MATH 240: Discrete Structures - Assignment 2

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Problem 1

To do this we can simplify the right-hand side using set identities, ultimately showing that it is equivalent to the left-hand side.

$$
\begin{aligned}
C \setminus B &= (C \setminus B) \setminus (B \setminus A) && \\
&= (C \cap \overline{B}) \cap \overline{(B \cap \overline{A})} && [\text{Definition } A \setminus B = A \cap \overline{B}] \\
&= (C \cap \overline{B}) \cap (\overline{B} \cup A) && [\text{De Morgan's Law: } \overline{A \cap B} = \overline{A} \cup \overline{B}] \\
&= C \cap \overline{B} \cap (\overline{B} \cup A) && [\text{Associative Law: } (A \cap B) \cap C = A \cap (B \cap C)] \\
&= C \cap \overline{B} && [\text{Absorption Law: } A \cap (A \cup B) = A] \\
&= C \setminus B
\end{aligned}
$$

Problem 2

Recall: A tautology is a statement which contains all true in the last column of its truth table.

(a)

p	q	$\overline{q}$	$p \implies q$	$\overline{p \implies q}$	$\overline{q} \wedge (\overline{p \implies q})$	$(\overline{q} \wedge (\overline{p \implies q})) \iff p$
T	T	F	T	F	F	F
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

$\therefore (\overline{q} \wedge (\overline{p \implies q})) \iff p$ is *not* a tautology.

(b)

p	q	r	$q \implies r$	$p \implies (q \implies r)$	$p \implies q$	$(p \implies q) \implies r$	$(p \implies (q \implies r)) \iff ((p \implies q) \implies r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

$\therefore (p \implies (q \implies r)) \iff ((p \implies q) \implies r)$ is *not* tautology.

Problem 3

To prove logical equivalence we can simplify either the left-hand side or the right-hand side using the laws of logic, ultimately showing that one side is equivalent to the other.

(a)

$$
\begin{aligned}
(p \iff q) &\equiv (p \implies q) \wedge (q \implies p) && [\text{Definition } (p \iff q) \equiv (p \implies q) \wedge (q \implies p)] \\
&\equiv (\overline{p} \vee q) \wedge (\overline{q} \vee p) && [\text{Definition } (p \implies q) \equiv \overline{p} \vee q]
\end{aligned}
$$

$$
\begin{aligned}
\overline{p \oplus q} &\equiv \overline{(p \wedge \overline{q}) \vee (\overline{p} \wedge q)} && [\text{Definition } p \oplus q \equiv (p \wedge \overline{q}) \vee (\overline{p} \wedge q)] \\
&\equiv \overline{(p \wedge \overline{q})} \wedge \overline{(\overline{p} \wedge q)} && [\text{De Morgan's Law: } \overline{p \vee q} \equiv \overline{p} \wedge \overline{q}] \\
&\equiv (\overline{p} \vee q) \wedge (q \vee \overline{p}) && [\text{De Morgan's Law: } \overline{p \wedge q} \equiv \overline{p} \vee \overline{q}]
\end{aligned}
$$

$$\therefore p \iff q \equiv \overline{p \oplus q}$$

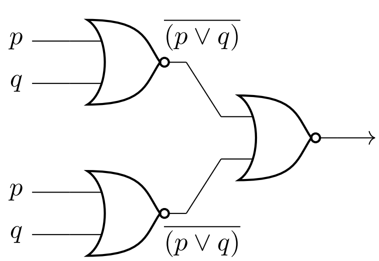
(b)

$$
\begin{aligned}
\overline{p} &\equiv (\overline{p \vee \overline{q}}) \vee (\overline{p} \wedge \overline{q}) && \\
&\equiv (\overline{p} \wedge q) \vee (\overline{p} \wedge \overline{q}) && [\text{De Morgan's Law: } \overline{p \vee q} \equiv \overline{p} \wedge \overline{q}] \\
&\equiv \overline{p} \wedge (q \vee \overline{q}) && [\text{Distributive Law: } p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)] \\
&\equiv \overline{p} \wedge 1 && [\text{Tautology: } p \vee \overline{p} \equiv 1] \\
&\equiv \overline{p} && [\text{Identity Law: } p \wedge 1 \equiv p]
\end{aligned}
$$

Problem 4

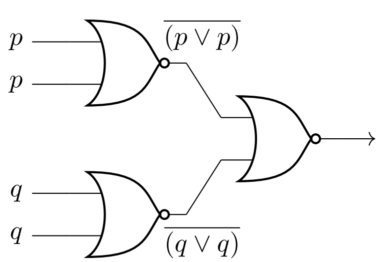
Recall **NOR**: (p $\downarrow q) \equiv \overline{p} \wedge \overline{q} \equiv \overline{(p \vee q)}$ *as seen in class

(a) OR $(p \vee q)$



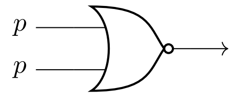
$\overline{\overline{(p \vee q)} \vee \overline{(p \vee q)}} \equiv (p \vee q) \wedge (p \vee q) \equiv p \vee q$

(b) AND $(p \wedge q)$



$\overline{\overline{(p \vee p)} \vee \overline{(q \vee q)}} \equiv (p \vee p) \wedge (q \vee q) \equiv p \wedge q$

(c) NOT ($\overline{p}$)



$$\overline{p \vee p} \equiv \overline{p} \wedge \overline{p} \equiv \overline{p}$$