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CS181 Winter 2019 – Problem Set 2

Due Tuesday, February 5, 11:59 pm

- Please write your student ID **and the names of anyone you collaborated with** in the spaces provided and attach this sheet to the front of your solutions. **Please do not include your name anywhere since the homework will be blind graded.**
- An extra credit of **5%** will be granted to solutions written using LaTeX. Here is one place where you can create LaTeXdocuments for free: `https://www.sharelatex.com/`. The link also has tutorials to get you started. There are several other editors you can use.
- If you are writing solutions by hand, please write your answers in a neat and readable hand-writing.
- Always explain your answers. When a proof is requested, you should provide a rigorous proof.
- If you don't know the answer, write "I don't know" along with a clear explanation of what you tried. For example: "I couldn't figure this out. I think the following is a start, that is correct, but I couldn't figure out what to do next. [[Write down a start to the answer that you are sure makes sense.]] Also, I had the following vague idea, but I couldn't figure out how to make it work. [[Write down vague ideas.]]" At least 20% will be given for such an answer.

  Note that if you write things that do not make any sense, no points will be given.
- The homework is expected to take anywhere between 8 to 14 hours. You are advised to start early.
- Submit your homework online on the course webpage on CCLE. You can also hand it in at the end of any class before the deadline.

Note: *All questions in the problem sets are challenging; you should not expect to know how to answer any question before trying to come up with innovative ideas and insights to tackle the question. If you want to do some practice problems before trying the questions on the problem set, we suggest trying problems 1.17 and 1.23 from the book. Do not turn in solutions to problems from the book.*

1 Problem1

If both shuffle (L_1, L_2) and shuffle $(L_1, \overline{L_2})$ are regular. Then because the regular class is closed under union, so shuffle $(L_1, L_2) \cup$ shuffle $(L_1, \overline{L_2}$ is regular. Because $L_2 \cup \overline{L_2} = \Sigma^*$, this is shuffle (L_1, Σ^*). In discussion, we show that L_{alt} is regular if L is regular where $L_{alt} = \{x | \exists$ y such that $x_1x_2x_3... = y_1y_3y_5...\}$. In this case shuffle $(L_1, \Sigma^*)_{alt}$ is just L_1. So L_1 is regular which contradicts with the condition that L_1 is non-regular.
By contradiction, both shuffle (L_1, L_2) and shuffle $(L_1, \overline{L_2})$ cannot be regular.

2 Problem2

(a) Let the pumping length q = 1. $\forall a \in L_2$,$a \in L_1$ or $a \in b^*$. If $a \in b^* = b^m$, then x = ϵ, y=b, z = b^{m-1}. Then $\forall i > 0, xy^iz = b^{i+m-1} \in b^* \subset L_2$. If $a \in L_1 = a^mb^n(m > 0)$,then because q=1, then x= ϵ, y=a, z= $a^{m-1}b^n$, so $\forall i > 0, xy^iz = a^{m+i-1}b^n \in L_1 \subset L_2$.If m=0, then this is b^n which definitely satisfies pumping lemma because it will be in b^*. So L_2 satisfies pumping lemma if q=1.

(b) Let L be a regular language, and M the DFA for L with $|Q|$ number of states. Let p = $|Q|$, and $\omega \in L$ such that $\omega = xyz, |y| \geq p$. Let M be at $q_{|x|}$ when it finishes reading x, at $q_{|x|+|y|}$ when it finishes reading y. Then there are $|y| + 1$ i between, by pigeonhole theorem, $\exists i, j$ between $|x|$ and $|x| + |y|$ such that $q_i = q_j$. Let a = the string it reads from $q_{|x|}$ to q_i, b = the string it reads from q_i to q_j, c = the string it reads from q_j to $q_{|x|+|y|}$. Then $\forall i > 0, xab^icz \in L$.

(c)Assume L_2 is regular and let p the pumping length of general pumping lemma. Let $x \in L_2 = ab^q(q > p)$. x = ab^{q-p}, $y = b^p$ as $|y| \geq p$, $z = \epsilon$. Then there is $a_1b_1c_1$ such that $a_1 = b^m, b_1 = b^n, c_1 = b^{p-m-n}$. So $xab^icz = ab^{q+ni-n}$,lei i = q+1, $xab^icz = ab^{q+qn} = ab^{q(n+1)}$. As q,n+1 ¿ 1, q(n+1) is not prime. So $xab^icz \notin L_2$. It contradicts with pumping lemma, so L_2 is not regular.

3 Problem3

Let L be the language 0^*21^*, this is regular because it can be recognized by a simple DFA with

Q = $\{q_0, q_1\}$
$\Sigma = \{0, 1, 2\}$
$q_0 = q_0$
$F = \{q_1\}$
$\delta(q_0, 0) = q_0$, $\delta(q_0, 2) = q_1$, $\delta(q_1, 1) = q_1$

There are 3 conditions for $L_{\frac{1}{3}-\frac{1}{3}}$.
If 2 is in y, then x must be all 0s and z must be all 1s, and $|x| = |z|$. In this case, $L_{\frac{1}{3}-\frac{1}{3}} = \{0^n1^n \| n \geq 1\}$ as we can just have 2 and add 0 to the front, 1 to the back until $|y| = |x|$.
If 2 is in x, then y and z must be all 1s. So x is 0^a21^b, y and z are 1^{a+b+1}, $L_{\frac{1}{3}-\frac{1}{3}}$ in this case is $\{0^a21^{a+2b+1} | a, b \geq 0\}$. If 2 is in z, then x and y must be all 0s. So z is 0^a21^b, x and y are 0^{a+b+1}, $L_{\frac{1}{3}-\frac{1}{3}}$ in this case is $\{0^{2a+b+1}21^b | a, b \geq 0\}$.

So $L_{\frac{1}{3}-\frac{1}{3}}$ is the union of these 3 cases. $L_{\frac{1}{3}-\frac{1}{3}} \cap (0, 1)^* = \{0^n1^n \| n \geq 1\} = L_{eq}$ as the other 2 cases all have 2 in the language. Two intersection of regular languages must be regular, and so if $L_{\frac{1}{3}-\frac{1}{3}}$ is regular, L_{qe} is regular which is obviously contradiction to what we proved before in class. So $L_{\frac{1}{3}-\frac{1}{3}}$ need not be regular if L is regular.