UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2022

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1. Solve for w if $\sqrt{3w+3}-6=0$.

(A) 9 **(B) 10** **(C) 11** **(D) 12** **(E) 13**

2. Rickie's Roller Palace charges $6.00 per session. They rent skates for $4.00 and knee pads for $5.00. Ted took Martha skating Friday night. They both needed skates and Ted also rented knee pads for himself, as he is a bit clumsy. If the tax rate is 8.25%, what was the total cost?

(A) $27.06 **(B) $27.20** **(C) $27.34** **(D) $27.48** **(E) $27.62**

3. Line L_1 contains the points $(-5, 6)$ and $(3, -2)$. Line L_2 is parallel to L_1 and contains the point $(-2, -4)$. Which of the following points lies on L_2?

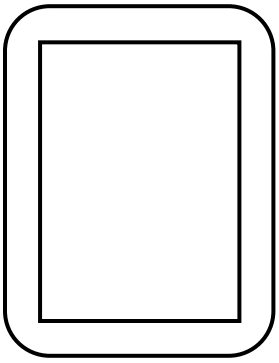
(A) $(-8, 3)$ **(B) $(-4, -1)$** **(C) $(2, -6)$** **(D) $(4, -9)$** **(E) $(8, -14)$**

4. Danny the deer has a top speed of 30 mph and Charlie the cougar has a top speed of 45 mph. Charlie was 150 feet west of Danny when Danny spotted Charlie sneaking up. They both took off at top speed heading east. How long will it take Charlie to catch Danny? (nearest tenth)

(A) 6.6 sec **(B) 6.8 sec** **(C) 7.0 sec** **(D) 7.2 sec** **(E) 7.4 sec**

5. The Tigard City Park has a jogging path along its perimeter. The rectangular shaped park measures 0.45 mile by 0.75 mile when measuring inside the jogging path. If the path is two yards wide, even at the corners, find the total area of the jogging path. (nearest square foot)

(A) 76,010 ft^2
(B) 76,055 ft^2
(C) 76,100 ft^2
(D) 76,145 ft^2
(E) 76,190 ft^2



Problem 5

6. The number of students attending LWU near Eden was 1014 in 1998. Since then, enrollment has increased by 4% each year. The projected enrollment in 2024 is _____ students.

(A) 2802 **(B) 2811** **(C) 2820** **(D) 2829** **(E) 2838**

7. Sarah's Gym in Seymour charges members $10.00 per month, plus $1.50 every time a member comes in to exercise. They also charge $1.00 if you need a towel. In a typical month, Katie exercises 12 times and she needs a towel six times. What is Katie's annual cost to exercise at Sarah's gym?

(A) $408.00 **(B) $412.00** **(C) $416.00** **(D) $420.00** **(E) $424.00**

8. If $\frac{x+4}{x^2-9x+20}-\frac{x-5}{x^2-16}=\frac{ax^2+bx+c}{dx^4+ex^3+fx^2+gx+h}$, then $\frac{a+b+c}{d+e+f+g+h}=$ _____.

(A) 0.09 **(B) 0.12** **(C) 0.15** **(D) 0.18** **(E) 0.21**

9. Jake starts at the Square in Jackson and walks 4 blocks north. Then he turns and heads 6 blocks east. Next, he turns and walks 12 blocks south and stops for lunch at The Mighty Moose Pizzeria. If each block in Jackson is one-eighth of a mile long, how far is The Mighty Moose from the Square?

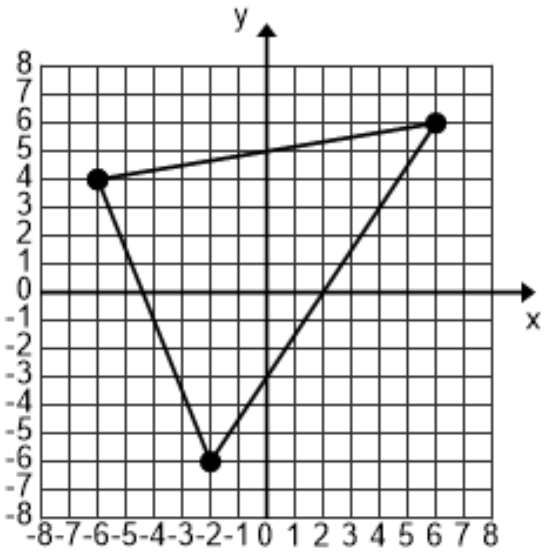
(A) 6400 ft (B) 6450 ft (C) 6500 ft (D) 6550 ft (E) 6600 ft

10. Find the area of the triangle on the right.

(A) 58
(B) 60
(C) 62
(D) 64
(E) 66

11. Find the perimeter of the triangle on the right. (nearest tenth)

(A) 37.1
(B) 37.4
(C) 37.7
(D) 38.0
(E) 38.3



Problems 10, 11

12. Consider the conditional statement "If Vidit has a good day, then he breaks 300." Also, consider the statement "If Vidit does not break 300, then he does not have a good day." This is called the _____ of the conditional statement.

(A) Inverse (B) Contrapositive (C) Converse (D) Transpose (E) Antithesis

13. Consider triangle ABC with $m\angle A = 13x$, $m\angle B = 5x$, and $m\angle C = 4x + 4$. Find the value of $m\angle A - m\angle C$.

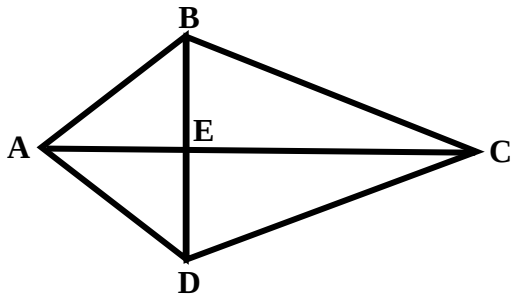
(A) 64° (B) 66° (C) 68° (D) 70° (E) 72°

14. Consider acute triangle DEF with point G on $\overline{DE}$ such that G is the midpoint of $\overline{DE}$. If point P is the centroid of the triangle and PG = 12, then FG = _____.

(A) 18 (B) 24 (C) 30 (D) 36 (E) 42

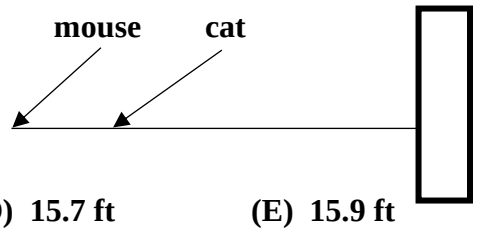
15. Kite ABCD has an area of 120 cm^2. If BE = 6 cm, then AC = _____ cm.

(A) 18
(B) 20
(C) 22
(D) 24
(E) 26



Problem 15

16. The angle of depression from the roof to the cat is 20°. The angle of depression from the roof to the mouse is 18°. If the distance from the cat to the building is 132 ft, find the distance from the mouse to the cat. (nearest tenth)



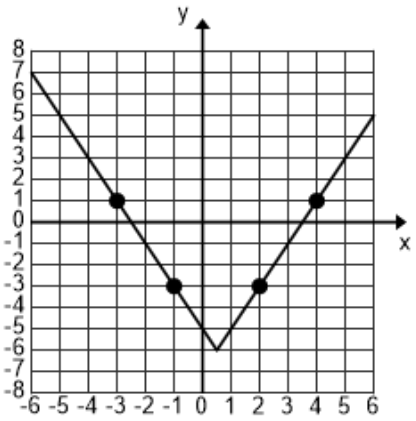
(A) 15.1 ft (B) 15.3 ft (C) 15.5 ft (D) 15.7 ft (E) 15.9 ft

17. By road, it is 670 miles from Bonner's Ferry to Preston. Lura leaves Preston at 8:00 AM and begins driving at an average speed of 64 mph towards Bonner's Ferry. Phil leaves Bonner's Ferry at 10:00 AM and begins driving towards Preston at an average speed of 60 mph. How far from Preston are they when they meet? (nearest mile)

(A) 400 mi (B) 404 mi (C) 408 mi (D) 412 mi (E) 416 mi

18. Which point lies on the curve shown on the right?

(A) (11, 16)
(B) (14, 20)
(C) (17, 25)
(D) (20, 33)
(E) (23, 41)



Problem 18

19. The diagonal of a television screen measures 51 inches. The width of the rectangularly shaped television screen is 21 inches greater than the height. Find the area of the television screen.

(A) 1072 in^2 (B) 1080 in^2 (C) 1088 in^2 (D) 1096 in^2 (E) 1104 in^2

20. Consider the parent function $f(x) = x^2$. If the graph of $y = f(x)$ is reflected across the x-axis, shifted horizontally 6 units left, and then shifted vertically 4 units down, the point $(-1, b)$ lies on the graph. Find the value of b.

(A) –29 (B) –28 (C) –27 (D) –26 (E) –24

21. Margarita deposits $10,000 into an account that earns 6.25% annual interest compounded quarterly. Juan deposits $12,000 into an account that earns 3.75% annual interest compounded monthly. How many years are required for the balance in Margarita's account to equal the balance in Juan's account? (nearest tenth)

(A) 6.6 (B) 6.8 (C) 7.0 (D) 7.2 (E) 7.4

22. The graph of $f(x) = \dfrac{4x^2}{x^2 - 16}$ has _____ asymptotes.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

23. The graph of the circle $x^2 + y^2 = 25$ and the graph of the line $y = .8x + 2$ intersect at points A and B. Find the length of $\overline{AB}$. (nearest tenth)

(A) 8.7 **(B) 8.9** **(C) 9.1** **(D) 9.3** **(E) 9.5**

24. Let g(x) be the inverse function of $f(x) = 5x\sqrt{x}$. Find the smallest positive integer value of x such that $g(x) > 2$.

(A) 13 **(B) 14** **(C) 15** **(D) 16** **(E) 17**

25. Let w and m be the complex solutions to the equation $3x^2 - 2x + 15 = 0$. If $w^2 - m^2 = \frac{a\sqrt{b}}{c}i$ where a, b and c are positive integers then the value of a + b + c = _____.

(A) 24 **(B) 26** **(C) 28** **(D) 30** **(E) 32**

26. Find x if $4\left(3^{2x-1}\right) - 5 = 6$.

(A) $\frac{1}{2} + \frac{2\ln\left(\frac{11}{4}\right)}{\ln(9)}$ **(B)** $\frac{\ln\left(\frac{33}{4}\right)}{\ln(3)}$ **(C)** $\frac{1}{2} + \frac{\ln\left(\frac{11}{4}\right)}{\ln(3)}$ **(D)** $\frac{\ln\left(\frac{33}{4}\right)}{\ln(6)}$ **(E)** $\frac{1}{2} + \frac{\ln\left(\frac{11}{4}\right)}{\ln(9)}$

27. The graph of $y = \frac{1}{4}\tan(3x)$ has a vertical asymptote at x = _____.

(A) $\frac{3\pi}{4}$ **(B)** $\frac{4\pi}{3}$ **(C)** 2π **(D)** $\frac{13\pi}{6}$ **(E)** $\frac{29\pi}{12}$

28. The vector $u = \langle 6, -4 \rangle$ is orthogonal to the vector v = _____.

(A) $\langle 2, 3 \rangle$ **(B)** $\langle -2, 3 \rangle$ **(C)** $\langle 2, -3 \rangle$ **(D)** $\langle 3, -2 \rangle$ **(E)** $\langle -4, 6 \rangle$

29. Consider the system of inequalities $\begin{matrix} x^2 - y \le 4 \\ y - x \le 6 \end{matrix}$ Which of the following points does not satisfy the system?

(A) (−2.6, 3.3) **(B) (−1.9, − 0.3)** **(C) (1.1, − 2.7)** **(D) (2.2, 0.8)** **(E) (3.6, 9.6)**

30. Phillip Physics was finding the current (in amps) in different parts of a circuit. He came up with the following system: $6 - 12I_1 - 24I_2 = 0$, $6 - 12I_1 - 6I_3 - 3 = 0$, $I_1 = I_2 + I_3$
Find I_3. (nearest hundredth)

(A) 0.07 A **(B) 0.11 A** **(C) 0.14 A** **(D) 0.18 A** **(E) 0.21 A**

31. Assume the license plates in a small country must consist of 4 letters followed by 3 digits. How many distinct license plates can be formed in this country?

(A) 11,232,000 **(B) 122,668,000** **(C) 234,104,000** **(D) 345,540,000** **(E) 456,976,000**

32. Consider the graph of $x^2 - 4y^2 - 4x - 24y - 33 = 0$. The vertices of the graph are (a, b) and (c, b). If $a < c$, then a = _____.

(A) 1 **(B) 1.25** **(C) 1.5** **(D) 1.75** **(E) 2**

33. Find the angle between the vectors $\mathbf{u} = \langle 2, -3, 4 \rangle$ and $\mathbf{v} = \langle -1, -2, 5 \rangle$. (nearest tenth)

(A) 35.2° **(B) 35.5°** **(C) 35.8°** **(D) 36.1°** **(E) 36.4°**

34. Juanita is flying a kite. The kite string has a length of 333 ft and the angle of elevation from Juanita to the kite is 58.6°. If the end of the string Juanita is holding is 4 ft above the ground, how high above the ground is the kite? (nearest foot)

(A) 282 ft **(B) 284 ft** **(C) 286 ft** **(D) 288 ft** **(E) 300 ft**

35. Simplify: $\dfrac{a}{b + \dfrac{c}{d + \dfrac{e}{f}}}$

(A) $\dfrac{af + ae}{bdf + be + cf}$ **(B)** $\dfrac{adf + ae}{bd + be + cf}$ **(C)** $\dfrac{ad + aef}{bdf + be + cf}$ **(D)** $\dfrac{ad + ae}{bf + be + cf}$ **(E)** $\dfrac{adf + ae}{bdf + be + cf}$

36. Five physics books and six chemistry books are on a shelf. How many ways can they be arranged if the physics books are kept together and the chemistry books are kept together?

(A) 86,400 **(B) 129,600** **(C) 172,800** **(D) 216,000** **(E) 259,200**

37. Simplify: $39^{\log_{39}(7)}$

(A) 0 **(B) 1** **(C) 7** **(D) 39** **(E) 49**

38. Twelve men can do 20 jobs in five days. How many days would it take forty men to do 180 jobs?

(A) 12 **(B) 13.5** **(C) 15** **(D) 16.5** **(E) 18**

39. Find the slope of the line tangent to the graph of $f(x) = x + \dfrac{6}{x}$ at the point $(6, 7)$.

(A) $\dfrac{1}{2}$ **(B)** $\dfrac{2}{3}$ **(C)** $\dfrac{5}{6}$ **(D) 1** **(E)** $\dfrac{7}{6}$

40. Consider the graph of $f(x) = 5x^5 - 3x^3$. Find the sum of the y-values of the points of inflection. (nearest hundredth)

(A) –0.32 (B) –0.16 (C) 0.00 (D) 0.16 (E) 0.32

41. Use four rectangles of equal widths to find an approximation of the area of the first quadrant region bounded by the curves $y_1 = 4 - x^2$, $y_2 = 0$, and $x = 0$. Use left endpoints of each interval to find an upper sum.

(A) 5.5 (B) 5.75 (C) 6 (D) 6.25 (E) 6.5

42. A group of 100 mice was accidentally released on a small island west of the state of Washington on March 1, 2018. A University of Washington professor estimates that the island can support no more than 5,000 mice. On March 1, 2020, the mice population had reached 1,200. The professor used a logistic model to predict the population on March 1, 2025. He predicted _____ mice.

(A) 4979 (B) 4981 (C) 4983 (D) 4985 (E) 4987

43. When evaluating $\int \left(x^2 \ln(x) \right) dx$ using the method of integration by parts, the best choice for dv is _____.

(A) $x\,dx$ (B) x^2 (C) $x^2\,dx$ (D) $\ln(x)$ (E) $x^3\,dx$

44. Let $\sum a_n$ be a series with nonzero terms. The Ratio Test for determining the convergence or divergence of the series fails when $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| =$ _____.

(A) 0 (B) 0.5 (C) 1 (D) 1.5 (E) 2

45. Find the area of the region bounded by the curves $y_1 = 8 - 2x$ and $y_2 = \frac{1}{x}$. (nearest tenth)

(A) 10.8 (B) 11.0 (C) 11.2 (D) 11.4 (E) 11.6

46. A particle was traveling along the x-axis. It was accelerating to the right at a constant 2.44 m/s^2. At t = 0, the position of the particle was at x = 3.00 m and the velocity of the particle was –5.66 m/s. The position of the particle at t = 4.22 s is at x = _____ m. (nearest hundredth)

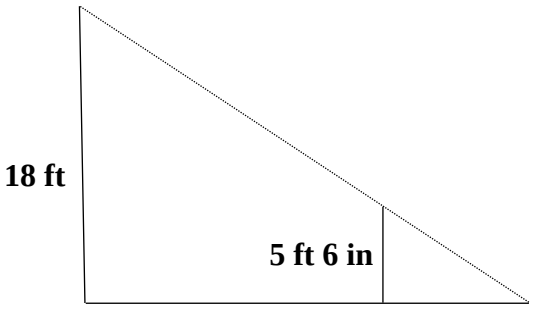
(A) 0.62 (B) 0.84 (C) 1.06 (D) 1.28 (E) 1.30

47. A rectangular sheet of cardboard is to be used to make an open top box by cutting out equal sized squares from each corner. If the dimensions of the cardboard sheet are 36 in by 48 in, then the maximum volume possible of the open top box is _____ in^3. (nearest whole number)

(A) 5240 (B) 5244 (C) 5248 (D) 5252 (E) 5256

48. A woman is 5 ft 6 in tall and walks at a rate of 6 ft/s away from a streetlight that is 18 ft above the ground. When she is 20 ft from the base of the light, at what rate is the length of her shadow increasing? (nearest hundredth)

(A) 2.64 ft/s
(B) 2.75 ft/s
(C) 2.86 ft/s
(D) 2.97 ft/s
(E) 3.03 ft/s



Problem 48

49. Northern Arizona won the NCAA DI men's cross-country race held on March 15, 2021. The 10-K times of their top five runners were 29:58, 30:02, 30:05, 30:10 and 30:50. Coach Smith analyzed the times and found that the difference between the mean and median was _____. (nearest tenth)

(A) 7.1 sec **(B) 7.4 sec** **(C) 7.7 sec** **(D) 8.0 sec** **(E) 8.3 sec**

50. In a normal distribution with mean μ and standard deviation σ, _____% of the observations fall within 2σ of μ. (nearest whole number)

(A) 91 **(B) 93** **(C) 95** **(D) 97** **(E) 99**

51. Maria received some data for 100 girls attending Chavez Middle School. She entered the data into a computer and the statistical software on the computer generated a Least Squares Regression Line with a correlation of r = 0.825. The measurements of the girls' heights and weights were made in centimeters and pounds. Her teacher told her to convert the heights to inches and recompute the LSRL. What effect did this have on the value of the correlation, r?

(A) r decreased by a factor of $\frac{2.54}{100}$ **(B) r decreased by a factor of $\frac{2.54}{\sqrt{100}}$** **(C) no effect**

(D) r increased by a factor of $\frac{2.54}{100}$ **(E) r increased by a factor of $\frac{2.54}{\sqrt{100}}$**

AP courses	0	1	2	3	4
Probability	.02	.14	.28	.32	.24

52. Students at the Texline Young Men's Academy are encouraged to take AP courses their senior year. They are not allowed to take more than four AP courses. Use the table above to find the expected number of AP courses a student at TYMA will take his senior year.

(A) 2.62 **(B) 2.64** **(C) 2.66** **(D) 2.68** **(E) 2.70**

53. Mr. Stat Teacher brought a huge jar containing 5000 marbles to class. Each marble was either red or blue. Michael took an SRS of 200 marbles from the jar. He separated the marbles into two bowls and ended up with 112 blue marbles in one bowl and 88 red marbles in the other bowl. Construct a 95% confidence interval for p, the proportion of blue marbles in the jar. (nearest ten-thousandth)

(A) (.4835, .6365) **(B) (.4912, .6288)** **(C) (.5066, .6134)** **(D) (.5176, .6024)** **(E) (.5312, .5888)**

54. Coach uses one of three players to bat lead-off in games. Joe is selected to lead off 20% of the time and he leads off with a hit 30% of the time. Bob is selected to lead off 30% of the time and he leads off with a hit 20% of the time. Mike is selected to lead off 50% of the time and leads off with a hit 40% of the time. Find the probability that the lead-off hitter for this team leads off with a hit. (nearest hundredth)

(A) 0.28 (B) 0.30 (C) 0.32 (D) 0.34 (E) 0.36

55. Suppose the distribution of the length of drives by Golfer George in his long career is approximately normal with a mean of 315 yards and a standard deviation of 15 yards. What percent of his drives are between 330 yards and 345 yards? (nearest tenth)

(A) 12.5% (B) 13.6% (C) 14.7% (D) 15.8% (E) 16.9%

56. This mathematician is credited for developing differential and integral calculus independently of Isaac Newton.

(A) George Boole (B) John Napier (C) John Venn (D) Gottfried Leibniz (E) Alicia Stott

57. Which of the following numbers are classified as "perfect" numbers?
I. 6 II. 28 III. 496 IV. 8,132

(A) I, II only (B) I, III only (C) II, III only (D) I, II, II only (E) I, II, III, IV

58. Find the focus of the parabola shown on the right.

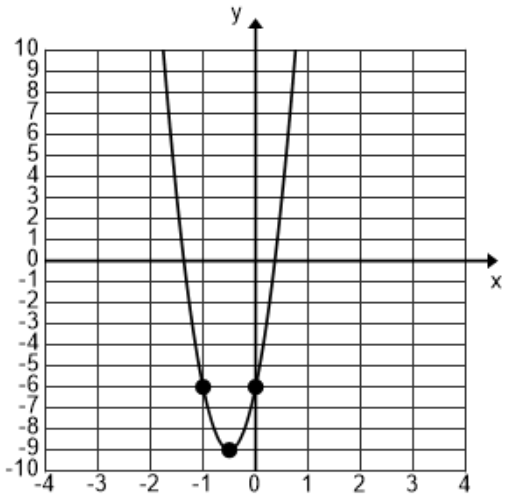
(A) $\left(-\frac{1}{2}, -8\right)$

(B) $\left(-\frac{1}{2}, -8\frac{3}{4}\right)$

(C) $\left(-\frac{1}{2}, -8\frac{11}{12}\right)$

(D) $\left(-\frac{1}{2}, -8\frac{23}{24}\right)$

(E) $\left(-\frac{1}{2}, -8\frac{47}{48}\right)$



Problems 58, 59

59. Suppose the graph on the right is the graph of $f''(x)$. If $f'(1) = 4$ and $f(-1) = 0$, then $f(-2) =$ _____.

(A) −14 (B) −12 (C) −10 (D) −8 (E) −6

60. $ABC6_{14} + ABC6_{15} =$ _____$_{16}$

(A) 106C5 (B) 105B5 (C) 104A5 (D) 10395 (E) 10285

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational B ● 2022
Answer Key

1. C	**21. E**	**41. D**
2. A	**22. D**	**42. C**
3. E	**23. E**	**43. C**
4. B	**24. C**	**44. C**
5. D	**25. C**	**45. E**
6. B	**26. E**	**46. B**
7. A	**27. D**	**47. A**
8. C	**28. A**	**48. A**
9. E	**29. D**	**49. D**
10. D	**30. A**	**50. C**
11. B	**31. E**	**51. C**
12. B	**32. A**	**52. A**
13. C	**33. B**	**53. B**
14. D	**34. D**	**54. C**
15. B	**35. E**	**55. B**
16. E	**36. C**	**56. D**
17. C	**37. C**	**57. D**
18. D	**38. B**	**58. E**
19. B	**39. C**	**59. D**
20. A	**40. C**	**60. E**