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GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. $817 + (-451) + (-377)$ = _____

A. -17 B. -11 C. 21 D. -19 E. -13

2. $611\frac{5}{8} - 276\frac{4}{5}$ = _____

A. $334\frac{33}{40}$ B. $336\frac{33}{40}$ C. $333\frac{37}{40}$ D. $334\frac{19}{40}$ E. $333\frac{21}{40}$

3. $992 \times 997 \times 0.01$ = _____ (nearest ten)

A. 9,800 B. 9,900 C. 9,880 D. 9,980 E. 9,890

4. $\frac{17}{8} \div 4$ = _____ (nearest thousandth)

A. 0.532 B. 0.53 C. 1.531 D. 1.532 E. 0.531

5. 1,400,000,000 millimeters = _____ kilometers

A. 140 B. 1,400 C. 14,000 D. 14 E. 1.4

6. 14^2 = _____

A. 176 B. 186 C. 192 D. 196 E. 216

7. If A = 99 + 317, B = 1,000 – 611, and C = 631 + 17 – 187, then $A + B - C$ = _____ (Arabic numeral).

A. 366 B. 394 C. 362 D. 344 E. 372

8. Simplify: $-\frac{3}{8}(24a + 16b) - (-4a + 3b) + 3(-5b - a)$

A. $8a - 12b$ B. $-8a - 12b$ C. $-8a - 24b$ D. $8a - 24b$ E. $-8a + 12b$

9. 28.8 is what percent of 64?

A. 45% B. 35% C. 27.5% D. 45.5% E. 31.75%

10. If $A = 3$ and $B = \sqrt{800}$, then the product of AB lies between which two integers?

A. 28 and 29 B. 29 and 30 C. 31 and 32 D. 83 and 84 E. 84 and 85

11. What number when divided by 12 gives a quotient of 376 with a remainder of 9?

A. 4,561 B. 4,581 C. 4,391 D. 4,481 E. 4,521

12. The _____ term of the sequence $5, \frac{19}{3}, \frac{23}{3}, \ldots$ is $\frac{55}{3}$.

A. 11^{th} B. 10^{th} C. 9^{th} D. 12^{th} E. 13^{th}

13. $3^2 \cdot 11 \cdot 23$ is the prime factorization of which number?

A. 2,437 B. 2,117 C. 2,277 D. 2,357 E. 2,187

14. Shamika has five US coins that are worth a total of 35¢. How many of Shamika's coins are nickels?

A. 4 B. 3 C. 2 D. 1 E. 0

15. What is the positive difference between 13^2 and 21^2?

A. 64 B. 127 C. 415 D. 272 E. 233

16. If A = 1, B = 2, C = 3, …, X = 24, Y = 25, and Z = 26, what is the sum of the letters of the word *CHAMPIONSHIP*?

A. 127 B. 129 C. 131 D. 133 E. 135

17. Evaluate $\frac{-3\left|\frac{1}{2}a\right|-6b}{-(ab)^2}$ for $a = -8$ and $b = -\frac{1}{2}$.

A. $-\frac{7}{16}$ B. $-\frac{3}{4}$ C. $\frac{9}{16}$ D. $\frac{3}{4}$ E. $\frac{7}{8}$

18. Two positive integers have a sum of 140 and are in a ratio of 12:23. What is the value of the larger integer tripled?

A. 144 B. 192 C. 240 D. 276 E. 324

19. What is the product of the median and range of the data in the stem-and-leaf plot?

4	6 7 7
5	0 1 3 4 5 6 6
6	0 0 4

key: 4|6 = 46

A. 972 B. 1,008 C. 3,456 D. 1,045 E. 952

20. What is the LCM of the numbers 90, 75, and 165?

A. 4,750 B. 5,250 C. 5,150 D. 4,950 E. 4,650

21. If 55% of a number *m* is 121, what is $\frac{5}{44}$ of *m*?

A. 35 B. 20 C. 30 D. 45 E. 25

22. What is the value of $(-2 \blacklozenge 1) \blacklozenge (3 \blacklozenge 2)$, if $a \blacklozenge b = 2(a-b)^2$?

A. 144 B. 578 C. 432 D. 512 E. 576

23. What is the greatest prime factor of 15! ?

A. 11 B. 13 C. 17 D. 7 E. 3

24. Point *A* has coordinates $(-19, -6)$. What are the coordinates of point *A* if it is rotated clockwise about the origin by 270°?

A. (6, 19) B. $(-19, 6)$ C. $(19, 6)$ D. $(6, -19)$ E. $(-6, -19)$

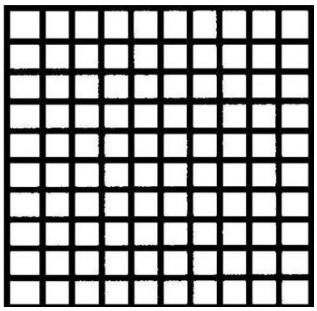
25. A math tutor's fee increased by \$13 an hour. This is a 26% increase from her original fee. What is the tutor's new hourly fee?

A. \$55 an hour B. \$57 an hour C. \$65 an hour D. \$58 an hour E. \$63 an hour

26. $\frac{1-3+5-7+9}{11-13+15-17+19} + \frac{5^2-4^2+3^2-2^2+1^2}{1+2+3+4+5} + \frac{1+3+5}{7+9+11} =$ _____

A. 1⅓ B. 2⅓ C. 1 D. 2⅔ E. 1⅔

27. How many squares can be found in the 10×10 square grid below?



A. 385 B. 1,000 C. 295 D. 415 E. 225

28. 118_{14} = _____ (base 11)
A. 177 B. 183 C. 181 D. 175 E. 189

29. 33^2 = _____
A. 1,229 B. 1,029 C. 1,089 D. 1,129 E. 969

30. Two angles are supplementary. If one angle is 36° less than twice the other angle, what is the complement of the smaller angle?
A. 16° B. 12° C. 20° D. 18° E. 22°

31. If $f(x) = -x^2 - 4$ and $g(x) = 6x - 1$, what is the value of $f(-3) - g(-2)$?
A. 4 B. 17 C. 0 D. -6 E. -7

32. In a hotdog eating contest, only whole hotdogs were counted as being eaten. The 1st place participant ate twice as many hotdogs as the 2nd place contestant, 3 times as many as the 3rd place contestant, and 4 times as many as the 4th place contestant. Altogether, the four contestants ate fewer than 60 hotdogs. What is the greatest number of hotdogs the 1st place contestant could have eaten?
A. 24 B. 12 C. 36 D. 18 E. 28

33. What are the roots of the equation $9x = 5x^2 + 4$?
A. $\left\{\frac{3}{5}, 1\right\}$ B. $\left\{-\frac{4}{5}, -1\right\}$ C. $\left\{-3, \frac{3}{5}\right\}$ D. $\left\{\frac{4}{5}, 1\right\}$ E. $\left\{-\frac{3}{5}, \frac{4}{5}\right\}$

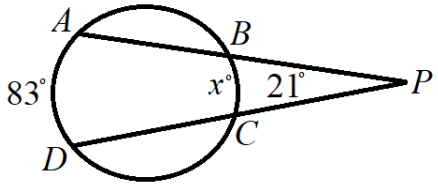
34. What is the product of the number of faces, edges, and vertices of a dodecagonal prism?
A. 12,256 B. 17,576 C. 10,796 D. 14,056 E. 12,096

35. Using the set of numbers, $-2, 5, 7, -2, 21, 0, -1$, and 4, which of the following is ordered from least to greatest?
A. mean, median, mode B. mean, mode, median C. median, mode, mean D. mode, median, mean E. mode, mean, median

36. A rectangular piece of land will be set aside and used as an animal rehabilitation refuge. The rectangular piece of land measures 6×10^5 meters by 5×10^4 meters. In scientific notation, what will be the area of the rehabilitation refuge be in squared kilometers?
A. $3 \times 10^4\ km^2$ B. $3 \times 10^{10}\ km^2$ C. $3 \times 10^9\ km^2$ D. $3 \times 10^6\ km^2$ E. $3 \times 10^{21}\ km^2$

37. Line *A* passes through the points $(-3, 7)$ and $(-1, 5)$. Line *B* passes through the points (8, 18) and (6, *y*). Line *C* passes through the points (11, -1) and (*x*, 5). If lines *A* and *B* are perpendicular and lines *A* and *C* are parallel, what is the value of *x* – *y*?
A. -11 B. -5 C. -7 D. -13 E. -9

38. What is the value of *x* below?



A. 62 B. 31 C. 41 D. 52 E. 76

39. Solve for *x*: $\frac{n-3}{x+4} = \frac{5}{6}$
A. $x = \frac{6n-38}{5}$ B. $x = \frac{6n+38}{5}$ C. $x = \frac{5n-38}{6}$ D. $x = \frac{5}{6}(n-3)$ E. $x = \frac{5}{6}(n+3)$

40. What is the area of the quadrilateral with its vertices located at $(4, -4), (-4, 6), (-8, -5)$, and $(8, 4)$?
A. 112 units^2 B. 124 units^2 C. 126 units^2 D. 116 units^2 E. 120 units^2

41. Which of the following is equivalent to $5\log_4(7) + 2\log_4(3) - \log_4(6)$?
A. $\log_4\left(\frac{5^7 \cdot 2^3}{6}\right)$ B. $\log_4(35 + 6 - 6)$ C. $\log_4\left(\frac{7^5 \cdot 3^2}{6}\right)$ D. $\log_4(7^5 + 3^2 - 6)$ E. $\log_4(35 + 9 - 6)$

42. $\sqrt{\sqrt[3]{343} + \sqrt[3]{729}}$ = _____
A. 19 B. 17 C. 7 D. 13 E. 4

43. If $\pi = 3$, what is the diameter of the base of a cone with a total surface area of 432 cm^2 and a slant height that is three times the length of the radius?
A. 18 cm B. 16 cm C. 10 cm D. 14 cm E. 12 cm

44. $\frac{(3a^{-2}b)^3}{(2a^{-5}b^2)^2} \cdot \frac{16a^{-2}b^3}{9ab^2} \div \left(\frac{3a^3b^2}{2ab}\right)^2 =$ _____
A. $\frac{16}{3a^3b^2}$ B. $\frac{16a^3}{3b^2}$ C. $\frac{8a^2}{9b^3}$ D. $\frac{8}{9a^2b^3}$ E. $\frac{16a}{3b^2}$

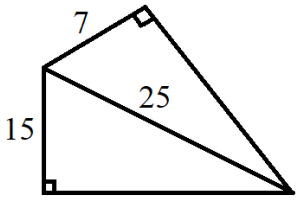
45. What is the sum of the first 20 triangular numbers?
A. 1,260 B. 1,540 C. 1,480 D. 1,420 E. 1,360

46. Simplify by rationalizing the denominator: $\frac{12+2\sqrt{3}}{4-3\sqrt{3}}$
A. $\frac{8+5\sqrt{3}}{4}$ B. $\frac{8-5\sqrt{3}}{4}$ C. $6 - 4\sqrt{3}$ D. $-6 + 4\sqrt{3}$ E. $-6 - 4\sqrt{3}$

47. If $A = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -4 & 2 \end{bmatrix}$, then $A^2 - B^2 = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$. What is the value of $wx + yz$?
A. -78 B. -96 C. 72 D. -30 E. -44

48. $103_4 \times 32_4$ = _____ (base 4)
A. 10022 B. 11032 C. 11012 D. 21012 E. 10312

49. What is the perimeter of the quadrilateral below?



A. 47 units B. 71 units C. 67 units D. 66 units E. 91 units

50. The geometric mean of *a*, *b*, and *c* is $8\sqrt[3]{3}$. If $a = 12$ and $b = 8$, what is the value of *c*?
A. 22 B. 18 C. 24 D. 12 E. 16

2020 – 2021 TMSCA Middle School State Mathematics Championship Test Answer Key

1. B
2. A
3. E
4. E
5. B
6. D
7. D
8. C
9. A
10. E
11. E
12. A
13. C
14. B
15. D
16. C
17. C
18. D
19. A
20. D
21. E
22. D
23. B
24. D
25. E
26. E
27. A
28. E
29. C
30. D
31. C
32. A
33. D
34. E
35. D
36. A
37. A
38. C
39. A
40. D
41. C
42. E
43. E
44. A
45. B
46. E
47. B
48. A
49. D
50. E

17. If $a = -8$ and $b = -\frac{1}{2}$, then $\frac{-3\left|\frac{1}{2}a\right|-6b}{-(ab)^2} = \frac{-3\left|\frac{1}{2}(-8)\right|-6\left(-\frac{1}{2}\right)}{-\left((-8)\left(-\frac{1}{2}\right)\right)^2} = \frac{-3|-4|+3}{-(4)^2} = \frac{-3(4)+3}{-16} = \frac{-12+3}{-16} = \frac{-9}{-16} = \frac{9}{16}$.

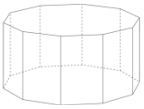
23. 15! = $15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Breaking up all these factors into their prime factorization, then 15! = $2^{11} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 13$. This shows that 13 is the largest prime factor of 15!.

27. The number of squares found in a square array of *n* squares is equal to the sum of all the perfect square numbers starting at *n* all the way down to 1. Therefore, in a 10×10 grid of squares, there are $10^2 + 9^2 + 8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 100 + 81 + 64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = 385$ squares.

31. If $f(x) = -x^2 - 4$ and $g(x) = 6x - 1$, then value of $f(-3) = -(-3)^2 - 4 = -9 - 4 = -13$ and the value of $g(-2) = 6(-2) - 1 = -12 - 1 = -13$. Therefore, the value of $f(-3) - g(-2) = -13 - (-13) = -13 + 13 = 0$.

33. First, put the given equation $9x = 5x^2 + 4$ into standard form, $Ax^2 + Bx + C = 0$, which would be $5x^2 - 9x + 4 = 0$. You can solve this quadratic equation by using the quadratic formula, which is $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$. From our equation, *A* = 5, *B* = −9, and *C* = 4. Substituting into the formula and we get $x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(4)}}{2(5)} = \frac{9 \pm \sqrt{81-80}}{10} = \frac{9 \pm \sqrt{1}}{10} = \frac{9 \pm 1}{10} = \left\{\frac{4}{5}, 1\right\}$.

34. From the picture, we see that a dodecagonal prism has 24 vertices, 36 edges and 14 faces. Therefore, the product of the number of vertices, edges and faces of a dodecagonal prism is equal to 24(36)(14) = 12,096.



35. Arrange the numbers in order from least to greatest to get $-2, -2, -1, 0, 4, 5, 7$, and 21. The mean is $\frac{-2+(-2)+(-1)+0+4+5+7+21}{8} = 4$. The median is $\frac{0+4}{2} = 2$. The mode is -2. Therefore, from least to greatest would be -2, 2, and 4, which is mode, median, and mean, respectively.

39. To solve for *x* in the equation $\frac{n-3}{x+4} = \frac{5}{6}$, first cross multiply to get 5*x* + 20 = 6*n* – 18. Next, subtract 20 from both sides of the equation to get 5*x* = 6*n* – 38. Finally divide both sides of the equation by 5 to get $x = \frac{6n-38}{5}$.

45. One way to find the sum of the first 20 triangular numbers is to list them all out and then find their sum. Another way is to know the formula for the sum of the first n^{th} triangular numbers is $\frac{n(n+1)(n+2)}{6}$. Since we are finding the sum of the first 20 triangular numbers, *n* = 20. Substitute in the formula and get $\frac{20(21)(22)}{6} = 1{,}540$.

50. The geometric mean of three numbers *a*, *b*, and *c* is $\sqrt[3]{abc}$. So, if *a* =12 and *b* = 8, then we know that $\sqrt[3]{12 \cdot 8 \cdot c} = 8\sqrt[3]{3}$. Since we know that $8\sqrt[3]{3} = \sqrt[3]{512 \cdot 3} = \sqrt[3]{1{,}536}$, then $\sqrt[3]{12 \cdot 8 \cdot c} = \sqrt[3]{1{,}536}$. $12 \cdot 8 = 96$, so $\sqrt[3]{96c} = \sqrt[3]{1{,}536}$. We can then see that $96c = 1{,}536$. Dividing both sides of the equation by 96 gives us the solution *c* = 16.