



有事大量 Homework II Solutions

5.1.2 A 1s a 3×3 motrix,
$$det(A) = -1$$
 $det(\pm A) = (\pm 1)^3 det(A) = \pm (-1) = -\frac{1}{8}$
 $det(-A) = (-1)^3 det(A) = (-1)(-1) = 1$
 $det(A^2) = det(A) det(A) = (-1)(-1) = 1$
 $det(A^{-1}) = \frac{1}{det(A)} = \frac{1}{-1} = -1$

5.1.7 Rotation:
$$|\cos \theta - \sin \theta| = \cos^2 \theta - (-\sin \theta)(\sin \theta)$$

$$= \cos^2 \theta + \sin^2 \theta = 1$$

Reflection =
$$|1-2\cos^2\theta|$$
 - $2\sin\theta\cos\theta$ = $|-2\cos\theta\sin\theta|$ | - $2\sin^2\theta$

$$(1-2\cos^2\theta)(1-2\sin^2\theta)-(-2\cos\theta\sin\theta)^2=$$

$$1-2\cos^{2}\theta-2\sin^{2}\theta+4\cos^{2}\theta\sin^{2}\theta-4\cos^{2}\theta\sin^{2}\theta=$$

$$1-2(\cos^{2}\theta+\sin^{2}\theta)=1-2=-1$$



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(b)
$$E_1 = 1$$
, $E_2 = 0$, $E_3 = E_2 - E_1 = -1$, $E_4 = E_3 - E_2 = -1$
 $E_5 = E_4 - E_3 = 0$, $E_6 = E_5 - E_4 = 1$, $E_7 = E_6 - E_5 = 1$
 $E_8 = E_7 - E_6 = 0$

NOTE (c) Notice $E_7 = E_1$ and $E_8 = E_2$ 50 the values will repeat in cycles of $6 = 1, 0, -1, -1, 0, 1, \dots$ $E_n = E_{n-6}$ for any $n = 1, 0, -1, -1, 0, 1, \dots$

So E100=E94=E88=E82=--==E10=E4=-1

 $\frac{5.2.19}{V_{4} = -1} \left(a \right) \text{ Expand across 4th row} = \frac{5.2.19}{V_{4} = -1} \left(a^{2} a^{3} a^{3} + x \right) \left(a^{2} a^{3} a^{3} a^{3} \right) \left(a^{2} a^{3} a^{3} a^{3} \right) \left(a^{2} a^{3} a$

The 3x3 determinants are just numbers, so V4 is a V3 cubic polynomial in X.

- (b) $V_4 = 0$ when x=a,b,c because then two rows of the motrix will be the same, so determinant will be 0. So $r_1=a$, $r_2=b$, $r_3=c$.
- (c) Any cubic polynomial can be factored as $A(x-r_1)(x-r_2)(x-r_3)$ where A is the coefficient of x^3 and r_1, r_2, r_3 are the three roots (which could be complex numbers in general). For V_4 , $V_4 = V_3(x-p)(x-b)(x-c)$ from parts (a) and (b)



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(d) Using formula for V_3 from 5.1.18, $V_4 = (b-a)(c-a)(c-b)(x-a)(x-b)(x-c)$

== (1) = - (1) () = - (1) () = - (1) () = - (1) ()

For "big formula," det P=sum of 24 terms ± p1i, p2i, p3i, P4, i4

The only non-zero term chooses

Col 4 for Row 1, Col 1 for Row 2, Col 2 for

different column

for each row

Row 3, and Col 3 for Row 4.

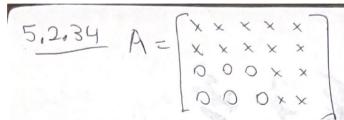
Then det P = ± (1)(1)(1)(1) + a bunch of 0's

1 (-1) # row switches to reorder 4,1,2,3 to 1,2,3,4

we condo 4,1,2,3 → 1,4,2,3 → 1,2,4,3 → 1,2,3,4 (3 row switches.

50 det P = (-1)3 (1)(1)(1)(1) = -1, same as before.

Then $det(P^2) = (det P)(det P) = (-1)(-1) = +1$ \$



6

(a) MM The last 3 rows are all contained in Span ([0,0,010], [0,0001]), which is 2 dimensional, So one of the last three rows has to be a linear combination of the other 2.

(b) Each of the 120 terms in the big formula looks like ± a, i, a2, i2 a3, i3 a4, i4 a5, i5, where Columns i, i2, i3, i4, is are all different. Support term For this to be non-zero, we'd have to choose either Col 4 or 5 for Row 3, and then the other one of Col 4 or 5 for Row 4. But then we have to choose one of Cols 1,2,3 for Row 5, 50 a5, i5=0 ~9 the whole





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Graded Problem 2
$$\begin{vmatrix} 1-1 & 1-1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & 0 & 0 & 0 \\ 4 & 0 & -2 & 0 & 1 \\ 0 & -2 & 0 & 2 & 0 \end{vmatrix}$$
 $\begin{vmatrix} 1 & 1-1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 3 & 4 & 5 \\ 4 & -2 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{vmatrix}$

$$=-2(13+19-14)=-36$$