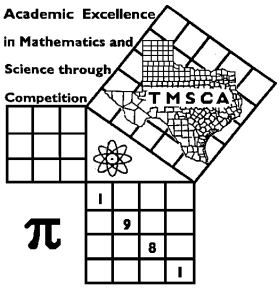


TMSCA HIGH SCHOOL MATHEMATICS DISTRICT WARM-UP (UIL G) ©

2022

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

2021-2022 TMSCA High School Math District Warmup Test

1. **Solve for k:** $\sqrt{3k+1}-1=\sqrt{k^2-16}$

   (A) 4.8 (B) 5.0 (C) 5.2 (D) 5.4 (E) 5.6

2. **Consider four consecutive positive integers such that three times the sum of the first and third is 168 greater than twice the sum of the second and fourth. What is the smallest of the four integers?**

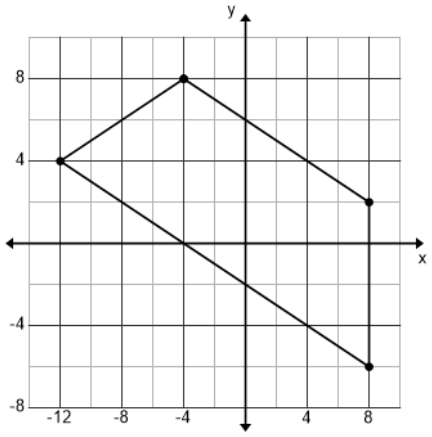
   (A) 79 (B) 82 (C) 85 (D) 88 (E) 91

3. **Find the perimeter of the quadrilateral on the right. (nearest tenth)**

   (A) 52.1 (B) 52.4 (C) 52.7 (D) 53.0 (E) 53.3

4. **Find the area of the quadrilateral shown on the right. (nearest whole number)**

   (A) 124 (B) 126 (C) 128 (D) 130 (E) 132



Problems 3, 4

5. **Charlie went to Mazo's Burgers and ordered 2 cheeseburgers, one vanilla shake and one peach pie for $13.66. Bill went to Mazo's Burgers and ordered 4 cheeseburgers, 4 vanilla shakes and 4 peach pies for $35.60. A cheeseburger cost four times as much as a peach pie. What will Salem pay for 6 cheeseburgers, 5 vanilla shakes and 4 peach pies?**

   (A) $47.95 (B) $47.98 (C) $48.01 (D) $48.04 (E) $48.07

6. **Julie can wash and wax a car by herself in 45 minutes. Mindy can wash and wax a car by herself in 40 minutes. On Saturday, they have 28 cars to wash and wax. Mindy starts work at 8:00 AM. She is joined by Julie at 9:30 AM and they work together, without taking a break, until all 28 cars are finished. What time did they finish? (nearest minute)**

   (A) 6:35 PM (B) 6:39 PM (C) 6:43 PM (D) 6:47 PM (E) 6:51 PM

7. **How many odd three-digit counting numbers less than 400 can be formed if the second digit cannot be a 4 or 6. Repetition is permitted.**

   (A) 60 (B) 80 (C) 100 (D) 120 (E) 150

8. **Steve lives in Anchorage, which is located at a latitude of 61.218° N. If the radius of the earth is 3960 miles, how far does Steve travel in 6 hours due to the rotation of the earth? (nearest mile)**

   (A) 2995 mi (B) 2999 mi (C) 3003 mi (D) 3007 mi (E) 3011 mi

9. **A circle is inscribed in a regular pentagon. The area of the circle is 224. Find the area of the pentagon. (nearest tenth)**

   (A) 256.6 (B) 257.8 (C) 259.0 (D) 260.2 (E) 261.4

10. Find the number that is $\frac{4}{7}$ of the way from $2\frac{1}{2}$ to $9\frac{2}{3}$.

(A) $6\frac{4}{7}$ **(B)** $6\frac{25}{42}$ **(C)** $6\frac{13}{21}$ **(D)** $6\frac{9}{14}$ **(E)** $6\frac{2}{3}$

11. Chad took three days to drive from Nampa to Valley View, a distance of 1640 miles. On the first day, he traveled 560 miles at an average speed of 66 mph. On the second day, he traveled 620 miles at an average speed of 68 mph. If the total drive time for the trip was 25 hours, what was his average speed on the third day? (nearest tenth)

(A) 61.6 mph (B) 61.8 mph (C) 62.0 mph (D) 62.2 mph (E) 62.4 mph

12. A ball is dropped from a height of 12 feet. On each bounce, it rebounds three-fifths of the distance it fell. How far does the ball fall on its seventh fall? (nearest tenth)

(A) 6.7 in (B) 6.9 in (C) 7.1 in (D) 7.3 in (E) 7.5 in

13. Russell has 6 guards, 6 wings and 3 posts on his basketball team. If his starting lineup has two guards, two wings and one post, how many ways can he select his starting lineup?

(A) 600 (B) 616 (C) 636 (D) 648 (E) 675

14. Two dice are rolled and the sum is noted. Find the probability that the sum is less than 6 or greater than 9?

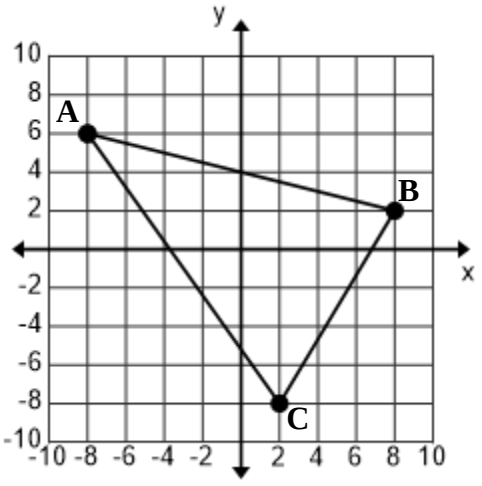
(A) $\frac{7}{18}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{17}{36}$ **(E)** $\frac{1}{2}$

15. Point D (not shown) is the centroid of ΔABC. AD = _____. (nearest tenth)

(A) 10.1 (B) 10.3 (C) 10.5
(D) 10.7 (E) 10.9

16. Find the distance from the midpoint of $\overline{AB}$ to the midpoint of $\overline{BC}$. (nearest tenth)

(A) 8.0 (B) 8.2 (C) 8.4
(D) 8.6 (E) 8.8



Problems 15, 16

17. Consider the graph of $h(x) = \frac{2x^2}{x^2 + x - 6}$. Which of the following are true?

I. x = 2 is a vertical asymptote II. x = 3 is a vertical asymptote III. y = 1 is a horizontal asymptote

(A) I only (B) II only (C) I, II only (D) II, III only (E) I, II, III

18. If $f(x)=\sqrt{25-x^2}$, $h(x)=\dfrac{6}{x-2}$, and $g(x)=\left(\dfrac{h}{f}\right)(x)$, then the domain of g(x) is _____.

(A) $[-5,2)\cup(2,5]$ (B) $[-5,5]$ (C) $(-\infty,\infty)$ (D) $(-5,5)$ (E) $(-5,2)\cup(2,5)$

19-20. The distribution of IQ scores of the students who attend the Western STEM Academy in Missoula is approximately normal with a mean score of 115 and a standard deviation of 15. Over 2,000 students attend the school.

19. To be allowed to take Mr. Newberry's Advanced Mathematics class, a student must have an IQ that is at the 95th percentile or higher of all students at the school. What is the minimum IQ score that a student is required to have in order to take this class? (nearest whole number)

(A) 140 (B) 142 (C) 144 (D) 146 (E) 148

20. Mr. Williams was asked to teach an Advanced Statistics class, but with no restrictions on who could take the class. The headmaster randomly selected 25 students to take the class. What is the probability that the mean IQ score of this group of 25 students is 120 or greater? (nearest thousandth)

(A) 0.048 (B) 0.128 (C) 0.208 (D) 0.289 (E) 0.369

21. Two different samples of radioactive materials are decaying. Sample A has a mass of 4.44 g with a half-life of 27 days. Sample B has a mass of 3.56 g with a half-life of 36 days. Find the time required for the amount remaining in sample A to equal the amount remaining in sample B. (nearest tenth)

(A) 32.2 days (B) 33.3 days (C) 34.4 days (D) 35.5 days (E) 36.6 days

22. Lisa is sitting on the roof of a 94.7-ft-tall building looking down at a park. The angle of depression from the roof to the far side of the park is 8.55° and the angle of depression to the near side of the park is 32.2°. Find the distance from the near side to the far side of the park. (nearest tenth)

(A) 475.1 ft (B) 476.2 ft (C) 477.3 ft (D) 478.4 ft (E) 479.5 ft

23. Brenda's new house has a large backyard. It is rectangular in shape and measures 90 ft by 120 ft. If she walks the perimeter of her yard every morning for 45 minutes at a speed of 3.25 mph, how many laps around her yard does she walk in a week? (nearest tenth)

(A) 211.2 (B) 212.3 (C) 213.4 (D) 214.5 (E) 215.6

24. Brenda also has a pool in the backyard, near the house. It is shaped like a large regular hexagon. Each side is 20 feet long. The pool has a constant depth of five feet. When Brenda fills the pool to 80% of its capacity, _____ gallons of water are required? (nearest gallon)

(A) 31,040 (B) 31,054 (C) 31,068 (D) 31,082 (E) 31,096

25. Two weeks before the final exam, Professor Bixler gave his students a list of 24 physics problems to solve. Twelve of these problems would be on the exam. James was able to solve 20 of the problems as he prepared for the exam. Find the probability that he was able to solve at least 11 of the problems on the final exam. (nearest thousandth)

(A) 0.284 **(B) 0.295** **(C) 0.306** **(D) 0.317** **(E) 0.328**

26-27. The equation of an ellipse is $25x^2+16y^2+50x-32y+16=0$.

26. Find the area of the ellipse. (nearest tenth)

(A) 3.7 **(B) 3.9** **(C) 4.1** **(D) 4.3** **(E) 4.5**

27. Find the eccentricity of the ellipse. (nearest hundredth)

(A) 0.42 **(B) 0.48** **(C) 0.54** **(D) 0.60** **(E) 0.66**

28. The equation of a hyperbola is $4x^2-y^2+8x-6y+4=0$. The y-intercept of the asymptote with negative slope is _____.

(A) $(0,-5.0)$ **(B) $(0,-4.5)$** **(C) $(0,-4.0)$** **(D) $(0,-3.5)$** **(E) $(0,-3.0)$**

29. A line intersects a circle, with center O, at points A and B. The equation of the line is $y=0.25x+3$ and the equation of the circle is $x^2+y^2=36$. Find the area of the sector with central angle $\angle AOB$. (nearest tenth)

(A) 38.3 **(B) 38.6** **(C) 38.9** **(D) 39.2** **(E) 39.5**

30. Convert the polar equation to rectangular form. $r=\csc\theta\cot^2\theta$.

(A) $y=-|\cos(2x)|$ **(B) $y=|.5x|$** **(C) $y^3=x^2$** **(D) $y=\sqrt{|x|}$** **(E) $y=|\sin(2x)|$**

31. The equation of a sphere is $4x^2+4y^2+4z^2-4x-32y+8z+5=0$. Find the volume of the sphere. (nearest whole number)

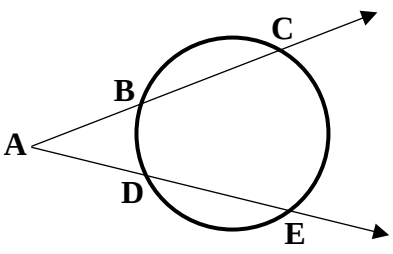
(A) 256 **(B) 259** **(C) 262** **(D) 265** **(E) 268**

32. Find the area of a triangle with vertices $(2,1,3)$, $(3,2,1)$, and $(5,2,4)$. (nearest tenth)

(A) 3.7 **(B) 3.9** **(C) 4.1** **(D) 4.3** **(E) 4.5**

33. Given: $mBC=97°$, $mCE=138°$, and $mED=91°$
Find the measure of $\angle CAE$.

(A) 51° **(B) 52°** **(C) 53°** **(D) 54°** **(E) 55°**



34. Consider a circle with center O. Points A and B lie on the circle with $m\angle AOB = 55°$. If the length of minor arc AB is 11, what is the area of the circular segment bounded by $\overline{AB}$ and minor arc AB? (nearest hundredth)

(A) 9.12 (B) 9.15 (C) 9.18 (D) 9.21 (E) 9.24

35. The length of each side of a square dartboard is 2 feet. An equilateral triangle is in the middle of the dartboard. Each side of the triangle is 6 inches long. If a dart lands in the triangle, you get 12 points. If a dart lands anywhere else on the dartboard, you get 2 points. If Jasmine throws a dart that lands on the dart board, what is the expected number of points that she will earn? (nearest hundredth)

(A) 2.27 (B) 2.34 (C) 2.41 (D) 2.48 (E) 2.55

36-37. A water trough is shaped like an inverted cone with a diameter of 6 feet and a height of 3 feet. Do not consider the thickness of the walls of the trough.

36. Rancher Rob needs to paint the outer lateral surface of the cone. If a one-gallon can of paint will cover 360 square feet, how many ounces of paint will he use? (nearest tenth)

(A) 13.6 (B) 13.8 (C) 14.0 (D) 14.2 (E) 14.4

37. After the paint dried, Rob filled the trough completely full and left to do other chores. While he is gone, a leak develops in the bottom of the cone. Water leaks out at a rate of 12 in^3 per second. Find the rate at which the height of the water in the cone is decreasing when the diameter of the surface of the water is only 4 feet. (nearest thousandth)

(A) 0.377 in/min (B) 0.384 in/min (C) 0.391 in/min (D) 0.398 in/min (E) 0.405 in/min

Short Size	Small	Medium	Large	Extra large
Waist (inches)	$30 \le \text{Waist} < 32$	$32 \le \text{Waist} < 34$	$34 \le \text{Waist} < 36$	$36 \le \text{Waist} \le 38$

38-39. The waist measurements of runners in Houston are approximately normally distributed with a mean of 33.8 inches and a standard deviation of 1.5 inches. Rick's Outlet has a large inventory of shorts in the sizes in the table above.

38. Rick's Outlet only stocks the sizes listed above. What percentage of runners in Houston will be able to find shorts in their size at Rick's Outlet? (nearest tenth)

(A) 98.6% (B) 98.9% (C) 99.2% (D) 99.5% (E) 99.8%

39. Out of thousands of Houston runners, 25 of them were randomly selected and asked their shorts size. Find the probability that exactly 6 of the 25 runners wear size small shorts. (nearest ten-thousandth)

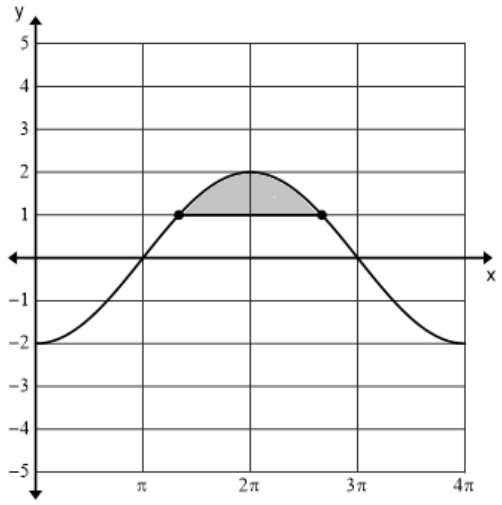
(A) 0.0336 (B) 0.0349 (C) 0.0362 (D) 0.0375 (E) 0.0388

40. Find the area of the shaded region. (nearest hundredth)

(A) 2.62 (B) 2.65 (C) 2.68 (D) 2.71 (E) 2.74

41. The shaded region is revolved about the x-axis. Find the volume of the solid generated. (nearest whole number)

(A) 21 (B) 22 (C) 23 (D) 24 (E) 25



Problems 40-41

42. Find the length of the path of a particle traveling along the graph of the parametric equations $x = \sin(t)$ and $y = e^t$ from $t = 0$ to $t = \pi$. (nearest tenth)

(A) 22.2 (B) 22.4 (C) 22.6 (D) 22.8 (E) 23.0

43. Find the sum of the series $1 + \ln 8 + \frac{(\ln 8)^2}{2} + \frac{(\ln 8)^3}{6} + \ldots + \frac{(\ln 8)^n}{n!} + \ldots$ (nearest thousandth)

(A) 7.996 (B) 7.998 (C) 8.000 (D) 8.002 (E) 8.004

44. $f(x) = \begin{cases} ax + b & \text{for } x \le 1 \\ x^2 + 1.5b & \text{for } x > 1 \end{cases}$. Given: a and b are constants and f is differentiable at x = 1. Find the value a + b.

(A) 2.5 (B) 3.0 (C) 3.5 (D) 4.0 (E) 4.5

45. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 8t + 2,\ t \ge 0$. At t = 1, the particle is at x = 7. Find the position of the particle at t = 2.

(A) 2.5 (B) 3.0 (C) 3.5 (D) 4.0 (E) 4.5

46. Rhonda's Computers carries a large supply of refurbished HP laptops with I-7 processors, 16 GB memory, and 256 GB SSD storage. The regular price is $550. All college students get the special price of $484.00. College students who competed in the UIL state meet when they were in high school get a 6% discount off of the special price. What is the final price for a college student who competed in the UIL state meet when she was in high school?

(A) $454.96 (B) $455.00 (C) $455.04 (D) $455.08 (E) $455.12

47. Consider the sequence $-16, -6, 4, 14, 24, \ldots$ Find the sum of the first 25 terms of the sequence.

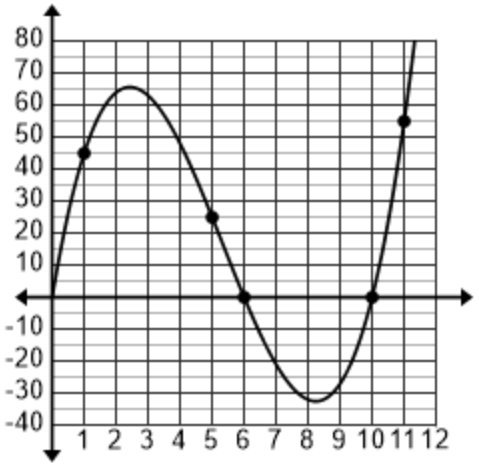
(A) 2,570 (B) 2,580 (C) 2,590 (D) 2,600 (E) 2,610

48. The equation of the curve on the right has the form $f(x)=ax^3+bx^2+cx+d$. $a+b+c+d=$ _____.

(A) 33 (B) 36 (C) 39 (D) 42 (E) 45

49. The graph of f is shown on the right. Let $h(x)=\int_0^x f(t)dt$. For which of the following values of x does the graph of h have a point of inflection? (nearest hundredth)

(A) 2.38 (B) 4.33 (C) 6.29 (D) 8.24 (E) 10.19



Problems 48, 49

50. Consider the sequence $2,4,7,10,14,19,26,36,a,b,110$. $a+b=$ _____.

(A) 123 (B) 124 (C) 125 (D) 126 (E) 127

51. Consider the sequence $16,-8,4,-2,1,-\frac{1}{2},\ldots$ Find the sum of the ninth and tenth terms.

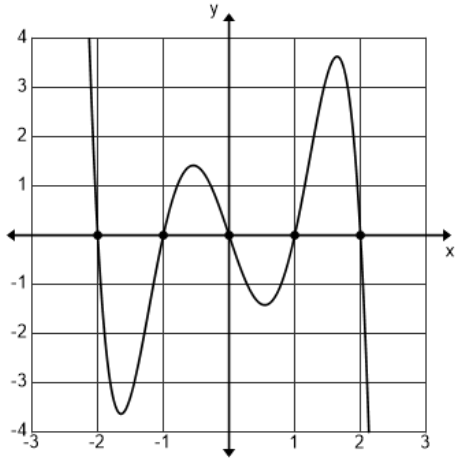
(A) $\frac{1}{64}$ (B) $-\frac{1}{64}$ (C) $\frac{1}{32}$ (D) $-\frac{1}{32}$ (E) $\frac{1}{16}$

52. The graph of f(x) is shown on the right. Which of the following are true?
I. f(x) is an odd function
II. f(x) is an even function
III. f(x) has 3 points of inflection
IV. f(x) has 5 points of inflection

(A) I only (B) II only (C) I, III only
(D) II, III only (E) I, IV only

53. For the function f(x) shown on the right, find the sum of the values of the two relative minima. (nearest hundredth)

(A) –5.05 (B) –4.94 (C) –4.83 (D) –4.72 (E) –4.61



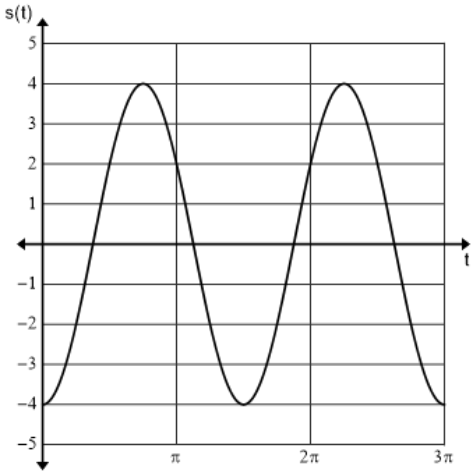
Problems 52, 53

Diameter	6 inches	8 inches	10 inches	12 inches	15 inches
Cost	$6.48	$11.52	$18.00	$25.92	?

54. Roma's Pizza makes five sizes of their supreme pizzas. They are very expensive, but they are the best pizzas in Denton County. Find the cost of a 15-inch pizza.

(A) $40.02 (B) $40.14 (C) $40.26 (D) $40.38 (E) $40.50

55-56. A mass is hung on a vertical spring and the mass is allowed to come to rest at its equilibrium position. Then the mass is pulled down 4 cm and released. The mass oscillates in simple harmonic motion. The position of the mass as a function of time is shown on the sinusoidal graph below, where s(t) is measured in centimeters and t is measured in seconds.



Problems 55, 56

55. Find the maximum speed of the oscillating mass.

(A) $4.\overline{6}$ cm/s (B) $4.8\overline{3}$ cm/s (C) 5.00 cm/s
(D) $5.1\overline{6}$ cm/s (E) $5.\overline{3}$ cm/s

56. Find the maximum acceleration of the oscillating mass.

(A) $6.\overline{7}$ cm/s^2 (B) $6.\overline{8}$ cm/s^2 (C) 7.00 cm/s^2
(D) $7.\overline{1}$ cm/s^2 (E) $7.\overline{2}$ cm/s^2

57. Paula leaves the Rock Springs airport at 2:00 PM in her Dassult Falcon 7X and flies at a constant speed of 500 mph on a bearing of 348° towards Butte. Paul also leaves the Rock Springs airport at 2:00 PM in his Cessna Citation X+ and flies at a constant speed of 500 mph on a bearing of 22°. He had been flying for thirty minutes when she called and invited him to meet her in Butte. He made a course correction, increased his speed to 700 mph, and headed directly toward Butte. He arrived at the Butte airport at the same time as Paula. Find the distance from Rock Springs to Butte. (nearest mile)

(A) 446 mi (B) 450 mi (C) 454 mi (D) 458 mi (E) 462 mi

58. Identify the conic $73x^2+72xy+52y^2+30x-40y-75=0$ and find the angle of rotation. (nearest tenth)
I. Hyperbola II. Ellipse III. Parabola IV. 36.9° V. 39.6°

(A) I, IV (B) I, V (C) II, IV (D) II, V (E) III, IV

59. Find the sum of the seventh triangular number, the seventh pentagonal number and the seventh octagonal number. Then, find the positive difference between the 14th Fibonacci number and this sum.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

60. Ty had a career batting average of 0.367 with a standard deviation of 0.0371. Rogers had a career batting average of 0.358 with a standard deviation of 0.0377. Assume that both players played during the same time period, but for different teams. Also assume that their career averages are independent of each other. For any given season, find the probability that Rogers had a higher batting average than Ty. (nearest hundredth)

(A) 0.41 (B) 0.43 (C) 0.45 (D) 0.47 (E) 0.49

2021-2022 TMSCA HSM District Warm Up Test Key

1. B	**21. C**	**41. D**
2. C	**22. E**	**42. B**
3. C	**23. D**	**43. C**
4. C	**24. E**	**44. D**
5. E	**25. B**	**45. D**
6. A	**26. B**	**46. A**
7. D	**27. D**	**47. D**
8. A	**28. A**	**48. E**
9. C	**29. A**	**49. D**
10. B	**30. C**	**50. C**
11. D	**31. E**	**51. C**
12. A	**32. B**	**52. C**
13. E	**33. B**	**53. A**
14. C	**34. E**	**54. E**
15. C	**35. A**	**55. E**
16. D	**36. D**	**56. D**
17. A	**37. D**	**57. B**
18. E	**38. C**	**58. C**
19. A	**39. A**	**59. C**
20. A	**40. E**	**60. B**