

# TMSCA HIGH SCHOOL MATHEMATICS TEST #13 (UIL D) © MARCH 5, 2022

## GENERAL DIRECTIONS

1. About this test:
  - A. You will be given 40 minutes to take this test.
  - B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

1. Consider the formula for Newton's Law of Universal Gravitation:  $F = \frac{Gm_1m_2}{r^2}$ . Solve for  $r$ , a positive number, if  $F = 3.20 \times 10^{-5}$ ,  $G = 6.67 \times 10^{-11}$ ,  $m_1 = 120$ , and  $m_2 = 250$ . (nearest hundredth)

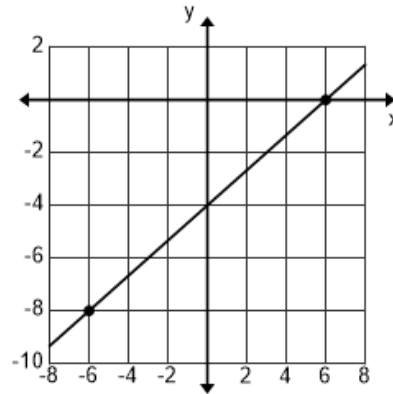
(A) 0.22                      (B) 0.25                      (C) 0.28                      (D) 0.31                      (E) 0.34

2. A motivational speaker was hired to speak at a school assembly. Her fee was \$1000. The price of a student ticket was \$4.00 and the price of an adult ticket was \$8.00. If 522 people bought tickets and the school made \$2620 after paying the speaker, how many adult tickets were sold?

(A) 371                      (B) 375                      (C) 379                      (D) 383                      (E) 387

3. Which of the following points does not lie on the line shown on the right?

(A)  $(-15, -14)$   
 (B)  $(-9, -10)$   
 (C)  $(-3, -6)$   
 (D)  $(9, 2)$   
 (E)  $(15, 8)$



4. Solve the inequality.  $-2x + 6 \leq 13$  and  $3x + 7 > 13$

(A)  $x > 2$                       (B)  $x \geq -3.5$                       (C)  $-3.5 \leq x < 2$                       (D)  $x \geq 2$                       (E)  $x > -3.5$

5. Galen is an elite distance runner. In a recent 10,000 m race, he ran the first 9,000 m at a pace where his average pulse was 174 beats/min. If he averaged 2 min 40.5 sec for each of the first 9 kilometers, how many times did his heart beat during the first 9,000 m of the race? (nearest whole number)

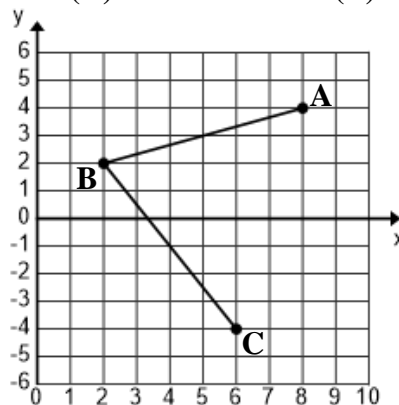
(A) 4183                      (B) 4189                      (C) 4195                      (D) 4201                      (E) 4207

6. The volume of a rectangular solid is  $980 \text{ cm}^3$ . The length is twice the width and the height is three more than the width. Find the total surface area of the solid.

(A)  $604 \text{ cm}^2$                       (B)  $608 \text{ cm}^2$                       (C)  $612 \text{ cm}^2$                       (D)  $616 \text{ cm}^2$                       (E)  $620 \text{ cm}^2$

7. The distance between consecutive grid lines is one mile. If Cindy ran from A to B to C, how far did she run? (nearest foot)

(A) 71,440 ft  
 (B) 71,447 ft  
 (C) 71,454 ft  
 (D) 71,461 ft  
 (E) 71,468 ft



8. The UT crew rowed 15 miles upstream in 2.5 hours. After a brief rest, they rowed 15 miles downstream back to their starting point in 1.5 hours. If they rowed at the same constant rate the entire 30 miles, what was the speed of the current? (nearest tenth)
- (A) 1.6 mph      (B) 1.8 mph      (C) 2.0 mph      (D) 2.2 mph      (E) 2.4 mph
9. Given :  $m\angle A = 6x + 2$  and  $m\angle B = 8x + 4$ . If  $\angle A$  and  $\angle B$  are complementary angles and  $\angle B$  and  $\angle C$  are supplementary angles, then  $m\angle C = \underline{\hspace{1cm}}^\circ$ .
- (A)  $122^\circ$       (B)  $124^\circ$       (C)  $126^\circ$       (D)  $128^\circ$       (E)  $130^\circ$
10. During the summer, Joe got up at 7:00 AM every morning and ran 14 laps in lane 8 on the high school track. If one lap in lane 8 is 453.7 meters, how many miles did he run each morning? (nearest hundredth)
- (A) 3.75      (B) 3.80      (C) 3.85      (D) 3.90      (E) 3.95
11. Consider triangle ABC with point D on ray  $\overrightarrow{AC}$ . If  $AB = 10$ ,  $BC = 8$ ,  $AC = 12$ , and  $AD = 18$ , then  $m\angle BCD = \underline{\hspace{1cm}}^\circ$ . (nearest whole number)
- (A)  $120^\circ$       (B)  $122^\circ$       (C)  $124^\circ$       (D)  $126^\circ$       (E)  $128^\circ$
12. Consider regular hexagon ABCDEF. If  $AC = 9\sqrt{3}$ , then the area of the hexagon is \_\_\_\_\_. (nearest whole number)
- (A) 210      (B) 213      (C) 216      (D) 219      (E) 222
13. Consider isosceles triangle DEF with  $DE = EF = 15$  and  $m\angle E = 48^\circ$ . Find the perimeter of triangle DEF. (nearest tenth)
- (A) 42.2      (B) 42.4      (C) 42.6      (D) 42.8      (E) 43.0
14. The \_\_\_\_\_ is the point of concurrency of the three altitudes of a triangle.
- (A) incenter      (B) centroid      (C) circumcenter      (D) orthocenter      (E) median
15. Triangle ABC is similar to triangle DEF. Given:  $AB = 42$ ,  $BC = 54$ ,  $AC = 78$ ,  $DE = 36$ ,  $EF = x$ , and  $DF = w - 12$ .  $x + w = \underline{\hspace{1cm}}$ . (nearest whole number)
- (A) 119      (B) 122      (C) 125      (D) 128      (E) 131
16. Quadrilateral ABCD is inscribed in a circle with center O. If  $m\angle C = 118^\circ$  and  $m\angle D = 82^\circ$ , then  $m\angle A = \underline{\hspace{1cm}}^\circ$ .
- (A) 62      (B) 72      (C) 82      (D) 98      (E) 118

17. Tom left City Park and cycled 16 miles on a bearing of  $148^\circ$ . From there, he cycled 8 miles on a bearing of  $118^\circ$ . If he wishes to head straight back to City Park, on what bearing should he head?
- (A)  $314^\circ$  (B)  $316^\circ$  (C)  $318^\circ$  (D)  $320^\circ$  (E)  $322^\circ$
18. The number of mice varies inversely as the number of cats and directly as the number of dogs squared. When there were 400 mice, there were 20 dogs and 40 cats. How many mice were there when there were 4 cats and 16 dogs?
- (A) 1280 (B) 1600 (C) 1920 (D) 2240 (E) 2560
19. Find the number that is  $\frac{4}{21}$  of the way from  $-6\frac{3}{4}$  to 9.
- (A)  $-4$  (B)  $-3\frac{3}{4}$  (C)  $-3\frac{1}{2}$  (D)  $-3\frac{1}{4}$  (E)  $-3$
20. There are six chairs in the conference room. How many ways can Abe, Ben, Cam, Dan, Eve and Francis be arranged in a circular seating arrangement?
- (A) 120 (B) 360 (C) 720 (D) 40,320 (E) 46,656
21. Which of the following is not a fourth root of  $16(\cos 120^\circ + i \sin 120^\circ)$ ?
- (A)  $\sqrt{3} + i$  (B)  $-1 + \sqrt{3}i$  (C)  $-\sqrt{3} - i$  (D)  $\sqrt{3} - i$  (E)  $1 - \sqrt{3}i$
22. The expression  $\frac{4\cot(x)}{\tan(2x)}$  simplifies to \_\_\_\_\_.
- (A)  $\sec^2(x) - 2$  (B)  $\cot(x) - \tan(x)$  (C)  $2\csc^2(x) - 4$  (D)  $\tan(x) - \cot(x)$  (E)  $2\sec^2(x) - 4$
23. Find the eccentricity of the ellipse.  $x^2 + 9y^2 - 6x + 36y + 44 = 0$  (nearest hundredth)
- (A) 0.33 (B) 0.48 (C) 0.63 (D) 0.79 (E) 0.94
24. If  $f(x) = x^2 - 16$  and  $h(x) = \sqrt{16 - x^2}$ , then the domain of  $(f(h(x)))$  is \_\_\_\_\_.
- (A)  $[-4, 4]$  (B)  $(-4, 4)$  (C)  $(-4, \infty)$  (D)  $(-\infty, 4)$  (E)  $(-\infty, \infty)$
25. Bank A offers a CD which earns 4.55% annual interest compounded quarterly. Bank B offers a CD which earns a negotiable rate of interest compounded monthly. If Maria places \$12,500 into a CD at bank A and leaves it for 8 years, what is the minimum rate that bank B will need to offer so that Maria will earn at least \$110 more in 8 years if she places \$12,500 in a CD in Bank B? (nearest hundredth)
- (A) 4.57% (B) 4.59% (C) 4.61% (D) 4.63% (E) 4.65%

26. Randy bought a new car in 1991 for \$85,500. In 2021, the book value of the car was \$60,000. If the value of the car depreciates exponentially, what will the book value of the car be in 2039?
- (A) \$46,464.35      (B) \$46,976.63      (C) \$47,488.91      (D) \$48,001.19      (E) \$48,513.47
27. Scott is driving on a road that goes straight to Big Butte. When he passes through Arco, the angle of elevation to the peak of Big Butte is  $3.7^\circ$ . Ten miles later, the angle of elevation to the peak is  $28.3^\circ$ . Find the height of Big Butte. (nearest foot)
- (A) 3880 ft      (B) 3892 ft      (C) 4004 ft      (D) 4016 ft      (E) 4028 ft
28. If  $\frac{x+9}{x^2-x-20} = \frac{A}{x-5} + \frac{B}{x+4}$ , then  $A + B =$  \_\_\_\_\_.
- (A)  $\frac{9}{19}$       (B)  $\frac{9}{14}$       (C) 1      (D)  $\frac{14}{9}$       (E)  $\frac{19}{9}$
29. Find the seventh term of the sequence. 2.5, 2.75, 2.875, 2.9375, 2.96875, ...
- (A) 2.9920125      (B) 2.9920275      (C) 2.9920925      (D) 2.9921875      (E) 2.9922125
30. Mr. White's combination lock will open when you select the correct three numbers in order. Each number must be a positive integer from 1 to 40, inclusive. If the integers can be repeated, how many different lock combinations are possible?
- (A) 56,220      (B) 56,840      (C) 57,660      (D) 59,280      (E) 64,000
31. Find the distance between the lines.  $5x + 12y = 2$  and  $5x + 12y = 18$
- (A)  $\frac{10}{13}$       (B)  $\frac{12}{13}$       (C)  $\frac{14}{13}$       (D)  $\frac{16}{13}$       (E)  $\frac{18}{13}$
32. Consider the hyperbola  $9y^2 - x^2 + 54y + 10x + 55 = 0$ . The vertices are (a, b) and (a, c). If  $b < c$ , then  $b =$  \_\_\_\_\_
- (A) -6      (B) -4      (C)  $-3\frac{1}{3}$       (D) -3      (E)  $-2\frac{2}{3}$
33. Convert the rectangular equation to polar form.  $5x + 3y - 4 = 0$
- (A)  $r = \frac{4}{5\cos\theta + 3\sin\theta}$       (B)  $r = 4\sec\theta$       (C)  $r = \frac{4}{3\cos\theta + 5\sin\theta}$   
(D)  $r = 4\csc\theta$       (E)  $r = \frac{4}{5\cos\theta - 3\sin\theta}$

34. Consider a triangle with the following vertices.  $(2, -6, -4)$ ,  $(10, -2, 4)$ ,  $(-2, 2, 4)$

Which of the following correctly describe the triangle?

I. Right      II. Isosceles      III. Scalene      IV. Acute      V. Obtuse

- (A) I, II      (B) II, IV      (C) II, V      (D) III, IV      (E) III, V

35. Find the phase shift of  $y = 6\cos\left(3x + \frac{3\pi}{4}\right)$

- (A)  $-\frac{3\pi}{4}$       (B)  $-\frac{\pi}{2}$       (C)  $-\frac{\pi}{4}$       (D)  $\frac{\pi}{2}$       (E)  $\frac{3\pi}{4}$

36. The diameter of a cylinder is half the height. If the volume of the cylinder is  $864\pi$ , the total surface area is \_\_\_\_\_.

- (A)  $354\pi$       (B)  $\frac{1070\pi}{3}$       (C)  $360\pi$       (D)  $\frac{1086\pi}{3}$       (E)  $366\pi$

37. The latitude of Millersview, Texas is  $31.41^\circ \text{N}$ . Find the Earth's rotational speed at Millersview if the radius of the Earth at the equator is 3960 miles. (nearest whole number)

- (A) 873 mph      (B) 877 mph      (C) 881 mph      (D) 885 mph      (E) 889 mph

38. Classify the conic and find the angle of rotation.  $x^2 - 2xy + y^2 - 2\sqrt{2}x - 2\sqrt{2}y = 0$

I. Parabola      II. Ellipse      III. Hyperbola      IV.  $\frac{\pi}{6}$       V.  $\frac{\pi}{4}$       VI.  $\frac{\pi}{3}$

- (A) I, IV      (B) II, V      (C) III, VI      (D) I, V      (E) II, VI

39. Find the sum of the solutions in the interval  $0 \leq \theta < 2\pi$  for  $\sin^2 \theta - \cos^2 \theta = \sin \theta$ .

- (A)  $\frac{10\pi}{3}$       (B)  $\frac{7\pi}{2}$       (C)  $\frac{11\pi}{3}$       (D)  $\frac{23\pi}{6}$       (E)  $4\pi$

40. An ant is traveling at constant speed due east from point A to point E. As it travels, it passes points B, C, and D in order. When it reaches point B, it is 10 feet from a flower at point F, and  $m\angle FBE$  is  $36^\circ$ . One minute later, the ant reaches point C, and  $m\angle FCE = 72^\circ$ . How long will it take the ant to travel from point C to point D if point D is 30 feet from the flower? (nearest hundredth)

- (A) 4.99 min      (B) 5.01 min      (C) 5.03 min      (D) 5.05 min      (E) 5.07 min

41. Consider the conditional statement "If  $f(x)$  is differentiable at  $x = c$ , then  $f(x)$  is continuous at  $x = c$ ." Which of the following statements is/are also true?

I. Converse      II. Inverse      III. Contrapositive

- (A) III only      (B) I, III only      (C) II, III only      (D) I, II only      (E) I, II, III

42. A 26-foot-long ladder is leaning against a wall. The base of the ladder is being pulled away from the wall at rate of 6 inches per second. Find the rate at which the angle between the ladder and the wall is changing when the base of the ladder is 10 ft from the wall.

(A)  $\frac{1}{72}$  rad/s      (B)  $\frac{1}{48}$  rad/s      (C)  $\frac{1}{36}$  rad/s      (D)  $\frac{1}{24}$  rad/s      (E)  $\frac{1}{12}$  rad/s

43. Let  $y = f(x)$  be the solution to the differential equation  $\frac{dy}{dx} = y + 1$  with the initial condition  $f(0) = 1$ .

What is the approximation for  $f(1)$  if Euler's method is used, starting at  $x = 0$  with a step size of 0.5?

(A) 3.45      (B) 3.50      (C) 3.55      (D) 3.60      (E) 3.65

44. Find the average value of the function  $f(x) = 4 + e^{-x}$  over the closed interval  $[2, 8]$ .  
(nearest hundredth)

(A) 4.00      (B) 4.01      (C) 4.02      (D) 4.03      (E) 4.04

45. Given:  $g''(x) = 12x - 12$ ,  $g(1) = 5$ ,  $g(-1) = 1$ .  $g(3) = \underline{\hspace{2cm}}$ .

(A) 9      (B) 10      (C) 11      (D) 12      (E) 13

46. Find the volume of the solid generated by revolving the region bounded by the graphs of  $y = x + 2$  and  $y = 2x^2$  about the line  $y = -1$ . (nearest whole number)

(A) 41      (B) 42      (C) 43      (D) 44      (E) 45

47. The function  $f$ , defined by  $f(x) = \int_0^x \cos(t^2) dt$  on the interval  $[1, 3]$ , has a local minimum at  $x = \underline{\hspace{2cm}}$ .  
(nearest hundredth)

(A) 1.25      (B) 1.96      (C) 2.17      (D) 2.33      (E) 2.81

48. Nick's house had a small water leak in an underground pipe in his yard. Water was leaking at a rate of  $L(t) = 2\ln(t+1)$  gallons per hour, where  $t$  is measured in hours. Nick noticed the leak at  $t = 100$  hours and shut off the water. How much water leaked between  $t = 0$  and  $t = 100$  hours?  
(nearest whole number)

(A) 720 gal      (B) 724 gal      (C) 728 gal      (D) 732 gal      (E) 736 gal

49. 

$x$	-3	-1	1	3	5
$g(x)$	-60	-2	0	-6	28

Evaluate  $\int_{-3}^5 (2 - 3g'(x)) dx$  using the table above.

(A) -264      (B) -256      (C) -248      (D) -240      (E) -232



50. Nico's weekly mileage for the last five weeks has been 72, 77, 80, 84 and 77. Find the sum of the mean, median and mode for this data.
- (A) 231                      (B) 232                      (C) 233                      (D) 234                      (E) 235
51. A poll was taken to measure the proportion of registered voters who plan to vote for Gina Thomas for mayor of Boise. A 90% confidence interval was constructed based on a sample survey of registered voters. Assume that all required conditions were met and the confidence interval calculated was (0.47, 0.53). Which of the following is a correct way to interpret this interval?
- (A) Between 47% and 53% of registered voters will vote for Gina.  
 (B) There is a 90% probability that about 50% of registered voters plan to vote for Gina.  
 (C) There is a 45% probability that Gina will win the election.  
 (D) It is likely that the true percentage of registered voters who will vote for Gina is between 47% and 53%.  
 (E) There is a 90% probability that at least 47% of the registered voters will vote for Gina.
52. Jerome and Malcom have both entered many long jump competitions. Jerome's distances follow a normal distribution with a mean of 25 feet and a standard deviation of 0.50 feet. Malcom's distances follow a normal distribution with a mean of 26 feet with a standard deviation of 0.75 feet. Assume that Jerome's distance on any randomly selected day is independent of Malcom's distance on that day. If Jerome and Malcom both enter a track meet on Saturday, what is the probability that Jerome will jump at least as far as Malcom? (nearest hundredth)
- (A) 0.05                      (B) 0.09                      (C) 0.13                      (D) 0.17                      (E) 0.21
53. Assume that the mean personal best long jump for elite male athletes is 26 feet with a standard deviation of 0.75 feet. Assume that the mean personal best long jump for elite female athletes is 22 feet with a standard deviation of 0.65 feet. If Carl has a personal best of 28 ft 6 in, what would Jackie need her personal best to be in order to have a standardized score equal to Carl's standardized score? (nearest inch)
- (A) 23 ft 6 in                      (B) 23 ft 8 in                      (C) 23 ft 10 in                      (D) 24 ft                      (E) 24 ft 2 in
54. Cynthia rolled a fair die six times. Find the probability that she got at least one 4. (nearest thousandth)
- (A) 0.665                      (B) 0.676                      (C) 0.687                      (D) 0.698                      (E) 0.709
55. Last week, Will tested to see how many times he could bench press different amounts of weight. The table above shows the number of times he was able to lift specified amounts of weight. Calculate the LSRL and find the correlation,  $r$ , of the line. (nearest thousandth)

<b>Pounds</b>	<b>100</b>	<b>120</b>	<b>140</b>	<b>160</b>	<b>180</b>	<b>200</b>
<b>Reps</b>	<b>20</b>	<b>17</b>	<b>13</b>	<b>10</b>	<b>6</b>	<b>4</b>

56.  $CAD_{16} - (321_{16} + 789_{16}) = \underline{\hspace{2cm}}_{16}$

- (A) 193                      (B) 203                      (C) 213                      (D) 223                      (E) 233

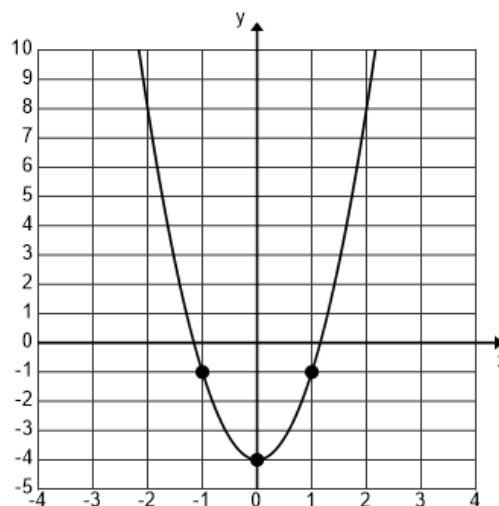
57. Keri baked a large batch of cinnamon rolls. She gave one-fourth of them to Carissa. Then she gave one-third of what remained to Rochelle. Next, she sold half of what remained for \$1.25 each. Finally, she gave 15 of what remained to her dad, leaving her with 6. How much did she make from the cinnamon rolls she sold?

- (A) \$21.25                      (B) \$22.50                      (C) \$23.75                      (D) \$25.00                      (E) \$26.25

58. The graph of  $f'(x)$  is shown on the right.

If  $f(2) = 2$ , then  $f(-3) = \underline{\hspace{2cm}}$

- (A) -14  
(B) -13  
(C) -12  
(D) -11  
(E) -10



59. Dr. Chuang created a list of 24 chemistry problems for his Organic Chemistry students. Twelve of these problems will be on the final exam. Michelle was able to solve 18 of these as she prepared for the exam. Find the probability that she will be able to correctly answer exactly 10 problems on the final exam. (nearest hundredth)

- (A) 0.22                      (B) 0.24                      (C) 0.26                      (D) 0.28                      (E) 0.30

60. The graph of the curve represented by the parametric equations  $x = 2 + 5\cos\theta$  and  $y = 3 + 4\sin\theta$  is an ellipse. Which of the following points is a focus of this ellipse?

- (A) (-3, 3)    (B) (-2, 3)    (C) (2, 3)    (D) (5, 3)    (E) (7, 3)

**2021-2022 TMSCA HSM Test 13 Answer Key**

<b>1. B</b>	<b>21. D</b>	<b>41. A</b>
<b>2. D</b>	<b>22. C</b>	<b>42. B</b>
<b>3. E</b>	<b>23. E</b>	<b>43. B</b>
<b>4. A</b>	<b>24. A</b>	<b>44. C</b>
<b>5. B</b>	<b>25. C</b>	<b>45. A</b>
<b>6. D</b>	<b>26. E</b>	<b>46. D</b>
<b>7. E</b>	<b>27. A</b>	<b>47. C</b>
<b>8. C</b>	<b>28. C</b>	<b>48. D</b>
<b>9. D</b>	<b>29. D</b>	<b>49. C</b>
<b>10. E</b>	<b>30. E</b>	<b>50. B</b>
<b>11. C</b>	<b>31. D</b>	<b>51. D</b>
<b>12. A</b>	<b>32. C</b>	<b>52. C</b>
<b>13. A</b>	<b>33. A</b>	<b>53. E</b>
<b>14. D</b>	<b>34. B</b>	<b>54. A</b>
<b>15. C</b>	<b>35. C</b>	<b>55. A</b>
<b>16. A</b>	<b>36. C</b>	<b>56. B</b>
<b>17. C</b>	<b>37. D</b>	<b>57. E</b>
<b>18. E</b>	<b>38. D</b>	<b>58. B</b>
<b>19. B</b>	<b>39. B</b>	<b>59. B</b>
<b>20. A</b>	<b>40. E</b>	<b>60. D</b>