OSE Lab Homework 2 DSGE

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Problem Set 1.

Exercise 1. We plug the steady states into the Euler equation in order to find A i.e.

$$\frac{1}{A^{\alpha/(1-\alpha)} - A^{1/(1-\alpha)}} = \beta \frac{\alpha A^{(\alpha-1)/(1-\alpha)}}{A^{\alpha/(1-\alpha)} - A^{1/(1-\alpha)}}$$

which means that $\beta\alpha A^{(\alpha-1)/(1-\alpha)} = 1$ so $A = \alpha\beta$.

Exercise 2. With the functional form assumption, we have $u_c = 1/c$ and $u_l = a/(l-1)$. And $F_K = \alpha e^z K^{\alpha-1} L^{1-\alpha}$ and $F_L = -(1-\alpha)e^z K^\alpha L^{-\alpha}$. The equations that are

$$\begin{aligned}
&c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \\
&1/c_t = \beta E_t\{[(r_{t+1} - \delta)(1-\tau) + 1]/c_{t+1}\} \\
&a/(l_t - 1) = w_t(1-\tau)/c_t \\
&r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha} \\
&w_t = -(1-\alpha)e^{z_t} K_t^\alpha L_t^{-\alpha} \\
&\tau[w_t l_t + (r_t - \delta)k_t] = T_t \\
&z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z.
\end{aligned}$$

I don't think we could use the same tricks as in Exercise 1 to solve for the policy function in this case.

Exercise 3. We have $F_K = \alpha e^z K^{\alpha-1} L^{1-\alpha}$ and $F_L = -(1-\alpha)e^z K^\alpha L^{-\alpha}$. And $u_c = c^{-\gamma}$ and

$u_l = a/(l-1)$. The equations that are

$$
\begin{aligned}
&c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \\
&1/c_t = \beta E_t\{[(r_{t+1} - \delta)(1-\tau) + 1]c_{t+1}^{-\gamma}\} \\
&a/(l_t - 1) = w_t(1-\tau)c_{t+1}^{-\gamma} \\
&r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha} \\
&w_t = -(1-\alpha)e^{z_t} K_t^{\alpha} L_t^{-\alpha} \\
&\tau[w_t l_t + (r_t - \delta)k_t] = T_t \\
&z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z.
\end{aligned}
$$

Exercise 4. From the functional form, we have $u_c = c^{-\gamma}$ and $u_l = -a/(l-1)^{-\xi}$ and $F_K = e^z/\eta[\alpha K^\eta + (1-\alpha)L^\eta]^{1/\eta - 1}\alpha\eta K^{\eta-1}$ and $F_K = e^z/\eta[\alpha K^\eta + (1-\alpha)L^\eta]^{1/\eta-1}(1-\alpha)\eta K^{\eta-1}$. The equations that are

$$
\begin{aligned}
&c_t = (1-\tau)[w_t l_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \\
&1/c_t = \beta E_t\{[(r_{t+1} - \delta)(1-\tau) + 1]c_{t+1}^{-\gamma}\} \\
&a/(l_t - 1)^{-\xi} = w_t(1-\tau)c_{t+1}^{-\gamma} \\
&r_t = \alpha e^{z_t} K_t^{\alpha-1} L_t^{1-\alpha} \\
&w_t = -(1-\alpha)e^{z_t} K_t^{\alpha} L_t^{-\alpha} \\
&\tau[w_t l_t + (r_t - \delta)k_t] = T_t \\
&z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z.
\end{aligned}
$$

Exercise 5. We have $u_c = c^{-\gamma}$ and $u_l = 0$. And $F_K = \alpha K^{\alpha-1}(Le^z)^{1-\alpha}$ and $F_L = e^{z(1-\alpha)}K^\alpha(1-\alpha)L^{-\alpha}$.

$$
\begin{aligned}
&c_t = (1-\tau)[w_t + (r_t - \delta)k_t] + k_t + T_t - k_{t+1} \\
&c_t^{-\gamma} = \beta E_t\{[(r_{t+1} - \delta)(1-\tau) + 1]c_{t+1}^{-\gamma}\} \\
&r_t = \alpha K_t^{\alpha-1}(e^{z_t})^{1-\alpha} \\
&w_t = e^{z_t(1-\alpha)} K_t^{\alpha}(1-\alpha) L_t^{-\alpha} \\
&\tau[w_t + (r_t - \delta)k_t] = T_t \\
&z_t = (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \epsilon_t^z.
\end{aligned}
$$

In steady states, we have

$$
\begin{aligned}
c &= (1-\tau)[w + (r-\delta)k] + T \\
1 &= \beta[(r-\delta)(1-\tau) + 1] \\
r &= \alpha K^{\alpha-1} \\
w &= K^{\alpha}(1-\alpha) \\
\tau[w + (r-\delta)k] &= T
\end{aligned}
$$

Therefore, we have $K^* = (\frac{\beta^{-1}-1}{\alpha(1-\tau)} + \frac{\delta}{\alpha})^{1/(\alpha-1)}$. My numerical solutions are: $c = 1.4845048188495245$, $w = 1.3279527683512735$, $r = 0.12148227712137499$, $K = 7.28749795069251$ and $T = 0.07422524094247622$. My analytical solution for the steady-state capital is 7.287497950692988.

Exercise 6. The difference from the last question is that $u_l = -a(1-l)^{-\xi}$. The steady-states equations are:

$$
\begin{aligned}
c &= (1-\tau)[wl + (r-\delta)k] + T \\
1 &= \beta[(r-\delta)(1-\tau) + 1] \\
a(1-l)^{-\xi} &= w(1-\tau)c^{-\gamma} \\
r &= \alpha K^{\alpha-1}(l)^{1-\alpha} \\
w &= K^{\alpha}(1-\alpha)l^{-\alpha} \\
\tau[wl + (r-\delta)k] &= T
\end{aligned}
$$

My numerical results are: $c = 0.8607032061896154$, $w = 1.3279527685608083$, $r = 0.1214822771213749$, $K = 4.225229022912356$, $T = 0.04303516030948078$ and $L = 0.5797914531042369$.

Problem Set 2.

Exercise 1. By differentiating with respect to K_{t+2}, K_{t+1}, K_t , z_{t+1} and z_t , and evaluating these at the steady state values we can recover the Uhlig matrices:

$$F = \frac{\alpha \bar{K}^{\alpha-1}}{\bar{K}^{\alpha} - \bar{K}}$$
$$G = -\frac{\alpha \bar{K}^{\alpha-1}(\alpha + \bar{K}^{\alpha-1})}{\bar{K}^{\alpha} - \bar{K}}$$
$$H = \frac{\alpha^2 \bar{K}^{2\alpha-2}}{\bar{K}^{\alpha} - \bar{K}}$$
$$L = -\frac{\alpha \bar{K}^{2\alpha-1}}{\bar{K}^{\alpha} - \bar{K}}$$
$$M = \frac{\alpha^2 \bar{K}^{2\alpha-2}}{\bar{K}^{\alpha} - \bar{K}}$$

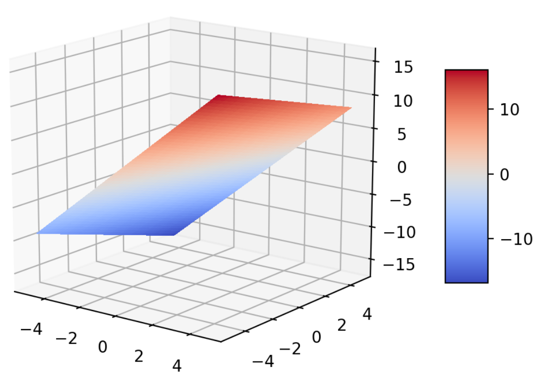
$N = I$, identity matrix in our case. Also we can then use the Riccatti and Sylveser equations to derive the scalar values P and Q.

$$P = \frac{-G \pm \sqrt{G^2 - 4FH}}{2F}$$
$$Q = -\frac{LN + M}{FN + FP + G}$$

The steady states of capital level is $\bar{K} = A^{1/(1-\alpha)}$ and $A = \alpha\beta$. The policy function is

$$K' = \bar{K} + P(K - \bar{K}) + Qz.$$

The policy function is plotted below.



Exercise 3. We have $\tilde{X}_t = X_t - \bar{X}$, $\tilde{Z}_t = Z_t - \bar{Z}$ and $\tilde{Z}_t = N\tilde{Z}_{t-1} + \varepsilon_t$. In addition, we have $\tilde{X}_t = P\tilde{X}_{t-1} + Q\tilde{Z}_t$. Plug $\tilde{X}_{t+1}$ and $\tilde{Z}_{t+1}$ into the expectation and use $E_t\varepsilon_t = 0$:

$$
\begin{aligned}
&E_t\{FP\tilde{X}_t + FQ\tilde{Z}_{t+1} + G\tilde{X}_t + H\tilde{X}_{t-1} + LN\tilde{Z}_t + M\tilde{Z}_t\} \\
=&E_t\{FPP\tilde{X}_{t-1} + FPQ\tilde{Z}_t + FQN\tilde{Z}_t + GP\tilde{X}_{t-1} + GQ\tilde{Z}_t + H\tilde{X}_{t-1} + LN\tilde{Z}_t + M\tilde{Z}_t\} \\
=&E_t\{[(FP+G)P+H]\tilde{X}_{t+1} + [(FQ+L)N + (FP+G) + M]\tilde{Z}_t\} = 0.
\end{aligned}
$$

Exercise 4. The steady-states equations are:

$$
\begin{aligned}
&c = (1-\tau)[wl + (r-\delta)k] + T \\
&1 = \beta[(r-\delta)(1-\tau) + 1] \\
&a(1-l)^{-\xi} = w(1-\tau)c^{-\gamma} \\
&r = \alpha K^{\alpha-1}(l)^{1-\alpha} \\
&w = K^{\alpha}(1-\alpha)l^{-\alpha} \\
&\tau[wl + (r-\delta)k] = T
\end{aligned}
$$

My numerical results are: $c = 0.8607032061896154$, $w = 1.3279527685608083$, $r = 0.1214822771213749$, $K = 4.225229022912356$, $T = 0.04303516030948078$ and $L = 0.5797914531042369$.

Exercise 6. I have the following results: $F = 3.1333792783220216$, $G = -9.24666619684417$, $H = 3.1973257942061446$, $L = -1.6783029001074112$, $M = 3.1973257942061446$, $P = 2.5510204081632657$ and $Q = -0.8079790534817285$.

Problem Set 6.

Exercise 1. For equation 5, we differentiate with respect to u again ($F_{...}$ shorts for derivative w.r.t. u/x)

$$
\begin{aligned}
&F_{xxx}x_ux_ux_u + 2F_{xx}x_ux_{uu} + F_{xxu}x_ux_u + F_{xux}x_ux_u + F_{xu}x_{uu} + F_{xuu}x_u \\
&+F_{xx}x_{uu}x_u + F_{xu}x_{uu} + F_xx_{uuu} + F_{xu}x_{uu} + F_{xux}x_ux_u + F_{xuu}x_u + F_{uuu} + F_{uux}x_u.
\end{aligned}
$$

Exercise 2. In equilibrium, we should have $n^s = n^d$. So we have two unknowns: π and w, and two equations, we are able to solve them. `fslove` gave me: $w = 0.6273617790446826$ and $\pi = 1.8856574119953569$.

To approximate the equilibrium condition, we could plug $n^d = n^s$ and π into the labor supply and differentiate both sides with respect to k. Note we have $\frac{\partial n^d}{\partial k} = 1/\alpha[\frac{(1-\alpha)z}{w}]^{1/\alpha - 1} - \frac{1}{\alpha}[(1-\alpha)z]^{1/\alpha} w^{-1/\alpha-1} k \frac{\partial w}{\partial k}$.

Exercise 3. The function can be viewed as $F(y(x), x)$. Take derivative w.r.t. x on both sides we have

$$-2.5(x^{0.35} + 0.9x - y)^{-3.5}(0.35x^{-0.65} + 0.9 - y') + 0.95 * 2.5(y^{0.35} + 0.9y)^{-3.5}(0.35y^{-0.65}y' + 0.9y') = 0.$$