Notes for 1/31/17

An inductive prood od a statement of a statement of the form $\forall \in N, P(n)$ goes as follows:

1. Prove P(1). (This is usually pretty obvious)
2. prove that $\forall n, (P(n) \Rightarrow P(n+1)$
   (a) Let n be any natural number
   (b) Assume P(n) is true
   (c) ...Juicy bits...
   (d) Conclude P(n+1) is true

claim 1 *The square of any odd number has a for 8k+1 for some interger k.*
Claim: (Case when the off number is at least 3)

Proof By Induction 1 $\forall n \in N \exists k : (2n+1)^2 = 8k+1$
We proceed by induction.
Base Case (n=1) Use k=1. Then $(2*1+1)^2 = 9 = 8k+1$
Induction step: Let n be a natural number. Assume $\exists k : (2n+1)^2 = 8k+1$.
(Scratch work)
$P(n) : \exists k : (2n+1)^2 = 8k+1$
$P(1) :$
$P(n) \Rightarrow P(n+1)$
$P(n+1) : \exists l : (2(n+1)+1)^2 = 8l+1$
(End Scratchwork)

$$\begin{aligned}
(2(n+1)+1)^2 &= (2n+2+1)^2 \\
&= ((2n+1)^+2)^2 \\
&= (2n+1)^2) + 4 + 4(2n+1) \\
&= 8k+1+4(2n+1+1) \\
&= 8k+1+4(2n+2) \\
&= 8k+1+8(n+1) \\
&= 8)k+n+1)+1 \\
&= 8l+1
\end{aligned}$$

Therefore, through mathematical induction, the claim is true.

claim 2 *For any* $n \in N, \frac{n^3}{3} + \frac{n^5}{5} + \frac{7n}{15}$ *is an integer.*

Proof By Induction 2 *NTS:* $\frac{1}{3} + \frac{1}{5} + \frac{1}{5} + \frac{7}{15} = 1$
Inductive Step: Let $n \in N$ *Assume* $\frac{n^3}{3} + \frac{n^5}{5} + \frac{7n}{15}$ *is an Integer.*
WE'll show $\frac{(n+1)^3}{3} + \frac{(n+1)^5}{5} + \frac{7(n+1)}{15}$ *is an integer.*

$$+7n+1\overline{}_{15=\frac{n^3}{3}+\frac{n^5}{5}+\frac{7n}{15}+n^2+n+n^4+2n^3+2n^2+n+\frac{1}{3}+\frac{1}{5}+\frac{7}{15}}$$

This is an Integer. By induction, this completes the proof.

claim 3 *Any finite collection of real numbers has a greatest element.*
For any $n \in N$, *any set of exactlty n real numbers has a greatest element.*

Proof By Induction 3 *(By induction on the size of set n)*
Base case n=1. Letn be a set of exactly 1 real number.

Let a be Franklin single element, then a is the greatest.
Inductive step. Assume that any set of exactly n distinct real numbers has a greatest element.
We'll show: any collection of exactly n+!distinct real numbers has a greatest element
Let B be a set of exactly n+1 distinct real numbers.
Let b be one of them. Let B' be B, but with b removed. Then B' has exactly n distinct elements.
So B' has a greatest element, Aisha.
Case: b¡Aisha Aisha is greatest in B
Case: Aisha¡b. b is greatest in B.