CS 556 Homework 3 - Vector Spaces and Subspaces

Solve the following questions. Type your solutions using Latex or any other program you prefer and submit your typed solutions as a PDF file.

Question 1 *(10 points)* Write the complete solution of the following linear system:

$$x + 2y - z = 1$$
$$3x + 5y + 2z = 3$$
$$2x + y + 13z = 2$$

Question 2 *(10 points)* Find the rank of the following matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{bmatrix}$$

Question 3 *(10 points)* Construct a matrix A whose column space contains vectors $\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$, and whose null space contains the vector $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

Question 4 *(10 points)* Compute the following matrix-vector multiplication as:

a) Linear combination of columns.

b) Dot product of rows.

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & 1 & 0 \\ 3 & 5 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

Question 5 *(10 points)* Find the value of k for which the matrix has:

a) Dependent columns

b) Independent columns

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 1 \\ 4 & 8 & k \end{bmatrix}$$

Question 6 *(20 points)* Find a basis for the four fundamental subspaces of the matrix.

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Question 7 *(10 points)* Consider the vectors $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

a) In the x-y plane mark all nine linear combinations $c\vec{v} + d\vec{w}$, with c = {-2,0,-2} and d = {0,1,2}.

b) What shape do all linear combinations $c\vec{v} + d\vec{w}$ fill? A line? The whole plane? Are the vectors $\vec{v}$ and $\vec{w}$ independent?

Question 8 *(10 points)* Consider the vectors $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

a) Can you solve the system $x\vec{u} + y\vec{v} + z\vec{w} = \vec{b}$, if $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$?

b) What if $\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$? How many solutions are there?

c) Are the vectors $\vec{u}, \vec{v}$ and $\vec{w}$ dependent or independent?

d) Use parts (a) - (c) to decide if $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is an invertible matrix or not.

Question 9 *(10 points)* Consider the linear system for some constants b and g:

$$x - 2y + 3z = 3$$
$$2x + y + bz = -4$$
$$x + 0y + 1z = g$$

a) What constant b makes the system singular (missing a pivot).

b) For the value of b found in Part (a), for which values of g, the system has infinitely many solutions?

c) Find two distinct solutions of the system for that g.