



# TMSCA MIDDLE SCHOOL MATHEMATICS

TEST #12 ©

FEBRUARY 19, 2022

## GENERAL DIRECTIONS

- About this test:
  - You will be given 40 minutes to take this test.
  - There are 50 problems on this test.
- All answers must be written on the answer sheet/Scantron form/Chatsworth card provided. If you are using an answer sheet be sure to use **BLOCK CAPITAL LETTERS**. Clean erasures are necessary for accurate grading on Scantrons and Chatsworth cards.
- If you are using a Chatsworth or Scantron card, please follow the specific instructions given at your particular meet.
- You may write anywhere on the test itself. You must write only answers on the answer sheet.
- You may use additional scratch paper provided by the contest director.
- All problems have **ONE** and **ONLY ONE** correct [BEST] answer. There is a penalty for all incorrect answers.
- Calculators **MAY NOT** be used on this test.
- All problems answered correctly are worth **FIVE** points. **TWO** points will be deducted for all problems answered incorrectly. No points will be added or subtracted for problems not answered.
- In case of ties, percent accuracy will be used as a tie breaker.

[illegible]

1.  $1,033 + (-455) =$  \_\_\_\_\_

- A. 1,488      B. 572      C. 578      D. 522      E. 1,422

2.  $3,423,611 - 3,408,655 =$  \_\_\_\_\_ (nearest ten)

- A. 14,950      B. 14,900      C. 15,000      D. 14,940      E. 14,960

3.  $-12\frac{3}{5} \times 4\frac{1}{6} =$  \_\_\_\_\_ (decimal)

- A. -55.5      B. -48.5      C. -49.5      D. -52.5      E. -54.5

4.  $89.32 \div 0.8 =$  \_\_\_\_\_

- A. 111.65      B. 113.35      C. 113.75      D. 109.45      E. 108.85

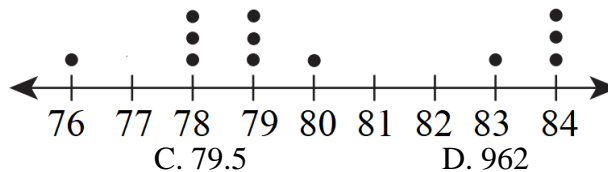
5. There are 72 boys and 90 girls on the local swim team. At the next swim meet, the swim coach would like to give all the competitors a motivational speech. The swim coach would like to arrange the members to sit in equal rows with only girls or only boys in each row. What is the greatest number of students that can be in each row?

- A. 9      B. 8      C. 162      D. 18      E. 36

6.  $10(45 - 38)^2 =$  \_\_\_\_\_ (Roman numeral)

- A. CDXC      B. CDLXX      C. CDXXX      D. CDXCL      E. CDXLV

7. What is the median of the data points plotted in the dot plot below?



- A. 78.5      B. 79      C. 79.5      D. 962      E. 80.5

8. If August is a month that contains Friday the 13<sup>th</sup>, what day of the week lands on August 31<sup>st</sup>?

- A. Monday      B. Tuesday      C. Wednesday      D. Thursday      E. Friday

9. Which integer below is a solution to the inequality  $8 - 2n > 24$ ?

- A. -8      B. -10      C. 0      D.  $\frac{1}{2}$       E. 74

10. 8.75 meters = \_\_\_\_\_ millimeters

- A. 87.5      B. 875      C. 0.875      D. 8,750      E. 87,500

11. Simplify:  $\frac{1}{2}(8n - 14) + 3(5 - n)$

- A.  $n - 29$       B.  $n - 22$       C.  $n + 8$       D.  $n + 22$       E.  $7n + 1$

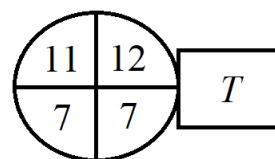
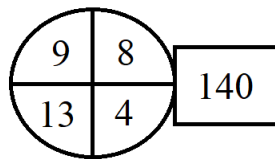
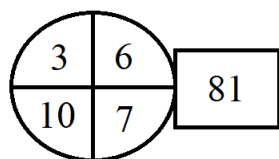
12. Which of the following integers is not divisible by 14?

- A. 196      B. 2,744      C. 742      D. 996      E. 518

13.  $9(2x - 3) - 8x$  is equivalent to which of the following, if  $x = 24$ ?

- A. 213      B. 237      C. 9,528      D. 528      E. 597

14. What are the new coordinates of the point  $(-4, 17)$  after it is translated to the left 4 units and down eight units and then reflected over the  $y$ -axis?  
 A.  $(-8, -9)$       B.  $(8, 9)$       C.  $(0, 25)$       D.  $(8, 25)$       E.  $(-8, 9)$
15. What is the sum of all the prime numbers less than 30?  
 A. 128      B. 159      C. 160      D. 129      E. 143
16.  $45\%$  of  $700 = 75\%$  of \_\_\_\_\_  
 A. 380      B. 360      C. 400      D. 540      E. 420
17. What is the 15<sup>th</sup> term in the sequence  $-34, -26, -18, -10, \dots$ ?  
 A. 64      B. 54      C. 62      D. 70      E. 78
18. There are 288 sprinkles on 3 dozen doughnuts. If there are the same number of sprinkles on each doughnut, how many sprinkles are on 7 doughnuts?  
 A. 336 sprinkles      B. 672 sprinkles      C. 96 sprinkles      D. 56 sprinkles      E. 84 sprinkles
19.  $240 \text{ dimes} + 320 \text{ nickels} = \underline{\hspace{1cm}} \text{ quarters} + 400 \text{ pennies}$   
 A. 124      B. 128      C. 136      D. 140      E. 144
20. Mencie is a painter that charges \$130 for paint supplies and \$45.00 for each hour he works. If Mencie is hired to do a job that will take 6 hours, how much will he be paid?  
 A. \$400.00      B. \$445.00      C. \$355.00      D. \$485.00      E. \$440.00
21. Given the set of numbers  $\{104, 111, 116, 134, 141, 145, 165\}$ , what is the value of the inter-quartile range?  
 A. 134      B. 34      C. 61      D. 31      E. 269
22. The area of a triangle is  $72 \text{ units}^2$ . If the height of the triangle is 6 units, what is the base length of the triangle?  
 A. 18 units      B. 16 units      C. 12 units      D. 20 units      E. 24 units
23. What are the range values of the function  $f(x) = x^2 - x + 3$ , with the domain of  $\{-2, 3, 8\}$ ?  
 A.  $\{9, 59\}$       B.  $\{9, 23, 59\}$       C.  $\{-9, 5, 75\}$       D.  $\{-9, 14, 33\}$       E.  $\{-3, 6, 11\}$
24. If  $12,100 = 2^a \cdot 5^b \cdot 11^c$ , find the value of  $a^2 b^2 c^2$ ?  
 A. 29      B. 144      C. 64      D. 729      E. 792
25. How many two-digit integers exist where the one's digit is greater than the ten's digit?  
 A. 45      B. 55      C. 50      D. 36      E. 32
26. Use the examples in the picture below to find the value of  $T$ .



- A. 176      B. 161      C. 177      D. 189      E. 184

27. What is the sum of  $3 \times 10^5$  and 60,000, expressed in scientific notation?

- A.  $9 \times 10^5$       B.  $9 \times 10^4$       C.  $3.6 \times 10^5$       D.  $3.6 \times 10^4$       E.  $6.3 \times 10^5$

28. Annalese is decorating a section of one of her walls in her bathroom with colored tiles. Annalese wants to use 4 inch  $\times$  4 inch tiles to cover a section that is 2 feet  $\times$  4 feet. How many tiles will Annalese need?

- A. 144      B. 72      C. 2      D. 108      E. 256

29. What is the value of  $y$ , if a line passes through the points  $(-24, 6)$  and  $(8, y)$  and has a slope of  $-\frac{5}{8}$ ?

- A. 24      B. 16      C. -16      D. -8      E. -14

30. If  $a \neq b \neq c$ , and  $a$ ,  $b$ , and  $c$  are positive integers, what is the least possible value of  $d$  for which  $a^1 + b^2 + c^3 = d$ ?

- A. 1      B. 8      C. 3      D. 32      E. 6

31.  $13_5 \times 24_5 =$  \_\_\_\_\_ (base 5)

- A. 132      B. 422      C. 432      D. 344      E. 334

32. Let  $U$  be a universal set and  $A$ ,  $B$ , and  $C$  be subsets of  $U$  defined as shown. Find  $A \cup (B \cap C)$ .

- $U = \{t, u, v, w, x, y, z\}$        $A = \{t, u, v, w\}$        $B = \{w, x, y, z\}$        $C = \{u, v, w, x, y\}$   
A.  $\{z\}$       B.  $\{t, u, v, w\}$       C.  $\{u, v, z\}$       D.  $\{w\}$       E.  $\{t, u, v, w, z\}$

33. If the vertex angle of an isosceles triangle measures  $128^\circ$ , what is the measure of one of the base angles?

- A.  $13^\circ$       B.  $26^\circ$       C.  $52^\circ$       D.  $38^\circ$       E.  $19^\circ$

34. If  $f(x) = -x^2 - 1$ ,  $g(x) = 15 - x$ , and  $h(x) = \frac{x}{4}$ , then what is the value of  $g(f(h(20)))$ ?

- A. -101      B. -11      C. -9      D. 41      E. 39

35. How many total diagonals can be drawn inside of a regular heptagon?

- A. 6      B. 12      C. 18      D. 14      E. 9

36. If the letters of the word *PRODUCT* were placed in a bag and two letters are drawn at random, what is the probability that the first letter is a vowel, and with replacement, the second letter is either a  $C$  or  $T$ ?

- A.  $\frac{4}{49}$       B.  $\frac{2}{343}$       C.  $\frac{4}{343}$       D.  $\frac{2}{7}$       E.  $\frac{4}{7}$

37. What is the range of the graph of the quadratic equation  $y = -3x^2 + 18x - 8$ ?

- A.  $y \leq 3$       B.  $y \leq 19$       C.  $y \geq 6$       D.  $y \leq -6$       E.  $y \geq -8$

38. A population of 400 insects is decreasing at a rate of 23% per month. If  $x$  represents the number of months, which function represents the population of insects after  $x$  months?

- A.  $y = 400(1.23)^x$       B.  $y = 400(0.23)^x$       C.  $y = 0.23(400)^x$       D.  $y = 23(400)^x$       E.  $y = 400(0.77)^x$

39.  $\left(\frac{2(a^3b^{-1})^3}{5a^{-2}b^4}\right) \cdot \left(\frac{10a^4b^{-4}}{(2a^5b)^2}\right) =$  \_\_\_\_\_

- A.  $\frac{a^2}{2b^7}$       B.  $\frac{a^5}{b^{12}}$       C.  $\frac{a^5}{b^{13}}$       D.  $\frac{a^5}{2b^{13}}$       E.  $\frac{a}{b^7}$

40. If  $1 - \frac{2}{3-\frac{4}{5}} = \frac{A}{B}$ , then what is the value of  $A + B$ ?

- A. 8                      B. 12                      C. 14                      D. 15                      E. 16

41. What is the next term in the sequence  $36, 12, 4, 1\frac{1}{3}, \dots$ ?

- A.  $1\frac{1}{6}$                       B.  $\frac{1}{6}$                       C.  $\frac{1}{12}$                       D.  $\frac{4}{9}$                       E.  $1\frac{1}{9}$

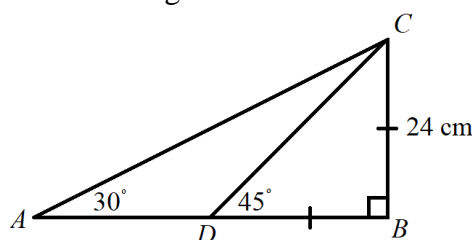
42. Paddling with the water current, Avery can travel 24 km in 3 hours. Against the water current, Avery travels the same distance in 4 hours. What is the rate of the water current?

- A. 7 km/hr                      B. 6 km/hr                      C. 3 km/hr                      D. 1 km/hr                      E. 2 km/hr

43. Shania is drawing a quadrilateral on a coordinate grid with vertices located at  $(-5, -8)$ ,  $(-11, 2)$ ,  $(5, 3)$ , and  $(2, -4)$ . If Shania colors two-thirds of the quadrilateral blue, how many square units of the quadrilateral are not blue?

- A.  $33.8\bar{3}$  units<sup>2</sup>                      B.  $33.\bar{6}$  units<sup>2</sup>                      C.  $67.8\bar{3}$  units<sup>2</sup>                      D.  $67.\bar{6}$  units<sup>2</sup>                      E.  $45.\bar{6}$  units<sup>2</sup>

44. Using the picture below, which of the following is the measure of  $\overline{AD}$ ?



- A. 48 cm                      B.  $24(\sqrt{3} - 1)$  cm                      C.  $24\sqrt{3}$  cm                      D.  $24(\sqrt{2} + 1)$  cm                      E.  $-24(\sqrt{3} - 2\sqrt{2})$  cm

45. If the solution of the equation  $(12 - x)^{\frac{1}{2}} = x$  is  $S$ , what is the value of  $2S^3$ ?

- A. 54                      B. 18                      C. 432                      D. 128                      E. 16

46. What is the solution to the inequality  $|n + 7| \leq 15$ ?

- A.  $[8, 22]$                       B.  $(8, 22)$                       C.  $(-22, 8)$                       D.  $[-8, 22]$                       E.  $[-22, 8]$

47. What is the name of the regular polygon with an interior angle of  $135^\circ$ ?

- A. hexagon                      B. octagon                      C. decagon                      D. undecagon                      E. dodecagon

48.  $\frac{2\sqrt{3}+3\sqrt{2}}{6}$  is the sum of which of the following expression?

- A.  $\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}$                       B.  $\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}$                       C.  $\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}}$                       D.  $\frac{2}{\sqrt{3}} + \frac{3}{\sqrt{2}}$                       E.  $\frac{3}{\sqrt{3}} + \frac{2}{\sqrt{2}}$

49. The sum of the two linear factors of  $x^2 - 10x + 25$  is subtracted from the sum of the two linear factors of  $x^2 - 25$ . What is the resulting difference?

- A. 5                      B. 10                      C. 15                      D. -5                      E. 50

50. If  $A$  is the multiplicative inverse of  $3i$ , then  $A$  is equal to which of the following?

- A.  $\frac{1}{3}$                       B.  $-\frac{1}{3}$                       C.  $-3i$                       D.  $-3$                       E.  $-\frac{i}{3}$

2021 – 2022 TMSCA Middle School Mathematics Test #12 Answer Key

1. C	18. D	35. D
2. E	19. E	36. A
3. D	20. A	37. B
4. A	21. B	38. E
5. D	22. E	39. C
6. A	23. A	40. B
7. B	24. C	41. D
8. B	25. D	42. D
9. B	26. B	43. A
10. D	27. C	44. B
11. C	28. B	45. A
12. D	29. E	46. E
13. A	30. B	47. B
14. B	31. B	48. C
15. D	32. E	49. B
16. E	33. B	50. E
17. E	34. D	

2021 – 2022 TMSCA Middle School Mathematics Test #12 Selected Answers

10. 1 meter = 1,000 millimeters, so 8.75 meters =  $8.75(1,000) = 8,750$  millimeters.

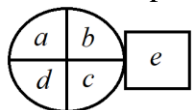
13. If  $x = 24$ , then  $9(2x - 3) - 8x = 9(2(24) - 3) - 8(24) = 9(48 - 3) - 192 = 9(45) - 192 = 405 - 192 = 213$ .

21. The set of numbers 104, 111, 116, 134, 141, 145, 165 has a median of 134. The lower quartile is the median of the lower half of the data, which is 111. The upper quartile is the median of the upper half of the data, which is 145. The interquartile range is the lower quartile subtracted from the upper quartile, which is  $145 - 111 = 34$ .

25. Create a list starting with 12 as shown:

12, 13, 14, 15, 16, 17, 18, 19	= 8
23, 24, 25, 26, 27, 28, 29	= 7
34, 35, 36, 37, 38, 39	= 6
...	
89	= 1

26. Label a picture as shown:



From the first example,  $3(7) + 6(10) = 81$ . From the second example,  $9(4) + 8(13) = 140$ .

So, the rule from the picture is  $ac + bd = e$ . Therefore,  $T = 11(7) + 12(7) = 77 + 84 = 161$ .

27.  $3 \times 10^5 = 300,000$  and  $300,000 + 60,000 = 360,000 = 3.6 \times 10^5$ .

28. A section that is 2 feet  $\times$  4 feet is equal to  $2(12)$  inches  $\times$   $4(12)$  inches, which is 24 inches  $\times$  48 inches. The section has an area of  $24(48) = 1,152$  inches<sup>2</sup>. If tiles are 4 inches  $\times$  4 inches, then their area is  $4(4) = 16$  inches<sup>2</sup>. Now,  $1,152 \div 16 = 72$ . It will take 72 tiles to cover a 2 feet  $\times$  4 feet section of wall.

30. If  $a \neq b \neq c$  and  $a$ ,  $b$ , and  $c$  are positive integers, the least possible value of  $d$  for which  $a^1 + b^2 + c^3 = d$  is therefore,  $3^1 + 2^2 + 1^3 = 8$ .

35. The formula for total diagonals that can be drawn inside a regular polygon is  $\frac{n(n-3)}{2}$ , where  $n$  is equal to the number of sides of the polygon. A heptagon has 7 sides, so  $n = 7$ . Substituting into the formula, we see a heptagon has a total of  $\frac{7(7-3)}{2} = \frac{7(4)}{2} = \frac{28}{2} = 14$  diagonals that can be drawn inside of it.

$$39. \left( \frac{2(a^3b^{-1})^3}{5a^{-2}b^4} \right) \cdot \left( \frac{10a^4b^{-4}}{(2a^5b)^2} \right) = \left( \frac{2a^9b^{-3}}{5a^{-2}b^4} \right) \cdot \left( \frac{10a^4b^{-4}}{4a^{10}b^2} \right) = \left( \frac{2a^{11}}{5b^7} \right) \cdot \left( \frac{5}{2a^6b^6} \right) = \frac{2 \cdot 5 \cdot a^{11}}{2 \cdot 5 \cdot a^6 \cdot b^{13}} = \frac{a^5}{b^{13}}$$

$$40. 1 - \frac{2}{3 - \frac{4}{5}} = 1 - \frac{2}{\frac{15}{5} - \frac{4}{5}} = 1 - \frac{2}{\frac{11}{5}} = 1 - 2 \div \frac{11}{5} = 1 - 2 \cdot \frac{5}{11} = 1 - \frac{10}{11} = \frac{1}{11}. \text{ If } \frac{1}{11} = \frac{A}{B}, \text{ then } 1 + 11 = 12.$$

49.  $x^2 - 25$  can be factored to  $(x + 5)(x - 5)$ . The sum of its linear factors is  $x - 5 + x + 5 = 2x$ .  $x^2 - 10x + 25$  can be factored to  $(x - 5)(x - 5)$ . The sum of the two linear factors is  $x - 5 + x - 5 = 2x - 10$ . If  $2x - 10$  is subtracted from  $2x$ , then the difference is  $2x - (2x - 10) = 2x - 2x + 10 = 10$ .