This is the definition for the language for this thesis project. It's based on "An Introduction to Algebraic Effects and Handlers" by Matija Pretnar. We're making the following changes:

- Functions and operations can take multiple values. This is because partial application is cumbersome with effects. Some operations (e.g. catch) also require multiple arguments.
- We add a global scope of functions and effects.
- We add type information to global declarations so they can be type-checked separately.
- We add more value types (integers, tuples, strings, unit) to make more interesting effects.
- We add elaborations for higher-order effects.
- The entrypoint of the program is the main function in the global scope.

1 Syntax definition

$$\text{program } p ::= d\ p$$

$$\begin{array}{rlr}
\text{value } v ::= & x & \text{variable} \\
& \mid () & \text{unit} \\
& \mid \textbf{true} \mid \textbf{false} & \text{boolean} \\
& \mid n & \text{integer} \\
& \mid s & \text{string} \\
& \mid h & \text{handler} \\
& \mid e & \text{elaboration} \\
& \mid \textbf{fun}\ (x_1, \ldots, x_n) \mapsto c & \text{anonymous function} \\
& \mid (v_1, \ldots, v_n) & \text{tuple}
\end{array}$$

$$\begin{array}{rl}
\text{handler } h ::= & \textbf{handler}\ \{ \\
& \quad \textbf{return}\ x \ \mapsto\ c_r, \\
& \quad op_1\ (x_1, \ldots, x_n; k) \mapsto c_1, \\
& \quad \ldots, \\
& \quad op_m\ (x_1, \ldots, x_n; k) \mapsto c_m, \\
& \}
\end{array}$$

$$
\begin{aligned}
\text{elaboration } e ::= {} & \textbf{elaboration } \{ \\
& \quad op_1\ (c_1, \ldots, c_n; k) \mapsto c_1, \\
& \quad \ldots, \\
& \quad op_m\ (c_1, \ldots, c_n; k) \mapsto c_m, \\
& \}
\end{aligned}
$$

$$
\begin{array}{rlr}
\text{computation } c ::= & \textbf{return } v & \text{pure value} \\
& \mid op\,(v_1, \ldots, v_n; y.c) & \text{algebraic operation} \\
& \mid op_h\,(c_1, \ldots, c_n; y.c) & \text{higher-order operation} \\
& \mid \textbf{do } x \leftarrow c_1 \textbf{ in } c_2 & \text{sequencing} \\
& \mid \textbf{if } v \textbf{ then } c_1 \textbf{ else } c_2 & \text{conditional} \\
& \mid v(v_1, \ldots, v_n) & \text{function application} \\
& \mid \textbf{with } v \textbf{ handle } c & \text{handle algebraic effects} \\
& \mid \textbf{with } v \textbf{ elaborate } c & \text{elaborate higher-order effects}
\end{array}
$$

$$
\begin{aligned}
\text{declaration } d ::= {} & \textbf{fun } ident(x_1, T_1, \ldots, x_n : T_n) : T_c \mapsto c \\
& \mid \textbf{effect } ident\ \{ \\
& \qquad op_1(A_1, \ldots, A_m) : A_{c_1}, \\
& \qquad \ldots \\
& \qquad op_n(A_1, \ldots, A_m) : A_{c_n}, \\
& \} \\
& \mid \textbf{heffect } ident\ \{ \\
& \qquad op_1(\underline{C}_1, \ldots, \underline{C}_{m_1}) : \underline{C}_1, \\
& \qquad \ldots, \\
& \qquad op_n(\underline{C}_1, \ldots, \underline{C}_{m_n}) : \underline{C}_n, \\
& \}
\end{aligned}
$$

$$
\begin{aligned}
\text{value type } A, B ::= {} & \textbf{bool} \\
& \mid \textbf{str} \\
& \mid \textbf{int} \\
& \mid () \\
& \mid (A_1, \ldots, A_n) \\
& \mid A \rightarrow \underline{C} \\
& \mid \underline{C} \Rightarrow \underline{D}
\end{aligned}
$$

$$
\text{computation type } \underline{C}, \underline{D} ::= A!\{op_1, \ldots, op_n\}
$$

2 Semantics

$$\frac{c_1 \leadsto c_1'}{\textbf{do } x \leftarrow c_1 \textbf{ in } c_2 \leadsto \textbf{do } x \leftarrow c_1' \textbf{ in } c_2}$$

$$\frac{}{\textbf{do } x \leftarrow \textbf{return } v \textbf{ in } c \leadsto c[v/x]}$$

$$\frac{}{\textbf{do } x \leftarrow \mathit{op}\,(v_1, \ldots, v_n; y.c_{op}) \textbf{ in } c_{ret} \leadsto \mathit{op}\,(v_1, \ldots, v_n; y.\textbf{do } x \leftarrow c_{op} \textbf{ in } c_{ret})}$$

$$\frac{}{\textbf{do } x \leftarrow \mathit{op}_h\,(c_1, \ldots, c_n; y.c_{op}) \textbf{ in } c_{ret} \leadsto \mathit{op}_h\,(c_1, \ldots, c_n; y.\textbf{do } x \leftarrow c_{op} \textbf{ in } c_{ret})}$$

$$\frac{}{\textbf{if true then } c_1 \textbf{ else } c_2 \leadsto c_1}$$

$$\frac{}{\textbf{if false then } c_1 \textbf{ else } c_2 \leadsto c_2}$$

$$\frac{}{(\textbf{fun } (x_1, \ldots, x_n) \mapsto c)(v_1, \ldots, v_n) \leadsto c[v_1/x_1, \ldots, v_n/x_n]}$$

$$\frac{c \leadsto c'}{\textbf{with } h \textbf{ handle } c \leadsto \textbf{with } h \textbf{ handle } c'}$$

$$\frac{}{\textbf{with } h \textbf{ handle } (\textbf{return } v) \leadsto c_r[v/x]}$$

$$\frac{}{\textbf{with } h \textbf{ handle } \mathit{op}_i\,(v_1, \ldots, v_n; y.c) \leadsto c_i[v_1/x_1, \ldots, v_n/x_n, (\textbf{fun } y \mapsto \textbf{with } h \textbf{ handle } c)/k]}$$

$$\frac{}{\textbf{with } h \textbf{ handle } \mathit{op}\,(v_1, \ldots, v_n; y.c) \leadsto \mathit{op}\,(v_1, \ldots, v_n; y.\textbf{with } h \textbf{ handle } c)}$$

$$\frac{c_i' = \textbf{with } h \textbf{ handle } c_i}{\textbf{with } h \textbf{ handle } \mathit{op}_h\,(c_1, \ldots, c_n; y.c) \leadsto \mathit{op}_h\,(c_1, \ldots, c_n; y.\textbf{with } h \textbf{ handle } c)}$$

$$\frac{c \leadsto c'}{\textbf{with } e \textbf{ elaborate } c \leadsto \textbf{with } e \textbf{ elaborate } c'}$$

$$\frac{}{\textbf{with } e \textbf{ elaborate } (\textbf{return } v) \leadsto \textbf{return } v}$$

$$\frac{c_i' = \textbf{with } e \textbf{ elaborate } c_i}{\textbf{with } e \textbf{ elaborate } \mathit{op}_i\,(c_1, \ldots, c_n; y.c) \leadsto c_i[c_1'/x_1, \ldots, c_n'/x_n, (\textbf{fun } y \mapsto \textbf{with } e \textbf{ elaborate } c)/k]}$$

$$\frac{}{\textbf{with } e \textbf{ elaborate } \mathit{op}\,(c_1, \ldots, c_n; y.c) \leadsto \mathit{op}\,(c_1, \ldots, c_n; y.\textbf{with } e \textbf{ elaborate } c)}$$

3 Typing judgements

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \qquad \frac{}{\Gamma \vdash \textbf{true} : \textbf{bool}} \qquad \frac{}{\Gamma \vdash \textbf{false} : \textbf{bool}}$$

$$\frac{\Gamma, x : A \vdash c : \underline{C}}{\Gamma \vdash \textbf{fun}\ x \mapsto c : A \to \underline{C}}$$

$$\frac{\begin{array}{c}\Gamma, x : A \vdash c_r : B!\Delta' \\ \big[(op_i : A_i \to B_i) \in \Sigma \quad \Gamma, x : A_i, k : B_i \to B!\Delta' \vdash c_i : B!\Delta'\big]_{1 \le i \le n} \\ \Delta \backslash \{op_i\}_{1 \le i \le n} \subseteq \Delta'\end{array}}{\Gamma \vdash \textbf{handler}\ \{\textbf{return}\ x \mapsto c_2, op_1\ (x; k) \mapsto c_1, \ldots, op_n\ (x; k) \mapsto c_n\} : A!\Delta \Rightarrow B!\Delta'}$$

$$\frac{\Gamma \vdash v : A}{\Gamma \vdash \textbf{return}\ v : A!\Delta}$$

$$\frac{(op : A_{op} \to B_{op}) \in \Sigma \quad \Gamma \vdash v : A_{op} \quad \Gamma, y : B_{op} \vdash c : A!\Delta \quad op \in \Delta}{\Gamma \vdash op\,(v; y.c) : A!\Delta}$$

$$\frac{\Gamma \vdash c_1 : A!\Delta \quad \Gamma x : A \vdash c_2 : B!\Delta}{\Gamma \vdash \textbf{do}\ x \leftarrow c_1\ \textbf{in}\ c_2 : B!\Delta} \qquad \frac{\Gamma \vdash : A \to \underline{C} \quad \Gamma \vdash v_2 : A}{\Gamma \vdash v_1 v_2 : \underline{C}}$$

$$\frac{\Gamma \vdash v : \textbf{bool} \quad \Gamma \vdash c_1 : \underline{C} \quad \Gamma \vdash c_2 : \underline{C}}{\Gamma \vdash \textbf{if}\ v\ \textbf{then}\ c_1\ \textbf{else}\ c_2 : \underline{C}} \qquad \frac{\Gamma \vdash : \underline{C} \Rightarrow \underline{D} \quad \Gamma \vdash c : \underline{C}}{\Gamma \vdash \textbf{with}\ v\ \textbf{handle}\ c : \underline{D}}$$