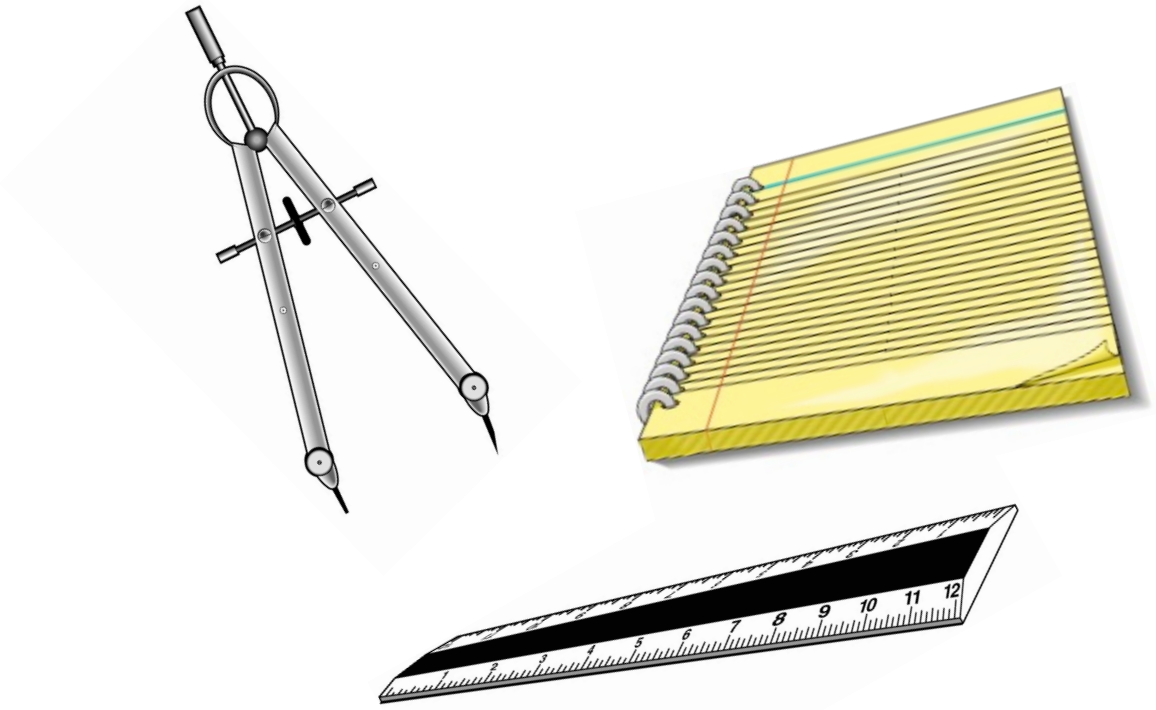
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

SAC • 2021



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YOU ARE INSTRUCTED TO DO SO!

25. Angles A and B are complementary angles while angles A and C are supplementary angles. If $m\angle A = 6x+1$ and $m\angle B = 9x-1$, then $m\angle C =$ _____°.

(A) 140 (B) 141 (C) 142 (D) 143 (E) 144

x	-3	-2	-1	0	1	2
f(x)	10	-9	-10	-5	-6	-25

Use the table above for problems 26 and 27.

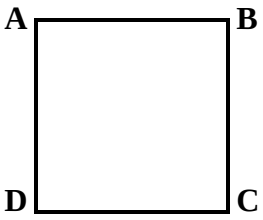
26. Find the value of $f(-4)$.

(A) 55 (B) 56 (C) 57 (D) 58 (E) 59

27. The point of inflection for the graph of $f(x)$ has coordinates (a, b). a + b = _____. (nearest tenth)

(A) –8.4 (B) –8.2 (C) –8.0 (D) –7.8 (E) –7.6

28. Quadrilateral ABCD shown on the right is a square. The midpoint of $\overline{AD}$ is point E and the midpoint of $\overline{AB}$ is point F. If EF = 18, then the area of the square is _____.



(A) 632 (B) 636 (C) 640 (D) 644 (E) 648

29. Consider a quadrilateral with vertices $A(-6, 4)$, $B(0, -8)$, $C(6, 4)$, and $D(0, 12)$. This quadrilateral can be classified as a _____.

(A) rhombus (B) rectangle (C) square (D) trapezoid (E) kite

30. Consider $\triangle ABC$ with point D on $\overline{AB}$ such that $\overline{CD} \perp \overline{AB}$. If $m\angle ACB = 78.28°$, AD = 9 and CD = 12, then DB = _____. (nearest tenth)

(A) 10.4 (B) 10.6 (C) 10.8 (D) 11.0 (E) 11.2

31. Mel is throwing darts at a circular target with a diameter of 24. On the target are two concentric circles with diameters of 8 and 16. A dart landing in the small circle earns 10 points. A dart landing inside the circle with a diameter of 16, but outside the small circle earns 6 points. A dart landing on the target outside of the two concentric circles earns 2 points. Find the expected value of the points earned on any randomly selected toss that lands on the target. (nearest tenth)

(A) 4.2 (B) 4.4 (C) 4.6 (D) 4.8 (E) 5.0

16. Consider a circle circumscribed about a regular pentagon. If the area of the circle is 452.4, then the area of the pentagon is _____. (nearest whole number)

(A) 334 (B) 336 (C) 338 (D) 340 (E) 342

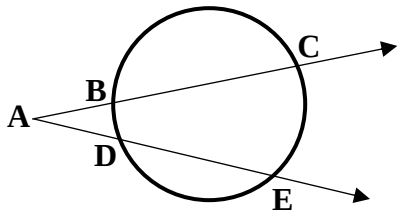
17. Which of the following is not one of the four fourth roots of $16(\cos 120° + i \sin 120°)$?

(A) $-\sqrt{3} - i$ (B) $\sqrt{3} + i$ (C) $1 - \sqrt{3}\,i$ (D) $-\sqrt{3} + i$ (E) $-1 + \sqrt{3}\,i$

Use the sketch on the right for problems 18 and 19. The information given in problem 18 does not carry over to problem 19.

18. If AB = 6, BC = 15, and AD = 8, then DE = _____. (nearest hundredth)

(A) 7.60 (B) 7.75 (C) 7.90 (D) 8.05 (E) 8.20



19. If $m\overset{\frown}{BD} = 28°$ and $m\overset{\frown}{CE} = 86°$, then $m\angle CAE =$ ____.°

(A) 29° (B) 30° (C) 31° (D) 32° (E) 33°

20. Find the eccentricity of the ellipse. $9x^2 + 16y^2 - 36x + 96y + 36 = 0$. (nearest hundredth)

(A) 0.62 (B) 0.64 (C) 0.66 (D) 0.68 (E) 0.70

21. The base of a pyramid is a square with each side equal to three-fifths of the height of the pyramid. If the volume of the pyramid is 700, what is the total area of the pyramid? (nearest whole number)

(A) 511 (B) 514 (C) 517 (D) 520 (E) 523

22. Consider the sequence 17, 21, 25, 29, 33, 37, …, 129, 133. Find the sum of the terms of the sequence.

(A) 2242 (B) 2246 (C) 2250 (D) 2254 (E) 2258

23. A fair die is rolled four times. What is the probability of getting an even number, a prime number, a Fibonacci number, and a perfect number, in that order?

(A) $\frac{17}{648}$ (B) $\frac{1}{36}$ (C) $\frac{19}{648}$ (D) $\frac{5}{162}$ (E) $\frac{7}{216}$

24. Simplify: $4\csc(2x)\cos(x)$

(A) 2csc(x) (B) sec(x) (C) csc(x)sec(x) (D) 2sec(x) (E) csc(x)

1. **Solve for k if $3k-4=28-5k$**

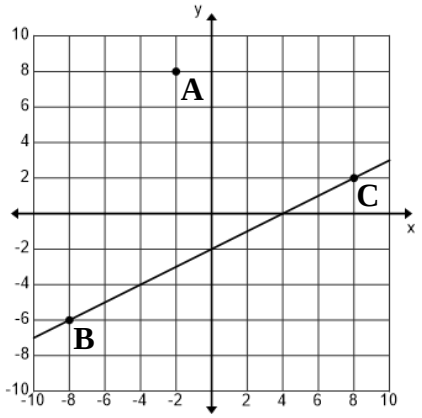
   (A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

2. **Joe's dad sent him to the Burger Barn with three twenty-dollar bills and one five-dollar bill. He ordered 6 cheeseburgers for $4.85 each, one basket of fries for $5.75, 6 large cokes for $2.19 each and 6 lemon pies for $1.25 each. The tax rate is 8.25%. How much change did he receive?**

   (A) $4.81 **(B) $4.84** **(C) $4.87** **(D) $4.90** **(E) $4.93**

3. **Consider a line that is perpendicular to $\overleftrightarrow{BC}$ and also contains point A. If the x-intercept of this line is (a, 0), then a = _____.**

   (A) 1.0
   (B) 1.5
   (C) 2.0
   (D) 2.5
   (E) 3.0



Problem 3

4. **The Reagan High math/science team brought in the Quebe Sisters for a UIL fundraiser. Their fee to appear was $5,000. Their version of "San Antonio Rose" is outstanding. A student ticket cost $8.00 and an adult ticket cost $15.00. A total of 2100 tickets were sold and $20,375 was raised after paying the fee. How many adult tickets were sold?**

   (A) 875 **(B) 963** **(C) 1050** **(D) 1137** **(E) 1225**

5. **Consider four consecutive even integers, all positive, such that five times the sum of the first two exceeds three times the sum of the first and fourth by 80. The third integer is _____.**

   (A) 26 **(B) 28** **(C) 30** **(D) 32** **(E) 34**

6. **Simplify:** $\dfrac{\dfrac{c}{w}+\dfrac{d}{w^2}}{\dfrac{m}{w^2}+\dfrac{k}{hw}}$

   (A) $\dfrac{cw+dh}{hm+kw}$ **(B)** $\dfrac{chw+dhm}{hm+kw}$ **(C)** $\dfrac{chw+dh}{hm+dk}$ **(D)** $\dfrac{chw+dh}{hm+kw}$ **(E)** $\dfrac{chw+dh}{m+kw}$

7. **If $f(x)=x^2+4$ and $h(x)=3x-1$, then $f\big(h(5)\big)=$ _____.**

   (A) 196 **(B) 200** **(C) 204** **(D) 208** **(E) 212**

32. Find the area of a triangle with vertices $(0,12)$, $(0,0)$ and $(12,0)$.

(A) 18 (B) 36 (C) 72 (D) 108 (E) 144

33. Polonium 221 has a half-life of 130 seconds. How long will it take a sample with a mass of 1.80 g to decay to a mass of 1.20 g? (nearest tenth)

(A) 75.4 s (B) 75.7 s (C) 76.0 s (D) 76.3 s (E) 76.6 s

34. If s(x) is the slant asymptote of $h(x) = \frac{x^3 + 6}{2x^2 + x - 1}$, then h(20) – s(20) = _____.

(nearest thousandth)

(A) 0.019 (B) 0.021 (C) 0.023 (D) 0.025 (E) 0.027

Temperature	64°	72°	86°	94°	96°	92°
Bottles Sold	420	450	500	530	540	520

Use the table above for problems 35 and 36. Karen owns the Kwik Stop in Sundown. She believes that the number of water bottles sold each day varies with the temperature. She made a table of the high temperature and the number of water bottles sold on the 15th day of the month, for the months of April through September.

35. Find the sum of the mean, median, and range for the number of water bottles sold on these six days.

(A) $1123.\overline{3}$ (B) $1125.\overline{3}$ (C) $1127.\overline{3}$ (D) $1129.\overline{3}$ (E) $1131.\overline{3}$

36. Use the data from the table to create an appropriate mathematical model and predict the high temperature on a day that Karen sold 354 water bottles. (nearest whole number)

(A) 40° (B) 42° (C) 44° (D) 46° (E) 48°

37. Assume the number of hours of daylight varies sinusoidally at the Clydehurst Christian Ranch in Montana. The longest day of the year has 15 hr 30 min of daylight and the shortest day has 8 hr 30 min of daylight. How many days during the year have at least 13 hours of daylight?

(A) 132 (B) 136 (C) 140 (D) 144 (E) 148

38. The Holiday Inn is across the street from the Hilton. The hotels are 120 feet apart. Joe looks out the window of his room at the Holiday Inn and notices that the angle of depression to the base of the Hilton is 36° and the angle of elevation to the top of the Hilton is 44°. How tall is the Hilton? (nearest foot)

(A) 191 ft (B) 194 ft (C) 197 ft (D) 200 ft (E) 203 ft

8. Find the number that is $\frac{5}{6}$ of the way from $-4\frac{1}{2}$ to $9\frac{3}{8}$.

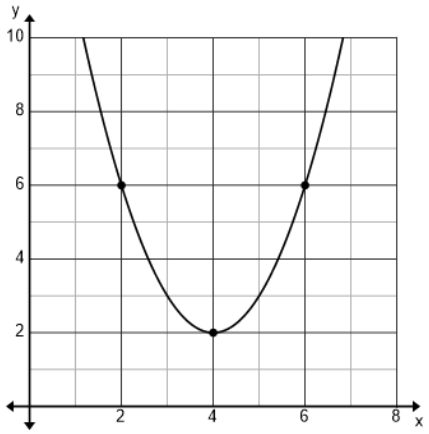
(A) $6\frac{7}{8}$ **(B)** $6\frac{15}{16}$ **(C)** 7 **(D)** $7\frac{1}{16}$ **(E)** $7\frac{1}{8}$

9. The focus of the parabola on the right has coordinates (a, b). $a + b =$ _____.

(A) 6 **(B)** $6\frac{1}{8}$ **(C)** $6\frac{1}{4}$ **(D)** $6\frac{3}{8}$ **(E)** $6\frac{1}{2}$

10. The graph of $f'(x)$ is shown on the right. If $f(1) = 2\frac{1}{3}$, then $f(2) =$ _____.

(A) $9\frac{1}{3}$ **(B)** $9\frac{2}{3}$ **(C)** 10 **(D)** $10\frac{1}{3}$ **(E)** $10\frac{2}{3}$



Problems 9, 10

11. Cindy rode her bike for 60 miles at 24 mph and then rode 36 miles at 30 mph. How fast does she need to ride the final 44 miles to have an overall speed of 28 mph? (nearest tenth)

(A) 33.6 mph (B) 33.8 mph (C) 34.0 mph (D) 34.2 mph (E) 34.4 mph

12. Given: $\triangle ABC \sim \triangle DEF$, AB = 15, AC = 12, $m\angle A = 62°$, DE = 10. EF = _____. (nearest tenth)

(A) 9.4 (B) 9.6 (C) 9.8 (D) 10.0 (E) 10.2

13. Consider the points $A(-6, 10)$ and $B(4, -6)$. Find the equation of a line that exists such that every point on the line is the same distance from A as it is from B.

(A) $5x - 8y = 21$ **(B)** $5x + 8y = -21$ **(C)** $5x - 8y = -21$ **(D)** $8x + 5y = 21$ **(E)** $5x + 8y = 21$

14. Over the last few years, the length of Randy's drives at the local driving range follows a normal distribution with a mean of 225 yards and a standard deviation of 6 yards. Approximately what percentage of his drives are between 219 yards and 231 yards? (nearest whole number)

(A) 68% (B) 70% (C) 72% (D) 74% (E) 76%

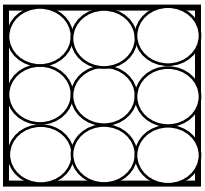
15. Points A and B lie on a circle with center O. The area of the circle is 531 and AB = 24. Find the distance from O to chord $\overline{AB}$. (nearest tenth)

(A) 5.0 (B) 5.1 (C) 5.2 (D) 5.3 (E) 5.4

55. If you were going to evaluate $\int \frac{\cos x}{\sin^3 x} dx$ using a u-substitution, the best choice for u is _____.

(A) $\cos x$ (B) $\sin x$ (C) $\sin^3 x$ (D) $\cos x dx$ (E) $\sin x dx$

56. If you cut nine circles out of a square piece of cardboard that measures 12 in by 12 in, how much cardboard is discarded? (nearest tenth)



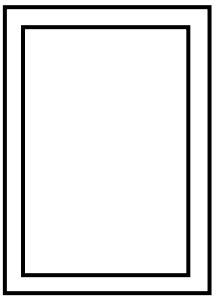
(A) 28.5 in^2 (B) 29.1 in^2 (C) 29.7 in^2
(D) 30.3 in^2 (E) 30.9 in^2

57. Jeremy has 49 coins with a total value of $7.05. He only has nickels, dimes and quarters. He has three more quarters than nickels. How many dimes does he have?

(A) 10 (B) 12 (C) 14 (D) 16 (E) 18

58. Russell's backyard pool is shaped like a rectangle that measures 30 ft by 50 ft. He decides to add a sidewalk that is 3 feet wide around the perimeter. Vedant, Caleb and Curtis will provide free labor, so he only has to pay for the concrete, which cost $6.00 per square foot. What will the sidewalk cost?

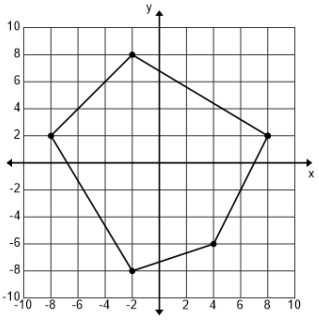
(A) $3080.00 (B) $3084.00 (C) $3088.00
(D) $3092.00 (E) $3096.00



Problem 58

59. Find the area of the polygon on the right. (nearest whole number)

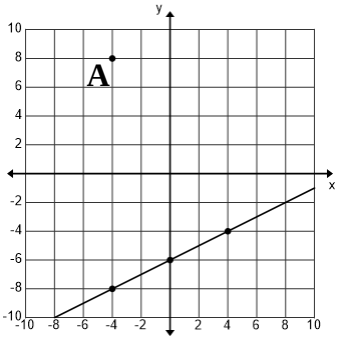
(A) 142
(B) 144
(C) 146
(D) 148
(E) 150



Problem 59

60. Find the distance between point A and the line shown on the right. (nearest tenth)

(A) 13.7
(B) 13.9
(C) 14.1
(D) 14.3
(E) 14.5



Problem 60

46. Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{4^n}$.

(A) $(-\infty, \infty)$ (B) $[-1, 1]$ (C) $(-1, 1)$ (D) $[-4, 4]$ (E) $(-4, 4)$

47. When analyzing data, statisticians often report the five-number summary. Which of the following are included in the five-number summary?
I. mean II. standard deviation III. median IV. quartiles V. maximum and minimum

(A) I, II, III, IV (B) I, II, III, V (C) I, IV, V (D) II, IV, V (E) III, IV, V

48. If $\left(x^3 - 9x^2 + kx - 12\right) \div (x - 1)$ has a remainder of zero, then k = _____.

(A) 14 (B) 16 (C) 18 (D) 20 (E) 22

49. Consider the sequence 3, 5, 8, 11, 15, 20, 27, 37, m, n, 111, … m + n = _____

(A) 127 (B) 128 (C) 129 (D) 130 (E) 131

50. $ABC1_{16} + ABC1_{15}$ = _____$_{14}$

(A) 21411 (B) 21421 (C) 21431 (D) 21441 (E) 21451

51. A shipment of twenty refurbished computers contains four defective computers. In how many ways can Rocket purchase five of these computers and get two defective ones?

(A) 2440 (B) 2744 (C) 3030 (D) 3360 (E) 3600

52. Find the distance between the points $(3, 5, 7)$ and $(-4, 1, -3)$. (nearest tenth)

(A) 12.5 (B) 12.8 (C) 13.1 (D) 13.4 (E) 13.7

53. The graph of the parametric equations $x = 2 + 3\cos\theta$ and $y = 1 + 2\sin\theta$ is an ellipse with vertices (a, b) and (c, b). a + c = _____.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

54. Find the value of c in the open interval $(-8, 2)$ that satisfies the mean value theorem for the function $f(x) = \sqrt{6 - x}$. (nearest hundredth)

(A) –2.68 (B) –2.57 (C) –2.46 (D) –2.35 (E) –2.24

39. The preferred swimming pool temperature of adult females follows a normal distribution with a mean of $82°\,F$ with a standard deviation of $3°\,F$. Find the probability that a randomly selected adult female will prefer a temperature between $26°\,C$ and $29°\,C$. (nearest thousandth)

(A) 0.600 (B) 0.617 (C) 0.625 (D) 0.633 (E) 0.650

40. A researcher took a random sample of 1,000 teenage males in order to estimate the mean number hours of sleep a typical teenage boy gets each night. A 90% confidence interval would be _____ than a 98% confidence interval and would involve _____ risk of being incorrect.

(A) wider, a smaller (B) wider, a greater (C) narrower, a smaller
(D) narrower, a greater (E) narrower, an equal

41. A one-sample t statistic from a sample of 40 observations for the two-sided test of $H_0 = 26$, $H_a \neq 26$ has the value $t = -1.44$. Find the p-value for this test. (nearest thousandth)

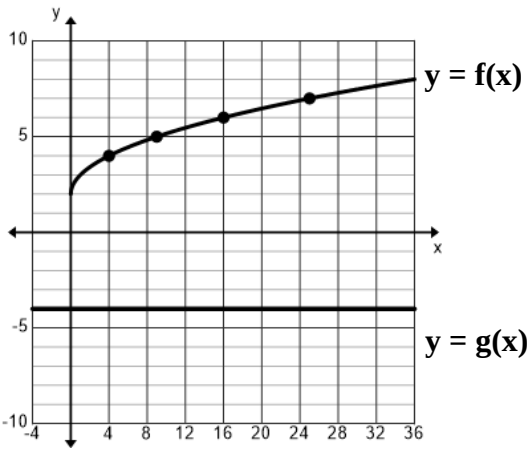
(A) 0.079 (B) 0.118 (C) 0.158 (D) 0.237 (E) 0.316

42. Find the area between the curves $y = f(x)$ and $y = g(x)$ shown on the right over the interval $[4, 24]$. (nearest whole number)

(A) 190 (B) 193 (C) 196 (D) 199 (E) 202

43. Find the volume of the solid generated by revolving the region bounded by $y = f(x)$, the x-axis, the line x = 4 and the line x = 24 about the line $y = g(x)$. (nearest whole number)

(A) 4890 (B) 4895 (C) 4900 (D) 4905 (E) 4910



Problems 42, 43

44. The graph of $r = 3 - 3\sin\theta$ is a _____.

(A) limacon with an inner loop (B) cardioid (C) dimpled limacon
(D) convex limacon (E) 3-petaled rose

45. Find the area of one petal of the rose curve $r = 6\cos(2\theta)$. (nearest tenth)

(A) 14.1 (B) 14.4 (C) 14.7 (D) 15.0 (E) 15.3