Fourier Transform

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1 Fourier Series

1.1 Real Coefficients

$$f(t) \quad | \quad f(t+T) = f(t) \quad \forall \quad t$$

$$g(t) = k + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$k = \frac{1}{T}\int_0^T f(t)dt$$

$$a_m = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi mt}{T}\right)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

1.2 Imaginary Coefficients

$$e^{it} = \cos(t) + i\sin(t); i = \sqrt{-1}$$

$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{i\frac{2\pi nt}{T}}$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-i\frac{2\pi nt}{T}}dt$$

2 Fourier Tranform

2.1 Continuous

$$\mathscr{F}(g(t)) = G(f) = \int_{-\infty}^{+\infty} g(t)e^{-i2\pi ft}dt$$

$$\mathscr{F}^{-1}(G(t)) = g(f) = \int_{-\infty}^{+\infty} G(t)e^{i2\pi ft}df$$