T M S C A   H I G H   S C H O O L

M A T H E M A T I C S

T E S T # 6 - UIL C ©

D E C E M B E R 1 1 , 2 0 2 1

## GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **may** be used on this test. Hand held computers may not be used.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Solve for x if $17x + 5! - 3^2(11 - 2) = (2x)^2 - 3$, $x < 0$.

   (A) −2.50   (B) −2.25   (C) −2.00   (D) −1.75   (E) −1.50

2. Jackie has $30 to spend on lunch for her and two friends. A hamburger cost $4.65, an order of fries cost $1.45, a coke cost $1.75 and an apple pie cost $1.25. The tax rate is 8.25%. If she orders three hamburgers, three orders of fries and three cokes, how many apple pies can she purchase?

   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

3. Consider three consecutive positive integers such that the product of the first and third exceeds twenty-four times the second by 531. The sum of the three integers is _____.

   (A) 114   (B) 144   (C) 174   (D) 204   (E) 234

4. Consider points $A(-6,4)$, $B(4,-2)$ and $C(-4,-8)$. Find the equation of the line that contains point C and is also parallel to $\overrightarrow{AB}$.

   (A) $3x + 5y + 12 = 0$ (B) $3x - 5y + 52 = 0$ (C) $3x + 5y + 28 = 0$
            (D) $3x - 5y + 28 = 0$ (E) $3x + 5y + 52 = 0$

5. The roots of $3x^2 + 9x - 120 = 0$ are b and c. Evaluate $|c - b|$.

   (A) 9   (B) 10   (C) 11   (D) 12   (E) 13

For problems 6, 7 and 8, use the graph on the right.
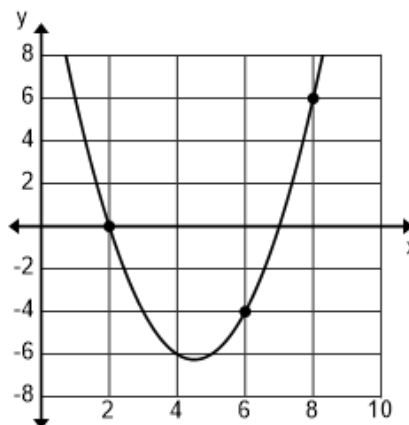
6. Find the y-coordinate of the vertex of the parabola.

   (A) −6   (B) −6.125   (C) −6.25   (D) $-6.\overline{3}$ (E) −6.4

7. The equation of the directrix of the parabola is $y =$ ____.

   (A) −6.375   (B) −6.5   (C) −6.625   (D) −6.75   (E) −6.875

8. The graph of $y = h'(x)$ is shown on the right. If $h(6) = 0$, then $h(8) =$ _____.

   (A) $\dfrac{1}{4}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{2}{3}$ (E) 1

9. Tom and Jill can mow a yard in four hours when they work together. Tom takes 1.25 times as long as Jill to mow the yard by himself. How long does it take Jill to mow the yard by herself?

   (A) 7 hr 12 min   (B) 7 hr 39 min   (C) 8 hr 6 min   (D) 8 hr 33 min   (E) 9 hr

For problems 10, 11 and 12, use the graph on the right.
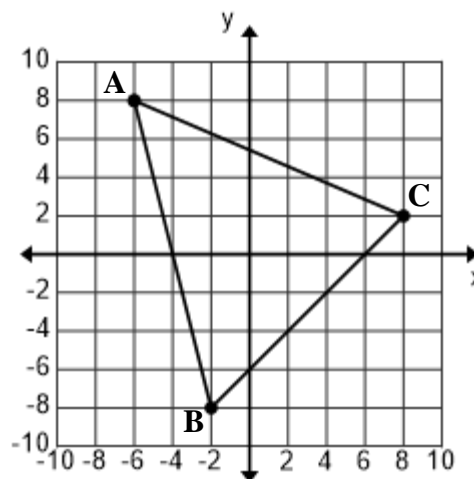
10. Find the perimeter of triangle ABC.
   (nearest whole number)

   (A) 40   (B) 42   (C) 44   (D) 46   (E) 48

11. Find the area of triangle ABC.
   (nearest whole number)

   (A) 94   (B) 96   (C) 98   (D) 100   (E) 102

12. Find the measure of $\angle ABC$. (nearest whole number)

   (A) 51°   (B) 53°   (C) 55°   (D) 57°   (E) 59°

13. Consider a circle with points A, B, C and D on the circle such that $\overline{AC}$ intersects $\overline{BD}$ at point P, inside the circle. If $AP = 8$, $AC = 26$ and $BP = 6$. Find BD. (nearest whole number)

   (A) 20   (B) 22   (C) 24   (D) 28   (E) 30

14. Consider triangle PQR with point T on $\overline{PR}$ such that $\overrightarrow{QT}$ bisects $\angle PQR$. If $PQ = 48$, $QR = 36$ and $RT = 24$, then $PT =$ _____. (nearest whole number)

   (A) 28   (B) 30   (C) 32   (D) 34   (E) 36

15. In a regular polygon, the ratio of the measure of an interior to the measure of an exterior angle is 3:1. How many sides does the polygon have?

   (A) 5   (B) 6   (C) 7   (D) 8   (E) 9

16. Consider a circle with center O. Diameter $\overline{AC}$ and chord $\overline{BD}$ intersect at point P, inside the circle. If $\overline{AC} \perp \overline{BD}$, $AP = 24$ and $PC = 8$, then $BD =$ _____. (nearest tenth)

   (A) 27.4   (B) 27.7   (C) 28.0   (D) 28.3   (E) 28.6

17. The volume of a right pyramid with a square base is 1296. If the height of the pyramid is 12, what is the total area?

   (A) 864   (B) 868   (C) 872   (D) 876   (E) 880

18. Consider the line that has a slope of 0.75 and that passes through the midpoint of a segment with endpoints $(-2, 8)$ and $(6, -4)$. Which of the following points does not lie on this line?

    (A) $(-6, -4)$ (B) $(-4, -2)$ (C) $(-2, -1)$ (D) $(2, 2)$ (E) $(6, 5)$

For problems 19 and 20, consider triangle ABC with $AB = 17$, $BC = 23$ and $m\angle ABC = 77°$.
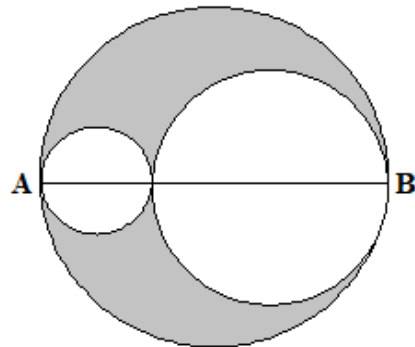
19. Find the area of triangle ABC. (nearest tenth)

    (A) 190.5   (B) 193.6   (C) 196.7   (D) 199.8   (E) 202.9

20. Find the area of the circle inscribed in triangle ABC. (nearest tenth)

    (A) 103.9   (B) 106.8   (C) 109.7   (D) 112.6   (E) 115.5

21. $\overline{AB}$ contains the center of all three circles. Find the shaded area if the radius of the smallest circle is 3 and $AB = 18$. (nearest whole number)

    (A) 101   (B) 104   (C) 107   (D) 110   (E) 113

22. The number of bots varies directly as the number of cots and inversely as the number of dots squared. When there were 12 bots, there were 8 cots and 15 dots. How many dots will there be when you have 27 bots and 2 cots?

    (A) 4   (B) 5   (C) 6   (D) 7   (E) 8

23. The latitude of York, Nebraska is N 40° 52′ 2.6112″. If the radius of the Earth is 3960 miles, how far is it along the Earth's surface from York to the equator? Assume the Earth is a sphere. (nearest mile)

    (A) 2809 mi   (B) 2813 mi   (C) 2817 mi   (D) 2821 mi   (E) 2825 mi

24. How many four-letter words, real or otherwise, can be made from the letters in the word POLARITY if repetition is not permitted?

    (A) 1080   (B) 1280   (C) 1480   (D) 1680   (E) 1880

25. The wheels of Caleb's toy firetruck have a diameter of 2 inches and they are revolving at 240 rpm. Find the velocity of the firetruck in mph. (nearest tenth)

    (A) 1.2 mph   (B) 1.4 mph   (C) 1.6 mph   (D) 1.8 mph   (E) 2.0 mph

26. Coach Taylor, Michael, Scottie, Addison, Rosa, Kaylee and Rodolfo are sitting around a circular table to practice number sense. How many seating arrangements are possible?

   (A) 360   (B) 720   (C) 2520   (D) 5040   (E) 823,543

27. How many ways can Andrew, Justin, Rich and Dhilan sit in a row of four chairs if Justin is not allowed to sit next to Andrew?

   (A) 8   (B) 12   (C) 16   (D) 20   (E) 24

28. Multiply: $\left[4\{(\cos(120°)+i\sin(120°)\}\right]\cdot\left[9\{\cos(270°)+i\sin(270°)\}\right] = $ _____ (rectangular form)

   (A) $6+6\sqrt{3}\,i$ (B) $6\sqrt{3}+6i$ (C) $18+18\sqrt{3}\,i$ (D) $18\sqrt{3}+18i$ (E) $36+36\sqrt{3}\,i$

29. If it is 10:00 PM, how long will it be until the hour hand and the minute hand are pointing in the same direction? (nearest second)

   (A) 54 min 24 sec   (B) 54 min 27 sec   (C) 54 min 30 sec   (D) 54 min 33 sec   (E) 54 min 36 sec

30. Simplify: $\dfrac{\sin^2\theta-\cos^2\theta}{\sin^4\theta-\cos^4\theta}$

   (A) 1   (B) $\sec\theta$ (C) $\csc\theta$ (D) $\sec\theta\csc\theta$ (E) -1

31. A ball is dropped from a height of 162 inches. On each bounce, the ball rebounds one third of the distance it fell. Find the total distance traveled by the ball. (nearest inch)

   (A) 324 in   (B) 365 in   (C) 405 in   (D) 446 in   (E) 486 in

32. Consider the hyperbola $49y^2-16x^2-98y-64x-799=0$. The coordinates of the vertices are (a,b) and (a,c). $a+b+c = $ _____.

   (A) -4 (B) -2 (C) 0   (D) 2   (E) 4

33. Which of the following are considered to be "resistant" measures?
   I. mean   II. median   III. standard deviation   IV. IQR

   (A) I, III only   (B) II, IV only   (C) I, II, III only   (D) I, II, IV only   (E) I, II, III, IV

34. If the points on a Normal probability plot lie close to _____, this indicates that the data are Normal.

   (A) a concave up curve   (B) a concave down curve   (C) a straight line
               (D) a parabolic curve   (E) a logarithmic curve

35. At Success STEM Academy, 45% of the seniors take AP Calculus, 30% take AP Statistics, and 10% take both courses. Find the probability that a randomly selected senior does not take either course.

(A) 0.15   (B) 0.20   (C) 0.25   (D) 0.30   (E) 0.35

36. If you reject H_0 when H_0 is true, you have committed a Type ___ error.

(A) I   (B) II   (C) III   (D) IV   (E) V

Age	13	14	15	16	17	18
Mileage	25	31	36	40	46	51

For problems 37 and 38, use the table above and the following information.
Brian Landeros was a cross-country runner at Valley View. The table above shows the average number of miles he ran each week of the cross-country season from his seventh-grade year, when he was 13 years old, through his senior year, when he was 18 years old.

37. Brian's average weekly mileage increased by about _____ miles each year that he ran for Valley View. (nearest tenth)

(A) 4.5   (B) 4.8   (C) 5.1   (D) 5.4   (E) 5.7

38. If Brian continues to increase his average weekly mileage at the same rate, he should average _____ miles per week when he is 24 years old. (nearest tenth)

(A) 75.0   (B) 77.2   (C) 79.4   (D) 81.6   (E) 83.8

For problems 39 and 40, consider the ellipse $9x^2 + 4y^2 - 54x + 32y + 1 = 0$.

39. Find the eccentricity of the ellipse. (nearest thousandth)

(A) 0.661   (B) 0.682   (C) 0.703   (D) 0.724   (E) 0.745

40. Find the slope, a negative value, of the line tangent to the ellipse when x = 5.

(A) $-\dfrac{\sqrt{2}}{2}$ (B) $-\dfrac{\sqrt{3}}{2}$ (C) -1 (D) $-\dfrac{\sqrt{5}}{2}$ (E) $-\dfrac{\sqrt{6}}{2}$

41. Find the sum of the first 10 terms of the sequence $4, 5, 9, 14, 23, 37, \ldots$

(A) 654   (B) 656   (C) 658   (D) 660   (E) 662

For problems 42, 43 and 44, consider the function $f(x) = x^3 - 6x^2 + x + 2$.

42. The graph of $y = f(x)$ has a point of inflection at the point (a, b). $a + b =$ _____.

  (A) −14   (B) −12   (C) −10   (D) −8   (E) −6

43. The local maximum of $f(x)$ over the interval $[1,5] = c$ and the local minimum of $f(x)$ over the interval $[1,5] = d$. Find $|c - d|$. (nearest tenth)
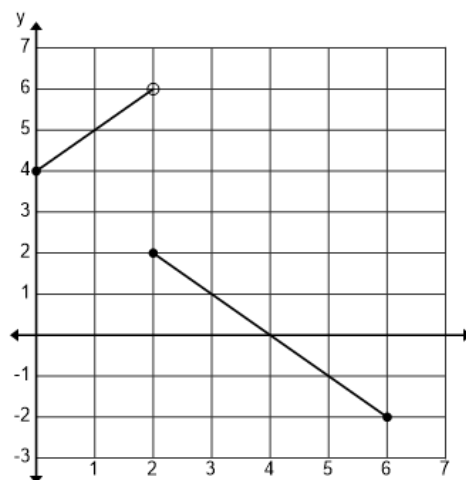
  (A) 20.0   (B) 22.0   (C) 24.0   (D) 26.0   (E) 28.0

44. Find the area of the fourth quadrant region bounded by the graph of $y = f(x)$ and $y = -2$. (nearest tenth)

  (A) 62.2   (B) 64.2   (C) 66.2   (D) 68.2   (E) 70.2

45. The graph of the function f consists of two line segments as shown on the right. Find the value of $\int_0^6 |f(x)| dx$? (nearest whole number)

  (A) 10
  (B) 11
  (C) 12
  (D) 13
  (E) 14

46. If $h(x) = \dfrac{(x-6)^3}{243} - \dfrac{(x-6)^2}{81} + \dfrac{(x-6)}{27} - \dfrac{1}{9}$, then $h'''(6) =$ _____.

  (A) $\dfrac{3 \cdot 2 \cdot 1}{27}$ (B) 0   (C) $\dfrac{3 \cdot 2 \cdot 1}{81}$ (D) 6   (E) $\dfrac{3 \cdot 2 \cdot 1}{243}$

47. If $\int_1^x f(t)dt = \dfrac{15x}{\sqrt{9x^2 + 6}} - 2$, then $\int_1^\infty f(t)dt =$ _____.

  (A) −2   (B) 0   (C) 3   (D) 9   (E) 13

Month	January	February	March	April	May	June
Amount	$125	$150	$100	$175	$200	$150

Use the table above and the following information for problems 48 and 49.
Landon makes a deposit into his savings account at the end of each month. The table above shows the deposits he made the first six months of 2021.

48. Find the sum of the mean, median, mode and range of the amounts of his deposits in the table.

   (A) $475　　　　(B) $500　　　　(C) $525　　　　(D) $550　　　　(E) $575

49. In July, Landon deposited $300 into his saving account. Which of the following values increased after this deposit?
   　I. mean　　　II. median　　　III. mode　　　IV. range

   (A) I, IV only　　(B) I, II, IV only　　(C) I only　　(D) II, IV only　　(E) I, II, III, IV

50. Professor Jenkins and two of his students spent a week in Brazil studying Spotted Peacock Bass. They collected and weighed 360 fish during the week. The five-number summary below is from this study.
   Minimum: 1.2 lb　　First Quartile: 6.6 lb　　Median: 9.8 lb　　Third Quartile: 14.2 lb
   Maximum: 27.6 lb
   Approximately how many fish weighed between 6.6 lb and 14.2 lb?

   (A) 90　　　　(B) 120　　　　(C) 150　　　　(D) 180　　　　(E) 210

51. Consider the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$. Points A, B, C and D lie on the graph of the ellipse such that quadrilateral ABCD is a rectangle. Find the maximum area of the rectangle. (nearest tenth)

   (A) 12.6　　　　(B) 14.8　　　　(C) 16.0　　　　(D) 17.2　　　　(E) 18.4

52. If $f(x) = x^2 - 16$ and $g(x) = 4x + 6$, then $f\big(g(12)\big) =$ _____.

   (A) 2894　　　　(B) 2900　　　　(C) 2906　　　　(D) 2912　　　　(E) 2918

53. Oscar's Lawn Service charges $28 per hour to mow and edge a lawn. If his crew takes an average of 2 hr 40 minutes to mow and edge my lawn, how much did I pay Oscar to take care of my yard last year? His crew came to my house 36 times last year.

   (A) $2684.00　　(B) $2688.00　　(C) $2692.00　　(D) $2696.00　　(E) $2700.00

54. A circle is inscribed in a square. The square has an area of 134 cm^2. Find the area of a circle inscribed in the square. (nearest whole number)
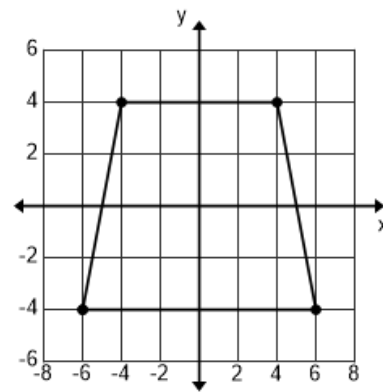
   (A) 97 cm^2　　(B) 99 cm^2　　(C) 101 cm^2　　(D) 103 cm^2　　(E) 105 cm^2

55. Find the area of the trapezoid on then right.
    (nearest whole number)

    (A) 72   (B) 74   (C) 76   (D) 78   (E) 80

56. Find the perimeter of the trapezoid on the right.
    (nearest tenth)

    (A) 36.5   (B) 36.8   (C) 37.1   (D) 37.4   (E) 37.7
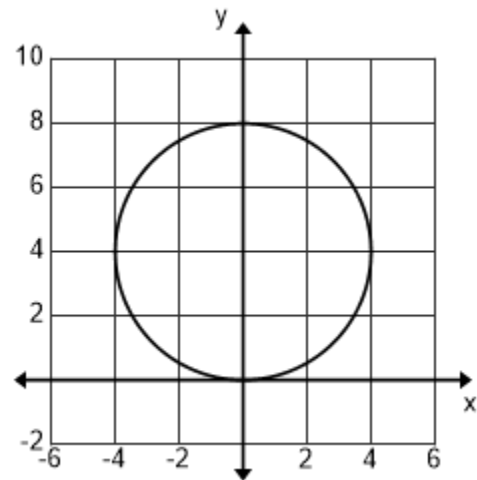
57. Find the area of the triangle with vertices $(2,4,6)$, $(5,3,1)$ and $(9,7,5)$. (nearest tenth)

    (A) 18.4   (B) 18.7   (C) 19.0   (D) 19.3   (E) 19.6

58. Find the polar equation for the circle shown on the right.

    (A) $r = 8\cos\theta$
    (B) $r = 8\sin\theta$
    (C) $r = 4\cos\theta$
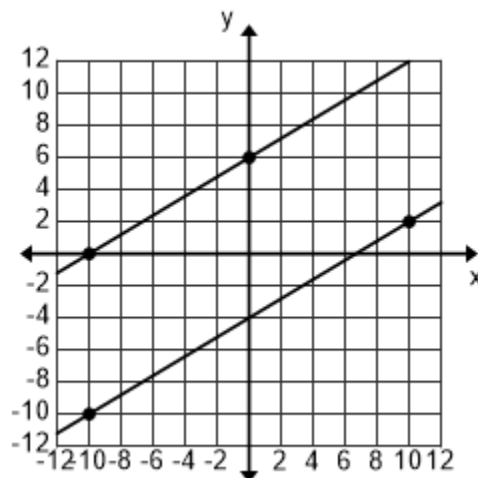   (D) $r = 4\sin\theta$
    (E) $r = 4\sin(2\theta)$

59. Find the parametric equations for the circle shown on the right.

    (A) $\begin{aligned} x(t) &= 4\cos(t) \\ y(t) &= 4\sin(t) \end{aligned}$
    (B) $\begin{aligned} x(t) &= 4\cos(t) \\ y(t) &= 8\sin(t) \end{aligned}$
    (C) $\begin{aligned} x(t) &= 8\cos(t) \\ y(t) &= 8\sin(t) \end{aligned}$
    (D) $\begin{aligned} x(t) &= 4 - 4\cos(t) \\ y(t) &= 4 + 4\sin(t) \end{aligned}$
    (E) $\begin{aligned} x(t) &= 4\cos(t) \\ y(t) &= 4 + 4\sin(t) \end{aligned}$

60. Find the distance between the parallel lines
    shown on the right. (nearest tenth)

    (A) 8.6
    (B) 9.0
    (C) 9.3
    (D) 9.7
    (E) 10.0

2021-2022 TMSCA HSM Test 6 Key

1. D	21. E	41. D
2. C	22. B	42. C
3. A	23. E	43. C
4. E	24. D	44. E
5. E	25. B	45. E
6. C	26. B	46. E
7. B	27. B	47. C
8. D	28. D	48. D
9. A	29. D	49. A
10. D	30. A	50. D
11. D	31. A	51. C
12. E	32. C	52. B
13. E	33. B	53. B
14. C	34. C	54. E
15. D	35. E	55. E
16. B	36. A	56. A
17. A	37. C	57. E
18. B	38. D	58. B
19. A	39. E	59. E
20. B	40. B	60. A

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