14/02/24

$$l=\int_a^b \sqrt{1+y'^2(x)}\,dx$$

2520 $y^2=2px$; Найти длину участка кривой от вершины до точки $(x,y)$. За переменную взять $x$

$$x=\frac{y^2}{2p} \qquad \operatorname{ch} t=\frac{e^t+e^{-t}}{2}$$

$$x'=\frac{y}{p}$$

$$l=\int_0^y \sqrt{1+\frac{y^2}{p^2}}\,dy=\left[\begin{array}{l} y=p\operatorname{sh}t \\ dy=p\operatorname{ch}t\,dt \end{array}\right]=$$

$$=\int_0^t \sqrt{1+\frac{p^2\operatorname{sh}^2t}{p^2}}\,p\operatorname{ch}t\,dt=\int_0^t \sqrt{1+\operatorname{sh}^2t}\,p\operatorname{ch}t\,dt=$$

$$=\int_0^t \sqrt{\operatorname{ch}^2 t}\;p\operatorname{ch}t\,dt=p\int_0^t (\operatorname{ch}t)\operatorname{ch}t\,dt=$$

$$=p\int_0^t \operatorname{ch}^2t\,dt=p\int_0^t \frac{1+\operatorname{ch}2t}{2}\,dt=$$

$$=\frac{p}{2}\left(t+\frac{\operatorname{sh}2t}{2}\right)\Big|_0^t=\frac{p}{2}\left(t+\frac{\operatorname{sh}2t}{2}\right)=$$

$$=\frac{p}{2}\left(\operatorname{arcsh}\frac{y}{p}+\operatorname{sh}t\operatorname{ch}t\right.=$$

$$= \frac{p}{2}\left(\operatorname{arsh}\frac{y}{p} + \frac{y}{p}\sqrt{1+\frac{y^2}{p^2}}\right)$$

**2522** $y = \ln(1-x^2)$; $x \in [0, \frac{1}{2}]$

Найти длину.

$$y' = \frac{1}{1-x^2}\cdot(-2x) = -\frac{2x}{1-x^2}$$

$$l = \int_0^{1/2}\sqrt{1+\frac{4x^2}{(1-x^2)^2}}\,dx = \int_0^{1/2}\sqrt{\frac{(1-x^2)^2+4x^2}{(1-x^2)^2}}\,dx =$$

$$= \int_0^{1/2}\sqrt{\frac{1-2x^2+x^4+4x^2}{(1-x^2)^2}}\,dx = \int_0^{1/2}\sqrt{\frac{x^4+2x^2+1}{(1-x^2)^2}} =$$

$$= \int_0^{1/2}\sqrt{\frac{(x^2+1)^2}{(x^2+1)^2}}\,dx = \int_0^{1/2}\left|\frac{x^2+1}{1-x^2}\right|dx =$$

$$= \int_0^{1/2}\frac{x^2+1}{1-x^2}\,dx = \int_0^{1/2}\frac{(x^2-1)+2}{1-x^2}\,dx =$$

$$= \int_0^{1/2}\left(-1-\frac{2}{x^2-1}\right)dx$$

$$\frac{2}{(x^2-1)} = \frac{2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \qquad A=1,\ B=-1$$

$$\int_0^{1/2}\left(-1 + \frac{1}{x-1} + \frac{1}{x+1}\right)dx = -x - \ln|x-1| + \ln|x+1|\Big|_0^{\frac{1}{2}}$$

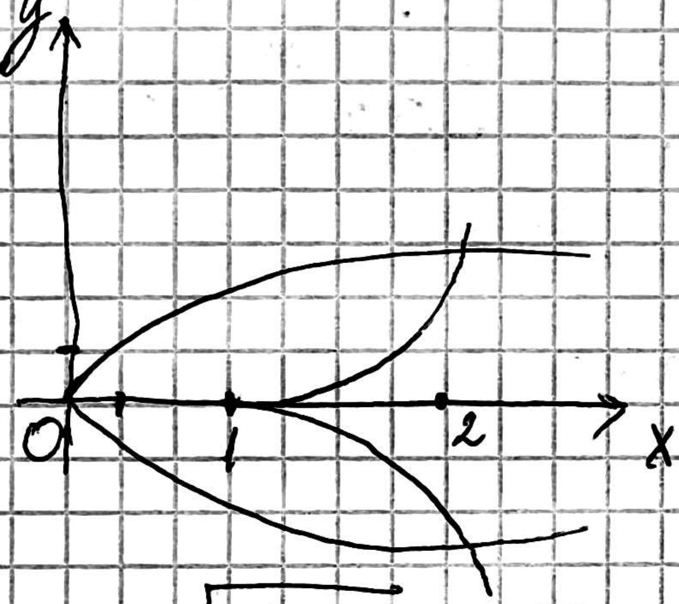
$= -\frac{1}{2} - \ln\frac{1}{2} + \ln 1\frac{1}{2} = \ln 3 - \frac{1}{2}$

2524

$$y^2 = \frac{2}{3}(x-1)^3$$

Найти длину куска отсекаемой параболой

$y^2 = \frac{x}{3}$



$\frac{2}{3}(x-1)^3 = \frac{x}{3}$

$2(x-1)^3 = x$

$x_1 = 2$

$y' = \frac{3}{2}(x-1)^{\frac{1}{2}} \cdot \frac{\sqrt{2}}{\sqrt{3}}$

$$y = \sqrt{\frac{2}{3}(x-1)^3} = \frac{\sqrt{2}}{\sqrt{3}} \cdot (x-1)^{\frac{3}{2}}$$

$$l = 2\int_1^2 \sqrt{1 + \left(\frac{3\sqrt{2}}{6}(x-1)^{\frac{1}{2}}\right)^2} = 2\int_1^2 \sqrt{1 + \frac{1}{2}(x-1)}\,dx =$$

$$= 2\int_1^2 \sqrt{\frac{1}{2} + \frac{x}{2}}\,dx = \sqrt{2}\int_1^2 \sqrt{1+x}\,dx =$$

$$= \sqrt{2} \cdot \frac{(1+x)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^2 = \frac{2\sqrt{2}}{3}\left(3^{\frac{3}{2}} - 2^{\frac{3}{2}}\right)$$

! Настойчиво решать головой труды

2526 $\quad 9ay^2 = x(x-3a)^2$

Найти длину петли.

$$y^2 = \frac{x(x-3a)^2}{9a} \qquad x \in [0, 3a]$$

$$|y| = \sqrt{\frac{x(x-3a)^2}{9a}} =$$

$$\cancel{y' = \frac{1}{2}\left(\frac{x(x-3a)}{9a}\right)} = \frac{\sqrt{x}\,|x-3a|}{3\sqrt{a}} = \frac{\sqrt{x}(-x+3a)}{3\sqrt{a}} =$$

$$= \frac{3ax^{\frac{1}{2}} - x^{\frac{3}{2}}}{3\sqrt{a}}$$

$$y' = \frac{\frac{3}{2}ax^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}}{3\sqrt{a}}$$

$$l = \int_0^{3a} \sqrt{1 + y'^2}\,dx = \int_0^{3a} \sqrt{1 + \frac{a^2x^{-1} - 2a + x}{4a}}\,dx =$$

$$= \int_0^{3a} \sqrt{\frac{a^2x^{-1} + 2a + x}{4a}}\,dx = \int_0^{3a} \sqrt{\frac{ax^{-\frac{1}{2}} + x^{\frac{1}{2}}}{4a}}\,dx =$$

$$= \int_0^{3a} \frac{ax^{-\frac{1}{2}} + x^{\frac{1}{2}}}{2\sqrt{a}}\,dx = \frac{1}{2\sqrt{a}}\left(2ax^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}}\right)\Bigg|_0^{3a} =$$

$$= \frac{\sqrt{x}^{\frac{4}{3}}}{\sqrt{a}} + \frac{\sqrt{x^3}}{3\sqrt{a}}\Bigg|_0^{a} = \frac{1}{2\sqrt{a}}\left(\frac{\cancel{2a}}{9a} + \frac{2}{3}(3a)^{\frac{3}{2}}\right)$$

2530 $(y-\arcsin x)^2 = 1-x^2$

Найти длину всей кривой.

$1-x^2 \geqslant 0$

$x^2 \geqslant 1$

$-1 \geqslant x \geqslant 1$

① $y - \arcsin x = \sqrt{1-x^2}$

② $y - \arcsin x = -\sqrt{1-x^2}$

$y = \sqrt{1-x^2} + \arcsin x$

$y = \arcsin x - \sqrt{1-x^2}$

$y' = \frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}} \cdot (-\cancel{2}x) + \frac{1}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} =$

$l_1 = \int_{-1}^{1} \sqrt{1 + \left(\frac{-x+1}{\sqrt{1-x^2}}\right)^2}\, dx = \int_{-1}^{1} \sqrt{1 + \frac{(1-x)^2}{1-x^2}}\, dx =$

$(1-x)(1+x)$

$= \int_{-1}^{1} \sqrt{1 + \frac{1-x}{1+x}}\, dx = \int_{-1}^{1} \sqrt{\frac{1+x+1-x}{1+x}}\, dx = \int_{-1}^{1} \sqrt{\frac{2}{1+x}}\, dx =$

$= \sqrt{2} \int_{-1}^{1} \frac{1}{\sqrt{1+x}}\, dx = \sqrt{2} \cdot 2\sqrt{1+x}\Big|_{-1}^{1} = \sqrt{2} \cdot 2\sqrt{2} = 4$

ДЗ 2530 – додомить

2529 – 2521 нечетные.