TMSCA HIGH SCHOOL MATHEMATICS TEST #5©

DECEMBER 2, 2023

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. Laplace's Law for surface tension of a spherical membrane with two surfaces is $P_i - P_0 = \frac{4\gamma}{r}$ where γ is the surface tension in $\frac{N}{m}$, r is the radius and $P_i - P_0$ is the gauge pressure in Pa. Calculate the gauge pressure inside a soap bubble 8 centimeters in diameter if the surface tension for soapy water is $\gamma = 0.025 \frac{N}{m}$.

(A) 1.5 Pa (B) 2.0 Pa (C) 2.5 Pa (D) 3.0 Pa (E) 3.5 Pa

2. Amy is looking at buying a rolling knife sharpener for her dad for his upcoming birthday. The rolling knife sharpener she is eying costs \$189 before a 8.25% tax. Amy currently works at Amy's Ice Cream and makes \$12.50 an hour before losing 20% of her paycheck to Social Security and taxes. How many hours will Amy need to work in order to be able to buy the knife sharpener? (nearest hour)

(A) 19 hours (B) 20 hours (C) 21 hours (D) 22 hours (E) 23 hours

3-4. Consider the points $A(-2,-6), B(0,7), C(2,3)$ and $D(m,4)$.

3. The line through points A and C is perpendicular to the line containing points B and D. What is the value of m?

(A) 6.45 (B) 6.55 (C) 6.65 (D) 6.75 (E) 6.85

4. Consider point $E(7,h)$ that lies on the perpendicular bisector of the line through points A and B. What is the value of h?

(A) $-\frac{23}{26}$ (B) $-\frac{11}{13}$ (C) $-\frac{21}{26}$ (D) $-\frac{10}{13}$ (E) $-\frac{19}{26}$

5. Sam is new to town and wants to go explore his neighborhood. Sam walks at an average pace of 5 miles per hour no matter the direction he is going. He first walks due north for 45 minutes then turns and walks due west for an hour and 15 minutes. How far is he from his starting point? (nearest tenth mile)

(A) 6.1 miles B) 6.4 miles (C) 6.7 miles (D) 7.0 miles (E) 7.3 miles

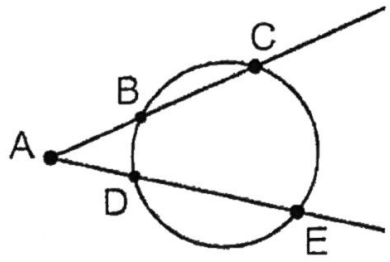
6. Elise is traveling from Lubbock to Tulsa, a distance of 449 miles. She travels 156 miles on the first day in 2 hours and 30 minutes. She travels 140 miles in 2 hours and 8 minutes on the second day. If the entire trip takes 6 hours and 45 minutes, what was Elise's speed on the third day if she completes her trip in three days? (nearest tenth mile per hour)

(A) 71.5 mph (B) 71.7 mph (C) 71.9 mph (D) 72.1 mph (E) 72.3 mph

7. Iliana and Logan are going to have a race. Iliana can run at a speed of 12.4 miles per hour and Logan can run 14.2 miles per hour. Logan gives Iliana a 40-meter head start. How many seconds will it be until Logan catches Iliana? (nearest tenth second)

(A) 49.7 sec (B) 49.9 sec (C) 50.1 sec (D) 50.3 sec (E) 50.5 sec

8-9. Consider the circle with two rays to the right.



8. If the measure of minor arc BD is $(5x-11)°$, the measure of minor arc CE is $(10x-13)°$ and $m\angle CAE = (2x+3.1)°$. What is the value of x?

(A) 8.1 (B) 8.2 (C) 8.3 (D) 8.4 (E) 8.5

9. If $\overline{AB} = 7.8$, $\overline{BC} = 8.2$, $\overline{AD} = m$, and $\overline{DE} = 10.4$, what is the length of $\overline{AE}$? (nearest tenth)

(A) 17.3 (B) 17.4 (C) 17.5 (D) 17.6 (E) 17.7

10. The perimeter of a regular octagon is 48 centimeters. What is the area of the octagon? (nearest square centimeter)

(A) 166 cm^2 (B) 168 cm^2 (C) 170 cm^2 (D) 172 cm^2 (E) 174 cm^2

11. Henry can build a desk in 3 hours and Rishi needs 4 hours to build the same desk. If Henry starts building desks 2 hours before Rishi starts working, how long will the two have to work together to build 11 desks? (nearest minute)

(A) 19 hr 34 min (B) 19 hr 37 min (C) 19 hr 40 min (D) 19 hr 43 min (E) 19 hr 46 min

12. If $s(x)$ is the oblique asymptote for the function $f(x) = \dfrac{3x^4+6x-2}{x-3}$, evaluate $s(-2)$.

(A) 45 (B) 46 (C) 47 (D) 48 (E) 49

13. A right circular cone has a volume of 482 and a total surface area of 533 and a radius greater than 5 units. What is the height of the cone? (nearest tenth)

(A) 6.0 (B) 6.1 (C) 6.2 (D) 6.3 (E) 6.4

14. What is the shortest distance from the point $(2,4,-9)$ to the line $2x-3y+8z=5$. (nearest tenth)

(A) 9.6 (B) 9.7 (C) 9.8 (D) 9.9 (E) 10.0

15. The base of a pyramid is a regular hexagon with a perimeter of 72 cm. The height of the pyramid is 16 centimeters. What is the volume of the pyramid? (nearest cubic centimeter)

(A) 1,915 cm^3 (B) 1,935 cm^3 (C) 1,955 cm^3 (D) 1,975 cm^3 (E) 1,995 cm^3

16. $2A32_{12} + 2A32_{13} = \_\_\_\_\_{}_{14}$

(A) $405B_{14}$ (B) $405C_{14}$ (C) $405D_{14}$ (D) $405E_{14}$ (E) 4060_{14}

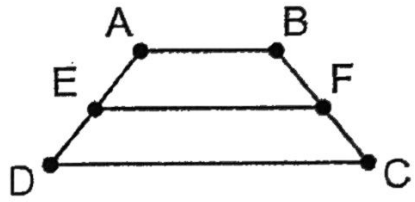
17. Three roots of $h(x)$, a fourth-degree polynomial, are $3, 4$ and $2-\sqrt{3}$. Find $b+c+d+e$ if $h(x)=x^4+bx^3+cx^2+dx+e$ and b, c, d, and e are rational numbers.

(A) −16 (B) −15 (C) −14 (D) −13 (E) −12

18. Consider a right triangle with vertices $A(-3,5), B(1,-3), C(5,b)$. If $m\angle ABC = 90°$, what is the value of b?

(A) −1.8 (B) −1.6 (C) −1.4 (D) −1.2 (E) −1.0

19-20. Consider the isosceles trapezoid to the right. $\overline{EF}$ is the median of trapezoid ABCD. $\overline{AB}=\overline{AD}=18$ and $m\angle ADC = 50°$.



19. What is the straight-line distance from point A to point F? (nearest tenth)

(A) 24.2 (B) 24.5 (C) 24.8 (D) 25.1 (E) 25.4

20. What is the area of trapezoid ABCD? (nearest integer)

(A) 408 (B) 410 (C) 412 (D) 414 (E) 416

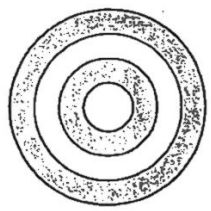
21. Find the domain of the function $f(x)=\dfrac{2x-7}{\sqrt{2x^2+7x-30}}$.

(A) $(-\infty,-6)\cup(5/2,\infty)$ (B) $(-\infty,-6]\cup(5/2,\infty)$ (C) $(-\infty,-6]\cup[5/2,\infty)$ (D) $(-6,5/2)$ (E) $[-6,5/2]$

22. Eric is at the top of a lighthouse at the edge of the ocean. His eyes are 180 feet above the bottom of the lighthouse and the lighthouse is on a cliff that is 100 feet above sea level. Eric spots a buoy floating at the top of the ocean that is slowly moving toward the horizon. How far would Eric have to travel by boat to reach the buoy just as it is past the horizon from his current viewpoint? (nearest tenth mile)

(A) 20.3 miles (B) 20.4 miles (C) 20.5 miles (D) 20.6 miles (E) 20.7 miles

23. Cale is throwing darts at four concentric circles as shown to the right. The radius of each circle is 1 inch, 2 inches, 3 inches and 4 inches, respectively. He is awarded points based on which circle his dart lands in. If the dart lands in the outermost shaded region, he will earn 1 point. The point values increase by 1 for each more inward circle until he gets to the center which is worth 5 points. What is the expected value of any throw, given the dart at least lands within the circle with radius 4 inches? (nearest tenth)



(A) 1.6 (B) 1.7 (C) 1.8 (D) 1.9 (E) 2.0

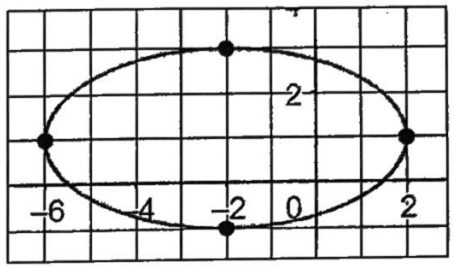
24. Find the area of the quadrilateral with vertices $A(-2,-3,14)$, $B(0,-4,8)$, $C(4,2,20)$ and $D(6,8,35)$. (nearest integer)

(A) 70 (B) 71 (C) 72 (D) 73 (E) 74

25. If the angle between the vectors $u=\langle 4,-2\rangle$ and $v=\langle -6,b\rangle$ is $110.2°$, what is the value of b if $b<0$? (nearest hundredth)

(A) −5.66 (B) −5.65 (C) −5.64 (D) −5.63 (E) −5.62

26-27. Consider the ellipse to the right.



26. What is the eccentricity of the ellipse? (nearest hundredth)

(A) 0.87 (B) 0.88 (C) 0.89 (D) 0.90 (E) 0.91

27. The line $4x-7y=-7$ intersects the ellipse at two points. What is the length of the chord created inside the ellipse by the intersecting line? (nearest hundredth)

(A) 5.60 (B) 5.62 (C) 5.64 (D) 5.66 (E) 5.68

28. A house is sitting on a rectangular lot that has a diagonal length of 301.6 feet and an area of 1 acre. What is the perimeter of the rectangular lot? (nearest foot)

(A) 836 feet (B) 838 feet (C) 840 feet (D) 842 feet (E) 844 feet

29. The graph of $r=5+5\cos\theta$ can be classified as a _____.

(A) Limaçon (B) Lemniscate (C) Circle (D) Cardioid (E) Rose Curve

30. Consider the sphere $x^2+y^2+z^2-6x+ky-8z=-20$. If the volume of the sphere is 288π cubic units, what is the value of k^2?

(A) 98 (B) 108 (C) 116 (D) 124 (E) 140

31. The graph of a curve represented by the parametric equations $x=3\sec\theta$ and $y=5\tan\theta$ has foci at the points (a,b) and (c,b). $|a-c|=$ ____. (nearest tenth)

(A) 11.5 (B) 11.6 (C) 11.7 (D) 11.8 (E) 11.9

32. Convert the rectangular equation $xy=4(x-y)$ to a polar equation.

(A) $r^2=8\sin 2\theta$ (B) $r=\dfrac{\cos\theta-\sin\theta}{4\sin\theta}$ (C) $r=\dfrac{8\cos\theta-8\sin\theta}{\sin 2\theta}$ (D) $r^2=\dfrac{4\sin\theta-4\cos\theta}{\cos 2\theta}$ (E) $r=8\tan 2\theta$

33. The temperature on a typical day in February in a city in Wyoming varies sinusoidally with a low of $-8°F$ at 4:00 AM and a high of $23°F$ at 4:00 PM. What is the expected temperature at 8:00 PM? (nearest tenth degree)

(A) $15.1°F$ (B) $15.2°F$ (C) $15.3°F$ (D) $15.4°F$ (E) $15.5°F$

34. Consider the graph of the function $f(x)=\frac{2}{3}-7\sin(3x-4)$. What is the sum of the amplitude, phase shift, vertical shift and period of the function? (nearest tenth)

(A) 10.9 (B) 11.0 (C) 11.1 (D) 11.2 (E) 11.3

35. A 3-foot piece of twine is cut to form a square and equilateral triangle of equal area. What is the perimeter of the triangle? (nearest tenth inch)

(A) 18.9 in (B) 19.0 in (C) 19.1 in (D) 19.2 in (E) 19.3 in

36. The partial fraction decomposition of $\frac{15x+9}{3x^2+5x-2}$ is $\frac{A}{x+c}+\frac{B}{3x+d}$. $A+B+c+d=$ _____.

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

37. A tank of water can be completely filled in 10 minutes and 12 seconds and completely drained in 13 minutes and 54 seconds if the plug is removed. If the tank is empty and the plug is open, how long will it take for the tank to overflow? (nearest tenth minute)

(A) 38.1 minutes (B) 38.3 minutes (C) 38.5 minutes (D) 38.7 minutes (E) 38.9 minutes

38. Consider an ellipse with vertices at $(-10,2)$ and $(4,2)$ with foci that are 5 units away from the center of the ellipse. The co-vertices are at (a,b) and (a,c). What is the value of $a+|b-c|$? (nearest tenth)

(A) 6.4 (B) 6.5 (C) 6.6 (D) 6.7 (E) 6.8

39. The unit vector orthogonal to both $u=3i+2j-k$ and $v=-2i-j+3k$ is the vector represented by $\frac{a}{\sqrt{75}}i+\frac{b}{\sqrt{75}}j+\frac{c}{\sqrt{75}}k$. $a+b+c=$ _____.

(A) -4 (B) -3 (C) -2 (D) -1 (E) 0

40. Derrick is going to his favorite ice cream parlor, Mark's Ice Cream Shop. Mark's has 16 unique flavors and 16 unique toppings. Derrick is planning on ordering a two-scoop cup and no more than 4 toppings on his ice cream cup and he will not double up on any toppings. How many unique cups of ice cream can Derrick order from Mark's Ice Cream Shop?

(A) 261,768 (B) 302,040 (C) 342,312 (D) 382,584 (E) 422,856

41. Consider the function $f(x)$ whose second derivative is $f''(x) = 30x - 4$. If $f(2) = 14$ and $f(4) = 10$, what is the value of $f'(-3)$?

(A) 15 (B) 17 (C) 19 (D) 21 (E) 23

42. Consider the region bounded by the function $f(x) = x - \sin x$, the x-axis and the lines $x = 2$ and $x = 8$. Using a right Riemann sum with six equal subintervals, approximate the area under the curve for the bounded region. (nearest tenth)

(A) 33.2 (B) 33.3 (C) 33.4 (D) 33.5 (E) 33.6

43. Find the exact value of the area of the region described in problem 42. (nearest tenth)

(A) 30.0 (B) 30.1 (C) 30.2 (D) 30.3 (E) 30.4

44-45. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \frac{3x+1}{2y}$ with the initial condition $f(2) = 3$.

44. Use Euler's method to approximate $f(3)$ using two steps of equal size from $x = 2$. (nearest thousandth)

(A) 4.160 (B) 4.164 (C) 4.168 (D) 4.172 (E) 4.176

45. Find the exact value of $f(3)$. (nearest thousandth)

(A) 4.174 (B) 4.177 (C) 4.180 (D) 4.183 (E) 4.186

46. A bubble is being blown and the volume is increasing at a constant 2 cubic inches per second. At the instant the radius of the bubble is 3 inches, at what rate is the surface area of the bubble increasing? (nearest hundredth square inch per second)

(A) 1.25 in^2/sec (B) 1.27 in^2/sec (C) 1.29 in^2/sec (D) 1.31 in^2/sec (E) 1.33 in^2/sec

47. The first derivative of the function f is given by $f'(x) = \frac{x}{5} - \cos(x)$. On the interval $(-4,4)$, for which of the following values is f concave down?

I. $x = -2.95$ II. $x = -1.5$ III. $x = 3.5$

(A) I & III (B) II & III (C) I only (D) III only (E) all of them

48. Consider the region bounded by the function $f(x)=\ln(6-x)$, the x-axis and the y-axis. This region serves as the base of a solid formed using cross sections. Each cross-section is perpendicular to the x-axis and is an isosceles right triangle with the hypotenuse as the base. Find the volume of the solid. (nearest hundredth)

(A) 1.94 (B) 1.95 (C) 1.96 (D) 1.97 (E) 1.98

49. Consider the series $f(x)=\sum_{n=1}^{\infty} a_n(x-2)^n$. It is known that $x=6$ is convergent and $x=-3$ is divergent. Which of the following statements must be true?

I. The series converges at $x=-2$
II. The series converges at $x=4$
III. The series is divergent at $x=7$

(A) I & II (B) II & III (C) I only (D) II only (E) III only

50-51. The position of a particle traveling along the y-axis can be given by $y(t)=\sin t+\cos t$ on the interval $[0,2\pi]$.

50. What is the sum of the velocity and acceleration of the particle at time $t=\frac{4\pi}{3}$?

(A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$ (E) $\sqrt{3}+1$

51. What is the total distance traveled by the particle from $t=0$ to $t=2\pi$? (nearest hundredth)

(A) 5.66 (B) 5.67 (C) 5.68 (D) 5.69 (E) 5.70

52. The assembly time required for manufacturing a baseball hat is normally distributed with a mean of 12 minutes and a standard deviation of 1 minute and 40 seconds. What is the probability that a randomly selected baseball hat took between 10 minutes and 13 minutes? (nearest thousandth)

(A) 0.605 (B) 0.607 (C) 0.609 (D) 0.611 (E) 0.613

53. Marble King, incorporated, located in West Virginia has the ability to make over one million marbles each day. The distribution of the diameter of each marble is approximately normal with a mean diameter of 13 millimeters and a standard deviation of 0.15 millimeters. Each day, a worker collects a random sample of 50 marbles from production to test for quality control. Find the probability that on any given day, that the mean diameter of the marbles collected is less than 12.97 millimeters. (nearest thousandth)

(A) 0.076 (B) 0.077 (C) 0.078 (D) 0.079 (E) 0.080

54-55. Use the table to the right for problems 54 and 55.

Years since 2000	Tuition at UMC (dollars)
1	6,460
2	6,940
4	7,420
6	7,850
8	8,170
10	8,420
12	8,750

54. Consider the years since 2000 as L_1 and the tuition at UMC in dollars as L_2. From these choices, the best way to linearize the data is to choose ___ as the independent variable and ___ as the dependent variable.

(A) $L_1, \sqrt{L_2}$ (B) $L_1, \frac{1}{L_2}$ (C) $\ln(L_1), L_2$ (D) $L_1, \ln(L_2)$ (E) $\ln(L_1), \ln(L_2)$

55. Use an appropriate model to predict the tuition at UMC 15 years after 2000. (nearest dollar)

(A) \$8,812 (B) \$8,817 (C) \$8,822 (D) \$8,827 (E) \$8,832

56. Suppose the mean SAT literature score for students in New York is a 583 with a standard deviation of 103 and the mean SAT math score is a 567 with a standard deviation of 121. George scored a 650 on the literature section of the SAT. Of the following, which is the lowest score he can make on the math section to have done at least as well on the math section as he did on the literature section?

(A) 620 (B) 630 (C) 640 (D) 650 (E) 660

57. Give the correct order of the following from least to greatest in a right-skewed distribution.
I. Mean II. Median III. Mode

(A) I, II, III (B) I, III, II (C) III, II, I (D) III, I, II (E) II, I, III

58. The average points per three darts thrown for a professional dart player is 87.3 with a standard deviation of 11.2. Assuming the average points per three darts is normally distributed, what average must a player throw to be in the 98th percentile of this distribution? (nearest tenth)

(A) 109.9 (B) 110.1 (C) 110.3 (D) 110.5 (E) 110.7

59. The probability that rain is within 10 miles of Seattle is 0.80. Given there is rain within 10 miles of Seattle, the probability that it will take Jean longer than an hour to drive to work in Seattle from Stanwood is 0.65. Find the joint probability that there is rain within 10 miles of Seattle and Jean takes longer than an hour to drive to work. (nearest hundredth)

(A) 0.44 (B) 0.46 (C) 0.48 (D) 0.50 (E) 0.52

60. Jessie recently opened a plant nursery in Louisiana that specializes in vegetable starters. She kept records of purchases and found that 76% of customers bought potato starters, 52% bought spinach starters and 28% bought both. If a customer is randomly selected, what is the probability that they order either potato starters or spinach starters, but not both?

(A) 70% (B) 71% (C) 72% (D) 73% (E) 74%

2023 – 2024 TMSCA High School Mathematics Test 5 Key

1. C
2. C
3. D
4. E
5. E
6. E
7. A
8. B
9. C
10. E
11. D
12. A
13. A
14. B
15. E
16. C
17. D
18. E
19. C
20. A
21. A
22. C
23. D
24. E
25. C
26. A
27. B
28. E
29. D
30. D
31. C
32. C
33. C
34. C
35. D
36. A
37. B
38. E
39. D
40. C
41. B
42. A
43. D
44. E
45. D
46. E
47. B
48. A
49. D
50. B
51. A
52. D
53. D
54. E
55. E
56. D
57. C
58. C
59. E
60. C

2023-2024 TMSCA High School Mathematics Test 5 Selected Solutions

$$m_{AC} = \frac{9}{4}, m_{BD} = -\frac{4}{9}$$

1. $P_i - P_0 = \dfrac{4(0.025)}{.04}$

2. $1.0825 \cdot 189 = 0.8(12.5t)$

3. $y_{BD} = 7 - \dfrac{4}{9}x$

   $y_{BD} = 4$

$$\text{midpt}_{AB} = \left(-1, \tfrac{1}{2}\right), m_{AB} = \frac{13}{2}$$

4. $y = \tfrac{1}{2} - \dfrac{2}{13}(x+1)$

   $y(7)$

5. $\sqrt{\left(\dfrac{45}{60} \cdot 5\right)^2 + \left(\dfrac{75}{60} \cdot 5\right)^2}$

6. $\dfrac{449 - 156 - 140}{6.75 - 2.5 - \left(2\dfrac{2}{15}\right)}$

7. $12.4t = 14.2t - 40\left(\dfrac{100}{2.54 \cdot 12 \cdot 5280}\right); t = 0.0138 \text{ hr}$

   $3600t$

8. $2(2x + 3.1) = (10x - 13) - (5x - 11)$

9. $7.8(7.8 + 8.2) = AD(AD + 10.4)$

   $AE = AD + 10.4$

10. $\text{side} = \dfrac{48}{8} = 6 \text{ cm}$

    $\text{Area} = 8 \cdot \dfrac{6^2}{4} \tan\left(\dfrac{180° \cdot 6}{16}\right)$

11. $\dfrac{1}{3}t + \dfrac{1}{4}(t-2) = 11$

    $t = 19.714 \text{ hr}$

$$h = \frac{1446}{\pi r^2}$$

12. $s(x) = 3x^3 + 9x^2 + 27x + 87$

13. $533 = \pi r^2 + \pi r\sqrt{r^2 + \left(\dfrac{1446}{\pi r^2}\right)}; r = 8.757$

    $h = 6.002$

14. $\dfrac{|4 - 12 - 72 - 5|}{\sqrt{4 + 9 + 64}}$

15. $V = \dfrac{1}{3}\left(6 \cdot \dfrac{12^2}{4} \tan(60°)\right)(16)$

16. $2(12)^3 + 10(12)^2 + 3(12) + 2 + 2(13)^3 + 10(13)^2 + 3(13) + 2 = 11059$

    $11059_{10} = 405D_{14}$

$$m_{AB} = \frac{-8}{4}, m_{BC} = \frac{1}{2}$$

17. $f(x) = (x-3)(x-4)\left(x - 2 + \sqrt{3}\right)\left(x - 2 - \sqrt{3}\right)$

    $f(1) - 1 = b + c + d + e$

18. $y_{BC} = -3 + \dfrac{1}{2}(x-1)$

    $y_{BC}(5) = -1$

19. $\overline{AF} = \sqrt{9^2 + 18^2 - 2(9 \cdot 18)\cos 130°}$

20. $\dfrac{18 \sin 50°(18 + 18 + 36\cos 50°)}{2}$

21. $(2x-5)(x+6) > 0$

2023-2024 TMSCA High School Mathematics Test 5 Selected Solutions

22. $\theta = \arccos\left(\dfrac{3960}{3960 + 280/5280}\right)$

$\text{dist} = 3960 \cdot \theta$

23. $1\left(\dfrac{16-9}{16}\right) + 2\left(\dfrac{9-4}{16}\right) + 3\left(\dfrac{4-1}{16}\right) + 5\left(\dfrac{1}{16}\right)$

Let vector $u_{AB} = \langle 2,-1,-6\rangle$, $v_{AC} = \langle 6,5,6\rangle$, $w_{AD} = \langle 8,11,21\rangle$

$\text{Area}_{ABCD} = \text{Area}_{BAC} + \text{Area}_{CAD}$

24. $\text{Area}_{BAC} = \dfrac{1}{2}\sqrt{41\cdot 97}\sin\left(\arccos\left(\dfrac{-29}{\sqrt{41\cdot 97}}\right)\right)$

$\text{Area}_{CAD} = \dfrac{1}{2}\sqrt{97\cdot 626}\sin\left(\arccos\left(\dfrac{229}{\sqrt{97\cdot 626}}\right)\right)$

25. $110.2^\circ = \arccos\left(\dfrac{-24-2b}{\sqrt{20\left(36+b^2\right)}}\right)$

$\dfrac{(x+2)^2}{16} + \dfrac{(y-1)^2}{4} = 1$

26. $\dfrac{\sqrt{12}}{4}$

27. $\text{ellipse} \rightarrow y = \pm\sqrt{4 - \dfrac{(x+2)^2}{4}} + 1$

$\text{line} \rightarrow y = \dfrac{4}{7}x + 1$

$\text{pts} \rightarrow (-3.307,-0.890) \text{ and } (1.573,1.899)$

28. $1 \text{ acre} = 43{,}560 \text{ ft}^2$

$ab = 43560$

$a^2 + b^2 = 301.6^2$

180 ft by 242 ft

$(x-3)^2 + \left(y + k/2\right)^2 + (z-4)^2 = -20 + 25 + k^2/4$

$\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$

$\text{center} \rightarrow (0,0)$

30. $288\pi = \dfrac{4}{3}\pi r^3; r = 6$

$5 + k^2/4 = 36$

$k^2 = 124$

31. $a = 3$

$b = 5$

$c = \sqrt{34}$

$\text{foci}: \left(\pm\sqrt{34}, 0\right)$

$r^2\cos\theta\sin\theta = 4r\cos\theta - 4r\sin\theta$

32. $r = \dfrac{4\cos\theta - 4\sin\theta}{\frac{1}{2}\sin 2\theta}$

let 4AM be $t = 0$

33. $f(t) = \dfrac{-8+23}{2} - \dfrac{23+8}{2}\cos\left(\dfrac{\pi t}{12}\right)$

$f(16)$

34. $7 + \dfrac{4}{3} + \dfrac{2}{3} + \dfrac{2\pi}{3}$

2023-2024 TMSCA High School Mathematics Test 5 Selected Solutions

$36 = 4s + 3t$

35. $s^2 = \dfrac{\sqrt{3}t^2}{4}$

$\left(\dfrac{36-3t}{4}\right)^2 = \dfrac{\sqrt{3}t^2}{4}$; $t = 6.392$ in

36. $c = 2, d = -1$

$15x + 9 = A(3x-1) + B(x+2)$

let $x = -2$ and $\frac{1}{3}$ to solve for A and B

$A = 3, B = 6$

37. $\dfrac{1}{10 + \frac{12}{60}}t - \dfrac{1}{13 + \frac{54}{60}}t = 1$

38. $5^2 = 7^2 - b^2$; $b = \pm\sqrt{24}$

center $\rightarrow (-3, 2)$

co-vertices $\rightarrow (-3, 2 \pm \sqrt{24})$

39. $a = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 5$

$b = -\begin{vmatrix} 3 & -1 \\ -2 & 3 \end{vmatrix} = -7$

$c = \begin{vmatrix} 3 & 2 \\ -2 & -1 \end{vmatrix} = 1$

40. ${}_{17}C_2\left(\sum_{n=0}^{4} {}_{16}C_n\right)$

41. $f(x) = 5x^3 - 2x^2 + Cx + D$

$14 = 5(8) - 2(4) + 2C + D$

$10 = 5(64) - 2(16) + 4C + D$

$C = -130, D = 242$

$f'(x) = 15x^2 - 4x - 130$

42. $\sum_{n=3}^{8} f(x)$

43. $\int_2^8 f(x)\,dx$

44. $f(2.5) \approx 3 + \dfrac{3(2)+1}{2(3)}(0.5) = \dfrac{43}{12}$

$f(3) \approx \dfrac{43}{12} + \dfrac{3(2.5)+1}{2\left(\frac{43}{12}\right)}(0.5)$

$y^2 = \dfrac{3}{2}x^2 + x + C$; $C = 1$

45. $y = \sqrt{\dfrac{3}{2}x^2 + x + 1}$

$y(3)$

46. $\dfrac{dV}{dt} = 4\pi r^2 \cdot \dfrac{dr}{dt}$

$2 = 4\pi(3)^2 \cdot \dfrac{dr}{dt}$

$\dfrac{dr}{dt} = \dfrac{1}{18\pi}$

$\dfrac{dSA}{dt} = 8\pi r \cdot \dfrac{dr}{dt}$

47. f is CC↓ when f′ is dec

48. $\dfrac{1}{4}\int_0^5 f(x)^2\,dx$

50. $y'\left(\dfrac{4\pi}{3}\right) + y''\left(\dfrac{4\pi}{3}\right)$

51. $\int_0^{2\pi} |y'(t)|\,dt$

52. normalcdf$\left(10, 13, 12, \frac{5}{3}\right)$

53. z-Test

$\mu_0 : 13$

$\sigma : 0.15$

$\bar{x} : 12.97$

$n : 50$

$\mu : < \mu_0$

55. $\ln \hat{y} = 0.1194 \ln \hat{x} + 8.7629$

reg eq $\rightarrow Y_1$

$e^{Y_1(\ln 15)}$

56. $\dfrac{650-583}{103} = \dfrac{a-567}{121}$

58. invNorm$(0.98, 0, 1) = \dfrac{s-87.3}{11.2}$

59. $0.8(0.65)$

60. $0.76 + 0.52 - 2(0.28)$