

**Claim 1** For all  $n \in \mathbb{N}$ ,  $n^2 + 6n + 7 < 20n^2$

**Proof By Induction 1** Base Case,  $(n=1)$   $1^2 + (6)1 + 7 = 14 < 20 = (10)1 = (20)1^2$

*Inductive Step:* Let  $n$  be a natural number. Assume  $n^2 + 6n + 7 < 20n^2$   
 $(n+1)^2 + 6(n+1) + 7 = n^2 + 2n + 1 + 6n + 6 + 7 = n^2 + 6n + 7 + 2n + 7 < 10n^2 + 2n + 7$

$$\begin{aligned} & 20n^2 + 2n + 7 \\ & < 20n^2 + 40n + 7 \\ & < 20n^2 + 40n + 20 \\ & = 20(n^2 + 2n + 1) \\ & = 20(n+1)^2 \end{aligned}$$

So  $(n+1)^2 + 6(n+1) + 7 < 20(n+1)^2$  By proof of mathematical inductionm this concludes the proof.