# Network Analysis

**Class 01 – Networks & Graphs: Basic Measures**

Prof. Patrick Terrematte

Chapter 2

# Networks & Graphs: Basic Measures

**Summary**

- Graph representations
- Type of Networks
- Degree distribution
- Paths & Connectedness
- Clustering

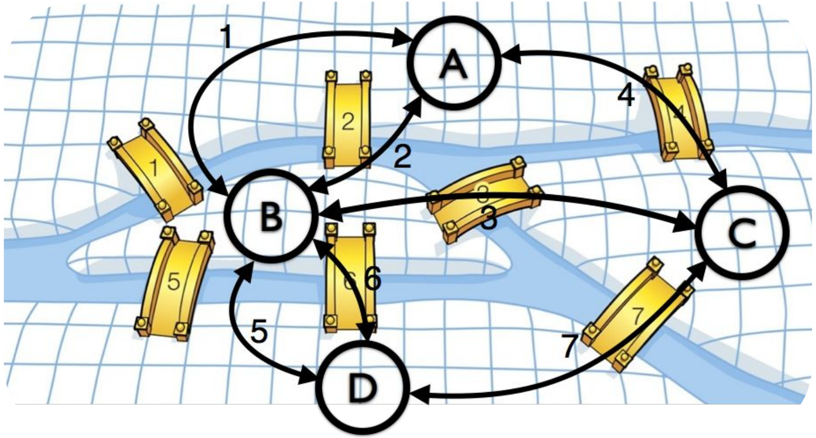
**Reading**

- Chapter 2 of Barabasi's book

# The Bridges of Konigsberg

Can one walk across the seven bridges and never cross the same bridge twice?

**Famous Konigsberg Citizens**
Immanuel Kant (philosopher, 1724-1804)




Euler's theorem (1735)

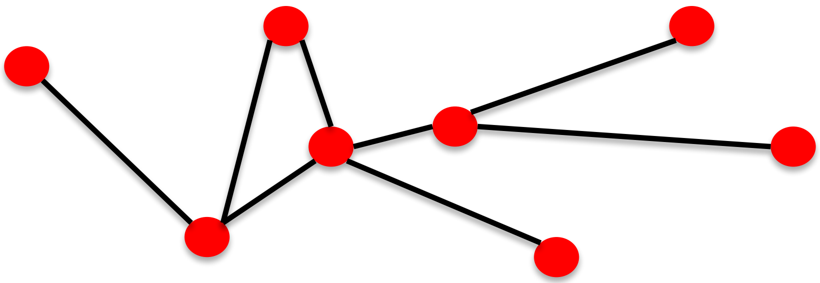
- If a graph has more than two nodes of odd degree, there is no path/cycle that crosses each bridge exactly once.
- If a graph is connected and has no odd degree nodes, it has at least one path.

# Components of a Complex System

Networks or Graphs?

Network *<nodes, links>*
refers to real systems
(www, social network, metabolic network)

Graph *<vertices, edges>*
mathematical representation of a network
(web graph, social graph)



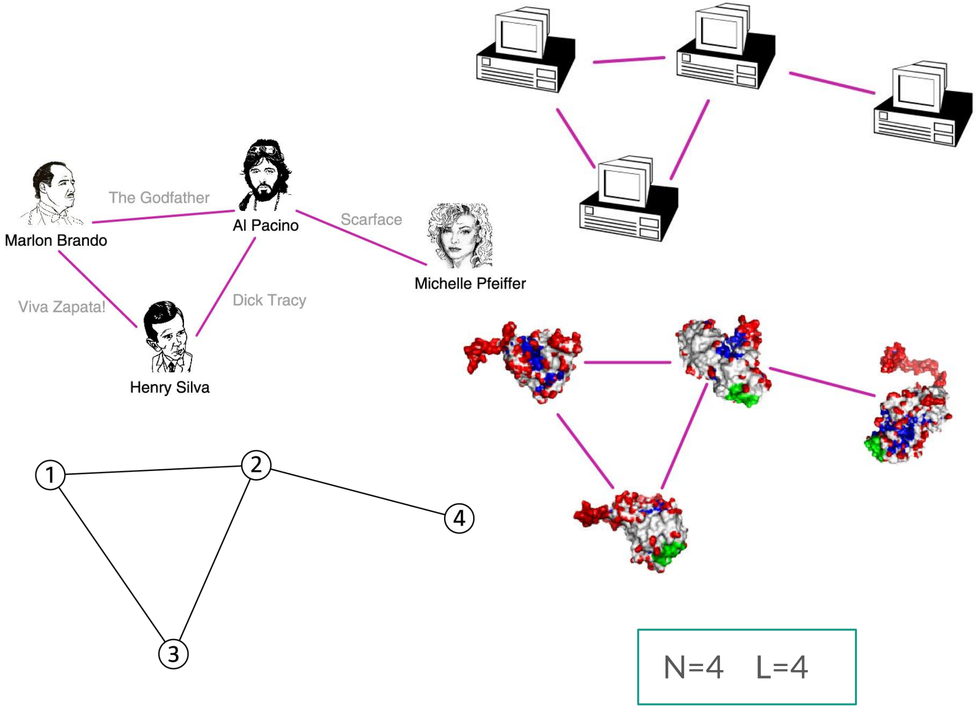
| | | Symbol |
|---|---|---|
| Components | nodes, vertices | N |
| Interactions | edges, links | L |
| System | network, graph | (N,L) |

# A Common Language

The choice of the proper network representation determines our ability to use network theory successfully.

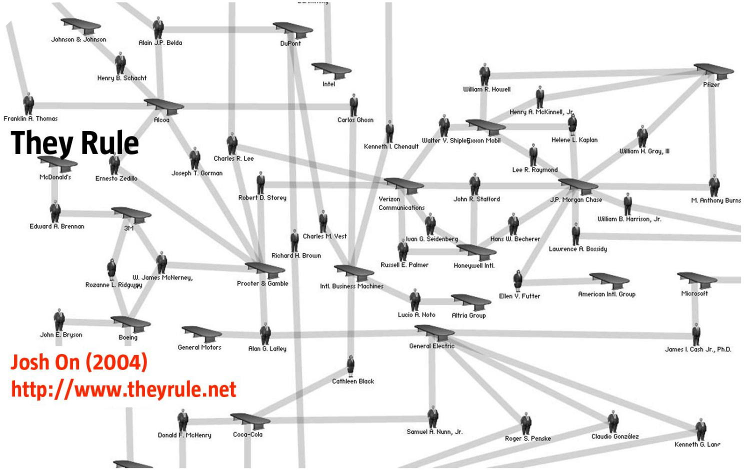
In some cases there is a unique, unambiguous representation. In other cases, the representation is by no means unique.

*The way we assign the links between a group of individuals will determine the nature of the question we can study.*



# Proper representations (examples)

If you connect individuals that work with each other, you will explore the *professional network*.



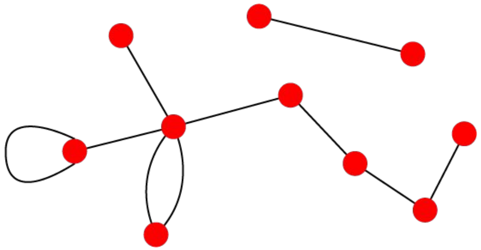
If you connect individuals based on their first name (e.g., *all Peters connected to each other*), you will be exploring what?

It is a network, *nevertheless*.

# Directedness

## Undirected graphs

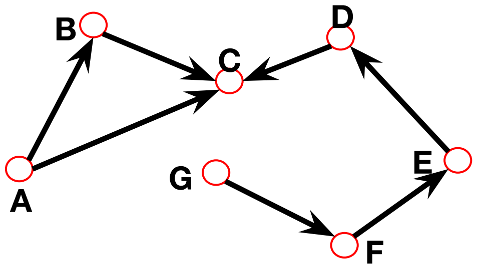
Links: undirected (*symmetrical*)



Examples of Undirected links
Co-authorship links
Actor network
Protein interactions

## Directed graphs (DiGraphs)

Links: directed (*arcs*).




Example of Directed links
URLs on the www
Phone calls
Metabolic reactions

| NETWORK | NODES | LINKS | DIRECTED UNDIRECTED | N | L |
|---|---|---|---|---|---|
| **Internet** | Routers | Internet connections | Undirected | 192,244 | 609,066 |
| **WWW** | Webpages | Links | Directed | 325,729 | 1,497,134 |
| **Power Grid** | Power plants, transformers | Cables | Undirected | 4,941 | 6,594 |
| **Mobile Phone Calls** | Subscribers | Calls | Directed | 36,595 | 91,826 |
| **Email** | Email addresses | Emails | Directed | 57,194 | 103,731 |
| **Science Collaboration** | Scientists | Co-authorship | Undirected | 23,133 | 93,439 |
| **Actor Network** | Actors | Co-acting | Undirected | 702,388 | 29,397,908 |
| **Citation Network** | Paper | Citations | Directed | 449,673 | 4,689,479 |
| **E. Coli Metabolism** | Metabolites | Chemical reactions | Directed | 1,039 | 5,802 |
| **Protein Interactions** | Proteins | Binding interactions | Undirected | 2,018 | 2,930 |

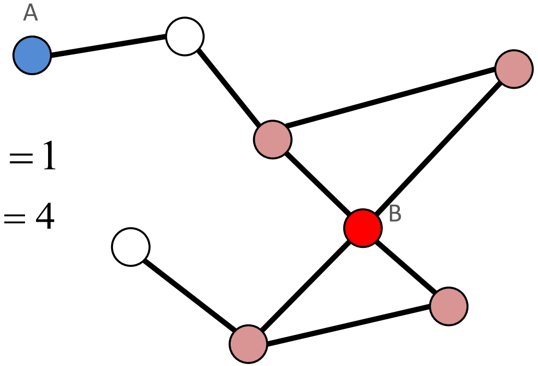
Reference Networks

# Degree,
# Average Degree,
# Degree Distribution

# Node Degree

## Undirected graphs

the number of links connected to the node

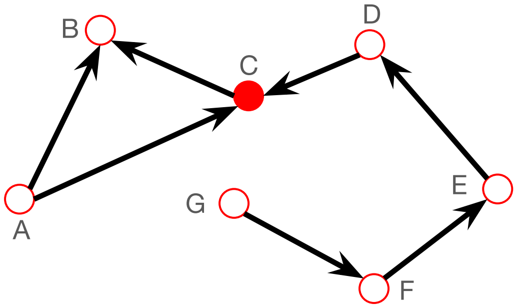



$$k_A = 1$$

$$k_B = 4$$

## Directed graphs (DiGraphs)

we can define an in-degree and out-degree. The (total) degree is the sum of in- and out-degree.




$$k_C^{in} = 2$$

$$k_C^{out} = 1$$

$$k_C = 3$$

Source: a node with $k^{in} = 0$;

Sink: a node with $k^{out} = 0$.

# A Bit of Statistics

Four key quantities characterize a sample of N values $x_1,...,x_n$

Average (mean):

$$\langle x \rangle = \frac{x_1 + x_2 + \ldots .1x_3}{N} = \frac{1}{N}\sum_{i=1}^{n} x_i$$

The $n^{th}$ moment:

$$\langle x^a \rangle = \frac{x_1^n + x_2^n + \ldots - x_n^n}{N} = \frac{1}{N}\sum_{k=1}^{N} x_i^n$$

The Standard deviation:

$$\sigma_\lambda = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \langle x \rangle)^2}$$

The Distribution of *x*:

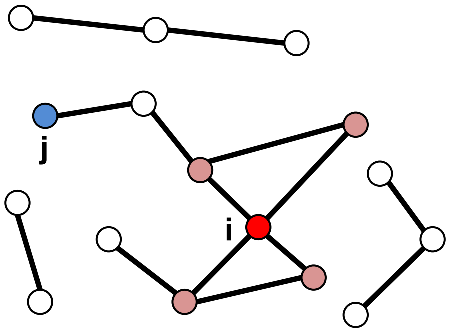
$$p_x = \frac{1}{N}\sum_i \delta_{x,x_i}$$

where $p_x$ f $\sum_x p_x = 1 \left( \int p_x dx = 1 \right)$

# Average Degree

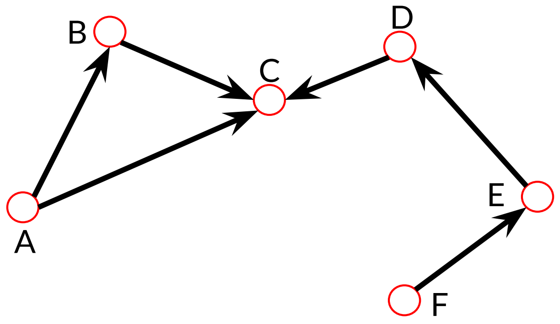
## Undirected graphs

N - the number of nodes in the graph




$$\langle k \rangle \equiv \frac{1}{N}\sum_{i=1}^{N} k_i \qquad \langle k \rangle \equiv \frac{2L}{N}$$

## Directed graphs (DiGraphs)




$$\langle k^{in} \rangle \equiv \frac{1}{N}\sum_{i=1}^{N} k_i^{in}, \qquad \langle k^{out} \rangle \equiv \frac{1}{N}\sum_{i=1}^{N} k_i^{out},$$

$$\langle k^{in} \rangle = \langle k^{out} \rangle \qquad \langle k \rangle = \frac{L}{N}$$

| NETWORK | NODES | LINKS | DIRECTED UNDIRECTED | N | L | ⟨k⟩ |
|---|---|---|---|---|---|---|
| **Internet** | Routers | Internet connections | Undirected | 192,244 | 609,066 | 6.33 |
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| **E. Coli Metabolism** | Metabolites | Chemical reactions | Directed | 1,039 | 5,802 | 5.58 |
| **Protein Interactions** | Proteins | Binding interactions | Undirected | 2,018 | 2,930 | 2.90 |

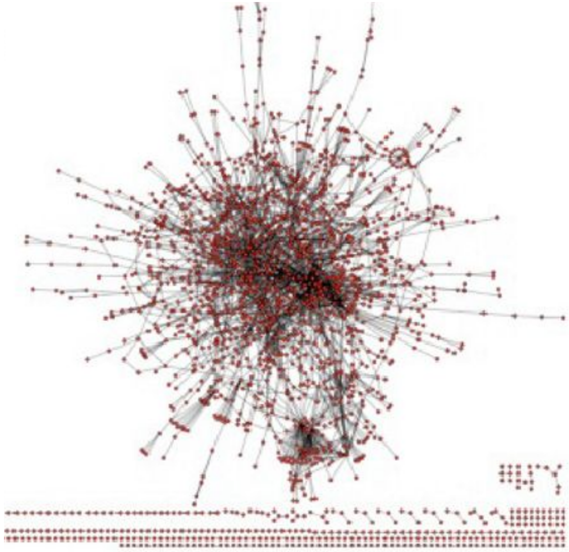
## Reference Networks: Average Degree

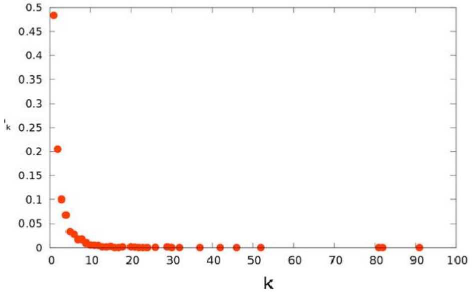
# Degree Distribution

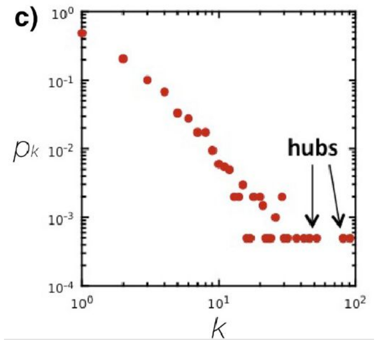
**P(k)**: probability that a randomly chosen node has degree *k*

$N_k$ = # nodes with degree k

$P(k) = N_k / N$ ➔ plot





# Degree Distribution (cont'd)

**Discrete Representation:** $p_k$ is the probability that a node has degree k.

**Continuum Description:** p(k) is the pdf of the degrees, where

$$\int_{k_1}^{k} p(k)dk$$

represents the probability that a node's degree is between $k_1$ and $k_2$.

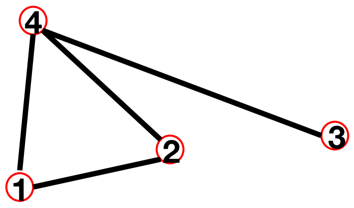
**Normalization condition:**

$$\sum_{a}^{\infty} p_k = 1 \quad \int_{K_{min}} p(k)dk = 1$$

where $K_{min}$ is the minimal degree in the network.

# Adjacency matrix

## Undirected graphs




$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

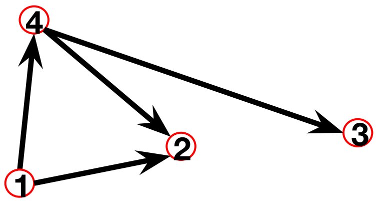
$$\boxed{k_i = \sum_{j=1}^{N} A_{ij}} \qquad \boxed{k_j = \sum_{i=1}^{N} A_{ij}}$$

$$A_{ij} = A_{ji}$$

$$A_{ii} = 0$$

$$L = \frac{1}{2}\sum_{i=1}^{N} k_i = \frac{1}{2}\sum_{ij}^{N} A_{ij}$$

## Directed graphs (DiGraphs)




$$A_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{k_i^{in} = \sum_{j=1}^{N} A_{ij}} \qquad \boxed{k_j^{out} = \sum_{i=1}^{N} A_{ij}}$$

$$A_{ij} \neq A_{ji}$$

$$A_{ii} = 0$$

$$L = \sum_{i=1}^{N} k_i^{in} = \sum_{j=1}^{N} k_j^{out} = \sum_{i,j}^{N} A_{ij}$$

# Paths and Connectedness

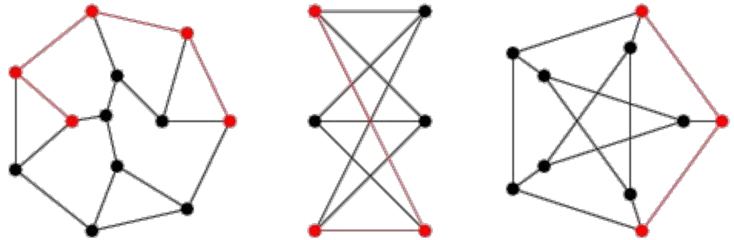
# Paths

A *path* *is* a sequence of nodes in which each node is adjacent to the next one

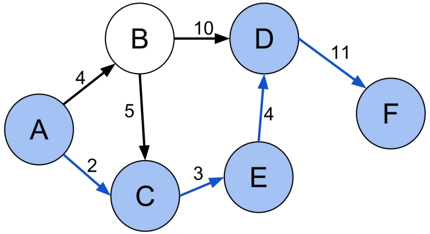
$P_{i0,in}$ of length $n$ between nodes $i_0$ and $i_n$ is an ordered collection of $n+1$ nodes and $n$ links

$$P_n = \{i_0, i_1, i_2, ..., i_n\}$$

$$P_n = \{(i_0, i_1), (i_1, i_2), (i_2, i_3), ..., (i_{n-1}, i_n)\}$$



Examples of phats in an undirected graph.

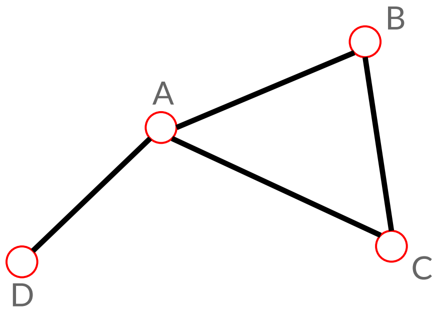



In a directed graph, the path can follow only the direction of an arrow.

# Distance in a Graph

## Undirected graphs

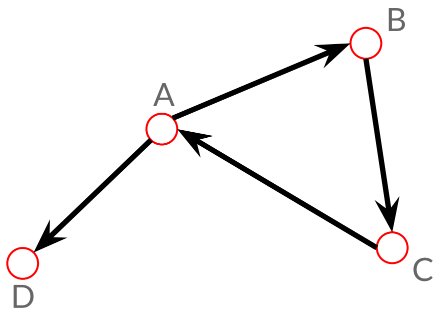
The *distance* (*shortest path, geodesic path*) between two nodes is defined as the number of edges along the shortest path connecting them.




*If the two nodes are disconnected, the distance is infinity.

## Directed graphs (DiGraphs)

Each path needs to follow the direction of the arrows.




Thus in a digraph the distance from node A to B (on an AB path) is generally different from the distance from node B to A (on a BCA path).

# Number of paths between two nodes

$N_{ij}$, number of paths between any two nodes ***i*** and ***j***:

***Length n=1:***
If there is a link between *i* and *j*, then $A_{ij}$=1 and $A_{ij}$=0 otherwise.

***Length n=2:***
If there is a path of length two between *i* and *j*, then $A_{ik}A_{kj}$=1, and $A_{ik}A_{kj}$=0 otherwise.

The number of paths of length 2:

$$N_{ij}^{(2)} = \sum_{k=1}^{N} A_{ik}A_{kj} = [A^2]_{ij}$$

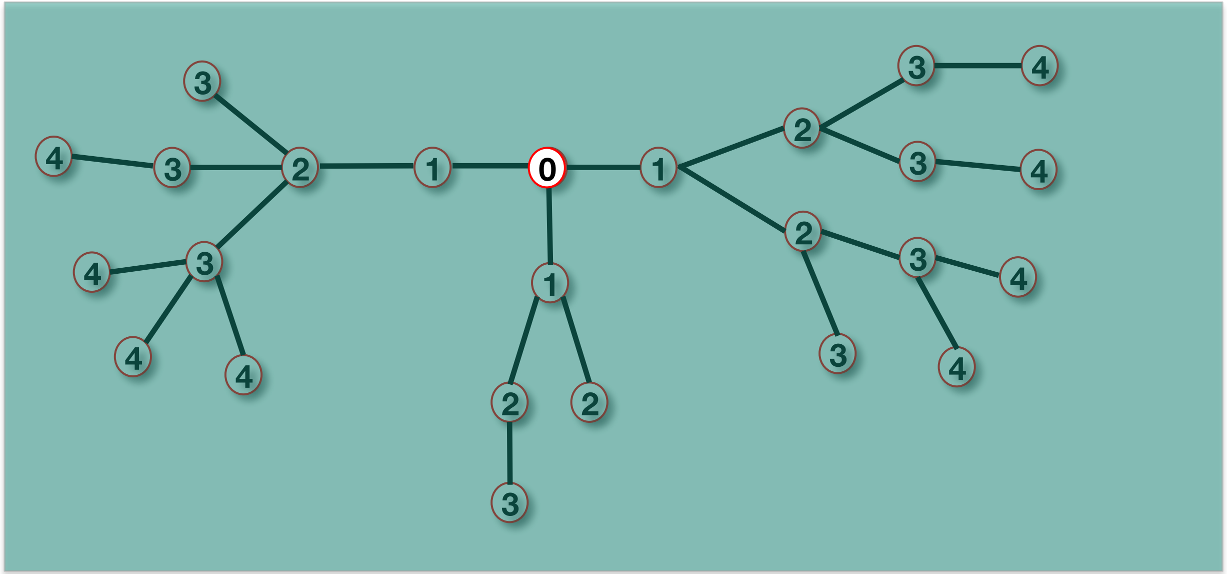
***Length n:***
In general, if there is a path of length *n* between *i* and *j*, then $A_{ik}...A_{lj}$=1 and $A_{ik}...A_{lj}$=0 otherwise.

The number of paths of length *n* between *i* and *j* is(*)

$$N_{ij}^{(n)} = [A^n]_{ij}$$

(*) *Holds for both directed and undirected networks.*

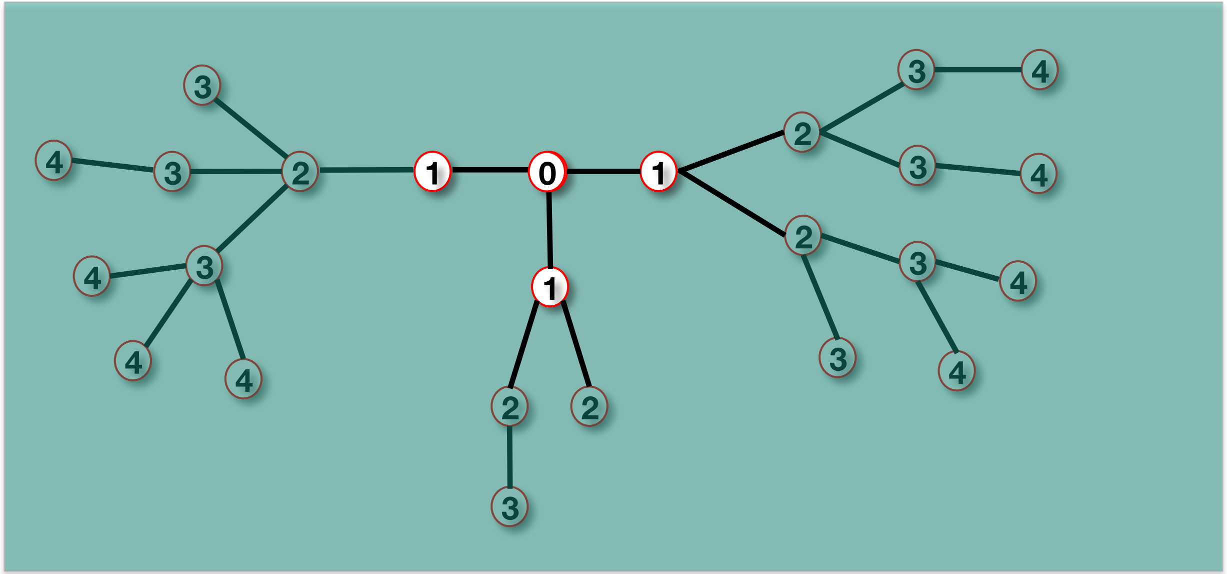
# Finding Distances: BFS




Distance between node 0 and node 4:

1. Start at 0.

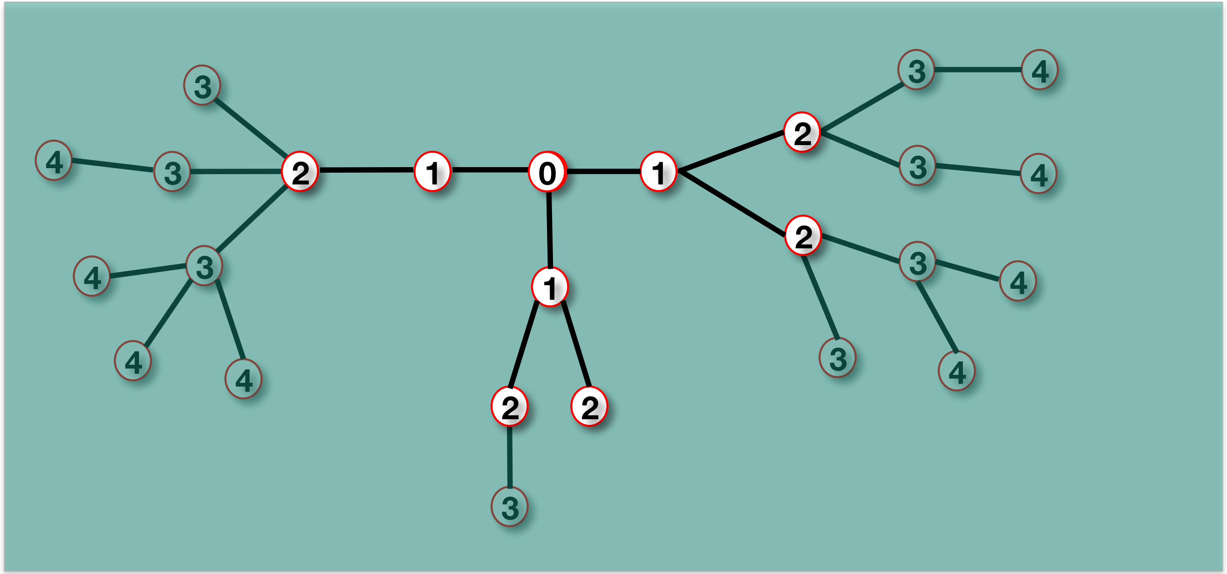
# Finding Distances: BFS




Distance between node 0 and node 4:

1. Start at 0.
2. Find the nodes adjacent to 1. Mark them as at distance 1. Put them in a queue.

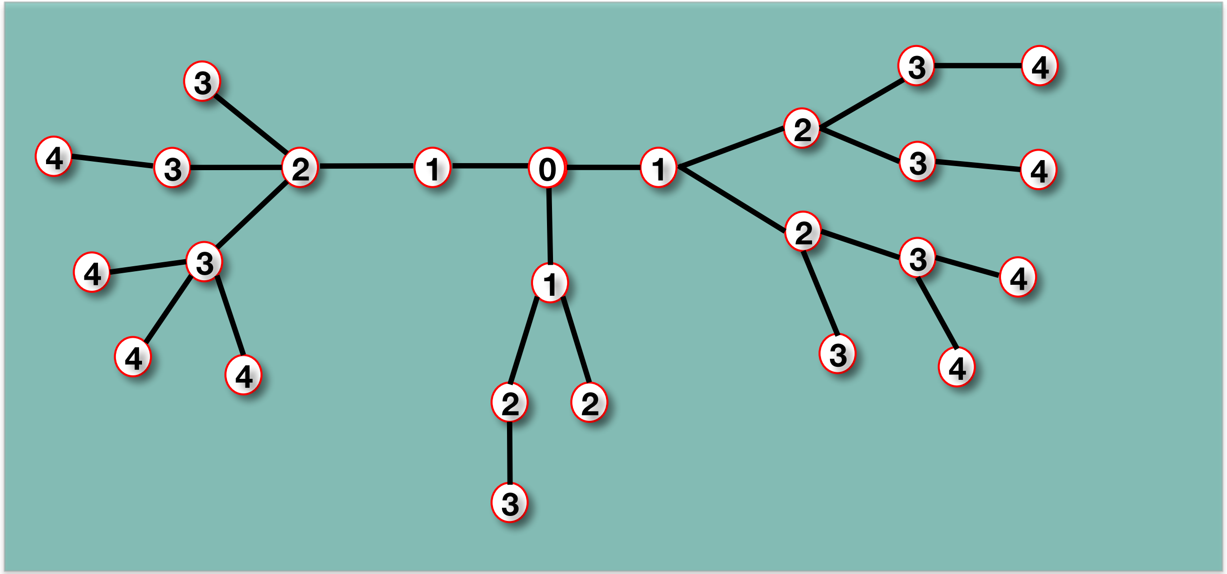
# Finding Distances: BFS




Distance between node 0 and node 4:

1. Start at 0.
2. Find the nodes adjacent to 1. Mark them as at distance 1. Put them in a queue.
3. Take the first node out of the queue. Find the unmarked nodes adjacent to it in the graph. Mark them with the label of 2. Put them in the queue.

# Finding Distances: BFS




Distance between node 0 and node 4:

Repeat until you find node 4 or there are no more nodes in the queue.

The distance between 0 and 4 is the label of 4 or, if 4 does not have a label, infinity.

# Diameter and Average distance

*Diameter* (***$d_{max}$***):

the maximum distance between any pair of nodes in the graph.

*Average path length/distance*, *<d>*, for a connected graph:

$$\langle d \rangle \equiv \frac{1}{2L_{\max}} \sum_{i,j \neq i} d_{ij}$$

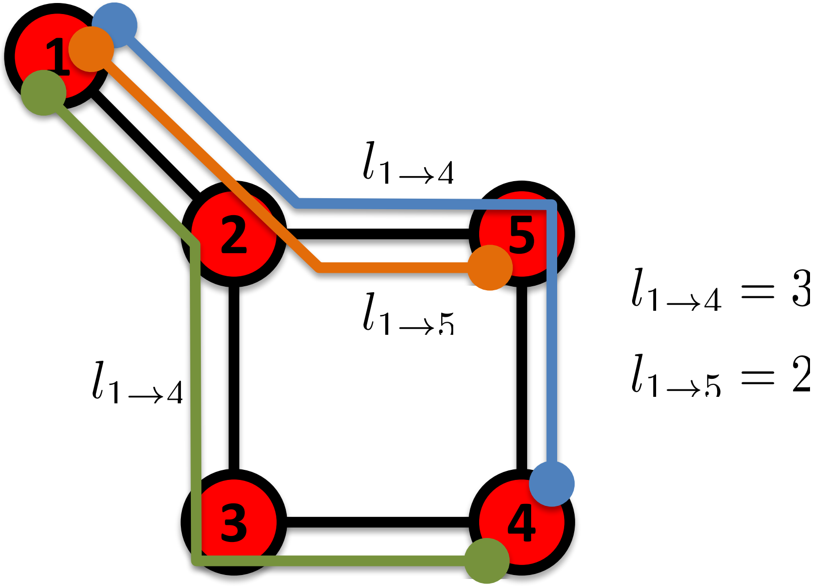
where $d_{ij}$ is the distance from node *i* to node j

In an *undirected graph* $d_{ij} = d_{ji}$, *so* we only need to count them once:

$$\langle d \rangle \equiv \frac{1}{L_{\max}} \sum_{i,j > i} d_{ij}$$

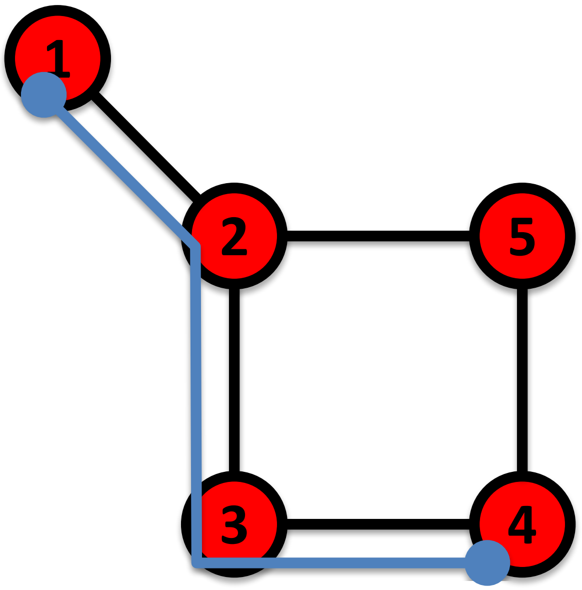
# Paths: a summary

**Shortest Path**
The path with the shortest length between two nodes (distance).



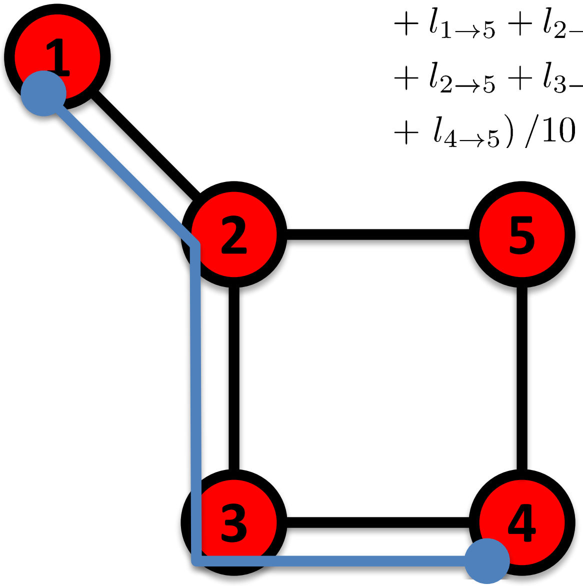
# Paths: a summary

**Diameter**
The longest shortest path in a graph.




$$l_{1 \to 4} = 3$$

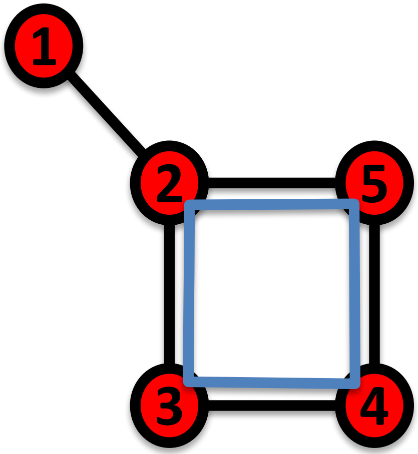
# Paths: a summary



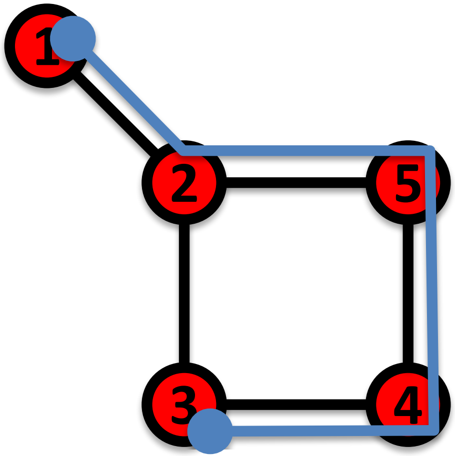

$$\left(l_{1\to 2} + l_{1\to 3} + l_{1\to 4} + \right.$$
$$+ l_{1\to 5} + l_{2\to 3} + l_{2\to 4} +$$
$$+ l_{2\to 5} + l_{3\to 4} + l_{3\to 5} +$$
$$\left. + l_{4\to 5}\right)/10 = 1.6$$

**Average Path Length**
The average of the shortest paths for all pairs of nodes.

# Paths: a summary

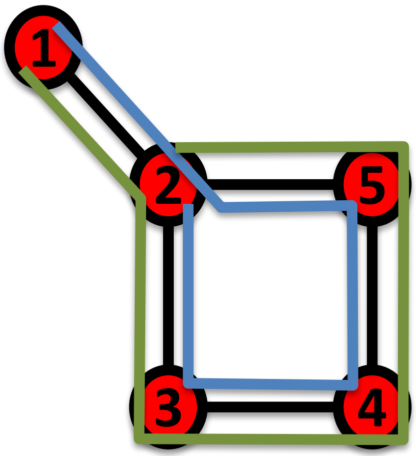



**Cycle**
A path with the same start and end node.

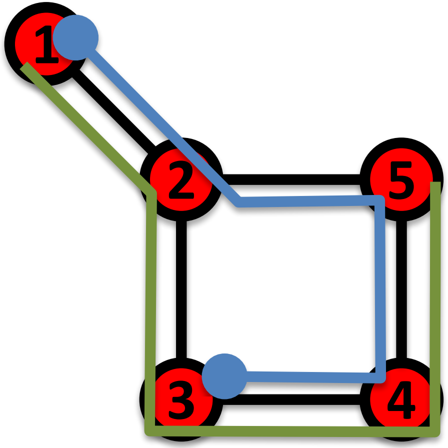



**Self-Avoiding Path**
A path that does not intersect itself.

# Paths: a summary




**Eulerian Path/Cycle**
A path that traverses each link exactly once.

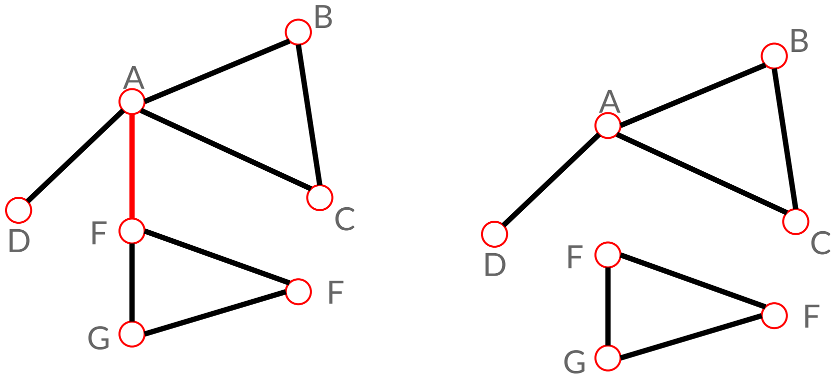



**Hamiltonian Path/Cycle**
A path that visits each node exactly once.

# Connectivity of undirected graphs

**Connected (undirected) graph:**
any two vertices can be joined by a path.

A **disconnected graph** is made up by two or more connected components.

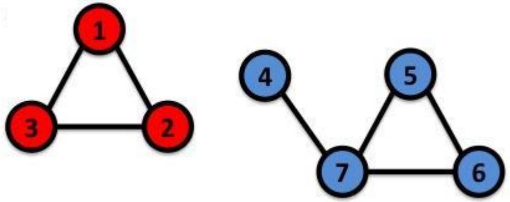



**Bridge**:
if we erase it, the graph becomes disconnected.
Example (A,F)

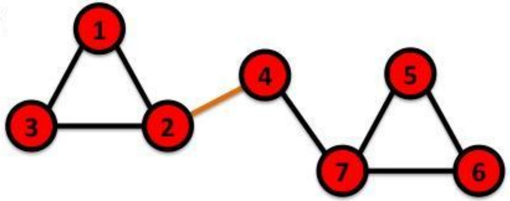
Largest Component: **Giant Component**
The rest: **Isolates**

# Connectivity of undirected graphs

The adjacency matrix of a network with several components can be written in a block-diagonal form, so that nonzero elements are confined to squares, with all other elements being zero.




$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$



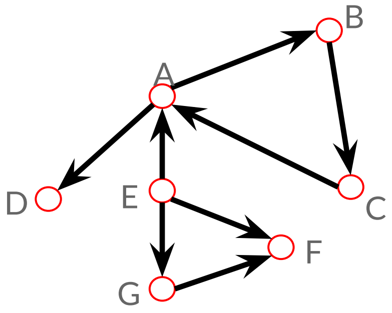

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

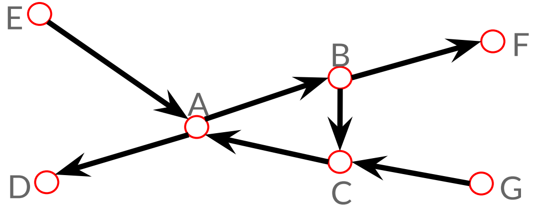
# Connectivity of directed graphs

**Strongly connected directed graph (SCC):**
has a path from each node to every other node and vice versa (e.g. AB path and BA path).

**Weakly connected directed graph (WCC):**
it is connected if we disregard the edge directions.

Strongly connected components can be identified, but not every node is part of a nontrivial strongly connected component.




**In-component:**
nodes that can reach the scc,

**Out-component:**
nodes that can be reached from the scc.

# Network Density

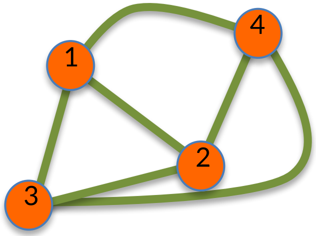
# Complete Graph

A graph with degree

$$L=L_{max}$$

is called a complete graph, and its average degree is

$$\langle k\rangle = N-1$$




$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

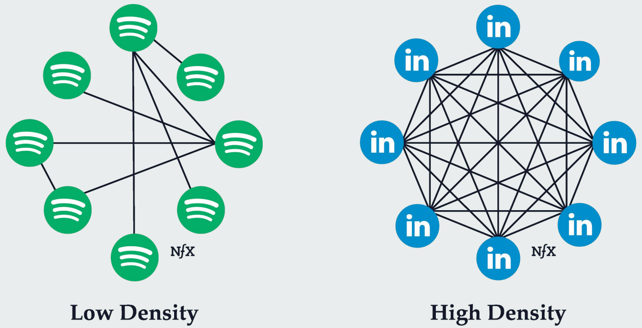
The maximum number of links an undirected network of N nodes can have is:

$$\boxed{L_{\max} = \binom{N}{2} = \frac{N(N-1)}{2}}$$

What about directed networks?

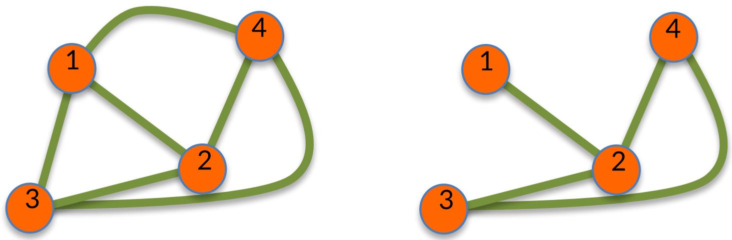
# Network Density

Ratio of existing edges over possible ones.




$$\boxed{d(G) = \frac{L}{L_{max}}}$$

Examples




Complete graph
d(G) = 1

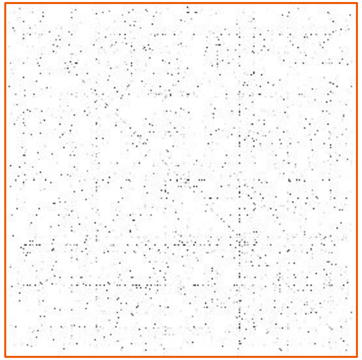
Sparse graph
d(G) = 2/3

# Most networks observed in real systems are sparse

$L \ll L_{max}$

$\langle k \rangle \ll N-1$

$d(G) \ll 1$



Sparse
Adjacency matrix

| | | | | |
|---|---|---|---|---|
| WWW (ND Sample): | N=325,729; | $L=1.4\ 10^6$ | $L_{max}=10^{12}$ | $\langle k\rangle=4.51$ |
| Protein (*S. Cerevisiae*): | N= 1,870; | L=4,470 | $L_{max}=10^7$ | $\langle k\rangle=2.39$ |
| Coauthorship (Math): | N= 70,975; | $L=2\ 10^5$ | $L_{max}=3\ 10^{10}$ | $\langle k\rangle=3.9$ |
| Movie Actors: | N=212,250; | $L=6\ 10^6$ | $L_{max}=1.8\ 10^{13}$ | $\langle k\rangle=28.78$ |

*(Source: Albert, Barabasi, RMP2002)*

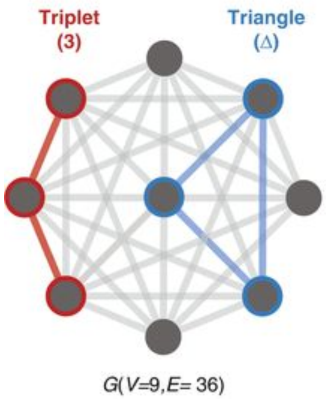
# Clustering Coefficient

# Clustering Coefficient

How "clustered" is my network?

**Global Clustering coefficient**

- Triangles and triplets
- C in [0,1]




$$C = \frac{3 \times \text{ number of triangles}}{\text{number of all triplets}}$$

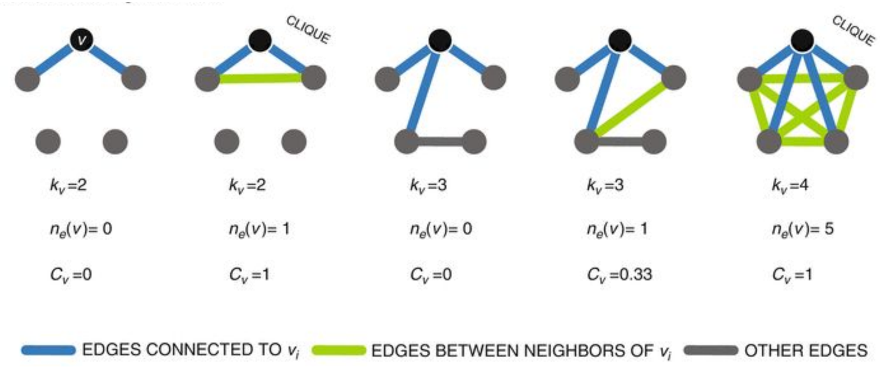
*Watts & Strogatz, Nature (1998)*

# Clustering Coefficient

What fraction of your neighbors are connected?

**Local Clustering coefficient**

- Node i with degree $k_i$
- $C_i$ in [0,1]




$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$

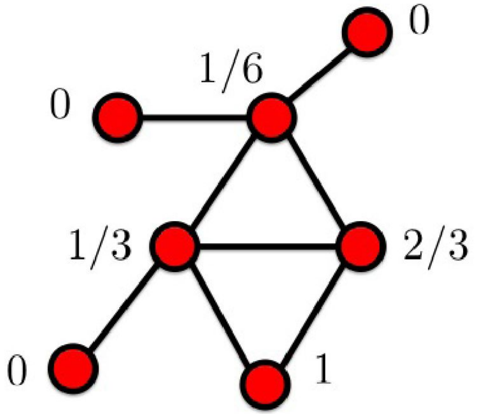
*Watts & Strogatz, Nature (1998)*

# Clustering Coefficient

What fraction of your neighbors are connected?

**Local Clustering coefficient**

- Node i with degree $k_i$
- $C_i$ in [0,1]




$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$

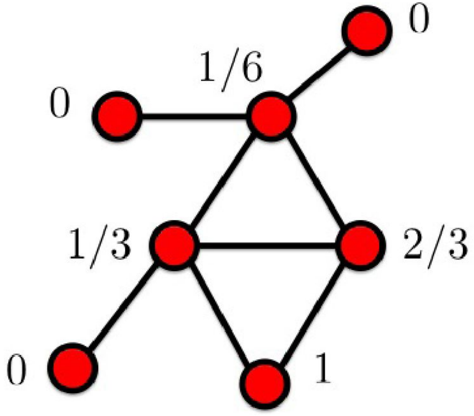
*Watts & Strogatz, Nature (1998)*

# Clustering Coefficient

What fraction of your neighbors are connected on average?

**Average Clustering coefficient**

- Average of local clustering coefficients
- C in [0,1]




$$\bar{C} \simeq 0.31$$

$$\bar{C} = \frac{1}{n} \sum_{i=1}^{n} C_i$$

*Watts & Strogatz, Nature (1998)*

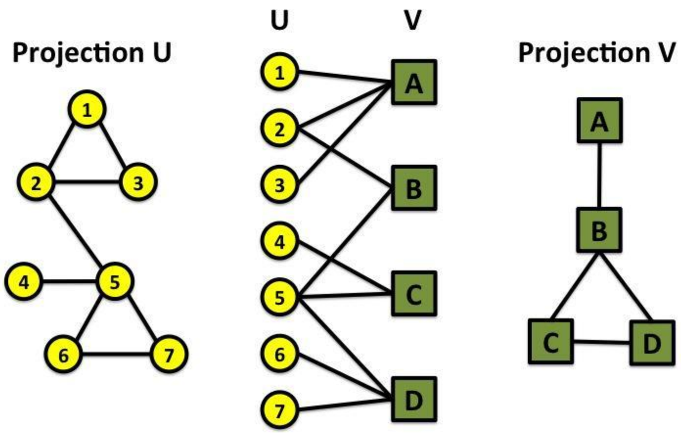
# Bipartite Networks

# Bipartite Graphs

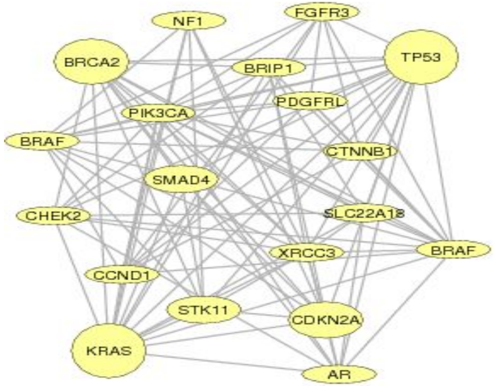
**Bipartite graph** (or bigraph)

a graph whose nodes can be divided into two disjoint sets *U* and *V* such that every link connects a node in *U* to one in *V*.

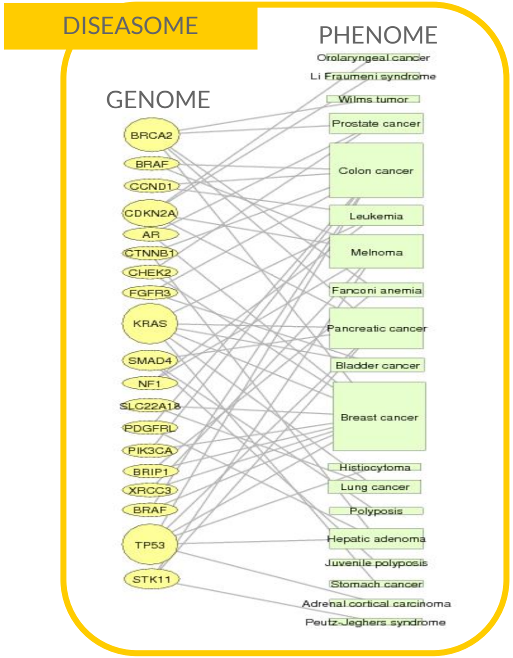
**Examples**
- Hollywood actor network
- Collaboration networks
- Disease network (diseasome)

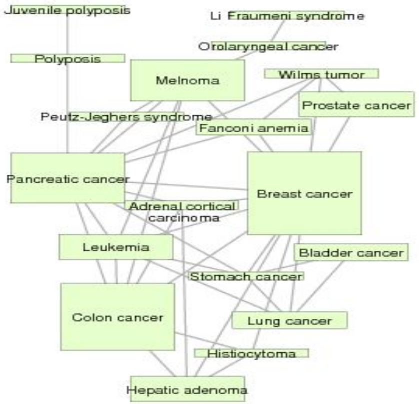



**Projection**
Two nodes of the **same class** are connected by a (weighted) edge if they share at least a **common neighbor**




Gene network
(projection)

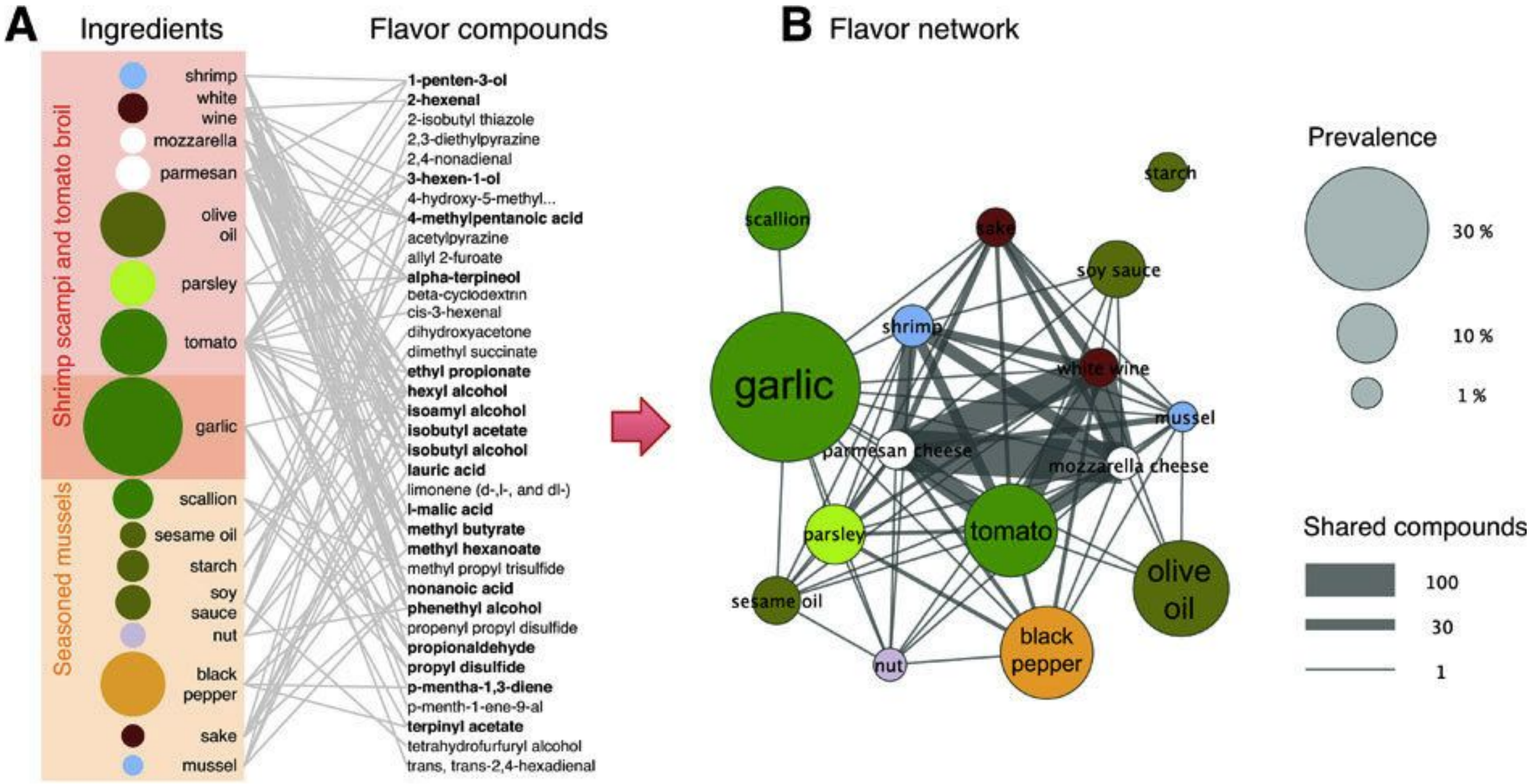



Disease network
(projection)

# Gene - DiseNetwork ase Network

*Goh, Cusick, Valle, Childs, Vidal & Barabási, PNAS (2007)*




Ingredient-Flavor Bipartite Network

Y.-Y. Ahn, S. E. Ahnert, J. P. Bagrow, A.-L. Barabási
Flavor network and the principles of food pairing,
Scientific Reports 196, (2011).

# Summarizing...

# Central quantities in Network Science

Degree Distribution

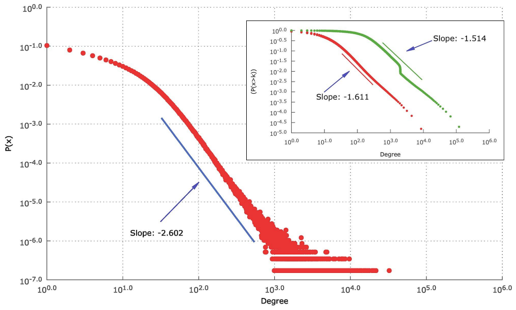
$$P(k)$$

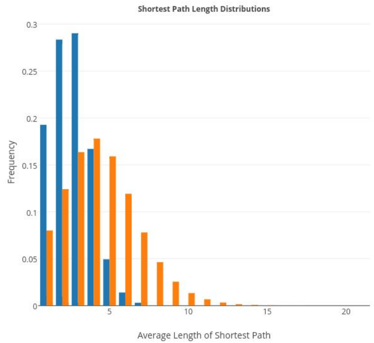
Path length

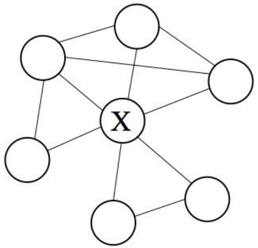
$$<d>$$

Clustering Coefficient

$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$



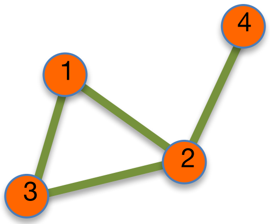



Type of graphs

# Directedness

### *Undirected graph*



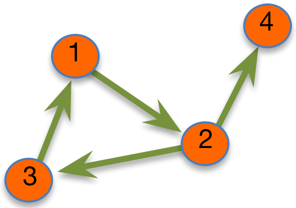

$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{2L}{N}$$

*Actor network, protein-protein interactions*

### *Directed graph*




$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} \neq A_{ji}$$

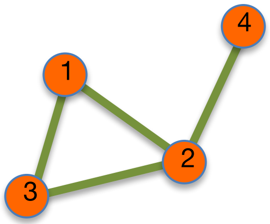
$$L = \sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{L}{N}$$

*WWW, citation networks*

Type of graphs

# Weightedness

*Unweighted graph*



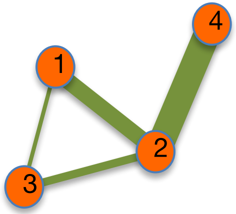

$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{2L}{N}$$

*protein-protein interactions, WWW*

*Weighted graph*




$$A_{y} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

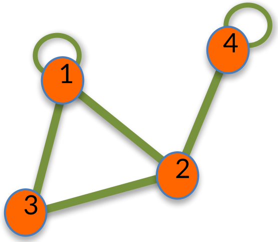
$$L = \frac{1}{2}\sum_{i,j=1}^{N} nonzero(A_{ij}) \qquad < k >= \frac{2L}{N}$$

*Call Graph, metabolic networks*

Type of graphs

# Loops & Multigraphs

## *Self Interactions*



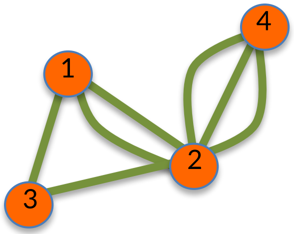

$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$A_{ii} \neq 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1, i\neq j}^{N} A_{ij} + \sum_{i=1}^{N} A_{ii} \qquad ?$$

*protein-protein interactions, WWW*

## *Multigraph (undirected)*




$$A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} nonzero(A_{ij}) \qquad \langle k \rangle = \frac{2L}{N}$$

*Social Network, Collaboration Network*

| Network | Directed | Weighted | Multigraph | Self-loops |
|---|---|---|---|---|
| WWW | yes | no | yes | yes |
| Protein interactions | no | no | no | yes |
| Collaboration network | no | yes | yes | no |
| Mobile phone calls | yes | yes | no | no |
| Facebook Friendship | no | no | no | no |

Real Networks can have multiple characteristics

Case Study:

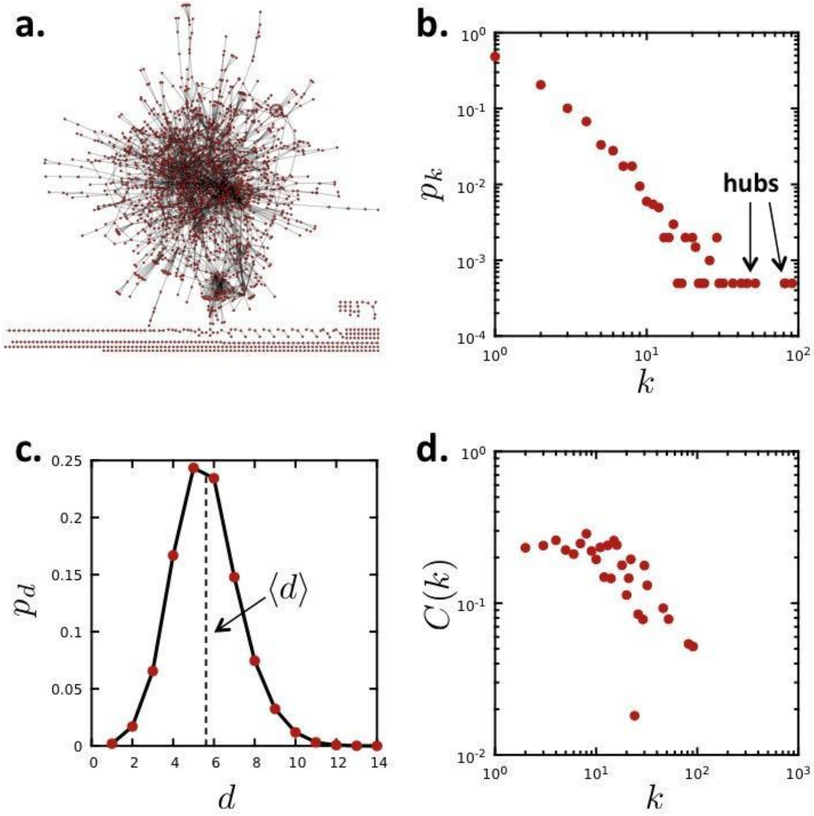
# Protein-Protein Interaction Network

Case study

# Protein-protein interaction

Undirected network
N=2,018 proteins as nodes
L=2,930 binding interactions as links.
Average degree <k>=2.90.

Not connected: 185 components
Largest (giant component) 1,647 nodes



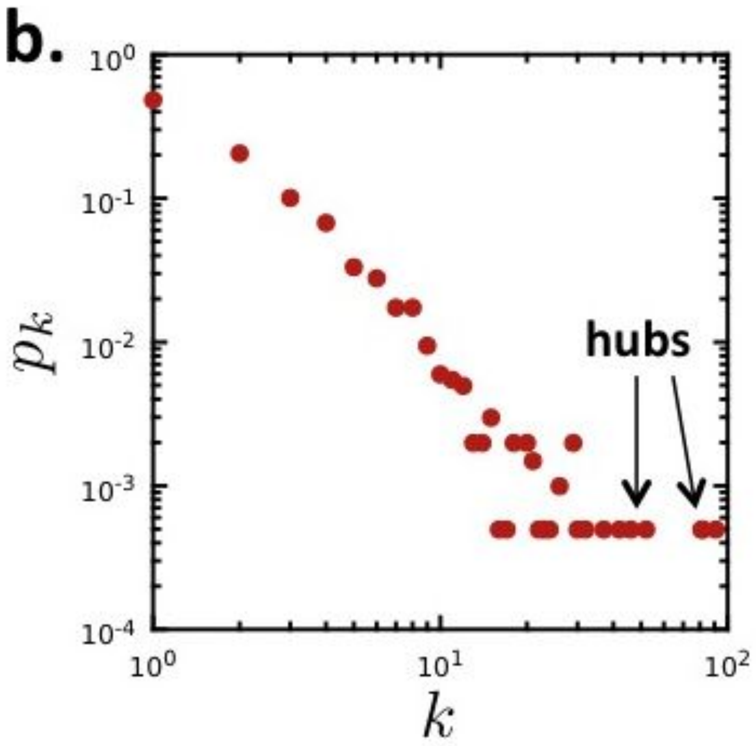
Case study

# Protein-protein interaction

$p_k$ is the probability that a node has degree k

$N_k$ = # nodes with degree k

$$p_k = N_k / N$$



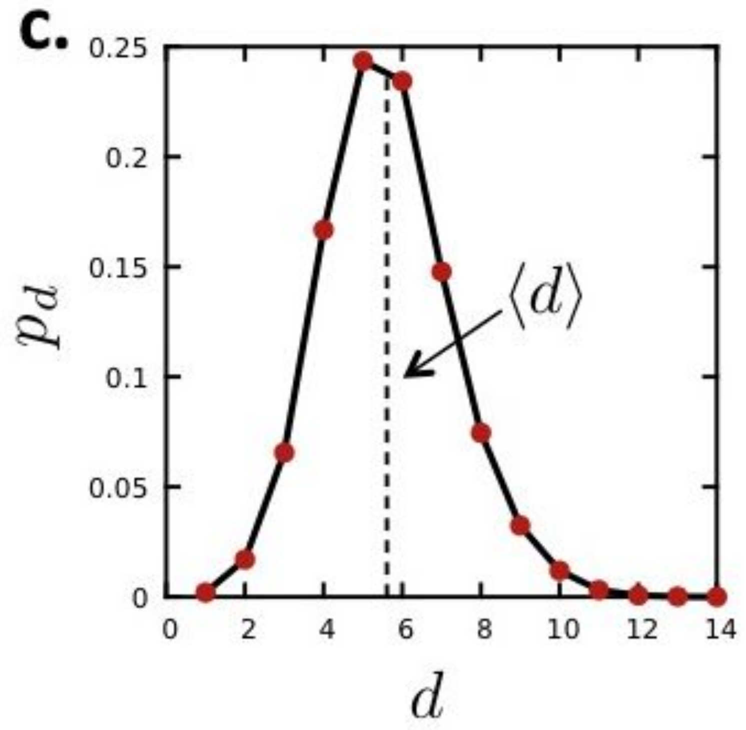
Case study

# Protein-protein interaction

Path length distribution

$d_{max}$=14

<d>=5.61



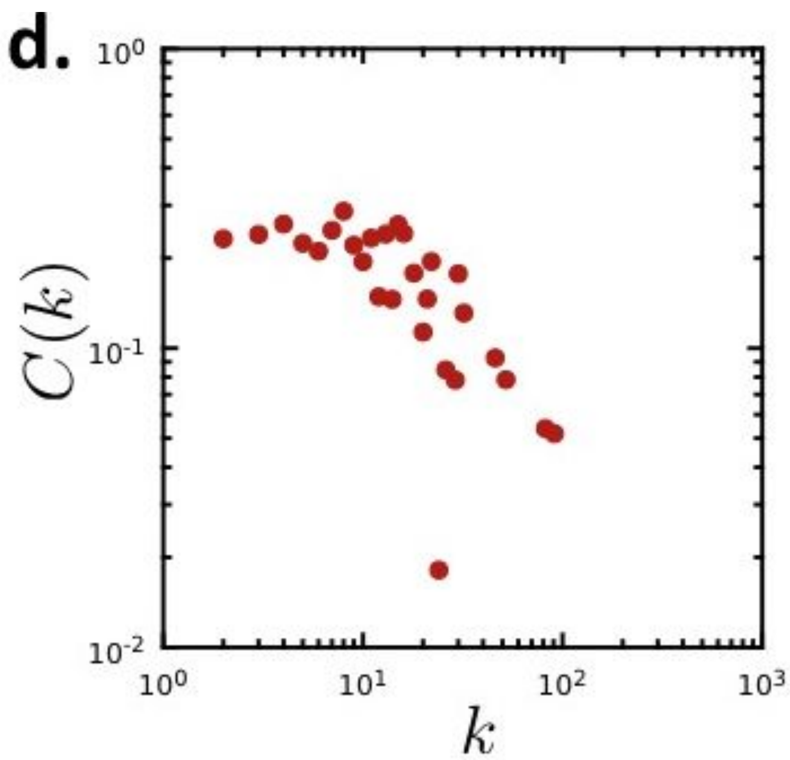
Case study

# Protein-protein interaction

Clustering coefficient vs. node degree

Average Clustering Coeff.

<C>=0.12



Chapter 2

# Conclusion

## Take Away Messages

1. Semantic shapes graph topology
2. Network properties can be measured
3. Degree distribution
4. Paths & Connectivity
5. Clustering Coefficient

## Suggested Readings

- Chapter 2 of Barabasi's book
- Chapter 2 of Kleinberg’s book

## What’s Next

Chapter 3:
**Random Networks**

## Notebook

Chapter 2: Basic Measures
https://github.com/sna-unipi/SNA_lectures_notebooks