

1 Introduction

2 Theory

3 Waveguides

In order to understand cavities, we start off with the discussion on waveguides. An electromagnetic wave, when confined to the interior of a hollow pipe is called a waveguide. We shall closely follow the derivation from [1]. The generalized Maxwell equations in terms of E-field and H-field are given by:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (1)$$

$$\nabla \cdot \mathbf{D} = \rho \quad (2)$$

$$\nabla \times \mathbf{H} = \vec{j} + \frac{\partial \mathbf{D}}{\partial t} \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4)$$

In vacuum, these equations become:

$$\nabla \times \mathbf{E} = \mu_0 \cdot \frac{\partial \mathbf{H}}{\partial t} \quad (5)$$

$$\nabla \cdot \mathbf{E} = 0 \quad (6)$$

$$\nabla \times \mathbf{H} = \epsilon_0 \cdot \frac{\partial \mathbf{E}}{\partial t} \quad (7)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (8)$$

Upon solving these equations, we get plane wave solutions which looks like this:

$$\Delta \vec{E}(\vec{r}) + \frac{\omega^2}{c^2} \vec{E}(\vec{r}) = 0 \quad (9)$$

$$\Delta \vec{B}(\vec{r}) + \frac{\omega^2}{c^2} \vec{B}(\vec{r}) = 0 \quad (10)$$

Now, let us look at a waveguide which is aligned along the z-direction. Taking the ansatz $\vec{E} = \vec{E}(x, y) \cdot e^{i(\omega t - kz)}$ and the separation $\Delta = \Delta_{\perp} + \frac{\partial^2}{\partial z^2}$, for the longitudinal fields yields:

$$\Delta_{\perp} E_z + k_c^2 E_z = 0 \quad (11)$$

$$\Delta_{\perp} H_z + k_c^2 H_z = 0 \quad (12)$$

where

$$k_c^2 = \frac{\omega^2}{c^2} - k^2$$

The quantity k_c is called the critical wave number and is a characteristic property of the cavity, as we shall see. Solving these equations further, we see that it is sufficient to know the longitudinal fields, E_z and B_z because we can easily determine the transverse components from them. Now, there are two waves to classify these waves:

1. $k_c^2 = 0$
 - (a) $\vec{\Delta}_\perp E_z \neq 0$ and $\vec{\Delta}_\perp H_z \neq 0$: HE or EH hybrid waves.
 - (b) $\vec{\Delta}_\perp E_z = 0$ and $\vec{\Delta}_\perp H_z = 0$: TEM waves.
2. $k_c^2 \neq 0$ In this case we do not get any propagation if $\omega \leq c \cdot k_c$. These waves are called evanescent waves or the cut off condition. The possible propagation modes are:
 - (a) $E_z = 0$: TE (transversal electric) or H waves.
 - (b) $H_z = 0$: TM (transversal magnetic) or E waves.

Corresponding to this critical wave, we have a critical frequency, which is $\omega_c = k_c \cdot c$, below which there is no propagation. One interesting thing to note here is that for a hollow waveguide, only TE or TM modes are possible, TEM is not, because no wave would exist in this case []. But for a coaxial cable, which consists of straight wire surrounded by a conduction sheath, we can get TEM modes.

3.1 Cylindrical waveguides