DUE Oct. 31, 2019

16.07 Orbital Dynamics Lab: Mission to Mars and Return to Earth

Introduction

This laboratory deals with celestial and spacecraft dynamics. You will formulate and program the equations governing the motion of planets in the gravitational field of the Sun. You will use your MATLAB code to predict the motion of the Earth and Mars in orbit about the Sun, as well as spacecraft in transfer orbits between them. You will design missions from Earth to Mars and also figure out how to get back to Earth, and how long you have to stay on Mars before returning. We are interested in launch dates, return dates, mission length, and ΔV requirements for various phases of the missions. The Hohmann transfer will be our reference mission. We will not consider the "escape" orbits within the SOI of the Earth or Mars, but rather only the elliptical transfer orbits about the Sun. To determine flight time and launch opportunities we will directly simulate these transfer orbits using MATLAB and appropriate initial conditions.

Table 1 lists physical parameters for the Sun, Earth, and Mars. Because of the extremely large mass of the Sun, we will consider it to be stationary in our coordinate system. When calculating planetary orbits around the Sun, the effect of other celestial bodies are ignored.

Table 1: Physical and Orbital Data

	mass (kg)	semimajor axis (km)	radius (km)	orbital velocity (m/s)
Sun	1.989×10^{30}	–	—	—
Earth	5.975×10^{24}	149×10^{6}	6378	29,790
Mars	6.441×10^{23}	228×10^{6}	3397	24,140

The orbits of Earth and Mars are slightly elliptical and although each orbit is planar, the orbits are misaligned in angle with respect to one another. As an approximation, however, we assume these orbits to be *circular* and *co-planar*.

In this lab, you will create a MATLAB code that will allow you to numerically simulate the motion of a celestial body under the influence of the Sun, taken as a fixed body at rest. In order to accomplish these tasks, your MATLAB code will integrate the equations of motion in time, starting from an initial condition. Then you will later add features like ΔV spacecraft thrusting, which will allow you to simulate orbital transfer missions.

The first step is to formulate the equations we would like to solve with the Matlab code. Our desired result is to calculate the positions and velocities of a planet or a spacecraft as a result of its interaction with the gravity field of the Sun. For instance, the attractive force that the gravity field of the Sun exerts on a body is given by,

$$\boldsymbol{F} = -\frac{\mu m}{r^3}\boldsymbol{r}. \tag{1}$$

Here, $\mu = GM$, the gravitational parameter of the Sun, where G is the universal gravitational constant, $G = 6.673 \times 10^{-11}$ m^3/kg-s^2, M is the mass of the Sun and m is the mass of the planet or spacecraft. r is the magnitude of the distance vector that points from the Sun to the body.

When working with problems involving the Sun and planets or spacecraft, it is usual to non-dimensionalize distances with respect to the distance between the Sun and the Earth: 149×10^6 km, defined as 1 AU (astronomical unit), and time by 24*60*60 sec = 1 day. What is the numerical value of μ in this non-dimensionalized system?

For numerical purposes it is convenient to write the equation of motion $\ddot{\boldsymbol{r}} = \boldsymbol{F}/m$ for the body as a set of first order state equations. That is, we write,

$$\dot{\boldsymbol{r}} = \boldsymbol{v}, \tag{2a}$$

$$\dot{\boldsymbol{v}} = \boldsymbol{F}/m. \tag{2b}$$

Therefore, to describe the motion of a body in two dimensions, four state variables are required – two for position and two for velocity. We will use a Cartesian coordinate system which means that the equations should be separated into Cartesian x and y components. The state vector **X** contains all variables that completely describe the state of the system. For purposes of uniformity, you should order the states as follows,

$$\mathbf{X} = [x,\ y,\ u,\ v\ ] \tag{3}$$

where x and y are position components, u is the x-component of velocity and v is the y-component of velocity.

The resulting equations in state variables are

$$\begin{aligned}
\dot{x} &= u \\
\dot{y} &= v \\
\dot{u} &= -\frac{\mu x}{(x^2+y^2)^{3/2}} \\
\dot{v} &= -\frac{\mu y}{(x^2+y^2)^{3/2}}
\end{aligned}$$

Problems

1. **Motion of the Earth and Mars**

   The first task is to program in MATLAB the equations governing the motion of the Earth and Mars about the Sun.

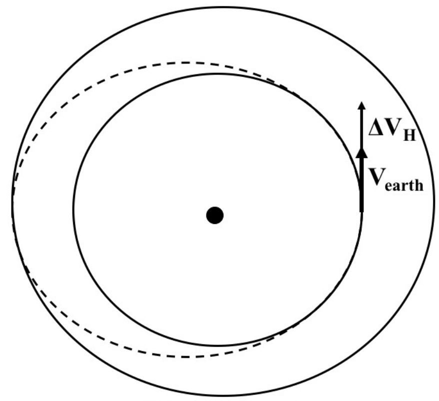
   Since we are only considering the effect of the Sun on each body, these calculation are separately done for each planet. What boundary conditions for x, y, u and v are applied to calculate the motion of the Earth about the Sun? What boundary conditions for x, y, u and v are applied to calculate the motion of Mars about the Sun?

   Run your calculations and plot the orbits of the Earth and Mars about the Sun. With proper boundary conditions, you should obtain circular orbits. What is the period of the orbit of Earth in days? What is the period of the orbit of Mars in days? Compare these numerical results to the orbital period computed analytically from the semi-major axis of the orbits.

2. **Hohmann Transfer, Earth to Mars; Mars to Earth**

   We will first analyze Hohmann transfers from Earth to Mars and back. This will give us reference values of ΔV's as well as reference times for the start, end and length of various phases of the mission.

   This is the basic reference mission for launch and arrival times, and the ΔV_H required for the mission.

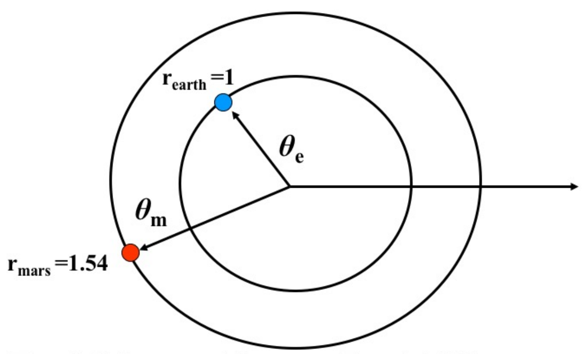


Hohmann Transfer

Even though the Earth is not always located at $y = 0$, $x = a_{Earth}$ as sketched in the figure, it is easier to apply the boundary conditions at this point and apply the effects of actual planetary positions later. What are the required boundary conditions for the Hohmann transfer?

Calculate the orbit obtained from the Hohmann transfer. What is the time required for the Hohmann transfer to Mars? What is the ΔV required? What is the time required for the transfer from Mars to Earth? What is the ΔV required?

To begin our study of possible trajectories from Earth to Mars and return, we must locate Earth and Mars in the solar system. On Jan 1, 2020, the positions of Earth and Mars will be as shown below: Earth at 104 degrees from the $\theta = 0$ reference line, and Mars at 216 degrees from the $\theta = 0$ reference line. You will use this information to locate the position of Earth and Mars with time. When asked for specific dates, you may present your answers as decimal years, such as 2020.53.

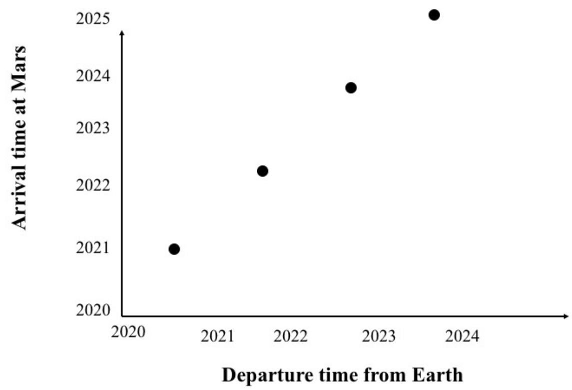


Earth/Mars positions on Jan 1 2020

(a) Now that you know where Earth and Mars are at all times, you should be able to calculate what launch dates can be used to launch a Hohmann transfer mission to Mars. What angle between Mars and Earth is required at the instant of launch for a successful interception of Mars from Earth on a Hohmann transfer orbit?

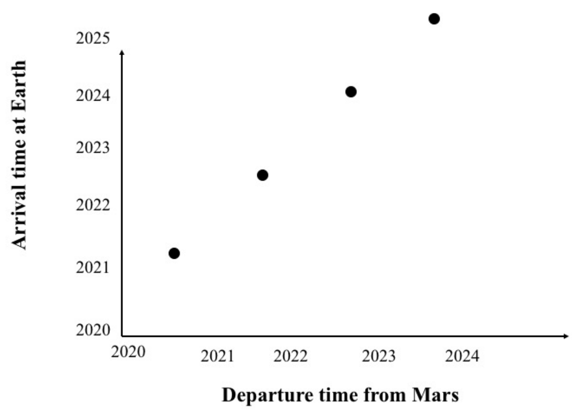
What are the dates when these conditions occur? For each launch date, calculate the arrival date at Mars.

Present your results as shown in the figure below: arrival dates at Mars as a function of launch dates from Earth.



(b) How often, in years, do launch opportunities occur?

(c) Now, compute the return Hohmann transfer from Mars to Earth. What angle is required between Mars and Earth at the instant of launch for a successful interception of Earth on a Hohmann transfer from Mars? Identify what launch dates can be used to return to Earth. Present your results as shown below: arrival dates at Earth as a function of launch dates from Mars.



(d) How long must we remain on Mars before heading back to Earth?

(e) How long is the minimum round-trip mission from Earth to Mars to Earth, using the Hohmann transfer?

(f) Report $\Delta V's$ (in m/s) for the following portions of Hohmann transfers to and from Mars. These values will serve as reference values for more complex missions.

i. ΔV to leave Earth's orbit and enter a Hohmann tranfer to Mars

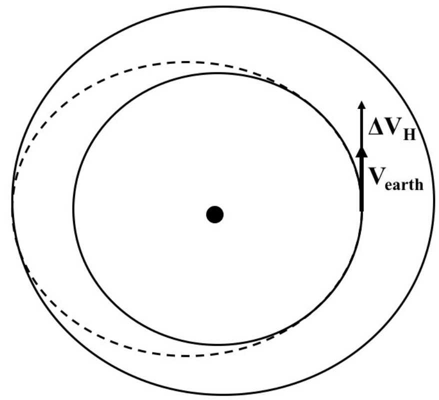
ii. ΔV to join Mars in its orbit from the Hohmann transfer

iii. ΔV to leave the Mars orbit and enter a Hohmann transfer back to Earth

iv. ΔV to rejoin Earth in its orbit

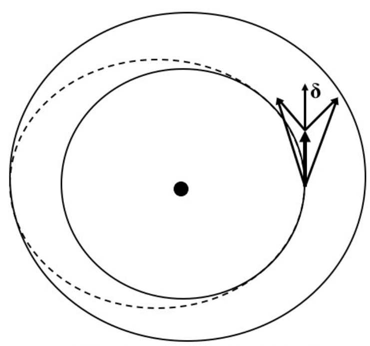
v. The total ΔV for the Earth — Mars — Earth Hohmann transfers.

3. **Other transfer orbits to Mars**
The Hohmann transfer is the basic reference mission for launch and arrival times, and the ΔV_H required.



Hohmann Transfer

Now that we have obtained the results for launch opportunities and flight times for Hohmann transfer orbits to Mars and returning to Earth, we will use the orbital solver to study more general transfer orbits to Mars, by exploring a range of ΔV's including different magnitudes and different angles, as shown below. Note that you can check your orbital solver is working properly by making sure you arrive at Mars orbit when leaving Earth with the ΔV found analytically in question 2, part (f), question (i).



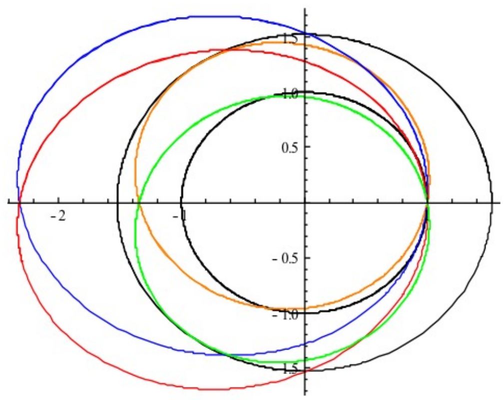
ΔV options for Lambert Splash

We will confine our studies to a ΔV twice the Hohmann transfer value in magnitude, $|\Delta V| = 2|\Delta V_H|$, but we will allow it to be launched at an angle δ from the Hohmann transfer direction. The total velocity of the spacecraft will be the vector sum of the earth's orbital velocity and the applied ΔV. The general solution to the orbital problem of arbitrary transfer between two orbits is called Lambert's problem. Therefore we will call our approach a Lambert splash. We will study the various orbits created by this "splash" and determine at what time after launch the spacecraft will intersect the orbit of Mars. If Mars happens to be there (and that is one of your jobs), our mission is successful. As mentioned in class, to do a detailed rendezvous at Mars requires application of the method of patched conics, or a special numerical approach, neither of which is part of this lab. We will only do this analysis for the outbound mission orbits Earth to Mars. These results should allow you to understand more complex discussions of possible round-trip Earth-to-Mars missions.

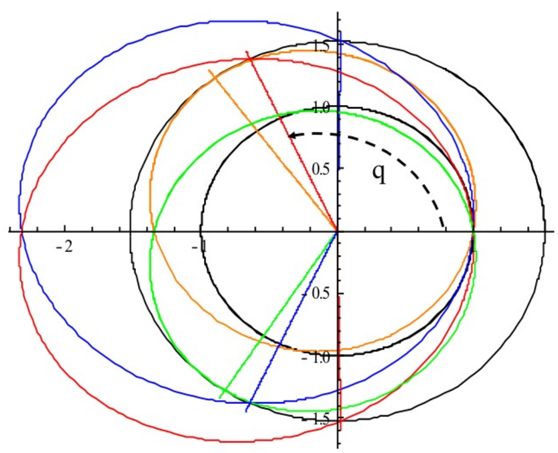
Not all values of δ will produce an orbit which intersects the orbit of Mars — a necessary condition for a successful mission. Many of these orbits will fall inside of the orbit of Mars.

(a) You should first identify by numerical solution the range of δ which will produce transfer orbits intersecting the orbit of Mars.

The figure below shows a range of transfer orbits produced by a ΔV of twice the Hohmann value for a range of $\delta's$. The orbits of Earth and Mars are also shown for reference. Of interest are the limiting orbits in the second and third quadrants which just skim Mars' orbit. **These are the limiting cases for δ; you should find and report something similar.** Also note that some of these orbits cross the orbit of Mars twice. Therefore, depending on our goals, there will be two launch opportunities for each orbit, and two flight times, a short trip and a long trip.



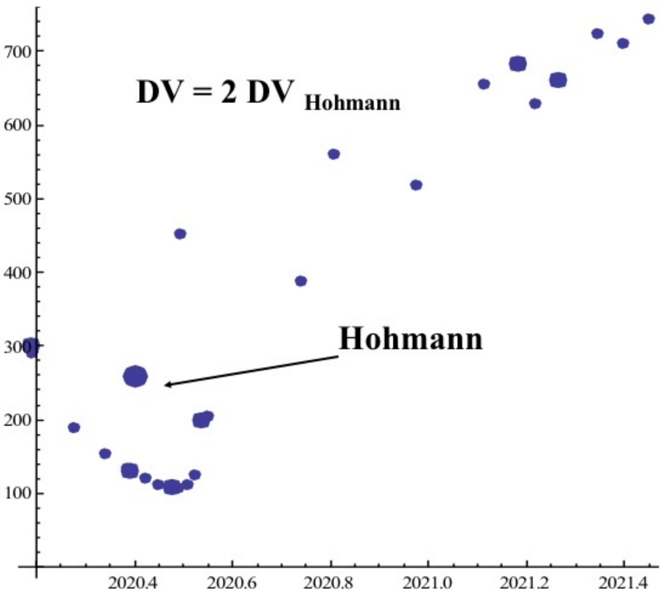
All of these orbits have the same ΔV; they differ in the time taken to reach Mars. They also differ in the angle θ traveled from launch at earth to the intersection with the orbit of Mars. These various angles are shown below.



Both of these later properties affect the time at which this particular orbit can be ***successfully launched*** since Mars must be there when we arrive.

(b) For a range of orbits with δ between the limiting cases identified in part (a), identify numerically both the time of flight and the angle traveled during the flight. Make a table of your results. From this information determine 1) the angle of Mars relative to Earth at the time of launch to complete the mission and 2) the total flight time from Earth to Mars.

(c) From your knowledge of the relative positions of Earth and Mars, determine the possible launch dates for each particular orbit. Note that since some of the orbits cross the orbit of Mars twice, two possible launch dates and time-of-flight's will be obtained. Plot the various mission options as time of flight vs. launch time for this family of orbits/missions. These results "surround" the reference Hohmann mission which has a flight time of 259 days and a launch date of 2020.4. Your points should look like that sketched below. This curve is called a porkchop plot.



This shows the points you would obtain from your analysis as well as additional points obtained from an 18 orbit "splash" at the same ΔV.

(d) What is the minimum time of flight to Mars for $|\Delta V| = 2|\Delta V_H|$?

Report Preparation

There is no need to produce a fancy report. You should include all the figures requested and answer all the questions posed and **submit a paper copy in recitation**. If your write-up is neat, organized, and readable, then there is no need to type it.

Your solver should consist of a MATLAB script file that sets the solver options, sets the initial state, and then calls `ode45`. One of the input parameters to `ode45` is a function that calculates the state derivative. In addition to the script file, you will need to write a function to define your system of equations. **Submit both files as a zip file to the 16.07 stellar website.**