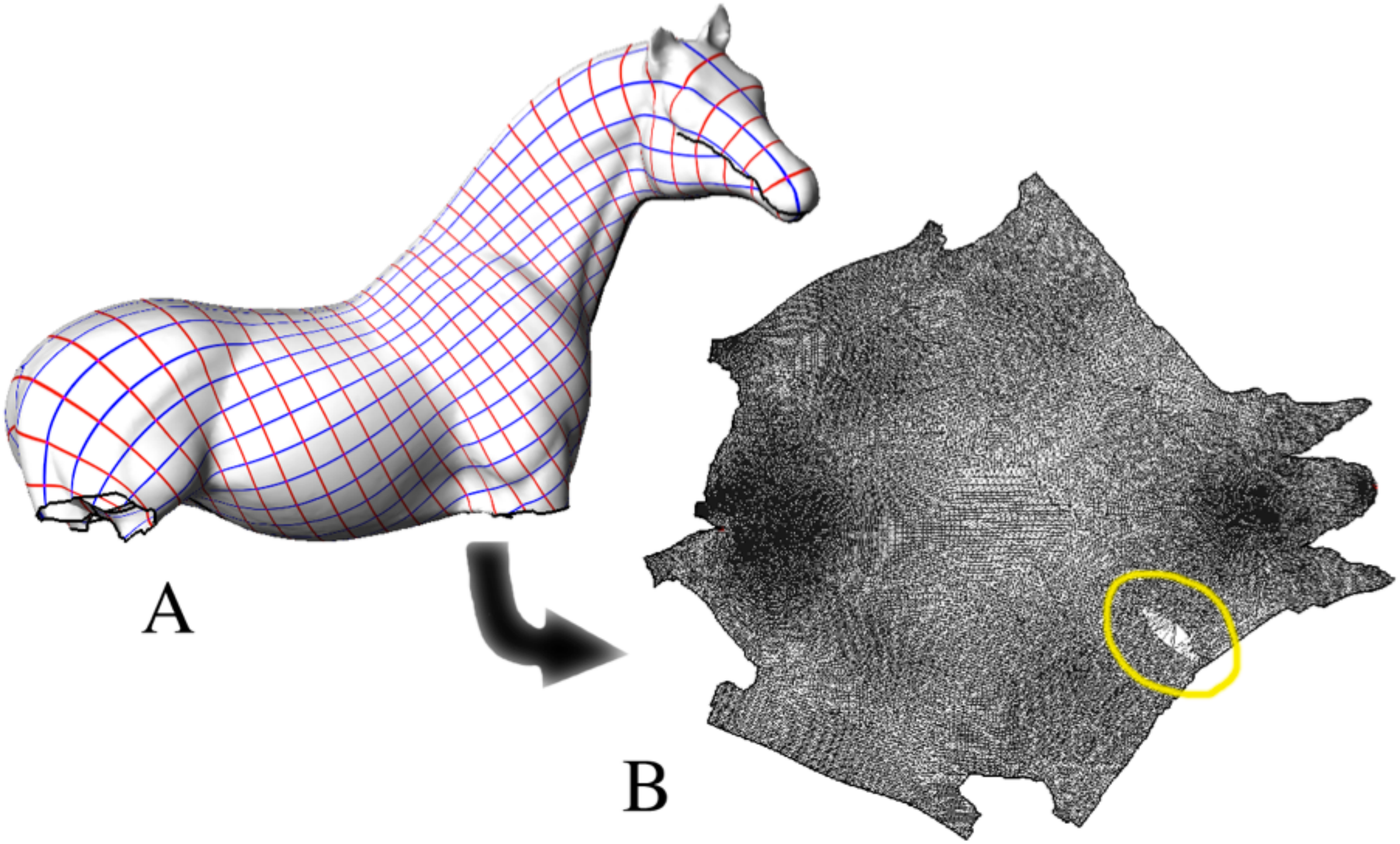
Global Parametrization

Acknowledgements: Olga Sorkine-Hornung

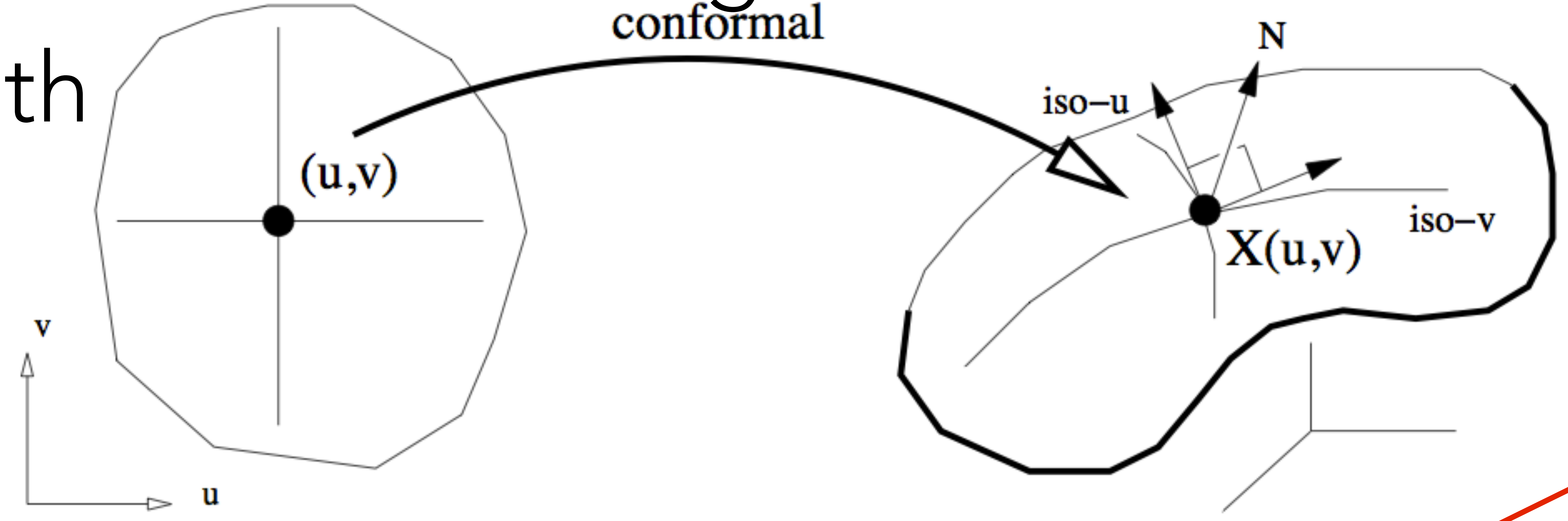
University of Victoria | Computer Science

Boundary-Free Parametrization



Conformal Maps

- In a conformal map the tangent vector to the iso-u and iso-v are orthogonal and have the same length



Complex numbers!

$$N(u,v) \quad \times \quad \frac{\partial \mathcal{X}}{\partial u}(u,v) \quad = \quad \frac{\partial \mathcal{X}}{\partial v}(u,v) \quad \longrightarrow \frac{\partial \mathcal{X}}{\partial u} - i\frac{\partial \mathcal{X}}{\partial v} = 0$$

Conformality on a Triangulation

- The conformality condition can be rewritten in the local frame of every triangle:

$$\left(N(u,v) \quad \times \quad \frac{\partial \mathcal{X}}{\partial u}(u,v) \quad = \quad \frac{\partial \mathcal{X}}{\partial v}(u,v) \right) \longrightarrow \frac{\partial \mathcal{X}}{\partial u} - i\frac{\partial \mathcal{X}}{\partial v} = 0$$

$$\frac{\partial \mathcal{U}}{\partial x} + i\frac{\partial \mathcal{U}}{\partial y} = 0$$

- Using the theorem of the derivatives of the inverse functions:

Least-Square Conformality

- This condition cannot be strictly enforced for every triangle, but we can minimize it:

$$C(T) = \int_T \left| \frac{\partial \mathcal{U}}{\partial x} + i \frac{\partial \mathcal{U}}{\partial y} \right|^2 dA = \left| \frac{\partial \mathcal{U}}{\partial x} + i \frac{\partial \mathcal{U}}{\partial y} \right|^2 A_T$$

- Summed over the triangulation:

$$C(\mathcal{T}) = \sum_{T \in \mathcal{T}} C(T)$$

Computation of LSCM

- Writing the previous equations with respect to the positions of the vertices in the plane result in a simple least-square system:

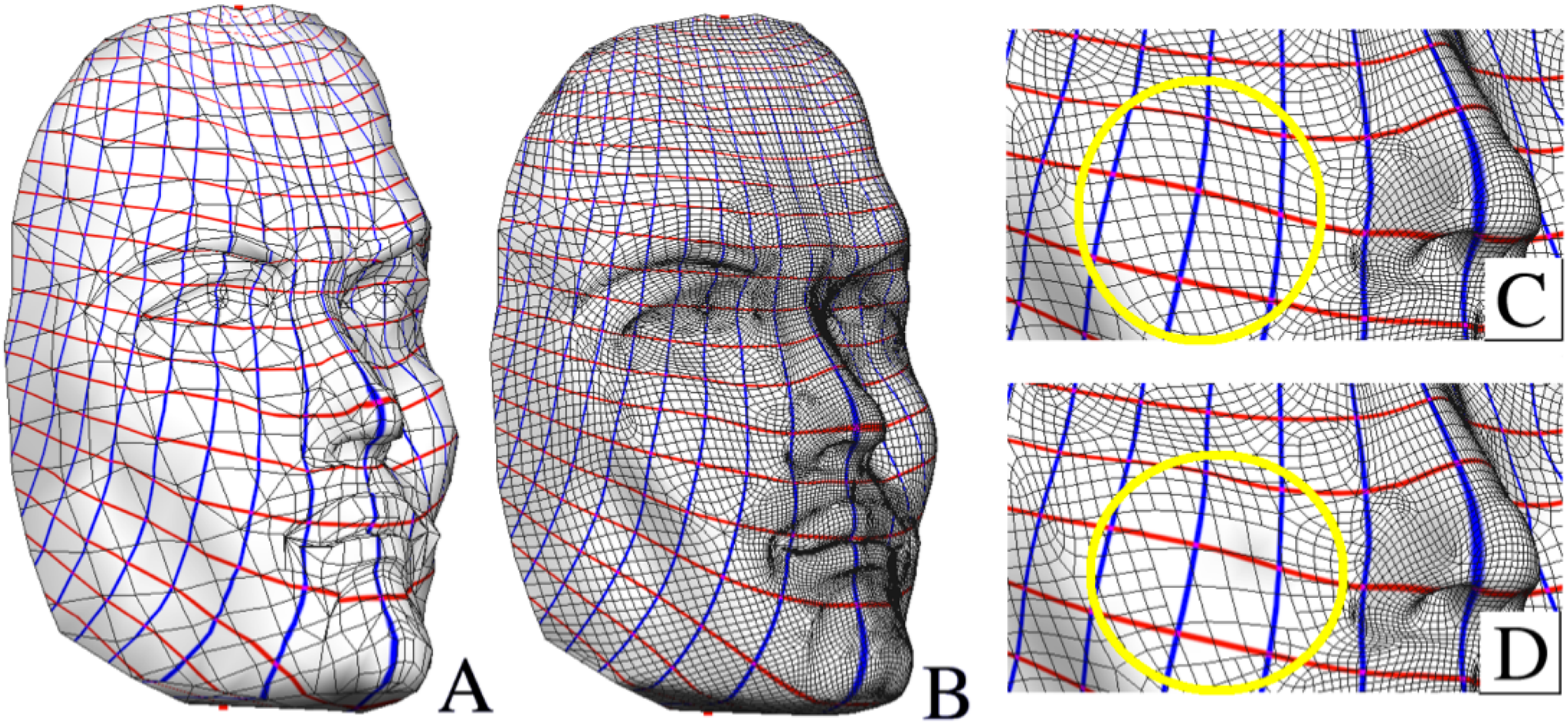
$$C(\mathbf{x}) = \|\mathcal{A}\mathbf{x} - \mathbf{b}\|^2$$

- At least two points must be fixed to make **A** full-rank

Properties of LSCM

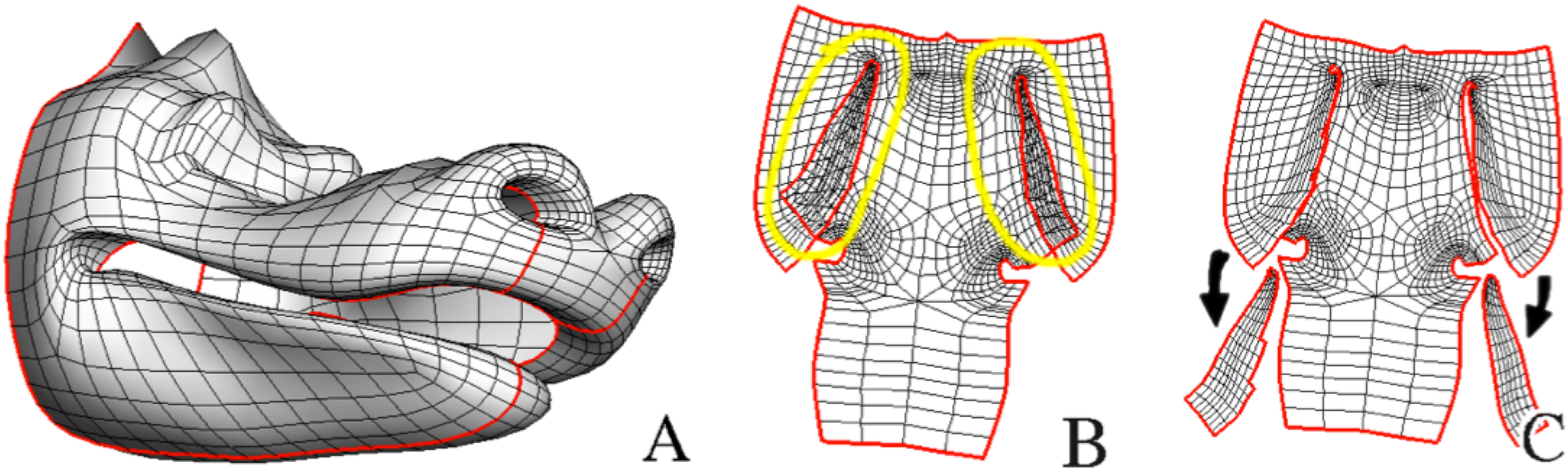
- The solution is unique
- The solution is invariant to similarity in texture space
- The solution is independent of the resolution
- Triangle flips are “rare”

Resolution independence



University of Victoria | Computer Science

Global overlaps are not prevented



Demo

- https://libigl.github.io/libigl-python-bindings/tut-chapter4/

Commercial applications

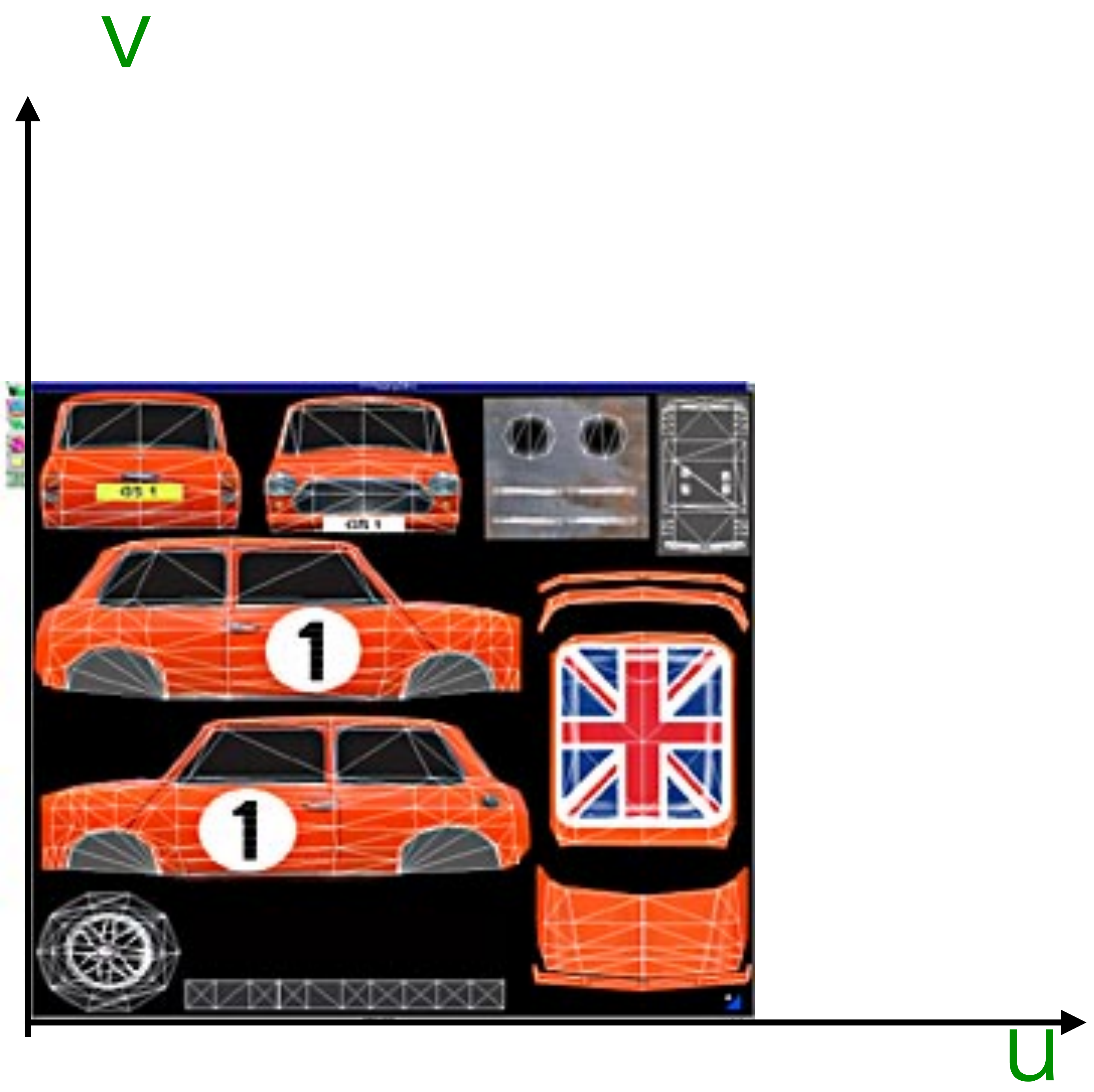
- Blender implements LSCM
- http://www.blender.org/download/sandbox/lscm-basics/

Global Parametrization

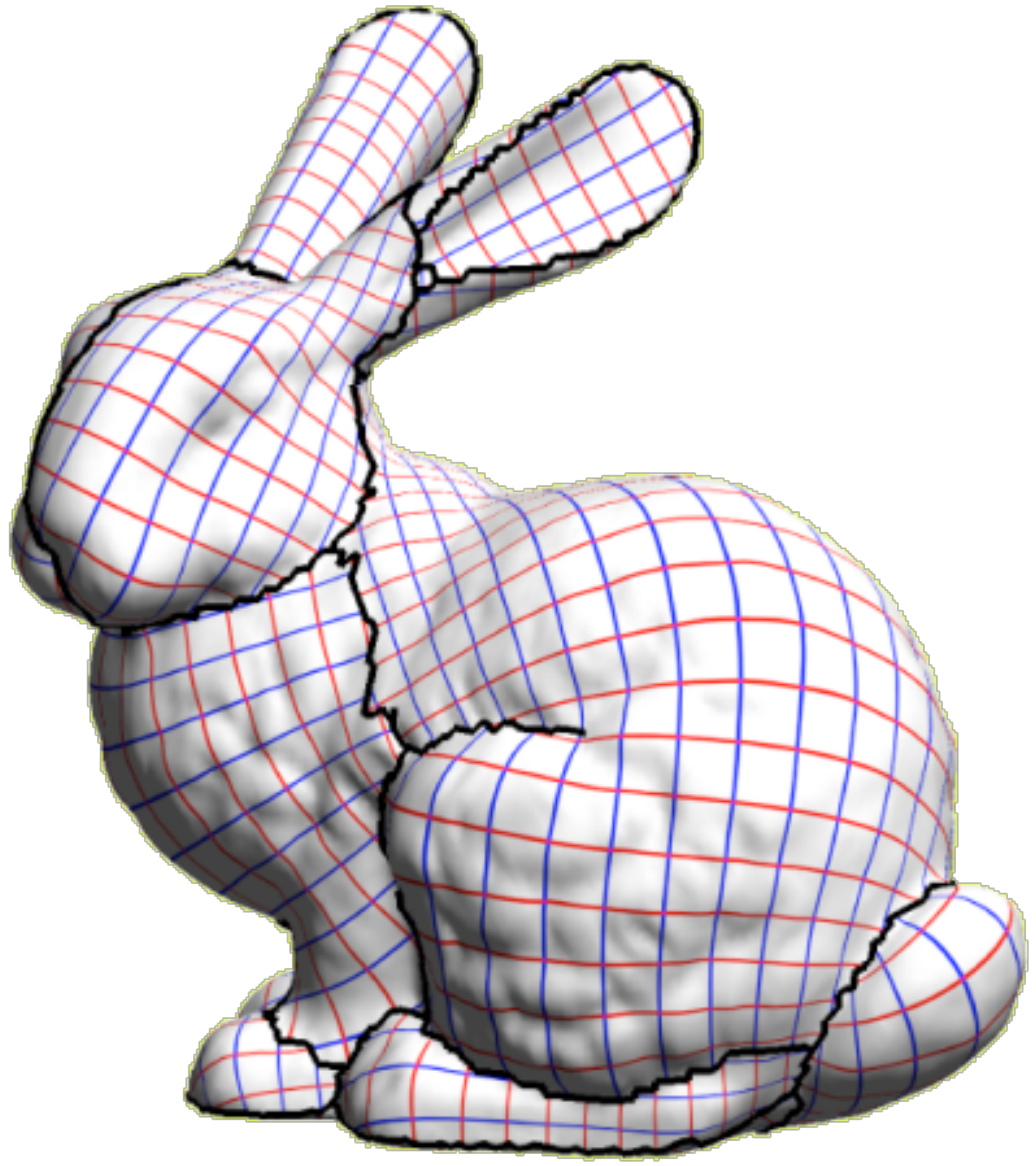
- All the algorithms that we studied till now are able to parametrize a part of a mesh homeomorphic to a disk

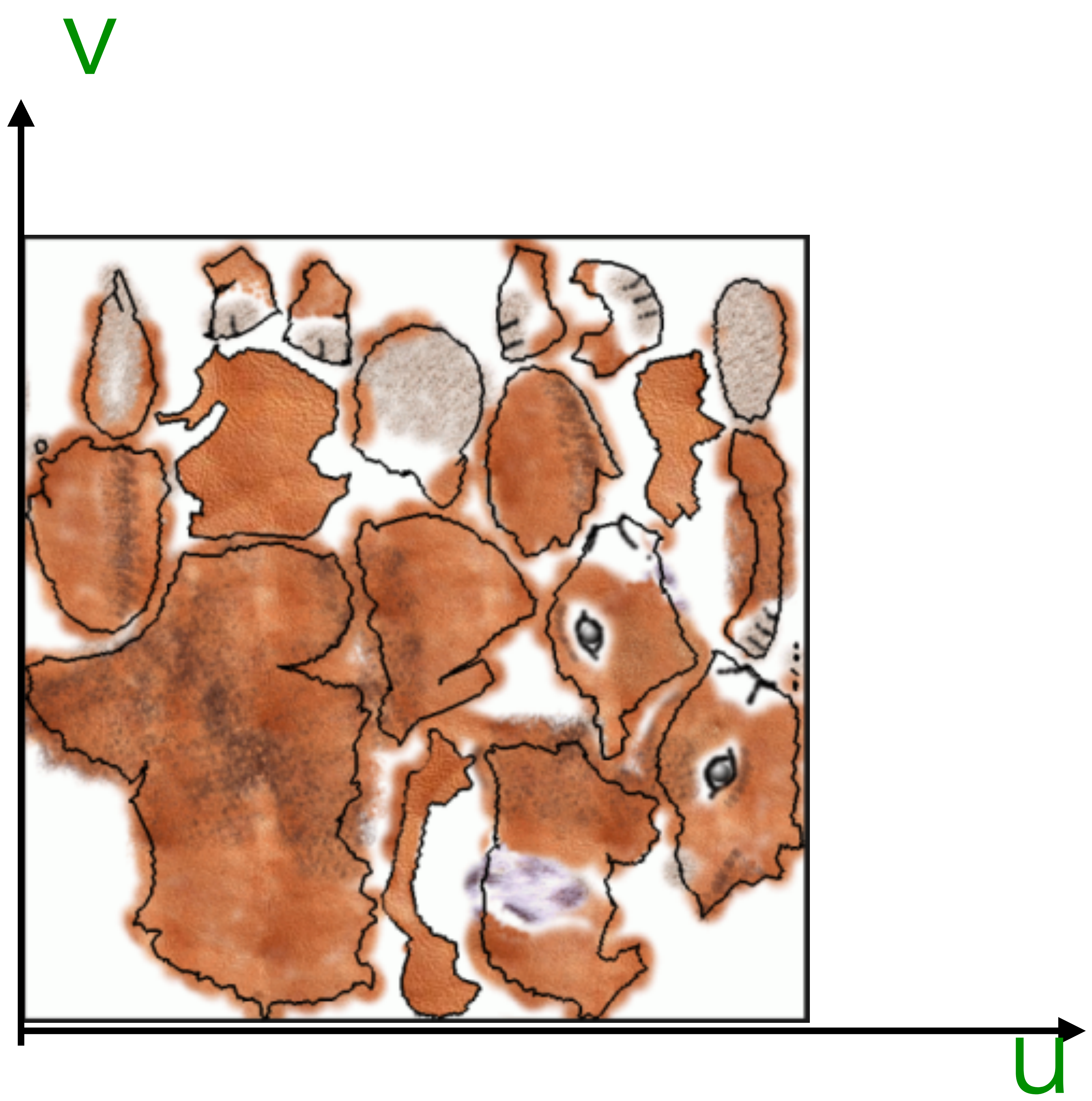
- To be able to parametrize arbitrary shapes we “cut” a mesh into parts, and we parametrize every part independently

Manual Segmentation

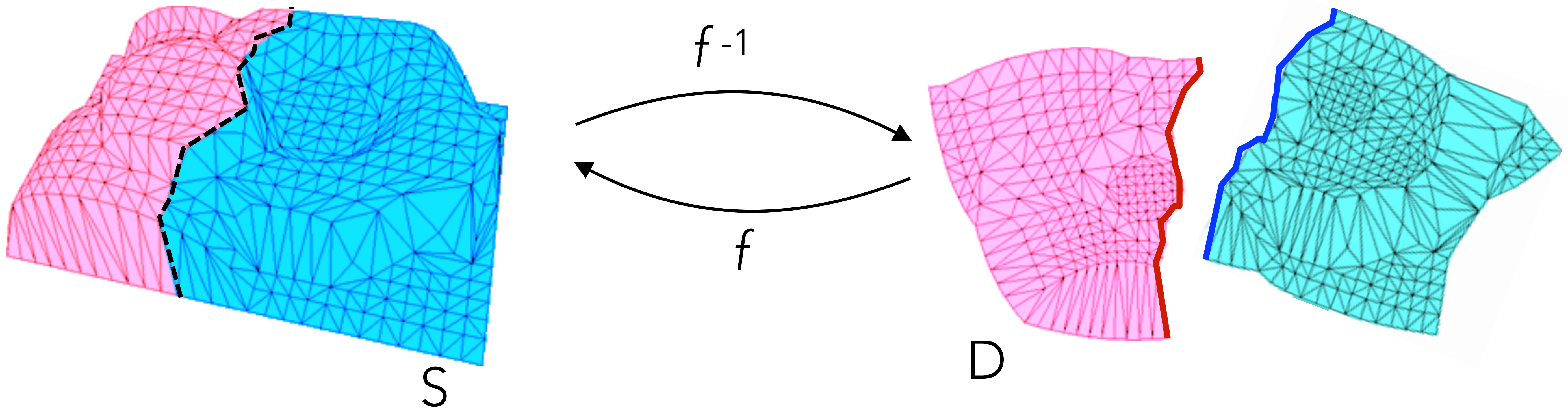


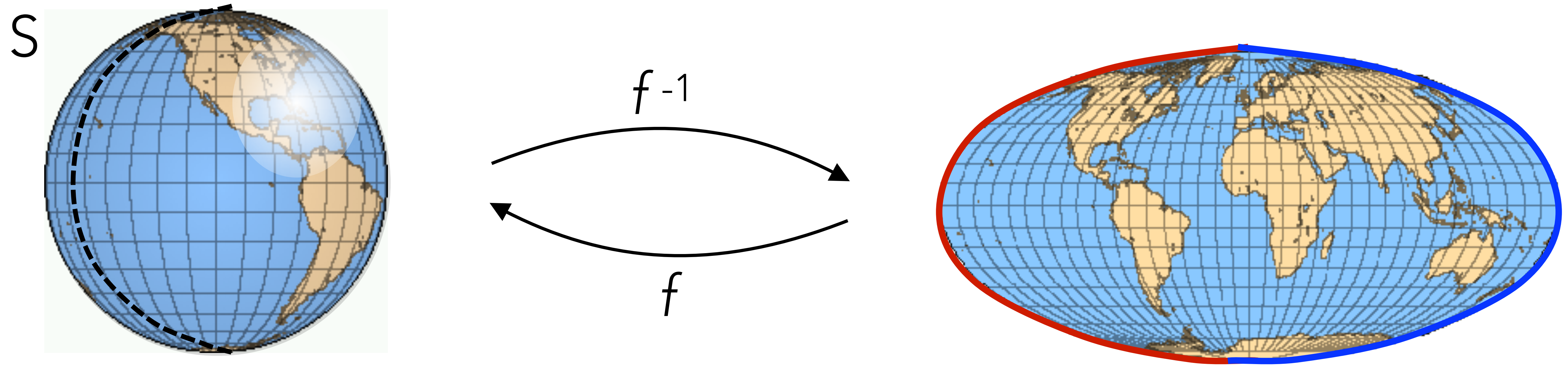
Automatic Segmentation





Good = "fewer cuts"?



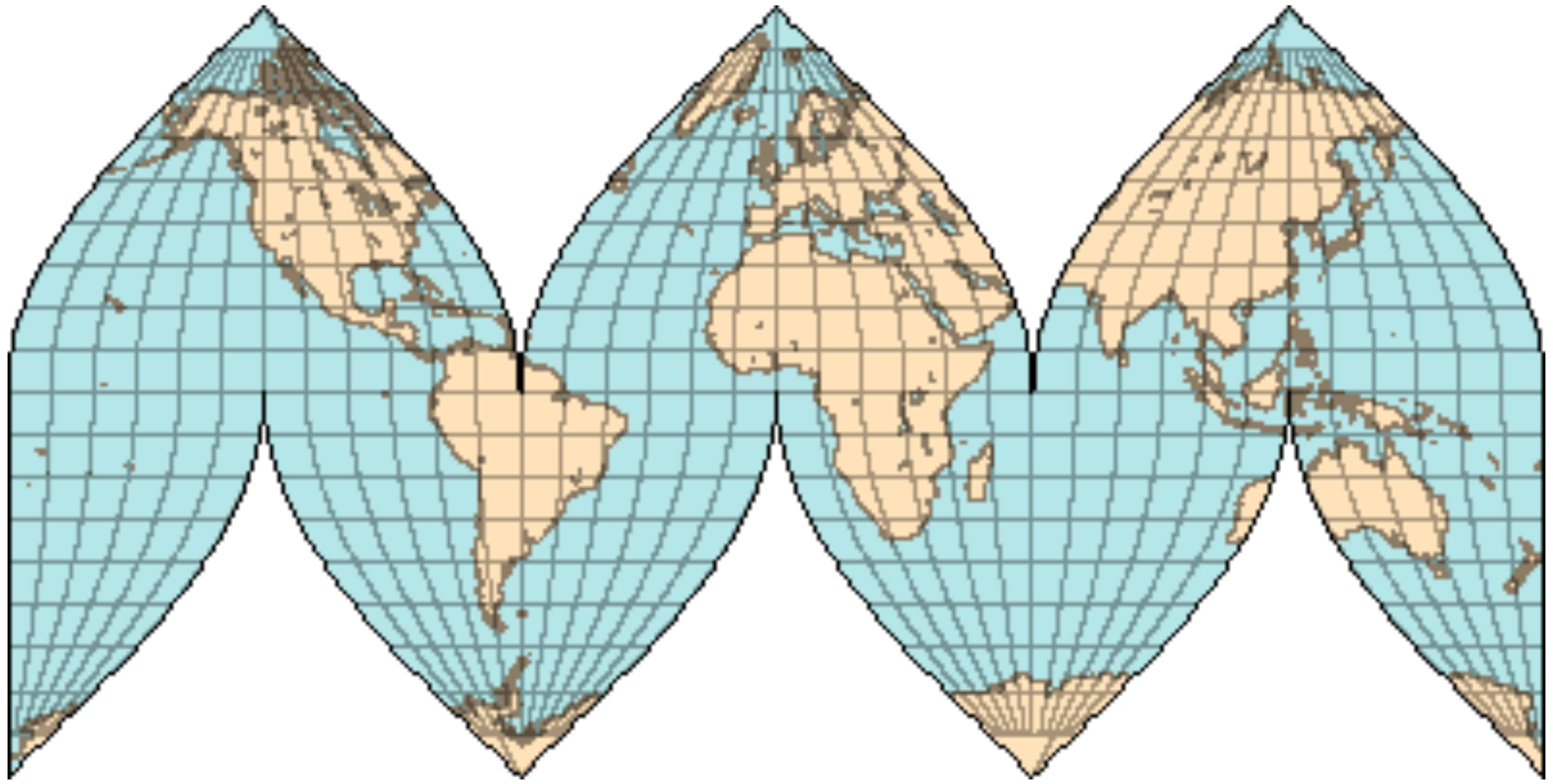


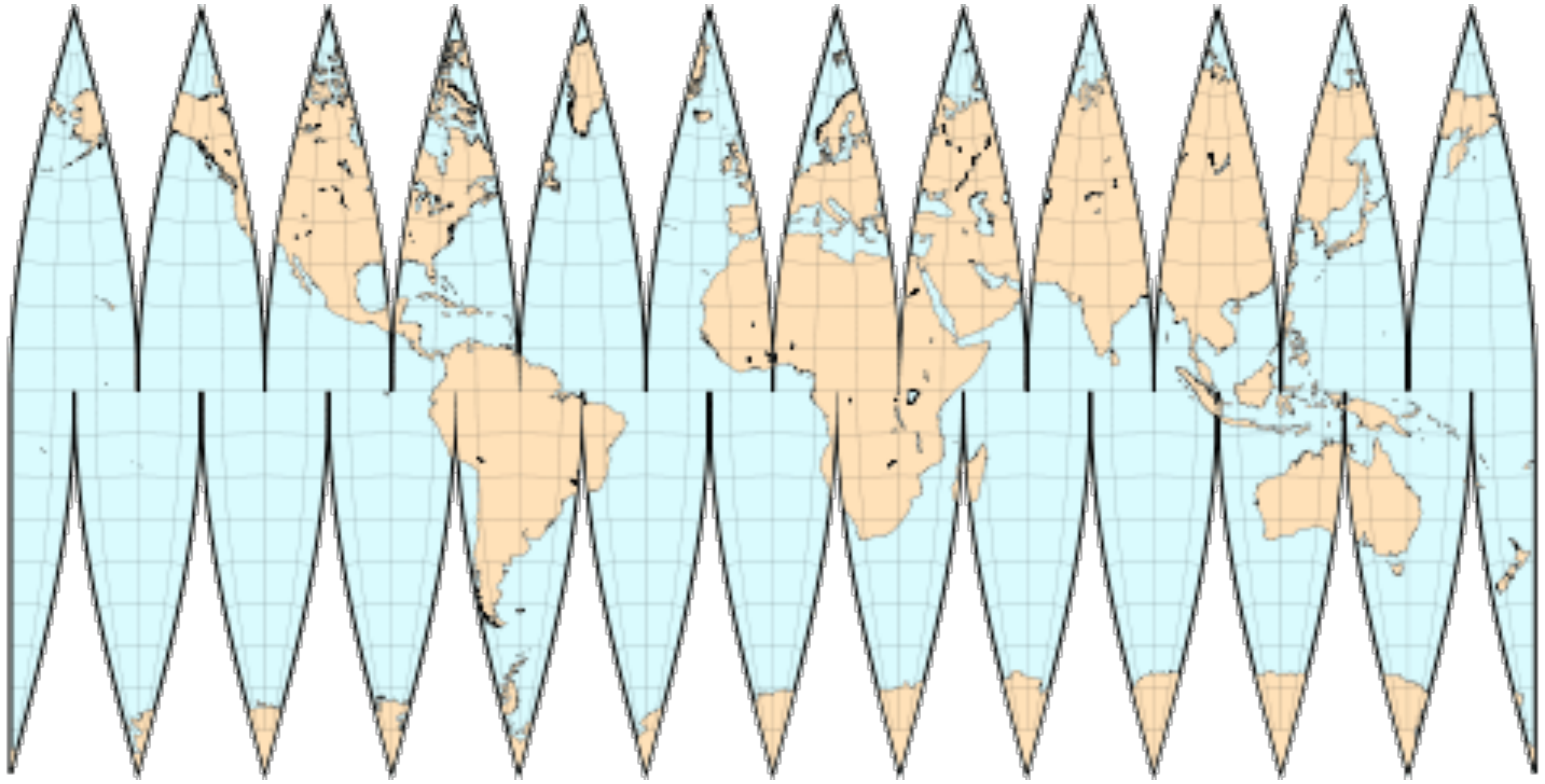
sphere in 3D

2D surface disk

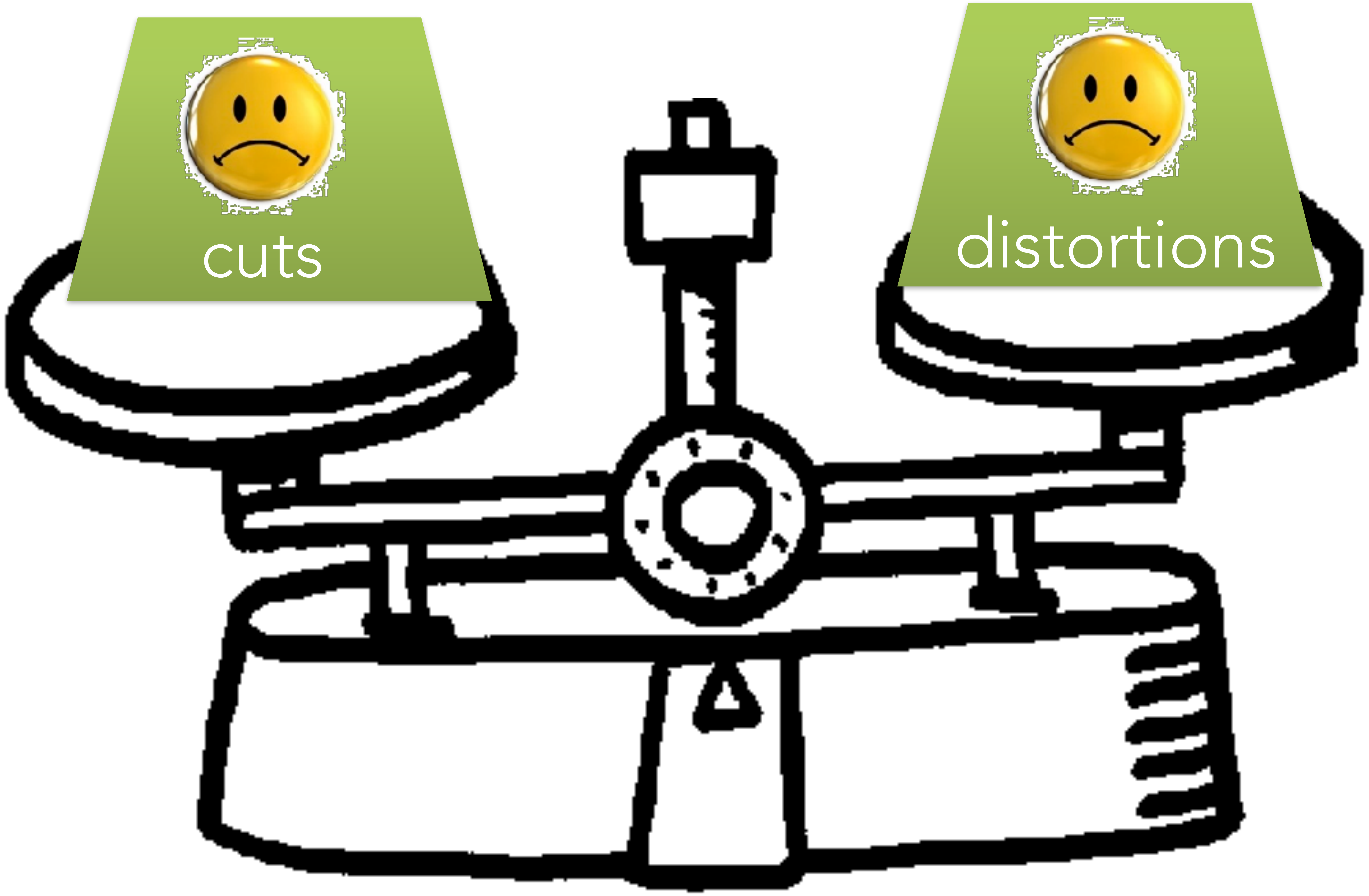
Good = “fewer cuts”?

but… more cuts => less distortion





A difficult balance



University of Victoria | Computer Science

How to handle cuts?

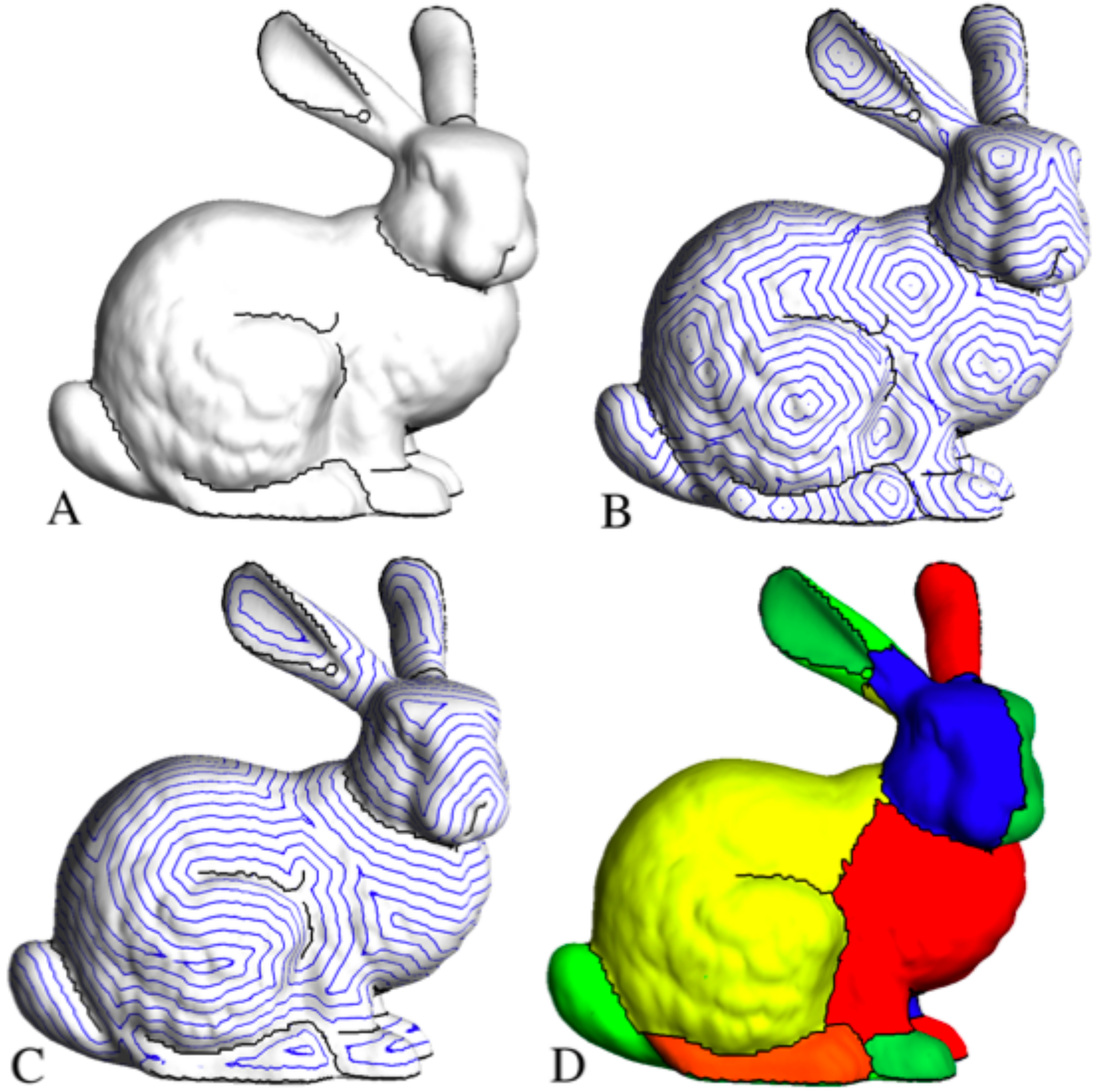
- We can ignore them and parametrize every chart separately
- We want to impose continuity of the derivatives of the parametrization across cuts
- Both have their own pros and cons, we will see an example for both cases

No continuity between charts

- Every chart can be parametrized separately
- The collection of the separate chart can be grouped in a square with a packing algorithm
- We study the method proposed in the LSCM paper

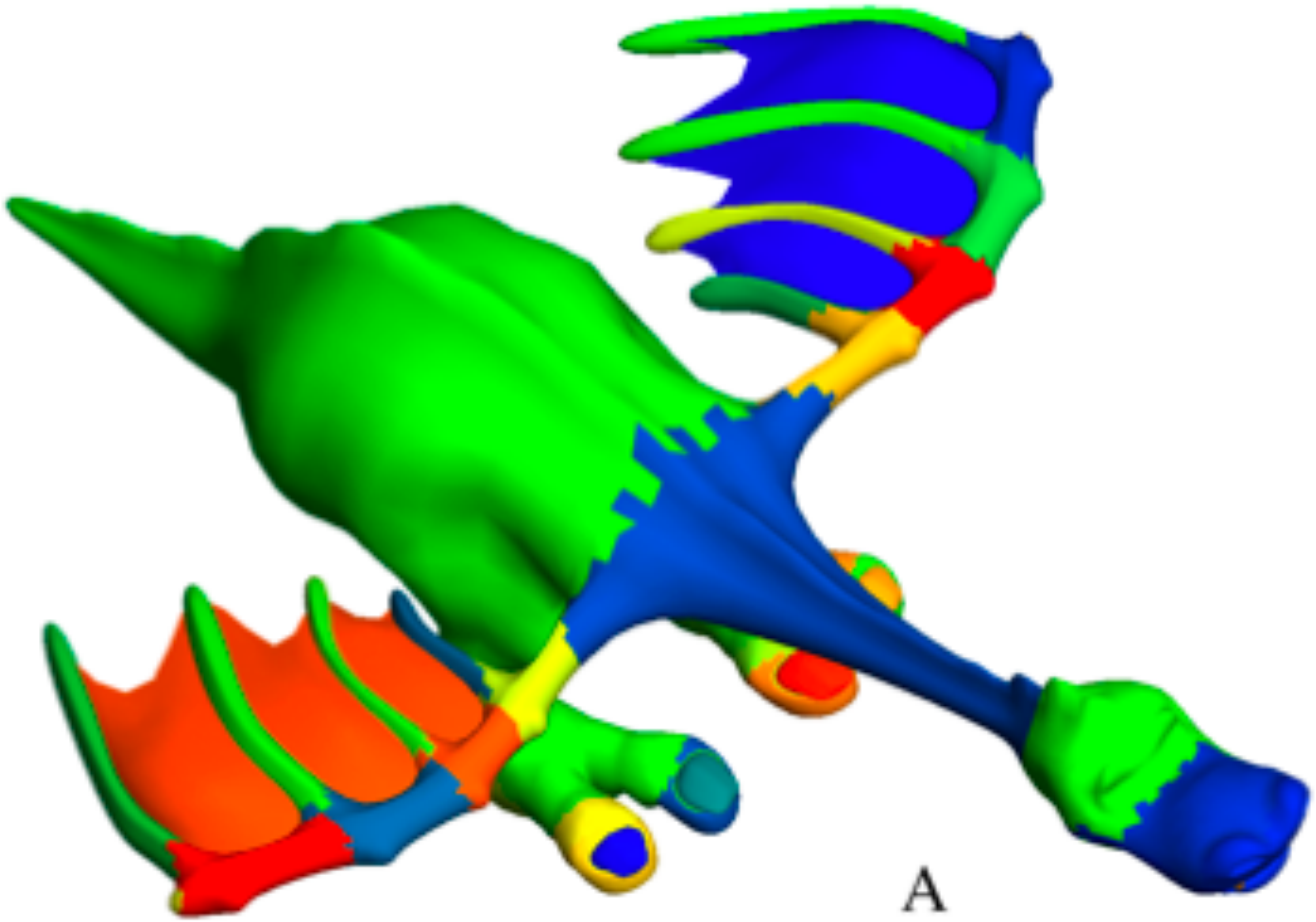
Segmentation

- Identify high curvature areas
- Seeds are the maxima of the distance to feature function
- Grow seeds into regions
- Merge regions if possible



University of Victoria | Computer Science

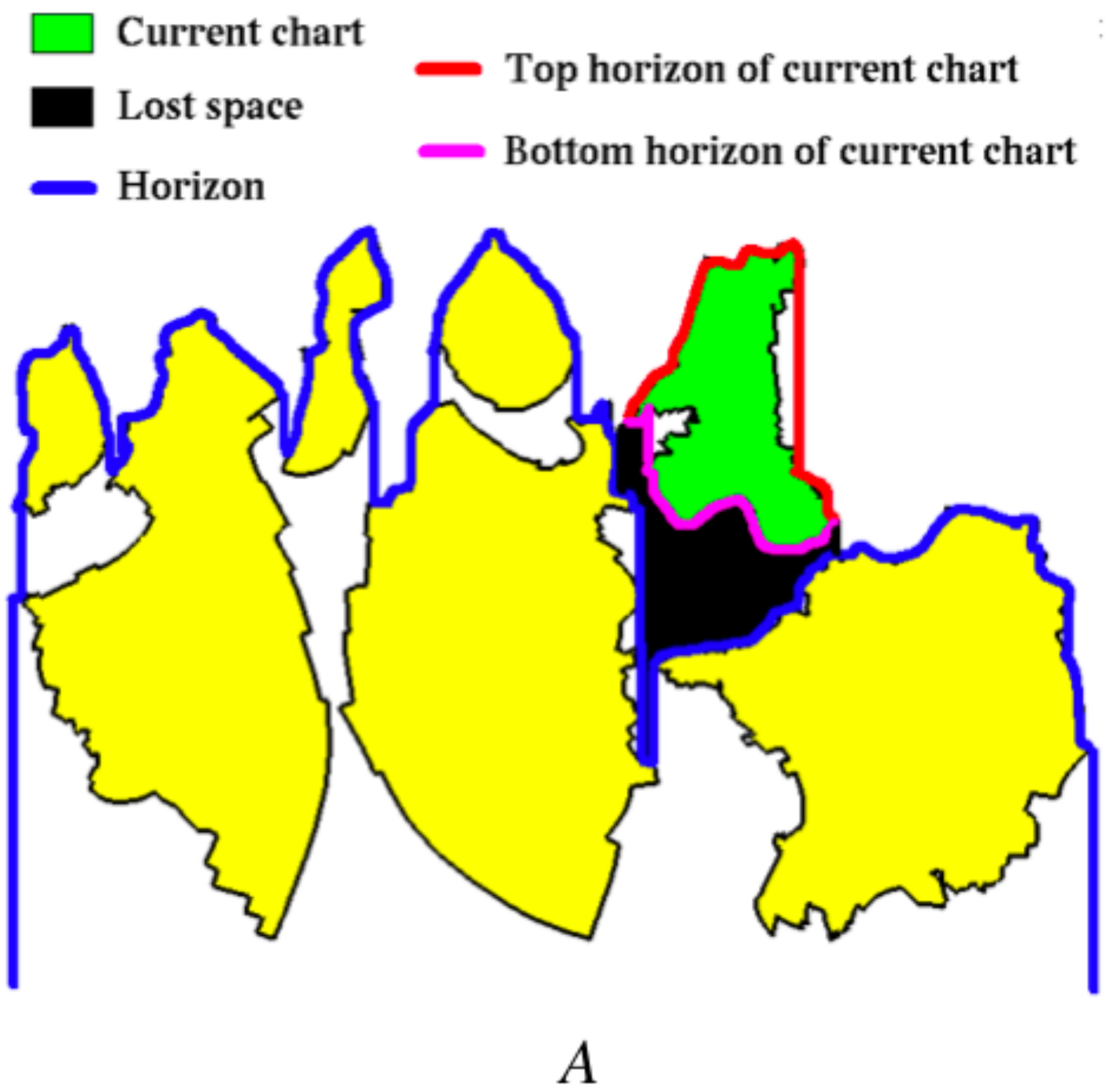
Segmentation result

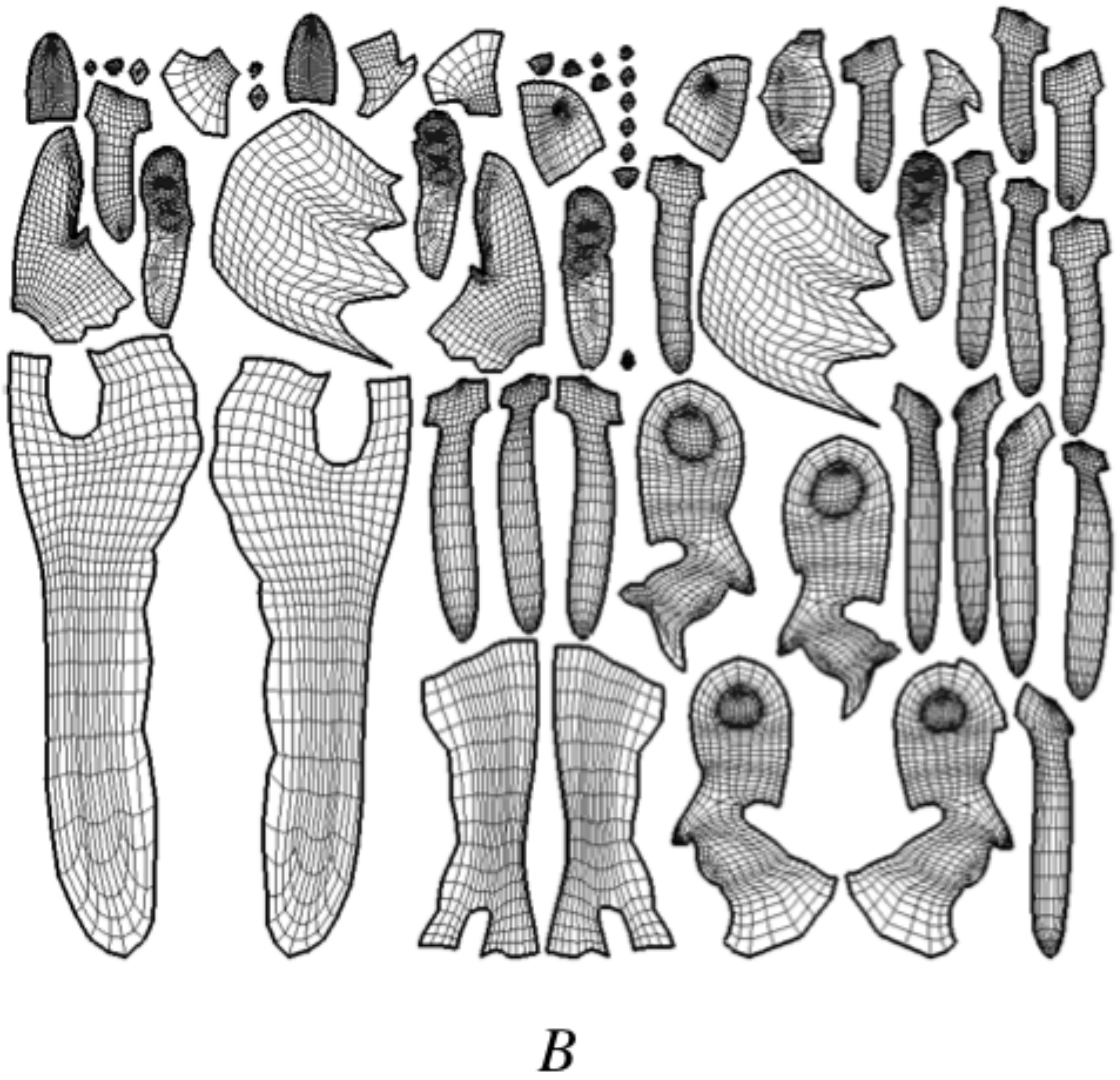


Packing

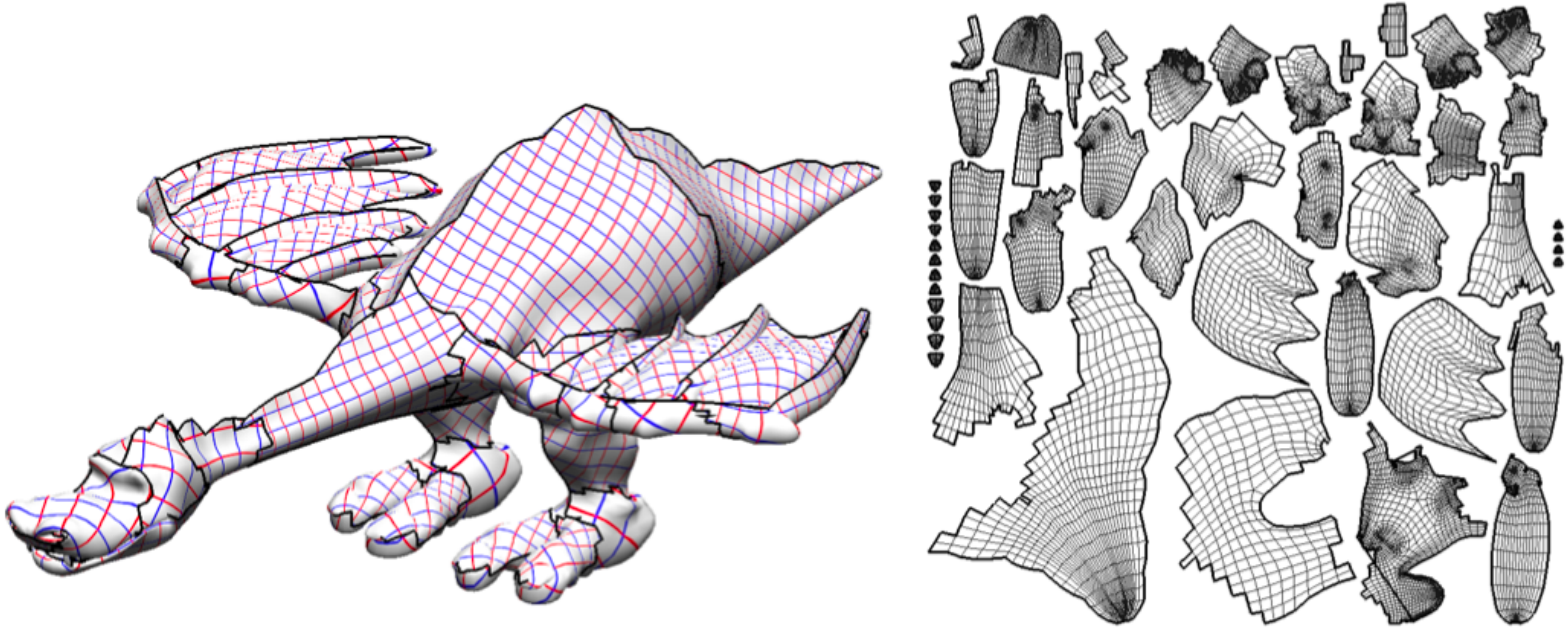
- The computation the optimal packing is NP-complete
- We need an heuristic
- The problem has been studied extensively in computational geometry, with the goal of computing high-quality results but only for a small number of objects
- The authors propose an heuristic based on the famous "Tetris" game that is extremely fast and generates good results

Tetris packing

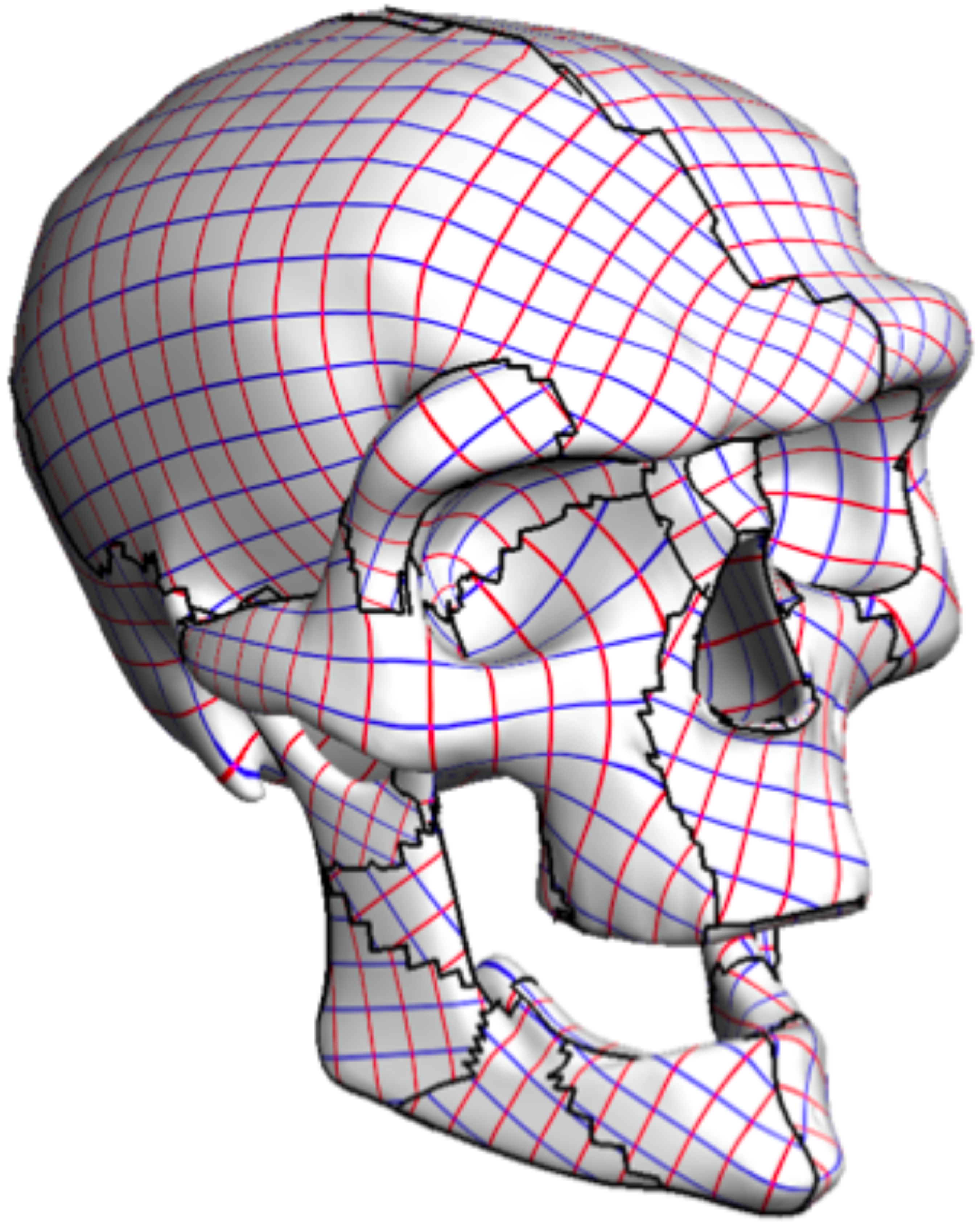


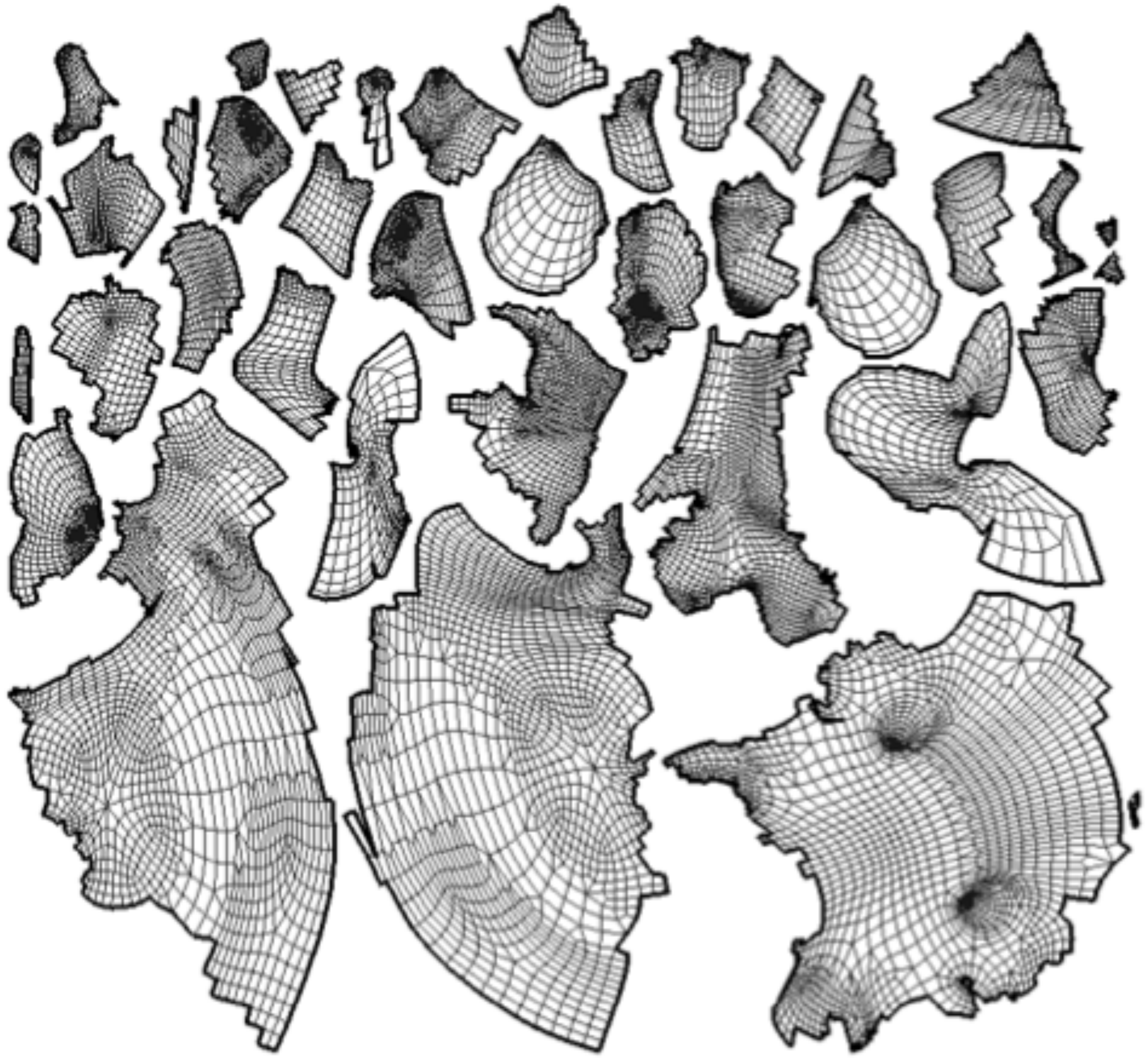


Results

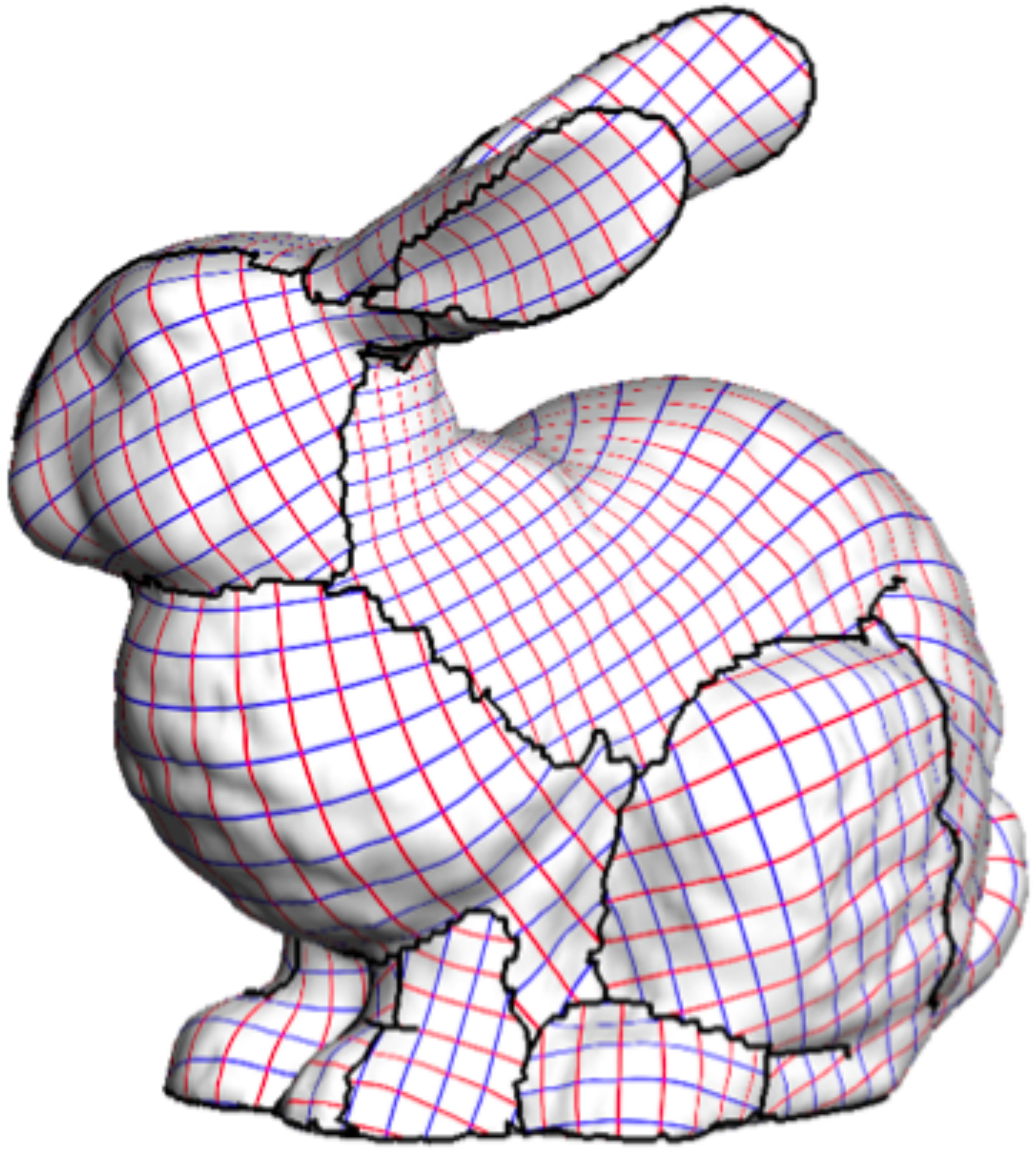


Results

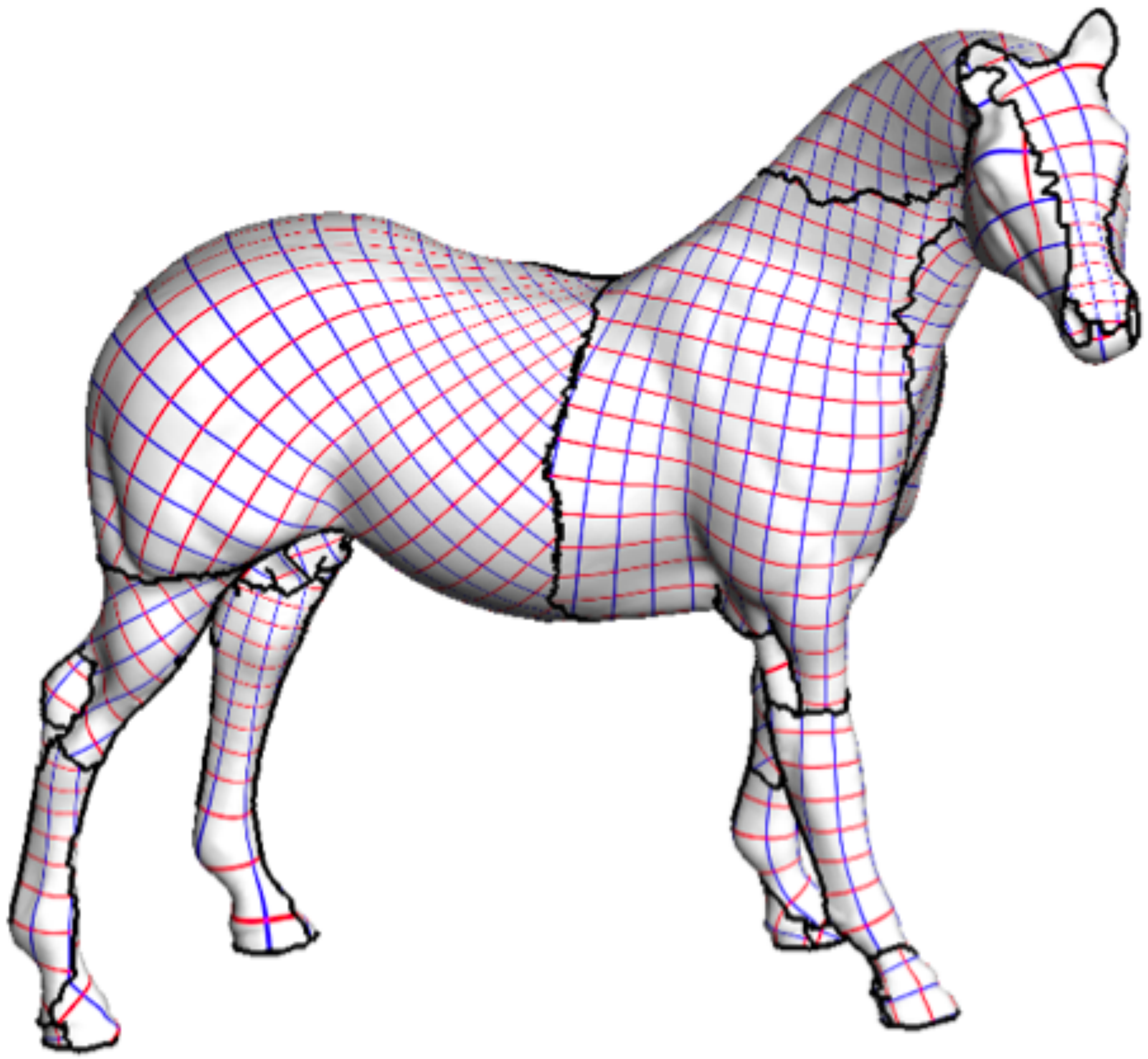


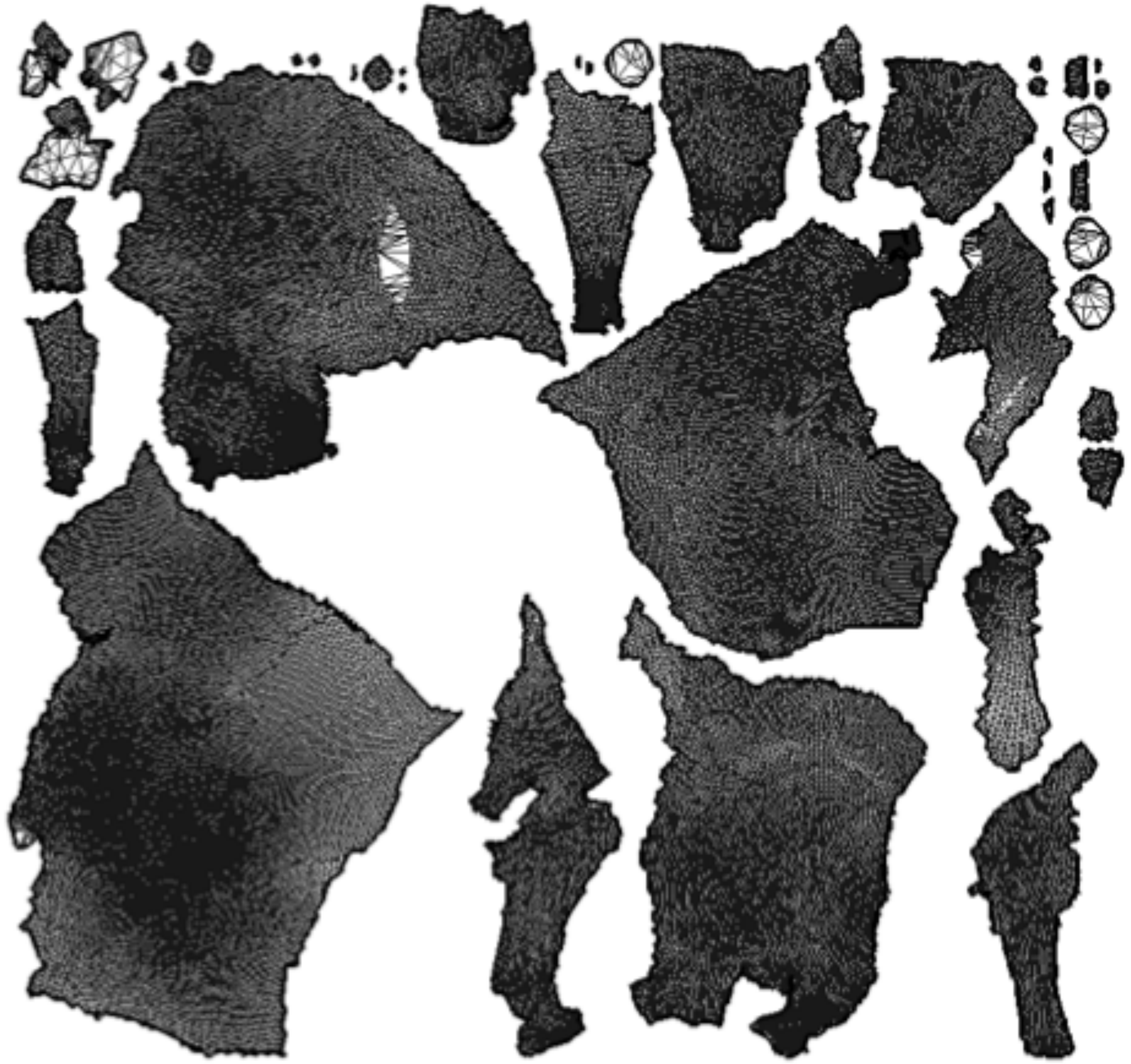


Results



Results





Applications

Continuity between charts

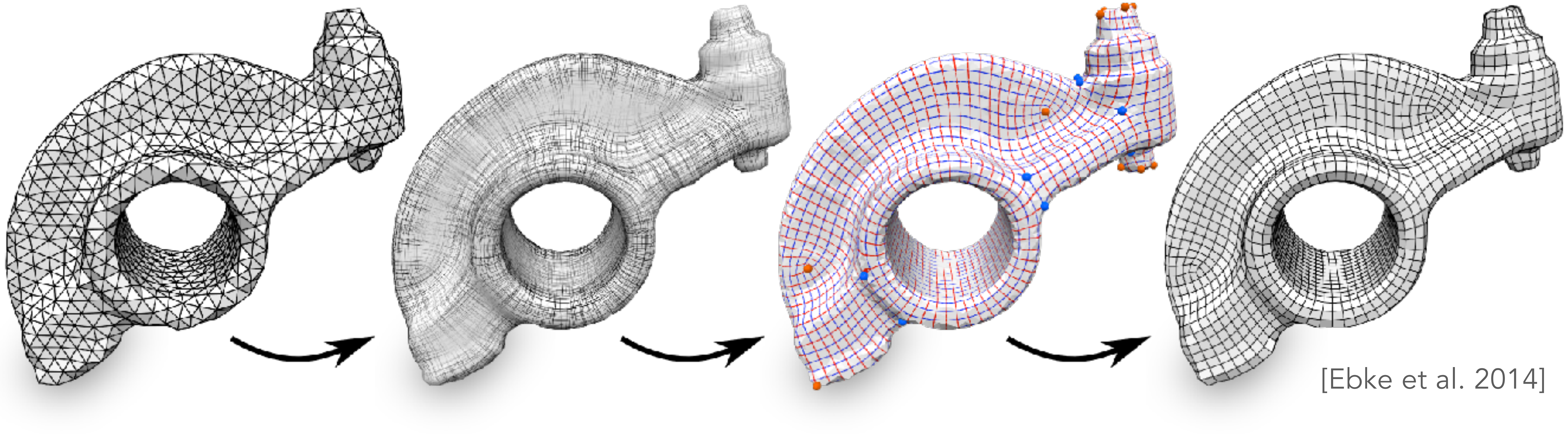
- For some applications (i.e. remeshing) continuity of the derivatives between charts is required

- If you are interested, I suggest to start from this paper:

Mixed-Integer Quadrangulation - David Bommes, Henrik Zimmer, Leif Kobbelt

Siggraph 2009

Quadrilateral Meshing



[Ebke et al. 2014]

[courtesy of Hans-Christian Ebke]

[courtesy of Hans-Christian Ebke]


[courtesy of Hans-Christian Ebke]

[courtesy of Hans-Christian Ebke]

References

- Surface Parameterization: a Tutorial and Survey: http://graphics.stanford.edu/courses/cs468-05-fall/Papers/param-survey.pdf
- Polygon Mesh Processing: Chapter 5
- Mixed Integer Quadrangulation: David Bommes, Henrik Zimmer, Leif Kobbelt. SIGGRAPH 2009