fysOblig2

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1 Oblig 2

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1.1 a) Show that the given V cannot exist in a vacuum

$$V(x,y,z) = A(x^2+y^2+z^2)$$

bruker at $\nabla^2 V = \frac{\rho}{\epsilon} = A(2+2+2)$

in a vacuum $\rho = 0$, so only way for $\nabla^2 V$ to be zero is if $A = 0$ and there is no potential

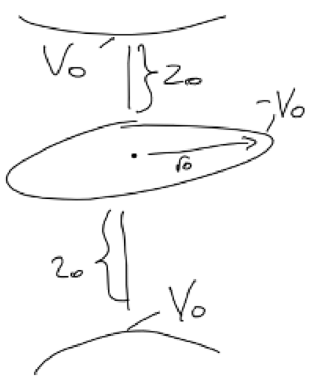
1.2 b) Find constants to make it possible

$$\nabla^2 V = A(2\alpha + 2\beta + 2\gamma) = 0$$

if $A \neq 0$, then $\alpha + \beta + \gamma = 0$

any choice of these that satisfies this equation

1.3 c) Find A



Ring $r=r_0$, $z=0$

$$V(r,z) = A(r^2 - 2z^2)$$

setter inn grensebetingelser og løser

$$V(r_0,0) = A r_0^2 = -V_0$$

$$A = -\frac{V_0}{r_0^2}$$

$$-\frac{V_0}{r_0^2}(r^2 - 2z^2) = \frac{V_0}{r_0^2}(2z^2 - r^2)$$

Tips of caps, $r=0$, $z=\pm z_0$

$$A(r^2 - 2z^2) = V_0$$

since it's z^2 it doesnt matter if

$z=-z_0$ or $z=z_0$

$$V(0,z_0) = -2z_0^2 A = V_0$$

$$A = -\frac{V_0}{2z_0^2}$$

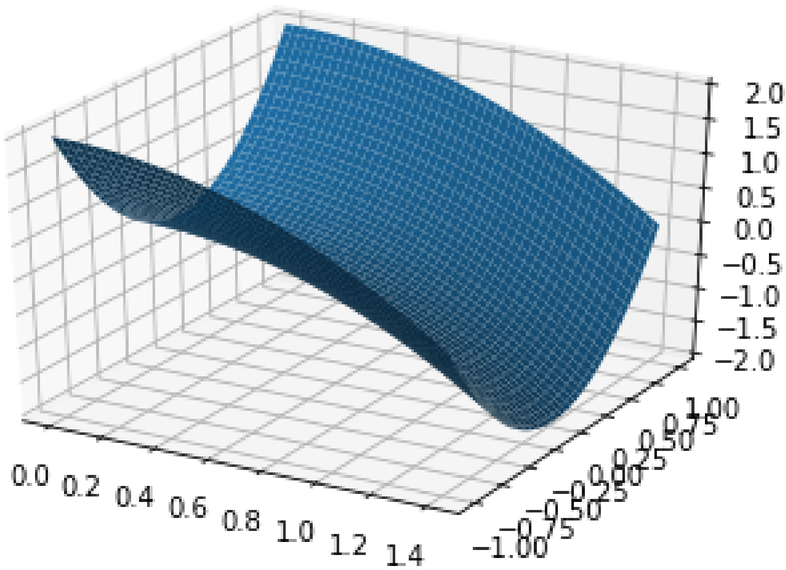
$$V_{caps} = \frac{V_0}{2z_0^2}(2z^2 - r^2)$$

if $A=A$ then we choose r_0 and z_0 so that $2z_0^2 = r_0^2$

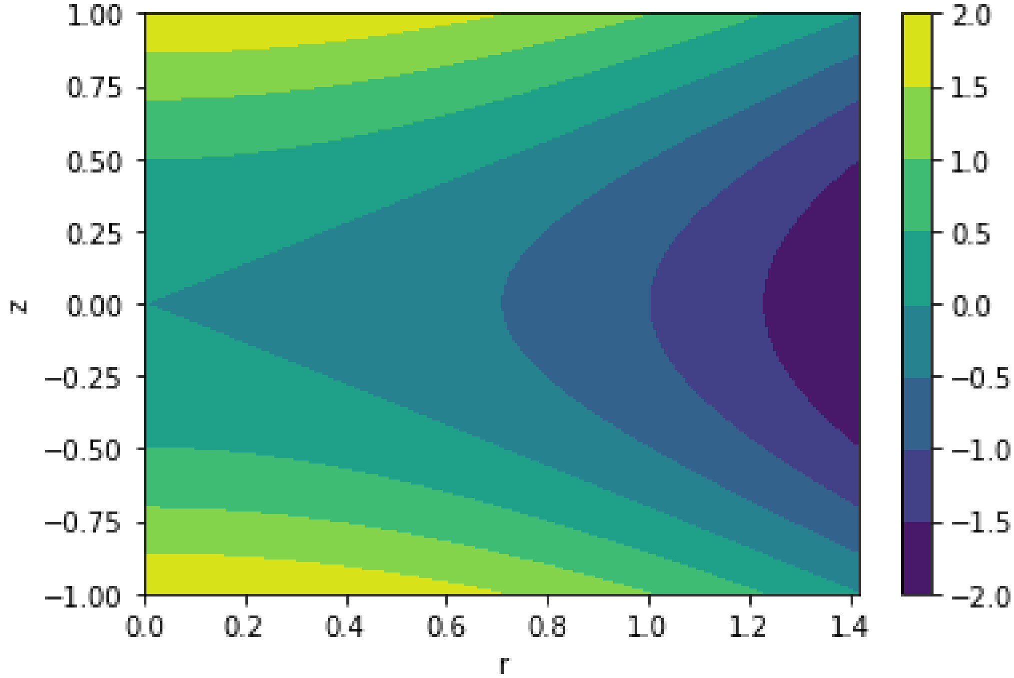
1.4 d) why we cant trap a positive particle

[2]:
```python
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
import numpy as np
z0 = 1
r0 = np.sqrt(2)*z0
r = np.linspace(0, r0, 1001)
z = np.linspace(-z0, z0, 1001)
r,z = np.meshgrid(r,z)
V = 2*z*z - r*r
# Plot a surface
fig = plt.figure()
ax = fig.gca(projection="3d")
```

```
surf = ax.plot_surface(r,z,V)
plt.show()
# Plot a contour plot
plt.contourf(r,z,V)
plt.xlabel("r")
plt.ylabel("z")
plt.colorbar()
```



[2]: <matplotlib.colorbar.Colorbar at 0x241a21c9e08>



The E field points from areas of higher potential to lower. A positively charged particle would move with the field. Lets look at r=0. We can see that increasing or decreasing Z would cause the particle to experience a net force in the other direction, accelerating it towards z=0. In vacuum it seems like this could be an infinite loop, moving it up and down. But anything other than r=0 would break that symmetry of potential and cause the particle to move to the right(on the graphs), towards r>0 and lower potential.

1.5 e) Find F

$$V(x,y,z) = \frac{V_0}{r_0^2}\left(2z^2 - x^2 - y^2\right)$$

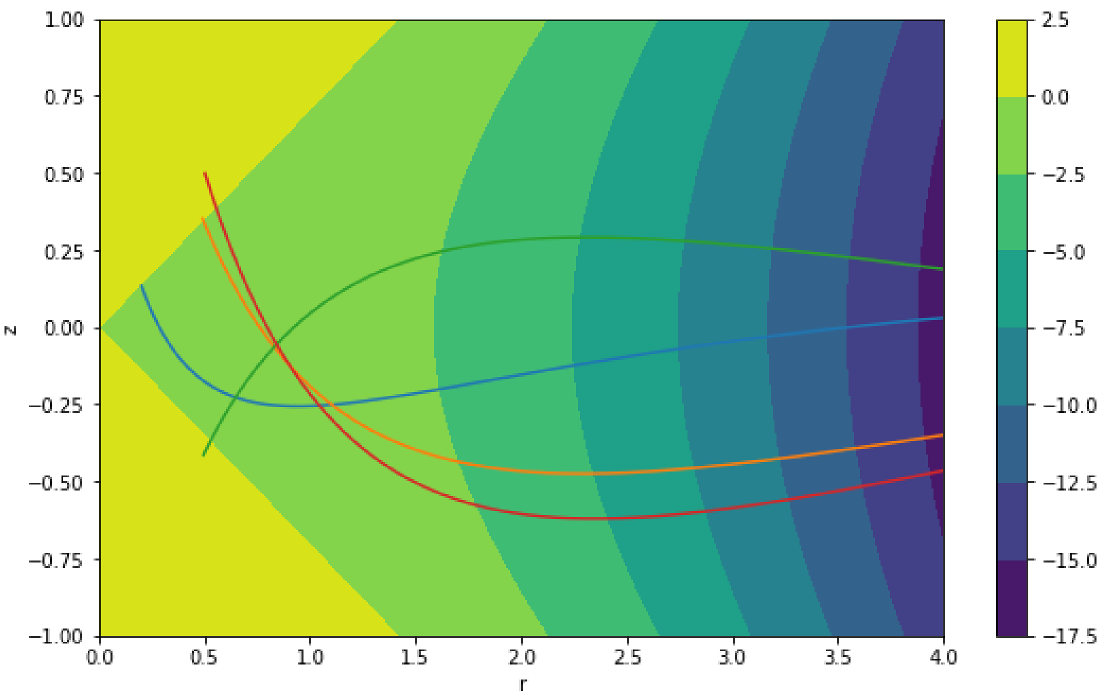
$$\vec{E} = -\nabla V = \frac{-V_0}{r_0^2}\left(4z\hat{z} - 2x\hat{x} - 2y\hat{y}\right)$$

$$\vec{F} = q\vec{E} = \frac{2qV_0}{r_0^2}(x, y, -2z)$$

1.6 f) Plot simulated trajectories

[10]:
```
import numpy as np
import matplotlib.pyplot as plt


def particle_trajectory(Nsteps):
    g = np.array([0, 0, -9.81])
    m = 5e-5  # mass of grain of cinnamon
    V0 = 4000  # V
    q_per_m = 1e-4  # C/kg
    q = m * q_per_m
    z0 = 0.005  # Typical size of trap
    r0 = np.sqrt(2) * z0

    dt = 1e-5  # Timestep

    # Initialize trajectory
    r = np.zeros((Nsteps, 3))
    v = np.zeros((Nsteps, 3))
    t = np.zeros(Nsteps)
    r[0, :] = np.random.uniform(-0.5 * z0, 0.5 * z0, size=3)

    # Simulate motion
    for i in range(Nsteps - 1):
        a = 2 * q * V0 / (m * r0 ** 2) * np.array([r[i, 0], r[i, 1], -2 * r[i,␣
↪2]]) + g
        v[i + 1, :] = v[i, :] + a * dt
        r[i + 1, :] = r[i, :] + v[i + 1, :] * dt
        t[i + 1] = t[i] + dt
    return r, v, t, z0


#---------------------------
plt.figure(figsize=(10, 6))

for k in range(4):

    stepcount = 4000
    r, v, t, z0 = particle_trajectory(stepcount)
    r = r / z0

    # the plot goes from r=0 to r=4*z0 so here we filter out excess data
    newPoints = []

    for i in range(stepcount):
        rlen = np.sqrt(r[i, 0] ** 2 + r[i, 1] ** 2)
```

```
        if rlen <= 4:  # z0=1 in plots
            newPoints.append(r[i])

    newPoints = np.array(newPoints)

    rPoints = np.array(  # compute r values, = x**2 + y**2
        [
            np.sqrt(newPoints[i, 0] ** 2 + newPoints[i, 1] ** 2)
            for i in range(len(newPoints))
        ]
    )

    plt.plot(rPoints, newPoints[:, 2])  # plot(r, z)
#----------------------


z0 = 1
rs = np.linspace(0, 4 * z0, 1001)
zs = np.linspace(-z0, z0, 1001)
rs, zs = np.meshgrid(rs, zs)
Vs = 2 * zs * zs - rs * rs
plt.xlabel("r")
plt.ylabel("z")
plt.contourf(rs, zs, Vs)
plt.colorbar()
plt.show()
```



As explained, the positive particle is accelerated in the z-direction towards z=0 when z!=0. And in the r-direction towards more positive r's when r != 0. But the graph seems skewed overall and wrong for r>r0. The potential should go to 0 as r increases further, not become more and more negative. We see here that the expression for V isnt meant to be used outside the trap.

1.7 Improving the trap

1.8 g) h)

$$V(x,y,z,t) = \frac{V_0 \cos(\Omega t)}{r_0^2}(2z^2 - x^2 - y^2)$$

$$E = -\nabla V = -\frac{V_0 \cos(\Omega t)}{r_0^2}(4z\hat{z} - 2x\hat{x} - 2y\hat{y})$$

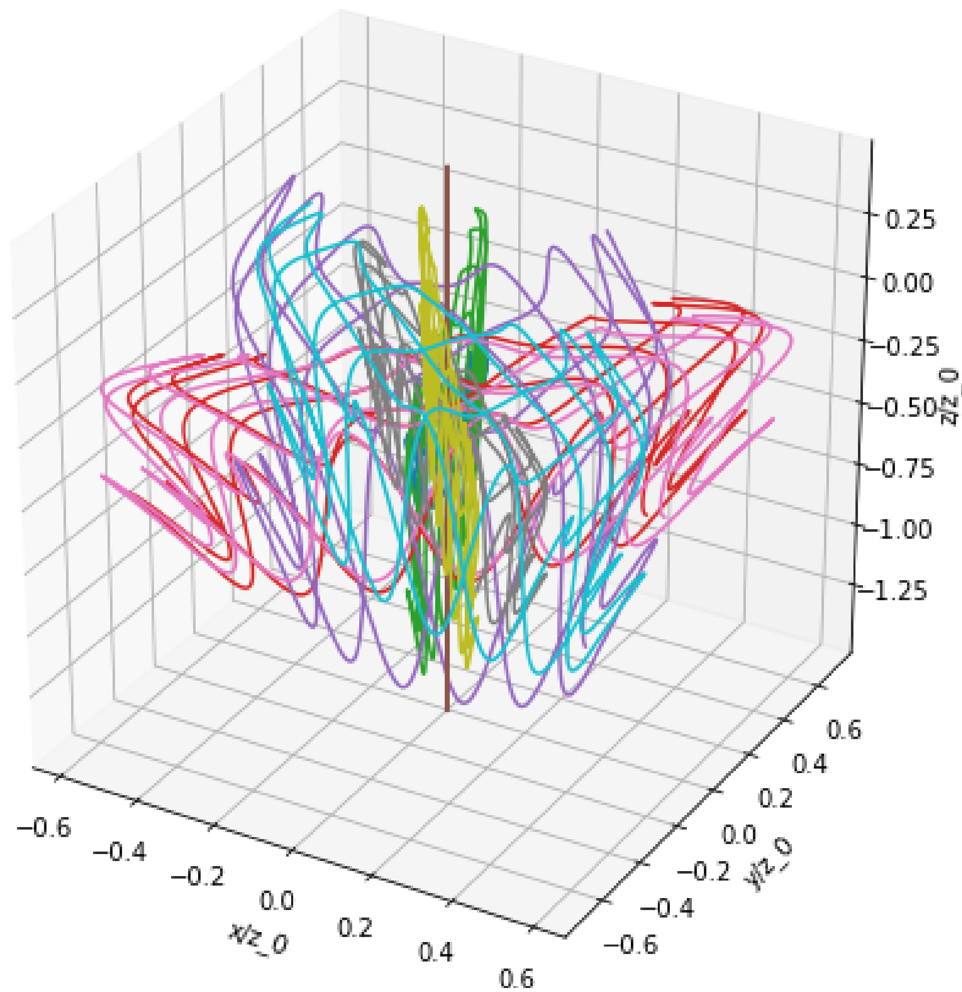
$$\vec{E} = \frac{2V_0 \cos(\Omega t)}{r_0^2}(x, y, -2z)$$

$$\vec{F} = q\vec{E} = q \cdot \frac{2V_0 \cos(\Omega t)}{r_0^2}(x, y, -2z)$$

1.9 i) Plot trajectory

[13]:
```
import numpy as np
import matplotlib.pyplot as plt


def particle_trajectory(Nsteps):
    g = np.array([0, 0, -9.81])
    m = 5e-5  # mass of grain of cinnamon
    V0 = 4000  # V
    q_per_m = 1e-4  # C/kg
    q = m * q_per_m
    z0 = 0.005  # Typical size of trap
    r0 = np.sqrt(2) * z0
    omega = 100 * np.pi
```

```
    dt = 1e-5  # Timestep

    # Initialize trajectory
    r = np.zeros((Nsteps, 3))
    v = np.zeros((Nsteps, 3))
    t = np.zeros(Nsteps)
    r[0, :] = np.random.uniform(-0.5 * z0, 0.5 * z0, size=3)

    # Simulate motion
    for i in range(Nsteps - 1):
        a = (
            np.cos(omega * t[i])
            * 2
            * q
            * V0
            / (m * r0 ** 2)
            * np.array([r[i, 0], r[i, 1], -2 * r[i, 2]])
            + g
        )
        v[i + 1, :] = v[i, :] + a * dt
        r[i + 1, :] = r[i, :] + v[i + 1, :] * dt
        t[i + 1] = t[i] + dt
    return r, v, t, z0


fig = plt.figure(figsize=(8, 8))
ax = fig.gca(projection="3d")
ax.set_xlabel("x/z_0")
ax.set_ylabel("y/z_0")
ax.set_zlabel("z/z_0")
for i in range(10):

    r, v, t, z0 = particle_trajectory(50000) #step count
    r = r / z0
    ax.plot(r[:, 0], r[:, 1], r[:, 2])

plt.show()
```



Looking at the range of x, y (-0.6 to 0.6) and z axes we can see that the particles are indeed stabilized. Plotting 10 of them and the step count of 50000 ensures that this isnt a fluke and that they actually stay inside for longer periods of time.