

1. Find an equation for the line tangent to the curve at the point defined by the given value of  $t$ .

(a).  $x = \sec^2 t - 1$        $y = \tan t$        $t = -\frac{\pi}{4}$

(b).  $x = -\sqrt{t+1}$        $y = \sqrt{3t}$        $t = 3$

2. Find  $dy$

(a).  $y = x + 2x^2 - \frac{1}{3}x^3 + x^4$

(b).  $y = x \cdot \cot x$

3. The curve  $L : y = y(x)$  is defined by the equation  $y^3 + y^2 = 2x^2$ . Find the equation for the line tangent to  $L$  at  $(1, 1)$ .

4. Let  $f(x)$  be a differentiable function on  $\mathbb{R}$ ,  $g(x) = f(x^2)$ ,  $h(x) = f(x)^2$ , and  $\frac{d}{dx}g(x) = \frac{d}{dx}h(x)$  when  $x = 1$ , prove that:  $f'(1) = 0$  or  $f(1) = 1$ .

5. Let  $f(x) = \frac{x^2 - 2}{x^2 - x - 2}$ , find  $f^{(n)}(x)$ .

6. Let  $f(x)$  be a twice-differentiable function on  $[-2, 2]$ , and  $|f(x)| < 1$ ,  $f(0)^2 + f'(0)^2 = 4$ .

Prove: There exist  $\lambda \in [-2, 2]$ , such that  $f(\lambda) + f'(\lambda) = 0$ .

Hint: Consider  $F(x) = f(x) \cdot \sin x + f'(x) \cdot \cos x$ , and use the first derivative theorem for local extreme values.