E106 Cavities

March 15, 2022

1 Cavities Assignment

The cylindrical cavity has an inner diameter of 78.5mm and length of 20mm. From the lab script, we have the formula for resonant frequency as:

$$(d\nu)^2 = \left(\frac{cj'^{(')}_{mn}}{\pi}\right)^2 + \left(\frac{c}{2}\right)^2 p^2 \left(\frac{d}{l}\right)^2 \tag{1}$$

Here $d = 2 \cdot a$ is the diameter of the cavity and $j'^{(')}_{mn}$ denotes the zero point Bessel function or its derivative. For TM modes it is zero point and for the TE modes, it is the derivative. So we have, $(d/l)^2 = 15.405625$. Therefore, from the modemap, the ten lowest eigenmodes are:

1. $TM_{010} = 0.5\text{e+}17$
   $\nu = 2.848GHz$
2. $TM_{110} = 1.3\text{e+}17$
   $\nu = 4.593GHz$
3. $TM_{210} = 2.4\text{e+}17$
   $\nu = 6.240GHz$
4. $TM_{020} = 2.8\text{e+}17$
   $\nu = 6.740GHz$
5. $TE_{111} = 3.6\text{e+}17$
   $\nu = 7.643GHz$
6. $TM_{310} = 3.7\text{e+}17$
   $\nu = 7.748GHz$
7. $TM_{011} = 3.9\text{e+}17$
   $\nu = 7.955GHz$
8. $TE_{211} = 4.3\text{e+}17$
   $\nu = 8.353GHz$
9. $TM_{120} = 4.5\text{e+}17$
   $\nu = 8.545GHz$
10. $TM_{111}/TE_{011} = 4.7\text{e+}17$
    $\nu = 8.733GHz$

Now, we will calculate the resonant frequencies using Eq.1 and explicit values of the zeros of the Bessel function and its derivative below.

Cavities Assignment Code

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[1]:
```
import numpy as np
from scipy.special import jn_zeros, jnp_zeros

d = 78.5e-3 #m
l = 20e-3 #m
c = 3e8  # m

def res_freq(m, n, p, mode="TM", Nzeros=5):

    jmn = jn_zeros(m,Nzeros)[n-1] if mode == "TM" else jnp_zeros(m,Nzeros)[n-1]

    return np.sqrt((c * jmn / np.pi)**2. + (c * p * d / (2 * l))**2) / d
```

[2]:
```
# ten lowest eigenmodes, listing (m,n,p,mode)
modes = [(0,1,0,"TM"), (1,1,0,"TM"), (2,1,0,"TM"), (0,2,0,"TM"),(1,1,1,"TE"),
 ↪(3,1,0,"TM"),(0,1,1,"TM"),(2,1,1,"TE"),(1,2,0,"TM"),(1,1,1,"TM"),(0,1,1,"TE")]

freqs = []

for mode in modes:
    freq = res_freq(*mode)
    freqs.append(freq)

    print("m,n,p: ({0}, {1}, {2}), mode:{3}, freq:{4:.4f}GHz".format(*mode,␣
 ↪freq*1e-9))
```

```
m,n,p: (0, 1, 0), mode:TM, freq:2.9254GHz
m,n,p: (1, 1, 0), mode:TM, freq:4.6612GHz
m,n,p: (2, 1, 0), mode:TM, freq:6.2473GHz
m,n,p: (0, 2, 0), mode:TM, freq:6.7150GHz
m,n,p: (1, 1, 1), mode:TE, freq:7.8273GHz
m,n,p: (3, 1, 0), mode:TM, freq:7.7613GHz
m,n,p: (0, 1, 1), mode:TM, freq:8.0503GHz
m,n,p: (2, 1, 1), mode:TE, freq:8.3698GHz
m,n,p: (1, 2, 0), mode:TM, freq:8.5343GHz
m,n,p: (1, 1, 1), mode:TM, freq:8.8304GHz
m,n,p: (0, 1, 1), mode:TE, freq:8.8304GHz
```