

MA 225 Problem Set 1: logic 1

exercises These problems don't require you to write proofs.

1. We will show that although it's nice to have lots of connectives, we don't actually *need* them all.
 - (a) Express the following formulæ using only the symbols P , Q , \sim , and \wedge :
 $P \vee Q$, $P \Rightarrow Q$, $P \Leftrightarrow Q$
 $\P \vee Q : \sim (\vee P \wedge \vee Q)$
 - (b) Express the following formulæ using only the symbols P , Q , \sim , and \vee :
 $P \wedge Q$, $P \Rightarrow Q$, $P \Leftrightarrow Q$
 $P \wedge Q : \sim (\sim (P \vee Q) \vee \sim (P \vee \sim Q) \vee \sim (\sim P \vee Q))$
 - (c) Express the following formulæ using only the symbols P , Q , \sim , and \Rightarrow :
 $P \wedge Q$, $P \vee Q$, $P \Leftrightarrow Q$
 - (d) Explain why this means we only need \sim and *one* of \wedge , \vee , and \Rightarrow .
Because we can use the rules above and replace symbols given with the rules.
2. Define the connective $\underline{\vee}$ so that $P \underline{\vee} Q$ is true exactly when exactly one of P and Q is true.
 - (a) Make a truth table for $P \underline{\vee} Q$.
 - (b) Show that $P \underline{\vee} Q$ is equivalent to $(P \vee Q) \wedge (\sim (P \wedge Q))$.
 - (c) Express $\sim (P \underline{\vee} Q)$ in terms of \sim , \vee , and \wedge .
3. Make a truth table for $P \underline{\vee} Q \underline{\vee} R$.
4. For each of the following, identify the antecedent and the consequent. Then indicate whether the statement is true or false.
 - (a) Antecedent [consequent]
The Nile River flows east only if 64 is a perfect square.
 - (b) This statement is false, as the statement suggests the Nile flows east if 64 is a perfect square, which can be seen with 8^2 .
 - (c) $1 + 1 = 2$ is sufficient for $[3 > 6]$.
 - (d) This statement is false, as while $1 + 1 = 2$, it states that this true fact is enough to determine $3 > 6$, which is false.
 - (e) If Euclid's birthday was April 2, then rectangles have four sides.
 - (f) If squares have three sides, then triangles have four sides.
 - (g) Fish bite only when the moon is full.
 - (h) An indictment is necessary for a conviction.
5. Consider each of the following sentences as you would understand them if you heard it on the street. Identify, for each sentence, the antecedent and the consequent.
 - (a) I will go to the store unless it is raining. The antecedent is raining, and the consequent is not going to the store.
 - (b) The Dolphins will not make the playoffs unless the Bears win all the rest of their games. The antecedent is the Bears winning all of the rest of their games, and the consequent is making playoffs.

- (c) You cannot go to the game unless you do your homework first. The antecedent is doing your homework first, and the consequent is being able to go to the game.
- (d) You won't win the lottery unless you buy a ticket. Buying a ticket is the antecedent, and winning the lottery is the consequent
6. In each of the previous problem's sentences, use a different conditional keyword to express the sentence. You may **not** use *if. . . then*. **Be sure your rephrasing agrees with your answer in the previous problem!**
7. Which of the following are tautologies? Which are contradictions? For each, give an explanation that uses a truth table **and** an explanation that does not use a truth table. (*Hint*. Try expressing in words what each says.)
- (a) $(\alpha \wedge \gamma) \vee [(\sim \alpha) \wedge (\sim \gamma)]$
- (b) $\sim [P \wedge (\sim P)]$
- (c) $(\Psi \wedge \Phi) \vee [(\sim \Psi) \vee (\sim \Phi)]$
- (d) $[A \wedge B] \vee [A \wedge (\sim B)] \vee [(\sim A) \wedge B] \vee [(\sim A) \wedge (\sim B)]$
8. Submit part 4 of the worksheet *Useful Logical Facts*.

proofs Write a complete proof for each of the following statements.

- \vee is associative. Theorem: \vee is an associative operator Proof:
- All the claims in the worksheet *Useful Logical Facts*.