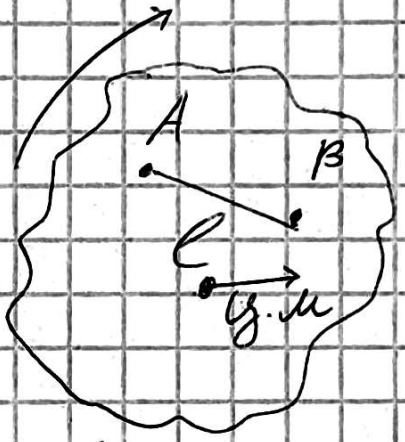
2/3/24

(11) Момент инерции твёрдого тела.

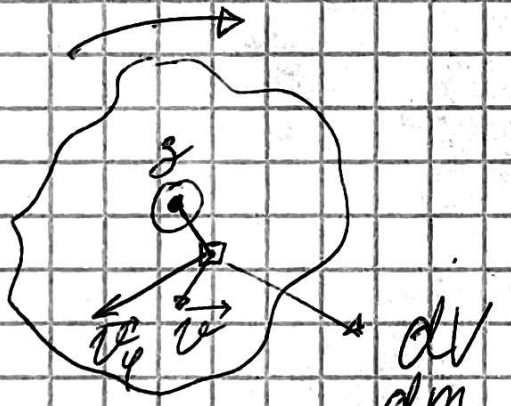
$\dot{\vec{L}} = \vec{M}$ $\quad \frac{d\omega}{dt} = \dot{\omega}$ (дифф. по времени)

$\frac{d\vec{L}}{dt} = \vec{M}$

• м.т

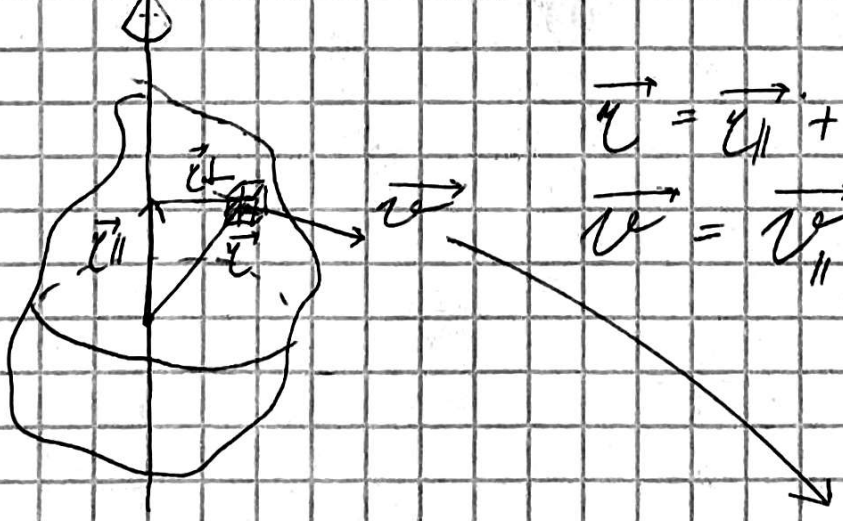


• Если $I = const \Rightarrow$ твёрдое тело.



$\frac{dV}{dm}$

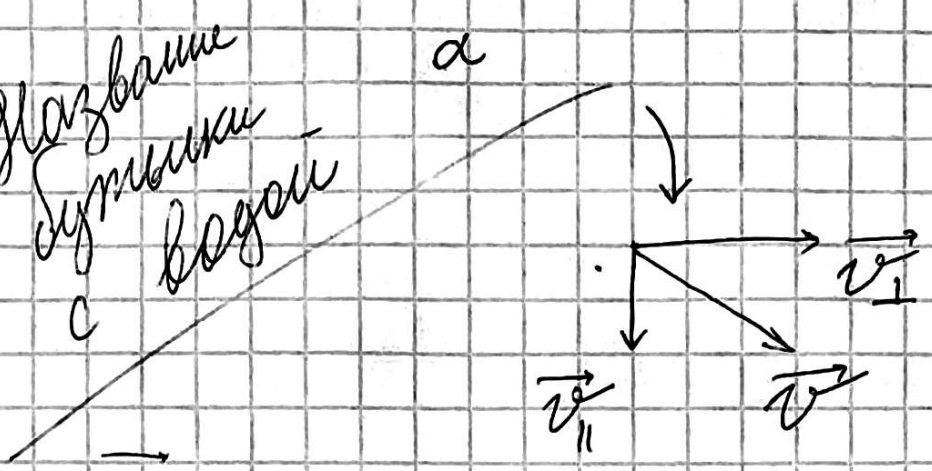
$|\vec{L}|_z = L_z$



$\vec{v} = \vec{v}_\parallel + \vec{v}_\perp$

$\vec{v} = \vec{v}_\parallel + \vec{v}_\perp$

Название струйки с водой



$$\vec{L} = [\vec{r} \times m\vec{v}] = m([\vec{r}_1 \times \vec{v}_{\parallel}] + [\vec{r}_{\perp} \times \vec{v}_{\parallel}] + [\vec{v}_1 \times \vec{r}_{\parallel}] + [\vec{r}_1 \times \vec{r}_{\perp}]) =$$

($\vec{v}_{\parallel} = 0$)

$$= \left| m([\vec{r}_{\perp} \times \vec{v}_{\parallel}] + [\vec{v}_{\perp} \times \vec{r}_{\parallel}] + [\vec{r}_{\perp} \times \vec{r}_{\perp}]) \right|_z = L_z$$

($[\vec{r}_{\perp} \times \vec{v}_{\parallel}] = 0$, $[\vec{v}_{\perp} \times \vec{r}_{\parallel}] = 0$)

$$L_z = m r_{\perp} r_{\parallel} \sin\alpha = m r_{\perp} r_{\parallel}$$

$$dL_z = dm \cdot r_{\perp} \cdot v_{\varphi}$$

$$v_{\varphi} = r_{\perp} \cdot \omega \Rightarrow v_{\varphi} = r_{\perp} \cdot \frac{d\varphi}{dt}$$

$$dm \cdot r_{\perp}^2 \cdot \frac{d\varphi}{dt} = dL_z \Big| \int$$

$$\int_m r_{\perp}^2 \cdot \frac{d\varphi}{dt} \, dm = L_z$$

$$\frac{d\varphi}{dt} \cdot \int_m r_{\perp}^2 \, dm = L_z$$

$\int_m r_{\perp}^2 \, dm$ - момент інерції твірдого тіла.

$L_z = J\frac{d\omega}{dt}$

$\frac{dL_z}{dt} = dM_z$

$\boxed{J\frac{d^2\varphi}{dt^2} = dM_z}$

$m\frac{d^2x}{dt^2} = F$

m d² T. Кёнига.

$E_k = E_k' + \frac{mv_0^2}{2}$

$J = \int_m r_\perp^2\, dm = \int_V r_\perp^2 \rho\, dV$

$[dm = \rho dV] =$

$= \iiint_V r_\perp^2 \rho\, dx\,dy\,dz$

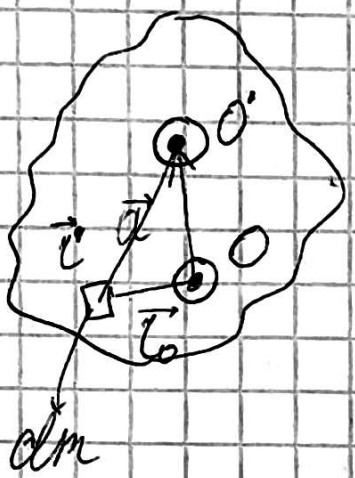
$dE_k' = \frac{dm\, v_\varphi^2}{2} = dm\, r_\perp^2 \left(\frac{d\varphi}{dt}\right)^2$ $\quad v_\varphi = r_\perp \frac{d\varphi}{dt}$

$E_k' = \int_m \frac{r_\perp^2 \left(\frac{d\varphi}{dt}\right)^2}{2} dm = \frac{\left(\int_m r_\perp^2\, dm\right) \cdot \left(\frac{d\varphi}{dt}\right)^2}{2} =$

$= \frac{J \cdot \left(\frac{d\varphi}{dt}\right)^2}{2}$

(12) Т. Гюйгенса-Штейнера.

$\vec{r} = \vec{a} + \vec{r}'$ $\quad \vec{r}' = \vec{r} - \vec{a}$



$J' = \int |\vec{r}'|^2 dm$

$|\vec{r}'|^2 = r^2 - 2(\vec{r}\vec{a}) + a^2$

$J' = \int_m r^2 dm - 2\int_m (\vec{r}\vec{a}) dm +$

$\int_m r^2 dm$ $\quad \int_m r^2 dm = J_{з.м}$

$+ \int_m a^2 dm$ $\quad \int_m a^2 dm = a^2 m$

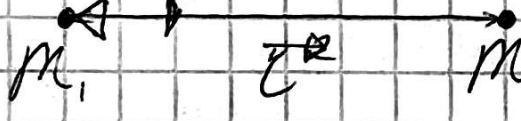
$2\int_m (\vec{r}\vec{a}) dm = 2\int\int \left(\underbrace{\vec{r} dm}_{=0} \cdot \vec{a} \right) = 0$

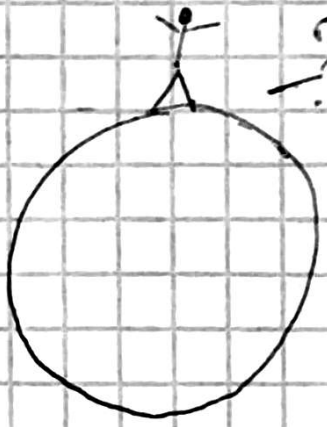
Ось O - проходит через з.м.

$J' = J_{з.м} + Ma^2$

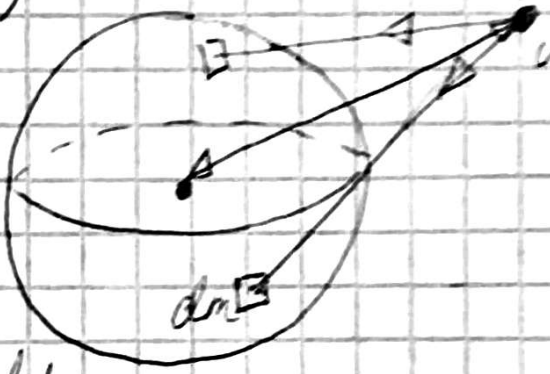
(13) Закон всемирного тяготения.

$\vec{F} = G \frac{m_1 m_2}{r^2} \cdot \left(\frac{\vec{r}}{r} \right)$





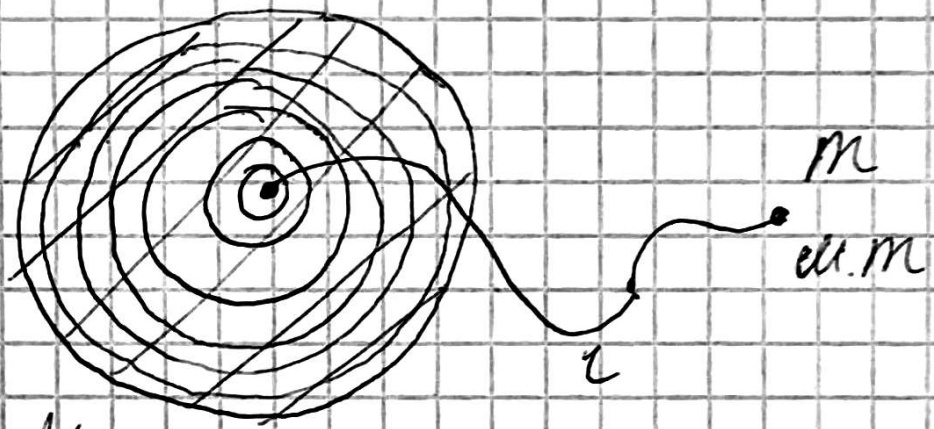
1)



M F-?

$$F = G\frac{Mm}{r^2}$$

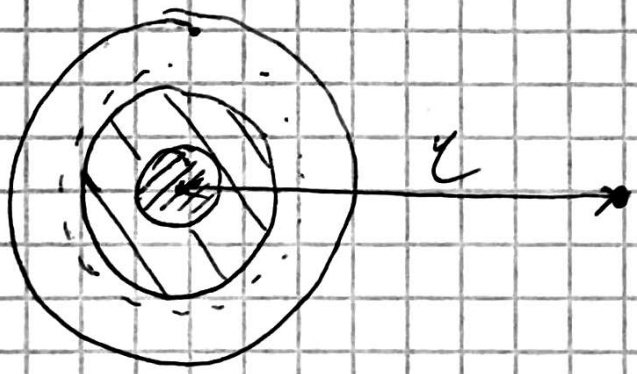
2)



M

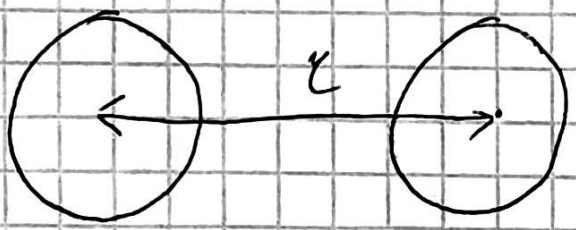
$$F = G\frac{Mm}{r^2}$$

3)

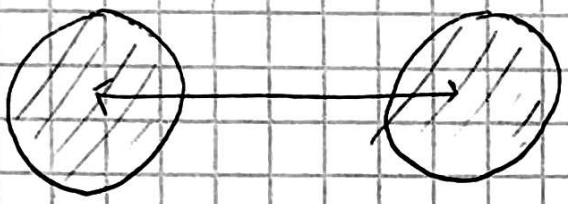


$$F = G\frac{Mm}{r^2}$$

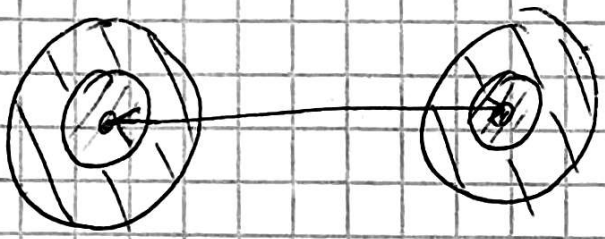
4)



5)



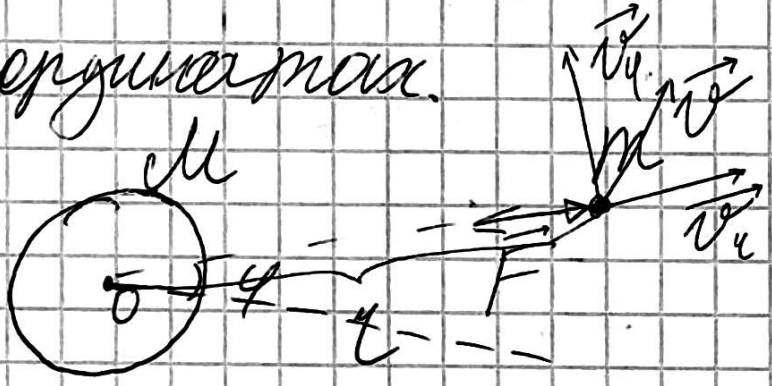
6)



$$\iint G \frac{dm_1\, dm_2}{r_{12}^2} \frac{\vec{r}_{12}}{r_{12}}$$

6D

(14) Движение планет в полярных координатах.



⊙ - the Sun

⊕ - the Earth

☽ - the Moon

1) $F_{грав.}$ - центральная сила.

2) З.С.Э $\quad E_к + E_п = E_0 = const$

3) $\vec{L} = const \quad 3D \to 2D$

4) $(v_r, v_\varphi) \quad \vec{v} = \vec{v}_r + \vec{v}_\varphi : \vec{v}_r \perp \vec{v}_\varphi$

5) $E_п = -G\frac{Mm}{r}$

$E_п^{\text{на}}(r=\infty) = 0$

6) $m \cdot r \cdot v \cdot \sin\alpha = L_0 = const$

$m \cdot r \cdot v_\tau = L_0$

7) $E_к = \frac{mv^2}{2}$ $\quad v^2 = v_r^2 + v_\tau^2$

$\frac{mv_r^2}{2} + \frac{mv_\tau^2}{2}$

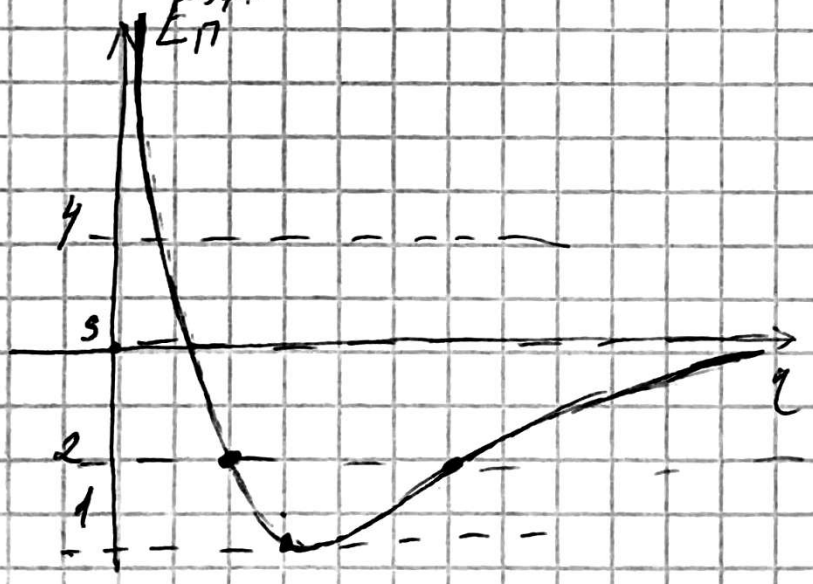
8) $v_\tau = \frac{L_0}{mr}$ $\quad \frac{mv_\tau^2}{2} = \frac{L_0^2}{2mr^2}$

$$\boxed{\frac{mv_r^2}{2} + \frac{L_0^2}{2mr^2} - G\frac{Mm}{r} = E_0}$$

(15.) Траектория движения.

r, v_r

$E_{кr} = \frac{mv_r^2}{2}$ $\quad E_п^{эфф} = \frac{L_0^2}{2mr^2} - G\frac{Mm}{r}$



$E_п^{эфф} = -\frac{2L_0}{2mr^3} + \frac{GmM}{r^2}$

$E_п^{эфф} = 0$

$\frac{GmM}{r^2} - \frac{L_0^2}{mr^3} = 0 \mid \cdot r^3$

$GmMr = \frac{L_0^2}{m}$

$r = \frac{L_0^2}{Gm^2M}$

1) $E_0 = (E_п^{эфф})_{min}$ (круг)

$v_ч - ?$ $v = v_{I к.}$

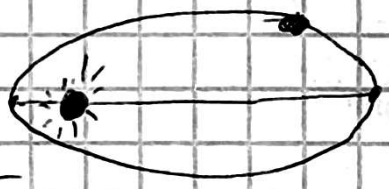
M - земля m - Гагарин. $R_⊕ = \frac{m^2 v^2 R_⊕^2}{G m M}$

$R_⊕ = \frac{v_{I кс}^2}{g}$ $\quad = g$

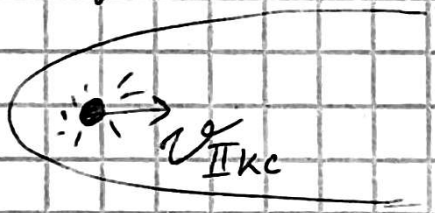
$v_{I кс} = \sqrt{R_⊕ \cdot g} \simeq \sqrt{6400 \cdot 10^3 \cdot 10} =$

$= 8000 м/с$

2) Эллипс



3) $E_0 = 0$



$v_{II кс} = \sqrt{2} v_{I кс}$

4)