuyers)
        j = np.random.randint(0,nSellers)
        dealPrices.append((sellerPriceList[j]+buyerPriceList[i]/0.5))
        if np.random.uniform(0,1)>=0.5:
             buyerPriceList[i] *=1/(1+np.random.uniform(0,d1))
             sellerPriceList[j]*=1+np.random.uniform(0,d2)
         else:
             buyerPriceList[i] *=1+np.random.uniform(0,d1)
             sellerPriceList[j]*=1/(1+np.random.uniform(0,d2))
    if len(dealPrices) > 2:
        meanPrice_ts.append(s.mean(dealPrices))
        meanPriceVar_ts.append(s.variance(dealPrices))
        meanPriceStDev_ts.append(s.stdev(dealPrices))
        meanPrice_ts.append(np.nan)
        meanPriceStDev_ts.append(np.nan)
    if t % 1000==0:
        with out:
             clear_output()
        with out:
             print('time', t, 'and_{\square}n._{\square}of_{\square}exchanges_{\square}in_{\square}the_{\square}last_{\square}cycle', \
               len(dealPrices))
             print(\
         'meanuanduvaruofuexchangeupricesuinutheulastucycle:u%1.3e,u%1.3e' %\
               (meanPrice_ts[-1], meanPriceVar_ts[-1]))
        plt.figure(3,figsize=(7,15),clear=True)
        plt.subplot(311)
        plt.plot(sorted(buyerPriceList,reverse=True),"r")
        plt.plot(sorted(sellerPriceList),"b")
        plt.title(\
             "buyers' _{\sqcup} prices _{\sqcup} (red) _{\sqcup} and _{\sqcup} sellers' _{\sqcup} prices _{\sqcup} (blue) ")
        xlabel("the_agents")
        ylabel("agents', reservation prices")
        plt.subplot(312)
        {\tt plt.title("mean\_price\_of\_each\_cycle")}
        xlabel("t")
        ylabel("meanupriceuofueachucycle")
        plt.plot(meanPrice_ts,"g")
```

```
plt.subplot(313)
         \verb|plt.title("price_{\sqcup}coef._{\sqcup}of_{\sqcup}variation_{\sqcup}within_{\sqcup}each_{\sqcup}cycle")|
         coefOfVariation=[]
         for m in range(len(meanPriceStDev_ts)):
              {\tt coefOfVariation.append(meanPriceStDev\_ts[m]/}
                                        meanPrice_ts[m])
         plt.plot(coefOfVariation,".",markersize=0.1)
         xlabel("t")
         ylabel("price_coef._of_variation_within_each_cycle")
         \mbox{\#plt.show()} \mbox{\#activate to see intermadiate plots}
         \#time.sleep(0.1)
# hist crashes with NaN
meanPrice_ts_hist=[]
for k in range(len(meanPrice_ts)):
    if not math.isnan(meanPrice_ts[k]):
         meanPrice_ts_hist.append(meanPrice_ts[k])
meanPriceStDev_ts_hist=[]
for k in range(len(meanPriceStDev_ts)):
    if not math.isnan(meanPriceStDev_ts[k]):
         meanPriceStDev_ts_hist.append(meanPriceStDev_ts[k])
plt.figure(4,figsize=(7,9))
plt.subplot(211)
if meanPrice_ts_hist != []:
    plt.title("mean_{\perp}price_{\perp}histogram")
    plt.hist(meanPrice_ts_hist,100,color="g");
plt.subplot(212)
if meanPriceStDev_ts_hist != []:
    \verb|plt.title("price_{\sqcup} standard_{\sqcup} deviation_{\sqcup}(within_{\sqcup} each_{\sqcup} cycle)_{\sqcup} histogram")|
    plt.hist(meanPriceStDev_ts_hist,100);
```

Appendices

Appendix A

Two triple cases of not balancing numbers of buyers and sellers

A.1 Case $nBuyers \gg nSellers$

With $nBuyers \gg nSellers$ (e.g., nBuyers = 100 and nSellers = 50, as in Fig. A.1), we have three possible paths of analysis.

A.1.a Case $nBuyers \gg nSellers$, with different rates of per-capita correction

If $nBuyers \gg nSellers$, we have in each cycle one call—in mean—to a *seller* from each *buyer*, the number of per-capita actions of the *sellers* in each cycle is greater of the number of per-capita actions of the *buyers*.

As a consequence, the probability that a *seller* decreases her price to meet that of a *buyer* is greater than the probability that a *buyer* increases her price to meet that of a *seller*.

We can observe that in Figs. A.2 and A.3 the prices are—in the end—lower than in Figs. 2 and 3 and, must of all, the price tendency has a strong negative slope. We always have $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, and seed = 111.

This result is inconsistent with the microeconomic theory, where we could expect that an excess of demand will generate the rise of the prices.

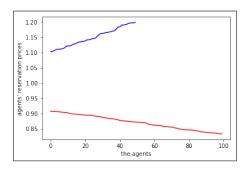


Figure A.1: An example of initial not overlapping demand curve and offer curve, case $nBuyers\gg nSellers$

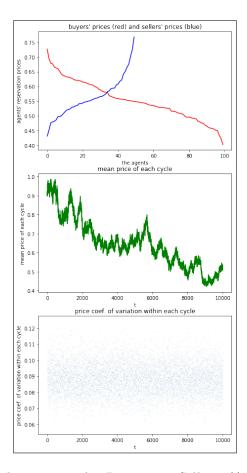


Figure A.2: Simplified Hayekian case, with $nBuyers \gg nSellers$: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

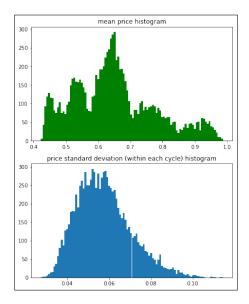


Figure A.3: Simplified Hayekian case, with $nBuyers \gg nSellers$: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A.1.b Case $nBuyers \gg nSellers$, with unequal rates of per-capita correction, with equivalent effects

Again, with $nBuyers \gg nSellers$, and always having in each cycle one call—in mean—to a seller from each buyer, the number of per-capita actions of the sellers in each cycle is greater of the number of per-capita actions of the buyers.

In this second version of the case $nBuyers \gg nSellers$, we always have $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, and seed = 111.

The novelty is that of setting usingRatios = True, so we are activating limitations to d_1 or d_2 .

The limitations work as follow:

- if the $\frac{nBuyers}{nSellers} > 1$ (our case in this example), d_2 , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by $\frac{nSellers}{nBuyers}$;8
- if the $\frac{nSellers}{nBuyers} > 1$, d_1 , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by $\frac{nBuyers}{nSellers}$.

We have now unequal rates of per-capita correction, with equivalent effects. The interpretation is that if the number of sellers is smaller than the number of buyers, the sellers act with a slow pace of price correction (proportional to $\frac{nSellers}{nBuyers}$) because in this way they can cherry-pick the best buyers (those with the higher reservation price). In this way, they avoid to contribute to the fall of the prices.

⁸An example to clarify: in this Section we have nBuyers=100 and nSellers=50, so $\frac{nBuyers}{nSellers}\equiv 2$ and $\frac{nSellers}{nBuyers}\equiv 0.5$; d_2 is reduced of the 50%.

Always with Fig. A.1 as the starting configuration of the prices, in Fig.s A.4 and A.5 we see now interesting price oscillations roughly confined between the limits of Fig. A.1.

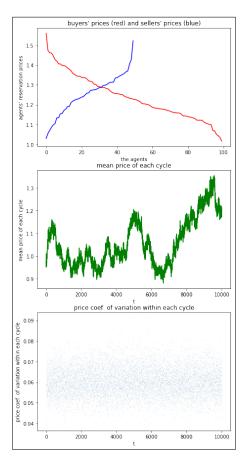


Figure A.4: Simplified Hayekian case, with $nBuyers \gg nSellers$ but with equivalent effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

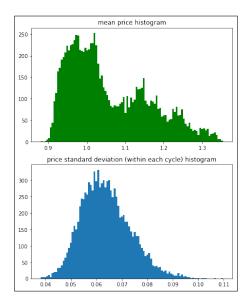


Figure A.5: Simplified Hayekian case, with $nBuyers \gg nSellers$ but with equivalent effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A.1.c Case $nBuyers \gg nSellers$, with unequal rates of per-capita correction, but squeezing the effects

In this third version of the case $nBuyers \gg nSellers$, we always have $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, and seed = 111.

The second novelty, after that of Section A.1.b, is that of setting usingSqueeze = True (and setting usingRatios = True as in Section A.1.b), so we are activating further limitations to d_1 or d_2 . We also have squeezeRate = 0.3.

- if the $\frac{nBuyers}{nSellers} > 1$ (our case in this example), d_2 , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by squeezeRate;
- if the $\frac{nSellers}{nBuyers} > 1$, d_1 , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by squeezeRate;

Always with Fig. A.1 as the starting configuration of the prices, in Fig.s A.6 and A.7 we see a limited price dynamics, very close to the top band of Fig. A.1. This result is perfectly consistent with microeconomic theory.

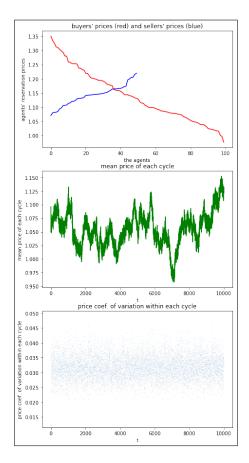


Figure A.6: Simplified Hayekian case, with $nBuyers \gg nSellers$ but squeezing the effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

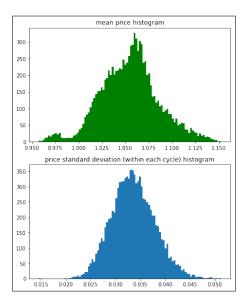


Figure A.7: Simplified Hayekian case, with $nBuyers \gg nSellers$ but squeezing the effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A.2 Case $nBuyers \ll nSellers$

With $nBuyers \ll nSellers$ (e.g., nBuyers = 50 and nSellers = 100, as in Fig. A.8), we again have three possible paths of analysis.

A.2.a Case $nBuyers \ll nSellers$, with different rates of per-capita correction

If $nBuyers \ll nSellers$, we have in each cycle one call—in mean—to a buyer from each seller, the number of per-capita actions of the buyers in each cycle is greater of the number of per-capita actions of the sellers.

As a consequence, the probability that a *buyer* increases her price to meet that of a *seller* is greater than the probability that a *seller* decreases her price to meet that of a *buyer*.

We can observe that in Figs. A.9 and A.10 the prices are—in the end—greater than in Figs. 2 and 3 and, must of all, the price tendence has a strong positive slope. We always have $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, and seed = 111.

This result is inconsistent with the microeconomic theory, where we could expect that an excess of offer will generate the fall of the prices.

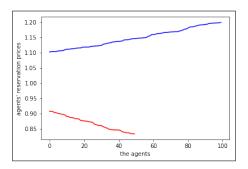


Figure A.8: An example of initial not overlapping demand curve and offer curve, case $nBuyers \ll nSellers$

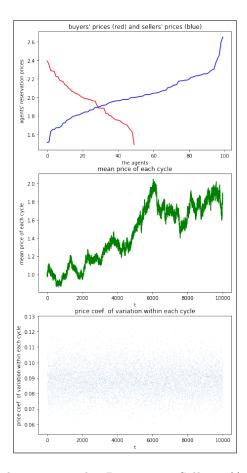


Figure A.9: Simplified Hayekian case, with $nBuyers \ll nSellers$: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

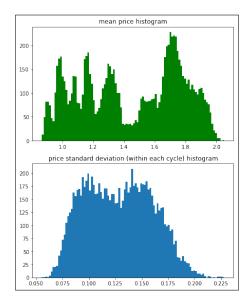


Figure A.10: Simplified Hayekian case, with $nBuyers \ll nSellers$: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A.2.b Case $nBuyers \ll nSellers$, with unequal rates of per-capita correction, with equivalent effects

In this second version of the case $nBuyers \ll nSellers$, we always have $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, and seed = 111.

As in Section A.1.b, the novelty is that of setting usingRatios = True, so we are activating limitations to d_1 or d_2 .

The limitations work as follow:

- if the $\frac{nBuyers}{nSellers} > 1$, d_2 , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by $\frac{nSellers}{nBuyers}$;
- if the $\frac{nSellers}{nBuyers} > 1$ (our case in this example), d_1 , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by $\frac{nBuyers}{nSellers}$.

We have now unequal rates of per-capita correction, with equivalent effects. The interpretation is that if the number of buyers is smaller than the number of sellers, the buyers act with a slow pace of price correction (proportional to $\frac{nBuyers}{nSellers}$) because in this way they can cherry-pick the best sellers (those with the lower reservation price). In this way, they avoid to contribute to the rise of the prices.

Always with Fig. A.8 as the starting configuration of the prices, in Fig.s A.11 and A.12 we see now a compressed price oscillations roughly close to the bottom limits of Fig. A.8.

⁹An example to clarify: in this Section we have nBuyers = 50 and nSellers = 100, so $\frac{nSellers}{nBuyers} \equiv 2$ and $\frac{nBuyers}{nSellers} \equiv 0.5$; d_1 is reduced of the 50%.

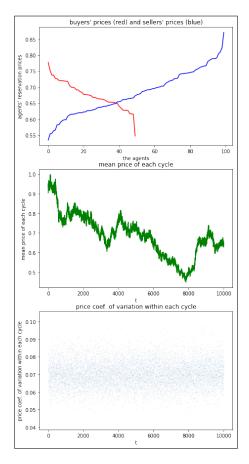


Figure A.11: Simplified Hayekian case, with $nBuyers \gg nSellers$ with equivalent effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

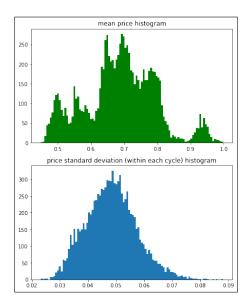


Figure A.12: Simplified Hayekian case, with $nBuyers \gg nSellers$ but with equivalent effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

A.2.c Case $nBuyers \ll nSellers$, with unequal rates of per-capita correction, but squeezing the effects

In this third version of the case $nBuyers \ll nSellers$, we always have $d_0 = 0.1$, $d_1 = 0.2$, $d_2 = 0.2$, and seed = 111.

The second novelty, after that of Section A.2.b, is that of setting usingSqueeze = True (and setting usingRatios = True as in Section A.2.b), so we are activating further limitations to d_1 or d_2 . We also have squeezeRate = 0.3.

- if the $\frac{nBuyers}{nSellers} > 1$, d_2 , i.e. the upper limit of the rate of correction of the price of the sellers, is multiplied by squeezeRate;
- if the $\frac{nSellers}{nBuyers} > 1$ (our case in this example), d_1 , i.e. the upper limit of the rate of correction of the price of the buyers, is multiplied by squeezeRate;

Always with Fig. A.8 as the starting configuration of the prices, in Fig.s A.13 and A.14 we see a limited price dynamics, very close to the bottom band of Fig. A.8. This result is perfectly consistent with microeconomic theory.

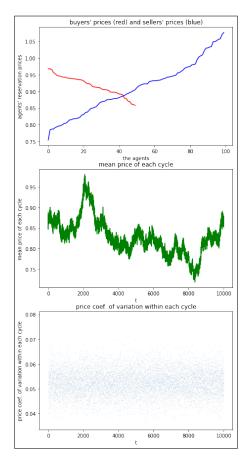


Figure A.13: Simplified Hayekian case, with $nBuyers \gg nSellers$ but squeezing the effects: (i) an example of final demand and offer curves, (ii) the history of mean prices tick-by-tick, (iii) their coefficients of variation within each tick (cycle)

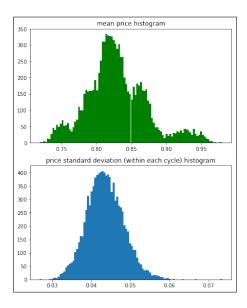


Figure A.14: Simplified Hayekian case, with $nBuyers \gg nSellers$ but squeezing the effects: (i) the distribution of mean prices of each cycle (i.e., tick-by-tick) and (ii) that of their standard deviations within each tick (cycle)

Appendix B

Activating idle agents

Fixed!!!!

prima presentazione, on nB=nS per cui nessuna correzione base, usa una iper correzione con sogli di non acquisto, l'effetto e' nullo ai nostri fini, ma ...

seconda presentazione con la scoperta che sempre tutti comperano solo circa il 50% dei casi

figure da folder sul desktop

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