1 Basics of Calculus

1.1 Derivatives

Chain Rule	Product Rule	Quotient Rule
$y = f(g(x))$	$y = f(x)g(x)$	$y = \frac{f(x)}{g(x)}$
$g'(x)f'(g(x))$	$g'(x)f(x) + f'(x)g(x)$	$\frac{g'(x)f(x) - f'(x)g(x)}{g(x)^2}$

1.2 Integrals

$$\int x^{-1}dx = \ln|x| + c \qquad \int e^x dx = e^x + c$$

$$\int \cos x dx = \sin x + c \qquad \int \sin x dx = -\cos x + c$$

$$\int \tan x dx = \ln|\sec^2 x| + c \qquad \int \sec x \tan x dx = \sec x + c$$

$$\int \sec^2 x dx = \tan x + c$$

1.2.1 u Substitution

$$\int \cos x e^{\sin x} dx$$

let $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x dx$$

$$\int e^u du$$

$$= e^u + c$$

$$= e^{\sin x} + c$$

1.2.2 Integration By Parts

$$\int u dv = uv - \int v du$$

let $u = 2t$ let $dv = e^t dt$

let $du = 2dt$ let $v = e^t$

$$\int 2te^t dt$$

$$\int 2te^t dt = 2te^t - \int 2e^t dt$$

$$= 2te^t - 2e^t + c$$

1.2.3 Trig Substitution

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

$$\int \frac{2}{x^2\sqrt{x^2 - 16}}$$

let $x = 4\sec\theta$

$$dx = 4\sec\theta\tan\theta d\theta$$

$$= \int \frac{8\sec\theta\tan\theta}{16\sec^2\theta\sqrt{16\sec^2\theta - 16}}d\theta$$

$$\int \frac{8\sec\theta\tan\theta}{16\sec^2\theta 4\tan\theta}$$

$$= \int \frac{1}{8\sec\theta}d\theta$$

$$\frac{1}{8}\int \cos\theta d\theta$$

$$= \frac{1}{8}\sin\theta + c$$

$$= \frac{1}{8}\frac{\sqrt{x^2 - 16}}{x} + c$$

1.2.4 Partial Fraction Decomposition

$$\int \frac{x^3 - 4x - 10}{x^2 - x - 6}dx$$

$$\begin{array}{r|rrrr}
 & & & x & +1 \\ \hline
x^2 - x - 6 & x^3 & +0x^2 & -4x & -10 \\
- & (x^3 & -x^2 & -6x & +0) \\ \hline
 & & x^2 & +2x & -10 \\
 & - & (x^2 & -x & -6) \\ \hline
 & & & 3x & -4
\end{array}$$

$$= \int x + 1 + \frac{3x - 4}{x^2 - x - 6}dx$$

$$= \frac{x^2}{2} + x + \int \frac{3x - 4}{x^2 - x - 6}dx$$

$$x^2 - x - 6 = (x - 3)(x + 2)$$

$$\frac{3x - 4}{(x - 3)(x + 2)}$$

$$= \frac{A}{x - 3} + \frac{B}{x + 2}$$

given $x = 3, x = -2$

$$A = 1, B = 2$$

$$= \int \frac{x^2}{2} + x + \int \frac{1}{x - 3} + \frac{2}{x + 2}dx$$

$$= \frac{x^2}{2} + x + \ln|x + 3| + 2\ln|x + 2| + c$$

1.3 Trig Identities

Pythagorean identities	Negative identities
$\sin^2\theta + \cos^2\theta = 1$	$\sin(-\theta) = -\sin\theta$
$1 - \sin^2\theta = \cos^2\theta$	$\cos(-\theta) = \cos\theta$
$1 - \cos^2\theta = \sin^2\theta$	$\tan(-\theta) = -\tan\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\cot(-\theta) = -\cot\theta$
$1 + \cot^2\theta = \csc^2\theta$	$\sec(-\theta) = \sec\theta$
$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$	$\csc(-\theta) = -\csc\theta$
Double Angle formulas	**Sum & difference formulas**
$\sin(2\theta) = 2\sin\theta\cos\theta$	$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - \sin^2\theta = 2\cos^2\theta - 1$	$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$
$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$	$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$

2 Differential Equations

2.1 Separable Differential Equations

If the De is in the form

$$\frac{dy}{dx} = f(x) \cdot g(y)$$

Then there exists a solution

$$\int \frac{1}{g(y)}dy = \int f(x)dx$$

2.2 First Order Linear Differential Equations

$$y' + yP(x) = Q(x)$$

The first stage of solving a First Order Linear DE, is finding the integrating factor ($I(x)$)

$$I(x) = Ce^{\int P(x)dx}$$

Then multiply everything by $I(x)$

$$I(x)y' + I(x)yP(x) = I(x)Q(x)$$

$$(yI(x))' = I(x)Q(x)$$

$$yI(x) = \int I(x)Q(x)dx$$

Get y by itself

2.3 Bernoulli's Differential Equation

$$y' + P(x)y = Q(x)y^n$$

1. Multiply everything by $(1 - n)y^{-n}$
2. let $u = y^{1-n}$, $\frac{du}{dx} = (1 - n)y^{-n} \cdot \frac{dy}{dx}$

$$xy' + y = -xy^2$$

$$y' + \frac{1}{x}y = -y^2$$

$$-y^2y' - \frac{1}{x}\frac{1}{y} = 1$$

let $u = y^{-2}$

$$\frac{du}{dx} = -y^{-2}\frac{dy}{dx}$$

$$u' - \frac{1}{x}u = 1$$

Solve as if 2.2

2.4 Second Order Homogenous Linear DE

$$ay'' + by' + cy = 0$$

Has Solutions:

$$y(x) = c_1y_1(x) + c_2y_2(x)$$

Using the Characteristic Equation:

$$ar^2 + br + c = 0$$

Solve for r which gives three possible cases:

1. Real roots r_1, r_2: $y_1(x) = e^{r_1x}, y_2(x) = e^{r_2x}$
2. Repeated Roots r, r: $y_1(x) = e^{rx}, y_2(x) = xe^{rx}$
3. Complex Conjugate $\alpha + \mathrm{B}i, \alpha - \mathrm{B}i$:

$$y(x) = e^{\alpha x}(C_1\cos(\mathrm{B}x) + C_2\sin(\mathrm{B}x))$$

2.5 Second Order non-Homogeneous DE

A Second order non-Homogeneous DE takes the form of

$$ay'' + by' + cy = f(x)$$

The complementary solution still exists

$$y_{c(x)} = C_1y_1(x) + C_2y_2(x)$$

But there is a particular solution which helps form the exact solution, which is found using 2 methods

$$y(x) = y_{c(x)} + y_{p(x)}$$

⚠

Remember to use the method asked for in the question even if there is an easier way

2.5.1 Undetermined Coefficients

This method is suitable for polynomials, exponents, sin or cos

$f(x)$: Polynomial - Guess polynomial of the same degree; e.g. $f(x) = x^2, y_{p(x)} = Ax^2 + Bx + C$

$f(x)$: Exponent - Guess $y_{p(x)} = Ae^{bx}$ e.g. $f(x) = 6e^{2x+1}, y_{p(x)} = Ae^{2x+1} = Ae^{2x}$

$f(x)$: $\sin(bx)$ or $\cos(bx)$ - Guess $y_{p(x)} = A\sin(bx) + B\cos(bx)$

If two terms in $f(x)$ are added, add the guesses, similarly if they are multiplied, multiply the guesses, e.g.

$$f(x) = x^2\sin(x) + e^{2x}$$

$$y_{p(x)} = (Ax^2 + Bx + C)(D\sin(x) + E\cos(x)) + Fe^{2x}$$

Finally find y_p' and y_p'' and equate the coefficients - Some expansion and simplification will be required.

2.5.2 Variation of Parameters

Given $ay'' + by' + cy = f(x)$ and the complementary solution $y_{c(x)} = C_1y_1(x) + C_1y_2(x)$ the particular solution is $y_{p(x)} = u_1(x)y_1(x) + u_2(x)y_2(x)$

$$u_1(x) = -\frac{1}{a}\int \frac{y_2(x)f(x)}{W(y_1, y_2)}dx$$

$$u_2(x) = \frac{1}{a}\int \frac{y_1(x)f(x)}{W(y_1, y_2)}dx$$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$$

3 Statistics

3.1 Types of Data

Qualitative
- Nominal - Unordered
- Ordinal - Ordered

Quantitative
- Discrete - Counts
- Continous - Measures

3.2 Probability

A number between 0 and 1, that describes how likely an event is to happen

If probability of an even t is p then the probability of that even not happening is $1 - p$

3.3 Frequency

Favourite ice cream flavours:

C, S, C, C, V, V, S, C, M

Chocolate	4
Strawberry	2
Vanilla	2
Mint	1
Total	9

Put into bar graph (write numbers into bars)

^ For Discrete data

For Continous data:

Break into "bins" e.g. $10 < X \leq 15$, Then produce a frequency table, and graph with histogram (the bars must touch)

3.4 Averages

3.4.1 Mean

Arithmatic Mean:

$$\overline{X} = \frac{\sum X}{n}$$

Sometimes can't be possible, e.g mean goals in a game 3.4 is not possible. Not great, susceptible to outliars.

3.4.2 Median

The middle value of the ordered set of values. If there is no middle (e.g. even number of values) you take the mean of the 2 middle values

Is not as susceptible to outliars, good way at assessing real world situations, and for skewed data.

3.4.3 Mode

Most common value.

Can be useful but its often not. Can be multiple Modes, does tell you the nature of the data, e.g. "bimodal" if there are 2 modes.

3.5 Variability

Average doesn't always give the full story

Range: biggest - smallest

Range can be easily thrown off by outliar data

IQR = Q3 - Q1

Q1 is the median of the first half

Q3 is the median of the second half

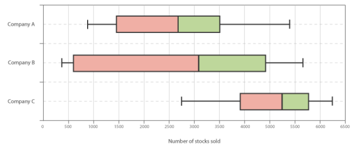
3.5.1 Box and Whisker plot

Take 5 number summary:

min, Q1, median, Q3, max

Draw a line where these values are on a number line

Draw a box between Q1 & Q3



3.6 Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N}}$$

Variance is σ^2

Larger variance = the data is more varied

3.7 Normal Distribution

μ Defines where the middle of the bell is

σ Defines how wide/narrow the bell is