Multiple Shock Impulse Response Functions

Terri van der Zwan

Erasmus University Rotterdam and Tinbergen Institute
t.vanderzwan@ese.eur.nl

Introduction

- Impulse response analysis is a widely employed tool in the field of macroeconomics and econometrics, popularized by Sims (1980).
- Identification issues/what comes first? Koop et al. (1996) introduce generalized impulse response functions.

We propose: Multiple shock impulse response functions, which take into account the correlation between the shocks. Incorporates:

* Contagion between shocks
* Temporal aggregation

Multiple impulse response functions can shed light on:

- The interaction and impact of financial shocks.
- The effects of multiple uncertainty sources on economic variables.
- The transmission of shocks across countries and assessing global macroeconomic linkages.

General Framework

Let $\boldsymbol{y}_t$ be a vector with n endogenous variables, modeled by a function of historical values of $\boldsymbol{y}_t$ and variables $\boldsymbol{z}_t$, and a function of n shocks $\boldsymbol{\nu}_t$:

$$\boldsymbol{y}_t = f(\boldsymbol{y}_{t-1}, ..., \boldsymbol{y}_{t-p}, \boldsymbol{z}_t, ..., \boldsymbol{z}_{t-q}) + g(\boldsymbol{\nu}_t), \qquad (1)$$

where $\boldsymbol{\nu}_t$ have mean zero and finite variances.

Impulse Response Concepts

Let $\boldsymbol{y}_t$ follow a process in accordance with Equation (1).

Definition 1: Traditional impulse response functions

The traditional impulse response functions of $\boldsymbol{y}_{t+h}$ to the s-th shock $\nu_{s,t}$ of size δ_s are defined as

$$\begin{aligned}\boldsymbol{\Psi}(h, \delta_s, \boldsymbol{\omega}_{t-1}) = \mathbb{E}[\boldsymbol{y}_{t+h} \mid \nu_{s,t} = \delta_s, \nu_{j,t} = 0\, \forall j \neq s,\\ \boldsymbol{\nu}_{t+1} = ... = \boldsymbol{\nu}_{t+h} = \boldsymbol{0}, \boldsymbol{\omega}_{t-1}]\\ - \mathbb{E}[\boldsymbol{y}_{t+h} \mid \boldsymbol{\nu}_t = \boldsymbol{\nu}_{t+1} = ... = \boldsymbol{\nu}_{t+h} = \boldsymbol{0}, \boldsymbol{\omega}_{t-1}],\end{aligned}$$

for horizon $h = 0, 1, ..., H$, where $\boldsymbol{\omega}_{t-1}$ denotes an historical path realization of the stochastic process that generates $\boldsymbol{y}_{t+h}$. This definition implies a linear function of $g(\cdot)$ and requires identification of the structural relations between shocks.

Definition 2: Generalized impulse response functions

The one shock generalized impulse response functions (Koop et al., 1996; Pesaran and Shin, 1998) of $\boldsymbol{y}_{t+h}$ to the s-th shock $\nu_{s,t}$ of size δ_s are defined as

$$\boldsymbol{\Psi}^g(h, \delta_s, \mathcal{I}_{t-1}) = \mathbb{E}[\boldsymbol{y}_{t+h} \mid \nu_{s,t} = \delta_s, \mathcal{I}_{t-1}] - \mathbb{E}[\boldsymbol{y}_{t+h} \mid \mathcal{I}_{t-1}],$$

for horizon $h = 0, 1, ..., H$, where $\mathcal{I}_{t-1}$ denotes the information set available at $t-1$. Here, the history is treated random and does not require identification of the structural relations.

Definition 3: Multiple shock impulse response functions

Let $\mathcal{S}$ be a set of indices corresponding to the $1 < m \leq n$ shocks of interest, where $|\mathcal{S}| > 1$. The multiple shock impulse response functions of $\boldsymbol{y}_{t+h}$ to a set of shocks $\boldsymbol{\nu}_{\mathcal{S},t}$ of size $\boldsymbol{\delta}_{\mathcal{S}}$ are defined as

$$\boldsymbol{\Psi}^{\mathcal{S}}(h, \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1}) = \mathbb{E}[\boldsymbol{y}_{t+h} \mid \boldsymbol{\nu}_{\mathcal{S},t} = \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1}] - \mathbb{E}[\boldsymbol{y}_{t+h} \mid \mathcal{I}_{t-1}],$$

for horizon $h = 0, 1, ..., H$.

Illustration: VAR(1) process

Let $\boldsymbol{y}_t$ denote the n variables of interest. The vector autoregression (VAR) with one lag is then

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{y}_{t-1} + \boldsymbol{u}_t, \qquad \boldsymbol{u}_t \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}). \qquad (2)$$

We assume i.i.d. residuals $\boldsymbol{u}_t$ and stability of the VAR.

Impulse response functions for Equation (2)

Let σ_{ss} be the (s, s)-th element of $\boldsymbol{\Sigma}$, $\boldsymbol{e}_s$ an s-th element unit vector, and $\boldsymbol{P}$ an $n \times m$ permutation matrix, with m unit vectors, then:

Generalized impulse response functions (GIRF) for one shock s:

$$\boldsymbol{\Psi}^g(h, \delta_s, \mathcal{I}_{t-1}) = \boldsymbol{B}^h \boldsymbol{\Sigma} \boldsymbol{e}_s (\sigma_{ss})^{-1} \delta_s. \qquad (3)$$

Multiple shock impulse response functions for $m > 1$ shocks:

$$\boldsymbol{\Psi}^{\mathcal{S}}(h, \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1}) = \boldsymbol{B}^h \boldsymbol{\Sigma}\boldsymbol{P}(\boldsymbol{P}'\boldsymbol{\Sigma}\boldsymbol{P})^{-1}\boldsymbol{\delta}_{\mathcal{S}}. \qquad (4)$$

Simulation

Consider a data generating process (DGP):

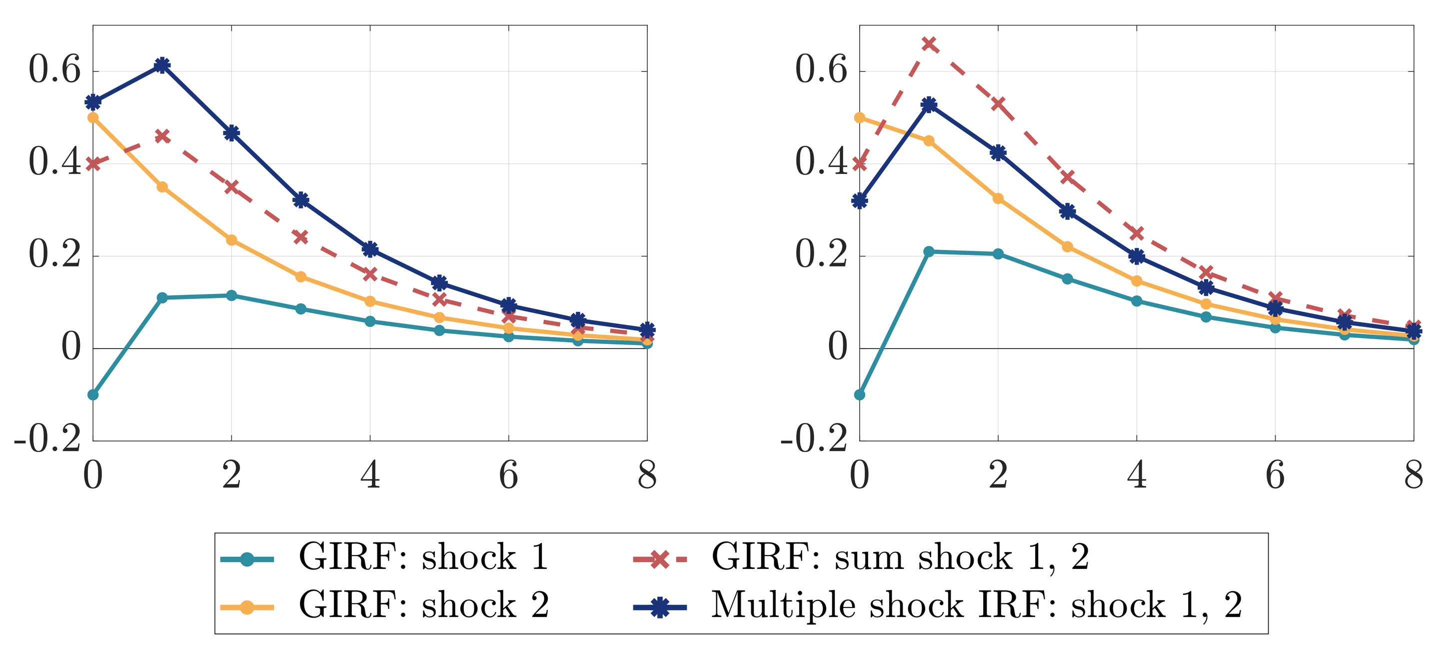
- DGP: $n = 3$ variables, Equation (2), with $\boldsymbol{B} = \begin{bmatrix} 0.4 & 0.1 & 0.1 \\ 0.1 & 0.4 & 0.1 \\ 0.2 & 0.2 & 0.4 \end{bmatrix}$.
- Consider 2 cases:

$$\boldsymbol{\Sigma}^{(1)} = \begin{bmatrix} 1 & -0.25 & -0.1 \\ -0.25 & 1 & 0.5 \\ -0.1 & 0.5 & 1 \end{bmatrix}, \text{ and } \boldsymbol{\Sigma}^{(2)} = \begin{bmatrix} 1 & 0.25 & -0.1 \\ 0.25 & 1 & -0.5 \\ -0.1 & -0.5 & 1 \end{bmatrix}.$$

- Analyze effect of first two shocks $\mathcal{S} = \{1, 2\}$ on variable 3.

Figure: Impulse Response Functions

(a) Case 1, DGP with $\boldsymbol{\Sigma}^{(1)}$ (b) Case 2, DGP with $\boldsymbol{\Sigma}^{(2)}$



The sum of the one-shock GIRFs $\sum_{\ell \in \mathcal{S}} \boldsymbol{\Psi}^g(h, \delta_\ell, \mathcal{I}_{t-1})$ (dashed red line) **underestimates** (case 1) or **overestimates** (case 2) the total effect, $\boldsymbol{\Psi}^{\mathcal{S}}(h, \boldsymbol{\delta}_{\mathcal{S}}, \mathcal{I}_{t-1})$ (solid blue line).

Summary and Further Research

- **Multiple shock impulse response functions are necessary to accurately analyze the combined effect of shocks.**
- Summing the one-shock generalized impulse response functions can lead to either over- or underestimation of the total effect.
- **Further research**:
  * Non-linear specifications, second order dynamics
  * Empirical analysis

References

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