CEMRACS 2018 project: Mathematical modelling of cell aggregation and segregation.

Kevin Atsou, [1] Marta Marulli, [2] Remi Tesson. [3]

[1] Laboratoire J.A. Dieudonné, Université de Nice Sophia-Antipolis,

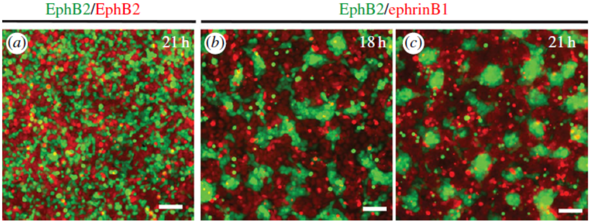
[2] LAGA, Université Paris 13, Università di Bologna,

[3] Institut Mathématiques de Marseille, Aix-Marseille Université.

August 22th 2018

Biological context

Cell segregation and border sharpening in two-species systems



Working hypothesis: inter(heterotypic) and intra(homotypic) species repulsion control cell segregation and border sharpening. They have more influence than inter- or intra- species adhesion.
Goal: to understand the mechanisms of morphogenesis.

blabla

blabla

Microscopic framework

Individual Based Model for particles interacting through repulsion interactions:

$$\begin{cases} dX_i^A = -\mu \nabla_{X_i^A} W^A(X^A, X^B)dt + \sqrt{2D_A} dB_i, & \forall i \in \{1, \dots, N_A\} \\ dX_i^B = -\mu \nabla_{X_\ell^A} W^B(X^A, X^B)dt + \sqrt{2D_B} dB_\ell, & \forall \ell \in \{1, \dots, N_B\} \end{cases} \tag{1}$$

where $\mu > 0$ is the mobility coefficient and B_i is a 2-dimensional Brownian motion $B_i = (B_i^1, B_i^2)$ of intensity $D_A, D_B > 0$ respectively for species A and B.

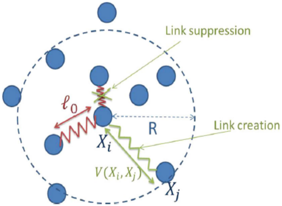
Macroscopic framework

$$\begin{cases} \partial_t f^A = \nabla \cdot \underbrace{\left(f^A \nabla_x (\Phi^{AA} * f^A) + f^A \nabla_x (\Phi^{AB} * f^B)\right)}_{\textit{interaction potential}} + \underbrace{D_A \Delta_x f^A}_{\textit{diffusion}} + \underbrace{\nu^A f^A \left(1 - \frac{f^A + f^B}{f^*}\right)}_{\textit{logistic term}} \\ \partial_t f^B = \nabla \cdot \left(f^B \nabla_x (\Phi^{BB} * f^B) + f^B \nabla_x (\Phi^{BA} * f^A)\right) + D_A \Delta_x f^A + \nu^B f^B \left(1 - \frac{f^A + f^B}{f^*}\right) \end{cases} \quad (2)$$

with $f^S(x,t)$ particle distributions of type S that give the probability to find a particle of type S at a point x and time t, $S \in \{A, B\}$.

Macroscopic framework

We suppose that the homo-heterotypic species links act as a springs of rest length R between the particles that it is also detection radius for the interaction.



Case of Hookean springs

$$\Phi^{ST}(x) = \frac{\nu_c^{ST}}{\nu_d^{ST}} \frac{\kappa^{ST}}{2} \begin{cases} (|x| - R)^2, & \text{for } |x| \leq R \\ 0, & \text{for } |x| > R \end{cases}$$

pp

blabla

Analysis of the macroscopic model

$$\begin{cases} \partial_t f^A = \nabla \cdot (f^A \nabla_x (\Phi^{AA} * f^A) + f^A \nabla_x (\Phi^{AB} * f^B)) + D_A \Delta_x f^A + \nu^A f^A \left(1 - \frac{f^A + f^B}{f^*}\right) \\ \partial_t f^B = \nabla \cdot (f^B \nabla_x (\Phi^{BB} * f^B) + f^B \nabla_x (\Phi^{BA} * f^A)) + D_A \Delta_x f^A + \nu^B f^B \left(1 - \frac{f^A + f^B}{f^*}\right) \end{cases} \tag{3}$$

Linearization around constant steady states $\bar{f}^A, \bar{f}^B$ and Fourier transform:

$$\partial_t \begin{pmatrix} \hat{f}^A \\ \hat{f}^B \end{pmatrix} = \underbrace{\begin{pmatrix} -|y|^2(2\pi \bar{f}^A \hat{\Phi}^{AA}(y) + D_A) - \nu_b^A \frac{\bar{f}^A}{f^*} & -|y|^2 2\pi \bar{f}^A \hat{\Phi}^{AB}(y) - \nu_b^A \frac{\bar{f}^A}{f^*} \\ -|y|^2 \bar{f}^B \hat{\Phi}^{BA}(y) - \nu_b^B \frac{\bar{f}^B}{f^*} & -|y|^2(2\pi \bar{f}^B \hat{\Phi}^{BB}(y) + D_B) - \nu_b^B \frac{\bar{f}^B}{f^*} \end{pmatrix}}_{M(y)} \begin{pmatrix} \hat{f}^A \\ \hat{f}^B \end{pmatrix} \tag{4}$$

The constant steady states will be unstable if:

-

Logistic vs. no logistic

Left Part

Right Part