110-1 Final, CSIE, NTPU

Advanced Algorithms (高等演算法)

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1. (15%) Given 4 matrices A_1 (with dimension 35×15), A_2 (15×5), A_3 (5×10), A_4 (10×20), we want to compute the matrix-chain product $A_1A_2A_3A_4$. It is known that different ways to parenthesize the product may need different numbers of scalar multiplications.
   (a) List all possible ways in which we can parenthesize the product $A_1A_2A_3A_4$.
   (b) Find the minimal number of scalar multiplications needed to compute the product $A_1A_2A_3A_4$.
   (c) Find the optimal parenthesizing to minimize the number of scalar multiplications.

35
15
175
35
525
5
2625
3500
8125

2. (10%) Use dynamic programming to find ***all*** Longest Common Subsequence of the two sequences X = (A, B, C, B, D, A) and Y = (B, D, C, A, B, A).

750+ 5250+ 7000 = 1300
1500 + 1000+ 10500 = 13000

3. (15%) For Minimum Spanning Trees,
   (a) Describe Prim's algorithm. (7%)
   (b) Use Prim's algorithm to find (show your steps) a minimum spanning tree for the graph given in Figure 1. (8%)

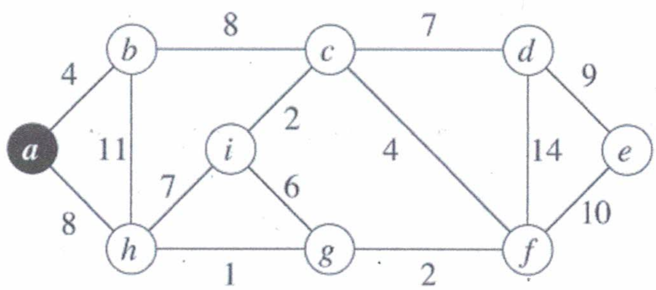

Figure 1

4. (10%) Given an initially empty hash table of size 7 (storage indexed from 0, 1, …, 6), suppose collisions are resolved by **linear probing** with hash function $h(k) = k \bmod 7$. Find the contents of all locations of the hash table after keys 50, 12, 35, 24, 40, 73, 69 are inserted in order.

5. (10%) Given an initially empty hash table of size 13, suppose collisions are resolved by **double hashing** with hash functions $h_1(k) = k \bmod 13$ and $h_2(k) = 1 + (k \bmod 11)$. Find the contents of all locations of the hash table after keys 69, 79, 72, 98, 14 are inserted in order.

6. (10%) Answer the following questions about amortized analysis.
   (a) What is the main idea of **aggregate** method?
   (b) What is the main idea of **accounting** method?

7. (15%) A sequence of n operations is performed on a data structure. The i-th operation costs i if i is an exact power of 2, and 1 otherwise.
   (a) Use worst-case analysis to determine the worst-case cost of an operation. (5%)
   (b) Use **aggregate** method to determine the amortized cost per operation. (5%)
   (c) Use **accounting** method to determine the amortized cost per operation. (5%)

8. (15%) Consider a sequence of n INCREMENT operations on an initially zero counter with k binary bits.
   (a) Use worst-case analysis to determine the worst-case cost of an operation. (5%)
   (b) Use **aggregate** method to determine the amortized cost per operation. (5%)
   (c) Use **potential** method to determine the amortized cost per operation. (5%)