

Student ID: _____
Collaborators: _____

CS181 Winter 2019 – Problem Set 2

Due Tuesday, February 5, 11:59 pm

- Please write your student ID **and the names of anyone you collaborated with** in the spaces provided and attach this sheet to the front of your solutions. **Please do not include your name anywhere since the homework will be blind graded.**
- An extra credit of **5%** will be granted to solutions written using L^AT_EX. Here is one place where you can create L^AT_EX documents for free: <https://www.sharelatex.com/>. The link also has tutorials to get you started. There are several other editors you can use.
- If you are writing solutions by hand, please write your answers in a neat and readable hand-writing.
- Always explain your answers. When a proof is requested, you should provide a rigorous proof.
- If you don't know the answer, write "I don't know" along with a clear explanation of what you tried. For example: "I couldn't figure this out. I think the following is a start, that is correct, but I couldn't figure out what to do next. [[Write down a start to the answer that you are sure makes sense.]] Also, I had the following vague idea, but I couldn't figure out how to make it work. [[Write down vague ideas.]]" At least 20% will be given for such an answer.
Note that if you write things that do not make any sense, no points will be given.
- The homework is expected to take anywhere between 8 to 14 hours. You are advised to start early.
- Submit your homework online on the course webpage on CCLE. You can also hand it in at the end of any class before the deadline.

Note: *All questions in the problem sets are challenging; you should not expect to know how to answer any question before trying to come up with innovative ideas and insights to tackle the question. If you want to do some practice problems before trying the questions on the problem set, we suggest trying problems 1.17 and 1.23 from the book. Do not turn in solutions to problems from the book.*

1. **(20 points)** Let L_1 and L_2 be languages and define

$$\text{shuffle}(L_1, L_2) = \{x_1 y_1 x_2 y_2 \dots x_n y_n \mid x_1 \dots x_n \in L_1, y_1 \dots y_n \in L_2\}.$$

Show that if the language L_1 is not regular and L_2 is any language then the languages $\text{shuffle}(L_1, L_2)$ and $\text{shuffle}(L_1, \overline{L_2})$ cannot both be regular. Recall for any language L , $\overline{L} = \Sigma^* \setminus L$ denotes the complement language of L .

Hint: Recall closure properties of regular languages.

2. **(40 points)** In this problem we investigate the limits of the Pumping Lemma as it was stated in class and look for an alternative that remedies one of these shortcomings.

- (a) **(10 points)** Let L_1 be the language

$$L_1 = \{a^i b^p \mid i \geq 0 \text{ and } p \text{ is a prime}\}.$$

Prove that the language $L_2 = b^* \cup L_1$ satisfies the conditions of the Pumping Lemma. I.e. show that there exists a $q \in \mathbb{N}$ such that for every word $w \in L_2$ with $|w| \geq q$ we can write $w = xyz$ such that $|xy| \leq q$, $|y| > 0$, and for every $i \geq 0$, $xy^i z \in L_2$.

- (b) **(20 points)** Prove the following generalization of the Pumping Lemma:

Let L be a regular language. There exists a $q \in \mathbb{N}$ such that for every $w \in L$ and every partition of w into $w = xyz$ with $|y| \geq q$ there are strings a, b, c such that $y = abc$, $|b| > 0$, and for all $i \geq 0$, $xab^i cz \in L$.

- (c) **(10 points)** Prove that the language L_2 is not regular.

3. **(40 points)** For a language L over alphabet Σ , we define

$$L_{\frac{1}{3}-\frac{1}{3}} = \{xz \in \Sigma^* \mid \exists y \in \Sigma^* \text{ with } |x| = |y| = |z| \text{ such that } xyz \in L\}.$$

Prove that if L is regular, then $L_{\frac{1}{3}-\frac{1}{3}}$ need not be regular.

Hint: Consider the language $0^ 2 1^*$ and recall closure properties of regular languages*