

```

(define tak (lambda (x y z)
  (if (< y x)
    (tak (tak (- x 1) y z)
      (tak (- y 1) z x)
      (tak (- z 1) x y))
    z)))
(tak 18 12 6)

```

=> 7

CPS convert by hand , every call is a tail call

```

(define (k< a b k1 k2) (if (< a b) (k1) (k2)))

(define (k- a b k) (k (- a b)))

(define (ktak x y z k)
  (k< y x
    (lambda () ; (< y x)
      (k- x 1
        (lambda (x1)
          (ktak x1 y z
            (lambda (tak1v)
              (k- y 1
                (lambda (y1)
                  (ktak y1 z x
                    (lambda (tak2v)
                      (k- z 1
                        (lambda (z1)
                          (ktak z1 x y
                            (lambda (tak3v)
                              (ktak tak1v tak2v tak3v k))))))))))))
                        (lambda () (k z))))))

```

```

;; identity continuation
(define kid (lambda (x) x))

```

```

(ktak 18 12 6 kid)

```

Lisp.scm
Minimal.scm
Minimal version is supposed to be absolute CORE of SPECIAL FORMS.
IF is a conditional . Cond can be built on if construct.
QUOTE allows arbitrary lisp expressions .
BEGIN allows sequencing of operations .
DEFINE allows mutate the environment , it also redefines the first found definition if it exists, or creates a new definition. the original environment is mutated in place.
SET! should alter existing binding in the environment.
LAMBDA allows abstraction .
There is another APPLICATION
.....
so these 7 special forms QUOTE , IF , BEGIN , DEFINE , SET! , LAMBDA , APPLICATION .
so we notice there is no READER and no pretty PRINTER.
we can make the low level code FAST , if it does little or no checks .
.....
Okay , do we need some PRIMITIVES .
.....
'() : nil : the empty list
(null? x) : type predicate
.....
Pairs
(cons x y) : constructor
(car x) : selector
(cdr x) : selector
(pair? x) : type predicate
.....
Symbols
(make-symbol x) : constructor
(eq? x y) : are two symbols the same memory location ?
(symbol? x) : type predicate
(gensym) : special symbol constructor
.....
Strings
(string-length x) : length of string
(string? x) : type predicate
.....
Self evaluating items - #a characters , "a string" strings , 123 numbers , #T #F booleans.

Symbols are looked up in the environment.
also hash tables , vectors ...

.....

Boolean

#T true : constructor

#F false : constructor

(boolean? x) : type predicate

.....

Compiler

If compiler can see an expression that can be simplified it is probably a good idea to replace it with an equivalent but more efficient expression.

```
(if #f 1 2) => 2
```

Lets assume got debugged code and want it to run fast. Here is where the compiler can help.

Lets build a compiler.

Lets build a correct interpreter first then .

In the following pages are the basis of a scheme interpreter using 7 registers argl , expr , val , et al .

The interpreter does not include call-with-current-continuation , but this is a small addition with another primitive that wraps continuation much like a closure wraps lambda and environment. * TO DO * .

Condition system needs to be written in also , as we do not yet have any way to signal a condition, let alone handle it or pass it off.

Garbage collector needs to be written up , that is also missing.

Evaluation of arguments left to right means that building up argument list requires a reversal at end of ev-operands.

* repl *

because in a repl we do not know when last expression will be ,
we treat it like an endless begin sequence.
thus we need to

MAYBE - avoid saving and restoring cont , have ev-read-3 do restore and continue
so save cont register before entering ev-read proper ,

```
(define (eval-repl exp env cont)
  (newline)
  (display ";; Ready > ")
  (let ((input (read)))
    (base-eval input env
      (lambda (result)
        (display ";; Value > ")
        (display result)
        (newline)
        (eval-repl #f env cont))))))
```

ev-read :
 save cont
 goto ev-read-2

ev-read-2 : display ";; Ready > "
 exp = read-primitive
 if eof-object?
 goto ev-read-4
 save env
 cont = ev-read-2
 goto eval-dispatch

ev-read-3 :
 restore env
 display ";; Value > " val
 goto ev-read

ev-read-4 :
 restore cont

goto continue

in this version , shorter but makes 2 saves and restores , instead of just 1 for each round trip of the read eval print loop.

```
-----  
          *    repl    *  
-----  
ev-read :  
  display ";; Ready > "  
  exp = read-primitive  
  if eof-object?  
    goto continue  
  save cont  
  save env  
  cont = ev-read-2  
  goto eval-dispatch  
  
ev-read-2 :  
  restore env  
  restore cont  
  display ";; Value > " val  
  goto ev-read  
  
-----
```

```

-----
*      apply-dispatch      *
-----
inputs      : proc = evaluated procedure , argl = evaluated arguments
outputs     :
clobber     :
preserved   :
-----
apply-dispatch :
  if primitive-procedure? proc
    goto primitive-apply
  if user-procedure? proc
    goto user-apply
  otherwise
    eval-error " unknown procedure type "
-----
*      primitive-apply      *
-----
primitive-apply :
save continue ;; allows primitive to use all registers
argl = reverse argl
val = call the primitive proc with arguments argl
restore continue ;; restore
goto continue
-----
*      user-apply      *
-----
user-apply :
argl = reverse argl
unev = user-lambda-params proc
env  = user-lambda-env proc
extend the environment [env] with [unev : params ] and [ argl : values ]
unev = user-lambda-body proc
goto ev-sequence ;; implicit sequence on body of lambda
-----

```

```

-----
                *   eval-dispatch   *
-----

example input  :    exp = < thing to be evaluated >
example output :    val = < what it evaluates to >
-----

inputs      : exp
outputs     : val
clobber     : argl , exp , unev , val , proc
preserved   : cont , env
-----

eval-dispatch :
  if self-evaluating? exp
    val = exp
    goto continue
  if symbol? exp
    val = lookup symbol exp env
    goto continue
  if pair? exp and car [exp] = 'if
    goto ev-if
  if pair? exp and car [exp] = 'quote
    val = car [ cdr exp ]
    goto continue
  if pair? exp and car [exp] = 'begin
    goto ev-begin
  if pair? exp and car [exp] = 'lambda
    goto ev-lambda
  if pair? exp and car [exp] = 'define
    goto ev-definition
  if pair? exp and car [exp] = 'set!
    goto ev-assignment
  if pair? exp
    goto ev-application
  otherwise
    eval-error " what now ? "
-----

```

```
-----  
self-evaluating ?  
-----
```

```
if boolean? exp  
  string? exp  
  vector? exp  
  procedure? exp  
then return true  
else return false  
-----
```

```
      *      ev-lambda      *  
-----
```

```
ev-lambda :  
  unev = car [ cdr exp ] ; params of lambda  
  exp = cdr [ cdr exp ] ; body of lambda  
  val = #[ closure unev exp env ]  
  goto cont  
-----
```

```
(define (user-procedure? x)  
  (and (vector? x)  
        (eq? 'closure (vector-ref x 0))))  
(define (user-lambda-params x) (vector-ref x 1))  
(define (user-lambda-body x) (vector-ref x 2))  
(define (user-lambda-env x) (vector-ref x 3))  
-----
```

```

-----
                *      ev-begin      *
-----

example input  :      exp = '(begin 1 2 3)
example output :      val = 3
-----

inputs       : exp
outputs      : val
clobber      : argl , exp , unev , val , proc
preserved    : cont , env
-----

ev-begin :
  unev = cdr [ exp ]
  goto ev-sequence

ev-sequence :
  if null? unev
    set val \# f
    goto continue
  exp = car [ unev ]
  if null? cdr [ unev ]
    ev-sequence-3
  save cont
  save env
  save unev
  cont = ev-sequence-2
  goto eval-dispatch
ev-sequence-2 :
  restore unev
  restore env
  restore cont
  unev = cdr unev
  goto ev-sequence
ev-sequence-3 :
  goto eval-dispatch
-----

```

```

-----
          *      ev-definition      *
-----
example input  :      exp = '(define a 5)
example output :      val = 'ok
-----

inputs       : exp
outputs      : val
clobber      : argl , exp , unev , val , proc
preserved    : cont , env
-----

ev-definition:
  unev = car [ cdr exp ] ; variable
  save unev
  save env ; save environment define will need that
  save cont
  exp = car [cdr [ cdr exp ]] ;; value to eval
  cont = ev-definition-2
  eval-dispatch

ev-definition-2:
  restore cont
  restore env
  restore unev
  ... primitive ... (env-define! exp val env) ...
  val = ok
  goto cont
-----

```

```

-----
                *      ev-assignment      *
-----

example input  :      exp = '(set! a 5)
example output :      val = 'ok
-----

inputs       : exp
outputs      : val
clobber      : argl , exp , unev , val , proc
preserved    : cont , env
-----

ev-assignment:
  save exp
  save env
  save cont
  exp = car [cdr [ cdr exp ]] ;; y
  cont = ev-assignment-2
  eval-dispatch

ev-assignment-2:
  restore cont
  restore env
  restore exp
  exp = car [ cdr exp ] ;; x
  ... primitive operation (env-set! exp val env) ...
  val = ok
  goto cont
-----

```

```

-----
                *      ev-if      *
-----
example input  :      exp = '(if #f 1 2)
example output :      val = 2
-----

inputs       : exp
outputs      : val
clobber      : argl , exp , unev , val , proc
preserved    : cont , env
-----

ev-if:
  unev = cdr [ cdr exp ]
  save unev
  save env
  save cont
  exp = car [cdr exp ] ;; x
  cont = ev-if-2
  eval-dispatch
ev-if-2:
  restore cont
  restore env
  restore unev
  if val is true
    goto ev-if-3
    goto ev if-4
ev-if-3:
  exp = car unev ;; y
  goto eval-dispatch
ev-if-4:
  unev = cdr unev
  if null? unev
    val = \#f ;; false
    goto continue
  otherwise
    exp = car unev
    goto eval-dispatch
-----

```



```

-----
                *      ev-application      *
-----
example input  :      exp = '(factorial 5)
example output :      val = 120
-----

inputs       : exp
outputs      : val
clobber      : argl , exp , unev , val , proc
preserved    : cont , env
-----

ev-application :
unev = cdr exp ; operands
save unev
save env
save cont
exp = car exp ; the operator
cont = ev-application-2
goto eval-dispatch

ev-application-2 :
restore cont
restore env
restore unev
proc = val ;
save proc
save env
save cont
cont = ev-application-3
argl = '() ; the empty list
goto ev-operands

ev-application-3 :
restore cont
restore env
restore proc
goto apply-dispatch
-----

```

```

-----
                *      ev-operands usage      *
-----
example input  :      unev = '((+ 10 20 ) (+ 30 40) (+ 50 60)
example output :      argl = '( 110  70  30 )
notes : argl result is REVERSEd .
-----

inputs      : unev
outputs     : argl , the arguments are in REVERSE order
clobber     : argl , exp , unev , val , maybe proc
preserved   : cont , env
-----

ev-operands :
    argl = '() ; empty list
    goto ev-operands-2

ev-operands-2 :
    if null? unev
        goto cont
    exp = car unev
    unev = cdr unev
    save env
    save unev
    save cont
    save argl
    cont = ev-operands-3
    goto eval-dispatch

ev-operands-3 :
    restore argl
    restore cont
    restore unev
    restore env
    argl = cons val argl ; adjoin
    goto ev-operands-2
-----

```

When we call `ev-operands` , we want to be sure `argl` is properly initialised. We observe that the arguments are accumulated in reverse . `ARGL` values are reversed

The rest of this document contains old notes that may be useful

```
-----  
          *      reverse      *  
piggy back on ev-operands  
-----  
ev-reverse :  
  unev = car [ cdr exp ]  
  save-cont  
  cont = ev-reverse-2  
  goto ev-operands  
  
ev-reverse-2 :  
  restore-cont  
  val = arg1  
  goto cont  
-----
```

Ev-operands takes unevaluated arguments in `unev` , we want evaluated arguments in `argl`

if no operands are to be evaluated we are done

```
(define (ev-operands exp env cont)
  (if (null? exp)
      (cont '())
      (base-eval (car exp) env
EV-3 :   (lambda (val1)
            (ev-operands (cdr exp) env
EV-4 :   (lambda (val2)
            (cont (cons val1 val2))))))))
```

Motivation

We have a working continuation passing style lisp interpreter and have successfully written call-with-current-continuation.

since callcc is crucial measure of a scheme system.

Also written trace routine that works especially well in cps format , want to see this translated to SICP register machine version

want to prove the simple CPS to register machine translation that we devised as a simple and trustworthy mechanism that requires mental overhead.

eval-application
left to right evaluation of arguments
many ways to do this , here we evaluate the operator and then the
operands then apply operator to operands , simple really.

```
(define (eval-application exp env cont)
  (let ((operator (car exp))
        (operands (cdr exp)))
    (base-eval operator env
CONT-2:      (lambda (eop)
              (eval-operands operands env
CONT-3:      (lambda (eargs)
                (base-apply eop ears args env cont)))))))
```

cont1 and cont2 need environment env and continuation cont
if we have unev be unevaluated arguments then exp can be operator in
first call to base eval
eop ends up in proc register
eargs ends up in argl register

```
ev-application :
unev = cdr exp ; operands
save unev
save env
save cont
exp = car exp ; the operator
cont = ev-application-2
goto eval-dispatch
```

```
ev-application-2 :
restore cont
restore env
restore unev
proc = val ;
...
```

now we have the procedure evaluated into proc register , we need to
preserve it over the operands evaluation , so we stash it on the stack.

```
save proc
save env
save cont
cont = ev-application-3
argl = '() ; the empty list
goto ev-operands
```

Now want operands to be evaluated and result placed in argl register

```
ev-application-3 :
restore cont
restore env
restore proc
goto apply-dispatch
```


Self evaluating forms just return themselves
evaluating quoted expression is just return the thing that is quoted

```
1 (define (eval-quote exp env cont)
2   (let ((thing-quoted (car (cdr exp))))
3     (cont the-thing-quoted)
```

```

1 (define (eval-if exp env cont)
2   (let ((condition (car (cdr exp))))
3     (consequent (car (cdr (cdr exp)))))
4   (alternative (cdr (cdr (cdr exp)))))
5   (base-eval condition env
6     (lambda (bool)
7       (if bool
8         (base-eval consequent env cont)
9         (if (null? alternative)
10            (cont #f)
11            (base-eval (car alternative) env cont))))))

```

Splits if expression apart , evaluates the condition , if the condition is true then go on to evaluate the consequent. If the condition is false and there is no alternative , continue with dummy #f false value. otherwise evaluate the alternative in current environment.

continuation at line 6 (lambda (bool)) need original env and continuation cont, also need original expression.

ev-if is not called from any other site other than eval-dispatch , good modularity.

```
;; (if x y [z]) - conditional where z is optional
```

```
ev-if:
```

```
  save exp
```

```
  save env
```

```
  save cont
```

```
  exp = car [cdr exp ]   ;; x
```

```
  cont = ev-if-2
```

```
  eval-dispatch
```

```
ev-if-2:
```

```
  restore cont
```

```
  restore env
```

```
  restore exp
```

```
  if val is true
```

```
    goto ev-if-3
```

```
  exp = cdr [ cdr [cdr exp ]
```

```
  if null? exp
```

```
    val = \#f   ;; false
```

```
    goto continue
```

```
  otherwise
```

```
    exp = car [ exp ]
```

```
    goto eval-dispatch
```

```
ev-if-3:
```

```
  exp = car [ cdr [cdr exp]]   ;; y
```

```
  goto eval-dispatch
```

```

1 (define (eval-set exp env cont)
2   (let ((var (car (cdr exp))))
3     (val (car (cdr (cdr exp)))))
4     (base-eval val env (lambda (data)
5                       (env-set! var data env)
6                       (cont 'ok)))))

```

Line 1 :

Line 2 : break up set! expression into variable and the value components

Line 3 :

Line 4 : evaluate the value that will become the result to be assigned to var in the environment

Line 5 : use a primitive procedure that alters the environment

Line 6 : passing a nominal result to show the assignment has been completed. Also useful to prevent code from depending on the result of assignment as assignment can return anything it likes

Looking at the routine we can see that continuation is (lambda (data) ...)

VAR : need that symbol

ENV : environment env is used in the env-set! primitive

CONT : original cont is required also

```

;; (set! x y)
ev-assignment:
  save exp
  save env
  save cont
  exp = car [cdr [ cdr exp ]] ;; y
  cont = ev-assignment-2
  eval-dispatch

ev-assignment-2:
  restore cont
  restore env
  restore exp
  exp = car [ cdr exp ] ;; x
  ... primitive operation (env-set! exp val env) ...
  val = ok
  goto cont

```

SICP version uses unev

```
;; (set! x y)
ev-assignment:
  unev = car [ cdr exp ]
  save unev
  save env
  save cont
  exp = car [cdr [ cdr exp ]]  ;; y
  cont = ev-assignment-2
  eval-dispatch

ev-assignment-2:
  restore cont
  restore env
  restore unev
  ;; unev = x , val = evaluated y
  ... primitive operation (env-set! exp val env) ...
  val = ok
  goto cont
```

```

1 (define (eval-define exp env cont)
2   (let ((var (car (cdr exp))))
3     (val (car (cdr (cdr exp)))))
4     (base-eval val env (lambda (data)
5                       (env-define! var data env)
6                       (cont 'ok)))))

```

Line 1 :

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Looking at the routine we can see that continuation is (lambda (data) ...)

VAR : need that symbol

ENV : environment env is used in the env-set! primitive

CONT : original cont is required also

```

;; (define f y)
ev-definition:
  save exp
  save env
  save cont
  exp = car [cdr [ cdr exp ]] ;; y
  cont = ev-definition-2
  eval-dispatch

ev-definition-2:
  restore cont
  restore env
  restore exp
  exp = car [ cdr exp ] ;; f
  ... primitive operation (env-define! exp val env) ...
  val = ok
  goto cont

```

this version is new tail recursive as last expression in a begin sequence is evaluated without any further added continuations also incorporated a safety valve for empty begin expressions

```

1 (define (eval-begin exp env cont)
2   (let ((body (cdr exp)))
3     (eval-sequence body env cont)))
4
5 (define (eval-sequence body env cont)
6   (cond
7     ;; safety valve - just means (begin) evaluates to false
8     ((null? body) (cont \#f))
9     ;; last expression has no need to return -- key to tail recursion
10    ((null? (cdr body))
11     (base-eval (car body) env cont))
12    (else
13     (base-eval (car body) env
14                 (lambda (ignored)
15                   ;; totally ignore the result
16                   (eval-sequence (cdr body) env cont))))))

```

lines 1 to 3 going evaluate the expressions of the begin sequence so seems appropriate to set `unev` to be the body of the begin sequence Line 3 : simple jump to eval sequence

`eval-begin` : (set! `unev` (cdr `exp`)) goto `eval-sequence`

`ev-sequence` : Line 8 : if `unev` is null , meaning nothing to evaluate then the result of whole begin sequence is undefined. let it be false `#f` . (if (null? `unev`) (cont `#f`))

set! val `#f` goto continue

Line 10 : if this is the last expression , then all we need to do is pass this to `base-eval` , it never returns.

set! `exp` (car `unev`) goto `eval-dispatch` ; which is `base-eval`

Line 13 : otherwise evaluate the expression in the begin sequence and remember to come back to evaluate the rest

set! `exp` (car `unev`) we want to come back to somewhere where we can tell it to evaluate the rest , `ev-sequence-2` seems appropriate set! cont `ev-sequence-2`

but we need original `cont` to carry on evaluating the sequence we also need original environment we also need the other unevaluated arguments also

save-cont save-env save-unev set! `exp` (car `unev`) set! cont `ev-sequence-2` goto `eval-dispatch`

ev-sequence-2 : line 14 : we observe see each lambda in continuation passing style becomes a new goto label location in register machine basic stack discipline , we restore the stack to its original state

restore-unev restore-env restore-cont

(cdr body) set! unev (cdr unev)

line 16 : environment and continuation are the same as they are now , so we jump to ev-sequence goto ev-sequence

Here is what the translation looks like

```
(define (ev-begin)
  (set! unev (cdr exp))
  (ev-sequence))

(define (ev-sequence)
  (set! exp (car unev))
  (cond
    ((null? unev)
     (set! val \#f)
     (cont))
    ((null? (cdr unev))
     (ev-sequence-last-exp))
    (else
     S1:   (save-cont)      ;; ***
     S2:   (save-env)      ;; ***
     S3:   (save-unev)     ;; ***
           (set! cont ev-sequence-2)
           (eval-dispatch))))

(define (ev-sequence-2)
  R3:   (restore-unev)     ;; ***
  R2:   (restore-env)     ;; ***
  R1:   (restore-cont)    ;; ***
        (set! unev (cdr unev))
        (ev-sequence))

(define (ev-sequence-last-exp)
  (eval-dispatch))
```


ev-sequence HOWEVER is called from other sites in addition to eval-dispatch. This is AWFUL modularity , means we cannot think about ev-sequence in isolation but must then look at who calls ev-sequence .

notice that the saves S1 S2 S3 and restores R3 R2 R1 must happen in reverse order to each other. it doesnt matter if we save any cont env or unev , but they must be preserved on restore to their original values – stack discipline.

Observed in SICP book version saves S1 and restores R1 have been hoisted out but this has far reaching ramifications to everything that uses ev-sequence - due to stack discipline.

```
(define (ev-begin)
  (set! unev (cdr exp))
S1:  (save-cont)          ;; *****
      (ev-sequence))

(define (ev-sequence)
  (set! exp (car unev))
  (cond
    ((null? unev)
     (set! val \#f)
     (cont))
    ((null? (cdr unev))
     (ev-sequence-last-exp))
    (else
     S2:  (save-env)
     S3:  (save-unev)
          (set! cont ev-sequence-2)
          (eval-dispatch))))

(define (ev-sequence-2)
R3:  (restore-unev)
R2:  (restore-env)
      (set! unev (cdr unev))
      (ev-sequence))

(define (ev-sequence-last-exp)
R1:  (restore-cont)      ;; *****
      (eval-dispatch))
```