Interference Transform: Estimating the frequency and phase of low resolution samples

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Abstract

TODO low res samples, interest in best sinusoidal approximation The Discrete Fourier Transform decomposes an periodic wave in a finite set of predefined sinusoids, commonly with frequencies coinciding with the natural numbers, offering no guarantees that one of those frequencies is the best one, in the sense of the frequency that generates the closest sinusoid to the original wave. Starting from the observation of the intersection of plane waves in the frequency versus phase space we propose a mixed-integer programming formulation of the problem. The algorithm is based on the representation of the set of samples in a frequency versus phase space and the observation of the interference pattern thus generated. Each sample defines a sinusoidal plane wave in this space, with a particular amplitude, direction and frequency. The determination of the point in which maximum constructive interference occurs translates to the determination of the closest point to the lines that represent the maxima of each plane wave. TODO: better accuracy than FFT, more time consuming TODO: an alternative to wavelet and short-time Fourier transform

Keywords: Fast Fourier Transform, Digital Signal Processing, Wavelet Transform

1 Introduction

Despite the ubiquitous presence of the DFT algorithm, some of its shortcomings, specially with respect to low resolution waves, are yet to be addressed. Nevertheless, areas such as audio compression would greatly benefit from improved analysis of the characteristics of low resolution samples, enabling, for example, the extraction of parameters to be used in machine learning algorithms or sound restoration.

This neglect is felt in the virtual inexistence of research in this specific topic.

1.1 Literature Review

2 Methodology

2.1 Theory

One can represent the discrete wave as an an array $W[i]$ of $n-1$ real numbers. Such wave is related, as illustrated in figure 1, to an hypothetical continuous wave $S(t)$ of which its components are sampled in uniform time intervals at a rate of fps frames per second.

Thus, we have: $W[i] \in \mathbb{R}\ \forall\ i \in \mathbb{N}, 0 \leq i < n$ $S(t) \in \mathbb{R}\ \forall\ t \in \mathbb{R}, 0 \leq t < t_f$ where $t_f = \frac{n}{\text{fps}}$.

The relationship is better illustrated comparing two equivalent simple sinusoids; a continuous one and its discrete counterpart. Those can be described, respectively as

$$S(t) = a\cos\left(\phi + 2\pi\mathcal{F}t\right)$$

and

$$W[i] = a\cos\left(\phi + 2\pi f\frac{i}{n}\right)$$

where:

$\mathcal{F} \in \mathbb{R}^+$, absolute frequency of the continuous wave in Hz;
$f \in \mathbb{R}^+$, relative frequency of the discrete wave in cycles per n samples;
$f = \frac{n}{\text{fps}}\mathcal{F}$;
$i = t\ \text{fps}, i \in \mathbb{N}$, frame number;
$t \in \mathbb{R}$, time in seconds;
$n \in \mathbb{N}$, number os discrete samples;
$0 \leq \frac{i}{n} < 1$;

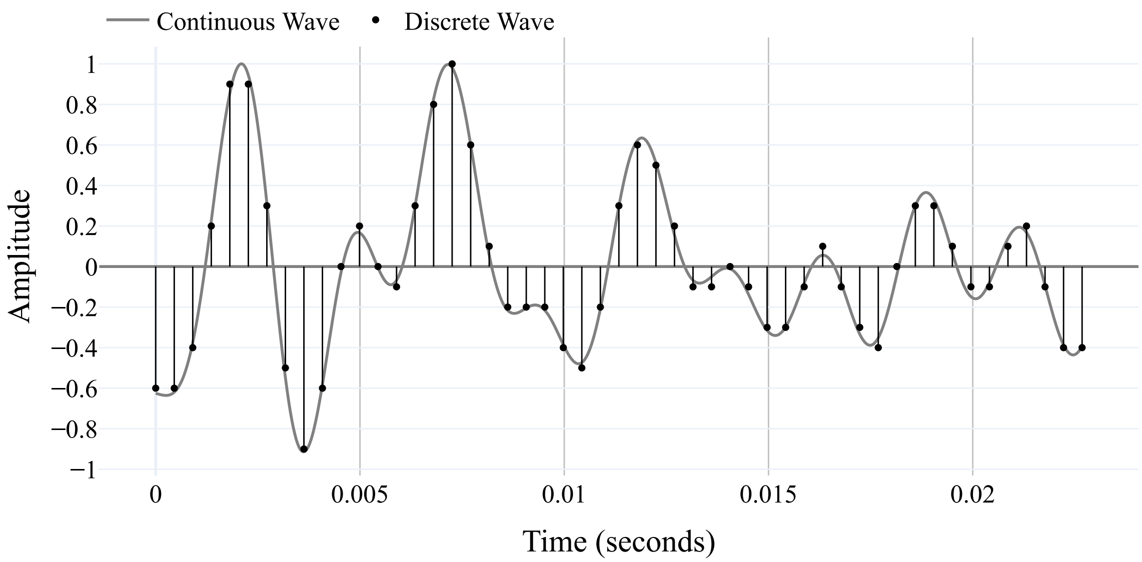

Figure 1: Discrete samples (black points) from a continuous wave (grey line). Note that the amplitude is also truncated

$a \in \mathbb{R}^+$, amplitude;
$\phi \in \mathbb{R}^+, 0 \leq \phi < 2\pi$, phase;

Figure 2 illustrates the relationship between the two.

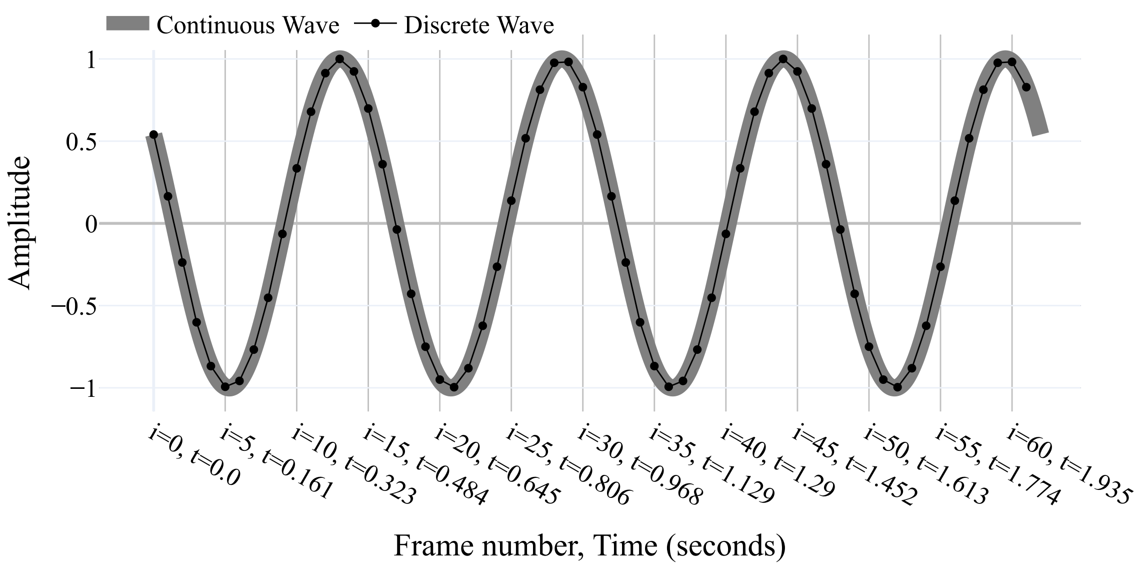

Figure 2: Continuous sinusoid (grey line) and the correspondent Discrete sinusoid (black points) illustrating the correspondence between frames i and time t

So it's easy to see that, for each frame of a given discrete wave $W[i]$, there exists an infinite family of continuous sinusoids that pass exactly through the point defined by i and $W[i]$. In fact, almost any sinusoid of the form $a\cos\left(\phi + 2\pi f \frac{i}{n}\right)$ can be forced to contain the point $(i, W[i])$, given a suitable choice of a, with the exception of the particular case where $W[i] \neq 0$ and $\cos\left(\phi + 2\pi \mathcal{F} \frac{i}{\text{fps}}\right)$.

Of interest to this work are those sinusoids, as shown in Figure 3, that have one of their infinite extrema, either maximum or minimum, coinciding with the abscissa i or, setting $a = W[t]$, coincident with the point $(i, W[i])$; those sinusoids represent the combinations of phase an frequency that would maximize the value of $W[t]S[t]$ for frame t.

One can define, for each frame, those waves mathematically as the family of continuous sinusoids that satisfy the equality $\cos(\phi + 2\pi\mathcal{F}t) = \operatorname{sgn}(W[t])$ or, in terms of discrete sinusoids, $\cos\left(\phi + 2\pi f \frac{i}{n}\right) = \operatorname{sgn}(W[t])$ where the sign function is defined as $\operatorname{sgn}(x) = \{-1 \text{ if } x < 0,\ 0 \text{ if } x = 0,\ 1 \text{ if } x > 0\}$. TODO We can plot

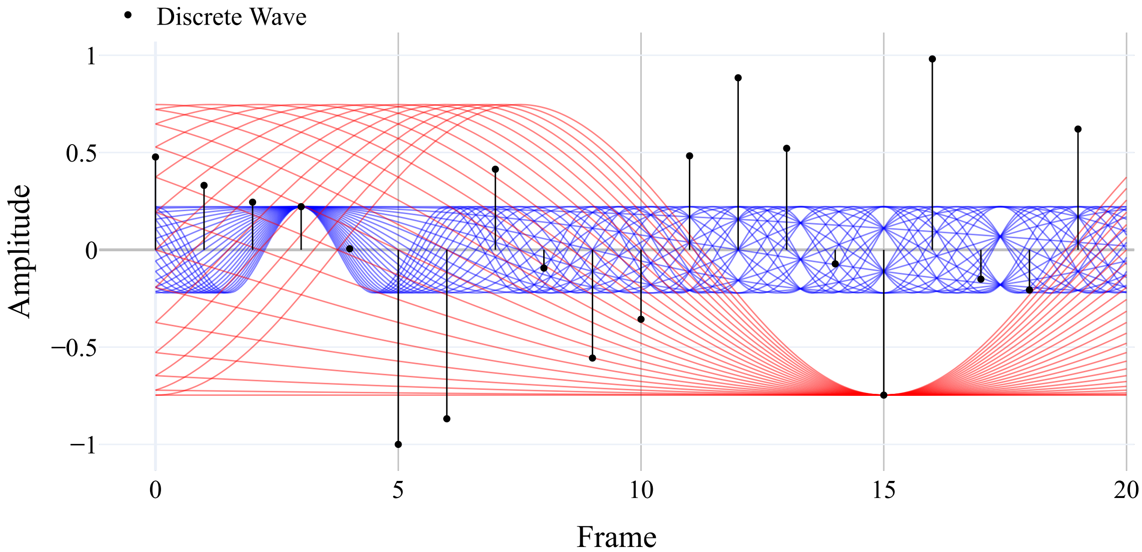

Figure 3: Sinusoids with maximum coinciding with frames i=3 and i=28

3 Results

4 Discussion + Conclusion

5 References