1. Find the definite integrals

a. $\int_{-2}^{2} x^3 \cdot \cos x + x^2 \, dx$

b. $\int_{3}^{1} x^2 - x + 1 \, dx$

c. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \cos x} \, dx$

d. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \csc x \cot x \, dx$

e. $\int_{0}^{\pi} \frac{1}{2}(\cos x + |\cos x|) \, dx$

f. $\int_{0}^{\frac{\pi}{3}} 4 \sec x \tan x \, dx$

2. Find the limits as follows

a. $\lim_{n \to \infty} \frac{1}{n^2} \cdot \sum_{k=1}^{n} k^2$

b. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{n}{n^2 + 2nk + k^2}$

3. Let f be continuous on $[a, b]$, $F(x) = \int_a^x f(t)(x - t) \, dt$,

prove that $F''(x) = f(x) \qquad x \in [a, b]$

4. Let f be continuous on $[0, +\infty)$, $\lim_{x \to +\infty} f(x) = A \in \mathbb{R}$, prove that:

$$\lim_{x \to +\infty} \frac{1}{x} \int_0^x f(t) \, dt = A$$