1. Evaluate the limits as follows: (They might be infinite limits)

a. $\lim\limits_{x\to 0} \frac{\sin(x^3)}{(\sin x)^2}$

b. $\lim\limits_{x\to 2^+} \frac{[x]^2-4}{x^2-4}$

c. $\lim\limits_{x\to 1} \frac{x+x^2+\cdots+x^m-m}{x-1}$ $(m\in \mathbb{N}^*)$

d. $\lim\limits_{x\to 3} \frac{x^3-2x^2+x-12}{x^4-3x^3-8x^2+21x+9}$

2. Find asymptotes of the following function:

a. $f(x) = \frac{3x^3+4x^2-x+2}{2x^2+x+7}$

3. Let f, g be two continuous functions on $\mathbb{R}$, let

$F(x) = \max(f(x), g(x))$ $\quad$ $G(x) = \min(f(x), g(x))$

Prove that: F and G are continuous functions on $\mathbb{R}$.

4. Let $D(x)$ be a function on $\mathbb{R}$, $D(x) = \begin{cases} 0 & x \text{ is irrational} \\ 1 & x \text{ is rational} \end{cases}$

Prove that: for all $x_0\in\mathbb{R}$, $D(x)$ is not continuous at x_0.
($D(x)$ is called Dirichlet function)

5. Let f be a continuous function on $[a,b]$, $x_i\in[a,b]$, $i=1,2,\cdots,n$.
Show that there exists $\eta\in[a,b]$ such that

$$f(\eta) = \frac{1}{n}[f(x_1)+f(x_2)+\cdots+f(x_n)].$$