



# TMSCA MIDDLE SCHOOL MATHEMATICS REGIONAL TEST © MARCH 5, 2022

## GENERAL DIRECTIONS

1. About this test:
  - A. You will be given 40 minutes to take this test.
  - B. There are 50 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
3. If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators **MAY NOT** be used on this test.
8. All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

2021 – 2022 TMSCA Middle School Mathematics Regional Test

1.  $46 + (-323) + (-743) =$  \_\_\_\_\_  
 A. -1,020      B. -974      C. -1,066      D. -1,112      E. -374

2.  $42\frac{3}{5} - 19\frac{1}{3} =$  \_\_\_\_\_  
 A.  $23\frac{4}{15}$       B.  $23\frac{2}{5}$       C.  $22\frac{7}{15}$       D.  $23\frac{7}{30}$       E.  $22\frac{1}{5}$

3.  $55.34 \times 4.5 \times 6.2 =$  \_\_\_\_\_ (nearest whole number)  
 A. 1,540      B. 1,542      C. 1,541      D. 1,543      E. 1,544

4.  $\frac{18}{5} \div \frac{3}{5} \div \frac{6}{5} =$  \_\_\_\_\_  
 A.  $5\frac{1}{5}$       B.  $5\frac{4}{5}$       C.  $5\frac{2}{5}$       D.  $5\frac{3}{5}$       E. 5

5. If  $\pi = 3$ , what is the radius of a circle with a circumference of 102 inches?  
 A. 51 inches      B. 34 inches      C. 17 inches      D. 6 inches      E. 12 inches

6.  $\sqrt{811}$  lies between which pair of integers?  
 A. 28 and 29      B. 29 and 30      C. 31 and 32      D. 27 and 28      E. 42 and 43

7. What is the product of the sum of the number of edges and faces of a rectangular prism and the sum of the number of edges and vertices of a hexagonal prism?  
 A. 216      B. 540      C. 360      D. 720      E. 288

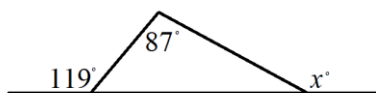
8. What is the GCF of 12, 16, 26, and 38?  
 A. 92      B. 46      C. 4      D. 2      E. 6

9.  $9 + 10 - 11 + 12 - 13 + 14 - 15 + 16 - 17 + 18 =$  \_\_\_\_\_  
 A. 23      B. 31      C. 25      D. 37      E. 33

10. Which of the following expressions matches, “six less than twice the sum of three numbers”.  
 A.  $6 - a + b + c$       B.  $6 - 2a - 2b - 2c$       C.  $2(a + b + c) - 6$       D.  $2(abc) - 6$       E.  $2(abc) \div 6$

11.  $76^2 =$  \_\_\_\_\_  
 A. 4,486      B. 5,776      C. 5,286      D. 5,296      E. 4,926

12. What is the value of  $x$  in the picture below?



A. 161      B. 133      C. 132      D. 148      E. 154

13. If  $a \heartsuit b = ab - a + b$ , then what is the value of  $(8 \heartsuit 5) \heartsuit 7$ ?  
 A. 85      B. 169      C. 325      D. 125      E. 229

14. 65,000,000 centimeters = \_\_\_\_\_ hectometers  
 A. 650      B. 65      C. 6.5      D. 6,500      E. 65,000

15. Connor starts with \$300.00. He gives 65% of his money to his friend Fred, and then Fred gives his mother 20% of his money. How much money does Fred's mother receive?  
 A. \$105.00      B. \$43.00      C. \$39.00      D. \$45.00      E. \$36.00

16. If  $A = 1$ ,  $B = 2$ ,  $C = 3$ , ...,  $Y = 25$  and  $Z = 26$ , what is the sum of the letters of the word *CALCULATE*?  
 A. 78      B. 82      C. 77      D. 80      E. 79

17. 51 nickels + 17 quarters = \_\_\_\_\_ dimes + 440 pennies  
 A. 32      B. 30      C. 28      D. 26      E. 24

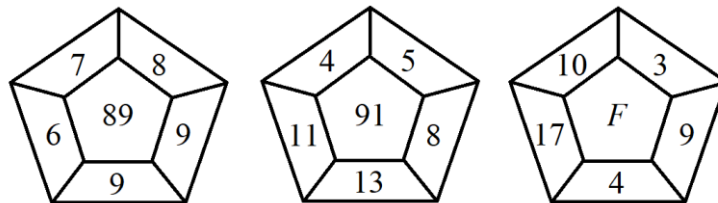
18. What is the sum of the lower quartile and upper quartile of the set of numbers 43, 21, 44, 52, 17, and 39?  
 A. 65      B. 69      C. 67      D. 71      E. 67.5

19. The sum of four consecutive positive odd integers is 232. What value is ten more than the smallest of the four integers?  
 A. 62      B. 63      C. 61      D. 64      E. 65

20. What is the 21<sup>st</sup> term of the arithmetic sequence  $-13, -9, -5, -1, \dots$ ?  
 A. 57      B. 67      C. 61      D. 53      E. 71

21.  $\text{MMDLXIX} + \text{XIII} - \text{DCCCLXXXVIII} = \underline{\hspace{2cm}}$  (Arabic number)  
 A. 1,694      B. 1,723      C. 1,724      D. 1,682      E. 1,718

22. Use the examples in the picture below to find the value of  $F$ .



A. 104      B. 103      C. 97      D. 93      E. 83

23. What is the value of  $a$ , if the mean of  $a$ , 76, and 83 is 81?  
 A. 79      B. 80      C. 82      D. 84      E. 85

24. What is the probability of rolling a pair of dice and getting a sum of 9?  
 A.  $\frac{1}{3}$       B.  $\frac{4}{9}$       C.  $\frac{1}{9}$       D.  $\frac{1}{12}$       E.  $\frac{5}{12}$

25. What is the unit rate of purchasing 9 movie tickets for \$93.15?  
 A. \$11.75      B. \$11.55      C. \$10.35      D. \$12.15      E. \$10.15

26. If a digit cannot repeat, how many two-digit numbers can be written using prime numbers less than 10?  
 A. 16      B. 25      C. 12      D. 24      E. 20

27. Two integers in the ratio of 7:11 has a sum of 396. What is the greater of the two integers?  
 A. 272      B. 256      C. 154      D. 304      E. 242

28. A base angle of an isosceles triangle measures  $22^\circ$ . What is the measure of the vertex angle of the triangle?  
 A.  $148^\circ$       B.  $22^\circ$       C.  $158^\circ$       D.  $136^\circ$       E.  $68^\circ$

29. Let  $U$  be a universal set and  $A$ ,  $B$ , and  $C$  be subsets of  $U$  defined as shown. Find  $(A' \cup C') \cap B$ .

$$U = \{c, o, m, p, u, t, e, r\}$$

$$A = \{c, o, m, p, u, t, e\}$$

$$B = \{p, u, t, e, r\}$$

$$C = \{o, u, e\}$$

A.  $\{c, m, p, t, r\}$

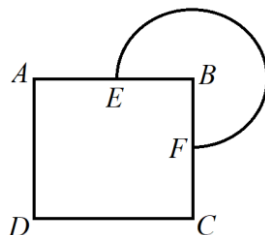
B.  $\{c, m, r\}$

C.  $\{p, t, r\}$

D.  $\{a, e, r\}$

E.  $\{o, m, p\}$

30. In the picture below, square  $ABCD$  overlaps  $\odot B$  as shown.  $E$  is the midpoint of  $\overline{AB}$ ,  $F$  is the midpoint of  $\overline{BC}$ , and  $\overline{EB}$  and  $\overline{FB}$  are both radii of  $\odot B$ . If  $\pi = 3$  and  $AB = 24$  units, what is the perimeter of the shape?



A. 142 units

B. 126 units

C. 144 units

D. 132 units

E. 136 units

31. Simplify:  $((a^4b^9)^2)^3)^4$

A.  $a^{13}b^{18}$

B.  $a^{96}b^{216}$

C.  $a^{15}b^{25}$

D.  $a^{192}b^{972}$

E.  $a^{169}b^{324}$

32. Molly drew a polygon with 15 sides. How many total degrees are in Molly's polygon?

A.  $2,340^\circ$

B.  $2,520^\circ$

C.  $2,700^\circ$

D.  $3,060^\circ$

E.  $2,680^\circ$

33. What is the equation of the axis of symmetry for the graph of the equation  $y = -3x^2 - 9 + 12x$ ?

A.  $x = 3$

B.  $x = -3$

C.  $x = \frac{1}{3}$

D.  $x = 2$

E.  $x = \frac{1}{4}$

34. What is the slope of the line passing through the points  $(5a + 3, 9a + 4)$  and  $(7 - 3a, 6 + a)$ ?

A.  $\frac{4a-1}{4a-2}$

B.  $\frac{4a+1}{4a-2}$

C.  $\frac{2a-1}{2a+1}$

D.  $\frac{8a+2}{2a+10}$

E.  $\frac{a-1}{a-2}$

35. If  $A = 2^2 \cdot 5$ ,  $B = 2 \cdot 5^2$ , and  $C = 4^3$ , what is the geometric mean of  $ABC$ ?

A. 200

B. 60

C. 80

D. 20

E. 40

36. The point  $(9, 2)$  is a solution to which system of linear equations?

A.  $\begin{cases} 4x - y = 34 \\ x + y = 2 \end{cases}$

B.  $\begin{cases} 3x - 2y = 23 \\ 2x + y = 6 \end{cases}$

C.  $\begin{cases} x - y = 7 \\ 5x - 7y = 38 \end{cases}$

D.  $\begin{cases} 2x - 4y = 10 \\ 2x + 4y = 26 \end{cases}$

E.  $\begin{cases} x - 3y = 3 \\ x - 2y = 13 \end{cases}$

37.  $\frac{(8.4 \times 10^{-4})(5 \times 10^6)}{2 \times 10^0} =$  \_\_\_\_\_ (scientific notation)

A.  $6.7 \times 10^2$

B.  $2.1 \times 10^3$

C.  $4.2 \times 10^2$

D.  $4.2 \times 10^3$

E.  $2.05 \times 10^2$

38.  $11011_2 \times 1111_2 =$  \_\_\_\_\_ (base 4)

A. 11122

B. 12121

C. 12111

D. 12201

E. 12221

39. If  $f(x) = x^2 + 3x - 8$ , find  $f(a - 5)$ .

A.  $a^2 - 15a - 13$

B.  $a^2 + 10a + 25$

C.  $a^2 - 7a + 2$

D.  $a^2 - 10a + 2$

E.  $a^2 - 15a + 2$

40. If  $\sqrt{3x + 1} + 1 = x$ , then  $3x - 6$  is equal to which of the following?

- A. 12                      B. 9                      C. -6                      D. -12                      E. 3

41. If  $1 + \frac{2^2}{1 + \frac{3^2}{5}} = \frac{A}{B}$ , then what is the value of  $A - B$ ?

- A. 5                      B. 12                      C. 9                      D. 10                      E. 8

42. What value of  $C$  is necessary to solve the quadratic equation  $x^2 - 18x + C = 36 + C$  by the method of completing the square?

- A. 81                      B. -9                      C. 36                      D. -81                      E. -36

43. What is the perimeter of a regular hexagon with an apothem of  $7\sqrt{3}$  cm?

- A. 126 cm                      B. 42 cm                      C. 84 cm                      D.  $84\sqrt{3}$  cm                      E.  $42\sqrt{3}$  cm

44. Which of the following expressions is equivalent to  $\frac{2a^3 + 4a^2}{2a^2}$ ?

- A.  $a + 2$                       B.  $2a(a + 1)$                       C.  $2a^2(a + 1)$                       D.  $2(a + 1)$                       E.  $2a^2(a + 2)$

45. What is the solution to the inequality  $-1 < 9 - n < 20$ ?

- A.  $(-10, 11)$                       B.  $[-10, 11]$                       C.  $(10, 11)$                       D.  $(-11, 10)$                       E.  $[-11, 10]$

46. The sum of the 2 linear factors of  $4x^2 - 24x + 27$  is subtracted from the sum of the two linear factors of  $4x^2 + 24x + 35$ . What is the value of the result?

- A. -48                      B. 48                      C. -8                      D. 16                      E. 24

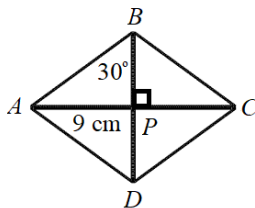
47. If  $64x + 64y = 65(x + y)$ , and  $xy \neq 0$ , what is the value of  $\frac{2y}{2x}$ ?

- A.  $-\frac{1}{2}$                       B. -2                      C. 2                      D. -4                      E. -1

48. The sum of  $\sqrt{2a}$  and  $\sqrt{18a}$  is equal to which of the following?

- A.  $2a\sqrt{2a}$                       B.  $4a\sqrt{2a}$                       C.  $6\sqrt{3a}$                       D.  $-6\sqrt{2a}$                       E.  $4\sqrt{2a}$

49. In the picture below,  $AP = 9$  cm and  $m\angle ABP = 30^\circ$ . What is the perimeter of the rhombus?



- A.  $36\sqrt{2}$  cm                      B. 72 cm                      C. 36 cm                      D.  $72\sqrt{3}$  cm                      E.  $36\sqrt{3}$  cm

50. Lesley plotted the points  $A(-8, -5)$  and  $B(6, -5)$ . If Lesley reflects points  $A$  and  $B$  over the line  $y = 4$  and connects the four points to create quadrilateral  $ABB'A'$ , what is the area of quadrilateral  $ABB'A'$ ?

- A. 224 units<sup>2</sup>                      B. 408 units<sup>2</sup>                      C. 274 units<sup>2</sup>                      D. 252 units<sup>2</sup>                      E. 195 units<sup>2</sup>

2021 – 2022 TMSCA Middle School Mathematics Regional Test Answer Key

1. A	18. A	35. E
2. A	19. E	36. D
3. E	20. B	37. B
4. E	21. A	38. C
5. C	22. D	39. C
6. A	23. D	40. B
7. B	24. C	41. D
8. D	25. C	42. A
9. A	26. C	43. C
10. C	27. E	44. A
11. B	28. D	45. D
12. D	29. C	46. E
13. E	30. B	47. E
14. D	31. B	48. E
15. C	32. A	49. B
16. A	33. D	50. D
17. E	34. A	

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7. A rectangular prism has 12 edges and 6 faces, so the sum is  $12 + 6 = 18$ . A hexagonal prism has 18 edges and 12 vertices, so the sum is  $18 + 12 = 30$ . Therefore, the product of 18 and 30 is  $18(30) = 540$ .

15. Fred gets 65% of  $\$300.00 = 0.65(300) = \$195.00$  from Connor. Fred then gives 20% of  $\$195.00$ , which is  $(0.2)(195) = \$39.00$  to his mother.

20. To find the  $n^{\text{th}}$  term of an arithmetic sequence, use the formula  $a_n = a_1 + (n - 1)d$ , where  $a_1$  is the first term of the sequence,  $d$  is the common difference, and  $n$  is the desired term. Using the sequence  $-13, -9, -5, -1, \dots$ , the  $21^{\text{st}}$  term would be  $a_{21} = -13 + (21 - 1)(4) = -13 + (20)(4) = -13 + 80 = 67$ .

23. If 81 is the mean of  $a$ , 76, and 83, then  $\frac{a+76+83}{3} = 81$ . This simplifies to  $\frac{a+159}{3} = 81$ . To solve for  $a$ , first multiply both sides of the equation by 3, which gives us  $a + 159 = 81(3) = 243$ . Subtracting 159 from both sides of the equation gives us  $a = 243 - 159 = 84$ .

34. Given two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , the slope formula is  $\frac{y_2 - y_1}{x_2 - x_1}$ . We are given the two points  $(5a + 3, 9a + 4)$  and  $(7 - 3a, 6 + a)$ , so  $y_2 = 6 + a$ ,  $y_1 = 9a + 4$ ,  $x_2 = 7 - 3a$ , and  $x_1 = 5a + 3$ . Substituting into our formula and we get  $\frac{6+a-(9a+4)}{7-3a-(5a+3)} = \frac{6+a-9a-4}{7-3a-5a-3} = \frac{-8a+2}{-8a+4} = \frac{-2(4a-1)}{-4(2a-1)} = \frac{4a-1}{2(2a-1)} = \frac{4a-1}{4a-2}$ .

35. The geometric mean of  $A$ ,  $B$ , and  $C$  is equal to  $\sqrt[3]{ABC}$ . Since  $A = 2^2 \cdot 5$ ,  $B = 2 \cdot 5^2$ , and  $C = 4^3$ , then  $\sqrt[3]{ABC} = \sqrt[3]{2^2 \cdot 5 \cdot 2 \cdot 5^2 \cdot 4^3} = \sqrt[3]{2^3 \cdot 4^3 \cdot 5^3} = \sqrt[3]{2^3} \cdot \sqrt[3]{4^3} \cdot \sqrt[3]{5^3} = 2 \cdot 4 \cdot 5 = 40$ .

39. If  $f(x) = x^2 + 3x - 8$ , then  $f(a - 5) = (a - 5)^2 + 3(a - 5) - 8 = a^2 - 10a + 25 + 3a - 15 - 8 = a^2 - 7a + 2$ .

40. To solve for  $x$  in the equation  $\sqrt{3x + 1} + 1 = x$ , first subtract 1 from both sides of the equation to get  $\sqrt{3x + 1} = x - 1$ . Now, squaring both sides of the equation gives us  $3x + 1 = (x - 1)^2$ . Multiply out the binomial to get  $3x + 1 = x^2 - 2x + 1$ . Subtract  $3x$  and 1 from both sides of the equation to get  $x^2 - 5x = 0$ . Factor this out to get  $x(x - 5) = 0$ . Setting both equations equal to 0 gives us  $x = 0$  and  $x - 5 = 0$ . Solving the equations gives us  $x = 0$  and 5.  $x$  cannot be equal to 0 because  $\sqrt{3(0) + 1} + 1 \neq 0$ .  $x = 5$  because  $\sqrt{3(5) + 1} + 1 = 5$ . Therefore,  $3(5) - 6 = 15 - 6 = 9$ .

$$44. \frac{2a^3 + 4a^2}{2a^2} = \frac{2a^2(a+2)}{2a^2} = a + 2.$$

$$41. 1 + \frac{2^2}{1 + \frac{3^2}{5}} = 1 + \frac{4}{1 + \frac{9}{5}} = 1 + \frac{4}{\frac{5+9}{5}} = 1 + \frac{4}{\frac{14}{5}} = 1 + 4 \div \frac{14}{5} = 1 + 4 \cdot \frac{5}{14} = 1 + \frac{20}{14} = 1 + \frac{10}{7} = \frac{7}{7} + \frac{10}{7} = \frac{17}{7} = \frac{A}{B}.$$

Therefore,  $17 - 7 = 10$ .

45. To solve  $-1 < 9 - n < 20$ , subtract 9 from all sides of the equation to get  $-10 < -n < 11$ . Dividing by  $-1$  to both sides of the equation gives us  $10 > n > -11$ . This is rewritten as  $-11 < n < 10$ . Using interval notation,  $-11 < n < 10$  is written as  $(-11, 10)$ .