Fit a 2D ellipse to given data points

This Python program is used for fitting a 2D ellipse to data points (x, y). Several different approaches are used and compared to each other. It is programmed in Python 3.7.9. (windows version). The parameters of the found regression ellipse can be edited, loaded and saved.
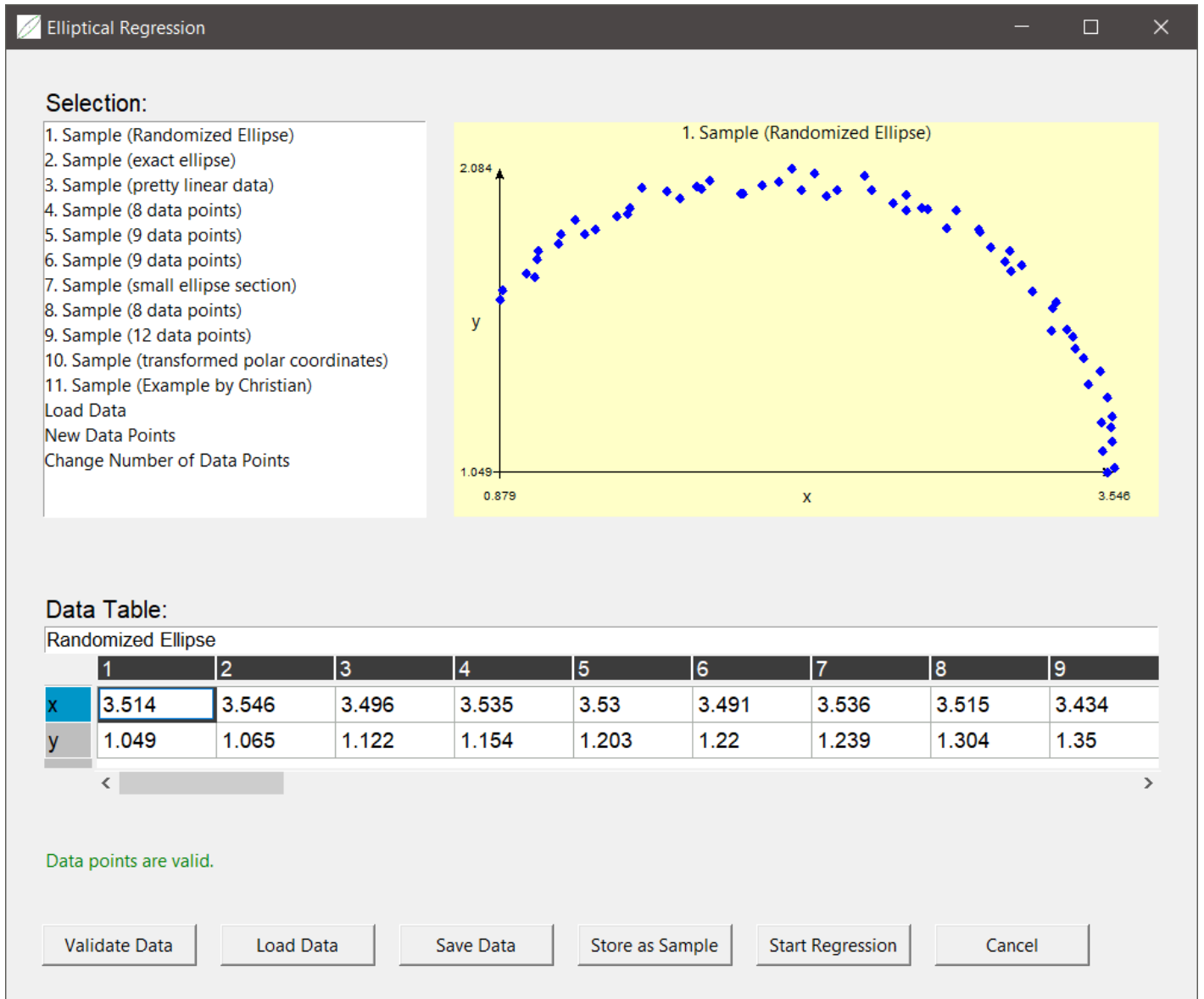
Files

File	Description
fitEllipse.py	Main program with the fitting algorithms (Normal start)
start_fitEllipse.py	Start of program in a venv environment (it will install all requirements in a venv environment)
requirements.txt	List of requirements (tkintertable is optional but highly recommended)
mainDialog.py	Main dialog (using tkinter + tkintertable)
editor.py	Result and editor dialog (using matplotlib)
editorConfig.py	Editor configuration file (global variables)
fileDialogs.py	File dialogs (open, save, ...)
quarticFormula.py	Formulas for quartic equations (including cardanic and quadratic cases)
Icon_fitEllipse.ico	Icon file for main dialog
Icon	Icon file for Pyinstaller Packager
fitEllipse.spec	Spec file for Pyinstaller Packager (windows version)
Docu (directory)	Documentation files (start ReadMe.html or Readme.md)
Package (directory)	Packaged version of program (windows EXE only)
Sample_Data (directory)	Examples for data points and parameter sets (including data points)

Start with *fitEllipse.py*

The program is started directly with fitEllipse.py (requirements see above). Dialogs can be closed by pressing *<ESC>* key.
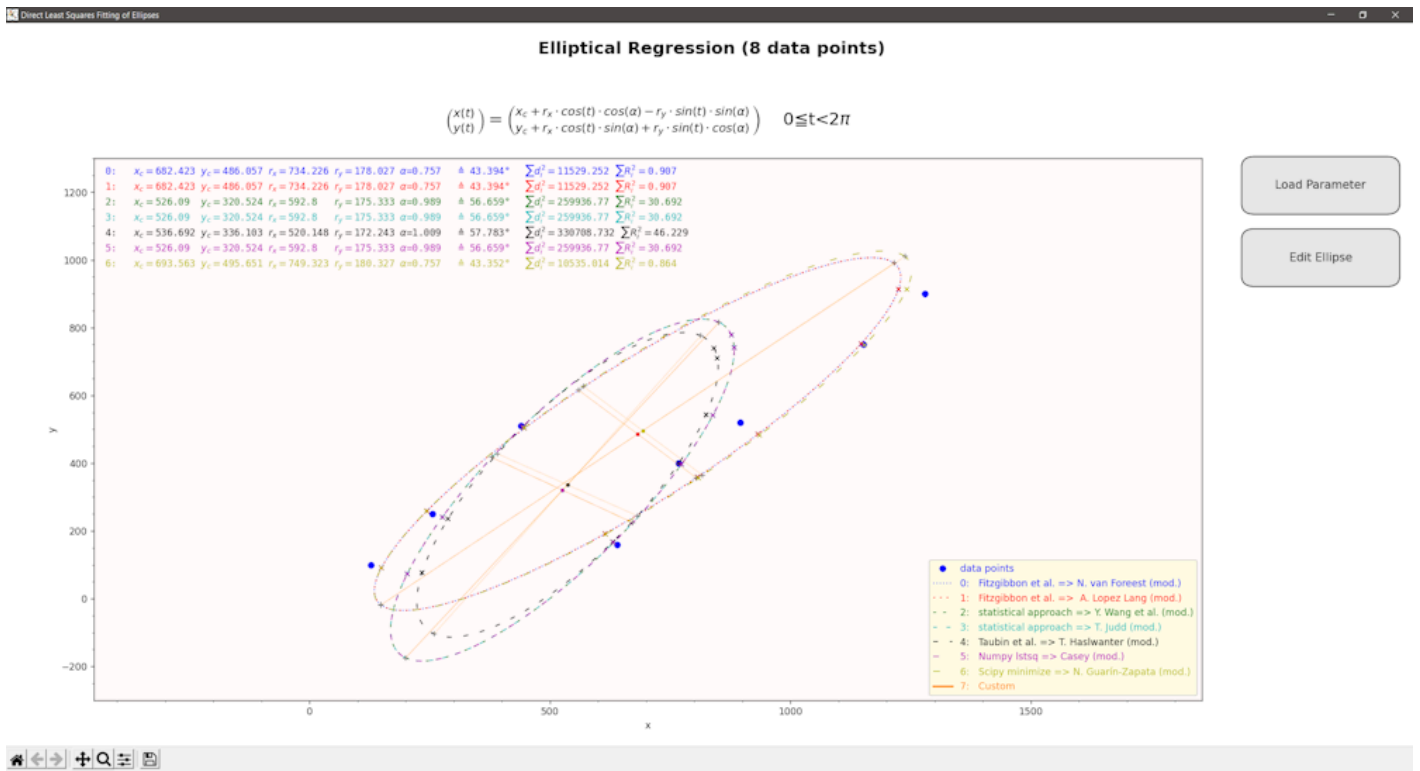
If *tkintertable* is installed a GUI is shown:

Elliptical Regression

Selection:

1. Sample (Randomized Ellipse)
2. Sample (exact ellipse)
3. Sample (pretty linear data)
4. Sample (8 data points)
5. Sample (9 data points)
6. Sample (9 data points)
7. Sample (small ellipse section)
8. Sample (8 data points)
9. Sample (12 data points)
10. Sample (transformed polar coordinates)
11. Sample (Example by Christian)
Load Data
New Data Points
Change Number of Data Points

1. Sample (Randomized Ellipse)

Data Table:

Randomized Ellipse

	1	2	3	4	5	6	7	8	9
x	3.514	3.546	3.496	3.535	3.53	3.491	3.536	3.515	3.434
y	1.049	1.065	1.122	1.154	1.203	1.22	1.239	1.304	1.35

Data points are valid.

| Validate Data | Load Data | Save Data | Store as Sample | Start Regression | Cancel |

After the regression is started with button

> Start Regression

the results are displayed via matplotlib:

If *tkintertable* is not installed a console is used (lesser options):

```
C:\WINDOWS\py.exe

tkintertable not installed!

scikit not installed!

Please enter sample number (1 ... 11) or L (load data file) or Q (Quit):
```

After the regression results via matplotlib are closed a summary is shown in the console:

```
C:\WINDOWS\py.exe

Data Points
x:
 [ 3.08  2.9  2.77  2.65  2.54  2.44]
y:
 [ 3.22  3.13  3.06  3  2.94  2.87]

Ellipse fitting
ax² + bxy + cy² + dx + ey + f = 0

i|a        b      c       d       e      f      |xc    yc     rx     ry     phi[-]  |Dist.²     Res.²
0|-2.62    7.89   -7.51   -6.75   18.7   -14.9  |2.8   2.71   1.53   0.321  0.508   |2.00e⁻⁹    8.90e⁻⁸
1|-2.62    7.89   -7.51   -6.75   18.7   -14.9  |2.8   2.71   1.53   0.321  0.508   |2.00e⁻⁹    8.90e⁻⁸
2|-0.651   1.73   -1.96   -0.248  4.51   -4.4   |3.22  2.57   2.12   0.646  0.461   |8.16e⁻⁷    7.33e⁻⁶
3|-0.651   1.73   -1.96   -0.248  4.51   -4.4   |3.22  2.57   2.12   0.646  0.461   |8.16e⁻⁷    7.33e⁻⁶
4|-0.657   1.75   -1.98   -0.263  4.56   -4.44  |3.22  2.57   2.12   0.644  0.461   |8.13e⁻⁷    7.37e⁻⁶
5|-0.651   1.73   -1.96   -0.248  4.51   -4.4   |3.22  2.57   2.12   0.646  0.461   |8.16e⁻⁷    7.33e⁻⁶
6|-2.62    7.89   -7.51   -6.75   18.7   -14.9  |2.8   2.71   1.53   0.321  0.508   |2.00e⁻⁹    8.90e⁻⁸

Legend:
0:   Fitzgibbon et al. => N. van Foreest (mod.)
1:   Fitzgibbon et al. =>  A. Lopez Lang (mod.)
2:   statistical approach => Y. Wang et al. (mod.)
3:   statistical approach => T. Judd (mod.)
4:   Taubin et al. => T. Haslwanter (mod.)
5:   Numpy lstsq => Casey (mod.)
6:   Scipy minimize => N. Guarín-Zapata (mod.)
7:   Custom
```

Start with *Start_fitEllipse.py*

The program is started within a venv enviropnment. The requirements are installed via pip (downloading libraries). This kind of start will take its time...

The advantage of this start type is that no manual installation of Python modules is neccessary. Extra installation is performed automatically in a separate virtual envorinment (venv). Internet Access is needed.
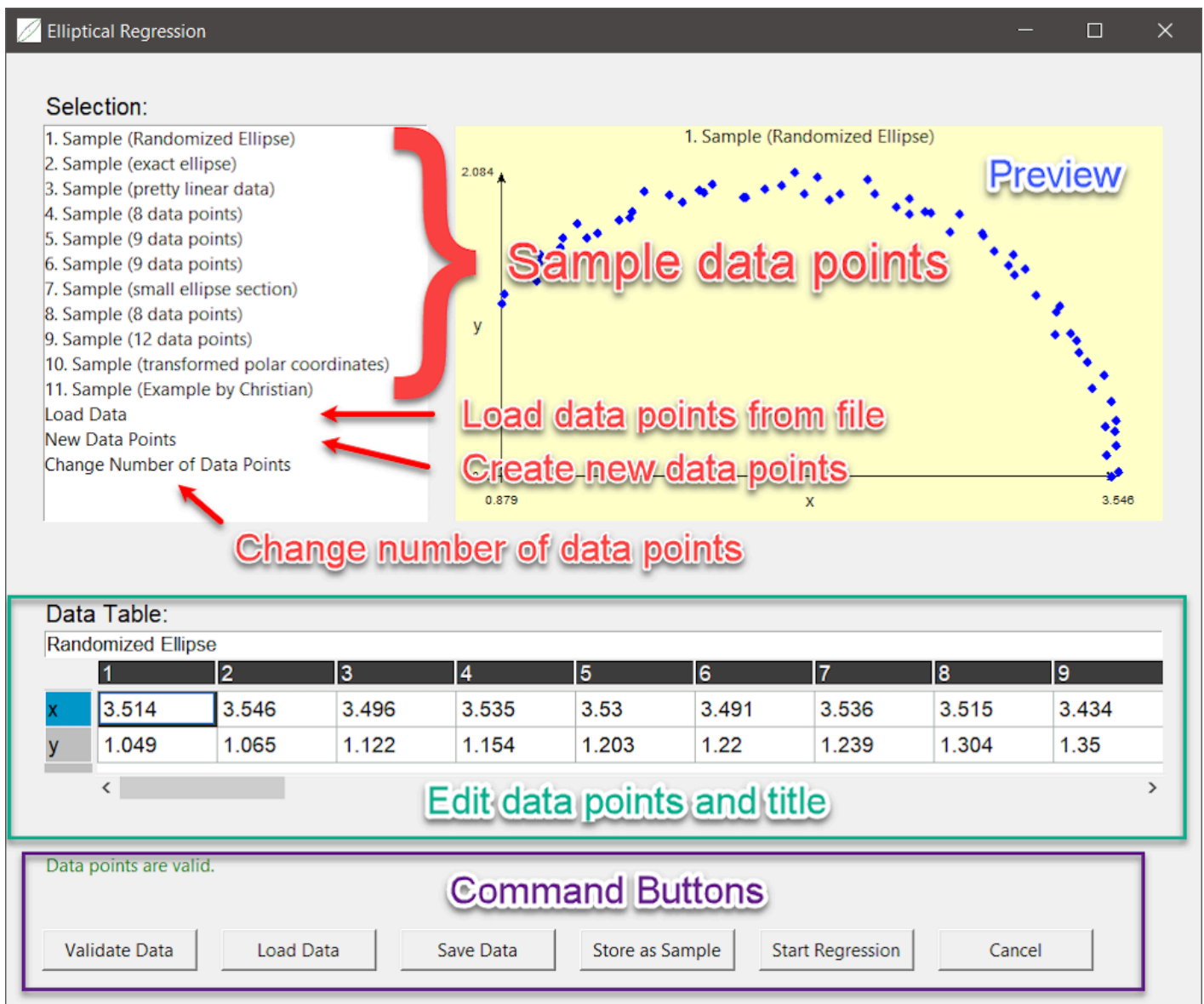
Start with *fitEllipse.exe* (Package directory)

The program is started as standalone EXE (windows only). No Python is neccessary. The EXE was packaged with pyinstaller. There is a spec file for Pyinstaller Packager (windows version):

fitEllipse.spec

Main dialog

The main dialog:

is divided into 4 areas:

Selection area

In this area you can:

- select different example point sets (stored for the session),
- load data points from csv- or txt-file (examples below),
- create a new set of data points or
- change number of data points (appending empty points or deleting points).

Preview area

In this area you will see a preview of the data points if the are validated (see below).

Data table area

In this are you can edit data points (even scientific format is accepted, commas are translated into points). In normal cases the input is validated automatically otherways *Validate Data* command can be issued manually.

Command buttons

In this area you can:

- validate data (check wheter table contains numbers and is complete),
- load data points from csv- or txt-file (examples below),
- save data points to csv-file,
- store current data points as sample (only for this session),
- start regression and show results (*<ENTER>* key)
- cancel (exit programm, *<ESC>* key)

Result dialog

The result dialog:
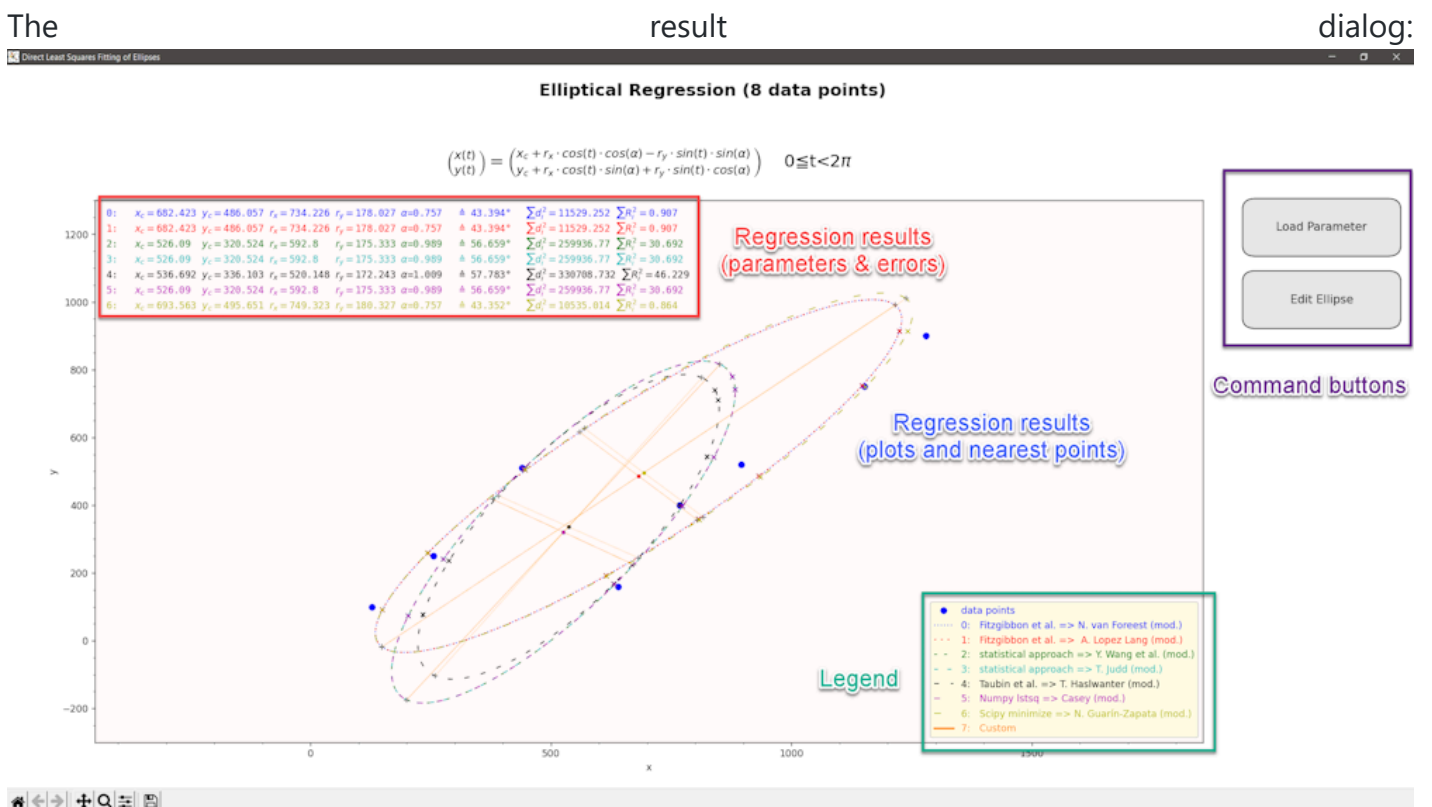
is divided into 4 areas:

Regression results (parameters & errors)

Next to the Regression results (parameters of regression ellipses) also the geometric Error:

$$\sum_{i=1}^{n} d_i^2$$

and the residual error:

$$\sum_{i=1}^{n} R_i^2$$
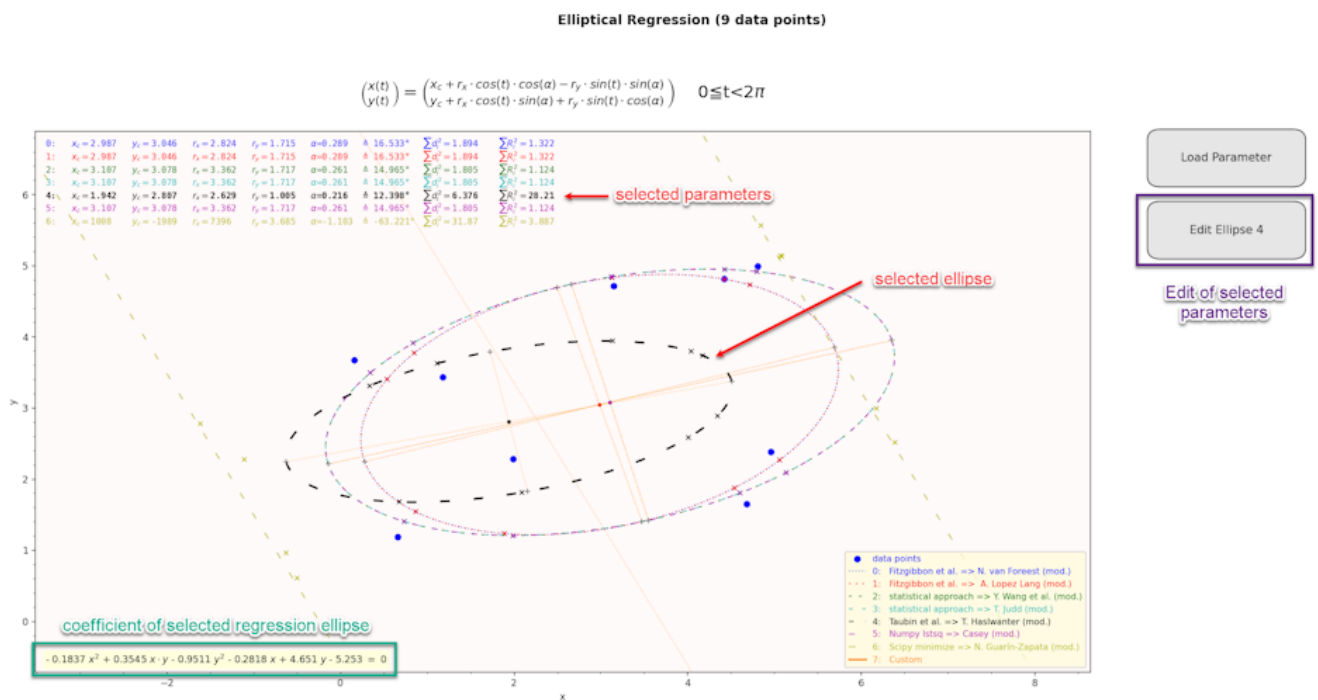
with

$$R(x_i, y_i) = a \cdot x_i^2 + b \cdot x_i \cdot y_i + c \cdot y_i^2 + d \cdot x_i + e \cdot y_i + f$$

are shown. In order to get comparable redidual errors the coefficients are normalized with:

$$R(x_c, y_c) \stackrel{!}{=} 1$$

If you click (pick) one result, the corresponding coefficients are shown and the Edit selection is changed to the selected regression parameters:

Regression results (plots and nearest points)

The regression ellipses are plotted (including center, axes and nearest points). If you click (pick) one result plot, the corresponding coefficients are shown and the Edit selection is changed to the selected regression parameters (see above).

Coefficients of selected regression ellipse (only shown during selection of results)

The coefficients of the selected regression ellipse are shown. Selection can be canceled by clicking into the space between results (not available in **editor mode**).
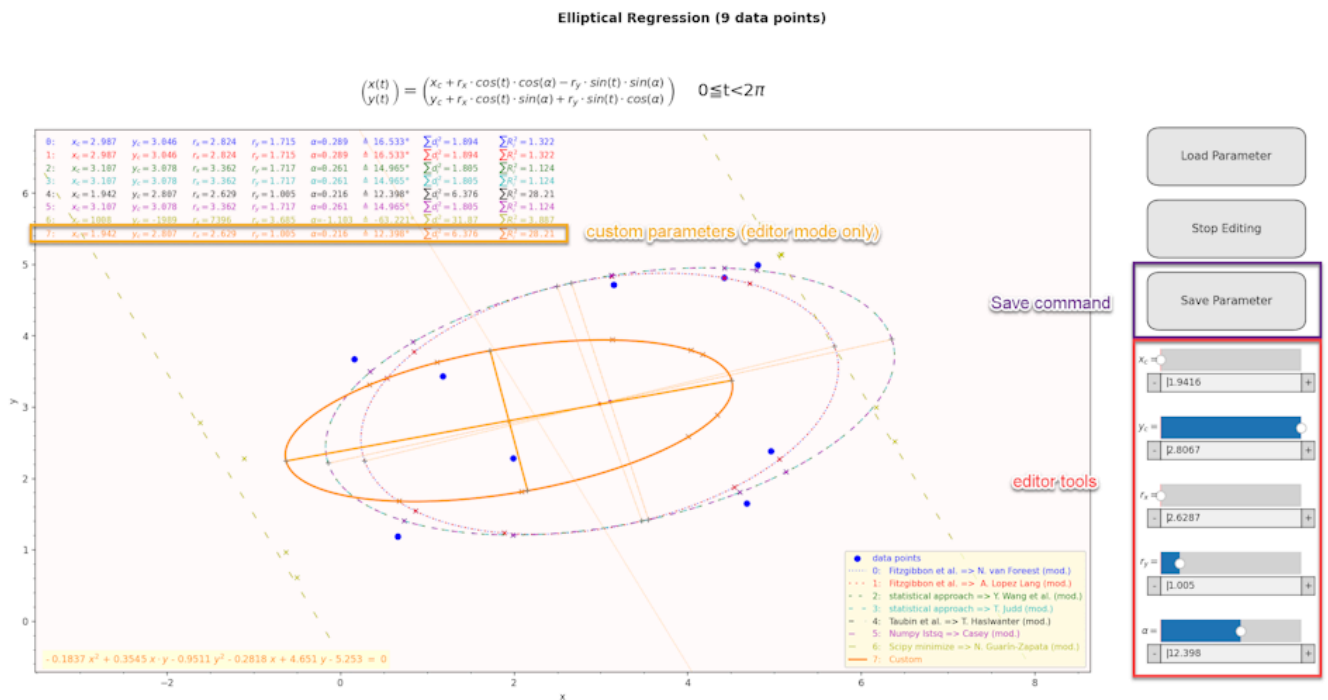
Command buttons

In this area you can:

- load a paramater set from csv-file (example see below) as custom ellipse. Automatically the **editor mode** will be activated.
- edit ellipse will activate **editor mode** with custom parameters of preselected parameters / last edited parameters.

Editor mode

The editor mode:

has an additional parameter set (custom), a command button and editor tools.

Additional custom parameter set (editor mode only)

An additional parameter set (custom) is shown in **editor mode** only. All changes are updated in realtime (depending on computer). The paramters are changes via editor tools or loading of parameters (see above)

Additional Save button (editor mode only)

The current custom ellipse parameters including errors and data points will be saved as csv-file.

Editor tools (editor mode only)

The custom parameters are changed via sliders, text inputs, decrements and increments. Custom parameters, coefficients and the plot is updated in realtime (depending on computer).

Files: Data points and parameters

Data points can be save from the **main dialog** or as part of the parameter file in **editor mode**.

Standalone data point files should be formatted as american csv-files (x and y in 2 columns, no header):

[Sample](#)

For compatibility reason headers are accepted:

[Sample Header](#)

as well as x and y in rows with other separators (tab, space or semicolon):

[Sample Rows](#)

as well as German csv format (preview is disturbed!):

[Sample German](#)

Autodetection will detect correct format in most cases.

Parameter files are separated into the parameter and the data point section (don't change header lines).

[Sample Parameter](#)

It is possible to use parameter files as input for data files (**main dialog**).

Requirements
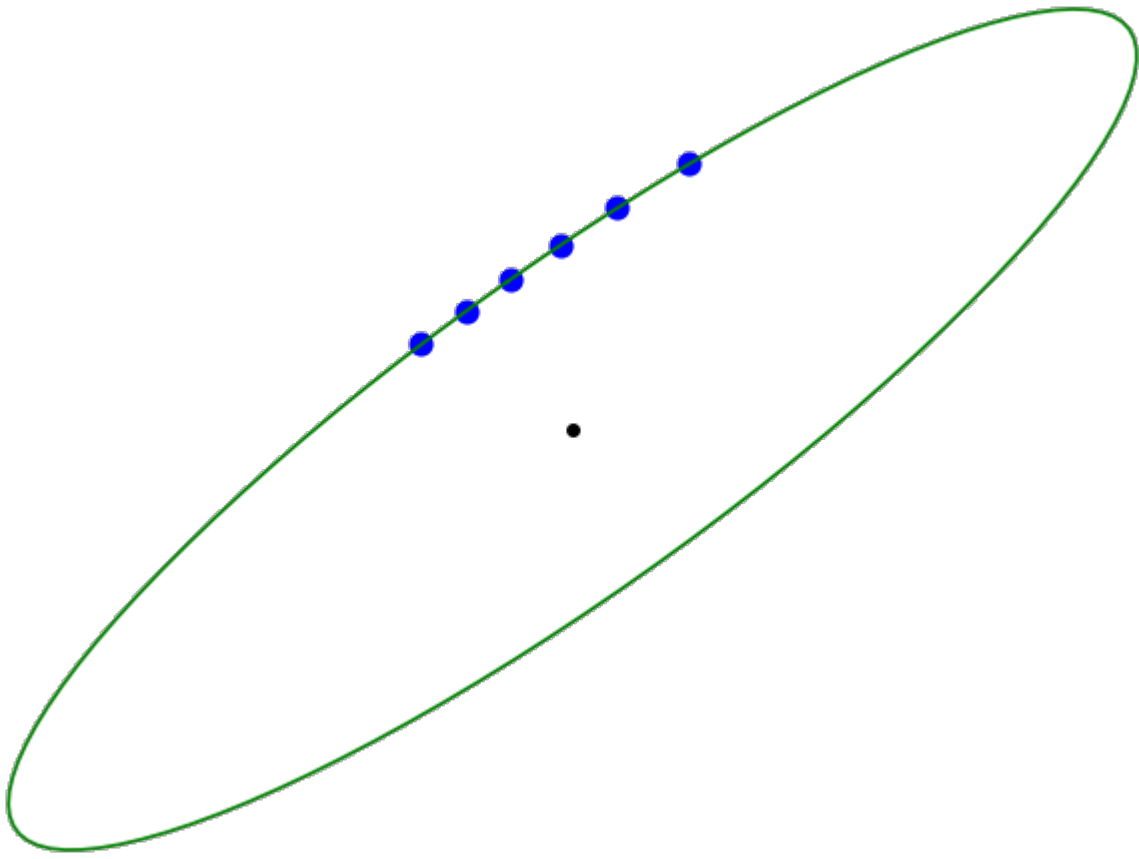
The requirements are listed in:

[requirements.txt](#)

tkintertable is optional but highly recommended (no **main dialog** without, see above).

scikit-image is purely optional and will add an algorithm of scikit (same results as Fitzgibbon et. al).

Attachments

```
matplotlib==3.5.3
numpy==1.21.6
scipy==1.7.3
tkintertable==1.3.3
```

```python
# -*- mode: python ; coding: utf-8 -*-


block_cipher = None


a = Analysis(
    ['fitEllipse.py'],
    pathex=[],
    binaries=[],
    datas=[('./Sample_Data', 'Sample_Data'),('./Icon_fitEllipse.ico', '.') ],
    hiddenimports=[],
    hookspath=[],
    hooksconfig={},
    runtime_hooks=[],
    excludes=[],
    win_no_prefer_redirects=False,
    win_private_assemblies=False,
    cipher=block_cipher,
    noarchive=False,
)
pyz = PYZ(a.pure, a.zipped_data, cipher=block_cipher)

exe = EXE(
    pyz,
    a.scripts,
    a.binaries,
    a.zipfiles,
    a.datas,
    [],
    name='fitEllipse',
    debug=False,
    bootloader_ignore_signals=False,
    strip=False,
    upx=True,
    upx_exclude=[],
    runtime_tmpdir=None,
    console=True,
    disable_windowed_traceback=False,
    argv_emulation=False,
    target_arch=None,
    codesign_identity=None,
    entitlements_file=None,
    icon=['Icon.png'],
)
```

-1.3046,0.2442
-1.7816,1.3049
-1.1218,-1.3168
-0.755,0.7056
1.5613,1.1787
0.9638,1.3228
-1.0693,1.7625
1.4383,1.2437
-0.2293,1.0088
-1.5728,-1.1709
0.73,1.5918
-1.7642,-0.4769
-1.7809,-0.8841

x,y
-1.3046,0.2442
-1.7816,1.3049
-1.1218,-1.3168
-0.755,0.7056
1.5613,1.1787
0.9638,1.3228
-1.0693,1.7625
1.4383,1.2437
-0.2293,1.0088
-1.5728,-1.1709
0.73,1.5918
-1.7642,-0.4769
-1.7809,-0.8841

3.587;3.576;3.57;3.587;3.571;3.559
1.032;1.083;1.082;1.217;1.167;1.227

3,587;3,576;3,57;3,587;3,571;3,559
1,032;1,083;1,082;1,217;1,167;1,227

a,b,c,d,e,f,xc,yc,rx,ry,phi[-],Dist.²,Res.²
0.416175,0.0494644,0.970661,-1.78794,-2.20895,2.16613,2.0836,1.08477,1.46812,0.959497,-0.044486,0.0478915,
0.123203
Points (x,y)
3.588,1.021
3.507,1.094
3.572,1.084
3.524,1.145
3.506,1.238
3.568,1.227
3.533,1.258
3.459,1.314
3.433,1.389
3.427,1.45
3.389,1.406
3.425,1.466
3.398,1.539
3.391,1.585
3.363,1.551
3.3,1.571
3.217,1.606
3.206,1.691
3.166,1.75
3.129,1.755
3.092,1.756
3.024,1.746
2.999,1.795
2.95,1.855
2.94,1.826
2.825,1.895
2.817,1.868
2.772,1.91
2.727,1.918
2.609,1.961
2.561,1.962
2.52,1.968
2.46,1.981
2.442,2.053
2.358,2.067
2.305,2.026
2.261,1.998
2.225,2.06
2.114,2.038
2.106,2.068
2.03,2.047
1.964,2.075
1.871,2.024
1.789,2.081
1.811,2.017
1.748,2.015
1.63,2.024
1.559,2.036
1.554,1.993
1.524,1.93
1.392,1.915
1.402,1.953
1.328,1.922

1.267,1.932
1.183,1.87
1.125,1.821
1.098,1.79
1.024,1.827
1.034,1.767
0.942,1.747
0.933,1.758
0.912,1.691
0.874,1.618