TMSCA HIGH SCHOOL

MATHEMATICS

TEST #6 (UIL C) ©

DECEMBER 10, 2022

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

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1. Des stopped at the Collin Street Bakery in Waco while she charged her EV. She purchased a Turkey Club Sandwich for $8.78, a coke for $2.25, three cookies for $1.15 each, and a large fruitcake for $36.55. If the tax rate was 8.125%, what was her final bill?

   (A) $54.94   (B) $55.06   (C) $55.18   (D) $55.30   (E) $55.42

2. Consider the line that contains points $A(8, -4)$ and $B(-6, 2)$. The sum of the x-intercept and the y-intercept of this line is _____.

   (A) $-2\dfrac{1}{21}$ (B) $-1\dfrac{19}{21}$ (C) $-1\dfrac{16}{21}$ (D) $-1\dfrac{13}{21}$ (E) $-1\dfrac{10}{21}$

3. Russell has 56 coins in a jar. All of the coins are nickels or dimes. The total value of the coins is $4.15. How many nickels does he have in his jar?

   (A) 29   (B) 31   (C) 33   (D) 35   (E) 37

4. Curtis has filled his swimming pool to a depth of 4 ft 6 in. The length of the rectangularly shaped pool is 36 ft and the width is 20 ft. If he adds 1000 gallons to his pool, what is the resulting depth? (nearest inch)

   (A) 4 ft 8 in   (B) 4 ft 10 in   (C) 5 ft 0 in   (D) 5 ft 2 in   (E) 5 ft 4 in

5. James ordered a RAV4 EV. The cost was $48,500 and he was required to make a down payment of $5,000. If he pays off the remaining balance in equal monthly payments over five years, what will each monthly payment be?

   (A) $717.25   (B) $719.75   (C) $721.44   (D) $723.67   (E) $725.00

6. Find the number that is halfway from $-3\dfrac{7}{8}$ to $5\dfrac{1}{3}$.

   (A) $\dfrac{17}{24}$ (B) $\dfrac{35}{48}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{37}{48}$ (E) $\dfrac{19}{24}$

7. Carolyn worked at the Forth Worth Zoo last summer. Her job included feeding the lions. If each lion needs 16 pounds of meat each day and there are eight lions at the zoo, how many pounds of meat are required to feed the lions in the month of March?

   (A) 3964   (B) 3968   (C) 3972   (D) 3976   (E) 3980

8. The radius of each tire on my car is 14 inches. If I drive at a constant speed of 75 mph for 12 minutes, how many revolutions will each tire make during this 12 minute period? (nearest whole number)

   (A) 10,786   (B) 10,792   (C) 10,798   (D) 10,804   (E) 10,810

9. Find the positive difference of the solutions to the equation $\frac{1}{3}|2x-5|+4=8$. (nearest tenth)

(A) 11.4   (B) 11.7   (C) 12.0   (D) 12.3   (E) 12.6

10. Vidit went to Whataburger to buy lunch for the math team. He purchased 4 cheeseburgers, 6 orders of fries, and 5 cokes for $41.15. Isaac also went to Whataburger to buy lunch for the science team. He purchased 6 cheeseburgers, 8 orders of fries, and 5 cokes for $54.31. If Kim goes to Whataburger and purchases 1 cheeseburger, 1 order of fries and 1 coke, how much will it cost? A cheeseburger cost $3.00 more than an order of fries.

(A) $8.75   (B) $8.79   (C) $8.83   (D) $8.87   (E) $8.91

11. Given: $\overline{ER} \perp \overline{AC}$, $m\angle BRC = 9x-9$ and $m\angle DRC = 3x-3$.
$m\angle ERD = $ _____ °

(A) 41°   (B) 43°   (C) 45°
(D) 47°   (E) 49°
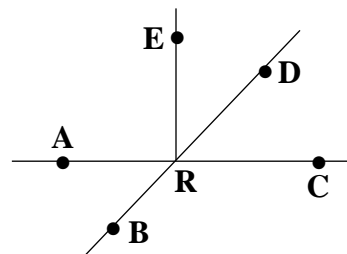
Problem 11

12. Find the area of an equilateral triangle with a perimeter of 47 cm. (nearest tenth)

(A) 105.4 cm²   (B) 105.7 cm²   (C) 106.0 cm²   (D) 106.3 cm²   (E) 106.6 cm²

13. Consider regular pentagon ABCDE. Point M is the midpoint of $\overline{AB}$.
Find the measure of $m\angle MED$. (nearest tenth)

(A) 84.0°   (B) 84.4°   (C) 84.8°   (D) 85.2°   (E) 85.6°

14. $\triangle ABC \sim \triangle DEF$. AB = 20, AC = 16, and DE = 17. DF = _____. (nearest tenth)

(A) 13.6   (B) 13.8   (C) 14.0   (D) 14.2   (E) 14.4

15. Consider $\triangle ABC$ with AB = 19, BC = 23, and AC = 26.5. Point D lies on $\overline{AC}$ such that $\overrightarrow{BD}$ bisects $\angle ABC$. AD = _____. (nearest tenth)

(A) 12.0   (B) 12.2   (C) 12.4   (D) 12.6   (E) 12.8

16. Consider equilateral $\triangle ABC$ with point R the midpoint of $\overline{AB}$ and point S the midpoint of $\overline{BC}$. If AC = 17.7, what is the area of $\triangle RBS$? (nearest tenth)

(A) 32.7   (B) 33.0   (C) 33.3   (D) 33.6   (E) 33.9

17. Consider trapezoid ABCD with median $\overline{EF}$. If the bases are $\overline{BC}$ and $\overline{AD}$ with
BC = 4x+1, EF = 24, and AD = 6x-3, then $AD - BC = $ _____. (nearest tenth)

(A) 5.4   (B) 5.6   (C) 5.8   (D) 6.0   (E) 6.2

18. Consider a cube in which the length of each edge is 12. Let BW be the length of one of the long diagonals and let DB be the length of a diagonal of one of the square faces. BW – DB = _____. (nearest tenth)

    (A) 3.6   (B) 3.8   (C) 4.0   (D) 4.2   (E) 4.4

19. A 15-ft-long string is cut into three equal pieces. One piece is used to form an isosceles right triangle, the second piece is used to form a square, and the third piece is used to form a circle. The sum of the areas of the three figures is _____. (nearest tenth)

    (A) 3.8 ft^2 (B) 4.0 ft^2 (C) 4.2 ft^2 (D) 4.4 ft^2 (E) 4.6 ft^2

20. Consider a circle with points A, B, C, and D lying on the circle such that chords $\overline{AB}$ and $\overline{CD}$ do not intersect. $AB = CD = 9$, $mAB = 128°$, and $mBC = 41°$. $mAD =$ _____.

    (A) 61°   (B) 63°   (C) 65°   (D) 67°   (E) 69°

21. George has a collection of blue marbles and red marbles. Seventy percent of the blues is added to thirty percent of the reds and the total is 66. Three times the number of blues exceeds twice the number of reds by 20. How many marbles does George have?

    (A) 132   (B) 134   (C) 136   (D) 138   (E) 140

22. The line $x - y = -2$ intersects the hyperbola $xy = 4$ in two points. Find the distance between the two points. (nearest tenth)

    (A) 5.7   (B) 5.9   (C) 6.1   (D) 6.3   (E) 6.5

23. If $f(x) = \dfrac{1}{x}$, then $\dfrac{f(x+h) - f(x)}{h} =$ _____.

    (A) $\dfrac{1}{hx}$ (B) $\dfrac{1}{x(x+h)}$ (C) $\dfrac{-1}{x(x+h)}$ (D) $\dfrac{1}{x^2}$ (E) $\dfrac{-1}{x^2}$

24. Find the eccentricity of the ellipse. $2x^2 + 5y^2 - 8x + 20y + 8 = 0$. (nearest hundredth)

    (A) 0.65   (B) 0.71   (C) 0.77   (D) 0.83   (E) 0.89

25. Consider the greatest integer function $f(x) = 2x - 3$. $f(5) + f(-5) =$ _____.

    (A) –6   (B) –3   (C) 0   (D) 3   (E) 6

26. A regular octagon is inscribed in a circle. If the area of the octagon is 181 cm^2, what is the area of the circle? (nearest whole number)

    (A) 189 cm^2 (B) 192 cm^2 (C) 195 cm^2 (D) 198 cm^2 (E) 201 cm^2

27. Find the distance from the point $(6, 8)$ to the line $y = -\dfrac{3}{5}x - 4$. (nearest tenth)

(A) 12.2   (B) 12.5   (C) 12.8   (D) 13.1   (E) 13.4

28. If $f(x) = 6x + 5$, $g(x) = \sqrt{x - 3}$, and $(f \circ g)(x) = 7$, then x = _____.

(A) 3   (B) $\dfrac{28}{9}$ (C) $\dfrac{29}{9}$ (D) $\dfrac{10}{3}$ (E) $\dfrac{31}{3}$

29. How many even three digit counting numbers less than 500 are there such that the second digit cannot be 3 or 5?

(A) 100   (B) 120   (C) 140   (D) 160   (E) 180

30. Eight chairs are arranged in a circle. If eight girls enter the room, how many different seating arrangements are possible?

(A) 2,520   (B) 5,040   (C) 16,800   (D) 28,560   (E) 40,320

31. The point P is the focus of the parabola shown on the right. If the equation of the parabola is $y = f(x)$, $f(-12) =$ _____.

(A) 24.000
(B) 24.125
(C) 24.250
(D) 24.375
(E) 24.500
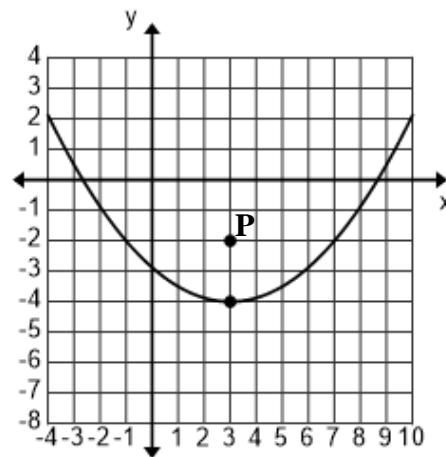
Problem 31

32. The angle of depression from an owl perched on the top of a tree to a mouse on the ground is 38°. The mouse cautiously moves 16 feet towards the tree and the new angle of depression is 52°. How tall is the tree? (nearest tenth)

(A) 31.3 ft   (B) 31.7 ft   (C) 32.1 ft   (D) 32.5 ft   (E) 32.9 ft

33. If 9 points lie evenly spaced on a circle, how many different triangles can be drawn using these points as vertices?

(A) 54   (B) 60   (C) 68   (D) 76   (E) 84

34. Which of the following is one of the three cube roots of -8?

(A) $\sqrt{3}+i$ (B) $-1-\sqrt{3}\,i$ (C) $-1+\sqrt{3}\,i$ (D) $1-\sqrt{3}\,i$ (E) $\sqrt{3}-i$

35. The fifth term of an arithmetic sequence is 29 and the twelfth term is 71. Find the sum of the first four terms.

(A) 56   (B) 58   (C) 60   (D) 62   (E) 64

36. Mrs. Warden brought 120 cookies to math practice, which started at 4:00 PM. At 4:12, the number of cookies had decreased to 96. If the rate of decrease was exponential, how many cookies were there when practiced ended at 5:20 PM? (nearest whole number)

(A) 25   (B) 27   (C) 29   (D) 31   (E) 33

37. A horizontal ellipse has a major axis of length 10 and a minor axis of length 8. The center of the ellipse is the point $(3,-2)$. The point $(0,b)$ lies on the ellipse. If $b>0$, then $b =$ _____. (nearest tenth)

(A) 1.0   (B) 1.2   (C) 1.4   (D) 1.6   (E) 1.8

38. The graph of $r = 5 + 2\sin(\theta)$ is classified as a _____.

(A) limacon with an inner loop
(B) cardioid
(C) dimpled limacon
(D) convex limacon
(E) lemniscate

39. Find the distance between the points $(3,-4,5)$ and $(-6,-2,-5)$. (nearest tenth)
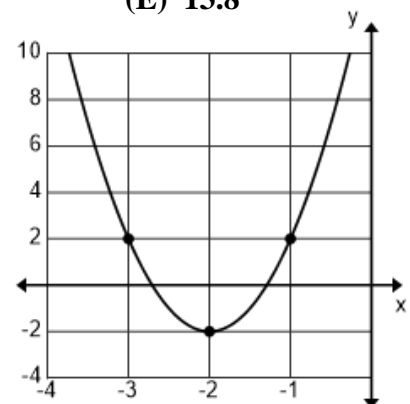
(A) 13.0   (B) 13.2   (C) 13.4   (D) 13.6   (E) 13.8

40. Find the length of the latus rectum of the parabola shown on the right. (nearest tenth)

(A) $\dfrac{1}{16}$ (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{4}$

(D) $\dfrac{1}{2}$ (E) 1

Problems 40, 41

41. The graph shown on the right is the graph of $y = f'(x)$. If $f(0) = 6$, then $f(-3) =$ _____. (nearest tenth)

(A) -6 (B) -3 (C) 0
(D) 3   (E) 6

42. Evaluate: $\displaystyle\lim_{x\to 2^+}\left(\frac{x^2-4x}{x-2}\right)=$ _____ .

   (A) $-\infty$ (B) -4 (C) -2 (D) 2 (E) ∞

43. Consider the function $f(x)=\dfrac{2}{\sqrt{x-2}}$. The y-intercept of the line tangent to the graph of $y=f(x)$ when $x=3$ is $(0,b)$. $b=$ _____ . (nearest tenth)

   (A) 4.8   (B) 5.0   (C) 5.2   (D) 5.4   (E) 5.6

44. Given: $x^2+y^2=36$. Find the value of $\dfrac{d^2y}{dx^2}$ at the point $\left(4,2\sqrt{5}\right)$. (nearest hundredth)

   (A) -0.52 (B) -0.48 (C) -0.44 (D) -0.40 (E) -0.36

45. Farmer Phil plans to build a fenced in grazing area for his goats next to a river. The area will be rectangular in shape with one side of the rectangle being the river itself. If the grazing area must be 735,000 square feet, find the minimum length of fence he needs for the three sides he will build? (nearest whole number)
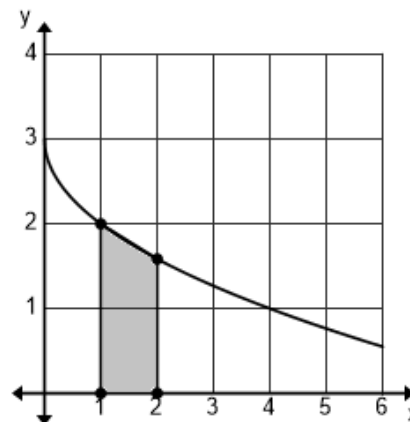
RIVER
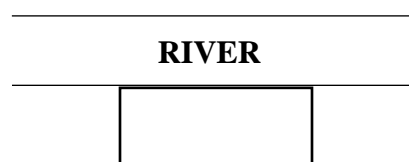
   (A) 2425 ft   (B) 2436 ft   (C) 2447 ft   (D) 2458 ft   (E) 2469 ft

46. If $\displaystyle\int_0^8 f(x)dx=12$ and $\displaystyle\int_{12}^8 f(x)dx=-6$, then $\displaystyle\int_0^{12} f(x)dx=$ _____ .

   (A) 6   (B) 12   (C) 18   (D) 24   (E) 30

47. Approximate the area bounded by the curves $y_1=3-\sqrt{x}$, $y_2=0$, $x_1=1$, and $x_2=5$ using four trapezoids of equal heights. One of the trapezoids is shown on the right. (nearest ten-thousandth)

   (A) 5.2261
   (B) 5.2293
   (C) 5.2325
   (D) 5.2357
   (E) 5.2389

Problems 47, 48

48. Find the exact area of the region described in problem 47. (nearest ten-thousandth)

   (A) 5.2111   (B) 5.2121   (C) 5.2131   (D) 5.2141   (E) 5.2151

49. The projected population of Frisco is modeled by $P(t) = 236,604e^{kt}$ where 236,604 was the population in 2020 and k is a constant. If the population reached 245,882 in 2022, at what rate will the population be increasing in 2030? (people per year, nearest whole number)

    (A) 5506   (B) 5509   (C) 5512   (D) 5515   (E) 5518

50. Find the area bounded by the curves $y = f(x)$ and $y = g(x)$. (nearest tenth)

    (A) 4.8   (B) 5.0   (C) 5.2
    (D) 5.4   (E) 5.6

51. Find the volume generated when the region bounded by the curves $y = f(x)$ and $y = g(x)$ is revolved about the line $y = -2$. (nearest whole number)
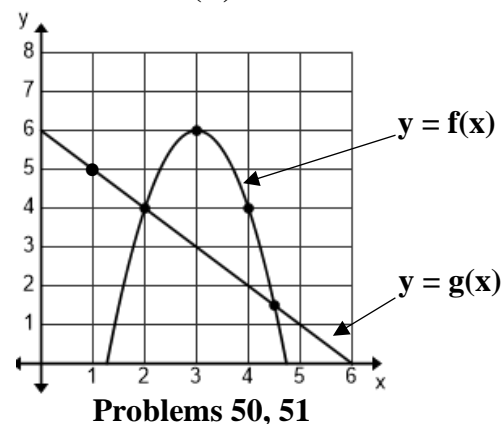
    Problems 50, 51

    (A) 180   (B) 184   (C) 188   (D) 192   (E) 196

Test #	1	2	3	4	5	6
Score	86	84	89	93	96	98

Use this table for problems 52 and 53.
Nickolas took Calculus II online from MIT. There were six tests administered during the course. He started a bit slow, but finished strong. The table above contains his grades on the tests. The average of the six tests count 80% of his grade and the final exam counts 20%. The professor requires an overall average of 92.0 or higher to receive an A.

52. Of the following scores, which is the minimum score will Nickolas need to make on the final exam in order to receive an A?

    (A) 96   (B) 97   (C) 98   (D) 99   (E) 100

53. Assume that Nickolas had four zoom sessions with Javier and Juan and scored high enough on the final exam to earn an A. The professor revealed that the mean grade for the semester was 78 with a standard deviation of 8. Assume that he scored the minimum score on the final exam required to earn an A. This resulted in an overall average that placed him at the _____ th percentile of the students enrolled in this course. (nearest whole number)

    (A) 90   (B) 92   (C) 94   (D) 96   (E) 98

54. Joe rolls a fair die 12 times. What is the probability that he rolled a 4 exactly three times? (nearest thousandth)

    (A) 0.197   (B) 0.203   (C) 0.212   (D) 0.221   (E) 0.230

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55. Over his professional career, the length of Arnold's drives followed an approximately normal distribution with a mean of 288 yd and a standard deviation of 9.3 yd. What percentage of his drives were longer than 300 yd? (nearest tenth)

(A) 9.0%   (B) 9.2%   (C) 9.4%   (D) 9.6%   (E) 9.8%

The table below contains some data from the Montana High School Baseball Association. It contains an analysis of the batting averages for all players who had at least 75 at bats during the season.

Minimum	Q1	Median	Q3	Maximum
.143	.266	.298	.377	.656

56. Of the following, which is the lowest batting average that would be classified as an outlier?

(A) 0.511   (B) 0.522   (C) 0.533   (D) 0.544   (E) 0.555

57. The East Texas STEM Academy has 160 seniors. If 90 students are taking AP Calculus, 68 students are taking AP Statistics, and 20 students are not taking either course, how many are taking both of these highly recommended courses.

(A) 16   (B) 18   (C) 20   (D) 22   (E) 24

Month	Sept (test 1)	Oct (test 2)	Nov (test 3)	Dec (test 4)
Problems worked	22	26	31	34

58. Audrey is tracking the number of problems worked by Superfresh on practice math tests administered on the first day of the month during the school year. She believes that there is a linear relationship between the test number and the number of problems worked. How many problems does she predict Superfresh will work on a practice test given on March 1st ? (nearest whole number)

(A) 43   (B) 45   (C) 47   (D) 49   (E) 51

	100m 200 m	800 m 1500 m	5,000 m 10,0000 m	Long Jump Triple Jump	Total
Professional	86	62	66	60	274
Division I	26	12	10	14	62
Division II	8	6	4	6	24
Total	120	80	80	80	360

59. The classifications of the athletes that made the national team in various track and field events over a ten year period were recorded and are shown in the table above. Coach analyzed the data with a Chi-Square test. What is the expected count of the Division I/800 m, 1500 m cell? (nearest tenth)

(A) 13.8   (B) 14.0   (C) 14.2   (D) 14.4   (E) 14.6

60. What is the contribution of the Division I/800 m, 1500 m cell to the Chi-Square statistic? (nearest thousandth)

(A) 0.185   (B) 0.196   (C) 0.207   (D) 0.218   (E) 0.229

TMSCA ● HSM Test 6 ● 2022
Answer Key

1. C	21. E	41. C
2. B	22. D	42. A
3. A	23. C	43. B
4. A	24. C	44. D
5. E	25. A	45. A
6. B	26. E	46. C
7. B	27. E	47. D
8. D	28. B	48. C
9. C	29. D	49. D
10. C	30. B	50. C
11. C	31. B	51. E
12. D	32. C	52. A
13. E	33. E	53. D
14. A	34. D	54. A
15. A	35. A	55. E
16. E	36. B	56. D
17. D	37. B	57. B
18. B	38. D	58. C
19. E	39. D	59. A
20. B	40. C	60. E