MQE: Economic Inference from Data: Module 4: Randomized Control Trials

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6/9/2020

```
library(knitr)
hook_output = knit_hooks$get('output')
knit_hooks$set(output = function(x, options) {
  # this hook is used only when the linewidth option is no
  if (!is.null(n <- options$linewidth)) {
    x = knitr:::split_lines(x)
    # any lines wider than n should be wrapped
    if (any(nchar(x) > n)) x = strwrap(x, width = n)
    x = paste(x, collapse = '\n')
  }
  hook_output(x, options)
})
```

Module 4: Randomized Control Trials

The Experimental Ideal:

Getting causal effects is HARD!

The Experimental Ideal:

Getting causal effects is HARD!

So where do we go from here?

Randomized control trials (RCT's) aka the "Gold standard" of experimental designs

Random assignment and the selection problem

The idea: use random assignment remove selection bias

Suppose a constant treatment effect: $Y_i(1) - Y_i(0) = \tau$, a constant. For observation i we have that

$$
\begin{aligned}
Y_i &= Y_i(0) + \tau D_i \\
Y_i &= E[Y_i(0)] + \tau D_i + Y_i(0) - E[Y_i(0)] \\
Y_i &= \alpha + \tau D_i + \eta_i
\end{aligned}
$$

where $\alpha = E[Y_i(0)]$, $\tau = Y_i(1) - Y_i(0)$, and η_i is the random part of $Y_i(0)$ since $\eta_i = Y_i(0) - E[Y_i(0)]$.

Random assignment and the selection problem

The expected outcomes for someone with treatment ($D_i = 1$), and without treatment ($D_i = 0$) are given by

$$E[Y_i(1)] = \alpha + \tau + E[\eta_i | D_i = 1]$$
$$E[Y_i(0)] = \alpha + E[\eta_i | D_i = 0]$$

so that we can break down the difference between these outcomes as

$$E[Y_i(1)] - E[Y_i(0)] = \underbrace{\tau}_{\text{treatment effect}} + \underbrace{E[\eta_i | D_i = 1] - E[\eta_i | D_i = 0]}_{\text{selection bias}}.$$

Random assignment and the selection problem

Selection bias will bias our estimate of τ if those who select into treatment have a different expected outcome compared to those who do not select into treatment:

$$E[Y_i(0)|D_i = 1] \neq E[Y_i(0)|D_i = 0].$$

This is because treatment is not random: $\{Y_i(1), Y_i(0)\} \not\perp D_i$.

There is no reason to believe that those who select into treatment have the same expected outcome as those who do not, if they were to be treated, that is to say, it is possible (and even likely) that

$$\underbrace{E[Y_i(0)|D_i = 0]}_{\text{observed}} \neq \underbrace{E[Y_i(0)|D_i = 1]}_{\text{unobserved}} \neq E[Y_i(0)]$$

Random assignment and the selection problem

The conditional independence assumption allows us to control for selection bias by conditioning on **observed characteristics**. . .

. . . **unobserved characteristics** that we cannot control for will often also bias our estimates.

Random assignment solves all of these selection bias problems.

Random assignment and the selection problem

Random assignment makes D_i independent of potential outcomes:

$$\{Y_i(1), Y_i(0)\} \perp D_i.$$

With random assignment, we know that in expectiation,

$$\underbrace{E[Y_i(0)|D_i = 0]}_{\text{observed}} = \underbrace{E[Y_i(0)|D_i = 1]}_{\text{unobserved}} = E[Y_i(0)]$$

and

$$\underbrace{E[Y_i(1)|D_i = 0]}_{\text{unobserved}} = \underbrace{E[Y_i(1)|D_i = 1]}_{\text{observed}} = E[Y_i(1)]$$

Random assignment and the selection problem

Thus, the causal **Average Treatment Effect (ATE)**, $\bar{\tau}$, is

$$\bar{\tau} = E[Y_i(1)] - E[Y_i(0)] = \underbrace{E[Y_i(1)|D_i = 1]}_{\text{observed}} - \underbrace{E[Y_i(0)|D_i = 0]}_{\text{observed}}$$

$$= E[Y_i|D_i = 1] - E[Y_i|D_i = 0].$$

and we can easily estimate $\bar{\tau}$, by taking the difference between the average value of Y_i in the treatment group and the average value of Y_i in the control group.

RCT estimation

RCT regressions are about as straigtforward as it gets.

As modeled above, you can estimate

$$Y_i = \alpha + \tau D_i + \eta_i$$

where $\alpha = E[Y_i(0)]$, $\tau = Y_i(1) - Y_i(0)$, and η_i is the random error term.

RCT estimation

The treatment effect will be given by

$$E[Y_i(1)] - E[Y_i(0)] = \underbrace{\tau}_{\text{treatment effect}} + \underbrace{E[\eta_i|D_i = 1] - E[\eta_i|D_i = 0]}_{\text{selection bias}}.$$

With proper randomizating,

$$E[\eta_i|D_i = 1] = E[\eta_i|D_i = 0]$$

so there is no selection bias giving us an unbiased estimate of τ.

RCT Simulation

I am a principal of a large school and I want to evaluate how access to small reading groups with a paraprofessional helps improve 4th grade test scores.

I take all the 4th graders and randomly assign 30 percent of them to treatment (participating in the reading groups) with therest to the control group which continued with class as normal.

RCT Simulation

I generate a set of simualted data:

```
set.seed(1999)

scores5<-as.data.frame(rep(c(1,2,3,4,5,6,7,8,9,10),times=30))
names(scores5)<-c("class")
scores5 <- fastDummies::dummy_cols(scores5, select_columns = "class")
scores5$error<-rnorm(300, mean=0, sd=10)

#treatment indicator
scores5$treat<-rbinom(300,1,0.3)

#mean reading score
alpha=75

#treatment effect
tau=10

#the data generating process: notice the class does affect a student's score
scores5$read4<-alpha+tau*scores5$treat+scores5$error+4*scores5$class_1+(-6)*scores5$class_2+8*scores5$clas
```

RCT Simulation

```
rct1<-felm(read4~treat,scores5)
stargazer(rct1, type="latex", header=FALSE)
```

Table 1

	Dependent variable:
	read4
treat	11.229***
	(1.442)
Constant	77.150***
	(0.750)
Observations	300
R^2	0.169
Adjusted R^2	0.166
Residual Std. Error	11.092 (df = 298)
Note:	*p<0.1; **p<0.05; ***p<0.01

Because treatment was randomized, even though the class the student is in does affect their score, we recover an unbiased estimate of the true treatment effect ($\tau = 10$).

RCT Key assumption

The key assumption is that

$$E[\eta_i|D_i = 1] = E[\eta_i|D_i = 0] = 0.$$

- we cannot test this assumption directly
- BUT we can do a **balance test**: we check to see if observable characteristics among treatment and control groups are the same on average.

RCT Balance tests

- can be presented as a table of the following regressions $X_i = \beta_0 + \beta_1 D_i + \epsilon_i$ where X_i is a vector of characteristics being tested.
- are often presented as simple t-test tables testing the difference in means between the treatment and control groups.

RCT Balance Tests

Balance test variables should be

- characteristics at baseline, prior to treatment,
- or characteristics that would be unaffected by treatment.

Balance tests are often run on many variables:

- some may come up with a statistically significant difference by simple random chance
- if many are significantly different, this is a red flag that the key assumption does not hold
- There are corrections that can be implemented if the unbalanced variables are are of particular concern (look up Bonferroni correction)

RCT Simulation

Suppose the principal is concerned that there were some problems with the randomization.

She has access to some additional data. She adds it to her data set and does a balance test.

RCT Simulation

```
#simulating covariates

#third grade test scores. Notice I am generateing simulated academic scores that have a correlation to thei
scores5$read3<-alpha+scores5$error+rnorm(300,3,2)
scores5$math3<-alpha+scores5$error+rnorm(300,15,2)
scores5$hist3<-alpha+scores5$error+rnorm(300,5,2)
scores5$pe3<-rnorm(300,90,2)

#other 4th grade test scores: notice I am generating scores that correlated with their subject performance
scores5$hist4<-4*scores5$treat+scores5$hist3+rnorm(300,-2,2)
scores5$pe4<-scores5$pe3+rnorm(300,0,5)
scores5$math4<-2*scores5$treat+scores5$math3+rnorm(300,-5,3)

#student characteristics
scores5$female<-rbinom(300,1,0.5)
scores5$age<-runif(300,9,10)
scores5$height<-rnorm(300,1.3,0.2)

scoresmini<-scores5[,c("treat", "read4", "read3", "math3","hist3","pe3","hist4","pe4","math4","female", "a
knitr::kable(head(scoresmini))
```

treat	read4	read3	math3	hist3	pe3	hist4	pe4	math4	fema
0	86.32672	83.27039	96.31086	87.68543	91.03541	88.16632	93.95379	94.98303	
0	68.62170	79.22211	88.71313	79.91069	93.06738	76.14878	94.28598	81.82348	
1	105.03009	87.89982	102.82710	92.07489	86.74861	95.05262	82.04735	102.39419	
0	85.69802	94.08272	101.67523	95.73204	88.38979	94.73489	76.42173	97.08292	
0	83.33690	74.47617	90.96568	80.65065	93.68531	77.05651	86.33431	84.26472	
0	78.19827	80.55488	96.59311	84.92801	87.12997	83.51559	93.56600	92.40844	

RCT Simulation

```
#as you can see, we have simulated some complex interrelationships between theses variables.
cor(scoresmini)
```

```
##                treat        read4       read3        math3        hist3
## treat   1.000000000  0.411103370  0.06210422  0.043810660  0.048985855
## read4   0.411103370  1.000000000  0.76370646  0.756595945  0.767164090
## read3   0.062104220  0.763706465  1.00000000  0.954947392  0.957691240
## math3   0.043810660  0.756595945  0.95494739  1.000000000  0.951163232
## hist3   0.048985855  0.767164090  0.95769124  0.951163232  1.000000000
## pe3    -0.134794987 -0.078928737  0.01977274  0.025802541  0.002345894
## hist4   0.210104738  0.799439890  0.92717436  0.910566252  0.965319194
## pe4    -0.043975444  0.001522679  0.06059868  0.067988360  0.044767865
## math4   0.140564617  0.764954614  0.92332515  0.951489208  0.917393931
## female  0.003009974 -0.019753464  0.01298834  0.002850811 -0.014423484
## age     0.069344630 -0.046119854 -0.09536580 -0.096909858 -0.103607528
## height -0.049395136  0.055676186  0.04243414  0.032683477  0.040917764
##                 pe3       hist4          pe4       math4       female
## treat  -0.134794987  0.21010474 -0.043975444  0.14056462  0.003009974
## read4  -0.078928737  0.79943989  0.001522679  0.76495461 -0.019753464
## read3   0.019772739  0.92717436  0.060598681  0.92332515  0.012988341
## math3   0.025802541  0.91056625  0.067988360  0.95148921  0.002850811
## hist3   0.002345894  0.96531919  0.044767865  0.91739393 -0.014423484
## pe3     1.000000000 -0.02968731  0.472814327  0.02354280  0.026070397
## hist4  -0.029687311  1.00000000  0.017628038  0.89351172 -0.017902077
## pe4     0.472814327  0.01762804  1.000000000  0.05519176 -0.013560064
## math4   0.023542799  0.89351172  0.055191759  1.00000000  0.034182940
## female  0.026070397 -0.01790208 -0.013560064  0.03418294  1.000000000
## age    -0.045765250 -0.08045850 -0.036665854 -0.11731677 -0.070468503
## height  0.046935386  0.03323997 -0.019725022  0.01223729 -0.079529570
##                age      height
## treat   0.06934463 -0.04939514
## read4  -0.04611985  0.05567619
## read3  -0.09536580  0.04243414
## math3  -0.09690986  0.03268348
## hist3  -0.10360753  0.04091776
## pe3    -0.04576525  0.04693539
```

RCT Simulation

```
namevec<-names(scores5)
#selecting variables to test
namevec<-namevec[!namevec%in%c("class","error", "treat","read4")]

#generating the syntax that will go in the lm function with a loop
allModelsList <- lapply(paste(namevec,"~treat"), as.formula)

#running all of the balance tests with a loop
allModelsResults <- lapply(allModelsList, function(x) lm(x, scores5))
```

Some of the variables you included in the balance test are problematic. Which ones?

$\Rightarrow$ Top Hat

RCT Simulation

```
stargazer(allModelsResults[[1]],allModelsResults[[2]],allModelsResults[[3]],allModelsResults[[4]], allMode
```

Table 3

	Dependent variable:				
	class_1	class_2	class_3	class_4	class_5
	(1)	(2)	(3)	(4)	(5)
treat	−0.019	0.049	−0.052	−0.052	−0.036
	(0.039)	(0.039)	(0.039)	(0.039)	(0.039)
Constant	0.105***	0.087***	0.114***	0.114***	0.110***
	(0.020)	(0.020)	(0.020)	(0.020)	(0.020)
Observations	300	300	300	300	300
R^2	0.001	0.005	0.006	0.006	0.003
Adjusted R^2	−0.003	0.002	0.003	0.003	−0.001
Residual Std. Error (df = 298)	0.301	0.300	0.300	0.300	0.301
F Statistic (df = 1; 298)	0.226	1.578	1.805	1.805	0.825

Note: *p<0.1; **p<0.05; ***p<0.01

RCT Simulation

```
stargazer(allModelsResults[[6]],allModelsResults[[7]],allModelsResults[[8]],allModelsResults[[9]], allMode
```

Table 4

	Dependent variable:				
	class_6	class_7	class_8	class_9	class_10
	(1)	(2)	(3)	(4)	(5)
treat	0.066*	0.015	−0.052	0.015	0.066*
	(0.039)	(0.039)	(0.039)	(0.039)	(0.039)
Constant	0.082***	0.096***	0.114***	0.096***	0.082***
	(0.020)	(0.020)	(0.020)	(0.020)	(0.020)
Observations	300	300	300	300	300
R^2	0.010	0.001	0.006	0.001	0.010
Adjusted R^2	0.006	−0.003	0.003	−0.003	0.006
Residual Std. Error (df = 298)	0.300	0.301	0.300	0.301	0.300
F Statistic (df = 1; 298)	2.866*	0.151	1.805	0.151	2.866*

Note: *p<0.1; **p<0.05; ***p<0.01

RCT Simulation

```
stargazer(allModelsResults[[11]],allModelsResults[[12]],allModelsResults[[13]],allModelsResults[[14]], all
```

Table 5

	Dependent variable:				
	read3	math3	hist3	pe3	hist4
	(1)	(2)	(3)	(4)	(5)
treat	1.329	0.934	1.038	−0.613**	4.672***
	(1.237)	(1.233)	(1.226)	(0.261)	(1.260)
Constant	78.760***	90.965***	80.911***	90.044***	79.153***
	(0.643)	(0.641)	(0.637)	(0.136)	(0.654)
Observations	300	300	300	300	300
R^2	0.004	0.002	0.002	0.018	0.044
Adjusted R^2	0.001	−0.001	−0.001	0.015	0.041
Residual Std. Error (df = 298)	9.511	9.483	9.425	2.006	9.685
F Statistic (df = 1; 298)	1.154	0.573	0.717	5.515**	13.762***

Note: *p<0.1; **p<0.05; ***p<0.01

RCT Simulation

```
stargazer(allModelsResults[[16]],allModelsResults[[17]],allModelsResults[[18]],allModelsResults[[19]], all
```

Table 6

	Dependent variable:				
	pe4	math4	female	age	height
	(1)	(2)	(3)	(4)	(5)
treat	−0.548	3.122**	0.003	0.043	−0.022
	(0.722)	(1.274)	(0.065)	(0.036)	(0.026)
Constant	89.953***	86.029***	0.466***	9.490***	1.295***
	(0.375)	(0.662)	(0.034)	(0.019)	(0.013)
Observations	300	300	300	300	300
R^2	0.002	0.020	0.00001	0.005	0.002
Adjusted R^2	−0.001	0.016	−0.003	0.001	−0.001
Residual Std. Error (df = 298)	5.549	9.794	0.501	0.279	0.198
F Statistic (df = 1; 298)	0.577	6.007**	0.003	1.440	0.729

Note: *p<0.1; **p<0.05; ***p<0.01

RCT Simulation

Our data seems reasonably balanced:

- a few come out as statistically significant: class_6 and class_10 at 10%,
- pe_3 at 5%.

This is the result of random chance as discussed above (we know this for certain since we modeled the data).

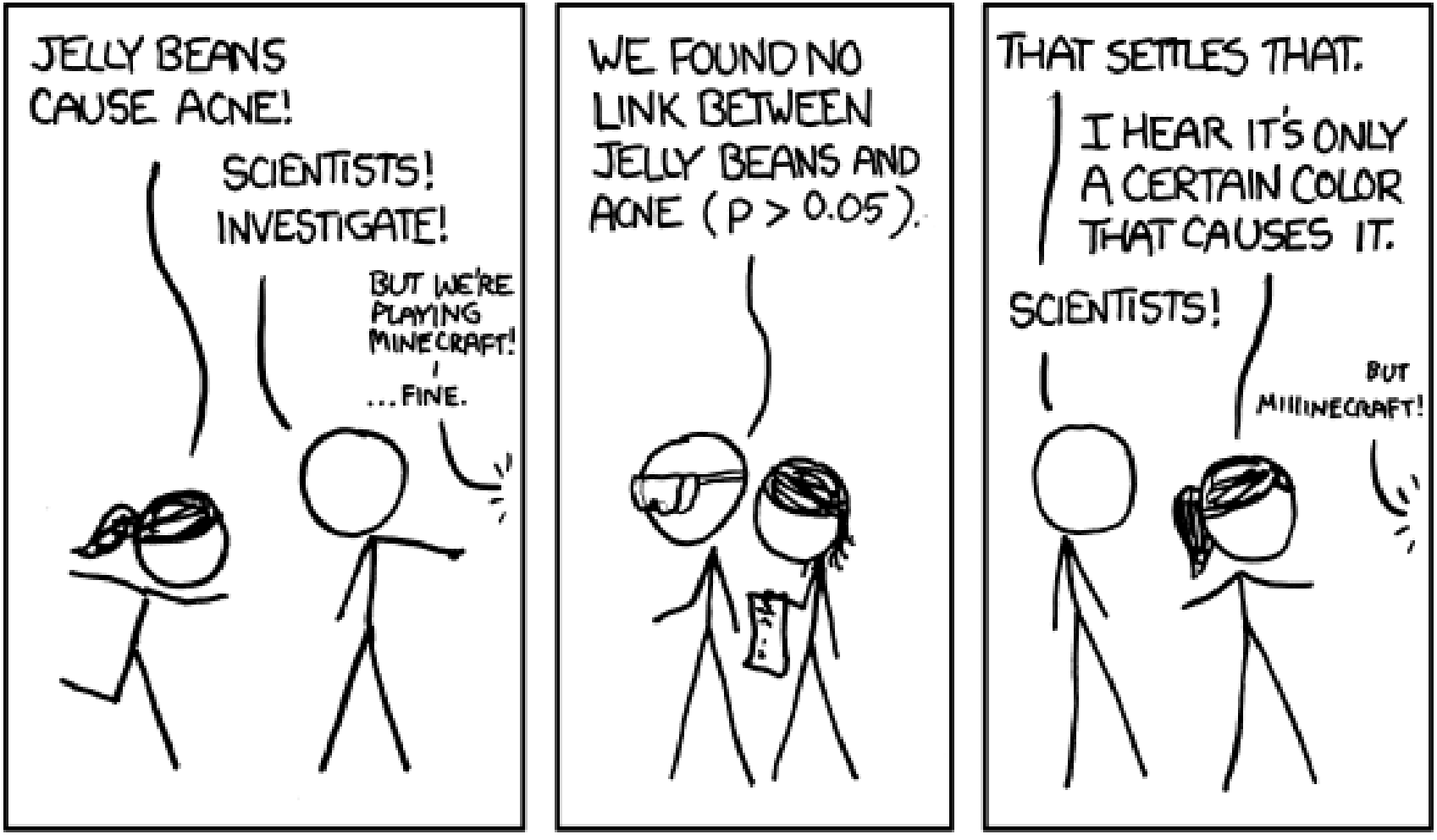
If you had not modeled the data, would you be concerned?

RCT Simulation

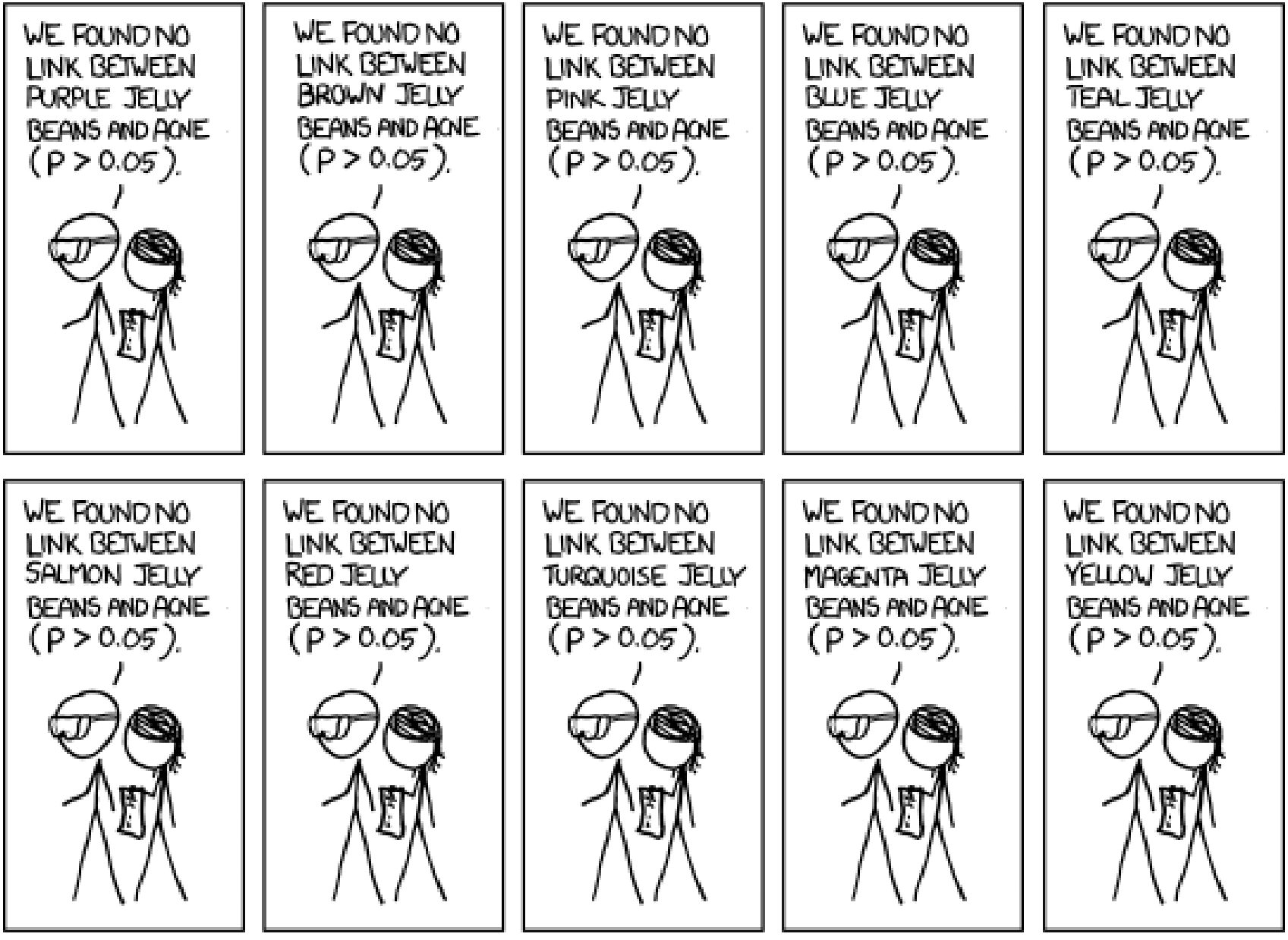
- pe_3:
  - was determined prior to treatment
  - is not generally a variable we would expect to correlated with reading scores
  - should reassure you that it is the result of random chance.
- Class_6 and Class_10 would be more concerning:
  - it might signal that some teachers were better able to get their students into the small groups
  - but the coefficients are not large, nor are they highly significant
  - should reassure you that they are the result of random chance.

If I change the seed in the simulation (try 5000), some other variables will likely be significant due to random chance.

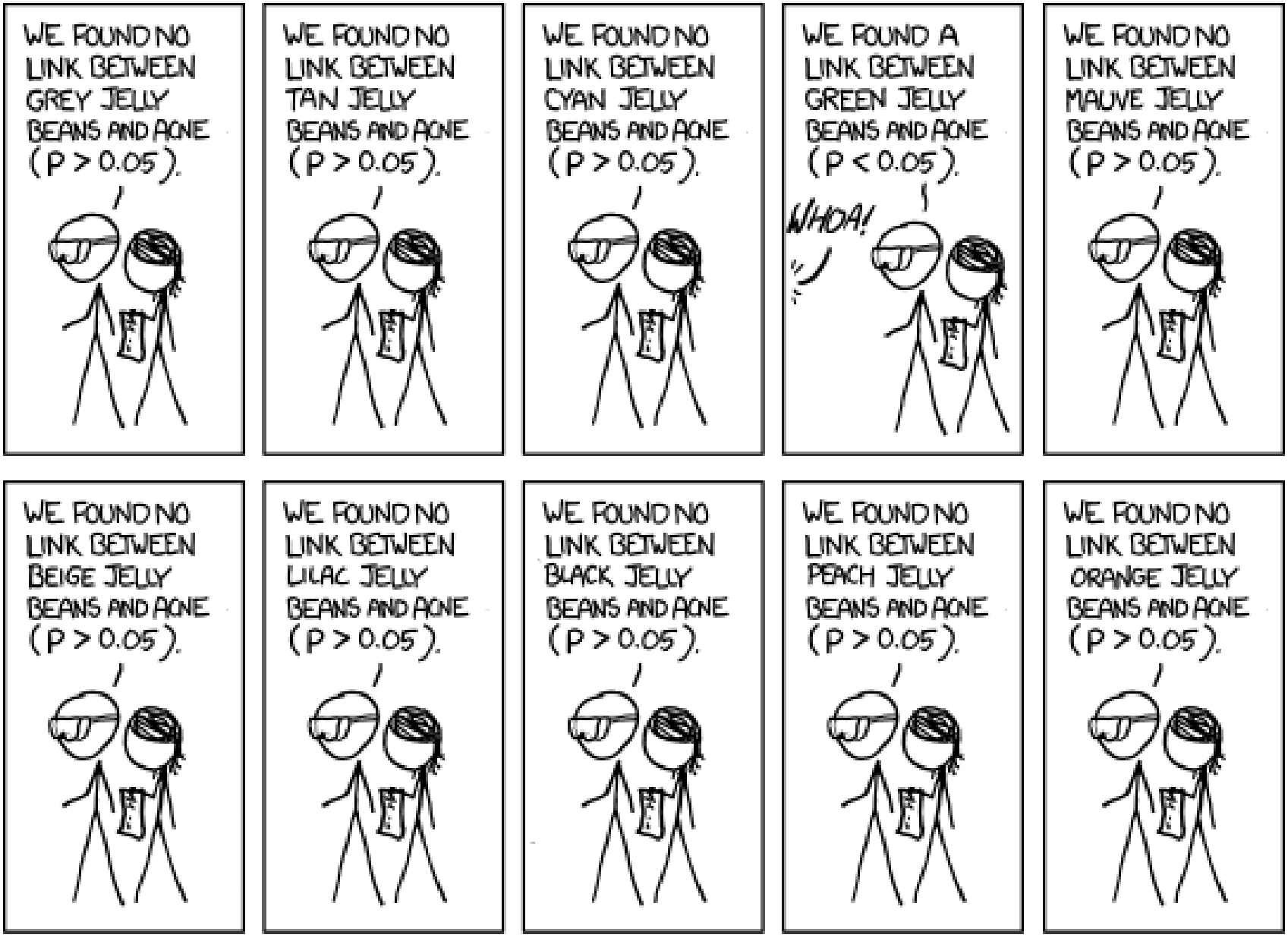
The laws of probability



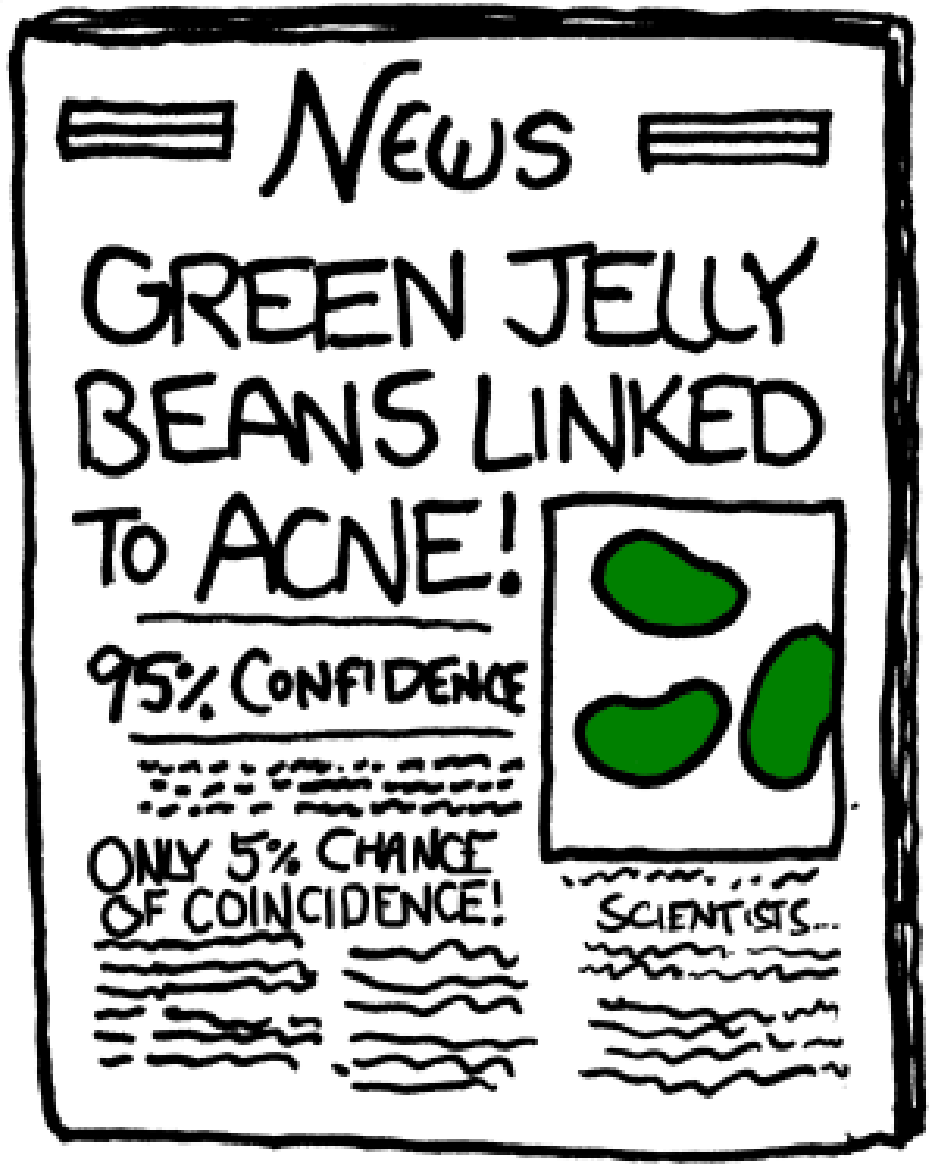
The laws of probability



The laws of probability



The laws of probability



Controls in RCT specifications

Because the treatment was randomized, estimating

$$Y_i = \alpha + \tau D_i + \epsilon$$

- gives us an unbiased estimate of τ
- controlling for omitted variables is not necessary

That said, it is common to see specifications in RCT projects that include a vector of control variables. Why?

- verify that estimated coefficient does not change significantly when controls are added
- adding controls can make our estimated more precise and shrink the standard errors.

Controls in RCT specifications

```
rct1<-felm(read4~treat,scores5)
rct2<-felm(read4~treat+read3+female+pe3+math3+hist3,scores5)
rct3<-felm(read4~treat+read3+female+pe3+math3+hist3|class,scores5)
```

Controls in RCT specifications

```
stargazer(rct1, rct2, rct3, type="latex", header=FALSE, omit.stat = "all")
```

Table 7

	Dependent variable:		
	read4		
	(1)	(2)	(3)
treat	11.229***	10.028***	10.152***
	(1.442)	(0.824)	(0.153)
read3		0.227	0.319***
		(0.153)	(0.028)
female		−0.426	0.120
		(0.727)	(0.135)
pe3		−0.235	0.074**
		(0.181)	(0.034)
math3		0.287**	0.339***
		(0.143)	(0.026)
hist3		0.472***	0.339***
		(0.149)	(0.027)
Constant	77.150***	16.382	
	(0.750)	(16.606)	
Note:			*p<0.1; **p<0.05; ***p<0.01

Controls in RCT specifications

Why does adding control variables add precision? Think about the formula for the variance/standard error of our estimator:

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST_x(1 - R_j^2)}$$

$$se(\beta_1) = \frac{\hat{\sigma}}{\sqrt{SST_x(1 - R_j^2)}}$$

$$\hat{\sigma}^2 = \frac{1}{n - k - 1} \sum_i^n \hat{u}_i^2$$

If we include more x's in our regression,

- we can reduce $\hat{u}_1^2$, i.e. the unexplained variation in Y goes down
- $\Rightarrow se(\hat{\beta}_j)$ decreases
- $\Rightarrow \hat{\beta}$ can be estimated more precisely.

Heterogeneity

We can measure heterogeneity of the program effects for individuals with specific characteristics by interacting these characteristics with the treatment variable.

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 x_i + \beta_3 D_i \times x_i + \epsilon,$$

Example:

- x_i could be an indicator variable for being female
- β_3 gives us the differential effect of the treatment for females relative to non-females.

Simulation:

I start by searching for heterogeneity by gender using our existing simulation data.

```
rct1<-felm(read4~treat,scores5)
rcthet1<-felm(read4~treat+female+female*treat,scores5)
stargazer(rct1,rcthet1, type="latex", header=FALSE,  omit.stat="ser")
```

Table 8

	Dependent variable:	
	read4	
	(1)	(2)
treat	11.229***	12.829***
	(1.442)	(1.980)
female		0.412
		(1.504)
treat:female		−3.413
		(2.893)
Constant	77.150***	76.959***
	(0.750)	(1.026)
Observations	300	300
R^2	0.169	0.173
Adjusted R^2	0.166	0.165
Note:	*p<0.1; **p<0.05; ***p<0.01	

Simulation:

I now will simulate a DGP with heterogeneous treatment effects by gender

```
#the second data generating process
nf<-20
scores5$read4het1<-(alpha+nf*scores5$treat+scores5$error+4*scores5$class_1+(-6)*scores5$class_2
                    +8*scores5$class_3+(-4)*scores5$class_4+7*scores5$class_5
                    +(-2)*scores5$class_6+5*scores5$class_7+(-10)*scores5$class_8
                    +8*scores5$class_9+4*scores5$class_10+(-20)*scores5$female*scores5$treat)

rct2<-felm(read4het1~treat,scores5)
rcthet2<-felm(read4het1~treat+female+female*treat,scores5)

#the third data generating process
nf2<-30
scores5$read4het2<-(alpha+nf2*scores5$treat+scores5$error+4*scores5$class_1+(-6)*scores5$class_2
                    +8*scores5$class_3+(-4)*scores5$class_4+7*scores5$class_5
                    +(-2)*scores5$class_6+5*scores5$class_7+(-10)*scores5$class_8
                    +8*scores5$class_9+4*scores5$class_10+(-40)*scores5$female*scores5$treat)

rct3<-felm(read4het2~treat,scores5)
rcthet3<-felm(read4het2~treat+female+female*treat,scores5)
```

Simulation:

```
stargazer(rct1,rcthet1,rct2,rcthet2,rct3,rcthet3, type="latex",header=FALSE, omit.stat="ser")
```

Table 9

	Dependent variable:					
	read4		read4het1		read4het2	
	(1)	(2)	(3)	(4)	(5)	(6)
treat	11.229^{***}	12.829^{***}	11.847^{***}	22.829^{***}	12.464^{***}	32.829^{***}
	(1.442)	(1.980)	(1.636)	(1.980)	(2.046)	(1.980)
female		0.412		0.412		0.412
		(1.504)		(1.504)		(1.504)
treat:female		-3.413		-23.413^{***}		-43.413^{***}
		(2.893)		(2.893)		(2.893)
Constant	77.150^{***}	76.959^{***}	77.150^{***}	76.959^{***}	77.150^{***}	76.959^{***}
	(0.750)	(1.026)	(0.850)	(1.026)	(1.063)	(1.026)
Observations	300	300	300	300	300	300
R^2	0.169	0.173	0.150	0.342	0.111	0.560
Adjusted R^2	0.166	0.165	0.147	0.336	0.108	0.556
Note:						*p<0.1; **p<0.05; ***p<0.01

Simulation:

DGP 1, 2 and 3 return similar estimates of the ATE (average treatment effect).

This ATE hides important heterogenety that is quite different for DGP 1, 2 and 3:

- No heterogeneity in DGP 1
- A positive effect on non-females and no effect on females in DGP 2
- A positive effect on non-females and negative effect on females in DGP 3