Homework for Lecture 8

1. Write a module called solver.py that contains well written codes with docstrings for the following three algorithms:

   - Newton's method
   - The Secant Method
   - The bisection Method

   Each algorithm should be written with a while loop that stops when a certain tolerance is reached (each algorithm should take an input called TOL). Then, for each algorithm, find the first n for which

   $$|x_n - x_{n-1}| < 10^{-10}$$

   Use the following initializations:
   Newton: $x_0 = 0$
   Secant: $x_0 = 0$ and $x_1 = 0.1$
   Bisection: $[a, b] = [0, 1]$

   Complete the table below:

	Newton	Secant	Bisection
nb of iterations needed to reach 10^{-10} tolerance			

2. Let's consider the equation

   $$x - \sqrt{x} = 0$$

   Obviously this can be solved by hand (what are the true solutions?). Let x^* be one of the solutions, the goal will be to find what is the smallest n such that

   $$|x_n - x^*| < 10^{-10}$$

   Note that this is different from the previous problem in which the stopping criterion was $|x_n - x_{n-1}| < 10^{-10}$.

   (a) Compute the 2 solutions of $x - \sqrt{x} = 0$ with pen and paper.

   (b) Plot $y = x - \sqrt{x}$. Answer the following question just by watching the picture.

   i. Toward which solution will Newton's method converge if the initial iterate is $p_0 = 0.5$?

   ii. Explain what is going to happen if you use Newton's method with initial iterate $p_0 = 0.1$.

   iii. Toward which solution will the secant method converge if the initial iterates are $p_0 = 0.5$ and $p_1 = 0.55$?

   (c) This is the important question. Initialize each of the four methods as follow:

   Newton: $x_0 = 0.5$
   Secant: $x_0 = 0.5$ and $x_1 = 0.55$
   Fixed point[1]: $x_0 = 0.5$
   Bisection: $[a, b] = [0.5, 1.1]$
   Note that with these initializations, each method will converges to the same solution x^*. For each algorithm, find the number of iterations needed to be within 10^{-10} of the true solution. That is, what is the smallest n so that $|x_n - x^*| < 10^{-10}$.

	Newton	Secant	fixed point	Bisection
nb of iterations needed to reach desired accuracy				

   Which method converge the fastest? the slowest?

[1]The successive iterates of the fixed point method are defined by $x_{n+1} = \sqrt{x_n}$. It turns out that these iterates will converge toward the desired solution (we haven't discuss this method in class, just trust me).