MA 225 Problem Set 1: logic 1

exercises These problems don't require you to write proofs.

1. We will show that although it's nice to have lots of connectives, we don't actually *need* them all.
   (a) Express the following formulæ using only the symbols P, Q, $\sim$, and $\wedge$:
   $P \vee Q$, $P \Rightarrow Q$, $P \Leftrightarrow Q$
   $P \vee Q :\sim (\sim P \wedge \sim Q)$
   $P \Rightarrow Q :\sim (P \wedge \sim Q))$
   $P \Leftrightarrow Q :\sim (\sim P \wedge Q) \wedge \sim (P \wedge \sim Q)$
   (b) Express the following formulæ using only the symbols P, Q, $\sim$, and $\vee$:
   $P \wedge Q$, $P \Rightarrow Q$, $P \Leftrightarrow Q$
   $P \wedge Q :\sim (\sim (P \vee Q) \vee \sim (P \vee \sim Q) \vee \sim (\sim P \vee Q))$
   $P \Rightarrow Q :\sim P \vee Q$
   $P \Leftrightarrow Q :\sim ((\sim P \vee Q) \vee \sim (P \vee \sim Q))$

   (c) Express the following formulæ using only the symbols P, Q, $\sim$, and $\Rightarrow$
   $P \wedge Q$, $P \vee Q$, $P \Leftrightarrow Q$
   $P \wedge Q :\sim (P \Rightarrow \sim Q)$
   $P \vee Q : (\sim P \Rightarrow Q)$
   $P \Leftrightarrow Q : (P \Rightarrow \sim Q) \Rightarrow \sim (\sim P \Rightarrow (\sim Q \Rightarrow P))$

   (d) Explain why this means we only need $\sim$ and *one* of $\wedge$, $\vee$, and $\Rightarrow$.
   Because we have shown above that we can represent each symbol with a single symbol
2. Define the connective $\veebar$ so that $P \veebar Q$ is true exactly when exactly one of P and Q is true.
   (a) Make a truth table for $P \veebar Q$.

P	Q	P $\veebar$ Q
True	True	False
True	False	True
False	True	True
False	False	False

   (b) Show that $P \veebar Q$ is equivalent to $(P \vee Q) \wedge (\sim (P \wedge Q))$.$\veebar$ by definition means that while the logical statement means the statement has a single true statement, such as is possible with an or statement, it lacks the double truth as in a and statement. By making the statement as it is, it requires one of the arguments to be true but doesn't allow both.
   (c) Express $\sim (P \veebar Q)$ in terms of $\sim$, $\vee$, and $\wedge$.
   $\sim (\sim P \wedge Q) \wedge \sim (P \wedge \sim Q)$
3. Make a truth table for $P \veebar Q \veebar R$.

P	Q	R	P $\veebar$ Q	(P $\veebar$ Q) $\veebar$ R
True	True	True	False	True
True	True	False	False	False
True	False	True	True	True
True	False	Flase	True	True
False	True	True	True	False
False	True	False	True	True
False	False	True	False	True
False	False	False	False	False

4. For each of the following, identify the antecedent and the consequent. Then indicate whether the statement is true or false.

   ++Antecedent++ –consequent–

(a) –The Nile River flows east– only if ++64 is a perfect square++.
This statement is false, as the statement suggests the Nile flows east if 64 is a perfect square, which can be seen with 8^2.

(b) $++1+1=2++$ is sufficient for – 3>6 –.
This statement is false, as while 1+1=2, it states that this true fact is enough to determine $3 > 6$, which is false.

(c) If ++Euclid's birthday was April 2++, then –rectangles have four sides.–
This statement is true, as it does not say that this is the only time when rectangles have four sides, and since they have four sides, this statement is true.

(d) ++If squares have three sides++, then –triangles have four sides–.
This statement is true, as while triangles don't have four sides, this only applies when squares have three sides, which is false.

(e) Fish bite– only when ++the moon is full++.
This is false, as plenty of people catch fish during the day when the moon isn't even out.

(f) An indictment++ is necessary for –a conviction–.
This is true, as without an indictment, there would be nothing to convict the person of.

5. Consider each of the following sentences as you would understand them if you heard it on the street. Identify, for each sentence, the antecedent and the consequent.
   (a) –I will go to the store– ++unless it is raining++. On the condition that it is not raining, I will go to the store.

   (b) –The Dolphins will not make the playoffs– ++unless the Bears win all the rest of their games++. The only way the Dolphins will make the play Offs is given they win the rest of their games.

   (c) –You cannot go to the game– ++unless you do your homework first++. Doing your homework is required to go to the game.

   (d) –You won't win the lottery– ++unless you buy a ticket++. You need to buy a ticket in order to win the lottery.

6. In each of the previous problem's sentences, use a different conditional keyword to express the sentence. You may **not** use *if. . . then.* **Be sure your rephrasing agrees with your answer in the previous problem!**

7. Which of the following are tautologies? Which are contradictions? For each, give an explanation that uses a truth table **and** an explanation that does not use a truth table. (*Hint.* Try expressing in words what each says.)
   (a) $(\alpha \wedge \gamma) \vee [(\sim \alpha) \wedge (\sim \gamma)]$

α	γ	$(\alpha \wedge \gamma) \vee [(\sim \alpha) \wedge (\sim \gamma)]$
True	True	True
True	False	False
False	True	False
False	False	True

The expression is neither, as seen above, as all of the values aren't the same. This is because all the expression did was checked the and operator and it's inverse, or a $\Rightarrow$

(b) $\sim [P \wedge (\sim P)]$

P	$\sim [P \wedge (\sim P)]$
True	True
True	True

The expression is a Tautologie, as all of the inputs produce True. This expression is essentially expressing how the value is binary, with the value being either true or false.

(c) $(\Psi \wedge \Phi) \vee [(\sim \Psi) \vee (\sim \Phi)]$

Ψ	Φ	$(\Psi \wedge \Phi) \vee [(\sim \Psi) \vee (\sim \Phi)]$
True	True	True
True	False	True
False	True	True
False	False	True

The above expresion was a Tautologie, as all of the values were true regardless of input. The expression essentially checks if its inverse is true under or, which it is unless bothinput values are True, in which case the or matched with the inverse part checks to see if the is True, making the proposition always true.

(d) $[A \wedge B] \vee [A \wedge (\sim B)] \vee [(\sim A) \wedge B] \vee [(\sim A) \wedge (\sim B)]$

A	B	$[A \wedge B] \vee [A \wedge (\sim B)] \vee [(\sim A) \wedge B] \vee [(\sim A) \wedge (\sim B)]$
True	True	True
True	False	True
False	True	True
False	False	True

The proposition above is a tautologie because as seen above, regardless of input the result is True. This is because it surrounds each case with parentheses and uses an or to compare all of them, therefore making the statement always true.

8. Submit part 4 of the worksheet *Useful Logical Facts*.

proofs Write a complete proof for each of the following statements.

1. $\veebar$ is associative.
   Claim: $\veebar$ is an associative operator
   Proof: Let's consider the following truth table

1	2	3	4	5
P	Q	R	P $\veebar$ (Q $\veebar$ R)	(P $\veebar$ Q) $\veebar$ R
True	True	True	True	True
True	True	False	False	False
True	False	True	True	True
True	False	Flase	True	True
False	True	True	False	False
False	True	False	True	True
False	False	True	True	True
False	False	False	False	False

   As we can see from Columns 4 and 5, the shift in brackets created an identical equation, therefore the equations $P \veebar (Q \veebar R) and (P \veebar Q) \veebar R$ are equal, and the operator $\veebar$ is associative.

2. All the claims in the worksheet *Useful Logical Facts*.