Data given

The duration of the 15 print-heads is given as

1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

Step 1 :-

Calculate the sample mean and sample standard Diviation.

Calculate the mean ($\bar{x}$)

sample standard diviation (s)

mean Formula:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

n=15 (number of elements)

Sample standard diviation Formula:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Step 2 :-

Construct a 99%. confidence Interval using sample standard Diviation (t- distribution)

The small size sample ($n = 15$) we use the t- distribution instead of the normal distribution. The t-distribution is more appropriate for small size. because it accounts for a uncertainly in estimating the Population standard diviation from the sample.

Formulla for the confidence interval

$$\bar{x} \pm t_{\alpha/2,\, df} \times \frac{s}{\sqrt{n}}$$

$t_{\alpha/2}$ t-value for given 99%.
confidence level freedom ($df = n-1$)

$df = 15 - 1 \Rightarrow df = 14$

Step 3 :-

Construct a 99% confidence interval using unknown population standard deviation.

$$\bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

σ = 0.2 million character

①

$\bar{x}$ = 1.13 + 1.55 + 1.43 + 0.92 + 1.25 + 1.30 + 1.32 + 0.85 + 1.07 + 0.48 + 1.20 + 1.33 + 1.18 + 1.22 + 1.29 / 15

$\bar{x} = \frac{19.81}{15} \simeq 1.3207$ million character.

② sample standard diviation

$$\sum_{i=1}^{n} (x_i - \bar{x})^2$$

$(1.33 - 1.3207) = 0.0360$

$(1.55 - 1.3207) = 0.0533$

$(1.43 - 1.3207) = 0.0148$

$(1.25 - 1.3207) = 0.0015$

$(1.30 - 1.3207) = 0.0001$

$(1.32 - 1.3207) = 0.0001$

$(0.85 - 1.3207) = 0.1856$

$(1.07 - 1.3207) = 0.0457$

$(0.42 - 1.3207) = 0.0154$

$(1.20 - 1.3207) = 0.0135$

$(1.33 - 1.3207) = 0.0181$

$(1.18 - 1.3207) = 0.0103$

$(1.22 - 1.3207) = 0.0103$

$(1.29 - 1.3207) = 0.0001$ +

$\simeq 0.5623$

$df = n - 1$
$= 15 - 1 = 14$

$s^2 = \frac{0.5623}{14} \simeq 0.0402$

$s = \sqrt{0.0402} \simeq 0.2005$

million character.

Part (a) - Confidence Interval using sample standard deviation

For a 99% confidence level and df = 14, the t-value is approximately

<u>2.977</u>

Confidence Interval formula :-

$$CI = \bar{x} \pm t_{\alpha/2, df} \times \frac{s}{\sqrt{n}}$$

$$CI = 1.3207 \pm 2.977 \times \frac{0.2005}{\sqrt{15}}$$

$$CI = 1.3207 \pm 2.977 \times 0.0518$$

$$CI = 1.3207 \pm 0.1543$$

So 99% confidence Interval is

(1.1664, 1.4750) million character

Part (b) - Confidence Level using unknown Population Standard Deviation.

If the Population Standard diviation is unknown

(... million characters)

we use the Z-distribution. The Z-value for a 99% confidence interval is $\underline{\underline{2.576}}$

$$CI = \bar{x} \pm Z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}$$

$$CI = 1.3207 \pm 2.576 \times \frac{0.2}{\sqrt{15}}$$

$$CI = 1.3207 \pm 2.576 \times 0.0516$$

$$CI = 1.3207 \pm 0.1337$$

99% confidence interval is

(1.1870, 1.4544) million characters

Summary of result:-

1) 99% confidence interval using sample standard Deviation:

(1.1664, 1.4750) million characters

2) 99% confidence Interval using known Population standard Deviation:

(1.1870, 1.4544) million characters