A249: Laser Gyroscope

April 4, 2022

In here we will present the tasks that we have to complete before conducting the lab.

1 Getting Started with Gyroscopes

We downloaded the `phyphox` app made by RWTH Aachen University, and we played around with the Gyroscope function. We rotated the phone in several directions to observe the relationship between the orientation of the phone and the corresponding coordinates used in the application. This is shown in Fig. 1. Fig. 2 - 4 show the corresponding time series for the x, y, z coordinates for each rotation that we performed.

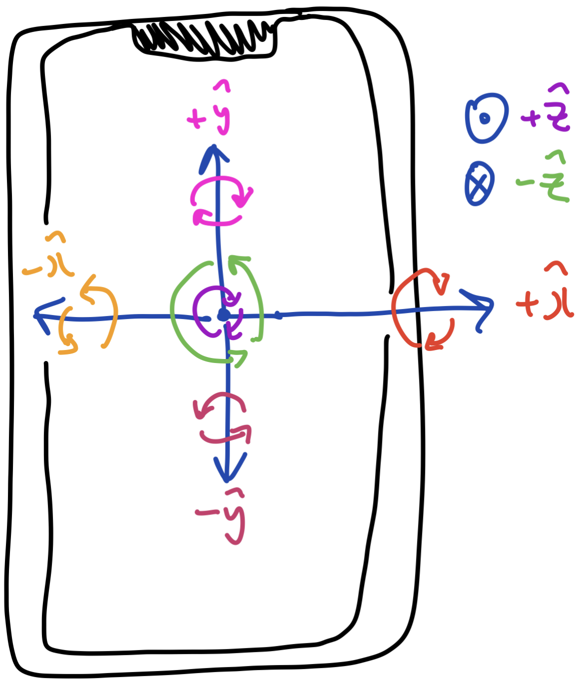
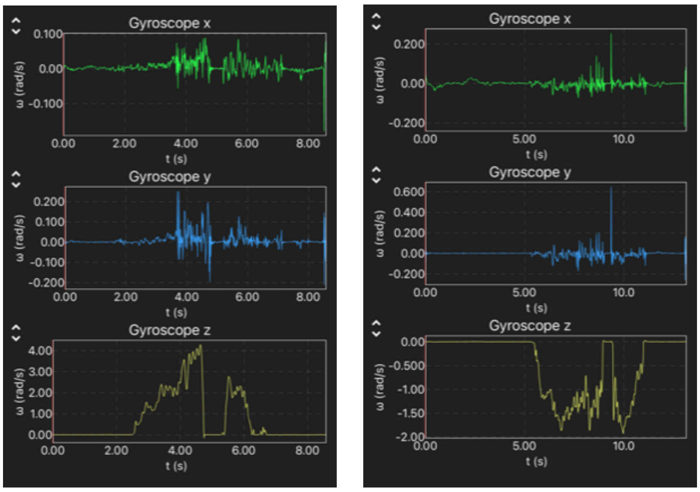


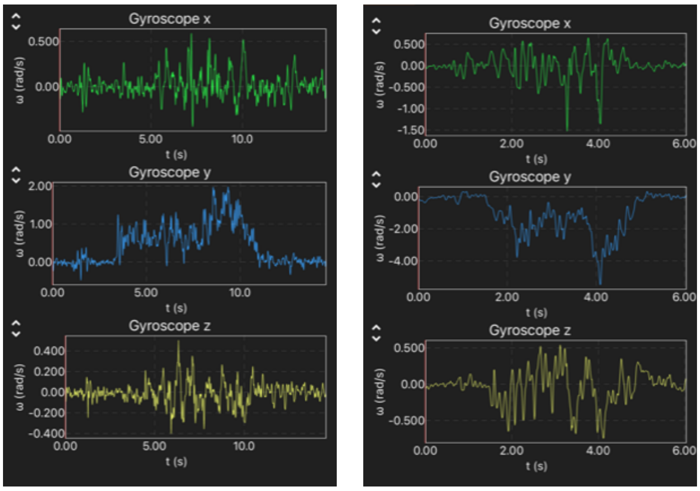
Figure 1: Sketch of the phone used to measure the gyroscope with its relevant x, y, z coordinates and sense of rotation. The sense of rotation is color coded with each rotation axis, and the direction of the axis is indicated by the blue arrows.



(a) $+z$ direction, anti-clockwise

(b) $-z$ direction, clockwise

Figure 2: The time series of the x, y, z coordinates shown from the `phyphox` application when the phone was rotated parallel to the surface. This corresponds to rotations in the z-direction. Note the large amplitudes in the time series, which indicates rotations in the relevant axis.

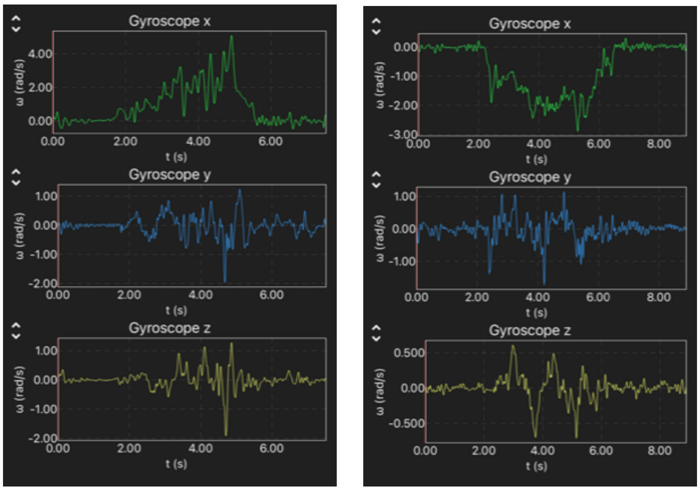


(a) $+y$ direction, into surface (anti-clockwise)

(b) $-y$ direction, out of surface (clockwise)

Figure 3: Same as Fig. 2 but for rotations into / out of the surface, corresponding to rotations in y-direction.

To save the rotation rates and import it into the computer for further analysis, the `Export Data` feature can be utilized. This will save the data as a `.zip` file that contains the raw data, the software and specifics regarding the device used, and the system time in which the experiment was started and finished. All such files are saved as `.csv` formats. An example for the raw data is shown in the Appendix section.

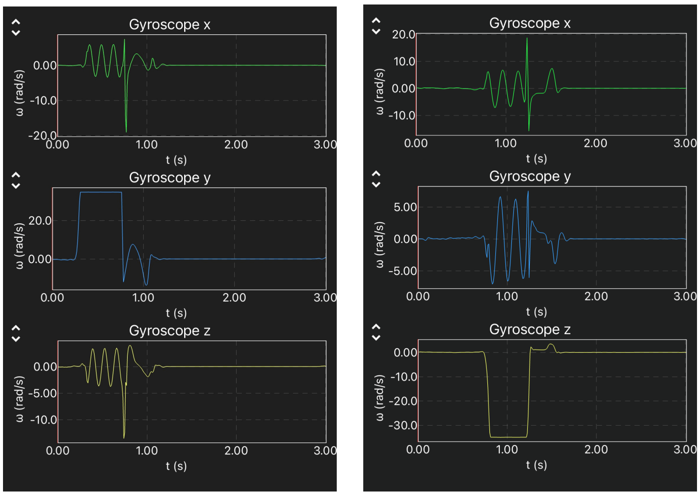


(a) $+x$ direction, rotation towards user (anti-clockwise)

(b) $-x$ direction, rotation away from user (clockwise)

Figure 4: Same as Fig. 2 but for rotations towards / away the user, corresponding to rotations in x-direction.

To obtain a faster rotation rate, we applied maximal torque on each end of the phone such that the rotation at each axis was maximal. This was done at an adequate height to ensure the rotation rate was properly measured. Several cushions were placed on top of a bed to ensure that the phone does not break. Fig. 5 show the time series of the x, y, z coordinates in the $+y$ and $-z$ directions. Performing the measurement in such a way yields a more notable and stable measurement of the rotation rate



(a) $+y$ direction

(b) $-z$ direction

Figure 5: Same as Fig. 2 but for fast rotations in the (a) $+y$ and (b) $-z$ direction.

2 Allan Deviation

We start off by keeping the phone flat on the ground for the duration of an hour and record the accelerometer of the phone. In Fig. 6, we see the raw data for individual axis of rotation.

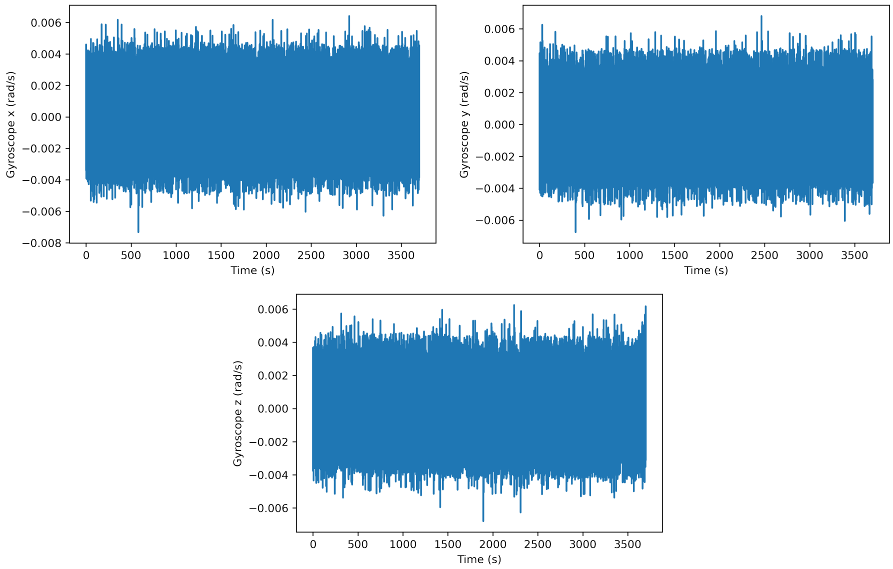


Figure 6: Raw data of all the three axes of gyroscope from phone.

From these figures, it is difficult to come to any conclusion because all we see in random noise. To analyze it further, we will take the Fourier Transoform. For plotting the data in the frequency domain, we use the `numpy` tool from the `Python`.

In Fig. 8, we see that in the x-axis Fourier transform, there is a noticeable peak at approximately 0.19 Hz. This could be a small disturbance that might have been caused by some random movement. But it would have to be periodic enough to be noticeable. It also could be because of the rotation of earth but we would have to investigate this further.

For computing the Allan deviation for all of these plots, the function `adev()` was used from the Python package, `allantools`.

We notice that around 700s, the drift becomes larger than the stability, and such indicates the shot-noise time interval. We also do a fit with a slope of -0.5 in order to see this better. Since the Allan deviation is used to quantify the stability of the system over a period of time, we use the value of 700s for doing the measurements for the next part of the homework. To calculate the shot noise limited sensitivity, $\mathcal{A}$, we use the formula:

$$\sigma_{ad}(\tau) = \frac{\mathcal{A}}{\sqrt{\tau}} \tag{1}$$

Using the fact that we plot the Allan deviation with log scale, we solve the Eqn. 1 to find out that the exponential of the y-intercept should give us the value of $\mathcal{A}$. Because we do not have any readings at the y-intercept, we take the average of the errors for all values of τ and calculate the error. The results are summarized in the table 1.

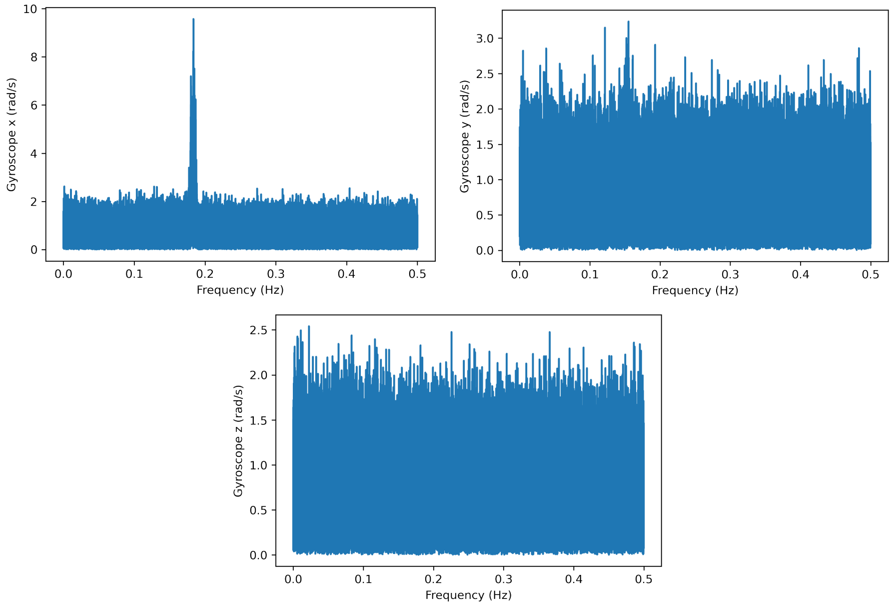


Figure 7: Fourier Transform of all 3 axes of the raw data.

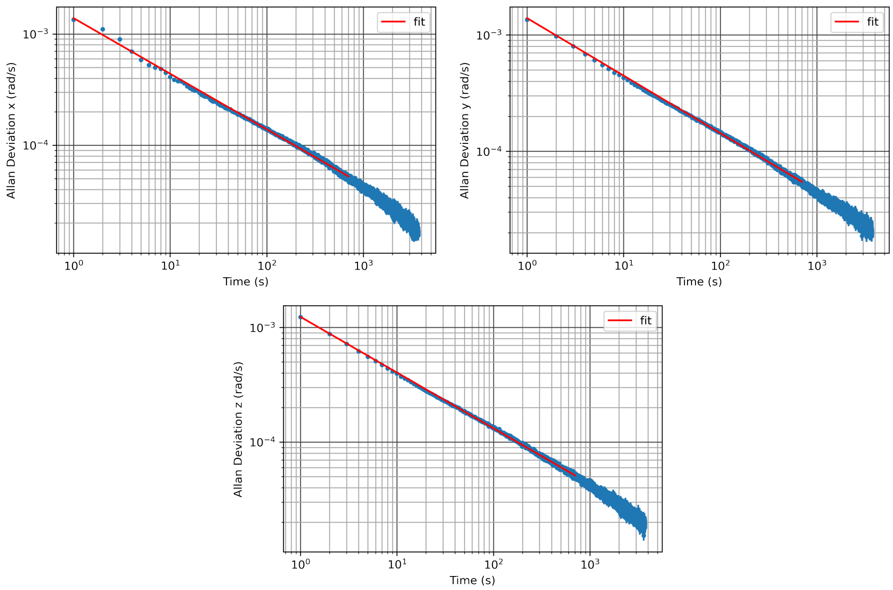


Figure 8: Allan deviation of 3 axes of rotation.

3 Earth Rotation Rate

To determine the rotation rate of the Earth, we utilize the same method as we have performed to measure the Allan deviation, but we perform this for the shot-time limited interval derived by the section above.

A_x	0.001389 ± 0.000004047
A_y	0.001394 ± 0.000004203
A_z	0.001238 ± 0.000003919

Table 1: Shot noise limited sensitivity for different components

We first measure the gyroscope x, y, z coordinates for the duration of the shot-time interval of $\tau = 700$s. After the data was recorded, we performed the same measurement but with the phone flipped upside down. This corresponds to a flip in the y direction. We then performed the same process but flipping in the z-direction instead (i.e. rotations parallel to the surface). This was performed for a shorter duration of $\tau = 300$s due to time constraints. The corresponding time series for both results are shown in Fig. 9, 10, 11 and 12.

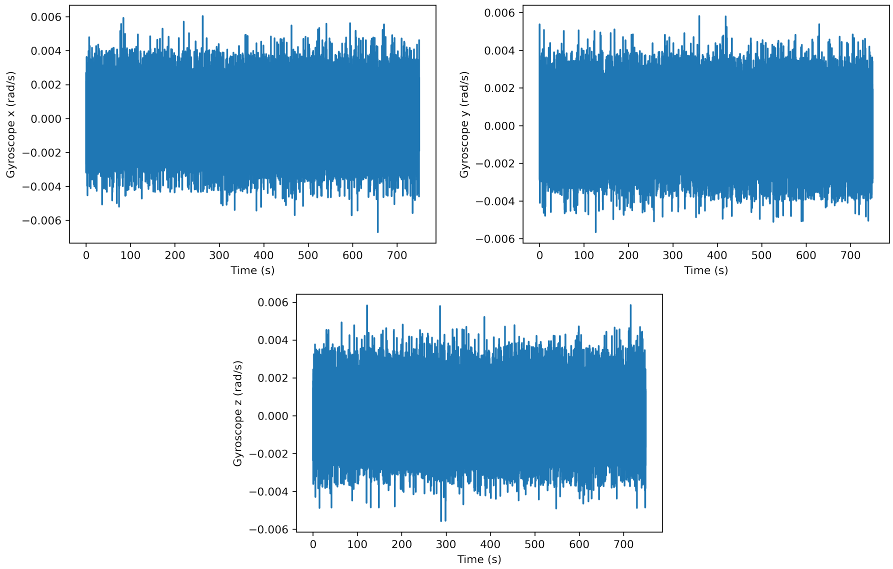


Figure 9: Raw data of all the three axes of gyroscope from phone for measurement of the Earth's rotation. The measurement was took with the phone face up.

After performing the FFT onto the resulting time series, we observed the sharp peak in the gyroscope x-direction. This shows that there is a clear bias in the x-direction of the phone gyroscope.

The temporal average of each measurement (both flipped and unflipped) was taken, and the difference between the unflipped and flipped measurements were taken (normalized by 2). For example, for the x coordinate we have:

$$\delta \bar{\omega_x} = \frac{|\bar{x}_{noflip} - \bar{x}_{flip}|}{2} \tag{2}$$

Such measurements for both rotations are tabulated on Table 2. The magnitude of the values are also evaluated. Note that for arbitrarily small differences, the deviations may be small and thus to properly normalize this, one needs to take into arround the uncertainty in the measurement as well.

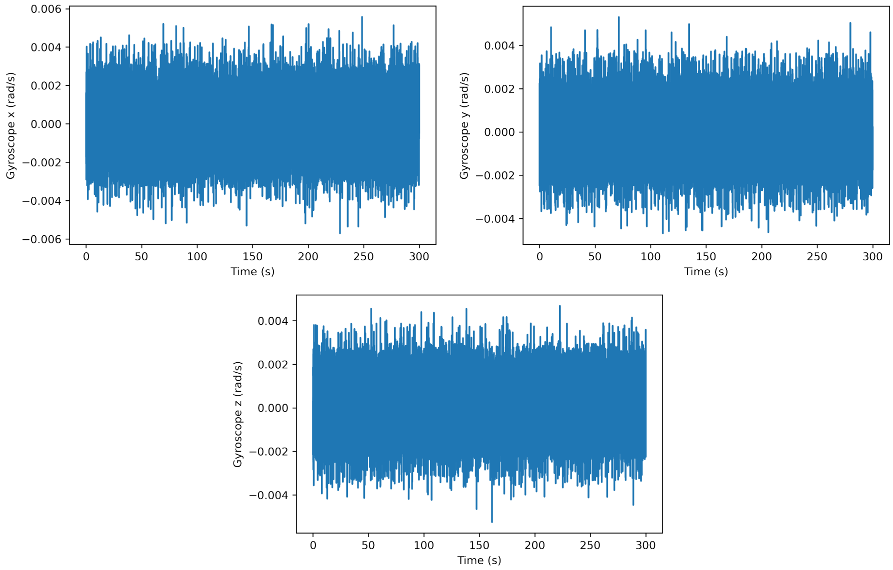


Figure 10: Same as Fig. 9 but for phone face down instead.

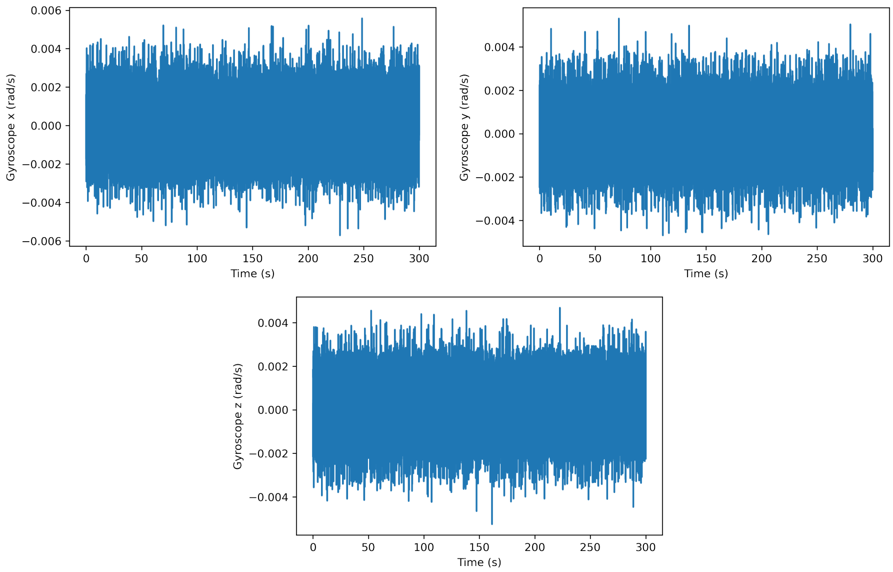


Figure 11: Same as Fig. 9 but for rotations in the z-direction. This was performed with the phone facing towards the South Pole.

Furthermore, to properly account for the Earth's rotation rate, we need to transform from the local frame in which we observe the phone's orientation in. This can be done by a simple coordinate transformation as such: $\omega = \Omega_E \cos\phi$, where ϕ is the geographic latitude. With $\phi = 39.257°$ (defined from the North Pole), we obtain our corrected rotation rate of the Earth as $\Omega_E = 3.083\mu\text{rads}^{-1}$ and $\Omega_E = 38.80\mu\text{rads}^{-1}$ for the up/down and North/South flip case

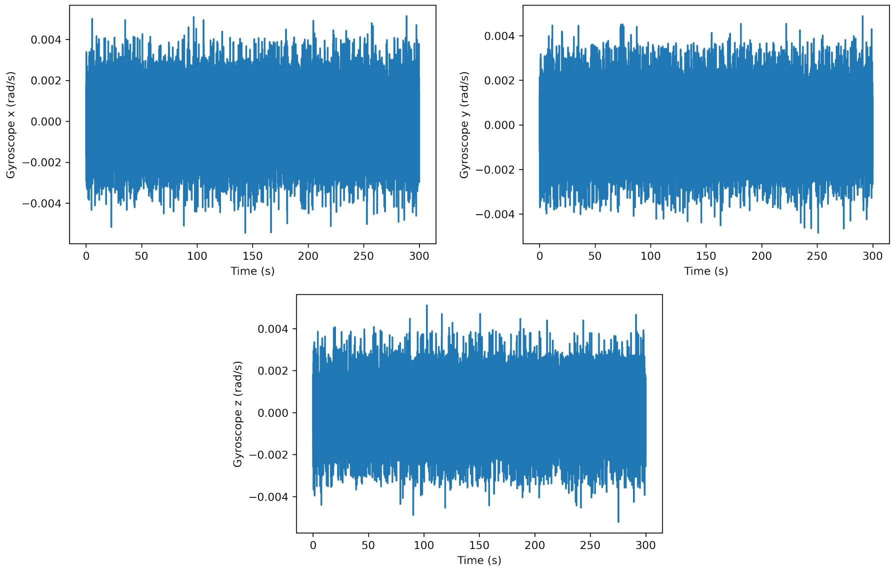


Figure 12: Same as Fig. 11 but for those facing towards the North Pole instead.

Coordinate	$\delta\bar{\omega}_i$, face up/down (rad/s)	$\delta\bar{\omega}_i$, face North/South (rad/s)
x	9.273×10^{-8}	9.956×10^{-6}
y	4.603×10^{-7}	2.814×10^{-5}
z	2.340×10^{-6}	3.357×10^{-6}
ω	2.386×10^{-6}	3.004×10^{-5}

Table 2: Difference of temporal average of rotation rate for each gyroscopic coordinate and its magnitude.

respectively.

When comparing the values of the rotation rate to the analytical value $\Omega_{E,thr} = 72.92\mu\text{rad}s^{-1}$, both yielded values are underestimated. However, the rate where the phone was flipped upside down yielded lower values compared to that when we flipped the phone from South to North. This indicates a clear bias in the rotation and how it impacts the measurement, which is clear due to the East-West rotation of the Earth.

Furthermore, the underestimation of the values are clear due to the various systematical uncertainties that need to be accounted for. The movement of nearby people, the wind, the temperature differences can all impact the phone gyroscope in some way. The MEMS gyroscope does not have as many tools that will calibrate for such deviations as compared to our experimental setup.