Calculus A(1): Homework 1

Each assigned exercise is worth 20 points. When we refer to a paragraph number (e.g. §1.3), we refer to the PDF of the textbook *Thomas Calculus* that you can find on the weblearn. The bonus exercises are optional and more difficult. We may (or may not) decide to grade one of your bonus exercises and use it to replace one assigned exercise (if it improves your total grade).

Routine exercises (do not hand-in)

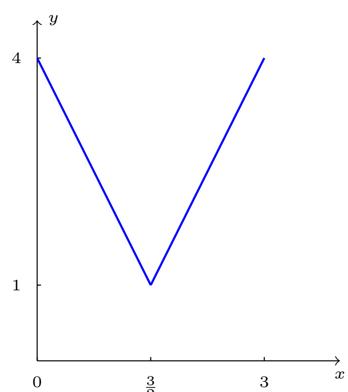
1. §1.1 Exercises 15, 27, 39, 41, 46, 48
2. §1.2 Exercises 9, 12, 18, 37
3. §1.3 Exercises 2, 6, 7, 10, 19, 23, 34
4. Sketch the graphs of the following power functions: $x^{1/3}$, $x^{2/3}$, $x^{1/4}$.

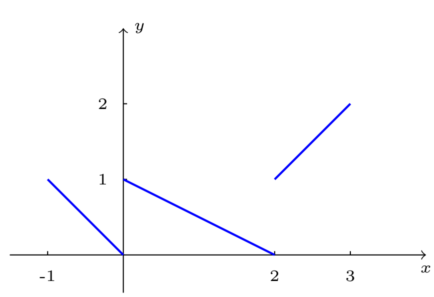
Assigned exercises (hand-in)

1. Graph the following equations and explain why they are not graphs of a function of x

$$\textbf{a}.\ \mid x \mid + \mid y \mid = 2 \qquad \textbf{b}.\ \mid x - y \mid = 1$$

2. Find a formula (possibly piecewise defined) for the functions whose graphs are given as follows:





3. Find all $x \in \mathbb{R}$ such that $\sqrt{(x-10)(x-7)} = \sqrt{30-4x}$.
4. Prove that for any $a, b \in \mathbb{R}$, we have $\mid a-b \mid \geq \Big| \mid a \mid - \mid b \mid \Big|$. Give an example (of values of a and b) such that the inequality is strict.
5. Prove that every function $f : \mathbb{R} \to \mathbb{R}$ can be expressed as the sum of an odd function and an even function. This means there exists $g : \mathbb{R} \to \mathbb{R}$ odd and $h : \mathbb{R} \to \mathbb{R}$ even such that $f = g + h$.

Bonus exercises (optional)

1. Prove that for any $a, b \in \mathbb{R}$ with $a, b \geq 0$, we have $\frac{a+b}{2} \geq \sqrt{ab}$.
2. Prove that $\sqrt{2}$ is not in $\mathbb{Q}$, that is there does not exist $x \in \mathbb{Q}$ such that $x^2 = 2$.
3. Prove that for any $a, b \in \mathbb{R}$ with $a < b$, there exists $x \in \mathbb{Q}$ such that $a < x < b$. You can use the *Archimedean property of* $\mathbb{R}$: for all $y \in \mathbb{R}$, there exists $n \in \mathbb{N}$ such that $n > y$.
4. (Hard) Find the functions $f : \mathbb{R} \to \mathbb{R}$, such that

$$\forall x \in \mathbb{R}, \quad f(x) + xf(1-x) = 1 + x$$