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A249: Laser Gyroscope

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Abstract

In this experiment, we attempt to measure the rotation rate of the Earth using a ring cavity laser gyroscope. We passed through a diode laser through a square cavity placed on a rotating platform, and the corresponding Sagnac frequency between the counter-propagating beams were observed. To quantify the performance of the ring laser gyroscope, we determined the free spectral range, the error signal due to Pound-Drever-Hall locking, the finesse and reflectivity of the cavity, and the Allan deviation of the gyroscope. We also determined the scale factor that converts between the obtained Sagnac frequency and rotation rate. The values that we obtained were mostly on the order of the magnitude of the expected values. Due to the lock-in threshold being 2 orders of magnitude larger than the rotation rate of the Earth, our setup is not viable to determine the rotation rate of the Earth. We suggest adjustments that can be performed in order to allow our laser gyroscope to measure the rotation rate.

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Chapter 1

Introduction

The rotation of the Earth has been investigated ever since the advent of calenders. The first measurements for the rotation of the Earth has been done by observing the apparent position of a fixed star, allowing the Mayans to obtain measurements with relative uncertainties of around 10^{-5} [1].

Today, the rotation of the Earth is measured using interferometry, where radio telescopes all around the world are linked to construct a very-long base-interferometer system (VLBI). Fig. 1.0.1 show the locations of all radio telescopes under the CONT17 campaign, a continuous VLBI session held for two weeks in 2017 [2]. The relative uncertainties that are obtained reach approximately 10^{-10} . Today, the rotation of the Earth is measured to be approximately $72.92 \text{ prad s}^{-1}$ along its rotation axis [1].

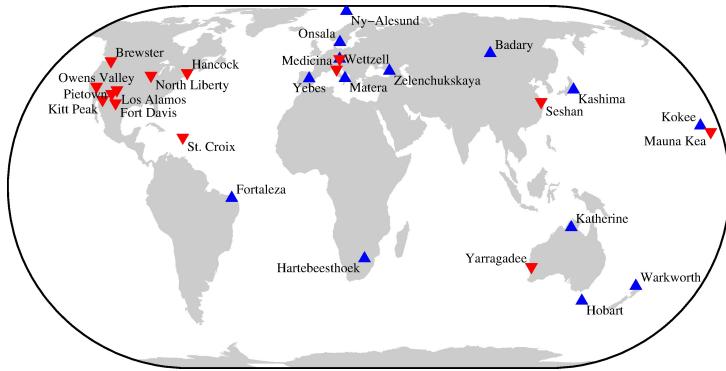


Figure 1.0.1: Radio telescopes used for CONT17. Both blue and red markers indicate legacy stations used for measurements in past sessions.

Determining the rotation of the Earth allows us to understand the vast phenomena that occur on Earth, including tidal breaking, seasonal variations, and the Chandler wobble, the nutation that occurs with Earth's rotation axis. Furthermore, climate change has a strong effect on the rotation rate as the melted ice from the polar ice caps gravitate towards the equator, increasing the angular momentum and thus the rotation period of Earth. This is shown to increase at a rate of $1.2 \mu\text{s}$ per year [1].

While the VLBI can determine the rotation rate with high precision, due to the operation times the temporal resolution is low. As such, transient events such as earthquakes and tides cannot be resolved. As of such, laser gyroscopes have been employed to resolve such details. While it lacks in the long-term stability of VLBI, they have a temporal resolution of approximately an hour or less [3]. The G-ring at the German Fundamentalstation Wettzell is one of the best ring laser gyroscopes with a sensitivity of around $12 \text{ prad/s}/\sqrt{\text{Hz}}$ [1]. See Fig. 1.0.2 for an image and the setup of the G-Ring gyroscope. Ring laser gyroscopes are still explored today to describe other phenomena from the Earth's rotation such as the Lense-Thirring Effect, and gyroscopes such as the GINGERino have been proposed to measured such quantities [4].

In our experiment, we use a ring laser gyroscope to attempt to measure the rotation rate of the Earth. This is done by employing the Sagnac effect, which allows us to determine the rotation of an inertial system by the path length difference between two light beams. To observe the quality of the constructed laser gyroscope, we measure optical cavity parameters such as the free spectral range, PDH locking error signals, lock-in threshold, and the finesse. We further quantify the stability and sensitivity

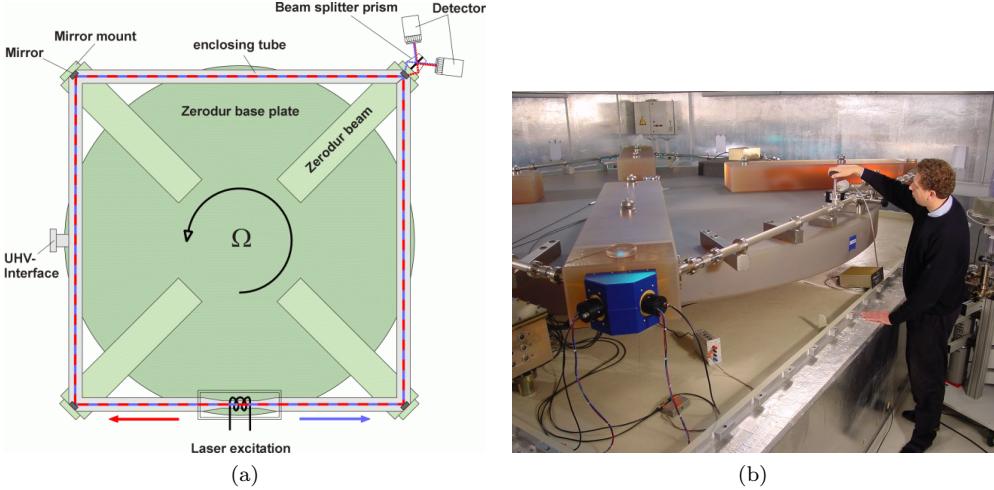


Figure 1.0.2: The G-Ring gyroscope at the Fundamentalstation Wettzell. (a): The setup of the gyroscope. (b): Image of the G-ring. Obtained from Ref. [3].

of the gyroscope by determining the Allan deviation.

This paper is structured as follows. Chapter 1 motivates the experiment by elaborating on the usage of ring laser gyroscopes to determine the rotation rate of the Earth. Chapter 2 describes the theory behind the quantities measured in the experiment. Chapter 3 describes the setup used in our experiment and the method used to determine the relevant quantities for each part of our experiment. Chapter 4 shows the pre-laboratory tasks that were required before starting the experiment. Chapter 5 shows the results that we have obtained and discussions related to the results. Finally, we summarize our findings and address possible outlooks for our experiment in Chapter 6. It is to be noted that throughout the report, we will be referencing to the manual from Thorsten and Stellmer (Ref. [1]).

Chapter 2

Theory

2.1 Gyroscopes

2.1.1 The Sagnac Effect

The Sagnac effect tells us that whilst the motion between two inertial frames cannot be distinguished, two rotating frames can be distinguished, allowing one to directly measure the rotation rate of an inertial system [1]. This effect was first observed by George Sagnac in 1913, whom believed that this experiment was a proof that aether exists in an inertial frame [5]. This, however, was disproven by Max von Laue in 1911 where he showed that the Sagnac effect was compatible with special relativity [6]. However, the interpretation of the Sagnac effect due to the general theory of relativity is still investigated today, even though it is already well-known in literature [7]. In our analysis, we utilize the Sagnac effect on a gyroscope to measure the rotation rate of the Earth.

To observe the Sagnac effect, we consider an interferometer setup with light propagating with wavelength λ enclosing an area \vec{A} with perimeter P . Placing such a setup onto a rotating platform with frequency $\vec{\Omega}$, we observe that the optical path that each light travels changes. For example, if the table rotates counter-clockwise, then the path of the co-rotating light increases, while that of the other light decreases (see Fig. 2.1.1). The Sagnac effect then tells us the resulting phase shift between the two lights:

$$\delta\phi = \frac{8\pi\vec{A} \cdot \vec{\Omega}}{c\lambda} \propto \vec{A} \cdot \vec{\Omega}. \quad (2.1.1)$$

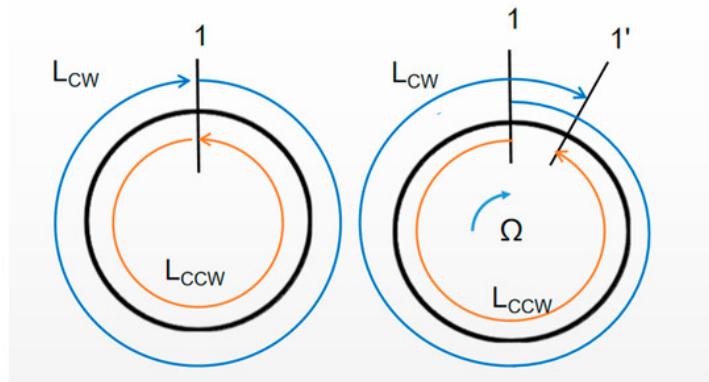


Figure 2.1.1: The Sagnac effect. *Left:* Setup without rotation. The beam moving clockwise (blue) and counter-clockwise (orange) have the same optical path length. *Right:* Setup with a clockwise rotation Ω . The path length of the clockwise beam is larger than that of the counter-clockwise beam. Obtained from Ref. [8].

A more detailed derivation using the relativistic law of velocity addition can be found in Ref. [7].

2.1.2 Ring Laser Gyroscopes

Active Ring Laser Gyroscopes

In order to incorporate the Sagnac effect within our experiment, we utilize ring laser gyroscopes. A laser is placed within an enclosed cavity, and emits two counter-propagating beams. Such beams reflect off mirrors and interfere at the end of their propagation. When rotating the platform in which such setup is placed, different interference patterns can be observed, and transforms the ring cavity system into a cavity resonator. The corresponding beat frequency $\delta\nu$ observed is then the Sagnac frequency, which is given as such:

$$\delta\nu = \frac{4\vec{A} \cdot \vec{\Omega}}{P\lambda}. \quad (2.1.2)$$

In our experiment, we only consider square ring cavities so that $A = L^2$ and $P = 4L$, where L is the path length of each arm. As such, we can simplify Eq. 2.1.2 as such:

$$\delta\nu = \frac{L\Omega}{\lambda} = n\Omega, \quad (2.1.3)$$

where $n = L/\lambda$ is the number of nodes of the light field in each arm. Thus the Sagnac frequency is proportional to the rotation rate of the inertial system [1].

Passive Ring Laser Gyroscopes

In contrast to the active ring laser gyroscope, in which the laser is contained within the ring cavity, the passive ring laser gyroscope places the laser source outside of the cavity system. In this system, the external laser is locked to the counter-propagating modes of the resonator. By placing the laser outside of the resonator, we can reduce the systematic effects due to the lasing medium and the lock-in effect (see Sec. 2.2.5), and also increase the available light power for the beam. This method, however, also introduces an added complexity of laser locking [1]. See Fig. 2.1.2 for a comparison between the two systems. In our experiment, we use the passive ring laser gyroscope and thus laser locking becomes an importance in our measurements.

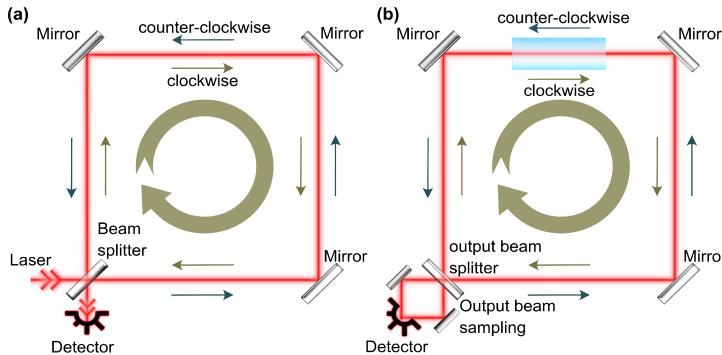


Figure 2.1.2: Ring laser gyroscope systems. *Left:* Passive, *Right:* Active. Obtained from Ref. [9].

Gyroscope Sensitivity

The sensitivity of the ring laser gyroscope depends on a variety of factors, namely the wavelength λ , arm length L , finesse F of the resonator (see Sec. 2.2.2), and the shot-noise limited detection given by the number of photons $N = P_{\text{opt}}/h\nu = P_{\text{opt}}\lambda/hc$ detected per unit time. P_{opt} represents the optical power given to the laser. Combining all such factors, we obtain the sensitivity of the ring laser gyroscope for an integration time τ as such:

$$\delta\Omega = \frac{1}{4} \frac{c}{L^2 F} \sqrt{\frac{ch\lambda}{P_{\text{opt}}}} \frac{1}{\sqrt{\tau}}. \quad (2.1.4)$$

This gyroscope sensitivity can be directly compared with the Allan deviation σ_{ad} that determines the instability of a measurement for some averaged integration time τ (see Sec. 2.3).

2.2 Optical Cavities

A cavity is a hollow conductor which contains electromagnetic waves going in and reflecting off the walls. At certain frequencies, these reflected waves form standing waves which correspond to the resonant modes of the cavity. An optical cavity consists of an arrangement of mirrors, which produces standing electromagnetic waves. In our setup, instead of the traditional Fabry-Pérot interferometer, which consists of two opposing flat mirrors, we use four mirrors. Nevertheless, the same principles of resonance modes applies.

2.2.1 Cavity Modes

The solution to optical resonators with parabolic mirrors and homogeneous medium is given by the Hermite-Gaussian modes. The Hermite-Gaussian modes are the solution to the paraxial Helmholtz equation, when considering Cartesian coordinates with the beam propagating along the z -axis (Fig. 2.2.1).

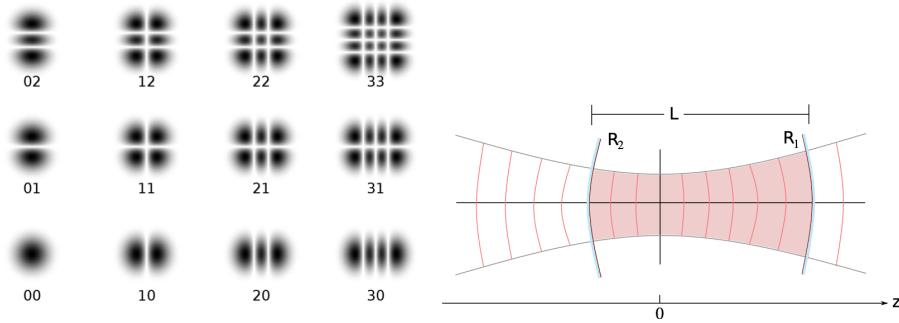


Figure 2.2.1: Hermite-Gaussian Transverse Electromagnetic Mode (TEM _{l,m}) (left) and the beam's wave-front between two mirrors of radii R_1 and R_2 with a separation of length L , inside a cavity [1].

The Hermite-Gaussian modes are also called Transverse Electromagnetic Mode (TEM) and are labelled as TEM _{m,n} , where $m, n \in \mathbb{N}^{\geq 0}$ are the integers, which represent the order of Hermite-polynomials along the x and y direction. The resonance frequency is given by

$$\nu_{l,m,n} = \delta\nu_{\text{FSR}} \left(l + (m + n + 1) \frac{\delta\zeta}{\pi} \right), \quad (2.2.1)$$

where $l \in \mathbb{N}^{\geq 0}$ represents the different longitudinal nodes. The spacing between these longitudinal modes is given by the free spectral range, $\delta\nu_{\text{FSR}}$,

$$\delta\nu_{\text{FSR}} = \frac{c}{P}. \quad (2.2.2)$$

The fundamental mode, TEM_{0,0}, is the Gaussian mode and is used for locking the lasers beam in our setup, as can be seen in Fig. 4.2.4. In order to achieve stability, the following condition on the radii of two mirrors, R_1 and R_2 , has to be met

$$0 \leq g_1 g_2 = \left(1 + \frac{L}{R_1} \right) \left(1 + \frac{L}{R_2} \right) \leq 1. \quad (2.2.3)$$

The above condition is true for a two mirror setup. In order to extend this to four mirror setup, we simply compute the magnitude of eigenvalues of ABCD transfer matrix for the roundtrip.

Airy Lineshape and Optical Cavity Ringing

We expect the power transmitted through the cavity to have a Lorentzian like lineshape, called an Airy function, when the laser is scans through the frequencies of coupled modes. This occurs even with cavities with high Finesse cavities because the laser sweeps through different frequencies faster than the storage time of the cavity. This does not allow the cavity to recover and fill itself when resonance is approached. This leads to a beat, as the evolving field inside and the incoming field start to interfere and create an oscillatory behaviour.

2.2.2 Finesse

The Finesse of a cavity essentially quantifies the number of bounces a beam makes before leaking out or being absorbed. Mathematically, it is the ratio of the free spectral range, $\delta\nu_{\text{FSR}}$, to the spectral linewidth $\delta\nu_{1/2}$, the Full Width Half Maximum (FWHM) of the transmission peak. Since it is inversely proportional to the FWHM, a high finesse equals a narrow resonance line and vice versa. A narrow resonance line is better for high precision frequency applications. The formula is given by

$$\mathcal{F} = \frac{\delta\nu_{\text{FSR}}}{\delta\nu_{1/2}} \approx \frac{\pi\sqrt{\mathcal{R}}}{1 - \mathcal{R}}, \quad (2.2.4)$$

where $\mathcal{R} = \sqrt{\mathcal{R}_1\mathcal{R}_2\mathcal{R}_3\mathcal{R}_4}$, where \mathcal{R}_i is the reflectivity of the cavity mirrors.

2.2.3 Cavity-Ring-Down Technique

To precisely compute the finesse of mirrors in the cavity, we employ ring-down technique. In this, we look at how the power in the cavity evolves temporally, when the cavity-locked laser is switched off quickly. In order to achieve the latter, we switch the RF signal driving the Acousto-Optical Modulator (AOM).

Neglecting the losses in the cavity medium, the time evolution of the laser intensity is given by

$$I(t) = I_0 \mathcal{R}^{2t/\tau_r} = I_0 \exp\left(-\frac{t}{\tau_{dec}}\right), \quad (2.2.5)$$

where $\tau_r = P/c$ is the roundtrip time and τ_{dec} is the decay time, which is given by

$$\tau_{dec} = -\frac{\tau_r}{\log \mathcal{R}^2} = -\frac{P}{c \log \mathcal{R}^2}. \quad (2.2.6)$$

2.2.4 Pound-Drever-Hall Locking

Pound-Drever-Hall (PDH) technique is a way to stabilize the frequency of light emitted by the laser by locking to a stable cavity. The central idea of this technique is the following - phase modulated light, typically by an Electro-Optical Modulator (EOM), consisting of carrier frequency ω and two side bands Ω is inserted into the cavity. The reflected light from the cavity is recorded with a fast photodiode, which consists of the two unaltered side bands and a phase shifted carrier. This phase shifted carrier is mixed with the local oscillator and passed through a low pass filter in order to remove high frequency components. The resulting signal shows how far off the laser carrier is from resonance. This signal is then fed into the Proportional-Integral-Derivative (PID) controller, which then applies an error signal (Fig. 2.2.2) and which is then fed back into the cavity, allowing the laser to remain locked.

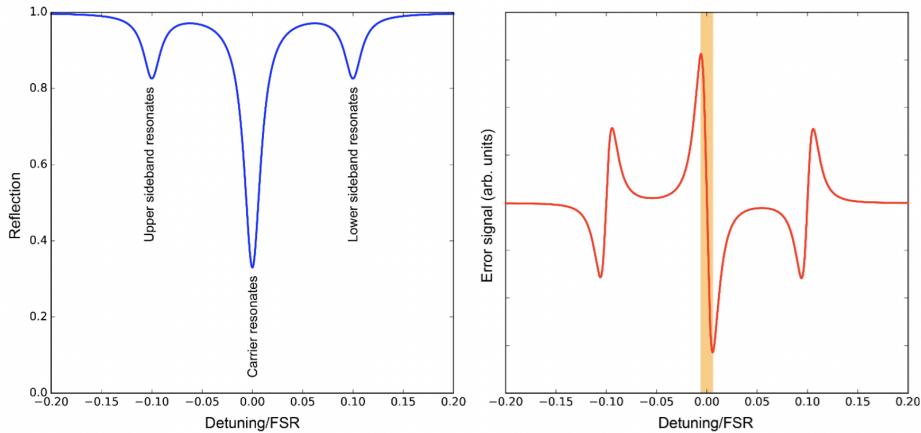


Figure 2.2.2: Pound-Drever-Hall (PDH) cavity reflection (left), which shows the ingoing carrier frequency along with two sidebands. The error signal after frequency demodulation (right) that is used for PID feedback to stabilize the laser [10].

To the first order, the frequency components can be approximated to:

$$E_{\text{in}} = E_0 e^{i(\omega t + \beta \sin(\Omega t))} \approx E_0 \left(J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\omega + \Omega)t} - J_1(\beta) e^{i(\omega - \Omega)t} \right), \quad (2.2.7)$$

where J_k are the Bessel functions and β is the modulation depth. The reflection coefficient is given by $F(\omega) = E_{\text{ref}}/E_{\text{in}}$.

The slope of the error signal is given by

$$\epsilon \propto -\frac{4}{\pi} \sqrt{P_c P_s} \delta\nu_{1/2}, \quad (2.2.8)$$

where P_c is the carrier power given by $P_c = J_0^2(\beta) |E_0^2|$ and P_s is the sideband power which is given by $P_s = J_1^2(\beta) |E_0^2|$. Selecting good PID parameters for correcting both fast and slow laser lock disturbances can help achieve frequency stabilization to a fraction of the cavity linewidth.

2.2.5 Lock-In Effect

The lock-in effect occurs when the two oscillator modes with a small delta frequency between them start oscillating at the same frequency due to weak coupling between them. This is a common occurrence in the field of physics and the best example are two oscillating pendulums hung upon the same wooden wall, oscillating with slightly different frequency eventually synchronize and start oscillating at the same frequency. In general, the frequency difference between two pendulums is,

$$\delta f = \sqrt{f_0^2 - f_L^2}, \quad (2.2.9)$$

where f_L is the lock-in threshold below which the two pendulums synchronize. This effect can also be seen in RF electronics and optics.

In our setup, we can encounter this phenomenon due to the back scattering losses in the mirrors of the cavity, which is usually only a few ppm, but can amplified due to presence of dust. These losses lead to a delta in the frequency, which leads to the lock-in effect. The lock-in threshold is given by

$$\Omega_L = \frac{c\lambda^2 r_s}{32\pi A d}, \quad (2.2.10)$$

where λ is the laser wavelength, A is the enclosed area of the cavity, d is the cavity mode beam diameter and r_s the fraction of backscattering as compared to other losses [11].

With the present setup, the lock-in threshold is much greater than the Earth rotation, which will limit our ability to measure it accurately. In order to reduce the locking threshold, a larger ring and reduced scattering losses will be required.

2.3 Allan Deviation

The Allan deviation is used to quantify the instability of any device that measures differences in frequencies. Assuming that the measurement is only limited by the photon shot-noise (amongst others), we can describe the Allan variance as the deviation between temporal averages of measurements y over some time interval or integration time τ :

$$\sigma_{\text{ad}}^2 = \frac{1}{2M} \sum_{n=1}^M (\bar{y}(\tau)_{n+1} - \bar{y}(\tau)_n)^2, \quad (2.3.1)$$

where M is the number of samples. The Allan deviation is then the square root of the variance.

The Allan deviation is especially helpful to understand the sensitivity in gyroscopes. Imposing the same assumptions as above, we can describe the Allan deviation with typical shot noise scaling $\propto 1/\sqrt{\tau}$ as such:

$$\sigma_{\text{ad}} = \mathcal{A} \tau^m = \frac{\mathcal{A}}{\sqrt{\tau}}, \quad (2.3.2)$$

where \mathcal{A} (given in rad/s/ $\sqrt{\text{Hz}}$) is known as the shot-noise limited sensitivity of the gyroscope and $m = -\frac{1}{2}$ [1].

To compare this value to the sensitivity of the laser gyroscope, we use the relation between the Sagnac frequency and the rotation rate given in Eq. 2.1.3 and evaluate the Allan deviation at the shot-noise time interval τ_{sn} :

$$\delta\Omega_{\text{ad}} = \frac{\sigma_{\text{ad}}(\tau_{\text{sn}})}{n} = \frac{\mathcal{A}}{\sqrt{\tau_{\text{sn}}} n}. \quad (2.3.3)$$

Chapter 3

Pre-Lab Exercises

Before conducting the experiment, we were required to determine the rotation rate of the Earth using our phone. Our phones contain a microelectromechanical system (MEMS), which is a portable and inexpensive inertial sensor that track the motion of the phone. Using the application **phyphox** constructed by RWTH Aachen University, we evaluated the capabilities of the MEMS gyroscope within our phones.

3.1 Task 1: Getting Started

We downloaded the **phyphox** app made by RWTH Aachen University, and we played around with the Gyroscope function. We rotated the phone in several directions to observe the relationship between the orientation of the phone and the corresponding coordinates used in the application. This is shown in Fig. 3.1.1. Fig. 3.1.2 - 3.1.4 show the corresponding time series for the x, y, z coordinates for each rotation that we performed.

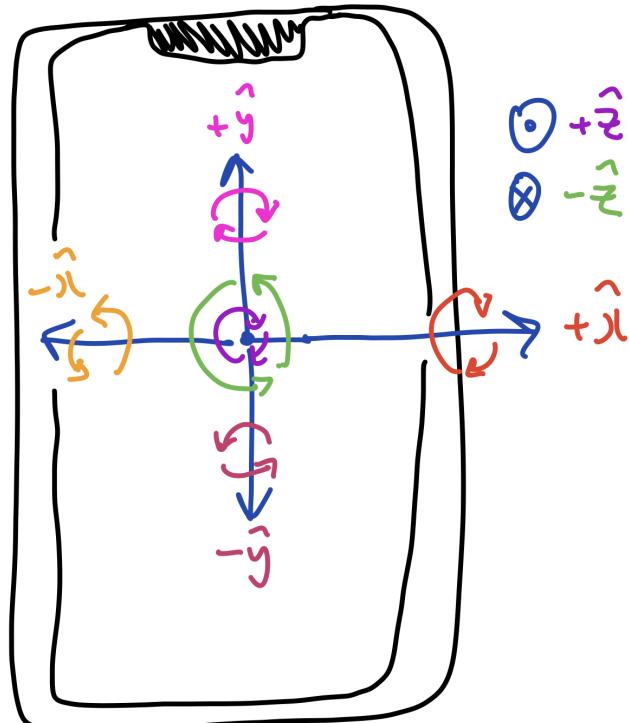
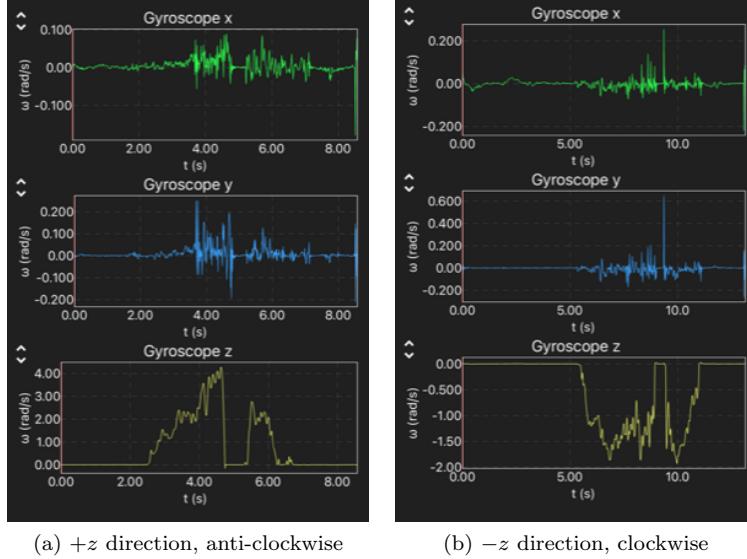


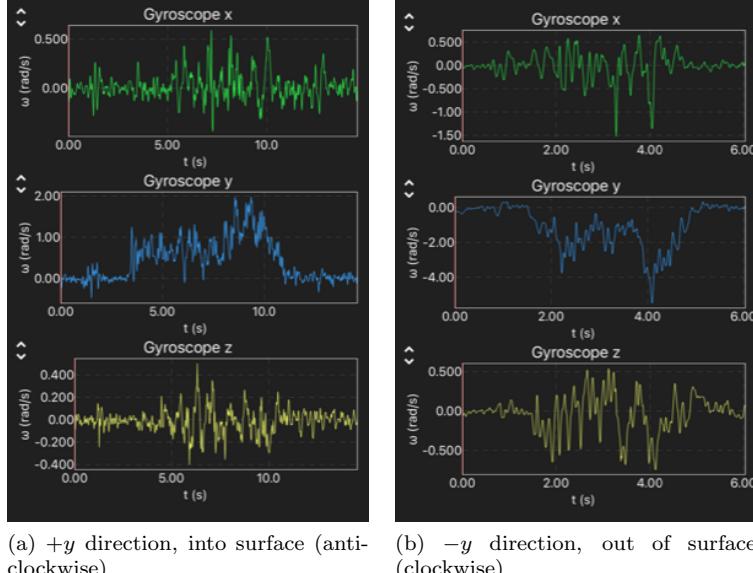
Figure 3.1.1: Sketch of the phone used to measure the gyroscope with its relevant x, y, z coordinates and sense of rotation. The sense of rotation is color coded with each rotation axis, and the direction of the axis is indicated by the blue arrows.



(a) $+z$ direction, anti-clockwise

(b) $-z$ direction, clockwise

Figure 3.1.2: The time series of the x, y, z coordinates shown from the `phyphox` application when the phone was rotated parallel to the surface. This corresponds to rotations in the z -direction. Note the large amplitudes in the time series, which indicates rotations in the relevant axis.



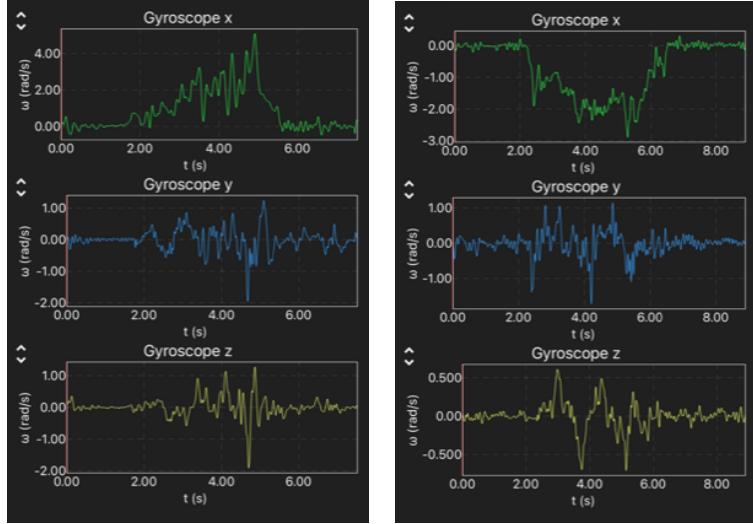
(a) $+y$ direction, into surface (anti-clockwise)

(b) $-y$ direction, out of surface (clockwise)

Figure 3.1.3: Same as Fig. 3.1.2 but for rotations into / out of the surface, corresponding to rotations in y -direction.

To save the rotation rates and import it into the computer for further analysis, the **Export Data** feature can be utilized. This will save the data as a `.zip` file that contains the raw data, the software and specifics regarding the device used, and the system time in which the experiment was started and finished. All such files are saved as `.csv` formats. An example for the raw data is shown in the Appendix section.

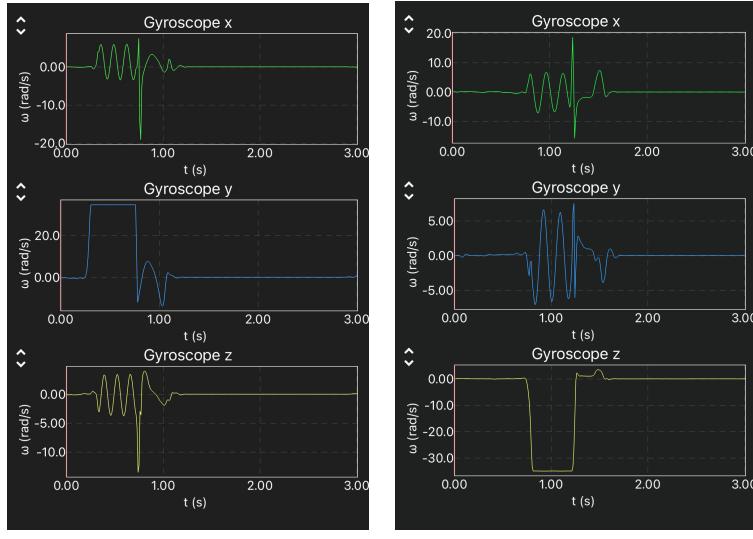
To obtain a faster rotation rate, we applied maximal torque on each end of the phone such that the rotation at each axis was maximal. This was done at an adequate height to ensure the rotation rate was properly measured. Several cushions were placed on top of a bed to ensure that the phone does not break. Fig. 3.1.5 show the time series of the x, y, z coordinates in the $+y$ and $-z$ directions. Performing the measurement in such a way yields a more notable and stable measurement of the rotation rate



(a) $+x$ direction, rotation towards user (anti-clockwise)

(b) $-x$ direction, rotation away from user (clockwise)

Figure 3.1.4: Same as Fig. 3.1.2 but for rotations towards / away the user, corresponding to rotations in x -direction.



(a) $+y$ direction

(b) $-z$ direction

Figure 3.1.5: Same as Fig. 3.1.2 but for fast rotations in the (a) $+y$ and (b) $-z$ direction.

3.2 Task 2: Allan Deviation

To determine the Allan deviation of the phone's MEMS gyroscope, we placed our phone on a platform with minimal disturbance and took the measurement for approximately one hour. Fig. 3.2.1 shows the time series for each gyroscopic component obtained. From the time series alone, we observe that there are no significant deviations in the measurements in each direction.

We then performed a Fast Fourier Transform (FFT) using the built-in package `scipy.fft` to obtain the plot in the frequency domain. Fig. 3.2.2 shows the Fourier spectrum in each gyroscopic component. We observe that up until around a frequency of 1.5 Hz, the frequency spectrum shows the noise of the measurement. After this point, we observe that the spectrum drops significantly, which indicates that the gyroscope readings occur after this point in the frequency domain. We observe that there is a dip in the spectrum at around 2.2 Hz as well as a small peak at approximately 18 Hz for the measurement in the x direction. This may be caused by the location in which the MEMS gyroscope is placed within

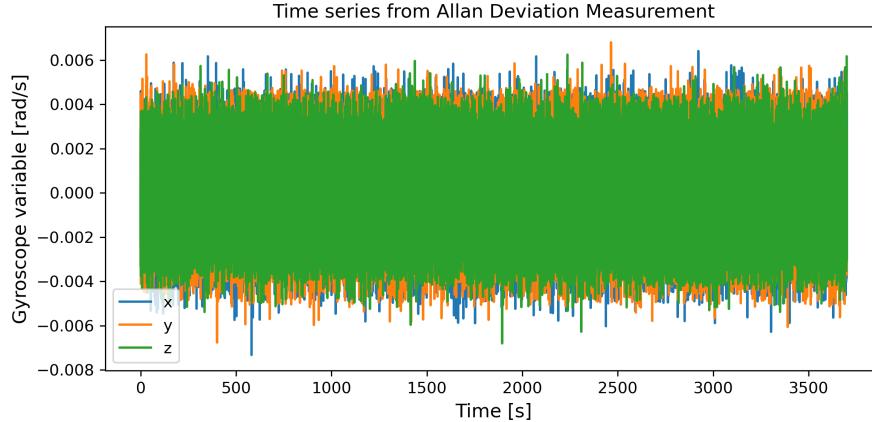


Figure 3.2.1: Time series of the Allan deviation measurement for each gyroscopic component.

the phone, which would account for the bias in the x direction. We observed the same phenomena when measuring for the rotation rate of the Earth, indicating a clear bias in the x direction.

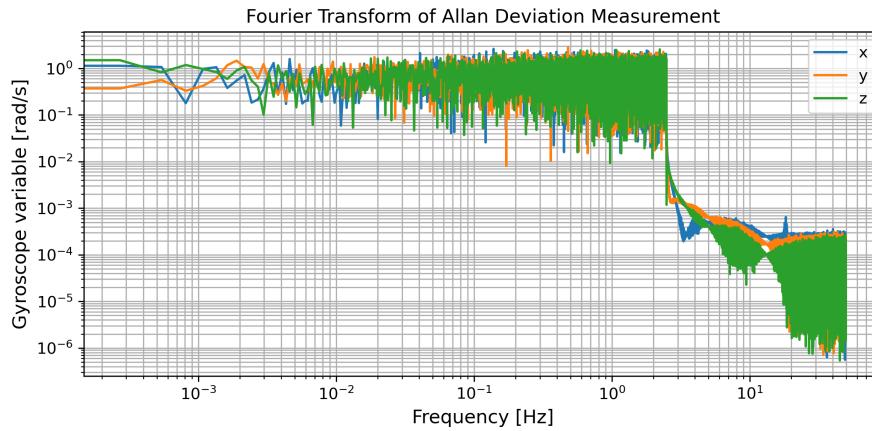


Figure 3.2.2: The Fourier spectrum of the Allan deviation measurement in each gyroscopic direction.

To determine the Allan deviation, we used the function `oadev` from the Python package `allantools` that computes the overlapping Allan deviation given the frequency data, the sample rate, and the time series of the measurement. This returns the Allan deviation for each integration time τ . By using `oadev` instead of the standard `adev`, the noise that is apparent after the point of instability is reduced, yielding a smooth curve at each integration time. The sampling rate was obtained by taking the inverse of the timestep in the measurement, yielding a sampling rate of 99.2 Hz. Fig. 3.2.3 shows the Allan deviation for each gyroscopic component at each integration time, displayed in a log-log format.

The shot-noise time interval, which is the integration time up until the point where the instability of the instrument or drift dominates, can be determined as the point where the first minima of the plot is observed. From Fig. 3.2.3, we observe that the point of instability of the x direction occurs at a much earlier integration time than compared to those from the y and z directions. This indicates that the bias from the x direction affected the Allan deviation measurement.

We also determine the shot-noise limited sensitivity \mathcal{A} by performing a log-linear fit according to Eq. 3.2.2. By taking the logarithm on both sides, we obtain an equation that can be linearized as such:

$$\log(\sigma_{\text{ad}}) = m \log(\tau) + \log \mathcal{A} \quad (3.2.1)$$

where m should yield -0.5 due to the shot-noise scaling. The shot-noise limited sensitivity can then be obtained by taking the exponential of the intercept from the log-linear fit. The data was truncated after the point of instability for each direction. Fig. 3.2.3 shows the log-linear fit for each component. Table 3.1 show the shot-noise limited time interval and the fit parameters (the slope m and the shot-noise limited sensitivity \mathcal{A}) for each component.

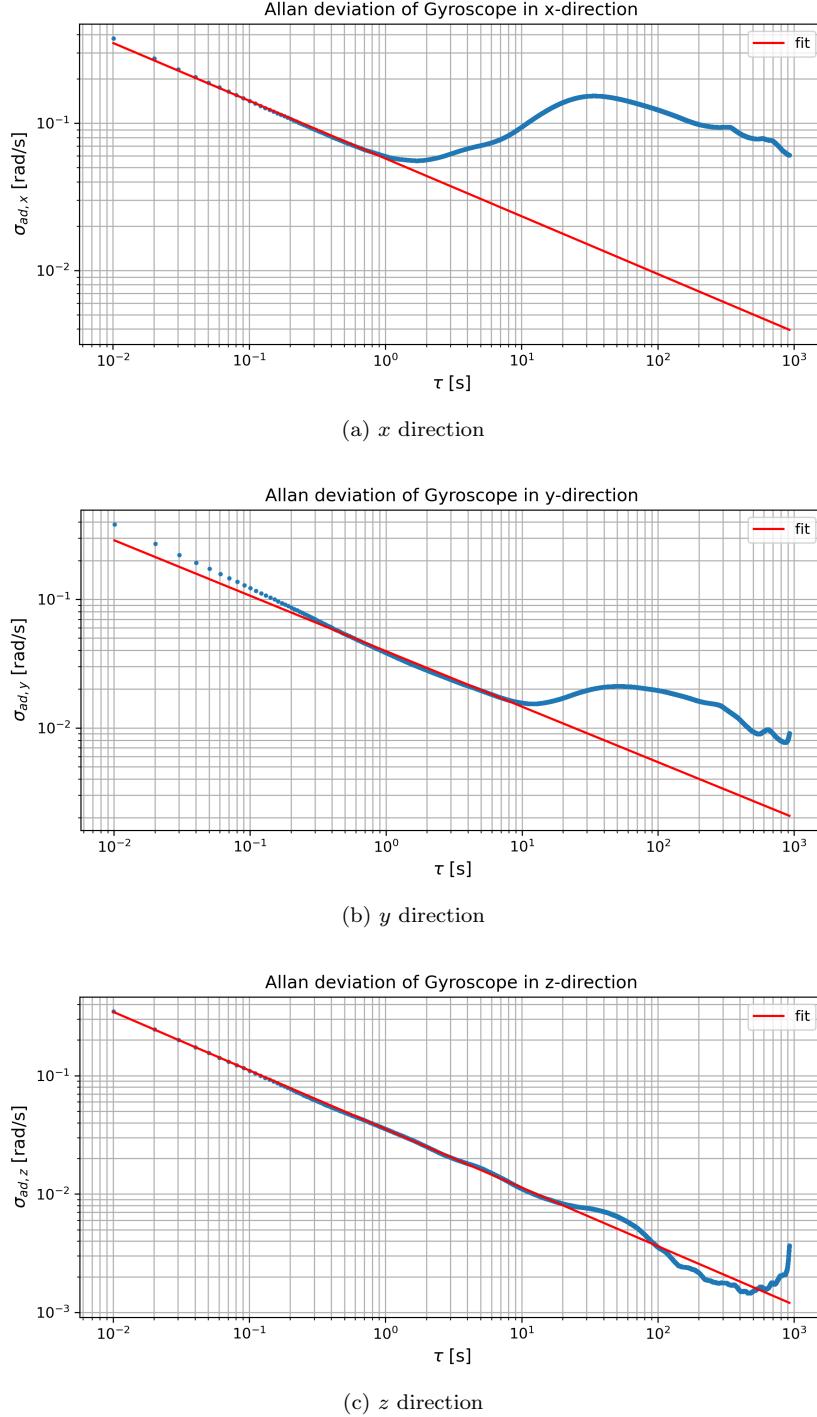


Figure 3.2.3: The Allan deviation for each gyroscopic component for each integration time τ . The log-linear fit is also shown.

Component	τ [s]	m	\mathcal{A} [rad/s/ $\sqrt{\text{Hz}}$]	$ m - (-0.5) $
x	1	-0.3925	0.0576	0.1106
y	10	-0.4318	0.0395	0.0677
z	20	-0.4952	0.0354	0.0048

Table 3.1: The shot-noise limited time interval and associated fit parameters from the log-linear fit of the Allan deviation in each component.

We observe that the time intervals vary between each component, however τ_x is one order of magnitude smaller than those from the y and z direction. This indicates that the bias observed in Fig. 3.2.2 influenced the measurement of the Allan deviation.

When comparing the fit parameters, we observe that the slope from the z component yield the value most closest to -0.5 , which may be due to the longer shot-noise time interval, which in turn allows more data points to be fit. However, all components yield slopes close to -0.5 , with the largest deviation from the x-direction with a residual of ~ 0.11 . We also observe that the shot-noise limited sensitivity from the x component is approximately 0.02 larger than that of the y and z components. This again indicates the influence of the bias of measurement in the x direction. This also shows that the phone's gyroscope is more sensitive to measurements in the y and z direction as compared to that of the x direction.

3.3 Task 3: Rotation Rate of Earth

To determine the rotation rate of the Earth, we utilize the same method as we have performed to measure the Allan deviation, but we perform this for the shot-time limited interval derived by the section above.

We first measure the gyroscope x, y, z coordinates for the duration of the shot-time interval of $\tau = 700\text{s}$. After the data was recorded, we performed the same measurement but with the phone flipped upside down. This corresponds to a flip in the y direction. We then performed the same process but flipping in the z -direction instead (i.e. rotations parallel to the surface). This was performed for a shorter duration of $\tau = 300\text{s}$ due to time constraints. The corresponding time series for both results are shown in Fig. 3.3.1, 3.3.2, 3.3.3 and 3.3.4.

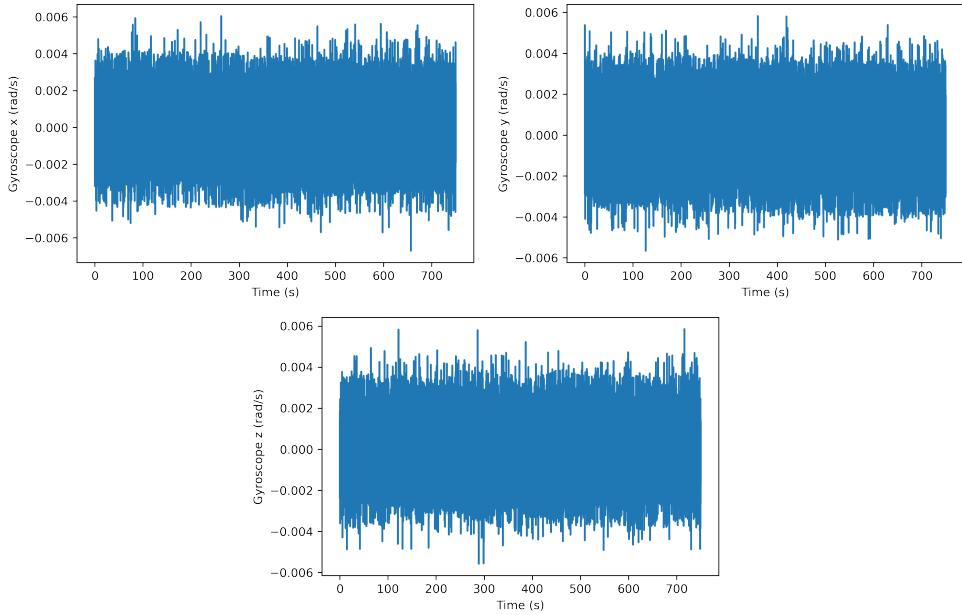


Figure 3.3.1: Raw data of all the three axes of gyroscope from phone for measurement of the Earth's rotation. The measurement was took with the phone face up.

After performing the FFT onto the resulting time series, we observed the sharp peak in the gyroscope x -direction. This shows that there is a clear bias in the x -direction of the phone gyroscope.

The temporal average of each measurement (both flipped and unflipped) was taken, and the difference between the unflipped and flipped measurements were taken (normalized by 2). For example, for the x coordinate we have:

$$\delta\bar{\omega}_x = \frac{|\bar{x}_{\text{noflip}} - \bar{x}_{\text{flip}}|}{2} \quad (3.3.1)$$

Such measurements for both rotations are tabulated on Table 3.2. The magnitude of the values are also evaluated. Note that for arbitrarily small differences, the deviations may be small and thus to properly normalize this, one needs to take into account the uncertainty in the measurement as well.

Furthermore, to properly account for the Earth's rotation rate, we need to transform from the local frame in which we observe the phone's orientation in. This can be done by a simple coordinate

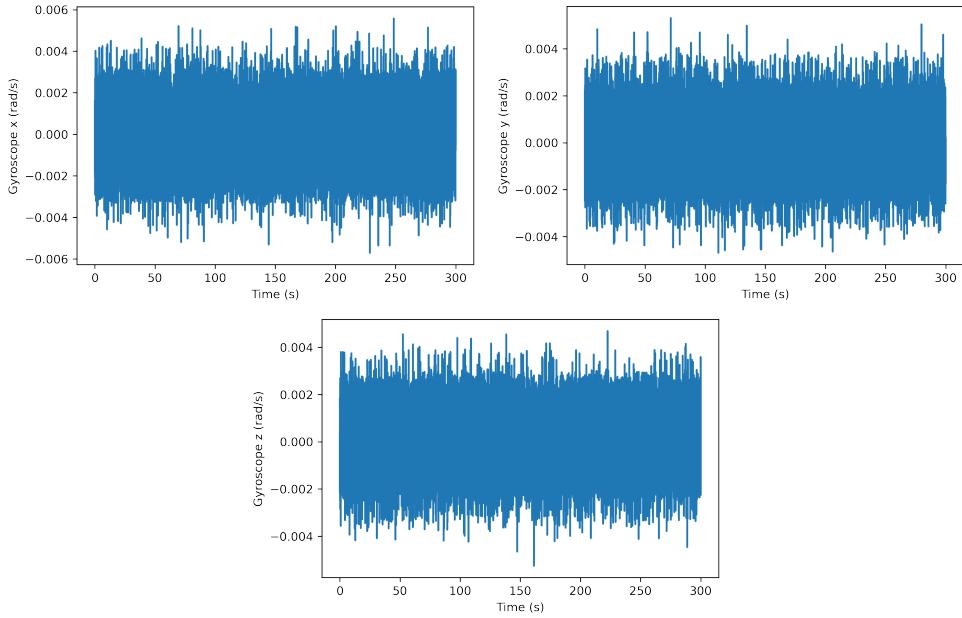


Figure 3.3.2: Same as Fig. 3.3.1 but for phone face down instead.

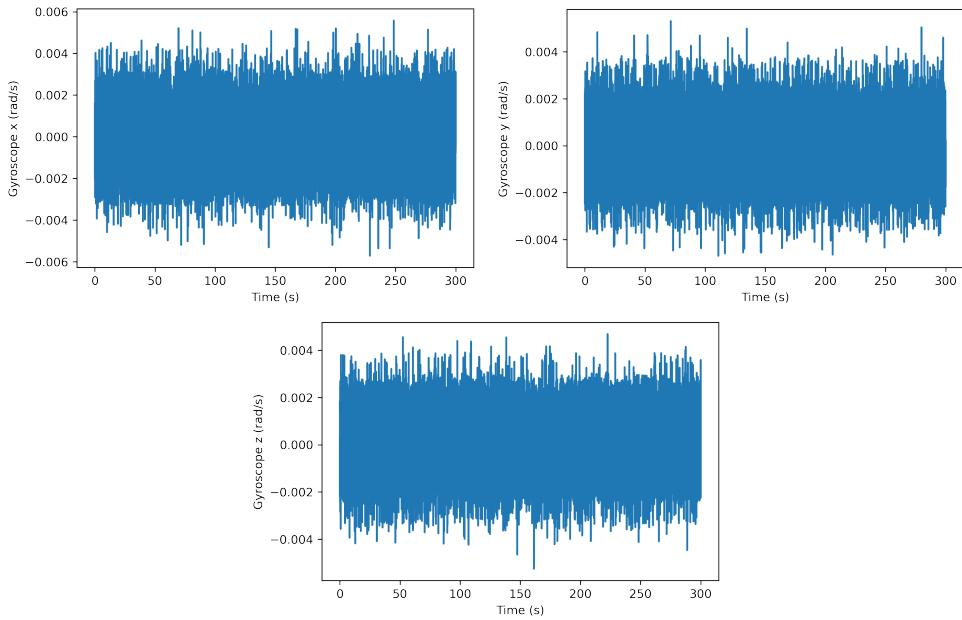


Figure 3.3.3: Same as Fig. 3.3.1 but for rotations in the z -direction. This was performed with the phone facing towards the South Pole.

transformation as such: $\omega = \Omega_E \cos \phi$, where ϕ is the geographic latitude. With $\phi = 39.257^\circ$ (defined from the North Pole), we obtain our corrected rotation rate of the Earth as $\Omega_E = 3.083\mu\text{rads}^{-1}$ and $\Omega_E = 38.80\mu\text{rads}^{-1}$ for the up/down and North/South flip case respectively.

When comparing the values of the rotation rate to the analytical value $\Omega_{E,thr} = 72.92\mu\text{rads}^{-1}$, both yielded values are underestimated. However, the rate where the phone was flipped upside down yielded lower values compared to that when we flipped the phone from South to North. This indicates a clear bias in the rotation and how it impacts the measurement, which is clear due to the East-West rotation of the Earth.

Furthermore, the underestimation of the values are clear due to the various systematical uncertainties that need to be accounted for. The movement of nearby people, the wind, the temperature differences can all impact the phone gyroscope in some way. The MEMS gyroscope does not have as many tools that will calibrate for such deviations as compared to our experimental setup.

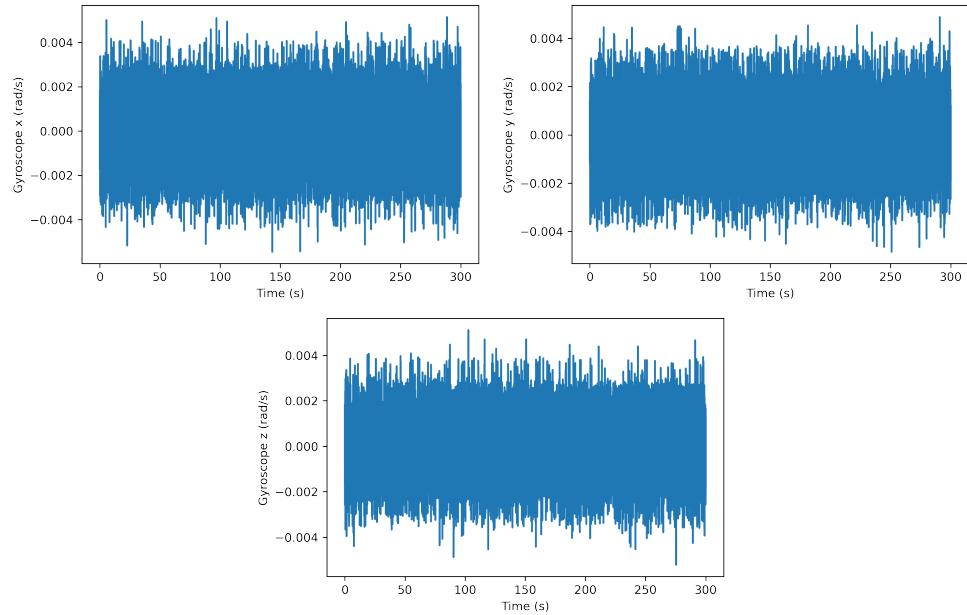


Figure 3.3.4: Same as Fig. 3.3.3 but for those facing towards the North Pole instead.

Coordinate	$\delta\bar{\omega}_i$, face up/down (rad/s)	$\delta\bar{\omega}_i$, face North/South (rad/s)
x	9.273×10^{-8}	9.956×10^{-6}
y	4.603×10^{-7}	2.814×10^{-5}
z	2.340×10^{-6}	3.357×10^{-6}
ω	2.386×10^{-6}	3.004×10^{-5}

Table 3.2: Difference of temporal average of rotation rate for each gyroscopic coordinate and its magnitude.

Chapter 4

Experimental Set-Up and Procedure

4.1 Experimental Set-Up

In this experiment, we use a interference filter laser diode with $\lambda = 1064\text{ nm}$ as our laser, placed outside of the ring cavity system. The signal is then transmitted through an electro-optical phase modulator (EOM). The EOM generates sidebands of $\Omega/(2\pi) = 10\text{ MHz}$. Each beam is then passed through an acousto-optical modulator (AOM), where the light is diffracted with a frequency of approximately 200 MHz. The beams are then passed through a square cavity with an arm length of approximately 0.25 cm, placed on a rotating table. The cavity consists of four mirrors with high reflectivity, and allows the laser to be passed through the cavity in a clockwise or counter-clockwise manner. A plexiglass covering is placed around the cavity to limit any degradation of the reflectivity due to dust [1]. Fig. 4.1.1 shows the setup of the experiment, and Fig. 4.1.2 shows the schematic of the set-up.

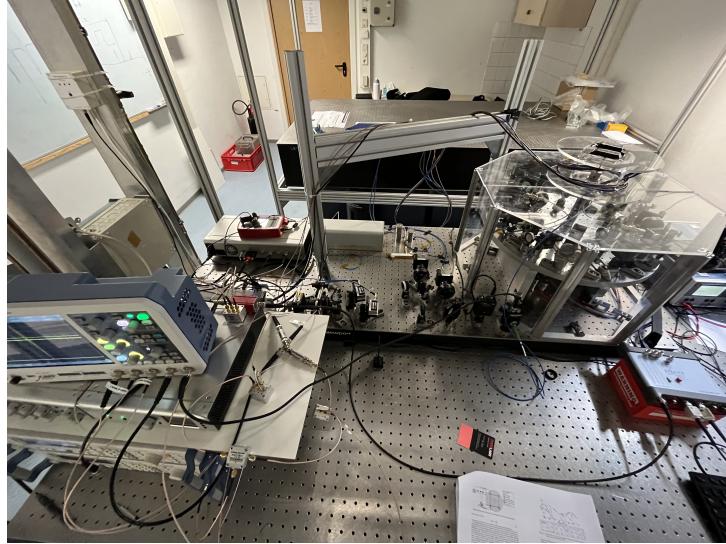


Figure 4.1.1: The set-up used in this experiment.

In order to perform the laser locking, the signal from the cavity is passed into the Moglabs laser lock box, in which the fast and slow PID controller controlled the laser frequency. For PID optimization, we configured the parameters given in the laser lock box until the lock was stable with a high intensity. To lock the laser, we turned the two switches located next to the parameter controls simultaneously. To observe the corresponding signal from the first laser, we connected the drivers to an oscilloscope. This setup was used to determine the free spectral range and the PDH error signal [1]. The obtained signal was exported through a .csv file via a USB port. Fig. 4.1.3 show the oscilloscope, Moglabs laser lock box, and EOM used in this experiment.

To lock the laser moving in the other direction, the signal was passed into the AOM driver control box, in which its build-in PID controller set the laser frequency. This was connected to an external AOM software in which the PID parameters can be set. Different drivers can be connected to the software so

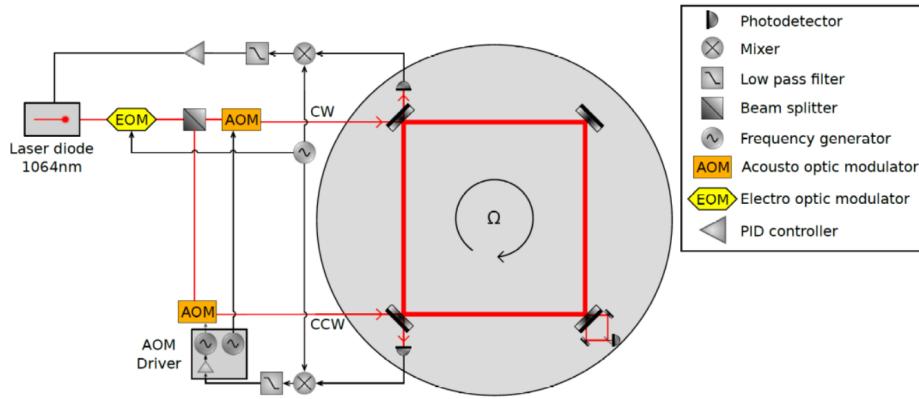


Figure 4.1.2: The schematic of the set-up. Obtained from Ref. [1].

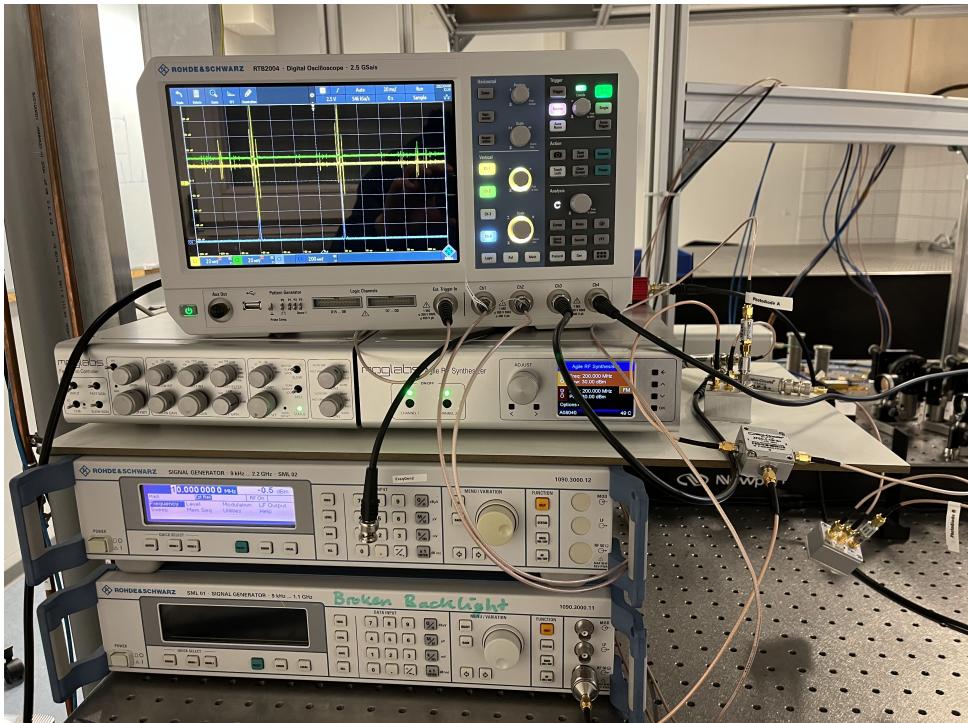


Figure 4.1.3: The oscilloscope (top), Moglabs laser lock box (middle), and EOM (bottom) used in this experiment.

that the amplitude and frequency of the signal can be monitored during runtime. The corresponding signals was saved as a .mat file. As the laser can easily be out of lock (due to clapping, vibrations, thermal fluctuations etc.), we performed the locking procedure multiple times throughout the experiment. To make sure that the signal was in lock, we used the camera placed on the rotation table which displayed the cavity transmission on the external monitor.

To rotate the table, we used a rotary encoder that sends a voltage signal that controls the rotation rate of the table. The output was connected to the AOM software so that the rotation rate can be observed simultaneously with the observed beat frequency when measuring the Sagnac frequency and the Allan deviation. Fig. 4.1.4 shows the gyroscope used in this experiment.

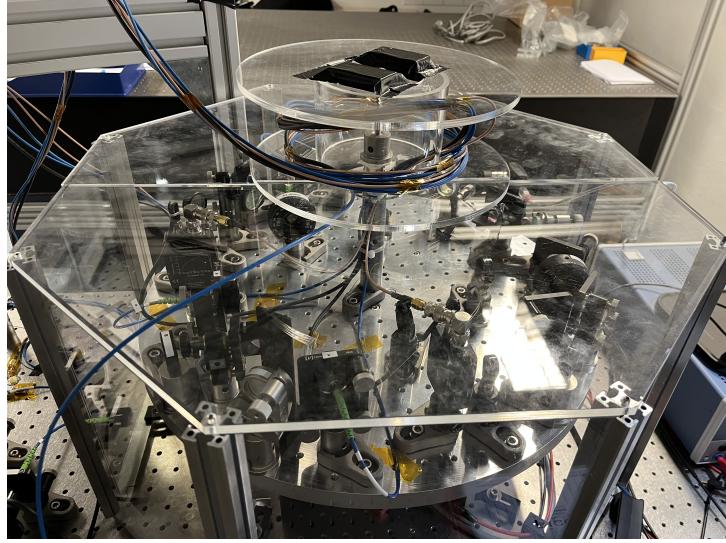


Figure 4.1.4: The ring cavity laser gyroscope used in this experiment.

4.2 Procedure

4.2.1 Free Spectral Range

We first measured both the signal of the laser input and after the laser has passed through the ring cavity system. The window was adjusted until three resonance peaks and their corresponding sidebands were observed. See Fig. 4.2.1 the observed raw signal.

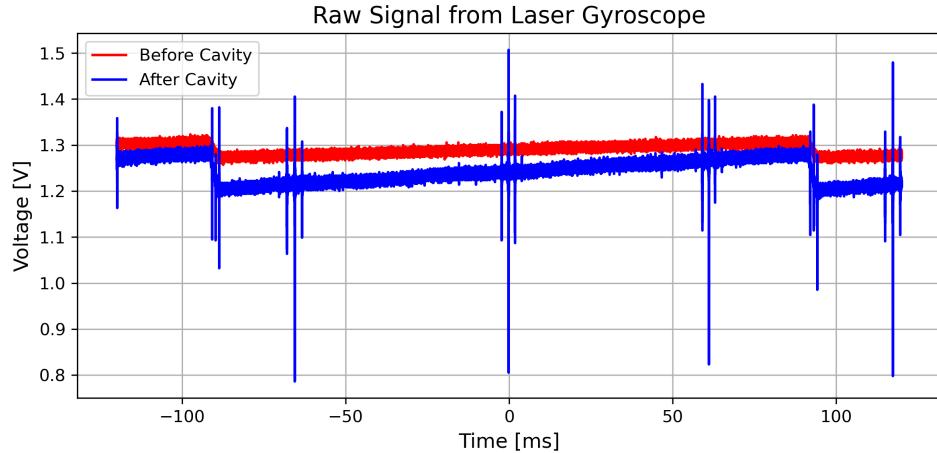


Figure 4.2.1: Raw signal input from the laser before and after passing through the ring cavity system.

As the raw data was provided in the time domain, we converted the time axis into a frequency axis. In order to do so, we use the fact that the modulation frequency from the EOM was $\Omega = 10$ MHz, and we determined the width in time between the peaks and the sidebands to obtain a conversion time between the two axes. The error of the conversion time was obtained purely statistically by using the standard error of the mean.

From this conversion time, we obtained the free spectral range as the peak-to-peak difference between the resonance peaks. The error in the free spectral range was considered by observing the approximate full-width half-maximum of each peak. We then took the weighted average of the free spectral ranges to obtain an averaged free spectral range.

We then evaluated the cavity perimeter from Eq. 2.2.2 for each free spectral range that we have observed. We compared our obtained results with the measured value of the cavity perimeter $P_{\text{meas}} = 0.990 \pm 0.005$ m obtained from Ref. [1].

4.2.2 PDH Error Signal and PID Optimization

To observe the error signal generated from PDH locking, we observed the combined signal generated from the fast and slow PID controller. Fig. 4.2.2 shows the raw signal obtained from the PDH error.

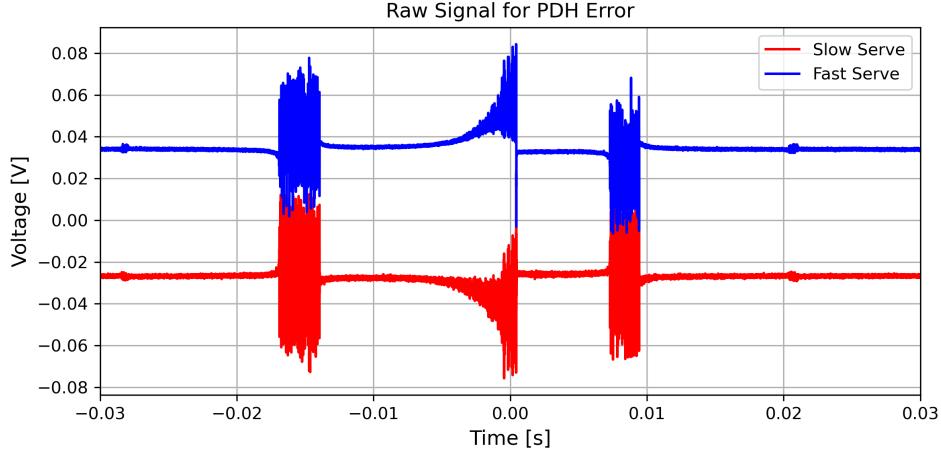


Figure 4.2.2: Raw Signal from PDH locking using the fast and slow PID controller.

The corresponding data was then used to determine the linear fit of the error signal where the zero crossing and linear slope is present (i.e. near the middle of the observed error signal). This was performed by first filtering the data using `scipy.signal.sosfilt`, then using a least-squares fit with `numpy.polyfit`. Fig. 4.2.3 shows the filtered data from the error signal generated by PDH locking. We observe that the shape of the signal closely resembles that of Fig. 2.2.2 as desired.

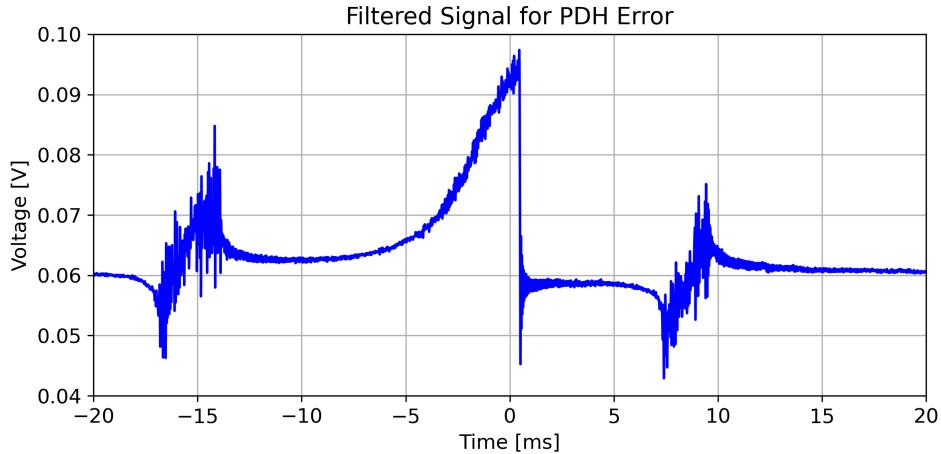


Figure 4.2.3: Filtered signal from PDH locking. The behavior of the signal closely resembles that of the signal shown in Fig. 2.2.2.

The error signal was then used to determine the optimal PID parameters that yield the most stable and large signal. In order to do this, the following parameters were modified: SLOW INT, FAST INT, DIFF GAIN, FAST DIFF / FILTER, SLOW GAIN, FAST GAIN, GAIN LIMIT, and OFFSET. The major parameters that influenced the signal were the SLOW INT, SLOW GAIN, FAST DIFF / FILTER and FAST GAIN. Modifying the SLOW INT and FAST DIFF caused the laser to be out of lock, whereas FAST GAIN decreased the noise observed in the signal. As we decreased the SLOW GAIN, we observed that the laser became more out of lock, allowing for the control of the laser locking. The OFFSET parameter adjusted the offset between the two signals. Table 4.1 shows the optimal PID parameters used in the subsequent measurements which were both stable and yielded a strong signal strength.

When performing laser locking, we further needed to consider the systematics due to the environment. Any small disturbance such as clapping, tapping the desk, moving the cables or even walking influenced

PID Parameter	Value
SLOW INT	25
FAST INT	20K
FAST DIFF / FILTER	10M
FAST GAIN	6 dB
GAIN LIMIT	30

Table 4.1: The optimal PID parameters used for this experiment. The SLOW GAIN and FAST GAIN parameter values are not noted as specific values are not marked on the Moglabs PID controller.

the degree of locking for the laser. Other factors such as the temperature fluctuations in the room may have caused the laser to be out of lock as well. In order to verify that we were obtained the correct resonant mode when optimizing the PID parameters (i.e. the TEM_{010} mode), we used a camera located on the rotation table. An image of the resulting mode obtained when locking can be seen in Fig. 4.2.4.

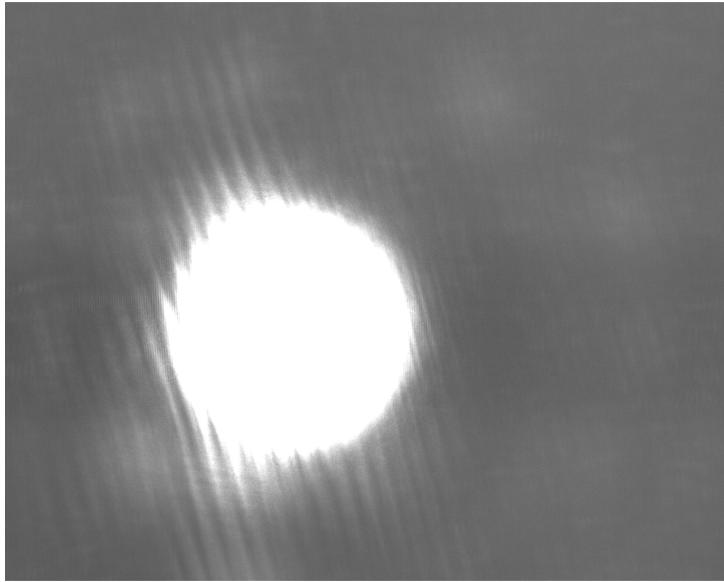


Figure 4.2.4: The Fundamental $\text{TEM}_{0,0}$ Gaussian mode.

4.2.3 Cavity Ring-Down

As discussed in the 2.2.3, here we measure the exponential decay of power within the cavity after the “switch” is turned off. After setting the photodiode channel to the appropriate height, trigger mode was set to single. The laser beam is then switched off using the front panel control of the AOM driver. We notice an exponential decay, as seen in Fig. 4.2.5. Several measurements were taken out which twelve were analyzed. Rest of the figures can be found in the Appendix 7. The data was ananlysed in Python and curve fitting was done using the following function

$$f(t) = a \exp\left(-\frac{t}{t_{dec}}\right) + c, \quad (4.2.1)$$

where a is the amplitude, t_{dec} is the decay constant and c is a constant. The fitting was done using the function `scipy.optimize.curve-fit`. This function also gives us the error associated with the fit. Using the fit parameters, the decay constant can be computed using the Eqn. 2.2.5. Using the value of decay constant, reflectivity was computed using the Eqn. 2.2.6 and finesse of the cavity was computed using Eqn. 2.2.4. The weighted mean and error was calculated for the twelve different measurements. Error propagation was used in order to calculate the error associated with these quantities.

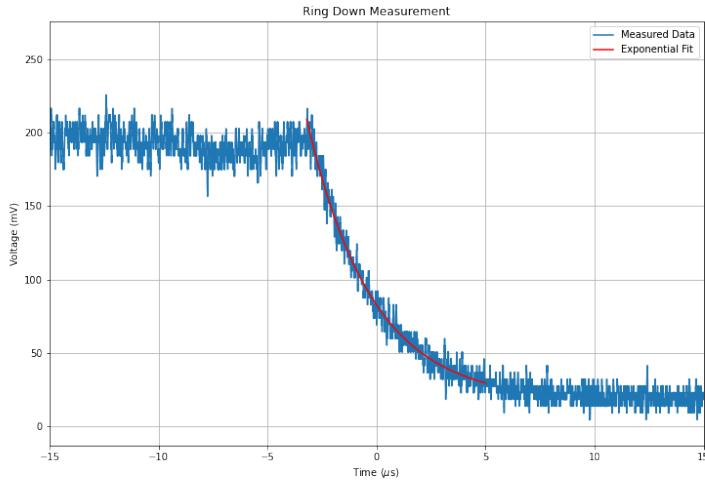
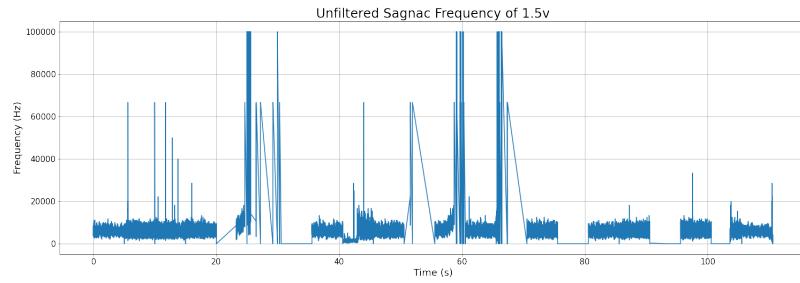
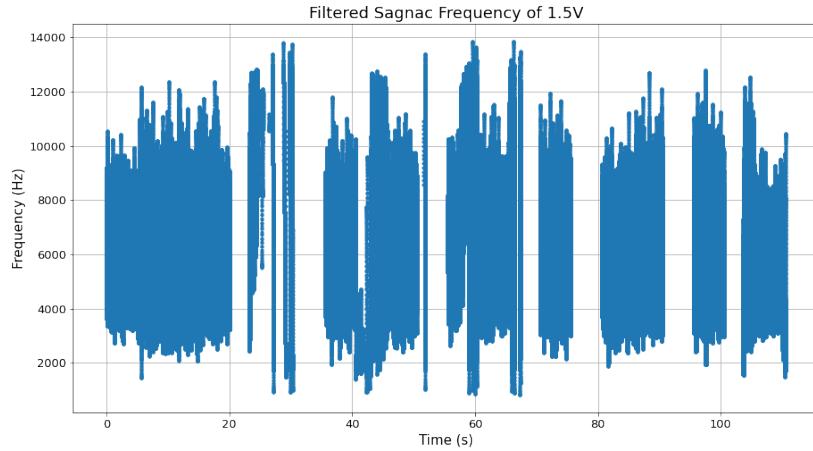


Figure 4.2.5: Exponential decay of power in the cavity after switching it off.



(a) Unfiltered data of Sagnac beat frequency with 1.5V applied to the rotation table.



(b) Filtered data of Sagnac beat frequency with 1.5V applied to the rotation table.

Figure 4.2.6: Unfiltered and filtered data for 1.5V.

4.2.4 Scale Factor

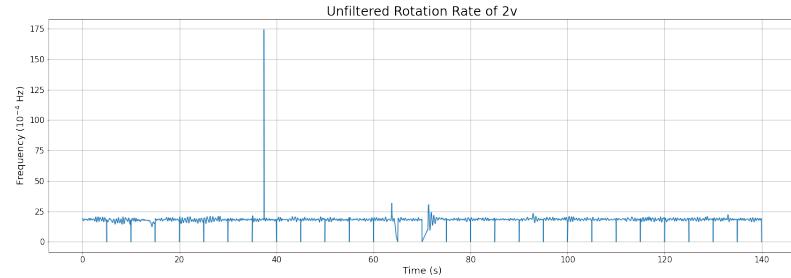
The gyroscope table was set into rotation, all the while making sure that the cavity was still locked. The oscilloscope channel was used to read the frequency. In order to get the frequency in Hz, the encoder rate of 10,000 needs to be taken into account. Since one complete rotation corresponds to 10,000 counts, the recorded values need to be divided by 10,000. Since this is the angular frequency of the table, in

order to get the rotation rate in Hz, the final fit value is divided by a factor of 2π in order to get the correct dimensions.

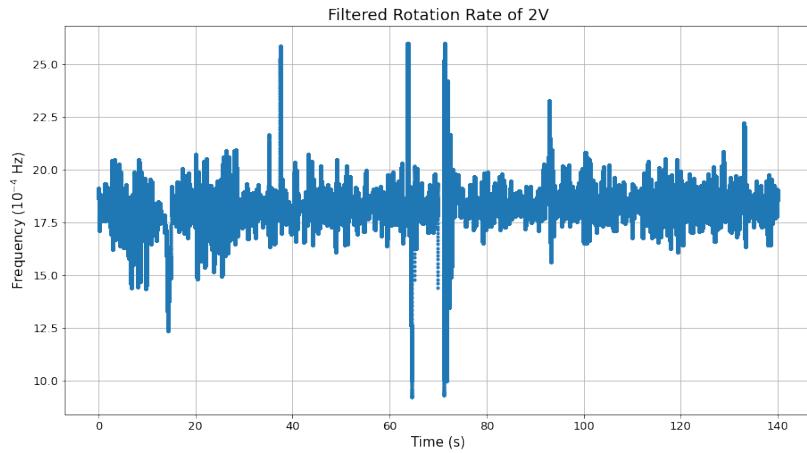
The beat signal was recorded from another channel and evaluated for five different voltages, which correspond to five different rotation rates. Higher rotation rates lead to highly unstable cavity, with stability of the lock not lasting more than 2-3s. Hence, lower rotation rates were preferred for a much more reliable data. Nevertheless, even at low rotation rates, the cavity falls out of lock and we had to re-lock it and continue taking readings. Another interesting thing to note is that this lock could be broken with slight disturbances like banging the table or moving the chair.

As an example, the unfiltered data for beat frequency at 1.5V is shown in Fig. 4.2.7a. In order to extract usable data from this, first loose upper and lower bounds were set to make sure that unreasonably high and low values do not contribute. From `scipy.signal`, the package `savgol-filter` was used to filter the data. The filtered data is shown in Fig. 4.2.7b.

Similarly, in order to get the rate of rotation of the table, the data was first filtered. As an example for this, we look at the data corresponding to 2V. This can be seen in Fig. 4.2.7.



(a) Unfiltered data of rate of rotation of table with 2V.



(b) Filtered data of rate of rotation of table with 2V.

Figure 4.2.7: Unfiltered and filtered data for 2V.

In order to extract the Sagnac frequency and the rate of rotation, the average of the filtered data was taken. Averages were calculated for six different rotation rates. These were then plotted against each other and a fit was made using the package `np.polyfit`. A standard systematic error of ± 2500 Hz was taken in order to account for the noise and fluctuations. Statistical error was computed using assuming a gaussian distribution and error propagation was done.

4.2.5 Allan Deviation

Finally, we measured the Allan deviation of the laser gyroscope. We performed the same procedure as with the scale factor measurement. Using the same PID parameters as before, we set the rotation frequency to rotate with 0.75 V, so that the laser will remain locked as long as possible. The gyroscope was rotated until the cable connecting the gyroscope did not extend or contract any further, then the

same analysis was performed in the opposite rotation direction. The measurement was taken continuously for approximately 1.5 hours. The unprocessed time series for the frequency measurement can be seen in Fig. 4.2.8.

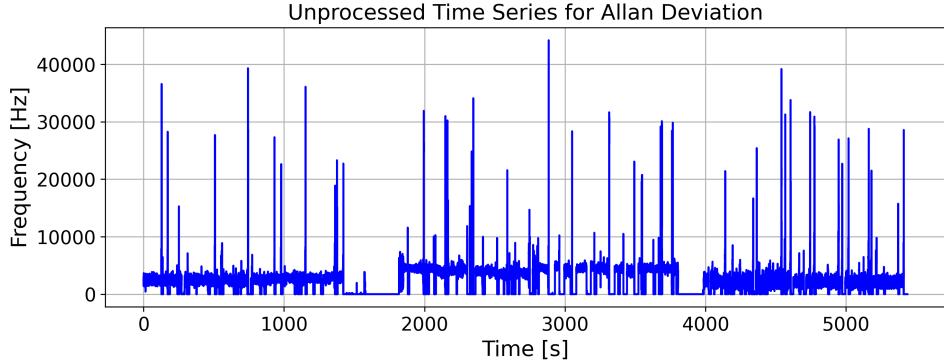


Figure 4.2.8: Unprocessed time series obtained from Allan deviation measurements. We observe large spikes at numerous locations as well as regions with zero frequency.

From the raw data, we first removed any measurements that were taken while the laser was unlocked and when the rotation of the table was switched. We then apply filtering towards our data using `scipy.signal.sosfilt` to remove fluctuations of the signal. Fig. 4.2.9 shows the measured data after filtering. We observe that the signal of the Sagnac frequency differs in the region $t \in [1800, 3800]$ as compared to the other regions, yielding an average frequency of ~ 2000 Hz larger than the others. This is due to the fact that we have rotated the table counter-clockwise for that duration. The rotation rate of the table, however, was kept the same, and as such this has no impact in the calculation of the Allan deviation.

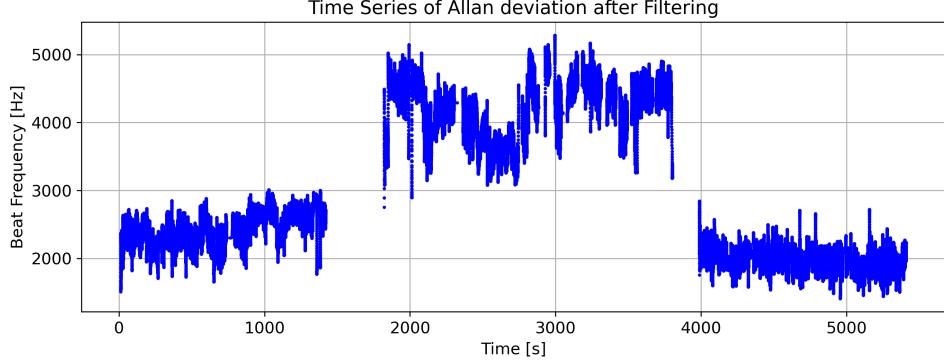


Figure 4.2.9: Time series of Allan deviation measurement after filtering and removing unwanted data measurements.

We then performed the same procedure as with the Pre-lab tasks in Sec. 3.2 to determine the Allan deviation, namely to first determine the frequency spectrum of the signal using `scipy.fft` before passing through the frequency data into `allantools.oadev` to obtain the Allan deviation for each integration time τ . We also determined the shot-noise limited time τ by observing the time duration in which the instability of the measurement starts. For the rotation frequency of the table, we followed the same procedure from Sec. 4.2.4.

Chapter 5

Results and Discussion

5.1 Free Spectral Range

Table 5.1 shows the corresponding conversion time obtained from each sideband using the modulation frequency of 10 MHz. The mean conversion time that we obtained was 2.104 ± 0.066 ms which is the conversion time that we used to determine the free spectral range. Fig. 5.1.1 shows the corresponding resonant peaks in both the time and frequency axis.

Sideband	Conversion time [ms]
1	2.385
2	2.226
3	2.110
4	2.003
5	1.998
6	1.900
Mean	2.104 ± 0.066

Table 5.1: The conversion time determined from each sideband with the three resonant peaks obtained from Fig. 4.2.1.

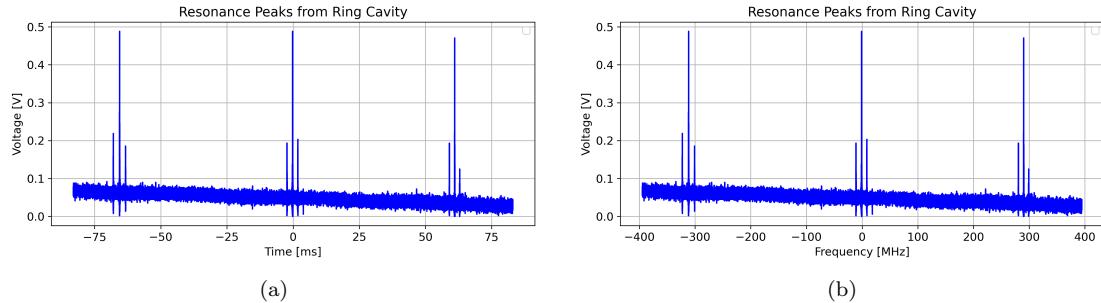


Figure 5.1.1: The observed resonant peaks from the ring cavity in the laser gyroscope. (a) and (b) show the same plot in the time and frequency domain respectively.

From the obtained conversion time, we obtained two values of the free spectral range from the peak-to-peak difference between each resonance peak. Correspondingly we also obtained two values for the cavity perimeter. Table 5.2 shows the obtained free spectral range and cavity perimeter for each peak-to-peak difference as well as their weighted average.

When comparing the average cavity perimeter to the measured values given in Ref. [1], we observed that the experimental value were 1.52σ away. This indicates that the result we have obtained is in good, but not great agreement with the measured value. Such results may be caused by

Peak	$\delta\nu_{\text{FSR}}$ [MHz]	Cavity Perimeter [cm]
1-2	310.60 ± 0.12	96.587 ± 0.039
2-3	291.12 ± 0.11	103.050 ± 0.040
Mean	299.778 ± 0.083	99.760 ± 0.028

Table 5.2: The free spectral range $\delta\nu_{\text{FSR}}$ and the cavity perimeter P obtained for each peak-to-peak. The weighted average is also shown.

5.2 PDH Error Signal

Once applying the linear fit within the region of interest, we obtain the following parameters: $a = -1168 \pm 580 \text{ J s}^{-2}$, $b = 639.82 \pm 0.13 \text{ m J s}^{-1}$ for a linear fit of $\epsilon_{\text{fit}} = at + b$. The filtered data within the region of interest with the linear fit is shown on Fig. 5.2.1. As we can see, the slope is not truly vertical, indicating that there is some broadening of the cavity linewidth, as $a \sim \sqrt{P_c P_s} \delta\nu_{\frac{1}{2}}$. This is to be expected as thermal fluctuations as well as other disturbances can cause such broadening of the spectrum.

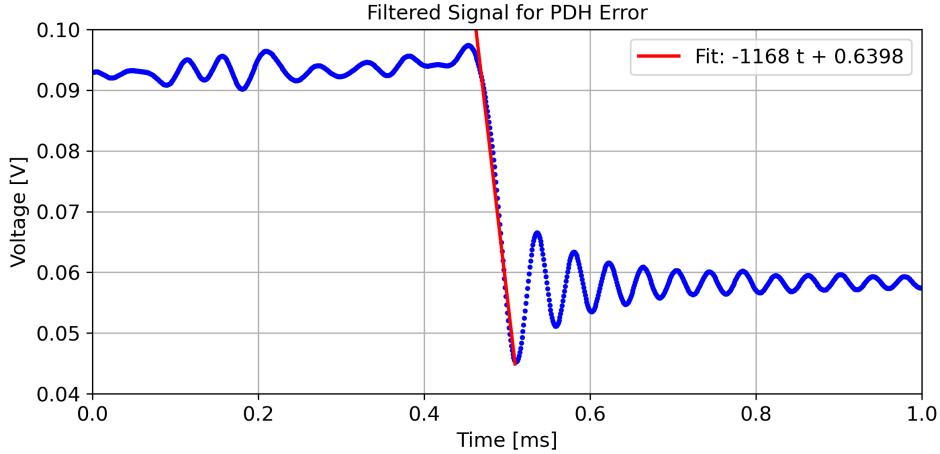


Figure 5.2.1: The filtered signal from PDH locking in a zoomed-in region. The linear fit with associated fit parameters are also shown.

As we can observe from Fig. 4.2.3, the signal cannot truly exhibit the sharp peaks as observed in the Fig. 2.2.2. Such behavior is caused by the noise of the signal generated by the surrounding environment, as well as the oscillatory behavior of the signal, which causes the zero values to be oscillatory. Thus true peaks cannot be observed which prevents us from applying a true linear fit to the data.

5.3 Cavity Ring-down

The fit parameters and the errors associated with them are shown in Table 5.3. The corresponding values of reflectivity and finesse is summarized in Table 5.4. Lastly, the weighted mean and error were calculated and are shown in Table 5.5.

5.4 Scale Factor

From the filtered data, the average Sagnac frequencies and the rotation rate were computed. These values can be found in Table 5.6.

The scale factor is shown in Fig. 5.4.1. The slope of the fit function is 941102.493 ± 0.203 . The theoretical value is estimated using the arm length which was calculated to be 0.24m and the wavelength of 1064nm, giving 225563.909. Our estimated value deviates from the expected value by 31.52%. This could be because the intensity of the lock was unable to reach an the ideal value of approximately 1V. It could also be due to the fact that our data did not include the direction of rotation (CW or CCW).

Amplitude	Decay constant (μs)	Constant
66.121 ± 0.200	$3.049 \pm 3.504 \times 10^{-6}$	17.995 ± 0.132
64.770 ± 0.152	$2.911 \pm 2.908 \times 10^{-6}$	20.862 ± 0.098
61.846 ± 0.144	$2.978 \pm 2.954 \times 10^{-6}$	18.201 ± 0.094
61.787 ± 0.124	$2.915 \pm 2.605 \times 10^{-6}$	18.501 ± 0.080
59.468 ± 0.136	$3.098 \pm 2.899 \times 10^{-6}$	18.823 ± 0.090
56.683 ± 0.129	$2.887 \pm 3.271 \times 10^{-6}$	19.797 ± 0.083
63.604 ± 0.156	$3.031 \pm 2.972 \times 10^{-6}$	16.951 ± 0.103
59.921 ± 0.167	$2.899 \pm 3.755 \times 10^{-6}$	19.726 ± 0.108
58.560 ± 0.140	$2.959 \pm 3.238 \times 10^{-6}$	19.142 ± 0.091
59.752 ± 0.142	$2.839 \pm 3.272 \times 10^{-6}$	20.856 ± 0.091
63.945 ± 0.160	$3.008 \pm 3.042 \times 10^{-6}$	17.469 ± 0.105
65.165 ± 0.132	$2.970 \pm 2.461 \times 10^{-6}$	17.501 ± 0.087

Table 5.3: Fit parameters.

Reflectivity	Finesse
$0.999454 \pm 1.516 \times 10^{-7}$	5757.144 ± 0.509
$0.999428 \pm 1.587 \times 10^{-7}$	5498.087 ± 0.486
$0.999441 \pm 1.552 \times 10^{-7}$	5623.439 ± 0.497
$0.999429 \pm 1.585 \times 10^{-7}$	5505.279 ± 0.487
$0.999463 \pm 1.491 \times 10^{-7}$	5850.284 ± 0.517
$0.999423 \pm 1.600 \times 10^{-7}$	5452.519 ± 0.482
$0.999451 \pm 1.524 \times 10^{-7}$	5723.937 ± 0.506
$0.999426 \pm 1.594 \times 10^{-7}$	5474.940 ± 0.484
$0.999437 \pm 1.562 \times 10^{-7}$	5587.268 ± 0.494
$0.999414 \pm 1.627 \times 10^{-7}$	5362.187 ± 0.474
$0.999447 \pm 1.536 \times 10^{-7}$	5680.210 ± 0.502
$0.999440 \pm 1.556 \times 10^{-7}$	5608.913 ± 0.496

Table 5.4: Reflectivity and Finesse of each measurement.

Mean Reflectivity	Mean Finesse
$0.999438 \pm 4.503 \times 10^{-8}$	5587.0880 ± 0.142

Table 5.5: Weighted mean of reflectivity and finesse.

Voltage	Average Sagnac Frequency (Hz)	Rotation Rate (rad/s)
1V	3693.55 ± 0.27	8.44 ± 0.00012
1.5V	6062.34 ± 0.39	13.41 ± 0.00019
2V	9143.27 ± 0.63	18.25 ± 0.00020
2.5V	14975.54 ± 0.71	22.86 ± 0.00026
3V	14935.25 ± 2.36	28.20 ± 0.00028
5V	26884.50 ± 2.51	47.95 ± 0.00031

Table 5.6: Average Sagnac Frequency and corresponding Rotation Rate of the table.

This could lead to higher or lower Sagnac beat frequencies, depending on the direction of rotation, which should be considered separately.

5.5 Allan Deviation

In Fig. 5.5.1, we show the frequency spectrum of the measurement obtained for the Allan deviation. We observe that the laser gyroscope is at least sensitive to rotations starting from $\sim 5 \times 10^{-4}$. This can be noted due to the cutoff between the noise and the actual signal.

Fig. 5.5.2 shows the Allan deviation obtained for each integration time. We observe that the shot-

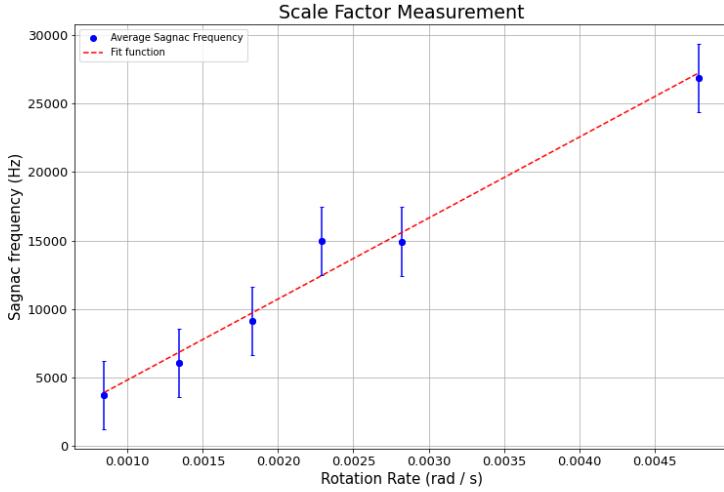


Figure 5.4.1: The Sagnac frequency and their linear fit, which gives the scale factor.

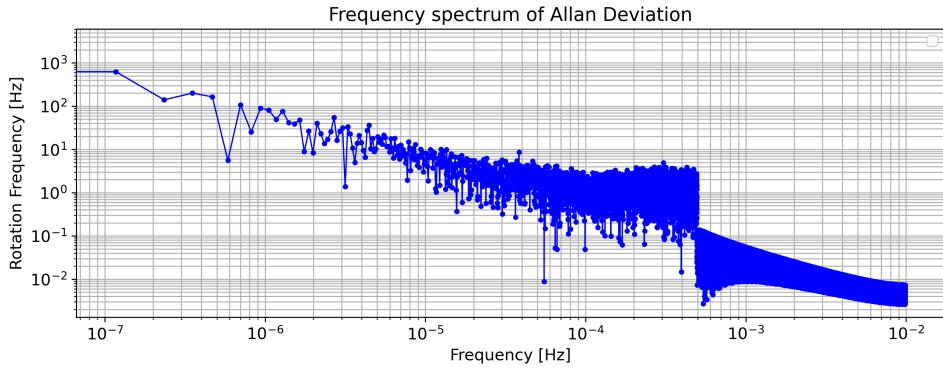


Figure 5.5.1: The frequency spectrum of the Allan deviation measurement.

noise time interval is approximately $\tau_{\text{sn}} = 4\text{s}$, which is extensively small compared to those in typical experiments, which can yield values as high as $\tau_{\text{sn}} \sim 10^5$ [1]. Further, when we perform the linear fit, we obtain the following values for our fit parameters:

$$m = -0.6966511 \pm 0.0000016 \quad A = 51163.064 \pm 0.094. \quad (5.5.1)$$

When comparing with the expected power of -0.5 , we see that the residual gives 0.196 , giving a relatively large deviation. A possible cause for this may be due to the lack of errors that needed to be taken into account. As we can see from the errors in the fit parameters, the errors are largely underestimated, indicating that we have neglected to consider other errors into account. A possible error that we have not accounted for may be due to the temperature fluctuations in the room. As the measurement was performed over an extensive period of time, the difference in the temperature in the room can affect the degree of locking, causing larger deviations in the measurement. Other possible causes may be due to the dust accumulation on the mirrors, decreasing the reflectivity of the mirrors.

To obtain the gyroscopic sensitivity, we use the scale factor obtained from Sec. 4.2.4 using Eq. 2.3.3. Evaluating the Allan deviation at the shot-noise time interval, we then obtain the shot-noise limited sensitivity of our laser gyroscope as $\delta\Omega_{\text{ad}} = 0.089\,621\,1 \pm 0.000\,002\,5 \mu\text{rad s}^{-1}$. We compare this value to the theoretical one, assuming that $1\,\mu\text{W}$ of optical power is available for detection. With this assumption, along with the values for the finesse of the cavity, the laser wavelength, and area of the cavity, we obtain a theoretical gyroscopic sensitivity of $\delta\Omega_{\text{omega}} = 0.031\,357 \pm 0.000\,018 \mu\text{rad s}^{-1}$. Comparing the two sensitivity values, we observe that the relative difference is approximately 0.650 , yielding relatively accurate results. When comparing the values, however, we did not take the values of the errors into account as they are too small. This is again caused by the underestimation of the possible errors in our

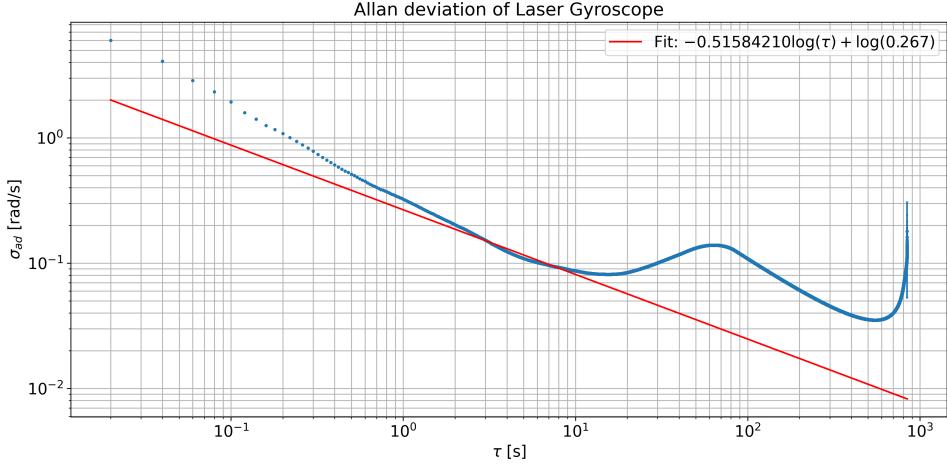


Figure 5.5.2: The Allan deviation of the laser gyroscope, plotted against the integration time in a log-log plot. The log-linear fit is also shown.

experiment.

5.6 Rotation Rate of the Earth

In theory, the methods that we have performed above is sufficient to determine the rotation rate of the Earth. As we are working with a ring cavity system, however, we need to consider the lock-in effect as described in Sec. 2.2.5. To determine the lock-in threshold, we use our known laser wavelength of $\lambda = 1064\text{ nm}$ and assume that the beam diameter is $d = 300\text{ }\mu\text{m}$ as given in Ref. [1]. We also use the value of the cavity area from the obtained cavity perimeter from Sec. 5.1, yielding $A = L^2 = P/16 = 622.01 \pm 0.35\text{ cm}^2$. To consider the fraction of backscattering, we assume that the fraction has an upper limit of $r_s \leq \sqrt{4 - \text{sqrt}(\mathcal{R})}$, giving us $r_{s,\text{max}} = 0.047377 \pm 0.0000019$.

Considering all such values, we evaluate the lock-in threshold using Eq. 2.2.10 to be $\Omega_L = 8.5774 \pm 0.0049\text{ mrad s}^{-1}$. Comparing this to the rotation rate of the Earth of $72.92\text{ }\mu\text{rad s}^{-1}$, we observe that the lock-in effect will prevent us to measure the rotation rate. As of such, our laser gyroscope is not capable to measure the rotation rate of the Earth. To be able to measure such quantity, the lock-in threshold must be lowered. This can be done by either increasing the effective area of the cavity or decreasing the laser wavelength. The mirrors should also have higher reflectivity to prevent backscattering from influencing the beat frequency pattern.

Chapter 6

Conclusion and Outlook

In this experiment, we attempted to measure the rotation rate of the Earth using a ring laser gyroscope. A 1064 nm diode laser was passed through a square cavity, constructed with four highly reflective mirrors. This allowed the laser to be divided into two counter-propagating beams. The cavity was placed on a rotating table, and the Sagnac frequency between the beams was measured. In order to quantify the performance of the gyroscope and the cavity system, we determined and investigated the free spectral range, PDH error signal, finesse and reflectivity of the cavity, and the Allan deviation. The scale factor between the Sagnac frequency and rotation rate was also measured. Each of these quantities were compared with their expected values.

To determine the free spectral range, the cavity resonance signal was obtained by connecting the output signal to an oscilloscope. The free spectral range was obtained by determining the peak-to-peak difference between the carrier signal, which was correspondingly obtained by converting the time to frequency axis via the modulation frequency of 10 MHz. The cavity perimeter was also calculated based from the free spectral range. From this, we obtained a free spectral range of 299.778 ± 0.083 MHz and a cavity perimeter of 99.760 ± 0.028 cm, obtaining values that are on good agreement with the measured values.

The PDH error signal was obtained by taking the error signal from the fast and slow PID controller. The combined signal was first filtered before obtaining a linear fit, obtaining $a = 1168 \pm 580$ s $^{-2}$, $b = 639.82 \pm 0.13$ mJ s $^{-1}$. We further saw that we cannot truly perform a linear fit of the error signal due to the oscillatory behavior of the signal.

The ring down technique allows us to compute the reflectivity and the finesse of the cavity. The power loss in the cavity after the laser is switched off follows an exponential decay curve. Using analysis methods to fit a the curve, gives the corresponding parameters the decay constant, using which we can get the reflectivity and finesse. The average reflectivity was measured to be $0.999438 \pm 4.503 \times 10^{-8}$, the ideal value being 1. The average finesse was measured to be 5587.0880 ± 0.142 , with the ideal value approaching infinity. Since there is no way to compute the theoretical values for our equipment, we cannot quantify the deviation from a standard value.

The scale factor is measured by taking the slope of linear fit of between the Sagnac frequency and rotation rate of the table. In order to achieve this, we take several measurements at different rate of rotations. Since the cavity falls out of lock multiple times, these “breaks” in the data need to be removed before taking the average. After taking the average, the Sagnac frequency and rotation rate were plotted against each other and a value of 941102.493 ± 0.203 was obtained. The theoretical value is 225563.909, which deviates from our value by 31.52%. This deviation could be because of not being able to achieve a higher intensity lock. It could also be because the data obtained does not include whether the table was rotating CW or CCW, further affecting our analysis.

The Allan deviation was determined by taking an extensive measurement of the Sagnac frequency with a slow rotation rate of the table. The Fourier spectrum of the resulting signal was taken, and this was passed through `allantools.oadev` to obtain the Allan deviation. The shot-noise time interval was shown to be $\tau_{\text{sn}} = 10$ s. We performed a log-linear fit and obtained a gyroscopic sensitivity that was relatively close to the theoretical value.

The current setup can be improved in several ways so that it would be possible to measure of the rate of rotation of earth. With the current setup, in order to overcome the lock-in effect, the backscattering from the mirrors needs to be reduced. To do that, the set up can be completely sealed and additionally, can be sealed in vacuum. This should reduce the locking threshold. Further, we can introduce some

forced dither, which is an intentional noise applied to randomize the quantization error, which can help overcome this problem. This is done by rotating the laser cavity clockwise and counter-clockwise about its axis at the resonance frequency. This method ensures that the angular velocity is not close to the lock-in threshold. But this does not fix the problem completely, as every time the direction of rotation is switched, for a brief moment, the rotation rate is almost zero, allowing for the lock-in effect to occur briefly. To fix this, noise can be introduced to the vibrations.

Further ways to improve the set up would be to make the base more stable, as it was still susceptible to disturbances from the surrounding, for example, the opening and closing of door. Furthermore, a larger ring would increase the effective perimeter and the area which would further reduce the locking threshold.

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Chapter 7

Appendix

In the following section, we include additional plots and figures for the sake of completeness.

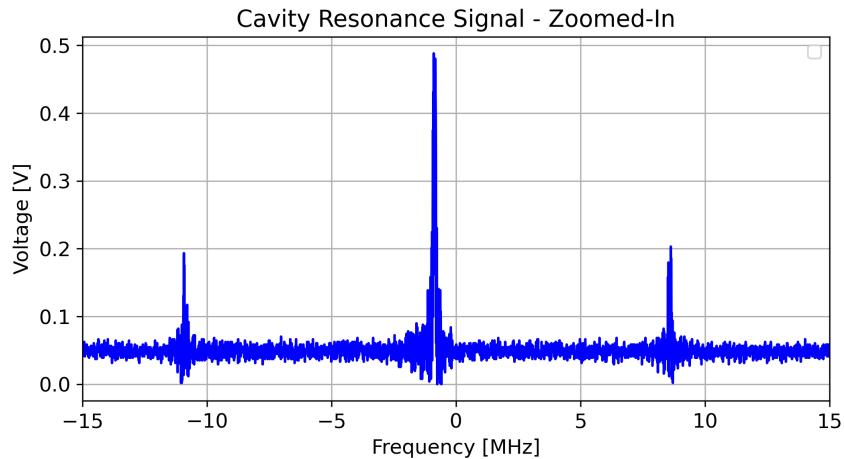


Figure 7.0.1: The cavity resonance signal in frequency domain, zoomed into a single cavity resonance. We observe that the difference between the sidebands and the carrier signal is ~ 10 MHz, which indicates that we have correctly converted our axes.

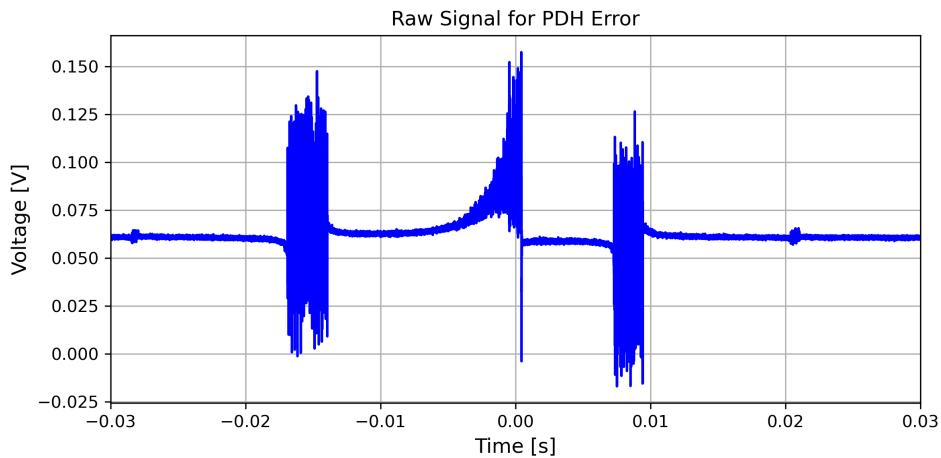


Figure 7.0.2: The PDH signal obtained from subtracting the contribution from the laser input frequency. This allows the error signal to look similar to that from Fig. 2.2.2. This signal was then used to determine the linear fit of the error signal.

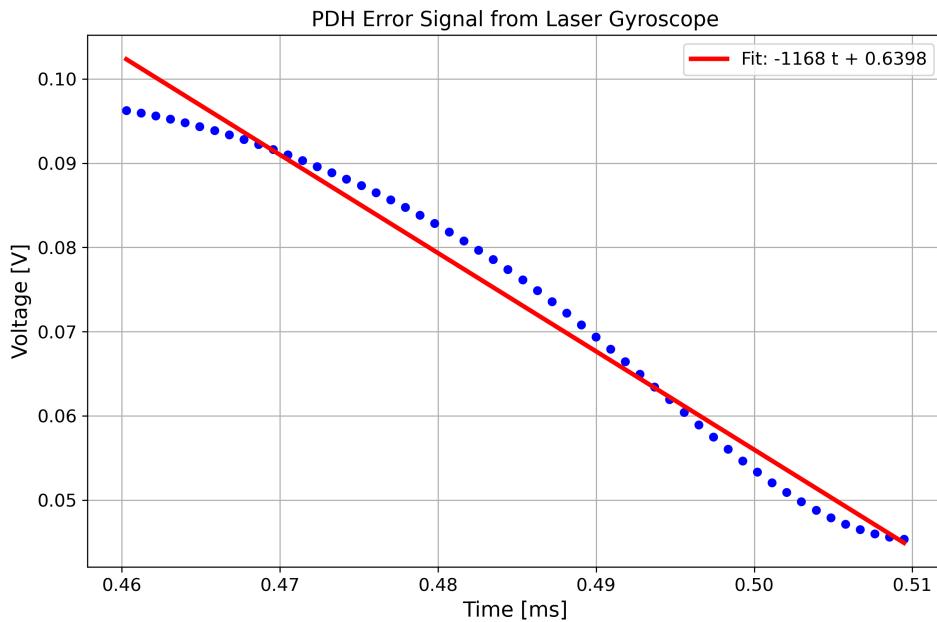


Figure 7.0.3: The PDH error signal, zoomed into a region in which the linear fit was performed.

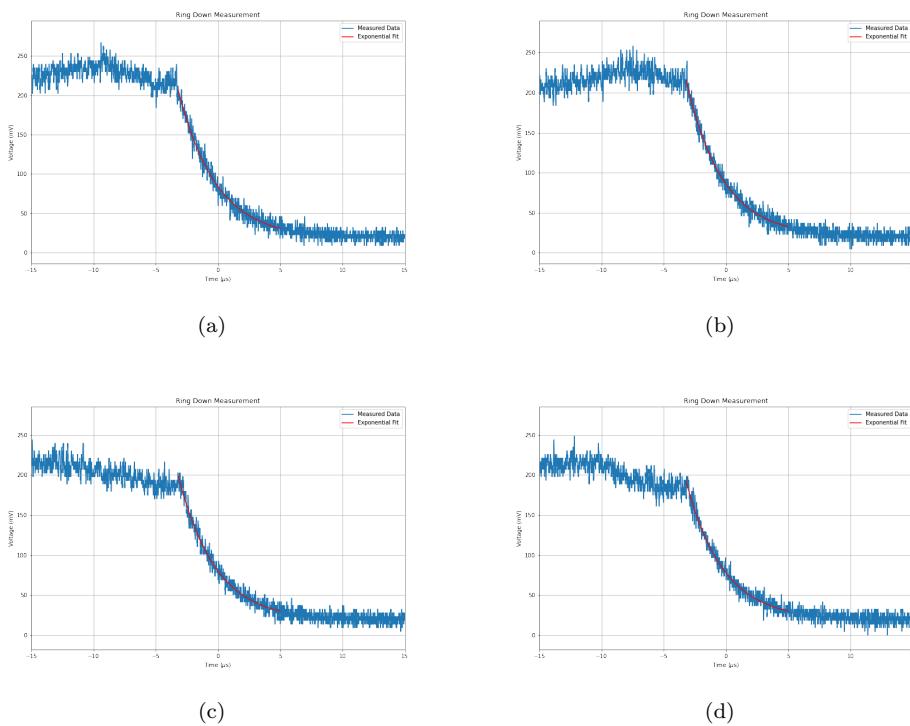
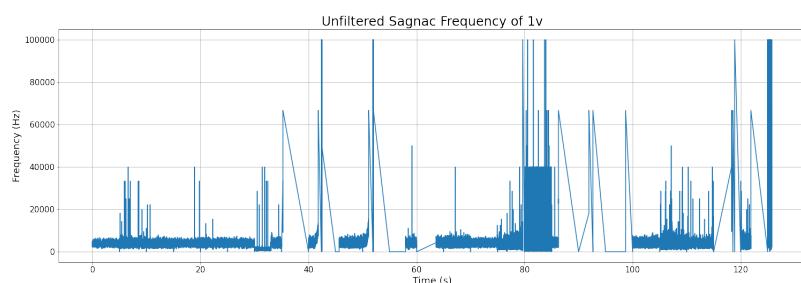
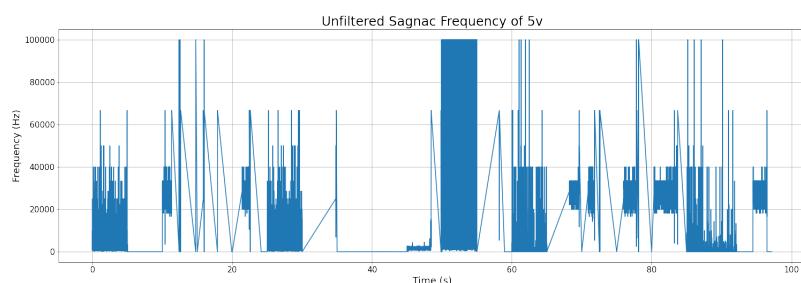


Figure 7.0.4: Additional plots showing the exponential decay, with fit, in the ring down technique.

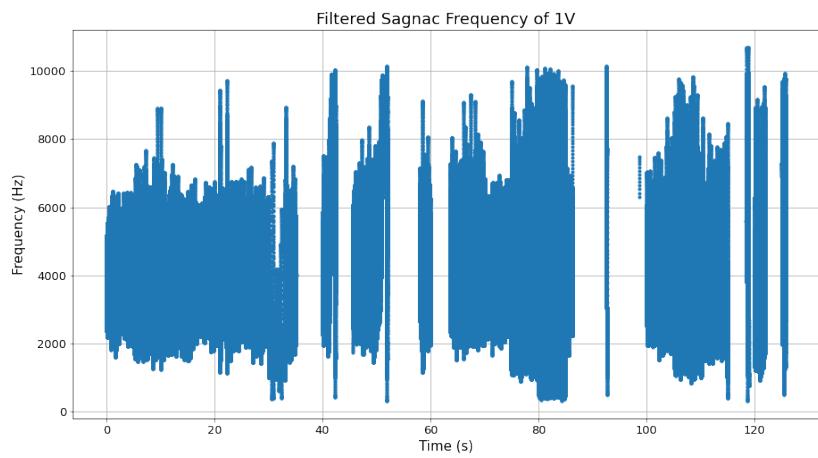


(a)

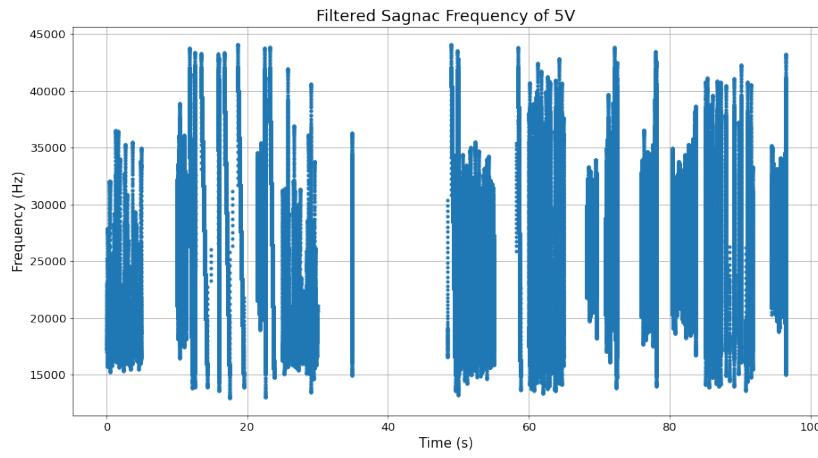


(b)

Figure 7.0.5: Unfiltered Sagnac frequency corresponding to 1V and 5V being applied to the rotation table.



(a)



(b)

Figure 7.0.6: Filtered Sagnac frequency corresponding to 1V and 5V being applied to the rotation table.

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