rosdyn EKF

1 Dynamics

Robot dynamics is normally model with an additive static friction (namely, the breakaway friction depends on $\text{sign}(q)$. However, static friction is not derivable when $\text{sign}(q_i) = 0$, which is required by the EKF. To overcome this issue, a LuGre model is introduced. The LuGre model's internal state is not considered part of the observed state.

The motor torque is:

$$\tau = \tau_d + \tau_f(z, \dot{q}) \tag{1}$$

where $\tau_f(z, \dot{q})$ is the friction torque, τ_d is the component due to gravitation, Coriolis and inertial terms:

$$\tau_d = B(q)\ddot{q} + NL(q, \dot{q}) \tag{2}$$

where $B(q)$ is the inertia matrix, $NL(q, \dot{q})$ is the sum of gravitational part and Coriolis effect.

The forward dynamics equation is:

$$\ddot{q} = B(q)^{-1} (\tau_d - NL(q, \dot{q}) - \tau_f(z, \dot{q})) \tag{3}$$

The state equation is

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ B(q)^{-1} (\tau_d - NL(q, \dot{q}) - \tau_f(z, \dot{q})) \end{bmatrix} \tag{4}$$

1.1 Jacobians

Jacobians of (2) w.r.t. q and $\dot{q}$ are to complex to be computed analytically, therefore a numerical derivation is needed.

$$\frac{\partial \ddot{q}}{\partial q} = NUMERIAL \tag{5}$$

$$\frac{\partial \ddot{q}}{\partial \dot{q}} = NUMERIAL \tag{6}$$

Jacobian of (2) w.r.t $\ddot{q}$ is the inertia matrix.

$$\frac{\partial \tau_d}{\partial \ddot{q}} = B(q) \tag{7}$$

2 LuGre Model

Note: z is supposed to be know. It is not part of observed states

$$\begin{cases} g(\dot{q}) = c_0 \\ f(\dot{q}) = c_1\dot{q} \end{cases} \tag{8}$$

$$\begin{cases} \dot{z} = \dot{q} - \sigma_0 \dfrac{|\dot{q}|}{c_0} z \\ \tau_f(z, \dot{q}) = \sigma_0 z + \sigma_1 \dot{z} + c_1 \dot{q}. \end{cases} \tag{9}$$

$$\begin{cases} \dot{z} = \dot{q} - \sigma_0 \dfrac{|\dot{q}|}{c_0} z \\ \tau_f(z, \dot{q}) = \sigma_0 z - \sigma_1 \sigma_0 \dfrac{|\dot{q}|}{c_0} z + \sigma_1 \dot{q} + c_1 \dot{q}. \end{cases} \tag{10}$$

$$\begin{cases} \dot{z} = \dot{q} - \sigma_0 \dfrac{|\dot{q}|}{c_0} z \\ \tau_f(z, \dot{q}) = \sigma_0 z \left(I - \sigma_1 \dfrac{|\dot{q}|}{c_0} \right) + \sigma_1 \dot{q} + c_1 \dot{q}. \end{cases} \tag{11}$$

2.1 Jacobians

$$\begin{aligned} \frac{\partial z}{\partial \dot{q}} &= I - \operatorname{sign} \dot{q} \frac{\sigma_0}{c_0} z \\ \frac{\partial z}{\partial z} &= -\sigma_0 \frac{|\dot{q}|}{c_0} \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{\partial F}{\partial \dot{q}} &= -\frac{\sigma_0 \sigma_1}{c_0} \operatorname{sign}(\dot{q}) z + \sigma_1 + c_1 \\ \frac{\partial F}{\partial z} &= \sigma_0 \left(I - \sigma_1 \frac{|\dot{q}|}{c_0} \right) \end{aligned} \tag{13}$$

2.2 Equilibrium point

$$\dot{z} = \dot{q} - \sigma_0 \frac{|\dot{q}|}{c_0} z = 0 \tag{14}$$

$$z = \frac{c_0 \operatorname{sign}(\dot{q})}{\sigma_0} \tag{15}$$

$$\begin{aligned} \tau_f(z, \dot{q}) &= c_0 \operatorname{sign}(\dot{q}) \left(I - \sigma_1 \frac{|\dot{q}|}{c_0} \right) + \sigma_1 \dot{q} + c_1 \dot{q} \\ &= c_0 \operatorname{sign}(\dot{q}) - \sigma_1 \dot{q} + \sigma_1 \dot{q} + c_1 \dot{q} \\ &= c_0 \operatorname{sign}(\dot{q}) + c_1 \dot{q} \end{aligned} \tag{16}$$