$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & -2 & 1 & 3 \\ -2 & -3 & 6 & 3 & 2 & 0 \\ 3 & 5 & -6 & 1 & 3 & 4 \end{array}\right) \begin{array}{l}(2)+2(1) \\ (3)-3(1)\end{array} \sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & -2 & 1 & 3 \\ 0 & 1 & 4 & -1 & 4 & 6 \\ 0 & -1 & -9 & 7 & 0 & -5 \end{array}\right) (2)+(2) \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & -2 & 1 & 3 \\ 0 & 1 & 4 & -1 & 4 & 6 \\ 0 & 0 & 1 & 6 & 4 & 1 \end{array}\right) (2)-4(3) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & -2 & 1 & 3 \\ 0 & 1 & 0 & -25 & -12 & 2 \\ 0 & 0 & 1 & 6 & 4 & 1 \end{array}\right) \sim$$

$$\begin{array}{l}(1)=2(2) \\ (1)+(3)\end{array} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & 5 & 4 \\ 0 & 1 & 0 & -25 & -12 & 2 \\ 0 & 0 & 1 & 6 & 4 & 1 \end{array}\right) (1)-2(2) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 54 & 29 & 0 \\ 0 & 1 & 0 & -25 & -12 & 2 \\ 0 & 0 & 1 & 6 & 4 & 1 \end{array}\right)$$

$$T = \begin{pmatrix} 54 & 29 & 0 \\ -25 & -12 & 2 \\ 6 & 4 & 1 \end{pmatrix}$$

② $\ell = (\ell_1, \ell_2, \ell_3)$

$\ell_1 = (1, -2, 3)$

$\ell_2 = (2, -3, 5)$

$\ell_3 = (-1, 6, -6)$

$x_\ell = (2, 2, 1)$

$x_\ell - ?$

$\ell = E \cdot T$

$\boxed{T \cdot X_\ell = X_\ell}$

$(T \mid X_\ell) \longrightarrow (E \mid X_\ell)$

③ $L = \langle a_1, a_2, a_3, a_4 \rangle$ Найти размерность $\overline{L}$

$a_1 = (-5, 8, -4, -1)$

$a_2 = (1, 2, -1, 3)$

$a_3 = (-2, 5, -4, 1)$

$a_4 = (-4, 19, -14, 9)$

$$A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 8 & 5 & 19 \\ -1 & -4 & -4 & -14 \\ 3 & -1 & 1 & 9 \end{pmatrix} \begin{matrix} (2)-2(1) \\ (3)+(1) \\ (4)-3(1) \end{matrix} \sim$$

$a_2 \; a_1 \; a_3 \; a_4$

$$\begin{pmatrix} 1 & -5 & -2 & -4 \\ 0 & 18 & 9 & 27 \\ 0 & -12 & -6 & -18 \\ 0 & 14 & 7 & 21 \end{pmatrix} \begin{matrix} \\ :9 \\ :-6 \\ :1 \end{matrix} \sim \begin{pmatrix} 1 & -5 & -2 & -4 \\ 0 & 2 & 1 & 3 \\ 0 & 2 & 1 & 3 \\ 0 & 2 & 1 & 3 \end{pmatrix}$$

$\dim L = \operatorname{rang}(A) = 2 \Rightarrow$ 2 линейно независимых вектора

$L = \langle a_1, a_2 \rangle$

④ $L = \langle a_1, a_2, a_3 \rangle$ Систему однор. уравнений задающую подпр-во L

$a_1 = (1, -2, -1, 3)$

$a_2 = (3, 5, 4, -2)$

$a_3 = (5, 1, 2, 4)$

$L = \{ X = (x_1, x_2, x_3, x_4) \mid x = \alpha a_1 + \beta a_2 + \gamma a_3 \}$

$$\left(\begin{array}{ccc|c} 1 & 3 & 5 & X_1 \\ -2 & 5 & 1 & X_2 \\ -1 & 4 & 2 & X_3 \\ 3 & -2 & 4 & X_4 \end{array}\right) \xrightarrow{\begin{array}{l}(2)+2(1)\\(3)+(1)\\(4)+3(1)\end{array}} \left(\begin{array}{ccc|c} 1 & 3 & 5 & X_1 \\ 0 & 11 & 11 & X_2+2X_1 \\ 0 & 7 & 7 & X_1+X_3 \\ 0 & -11 & -11 & X_4-3X_1 \end{array}\right) \begin{array}{l}(4)+(2)\\(3)\cdot 11-7(2)\end{array} \sim$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 5 & X_1 \\ 0 & 11 & 11 & 2X_1+X_2 \\ 0 & 0 & 0 & 11(X_3+X_1)-7(X_2+2X_1) \\ 0 & 0 & 0 & X_2+2X_1-3X_1+X_4 \end{array}\right)$$

$$L: \begin{cases} 11(X_3+X_1)-7(X_2+2X_1)=0, \\ X_2+2X_1-3X_1+X_4=0. \end{cases} \longrightarrow$$

$$\begin{cases} 11X_3+11X_1-7X_2+14X_1=0, \\ X_2-X_1+X_4=0 \end{cases} \longrightarrow \begin{cases} -3X_1-7X_2+11X_3=0, \\ -X_1+X_2+X_4=0 \end{cases}$$

⑤ $L_1=\langle a_1, a_2, a_3\rangle$! Кома бела

$L_2=\langle b_1, b_2, b_3\rangle$ ЛВ

$a_1=(2,1,1,-1)$ $b_1=(-1,1,2,1)$

$a_2=(-3,0,1,2)$ $b_2=(5,1,0,-3)$

$a_3=(1,-2,1,0)$ $b_3=(0,1,1,1)$

1) $L_1+L_2=?$ 2) $L_1 \cap L_2=?$

$L_1 + L_2 = \langle L_1 \cup L_2 \rangle = \langle a_1, a_2, a_3, b_1, b_2, b_3 \rangle$

$$A = \begin{pmatrix} 2 & -3 & 1 & -1 & 5 & 0 \\ 1 & 0 & -2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 0 & 1 \\ -1 & 2 & 0 & 1 & -3 & 1 \end{pmatrix} \begin{matrix} (1)-2(2) \\ (3)-(2) \\ (4)+(2) \end{matrix} \sim \begin{pmatrix} 0 & -3 & -5 & -3 & 3 & -2 \\ 1 & 0 & -2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 & -1 & 0 \\ 0 & 2 & -2 & 2 & -2 & 2 \end{pmatrix}$$

$$\begin{matrix} (1)+3(3) \\ (4)-2(3) \end{matrix} \sim \begin{pmatrix} 0 & 0 & 14 & 0 & 0 & -2 \\ 1 & 0 & -2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 & -1 & 0 \\ 0 & 0 & -8 & 0 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 & -1 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 14 & 0 & 0 & -2 \end{pmatrix} : 6 \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{array}\right) \quad \operatorname{rang}(A) = 4 = \dim(L_1 + L_2)$$

$L_1 + L_2 = V$

L_1: {(1)

L_2: {(2)

$L_1 \cap L_2$: { (1), (2) $\xrightarrow{\text{ФСР}}$

$$L_1: \left(\begin{array}{ccc|c} 2 & -3 & 1 & x_1 \\ 1 & 0 & -2 & x_2 \\ 1 & 1 & 1 & x_3 \\ -1 & 2 & 0 & x_4 \end{array}\right) \begin{matrix} (1)-2(2) \\ (3)-(2) \\ (4)+(2) \end{matrix} \left(\begin{array}{ccc|c} 0 & -3 & 5 & x_1-2x_2 \\ 1 & 0 & -2 & x_2 \\ 0 & 1 & 3 & x_3-x_2 \\ 0 & 2 & -2 & x_4+x_2 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & x_2 \\ 0 & -3 & 5 & x_1-2x_2 \\ 0 & 1 & 3 & x_3-x_2 \\ 0 & 2 & -2 & x_2+x_4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & x_2 \\ 0 & 1 & 3 & -x_2+x_3 \\ 0 & 2 & -2 & x_2+x_4 \\ 0 & -3 & 5 & x_1-2x_2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & \\ 0 & 1 & 3 & \\ 0 & 0 & -8 & \\ 0 & 0 & 14 & (L_1 - 2x_2 = \quad x_2) \end{array}\right) =$$

$$L_2 : \begin{cases} X_1 - 2X_2 + X_3 + X_4 = 0 \end{cases}$$