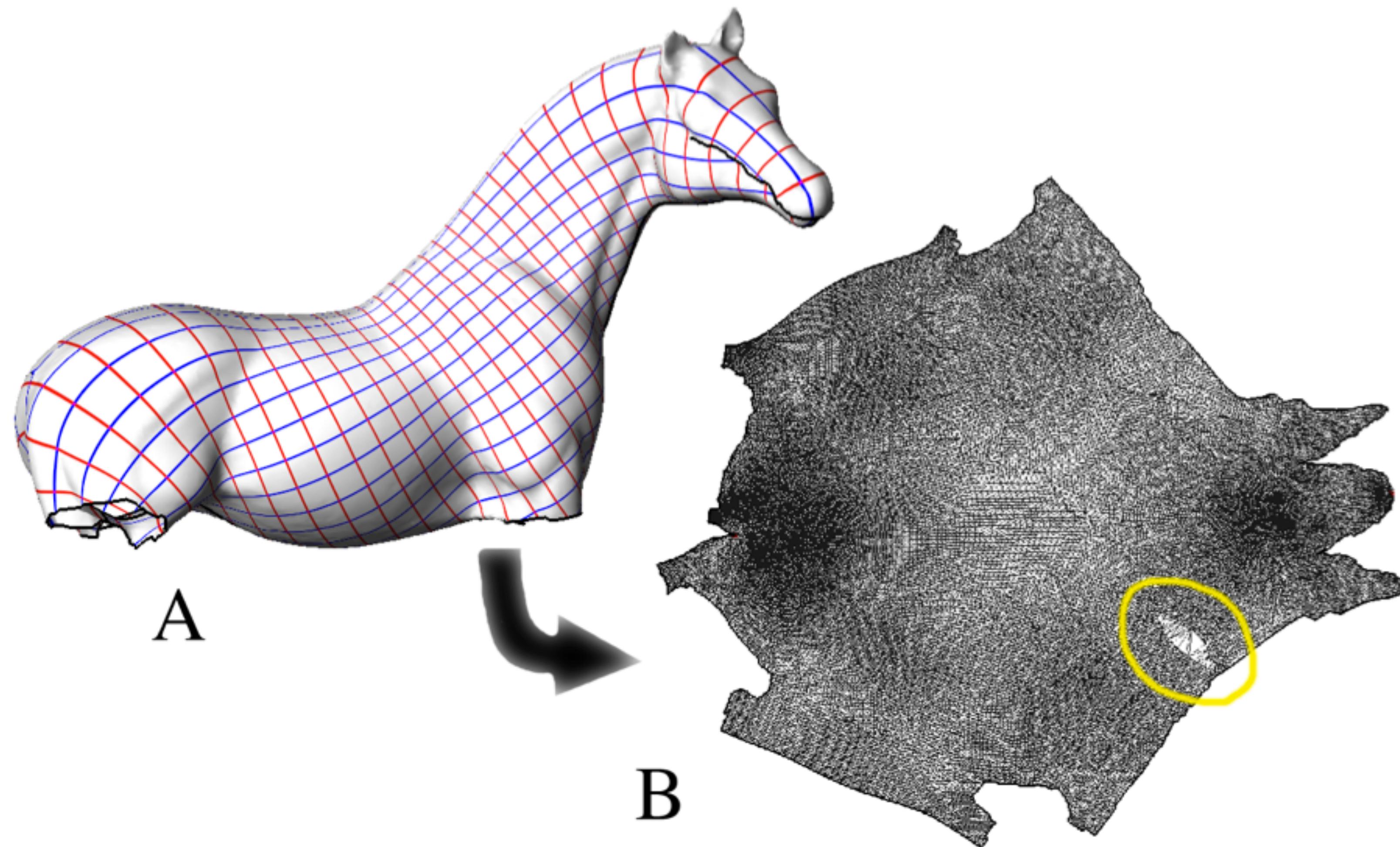


Global Parametrization

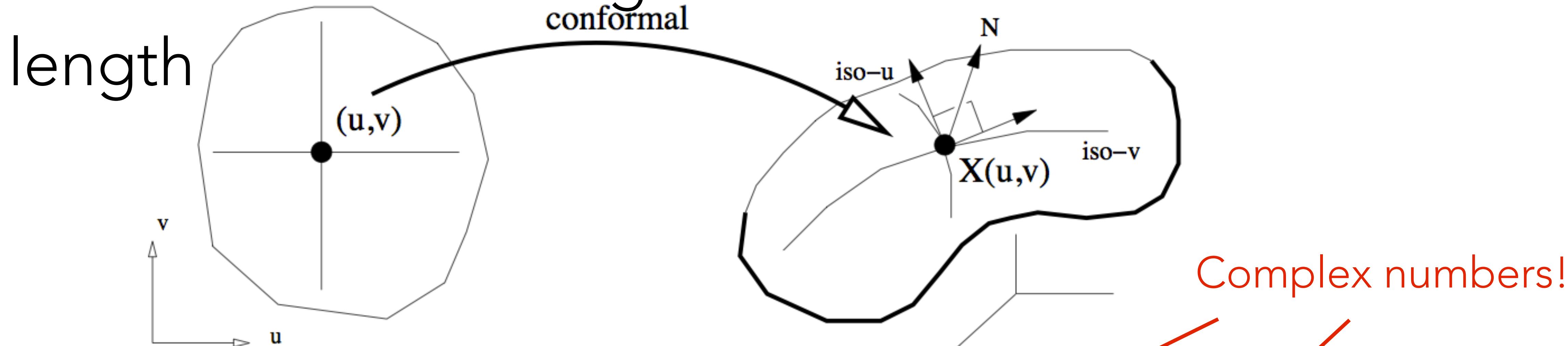
Acknowledgements: Olga Sorkine-Hornung
CSC 486B/586B - Geometric Modeling - Teseo Schneider

Boundary-Free Parametrization



Conformal Maps

- In a conformal map the tangent vector to the iso-u and iso-v are orthogonal and have the same length



$$N(u, v) \times \frac{\partial \mathcal{X}}{\partial u}(u, v) = \frac{\partial \mathcal{X}}{\partial v}(u, v) \rightarrow \frac{\partial \mathcal{X}}{\partial u} - i \frac{\partial \mathcal{X}}{\partial v} = 0$$

Complex numbers!

Conformality on a Triangulation

- The conformality condition can be rewritten in the local frame of every triangle:

$$\left(N(u, v) \times \frac{\partial \mathcal{X}}{\partial u}(u, v) = \frac{\partial \mathcal{X}}{\partial v}(u, v) \right) \rightarrow \frac{\partial \mathcal{X}}{\partial u} - i \frac{\partial \mathcal{X}}{\partial v} = 0.$$

$$\frac{\partial U}{\partial x} + i \frac{\partial U}{\partial y} = 0$$

- Using the theorem of the derivatives of the inverse functions:

Least-Square Conformality

- This condition cannot be strictly enforced for every triangle, but we can minimize it:

$$C(T) = \int_T \left| \frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y} \right|^2 dA = \left| \frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y} \right|^2 A_T$$

- Summed over the triangulation:

$$C(\mathcal{T}) = \sum_{T \in \mathcal{T}} C(T)$$

Computation of LSCM

- Writing the previous equations with respect to the positions of the vertices in the plane result in a simple least-square system:

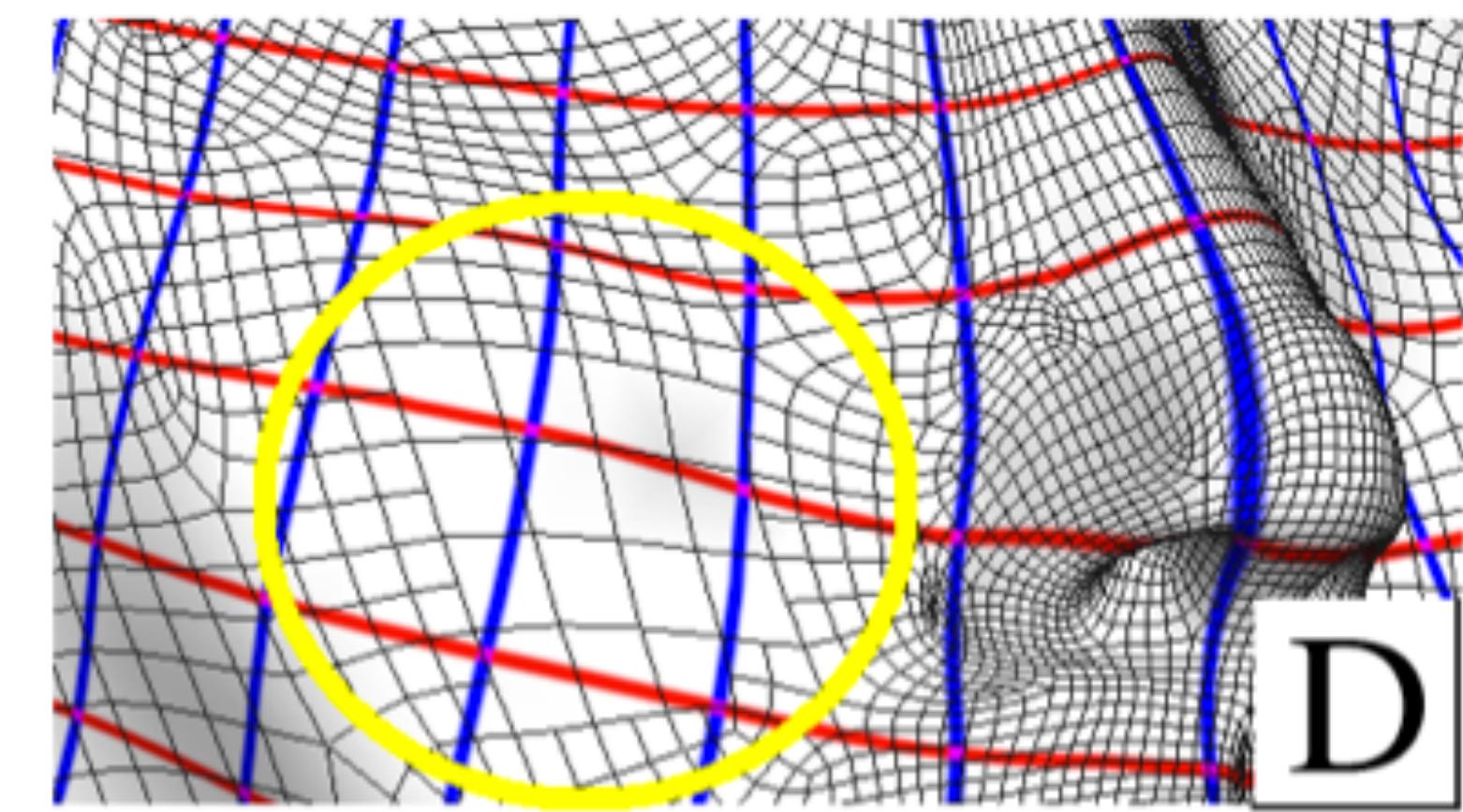
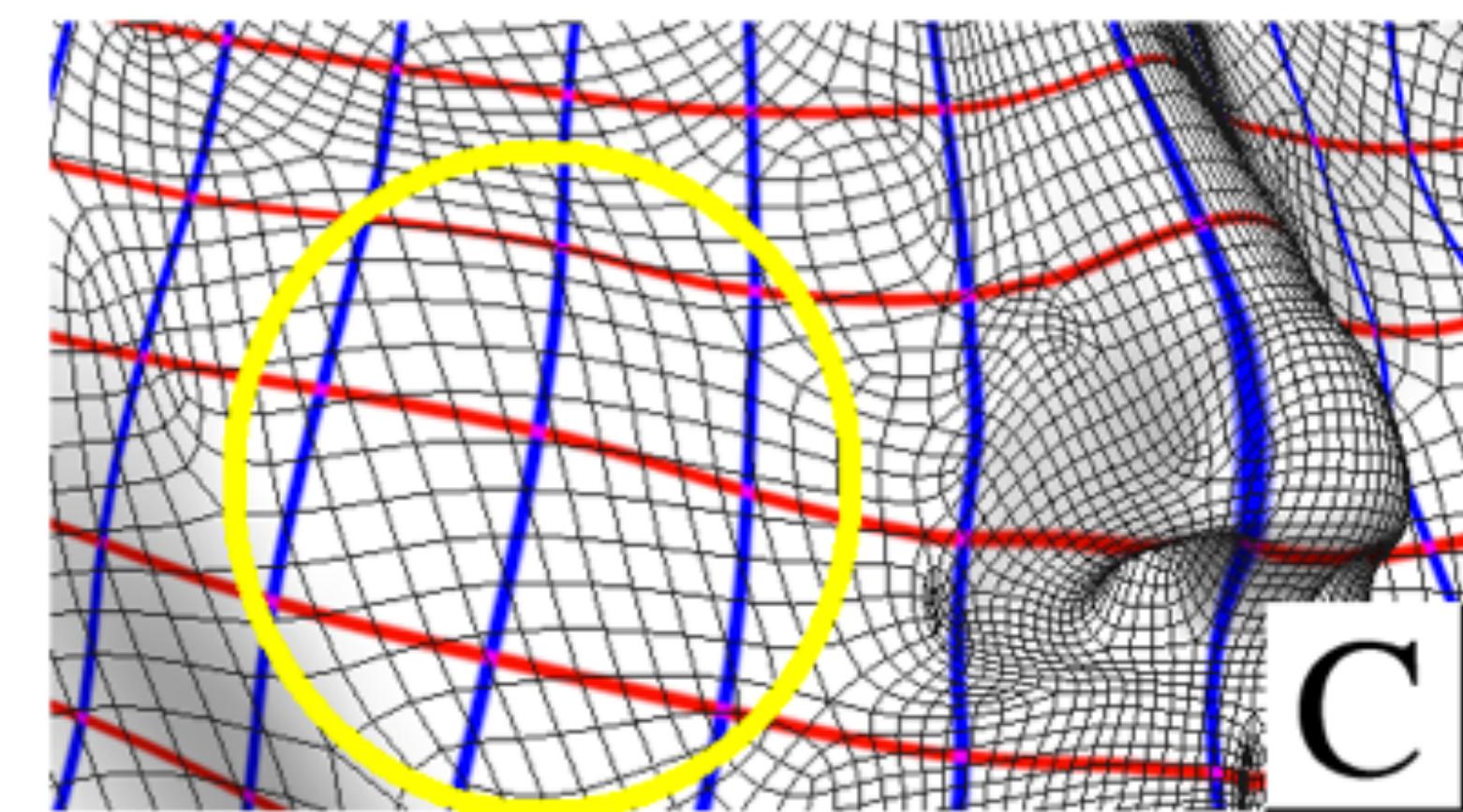
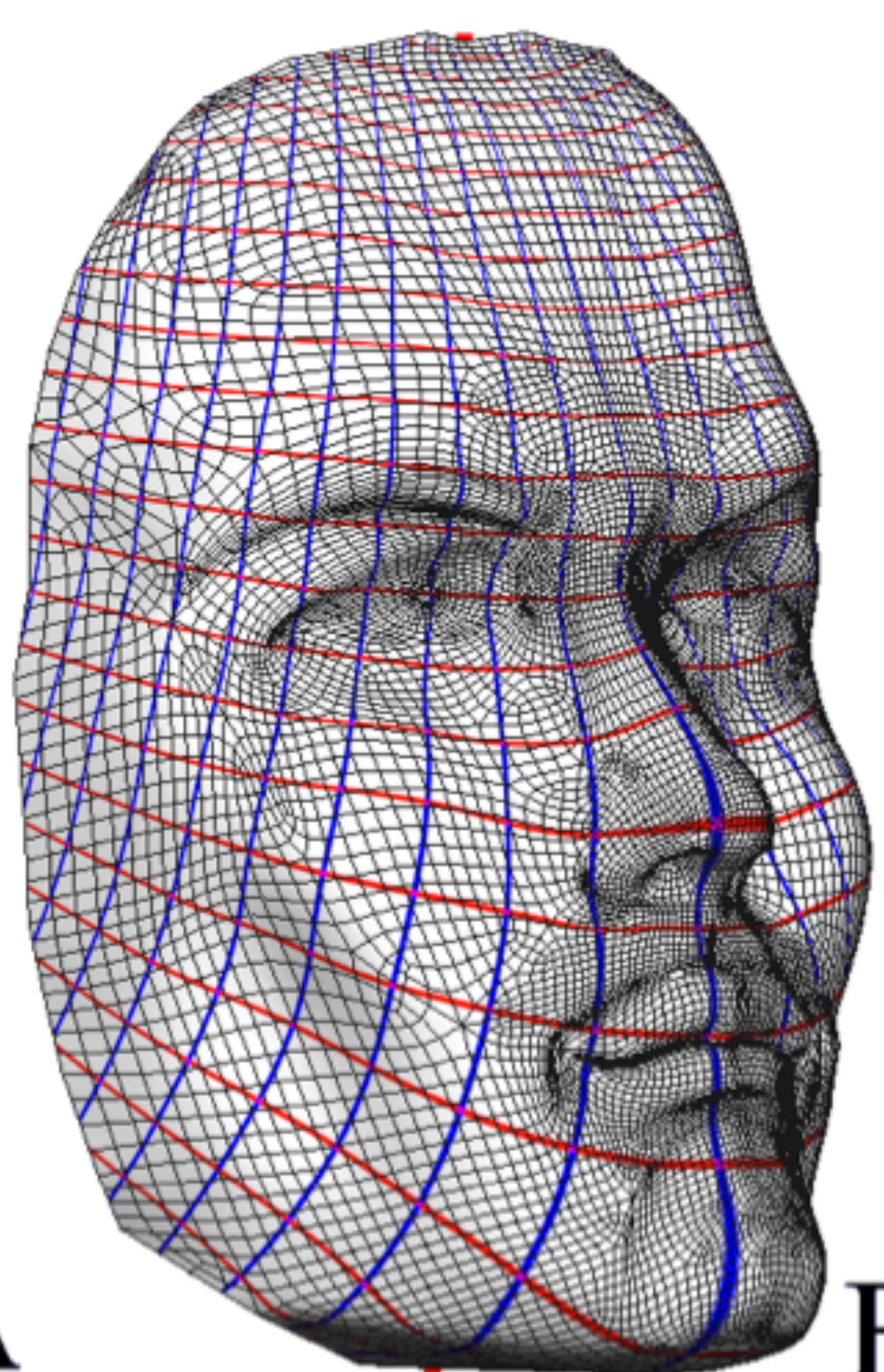
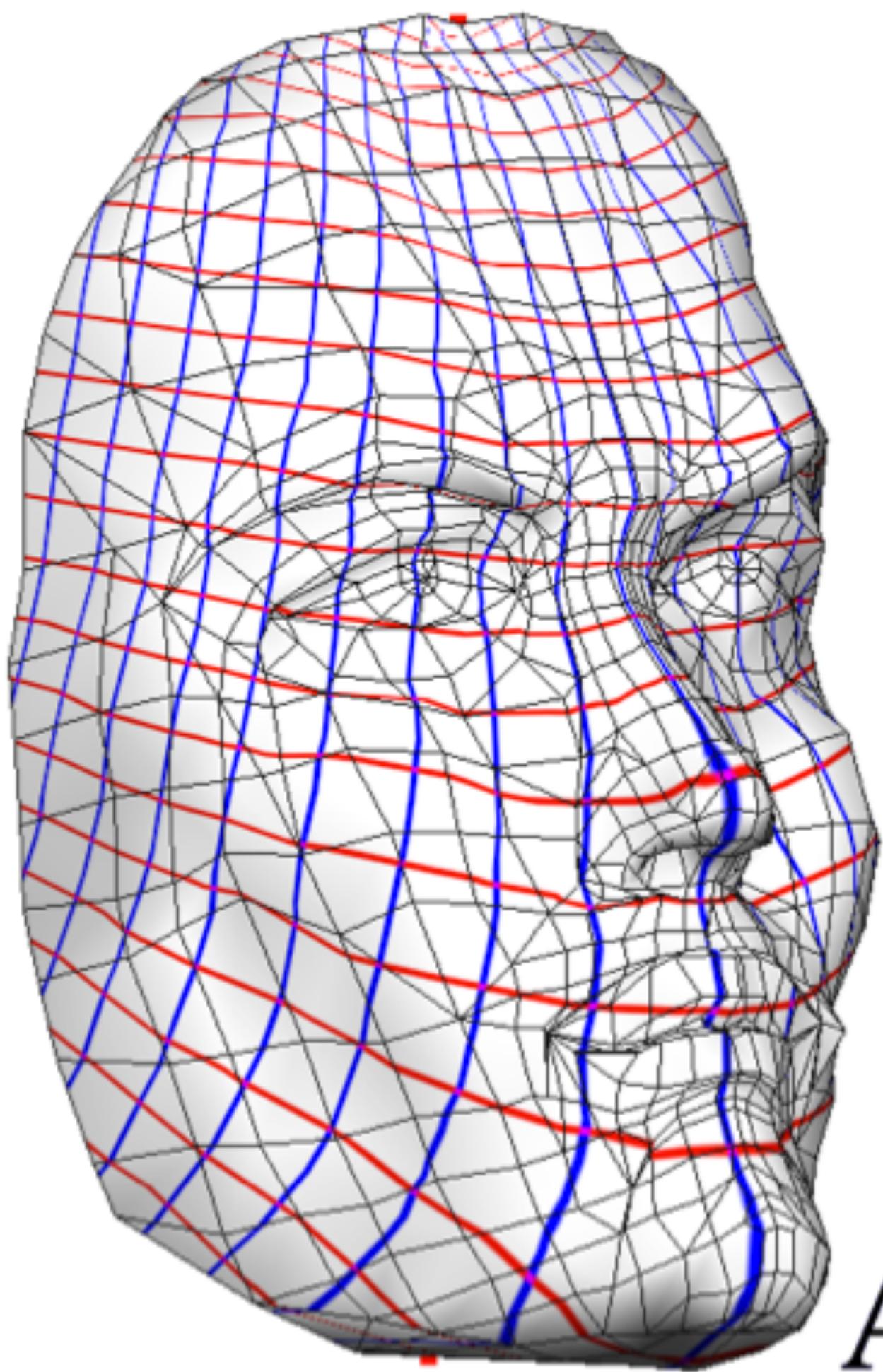
$$C(\mathbf{x}) = \|\mathcal{A}\mathbf{x} - \mathbf{b}\|^2$$

- At least two points must be fixed to make \mathbf{A} full-rank

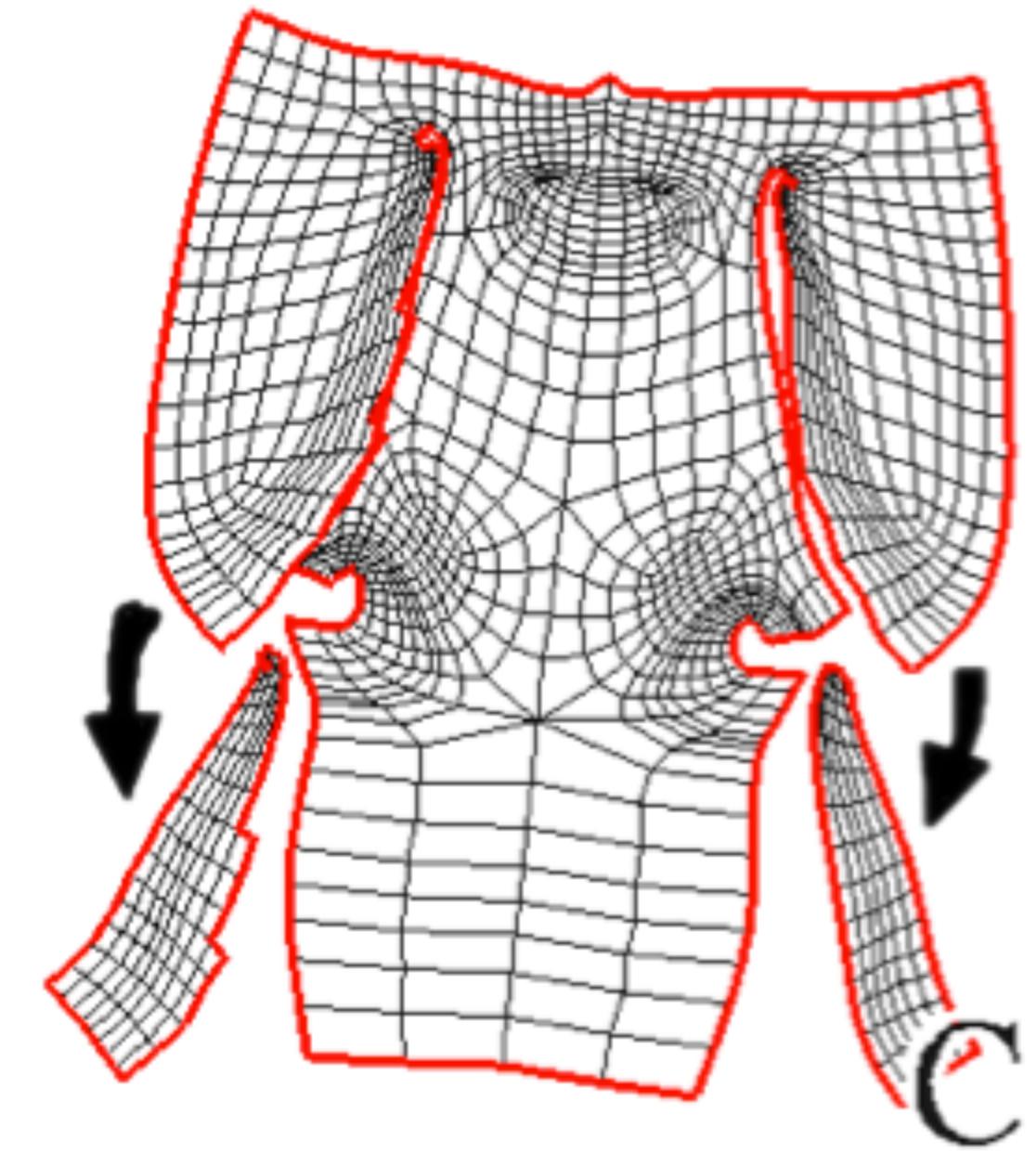
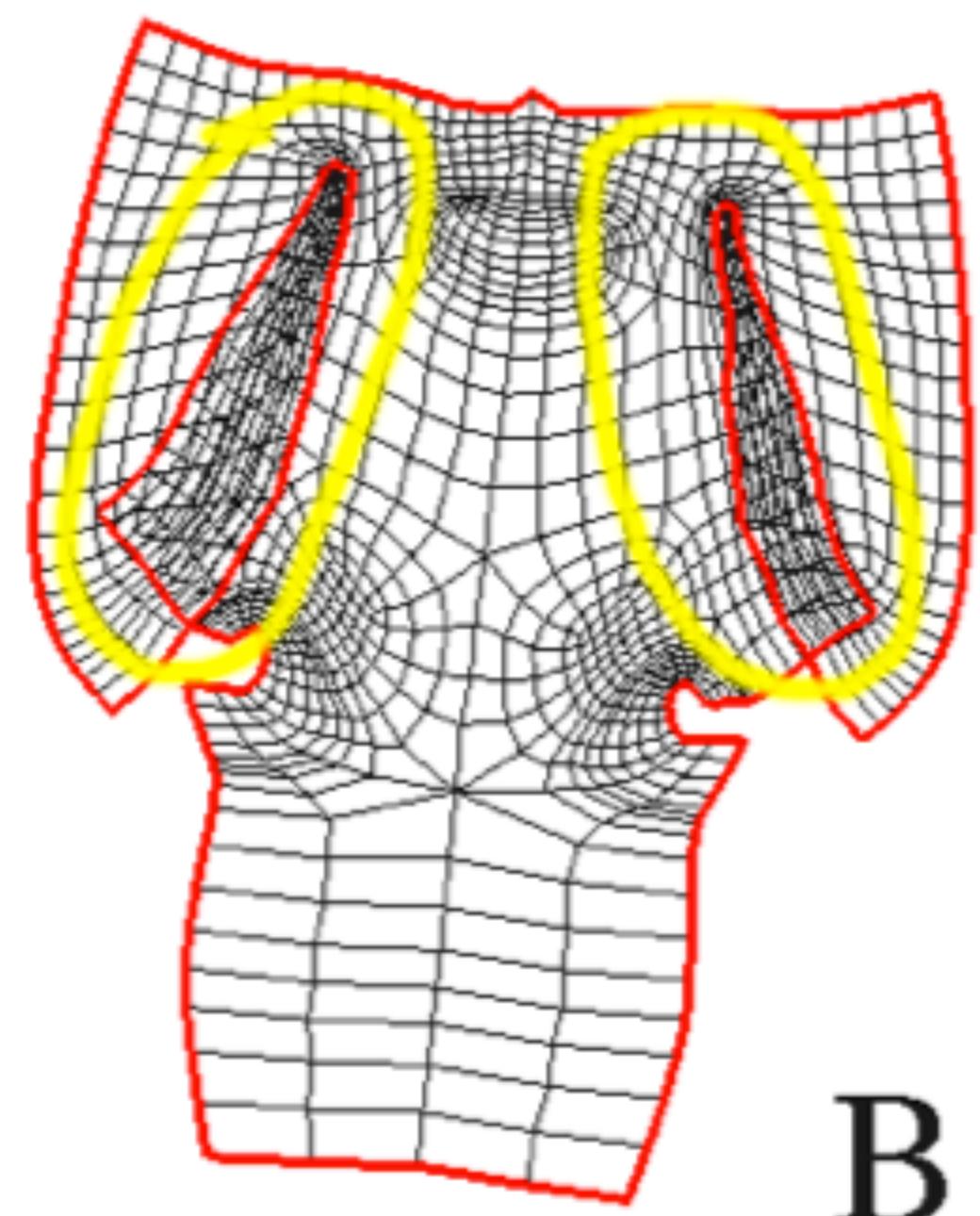
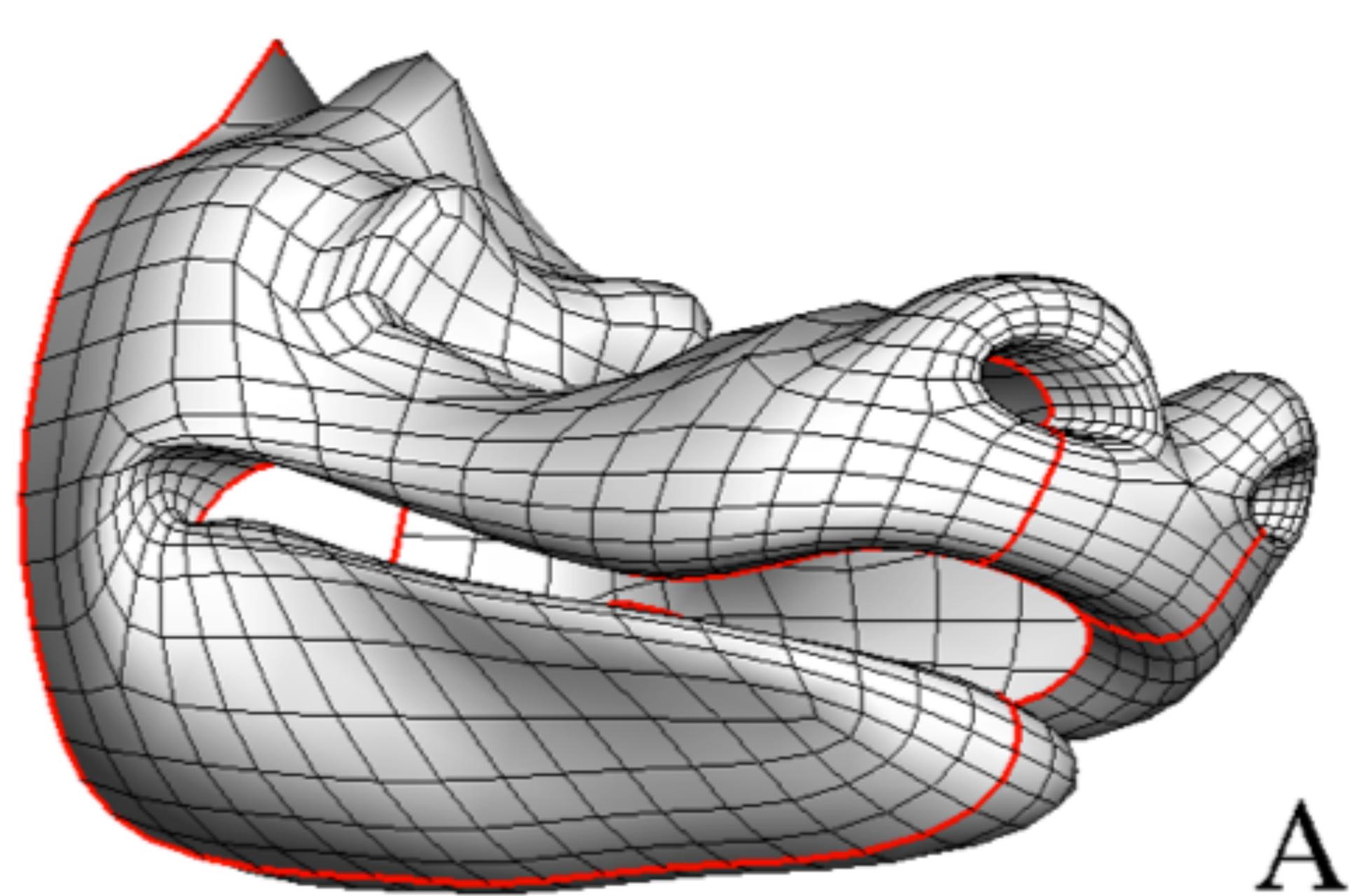
Properties of LSCM

- The solution is unique
- The solution is invariant to similarity in texture space
- The solution is independent of the resolution
- Triangle flips are “rare”

Resolution independence



Global overlaps are not prevented



Demo

- <https://libigl.github.io/libigl-python-bindings/tut-chapter4/>

Commercial applications

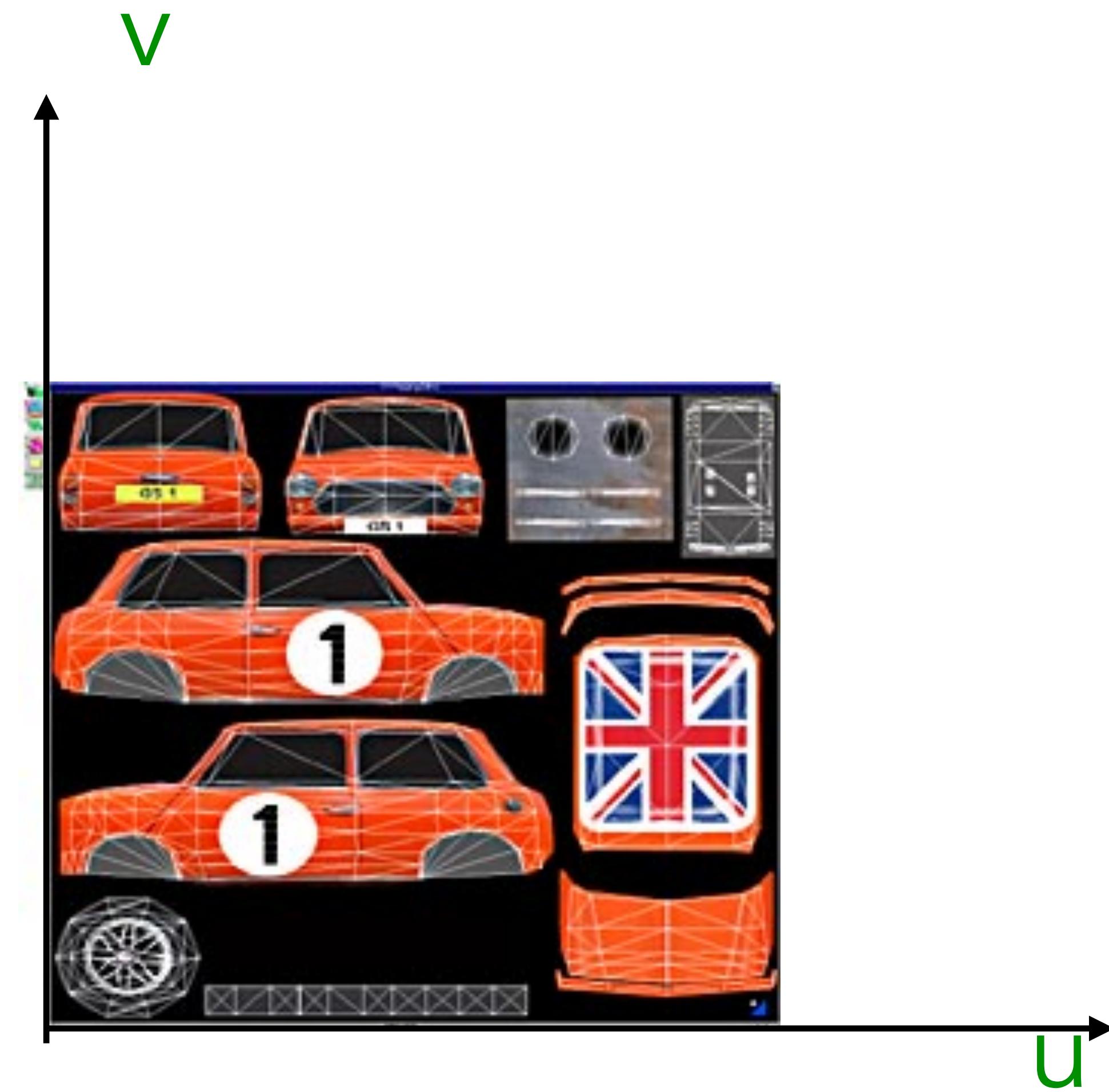
- Blender implements LSCM
- <http://www.blender.org/download/sandbox/lscm-basics/>



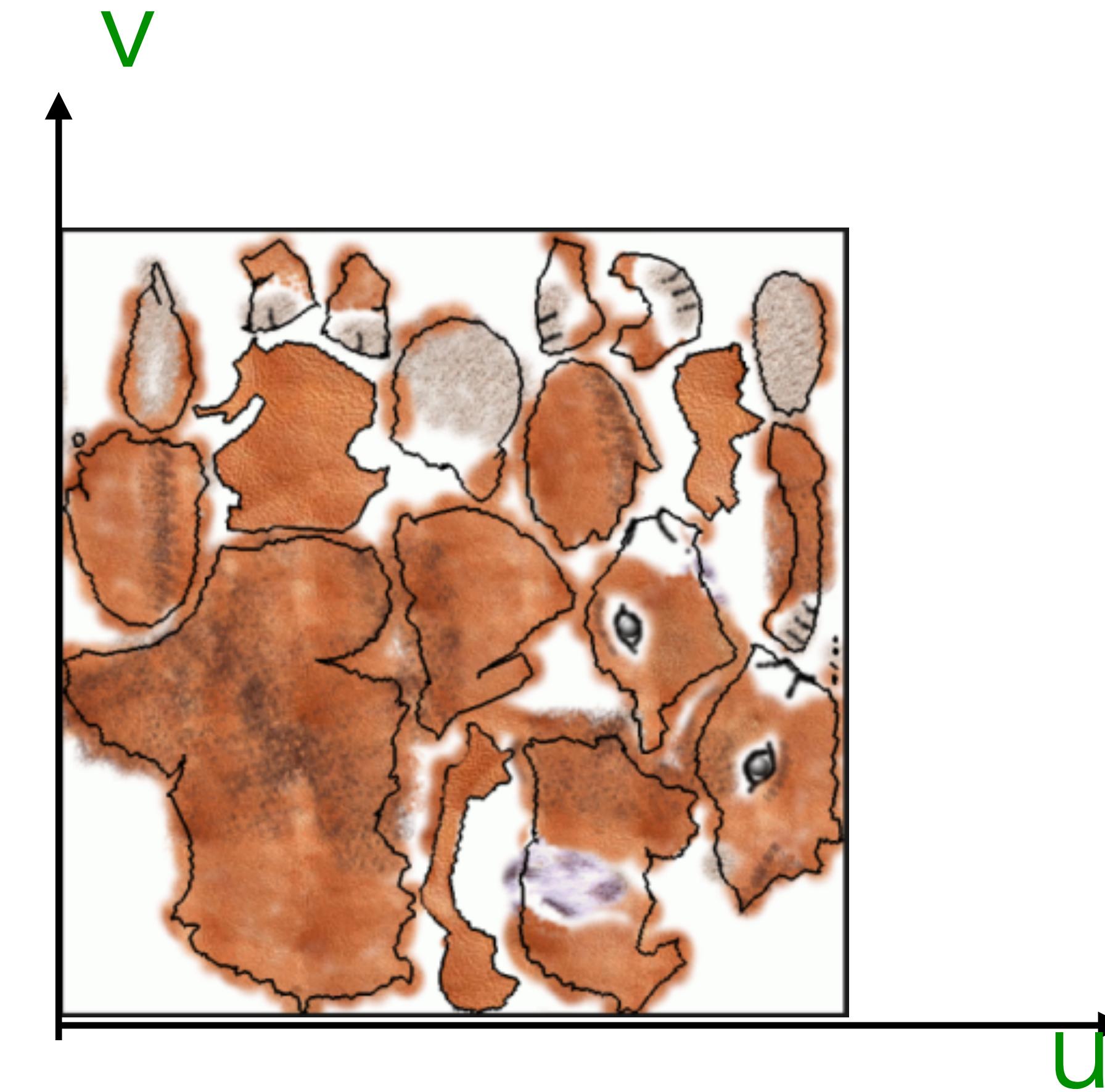
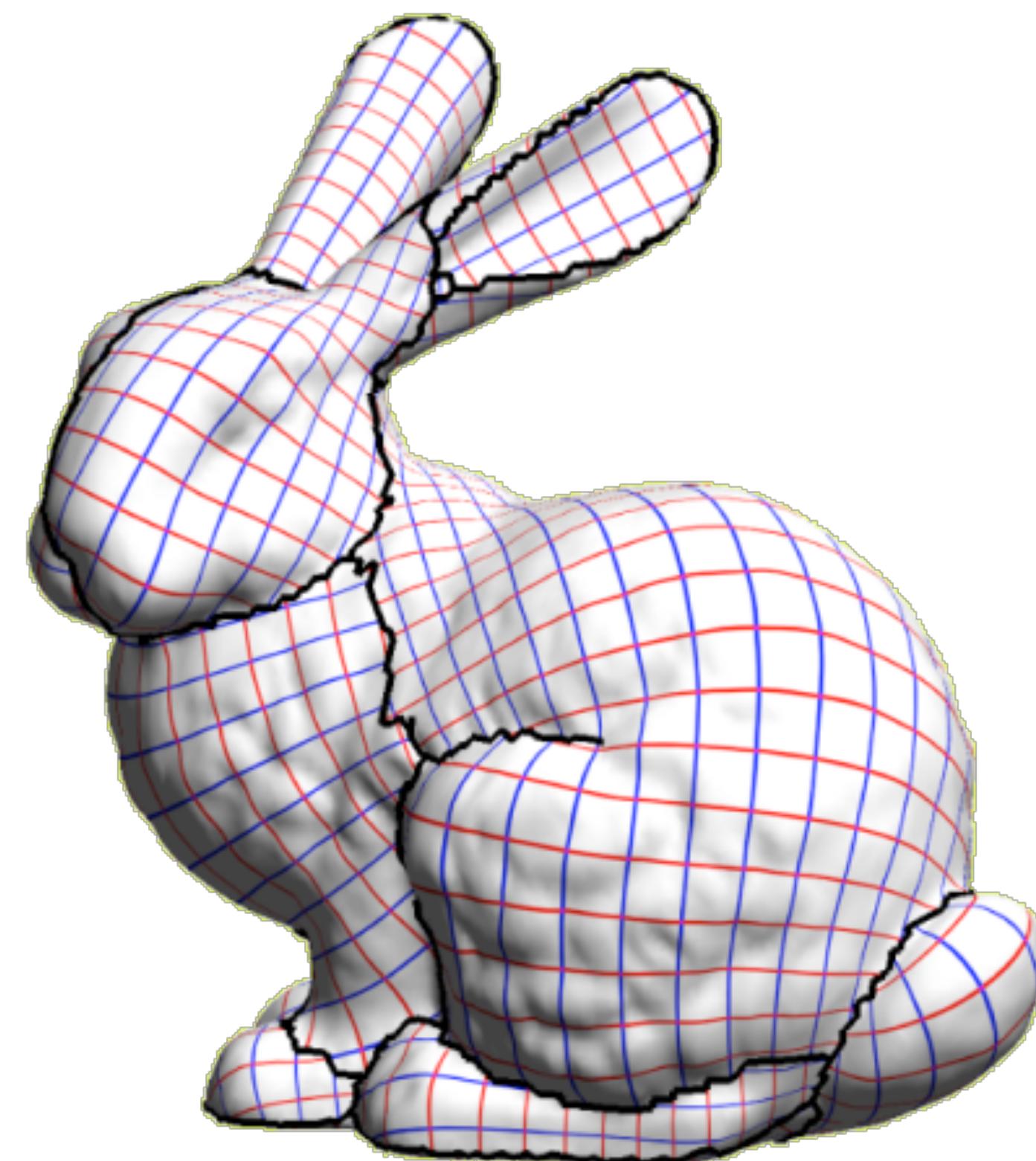
Global Parametrization

- All the algorithms that we studied till now are able to parametrize a part of a mesh homeomorphic to a disk
- To be able to parametrize arbitrary shapes we “cut” a mesh into parts, and we parametrize every part independently

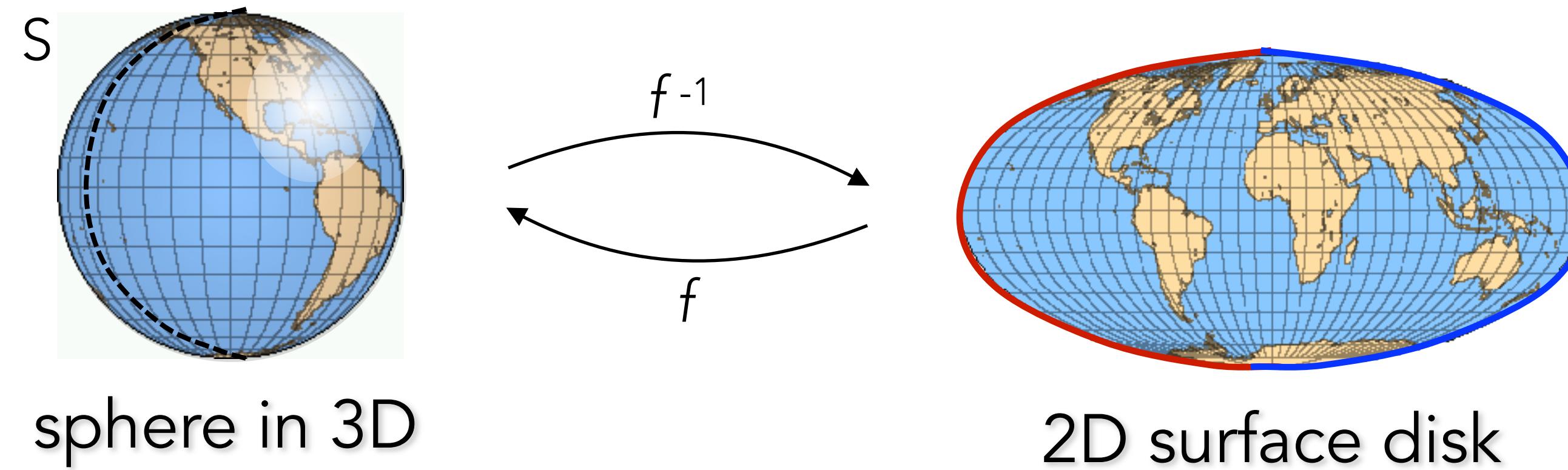
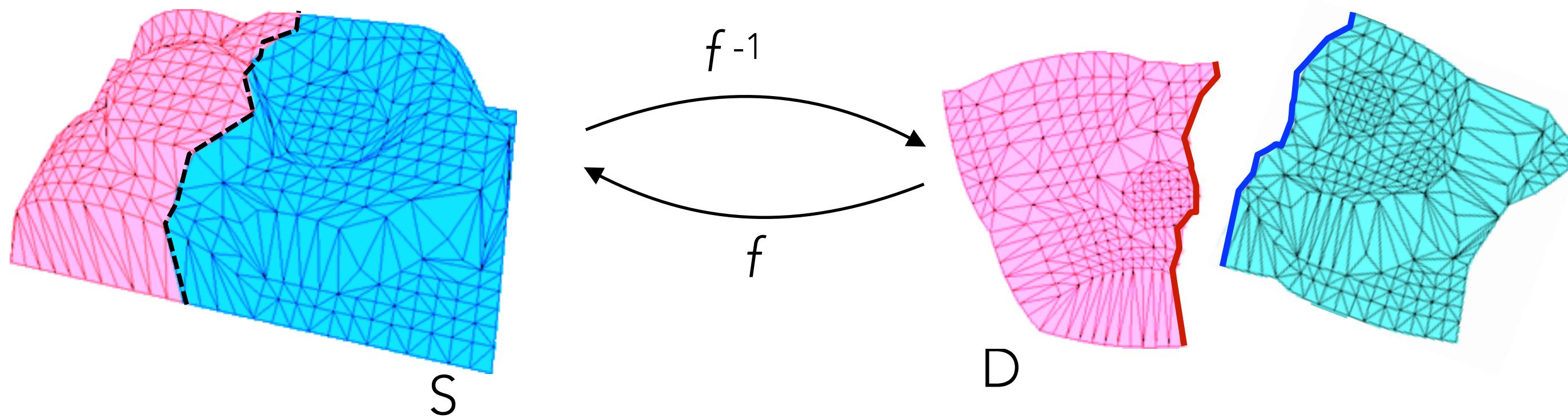
Manual Segmentation



Automatic Segmentation

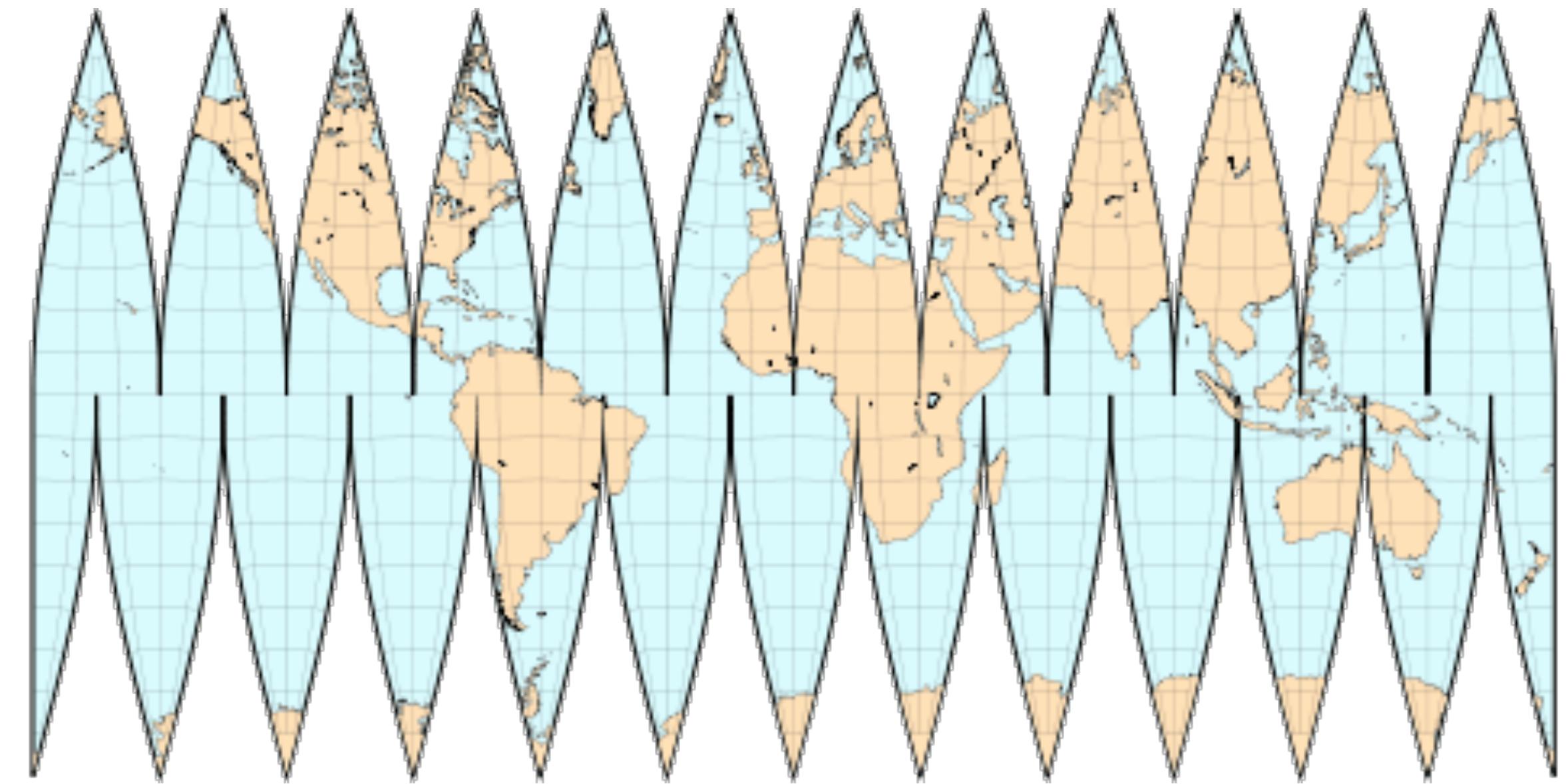
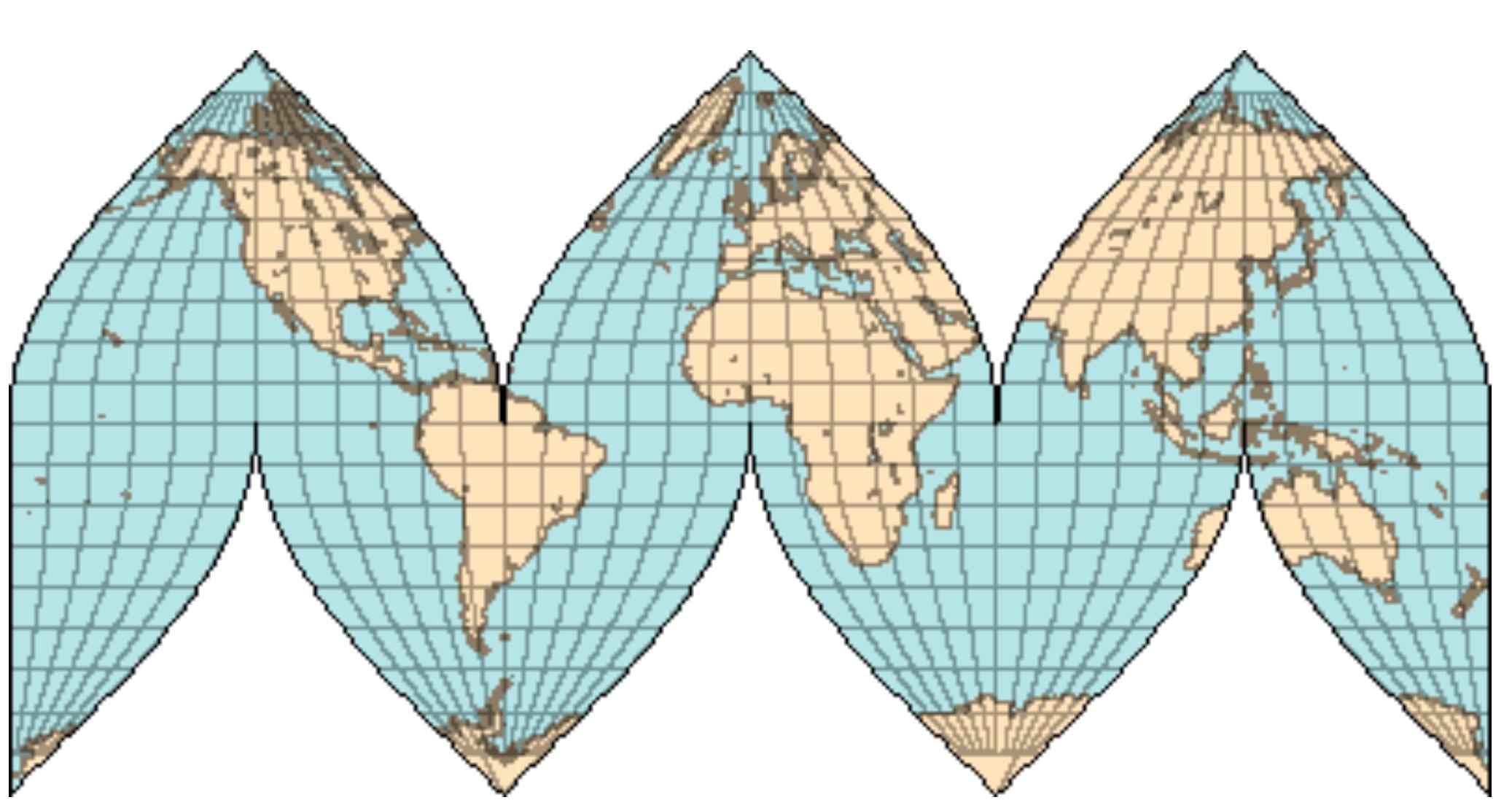


Good = “fewer cuts”?

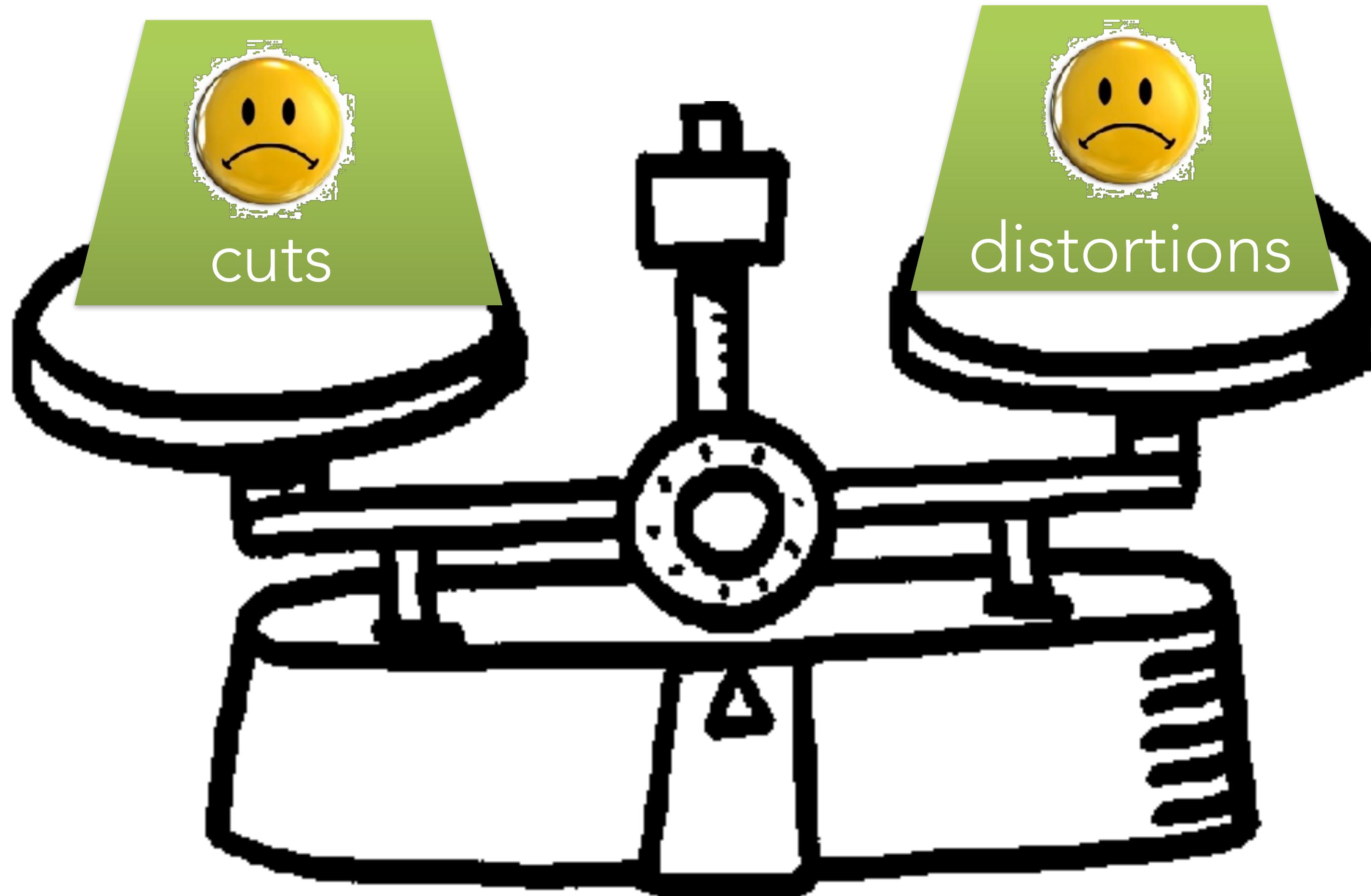


Good = “fewer cuts”?

but... more cuts => less distortion



A difficult balance



How to handle cuts?

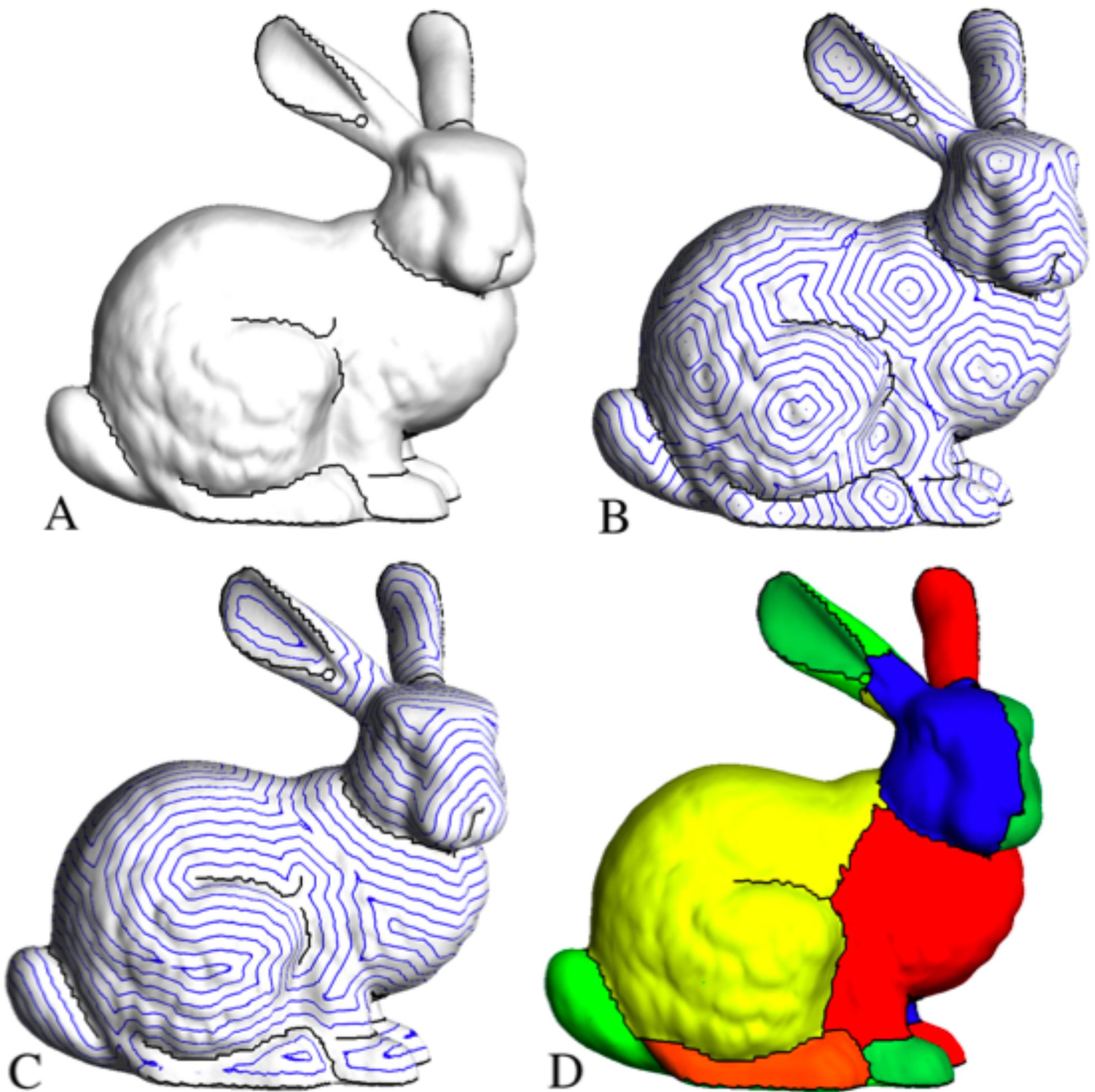
- We can ignore them and parametrize every chart separately
- We want to impose continuity of the derivatives of the parametrization across cuts
- Both have their own pros and cons, we will see an example for both cases

No continuity between charts

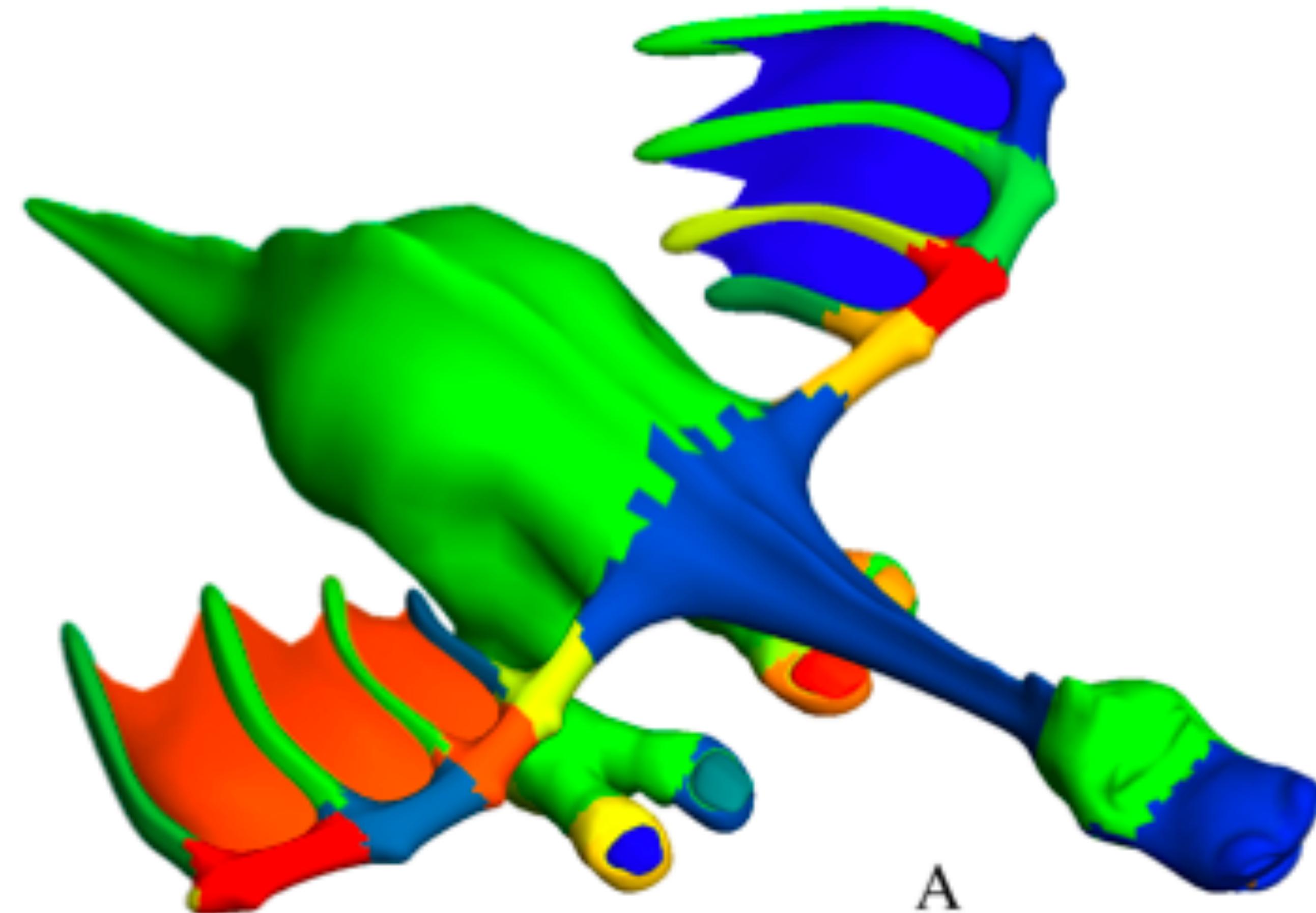
- Every chart can be parametrized separately
- The collection of the separate chart can be grouped in a square with a packing algorithm
- We study the method proposed in the LSCM paper

Segmentation

- Identify high curvature areas
- Seeds are the maxima of the distance to feature function
- Grow seeds into regions
- Merge regions if possible



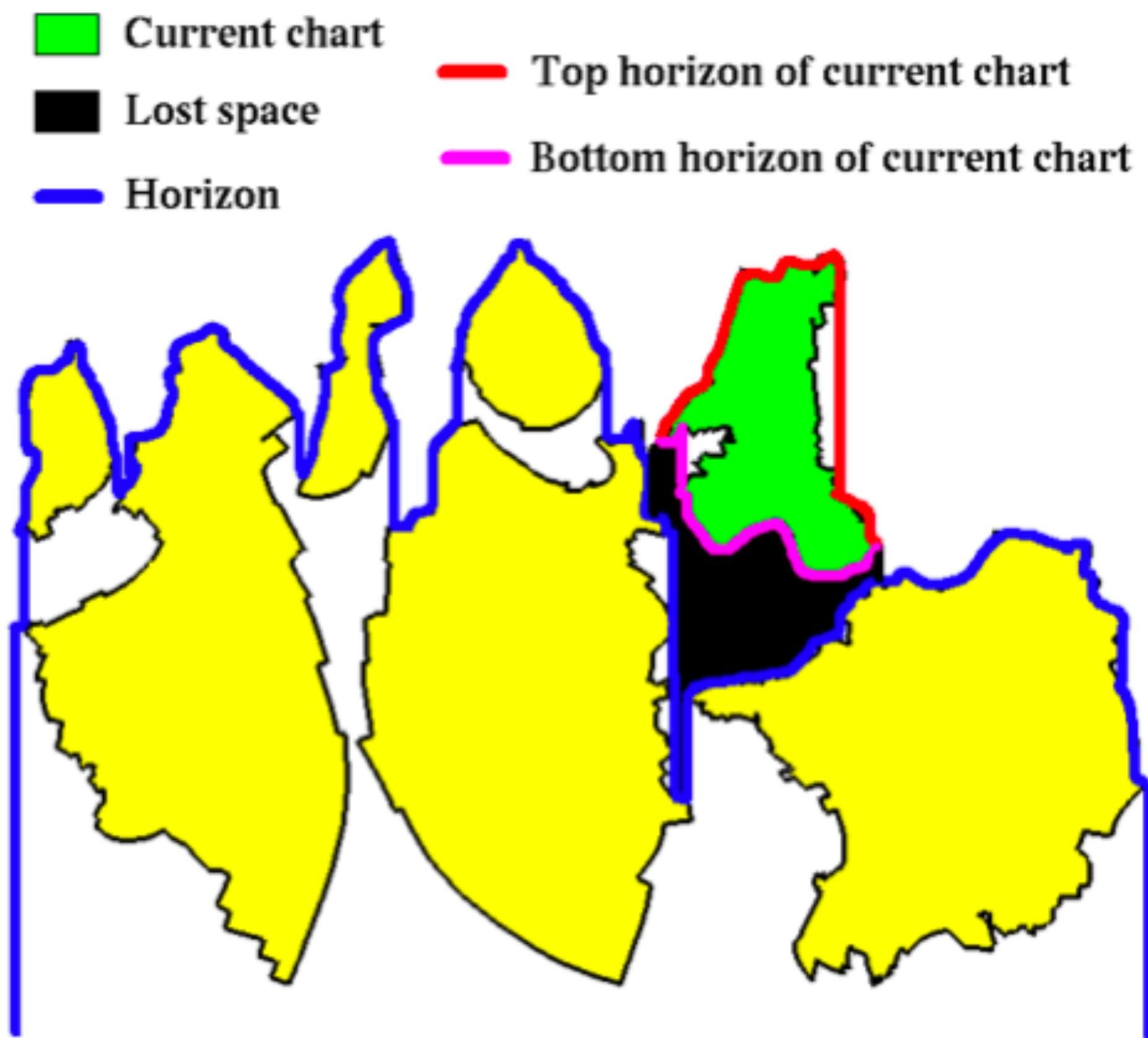
Segmentation result



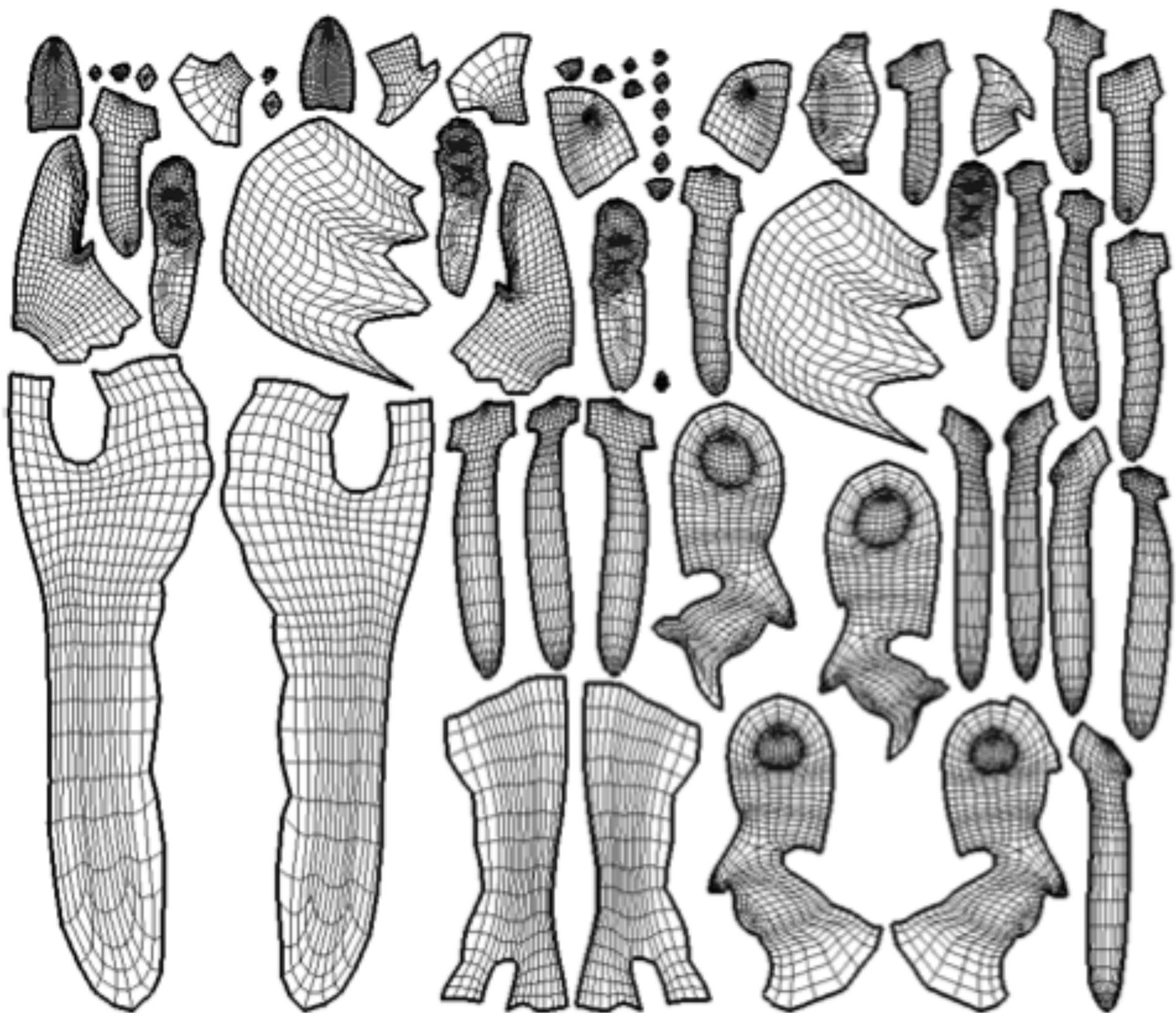
Packing

- The computation the optimal packing is NP-complete
- We need an heuristic
- The problem has been studied extensively in computational geometry, with the goal of computing high-quality results but only for a small number of objects
- The authors propose an heuristic based on the famous “Tetris” game that is extremely fast and generates good results

Tetris packing

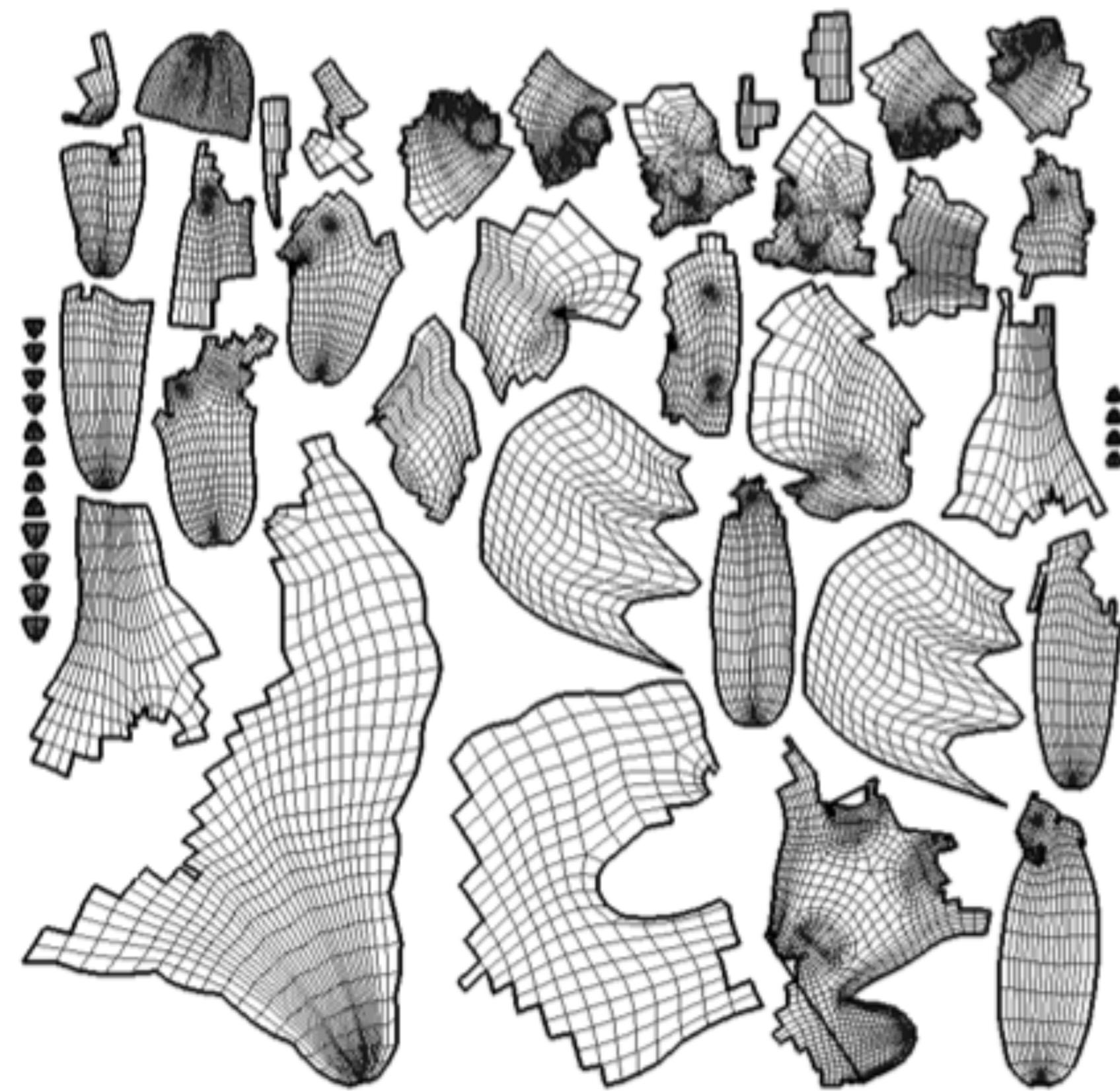
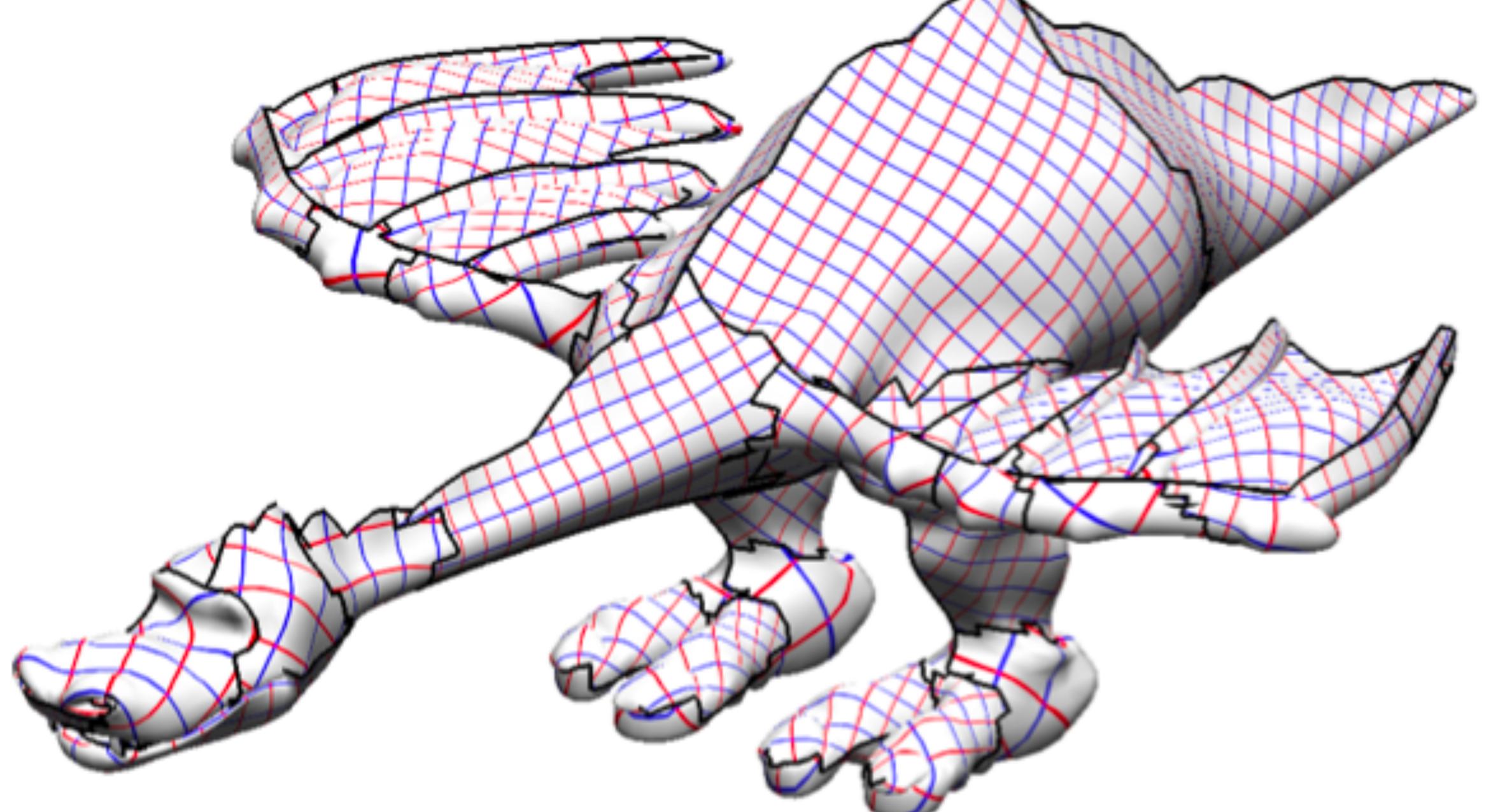


A

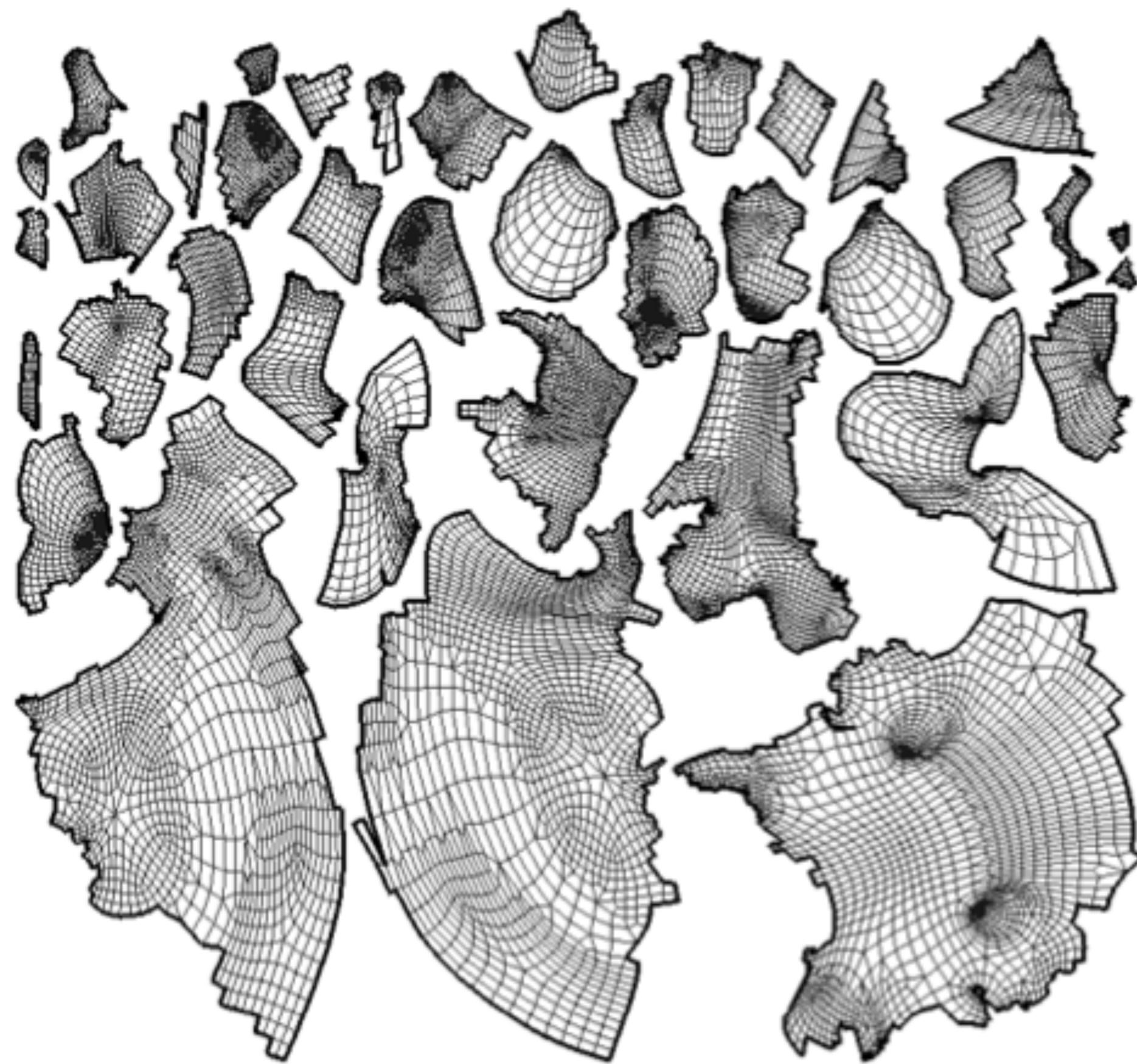
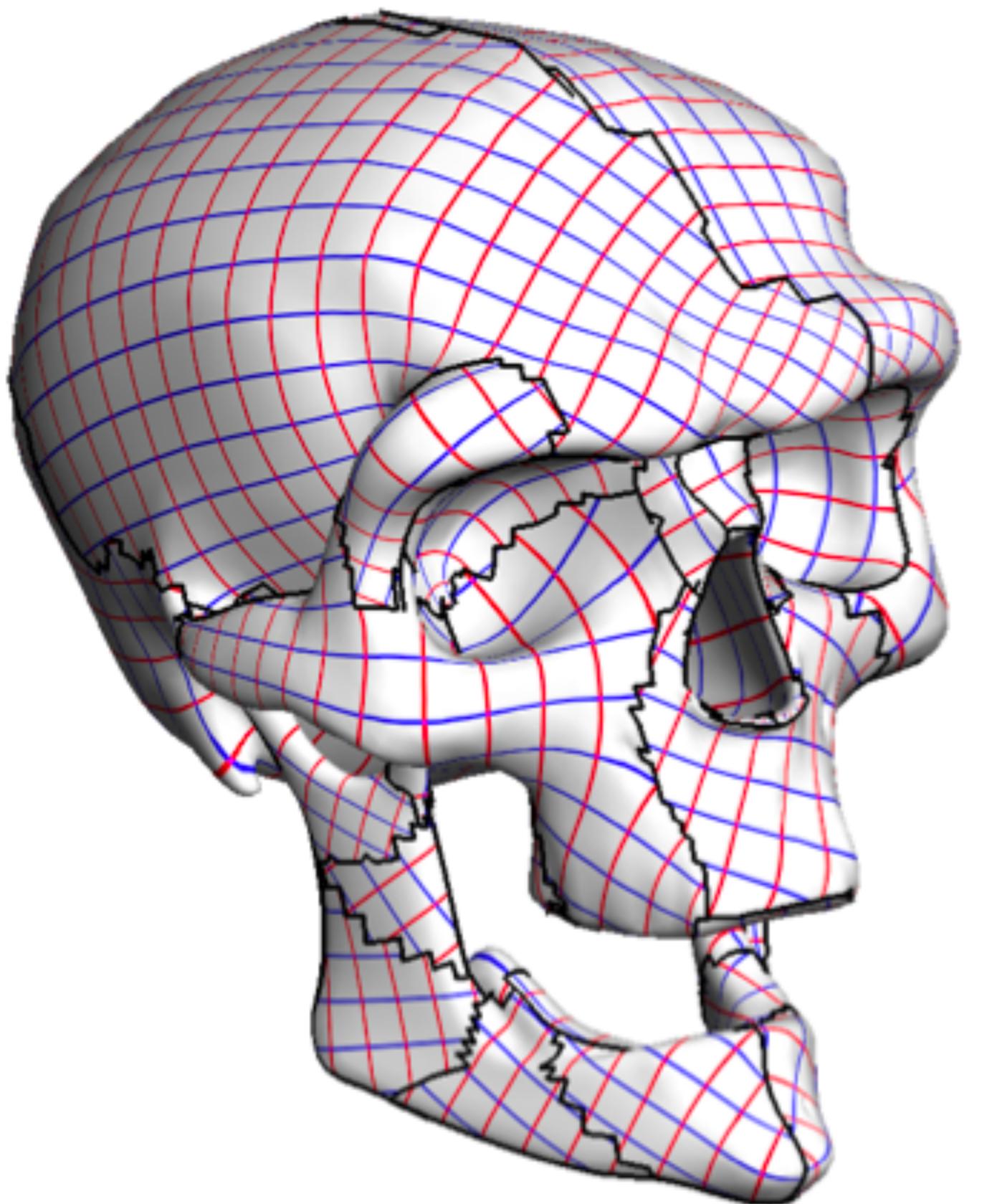


B

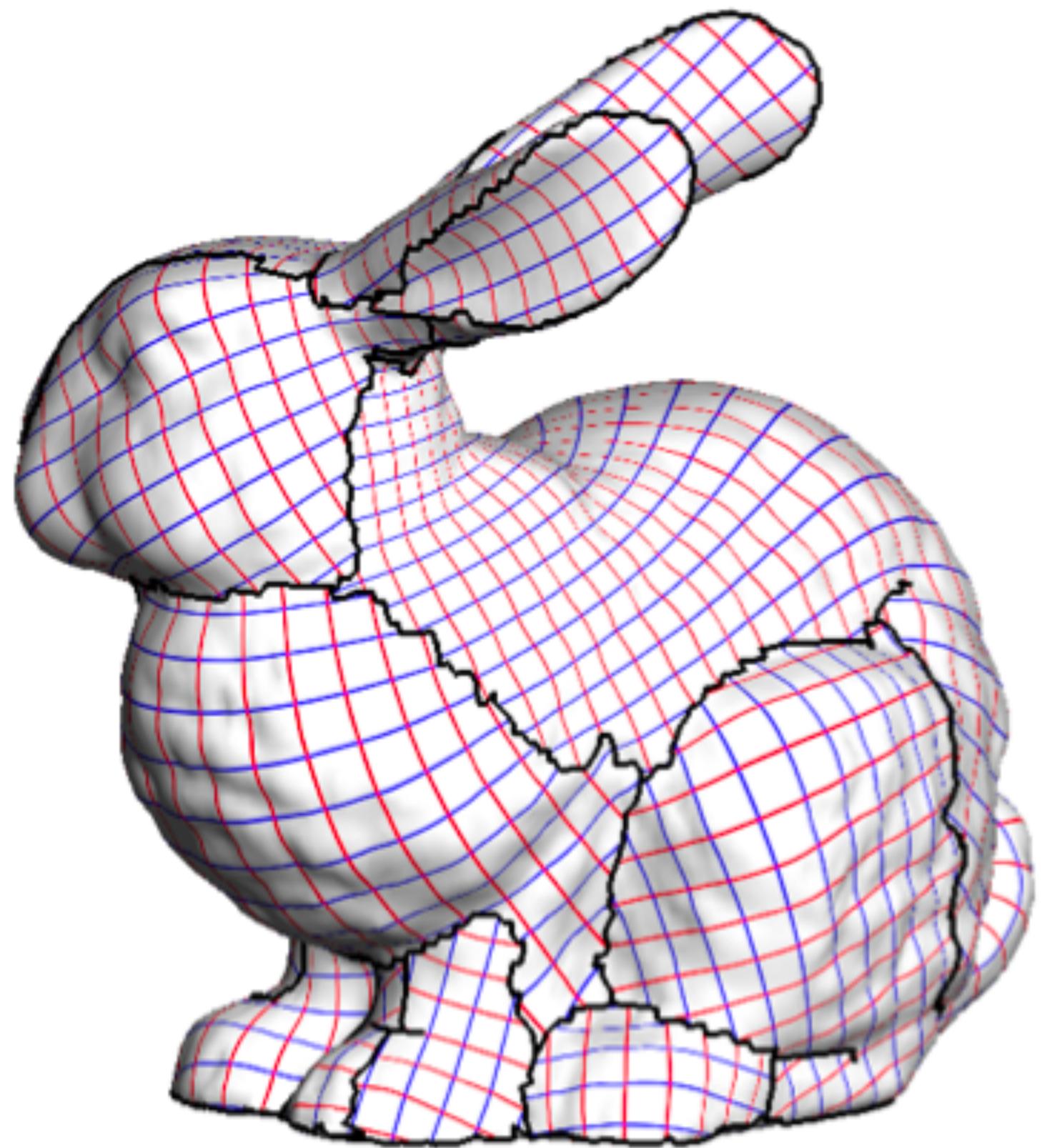
Results



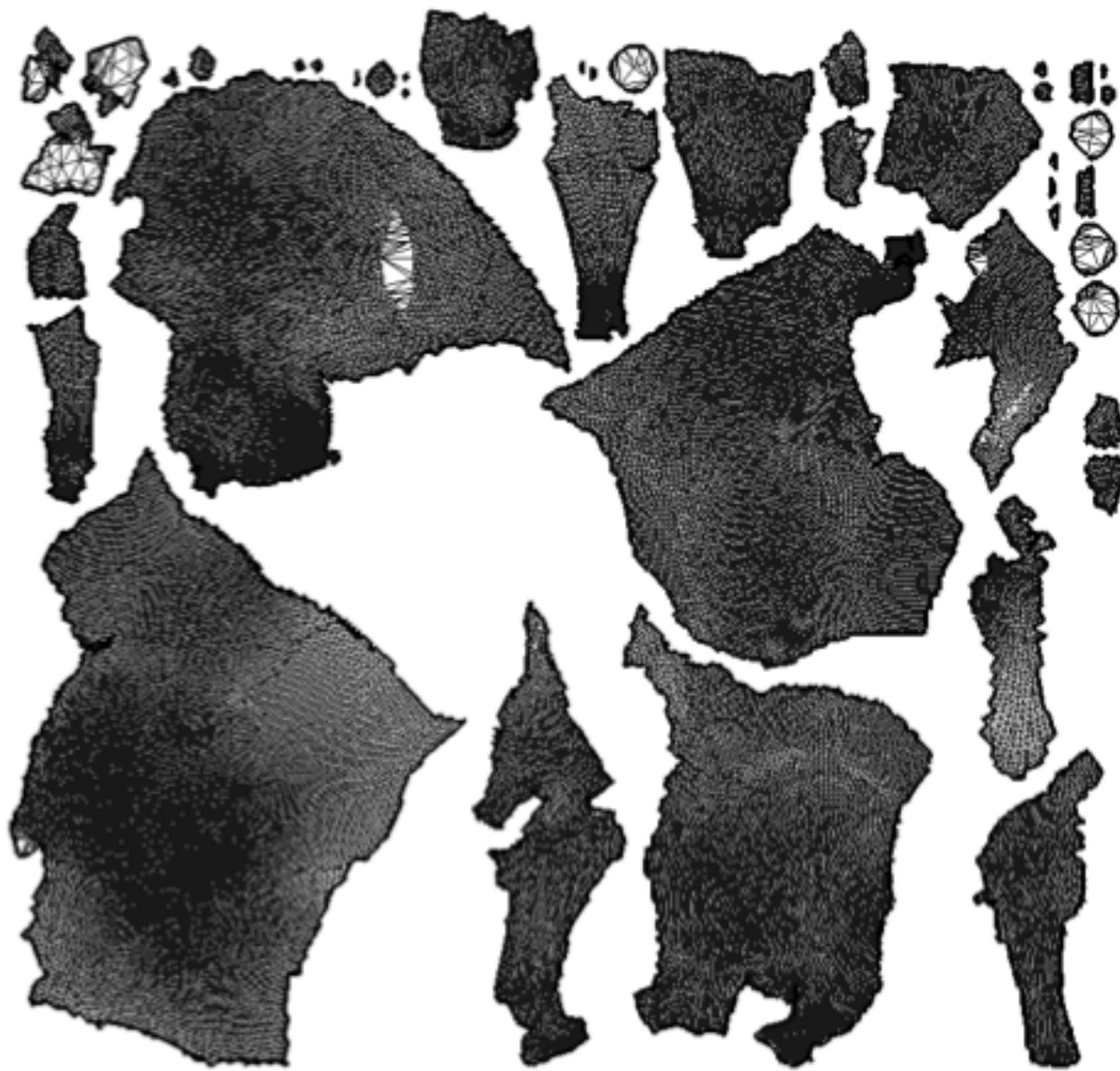
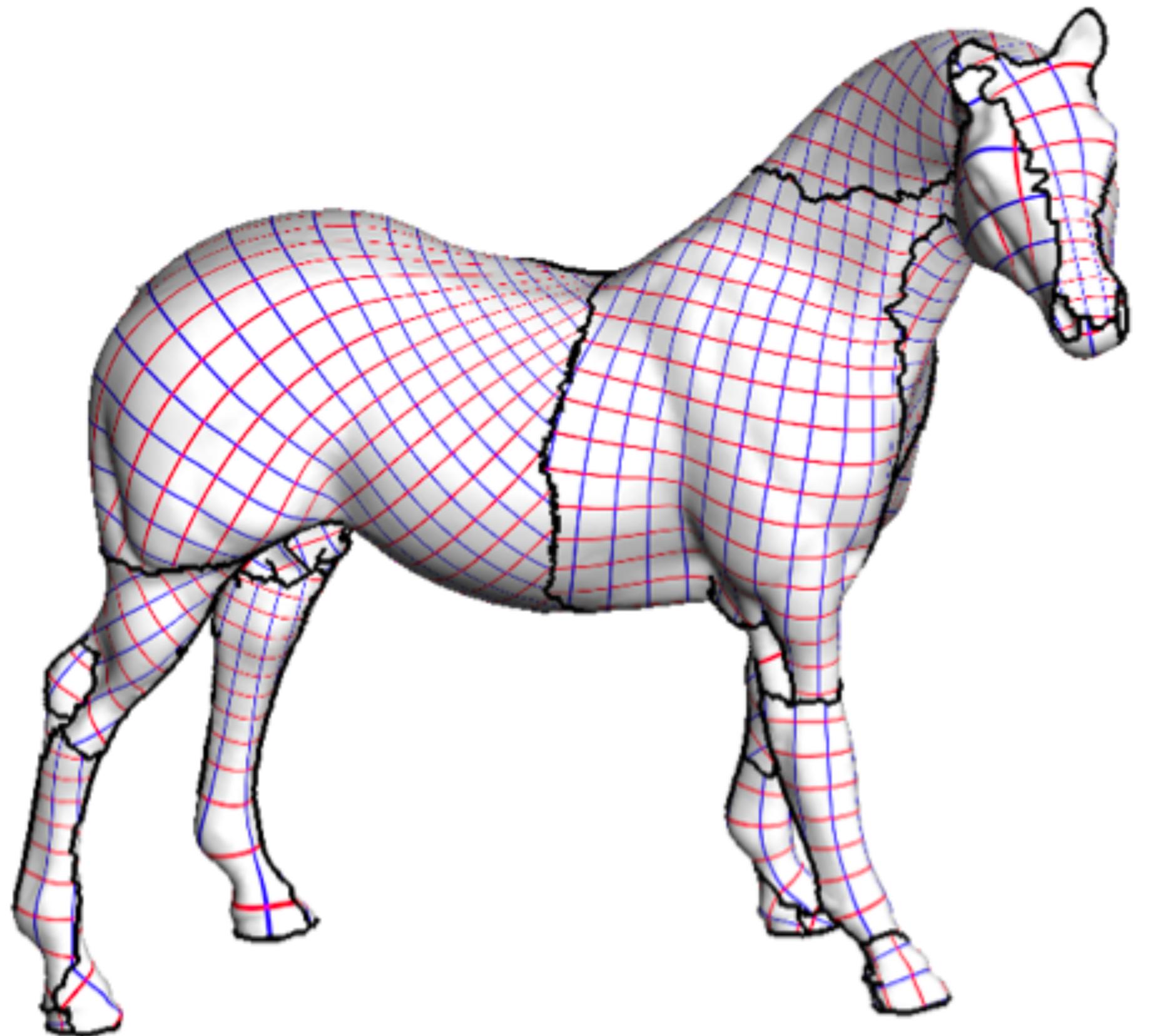
Results



Results



Results



Applications

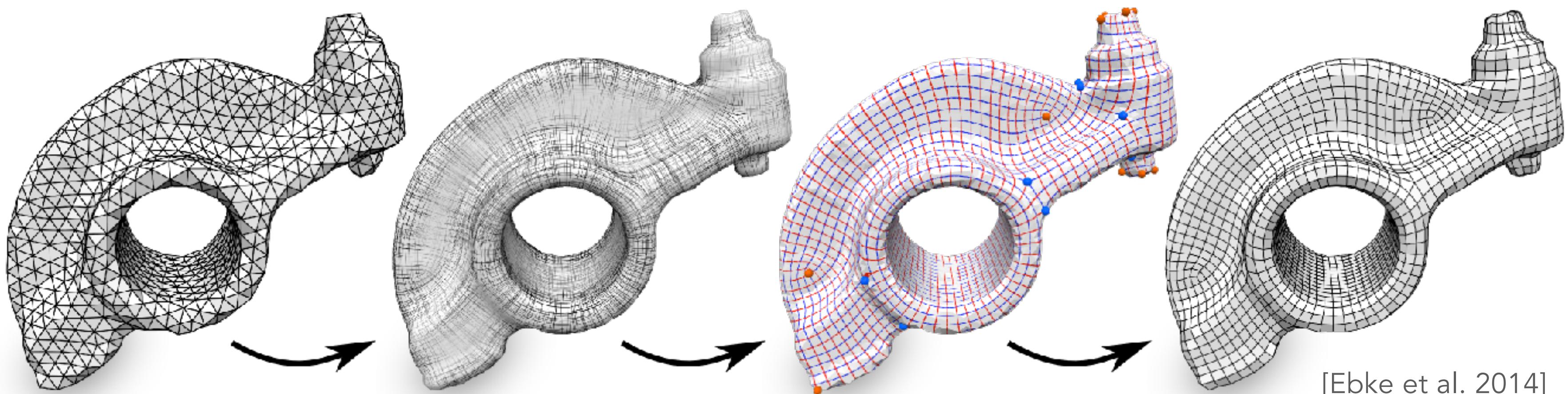


Continuity between charts

- For some applications (i.e. remeshing) continuity of the derivatives between charts is required
- If you are interested, I suggest to start from this paper:

**Mixed-Integer Quadrangulation - David Bommes, Henrik Zimmer, Leif
Kobbelt
Siggraph 2009**

Quadrilateral Meshing



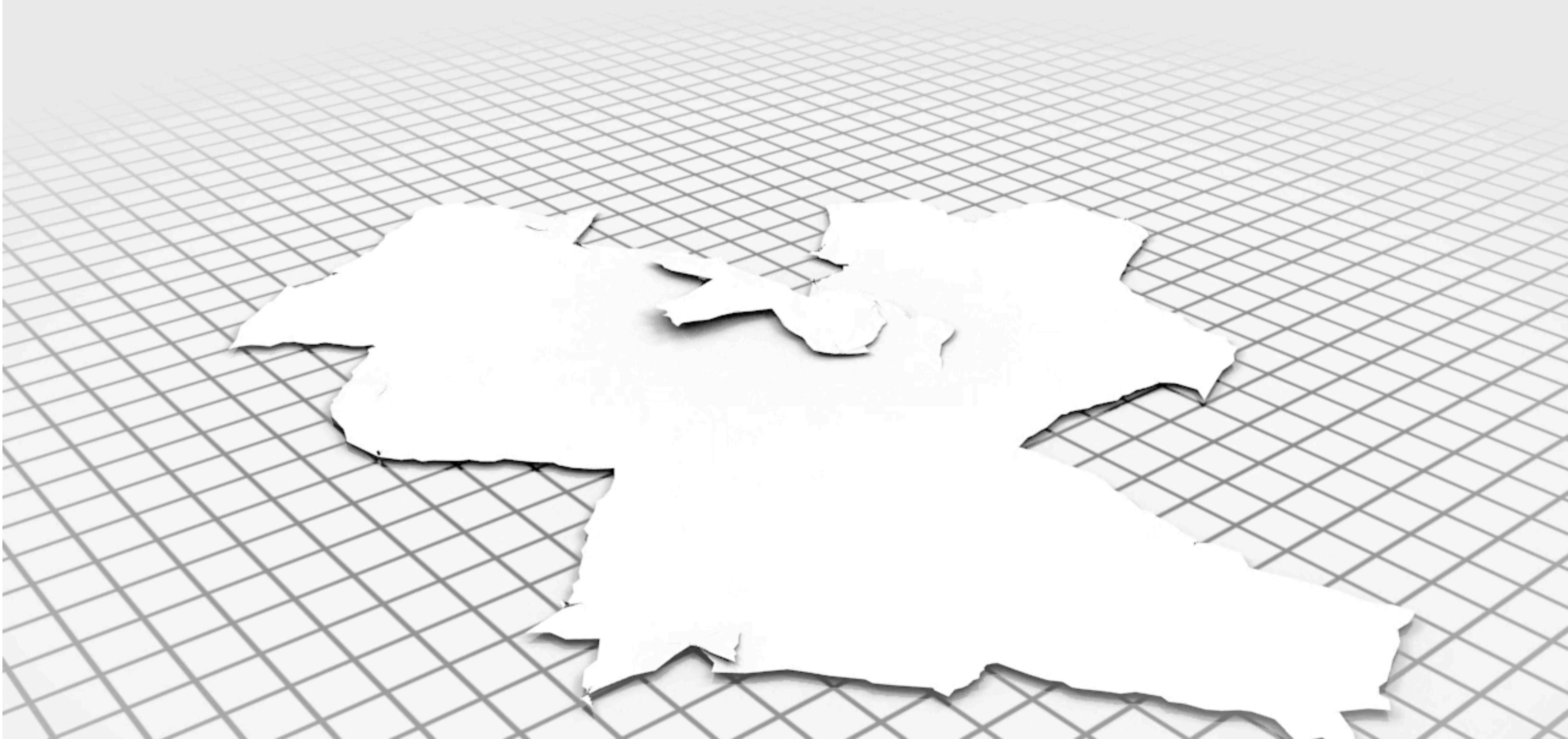
[courtesy of Hans-Christian Ebke]



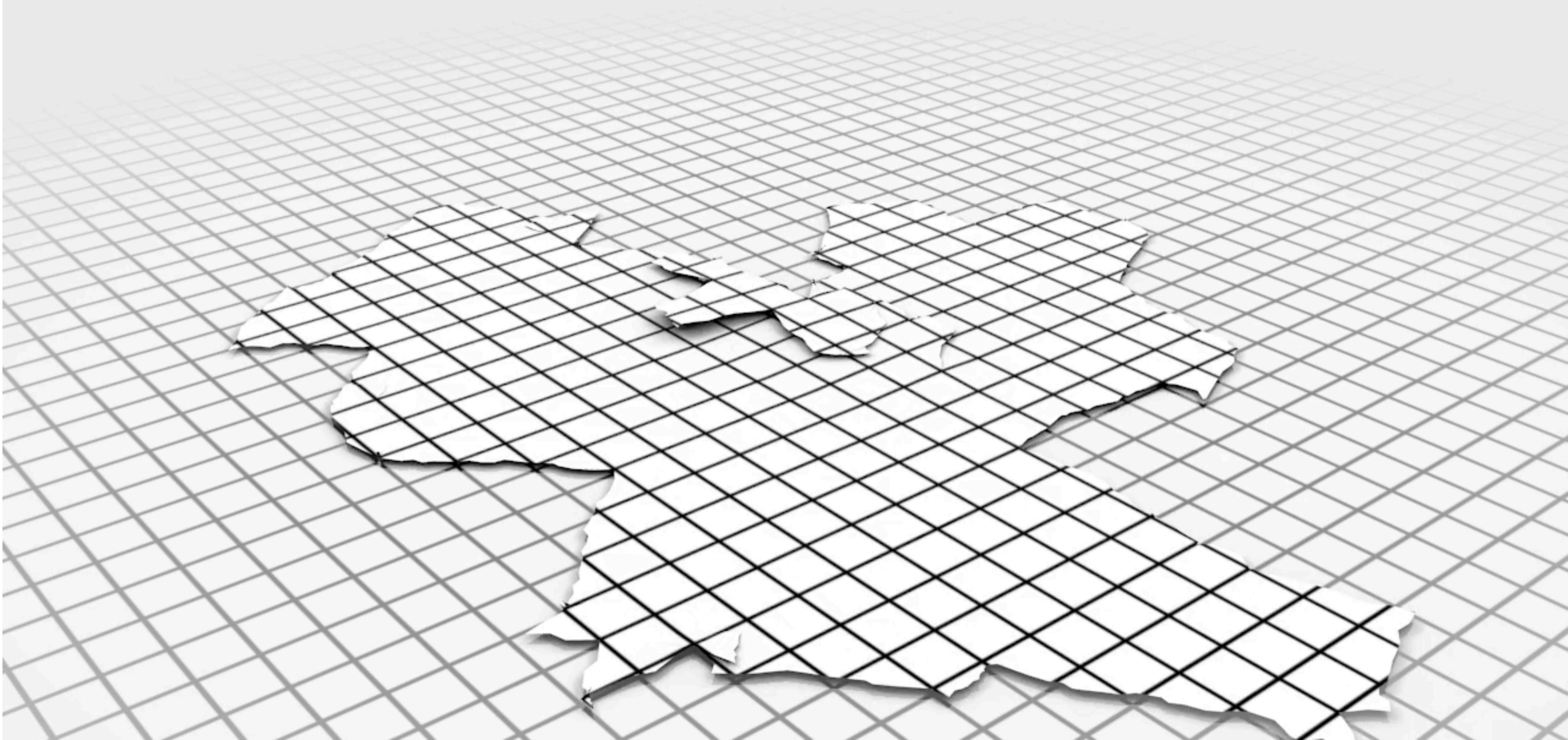
[courtesy of Hans-Christian Ebke]



[courtesy of Hans-Christian Ebke]



[courtesy of Hans-Christian Ebke]



References

- Surface Parameterization: a Tutorial and Survey: <http://graphics.stanford.edu/courses/cs468-05-fall/Papers/param-survey.pdf>
- Polygon Mesh Processing: Chapter 5
- Mixed Integer Quadrangulation: David Bommes, Henrik Zimmer, Leif Kobbelt. SIGGRAPH 2009