Calculus A(1): Homework 7

The total is 100 points. When we refer to a paragraph number (e.g. §5.6), we refer to the PDF of the textbook *Thomas Calculus* that you can find on the weblearn. The bonus exercises are optional and more difficult. We may (or may not) decide to grade one of your bonus exercises and use it to replace one assigned exercise (if it improves your total grade).

Routine exercises (do not hand-in)

§4.7, Exercise 3, 6, 11
§4.8, Exercise 1, 4, 6, 11, 16, 20, 36, 55
§5.1, Exercise 4, 17
§5.2, Exercises 1, 4, 9, 17, 25, 38
§5.3, Exercises 1, 9, 11, 15, 18, 21, 22, 30, 59, 65, 69, 71, 73
§5.4, Exercises 8, 20, 26, 27, 34, 61
§5.5, Exercises 2, 4, 10, 50
§5.6, Exercises 8, 12, 24, 30, 33, 73, 80, 81, 84
p. 394, Exercise 23

Assigned exercises (hand-in)

1. (10pts) Using L'Hopital's rule, find the following limits.

   a. $\lim_{x\to 0} \frac{x\sin(x)}{x^3+2x^2+\sin(x)}$ **b.** $\lim_{x\to 2} \frac{x^2-(2x+1)\sqrt{2x}+6}{x-2}$

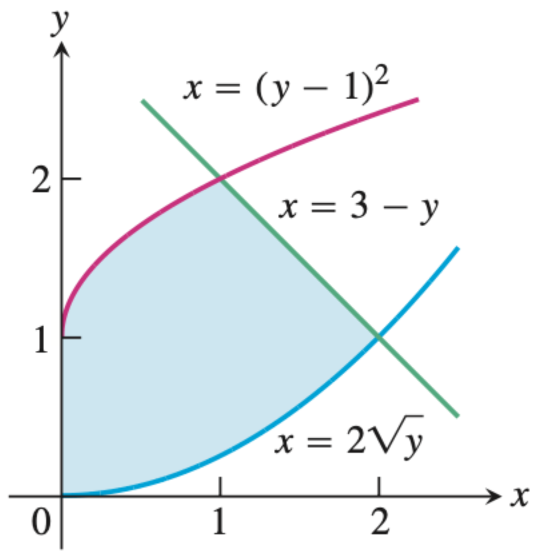
2. (10pts) Compute the following definite integral:

$$\int_0^3 \sqrt{9-x^2}\mathrm{d}x$$

3. (10pts) Compute the derivative of $f:[0,1]\to\mathbb{R}$ given by $f(x)=\int_0^{x^3}\frac{\mathrm{d}t}{t^2+1}$.

4. (10pts) What values of a and b (with $a \leq b$) minimize the value of the definite integral $\int_a^b (x^2-2x)\mathrm{d}x$? (Hint: where is the function x^2-2x positive or negative?)

5. (20pts) (This is Exercise 78 of §5.6) Find the area of the blue region below:



6. (20pts) Compute the following indefinite integrals.

a. $\displaystyle\int \frac{1}{\cos^2(x)\sin^2(x)}\mathrm{d}x$ **b.** $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x\cdot\cos^3(\sqrt{x})}}\mathrm{d}x$

7. (20pts) Let $a > 0$ and $f : [0, a] \to \mathbb{R}$ be continuous. Find the value of the following definite integral:

$$I = \int_0^a \frac{f(x)}{f(x) + f(a-x)}\mathrm{d}x\ .$$

(Hint: Make the substitution $u = a - x$.)

Bonus exercises (optional)

1. Let $f : [a, b] \to \mathbb{R}$ be a continuous function, and let $F : [a, b] \to \mathbb{R}$ be an anti-derivative of f.

a. Let $a = x_0 < x_1 < ... < x_n = b$ be a partition of $[a, b]$. Show that

$$F(b) - F(a) = \sum_{i=1}^{n} (F(x_i) - F(x_{i-1}))\ .$$

b. Use the Mean Value Theorem for derivatives to show that $\sum_{i=1}^{n}(F(x_i) - F(x_{i-1}))$ is a Riemann sum for f (corresponding the the above partition).

c. Use the result of **b.** to prove that

$$F(b) - F(a) = \int_a^b f(x)dx\ .$$

(You are not allowed for this question to use the fundamental theorem of calculus, but you can use the fact that f is integrable, since f is continuous.)