Assignment 3

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Problem 1:

(a) Methods are implemented in the following source files:
   (i) 'Fib_naive.cpp' → naive approach with STDIN and STDOUT
   (ii) 'Fib_naive_generator.cpp' → naive approach with generating increasing N
   (iii) 'Fib_bottom_up.cpp' → Bottom up approach with STDIN and STDOUT
   (iv) 'Fib_bottom_up_generator.cpp' → Bottom up approach with generating increasing N
   (v) 'Fib_matrix.cpp' → Matrix approach with STDIN and STDOUT
   (vi) 'Fib_matrix_generator.cpp' → Matrix approach with generating increasing N
   (vii) 'Fib_closed_form.cpp' → Closed form approach with STDIN and STDOUT
   (viii) 'Fib_closed_form_generator.cpp' → Closed form approach with generating increasing N
   There is a 'Makefile', which you can run using the command line by typing 'make' in the directory of the source files and the 'Makefile'. Then run the programs by typing ./[name_of_the_program] in the command line (**NOTE:** Generator programs might take a while, they were initially used for generating run-time data).

(b) Table can be found in 'table.pdf'

(c) In practice yes. However, if not properly regulated, **closed form approach** might not give the correct n-th fibonacci number. The reason for this is floating point precision. In my program, I used round(x) function from C++ library <cmath>.

(d) Plot can be found in 'plot.pdf'.

Problem 2:

a) Brute force multiplication in real life (3rd grade multiplicat algorithm has running time of $\Theta(n^2)$. In our case, a an have n-bits each. Brute-force algorithm is ~~pretty~~ straight forwa Multiply each bit ~~with each~~ from a with each bit of b Each time shift the ~~previous~~ next one for one place to the left relative to the first one. Consider the example:

$$\begin{array}{r} \overset{\curvearrowright 11}{1011} \cdot 1101 \curvearrowright 13 \\ \hline 1011 \\ 0000 \\ 101100 \\ 1011000 \\ \hline 10001111 \end{array}$$

$10001111 \longrightarrow 143 \Rightarrow 11 \cdot 13 = 143$ ✓

After doing that, we add all our results.

First multiplying each bit of a with each bit of b takes n^2 operations $\therefore T_1(n) = \Theta(n^2)$. Adding ~~n bit numbers~~ n bit numbers n times also takes n^2 operations $\therefore T_2(n) = \Theta(n^2)$ (Since each time we shift the number to the left one time, that can be considered constant).

The total time is $T(n) = T_1(n) + T_2(n) = \Theta(n^2) + \Theta(n^2)$

$$\Rightarrow T(n) = \Theta(2n^2) \Rightarrow \boxed{T(n) = \Theta(n^2)}$$

b) Let us do a little magic with numbers in base 10. Say, we want to ~~add~~ multiply 32 and 21.

$32 \cdot 21 = 672$, but

$32 = 10 \cdot 3 + 2$ and $21 = 10 \cdot 2 + 1 \Rightarrow 32 \cdot 21 = (3 \cdot 10 + 2)(2 \cdot 10 + 1)$

$\Rightarrow \quad 6 \cdot 10^2 + 7 \cdot 10 + 2 = 672$

Another example: $2347 \cdot 3732 = 8759004$

Also: $23 \cdot 10^2 + 47 = 2347$

$37 \cdot 10^2 + 32 = 3732$

$2347 \cdot 3732 = (23 \cdot 10^2 + 47)(37 \cdot 10^2 + 32) = 851 \cdot 10^4 + 2475 \cdot 10^2 + 1504 =$

$= 8759004 \quad \checkmark$

We can separate those terms recursively one more time.

We can do this algorith also for numbers in base 2.

$$a \cdot b = (a_l \cdot 2^{\frac{n}{2}} + a_r)(b_l \cdot 2^{\frac{n}{2}} \cdot b_r) =$$

a_l is left

$$= \textcircled{1} a_l b_l \cdot 2^n + 2^{\frac{n}{2}} \left(\textcircled{2} a_l b_r + \textcircled{3} a_r b_l \right) + \textcircled{4} a_r b_r$$

$b_l / a_l \rightarrow \frac{n}{2}$ left most bits of b/a

$b_r / a_r \rightarrow \frac{n}{2}$ rightmost bits of b/a

In the equation we still have four recursive calls (labeled 1→4)

$$\Rightarrow \quad T(n) = 4T\left(\frac{n}{2}\right) + \Theta(n) \xrightarrow{\text{master method}} T(n) = \Theta(n^2)$$

However: $a_l b_r + b_l a_r = (a_l + a_r)(b_l + b_r) - a_l b_l - a_r b_r$

$$\Rightarrow \quad a \cdot b = \textcircled{1} a_l b_l \cdot 2^n + 2^{\frac{n}{2}} \left[\textcircled{3} (a_l + a_r)(b_l + b_r) - a_l b_l - a_r b_r \right] + \textcircled{2} a_r b_r$$

$$\Rightarrow \quad T(n) = 3T\left(\frac{n}{2}\right) + \Theta(n)$$

$f(n) = \Theta(n)$

$n^{\log_b a} = n^{\log_2 3} \approx n^{1,58}$

Since

Considering $\Theta(n^{\log_b a + \varepsilon})$ for $\varepsilon \approx 0.58$ we get:

$T(n) = \Theta(n^{1,58})$

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c) $T(n) = 3T(\frac{n}{2}) + \Theta(n)$

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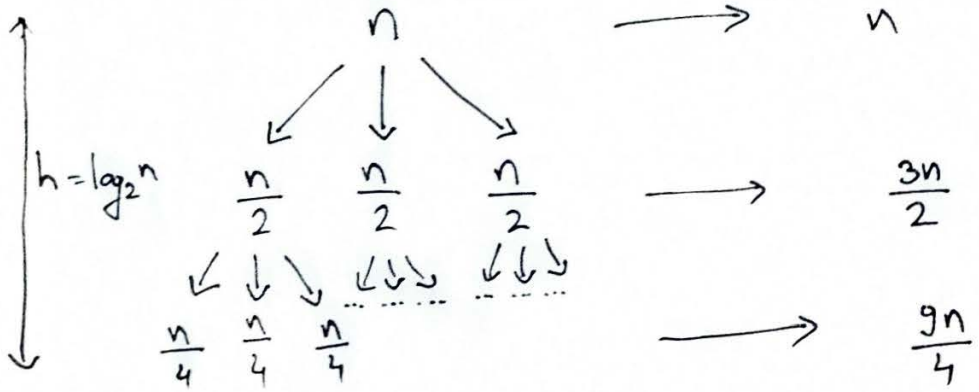
d)



$$\Rightarrow \quad n + \frac{3n}{2} + \frac{9n}{4} + \frac{27n}{8} + \dots = \sum_{k=0}^{h} \frac{3^k n}{2^k} = \frac{\frac{1}{2}(3^h - 1)}{2^{h+1} - 1} n$$

$$\Rightarrow \quad \frac{(3^h - 1)}{2^{h+2} - 2} n = \frac{3^h - 1}{2(2^{h+1} - 1)} n \approx n^{1,58}$$

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e) $T(n) = 3T(\frac{n}{2}) + \Theta(n)$

$f(n) = n$

$n^{\log_b a} = n^{\log_2 3} \approx n^{1,58}$

So using case 3, $\varepsilon > 0 \Leftrightarrow \varepsilon = 0,58$ and $2\frac{n}{3} \leq cn$ for $c < 0$, i.e., $c = \frac{2}{3} \Rightarrow T(n) = \Theta(n^{1,68})$ □