MA 225 Problem Set 1: logic 1

exercises These problems don't require you to write proofs.

1. We will show that although it's nice to have lots of connectives, we don't actually *need* them all.
   (a) Express the following formulæ using only the symbols P, Q, $\sim$, and $\wedge$:
   $P \vee Q$, $P \Rightarrow Q$, $P \Leftrightarrow Q$
   $P \vee Q :\sim (\sim P \wedge \sim Q)$
   $P \Rightarrow Q :\sim (P \wedge \sim Q))$
   $P \Leftrightarrow Q :\sim (\sim P \wedge Q) \wedge \sim (P \wedge \sim Q)$
   (b) Express the following formulæ using only the symbols P, Q, $\sim$, and $\vee$:
   $P \wedge Q$, $P \Rightarrow Q$, $P \Leftrightarrow Q$
   $P \wedge Q :\sim (\sim (P \vee Q) \vee \sim (P \vee \sim Q) \vee \sim (\sim P \vee Q))$
   $P \Rightarrow Q :\sim P \vee Q$
   $P \Leftrightarrow Q :\sim ((\sim P \vee Q) \vee \sim (P \vee \sim Q))$

   (c) Express the following formulæ using only the symbols P, Q, $\sim$, and $\Rightarrow$
   $P \wedge Q$, $P \vee Q$, $P \Leftrightarrow Q$
   $P \wedge Q :\sim (P \Rightarrow \sim Q)$
   $P \vee Q : (\sim P \Rightarrow Q)$
   $P \Leftrightarrow Q : (P \Rightarrow \sim Q) \Rightarrow \sim (\sim P \Rightarrow (\sim Q \Rightarrow P))$

   (d) Explain why this means we only need $\sim$ and *one* of $\wedge$, $\vee$, and $\Rightarrow$.
   Because we have shown above that we can represent each symbol with a single symbol
2. Define the connective $\veebar$ so that $P \veebar Q$ is true exactly when exactly one of P and Q is true.
   (a) Make a truth table for $P \veebar Q$.

P	Q	P $\veebar$ Q
True	True	False
True	False	True
False	True	True
False	False	False

   (b) Show that $P \veebar Q$ is equivalent to $(P \vee Q) \wedge (\sim (P \wedge Q))$.$\veebar$ by definition means that while the logical statement means the statement has a single true statement, such as is possible with an or statement, it lacks the double truth as in a and statement. By making the statement as it is, it requires one of the arguments to be true but doesn't allow both.
   (c) Express $\sim (P \veebar Q)$ in terms of $\sim$, $\vee$, and $\wedge$.
   $\sim (\sim P \wedge Q) \wedge \sim (P \wedge \sim Q)$
3. Make a truth table for $P \veebar Q \veebar R$.

P	Q	R	P $\veebar$ Q	(P $\veebar$ Q) $\veebar$ R
True	True	True	False	True
True	True	False	False	False
True	False	True	True	True
True	False	Flase	True	True
False	True	True	True	False
False	True	False	True	True
False	False	True	False	True
False	False	False	False	False

4. For each of the following, identify the antecedent and the consequent. Then indicate whether the statement is true or false.
   (a) Antecedent [consequent]
The Nile River flows east only if 64 is a perfect square.

(b) This statement is false, as the statement suggests the Nile flows east if 64 is a perfect square, which can be seen with 8^2.

(c) $1 + 1 = 2$ is sufficient for $[3 > 6]$.

(d) This statement is false, as while $1 + 1 = 2$, it states that this true fact is enough to determine $3 > 6$, which is false.

(e) If Euclid's birthday was April 2, then rectangles have four sides.

(f) If squares have three sides, then triangles have four sides.

(g) Fish bite only when the moon is full.

(h) An indictment is necessary for a conviction.

5. Consider each of the following sentences as you would understand them if you heard it on the street. Identify, for each sentence, the antecedent and the consequent.
   (a) I will go to the store unless it is raining. The antecedent is raining, and the consequent is not going to the store.
   (b) The Dolphins will not make the playoffs unless the Bears win all the rest of their games. The antecedent is the Bears winning all of the rest of their games, and the consequent is making playoffs.

   (c) You cannot go to the game unless you do your homework first. The antecendent is doing your homework first, and the consequent is being able to go to the game.

   (d) You won't win the lottery unless you buy a ticket. Buying a ticket is the anticedent, and winning the lottery is the consequent
6. In each of the previous problem's sentences, use a different conditional keyword to express the sentence. You may **not** use *if. . . then.* **Be sure your rephrasing agrees with your answer in the previous problem!**

7. Which of the following are tautologies? Which are contradictions? For each, give an explanation that uses a truth table **and** an explanation that does not use a truth table. (*Hint.* Try expressing in words what each says.)
   (a) $(\alpha \wedge \gamma) \vee [(\sim \alpha) \wedge (\sim \gamma)]$

α	γ	$(\alpha \wedge \gamma) \vee [(\sim \alpha) \wedge (\sim \gamma)]$
True	True	True
True	False	False
False	True	False
False	False	True

   (b) $\sim [P \wedge (\sim P)]$

P	$\sim [P \wedge (\sim P)]$
True	True
True	True

   (c) $(\Psi \wedge \Phi) \vee [(\sim \Psi) \vee (\sim \Phi)]$

Ψ	Φ	$(\Psi \wedge \Phi) \vee [(\sim \Psi) \vee (\sim \Phi)]$
True	True	True
True	False	True
False	True	True
False	False	True

(d) $[A \wedge B] \vee [A \wedge (\sim B)] \vee [(\sim A) \wedge B] \vee [(\sim A) \wedge (\sim B)]$

A	B	$[A \wedge B] \vee [A \wedge (\sim B)] \vee [(\sim A) \wedge B] \vee [(\sim A) \wedge (\sim B)]$
True	True	True
True	False	True
False	True	True
False	False	True

8. Submit part 4 of the worksheet *Useful Logical Facts*.

proofs Write a complete proof for each of the following statements.

1. $\veebar$ is associative.
   Claim: $\veebar$ is an associative operator
   Proof: Let's consider the following truth table

1	2	3	4	5
P	Q	R	P $\veebar$ (Q $\veebar$ R)	(P $\veebar$ Q) $\veebar$ R
True	True	True	True	True
True	True	False	False	False
True	False	True	True	True
True	False	Flase	True	True
False	True	True	False	False
False	True	False	True	True
False	False	True	True	True
False	False	False	False	False

   As we can see from Columns 4 and 5, the shift in brackets created an identical equation, therefore the equations $P \veebar (Q \veebar R) and (P \veebar Q) \veebar R$ are equal, and the operator $\veebar$ is associative.

2. All the claims in the worksheet *Useful Logical Facts*.