MATH 240: Discrete Structures - Assignment 1

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Problem 1

To do this we can simplify the right-hand side using set identities, ultimately showing that it is equivalent to the left-hand side.

$$\begin{aligned}
C \setminus B &= (C \setminus B) \setminus (B \setminus A) \\
&= (C \cap \overline{B}) \cap \overline{(B \cap \overline{A})} && [\text{Definition } A \setminus B = A \cap \overline{B}] \\
&= (C \cap \overline{B}) \cap (\overline{B} \cup A) && [\text{De Morgan's Law: } \overline{A \cap B} = \overline{A} \cup \overline{B}] \\
&= C \cap \overline{B} \cap (\overline{B} \cup A) && [\text{Associative Law: } (A \cap B) \cap C = A \cap (B \cap C)] \\
&= C \cap \overline{B} && [\text{Absorption Law: } A \cap (A \cup B) = A] \\
&= C \setminus B
\end{aligned}$$

Problem 2

Recall: A tautology is a statement which contains all true in the last column of its truth table.

(a)

p	q	$\overline{q}$	$p \implies q$	$\overline{p \implies q}$	$\overline{q} \wedge (\overline{p \implies q})$	$(\overline{q} \wedge (\overline{p \implies q})) \iff p$
T	T	F	T	F	F	F
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

$\therefore (\overline{q} \wedge (\overline{p \implies q})) \iff p$ is *not* a tautology.

(b)

p	q	r	$q \implies r$	$p \implies (q \implies r)$	$p \implies q$	$(p \implies q) \implies r$	$(p \implies (q \implies r)) \iff ((p \implies q) \implies r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

$\therefore (p \implies (q \implies r)) \iff ((p \implies q) \implies r)$ is *not* tautology.

Problem 3

To prove logical equivalence we can simplify either the left-hand side or the right-hand side using the laws of logic, ultimately showing that one side is equivalent to the other.

(a)

$$
\begin{aligned}
(p \iff q) &\equiv (p \implies q) \land (q \implies p) && \text{[Definition } (p \iff q) \equiv (p \implies q) \land (q \implies p)\text{]} \\
&\equiv (\overline{p} \lor q) \land (\overline{q} \lor p) && \text{[Definition } (p \implies q) \equiv \overline{p} \lor q\text{]}
\end{aligned}
$$

$$
\begin{aligned}
\overline{p \oplus q} &\equiv \overline{(p \land \overline{q}) \lor (\overline{p} \land q)} && \text{[Definition } p \oplus q \equiv (p \land \overline{q}) \lor (\overline{p} \land q)\text{]} \\
&\equiv \overline{(p \land \overline{q})} \land \overline{(\overline{p} \land q)} && \text{[De Morgan's Law: } \overline{p \lor q} \equiv \overline{p} \land \overline{q}\text{]} \\
&\equiv (\overline{p} \lor q) \land (q \lor \overline{p}) && \text{[De Morgan's Law: } \overline{p \land q} \equiv \overline{p} \lor \overline{q}\text{]}
\end{aligned}
$$

$$\therefore p \iff q \equiv \overline{p \oplus q}$$

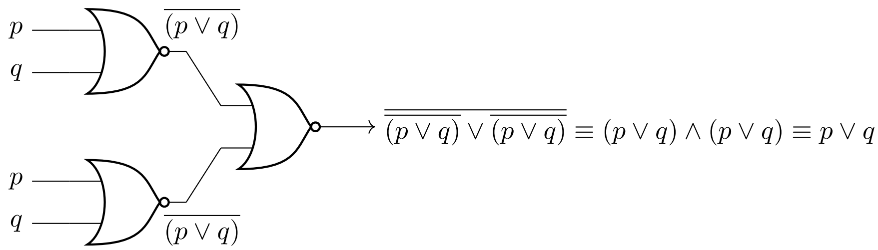
(b)

$$
\begin{aligned}
\overline{p} &\equiv (\overline{p \lor \overline{q}}) \lor (\overline{p} \land \overline{q}) && \\
&\equiv (\overline{p} \land q) \lor (\overline{p} \land \overline{q}) && \text{[De Morgan's Law: } \overline{p \lor q} \equiv \overline{p} \land \overline{q}\text{]} \\
&\equiv \overline{p} \land (q \lor \overline{q}) && \text{[Distributive Law: } p \land (q \lor r) \equiv (p \land q) \lor (p \land r)\text{]} \\
&\equiv \overline{p} \land 1 && \text{[Tautology: } p \lor \overline{p} \equiv 1\text{]} \\
&\equiv \overline{p} && \text{[Identity Law: } p \land 1 \equiv p\text{]}
\end{aligned}
$$

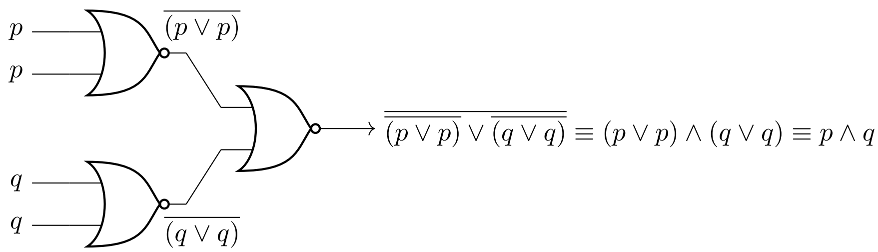
Problem 4

Recall **NOR**: $(p \downarrow q) \equiv \overline{p} \land \overline{q} \equiv \overline{(p \lor q)}$ *as seen in class

(a) OR $(p \lor q)$



(b) AND $(p \land q)$



(c) NOT $(\overline{p})$