(4) рекуррентные формулы! 19/2/24

$$\int \frac{dx}{\sin^3 x \cos^5 x} = \int \frac{\operatorname{tg} x \, dx}{\sin^4 x \cos^4 x} = 16 \int \frac{\operatorname{tg} x \, dx}{(\sin 2x)^4} =$$

$$= \left| \begin{matrix} t = \operatorname{tg} x & \sin 2x = \dfrac{2t}{1+t^2} \\ dx = \dfrac{dt}{1+t^2} & \end{matrix} \right| = 16 \int \frac{dt}{(1+t^2)\left(\frac{2t}{1+t^2}\right)^4} =$$

$$= \int \frac{(t^2+1)^3 \, dt}{t^4} = \ldots$$

(10) $$J_{2n+1} = \int \frac{dx}{\sin^{2n+1} x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^{2n+1} x} dx =$$

$$= \int \frac{dx}{\sin^{2n-1} x} + \int \frac{\cos^2 x \, dx}{\sin^{2n+1} x} = J_{2n-1} + I =$$

$$I = \int \frac{\cos x \cdot \cos x}{\sin^{2n+1}} dx = \left| \begin{matrix} u = \cos x \\ dv = \dfrac{\cos x \, dx}{\sin^{2n+1} x} \end{matrix} \right.$$

$$du = -\sin x \, dx$$

$$V = \int \frac{d(\sin x)}{\sin^{2n+1} x} = -\frac{1}{2n \sin^{2n} x}$$

$$= -\frac{\cos x}{2n \sin^{2n} x} - \frac{1}{2n} \int \frac{\sin x \, dx}{\sin^{2n} x}$$

$$= -\frac{\cos x}{2n \sin^{2n} x} - \frac{1}{2n} J_{2n-1}$$

$$J_{2n+1} = J_{2n-1} - \frac{\cos x}{2n \sin x} - \frac{1}{2n} J_{2n-1} =$$

$$= \frac{2n-1}{2n} \mathcal{I}_{2n-1} - \frac{\cos x}{2n \sin^{2n} x}$$

$$\int \frac{dx}{\sin x} = \ln\left|tg \frac{x}{2}\right|$$

$$C_{2n+1} = \int \frac{dx}{\cos^{2n+1} x} = \frac{\sin x}{2n \cos^{2n} x} + \frac{2n-1}{2n} C_{2n-1}$$

(10) $$\int \frac{dx}{\cos^3 3x} = \frac{1}{3}\int \frac{d(3x)}{\cos^3 3x} = \frac{1}{3}\int \frac{dt}{\cos^3 t} =$$

$$= \frac{1}{3}\left(\frac{\sin t}{2\cos^2 t} + \frac{1}{2}\int \frac{dt}{\cos t}\right) =$$

$$= \frac{1}{3}\left(\frac{\sin 3x}{2\cos^2 3x} + \frac{1}{2}\ln\left|tg\left(\frac{t}{2} + \frac{\pi}{4}\right)\right| + C =$$

$$= \frac{\sin 3x}{6\cos^2 3x} + \frac{1}{6}\ln\left|tg\left(\frac{t}{2} + \frac{\pi}{4}\right)\right| + C$$

(!) $$\left.\begin{array}{l} \int \sin \alpha x \sin \beta x\, dx \\ \int \cos \alpha x \cos \beta x\, dx \\ \int \sin \alpha x \cos \beta x\, dx \end{array}\right\} \Rightarrow$$

$$\sin a \sin b = \frac{1}{2}(\cos(a-b) - \cos(a+b))$$

$$\cos a \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b))$$

$$\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$$

$$\int \cos x \cos 2x \cos 5x \, dx = \frac{1}{2}\int \cos x \left(\cos(2x-5x) + \cos(7x) \right) dx$$

$$= \frac{1}{2}\int \cos x \left(\cos 3x + \cos 7x \right) dx =$$

$$= \frac{1}{4}\int \left(\cos(4x) + \cos(-2x) + \cos(8x) + \cos(-6x) \right) dx =$$

$$= \frac{1}{4}\int \left(\cos(2x) + \cos(4x) + \cos 6x + \cos(8x) \right) dx =$$

$$= \frac{\sin 2x}{8} + \frac{\sin 4x}{16} + \frac{\sin 6x}{24} + \frac{\sin 8x}{32} + C$$

Интегралы от иррациональных функций

(1) $$\int \frac{(1-6x)\,dx}{\sqrt{4x^2+4x+2}} = \int \frac{(1-6x)\,dx}{\sqrt{(2x+1)^2+1}} =$$

$$= \left| \begin{matrix} t = 2x+1 \\ dt = 2dx \\ x = \frac{t-1}{2} \end{matrix} \right| = \frac{1}{2}\int \frac{1 - 3t + 3}{\sqrt{t^2+1}} = \frac{1}{2}\int \frac{4-3t}{\sqrt{t^2+1}} dt =$$

$$= \frac{4}{2}\int \frac{1}{\sqrt{t^2+1}} dt - \frac{3}{2}\int \frac{t}{\sqrt{t^2+1}} dt =$$

$$= 2\ln\left| t + \sqrt{t^2+1} \right| - \frac{3}{4}\int \frac{d(t^2+1)}{\sqrt{t^2+1}} + C =$$

$$= 2\ln\left| t + \sqrt{t^2+1} \right| - \frac{3}{2}\sqrt{t^2+1} + C = 2\ln\left| 2x+1 + \sqrt{(2x+1)^2+1} \right| - \frac{3}{2}\sqrt{4x^2+4x+2}$$

! НОК и НОД

(2) $\int \frac{(2x+1)dx}{\sqrt{3+6x-9x^2}} = \int \frac{(2x+1)dx}{\sqrt{}}$

(II) $\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{p_1}{q_1}}, \ldots, \left(\frac{ax+b}{cx+d}\right)^{\frac{p_s}{q_s}}\right)dx$

$t^k = \frac{ax+b}{cx+d}$; $k = \text{НОК}(q_1, \ldots, q_s)$

(3) $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx = \left[\begin{array}{l} t^6 = x \\ t = \sqrt[6]{x} \\ dx = 6t^5 dt \end{array}\right] =$

$\frac{2}{3}, \frac{1}{6}, \frac{1}{3}$

$= \int \frac{t^4+t^4+t}{t^6(1+t^2)} 6t^5 dt = 6\int \frac{t^5+t^3+1}{1+t^2}dt =$

$= \vdots \quad \begin{array}{r|l} t^5+t^3+1 & 1+t^2 \\ -(t^5+t^3) & t^3 \\ 1 & \end{array} \quad \vdots = 6\int \left(t^3 + \frac{1}{t^2+1}\right)dt =$

$= \frac{6t^4}{4} + 6\,\text{arctg}\,t + C = 1{,}5\sqrt[3]{x^2} + 6\,\text{arctg}\sqrt[6]{x} + C$

(4) $\int \frac{dx}{\sqrt[3]{(2+x)(2-x)^5}} = \int \frac{dx}{(2+x)^{1/3}(2-x)^{5/3}} = \int \frac{(2-x)^{\frac{1}{3}}dx}{(2+x)^{\frac{1}{3}}(2-x)^2}$

$= \int \sqrt[3]{\frac{2-x}{2+x}} \frac{dx}{(2-x)^2} = \left| \begin{array}{l} t^3 = \frac{2-x}{2+x} \\ t = \sqrt[3]{\frac{2-x}{2+x}} \end{array} \right.$ $\quad 2t^3 + xt^3 = 2-x$

$x = \frac{2-2t^3}{t^3+1}$

$$dx = \frac{-(2+x)-(2-x)}{(2+x)^2} = \cancel{\frac{-2-x-2+x}{(2+x)^2}}$$

$$= \frac{-4}{(2+x)^2} \qquad dx = \left(\frac{2-2t^3}{t^3+1}\right)' dt =$$

$$= \frac{-6t^5-6t^2-6t^2+6t^5}{(t^3+1)^2}\,dt =$$

$$= \frac{-12\,t^2}{(t^3+1)^2}\,dt$$

$$2-x = 2-\frac{2-2t^3}{t^3+1} = \frac{2t^3+2-2+2t^3}{t^3+1} =$$

$$= \frac{4t^3}{t^3+1}$$

$$= \int t\cdot\frac{\cancel{4}t^3}{\cancel{t^3+1}}\cdot\frac{(t^3+1)^2}{\cancel{16}t^3}\,dt =$$

$$-\frac{3}{4}\int\frac{dt}{t^3} = -\frac{3}{4}\cdot\frac{t^{-2}}{-2}+C = \frac{3}{8t^2}+C =$$

$$= \frac{3}{8}\cdot\sqrt[3]{\left(\frac{2+x}{2-x}\right)^2}+C$$