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|---|---|---|---|---|
| Big players | | Herschel Rabitz | | Princeton University, h-index 96 |
| | | F. K. Wilhelm | | Saarland University, h-index 45 |
| | | | | |
| Algorithms | | | aka | **Yellow: Innovation, Blue: Comparison** |
| GRAPE | 2-Dec-04 | Steffen Glaser | | Optimal control of coupled spin dynamics: design of NMR pulse sequences by gradient ascent algorithms |
| | | | | minimize the time required to produce a given unitary propagator |
| | 4-Aug-11 | Ilya Kuprov | | Second order gradient ascent pulse engineering |
| | | | | more accurate gradients, convergence acceleration using the BFGS quasi-Newton algorithm, faster control derivative calculation algorithms |
| | 12-Oct-17 | Alexandre Blais | | Resonator reset in circuit QED by optimal control for large open quantum systems |
| | | | | large open quantum systems, avoids explicit matrix exponential calculations, polynomial speedup, reduced memory, empty the cavity from measurement photons 4 times faster than passive reset |
| | | | | |
| Automatic differentiation | 13-Apr-17 | David Schuster | | Speedup for quantum optimal control from automatic differentiation based on graphics processing units |
| | | | | advanced optimization criteria and incorporate them in the optimization process with ease, fine-grained evaluation of performance at each intermediate time step |
| | | | | intricate control on the evolution path, suppression of departures from the truncated model subspace, as well as minimization of the physical time |
| | 20-May-19 | David Schuster | | Gradient-based optimal control of open quantum systems using quantum trajectories and automatic differentiation |
| | | | | quantum trajectories, less computational cost than the regular density matrix approaches, an improved-sampling algorithm which minimizes the number of trajectories needed |
| | | | | the use of quantum trajectories significantly reduces the computation complexity while achieving a multitude of simultaneous optimization targets. |
| | | | | state-transfer fidelities despite dissipation, gate times, readout fidelity while maintaining the quantum nondemolition, fast resonator reset. |
| | 18-Feb-20 | David Schuster | | Universal gates for protected superconducting qubits using optimal control |
| | | | | heavy-fluxonium qubit and the 0-π qubit, disjoint support of low-lying wave functions prevents direct population transfer between the computational-basis states. |

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| | | | | Instead, optimal control favors dynamics involving higher-lying levels, effectively lifting the protection for a fraction of the gate duration. 0-π qubit, offset-charge dependence |
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| Krotov | 1992 | David J. Tannor | | Control of Photochemical Branching: Novel Procedures for Finding Optimal Pulses and Global Upper Bounds |
| | | | | NA |
| | 26-Nov-02 | David J. Tannor | | Loading a Bose-Einstein condensate onto an optical lattice: An application of optimal control theory to the nonlinear Schrödinger |
| | | | | nonlinear Schrodinger equation (NLSE), a BEC initially at rest in a harmonic trap, A phase develops across the BEC when an optical lattice potential is turned on, goal is to counter this effect |
| | 24-May-11 | David J. Tannor | | Optimal control with accelerated convergence: Combining the Krotov and quasi-Newton methods |
| | | | | monotonic increase of the objective, significant savings over gradient (first-order) methods, significantly faster growth of the objective at early iterations than in gradient methods |
| | | | | drawback: problematic when high fidelity is desired. Achievement: enhanced convergence (second-order or quasi-Newton) as the optimal solution is approached, controlling electron in the Na atom |
| | 9-Dec-03 | Ronnie Kosloff | | Optimal control theory for unitary transformations |
| | | | | implementation of Fourier transform in the Na2 molecule for up to five qubits |
| | 30-Oct-09 | S G Schirmer | | Implementation of fault-tolerant quantum logic gates via optimal control |
| | | | | extremely difficult to implement using conventional techniques, T-gate for the five-qubit stabilizer code |
| | 9-Mar-12 | Christiane P. Koch | | Monotonically convergent optimization in quantum control using Krotov's method |
| | | | | a large class of quantum control problems: nonlinear equations of motion, non-unitary time evolution, nonlinear dependencies of the Hamiltonian on the control, time-dependent targets. |
| | | | | optimization functionals that depend to higher than second order on the time-evolving states |
| | 16-Jul-18 | Kurt Jacobs | | Efficient optimization of state preparation in quantum networks using quantum trajectories |
| | | | | wave-function Monte-Carlo = quantum-jump trajectories, allows efficient simulation of open systems by independently tracking the evolution of many pure-state “trajectories”. |
| | | | | generating entangled states in a network consisting of systems coupled in a unidirectional chain |

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| | 13-Aug-08 | Regina de Vivie-Riedle | | Monotonic Convergent Optimal Control Theory with Strict Limitations on the Spectrum of Optimized Laser Fields |
| | | | | modified optimal control scheme based on the Krotov method, allows for strict limitations on the spectrum of the optimized laser fields |
| | | | | A frequency constraint is introduced and derived mathematically correct, without losing monotonic convergence of the algorithm. |
| | | | | challenging control of nonresonant Raman transitions, molecular vibrational qubits |
| | | | | |
| Closed-loop Nelder-Mead with ORBIT | 20-Jun-14 | John M. Martinis | | Optimal Quantum Control Using Randomized Benchmarking |
| | | | | closed-loop Nelder-Mead algorithm with ORBIT for automated tune-up as it is a gradient-free method, and therefore less sensitive to noise |
| | | | | improve singleand two-qubit gates, minimize gate bleedthrough, where a gate mechanism can cause errors on subsequent gates, and identify control crosstalk in superconducting qubits |
| | | | | |
| Ad-HOC | 20-Jun-14 | F. K. Wilhelm | | Adaptive Hybrid Optimal Quantum Control for Imprecisely Characterized Systems |
| | | | | quantum optimal control's experimental application, is hindered by imprecise knowledge of the input variables, the quantum system's parameters |
| | | | | adaptive hybrid optimal control, using a protocol named Ad-HOC. enhances gate fidelities by an order of magnitude, making optimal control theory applicable and useful. |
| | | | | combines open- and closed-loop optimal control by first performing a gradient search then an experimental fidelity estimation with a gradient-free method |
| | | | | combines a model based gradient search and the model free Nelder-Mead (NM) algorithm |
| | | | | |
| GOAT | 9-Apr-18 | F. K. Wilhelm | | Tunable, Flexible, and Efficient Optimization of Control Pulses for Practical Qubits |
| | | | | Gradient Optimization of Analytic conTrols (GOAT). experimental implementations require both the controls and the resultant dynamics to conform to hardware-specific constraints. |
| | | | | Superconducting qubits present the additional requirement that pulses must have simple parameterizations, so they can be further calibrated in the experiment |

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| | | | | fast coherence-limited pulses for two leading superconducting qubits architectures - flux-tunable transmons and fixed-frequency transmons with tunable couplers. |
| | | | | GRAPE and Krotov fail criterion (i) flexibility. In contrast, GOAT, which does not derive from the variational formulation, does not require back propagation |
| | | | | GRAPE and Krotov fail criterion (ii) numerical accuracy. GOAT, which allows arbitrary piecewise-continuous controls, does not suffer from this problem. |
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| Digital Single-Flux Quantum Pulses | 29-Aug-16 | F. K. Wilhelm | | Optimal Qubit Control Using Single-Flux Quantum Pulses |
| | | | | Single-flux quantum pulses are a natural candidate for on-chip control of superconducting qubits. |
| | | | | can drive high-fidelity single-qubit rotations—even in leaky transmon qubits—if the pulse sequence is suitably optimized. |
| | | | | We achieve this objective by showing that, for these restricted **all-digital pulses**, genetic algorithms can be made to converge to arbitrarily low error. |
| | | | | verified up to a reduction in gate error by 2 orders of magnitude compared to an evenly spaced pulse train. |
| | | | | |
| Rapid monotonic ally convergent (Rabitz) | 4-Jun-98 | Herschel Rabitz | | Rapidly convergent iteration methods for quantum optimal control of population |
| | | | | quadratic and monotonic convergence, within very few steps, the optimized objective functional comes close to its convergent limit |
| | 29-Jun-98 | Herschel Rabitz | | A rapid monotonically convergent iteration algorithm for quantum optimal control over the expectation value of a positive definite operator |
| | | | | quadratic and monotonic convergence, within very few steps, the optimized objective functional comes close to its convergent limit |
| | 12-Mar-04 | Herschel Rabitz | | Generalized monotonically convergent algorithms for solving quantum optimal control problems |
| | | | | many cost functionals can be reduced to two basic functionals by the introduction of product spaces. generalized pulse design equations can be derived from the basic functionals. |

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| | | | | four-level model systems employing stationary and/or nonstationary targets in the absence and/or presence of relaxation. |
| | | | | slow convergence may often be attributed to “trapping” and that relaxation processes may remove such unfavorable behavior |
| | 19-Mar-07 | Herschel Rabitz | | Monotonically convergent algorithms for solving quantum optimal control problems described by an integrodifferential equation of motion |
| | | | | inhomogeneous integrodifferential equation of motion, four-level model system under the influence of non-Markovian relaxation |
| | 3-Oct-07 | Herschel Rabitz | | Quantum Control Landscapes |
| | | | | this simplicity originates in universal properties of the solution sets to quantum control problems that are fundamentally different from their classical counterparts. |
| | | | | globally efficient quantum control algorithms |
| | 12-Jul-11 | Herschel Rabitz | | Exploring quantum control landscapes: Topology, features, and optimization scaling |
| | | | | effort required to find an optimal control field appears to be essentially invariant to the complexity of the system, number of states N ranging from 5 through 100 |
| | | | | topology of quantum control landscapes, 5000 individual optimization test cases, at least 99.9%, invariance of required search effort to system dimension N |
| | | | | distance traveled on the control landscape during a search and the magnitude of the control landscape slope |
| | 2-Dec-14 | Herschel Rabitz | | Characterization of control noise effects in optimal quantum unitary dynamics |
| | | | | a geometric interpretation of stochastic noise effects, more robust optimal controls are associated with regions of small overlap between landscape curvature and the noise correlation function |
| | | | | distinct noise spectral regimes that better support robust control solutions |
| | 23-Apr-03 | Gabriel Turinici | | New formulations of monotonically convergent quantum control algorithms |
| | | | | relationship between Krotov and Rabitz, a unified formulation that comprises both algorithms and that extends to a new class of monotonically convergent algorithms |
| | 8-May-07 | S.Volkwein | | Formulation and numerical solution of finite-level quantum optimal control problems |
| | | | | a cascadic non-linear conjugate gradient scheme and a monotonic scheme are discussed |
| | 7-Oct-07 | A.Borzi | | A cascadic monotonic time-discretized algorithm for finite-level quantum control computation |
| | | | | A computer package (CNMS) is presented |

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| Machine learning | 9-Mar-92 | Herschel Rabitz | | Teaching Lasers to Control Molecules |
| | | | | teach a laser pulse sequences to excite specified rotational states in a diatomic molecule. Over a series of pulses the algorithm learns an optimal sequence. |
| | 24-Apr-18 | Herschel Rabitz | | Data-driven gradient algorithm for high-precision quantum control |
| | | | | GRAPE's performance is often hindered by deterministic or random errors in the system model and the control electronics. |
| | | | | jointly learn from the design model and the experimental data obtained from process tomography, data-driven (d-GRAPE) |
| | | | | more powerful with broadband controls that involve a large number of control parameters, while other algorithms usually slow down due to the increased size of the search space. |
| | 27-Sep-18 | Pankaj Mehta | | Reinforcement Learning in Different Phases of Quantum Control |
| | | | | performance is comparable to optimal control methods |
| | | | | a single scalar reward (the fidelity of the resulting state), a spin-glass-like phase transition in the space of protocols as a function of the protocol duration |
| | 23-Apr-19 | Hartmut Neven | | Universal quantum control through deep reinforcement learning |
| | | | | trusted-region-policy-optimization, reinforcement learning harness non-local regularities of noisy control trajectories and facilitate transfer learning between tasks. |
| | | | | simultaneously optimize the speed and fidelity against leakage and stochastic control errors. |
| | | | | two-order-of-magnitude reduction in average-gate-error over GRAPE, one-order-of-magnitude reduction in gate time from **optimal gate synthesis** counterparts |
| | | | | |
| Open source packages | | | | |
| SIMPSON | 8-Dec-08 | Niels Chr.Nielsen | | Optimal control in NMR spectroscopy: Numerical implementation in SIMPSON |
| | | | | GRAPE |
| Spinach | 17-Nov-10 | Ilya Kuprov | | Spinach – A software library for simulation of spin dynamics in large spin systems |
| | | | | GRAPE |
| DYNAMO | 3-Aug-11 | S G Schirmer | | Comparing, optimizing, and benchmarking quantum-control algorithms in a unifying programming framework |
| | | | | GRAPE methods which update all controls concurrently, and Krotov-type methods which do so sequentially |

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| QuTiP | 28-Feb-12 | Franco Nori | | QuTiP: An open-source Python framework for the dynamics of open quantum systems |
| | | | | GRAPE |
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| Applications | | | | |
| determining optimized pulse sequences | 15-Sep-10 | David G.Cory | | Application of optimal control to CPMG refocusing pulse design |
| | | | | GRAPE |
| designing high-fidelity quantum gates | 2-Jan-07 | F. K. Wilhelm | | Optimal control of coupled Josephson qubits |
| | | | | GRAPE, two qubit CNOT gate fidelity 1-1e-9, leakage is below 1%,  five times faster than the pioneering experiment, |
| | | | | TOFFOLI gate in three linearly coupled charge qubits, 13 times faster than decomposing it into a circuit of nine CNOT gates |
| | 12-Feb-09 | F. K. Wilhelm | | Optimal control of a leaking qubit |
| | | | | GRAPE, We apply optimal control theory to the envelope of a resonant Rabi pulse in a qubit in the presence of a single weakly off-resonant leakage level. |
| | | | | The gate error of a spin-flip (NOT) operation reduces by orders of magnitude compared to simple pulse shapes. |
| | | | | Near-perfect gates can be achieved for any pulse duration longer than an intrinsic limit given by the nonlinearity. also discuss ways to improve the pulse shapes. |
| | 2-Mar-09 | F. K. Wilhelm | | Optimal Control of a Qubit Coupled to a Non-Markovian Environment |
| | | | | open systems GRAPE, decoupling the qubits from the intrinsic noise of the material, paradigmatic, non-Markovian model: a single-qubit coupled to a two-level system exposed to a heat bath |
| | 8-Sep-09 | F. K. Wilhelm | DRAG | Simple Pulses for Elimination of Leakage in Weakly Nonlinear Qubits |
| | | | | simple qubit control fails on short time scales because of coupling to leakage levels, analytic formula, adding a second control that is proportional to the time derivative of the first |
| | | | | These results show that even weak anharmonicity is sufficient and in general not a limiting factor for implementing quantum gates. |
| | 26-Nov-13 | F. K. Wilhelm | | Optimized controlled-Z gates for two superconducting qubits coupled through a resonator |

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| | | | | GRAPE, fast entangling gates to minimize the effects of decoherence |
| | 17-Dec-15 | F. K. Wilhelm | | Training Schrödinger's cat: quantum optimal control |
| | | | | state-of-the-art quantum control techniques are reviewed, address key challenges and sketch a roadmap for future developments |
| | 30-Oct-09 | S G Schirmer | | Implementation of fault-tolerant quantum logic gates via optimal control |
| | | | | GRAPE, all of the elementary logic gates for the five-qubit stabilizer code |
| preparing entangled states | 28-Feb-14 | Jörg Wrachtrup | | High-fidelity spin entanglement using optimal control |
| | | | | GRAPE, scalable room temperature spin-based quantum information devices |