Liam Scalzulli **MATH240_FALL2022**

Assignment 1._Set_Theory due 09/23/2022 at 11:59pm EDT

Problem 1. **(1 point)**

Match the following sets:

___1. $\{1,2,3,4,5,\dots\}$
___2. $\{1,4,9,16,25,\dots\}$
___3. $\{2,4,6,8,10,\dots\}$
___4. $\{\}$

A. $\{2n : n \in \mathbb{N}\}$
B. $\emptyset$
C. $\mathbb{N}$
D. $\{n : \sqrt{n} \in \mathbb{N}\}$

Answer(s) submitted:

- C
- D
- A
- B

(correct)

Problem 2. **(1 point)**

Set operations and comparisons

Suppose that A, B, C, D are sets. Suppose that $A \cap B \cap C = \emptyset$, that $A \cap B \supseteq D$, and that $C \subset D$. Answer the following T/F questions. (You must enter T or F for each answer.)

___1. Is $C = \emptyset$?
___2. If in addition $A \setminus D = B \setminus D$ is $A = B$?
___3. Is $D = \emptyset$?

Answer(s) submitted:

- T
- T
- F

(correct)

Problem 3. **(1 point)**

Order the following sets by inclusion.

$$A = \{n \in \mathbb{N} \mid n = 5m \text{ for some } m \in \mathbb{Q}\}$$
$$B = \{n \in \mathbb{N} \mid n = 65m \text{ for some } m \in \mathbb{N}\}$$
$$C = \{n \in \mathbb{N} \mid n = 5m \text{ for some } m \in \mathbb{N}\}$$
$$D = \{130, 390, 520\}$$
$$E = \mathbb{Z}$$

___ $\subseteq$ ___ $\subseteq$ ___ $\subseteq$ ___ $\subseteq$ ___

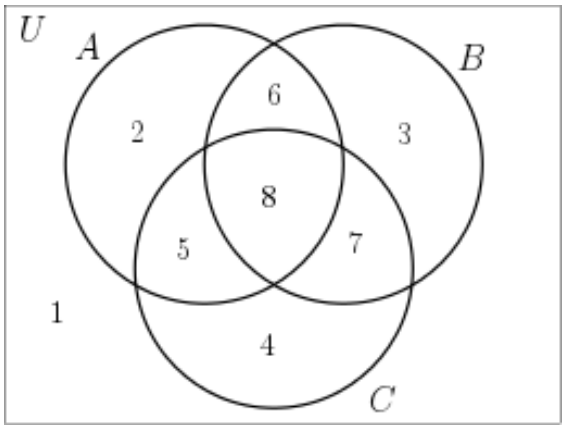
Answer(s) submitted:

- D
- B
- C
- A
- E

(correct)

Problem 4. **(1 point)**

For each set, list all regions (numbered 1 through 8) in the Venn diagram below that are part of the given set. Give your answer as a list of numbers separated by commas, for example $\boxed{3,2,8,1}$. If there are no regions, enter "none".



$C \cup A$ _____

$B \setminus C$ _____

$\overline{C \cup B}$ _____

$A \cup (C \setminus B)$ _____

$C \cap (A \setminus \overline{B})$ _____

Answer(s) submitted:

- `2,4,5,6,7,8`
- `3,6`
- `1,2`
- `2,4,5,6,8`
- `8`

(correct)

Problem 5. **(1 point)**

Define the following sets, which are subsets of the universe $U = \{1,2,3,\ldots,20\}$.

$$A = \{1,2,9,12,15,19\}$$
$$B = \{2,5,9,17,19\}$$
$$C = \{2,3,7,14,17,19,20\}$$

For each set below, list its elements. Use set braces in your answer, for example $\boxed{\{1,2,3\}}$, and if there are no elements write $\boxed{\{\}}$.

$A \cup C =$ _____

$B \setminus C =$ _____

$\overline{A} =$ _____

$A \cap (B \cup C) =$ _____

$\overline{A \setminus (B \cap C)} =$ _____

Answer(s) submitted:

- `{1,2,3,7,9,12,14,15,17,19,20}`
- `{5,9}`
- `{3,4,5,6,7,8,10,11,13,14,16,17,18,20}`
- `{2,9,19}`
- `{2,3,4,5,6,7,8,10,11,13,14,16,17,18,19,20}`

(correct)

Problem 6. **(1 point)**

Let $A = \{1,3,5,10\}$ and $B = \{5,9\}$. Write out all elements of the following sets. Give your answer as a list of points, for example $\boxed{(1,2),(2,3)}$. Do not include the set braces $\{\}$, they are already there.

$A \times B = \{$ _____ $\}$.

$B \times A = \{$ _____ $\}$.

$(A \setminus B) \times (B \setminus A) = \{$ _____ $\}$.

$(A \cup B) \times (A \cap B) = \{$ _____ $\}$.

Answer(s) submitted:

- `(1,5),(1,9),(3,5),(3,9),(5,5),(5,9),(10,5),(10,9)`
- `(5,1),(5,3),(5,5),(5,10),(9,1),(9,3),(9,5),(9,10)`
- `(1,9),(3,9),(10,9)`
- `(1,5),(3,5),(5,5),(9,5),(10,5)`

(correct)

Problem 7. **(1 point)**

Find the cardinality of each indicated set. If the set is infinite, write 'inf'. Assume $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

$A = \{n \in \mathbb{Z} \,|\, |n| < 63\}$. $|A| =$ _____

$B = \{n \in A \,|\, n = 3m \text{ for some } m \in \mathbb{N}\}$. $|B| =$_____. Note that A is the set define above.

$C = \{n \in \mathbb{Z} \,|\, n = m^2 \text{ for some } m \in \mathbb{Z} \text{ and } n \leq 117\}$. $|C| =$ _____

$D = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \,|\, 10x + 10y = 0\}$. $|D| =$ _____

Answer(s) submitted:

- `125`
- `21`
- `11`
- `inf`

(correct)

Problem 8. **(1 point)**

Let A and B be subsets of some ambient (universal) set U, and let $\overline{A} = U \setminus A$. The set

$$(((\overline{B} \setminus A) \cup \overline{B})) \cap B$$

is equal to which of the following sets? Hint: simplify using set identities.

- $\emptyset$
- A
- B
- $\overline{A}$
- $\overline{B}$
- $A \cap B$
- $\overline{A} \cap B$
- $A \cap \overline{B}$
- $\overline{A} \cap \overline{B}$
- $A \cup B$
- $\overline{A} \cup B$
- $A \cup \overline{B}$
- $\overline{A} \cup \overline{B}$
- $(A \cap \overline{B}) \cup (\overline{A} \cap B)$
- $(\overline{A} \cup B) \cap (A \cup \overline{B})$
- U

Answer(s) submitted:

- `Choice 1`

(correct)

Problem 9. **(1 point)**

The set

$$(\overline{A} \cap B) \cap (\overline{(\overline{B} \setminus (A \cup \overline{A}))})$$

is equal to which of the following sets? Hint: simplify using set identities.

- $\emptyset$
- A
- B
- $\overline{A}$
- $\overline{B}$
- $A \cap B$
- $\overline{A} \cap B$
- $A \cap \overline{B}$
- $\overline{A} \cap \overline{B}$
- $A \cup B$
- $\overline{A} \cup B$
- $A \cup \overline{B}$
- $\overline{A} \cup \overline{B}$
- $(A \cap \overline{B}) \cup (\overline{A} \cap B)$
- $(\overline{A} \cup B) \cap (A \cup \overline{B})$
- U

Answer(s) submitted:

- `Choice 7`

(correct)

Problem 10. **(1 point)**

Power sets

If a set A contains four elements, how many elements are in the power set $\mathcal{P}(A)$?

___

How many elements are in the power set $\mathcal{P}(\mathcal{P}(\emptyset))$?

___

Answer(s) submitted:

- `16`
- `2`

(correct)