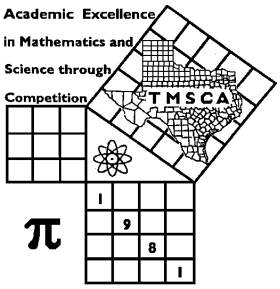

TMSCA HIGH SCHOOL MATHEMATICS STATE TEST (UIL E) ©

MARCH 19, 2022

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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2021-2022 TMSCA High School Math State Meet Test

1. **Consider the formula for the capacitance of a parallel-plate capacitor: $C = k\varepsilon_0 \frac{A}{d}$. Find k when $C = 1.62\times10^{-9}$, $\varepsilon_0 = 8.854\times10^{-12}$, $A = 0.122$, and $d = 0.00488$. (nearest tenth)**

   (A) 4.6 (B) 5.5 (C) 6.4 (D) 7.3 (E) 8.2

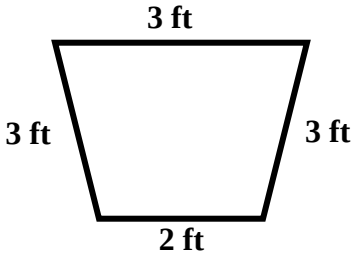
2. **All 22 of the lawns on 3rd Street are the same size. Dan can mow a lawn in 90 minutes. Jan can mow a lawn in 85 minutes. Nan can mow a lawn in 80 minutes. If they work together, how long would it take them to mow all 22 lawns? (nearest minute)**

   (A) 10 hr 14 min (B) 10 hr 18 min (C) 10 hr 22 min (D) 10 hr 26 min (E) 10 hr 30 min

3. **Consider a line segment with endpoints $(-4,6)$ and $(6,-8)$. Find the y-intercept of the perpendicular bisector of this line segment.**

   (A) $\left(0,-\frac{8}{7}\right)$ (B) $\left(0,-\frac{10}{7}\right)$ (C) $\left(0,-\frac{12}{7}\right)$ (D) $(0,-2)$ (E) $\left(0,-\frac{16}{7}\right)$

4. **A water trough on the Waterfield Ranch is full of water. A side view (isosceles trapezoid) of the trough is shown on the right. The length of the trough is 6 ft. How many gallons of water does trough hold when full? (nearest gallon)**



   (A) 332 (B) 336 (C) 340 (D) 344 (E) 348

5. **Rancher Rob has 410 feet of fencing to use to make a rectangularly shaped pen for his horses. If he uses all 410 feet of the fencing and the length is 1.5 times the width, what is the area of the pen?**

   (A) 10,030 ft^2 (B) 10,044 ft^2 (C) 10,058 ft^2 (D) 10,072 ft^2 (E) 10,086 ft^2

6. **Worley ate lunch at Furr's Cafeteria all 365 days last year. He only ate one of two lunches: the turkey plate which cost \$14.07 or the enchilada plate which cost \$14.87. If he spent a total of \$5,277.15 on lunch last year, how many times did he order the turkey plate?**

   (A) 177 (B) 180 (C) 184 (D) 188 (E) 192

7. **$5+18+31+44+57+...+434+447=$**

   (A) 7,897 (B) 7,910 (C) 7,923 (D) 7,936 (E) 7,949

8. **Consider triangle ΔABC. $AB = 17$, $BC = 20$, and $AC = 23$. $m\angle ABC - m\angle ACB =$ _____° (nearest tenth)**

   (A) 29.3 (B) 29.7 (C) 30.1 (D) 30.5 (E) 30.9

9. **$1+3+6+10+15+21+28+...+1176+1225=$ _____.**

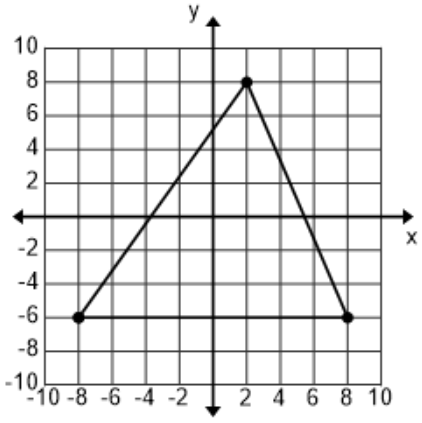
   (A) 20,792 (B) 20,801 (C) 20,809 (D) 20,817 (E) 20,825

10. Find the perimeter of the triangle shown on the right. (nearest tenth)

(A) 48.0 (B) 48.4 (C) 48.8 (D) 49.2 (E) 49.6

11. Find the area of the triangle shown on the right. (nearest tenth)

(A) 105.4 (B) 107.6 (C) 109.8 (D) 112.0 (E) 114.2



Problems 10, 11, 12

12. Find the centroid of the triangle shown on the right.

(A) $\left(\frac{2}{3},-\frac{4}{3}\right)$ **(B)** $\left(\frac{2}{5},-\frac{4}{5}\right)$ **(C)** $\left(\frac{1}{2},-1\right)$ **(D)** $\left(\frac{7}{10},-\frac{9}{7}\right)$ **(E)** $\left(\frac{4}{5},-\frac{6}{5}\right)$

13. License plates in Mathville must have two capital letters followed by four positive single-digit integers. How many different license plates are possible? Repetition is permitted.

(A) 1,965,600 (B) 3,276,000 (C) 4,435,236 (D) 5,638,760 (E) 6,760,000

14. The roots of $3x^3+bx^2+cx+d=0$ are $-4,-2$ and 5. $b+c+d=$ _____.

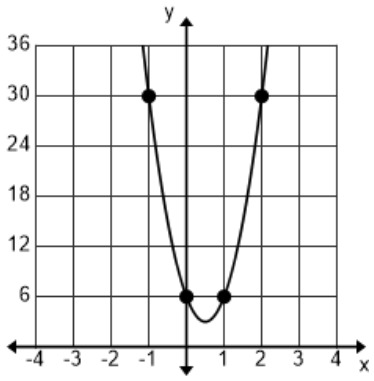
(A) –183 (B) –176 (C) –169 (D) –162 (E) –155

15. Consider the graph of $y=f''(x)$ shown on the right. If $f(1)=3$ and $f(-1)=15$, then $f(2)=$ _____. (nearest whole number)

(A) 6 (B) 9 (C) 12 (D) 15 (E) 18

16. The absolute minimum of $f(x)$ is _____. (nearest whole number)

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6



Problems 15 and 16

17. Kami is 5 ft 4 in tall. She is standing in the park across from the Holiday Inn. At 5:15 PM, the length of her shadow was 3 ft 6 in and the length of the shadow of the Holiday Inn was 96 ft. How tall is the Holiday Inn? (nearest inch)

(A) 142 ft 7 in (B) 143 ft 6 in (C) 144 ft 5 in (D) 145 ft 4 in (E) 146 ft 3 in

18. Consider a circle with points A, B, C and D on the circle. Chord $\overline{BD}$ intersects chord $\overline{AC}$ at point E. If $BE=18, BD=39$ and $CE=30$, then $AC=$ _____.

(A) 41.8 (B) 42.0 (C) 42.2 (D) 42.4 (E) 42.6

19. Consider the function $f(x)=\frac{4+7x}{3x-5}$. Evaluate $f(4)+f^{-1}(4)$. (nearest hundredth)

(A) 9.26 (B) 9.37 (C) 9.48 (D) 9.59 (E) 9.70

20. The frequency of vibration of a piano string varies directly as the square root of the tension that the string is under and inversely as the length of the string. A middle C string vibrates at 256 Hz. Find the frequency of a string that is under twice the tension and is twice as long. (nearest whole number)

(A) 181 Hz (B) 226 Hz (C) 272 Hz (D) 317 Hz (E) 362 Hz

21. Which of the following is/are asymptotes of the graph of $f(x)=\frac{x^2-16}{x^2-x-12}$?

I. $x=-4$ II. $x=-3$ III. $x=4$ IV. $y=1$

(A) 1, III only (B) I, IV only (C) II, IV only (D) II, III, IV only (E) I, II, III, IV

22. The new city park in Lindsay will have a rectangular athletic field. The area of the field must be at least 2000 m^2 and the perimeter must be exactly 200 m. Out of the following intervals, for the length in meters, which one is the widest interval that meets the criteria?

(A) 27.1, 72.9 (B) 27.3, 72.7 (C) 27.5, 72.5 (D) 27.7, 72.3 (E) 27.9, 72.1

23. Find the distance along the Earth's surface from a city located at a latitude of $57°\ 35'\ 22''$ N to a city located at a latitude of $18°\ 43'\ 19''$ N if both cities are located at the same longitude. Assume the Earth is a sphere with a radius of 3960 miles. (nearest mile)

(A) 2686 mi (B) 2689 mi (C) 2692 mi (D) 2695 mi (E) 2698 mi

24. A regular pentagon is inscribed in a circle. If the area of the circle is 3,420 cm^2, what is the area of the pentagon? (nearest cm^2)

(A) 2576 cm^2 (B) 2579 cm^2 (C) 2582 cm^2 (D) 2585 cm^2 (E) 2588 cm^2

25. Simplify: $\frac{\cos(2x)\cot(2x)}{1-\sin(2x)}-1$

(A) sec(2x) (B) csc(2x) (C) cos(2x) (D) sin(2x) (E) tan(2x)

26. Which of the following is a cube root of -216?

(A) $3\sqrt{3}-3i$ (B) $3-3\sqrt{3}i$ (C) $-3\sqrt{3}+3i$ (D) $-3\sqrt{3}-3i$ (E) $-3-3\sqrt{3}i$

27. Find the seventh term of the sequence. $1, 3, \frac{9}{2}, \frac{9}{2}, \frac{27}{8}, \frac{81}{40}, \ldots$

(A) $\frac{77}{80}$ (B) $\frac{79}{80}$ (C) $\frac{81}{80}$ (D) $\frac{83}{80}$ (E) $\frac{87}{80}$

28. The E&M class at Argyle was analyzing a circuit with two batteries and three resistors. They used the following system of equations to find the currents flowing through each resistor. Which of the following is the value of I_3?

$$24 - 12I_1 + 18 - 36I_2 = 0$$
$$24 - 12I_1 - 6I_3 = 0$$
$$I_1 = I_2 + I_3$$

(A) 0.65 (B) 0.9 (C) 1.15 (D) 1.3 (E) 1.55

29. Raytheon Technologies interviews 15 people for 7 engineering positions. Six of the people are graduates of Baylor University. If all 15 applicants are equally qualified, what is the probability that exactly three of the seven positions will be filled with Baylor graduates? (nearest hundredth)

(A) 0.31 (B) 0.33 (C) 0.35 (D) 0.37 (E) 0.39

30. Find the angle between the lines $3x - 4y = 5$ and $2x + 3y = 11$. (nearest tenth)

(A) 70.3° (B) 70.6° (C) 70.9° (D) 71.2° (E) 71.5°

31. The vertex of the parabola $2x^2 - 12x - y = -16$ is (a, b). $a + b =$ _____.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

32. The graph of $x^2 + 2xy + y^2 - 4 = 0$ is _____

(A) an ellipse (B) a hyperbola (C) a parabola (D) two parallel lines (E) a point

33-34 The center field fence at Hirschi High is 8 ft tall and it is 400 ft from home plate. Anthony hits a ball when it is 2 ft above home plate and the path the ball takes toward the centerfield bleachers is modeled by the parametric equations $x = \left(125\cos\left(\frac{\pi}{6}\right)\right)t$ and $y = 2 + \left(125\sin\left(\frac{\pi}{6}\right)\right)t - 16.087t^2$.

33. Find the elapsed time from the moment the ball left the bat and when the ball was directly over the outfield fence. (nearest tenth)

(A) 3.3 s (B) 3.5 s (C) 3.7 s (D) 3.9 s (E) 4.1 s

34. By how much did the ball clear the outfield fence? (nearest tenth)

(A) 3.7 ft (B) 4.1 ft (C) 4.5 ft (D) 4.9 ft (E) 5.3 ft

35. Find the area of a triangle with vertices $(3, -5, 6)$, $(-2, 1, 2)$, and $(4, -2, 4)$. (nearest tenth)

(A) 12.6 (B) 12.8 (C) 13.0 (D) 13.2 (E) 13.4

36. The windshield wiper on the rear window of my car rotates 140°. The total length of the wiper mechanism is 27 inches and the length of the wiper blade is 21 inches. Find the area covered by wiper blade. (nearest whole number)

(A) 841 in^2 (B) 847 in^2 (C) 853 in^2 (D) 859 in^2 (E) 866 in^2

37. Grant Fisher placed fifth in the Olympic 10,000 m. During the winter months, he concentrates on high mileage. Over a six-week period, he put in weekly totals of 108 mi, 112 mi, 120 mi, 116 mi, 122 mi and 116 mi. Find the sum of the mean, median, mode and range of the data. (nearest tenth)

(A) 361.7 mi (B) 362.7 mi (C) 363.7 mi (D) 364.7 mi (E) 365.7 mi

38. The average salary for teachers at Sundown High School was $56,380 in 2020 and the school board decided to give each teacher a $2,000 raise for 2021. All of the teachers from 2020 returned in 2021 and no new teachers were hired. The Statistics class at SHS analyzed all of the data from the salaries for 2020 and 2021. Which of the following did not change from 2020 to 2021?

I. mean II. median III. IQR IV. standard deviation V. quartiles

(A) I, II, V only (B) I, IV only (C) III, V (D) II, IV, V only (E) III, IV only

Year	2010	2012	2014	2016	2018	2020
Population	10,294	10,469	10,657	10,822	10,994	11,175

39. A plot of the data shows that a _____ regression is appropriate to use as an equation to model the population for the years from 2010 to 2020.

(A) linear (B) quadratic (C) cubic (D) exponential (E) logistic

40. It is risky to use the model you calculated for problem 39 to predict the population in 2050 because you would be estimating the population assuming the existing trend continues after 2020. The statistics term for making such predictions is _____.

(A) convergence (B) normal approximation (C) interpolation (D) extrapolation (E) correlation

41. The potatoes from Farmer Phil's farm have a mean weight of 7.5 oz with a standard deviation of 1.5 oz. Assume the weights are approximately normally distributed. If two potatoes are selected at random, what is the probability that both will weigh more than 9 oz? (nearest thousandth)

(A) 0.025 (B) 0.036 (C) 0.047 (D) 0.058 (E) 0.069

42. Suppose it is known that 22% of the seniors at Coronado High School will enroll at Texas Tech after graduating. If a group of 15 seniors are randomly selected, what is the probability that exactly 5 of the selected seniors will enroll at Texas Tech? (nearest thousandth)

(A) 0.118 (B) 0.129 (C) 0.140 (D) 0.151 (E) 0.162

43. Glen Polk Auto has two locations, one in Sanger and one in Gainesville. Sixty percent of all sales occur in Gainesville. At the Sanger location, 40% of the vehicles sold are pickups. At the Gainesville location, 55% of the vehicles sold are pickups. If a customer who purchased a vehicle is selected at random, what is the probability that he did not purchase a pickup?

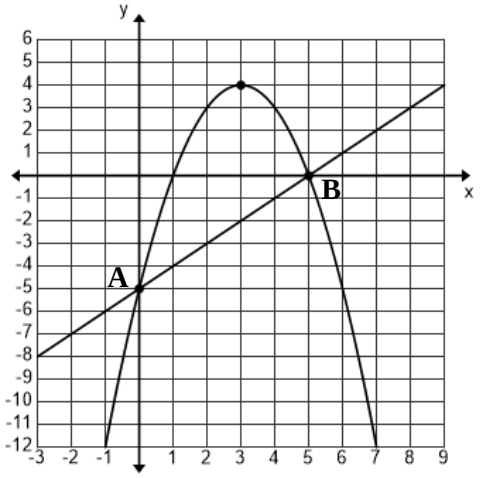
(A) 0.47 **(B) 0.49** **(C) 0.51** **(D) 0.53** **(E) 0.55**

44. The midpoint of line segment $\overline{AB}$ is the point (a, b).
$a + b =$ _____

(A) -1 **(B) -0.5** **(C) 0** **(D) 0.5** **(E) 1**

45. Find the area bounded by the graphs of the parabola and the line shown on the right.
(nearest tenth)

(A) $20\frac{1}{6}$ **(B) $20\frac{1}{3}$** **(C) $20\frac{1}{2}$** **(D) $20\frac{2}{3}$** **(E) $20\frac{5}{6}$**



Problems 44, 45

46. Nike is designing a 400 m track for their headquarters in Beaverton, Oregon. The design consists of a rectangle with a semicircle on each end. Find the value of the radius that will maximize the rectangular area. (nearest hundredth)

(A) 31.57 m **(B) 31.70 m** **(C) 31.83 m** **(D) 31.96 m** **(E) 32.09 m**

47. The function f is continuous on the interval $[-8, 8]$ and $\int_0^8 f(x)dx = 12$. $\int_{-4}^{4} f(x+4)dx =$ _____

(A) 0 **(B) 12** **(C) 24** **(D) 36** **(E) 48**

48. Find the volume of the solid generated by revolving the region bounded by the graphs of $y = e^{-x^2}$, $y = -2$, $x = 0$, $x = 4$ about the line $y = -5$. (nearest whole number)

(A) 207 **(B) 213** **(C) 219** **(D) 225** **(E) 231**

49. Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n (x+2)^n}{3^n}$

(A) $(-1, 1)$ **(B) $[-5, 1]$** **(C) $[-2, 2]$** **(D) $(-5, 1)$** **(E) $(-2, 2)$**

50. Consider the parabola $y^2 = 16x$. Find the y-intercept of the line tangent to the graph of the parabola at the point $(4, 8)$.

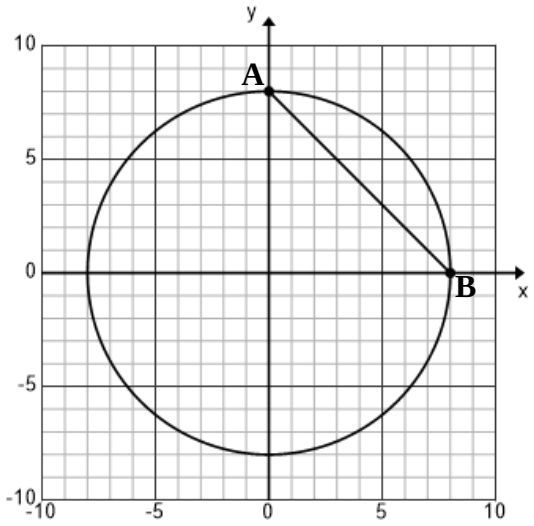
(A) $(0, 2)$ **(B) $(0, 3)$** **(C) $(0, 4)$** **(D) $(0, 5)$** **(E) $(0, 6)$**

51. Consider a circular sector with radius R and central angle θ. If the perimeter of the sector must be 48 inches, what is the maximum possible area of the sector? (nearest whole number)

(A) 144 in^2 (B) 146 in^2 (C) 148 in^2 (D) 150 in^2 (E) 152 in^2

52. Consider the circle shown on the right. Find the area of the region between chord $\overline{AB}$ and minor arc $\overset{\frown}{AB}$. (nearest tenth)

(A) 18.0
(B) 18.3
(C) 18.6
(D) 18.9
(E) 19.2



Problem 52

53. Melinda is hoping to attend Harvard University after she graduates from high school four years from now. Her grandma agreed to pay for her first year if she wins state in mathematics her senior year. To prepare for that eventuality, how much money should Grandma place into an account now if she can earn 5.75% annual interest compounded quarterly? The estimated total cost to attend Harvard for one year, four years from now, is $75,000.

(A) $59,202.57 (B) $59,335,56 (C) $59,462.78 (D) $59,577,34 (E) $59,687.68

54. A pyramid has a square base with each side = 14 cm and the height of the pyramid is 24 cm. Find the total area of the pyramid. (nearest whole number)

(A) 884 cm^2 (B) 888 cm^2 (C) 892 cm^2 (D) 896 cm^2 (E) 900 cm^2

55. The thickness of a quarter is 1.75 mm. Cindy has a stack of quarters that is 0.35 mm more than 28 feet tall. What is the value of her stack of quarters?

(A) $1,206.75 (B) $1,213.00 (C) $1,219.25 (D) $1,225.50 (E) $1,231.75

56. A circle has an area of 531 cm^2. Find the perimeter of an equilateral triangle that has the same area as the circle. (nearest whole number)

(A) 103 cm (B) 105 cm (C) 107 cm (D) 109 cm (E) 111 cm

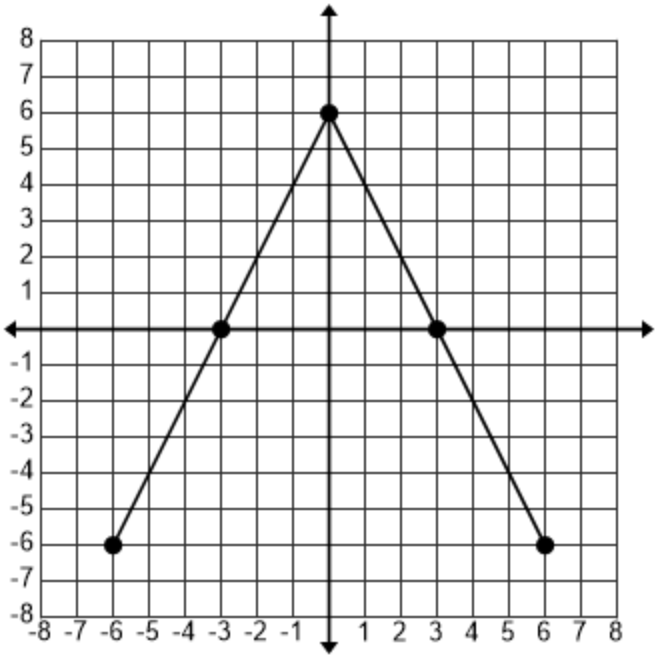
57. Dak received a $200 gift card from Academy Sports for being named the player of the week. He bought some shoes for $78.95, two shirts for $18.85 each, some shorts for $24.50, and some socks for $12.35. If the tax rate is 8.125%, how much remains on his gift card?

(A) $34.03 (B) $37.15 (C) $40.27 (D) $43.38 (E) $46.50

58. The function $y = f(x)$ is the solution to the differential equation $\frac{dy}{dx} = \frac{x}{4} + \frac{y}{3}$ with the initial condition $f(1) = 2$. Find the approximation for $f(2)$ if Euler's method is used, starting at x = 1 with a step size of 0.5. (nearest hundredth)

(A) 2.88 **(B) 2.94** **(C) 3.00** **(D) 3.06** **(E) 3.12**

59 – 60. The graph of the function f is shown on the right. The function h is defined as $h(x) = \int_0^x f(t)dt$.



59. $h(-3) =$ _____.

(A) –9 **(B) –2** **(C) 0** **(D) 2** **(E) 9**

60. $h''(-3) =$ _____.

(A) –9 **(B) –2** **(C) 0** **(D) 2** **(E) 9**

2021-2022 TMSCA HSM State Test Key

1. D	**21. C**	**41. A**
2. C	**22. D**	**42. B**
3. C	**23. A**	**43. C**
4. A	**24. E**	**44. C**
5. E	**25. B**	**45. E**
6. D	**26. B**	**46. C**
7. B	**27. C**	**47. B**
8. D	**28. B**	**48. E**
9. E	**29. E**	**49. D**
10. B	**30. B**	**50. C**
11. D	**31. A**	**51. A**
12. A	**32. D**	**52. B**
13. C	**33. C**	**53. E**
14. A	**34. E**	**54. D**
15. B	**35. A**	**55. C**
16. B	**36. B**	**56. B**
17. E	**37. A**	**57. A**
18. E	**38. E**	**58. D**
19. B	**39. A**	**59. A**
20. A	**40. D**	**60. D**