CS145 Howework 1

Important Note: HW1 is due on **11:59 PM PT, Oct 19 (Monday, Week 3)**. Please submit through GradeScope (you will receive an invite to Gradescope for CS145 Fall 2020.).

Print Out Your Name and UID

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Before You Start

You need to first create HW1 conda environment by the given `cs145hw1.yml` file, which provides the name and necessary packages for this tasks. If you have `conda` properly installed, you may create, activate or deactivate by the following commands:

```
conda env create -f cs145hw1.yml
conda activate hw1
conda deactivate
```

OR

```
conda env create --name NAMEOFYOURCHOICE -f cs145hw1.yml
conda activate NAMEOFYOURCHOICE
conda deactivate
```

To view the list of your environments, use the following command:

```
conda env list
```

More useful information about managing environments can be found [here (https://docs.conda.io/projects/conda/en/latest/user-guide/tasks/manage-environments.html)](https://docs.conda.io/projects/conda/en/latest/user-guide/tasks/manage-environments.html).

You may also quickly review the usage of basic Python and Numpy package, if needed in coding for matrix operations.

In this notebook, you must not delete any code cells in this notebook. If you change any code outside the blocks that you are allowed to edit (between `START/END YOUR CODE HERE`), you need to highlight these changes. You may add some additional cells to help explain your results and observations.

In [1]:
```
import numpy as np
import pandas as pd
import sys
import random as rd
import matplotlib.pyplot as plt
%load_ext autoreload
%autoreload 2
```

If you can successfully run the code above, there will be no problem for environment setting.

1. Linear regression

This workbook will walk you through a linear regression example.

In [2]:
```
from hw1code.linear_regression import LinearRegression

lm=LinearRegression()
lm.load_data('./data/linear-regression-train.csv','./data/linear-regression-test.
# As a sanity check, we print out the size of the training data (1000, 100) and t
print('Training data shape: ', lm.train_x.shape)
print('Training labels shape:', lm.train_y.shape)
```

```
Training data shape:  (1000, 100)
Training labels shape: (1000,)
```

In [3]:
```
def train_and_predict(lm, beta):
    train_predict_y = lm.predict(lm.train_x, beta)
    test_predict_y = lm.predict(lm.test_x, beta)
    training_error = lm.compute_mse(train_predict_y, lm.train_y)
    testing_error = lm.compute_mse(test_predict_y, lm.test_y)
    return training_error, testing_error
```

1.1 Closed form solution

In this section, complete the `getBeta` function in `linear_regression.py` which use the close for solution of $\hat{\beta}$.

Train you model by using `lm.train('0')` function.

Print the training error and the testing error using `lm.predict` and `lm.compute_mse` given.

In [4]:
```
from hw1code.linear_regression import LinearRegression

lm=LinearRegression()
lm.load_data('./data/linear-regression-train.csv','./data/linear-regression-test.
training_error= 0
testing_error= 0
#========================#
# STRART YOUR CODE HERE  #
#========================#
beta = lm.train('0')
training_error, testing_error = train_and_predict(lm, beta)
#========================#
#   END YOUR CODE HERE   #
#========================#
print('Training error is: ', training_error)
print('Testing error is: ', testing_error)
```

```
Learning Algorithm Type:  0
Training error is:  0.08693886675396784
Testing error is:  0.11017540281675803
```

1.2 Batch gradient descent

In this section, complete the `getBetaBatchGradient` function in `linear_regression.py` which compute the gradient of the objective fuction.

Train you model by using `lm.train('1')` function.

Print the training error and the testing error using `lm.predict` and `lm.compute_mse` given.

In [5]:
```
"""add import here so can just run this cell alone when update the source code""
from hw1code.linear_regression import LinearRegression

lm=LinearRegression()
lm.load_data('./data/linear-regression-train.csv','./data/linear-regression-test.
training_error= 0
testing_error= 0
#========================#
# STRART YOUR CODE HERE  #
#========================#
beta = lm.train('1')
training_error, testing_error = train_and_predict(lm, beta)
#========================#
#   END YOUR CODE HERE   #
#========================#
print('Training accuracy is: ', training_error)
print('Testing accuracy is: ', testing_error)
```

```
Learning Algorithm Type:  1
Training accuracy is:  0.08693895533150824
Testing accuracy is:  0.11016592170824556
```

1.3 Stochastic gadient descent

In this section, complete the `getBetaStochasticGradient` function in `linear_regression.py` , which use an estimated gradient of the objective function.

Train you model by using `lm.train('2')` function.

Print the training error and the testing error using `lm.predict` and `lm.compute_mse` given.

In [6]:
```
"""add import here so can just run this cell alone when update the source code""
from hw1code.linear_regression import LinearRegression

lm=LinearRegression()
lm.load_data('./data/linear-regression-train.csv','./data/linear-regression-test.
training_error= 0
testing_error= 0
#========================#
# STRART YOUR CODE HERE  #
#========================#
beta = lm.train('2')
training_error, testing_error = train_and_predict(lm, beta)
#========================#
#   END YOUR CODE HERE   #
#========================#
print('Training accuracy is: ', training_error)
print('Testing accuracy is: ', testing_error)
```

```
Learning Algorithm Type:  2
Training accuracy is:  0.09337211730536127
Testing accuracy is:  0.11828923255919811
```

In [7]:
```
"""Normalize data"""
from hw1code.linear_regression import LinearRegression

lm=LinearRegression()
lm.load_data('./data/linear-regression-train.csv','./data/linear-regression-test.
lm.normalize()
for train_type in ['0', '1', '2']:
    beta = lm.train(train_type)
    training_error, testing_error = train_and_predict(lm, beta)
    print('Training error for ' + train_type + ' = ', training_error )
    print('Testing error for ' + train_type + ' = ', testing_error )
```

```
Learning Algorithm Type:  0
Training error for 0 =  0.08693886675396784
Testing error for 0 =  0.11017540281675804
Learning Algorithm Type:  1
Training error for 1 =  0.10024222412685467
Testing error for 1 =  0.14039955364152018
Learning Algorithm Type:  2
Training error for 2 =  0.0869952008695027
Testing error for 2 =  0.1099647262725722
```

Questions:

1. Compare the MSE on the testing dataset for each version. Are they the same? Why or why not?
2. Apply z-score normalization for eachh featrure and comment whether or not it affect the three algorithm.
3. Ridge regression is adding an L2 regularization term to the original objective function of mean squared error. The objective function become following:

$$J(\beta) = \frac{1}{2n} \sum_i \left(x_i^T \beta - y_i\right)^2 + \frac{\lambda}{2n} \sum_j \beta_j^2,$$

where $\lambda \leq 0$, which is a hyper parameter that controls the trade off. Take the derivative of this provided objective function and derive the closed form solution for β.

Your answer here:

1. The testing errors are not the same and slightly different. The Closed Form and Batch Gradient performed similarly while the Stochastic Graident performed a bit worse. I think Closed Form and Batch Gradient all reached the optimum and the little difference is because Batch Gradient has learning rate which prevents it from achieving the absolute best in limited iterations. The Stochastic Graidient performed worse maybe because the learning rate is smaller and it converges much slower.
2. It does not affect the Closed Form algorithm but affects Batch Gradient and Stochastic Gradient algorithms.
3. The matrix form of objective function can be written as $\frac{1}{2n}(X\beta - y)^T(X\beta - y) + \frac{\lambda}{2n}\beta^T\beta$
Taking derivative of the matrix form as 0: $J(\beta)' = (X^TX\beta - X^Ty)/n + \frac{\lambda}{n}\beta = 0$
So $(X^TX + \lambda)\beta = X^Ty$, the closed form solution of β is

$$\beta = \quad (X^TX + \lambda I)^{-1}X^Ty$$

2. Logistic regression

This workbook will walk you through a logistic regression example.

In [8]:
```
from hw1code.logistic_regression import LogisticRegression

lm=LogisticRegression()
lm.load_data('./data/logistic-regression-train.csv','./data/logistic-regression-t
# As a sanity chech, we print out the size of the training data (1000, 5) and tra
print('Training data shape: ', lm.train_x.shape)
print('Training labels shape:', lm.train_y.shape)
```

```
Training data shape:  (1000, 5)
Training labels shape: (1000,)
```

In [9]:
```
def train_and_predict_logistic(lm, beta):
    train_predict_y = lm.predict(lm.train_x, beta)
    test_predict_y = lm.predict(lm.test_x, beta)
    training_accuracy = lm.compute_accuracy(train_predict_y, lm.train_y)
    testing_accuracy = lm.compute_accuracy(test_predict_y, lm.test_y)
    return training_accuracy, testing_accuracy
```

2.1 Batch gradiend descent

In this section, complete the `getBeta_BatchGradient` in `logistic_regression.py` , which compute the gradient of the log likelihoood function.

Complete the `compute_avglogL` function in `logistic_regression.py` for sanity check.

Train you model by using `lm.train('0')` function.

And print the training and testing accuracy using `lm.predict` and `lm.compute_accuracy` given.

In [10]:
```
"""add import here so can just run this cell alone when update the source code"""
from hw1code.logistic_regression import LogisticRegression

lm=LogisticRegression()
lm.load_data('./data/logistic-regression-train.csv','./data/logistic-regression-t
training_accuracy= 0
testing_accuracy= 0
#========================#
# STRART YOUR CODE HERE  #
#========================#
lm.normalize()
beta = lm.train('0')
training_accuracy, testing_accuracy = train_and_predict_logistic(lm, beta)
#========================#
#   END YOUR CODE HERE   #
#========================#
print('Training accuracy is: ', training_accuracy)
print('Testing accuracy is: ', testing_accuracy)
```

```
average logL for iteration 0: -0.48427563333314566
average logL for iteration 1000: -0.46010037535085324
average logL for iteration 2000: -0.46010037535085324
average logL for iteration 3000: -0.46010037535085324
average logL for iteration 4000: -0.46010037535085324
average logL for iteration 5000: -0.46010037535085324
average logL for iteration 6000: -0.46010037535085324
average logL for iteration 7000: -0.46010037535085324
average logL for iteration 8000: -0.46010037535085324
average logL for iteration 9000: -0.46010037535085324
Training avgLogL:  -0.46010037535085324
Training accuracy is:  0.797
Testing accuracy is:  0.7534791252485089
```

2.2 Newton Raphhson

In this section, complete the `getBeta_Newton` in `logistic_regression.py` , which make use of both first and second derivative.

Train you model by using `lm.train('1')` function.

Print the training and testing accuracy using `lm.predict` and `lm.compute_accuracy` given.

In [11]:
```
"""add import here so can just run this cell alone when update the source code"""
from hw1code.logistic_regression import LogisticRegression

lm=LogisticRegression()
lm.load_data('./data/logistic-regression-train.csv','./data/logistic-regression-t
training_accuracy= 0
testing_accuracy= 0
#========================#
# STRART YOUR CODE HERE  #
#========================#
lm.normalize()
beta = lm.train('1')
training_accuracy, testing_accuracy = train_and_predict_logistic(lm, beta)
#========================#
#   END YOUR CODE HERE   #
#========================#
print('Training accuracy is: ', training_accuracy)
print('Testing accuracy is: ', testing_accuracy)
```

```
average logL for iteration 0: -0.5564001859888966
average logL for iteration 500: -0.46010037535085324
average logL for iteration 1000: -0.46010037535085324
average logL for iteration 1500: -0.46010037535085324
average logL for iteration 2000: -0.46010037535085324
average logL for iteration 2500: -0.46010037535085324
average logL for iteration 3000: -0.46010037535085324
average logL for iteration 3500: -0.46010037535085324
average logL for iteration 4000: -0.46010037535085324
average logL for iteration 4500: -0.46010037535085324
average logL for iteration 5000: -0.46010037535085324
average logL for iteration 5500: -0.46010037535085324
average logL for iteration 6000: -0.46010037535085324
average logL for iteration 6500: -0.46010037535085324
average logL for iteration 7000: -0.46010037535085324
average logL for iteration 7500: -0.46010037535085324
average logL for iteration 8000: -0.46010037535085324
average logL for iteration 8500: -0.46010037535085324
average logL for iteration 9000: -0.46010037535085324
average logL for iteration 9500: -0.46010037535085324
Training avgLogL:  -0.46010037535085324
Training accuracy is:  0.797
Testing accuracy is:  0.7534791252485089
```

Questions:

1. Compare the accuracy on the testing dataset for each version. Are they the same? Why or why not?
2. Regularization. Similar to linear regression, an regularization term could be added to logistic regression. The objective function becomes following:

$$J(\beta) = -\frac{1}{n}\sum_i \left(y_i x_i^T \beta - \log\left(1 + \exp\{x_i^T \beta\}\right)\right) + \lambda \sum_j \beta_j^2,$$

where $\lambda \leq 0$, which is a hyper parameter that controls the trade off. Take the derivative $\frac{\partial J(\beta)}{\partial \beta_j}$ of this provided objective function and provide the batch gradient descent update.

Your answer here:

1. They are the same because both methods achieve the optimum for log likelihood and the result beta is unique.
2. The derivative of new objective function is the derivative of original J + derivate of regularization term.

$$\frac{\partial J(\beta)}{\partial \beta_j} = \sum_{i=1}^{N} x_{ij}(y_i - p_i(\beta)) + 2\lambda\beta_j$$

The batch gradient update is

$$\beta^{new} = \beta^{old} + \eta v(v_j = \sum_{i=1}^{N} x_{ij}(y_i - p_i(\beta)) + 2\lambda\beta_j)$$

2.3 Visualize the decision boundary on a toy dataset

In this subsection, you will use the same implementation for another small dataset with each datapoint x with only two features (x_1, x_2) to visualize the decision boundary of logistic regression model.

In [12]:
```
from hw1code.logistic_regression import LogisticRegression

lm=LogisticRegression(verbose = False)
lm.load_data('./data/logistic-regression-toy.csv','./data/logistic-regression-toy
# As a sanity chech, we print out the size of the training data (99,2) and traini
print('Training data shape: ', lm.train_x.shape)
print('Training labels shape:', lm.train_y.shape)
```

```
Training data shape:  (99, 2)
Training labels shape: (99,)
```

In the following block, you can apply the same implementation of logistic regression model (either in 2.1 or 2.2) to the toy dataset. Print out the $\hat{\beta}$ after training and accuracy on the train set.

In [13]:
```
training_accuracy= 0
#========================#
# STRART YOUR CODE HERE  #
#========================#
beta = lm.train('0')
train_predict_y = lm.predict(lm.train_x, beta)
training_accuracy = lm.compute_accuracy(train_predict_y, lm.train_y)
print(beta)
#========================#
#   END YOUR CODE HERE   #
#========================#
print('Training accuracy is: ', training_accuracy)
```

```
Training avgLogL:  -0.329147431295712
[-2.6205116   0.76037154  1.17194674]
Training accuracy is:  0.8888888888888888
```

Next, we try to plot the decision boundary of your learned logistic regression classifier. Generally, a decision boundary is the region of a space in which the output label of a classifier is ambiguous. That is, in the given toy data, given a datapoint $x = (x_1, x_2)$ on the decision boundary, the logistic regression classifier cannot decide whether $y = 0$ or $y = 1$.

Question

Is the decision boundary for logistic regression linear? Why or why not?

Your answer here:

Yes it is linear as there are only two variables involved with only first indexes.

Draw the decision boundary in the following cell. Note that the code to plot the raw data points are given. You may need `plt.plot` function (see [here (https://matplotlib.org/tutorials/introductory/pyplot.html)](https://matplotlib.org/tutorials/introductory/pyplot.html)).

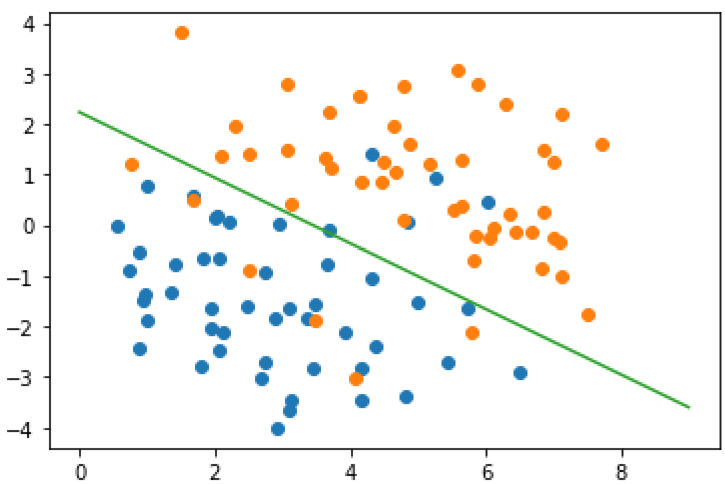
In [14]:
```
# scatter plot the raw data
df = pd.concat([lm.train_x, lm.train_y], axis=1)
groups = df.groupby("y")
for name, group in groups:
    plt.plot(group["x1"], group["x2"], marker="o", linestyle="", label=name)

# plot the decision boundary on top of the scattered points
#========================#
# STRART YOUR CODE HERE  #
#========================#

x1_values = [i for i in np.arange(0,9,0.01)]
x2_values = []

# boundary: x1*beta[1] + x2*beta[2] + beta[0] = 0
for x1 in x1_values:
    x2 = (-beta[0] - x1*beta[1]) / beta[2]
    x2_values.append(x2)

plt.plot(x1_values, x2_values)
#========================#
#   END YOUR CODE HERE   #
#========================#
plt.show()
```



End of Homework 1 :)

After you've finished the homework, please print out the entire `ipynb` notebook and two `py` files into one PDF file. Make sure you include the output of code cells and answers for questions. Prepare submit it to GradeScope.

```python
import pandas as pd
import numpy as np
import sys
import random as rd

#insert an all-one column as the first column
def addAllOneColumn(matrix):
    n = matrix.shape[0] #total of data points
    p = matrix.shape[1] #total number of attributes

    newMatrix = np.zeros((n,p+1))
    newMatrix[:,1:] = matrix
    newMatrix[:,0] = np.ones(n)

    return newMatrix

# Reads the data from CSV files, converts it into Dataframe and returns x and y dataframes
def getDataframe(filePath):
    dataframe = pd.read_csv(filePath)
    y = dataframe['y']
    x = dataframe.drop('y', axis=1)
    return x, y

# train_x and train_y are numpy arrays
# function returns value of beta calculated using (0) the formula beta = (X^T*X)^-1)*(X^T*Y)
def getBeta(train_x, train_y):
    n = train_x.shape[0] #total of data points
    p = train_x.shape[1] #total number of attributes

    beta = np.zeros(p)
    #========================#
    # STRART YOUR CODE HERE  #
    #========================#

    x_transpose = np.transpose(train_x)
    left_operand = np.linalg.inv(np.matmul(x_transpose, train_x))
    right_operand = np.matmul(x_transpose, train_y)
    beta = np.matmul(left_operand, right_operand)

    #========================#
    #   END YOUR CODE HERE   #
    #========================#
    return beta

# train_x and train_y are numpy arrays
# lr (learning rate) is a scalar
# function returns value of beta calculated using (1) batch gradient descent
def getBetaBatchGradient(train_x, train_y, lr, num_iter):
    beta = np.random.rand(train_x.shape[1])

    n = train_x.shape[0] #total of data points
    p = train_x.shape[1] #total number of attributes


    beta = np.random.rand(p)
    #update beta interatively
    for iter in range(0, num_iter):
       deriv = np.zeros(p)
```

```
       for i in range(n):
           #========================#
           # STRART YOUR CODE HERE  #
           #========================#

            # gradient of OLS(1/2 (yi-xi)^2)
            cur_deriv = train_x[i] * (np.dot(beta, np.transpose(train_x[i])) - train_y[i])
            deriv = np.add(deriv, cur_deriv)

           #========================#
           #   END YOUR CODE HERE   #
           #========================#
       deriv = deriv / n
       beta = beta - deriv.dot(lr)
    return beta

# train_x and train_y are numpy arrays
# lr (learning rate) is a scalar
# function returns value of beta calculated using (2) stochastic gradient descent
def getBetaStochasticGradient(train_x, train_y, lr):
    n = train_x.shape[0] #total of data points
    p = train_x.shape[1] #total number of attributes

    beta = np.random.rand(p)

    epoch = 100
    for iter in range(epoch):
        indices = list(range(n))
        rd.shuffle(indices)
        for i in range(n):
            idx = indices[i]
           #========================#
           # STRART YOUR CODE HERE  #
           #========================#

           # use np.multiply instead of * here to avoid overflow
           # needs to multiply lr first to avoid overflow
            coefficient = lr * (train_y[idx] - np.dot(np.transpose(train_x[idx]), beta))
            cur_update = np.multiply(coefficient, train_x[idx])
            beta = np.add(beta, cur_update)

           #========================#
           #   END YOUR CODE HERE   #
           #========================#
    return beta


# Linear Regression implementation
class LinearRegression(object):
    # Initializes by reading data, setting hyper-parameters, and forming linear model
    # Forms a linear model (learns the parameter) according to type of beta (0 - closed form, 1 - batch gradient, 2 - stochastic gradient)
    # Performs z-score normalization if z_score is 1
    def __init__(self,lr=0.005, num_iter=1000):
        self.lr = lr
        self.num_iter = num_iter
        self.train_x = pd.DataFrame()
        self.train_y = pd.DataFrame()
```

```python
        self.test_x = pd.DataFrame()
        self.test_y = pd.DataFrame()
        self.algType = 0
        self.isNormalized = 0

    def load_data(self, train_file, test_file):
        self.train_x, self.train_y = getDataframe(train_file)
        self.test_x, self.test_y = getDataframe(test_file)

    def normalize(self):
        # Applies z-score normalization to the dataframe and returns a normalized dataframe
        self.isNormalized = 1
        means = self.train_x.mean(0)
        std = self.train_x.std(0)
        self.train_x = (self.train_x - means).div(std)
        self.test_x = (self.test_x - means).div(std)

    # Gets the beta according to input
    def train(self, algType):
        self.algType = algType
        newTrain_x = addAllOneColumn(self.train_x.values) #insert an all-one column as the first column
        print('Learning Algorithm Type: ', algType)
        if(algType == '0'):
            beta = getBeta(newTrain_x, self.train_y.values)
            #print('Beta: ', beta)

        elif(algType == '1'):
            beta = getBetaBatchGradient(newTrain_x, self.train_y.values, self.lr, self.num_iter)
            #print('Beta: ', beta)
        elif(algType == '2'):
            # change learning rate to 0.0005 to converge
            beta = getBetaStochasticGradient(newTrain_x, self.train_y.values, 0.0005)
            #print('Beta: ', beta)
        else:
            print('Incorrect beta_type! Usage: 0 - closed form solution, 1 - batch gradient descent, 2 - stochastic gradient descent')


        return beta

    # Predicts the y values on given data and learned beta
    def predict(self,x, beta):
        newTest_x = addAllOneColumn(x)
        self.predicted_y = newTest_x.dot(beta)
        return self.predicted_y


    # predicted_y and y are the predicted and actual y values respectively as numpy arrays
    # function returns the mean squared error (MSE) value for the test dataset
    def compute_mse(self,predicted_y, y):
        mse = np.sum((predicted_y - y)**2)/predicted_y.shape[0]
        return mse
```

```python
# -*- coding: utf-8 -*-

import pandas as pd
import numpy as np
import sys
import random as rd

#insert an all-one column as the first column
def addAllOneColumn(matrix):
    n = matrix.shape[0] #total of data points
    p = matrix.shape[1] #total number of attributes

    newMatrix = np.zeros((n,p+1))
    newMatrix[:,0] = np.ones(n)
    newMatrix[:,1:] = matrix


    return newMatrix

# Reads the data from CSV files, converts it into Dataframe and returns x and y dataframes
def getDataframe(filePath):
    dataframe = pd.read_csv(filePath)
    y = dataframe['y']
    x = dataframe.drop('y', axis=1)
    return x, y

# sigmoid function
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

# compute average logL
def compute_avglogL(X,y,beta):
    eps = 1e-50
    n = y.shape[0]
    avglogL = 0
    #========================#
    # STRART YOUR CODE HERE  #
    #========================#
    total_logL = 0
    for i in range(n):
        # logL = yiXi^TB - log(1 + exp{xi^TB})
        XiT_beta = np.dot(np.transpose(X[i]), beta)
        left_oper = np.multiply(y[i], XiT_beta)
        right_oper = np.log(1 + np.exp(XiT_beta))
        total_logL += (left_oper - right_oper)
    avglogL = total_logL / n
    #========================#
    #   END YOUR CODE HERE   #
    #========================#
    return avglogL


# train_x and train_y are numpy arrays
# lr (learning rate) is a scalar
# function returns value of beta calculated using (0) batch gradient descent
def getBeta_BatchGradient(train_x, train_y, lr, num_iter, verbose):
    beta = np.random.rand(train_x.shape[1])

    n = train_x.shape[0] #total of data points
```

```python
    p = train_x.shape[1] #total number of attributes


    beta = np.random.rand(p)
    #update beta interatively
    for iter in range(0, num_iter):
        #========================#
        # STRART YOUR CODE HERE  #
        #========================#

        """
        logL = yiXi^TB - log(1 + exp{xi^TB})
        deriv = sum yixij - sum x_ij exp{b^Txo}/ (1+exp{bTxi})
        """

        deriv = np.zeros(p)
        for j in range(p):
            jth_deriv = 0
            for i in range(n):
                betaT_xi = np.dot(beta, train_x[i])
                yi_xij = train_y[i] * train_x[i][j]
                pi_xij = train_x[i][j] * np.exp(betaT_xi) / (1 + np.exp(betaT_xi))
                jth_deriv = jth_deriv + yi_xij - pi_xij
            deriv[j] += jth_deriv
        beta = beta + np.multiply(lr, deriv)
        #========================#
        #   END YOUR CODE HERE   #
        #========================#
        if(verbose == True and iter % 1000 == 0):
            avgLogL = compute_avglogL(train_x, train_y, beta)
            print(f'average logL for iteration {iter}: {avgLogL} \t')
    return beta

# train_x and train_y are numpy arrays
# function returns value of beta calculated using (1) Newton-Raphson method
def getBeta_Newton(train_x, train_y, num_iter, verbose):
    n = train_x.shape[0] #total of data points
    p = train_x.shape[1] #total number of attributes

    beta = np.random.rand(p)
    for iter in range(0, num_iter):
        #========================#
        # STRART YOUR CODE HERE  #
        #========================#

        # calculate hessian matrix
        # -sum X_ij X_in p_i(beta)(1 - p_i(beta))
        hessian = np.zeros((p, p))
        for row in range(p):
            for col in range(p):
                for i in range(n):
                    betaT_xi = np.dot(beta, train_x[i])
                    pi_beta = np.exp(betaT_xi) / (1 + np.exp(betaT_xi))
                    hessian[row][col] -= (train_x[i][row] * train_x[i][col] * pi_beta * (1 - pi_beta))

        # calculate first derivative same as getBeta_BatchGradient
        deriv = np.zeros(p)
        for j in range(p):
            jth_deriv = 0
```

```python
            for i in range(n):
                betaT_xi = np.dot(beta, train_x[i])
                yi_xij = train_y[i] * train_x[i][j]
                pi_xij = train_x[i][j] * np.exp(betaT_xi) / (1 + np.exp(betaT_xi))
                jth_deriv = jth_deriv + yi_xij - pi_xij
            deriv[j] += jth_deriv

        beta = beta - np.matmul(np.linalg.inv(hessian), deriv)
        #========================#
        #   END YOUR CODE HERE   #
        #========================#
        if(verbose == True and iter % 500 == 0):
            avgLogL = compute_avglogL(train_x, train_y, beta)
            print(f'average logL for iteration {iter}: {avgLogL} \t')
    return beta



# Logistic Regression implementation
class LogisticRegression(object):
    # Initializes by reading data, setting hyper-parameters
    # Learns the parameter using (0) Batch gradient (1) Newton-Raphson
    # Performs z-score normalization if isNormalized is 1
    # Print intermidate training loss if verbose = True
    def __init__(self,lr=0.005, num_iter=10000, verbose = True):
        self.lr = lr
        self.num_iter = num_iter
        self.verbose = verbose
        self.train_x = pd.DataFrame()
        self.train_y = pd.DataFrame()
        self.test_x = pd.DataFrame()
        self.test_y = pd.DataFrame()
        self.algType = 0
        self.isNormalized = 0


    def load_data(self, train_file, test_file):
        self.train_x, self.train_y = getDataframe(train_file)
        self.test_x, self.test_y = getDataframe(test_file)

    def normalize(self):
        # Applies z-score normalization to the dataframe and returns a normalized
dataframe
        self.isNormalized = 1
        data = np.append(self.train_x, self.test_x, axis = 0)
        means = data.mean(0)
        std = data.std(0)
        self.train_x = (self.train_x - means).div(std)
        self.test_x = (self.test_x - means).div(std)

    # Gets the beta according to input
    def train(self, algType):
        self.algType = algType
        newTrain_x = addAllOneColumn(self.train_x.values) #insert an all-one column
as the first column
        if(algType == '0'):
            beta = getBeta_BatchGradient(newTrain_x, self.train_y.values, self.lr,
self.num_iter, self.verbose)
            #print('Beta: ', beta)

```

```
        elif(algType == '1'):
            beta = getBeta_Newton(newTrain_x, self.train_y.values, self.num_iter, self.verbose)
            #print('Beta: ', beta)
        else:
            print('Incorrect beta_type! Usage: 0 - batch gradient descent, 1 - Newton-Raphson method')

        train_avglogL = compute_avglogL(newTrain_x, self.train_y.values, beta)
        print('Training avgLogL: ', train_avglogL)

        return beta

    # Predict on given data x with learned parameter beta
    def predict(self, x, beta):
        newTest_x = addAllOneColumn(x)
        self.predicted_y = (sigmoid(newTest_x.dot(beta))>=0.5)
        return self.predicted_y

    # predicted_y and y are the predicted and actual y values respectively as numpy arrays
    # function returns the accuracy
    def compute_accuracy(self,predicted_y, y):
        acc = np.sum(predicted_y == y)/predicted_y.shape[0]
        return acc


```