Sensing Matrices for KAGRA Main Optics Optical Lever (OpLev) Displacement Sensing System.

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1 Introduction

Optical lever is a device used to measure angular displacement of a reflective surface. [1] It consists of a light source (e.g. superluminescent LED), the reflective surface (e.g. the suspended main optics), and beam position sensing device (e.g. Quadphotodiode (QPD)). In KAGRA, the beam used by the optical lever can also be used to measure the longitudinal displacement (along the reflective normal) of the reflective surface. This is done by sensing the beam position behind a convex lens. Although the phrase "optical lever" refers to the angular sensing part of the whole device, we refer the term "optical lever" in KAGRA to the whole device which senses all three displacements, longitudinal, pitch, and yaw.

There are two types of optical levers that are used as displacement sensors in KAGRA, regular and folded (the one used in MCo). The regular optical lever system can be subdivided into two types, horizontal (e.g. those for Type-A and Type-Bp suspensions) and vertical (e.g. Those for Type-B suspensions). Without loss of generality, we will derive the optical lever sensing matrix with a tilted plane of incidence. The derivation of the sensing matrix of a regular type is slightly different then that of a folded optical lever system. We will first derive the sensing matrix of a regular type and then modify it to fit a folded optical lever system.

2 Derivation

This section is organized as follows, we will first derive a very basic sensing matrix, assuming there's no misalignment in the optical system. And then, we will derive the general sensing matrix that includes all sorts of misalignment. Lastly, we will modify the matrix for a folded optical lever configuration.

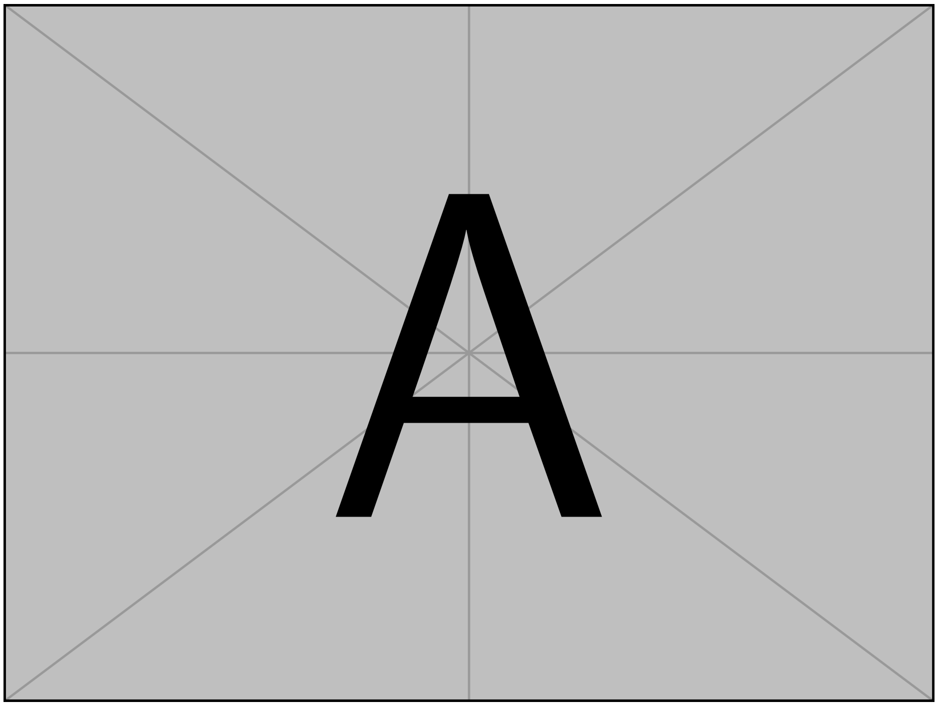

Figure 1: Different types of optical lever displacement sensing systems in KAGRA

2.1 Very basic derivation

In the simplest case, the beam position x_1 (along the incidence plane) is related to the angular displacement θ by

$$x_1 = (2r)\,\theta, \tag{1}$$

where r is the lever arm defined by the distance between the reflective surface and the sensing device. The same beam can be used to measure the longitudinal displacement of the reflective surface, if the light beam has an angle of incidence α. In this case, the beam displacement reads

$$x_1 = (2r)\,\theta + (2\sin\alpha)\,x_L, \tag{2}$$

where x_L is the longitudinal displacement of the reflective surface. As can be seen, equation. (2) shows a coupled sensor where it reads both the angular displacement and the longitudinal shift. In KAGRA, some optical levers have a second sensor measuring the beam displacement x_2 some distance d behind a convex lens with focal length f. In this case, we obtain the second beam displacement x_2 via ray transfer matrices [2]

$$\begin{pmatrix} x_2 \\ \cdot \end{pmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & r_{\text{lens}} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} (2\sin\alpha)\,x_L \\ 2\theta \end{pmatrix}, \tag{3}$$

where r_{lens} is the distance between the reflective surface and the lens. This gives

$$x_2 = (2\sin\alpha)\left(1 - \frac{d}{f}\right)x_L + 2\left[\left(1 - \frac{d}{f}\right)r_{\text{lens}} + d\right]\theta. \tag{4}$$

Furthermore, we can place the second beam displacement sensor distance behind the lens. So, if we set

$$d = \frac{r_{\text{lens}} f}{r_{\text{lens}} - f}, \tag{5}$$

then the angular coupling, i.e. the second term in Eqn. (4), becomes zero, effectively making the second beam displacement sensor a “length” (length as in longitudinal displacement) sensing device. If Eqn. (5) is satisfied, then the beam displacement measured by the second senor reads

$$x_2 = \frac{-2f\sin\alpha}{r_{\text{lens}} - f}\,x_L. \tag{6}$$

Then, from here, we can diagonalize the sensors. Consider a state-space model $\vec{y} = \mathbf{C}\vec{x}$, where $\vec{y} \equiv \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ are the measurements, and $\vec{x} \equiv \begin{pmatrix} x_L \\ \theta \end{pmatrix}$ are the states. Here, the $\mathbf{C}$ matrix is

$$\mathbf{C} = \begin{bmatrix} 2\sin\alpha & 2r \\ \frac{-2f\sin\alpha}{r_{\mathrm{lens}}-f} & 0 \end{bmatrix}, \tag{7}$$

which is not a diagonal matrix. The goal is to define another measurement basis $\vec{y'}$ such that $\vec{y'} = \mathbf{C}'\vec{x}$, where $\mathbf{C}'$ is a diagonal matrix. It's obvious that If we define $\vec{y'} \equiv \mathbf{C}^{-1}\vec{y}$, then $\mathbf{C}'$ becomes the identity, which is a diagonal matrix. Therefore, we define $\mathbf{C}^{-1}$ to be the sensing matrix, which maps sensor measurements to the displacements of the reflective surface.

2.2 Misalignment

References

[1] Tak Lun Terrence Tsang. BS and SR TM optical lever (oplev) diagonalization matrix. https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=10189.

[2] Wikipedia contributors. Ray transfer matrix analysis — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Ray_transfer_matrix_analysis&oldid=1018856234, 2021. [Online; accessed 27-April-2021].