21/2/24

$x = x(t)$

$y = y(t)$

$t \in [\alpha; \beta]$

$l = \int_{\alpha}^{\beta} \sqrt{x'^2(t) + y'^2(t)}\,dt$

2531 $x = a(t - \sin t)$

$y = a(1 - \cos t)$

Найти точку на циклоиде, которая делит первую арку циклоиды по длине в отношении 1:3

$t \in [0, 2\pi]$

2533 $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$

$\left(\left(\frac{x}{a}\right)^{\frac{1}{3}}\right)^2 + \left(\left(\frac{y}{b}\right)^{\frac{1}{3}}\right)^2 = 1$

$\left(\frac{x}{a}\right)^{\frac{1}{3}} = \cos t \quad \to \begin{cases} x = a\cos^3 t \\ y = b\sin^3 t \end{cases}$

$\left(\frac{y}{b}\right)^{\frac{1}{3}} = \sin t$

$\begin{cases} x' = 3a\cos^2 t(-\sin t) = -3a\sin t\cos^2 t, \\ y' = 3b\sin^2 t\cos t \end{cases}$

$$l = \int_0^{2\pi} \left(\sqrt{9a^2\sin^2 t\cos^4 t + 9b^2\sin^4 t\cos^2 t}\right) dt =$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t\cos^2 t\,(a^2\cos^2 t + b^2\sin^2 t)}\; dt =$$

$$\cancel{= 3\int_0^{2\pi} \sqrt{\left(\frac{1+\cos 2t}{2}\right)\left(\frac{1-\cos 2t}{2}\right)}}$$

$$= 3\int_0^{2\pi} \sqrt{\frac{1}{4}\sin^2 2t\left(a^2\cdot\left(\frac{1+\cos 2t}{2}\right) + b^2\left(\frac{1-\cos 2t}{2}\right)\right)} dt$$

$$= \frac{3}{2}\int_0^{2\pi} |\sin 2t| \sqrt{\left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^2-b^2}{2}\right)\cos 2t}\; dt =$$

= ! Для чётной ф-ции можно менять промежуток интегрирования в данном диапазоне.

$$= 3\int_0^{\pi} |\sin 2t| \sqrt{\frac{a^2+b^2}{2} + \left(\frac{a^2-b^2}{2}\right)\cos 2t}\; dt =$$

$$= 6\int_0^{\frac{\pi}{2}} \sin 2t \sqrt{\frac{a^2+b^2}{2} + \left(\frac{a^2-b^2}{2}\right)\cos 2t}\; dt =$$

$$= \left| \begin{array}{c} \frac{a^2+b^2}{2} + \left(\frac{a^2-b^2}{2}\right)\cos 2t = s^2 \\ \cancel{d\left(\frac{a^2+b^2}{2} + \left(\frac{a^2-b^2}{2}\right)\cos^2 t\right) = 2s\,ds} \end{array} \right| = \left| \begin{array}{c} (b^2-a^2)\sin t\, dt = \\ = 2s\,ds \end{array} \right|$$

$$= 6\int_a^b \frac{2s}{(b^2-a^2)}\,ds \cdot \sqrt{s^2} =$$

$$= 6\cdot\int_a^b \frac{2s^2}{(b^2-a^2)}\,ds = \frac{12}{b^2-a^2}\int_a^b s^2\,ds =$$

$$= \frac{12}{b^2-a^2}\cdot\frac{s^3}{3}\Big|_a^b = \frac{4s^3}{b^2-a^2}\Big|_a^b =$$

$$= \frac{4}{b^2-a^2}(b^3-a^3) = \frac{4(a^3-b^3)}{a^2-b^2} =$$

$$= \frac{4\cancel{(a-b)}(a^2+ab+b^2)}{\cancel{(a-b)}(a+b)} = \frac{4(a^2+ab+b^2)}{(a+b)}$$

$$\rho = a\sin^3\frac{\varphi}{3} \qquad \varphi\in[0, 6\pi]$$

$$\rho' = 3a\sin^2\frac{\varphi}{3}\cdot\cos\frac{\varphi}{3}\cdot\frac{1}{3} = a\sin^2\frac{\varphi}{3}\cos\frac{\varphi}{3}$$

$$l = \int^{6\pi}\sqrt{\left(a^2\sin^6\frac{\varphi}{3} + a^2\sin^4\frac{\varphi}{3}\cos^2\frac{\varphi}{3}\right)}\,d\varphi =$$

$$= a\int_0^{6\pi}\left|\sin^2\frac{\varphi}{3}\right|\sqrt{\sin^2\frac{\varphi}{3}+\cos^2\frac{\varphi}{3}}\,d\varphi = a\int_0^{6\pi}\left(\sin^2\frac{\varphi}{3}\cdot 1\right)d\varphi$$

$$= a\int_0^{6\pi}\left(\frac{1-\cos\frac{2}{3}\varphi}{2}\right)d\varphi =$$

$$= \frac{a}{2}\int_0^{6\pi}\left(1-\cos\frac{2}{3}\varphi\right)d\varphi = \frac{a}{2}\left(\varphi - \frac{3}{2}\sin\frac{2\varphi}{3}\right)\Big|_0^{6\pi} =$$

$$= \frac{a}{2}(6\pi) = 3a\pi$$

На дом 2534, 2538, 2546