Homework 8

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Problem 1c:

First consider a RBT with one node. Of course, it has no red nodes, since there is only root and the root is always black according to property 2. After inserting one new node, it is inserted as RED by default. It does not matter on which side is inserted in this case. Now consider inserting a new node. If it is inserted on empty side, then it is fine. If it is inserted on same side with the previous node, then:

1. We violate property 4. If we change it to black, we violate property 5.
2. In this case a rotation left/right is required.
3. After rotation, we violate property 2. We flip colors.
4. The root is black again and the leaves are red after flipping colors.

After this level, every insertion is the same as this one, only in a different subtree. In other words, if nodes are inserted on the left side, the deepest level of the right side would not be affected and it is already red, therefore we have at least one red node. It works also if we insert nodes on the right side (symmetrically).

However there might be an exception when there are deletions. Nevertheless, the problem does not state to argue in that case, it just states to when $n > 1$.