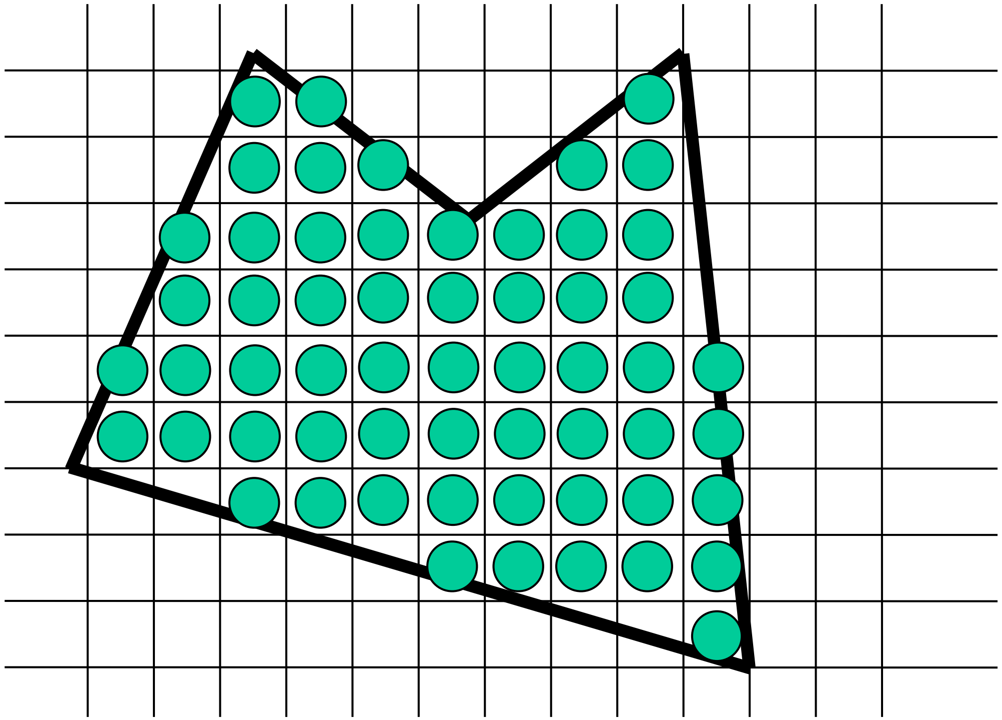
Lecture 7

Rasterizing polygons

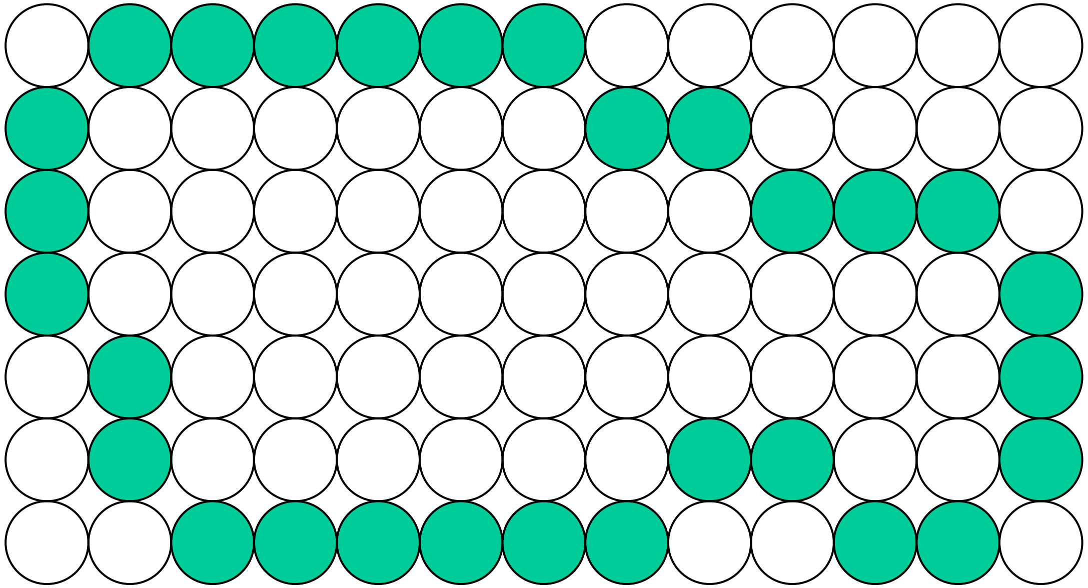
Rasterizing Polygons

Given a set of vertices and edges,
find the pixels that fill the polygon.



Flood Fill

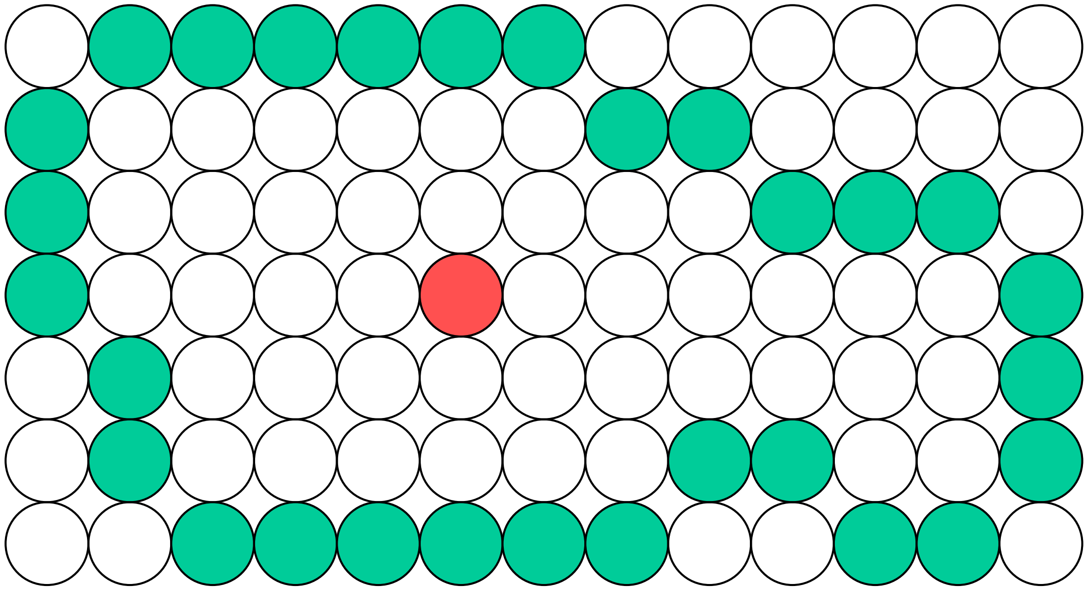
First, rasterizing its edges into the frame buffer using Bresenham’ s algorithm



How to fill polygons whose edges are already drawn?

Flood Fill

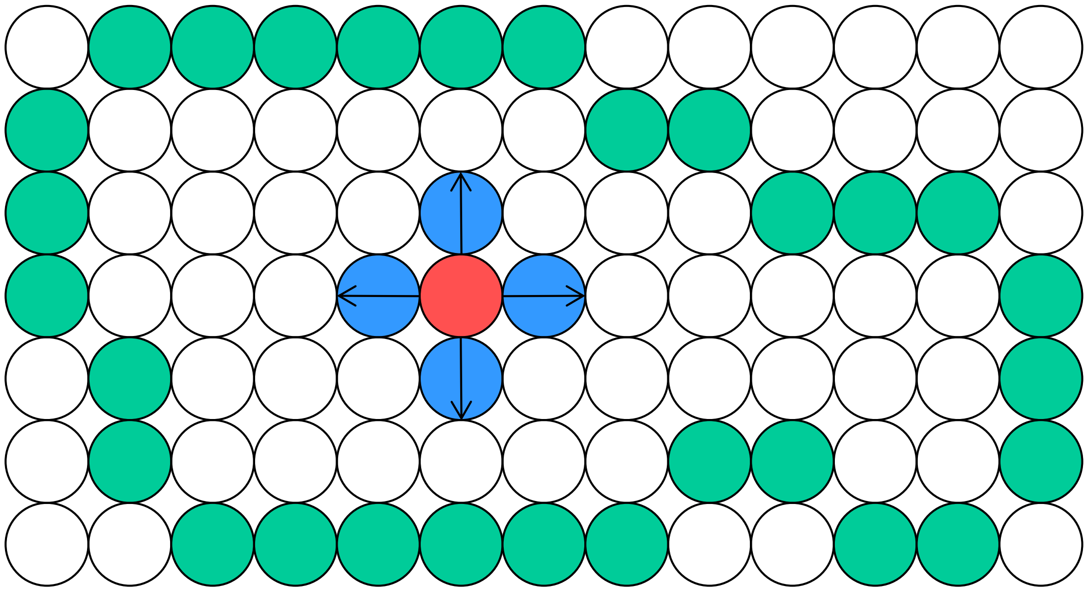
First, rasterizing its edges into the frame buffer using Bresenham’ s algorithm



Choose a point inside, and fill outwards

Flood Fill

First rasterizing its edges into the frame buffer using Bresenham’ s algorithm



Choose a point inside, and fill outwards

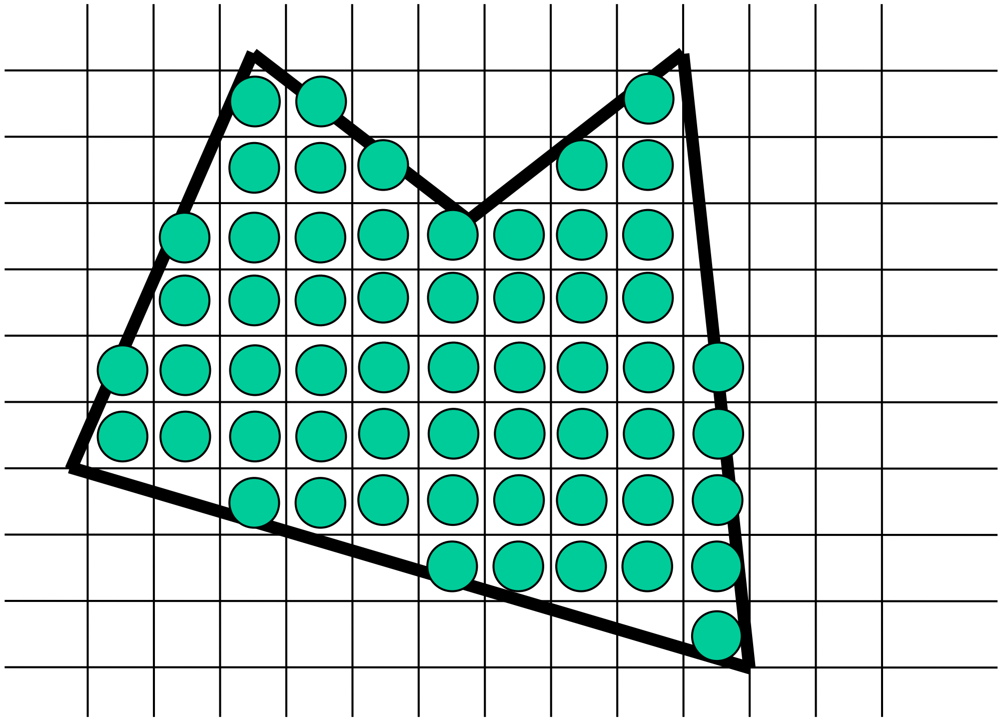
Flood Fill

Fill a point and recurse to all of its neighbors

```
floodFill(int x, int y, color c)
   if(stop(x,y,c))
      return;

   setPixel(x,y,c);
   floodFill(x-1,y,c);
   floodFill(x+1,y,c);
   floodFill(x,y-1,c);
   floodFill(x,y+1,c);

int stop(int x, int y, color c)
   return colorBuffer[x][y] == c;
```

Rasterizing Polygons

Given a set of vertices and edges,
find the pixels that fill the polygon.



Rasterizing Polygons

vList is an ordered list of the polygon's vertices

```
fillPoly(vertex vList[ ])
   boundingBox b = getBounds(vList);
   int xmin = b.minX;
   int xmax = b.maxX;
   int ymin = b.minY;
   int ymax = b.maxY;

   for(int y = ymin; y <= ymax; y++)
      for(int x = xmin; x <= xmax; x++)
         if(insidePoly(x,y,vList))
            setPixel(x,y);
```

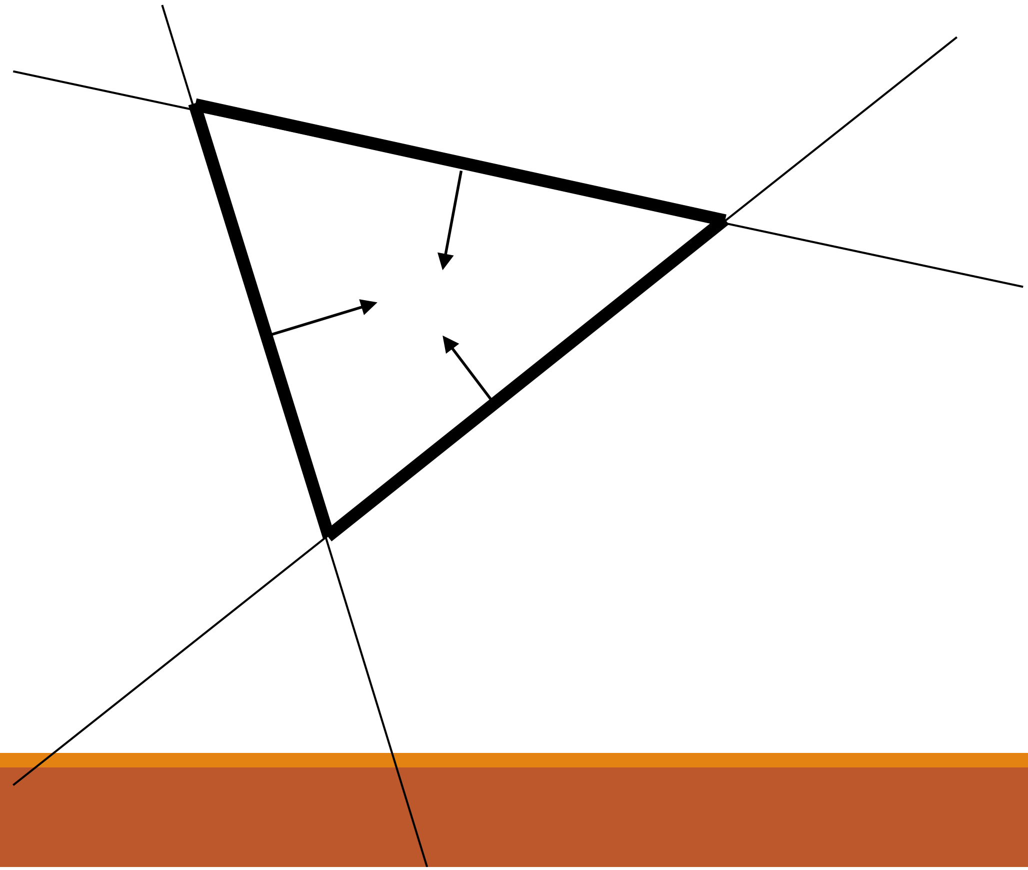
What does 'inside' mean?

How to test if a point is inside a polygon

1. Half-space tests
2. Jordan Curve Theorem (even/odd or +1/-1)
3. Winding number test

Half Space Tests

Given the edges of a triangle, the inside is
the intersection of half-spaces defined by the edges

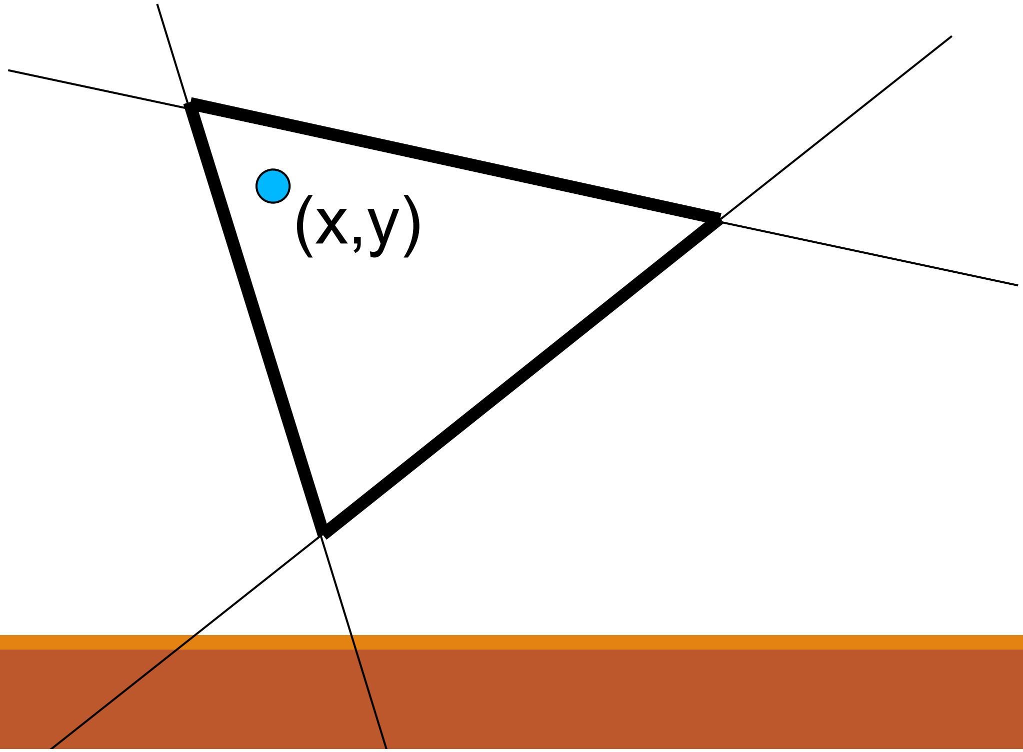


Half Space Tests

Easily computable:

$$l(x,y) = ax + by + c < 0$$

Iff (x,y) is inside



Half Space Tests

lineEq computes the implicit line value for 2 vertices & a point

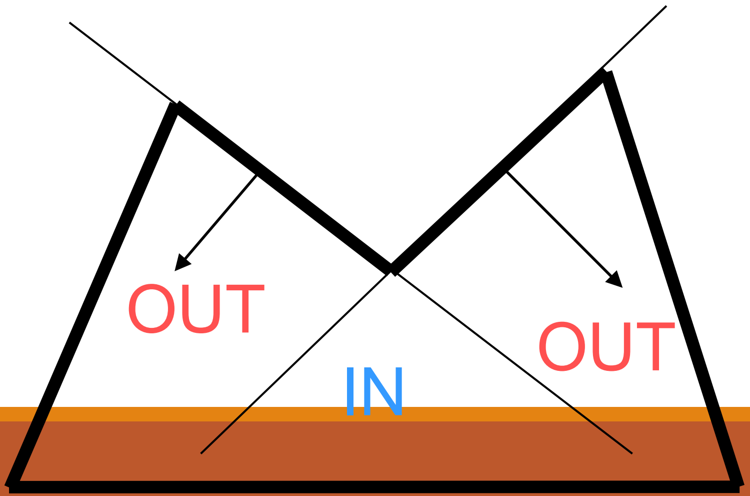
```
fillTriangle(vertex vList[3])
  //-- get the bounding box as before --//
  float e1 = lineEq(vList[0],vList[1],xmin,ymin);
  float e2 = lineEq(vList[1],vList[2],xmin,ymin);
  float e3 = lineEq(vList[2],vList[0],xmin,ymin);
  int xDim = xMax - xMin;

  for(int y = ymin; y <= ymax; y++)
    for(int x = xmin; x <= xmax; x++)
      if(e1<0 && e2<0 && e3<0)
        setPixel(x,y);
    e1 += a1; e2 += a2; e3+= a3;
  e1 += -xDim*a1+b1; e2 = -xDim*a2+b2; e3 = -xDim*a3+b3
```

Half Space Tests

Easily computable:

$$l(x,y) = ax + by + c < 0$$ Iff (x,y) is inside

Doesn’t work on concave objects!!
→ triangulate



What does 'inside' mean?

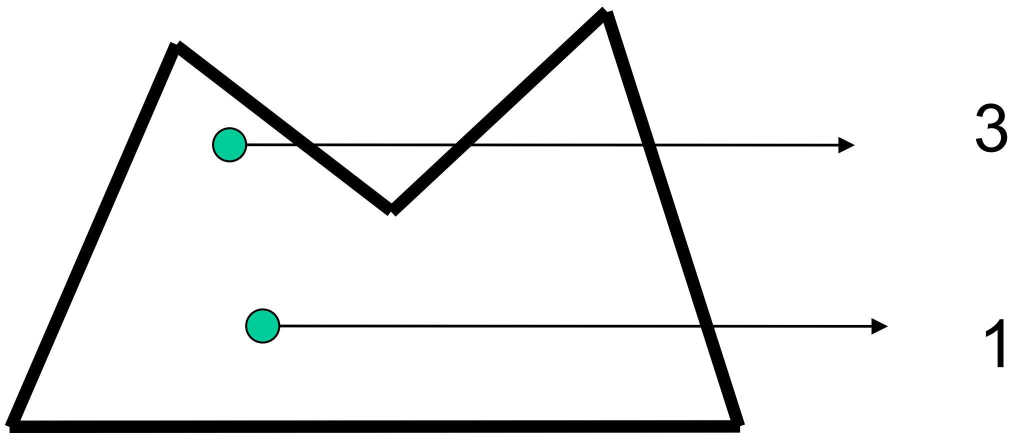
How to test if a point is inside a polygon

1. Half-space tests
2. Jordan Curve Theorem (even/odd or +1/-1)
   - Self-intersecting polygon OK
3. Winding number test

Jordan Curve Theorem

Even/odd approach

Hit test: inside or outside based on the number of intersected edges is even or odd

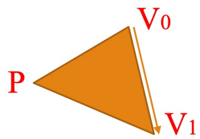


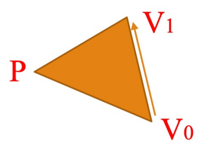
Any ray from a point inside a polygon will intersect the polygon's edges an odd number of times

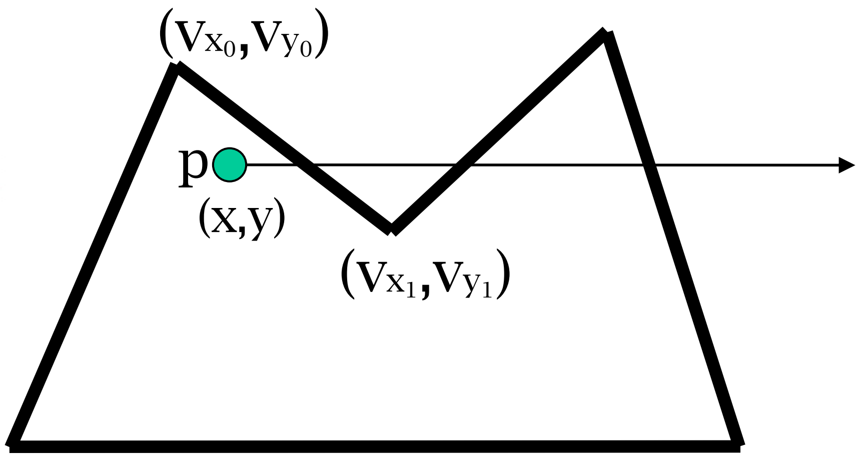
Jordan Curve Theorem

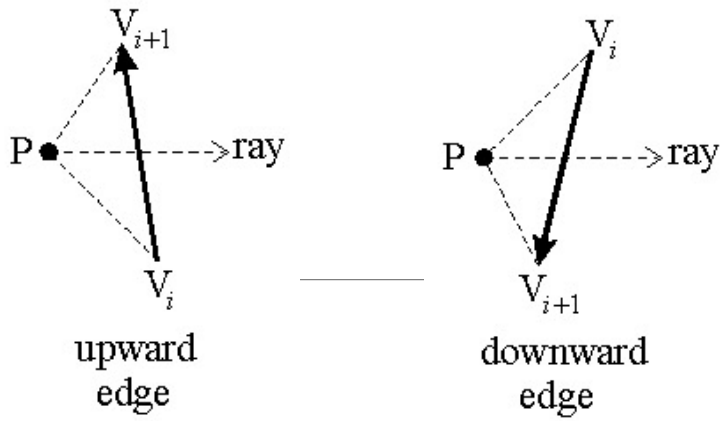
vList is an ordered list of the n polygon vertices

```
int jordanInside(vertex vList[ ], int n, float x, float y)
  int cross = 0;
  float x0, y0, x1, y1;

  x0 = vList[n-1].x – x;    y0 = vList[n-1].y – y;
  for(int i = 0; i < n; i++)
    x1 = vList[i].x – x;     y1 = vList[i].y – y;
    if(y0 > 0)
      if(y1 <= 0)
        if( x1*y0 > x0* y1)
          cross++
    else
      if(y1 > 0)
        if( x0*y1 > x1* y0)
          cross++
    x0 = x1; y0 = y1;

  return cross & 1;
```





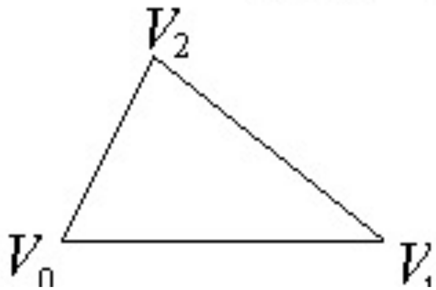




where $V_i = (x_i, y_i)$

$$2\,\mathrm{A}(\Delta) = \begin{vmatrix} (x_1 - x_0) & (x_2 - x_0) \\ (y_1 - y_0) & (y_2 - y_0) \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

$$= (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$$



i.e., x0y1-x1y0

Vi=(xi,yi)

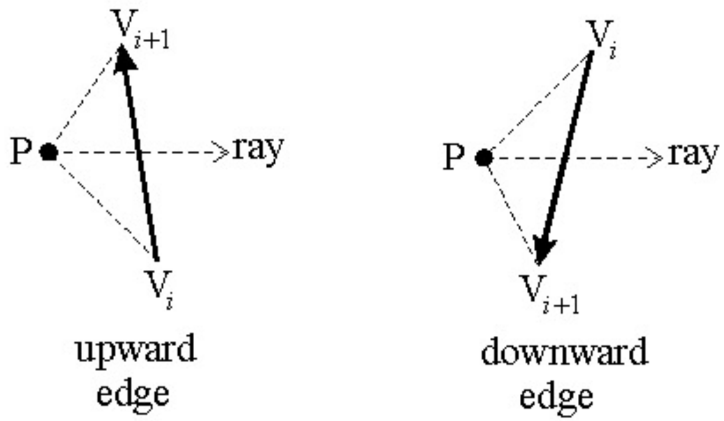
The signed area will be

-positive if the triangle *is* oriented counterclockwise

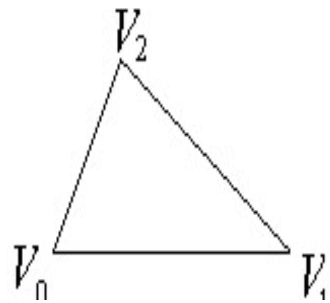
-negative if the triangle is oriented clockwise

http://geomalgorithms.com/a03-_inclusion.html

http://geomalgorithms.com/a01-_area.html



$$2A(\Delta) = \begin{vmatrix} (x_1 - x_0) & (x_2 - x_0) \\ (y_1 - y_0) & (y_2 - y_0) \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$
$$= (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$$



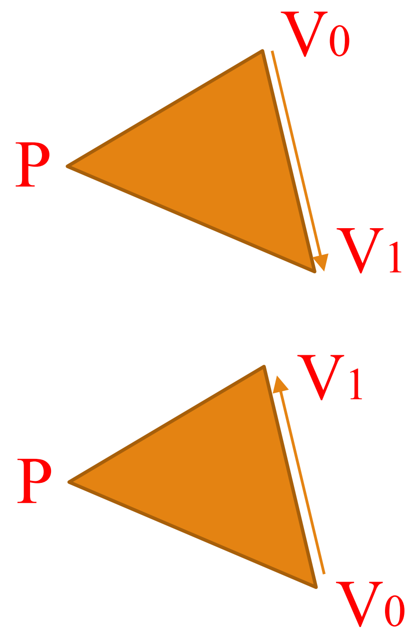
The signed area will be

-Positive, if the triangle *is* oriented counterclockwise

- x0y1>x1y0

-negative, if the triangle is oriented clockwise

- x1y0 >x0y1



2A = x0y1-x1y0

where
x0 = v0.x – p.x;
y0 = v0.y – p.y;

$2Area = x0y1-x1y0$

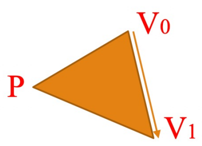
A> 0, if the triangle *is* oriented counterclockwise

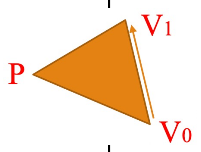
A< 0, if the triangle is oriented clockwise

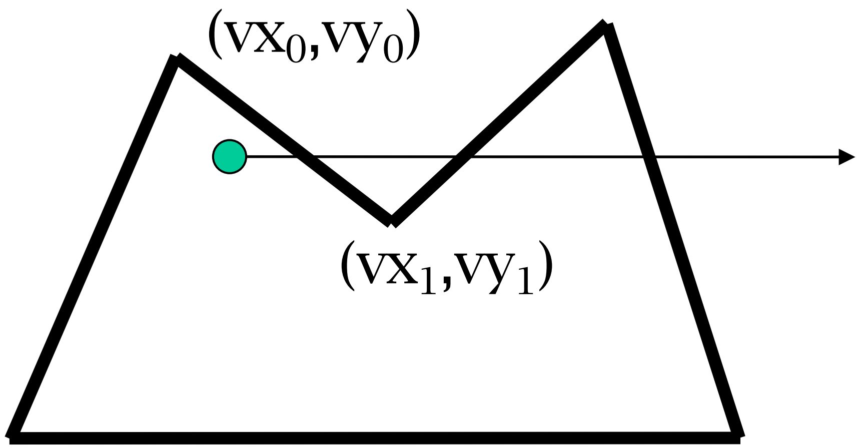
Jordan Curve Theorem

vList is an ordered list of the n polygon vertices

```
int jordanInside(vertex vList[ ], int n, float x, float y)
   int cross = 0;
   float x0, y0, x1, y1;

   x0 = vList[n-1].x – x;     y0 = vList[n-1].y – y;
   for(int i = 0; i < n; i++)
      x1 = vList[i].x – x;      y1 = vList[i].y – y;
      if(y0 > 0)
         if(y1 <= 0)
            if( x1*y0 > y1*x0)
               cross++
      else
         if(y1 > 0)
            if( x0*y1 > y0*x1)
               cross++
      x0 = x1; y0 = y1;

   return cross & 1;
```







Jordan Curve Theorem

What if it goes through a vertex?

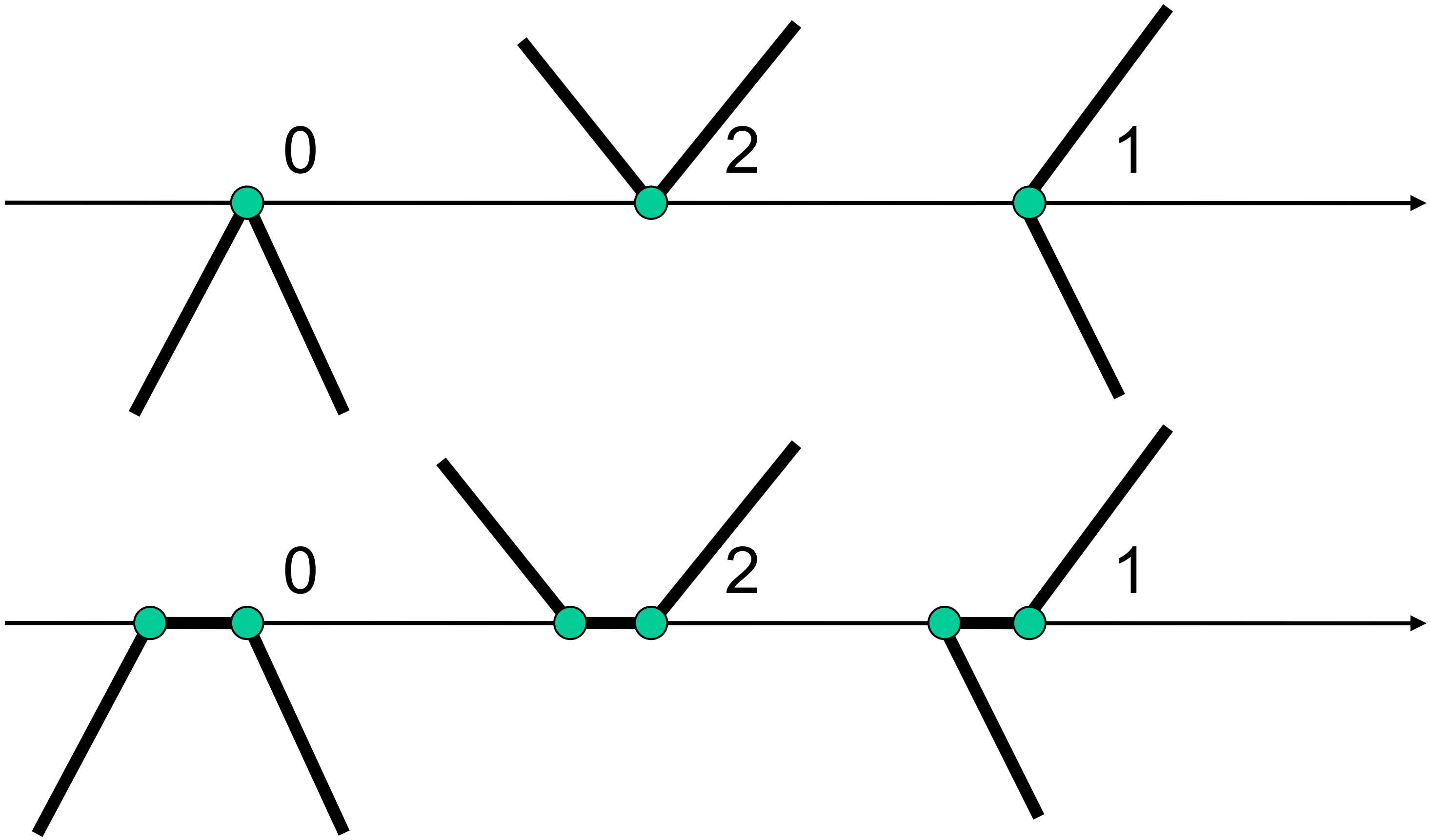
Treat these two cases differently:

- case (a): the vertex–scanline intersection is counted as either zero or two edge crossings
- case (b): the vertex–scanline intersection must be counted as one edge crossing.

(a) (b)

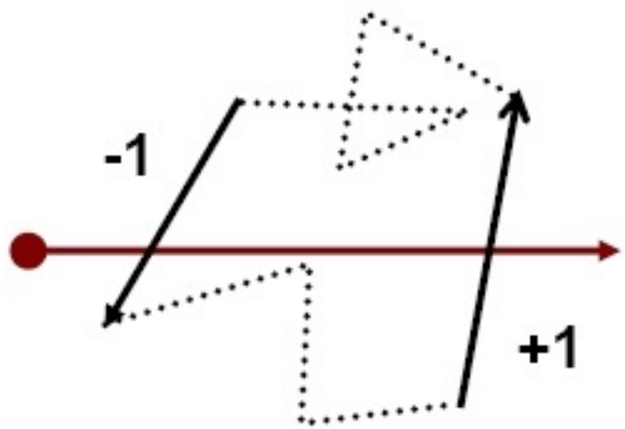
Jordan Curve Theorem

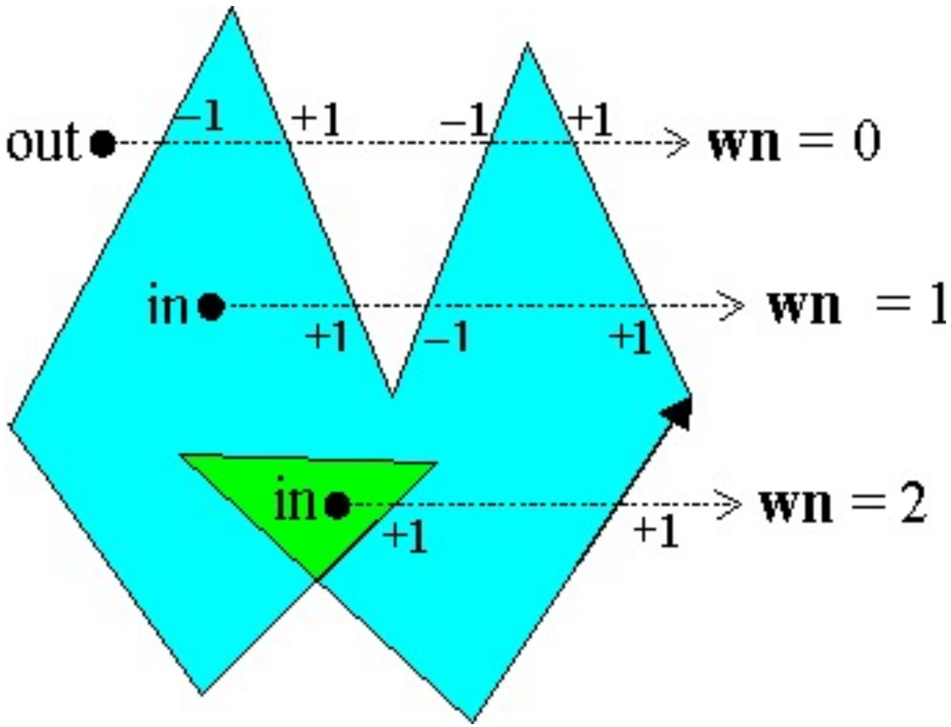
What if it goes through a vertex?

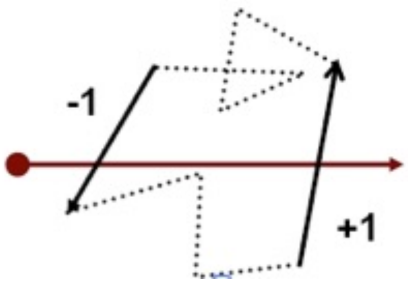


Jordan Curve Theorem

- Non-zero winding rule:
  - Draw a line from the test point to the outside
  - Count +1 if you cross an edge in an anti-clockwise sense
  - Count -1 if you cross an edge in a clockwise sense







count +1, if you cross an edge in an anti-clockwise sense
count -1, if you cross an edge in a clockwise sense
Inside point : non-zero

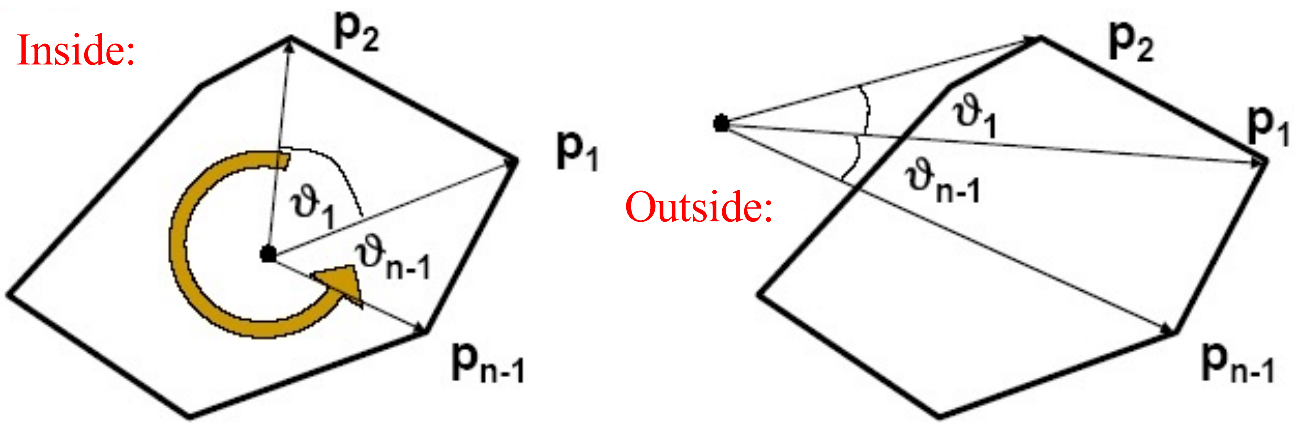
What does 'inside' mean?

How to test if a point is inside a polygon

1. Half-space tests
2. Jordan Curve Theorem (even/odd or +1/-1)
3. Winding number test

Winding Number Test-Method I

- Sum the angle subtended by the vertices



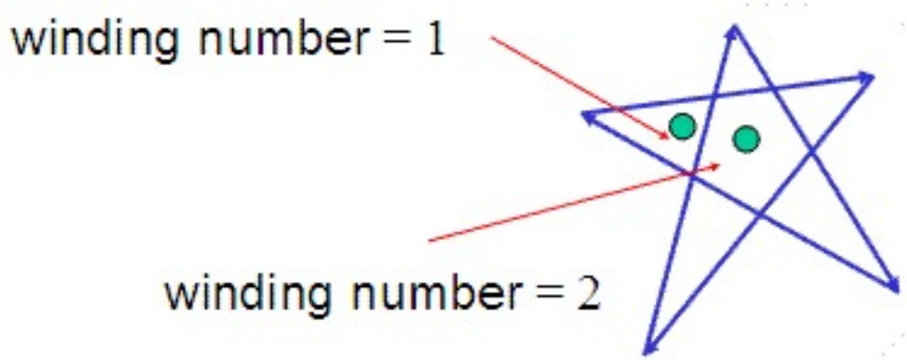
$$\sum_{i=1}^{n} \vartheta_i = 2\Pi$$

$$\sum_{i=1}^{n} \vartheta_i \neq 2\Pi$$

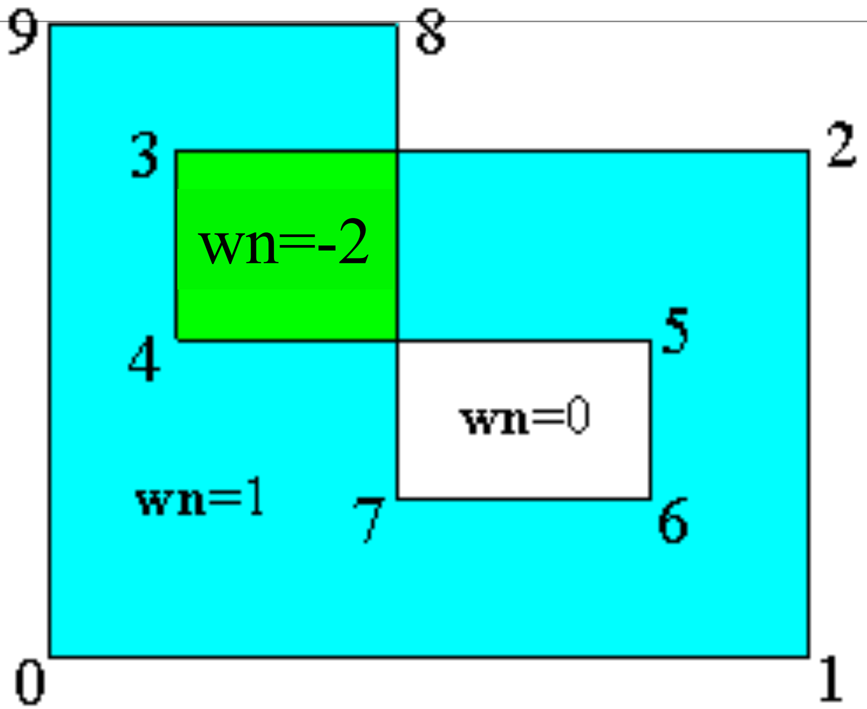
Winding Number Test – Method II

The number of times it is encircled by the edges of the polygon

- +1: if clockwise encirclements
- -1 : if counterclockwise encirclements
- a point is inside the polygon if its winding number is not zero



Winding Number Test II

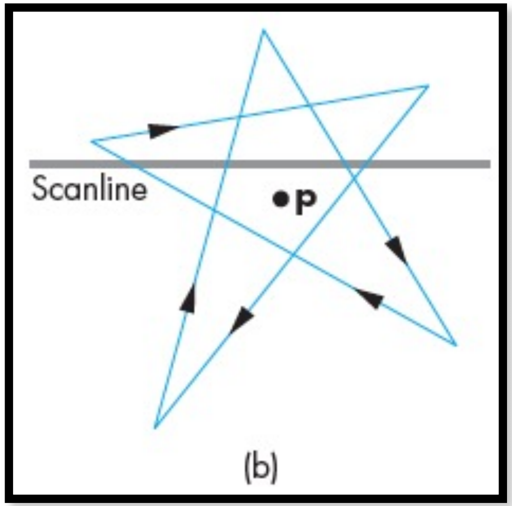


+1: if clockwise encirclements

-1 : if counterclockwise encirclements

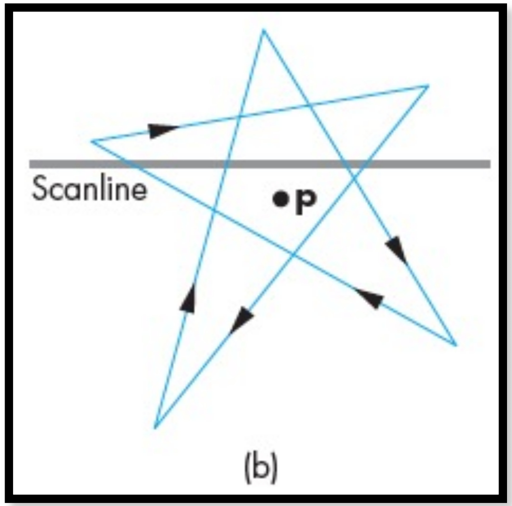
a point is inside the polygon if its winding number is not zero

Winding Number Test – Method II



+1: if clockwise encirclements
-1 : if counterclockwise encirclements
a point is inside the polygon if its winding number is not zero

Filling with the odd–even test？

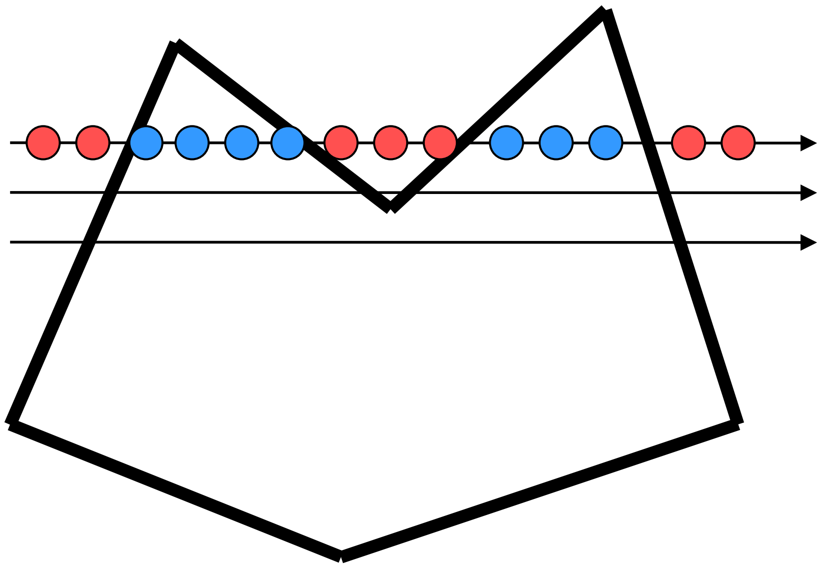


Hit test: inside or outside based on the number of intersected edges is even or odd

rasterization algorithms

Scan Line Algorithms

Take advantage of coherence in "insided-ness"

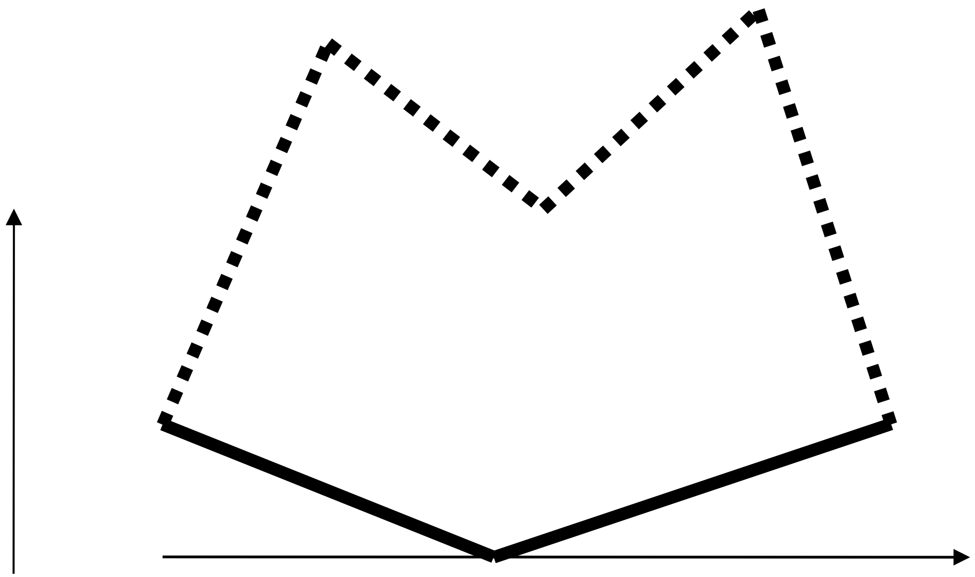


Inside/outside can only change at edge events

Current edges can only change at vertex events

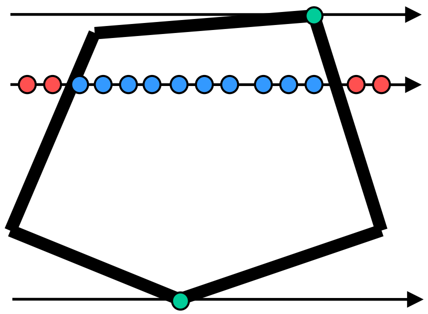
Scan Line Algorithms

Create a list of the edges intersecting the first scanline



Sort this list by the edge's x value on the first scanline

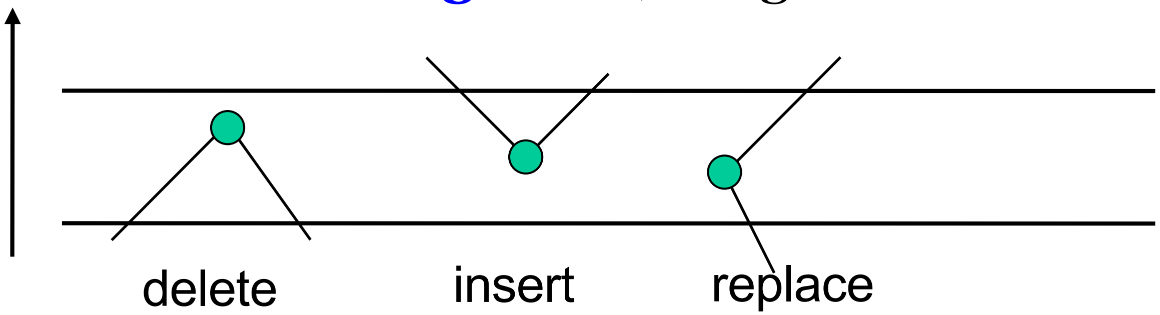
Call this the active edge list



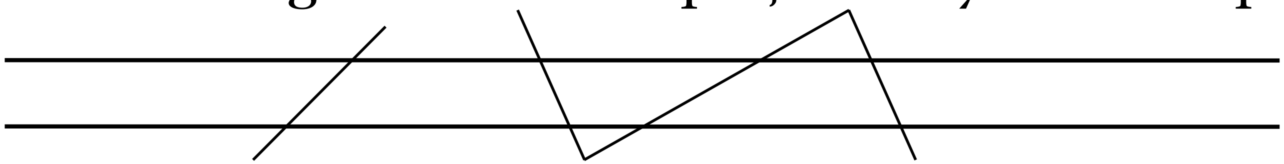
Scan Line Algorithms

For each scanline:

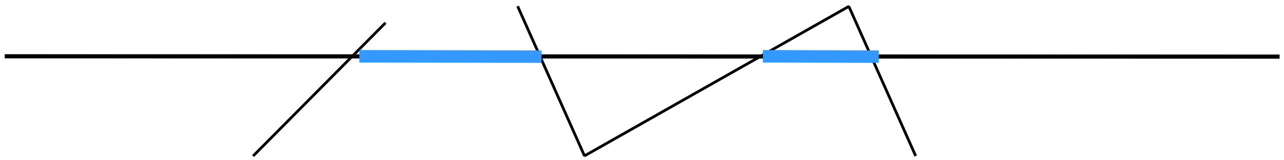
1. Maintain **active edge list** (using vertex events)



2. Increment edge' s x-intercepts, sort by x-intercepts

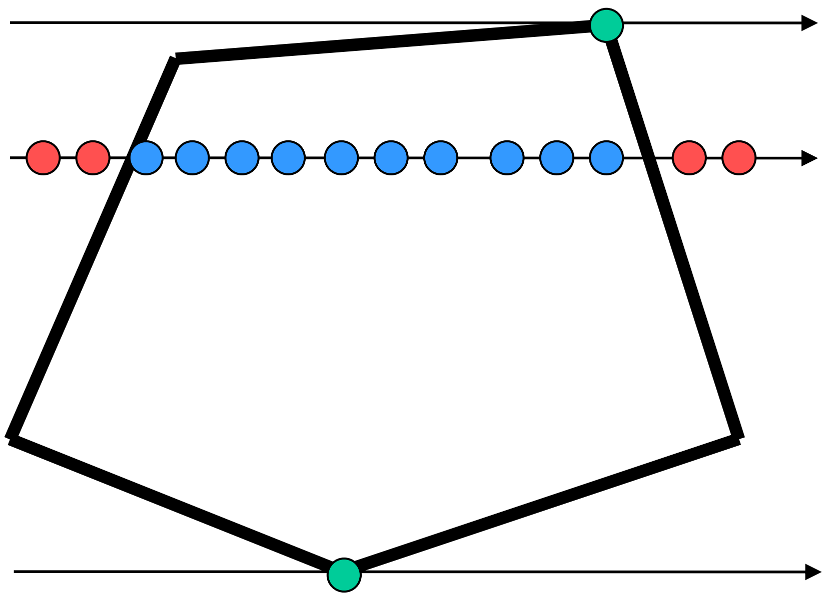


3. Output spans between left and right edges



Convex Polygons

Convex polygons only have 1 span

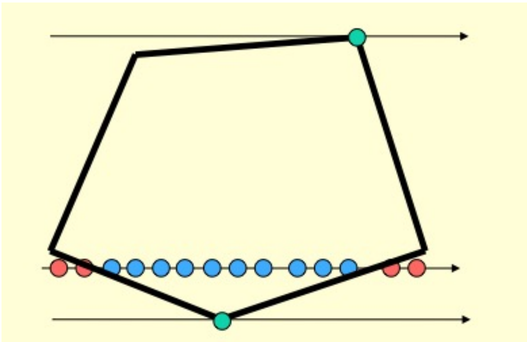


Insertion and deletion events happen only once

Crow's Algorithm

Step1: Find the vertex with the smallest y value to start

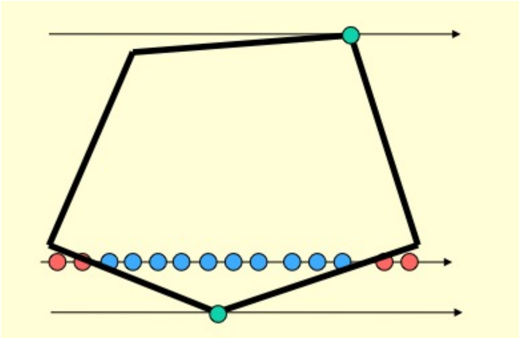
```
crow ( vertex vList[], int n)
        int iMin = 0;
        for(int i = 1; i < n; i++)
            if(vList[i].y < vList[iMin].y)
            iMin = i;

        scanY(vList,n,iMin);
```



Crow's Algorithm

Step2: Scan upward maintaining the **active edge list**

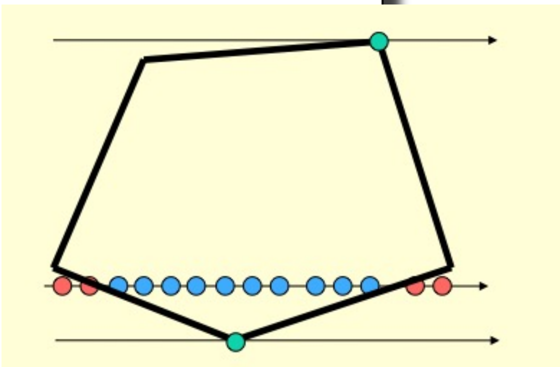
```
scanY(vertex vList[], int n, int i)
  int li, ri;    // left & right upper endpoint indices
  int ly, ry;    // left & right upper endpoint y values
  vertex l, dl;      // current left edge and delta
  vertex r, dr;      // current right edge and delta
  int rem;       // number of remaining vertices
  int y;         // current scanline

  li = ri = i;
  ly = ry = y = ceil(vList[i].y);

(1) for( rem = n; rem > 0)
  (2) // find appropriate left edge
      // find appropriate right edge
  (3) // while l & r span y (the current scanline)
        // draw the span
```



Crow's Algorithm

Find the appropriate next left edge

```
(2) while( ly < = y && rem > 0)
        rem--;
        i = li - 1;
        if(i < 0)
            i = n-1;  // go clockwise

        ly = ceil( v[i].y );
        if( ly > y ) // replace left edge
          differenceY( &vList[li], &vList[i], &l, &dl, y);
        li = i;  // index of the left endpoint
```



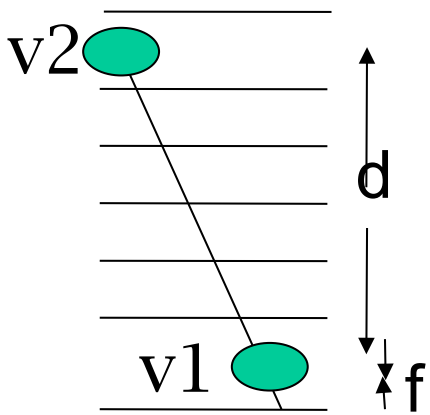
all vertices are in counter-clockwise order
index: 0~(n-1)

Crow's Algorithm

Calculate delta and starting values

```
differenceY(vertex *v1, vertex *v2, vertex *e, vertex *de, int y)
   difference(v1, v2, e, de, (v2.y – v1.y), y – v1.y);
```

```
difference(vertex *v1, vertex *v2, vertex *e, vertex *de, float d, float f)
   de.x = (v2.x – v1.x) / d;
   e.x   = v1.x + f * de.x;
```



Crow's Algorithm

Draw the spans

```
(3)for( ; y < ly && y < ry;  y++)
    // scan and interpolate edges
    scanX(&l, &r, y);
    increment(&l,&dl);
    increment(&r,&dr);
```

Increment the x value

```
  increment(vertex *edge, vertex *delta)
    edge.x += delta.x;
```

Crow's Algorithm

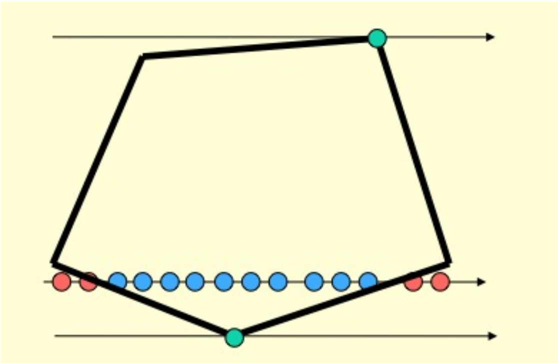
Draw the spans

```
scanX(vertex *l, vertex *r, int y)
   int x, lx, rx;
   vertex s, ds;

   lx = ceil(l.x);
   rx = ceil(r.x);
   if(lx < rx)
      for(x = lx, x < rx; x++)
         setPixel(x,y);
```

Crow's Algorithm

Step2: Scan upward maintaining the **active edge list**

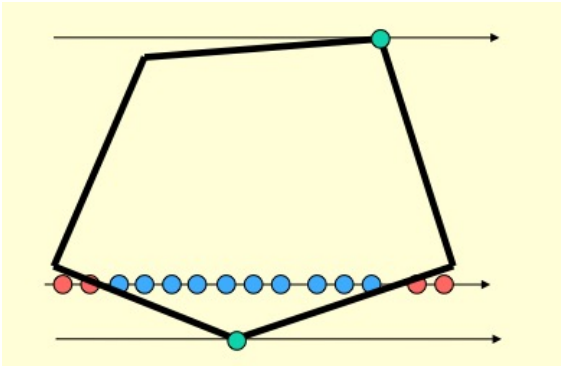
```
scanY(vertex vList[], int n, int i)
   int li, ri;     // left & right upper endpoint indices
   int ly, ry;     // left & right upper endpoint y values
   vertex l, dl;       // current left edge and delta
   vertex r, dr;       // current right edge and delta
   int rem;       // number of remaining vertices
   int y;          // current scanline

   li = ri = i;
   ly = ry = y = ceil(vList[i].y);

(1) for( rem = n; rem > 0)
(2)    // find appropriate left edge
       // find appropriate right edge
(3)    // while l & r span y (the current scanline)
          // draw the span
```



Crow's Algorithm

Step2: Scan upward maintaining the **active edge list**

```
scanY(vertex vList[], int n, int i)
   //…
   li = ri = i;
   ly = ry = y = ceil(vList[i].y);

(1) for( rem = n; rem > 0)
      // find appropriate left edge
 (2)  while( ly < = y && rem > 0)
           //…
      // find appropriate right edge
      while( ry < = y && rem > 0)
           //…
 (3)
     for( ; y < ly && y < ry;  y++) // while l & r span y (the current scanline)
      scanX(&l, &r, y);      // draw the span
      increment(&l,&dl);
      increment(&r,&dr);
```



Crow's Algorithm

Draw the spans

```
scanX(vertex *l, vertex *r, int y)
   int x, lx, rx;
   vertex s, ds;

   lx = ceil(l.x);
   rx = ceil(r.x);
   if(lx < rx)
      differenceX(l, r, &s, &ds, lx);
      for(x = lx, x < rx; x++)
         setPixel(x,y);
         increment(&s,&ds);
```

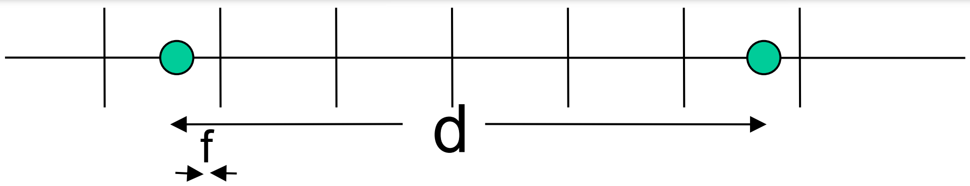
Interpolating other values
E.g, colors

Crow’s Algorithm

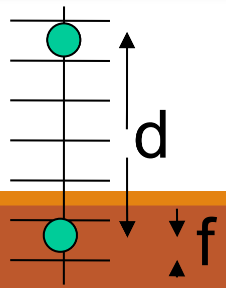
Calculate delta and starting values

```
differenceX(vertex *v1, vertex *v2, vertex *e, vertex *de, int x)
   difference(v1, v2, e, de, (v2.x – v1.x), x – v1.x);
```

```
difference(vertex *v1, vertex *v2, vertex *e, vertex *de, float d, float f)
   de.x = (v2.x – v1.x) / d;
   e.x   = v1.x + f * de.x;
```




```
differenceY(vertex *v1, vertex *v2, vertex *e, vertex *de, int y)
   difference(v1, v2, e, de, (v2.y – v1.y), y – v1.y);
```



Crow’s Algorithm

Draw the spans

```
scanX(vertex *l, vertex *r, int y)
   int x, lx, rx;
   vertex s, ds;
   lx = ceil(l->x);
   rx = ceil(r->x);
   if(lx < rx)
      differenceX(l, r, &s, &ds, lx);
      for(x = lx, x < rx; x++)
         setPixel(x,y);
         increment(&s,&ds);
```

```
increment(vertex *edge, vertex *delta)
   edge.x += delta.x;
```

```
differenceX(vertex *v1, vertex *v2, vertex *e, vertex *de, int x)
   difference(v1, v2, e, de, (v2.x – v1.x), x – v1.x);
```

```
difference(vertex *v1, vertex *v2, vertex *e, vertex *de, float d, float f)
   de->x = (v2.x – v1.x) / d;
   e->x   = v1.x + f * de.x;
```

Crow’s Algorithm

Interpolating other values

```
difference(vertex *v1, vertex *v2, veretx *e, vertex *de, float d, float f)
   de.x = (v2.x – v1.x) / d;
   e.x   = v1.x + f * de.x;
   de.r = (v2.r – v1.r) / d;
   e.r   = v1.r + f * de.r;
   de.g = (v2.g – v1.g) / d;
   e.g   = v1.g + f * de.g;
   de.b = (v2.b – v1.b) / d;
   e.b  = v1.b + f * de.b;
```

```
increment( vertex *v, vertex *dv)
   v.x += dv.x;
   v.r += dv.r;
   v.g += dv.g;
   v.b += dv.b;
```