

Probability Homework

Problem 1

We draw the top 7 cards from a well-shuffled standard 52-card deck. Find the probability that:

- (a) The 7 cards include exactly 3 aces.
- (b) The 7 cards include exactly 2 kings.
- (c) The probability that the 7 cards include exactly 3 aces. or exactly 2 kings, or both.

Problem 2

Alice and Bob have $2n+1$ coins, each coin with probability of heads equal to $1/2$. Bob tosses $n+1$ coins, while Alice tosses the remaining n coins. Assuming independent coin tosses, show that the probability that after all coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

Problem 3

We are given three coins: one has heads in both faces, the second has tails in both faces, and the third has a head in one face and a tail in the other. We choose a coin at random, toss it, and the result is heads. What is the probability that the opposite face is tails?

Problem 4

Each of k jars contain m white and n black balls. A ball is randomly chosen from jar 1 and transferred to jar 2, then a ball is randomly chosen from jar 2 and transferred to jar 3, etc. Finally a ball is randomly chosen from jar k . Show that the probability that the last ball is white is the same as probability that the first ball is white, i.e. it is $m/(m+n)$.

Problem 5

A power utility can supply electricity to a city from n different power plants. Power plant i fails with probability p_i , independent of the others.

- (a) Suppose that any one plant can produce enough electricity to supply the entire city. What is the probability that the city will experience a black-out?
- (b) Suppose that two power plants are necessary to keep the city from a black-out. Find the probability that the city will experience a black-out.