1. (a) (12 points) Find all solutions to the system of linear equations:

$$\begin{aligned} x_3 - \ x_4 - \ x_5 &= 4 \\ x_1 + 2x_2 + \ x_3 + 2x_4 + \ x_5 &= 2 \\ 2x_1 + 4x_2 + 3x_3 + 3x_4 + 3x_5 &= 4 \\ x_1 + 2x_2 + 2x_3 + \ x_4 + 2x_5 &= 2 \end{aligned}$$

(b) (3 points) Find the reduced row echelon form R of the coefficient matrix for the system of equations.

(a) Solve using elimination:

$$\left[\begin{array}{ccccc|c} 0 & 0 & 1 & -1 & -1 & 4 \\ 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 3 & 3 & 4 \\ 1 & 2 & 2 & 1 & 2 & 2 \end{array}\right] \xrightarrow{R1 \leftrightarrow R2} \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & -1 & -1 & 4 \\ 2 & 4 & 3 & 3 & 3 & 4 \\ 1 & 2 & 2 & 1 & 2 & 2 \end{array}\right] \xrightarrow[R4-R1]{R3-2R1}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & -1 & -1 & 4 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right] \xrightarrow[R4-R2]{R3-R2} \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & -1 & -1 & 4 \\ 0 & 0 & 0 & 0 & 2 & -4 \\ 0 & 0 & 0 & 0 & 2 & -4 \end{array}\right] \xrightarrow[\text{Then: } \frac{1}{2}R3]{R4-R3}$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & -1 & -1 & 4 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R1-R3 \quad R2+R3]{\text{Eliminate everything above leading 1's:}} \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R1-R2}$$

$$\underbrace{\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]}_{\text{RREF}} \longrightarrow \begin{array}{l} x_1 + 2x_2 + 3x_4 = 2 \\ x_3 - x_4 = 2 \\ x_5 = -2 \\ x_2, x_4 \text{ free} \end{array} \longrightarrow \begin{array}{l} x_1 = 2 - 2x_2 - 3x_4 \\ x_3 = 2 + x_4 \\ x_5 = -2 \\ x_2, x_4 \text{ free} \end{array}$$

(a) $$\begin{bmatrix} x_1 \\ x_2 \\ x_7 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 - 2x_2 - 3x_4 \\ x_2 \\ 2 + x_4 \\ x_4 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

(b) $$R = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. (a) (7 points) Consider the vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ in $\mathbb{R}^n$. Express the angle θ between $\mathbf{u}$ and $\mathbf{v}$ in terms of n, and find θ explicitly for $n = 2$ and $n = 4$.

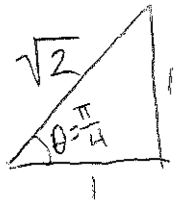
(b) (7 points) Find scalars a, b, c, d, e, f, g such that

$$\begin{bmatrix} a \\ d \\ f \end{bmatrix}, \quad \begin{bmatrix} b \\ 1 \\ g \end{bmatrix}, \quad \begin{bmatrix} c \\ e \\ 1/2 \end{bmatrix}$$

are orthogonal unit vectors, that is, every vector has length 1 and every two vectors are perpendicular to each other.

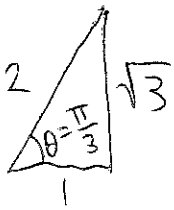
(a) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \dfrac{1(1)+1(0)+\dots+1(0)}{\sqrt{1^2+1^2+\dots+1^2}\sqrt{1^2+0^2+\dots+0^2}} = \dfrac{1}{\sqrt{n}}$

So $\boxed{\theta = \cos^{-1}\frac{1}{\sqrt{n}}}$ $n=2$:



$\rightsquigarrow \boxed{\theta = \frac{\pi}{4} \text{ or } 45^\circ}$

If $n=4$: $\theta = \cos^{-1}\frac{1}{2}$,



$\rightsquigarrow \boxed{\theta = \frac{\pi}{3} \text{ or } 60^\circ}$

(b) Look at 2nd vector first: Need $\sqrt{b^2+1^2+g^2} = 1$, so need $b=g=0$.

Then need $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} c \\ e \\ 1/2 \end{bmatrix} = 0 \rightsquigarrow e=0$, and then $\sqrt{c^2+0^2+\left(\frac{1}{2}\right)^2} = 1$

$\rightarrow c^2 = \frac{3}{4} \rightarrow c = \pm\frac{\sqrt{3}}{2}$

We also need $\begin{bmatrix} a \\ d \\ f \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0 \rightsquigarrow d=0$, and then $\begin{bmatrix} a \\ 0 \\ f \end{bmatrix} \cdot \begin{bmatrix} \pm\sqrt{3}/2 \\ 0 \\ 1/2 \end{bmatrix} = 0$

$\rightsquigarrow \pm\frac{\sqrt{3}}{2}a + \frac{1}{2}f = 0 \rightarrow f = \mp\sqrt{3}\,a$

Then $\sqrt{a^2+0^2+(\mp\sqrt{3}\,a)^2} = 1 \rightsquigarrow 4a^2 = 1 \rightarrow a = \pm\frac{1}{2}$

In conclusion, we get $b=d=e=g=0$,

four possible solutions for a, c, f:

$$c=\frac{\sqrt{3}}{2},\ a=\frac{1}{2},\ f=-\frac{\sqrt{3}}{2}$$

$$c=\frac{\sqrt{3}}{2},\ a=-\frac{1}{2},\ f=\frac{\sqrt{3}}{2}$$

$$c=-\frac{\sqrt{3}}{2},\ a=\frac{1}{2},\ f=\frac{\sqrt{3}}{2}$$

$$c=-\frac{\sqrt{3}}{2},\ a=-\frac{1}{2},\ f=-\frac{\sqrt{3}}{2}$$

3. (a) (5 points) Find a *nonzero* 2×2 matrix A such that $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

(b) (8 points) Find *all* invertible 2×2 matrices S such that $S^{-1}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} S = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

(a) Easy examples come from upper/lower triangular:

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix},\ a \neq 0: \quad \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0\cdot a + a\cdot 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A = \begin{bmatrix} 0 & 0 \\ b & 0 \end{bmatrix}$, $b \neq 0$, also works.

(b) If $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible, then $S^{-1}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} S =$

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} -b & d \\ a & -c \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \frac{1}{ad-bc}\begin{bmatrix} -ab+cd & -b^2+d^2 \\ a^2-c^2 & ab-cd \end{bmatrix} \overset{??}{=} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Need $b^2 = d^2 \rightarrow b = \pm d$

Need $a^2 = c^2 \rightarrow a = \pm c$

Need $\dfrac{ab-cd}{ad-bc} = -1 \rightarrow ab - cd = bc - ad$

$ab - (\pm a)(\pm b) = b(\pm a) - a(\pm b)$

$++ : 0 = 0$ ✓

$+- : 2ab = 2ab$ ✓

$-+ : 2ab = -2ab$ ×

$-- : 0 = 0$ ✓

$S = \cancel{\begin{bmatrix} a & b \\ +a & +b \end{bmatrix}}, \boxed{\begin{bmatrix} a & b \\ a & -b \end{bmatrix}}, \cancel{\begin{bmatrix} a & b \\ -a & -b \end{bmatrix}}$

not invertible

invertible if $a \neq 0, b \neq 0$

not invertible

Solution for S

4. (a) (10 points) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 7 & 14 \end{bmatrix}$.

(b) (4 points) Use the result from part (a) to solve the equation $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(a) $$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 7 & 0 & 1 & 0 \\ 3 & 7 & 14 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R3-3R1]{R2-2R1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 5 & -3 & 0 & 1 \end{array}\right] \xrightarrow{R2 \leftrightarrow R3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 5 & -3 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right] \xrightarrow[R2-5R3]{R1-3R3} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 7 & -3 & 0 \\ 0 & 1 & 0 & 7 & -5 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right] \xrightarrow{R1-2R2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7 & 7 & -2 \\ 0 & 1 & 0 & 7 & -5 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 \end{array}\right] \longrightarrow \boxed{A^{-1} = \begin{bmatrix} -7 & 7 & -2 \\ 7 & -5 & 1 \\ -2 & 1 & 0 \end{bmatrix}}$$

(b) $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -7 & 7 & -2 \\ 7 & -5 & 1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}$$

So $$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix}}$$

5. (a) (7 points) Find the LU decomposition of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}$.

(b) (7 points) Use the LU decomposition of A to solve $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$.

(a) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix} \xrightarrow[\substack{R3 - \boxed{1} R1 \\ R4 - \boxed{1} R1}]{R2 - \boxed{1} R1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 9 \\ 0 & 3 & 9 & 19 \end{bmatrix} \xrightarrow[R4 - \boxed{3} R2]{R3 - \boxed{2} R2}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 3 & 10 \end{bmatrix} \xrightarrow{R4 - \boxed{3} R3} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & +2 & 1 & 0 \\ 1 & +3 & 3 & 1 \end{bmatrix}$

So $\boxed{A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$

(b) $A\vec{x} = \vec{b} \rightsquigarrow L(\underbrace{U\vec{x}}_{\vec{y}}) = \vec{b}$ Solve: $\begin{cases} L\vec{y} = \vec{b} \\ U\vec{x} = \vec{y} \end{cases}$

$L\vec{y} = \vec{b}$ first: $y_1 = 1$

$y_1 + y_2 = 0 \longrightarrow y_2 = -1$

$y_1 + 2y_2 + y_3 = 1 \to y_3 = 1 - (1) - 2(-1) = 2$

$y_1 + 3y_2 + 3y_3 + y_4 = 0 \to y_4 = -(1) - 3(-1) - 3(2) = -4$

$\Longrightarrow \vec{y} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -4 \end{bmatrix}$

Now solve $U\vec{x} = \vec{y}$:

$$x_1 + x_2 + x_3 + x_4 = 1 \rightarrow x_1 = 1 - (-17) - 14 - (-4) = 8$$

$$x_2 + 2x_3 + 3x_4 = -1 \rightarrow x_2 = -1 - 2(14) - 3(-4) = -17$$

$$x_3 + 3x_4 = 2 \rightarrow x_3 = 2 - 3(-4) = 14$$

$$x_4 = -4$$

so $$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ -17 \\ 14 \\ -4 \end{bmatrix}}$$

6. (a) (9 points) Let S be the set of all vectors $\mathbf{x}$ in $\mathbb{R}^3$ such that $\mathbf{x}$ is orthogonal (i.e., perpendicular) to $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Show that S is a subspace of $\mathbb{R}^3$, and find a spanning set for S.

(b) (6 points) Let $\mathbb{R}^{2\times 2}$ be the vector space of all 2×2 matrices, and let T be the set of all *non*-invertible matrices in $\mathbb{R}^{2\times 2}$. Determine whether T is a subspace of $\mathbb{R}^{2\times 2}$.

(a) Note $\vec{x} \perp \vec{v} \Leftrightarrow x \cdot \vec{v} = 0$, so $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \vec{x} \cdot \vec{v} = 0 \right\}$

(1) Does S contain $\vec{0}$? Yes, because $\vec{0} \cdot \vec{v} = 0$.

(2) Is S closed under addition? Yes, because if $\vec{x}_1, \vec{x}_2$ are in S, then $(\vec{x}_1 + \vec{x}_2) \cdot \vec{v} = \vec{x}_1 \cdot \vec{v} + \vec{x}_2 \cdot \vec{v} = 0 + 0 = 0$

(3) Is S closed under scalar multiplication? Yes, because if $\vec{x}$ is in S, then $(c\vec{x}) \cdot \vec{v} = c(\vec{x} \cdot \vec{v}) = c(0) = 0$.

So S is a subspace.

For spanning set: Note $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$ means $x_1 + 2x_2 + 3x_3 = 0$,

so $x_1 = -2x_2 - 3x_3 \longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

So $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ would be a spanning set of S.

(b) T is <u>not</u> a subspace because it isn't closed under addition, for example: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are both non-invertible,

but $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ <u>is</u> invertible, so is not in T,

7. (15 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 5 & 4 & 3 & 2 \\ 1 & 6 & 6 & 6 & 6 \\ 1 & 7 & 8 & 10 & 12 \\ 1 & 6 & 6 & 7 & 8 \end{bmatrix}.$$

(a) Find a linear relation on b_1, b_2, b_3, b_4 that guarantees that $\mathbf{b} = (b_1, b_2, b_3, b_4)$ is a vector in the column space $\mathbf{C}(A)$.

(b) Find a spanning set (the special solutions) for the null space $\mathbf{N}(A)$.

(a)

$$\left[\begin{array}{ccccc|c} 1 & 5 & 4 & 3 & 2 & b_1 \\ 1 & 6 & 6 & 6 & 6 & b_2 \\ 1 & 7 & 8 & 10 & 12 & b_3 \\ 1 & 6 & 6 & 7 & 8 & b_4 \end{array}\right] \xrightarrow[\substack{R3-R1 \\ R4-R1}]{R2-R1} \left[\begin{array}{ccccc|c} 1 & 5 & 4 & 3 & 2 & b_1 \\ 0 & 1 & 2 & 3 & 4 & -b_1+b_2 \\ 0 & 2 & 4 & 7 & 10 & -b_1+b_3 \\ 0 & 1 & 2 & 4 & 6 & -b_1+b_4 \end{array}\right] \xrightarrow[R4-R2]{R3-2R2}$$

$$\left[\begin{array}{ccccc|c} 1 & 5 & 4 & 3 & 2 & b_1 \\ 0 & 1 & 2 & 3 & 4 & -b_1+b_2 \\ 0 & 0 & 0 & 1 & 2 & b_1-2b_2+b_3 \\ 0 & 0 & 0 & 1 & 2 & -b_2+b_4 \end{array}\right] \xrightarrow{R4-R3} \left[\begin{array}{ccccc|c} 1 & 5 & 4 & 3 & 2 & b_1 \\ 0 & 1 & 2 & 3 & 4 & -b_1+b_2 \\ 0 & 0 & 0 & 1 & 2 & b_1-2b_2+b_3 \\ 0 & 0 & 0 & 0 & 0 & -b_1+1b_2-b_3+b_4 \end{array}\right]$$

This needs to be 0 for $\vec{b}$ to be in $C(A)$, so the linear relation is $\boxed{-b_1+b_2-b_3+b_4=0}$

(b) To find $N(A)$, set $b_1=b_2=b_3=b_4=0$:

$$\left[\begin{array}{ccccc|c} 1 & 5 & 4 & 3 & 2 & 0 \\ 0 & 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R2-3R3]{R1-3R3} \left[\begin{array}{ccccc|c} 1 & 5 & 4 & 0 & -4 & 0 \\ 0 & 1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R1-5R2}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -6 & 0 & 6 & 0 \\ 0 & 1 & 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \begin{array}{l} x_1 = 6x_3 - 6x_5 \\ x_2 = -2x_3 + 2x_5 \\ x_4 = -2x_5 \\ x_3, x_5 \text{ free} \end{array} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} 6 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -6 \\ 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

This is the spanning set for $N(A)$