Calculus A(1): Homework 2

The total is 100 points. When we refer to a paragraph number (e.g. §1.6), we refer to the PDF of the textbook *Thomas Calculus* that you can find on the weblearn. The bonus exercises are optional and more difficult. We may (or may not) decide to grade one of your bonus exercises and use it to replace one assigned exercise (if it improves your total grade).

Routine exercises (do not hand-in)

1. §1.6 Exercises 13, 16, 54
2. §2.1 Exercises 1
3. §2.2 Exercises 16, 48, 53
4. §2.3 Exercises 9, 20, 45, 48
5. §2.4 Exercises 10, 16, 17, 22, 25, 33, 43, 46, 50, 51, 63, 68, 76, 78

Assigned exercises (hand-in)

1. (10 pts) Find the function values as follows.

$$\textbf{a}.\ \left(\sin(\frac{\pi}{8})\right)^2 \qquad\qquad \textbf{b}.\ \left(\cos(\frac{\pi}{12})\right)^2$$

2. (15 pts) (**The law of sines**) If a, b, and c are the sides opposite the angles A, B, and C in a triangle, show that

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

3. (15pts) Evaluate the limit as follows.

$$\textbf{a}.\ \lim_{x\to -5}\frac{x^2+3x-10}{x+5} \qquad \textbf{b}.\ \lim_{x\to 4}\frac{4x-x^2}{2-\sqrt{x}} \qquad \textbf{c}.\ \lim_{x\to -3}\frac{2-\sqrt{x^2-5}}{x+3}$$

4. (20pts) Prove that: $\lim_{x\to\infty}(\sin(\sqrt{x+1})-\sin(\sqrt{x-1}))=0$.

5. (20pts) Let $f:\mathbb{R}\to\mathbb{R}$ be such that $\lim_{x\to+\infty} f(x)=L\in\mathbb{R}$. Let $g:\mathbb{R}\to\mathbb{R}$ be defined by $g(x)=f(x+1)-f(x)$ for all $x\in\mathbb{R}$. Prove that $\lim_{x\to+\infty} g(x)$ exists and find it.

6. (20pts) $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ are two periodic functions with the same period $p>0$ and such that

$$\lim_{x\to+\infty}(f(x)-g(x))=0$$

Prove that for all $x\in\mathbb{R}$, we have $f(x)=g(x)$.

Bonus exercises (optional)

1. Evaluate the limit :
$$\lim_{x\to 0}\frac{(1+x)^{1/6}-1}{x}.$$

2. (Hard) Let $a\in\mathbb{R}$ and let $f:(a,+\infty)\to\mathbb{R}$ be a function. Assume that for any $b>a$, f is bounded on (a,b) (this means there exists $M>0$ such that for all $x\in(a,b)$, $\mid f(x)\mid\leq M$). Assume furthermore that $\lim\limits_{x\to+\infty}(f(x+1)-f(x))=A$. Prove that $\lim\limits_{x\to+\infty}\frac{f(x)}{x}=A$.