Calculus A(1): Homework 5

The total is 100 points. When we refer to a paragraph number (e.g. §3.6), we refer to the PDF of the textbook *Thomas Calculus* that you can find on the weblearn. The bonus exercises are optional and more difficult. We may (or may not) decide to grade one of your bonus exercises and use it to replace one assigned exercise (if it improves your total grade).

Routine exercises (do not hand-in)

1. §3.6, Exercises 10, 18, 25, 43, 67
2. §3.7, Exercises 1, 13, 24
3. §3.8, Exercises 4, 15, 31, 59, 62
4. §4.1, Exercises 4, 10, 23, 54, 61, 70

Assigned exercises (hand-in)

1. (10pts) Find an equation for the tangent line to the curve at the given point.

$$x^2 + xy + y^3 = 7 \quad \text{at } (2, 1).$$

2. (10pts) Find the differential dy.

a. $y = x + 2x^2 - \dfrac{1}{3}x^3 + x^4$ **b.** $y = x \cot x$

3. (20pts) Use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

a. $x^2y + y = \sin(x)$ **b.** $y^2 = \dfrac{\sin(x)}{x}$

c. $x = \tan(y)$ **d.** $y\sin(\dfrac{1}{x}) = 1 - xy$

4. (10pts) Use the approximation $(1+x)^k \approx 1 + kx$ to estimate the following:

a. $(1.0003)^{1000}$ **b.** $105^{\frac{1}{100}}$

5. (15 pts) Among all the rectangles whose diagonal has length equal to 1cm, which rectangle has the largest area? (You need to justify your answer.)

6. (20pts) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x^3 - 12x|$. Find all $c \in \mathbb{R}$ for which $f(x)$ have a **local** extremum at $x = c$. At which $c \in \mathbb{R}$ does $f(x)$ have a **global** extremum at $x = c$?

7. (15pts) Graph the function

$$f(x) = \begin{cases} x\sin(\frac{1}{x}), & x > 0 \\ 0 & x = 0 \end{cases}$$

and explain why $f(x)$ does not have a local extremum at $x = 0$.

Bonus exercises (optional)

1. Let $P(x) = x^n(1-x)^n$ and $n \in \mathbb{N}$. Prove that the nth derivative of P (which we denote by $P^{(n)}$) has n distinct roots in the interval $(0, 1)$ (recall that a **root** of function f is $c \in \mathbb{R}$ such that $f(c) = 0$). (Hint: consider the roots of $P^{(k)}$ for $k = 0, 1, ..., n-1$ and use Rolle's theorem.)