

9. ■ Fit a straight line and a quadratic to the data in the following table.

x	1.0	2.5	3.5	4.0	1.1	1.8	2.2	3.7
y	6.008	15.722	27.130	33.772	5.257	9.549	11.098	28.828

Which is a better fit?

10. ■ The following table displays thermal efficiencies of some early steam engines⁴. Use linear regression to predict the thermal efficiency in the year 2000.

Year	Efficiency (%)	Type
1718	0.5	Newcomen
1767	0.8	Smeaton
1774	1.4	Smeaton
1775	2.7	Watt
1792	4.5	Watt
1816	7.5	Woolf compound
1828	12.0	Improved Cornish
1834	17.0	Improved Cornish
1878	17.2	Corliss compound
1906	23.0	Triple expansion

11. The following table shows the variation of relative thermal conductivity k of sodium with temperature T . Find the quadratic that fits the data in the least-squares sense.

T (°C)	79	190	357	524	690
k	1.00	0.932	0.839	0.759	0.693

12. Let $f(x) = ax^b$ be the least-squares fit of the data (x_i, y_i) , $i = 0, 1, \dots, n$, and let $F(x) = \ln a + b \ln x$ be the least-squares fit of $(\ln x_i, \ln y_i)$ —see Table 3.4. Prove that $R_i \approx r_i/y_i$, where the residuals are $r_i = y_i - f(x_i)$ and $R_i = \ln y_i - F(x_i)$. Assume that $r_i < y_i$.
13. Determine a and b for which $f(x) = a \sin(\pi x/2) + b \cos(\pi x/2)$ fits the following data in the least-squares sense.

x	-0.5	-0.19	0.02	0.20	0.35	0.50
y	-3.558	-2.874	-1.995	-1.040	-0.068	0.677

14. Determine a and b so that $f(x) = ax^b$ fits the following data in the least-squares sense.

x	0.5	1.0	1.5	2.0	2.5
y	0.49	1.60	3.36	6.44	10.16

⁴ Source: Singer, C., Holmyard, E.J., Hall, A.R., and Williams, T.H., *A History of Technology*, Oxford Press, 1958.