Homework 2

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P1)

All information and files regarding this problem can be found in the ".zip" file.

P2)

a) $T(n) = 36\, T(\frac{n}{6}) + 2n$

We can apply master method for this recurrance.

Form: $T(n) = a\,T(\frac{n}{b}) + f(n)$, where

$a = 36$

$b = 6$

$f(n) = 2n$

$n^{\log_b a} = n^{\log_6 36} = n^2$

$f(n) = O(n^{\log_b a - \varepsilon})$ for $\varepsilon = 1$

$\Rightarrow T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_6 36}) = \Theta(n^2)$

The upper bound is $O(n^2)$

The lower bound is $\Omega(f(n)) = \Omega(n)$

b) $T(n) = 5T(\frac{n}{3}) + 17n^{1,2}$

Same as before we can use the master method, where

$a = 5$

$b = 3$

$f(n) = 17n^{1,2}$

$\log_3 5 \approx 1,46$

$n^{\log_3 5} = n^{1,46} \Rightarrow f(n) = O(n^{\log_a b - \varepsilon})$ for $\varepsilon \approx 0,26$

$\Rightarrow T(n) = \Theta(n^{1,46})$

$T(n) = O(n^{1,46})$

$T(n) = \Omega(n)$

c) $T(n) = 12T(\frac{n}{2}) + n^2 \lg n$

Substitution method:

Initial guess: $\boxed{T(n) = \Theta(n^3)}$

Induction:

Base case:

Since $T(n) = \Theta(1)$ for all n in the interval $1 \leq n \geq n_0$ where $n_0 = 2$, thus the base case is proven.

Assume that $T(n) \leq c_1 k^3 - c_2 k^2$ for $k < n$. Show $T(n) \leq c_1 n^3 - c_2 n^2$

$$
\begin{aligned}
T(n) &= 12T\left(\tfrac{n}{2}\right) + n^2 \lg n \\
&= 12\left(c_1\left(\tfrac{n}{2}\right)^3 - c_2\left(\tfrac{n}{2}\right)^2\right) + n^2 \lg n \\
&= \tfrac{3}{2}c_1 n^3 - 6c_2 n^2 + n^2 \lg n \\
&= \tfrac{3}{2}c_1 n^3 - \left(6c_2 n^2 - n^2 \lg n\right)
\end{aligned}
$$

$$\Rightarrow \tfrac{3}{2}c_1 n^3 - \left(6c_2 n^2 - n^2 \lg n\right) \le c_1 n^3 - c_2 n^2$$

$$\Rightarrow \tfrac{3}{2}c_1 n^3 - \left(6c_2 - \lg n\right) n^2 \le c_1 n^3 - c_2 n^2$$

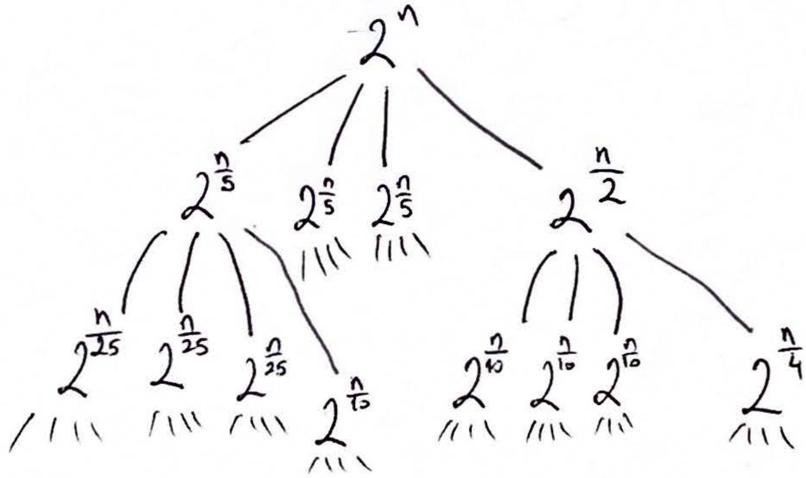
$\Theta(1) \le c_1 n^3 - c_2 n^2$ for $c_2 \ge 1$

$T(n) = \Theta(n^3)$ for $c_1 > 1$

$$\Rightarrow T(n) = O(n^3)$$
$$T(n) = \Omega(n^2)$$

d) $T(n) = 3T\left(\frac{n}{5}\right) + T\left(\frac{n}{2}\right) + 2^n$

Recursion tree method:



Cost:

2^n

$3 \cdot 2^{\frac{n}{5}} + 2^{\frac{n}{2}}$

$9 \cdot 2^{\frac{n}{25}} + 6 \cdot 2^{\frac{n}{10}} + 2^{\frac{n}{4}}$

$\cancel{\sum_{i=0}}$

The tree grows in a way that the leftmost branch is the shortest and the right most is the longest. Branches in-between vary in size because they are multiples of both 2 and 5.

Leftmost height $\Rightarrow h_1 = \log_5 n$

Rightmost height $\rightarrow h_2 = \log_2 n$

$$\sum_{i=0}^{h_1} 3^i \cdot 2^{\frac{n}{5^i}} + \sum_{j=0}^{h_2} 2^{\frac{n}{2^j}} =$$

$$= \left(2^n + 3 \cdot 2^{\frac{n}{5}} + 9 \cdot 2^{\frac{n}{25}} + \dots + 3^{h_1} \cdot \cancel{2}^{\frac{n}{25^{h_1}}}\right) + \left(2^n + 2^{\frac{n}{2}} + 2^{\frac{n}{4}} + \dots 2^{\frac{n}{h_2}}\right)$$

$$= 2^n \left(\dots \text{combined terms} \dots \right)$$

$\Rightarrow T(n) = \Theta(2^n)$ and in this case: $\cancel{\Theta}$

$T(n) = O(2^n)$ and $T(n) = \Omega(2^n)$

e) $T(n) = T\left(\frac{2n}{5}\right) + T\left(\frac{3n}{5}\right) + \Theta(n) \rightarrow \Theta(n) = f(n) = n$

$n \rightarrow n$

$\frac{2n}{5}$ $\quad \frac{3n}{5} \rightarrow n$

$\frac{4n}{25}$ $\quad \frac{6n}{25}$ $\quad \frac{6n}{25}$ $\quad \frac{9n}{25} \rightarrow n$

The height of the tree is $h = \log_5 3n = \log_5 3 + \log_5 n = k + \log_5 n$

where k is a constant.

$$\sum_{i=0}^{h} n = n \sum_{i=0}^{h} 1 = n \frac{h(h+1)}{2} = n \frac{h^2 + h}{2} = \frac{n(\log_5 n)^2 + n \log_5 n}{2}$$

As $n \to \infty$, the term $\frac{n(\log_5 n)^2}{2}$ grows faster, therefore,

$T(n) = \Theta\left(n(\log_5 n)^2\right)$

Also

$T(n) = O(n \log_5^2 n)$

$T(n) = \Omega(n \log_5^2 n)$