UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2022

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1. If $6 \times \sqrt{k-16} - 20 = 4$, then $k^2 =$ _____.

   (A) 4   (B) 16   (C) 32   (D) 256   (E) 1024

2. Helen Hoopster received a $250 gift card for her birthday. She purchased some shoes for $125.95, some shorts for $22.50, two t-shirts for $19.90 each, and some socks for $9.85. If the tax rate was 8.25%, how much remained on her gift card? (nearest cent)

   (A) $35.56   (B) $35.69   (C) $35.82   (D) $35.95   (E) $36.08

3. Biker Bob cycled due north for 45 minutes at 25 mph. Then he cycled due east for 30 minutes at 24 mph. Next, he cycled due south for 75 minutes at 28 mph. How far was Bob from where he started? (nearest tenth)

   (A) 19.6 mi   (B) 19.8 mi   (C) 20.0 mi   (D) 20.2 mi   (E) 20.4 mi

4. The public swimming pool in Crosbyton has two pipes that fill the pool. One of them can fill the pool by itself in 48 hours. The second one can fill the pool by itself in 36 hours. The drain can empty the pool in 60 hours if both fill pipes are shut off. If the pool is empty, how long will it take to fill the pool when both fill pipes are open and the drain is also functioning? (nearest tenth)

   (A) 31.3 hr   (B) 31.5 hr   (C) 31.7 hr   (D) 31.9 hr   (E) 32.1 hr

5. Carrie baked a batch of chocolate cookies. She gave one-sixth of them to Landon. Next, she gave one-fourth of what remained to Rose. Then she gave half of what remained to Caleb. Next, she gave 18 cookies to Elise. If she had twelve cookies left for herself, how many cookies did she give to Caleb?

   (A) 26   (B) 28   (C) 30   (D) 32   (E) 34

6. Find the number that is $\dfrac{7}{13}$ of the way from $-4\dfrac{7}{12}$ to $10\dfrac{3}{8}$.

   (A) $3\dfrac{6}{13}$ (B) $3\dfrac{49}{104}$ (C) $3\dfrac{25}{52}$ (D) $3\dfrac{51}{104}$ (E) $3\dfrac{1}{2}$

7. Turbo is an ultramarathoner. In a recent 100-mile race, he ran the first 28 miles at an average speed of 9.20 mph. He ran the next 38 miles at an average speed of 8.70 mph. If his goal was to average to average 9.00 mph for the entire race, at what average speed must he run for the remainder of the race to meet his goal pace? (nearest hundredth)

   (A) 9.05 mph   (B) 9.12 mph   (C) 9.19 mph   (D) 9.26 mph   (E) 9.33 mph

8. Cindy's Sporting Goods purchased 75 Tour Edition bowling balls for $60 each. The price of a bowling ball in June was $119.95 and they sold 15. In July, they reduced the price by 20% and they sold 20. In August, they reduced the price by 25% of the July price and they sold all of the remaining balls. What was the average profit on a single bowling ball? (nearest cent)

   (A) $27.96   (B) $28.02   (C) $28.08   (D) $28.14   (E) $28.20

9. Three times Juanita's age exceeds twice Carmen's age by 5. In fifteen years, four times Carmen's age will equal five times Juanita's age. How old is Carmen?

    (A) 20 yr   (B) 25 yr   (C) 30 yr   (D) 35 yr   (E) 40 yr

10. Consider positive integers a, b, and c such that ab = 96, ac = 144 and bc = 216. Find the value of $a+b+c$.
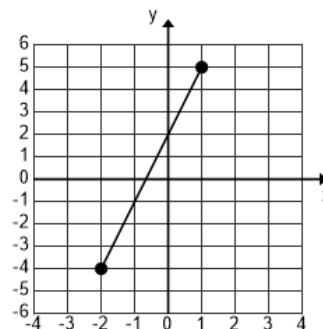
    (A) 36   (B) 38   (C) 40   (D) 42   (E) 44

11. Traffic can vary on my morning drive to work. On Thursday, I drove at an average speed of 45 mph and arrived 16 minutes late. On Saturday, I drove at an average speed of 90 mph and arrived 16 minutes early. At what average speed do I need to drive to arrive on time?

    (A) 57.5 mph   (B) 60 mph   (C) 62.5 mph   (D) 65 mph   (E) 67.5 mph

12. Find the y-intercept of the perpendicular bisector of line segment on the right.

    (A) $\left(0, \dfrac{1}{6}\right)$ (B) $\left(0, \dfrac{1}{4}\right)$ (C) $\left(0, \dfrac{1}{3}\right)$

           (D) $\left(0, \dfrac{1}{2}\right)$ (E) $\left(0, \dfrac{2}{3}\right)$

13. Consider triangle $\triangle ABC$ with $m\angle A = 66°$, $m\angle B = 56°$, and $AC = 12$ in.
    Point D is the midpoint of line segment $\overline{AB}$. Point F is the centroid. DF = _____. (nearest tenth)

    (A) 3.1 in   (B) 3.3 in   (C) 3.5 in   (D) 3.7 in   (E) 3.9 in

14. Consider equilateral triangle $\triangle ABC$ with BC = 9 in. Point D lies on ray $\overrightarrow{BC}$ with BD = 18 in. Find the area of triangle $\triangle ABD$. (nearest tenth)

    (A) 69.5 in^2 (B) 69.8 in^2 (C) 70.1 in^2 (D) 70.4 in^2 (E) 70.7 in^2

15. Consider triangle $\triangle ABC$ with AB = 16, AC = 24, and $m\angle BAC = 60°$. Point D lies on line segment $\overline{AC}$ and $\overline{AC} \perp \overline{DB}$. $m\angle DBC = $ _____. (nearest tenth)

   (A) 48.2°   (B) 48.5°   (C) 48.8°   (D) 49.1°   (E) 49.4°

16. Consider a circle with center O and chord $\overline{AB}$. Point C is the midpoint of $\overline{AB}$. If AB = 10 cm and CO = 12 cm, find the area of the circle. (nearest whole number)

    (A) 524 cm^2 (B) 531 cm^2 (C) 538 cm^2 (D) 545 cm^2 (E) 552 cm^2

17. A right circular cone has a radius of 12 cm and a slant height of 18 cm. Find the volume of the cone. (nearest whole number)

    (A) 2023 cm^3 (B) 2031 cm^3 (C) 2039 cm^3 (D) 2047 cm^3 (E) 2055 cm^3

18. $\overline{PT}$ is tangent to the circle on the right.
    If PT = 24 and PR = 18, then PQ = _____.
    (nearest tenth)
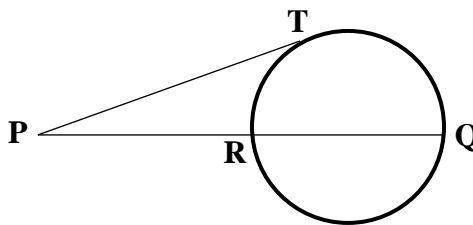
   (A) 31.6   (B) 31.8   (C) 32.0
              (D) 32.2   (E) 32.4

19. Consider $\triangle ABC$ with AB = 18 m, BC = 14 m, and AC = 16 m. If point D is the midpoint of $\overline{AB}$, E is
    the midpoint of $\overline{AC}$ and F is the midpoint of $\overline{BC}$, then the area of $\triangle DEF$ = _____. (nearest tenth)

   (A) 26.2 m²   (B) 26.4 m²   (C) 26.6 m²   (D) 26.8 m²   (E) 27.0 m²

20. One edge of a cube was increased by 2 in, one edge was decreased by 2 in, and the third edge was
    unchanged. If the volume of the rectangular solid is 32 in³ less than the cube, what is the volume of
    the cube?

   (A) 64 in³   (B) 216 in³   (C) 512 in³   (D) 1000 in³   (E) 1728 in³

21. An athletic field is rectangular in shape, has an area of 360 sq yd and a diagonal of 41 yd. What is
    the perimeter of the field? (nearest whole number)

   (A) 294 ft   (B) 297 ft   (C) 300 ft   (D) 303 ft   (E) 306 ft

22. Find the shortest distance between the lines $9x - 40y = 49$ and $18x - 80y = 16$. (nearest tenth)

   (A) 1.0   (B) 1.2   (C) 1.4   (D) 1.6   (E) 1.8

23. If $f(x) = \dfrac{1}{8}x - 3$ and $g(x) = 2x - 5$, then $\left(g^{-1} \circ f^{-1}\right)(2) = $ _____
   (A) 18   (B) 19.5   (C) 21   (D) 22.5   (E) 24

24. Monty has 5 red marbles, 7 green marbles, and 10 blue marbles. They are identical except for the
    color. How many distinguishable ways can Monty arrange them in a row?

   (A) 512,143,600   (B) 512,143,608   (C) 512,143,616   (D) 512,143,624   (E) 512,143,632

25. A company has received 16 applications from qualified applicants for six entry level analyst
    positions. Ten are from Montana and six are from Wyoming. In how many ways can the positions
    be filled if exactly two of the positions must be filled with applicants from Wyoming?

   (A) 720   (B) 3,150   (C) 12,000   (D) 84,600   (E) 151,200

26. A regular dodecagon is inscribed in a circle. If the area of the circle is 804 in², what is the area of the
    dodecagon? (nearest whole number)

   (A) 752 in²   (B) 756 in²   (C) 760 in²   (D) 764 in²   (E) 768 in²

27. Which are true for the function $g(x) = \dfrac{4}{x-5} + 6$?

   I. The domain is $(-\infty, 5) \cup (5, \infty)$.   II. The range is $(-\infty, 6) \cup (6, \infty)$.   III. The function is odd.

   (A) I only   (B) II only   (C) I, II only   (D) I, III only   (E) I, II, III

28. Which are true for the graph of $h(x) = \dfrac{x^2 - 16}{x^2 - 9x + 20}$?

   I. $(-4, 0)$ is an x-intercept.   II. $(4, 0)$ is an x-intercept.   III. $x = -5$ is a vertical asymptote.
   IV. $x = 4$ is a vertical asymptote.   V. $y = 1$ is a horizontal asymptote.

   (A) I, V only   (B) I, IV, V only   (C) I, III, IV only   (D) I, III, IV, V only   (E) III, IV, V only

29. $\dfrac{30(\cos 95° + i \sin 95°)}{5(\cos 125° + i \sin 125°)} = $ _____.

   (A) $6 - 6\sqrt{3}\,i$ (B) $-3\sqrt{3} - 3i$ (C) $6 + 6\sqrt{3}\,i$ (D) $3\sqrt{3} - 3i$ (E) $6\sqrt{3} - 6i$

30. If $\sin \theta = \dfrac{12}{13}$ and θ is in quadrant II, and if $\tan \alpha = -\dfrac{5}{12}$ and α is in quadrant IV, then what is
   the value of $\cos(\theta - \alpha)$?

   (A) $-\dfrac{120}{169}$ (B) $-\dfrac{90}{169}$ (C) $-\dfrac{60}{169}$ (D) $\dfrac{60}{169}$ (E) $\dfrac{90}{169}$

31. The sound level in decibels, β, is given by $\beta = 10 \log\left(\dfrac{I}{I_0}\right)$, where I is the intensity in watts per

   square meter and I_0 is the threshold of hearing, which equals 10^{-12} watts per square meter. If Kim
   is playing a violin that is producing sound at 87.0 dB, what is the sound level if 11 other musicians
   join Kim and they all play their violins at 87.0 dB? (nearest tenth)

   (A) 97.5 dB   (B) 97.8 dB   (C) 98.1 dB   (D) 98.4 dB   (E) 98.7 dB

32. The equations of the asymptotes of a hyperbola are $y = \dfrac{5}{9}x$ and $y = -\dfrac{5}{9}x$. There is a vertex

   at $(0, 5)$. The coordinates of the foci are (a, b) and (a, c). $|b - c| = $ _____. (nearest tenth)

   (A) 19.8   (B) 20.0   (C) 20.2   (D) 20.4   (E) 20.6

33. The graph of a curve represented by the parametric equations $x = 4\sec(\theta)$ and $y = 3\tan(\theta)$ has
   foci at (a, b) and (c, b). $|a - c| = $ _____. (nearest tenth)
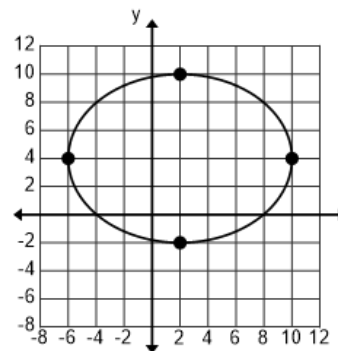
   (A) 5.3   (B) 6.5   (C) 7.6   (D) 8.8   (E) 10.0

34. Find the angle between the vectors $\mathbf{u} = \langle 2, -3, 4 \rangle$ and $\mathbf{v} = \langle -5, 1, -2 \rangle$. (nearest tenth)

    (A) 131.2°   (B) 133.3°   (C) 135.4°   (D) 137.5°   (E) 139.6°

35. The coordinates of the foci of the ellipse shown on the right are (a, b) and (c, b). $a + c =$ _____. (nearest tenth)

    (A) 3.8
    (B) 3.9
    (C) 4.0
    (D) 4.1
    (E) 4.2

36. Consider the sequence $2, 4, 8, 13, 20, 29, 41, 57, 79, k, 155, ...$ $k =$ _____.

    (A) 106   (B) 108   (C) 110   (D) 112   (E) 114

37. A new card game consists of 88 cards that are identical, except for color. There are 20 blue, 20 red, 20 green, and 20 yellow, and 8 purple cards. If you are dealt eight cards, what is the probability of being dealt 4 blue cards, 2 red cards, 1 green card and 1 purple card? (nearest hundred-thousandth)

    (A) 0.00118   (B) 0.00229   (C) 0.00340   (D) 0.00451   (E) 0.00562

38. Some professionals are playing poker. Poker Pete has a full house. This is when a player has a five-card combination with three cards of one type and two of another, such as three 5s and two 9s. Given that Pete has a full house, what is the probability that he has two kings and three queens?

    (A) $\dfrac{1}{624}$ (B) $\dfrac{1}{312}$ (C) $\dfrac{1}{156}$ (D) $\dfrac{1}{78}$ (E) $\dfrac{1}{39}$

39. Convert the rectangular equation $\left(x^2 + y^2\right)^3 = 64x^2y^2$ to a polar equation.
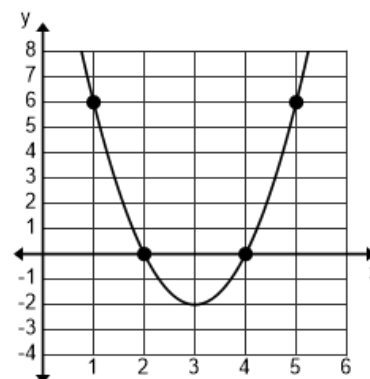
    (A) $r = 4\sin 2\theta$ (B) $r = 4\cos 2\theta$ (C) $r = 8\sin 2\theta$ (D) $r = 8\cos 2\theta$ (E) $r = 16\sin 2\theta$

40. If the focus of the parabola on the right is (a, b), then $a + b =$ _____.

    (A) $\dfrac{7}{8}$ (B) $\dfrac{15}{16}$ (C) 1   (D) $\dfrac{17}{16}$ (E) $\dfrac{9}{8}$

41. The graph on the right is the graph of $f'(x)$. If $f(1) = 1$, then $f(4) =$ _____.

    (A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) 1   (D) $1\dfrac{1}{3}$ (E) $1\dfrac{2}{3}$

Problems 40, 41

42. Consider the differential equation $\dfrac{dy}{dx} = 2x^2 - 3y$. **Let** $y = f(x)$ **be a particular solution to the differential equation with the initial condition** $f(1) = 2$. **Use Euler's method with two steps of equal size, starting at x = 1 to approximate** $f(2)$. **(nearest hundredth)**

(A) 2.25   (B) 2.30   (C) 2.35   (D) 2.40   (E) 2.45

43. Suppose g is a differentiable function and suppose f is a function defined by $f(x) = g\left(x^3 - 5\right)$.
$f'(2) =$ _____.

(A) $3g'(2)$ (B) $6g'(2)$ (C) $6g'(3)$ (D) $12g'(2)$ (E) $12g'(3)$

44. Consider the graph of the parabola $y = x^2 + x$ **in the x-y plane. A particle is moving at a constant speed of** $3\sqrt{26}$ **along the graph of the parabola. Given that** $\dfrac{dx}{dt} > 0$ **when the particle is at** $(2, 6)$, **find the value of** $\dfrac{dy}{dt}$ **when the particle is at** $(2, 6)$. **(nearest tenth)**
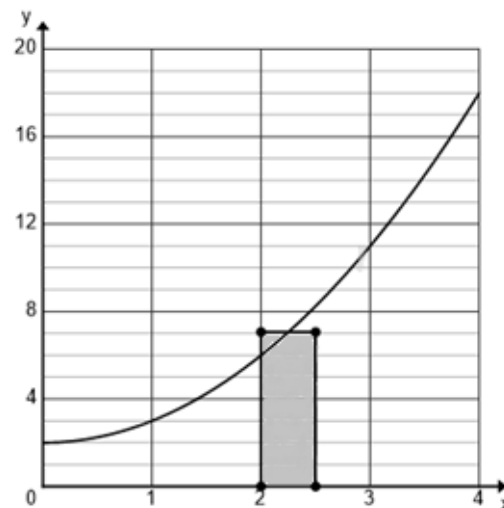
(A) 12.6   (B) 13.2   (C) 13.8   (D) 14.4   (E) 15.0

45. Approximate the area bounded by the curves $y = x^2 + 2$, $y = 0$, $x = 1$, **and** $x = 3$. **Divide the region into 4 subintervals of equal width. Then find the areas of four rectangles formed using the midpoints of each subinterval to find the heights. One of these rectangles is shown on the right. (nearest thousandth)**

(A) 12.600   (B) 12.625   (C) 12.650   (D) 12.675   (E) 12.700

46. Find the exact area of the region described in problem 45.

(A) $12\dfrac{1}{3}$ (B) $12\dfrac{1}{2}$ (C) $12\dfrac{2}{3}$ (D) $12\dfrac{5}{6}$ (E) 13   **Problems 45, 46**

x	2	4	6	10	16
f(x)	3	8	4	-12	2

47. Use the table above to evaluate $\displaystyle\int_{2}^{16}\left(4 - 3f'(x)\right)dx$.

(A) 51   (B) 53   (C) 55   (D) 57   (E) 59

48. Lauren is riding her bicycle from her house on Dove Ave to Amy's house, which is also on Dove Ave. On this day, it took her 10 minutes to go from her house to Amy's house. A graph of her velocity as a function of time is shown on the right. How far is it from Lauren's house to Amy's house? (nearest hundredth)

 (A) 1.10 mi   (B) 1.15 mi   (C) 1.20 mi
    (D) 1.25 mi   (E) 1.30 mi
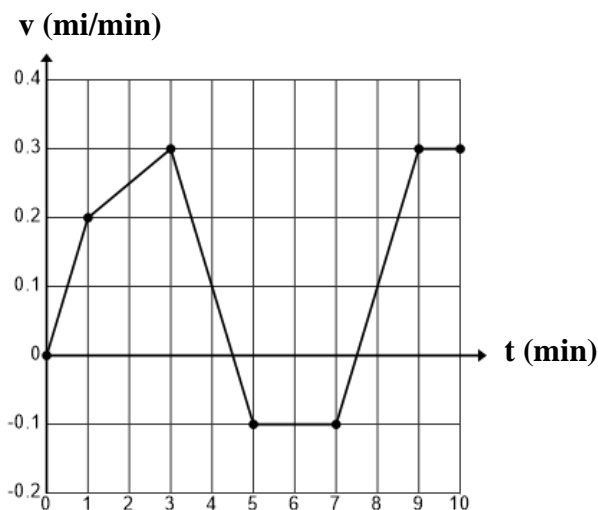
49. Use the graph on the right to find Lauren's acceleration at t = 8 min. (nearest thousandth)

 (A) 0.293 ft/s^2 (B) 0.313 ft/s^2 (C) 0.333 ft/s^2
    (D) 0.353 ft/s^2 (E) 0.373 ft/s^2

v (mi/min)

Problems 48, 49

50-51 A particle is traveling along the x-axis. The position of the particle is given by
 $x(t) = t^4 - 5t^3 + 5t^2 + 5t - 6$, $t \geq 0$, $x(t)$ in feet and t in seconds.

50. Find the total distance traveled by the particle from t = 0 to t = 4. (nearest tenth)

 (A) 36.0 ft   (B) 37.2 ft   (C) 38.3 ft   (D) 39.5 ft   (E) 40.6 ft

51. During the interval when the particle is traveling to the left, what is the maximum speed of the particle? (nearest tenth)

 (A) 2.5 ft/s   (B) 2.7 ft/s   (C) 2.9 ft/s   (D) 3.1 ft/s   (E) 3.3 ft/s

52. Suppose 85% of athletes who use steroids have a positive result when tested. Also, suppose 85% of athletes who do not use steroids have a negative result when tested. If 10% of athletes use steroids, what percent of athletes would test positive when tested? (nearest tenth)

 (A) 20.8%   (B) 21.2%   (C) 21.6%   (D) 22.0%   (E) 22.4%

 The following statistics represent a summary of the distribution in weights for a simple random sample of 1000 high school football players. All measures are in pounds.
 Mean: 210 Median: 190 Standard Deviation: 30 1st Quartile: 165 3rd Quartile: 220

53. Using the above information, which of the following values are considered outliers?
 I. 84 lb   II. 88 lb   III. 300 lb   IV. 306 lb

 (A) IV only (B) I, IV only (C) III, IV only (D) I, III, IV only (E) I, II, III, IV

54. Wildlife employees recently stocked Springfield Lake with 1,000 bass of various weights. Previously, the lake had no bass in it. The distribution of weights of the bass is approximately normal with a mean weight of 3.00 pounds and a standard deviation of 0.75 pounds. A researcher collected a random sample of 25 bass immediately after the lake was stocked. Find the probability that the mean weight of the fish he collected is greater than 3.25 pounds. (nearest thousandth)

 (A) 0.048   (B) 0.128   (C) 0.209   (D) 0.289   (E) 0.369

X	1	2	3	4	5
P(X)	0.02	0.06	0.24	0.32	0.36

55. Find the standard deviation of X from the probability distribution table above. (nearest hundredth)

(A) 0.98   (B) 1.01   (C) 1.04   (D) 1.07   (E) 1.10

56. Assume that the mean number of miles run each week by world class distance runners is 106. If the weekly mileages of world class distance runners are approximately normally distributed and a weekly mileage of 125 represents the 94th percentile, what is the approximate standard deviation of the weekly mileages of world class distance runners? (nearest tenth)

(A) 11.0 mi   (B) 11.3 mi   (C) 11.6 mi   (D) 11.9 mi   (E) 12.2 mi

57-58 Farmer Fred thinks the new sprinkler system he installed this year has increased the mean weight of his pears. Last year, the mean weight of his pears was 6.8 ounces. He took a random sample of 20 of this year's pears and found the mean weight was 7.2 ounces with a standard deviation of 0.9 ounces. Assume the weights of his pears are approximately normally distributed. Fred performed an appropriate statistical test on this data.

57. The proper null hypothesis is $H_0 : \mu = 6.8$ and the proper alternative hypothesis is _____.

(A) $H_a : \mu = 7.2$ (B) $H_a : \mu > 7.2$ (C) $H_a : \mu \neq 6.8$ (D) $H_a : \mu > 6.8$ (E) $H_a : \mu < 6.8$

58. At the $\alpha = .05$ level, he rejected H_0 because the P-value was _____. (nearest ten-thousandth)

(A) 0.0125   (B) 0.0216   (C) 0.0307   (D) 0.0398   (E) 0.0489

Use the table below for problems 59 and 60. AU = astronomical unit

Planet	Distance From the Sun (AU)	Period (Earth Years)
Mercury	0.387	0.241
Venus	0.722	0.615
Earth	1.00	1.00
Mars	1.52	1.88
Jupiter	5.20	11.9
Saturn	9.58	29.5
Uranus	19.2	84.0
Neptune	30.1	165

59. Consider the distance from the Sun as L_1 and the period as L_2. From these choices, the best way to linearize the data is to choose _____ as the independent variable and _____ as the dependent variable.

(A) $\ln(L_1)$, L_2 (B) L_1, $\log(L_2)$ (C) L_1, $(L_2)^2$ (D) L_1, $\ln(L_2)$ (E) $\log(L_1)$, $\log(L_2)$

60. Use an appropriate model to predict the period of Pluto if it is 39.5 AU from the Sun.
 (nearest whole number)

(A) 244 yr   (B) 248 yr   (C) 252 yr   (D) 256 yr   (E) 260 yr

University Interscholastic League
MATHEMATICS CONTEST
HS ● Region ● 2022
Answer Key

1. E	21. A	41. C
2. A	22. A	42. A
3. D	23. D	43. E
4. A	24. E	44. E
5. C	25. B	45. B
6. B	26. E	46. C
7. C	27. C	47. E
8. A	28. A	48. A
9. D	29. D	49. A
10. B	30. A	50. E
11. B	31. B	51. D
12. C	32. E	52. D
13. D	33. E	53. A
14. C	34. C	54. A
15. D	35. C	55. B
16. B	36. C	56. E
17. A	37. B	57. D
18. C	38. C	58. C
19. D	39. A	59. E
20. C	40. E	60. B

University Interscholastic League
MATHEMATICS CONTEST

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

Final _____ _____

2nd _____ _____

1st _____ _____

Contestant # _____

Conference _____

Score   **Initials**

1. _____   21. _____   41. _____

2. _____   22. _____   42. _____

3. _____   23. _____   43. _____

4. _____   24. _____   44. _____

5. _____   25. _____   45. _____

6. _____   26. _____   46. _____

7. _____   27. _____   47. _____

8. _____   28. _____   48. _____

9. _____   29. _____   49. _____

10. _____   30. _____   50. _____

11. _____   31. _____   51. _____

12. _____   32. _____   52. _____

13. _____   33. _____   53. _____

14. _____   34. _____   54. _____

15. _____   35. _____   55. _____

16. _____   36. _____   56. _____

17. _____   37. _____   57. _____

18. _____   38. _____   58. _____

19. _____   39. _____   59. _____

20. _____   40. _____   60. _____