Deep Dive Into Power Distributions in Microwave Ovens

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The distribution of power in microwaves are a pivotal component to their application, affecting their efficiency and performance. Understanding how power is distributed can lead to better use of the microwave in practice from science to at home use.

I. INTRODUCTION:

The roots of what we know as a microwave oven can be traced back to World War Two, with a magnetron being used in radar systems to create microwaves for detection of enemy vehicles. [1] Near the end of the war, an engineer Percy Spencer noticed that a chocolate bar began to melt when near the magnetrons used in the radar sets. [1] This idea led to the creation of commercialized microwave ovens, revolutionizing cooking for as we know it today. This paper will explore how power is distributed within commercial microwave ovens and examine how different food shapes affect how they are cooked, providing insight into the physics behind this common household appliance.

II. THEORY:

To begin, we must first define our system. For our microwave, we are going to assume that the walls of our system are perfectly reflective and are rectangular in shape. We also assume that the wave guide is a perfect conductor. We will generate our microwaves using a stimulated magnetron constrained to only one face and allow our waves to emit at any direction from said face. The microwave will only contain a given object, and will the empty space will be assumed to be a vacuum.

Due to the space being a vacuum, we then know that both the divergence of both the electric and magnetic fields produced from the microwaves will be zero [1]:

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$

Looking at the boundaries of the system, due to the wave guide being a perfect conductor, we can assume that at the walls, the parallel component of the electric field and the perpendicular component of the magnetic field to the walls are zero [2]:

$$E_{\parallel} = 0$$
$$B_{\perp} = 0$$

From these equations above, we can then produce our boundary conditions at every surface for the electric field[1]:

$$E_x(x,0,z) = E_x(x,L_y,z) = E_x(x,y,0) = E_x(x,y,L_z) = 0$$
$$E_y(0,y,z) = E_y(L_x,y,z) = E_y(x,y,0) = E_y(x,y,L_z) = 0$$
$$E_z(0,y,z) = E_z(L_x,y,z) = E_z(x,0,z) = E_z(x,L_y,z) = 0$$

As well as for the magnetic field:

$$B_x(0,y,z) = B_x(L_x,y,z) = 0$$
$$B_y(x,0,z) = B_y(x,L_y,z) = 0$$
$$B_z(xy,0) = B_z(x,y,L_z) = 0$$

Going into the form of our fields, we know that within this cavity, all field components must follow the general form of the wave equation [1]:

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} 1/c^2 = \nabla^2 \psi(x,t)$$

We also know that the solutions for any given A electric and magnetic field components, each will have separable solutions:

$$E_A = X(x)Y(y)Z(z)T(t)$$
$$B_A = X(x)Y(y)Z(z)T(t)$$

Now, from our known conditions and boundaries we obtain the solutions for the electric field components[1]:

$$E_x(x,y,z,t) = E_{ox} cos(k_1 x) sin(k_2 y) sin(k_3 z) e^{-iwt}$$
$$E_y(x,y,z,t) = E_{oy} sin(k_1 x) cos(k_2 y) sin(k_3 z) e^{-iwt}$$
$$E_z(x,y,z,t) = E_{oz} sin(k_1 x) sin(k_2 y) cos(k_3 z) e^{-iwt}$$

With the constants described as such [1]:

$$k_1 = \frac{l\pi}{L_x}, \quad k_2 = \frac{m\pi}{L_y}, \quad k_3 = \frac{n\pi}{L_z}$$

With the allowed frequencies of [1]:

$$\frac{\omega^2}{c^2} = k_1^2 + k_2^2 + k_3^2 = \pi^2 \left(\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2} \right)$$

From the relationship between the electric and magnetic field, we can gather the magnetic field components [2]:

$$\nabla \times \mathbf{E} = i\omega \mathbf{B}$$

$$B_x(x,y,z,t) = iw(E_{oz}k_2 sin(k_1x)cos(k_2y)cos(k_3z)e^{-iwt} - E_{oy}k_3 sin(k_1x)cosk_2y)cos(k_3z)e^{-iwt})$$
$$B_y(x,y,z,t) = iw(E_{ox}k_3 cos(k_1x)sin(k_2y)cos(k_3z)e^{-iwt} - E_{oz}k_1 cos(k_1x)sin(k_2y)cos(k_3z)e^{-iwt})$$
$$B_z(x,y,z,t) = iw(E_{oz}k_2 cos(k_1x)cos(k_2y)sin(k_3z)e^{-iwt} - E_{oy}k_1 cos(k_1x)cos(k_2y)sin(k_3z)e^{-iwt})$$

From these equations for the electric and magnetic fields, we can derive the poynting vector, although I am not explicitly writing it out here [2]:

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

Finally, we can arrive at our power distribution, described by the integral of our poynting vector dotted by the differentiable surface area [2]:

$$P = \int_S \mathbf{S} \cdot d\mathbf{A}$$

III. CODING:

For the coding portion of this project, I would like to assume that we know that the resulting poynting vector from our assumed electric and magnetic fields is correct. I would like to study different surface area shapes and how they are positioned inside of the microwave. The idea would be that even objects of different shapes could have the same surface area, but due to the power distribution within the microwave, the resulting power given to the object could change. I would first like to plot the hot spots within the cavity at many different frequencies. Then I would like to see where placing these different shapes could lead to the highest power transferred to them. If time permits, I would like to try different compositions within the shapes, to see how different materials cook.

IV. REFERENCES:

[1] Farside, *Electromagnetism - University of Texas*, 2024, `https://farside.ph.utexas.edu/teaching/jk1/Electromagnetism/node112.html#e6.4a`

[2] D. J. Griffiths, *Introduction to Electrodynamics*, 4th edition, Pearson, 2013.