Automatic Control Project

The dynamics of the drone is given by the equations

$$
\begin{aligned}
\dot{e}_z &= v_z \\
\dot{v}_z &= -g + \frac{f}{m_d} \\
\dot{e}_\psi &= \omega_z \\
\dot{\omega}_z &= \frac{\tau_c}{I_z}
\end{aligned}
$$

For the given dynamics' equations of the drone, the state space representation from presented as follows.

$$
\begin{bmatrix} \dot{e_z} \\ \dot{v_z} \\ \dot{e_\varphi} \\ \dot{w_z} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_z \\ v_z \\ e_\varphi \\ w_z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{\text{IZ}} \end{bmatrix} \begin{bmatrix} \widehat{f} \\ \tau \end{bmatrix}
$$

$$
\text{Z} = \begin{bmatrix} e_z \\ e_\varphi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_z \\ v_z \\ e_\varphi \\ w_z \end{bmatrix}
$$

Substitute given mass and moment of inertial values, md = 0.5kg and Iz = 0.05kgm^2

$$
\begin{bmatrix} \dot{e_z} \\ \dot{v_z} \\ \dot{e_\varphi} \\ \dot{w_z} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_z \\ v_z \\ e_\varphi \\ w_z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} \widehat{f} \\ \tau \end{bmatrix}
$$

$$
[z] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_z \\ v_z \\ e_\varphi \\ w_z \end{bmatrix}
$$

Checking the given systems whether controllable or not, the controllable matrix must have a full rank.

$$
\text{Ctrl} = \begin{bmatrix} B & \text{AB} & A^2B & A^3\text{B} \end{bmatrix}
$$

$$
\text{Ctrl} = \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

Rank (Ctrl) = 4

Since the rank of the controllability matrix ***Ctrl*** is equal to the number of states, which is 4 states, the system is controllable.

the gain matrix K found and the real part of the eigenvalues of the closed-loop system

$$\text{K\_c} = \begin{bmatrix} -1.1655 & -1 & 0 & 0 \\ 0 & 0 & -0.2002 & -0.1283 \end{bmatrix}$$

Real part of closed-loop eigenvalues of (A+B*K_c) become,

[-1, -1, -1.2827, -1.2827]

the complete dynamics of drone which includes perturbations and integral terms in state space representation form

$$\begin{bmatrix} \dot{\sigma_z} \\ \dot{e_z} \\ \dot{v_z} \\ \dot{\sigma_\psi} \\ \dot{e_\psi} \\ \dot{w_z} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_z \\ e_z \\ v_z \\ \sigma_\psi \\ e_\psi \\ w_z \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} \widehat{f} \\ \tau \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 20 \end{bmatrix} \begin{bmatrix} A_z \\ A_\psi \end{bmatrix}$$

$$[z] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \sigma_z \\ e_z \\ v_z \\ \sigma_\psi \\ e_\psi \\ w_z \end{bmatrix}$$

the optimality curve by minimizing gamma over the given interval

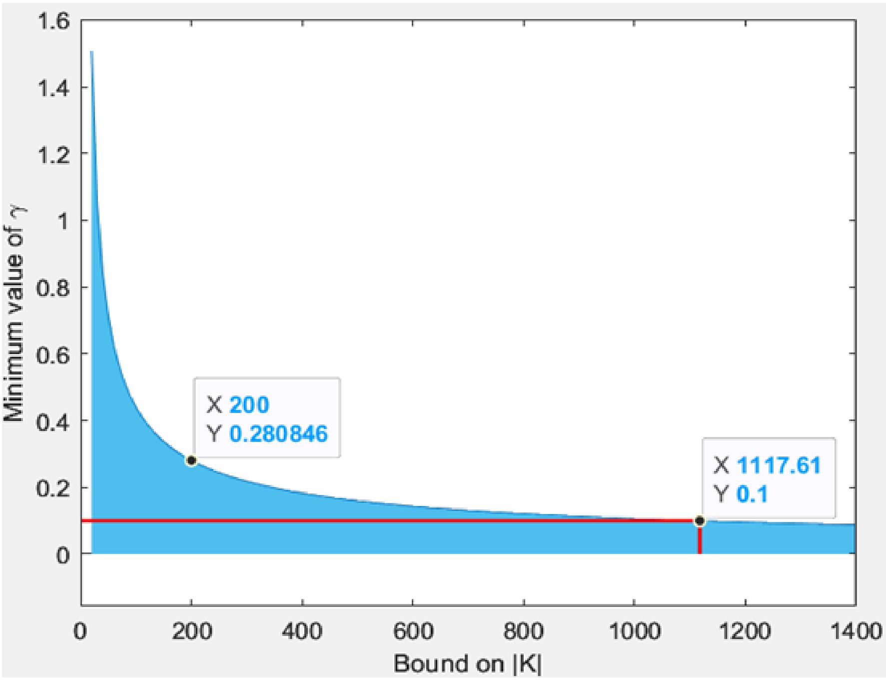

Fig. 1 Optimality curve

Over the given interval from k from 10:1400, it is not possible to guarantee gamma $(\gamma) less than 0.1 for the \|K\| < 200$, which is $\gamma > 0.1$.

$$K = \begin{bmatrix} -138.7582 & -134.2847 & -4.0834 & 0 & 0 & 0 \\ 0 & 0 & 0 & -35.7527 & -33.3610 & -0.9383 \end{bmatrix}$$

Norm of K is 193.1396.

a controller satisfying all the requirements and using k = 1200 in the LMI constraints, corresponding state feedback gain matrix

$$K = \begin{bmatrix} -122.8456 & -119.1515 & -3.9171 & 0 & 0 & 0 \\ 0 & 0 & 0 & -31.6632 & -29.5424 & -0.9180 \end{bmatrix}$$

The norm become 171.1826.

$\gamma(gamma) optimal value is 0.096 corresponding to k = 1200. real values of close loop system poles are:$

$$-1.0631 \quad -3.3855 \quad -3.3855 \quad -8.6264 \quad -8.6264 \quad -1.1076$$

Simulation results for the system starting from initial conditions $x(0) = \begin{bmatrix} 0 & 1 & 0.1 & 0 & 0.5 & 0 \end{bmatrix}^T$ with constant disturbances wz = Az = 2, w = A = 0.2 for 15 seconds.

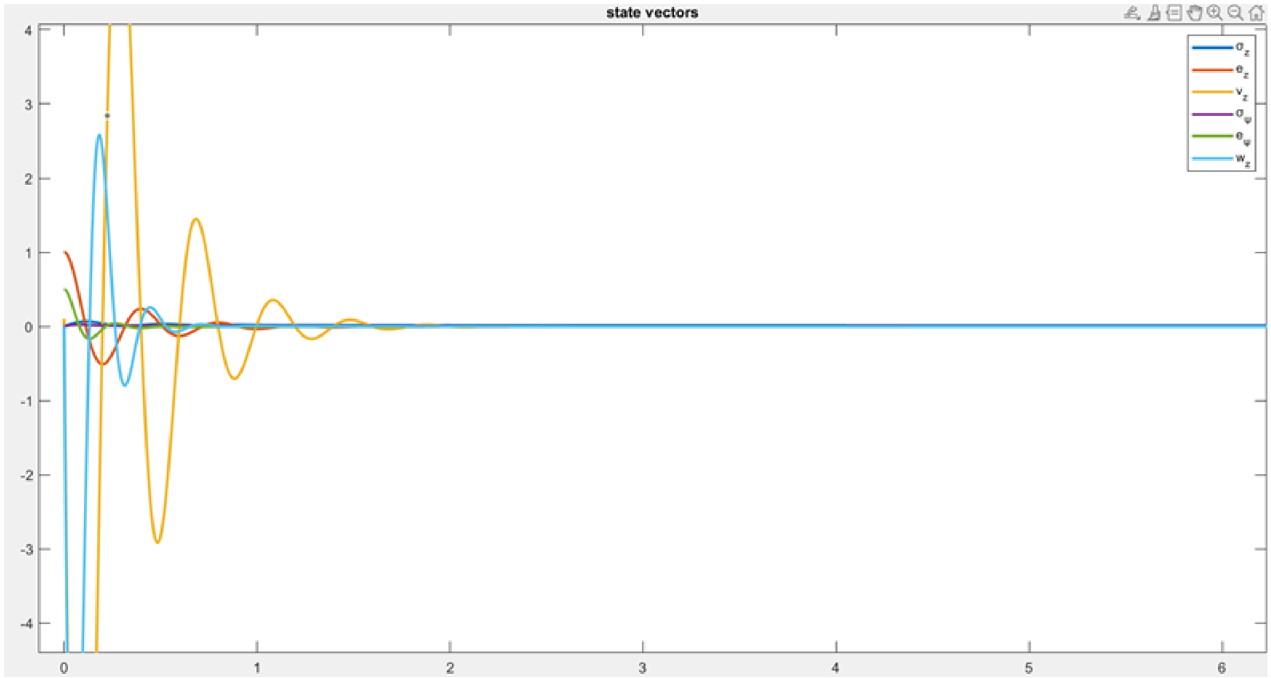

Fig. 2 individual state output graph in time domain

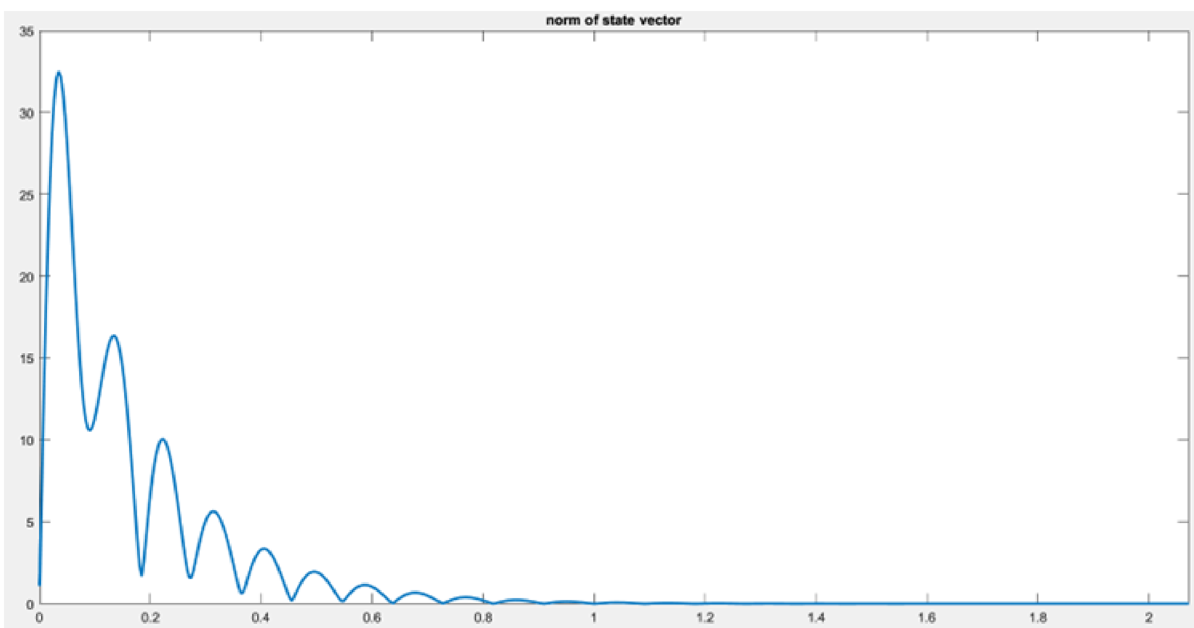

Fig. 3. Norm of state variables

As we can see in fig3, norm of state is converging to zero as time goes.

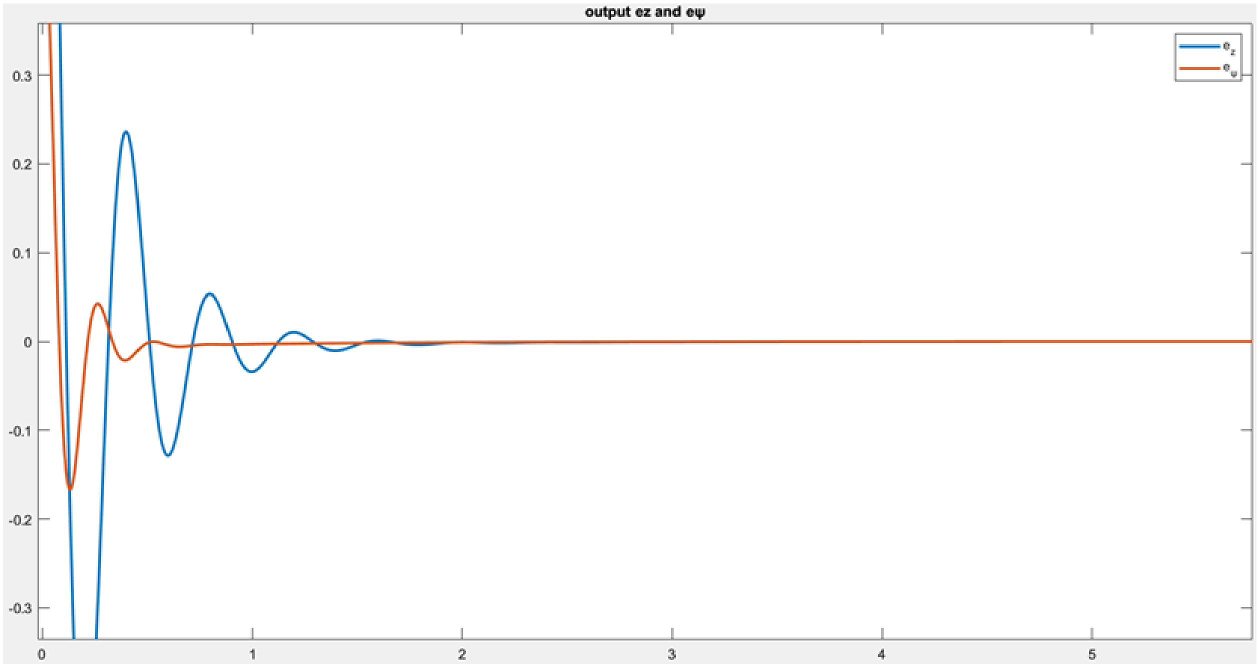

Fig 4. Outputs graph.