UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

District • 2022

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1. **Solve for k if $12|8k-27|-20=16$ and $k>3$.**

   (A) 3.25 (B) 3.5 (C) 3.75 (D) 4 (E) 4.25

2. **Teresa received a $125 Barnes and Noble gift card for her birthday. She used it to purchase two paperback novels for $11.95 each, one hardback travel book for $39.75, and two AP Calculus prep books for $23.65 each. If the tax rate was 8.25%, how much remained on her gift card? (nearest cent)**

   (A) $4.90 (B) $5.02 (C) $5.14 (D) $5.26 (E) $5.38

3. **Lisa made a 256 on Test A, 264 on Test B, 242 on Test C, 272 on Test D, and 248 on Test E. What score will she need to make on Test F to obtain an overall average of exactly 258?**

   (A) 260 (B) 262 (C) 264 (D) 266 (E) 268

4. **Jim earns $12.75 per hour for the first forty hours each week at his job at Oscar's Oil Change and Lube. He earns $19.15 for any hours more than forty, but not exceeding fifty. He earns $25.50 for any hours more than fifty. If Jim worked 62 hours last week, how much did he earn?**

   (A) $1002.50 (B) $1003.75 (C) $1005.00 (D) $1006.25 (E) $1007.50

5. **The Mendoza Construction Company is repaving a 16-mile stretch of highway between American Falls and Aberdeen. On a previous job, 22 workers repaved an 8 mile stretch of highway in 28 days. If 18 workers are used, how long will it take to repave the 16-mile stretch of highway? (nearest tenth)**

   (A) 67.5 days (B) 67.8 days (C) 68.1 days (D) 68.4 days (E) 68.7 days

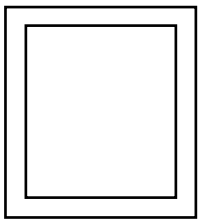
6. **Farmer Jones has a small farm with hogs, dogs and cows. The number of hogs exceeds four times the number of dogs by four. Seven times the number of dogs is twenty-eight less than four times the number of cows. Four times the sum of the number of dogs and cows is twelve greater than three times the number of hogs. How many animals are on Farmer Jones farm?**

   (A) 34 (B) 36 (C) 38 (D) 40 (E) 42

7. **Which ordered pair is a solution of the system? $y>-x-3$ and $y\le 2x+4$**

   (A) $(8,-11)$ (B) $(4,-8)$ (C) $(-2,-1)$ (D) $(4,13)$ (E) $(8,20)$

8. **The picture frame on the right is for an 8 in by 10 in picture. The width of the frame is the same on every side. If the total area of the frame, not including the picture, is 101.25 square inches, what is the width of the frame? (nearest hundredth)**



   (A) 2.05 in (B) 2.15 in (C) 2.25 in (D) 2.35 in (E) 2.45 in

9. **Bob's Burger Shack offers single and double meat burgers with the following optional condiments: mustard, ketchup, pickles, tomatoes, onions. How many different ways can I order a burger?**

   (A) 32 (B) 48 (C) 64 (D) 72 (E) 128

10. Simplify: $\dfrac{\dfrac{a^2}{b^2}+\dfrac{9c}{d}}{\dfrac{e^2}{bd}-\dfrac{5}{b^2}}$

(A) $\dfrac{a^2d+9bc}{be^2-5d}$ **(B)** $\dfrac{ad+9b^2c}{be^2-5d}$ **(C)** $\dfrac{a^2d+9b^2c}{be^2-5d}$ **(D)** $\dfrac{a^2d+9b^2c}{be-5d}$ **(E)** $\dfrac{a^2d+9b^2c}{be^2-5d^2}$

11. The midpoint of line segment $\overline{AB}$ is the point C with coordinates $(1,-1)$. The coordinates of point A are $(-3,-6)$. Find the length of line segment $\overline{AB}$. (nearest tenth)

(A) 12.2 **(B) 12.4** **(C) 12.6** **(D) 12.8** **(E) 13.0**

12. Three of the vertices of a parallelogram have coordinates $(2,-2)$, $(6,1)$ and $(-3,4)$. When finding the coordinates of the fourth vertex, you find that there are _____ distinct correct answers.

(A) 0 **(B) 1** **(C) 2** **(D) 3** **(E) 4**

13. Points A, B, C and D lie on a circle with center O. Chords $\overline{CD}$ and $\overline{AB}$ intersect at point P. If $AP=6x$, $BP=6x+9$, $DP=6x-3$ and $CD=18x+6$, then AB = _____. (nearest whole number)

(A) 26 **(B) 27** **(C) 28** **(D) 29** **(E) 30**

14. Given: Triangle ABC is similar to triangle DEF, $BC=19$, $m\angle ABC=48°$, and $m\angle BCA=73°$. If $DF=15$, then $AB-EF$ = _____. (nearest tenth)

(A) 3.9 **(B) 4.1** **(C) 4.3** **(D) 4.5** **(E) 4.7**

15. Consider the conditional statement "If a triangle is equilateral, then it is equiangular." Which of the following are true?
I. Converse II. Inverse III. Contrapositive

(A) I only **(B) III only** **(C) I, II only** **(D) I, III only** **(E) I, II, III**

16. Square ABCD has a perimeter of 60 and the center of a circle with a diameter of 30 is the point C. Find the area of the union of the two geometric figures. (nearest whole number)

(A) 743 **(B) 747** **(C) 751** **(D) 755** **(E) 759**

17. Find the domain of the function. $f(x)=\dfrac{\sqrt{4x-12}}{8x^2-8x+2}$

(A) $(-\infty,\infty)$ **(B)** $x\geq 2$ **(C)** $x\geq 3$ **(D)** $x>3$ **(E)** $x\geq 3,\ x\neq 4$

18. Suppose you could wrap a cable around the Earth along a path of constant latitude at 45° North. Assume the Earth is a sphere with a radius of 3960 miles. Find the length of the cable. (nearest whole number)

(A) 17,582 mi (B) 17,588 mi (C) 17,594 mi (D) 17,600 mi (E) 17,606 mi

19. If the area of the circle $x^2 + y^2 + 6x + 14y + k = 0$ is 201.062, what is the value of k? (nearest tenth)

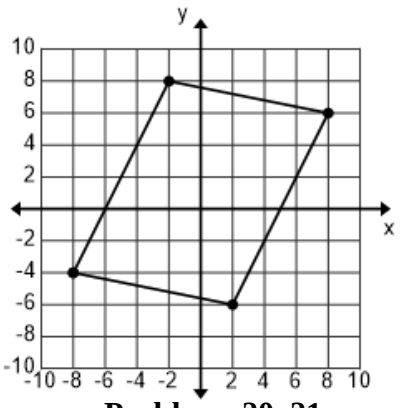
(A) –6.3 (B) –6.0 (C) –5.7 (D) –5.4 (E) –5.1

20. Find the perimeter of the quadrilateral shown on the right. (nearest tenth)

(A) 46.6 (B) 46.9 (C) 47.2 (D) 47.5 (E) 47.8

21. Find the area of the quadrilateral shown on the right. (nearest tenth)

(A) 130.8 (B) 131.1 (C) 131.4 (D) 131.7 (E) 132.0

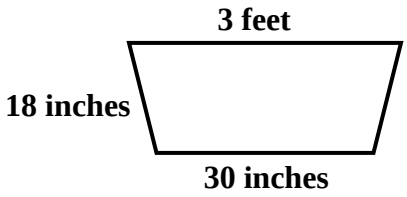


Problems 20, 21

22. Consider acute triangle $\triangle ABC$ with AB = 12 and BC = 15. If the area of the triangle equals 84.572, what is the measure of $\angle BCA$? (nearest whole number)

(A) 46° (B) 52° (C) 58° (D) 64° (E) 70°

23. Farmer Jones has a water trough that is shaped like a prism. The ends of the trough are isosceles trapezoids, and the other sides are rectangles six feet long. How many gallons of water will the trough hold when it is full? (nearest gallon)



(A) 175 (B) 177 (C) 179 (D) 181 (E) 183

24. Consider a regular octagon with an area of 204. Find the area of a circle inscribed in the octagon? (nearest whole number)

(A) 190 (B) 193 (C) 196 (D) 199 (E) 202

25. Consider the geometric sequence $6, a, b, c, 30\frac{3}{8}, d, \ldots$ If $a > 0$, then $a + b + c =$ _____.

(A) $42\frac{1}{4}$ (B) $42\frac{3}{8}$ (C) $42\frac{1}{2}$ (D) $42\frac{5}{8}$ (E) $42\frac{3}{4}$

26. Find the sum of the series $1 + 2 + 4 + 5 + 7 + 8 + 10 + 11 + 13 + 14 + 16 + 17 + \ldots + 304 + 305$

(A) 31,194 (B) 31,203 (C) 31,212 (D) 31,221 (E) 31,230

27. The amount of medicine in Norv's circulatory system when he takes a dose gradually dilutes with time. The half-life of his current medication is 36 hours. If he took an 8.00 mL dose at 8:00 AM on Monday, how much of the medicine is present in his circulatory system at 11:30 PM on Thursday of the same week? (nearest hundredth)

(A) 1.39 mL (B) 1.42 mL (C) 1.45 mL (D) 1.48 mL (E) 1.51 mL

28. Find the eccentricity of an ellipse with its center at $(-12,-5)$ and given that the ellipse is tangent to both axes. (nearest hundredth)

(A) 0.83 (B) 0.85 (C) 0.87 (D) 0.89 (E) 0.91

29. Consider the graph of the hyperbola $16x^2-9y^2-128x+18y+103=0$. The asymptotes of the graph have y-intercepts with coordinates $(0,a)$ and $(0,b)$. $a+b=$ _____.

(A) $\frac{2}{3}$ (B) 1 (C) $1\frac{1}{3}$ (D) $1\frac{2}{3}$ (E) 2

30. Consider the function $f(x)=\text{Sin}^{-1}(x)$. The domain of f(x) is _____.

(A) $-\frac{\pi}{2}<x<\frac{\pi}{2}$ (B) $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ (C) $-1<x<1$ (D) $-1\le x\le 1$ (E) $-\pi\le x\le\pi$

31. Tina is standing on a platform at the water's edge as she watches a Nimitz class carrier sail directly away from where she is watching. The high point of the carrier is 57 feet above the waterline and Tina's eye level is 32 feet above the waterline. How far is the high point of the carrier from Tina when the high point of the carrier disappears from her sight? The radius of the earth is 3960 miles. (nearest tenth)

(A) 15.9 mi (B) 16.2 mi (C) 16.5 mi (D) 16.8 mi (E) 17.1 mi

32. The period of $h(x)=\frac{1}{k}\sin(3\pi kx)$ is 4. Find the amplitude of $h(x)$.

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{2}$ (C) 2π (D) 3 (E) 6

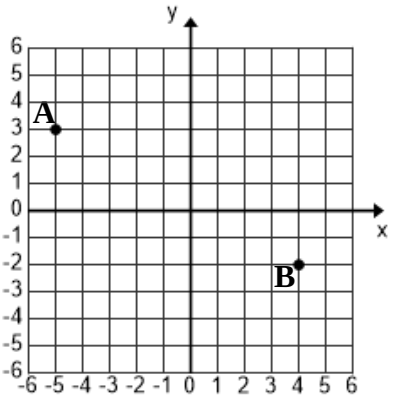
33. Consider the sequence $3,4,7,10,16,21,30,40,a,b,120,...$ $a+b=$ _____.

(A) 138 (B) 139 (C) 140 (D) 141 (E) 142

34. Consider the graph of the polar equation $r=\frac{3}{1+2\sin\theta}$. The coordinates of the foci are (a,b) and (a,c). $a+b+c=$ _____.

(A) 2 (B) $1+2\sqrt{2}$ (C) 4 (D) $2+2\sqrt{3}$ (E) 6

35. Find the equation of a line such that every point on the line is the same distance from point A as it is from point B.



(A) $18x-10y+15=0$
(B) $9x-5y+6=0$
(C) $9x-5y+7=0$
(D) $9x-5y+8=0$
(E) $18x-12y+13=0$

36. Find the distance between the plane $3x-5y+2z=6$ and the point $(2,3,4)$. (nearest tenth)

(A) 0.9 **(B)** 1.1 **(C)** 1.3 **(D)** 1.5 **(E)** 1.7

37. A shipment of 20 fuses contains 3 defective fuses. In how many ways can a person purchase 5 fuses and get at least 4 good fuses?

(A) 11,684 **(B)** 12,432 **(C)** 13,328 **(D)** 14,512 **(E)** 15,772

38. If $[x]=$ the greatest integer less than or equal to x, then
$[\sqrt{2}]+[\sqrt{3}]+[\sqrt{4}]+[\sqrt{5}]+[\sqrt{6}]+\ldots+[\sqrt{23}]+[\sqrt{24}]=$ _____.

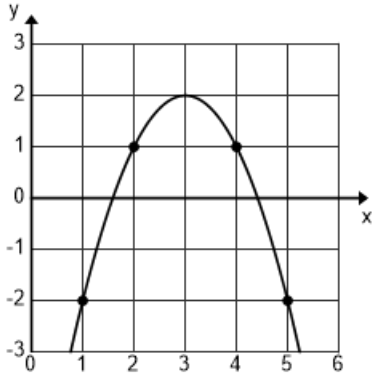
(A) 65 **(B)** 66 **(C)** 67 **(D)** 68 **(E)** 69

39. Identify the conic and calculate the angle of rotation for $4x^2-10xy+4y^2+10x+2y+1=0$.

I. Parabola **II.** Ellipse **III.** Hyperbola **IV.** 30° **V.** 45° **VI.** 60°

(A) I, IV **(B)** II, V **(C)** III, VI **(D)** II, VI **(E)** III, V

40. The coordinates of the focus of the parabola shown on the right are (a,b). $a+b=$ _____.



(A) 4 **(B)** $4\frac{1}{4}$ **(C)** $4\frac{1}{2}$ **(D)** $4\frac{3}{4}$ **(E)** $4\frac{7}{8}$

41. The graph of $f'(x)$ is shown on the right. If $f(3)=3$, then $f(1)=$ _____.

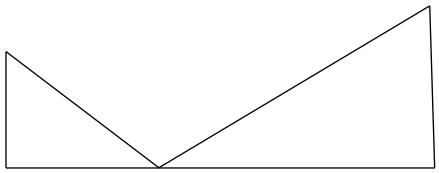
(A) 1 **(B)** $1\frac{1}{3}$ **(C)** $1\frac{1}{2}$ **(D)** $1\frac{2}{3}$ **(E)** 2

Problems 40, 41

42. Find the volume of the solid generated by revolving the region bounded by $y=-x^2+10x-17$ and $y=x^2-8x+19$ about the line $x=-6$. (nearest whole number)

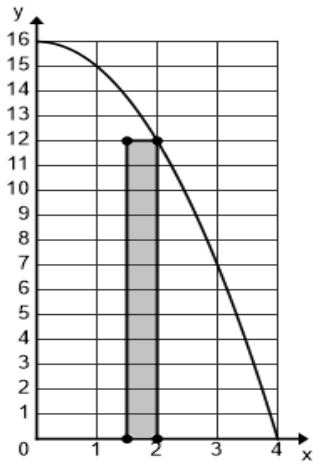
(A) 582 **(B)** 585 **(C)** 588 **(D)** 591 **(E)** 594

43. Consider two vertical posts that are 60 feet apart. Post A is 16 feet tall and post B is 24 feet tall. They are to be stayed by two wires attached to a single stake. A wire runs from ground level to the top of each post. Find the least amount of wire needed. (nearest inch)



(A) 71 ft 1 in (B) 71 ft 5 in (C) 71 ft 9 in (D) 72 ft 1 in (E) 72 ft 5 in

44. Consider the region bounded by the graph of the function $f(x) = 16 - x^2$, the x-axis, and the lines $x = 1$ and $x = 3$. Find an approximation of the area of this region by dividing the interval $[1, 3]$ into four subintervals of equal width and then constructing rectangles using the right endpoints of each subinterval. One of the rectangles is shown on the right. Find the value of this approximation.



Problems 44, 45

(A) 20.25 (B) 20.50 (C) 20.75 (D) 21.00 (E) 21.25

45. Find the exact value of the area of the region described in problem 44.

(A) $23\frac{1}{4}$ (B) $23\frac{1}{3}$ (C) $23\frac{1}{2}$ (D) $23\frac{2}{3}$ (E) $23\frac{3}{4}$

46. Consider the function $g(x)$, which is continuous on $[-4, 0]$, and with $g(-1) = 6$ and $g(-2) = 6$. If $g''(x)$ is continuous and negative on $[-4, 0]$, then which of the following must be true?

(A) $g'(-1.5) = 0$ (B) $g'(-1.5) > 0$ (C) $g'(-1.5) < 0$ (D) $g'(-3) > 0$ (E) $g'(-3) < 0$

47. A particle is moving along the x-axis. At t = 0, the particle is located at x = 0 and the acceleration of the particle is $a(t) = 12t,\ t \geq 0$. If the maximum displacement of the particle in the negative direction is -32, find the velocity of the particle at t = 0. (nearest whole number)

(A) −24 (B) −22 (C) −20 (D) −18 (E) −16

48. A conservation group releases 25 prairie dogs into a large park northeast of Lubbock at 1:00 AM on April 1st, 2022. The carrying capacity of the park is 500 prairie dogs. On April 1st, 2025, there were 100 prairie dogs in the park. Using a logistic model, the fastest rate of growth of the prairie dog population should occur on or about _____.

(A) 06 / 01 / 2027 (B) 09 / 01 / 2027 (C) 12 / 01 / 2027 (D) 03 / 01 / 2028 (E) 06 / 01 / 2028

49. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is a famous series in mathematics. Which of the following statements are true?

I. The series is known as the harmonic series.
II. The terms of the series tend to zero as n increases.
III. The series converges.

(A) II only (B) I, II only (C) II, III only (D) I, III only (E) I, II, III

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University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2022
Answer Key

1. C	**21. E**	**41. D**
2. A	**22. A**	**42. E**
3. D	**23. E**	**43. D**
4. E	**24. B**	**44. E**
5. D	**25. E**	**45. B**
6. C	**26. C**	**46. D**
7. E	**27. D**	**47. A**
8. C	**28. E**	**48. C**
9. C	**29. E**	**49. B**
10. C	**30. D**	**50. E**
11. D	**31. B**	**51. B**
12. D	**32. E**	**52. B**
13. B	**33. D**	**53. C**
14. A	**34. C**	**54. A**
15. E	**35. C**	**55. A**
16. D	**36. B**	**56. E**
17. C	**37. C**	**57. B**
18. C	**38. E**	**58. D**
19. B	**39. E**	**59. B**
20. C	**40. D**	**60. B**

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____

50. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

(A) $(-1, 1)$ **(B) $[-1, 1]$** **(C) $(0, 2)$** **(D) $[0, 2]$** **(E) $(-\infty, \infty)$**

51. The scatterplot of some data indicated a moderately strong linear relationship between two variables. A least-squares regression line was computed and the correlation coefficient was 0.80. However, the explanatory and response variables had been switched. A new regression line was computed using the correct explanatory and response variables. The new correlation coefficient was _____.

(A) –0.80 **(B) 0.80** **(C) –1.25** **(D) 1.25** **(E) 0.40**

52. There are 250 seniors at the High Plains Stem Academy in Borger. Twenty-eight students are taking Calculus, Physics and Statistics. Thirty-six students are taking Physics and Statistics, but not Calculus. Twenty-four students are taking Calculus and Statistics, but not Physics. Forty-four are taking Calculus and Physics, but not Statistics. If 126 students are taking Physics, 110 are taking Statistics, and 144 are taking Calculus, how many seniors are not taking any of these courses?

(A) 28 **(B) 30** **(C) 32** **(D) 34** **(E) 36**

53. Tim is 54 inches tall. This places him at the 95th percentile of all boys his age. The heights for boys this age form an approximately normal distribution with a mean of 51 inches and a standard deviation of _____ inches. (nearest hundredth)

(A) 1.70 **(B) 1.76** **(C) 1.82** **(D) 1.88** **(E) 1.94**

54. In the Deschutes County lottery, the cost of a lottery ticket is $20. Six positive integers, from 1 to 30, are randomly chosen without repetition. If you correctly pick all 6 numbers, you win $1,000,000. If you correctly pick exactly 5 of the 6 numbers, you win $10,000. If you correctly pick exactly 4 of the 6 numbers, you win $1,000. When the cost of the ticket is considered, what is the expected value of a lottery ticket?

(A) –$8.92 **(B) –$8.88** **(C) –$8.84** **(D) –$8.80** **(E) –$8.76**

55. A professor at Baylor took a random sample of students at Baylor to find the proportion of students who love math. She found that 67 out of the 150 students questioned love math. A professor at Rice took a similar random sample of students at Rice and he found that 53 out of 120 students questioned love math. Calculate a 95% confidence interval for the difference between the proportions of students at the two universities who love math. (nearest thousandth)

(A) (–.114, .124) **(B) (–.124, .134)** **(C) (–.134, .144)** **(D) (–.144, .154)** **(E) (–.154, .164)**

56. A linear regression was performed using the following data points.
$A(1, 17)$, $B(3, 56)$, $C(5, 80)$, $D(7, 126)$, $E(9, 142)$
The absolute value of the residual for point C is _____. (nearest tenth)

(A) 3.4 **(B) 3.6** **(C) 3.8** **(D) 4.0** **(E) 4.2**

	Under $40,000	$40,000 - $60,000	Over $60,000	Total
Mathematics	**12**	**40**	**108**	**160**
English	**77**	**29**	**14**	**120**
Political Science	**102**	**38**	**10**	**150**
Total	**191**	**107**	**132**	**430**

57. A professor at Texas Tech believes that graduates with math degrees are more likely to have good starting salaries than graduates with English degrees or Political Science degrees. The table above shows data collected from a random sample of Tech graduates. What is the expected cell count for the cell denoting English and $40,000 - $60,000 if there is no difference between the starting salaries of the three majors? (nearest hundredth)

(A) 28.44 (B) 29.86 (C) 31.28 (D) 32.70 (E) 34.12

58. Dr. Bixler determined that there is a strong, positive, linear relationship between student IQ scores and their scores on the physics section on the regional science test. He randomly selected 90 students who competed at region and found the IQ mean was 120 with a standard deviation of 9. The mean physics score was 50 with a standard deviation of 18. His analysis also found that $r^2 = 0.64$. What is the predicted physics score for a student with an IQ of 140? (nearest whole number)

(A) 70 (B) 74 (C) 78 (D) 82 (E) 86

59. A company produces barbells that have a mean weight of 45 pounds with a standard deviation of 0.25 pounds. The production process is such that the weight of each barbell produced is independent of the weights of other barbells. The weights are approximately normally distributed. If three barbells are randomly selected, what is the probability that all three will weigh more than 45.25 pounds? (nearest ten-thousandth)

(A) 0.0034 (B) 0.0040 (C) 0.0046 (D) 0.0052 (E) 0.0058

60. There are more than 30,000 children in Idaho needing a volunteer to listen to them read after school. The I.E.A. is attempting to estimate the proportion of all retired adults in Idaho who are willing to listen to children read after school. They plan to poll a random sample of retired adults to help them estimate this proportion. Of the following, which is the smallest sample size that will ensure a margin of error of no more than 4 percent for a 96% confidence interval estimate of the proportion of retired adults in Idaho who are willing to listen to children read after school?

(A) 435 (B) 670 (C) 905 (D) 1130 (E) 1355