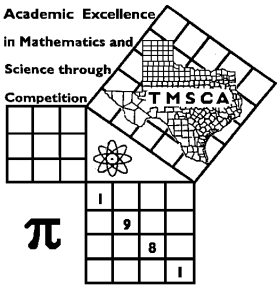


T M S C A H I G H S C H O O L

M A T H E M A T I C S

D I S D I N V I T A T I O N A L ©

2 0 2 3

GENERAL DIRECTIONS

1. About this test:
   A. You will be given 40 minutes to take this test.
   B. There are 60 problems on this test.
2. All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet, be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading.
3. If using a scantron answer form, be sure to correctly denote the number of problems not attempted.
4. You may write anywhere on the test itself. You must write only answers on the answer sheet.
5. You may use additional scratch paper provided by the contest director.
6. All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
7. Calculators used on this test must be conform to the UIL standards. Graphing calculators are allowed. Calculators need not be cleared.
8. All problems answered correctly are worth **SIX** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
9. In case of ties, percent accuracy will be used as a tie breaker.

1. An equation to find the maximum height of a projectile is $D_y = \frac{-v_0^{\,2}\sin(2\theta)}{g}$ where D_y is the maximum height, v_0 is the initial velocity, θ is the angle of the projectile release relative to horizontal and g is the gravitational constant $-9.81\ \frac{m}{s^2}$. What is the maximum height of a ball shot at $240\ \frac{m}{s}$ at an angle of $33°$? (nearest meter)

(A) 156 meters (B) 2,682 meters (C) 5,364 meters (D) 6,364 meters (E) 6,436 meters

2. What is the number that is $\frac{6}{7}$ of the way from $5\frac{2}{3}$ to $-12\frac{3}{4}$?

(A) $-10\frac{3}{14}$ (B) $-10\frac{4}{21}$ (C) $-10\frac{7}{42}$ (D) $-10\frac{3}{21}$ (E) $-10\frac{5}{42}$

3. Consider a line segment with endpoints $A(-2,9)$ and $(4,-3)$. Which of the following points lie on the perpendicular bisector of $\overline{AB}$?

(A) $(7,6.1)$ (B) $(7.5,6.3)$ (C) $(8,6.5)$ (D) $(8.5,6.85)$ (E) $(9,7.1)$

4. If $g(x) = -x^2 + bx + c$ and has zeros at $x = 6$ and $x = -3$, the maximum value of $g(x)$ is _____.

(A) 20.15 (B) 20.2 (C) 20.25 (D) 20.3 (E) 20.35

5. A private jet can be rented for a down payment of \$15,000 and \$7.50 per mile. If Bobby pays \$23,355 for a private jet flight from Seattle to Phoenix, what is the distance Bobby flew? (nearest mile)

(A) 1,114 miles (B) 1,124 miles (C) 1,134 miles (D) 1,144 miles (E) 1,154 miles

6. Twice Abel's age is 5 more than George's age. Three times George's age is 3 more than twice times Connor's age. In ten years, twice Abel's age will be two more than Connor's age. What is the sum of Abel's, George's and Connor's current ages?

(A) 84 (B) 85 (C) 86 (D) 87 (E) 88

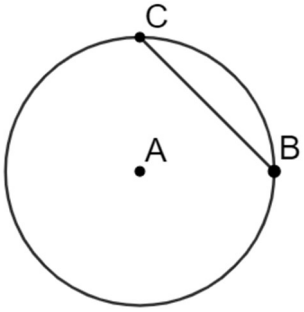
7. The local pound has a total of 52 dogs. The average age of the 20 poodles is 3.4 years old. The average age of the 14 bulldogs is 2.1 years old and the average age of the 18 dachshunds is 2.8 years old. What is the average age of all the dogs at the pound? (nearest tenth)

(A) 2.8 years (B) 2.9 years (C) 3.0 years (D) 3.1 years (E) 3.2 years

8. Consider a parabola with a vertex at $(3,-2)$. If the point $(-2,-8)$ lies on the graph of the parabola, which of the following does not lie on the graph of the parabola?

(A) $(-7,-26)$ (B) $(4,-2.24)$ (C) $(6,-4.16)$ (D) $(8,-8)$ (E) $(10,-13.74)$

9-10. Consider circle A to the right. The length of chord $\overline{BC}$ is 32 and the area of the circle is 1,868.547 square units.



9. Find the measure of major arc BC. (nearest tenth)

(A) 118.1 (B) 118.3 (C) 118.5 (D) 118.7 (E) 118.9

10. Find the area bounded by major arc BC and chord BC. (nearest integer)

(A) 1,722 (B) 1,727 (C) 1,732 (D) 1,737 (E) 1,742

11. Consider triangle ABC with $\overline{AB} = 14$ cm, $\overline{BC} = 16$ cm and $\overline{AC} = 21$ cm. If point D lies on side $\overline{AC}$ such that $\overline{BD}$ is the angle bisector of $m\angle ABC$, what is the length of segment $\overline{BD}$? (nearest tenth)

(A) 10.5 cm (B) 10.6 cm (C) 10.7 cm (D) 10.8 cm (E) 10.9 cm

12. Consider kite ABCD. $AB = BC = 14$ and $CD = AD = 18$. If $m\angle ABC = 50°$, find the area of the kite. (nearest integer)

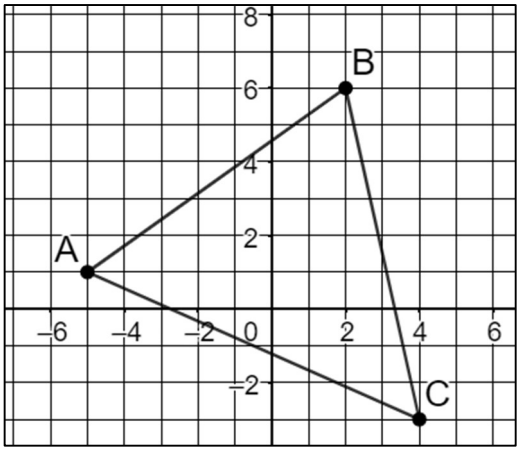
(A) 172 (B) 173 (C) 174 (D) 175 (E) 176

13. Consider the conditional statement "If Sonny has a smart phone, he has access to the internet." Which of the following are true?

I. Converse II. Contrapositive III. Inverse

(A) I only (B) II only (C) III only (D) I & II (E) II & III

14-17. Consider the graph of triangle ABC to the right.



14. The coordinate of the centroid of triangle ABC is (a,b). What is the value of $a+b$?

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

15. What is the area of the triangle?

(A) 35.5 (B) 36.0 (C) 36.5 (D) 37.0 (E) 37.5

16. What is the perimeter of the triangle? (nearest tenth)

(A) 27.1 (B) 27.3 (C) 27.5 (D) 27.7 (E) 27.9

17. Point D lies on segment AB such that $m\angle ADC = 90°$. What is the length of $\overline{CD}$? (nearest hundredth)

(A) 8.43 (B) 8.45 (C) 8.47 (D) 8.49 (E) 8.51

18. Consider right triangle XYZ with an area of 82 square units and $m\angle XYZ = 40°$. The perimeter of the triangle is ____ units. (nearest tenth)

(A) 43.9 (B) 44.0 (C) 44.1 (D) 44.2 (E) 44.3

19. Consider triangle DEF with point G on $\overline{DF}$ such that $\overline{EG} \perp \overline{DF}$. If $m\angle DEF = 90°$, $\overline{DG} = 11.2$ and $\overline{GF} = 15.7$, what is the length of segment $\overline{EG}$? (nearest tenth)

(A) 13.1 (B) 13.2 (C) 13.3 (D) 13.4 (E) 13.5

20. Consider the function $f(x) = \frac{ax+b}{cx+d}$ where a, b, c and d are all unique integers. If $g(x) = f^{-1}(x)$, then $g(x) =$ ___.

(A) $\frac{d-bx}{a-cx}$ (B) $\frac{b-dx}{a-cx}$ (C) $\frac{ax-b}{c-dx}$ (D) $\frac{bx-d}{a-cx}$ (E) $\frac{dx-b}{a-cx}$

21. Consider the quadratic functions $f(x)$ and $g(x)$. Both functions have vertices on the line $x = 3$ and a distance of 6 units from each other. Given: $f(x) \le g(x)$ when $x = 3$ and $f(3) = 1$ $g(x)$ opening down and $f(x)$ opening up. If the two parabolas have the same focus, what is the value of $g(-3)$?

(A) 3.95 (B) 4.00 (C) 4.05 (D) 4.10 (E) 4.15

22. A population of 30 howler monkeys is released in Madagascar on January 1, 2001. Scientists returned on January 1, 2010 to find the population had increased to 146 monkeys. If the population of the monkeys is increasing exponentially, what year is the population of the monkeys expected to be above 550 monkeys?

(A) 2014 (B) 2015 (C) 2016 (D) 2017 (E) 2018

23. What is the sum of the series given by: $\sqrt{10} + \sqrt{14} + \sqrt{18} + ... + \sqrt{206} + \sqrt{210}$. (nearest integer)

(A) 509 (B) 510 (C) 511 (D) 512 (E) 513

24. If the number of distinct diagonals for a polygon is 135, how many sides does the polygon have?

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21

25. Consider the graph of $f(x) = \frac{6x^2+11x-7}{x^2-x-2}$. Which of the following is true?

I. $(0.5, 0)$ is an x-intercept II. $\left(\frac{7}{3}, 0\right)$ is an x-intercept III. $x = 1$ is a vertical asymptote

IV. $f(x) \neq 6$ for all values of x V. There are three asymptotes

(A) I, V only (B) I, II, III only (C) I, II, III, V only (D) I, III, IV, V only (E) II, III, V only

26. Which of the following is not a fourth root of $-8-8\sqrt{3}i$?

(A) $-1+\sqrt{3}i$ (B) $1+\sqrt{3}i$ (C) $\sqrt{3}-i$ (D) $-1-\sqrt{3}i$ (E) $-\sqrt{3}+i$

27. Let w and m be the complex solutions to the equation $2x^2-3x+9=0$. If $w^2-m^2=\frac{a\sqrt{b}}{c}i$ where a, b and c are positive integers, then the value of $a+b+c=$ ____.

(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

28. A teacher that has been teaching in the same district for ten years has a salary of $83,000. The district increases the salary of the teacher by 3% each year. What sum of money will the teacher have been paid if they work in the same district for 35 years? (nearest dollar)

(A) $3,846,131 (B) $3,846,140 (C) $3,846,150 (D) $3,846,158 (E) $3,846,167

29. Consider the curve defined by the parametric equations $x(t)=\ln(t^2)$ and $y(t)=e^{t-2}$ for $t>0$. The rectangular equation that represents the curve is $y=f(x)$. Evaluate $f(2)$. (nearest hundredth)

(A) 2.05 (B) 2.06 (C) 2.07 (D) 2.08 (E) 2.09

30. Consider angle A in quadrant III and angle B in quadrant IV. If $\sin A=-\frac{4}{7}$ and $\cos B=\frac{8}{9}$, then $\cot(A-B)$ is _____. (nearest hundredth)

(A) 0.49 (B) 0.50 (C) 0.51 (D) 0.52 (E) 0.53

31. A ball is dropped from a height of 40 feet. If the ball bounces two-thirds of its previous height, find the total distance traveled by the ball.

(A) 200 feet (B) 203 feet (C) 206 feet (D) 209 feet (E) 212 feet

32. The graph of $3x^2-6xy-3y^2+2x-4y=0$ is _____.

(A) a parabola (B) an ellipse (C) a hyperbola (D) a line (E) two parallel lines

33. Sam is a forest ranger at a lookout tower to keep an eye on possible fires. Sam spots a fire on a bearing of $310°$ at a distance of 3 miles. Gina is in a lookout tower that also spots the fire at a bearing of $200°$ at a distance of 5 miles. What is the bearing Sam would have to walk to go directly to Gina? (nearest tenth)

(A) 354.8° (B) 354.9° (C) 355.0° (D) 355.1° (E) 355.2°

34. The conic with the equation $9x^2 - 90x - 5y^2 - 10y + 265 = 0$ has foci at the points (a,b) and (a,c). What is the value of $a \cdot b \cdot c$?

(A) -71 (B) -68 (C) -65 (D) -62 (E) -59

35. If the distance from the line $7x + 2y - kz = 4$ and the point $(3,-2,k)$ is $\frac{12}{\sqrt{78}}$, the value of k if $k > 0$ is:

(A) 4.7 (B) 4.8 (C) 4.9 (D) 5.0 (E) 5.1

36. Newton is an Algebra 2 teacher. He has Algebra 2 books from three different publishers. Newton has five copies of the first publication, seven copies of the second publication and six copies of the third publication. Newton wants to arrange the books on a shelf. In how many unique ways can Newton arrange the books?

(A) 14,702,688 (B) 119,891,016 (C) 225,079,344 (D) 330,267,672 (E) 435,456,000

37. Emily deposits \$10,000 into an account that earns 5.25% annual interest compounded monthly. Denny deposits \$8,000 into an account that earns 6.5% annual interest compounded daily. How many times must Emily's account compound for Emily and Denny to have the same account value?

(A) 209 (B) 210 (C) 211 (D) 212 (E) 213

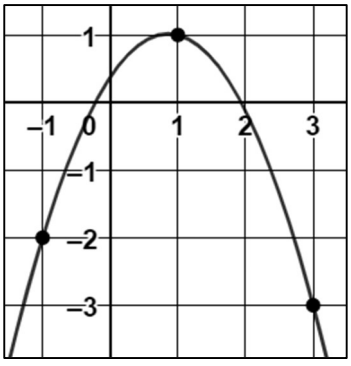
38. A surveyor is approaching a mountain that is 14,582 feet from ground level to the peak. Using a clinometer, the surveyor measures the current angle of elevation from where they are to the peak of the mountain as $9°$. The surveyor then walks some distance directly toward the peak of the mountain and measures the new angle of elevation as $12°$. How far did the surveyor walk between measuring the angle of elevation? (nearest hundredth mile)

(A) 4.43 miles (B) 4.44 miles (C) 4.45 miles (D) 4.46 miles (E) 4.47 miles

39. Consider the arithmetic sequence given by $a_n = 15, 19, 23, 27, \ldots$ and the geometric sequence given by $b_n = \frac{1}{16}, \frac{1}{12}, \frac{1}{9}, \ldots$. How many terms are required for the sum of the geometric sequence to be greater than the sum of the arithmetic sequence?

(A) 32 (B) 33 (C) 34 (D) 35 (E) 36

40. The focus of the parabola to the right is (a,b). $a + b =$ ___.



(A) $^{87}/_{56}$ (B) $^{11}/_{7}$ (C) $^{89}/_{56}$ (D) $^{45}/_{28}$ (E) $^{13}/_{8}$

41. What is the area in the first quadrant between the function and x-axis of the quadratic to the right? (nearest thousandth)

(A) 1.409 (B) 1.413 (C) 1.417 (D) 1.421 (E) 1.425

42. Line L is tangent to the graph of $y = e^x$ at the point (a, a^2). What is the positive value of a for which the y-intercept of the line is -1? (nearest thousandth)

(A) 0.738 (B) 0.741 (C) 0.744 (D) 0.747 (E) 0.750

43. Consider the function $f(x)$, a twice-differentiable function on the interval $[-2,6]$ with $f(1) = 0$ and $f'(3) = 0$. If $f''(x) < 0$ for $(-2,2)$ and $f''(x) > 0$ for $(2,6)$. Which of the following must be true?

I. $f(x)$ has a minimum at $x = 1$
II. $f(x)$ has a minimum at $x = 3$
III. $f'(x)$ is increasing at $x = 4$

(A) I only (B) II only (C) III only (D) I & III (E) II & III

44. What is the interval of convergence for the series given by $\sum_{n=2}^{\infty} \frac{(x-3)^{n+1}}{(n+1)\cdot 4^{n+1}}$?

(A) $[-3,4)$ (B) $(-3,4]$ (C) $[-1,7)$ (D) $(-1,7]$ (E) $(-1,7)$

45. Given $f(x) = \ln x + e^x$. If $g(x) = f^{-1}(x)$ for all $x > 0$, evaluate $g'(2)$. (nearest thousandth)

(A) 0.285 (B) 0.286 (C) 0.287 (D) 0.288 (E) 0.289

46. A population of 10 wolves have been released into a sanctuary on July 1st, 2022. The carrying capacity of the sanctuary is 120 wolves. On July 1st, 2023, the population had increased to 14 wolves. Using a logistic model, the population on July 1st, 2030 should be about _____ wolves.

(A) 69 (B) 71 (C) 73 (D) 75 (E) 77

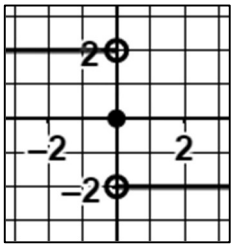
47. Rudy is about to fly his drone over a local park. Rudy sets his drone on the ground and steps back 14 feet. During takeoff, the drone rises perpendicular to the ground at a rate of 12 feet per second. At what rate is the angle of elevation from Rudy's eyes to the drone increasing when the drone is 16 feet above Rudy's eye level? (nearest hundredth radian per second)

(A) 0.368 $\frac{\text{rad}}{\text{sec}}$ (B) 0.370 $\frac{\text{rad}}{\text{sec}}$ (C) 0.372 $\frac{\text{rad}}{\text{sec}}$ (D) 0.374 $\frac{\text{rad}}{\text{sec}}$ (E) 0.376 $\frac{\text{rad}}{\text{sec}}$

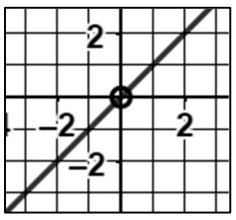
48. Consider the graphs of the functions $f(x)$ and $g(x)$ as shown to the right. Evaluate $\lim_{x\to 0} \frac{f(x)}{g(x)}$.

(A) ∞ (B) $-\infty$ (C) 0

(D) 1 (E) 2



$f(x)$



$g(x)$

49. Which value of c satisfies the Mean Value Theorem for the function $f(x)=\frac{x^3}{3}$ on the interval $[0,3]$?

(A) $-\sqrt{3}$ and $\sqrt{3}$ **(B) $\sqrt{3}$** **(C) 0** **(D) 3** **(E) $-\sqrt{3}$**

50. What is the value of $m+b$ if $y=mx+b$ is a solution to the differential equation $\frac{dy}{dx}=\frac{2}{3}x-y+2$?

(A) -1 **(B) 0** **(C) 1** **(D) 2** **(E) 3**

51. What is the coefficient of the x^2 term in the Maclaurin series for $\frac{1}{(1+x)^2}$ about $x=0$?

(A) $\frac{1}{6}$ **(B) $\frac{1}{3}$** **(C) 1** **(D) 3** **(E) 6**

52-53. The table below represents a large random sample of people from all ages. Researchers wanted to know if there is an association between a person's age and their favorite sport to watch. A Chi-Square Test for Association/Independence was performed.

	Baseball	Basketball	Football	Total
0-19	160	140	230	530
20-40	240	190	140	570
41-100	240	220	300	760
Total	640	550	670	1860

52. Find the expected count for the 20-40/Basketball cell. (nearest integer)

(A) 161 **(B) 163** **(C) 165** **(D) 167** **(E) 169**

53. A chi-square statistic of 50.23710148 and a p-value of _____.

(A) 3.222×10^{-10} **(B) 3.225×10^{-10}** **(C) 3.228×10^{-9}** **(D) 3.231×10^{-8}** **(E) 3.234×10^{-7}**

54. If Vincent rushes through a Number Sense test, he can finish the test in approximately 9 minutes and 10 seconds. Mr. Wagner wants to see if this is normal for Vincent, so he has Vincent take 100 tests over the course of 100 days. Mr. Wagner finds that the amount of time for Vincent to finish a Number Sense test follows a normal distribution. If the average amount of time it takes for Vincent to finish a Number Sense test is 9 minutes and 3.2 seconds with a standard deviation of 11.4 seconds, approximately how many of the 100 tests did Vincent finish in less than 8 minutes and 55 seconds?

(A) 17 **(B) 19** **(C) 21** **(D) 23** **(E) 25**

55. In a normal distribution _____ % of the observations fall within 1.25 standard deviations of the mean.

(A) 78.83 **(B) 78.87** **(C) 78.91** **(D) 78.95** **(E) 78.99**

56. Mrs. Wilson found there is a strong, positive, linear relationship between student average test scores and their average quiz scores. She randomly selected 100 students from classes at her school and found the average test score was an 83.1 with a standard deviation of 8.1 points. The average quiz score was an 85.4 with a standard deviation of 6.7 points. Her analysis also found that $r^2 = 0.83$. What is the predicted test average for a student who has a quiz average of 86.7? (nearest tenth)

(A) 83.8 (B) 83.9 (C) 84.1 (D) 84.3 (E) 84.5

57. Insurance company records indicate that 16% of houses have made hail damage claims and 10% of houses have made high wind damage claims. Also, 7% of houses have made claims on both types of structural damage. What is the probability that in a random sample of 30 houses, less than 4 houses claimed hail damage but not wind damage? (nearest thousandth)

(A) 0.711 (B) 0.713 (C) 0.715 (D) 0.717 (E) 0.719

58. If the linear correlation coefficient is 0.89, what percentage of variation in y is explained by x in the least squares regression line?

(A) 79.21% (B) 82.99% (C) 86.77% (D) 90.56% (E) 94.34%

59-60. A large random sample of 500 adults was surveyed about keeping their doctor appointments. A 95% confidence interval for the proportion of doctor visits canceled on the day of the appointment was constructed and found to be $(0.032, 0.120)$.

59. What is the point estimate for the proportion of doctor visits canceled on the day of the appointment from which this interval was constructed?

(A) 0.074 (B) 0.075 (C) 0.076 (D) 0.077 (E) 0.078

60. What is the margin error of the confidence interval? (nearest ten-thousandth)

(A) 0.0228 (B) 0.0229 (C) 0.0230 (D) 0.0231 (E) 0.0232

2023 – 2024 TMSCA High School Mathematics Invitational Test Key

1. C	**21. B**	**41. D**
2. E	**22. D**	**42. A**
3. C	**23. C**	**43. E**
4. C	**24. B**	**44. C**
5. A	**25. A**	**45. D**
6. E	**26. A**	**46. E**
7. A	**27. C**	**47. C**
8. E	**28. C**	**48. B**
9. B	**29. A**	**49. B**
10. D	**30. E**	**50. D**
11. C	**31. A**	**51. D**
12. E	**32. C**	**52. E**
13. B	**33. B**	**53. A**
14. E	**34. C**	**54. D**
15. C	**35. D**	**55. B**
16. D	**36. A**	**56. E**
17. D	**37. E**	**57. D**
18. B	**38. B**	**58. A**
19. C	**39. C**	**59. C**
20. E	**40. C**	**60. E**

2032-2024 TMSCA High School Mathematics Invitational Test Selected Solutions

1. $\dfrac{-240^2 \sin(66°)}{-9.81}$

2. $5\frac{2}{3} - \frac{6}{7}\left(5\frac{2}{3} + 12\frac{3}{4}\right)$

3. $\text{midpt} \to (1,3),\ m_{\perp} = \dfrac{1}{2}$

$y = 3 + \dfrac{1}{2}(x-1)$

4. $g(x) = -(x+3)(x-6)$

$g\left(\dfrac{3}{2}\right)$

5. $\dfrac{23355 - 15000}{7.5}$

6.
$2A - G = 5$
$3G - 2C = 3$
$2A - C = -8$
$A = 17,\ C = 42,\ G = 29$

7. $\dfrac{20(3.4) + 14(2.1) + 18(2.8)}{52}$

8.
$y = a(x-3)^2 - 2$
$-8 = a(-2-3)^2 - 2$
$a = -0.24$
$y = -0.24(x-3)^2 - 2$

$r = \sqrt{\dfrac{1868.547}{\pi}}$

$m\angle CAB = \theta$

9. $\sin\left(\theta/2\right) = \dfrac{16}{r}$

$\theta = 1.431\ldots \text{ rad}$

$\overset{\frown}{BC} = (2\pi - \theta)r$

same r and θ from #9

10. $\dfrac{1}{2}(2\pi - \theta)r^2 + \dfrac{1}{2}r^2 \sin\theta$

let $\overline{DC} = x$, let $\overline{BD} = a$, let $m\angle ACB = \theta$

$14^2 = 21^2 + 16^2 - 2(21)(16)\cos\theta$

11. $\dfrac{14}{21-x} = \dfrac{16}{x}$

$x = 11.2,\ \theta = 41.795°$

$a^2 = x^2 + 16^2 - 32x\cos\theta$

let E be where $\overline{AC}$ intersects $\overline{BD}$

$\overline{BE} = 14\cos(25°)$

12. $\overline{EC} = 14\sin(25°)$

$\overline{ED} = \sqrt{18^2 - \overline{EC}^2}$

$\text{Area} = \dfrac{\left(\overline{BE} + \overline{ED}\right)\left(2 \cdot \overline{EC}\right)}{2}$

14. $\left(\dfrac{-5+2+4}{3}, \dfrac{-3+1+6}{3}\right)$

15. $\dfrac{1}{2}\left|-30 - 6 + 4 - (2 + 24 + 15)\right|$

16. $\sqrt{74} + \sqrt{97} + \sqrt{85}$

17.
$85 = 74 + 97 - 2\sqrt{97 \cdot 74}\cos(m\angle BAC)$
$\overline{CD} = \sqrt{97}\sin(m\angle BAC)$

let hypotenuse = c

18. $\text{Area} = \dfrac{c^2 \sin 40° \cos 40°}{2} = 82$

$c = 18.249\ldots$

$c(\sin 40° + \cos 40° + 1)$

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19. $2\left(\overline{EG}\right)^2 + 15.7^2 + 11.2^2 = \left(15.7 + 11.2\right)^2$

20. $x = \dfrac{ay+b}{cy+d}$

$xcy + dx = ay + b$

$y\left(a - cx\right) = dx - b$

$g\left(x\right) = \dfrac{dx-b}{a-cx}$

21. $f\left(x\right) = a\left(x-3\right)^2 + 1$

$g\left(x\right) = -a\left(x-3\right)^2 + 7$

$\text{focus} : \left(-3, 4\right)$

$a = \dfrac{1}{12}$

22. $A = 30e^{rt}$

when $t = 9$, $A = 146$; $r = 0.1758...$

when $A = 550$, $t = 16.54...$

$2001 + t$

23. $\displaystyle\sum_{n=1}^{51} \sqrt{4n+6}$

24. $135 = \dfrac{n\left(n-3\right)}{2}$

25. $f\left(x\right) = \dfrac{\left(3x+7\right)\left(2x-1\right)}{\left(x-2\right)\left(x+1\right)}$

26. $-8 - 8\sqrt{3}i = 16\text{cis}\left(240°\right)$

$f\left(k\right) = 2\text{cis}\left(\dfrac{240°}{4} + \dfrac{360°k}{4}\right)$ for $k = 0, 1, 2, 3$

27. let $2x^2 + 3x + 9 = px^2 + qx + r$

$\dfrac{a\sqrt{b}}{c}i = \dfrac{-q\sqrt{q^2 - 4pr}}{p^2} = \dfrac{9\sqrt{7}}{4}i$

28. $\dfrac{83000}{1.03^9}\left(\dfrac{1 - \left(1.03\right)^{35}}{-0.03}\right)$

29. $2 = \ln\left(t^2\right)$

$t = \sqrt{e^2}$

$y\left(\sqrt{e^2}\right)$

30. $A = \pi + \arcsin\left(\dfrac{4}{7}\right)$

$B = 2\pi - \arccos\left(\dfrac{8}{9}\right)$

$\cot\left(A - B\right) = \dfrac{1}{\tan\left(A - B\right)}$

31. $40 + 2\left(\dfrac{\frac{2}{3}\left(40\right)}{1 - \frac{2}{3}}\right)$

32. $\left(-6\right)^2 - 4\left(3\right)\left(-3\right) > 0$

33. Let distance from Sam to Gina = d

$d = \sqrt{5^2 + 3^2 - 30\cos 110°}$

$\dfrac{\sin 110°}{d} = \dfrac{\sin\theta}{5}$

$\text{bearing} = 310° + \theta$

34. $\dfrac{\left(y-1\right)^2}{9} - \dfrac{\left(x-5\right)^2}{5} = 1$

foci at $\left(5, 1 \pm \sqrt{14}\right)$

$5\left(1 + \sqrt{14}\right)\left(1 - \sqrt{14}\right) = 5\left(-13\right)$

35. $\dfrac{\left|21 - 4 - k^2 - 4\right|}{\sqrt{49 + 4 + k^2}} = \dfrac{12}{\sqrt{78}}$

$10000\left(1 + \dfrac{0.0525}{12}\right)^{12t} = 8000\left(1 + \dfrac{0.065}{365.25}\right)^{365.25t}$

36. ${}_{18}C_5 \cdot {}_{13}C_7$

37. $t = 17.698$ years

$12t = 212.37...$

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$a = \dfrac{14582}{\sin 12°}$

38. $\dfrac{a}{\sin 9°} = \dfrac{d}{\sin 3°}$

$d = 23464.21 \text{ ft} = 4.44398 \text{ miles}$

$a_n = 4n + 11,\ b_n = \dfrac{1}{16}\left(\dfrac{4}{3}\right)^{n-1}$

39. $\dfrac{n}{2}(15 + 4n + 11) = \dfrac{1}{16}\left(\dfrac{1-\left(\frac{4}{3}\right)^n}{1-\left(\frac{4}{3}\right)}\right)$

$n = 33.2$

40. $y = -\dfrac{7}{8}x^2 + \dfrac{3}{2}x + \dfrac{3}{8}$

$\text{focus}: \left(\dfrac{6}{7}, \dfrac{57}{56} - \dfrac{2}{7}\right)$

41. $\int_0^{1.9357} y(x)\,dx$

42. $-1 = a^2 + e^a(0 - a)$

$\lim\limits_{n\to\infty}\left|\dfrac{(x-3)^{n+2}}{(n+2)\cdot 4^{n+2}} \cdot \dfrac{(n+1)\cdot 4^{n+1}}{(x-3)^{n+1}}\right| < 1$

$\dfrac{1}{4}|x-3| < 1$

44. $-1 < x < 7$

$CV \text{ at } x = -1$

$DV \text{ at } x = 7$

45. $g'(2) = \dfrac{1}{f'(g(2))}$

$g(2) = 0.79931...$

$f'(0.79931...) = 3.4751$

46. $P(t) = \dfrac{120}{1 + Ce^{-120kt}}$

When $P(0) = 10;\ C = 11$

When $P(1) = 14;\ k = 0.0031126...$

$P(8) = 77.20...$

47. $\tan\theta = \dfrac{h}{14}$

$\sec^2\theta \cdot \dfrac{d\theta}{dt} = \dfrac{1}{14} \cdot \dfrac{dh}{dt}$

$\left(\dfrac{452}{196}\right)\dfrac{d\theta}{dt} = \dfrac{1}{14}(12)$

49. $\dfrac{f(3) - f(0)}{3} = c^2$

$c = \sqrt{3}$, be mindful of interval

50. $m = \dfrac{2}{3}x - mx - b + 2$

$0 = \dfrac{2}{3} - m;\ m = \dfrac{2}{3}$

$m = -b + 2;\ b = \dfrac{4}{3}$

51. $f''(x) = \dfrac{6}{(1+x)^4}$

$f''(0) = 6$

$P_n(x) = ... + \dfrac{6x^2}{2!}$

52. $\dfrac{550 \cdot 570}{1860}$

53. $1 - \chi^2\text{cdf}(0, 50.23710148, 4)$

54. $100 \cdot \text{normalcdf}\left(0, 8\dfrac{55}{60}, 9\dfrac{3.2}{60}, \dfrac{11.4}{60}\right)$

55. $\text{normalcdf}(-1.25, 1.25, 0, 1)$

56. $\hat{y} = \sqrt{0.83}\left(\dfrac{8.1}{6.7}\right)\hat{x} - 10.9604...$

$\hat{y}(86.7)$

57. $p = 0.09$

$\text{binomcdf}(30, 0.09, 3)$

58. $(0.89)^2$

59. $\dfrac{0.12 + 0.032}{2}$

60. $\text{invNorm}(0.975, 0, 1)\sqrt{\dfrac{(0.076)(1-0.076)}{500}}$