

# LINEAR ALGEBRA – HOMEWORK 4

18 October, 2023  
Due: 26 October, 2023

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**Textbook Problems.** These problems will not be graded, but you must submit solutions to receive full credit for the homework.

**Problem 2.3.9.**

- (a) The elimination matrix  $E_{21}$  subtracts row 1 from row 2, and then  $P_{23}$  exchanges rows 2 and 3. What matrix  $M = P_{23}E_{21}$  does both steps at once?
- (b) The elimination matrix  $P_{23}$  exchanges rows 2 and 3, and then  $E_{31}$  subtracts row 1 from row 3. What matrix  $M = E_{31}P_{23}$  does both steps at once? Explain why the  $M$ 's are the same but the  $E$ 's are different.

**Problem 2.3.12.** Multiply these matrices:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \\ 1 & 4 & 0 \end{bmatrix}.$$

**Problem 2.3.17.** The parabola  $y = a + bx + cx^2$  goes through the points  $(x, y) = (1, 4)$  and  $(2, 8)$  and  $(3, 14)$ . Find and solve a matrix equation for the unknowns  $(a, b, c)$ .

**Problem 2.3.28.** If  $AB = I$  and  $BC = I$ , use the associative law to prove  $A = C$ .

**Problem 2.4.6.** Show that  $(A + B)^2$  is different from  $A^2 + 2AB + B^2$ , when

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$

Write down the correct rule for  $(A + B)(A + B) = A^2 + \underline{\hspace{2cm}} + B^2$ .

**Problem 2.4.15.** True or false:

- (a) If  $A^2$  is defined, then  $A$  has to be square.
- (b) If  $AB$  and  $BA$  are both defined, then  $A$  and  $B$  have to be square.
- (c) If  $AB$  and  $BA$  are both defined, then  $AB$  and  $BA$  are both square.
- (d) If  $AB = B$ , then  $A$  has to be the identity matrix  $I$ .

**Problem 2.4.18.** Write down the  $3 \times 3$  matrices whose entries are:

- (a)  $a_{ij} = \text{minimum of } i \text{ and } j$ .
- (b)  $a_{ij} = (-1)^{i+j}$ .
- (c)  $a_{ij} = i/j$ .

**Problem 2.4.21.** Compute  $A^2$ ,  $A^3$ ,  $A^4$ , and also  $A\mathbf{v}$ ,  $A^2\mathbf{v}$ ,  $A^3\mathbf{v}$ ,  $A^4\mathbf{v}$  for

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}.$$

**Problem 2.4.26.** Multiply  $AB$  using columns times rows:

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}.$$

**Problem 2.4.32.** Suppose you solve  $A\mathbf{x} = \mathbf{b}$  for three special right sides  $\mathbf{b}$ :

$$A\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad A\mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad A\mathbf{x}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

If the three solutions  $\mathbf{x}_1$ ,  $\mathbf{x}_2$ ,  $\mathbf{x}_3$  are the columns of a matrix  $X$ , what is  $A$  times  $X$ ?

### Graded Problems.

**Problem 1.** We say that two square matrices  $A$  and  $B$  *commute* if  $AB = BA$ . Find all  $2 \times 2$  matrices  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  that commute with the matrix  $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ . Also find all  $2 \times 2$  matrices that commute with  $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ . What matrices commute with *both*  $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ ?

**Problem 2.** Multiply the matrices:

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 1 & 1 \\ -1 & 0 \end{bmatrix}.$$